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A COMPARATIVE STUDY BETWEEN THE
KMV AND THE ZERO-PRICE PROBABILITY
FOR DEFAULT PREDICTION.

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ABSTRACT

This master's thesis is a comparative study between a structural model and a simulation based model for predicting default-probabilities. The structural model used is the KMV model and the simulation based is a new model called Zero-Price Probability. The main focus is on the new simulation based approach rather than the older established models.

The comparison is done to get an implicit idea of the power of both models. This was done by calculating the Zero-Price Probability using Monte Carlo simulations and also calculating the KMV, both models are presented for 500 observations and on a forecast horizon of one year. They are presented for several firms, both defaulted and of good financial health, from different industries and regions.

The results in this paper show that the Zero-Price Probability seems to be better at forecasting default for firms that have defaulted, however it also overestimates the default-probability of firms that haven't defaulted.

Keywords: Zero-Price Probability, ZPP, KMV, Credit risk, Monte Carlo Simulation, Default-probability

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Chapter 1. INTRODUCTION

This master's thesis is a comparative study between Merton type models and a new simulation based approach to finding default probabilities, which is one of the major parts in credit risk and credit risk management. Interest in credit risk has been increasing for a long time, starting with the significant increase in defaults experienced in the US in the 1980's. This sharp increase in defaults led to increased interest in predicting defaults and thus credit risk. Credit risk is the risk that a borrower will not be able to fulfil its obligations to the lender, which are pay back interests and amortizations on the outstanding loans. This is extremely visible in the credit derivatives market where credit risk is one of the main driving forces behind, for example, securitization using collateralized debt obligations or similar. Also the explosion in the credit default swap market is driven by the need to "insure" oneself from default risk.

In order to predict these defaults new theories were introduced, among which, the groundbreaking paper by Robert Merton in 1974 (Merton, 1974) which realizes that equity can be treated as a call option on the firm's assets where the strike price is the debt. His main focus was on debt valuation which also led to the fact that actually implementing the pure Merton model to predict default gave poor results, as shown by Jones, Mason and Rosenberg in 1984 (Jones, et al., 1984), among others, but even today as research continues using the same basic model, results are negative as Jarrow and Van Deventer showed in 1999 (Jarrow, et al., 1999).

To try to solve these problems, Vasicek in 1984 defined a modified framework based on earlier research which gave significantly better results (Vasicek, 1984). This approach was the basis behind the foundation of the KMV corporation and was expanded on by Kealhofer, as described in his paper written in 2003 (Kealhofer, 2003); this model is the one called KMV. Others have argued that the KMV model doesn't include all relevant information for default prediction for example Sobehart and Stein in 2000

argue that the KMV model can easily be improved on (Sobehart, et al., 2000). A different approach to both firm valuation and default prediction is suggested by Fantazzini, De Giuli and Maggi in 2007 (Fantazzini, et al., 2008) which not only removes several major assumptions for example the reliance on the Black-Scholes option pricing theory and all the drawbacks included in that, but means, for example, that it doesn't need the firm's asset volatility like all Merton style models. However, probably more important is the removal of the assumption of log-normality. This is the new simulation based approach which is investigated in this paper.

The effects of poor risk management can clearly be seen in the problems that have arisen from the unregulated sub-prime mortgage lending market in the USA (which collapsed in the summer of 2007) where the loans had been securitized into ever more complex securities, which when the housing prices stabilized and stopped increasing led to a huge increase in defaults of the underlying sub-prime mortgages. This, in turn, led to massive losses in the securities which had been sold. These losses dried up the liquidity of the markets and, as in the case of Northern Rock in the UK, almost led to a run on the banks and nationalization of the bank itself. Among other financial institutions that have suffered greatly from these problems are the US investment bank, Bear Sterns, and UBS of Switzerland, where the former was near default when JP Morgan Chase bank acquired them for an extremely low price supported by the US government (Federal Reserve). At the time of writing, UBS is still in operation, together with most other financial institutions, but more firms will likely end up in similar trouble as the aforementioned ones. This liquidity crisis is not only limited to the three above-mentioned countries, USA, UK and Switzerland, but has, and still is spreading around the world. In some cases it has even affected entire countries, for example, Iceland where banks can't finance themselves because of very large and growing credit default swap spreads, which indicate prices for lending, with spreads going from around 10bp to around 1200bp. This has resulted in the entire country getting decreases in ratings, as the Icelandic economy is currently highly dependent on the Icelandic

financial sector. The reasons mentioned above indicate an increased need to manage risk and in particular credit risk and default predictions.

To investigate the default predictions of firms, both of good and bad financial health, it is needed to have theoretical frameworks which rely on available data to calculate measures of default probabilities. In this paper two such frameworks are investigated in order to get an idea of the power of each of the two frameworks. Basically, the investigation is about which of the frameworks or models is better at predicting default. The two main frameworks or models which the paper is based on are the well-known KMV model and the Zero-Price Probability, ZPP, from the paper “A New Approach for Firm Value and Default Probability Estimation Beyond Merton Models” (Fantazzini, et al., 2008).

In this paper the basic results from the “A New Approach...” is confirmed, and the ZPP gives a higher estimate of default probabilities than the implementation of the KMV. Moreover, it seems that the ZPP gives a better prediction of default for firms that actually have defaulted but the KMV seems to give a better estimation for firms that do not default.

This paper is mostly directed at people either studying finance at the advanced level or generally interested in credit-risk, more specifically default prediction. Therefore some econometric and finance knowledge is needed to understand the implementation of the different models and interpretation of the results.

The main focus of this paper has been on comparing the default probabilities calculated from the KMV implementation of the Merton model with the default probabilities of the ZPP. Besides this, it also compared the default probabilities of a simplified version of the ZPP with the other two models. This paper, however, does not place much emphasis on asset values or firm valuation but looks mainly at default probabilities that in some cases can be derived from the firm valuation. Nor does the paper compare all available credit risk models, but focuses on the earlier mentioned ones.

The disposition of the paper is as follows. In section two, the theoretical frameworks for the Merton model, the KMV implementation of the Merton model and the zero-price probability method. In section three, the empirical

analysis is performed on the two main models (KMV and ZPP) but also on the simplified version of the ZPP. Also, in this section a short summary of the data used can be found, and the results from the calculations. Finally, in section four, the concluding discussion and conclusions can be found.

Chapter 2. THEORETICAL FRAMEWORK OF DEFAULT MODELS

In this section, the theoretical framework behind three credit risk models is discussed. Two of them are structural models and one is a new methodology that extracts default probabilities using only stock prices. All the models have one thing in common; they can all give predictions of default probabilities. First, the groundbreaking work done by Nobel laureate Robert Merton in 1974, the Merton model, is discussed. Secondly, the KMV model, which was developed by the KMV Corporation in the 1980s, and builds directly on the VK¹ model. The VK model builds on the Merton model but with some major improvements. The KMV Corporation was acquired by Moody's in April 2002 and its services are now sold to its subscribers. The third model is the ZPP model, which was first introduced in May 2007.

2.1 THE MERTON MODEL

Few models have made such an impact on credit risk as the model Robert Merton introduced in 1974. More than 30 years later the Merton model is still the basis of some of the most sophisticated credit risk models.

The Merton model (Merton, 1974) is a credit risk structural model whose main aim is to obtain a valuation equation for the company's debt. The model is a continuous one that requires a number of assumptions. Among other things, the model assumes that the underlying value of each firm follows a geometric Brownian motion (Bharath, et al., 2004). The model also assumes that the only source of uncertainty that influences the company's ability to pay in every time t is its asset value (Gheno, 2007).

¹ Vasicek-Kealhofer

The dynamics for the asset value (V) through time can be described by a diffusion-type stochastic process with the following stochastic differential equation:

$$dV = (aV - c)dt + \sigma Vdz \quad (1)$$

Where:

- a is the instantaneous expected rate of return of V per unit of time.
- c is the total dollar payout made by the company if positive and net dollars received negative.
- σ is the instantaneous variance of the return on the firm per unit of time.
- dz is the standard Gauss-Wiener process.

Under these assumptions, Merton modelled a firm's asset value as a lognormal process and assumed that the firm would default if the asset value falls below a certain default boundary. The Merton model assumes that the firm has only a single debt liability, one zero-coupon bond. The default is allowed at only one point in time, T. The model treated equity of the firm as a call option on the underlying value of the firm with the strike price equal to the face value of the firm's debt and time to maturity T. The value of the equity was derived by using the Black-Scholes (Black, et al., 1973) option valuation equation:

$$E = AN(d_1) - X_T e^{-rT} N(d_2) \quad (2)$$

Where E is the market value of equity, A is the asset value, X is the face value of the firm's debt, r is the risk free rate and N is the cumulative standard normal distribution function, d_1 is given by

$$d_1 = \frac{\ln A/X + (\mu + 1/2 \sigma^2)T}{\sigma\sqrt{T}} \quad (3)$$

Where μ is the drift of the asset return and σ is the volatility of the asset returns.

Then, d_2 is given by:

$$d_2 = d_1 - \sigma\sqrt{T} \quad (4)$$

The value of equity thus depends on, among other things, the market value of the company's assets, their volatility and the payment terms of the liabilities.

Merton concluded that the value of debt, D , was derived from the following expression:

$$D = A - E \quad (5)$$

Under the assumption that the market value of the company's assets evolves as a lognormal process, Merton showed that this model could be solved for a closed-form solution for the value of the company's debt (Kealhofer, 2003).

As noted above, Merton assumed the firm's only issued a single debt liability; each firm issued just one zero-coupon bond, B . Clearly, if at time T , the market value of the borrowing firm's asset is $A > B$, then the firm has an incentive to repay the loan and keep the rest as profit. If, however, the market value of the firm's assets is $A < B$ then the firm will not make the payment and default. This is illustrated in the figure below.

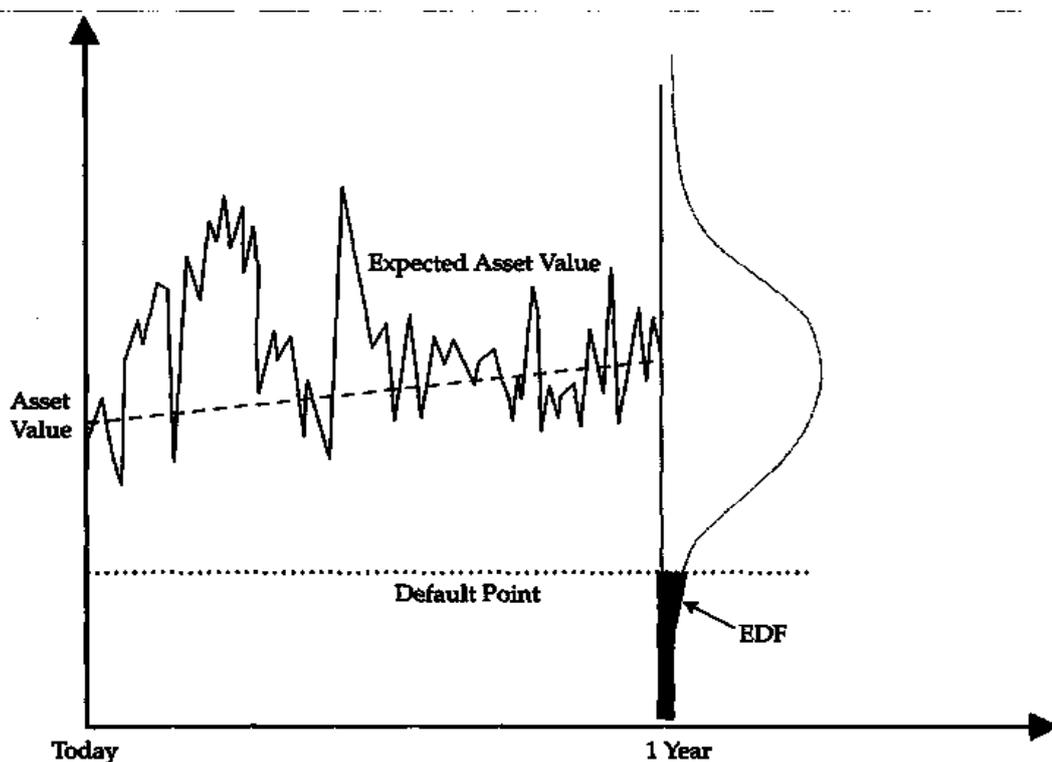


FIGURE 1, ILLUSTRATION OF PROBABILITY OF DEFAULT. QUANTIFYING CREDIT RISK I: DEFAULT PREDICTION - PAGE 31

Time is on the horizontal axis, going from the current period (today) to a future period (1 year). The vertical axis shows the market value of the company's assets. The company has a single, determinable value, in the current period but as time goes by the asset values can take on any number of different values, depending on the assets volatility. The more volatility of

the asset's values the more the likelihood of extreme outcomes. The mean asset value is shown by the dashed line. The frequency distribution shows the likelihood of various asset values one year in the future. The dotted horizontal line shows the amount of liability the company must pay back in one year's time. If the market value of assets goes below the dotted line, liabilities, in one year's time, the company will default. The area under the frequency distribution below the default point thus gives the probability of default. This represents the likelihood of the company's asset value in one year being less than the company owes. It is; therefore, clear that the probability of default will increase if the company's asset value will decrease, if the amount of liabilities increases or if the market value of assets increases (Kealhofer, 2003).

2.2 THE KMV MODEL

In 1984, Vasicek took a novel approach to implementation of the Merton model that has proven to have considerable success in measuring credit risk. The model, which is known as the Vasicek-Kealhofer (VK) model, builds on previous modifications made to the Merton model by other researchers. This model has been extended by the KMV Corporation to become one of the best measures of default risk. It is a default forecasting model that produces a probability of default for each firm at any given point in time. The main difference between the two models is that the KMV model primarily focuses on the probability of default of the company while the Merton model focuses on the valuation of debt.

The KMV model extends the Black-Scholes-Merton framework to produce a model of default probability. It assumes the firm's equity is a perpetual down-and-out option with the default point acting as the absorbing barrier for the firm's asset value. When the asset value hits the default point, the firm is assumed to default (Bohn, et al., 2003). In the Merton model, the default point is the single debt liability, the one zero coupon bond, while multiple classes of liabilities are modelled in the KMV model: short-term liabilities, long-term liabilities, convertible debt, preferred

equity and common equity. The value of this default option used in both the Merton and KMV models depends on the value of the following five variables:

$$\text{Value of a default option on a risky loan} = f(A, B, r, \sigma_A, t)$$

Where A is the market value of assets, B is the liabilities, r is the interest rate, σ_A is the volatility of the market value of assets and t is the time horizon.

The Merton model describes the unobserved value of this option as a function of four variables that are easily observed (A, B, r, t) and one variable that can be estimated (σ_A). However, in the KMV model the value of the option is observed as the total value of the firm's equity while the value of the underlying assets and the volatility of the underlying assets are unobservable (Bharath, et al., 2004). To solve the two unknowns, A and σ_A , the model first uses the relationship between the market value of a firm's equity and the market value of its assets to solve for A. Equity can be observed in the marketplace as by multiplying the firm's number of shares outstanding with its current stock price. Secondly, the model uses the relationship between the volatility of a firm's assets and the volatility of a firm's equity to solve for σ_A . It can be shown that equity volatility and asset volatility are related by the following equation:

$$\sigma_E = \frac{A}{E} \Delta \sigma_A \quad (6)$$

To derive the values for the two unknowns, the KMV model simultaneously solves equations (2), using an option pricing BSM² type model that allows for dividends, and (6), the volatility equation, to get numerical values for both A and σ_A (Saunders, et al., 2002). When the values for A, B and σ_A have been calculated and if it is assumed that future asset values are normally distributed around the firm's current asset value, the Distance to default (DD) is given by the following expression:

$$DD = \frac{A - B}{\sigma_A} \quad (7)$$

² Black-Scholes Merton

Distance to default shows how far a certain company is from defaulting. It shows the number of standard deviations to the default point. If the company's market value of assets falls below the default point the company will default on its obligations.

However, Crosbie and Bohn (Bohn, et al., 2003) realised that the relationship between asset volatility and equity volatility given by (6) only holds instantaneously. In practise, the market leverage moves around far too much for (6) to provide reasonable results. If market leverage decreases rapidly then (6) will tend to overestimate the asset volatility and thus overstate the firm's default probability. If the market leverage increases rapidly then (6) will underestimate the asset volatility and thus understate the default probability. Instead of using the relationship given by (6) the KMV model uses a more complex iterative procedure to solve for the asset volatility. This procedure guesses the initial value of asset volatility to determine the asset value and to deliver equity returns. The volatility of these asset returns are then used as inputs to the next iteration procedure that determines a new set of asset values and asset returns. This procedure continues in this manner until it converges. The KMV model then derives the asset volatility by combining the procedure above in a Bayesian manner with country, industry, and size averages to produce a more predictive estimate of the firm's asset volatility.

The distance to default is then calculated in the following way:

$$DD = \frac{\ln \frac{A}{B} + \left(\mu - \frac{1}{2} \sigma_A^2 \right) t}{\sigma_A \sqrt{t}} \quad (8)$$

Where μ is the expected market return to the assets per unit of time.

To illustrate how distance to default is converted into default probabilities assume that a company's asset value is 100 million, the value of liabilities is 80 million and its volatility is 10 million. Then the following is the distance to default:

$$DD = \frac{100m - 80m}{10m} = 2 \quad (9)$$

The assets value must decrease by 20 million, or two standard deviations, for the company to default. If the asset values are normally distributed then there is a 95% probability that the asset value will either increase or decrease by a factor of two standard deviations. In other words, there is a 2.5 % probability that the asset value will increase by two standard deviations and a 2.5% probability that the asset value will decrease by two standard deviations. The probability of default is therefore 2.5%.

However, the KMV model does not assume that asset values are normally distributed and does, therefore, not use the cumulative normal distribution to convert distance to default into default probabilities. Instead the KMV model uses its large historical database, which contains data on historical defaults and bankruptcy frequencies, to obtain a relationship between distance to default and default probabilities. This database contains over 30 years of information on over 6,000 public and 220,000 private company default events for a total of 60,000 public and 2.8 million private companies, healthy and distressed, around the world (Moody's KMV, 2008). From this data the KMV is able to generate a lookup table that relates the likelihood of default to various levels of distance to default. For example, suppose that determining the default probability over the next year of a company that is 5 standard deviations away from default, is the interesting problem. To determine the default probability, the KMV uses its database to find the number of firms, 5 standard deviations away from default, which defaulted over the next year. The KMV reports that 1% of all the companies in its database, which are 5 standard deviations away from default, defaulted over the next year. The probability of default is therefore 1%. The result of this process is the KMV EDF (expected default frequency) credit measure. The EDF is the probability that a company defaults within a given time period.

2.3 THE ZPP MODEL

Fantazzini, De Giuli and Maggi (Fantazzini, et al., 2008), provide a new method to predicting default probabilities and firm valuation. According to this approach the default probabilities can be calculated by estimating $Pr[E_T \leq 0]$ or $Pr[P_T \leq 0]$, given that equity (E) is equal to the number of shares multiplied with the stock price (P). Since $P_T = \max(P_T, 0)$ is a truncated variable, the default probability is the probability that P_T goes below the truncation level of zero. This method is denoted the Zero-Price Probability or simply the ZPP model.

As in the structural models the ZPP predicts default when equity becomes zero or negative, i.e. firm's asset values don't cover debts. This is given by the following equation:

$$E_T = A_T - B_T \quad (10)$$

However, this model uses E_T as an estimate of distance to default and the default probabilities are estimated by $Pr[E_T \leq 0]$.

In order to compute the firm value and an estimation of the ZPP the model must first solve a firm pricing function. This is done by treating the asset value as a bivariate contingent claim written on the traded securities, i.e. stocks and bond. Using this method the model obtains observable asset values and volatility, unlike the structural models.

The contingent claim is given by the following expression:

$$G(g(S_1(T), S_2(T)); T) \quad (11)$$

Where

- G is a univariate pay-off function which identifies the derivative.
- g is a bivariate function which describes the final cash flow.
- S is the price of the Ith underlying security.
- T is the time to maturity.

The model denotes the final value of the firm as:

$$A_T = G(E_T, B_T; T) = \max(E_T + B_T, 0) I_{[(E_T \geq 0), (0 \leq B_T \leq D)]} \quad (12)$$

where I is the indicator function, which means it is a function that takes on the value one if the conditions subscripted to it are fulfilled and zero otherwise.

However, solving this equation is extremely difficult and becomes even harder when the distribution departs from normality and markets become incomplete, because of ill-liquid assets such as bonds. The model resorts to copula theory to solve this problem. Using the copula theory a bivariate pricing kernel is written as a function of univariate pricing functions. The bivariate pricing function is then given by:

$$\begin{aligned} A_t &= g(E_t, B_t; t) \\ &= P(t, T) \int_0^\infty \int_0^\infty G(E_T, B_T; T) q(E_T, B_T | F_T) dE_T dB_T \end{aligned} \quad (13)$$

where $q(E, B | F_t)$ is a risk-neutral pdf³ which represents the market pricing kernel and $P(t, T)$ is the risk-free discount factor. The firm's value price can then be expressed as:

$$A_t = P(t, T) \int_0^\infty \int_0^\infty G(E_T, B_T; T) c_{E,B}(F_E, F_B | F_t) f_E(E | F_t) f_B(B | F_t) dE_T dB_T \quad (14)$$

Where $c_{E,B}$ is the bivariate copula density between stock and bond values, and f_E and f_B are the marginal densities. This approach has, however, one major drawback, which is the use of risk neutral probabilities for bond prices. This becomes very problematic because the bond market is very illiquid.

The model, however, recognises that there is a far better approach to the problem of pricing the firm. The model, therefore, makes two important assumptions. The first assumption is to consider the objective joint probability distribution for stocks and bonds, and discount the bivariate contingent claim with a risky discount factor, instead of using risk neutral probabilities. The second assumption is that when bonds are illiquid or not traded the bond prices can be expressed as a function of the risky interest rate. Given these assumption the firm pricing function can be modified as:

³ Probability density function

$$A'_t = P(t, T) \int_0^\infty \int_0^\infty G(E_T, B_T(i_T); T) c_{E,i}(F_E, F_i | F_t) f_E(E | F_t) f_i(i | F_t) dE_T di_T \quad (15)$$

This equation can be further simplified when considered that both B_T and $P_i(t, T)$ are known at time t . Therefore, only the distribution of stock prices is needed:

$$A''_t = P_i(t, T) \int_0^\infty G(E_T, B_T; T) f_E(E | F_t) dE_T \quad (16)$$

Finally, by using Monte Carlo simulation on the previous equation the firm pricing function is as follows:

$$\tilde{A}_t = P_i(t, T) \frac{1}{N} \sum_{i=1}^N G(\tilde{E}_{i,T}, B_T; T) \quad (17)$$

As soon as the firm pricing function is solved the model estimates the firm value and the ZZP, the probability that P_T goes below the truncation level of zero. To compute the probability that the future stock price will cross the zero barrier and the firm will default, i.e. $\Pr[P_{t+T} \leq 0]$, the following generic conditional model for the differences of prices levels $X_t = P_t - P_{t-1}$, without the log-transformation is used:

$$X_t = E(X_t | I_{t-1}) + \varepsilon_t = H_t^{1/2} \eta_t \quad \eta_t \sim iid(0,1) \quad (18)$$

Where $H_t^{1/2}$ is the conditional standard deviation, while I_t is the information set available at time t . Next using a Monte Carlo method a high number N of price trajectories are simulated using the estimated time series model above. Then the firm pricing function discussed above is estimated using $P_i(t, t+T)$ as a discount factor. To take the leverage effects into account, as well as leptokurtosis in the data, the model uses an AR(3)-Threshold-GARCH(1,1) model with a Student's t distribution. Which is an AR model with three lags using an asymmetric GARCH model for the errors, this asymmetric GARCH model is presented in (19) and is also called GJR after

the authors who described it first Glosten, Jagannathan and Runkle (Glosten, et al., 1993).

$$\sigma_t^2 = \alpha_0 + \alpha_1 \eta_{t-1}^2 + \gamma \eta_{t-1}^2 I_{t-1} + \beta \sigma_{t-1}^2 \quad (19)$$

Where $I_{t-1} = 1$ if $\eta_{t-1} < 0$ and otherwise 0

According to the model the default probability is simply the number of times n out of N when the price touched or crossed the barrier along the simulated trajectory. The firm value is then estimated only needing the stock prices and the face value of debt.

This approach has several advantages over the Merton-type structural models. For instance, this model does not need the firm's volatility, which has proven to be quite complicated to calculate. This approach does not use a log-normal distribution like almost every structural model. This method can estimate the default probability for any given time horizon, while the Merton-type models have been shown to give an almost zero probability of default when the maturity of debt is nearing its end. One thing that needs to be noted about the ZPP is that it is dependent on the efficiency of the markets. That is since it doesn't take any other data into account than stock prices it is essentially assuming that all information about a company is reflected in the price of its stock. This relies on the efficient market hypothesis being correct. However the stock-market is usually assumed to be near efficient for at least the more developed economies in the world. On less developed stock markets the ZPP approach might not be the best one, but considering that the KMV approach uses stock prices as well and data from reports which, in the case of less developed countries, probably are even less reliable, this comparative drawback of the ZPP isn't that important.

Chapter 3. EMPIRICAL STUDY

In this portion of the thesis, the two earlier presented theories and methods, KMV and ZPP, of calculating default probabilities are compared using real data to get an idea of the difference in predictive power of the two; by predictive power there are two-types of errors to look for.

1. False prediction off default within the prediction horizon, or Type I error.
2. Non-prediction of default when there is default within the prediction horizon, or Type II error

This thesis has not explicitly investigated the power of either of the models, as getting an explicit power would require several times the amount of investigated companies. However it is possible to get an implicit idea of the power of each model by comparing the different companies presented. In addition to the two models presented above, a different ZPP type model is suggested and tested, as it was considered interesting from an econometric viewpoint.

The results are presented in the graphs under each subsection. The data was collected from the DataStream database and consists of stock-prices and balance sheets for the different firms together with the interest rate of the United States one-year Treasury Bill, which was used as a estimation of the expected return of the firm's assets in the KMV model. The data horizon for most firms is at most about five years. Some firms however have less data and some use more, as presented in Table 1 below.

TABLE 1, DESCRIPTION OF DATA BY TIMESPAN

Company or Industry	Starting date of data	First date presented	Last date presented	Defaulted
<i>Airline industry</i> ⁴	1998-04-10	2001-04-24	2003-03-24	<i>No</i>
<i>Pan Am</i>	1988-04-08	1989-06-07	1991-04-03	<i>Yes</i>
<i>Alitalia</i>	2003-04-10	2006-05-12	2008-04-10	<i>No</i>
<i>Swissair</i>	1998-04-10	2000-03-02	2002-03-30	<i>Yes</i>
<i>Daimler-Chrysler</i>	2001-04-10	2004-05-11	2006-04-10	<i>No</i>
<i>Enron</i>	1998-04-10	2000-01-04	2001-12-03	<i>Yes</i>
<i>Asia Pulp & Paper</i>	1998-04-10	1998-04-10	2001-03-11	<i>Yes</i>
<i>Finova</i>	1998-04-10	1999-05-03	2001-03-20	<i>Yes</i>
<i>Parmalat</i>	1998-04-10	2002-01-22	2003-12-22	<i>Yes</i>
<i>WorldCom</i>	1998-04-10	2000-08-14	2002-07-02	<i>Yes</i>
<i>Remaining companies</i> ⁵	2003-04-10	2006-05-12	2008-04-10	<i>No</i>

Considering that the influence of the early samples in the data will be small, especially for the KMV which doesn't use them at all, the slightly differing samples were considered to be a very small issue. The reason behind using a unique time-span for Daimler-Chrysler was that the period when Daimler-Chrysler consisted of both companies it was in financial trouble due to Chrysler, and this was considered a more interesting period compared to the current one when the companies have separated. All estimations are on a one year horizon or assumed 250 trading days. This assumption is mainly done to simplify programming when calculating historical values, otherwise the amount of days would have to be updated between each historical value which for the ZPP becomes for the scope of this thesis unnecessarily complex.

3.1 MONTE CARLO SIMULATION OF ZPP

To get the ZPP measures, Monte Carlo simulations were used. Here, the basic idea behind Monte Carlo simulations will be overviewed and the

⁴ American Airlines, British Airways, Continental Airlines, Delta Airlines, KLM, Lufthansa, SAS

⁵ ABB, Astra-Zeneca, Bear-Sterns, BMW, BP, Exxon-Mobil, Ford, General Electric, General Motors, Intel, Kaupthing, A.P. Møller-Maersk, Nestlé, Pfizer, Philips, Royal Dutch Shell, Siemens

specific implementation regarding the simulations presented below in the results.

The basis behind Monte Carlo simulations is the use of repeated random number draws to represent the shocks in a process. In the case of ZPP the process is the return of a share in levels. This in turn gives a trajectory regarding the process that can be used to estimate how reality will look. To get an estimation of reality a very large amount of random trajectories is necessary - in this paper 10000 have been used. The random numbers in turn are interpreted as shocks to the process which has an expected value, in the case of the ZPP the expected value is assumed to be zero. This assumption follows the idea behind Martingale pricing which states that the best prediction of the price tomorrow is the same as the price today. These shocks are most commonly assumed to be asymptotically normally distributed.

An important note when using Monte Carlo simulations is the fact that true random numbers do not exist. Here, as in most other applications of it, the pseudo-random number generators in programming environments are used. Checking for bias in the random number generator or, for example, using more complicated techniques to produce the random number is not within the scope of this paper. Here, the random function included in MATLAB has been used which might lead to some bias in the results due to the random seeds and not true random function.

When the process is simulated a price trajectory will look similar Figure 2 below, in this case with 100 trajectories for British Petroleum.

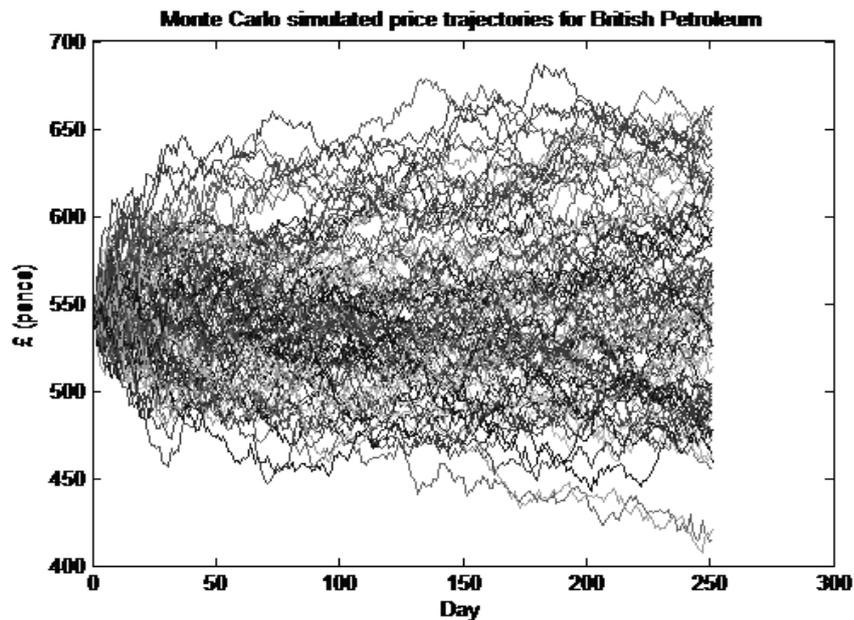


FIGURE 2, 100 SIMULATED PRICE TRAJECTORIES FOR BRITISH PETROLEUM

By looking at figure2 above it is possible to see that most of the trajectories don't give high deviations from the starting value, that is, it seems to follow the basic idea behind Martingale pricing.

The simulation of the ZPP is preformed in basically four steps to get the presented graphs. If historical values are of no interest, there are three basic steps.

- Calculate the conditional variance
- Calculate one price trajectory using the conditional variance
- Repeat step two for a set amount of times, in this case 10000
- If historical values are to be done repeat the three above for each time adapting the information for each time.

In the paper "A new Approach for Firm Value and Default Probability Estimation beyond the Merton Models" (Fantazzini, et al., 2008) where the ZPP is defined a GJR model for estimating the conditional variance was used. To estimate the ZPP the same model was used in this thesis, that is an AR(3)-TGARCH(1,1) with t-distribution, which in turn is a GJR type model also known as an asymmetric GARCH model.

However, the forecasting power of any volatility forecasting model is under a lot of debate in the econometric literature and research. That financial volatility can be forecasted is not questioned much but the possible

horizon of different models is questioned. GARCH type models estimate volatility well in the short term, measuring few observations. However the long term forecasting power, estimating for example the volatility one year ahead from daily observations, is not as good. This is due to, for example, the use of repeated substitution and errors, which in that case will be increased at every time point the substitution takes place. Or as stated by Christoffersen and Diebold (2000) *“Interestingly, however, much less is known about volatility forecastability at longer horizons, and, more generally, the pattern and speed of decay in volatility forecastability as we move from short to long horizons.”* (Christoffersen, et al., 2000)

In addition to this, a review by Poon and Granger in 2003 of the econometric research published on the subject showed that in 22 of 39 reviewed studies the historical volatility gave a better forecast than GARCH type models (Poon, et al., 2003).

The debate about the forecasting power thus resulted in it being interesting to try a different way to estimate the volatility used to estimate ZPP. This variant of the ZPP uses a constant historical volatility estimated from the period $t-T$, where T is the length of the forecast, here 250 days and t is the time point that is investigated usually “today”. This version of the ZPP was defined as constant variance Zero-Price Probability or cvZPP.

The variance was defined as:

$$\sigma^2 = \frac{1}{T} \sum_{t-T+i}^t (x_i - \bar{x}) \quad (20)$$

And in turn

$$x_t = \sigma \eta_t \quad \eta_t \sim iid(0,1) \quad (21)$$

This means that the only varying part in the simulation will be the shocks or random number draws. One advantage of this is that the estimation is not based around maximum likelihood which can give unstable results using the same data.

The choice of this very simple variance model is motivated by, as mentioned, the debate about GARCH type models and their forecasting power but more importantly by its ease of implementation into the

programming framework around the simulations. It only requires a moving estimation period to estimate.

When estimating several historical forecasts of ZPP (and cvZPP) the estimation windows are defined by which historical forecast is calculated. For example, if the forecast to be estimated is the one from one year ago, then the estimation window is the data up to that point. The next estimation then uses the same data but has the now known data between the first observation and the second one added to the original estimation window.

The original ZPP model uses all data available up to the estimation date, which in most cases in this thesis is between three and five years depending on which estimation is considered. The cvZPP, however, as stated only use 250 observations before “today” in the sample to calculate the historical variance.

3.2 IMPLEMENTATION OF THE KMV MODEL

In order to get an idea of the power of the ZPP-type models, they were compared with the very common KMV implementation of the Merton framework model. This is an analytical approach and therefore it doesn't rely on simulation or repeated random number draws. This also makes it a lot faster to compute for several time points compared to the ZPP-type models.

However the KMV model has a need for more data than the ZPP models. This data comes from the balance sheets of the company being assessed. The important piece is the development of debt over time; this debt is reported in balance sheets, at most, quarterly. That is, it is impossible to know the debt for the coming year at a near continuous interval. Also the amount of outstanding shares has to be known; this does not change as much and can therefore comfortably be assumed to be constant between annual reports.

In this thesis, it was decided that the KMV was not reasonable to calculate on a daily basis, as the approximations of debt values at that frequency from the data available was not good enough. Instead all

calculations were made at intervals of one month, or 20 trading days, meaning that debt and stock prices were monthly averages. Debt for the periods between annual reports (the database available did not have quarterly updated reports) was calculated as the average change for the previous reporting period. In some cases this led to negative or zero debt, in which case debt was decided to be constant. This happened for Siemens and General Motors, the reason behind the latter company's was the restructuring of the pension debt it went through to survive, and in the former the reason for it is unknown. It is also important to know how the debt is defined; in this paper, the debt for non banks was 100% of short term debt and 50% of long term debt. Banks also included 100% of deposits as they are, in a way, short term debt to the depositors. Also volatility for equity was calculated over the last month and then corrected to be yearly by multiplying with the square-root of the forecasting period, 250 days. As mentioned earlier the KMV uses the US one-year Treasury Bill as an approximation estimated return of the firm's assets.

Calculating KMV measures involves solving simultaneous equations for equity volatility and the option price on the company, as indicated by the price of equity. The function used in MATLAB to solve this was obtained from the presentation "A Real Option Approach to Credit Risk" (Gapen, 2007) by Michel Gapen and uses a fifth order polynomial approximation of the cumulative normal distribution that gives an accuracy around the decimal. It also requires a starting guess at what both company value and the volatility of the company value will be. Here, the starting guess was that the volatility was equal to the one from equity and the value of a company was twice that of the equity. For the volatility, it is reasonable to assume that the two things affecting a company's volatility are the volatility of equity and the volatility of the debt. Of these two origins of company volatility, the equity volatility is most likely much higher than the debt volatility and, therefore, the equity volatility is considered a good enough starting guess. More problematic was the guess for company value, where it was decided to use twice the value simply because it has to be significantly larger than the equity value. The `fsolve` function in MATLAB was sensitive to the starting

guesses. It was clear that by lowering the guess the default probability increased significantly, and by increasing it, the default probability was likewise decreased.

When the DD had been calculated, as described earlier, it was then used with the standard normal distribution and not the empirical KMV distribution. It should in either case give an approximation of the default probability that would result from having access to the empirical database. This estimate is the Expected Default Frequency or EDF presented in the result section.

3.3 EMPIRICAL RESULTS

This section contains the results from the explained three methods of calculating default probabilities. They are presented as graphs with all three of the methods in each. In some cases there were no data available to calculate the KMV measure and in these cases it is left out, the companies affected are Pan Am, WorldCom and Asia Pulp and Paper.

All companies are presented with 500 trading days, most commonly the 500 last from 2008-04-08 or 500 last from the company default date. However, to see the power of the models it was also interesting to see how the estimates react to an external shock that the companies cannot foresee. In this case, the most obvious was the 2001 September 11 attacks on the World Trade Centre and the Pentagon, which had a significant effect on the airline business. The airline business also included a scope to look closely at defaults as several airlines have defaulted or are very near default right now. Therefore, apart from Pan Am, Swissair and Alitalia, all airlines are estimated using a 500 day period around the shock. Swissair and Pan Am uses the 500 last days before default and Alitalia the 500 last from 2008-04-10 since that airline was at that time, and at the time of writing (2008-05-11), extremely close to defaulting with only months left before running out of money.

The companies have been divided up into three different sectors, airlines, financial firms and general industrial firms. The reason behind this

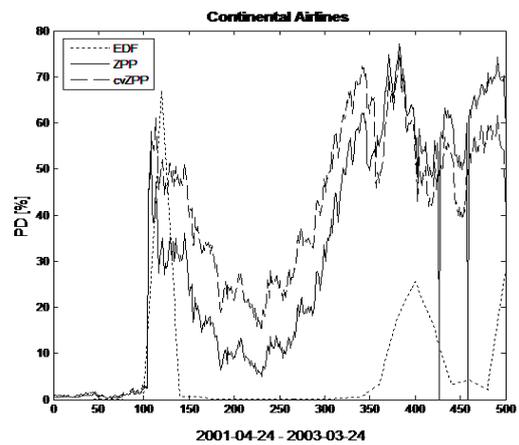
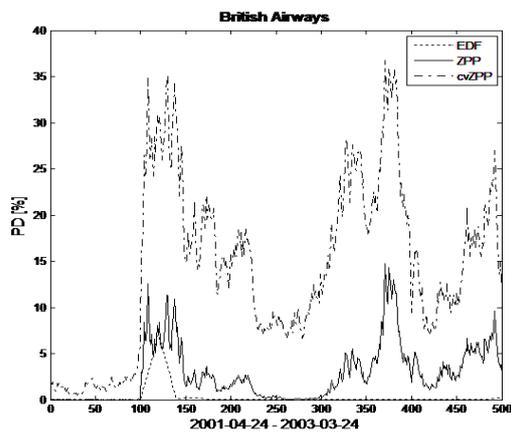
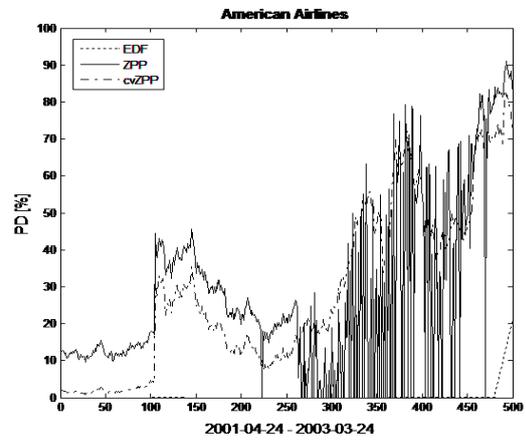
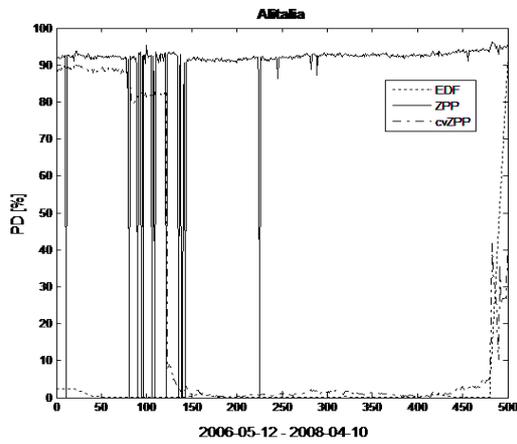
is, as stated, before airlines had a shock that affected the industry severely in 2001, and the financial firms had the ongoing sub-prime crisis which affected their default probabilities. The general industrial sector contains the rest of the firms in the investigation; they range from automotive firms in good financial health, such as BMW, to the defaulted company, Asia Pulp and Paper. Also included, as an indicator of false prediction of default, three of the largest oil companies in the world, Exxon-Mobil, Royal Dutch Shell and British Petroleum. These three oil companies are included since the real probability of these companies defaulting within a year is zero due to their financial strength.

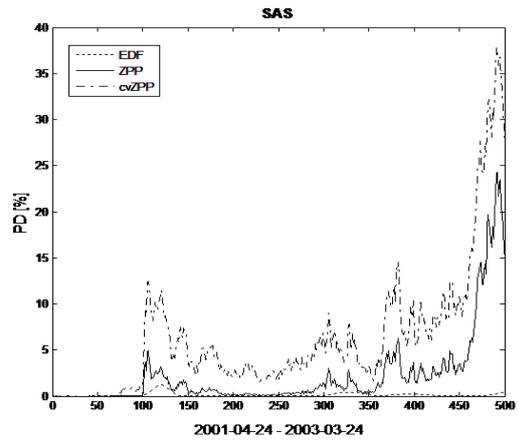
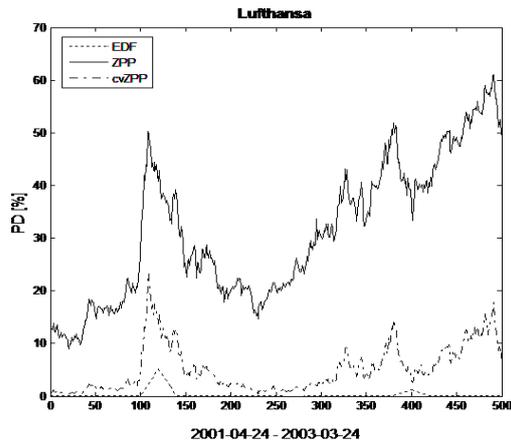
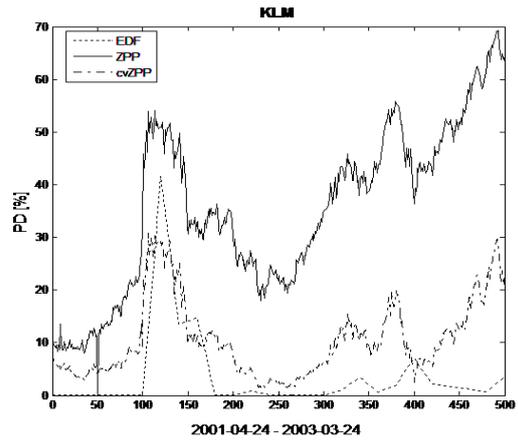
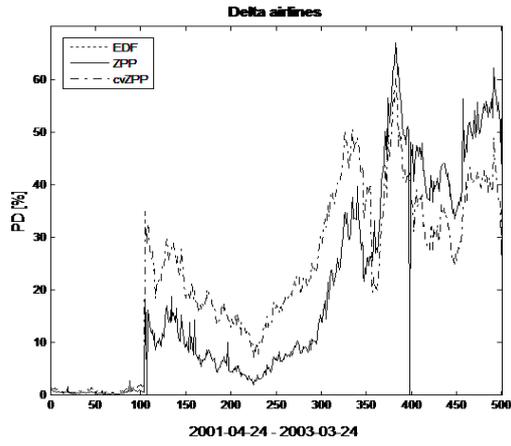
It should be noted that the calculation of one 500 day ZPP result takes a long time, in the region of 5 hours on a mid-range pc, the cvZPP takes less time, about 2 hours on the same computer. The total calculation times for these results are thus almost 240 hours or 10 days constant calculation. The analytical EDF is much faster and only takes a minute or so for all of the monthly values to be calculated for one firm.

3.3.1 AIRLINES

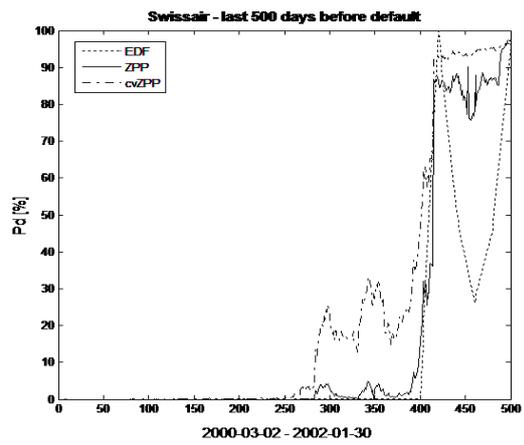
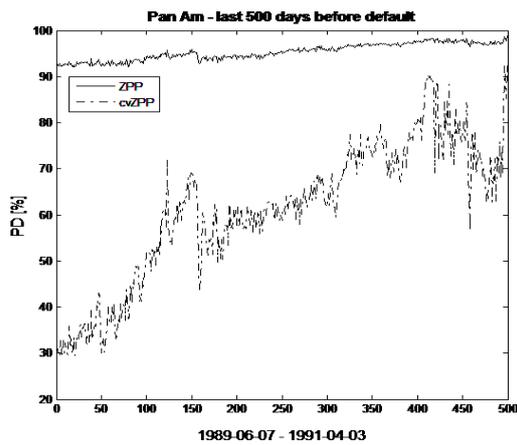
In this section the results for airlines are presented.

3.3.1.1 NON-DEFAULTED FIRMS





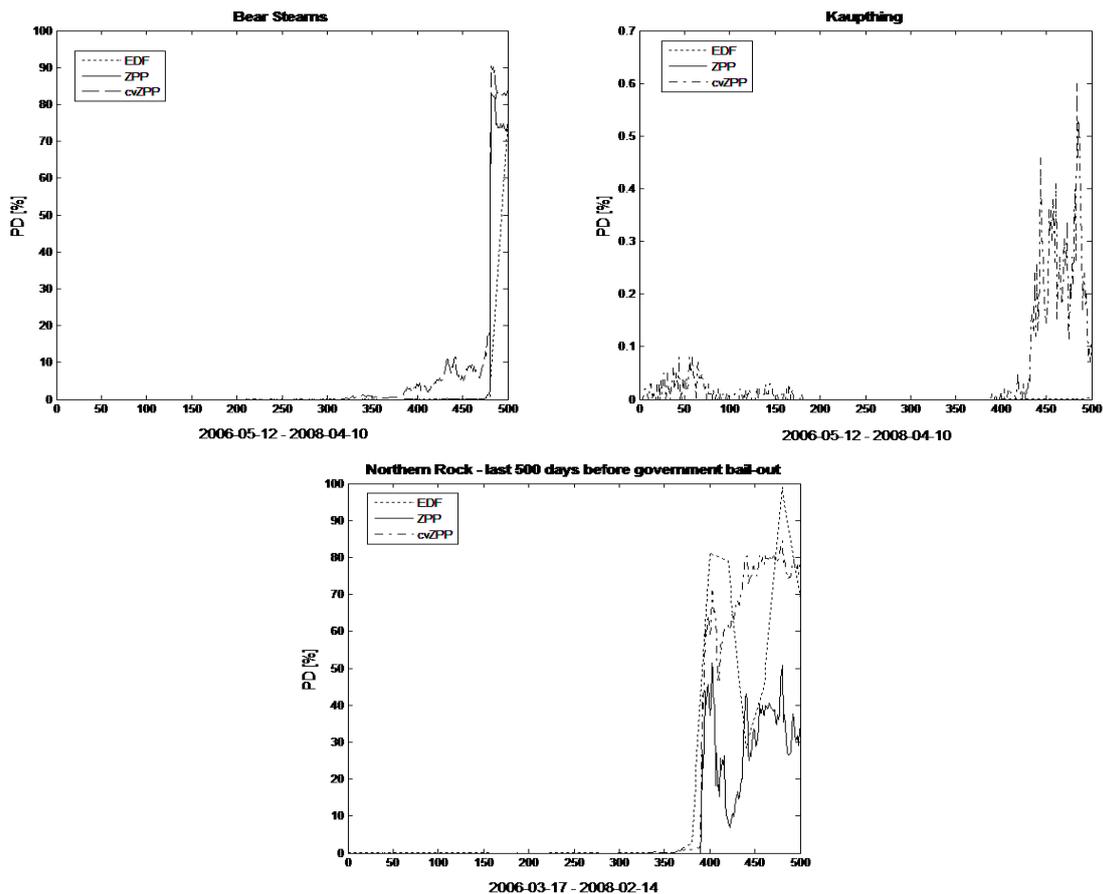
3.3.1.2 DEFAULTED FIRMS



From above figures it is clear that for the firms which included September 11 2001 in their presented data, that it had a significant effect. Also had the ZPP or cvZPP been used for PanAm the default-risk of that, at the time very reputable company, could have been foreseen.

3.3.2 FINANCIAL FIRMS

In this section the results for the financial firms are presented.

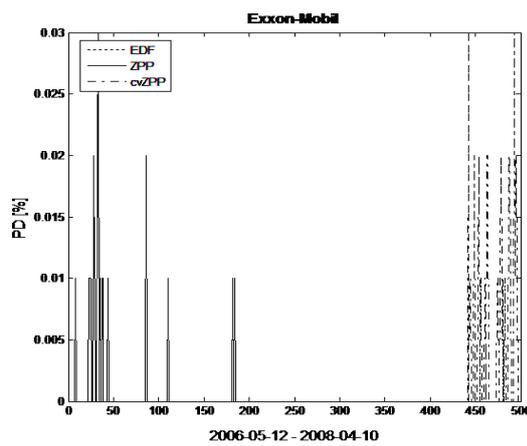
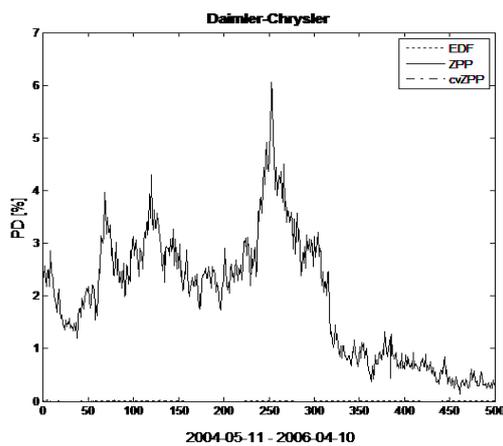
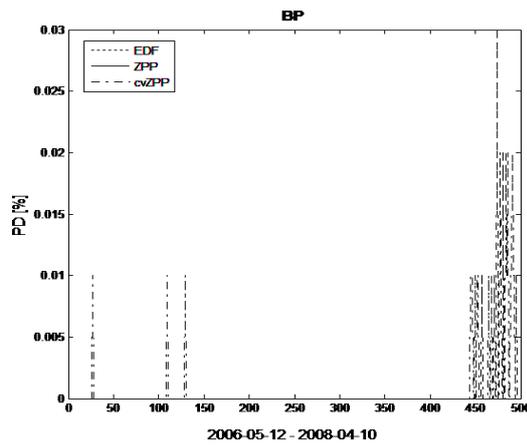
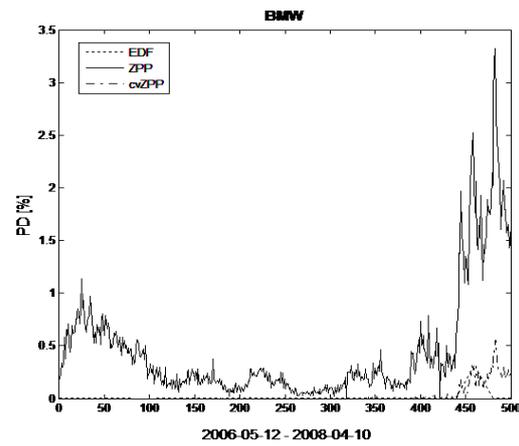
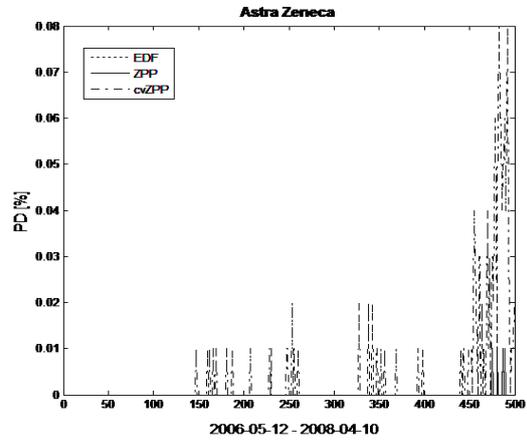
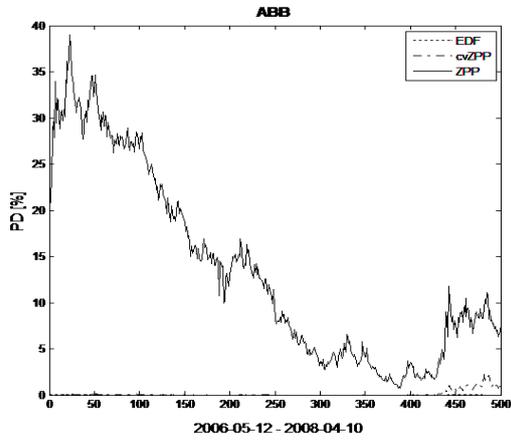


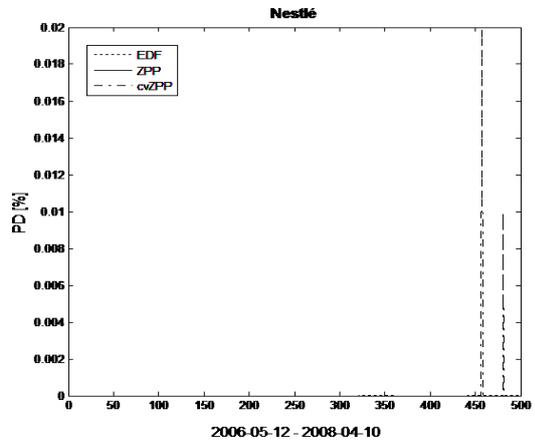
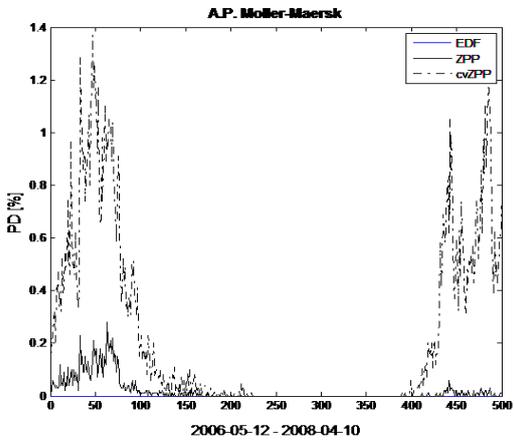
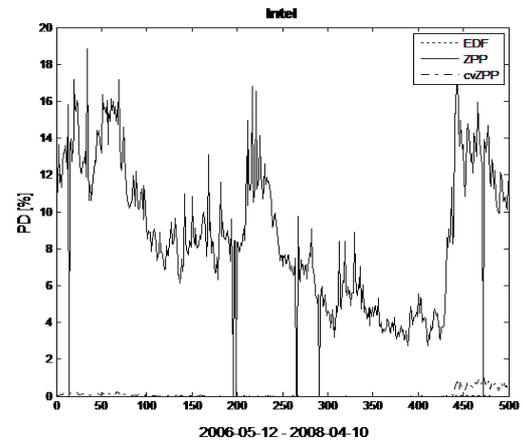
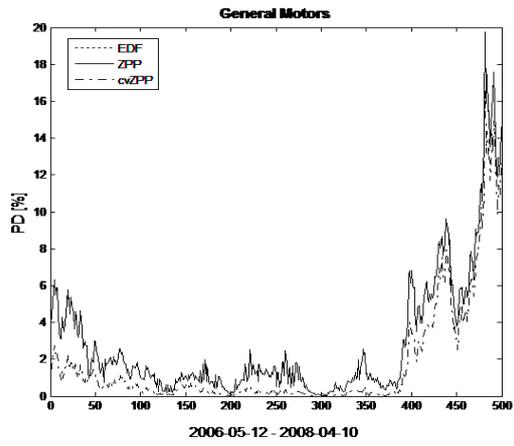
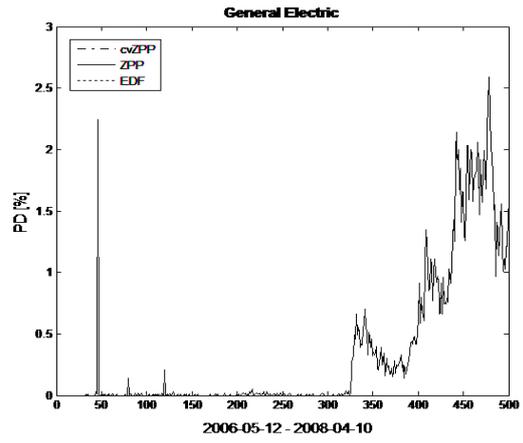
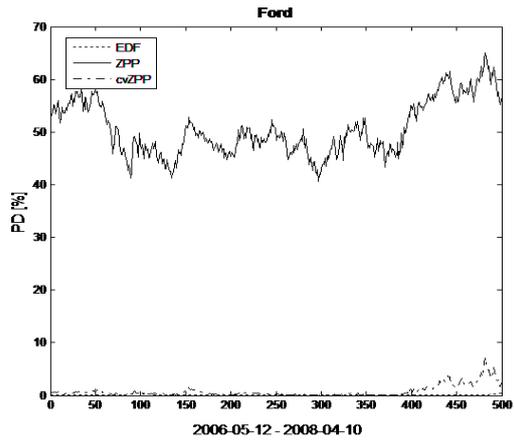
Before the summer 2007 when the sub-prime crisis became public knowledge none of these three firms had any default probability. However, as their exposure to this market was still there and the underlying default-risks still exists it means that all three calculated models underestimated the default risk at the earlier stages.

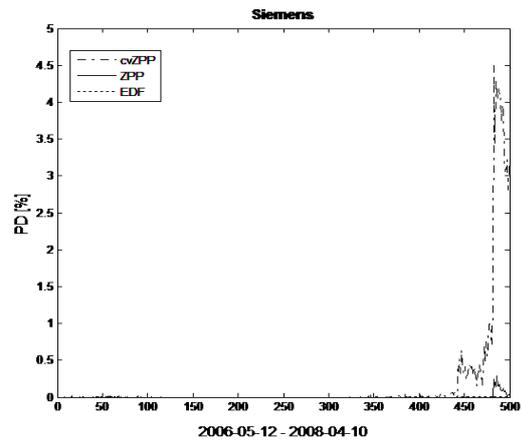
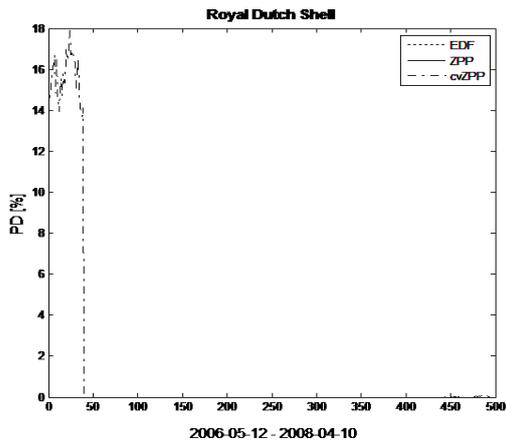
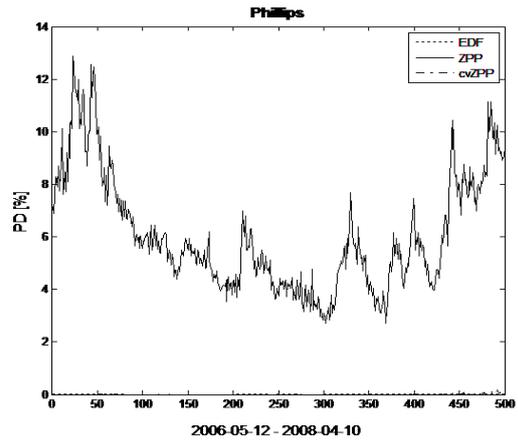
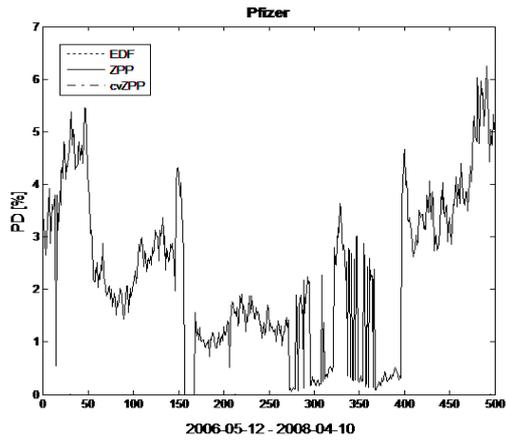
3.3.3 GENERAL INDUSTRIAL FIRMS

In this section the results for the rest of the firms investigated are presented.

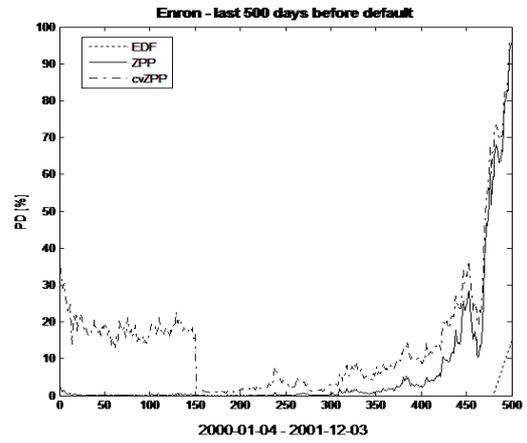
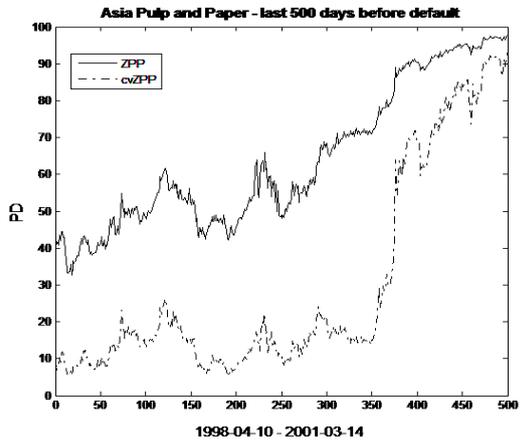
3.3.3.1 NON-DEFAULTED FIRMS

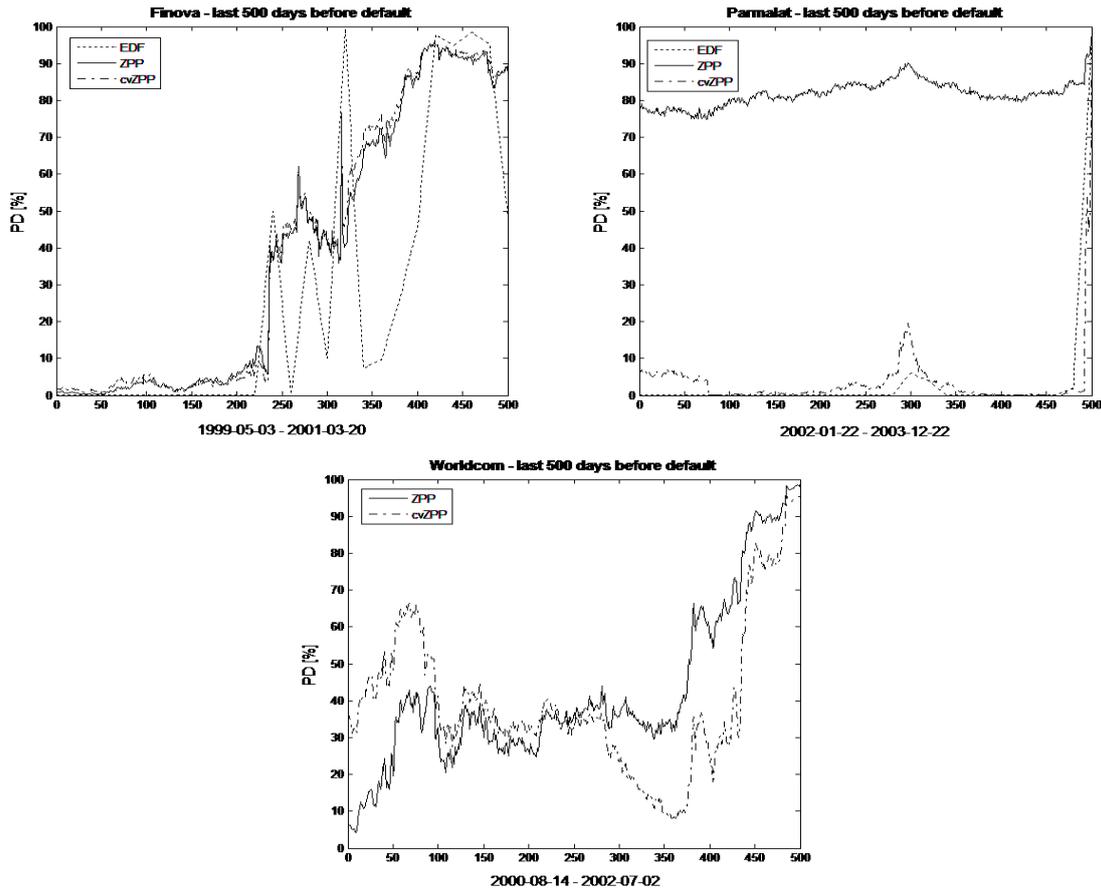






3.3.3.2 DEFAULTED FIRMS





The general industrial firms have very varied results. Main reason behind this is the large differences in both specific industry and also which market they operate or are listed in. Generally it can be said that firms in good health gets a overestimated default probability from the ZPP-type.

There are however some odd results that stand out. The oddest is the cvZPP estimation of Royal-Dutch Shell for the first 50 days; there is absolutely no reason why it would be that high. Notable is also that mostly the in this paper suggested cvZPP seems to follow the ZPP in trends, sometimes giving higher and sometimes lower results and only in the cases for Parmalat and Ford having significantly different levels.

3.3.4 SUMMARY OF RESULTS

The default probability as estimated by the ZPP is consistently higher than the EDF which is the same result as in the “A New Approach for Firm Value...” paper. Similarly the cvZPP almost always estimated a higher default probability than the EDF but compared to the ZPP it changed significantly. In some cases such as for Parmalat and Alitalia it followed the

EDF very closely and actually even reacted slower than the EDF which is the opposite as the ZPP which was high constantly. However in some cases the cvZPP seems to give a better estimate, for example it reacted more strongly than the ZPP in the Swissair case and also gave a significantly higher estimate for Northern Rock during the period just before the government bailout. Some obvious oddities can be seen, the cvZPP for Royal Dutch Shell gives a default probability of around 15% for the first 50 days in the calculated sample, the reason behind this is that the company was changed from one Dutch and one British company to one company in total which resulted in a very steep change in the price. It is, however, unclear why the British part of Royal Dutch Shell has that high default probability before that point when using the cvZPP. Another interesting result is the sharp leap the ZPP type models give for airlines at September 11 2001. Both of them react very quickly to the shock. The EDF measure mostly reacts as well but quickly falls back to the original level, thus implying a temporary increase in default probability, which by looking at the US airline industry is contradicted as several of the airlines have defaulted after 9/11.

What needs to be noted is that the EDF measure is most often not zero for all points, despite the appearance of it being that in the graphs above. It is, however, for most firms very low. For example Astra-Zeneca's EDF measure is, at most, around $9 \cdot 10^{-19}$ and the smallest non-zero point is about $8 \cdot 10^{-249}$, both of these observations are dwarfed by the ZPP and cvZPP. However, as the accuracy of the default distance calculations are around the decimal using results that have around 250 decimals is not reasonable since they are far smaller than the accuracy. In either case an accuracy of that magnitude is never interesting in finance.

Chapter 4. CONCLUSION

The main purpose of this paper has been to investigate the power of the ZPP-type models compared to an established standard structural model, namely the KMV. This was done by comparing the calculations of default probabilities for a number of firms varying from defaulted and near default to firms of extremely good financial health, like Exxon-Mobil. From interpreting the graphs in section three, the power of the different models can be seen. This power is as defined in the empirical section.

1. False prediction off default within the prediction horizon, or Type I error.
2. Non-prediction of default when there is default within the prediction horizon, or Type II error

The interest and significance of these two types of errors depends on the preferences of the lender (or investor). That is the type I error is mostly interesting for someone who is less afraid of default than the type II error. Clearly it can be seen in the results that the ZPP has a larger type I error than the Moody's KMV, however the KMV has a larger type II error than the ZPP. So a determination for each individual has to be made on what that person is more afraid of or more interested in.

Interestingly, the results for the cvZPP are not as clear-cut as for the ZPP. It tends to vary significantly and thus gives an idea that one very important factor is the volatility estimations which are required for ZPP estimation. As has been noted by, among others, Christofferson, the long term predictive power of GARCH type models might not be good. However, some of the results coming from the cvZPP are clearly incorrect, mainly the default probabilities calculated for the 50 first observations for Royal Dutch Shell which was, to say the least, high.

Also part of the investigation was to see how the measures reacted to external shocks that are impossible to foresee. This paper mentions the September 11 attacks on the World Trade Centre and the Pentagon, which

affected not only airlines but other firms as well, but airlines suffered the most, for example, due to increases in security costs and initially fewer travellers. It is in this case clear that both the EDF and ZPP-type models react to the shock. However the EDF mostly falls back to its original level within a very short time while the ZPP stays high. At least for the airline industry it could be argued that the ZPP is a better estimate as airlines both in the US and Europe defaulted after 9/11. Among the defaulted airlines was the “flying bank” Swissair where the cvZPP started rising a year before default compared to 100 days before default for the EDF measure. Several other airlines have since 9/11 either merged or defaulted, KLM merging with Air France and most recently the recovering Delta Airlines with Northwest Airlines; Lufthansa also took over the defaulted Swissair.

The financial sector is a more recent example of an industry in trouble due to the sub-prime crisis in the USA, where massive losses have been suffered by most major institutions in the world. Among them two large firms were close to default when being taken over by entities with better finances, namely Bear Sterns and Northern Rock. In the case of Northern Rock, the trouble they were in almost caused a run on the British banking system and this resulted in the bank being nationalized by the UK government. Bear Sterns was, in turn, acquired by the larger competitor, JP Morgan Chase bank, with the support of the US government. Interestingly, Kaupthing’s default probability for all of the three models is low, this is despite having a CDS spread around 1000 at the time the data was retrieved, which is a very high spread and should, in itself, implicitly indicate default probabilities.

For this paper, the interest was in the default predictions. Here, no model accurately predicts the defaults at any reasonable horizon. For Northern Rock, the horizon is about 100 days for all three tested models, which is not a bad horizon. For Bear Sterns, on the other hand, no model managed to predict defaults; the horizon here is about a month, 20 trading days, which for models that claim to give one year estimates is not a good result. That could have to do with the fact that no one managed to realize

the magnitude of the losses the major financial firms would suffer from the sub-prime crisis.

For the general industrial firms, some results stand out, mainly the difference between the cvZPP and ZPP. Most firms that are in “good” financial health do get low default probabilities for all estimates. Some do, however, get rather odd results. The one that stands out the most is for Ford where the ZPP gives a stable around 60% default probability while the cvZPP gives a few percent which is about the same as the EDF for the same firm. However, also some firms that are recovering from financial troubles, such as ABB, have an estimated default probability which indicates just that, meaning it is going down over time.

Also included among the general industrial firms were five known defaults, Enron, WorldCom, Asia Pulp and Paper, Parmalat and Finova. Enron being the largest default in history and is a commonly used example to compare different credit risk models and their “superiority” towards ratings and each other. If looking only at the Enron case the cvZPP performs best, it is constantly higher than the others. However, for Parmalat, the largest default in European history, the opposite is true. The cvZPP performs worst. In some cases, data for calculating the EDF was not available which brings to attention one of the major drawbacks of that model, namely the liabilities which are only publicly available at fixed intervals in either the quarterly or annual reports. This gives rise to the problem of discretization, which is, estimating it on a short interval is unwise, as how the liabilities develop over time is unknown. Also as reports are often “sugar-coated” to give potential investors an increased interest in investing in the firms, it can mean that the reports do not accurately describe the real state of the company, this was the huge problem for Enron, where the corporate leadership was dishonest about both liabilities and asset valuation and that, in turn, caused the eventual default of the firm.

However, it is, despite the obvious strengths of both types of models, not possible in this paper to come out and suggest that either should be seen as the “best”. This leads to the potential for future development and

research into not only models but also the often underlying volatility estimation.

Potential future venues for research are mainly to calculate some explicit power for each of the models. However, the ZPP has been shown to be greatly dependant on the volatility estimations, thus to determine if ZPP-type models are a good measure for default prediction it is needed to determine which volatility estimation method is best. It could also be of interest to try to adapt empirical databases to the ZPP to improve the results, meaning use the ZPP as an indicator similar to how KMV goes from default distance to EDF using their empirical database.

From the results presented and discussed above, it is clearly evident that the ZPP might have good predictive qualities. However, the problem with it is that it tends to overestimate the default probabilities of firms that haven't defaulted and thus if it is to be used, the user has to have a very high appetite for risk. For example, if the ZPP is the only reason, as it tends to overestimate default probability, not to give credit to a firm, it might exclude a large number of firms and thus limit the potential gains from lending if the sensitivity to risk is too great. This means that before it is possible to replace other credit risk measures, more research is needed. For now it can be a good option to complement other models since it tends to react more strongly slightly earlier compared to the EDF. However, getting higher default probabilities might not necessarily be a bad thing as limiting lending might not be a bad idea considering the current status of the financial markets following the sub-prime credit-crunch.

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