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How Gender-based Discrimination Affects Growth in an Extended Romer Model

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Abstract

The aim of the thesis is to formalize a relationship between gender-based discrimination in the research sector and economic growth in order to investigate whether discrimination affects GDP per capita and economic growth in the long run. The model has got its main features from the Romer model and has been elaborated in order to account for gender-based discrimination. Discrimination enters the model through the labour's input into the production function of GDP per capita – the final-goods sector and the research sector. The main findings of the model are that gender-based discrimination affects the level of GDP per capita negatively in the long run and that, even though the growth rate of GDP per capita was found to be unaffected in the long-run, gender-based discrimination affects the growth rate of transitory growth.

Keywords:

Discrimination, gender, endogenous growth, R&D, technological progress

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Denotations

D	discrimination
Y	output
y	output per capita
K	capital stock
L_y	labour employed in the final-goods sector
L_A	labour employed in the research sector
A	stock of ideas
s	savings
d	depreciation
n	population growth rate
s_R	share of labour force occupied in R&D
α	capital's share of total output
ϕ	how research productivity is affected by the stock of ideas already discovered
λ	parameter reflecting decreasing returns to scale in research
γ	elasticity measuring the importance of productivity in the research sector
p_y	the average productivity of the final-goods sector
p_A	the average productivity of the research sector
g_y	growth rate of GDP per capita
g_k	growth rate of the capital-labour ratio
g_A	growth rate of the stock of technology

Lower case letters denote variables in per capita.

1 Introduction

The endogenous growth model, which was developed by Paul Romer in a number of articles published in the mid-1980s and the 1990s, emphasizes on the importance of technological progress in the process of economic growth in the long-run. Technology is defined as the way input in the production of goods and services turns to output and technological progress refers to improvements of the production process. Technological progress is endogenized by the formalization of a relationship between economics of ideas and economic growth. The stock of ideas is equivalent to the stock of technology in the economy and it is the development of the idea-base over time that generates economic growth. The creation of ideas takes place in the research sector. The main input into the production of ideas is the employees of the research sector and the productivity of the research sector depends on the number of researchers employed. It may however also depend on the level of human capital accumulated by researchers, hence the researchers' talent for being innovative. In addition, there is proof of labour market discrimination related to gender in Western economies and the rationale of discrimination in economics is widely recognized in the literature. This implies that, under the assumption that productivity, i.e. talent for being innovative, is distributed uniformly between men and women, gender-based discrimination in the research sector leads to lower average productivity of the research workforce. This productivity loss of the research sector leads to a lower level of technological progress since the rate at which researchers produce new ideas has decreased due to discrimination, hence it should affect GDP per capita negatively in the long run.

1.1 Purpose

The aim of the thesis is to formalize a relationship between gender-based discrimination in the research sector and economic growth built upon the Romer model in order to answer the question of whether gender-based discrimination affects GDP per capita and economic growth in the long run. The impact of gender-based discrimination is to be investigated by the use of simulations.

1.2 Delimitations

The thesis takes stand point in prejudice against females, since discrimination of females is an evident feature of the labour market. The theories developed in the thesis can however just as well be applied to labour market discrimination in general, for example due to ethnicity, sexual orientation and looks. Hence the thesis does not seek to explain the psychology of gender-based discrimination, nor discuss discrimination in normative terms of right or wrong.

The term discrimination refers to *labour market discrimination*. Furthermore, the nature and the magnitude of gender-based labour market discrimination vary depending on a country's level of economic, social and political development. It is therefore important to note that the thesis seeks to explain the features of the labour market of Western economies. This is also the reason for the choice of the Romer model as a theoretical framework of the thesis. Hence the Romer model is most often applied to Western economies or the world as a whole.

In economics, growth is defined as the increase of goods and services produced by an economy and is measured by the percentage increase of gross domestic product, GDP. For more information about advantages and disadvantages of GDP as a measure of economic growth, see for example Fregert and Jonung (2003).

The model developed in the thesis is built upon the version of the Romer model developed by Jones (1995, 2002). There are small but crucial differences with regards to the technology accumulation function between the original Romer model and the version developed by Jones that results in a completely different solution to what determines economic growth in the long-run. The Romer model can be viewed as a special case of the Romer model developed by Jones. The reason for choosing Jones version of the Romer model is that the assumptions made by Jones are more plausible. The differences are to be discussed in more detail in section 3.3.1.

1.3 Related Literature

The rationale of discrimination in economics and the existence of discrimination of females in the labour market are, as will be discussed in section 2 of the thesis, widely acknowledged and visible in empirical studies. Until present time, research investigating the link between discrimination and economic growth is however limited. The following three articles model

discrimination and growth and are introduced briefly since they have been of great inspiration of the construction of the theoretical model of the thesis.

Esteve-Volvart (2000) investigates a model based on gender-based discrimination reflected in differentials in educational attainment. Discrimination reduces the potential level of human capital and is viewed as an inefficient practice in the economy. It is assumed that individuals are born with a given quantity of entrepreneurial talent and that talent is distributed uniformly between males and females. Individuals choose to become either managers or workers and their choice of schooling, which affects the position of each individual, depends on the level of discrimination of managerial positions. The empirical analysis finds that gender-based discrimination and growth are significantly related in a convex relationship.

Garcia-Minguez and Sanchez-Losada (2002) analyse the effect on growth of different governmental policies directed to eliminate statistical discrimination in wages by the use of an extended Romer model. Statistical discrimination is assumed to arise due to gender differences in quit rates and because individuals cannot signal whether they are quitters or not. Higher quit rates result in lower wages because of hiring costs. An overlapping generations model is used that divides the population into a young and an old generation, and each generation is divided into two groups with different quit rates. Garcia-Minguez and Sanchez-Losada conclude that transfer systems offsetting wage differentials increase the growth rate of an economy because of higher savings rates.

Sedgley and Elmslie (2005) modify the Romer model of endogenous growth to investigate how wage differentials at low skill levels and wage differentials at high skill levels affect per capita economic growth. Individuals choose their level of human capital based on comparing marginal benefits from acquiring additional human capital investments to the marginal costs. Discrimination is modelled as a tax or an iceberg transportation cost and the theory of dynastic equilibria and the Euler equation of the consumer optimization problem is used. Sedgley and Elmslie find that discrimination does not necessarily reduce economic growth. It depends on the distribution of discrimination across skills. If discrimination is reduced for high skill levels relative to low skill levels, growth is raised, and vice versa.

1.4 Outline of the Thesis

The thesis is organized as follows. Section 2 addresses the issue of how market forces allow discrimination to arise and persist in the economy and shows that the profitability of prejudice employers is lower than in the case of non-prejudice employers. In addition, a brief empirical overview of the research sector in terms of percentage shares of female employees is presented as well as the characteristics of females in the research sector. Section 3 outlays the gender-based economic growth model, which has got its main features from the Romer model and has been elaborated to account for discrimination in the two sectors – the final-goods sector and the research sector. In addition, the steady state solution to the endogenous growth model is discussed and analysed. Section 4 is dedicated to simulations of the gender-based economic growth model based on different levels of discrimination in the society and on different assumptions in connection to the importance of productivity in the research sector. The conclusions are presented in the 5th and final section of the thesis.

2 Discrimination

Labour market discrimination is defined as a situation where two equally productive individuals are treated unequally in the labour market and when the differences are related to observable characteristics such as gender, ethnicity, looks or sexual orientation.¹ The act of labour market discrimination causes differentials in labour force participation rates, wages, unemployment rates, occupational location, non-wage compensations, job characteristics, job mobility et cetera.² The objective of this section is to describe labour market discrimination in both its theoretical and empirical context in order to give a background to the concept of discrimination in economics and why it is both informative and relevant to formalize a relationship between gender-based discrimination and economic growth.

2.1 Economics of Discrimination

When labour market differentials are investigated it is important to separate the effect of discrimination from the effects of human capital differentials, i.e. attributes of workers in terms of educational attainment, years of labour market experience and years with current employer. It is however difficult to separate the effects. Differences in educational attainment may for example reflect both gender differentials in preference and choice and/or “pre-market” discrimination. It is therefore likely that when the effect of human capital differentials are taken into account, the effects of discrimination are underestimated, and vice versa.³ Labour market differentials related to gender have yet been investigated thoroughly in the literature and several studies have also found that gender differentials can most often only partly be explained by differences in the attributes of workers. Examples of creative and interesting studies are Lundberg and Rose (2000), Goldin and Rouse (2000), Johansson and Katz (2007) and Albrecht, Björklund and Vroman (2003).

What is the rationale of differences in demand of equally skilled workers? Nobel Laureate of 1992 Gary Becker is the theoretical father of *economics of discrimination*. Becker developed a theoretical framework of discrimination in economics that, since his doctoral dissertation “Economics of Discrimination” was published in 1957, have been widely acknowledged.

¹ OECD, 2008: 152

² Altonji and Blank, 1999: 3145

³ Ibid: 3146 - 53

Becker modelled discrimination as a taste or a preference of a majority group who gets disutility from interacting with a minority group. Several models of labour market discrimination have developed since, where the most commonly used models in economic analysis of discrimination are variations of Gary Becker's model of taste discrimination and models of statistical discrimination.

Taste discrimination is referring to a situation where discrimination arises on the behalf of the employer, the employee and/or the customers. In modelling tastes for discrimination, important concepts are the utility corrected wage and the discrimination coefficient, which give discrimination monetary value. In order to better understand the rationale of taste discrimination, employer taste discrimination is given as an example. Start by assuming that the labour market consists of two groups of workers, males and females. Assume thereafter that males and females are perfect substitutes and that a competitive employer faces a constant price of the two groups of w^F and w^M . If the employer has prejudice against women, the perceived wage of females is no longer w^F , but rather the utility corrected wage of $w^F(1+d)$, hence prejudice adds a positive percentage mark-up to the actual wage. The mark-up is the so-called discrimination coefficient and the greater the disutility of the employer the greater is the value of d . Firms' hiring decision are based on which labour is cheapest and under the profit-maximizing condition, depending on preferences, a prejudice employer end up with a completely homogenous workforce of either males or females. When the wage of males and females deviates, firms may forsake the opportunity to employ the labour with the lowest actual wage. Discrimination may therefore result in lower profitability of firms. Firms hire the "wrong gender" as well as too few workers, moving away from the profit-maximizing level of employment.⁴ Note that, in perfectly competitive markets, discrimination vanishes, since prejudice firms do not survive competition. Imperfect competition is therefore viewed as one of the main reasons for the existence of labour market discrimination.⁵

Employee discrimination reflects that discrimination might arise when co-workers at a firm get disutility from working alongside individuals of different gender, ethnicity or likewise, while customer discrimination describes a situation when purchasing decisions of prejudice individuals are influenced by a utility adjusted price.⁶

⁴ Borjes, 2005: 358, 362 - 263

⁵ OECD, 2008: 152

⁶ Borjes, 2005: 366 - 368

Statistical discrimination arises in the absence of prejudice and because employers cannot perfectly predict workers productivity and skills prior to a hiring decision. In the case of heterogeneous job applicants, employers have to base their hiring decision on beliefs about stereotypical behaviour of individuals of different gender or socio-economic background.⁷ If, for example, statistically females quit their jobs more often than males, a profit-maximizing employer chooses a male applicant. This is thus a choice made without knowing the true quitting probability of either the male or the female applicant. Statistical discrimination means that individuals from higher productivity groups benefit from discrimination at the same time as it creates gender differentials in wages and employment opportunities in the labour market.⁸

2.2 Empirical Overview

There is a world-wide trend of an increase of knowledge-based industries which has led to an increase in the demand for professional and highly educated workers. The growth rate of professional employment exceeds the total employment growth rate in developed countries and the members of the OECD reports growing numbers of researchers in the private sector as well as highly educated researchers. There is also evidence of a decline in young people's interest of a career in science and engineering and the growth rate of the number of graduates in fields of science and engineering are in some countries lower than the average growth rate of university graduates.⁹

This sub-section outlines a brief empirical overview of what characterizes the participation of females in R&D activities in Western economies. Data of female researcher participation rates in a selection of countries, with an additional focus on the Swedish experience, is presented as well as the horizontal and vertical segregation of females in the research sector. The reason for choosing Sweden as a "focus country" is that Sweden tops most rankings of gender equality made by international organizations, i.e. the Global Gender Gap Index 2007 published by the World Economic Forum¹⁰, and may therefore be of particular interest in the empirical overview. Are women in Sweden to a larger extent than in other countries represented in R&D?

⁷ OECD, 2008: 152

⁸ Botjes, 2005: 370

⁹ OECD, 2006 (1): 33 - 35

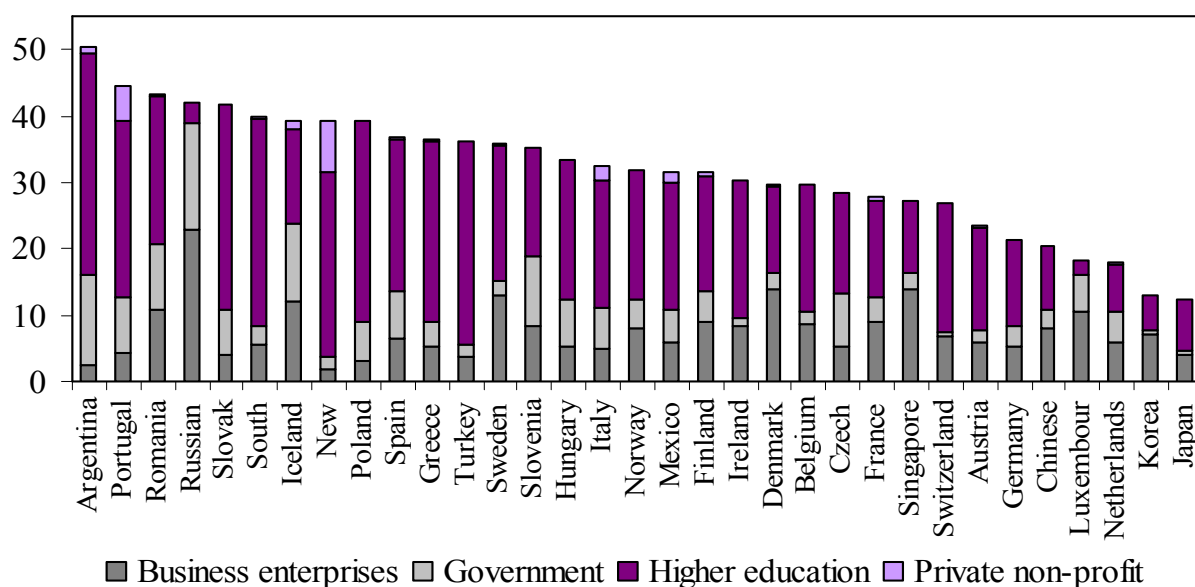
¹⁰ Hausmann, Tyson, Zahidi, 2007: 7

2.2.1 Female Researcher Participation Rates

The main actors of the R&D sector are found in the business sector, the higher education sector, the government sector and the private non-profit sector. The actors are either financiers, performers or both financiers and performers of research. It is oftentimes the case that the government sector and the private non-profit sector are financiers of R&D and that the higher education sector is performer of R&D. The business sector is most often both financier and performer of R&D.¹¹

A comparison of a sample of 33 selected countries in 2006 shows that, on average, 32 percent of all research employees are females. The percentage shares of females does however vary significantly, as is shown in figure 1, from a top notation at 50 percent in Argentina to 13 percent in Japan at the bottom.

Fig 1: The percentage share of female researchers of selected countries, 2006, by sector of employment¹²



In general, the share of female researchers is relatively high in the higher education sector and relatively low in the business sector. The latter is the sector that employs the largest amount of

¹¹ Statistics Sweden, 2007: 6

¹² OECD: Main Science and Technology Indicators, 2008. Notes: 2005 instead of 2006 for Belgium, Denmark, France, Germany, Greece, Iceland, Ireland, Italy, Luxembourg, the Netherlands, Norway, Poland, Portugal, Slovak Republic, Spain, Sweden, and South Africa; 2004 for Austria, and Switzerland; 2003 for Mexico; 2001 for New Zealand.

researchers in most countries. This feature can partly be explained by the distribution of male and female graduates in fields related to science and technology versus life sciences and social sciences. As will be discussed in the section 2.2.2, females are to a larger extent than males concentrated in the latter.¹³

Females' share of the research workforce in Sweden is by international comparison of an average level, see figure 1. Hence the fact that Sweden performs well in other fields related to gender does not affect the extent to which women are employed in the research sector. Figure 2 shows that the share of females increased from 33 percent in 1995 to 38 percent in 2005. The largest increase occurred in the higher education sector, where the share of female researchers increased by 3 percent. Furthermore, the amount of female researchers employed in the business sector increased by 2 percent and decreased by 0,5 percent in the government sector.

74 percent of the R&D in Sweden is performed in the business sector while the higher education sector performs 21 percent of the total R&D. One may also note that the private non-profit sector employs too few researchers in Sweden to be visible in the data.

Fig 2: The percentage share of female researchers in Sweden, 1995 - 2005, by sector of employment ¹⁴

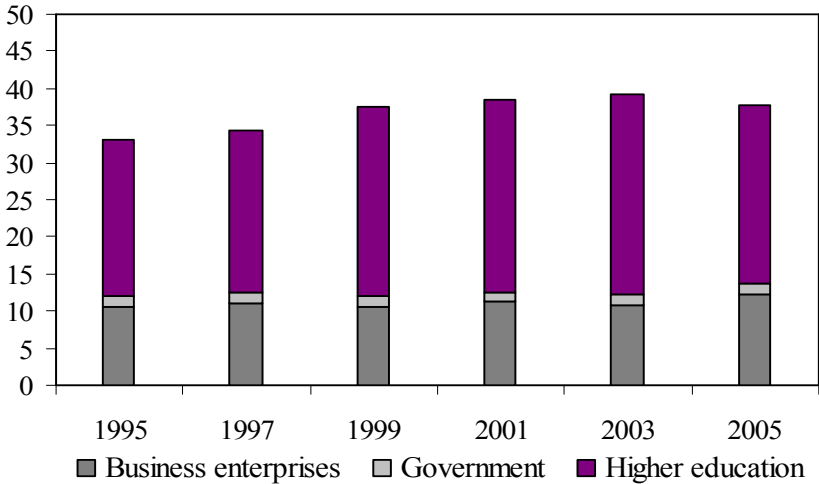


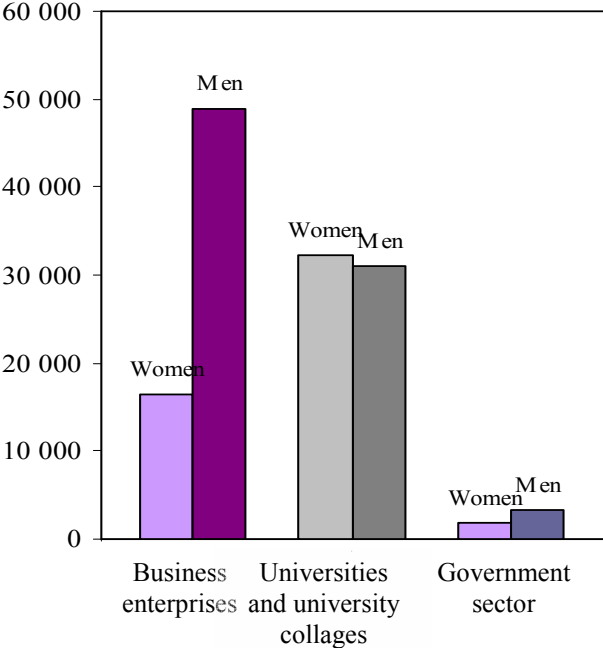
Figure 3 illustrates the amount of male and female employees in Sweden by sector of employment in 2005. The main part of all female researchers is employed in the higher education sector, while the business sector employs few females relative to its size. Large

¹³ OECD, 2006 (1): 36

¹⁴ Own calculations: Statistics Sweden, 2007: 39

corporations dominate R&D in the business sector while the 10 largest universities and university collages in Sweden dominate R&D in the higher education sector.¹⁵ Finally, in Sweden slightly more females than males are employed in the higher education sector.

*Fig 3: Male and female researchers in Sweden, 2005, by sectors of employment*¹⁶



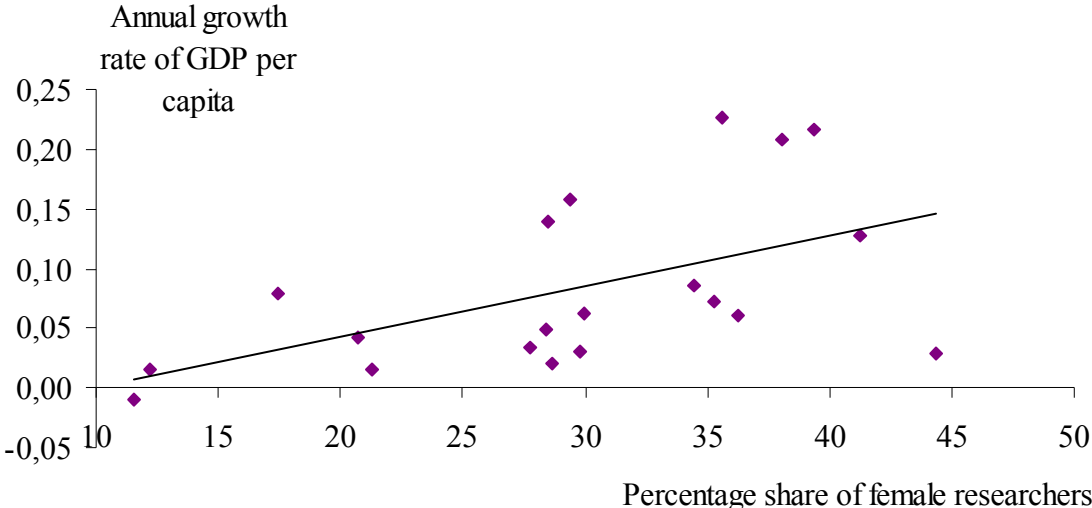
Despite the lower participation rates of females in the research sector in general, there is an overall trend in both the higher education related to science and technology and in the research sector of an increase in the number of participating females. However, the trend differs between countries, age groups et cetera. Women’s participation rates in research-related activities are higher in North America, the Nordic countries and in some European countries than in, for example, the Asian countries members of the OECD.¹⁷

One of the main predictions of the model developed in the thesis is that gender-based discrimination affects the level of GDP per capita negatively. This implies that a lower level of discrimination in the society results in a higher level of GDP per capita in the long-run. The percentage share of females in the research sector may be used as an approximation of discrimination, where equal shares of males and females in the research sector reflect absence of discrimination. Hence when the participation rate of females in the research sector increases,

¹⁵ Statistics Sweden, 2007: 12
¹⁶ Ibid: 39
¹⁷ OECD, 2006 (2): 11

the growth rate of the economy should increase as well, until the economy reaches the GDP per capita in steady state that is compatible with the new lower level of discrimination. From comparing the annual growth rate of GDP per capita with the total share of females in the research sector of 21 selected countries members of the OECD, it is found that the annual growth rate depends positively on the share of females in the research sector, see figure 4. The relationship is however unclear and the observations are too few to draw conclusions on, it may still be viewed as an indication of a relationship between growth and female R&D shares. Because there is evidence of increasing number of females in the research sector, the result corresponds well with the theoretical model developed in the thesis. It might however be problematic to use R&D shares as a proxy of discrimination, which becomes evident from the discussion held of horizontal and vertical segregation in the next subsection of the thesis.

Fig 4: Annual growth rate of GDP against the total share of female researchers, 2004, 21 selected countries ¹⁸



2.2.2 Horizontal and Vertical Segregation

With regard to gender, the scientific research sector in Western economies in general is characterized by two visible patterns. The first one is the so-called horizontal segregation effect. This means that women most often occupy certain fields of the research sector, such as biology, health, agriculture and pharmaceuticals, while few women are engaged in fields of for

¹⁸ OECD: Main Science and Technology Indicators database, EUROSTAT. Notes: The selected countries, in order of their level of discrimination in the research sector, are Portugal, Slovak Republic, Iceland, Poland, Spain, Turkey, Greece, Hungary, Ireland, Finland, Norway, Italy, Czech Republic, Denmark, France Switzerland, Austria, Luxembourg, Germany and Japan.

example physics, computing and engineering. The second pattern is the so-called vertical segregation effect, which refers to the fact that women's participation rates in higher positions are low. For example, only one-third of the university faculty in the United States are women, and it is even less in several European countries, and males are three times more likely than females to receive a professorship or likewise. In addition, there is evidence that more female students are enrolled in the higher education related to science and technology than there are females in the research sector.

Possible explanations of horizontal segregation are that preferences of males and females may differ (caused by either gender-specific motives or gender stereotypes) and that there are inadequate amounts of female role models and networking available for women in the fields of science and technology. The effect of the latter may explain why female participation rates in fields characterized by gender equality have increased more than in others fields. Other explanations may be the organizational structure of the research sector, the hiring procedures and the allocation of research funds and the culture at the research workplace, which may all restrain women's access to research-related jobs. Social attitudes may also play a part in affecting the extent to which women choose a career in male-dominated fields of work. Finally, discrimination against females in the research sector may affect horizontal segregation in two ways. Firstly through deliberate omissions of females in the hiring process of research jobs and secondly through discouraging women from choosing a career in a field related to science and technology in the first place.

In terms of vertical segregation, reasons may be the fact that women work part-time and on temporary job contracts more often than men, that there are fewer female role models in management in the research sector and that women oftentimes take a larger responsibility in terms of home- and childcare than men, which may be difficult to combine with a career in high position jobs. There is also evidence of a gender bias in the system for ranking research achievements, which may determine which person gets a promotion, as well as the research agenda, which is seldom set according to the interest of women. Finally, studies have found that the productivity of female researchers is lower than the one of male researchers and these possible differences may of course also affect the likelihood of women achieving high scientific positions.¹⁹ The presence of a gender productivity gap among academics has puzzled

¹⁹ OECD, 2006 (2): 12 - 14

researchers for a long time and several studies have been made on the topic. A recent study by sociologist Erin Leahey (2006) has found that the gender productivity gap, in addition to institutional factors, can be partly explained by gender differences in research specialization. Specialization is found to be strongly significantly related to productivity and, as shown by Leahey, females tend to have less specialized research programs than males.

Hence the causes behind the gender gap in the research sector are both multiple and related to each other in complex relationships, where perhaps only some of them, all of them, or none of them are related to discrimination. This makes it extremely difficult to measure the extent of discrimination in R&D. There is however a widespread understanding of the importance of research in the process of economic growth and the decreasing interest of a career in science and technology among the young generation is naturally of concern of policy makers. This trend has led policy makers around the world to turn their attention to groups underrepresented in the research sector, among them females, and was the main reason of why OECD in 2005 held a workshop called: “Women in scientific careers: Unleashing the potential”.

3 The Gender-based Endogenous Growth Model

This section describes the theoretical framework of the endogenous growth model developed by Paul Romer (1990) and adds to it assumptions of prejudice against females in the research sector. The Romer model which the thesis is built upon is the version of the model developed by Charles I. Jones (1995, 2002) and described in chapter 4 and 5 in his book “*Introduction to Economic Growth*”. In addition, parts of the theoretical framework of the Romer model, especially in connection to the concept of ideas and the presentation of the micro foundations of the model, originate from an introduction of the Romer model made by Sørensen and Whitta-Jacobsen (2005), see chapter 9 and 10 in “*Introducing Advanced Macroeconomics*”.

The first part of the section describes the concept of ideas in the model and relates it to the attributes of ideas as economic goods and imperfect competition. The second part addresses briefly the micro foundations of the Romer model in terms of the economy’s three sectors; the final-goods sector, the intermediate goods sector and the research sector. The macroeconomic endogenous growth model is developed in the third and final part of the section. The term endogenous refers to the fact that the driving force of economic growth is explained by the model itself. The opposite is true in the case of an exogenous growth model, i.e. economic growth is explained outside the model.

3.1 The Concept of Ideas

The Romer model emphasises on the importance of technological progress in the process of economic growth in the long-run, as will be explored in later parts of the thesis. Before moving on to the review of the Romer model, the concept of technology in the setting of the Romer model is examined in more detail.

Technology is defined as the way in which input to the production process turns to output. Furthermore, technological progress is the improvements of the production process originating from the creation of new technology. From the definitions it follows, which was first recognised by Romer and which enables for technology to be endogenized in the growth process, that the creation of technology is equivalent to the creation of ideas. An idea is a *design* of a capital good and can be interpreted as anything from a recipe, a method for organizing production or a code to a computer software programme. Ideas differ from other

economic goods in the way that they share features with public goods. In other words, ideas are non-rivalrous and more or less partly excludable. Non-rivalrous goods are goods that can be used by all individuals at the same time and where the degree of availability is unaffected. This implies that everyone with information of a non-rivalrous good can use it and that the good needs to be produced only once. The opposite is true for rival goods, for example private goods such as food and clothes, which has to be produced each time they are sold. Non-excludability refer to a situation where it is impossible to effectively prevent other individuals from using a good. Basic R&D is usually characterised by non-excludability while more “down-to-earth” ideas have a high degree of excludability. Basic R&D are oftentimes both non-rival and non-excludable, which implies that most research in basic R&D take place at public institutions or public enterprises and universities.

The research sector is assumed to consist of private, profit-motivated producers of ideas whose main motivation for creating new ideas is the opportunity to earn profit. The fact that ideas are non-rivalrous and partly excludable implies that production of idea-based technology will not take place in perfectly competitive markets. Hence there is free entry to the market and price is equal to marginal cost, firms receive negative profits equal to the fixed cost of producing an idea. Production of idea-based technology is thus linked to imperfect competition where imperfect competition enables firms to profit from their inventions. Important instruments for creating a market characterised by imperfect competition are patents and copyrights. These legal instruments ensure some excludability as well as monopoly power in the use of ideas. In the absence of legal systems, ideas could be copied more or less easily and eliminate the incentives of firms of producing idea-based technology.

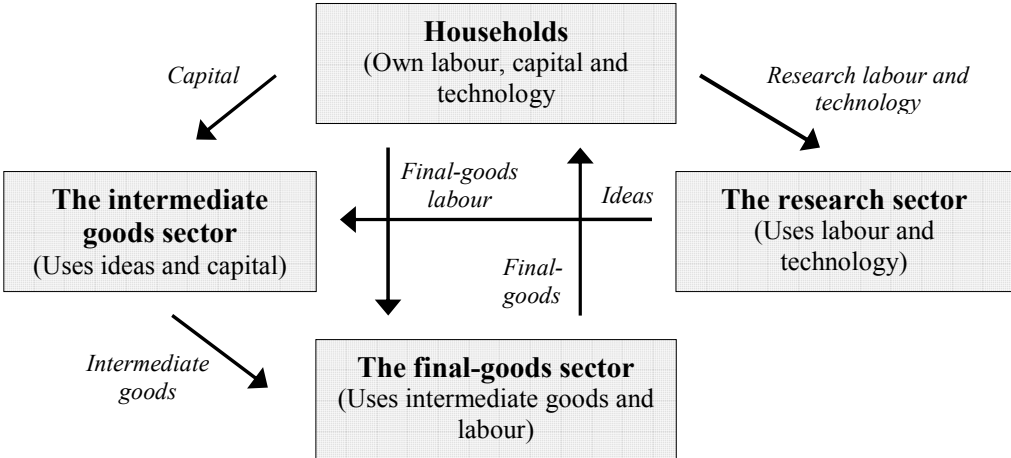
The market of ideas in the Romer model is thus characterised by imperfect competition maintained by a well-functioning patent system. Furthermore, the production of idea-based technology and imperfect competition are linked to the presence of increasing returns to scale, as will be explored in later parts of the thesis. Increasing returns to scale refers to a situation where a proportional increase of inputs into a production process results in a more than proportional increase in the output of that production process.

3.2 A Three Sector Economy

The micro foundations of the endogenous growth model are set up in an economy that consists of three sectors; the final-goods sector, the intermediate goods sector and the research sector. It is the presence of an intermediate goods sector that makes it possible to model imperfect competition and increasing returns to scale in a general equilibrium, which is, as aforementioned, a framework developed by Romer.

The interaction of the three sectors is explained as follows. *The research sector* uses labour and the existing stock of technology in the production of ideas. Firms of the research sector rent, sell or licence the right to use the ideas to the intermediate goods sector, e.g. in the form of patents. Firms of *the intermediate goods sector* require ideas and the stock of capital in the production of intermediate goods. Intermediate goods are goods used in the production of final output. The intermediate goods sector sells the intermediate goods to the final-goods sector. The inputs of *the final-goods sector* consist of intermediate goods and labour and the output produced by final-goods firms is either consumed or saved by *firms and households*, i.e. the consumers of the economy. The output that is saved increases the capital stock and households supply labour to either the research sector or the final-goods sector. Hence the consumers own the labour, the capital stock and the stock of technology and allocate these inputs in the production of ideas, intermediate goods or final-goods. The markets of ideas and intermediate goods are characterised by imperfect competition while the final-goods sector is perfectly competitive. The interactions between the sectors are summarized in the flow chart of figure 5:

Fig 5: The three sectors of the economy²⁰



²⁰ Sørensen and Whitta-Jacobsen (2005): 285

The market structure and the profit-maximizing behaviour of firms in each sector are described in detail below, for derivations see appendix A.

The final-goods sector: Firms in the final goods sector combine labour and intermediate goods in the production of a homogenous output, denoted Y . The production function of the final-goods sector is illustrated as follows:

$$Y = L_Y^{1-\alpha} \int_{j=0}^A x_j^\alpha \quad (1)$$

L_Y denotes the amount of labour working in the final goods sector and x_j denotes different intermediate goods. The first-order condition of the maximizing condition gives the solution to how much labour and intermediate goods final-goods firms use in production. Firms hire labour and buy intermediate goods until their value of marginal product is equal to the wage or price in accordance to equation 2 and 3:

$$w = (1-\alpha) \frac{Y}{L_Y} \quad (2)$$

$$p_j = \alpha L_Y^{1-\alpha} x_j^{\alpha-1} \quad (3)$$

The intermediate goods sector: Each intermediate firm buys the ideas from the research sector at a fixed cost. The ideas are protected by patents which make the intermediate good firms monopolists in their respective markets. One unit intermediate good is constructed from one unit capital good and the price of a unit of capital good is equal to the marginal cost, r . The price of the intermediate good is thus found from the solution to the profit-maximizing problem, which is simply a mark-up over the marginal cost:

$$p = \frac{1}{\alpha} r \quad (4)$$

The solution is equal for all intermediate goods firms, which means that all capital goods are sold at the same price, indicating that the demand of capital is equal to the stock of capital in the economy, see equation 5.

$$x = \frac{K}{A} \quad (5)$$

By inserting the above relationship into the final-goods production function, equation 1, gives the aggregate production function of the endogenous growth model which is going to be used the next section of the thesis:

$$Y = AL_y^{1-\alpha} A^{-\alpha} K^\alpha = K^\alpha (AL_Y)^{1-\alpha} \quad (6)$$

The research sector: The production function of ideas is equal to the stock of ideas and the number of people engaged in R&D:

$$\dot{A} = A^\phi L_A^\lambda \quad (7)$$

Inventors receive a patent for their designs and sell the right to produce capital based on their designs to intermediate goods firms. The price of a patent is determined by the present discounted value of the profits earned by the intermediate goods firm, P_A :

$$P_A = \frac{\pi}{r - n} \quad (8)$$

The solution is equal to the revenues from purchasing a patent, π , divided by the interest rate from purchasing a unit of capital, r , subtracted with the growth rate of the population, n .

3.3 Macroeconomic Modelling

The previous sections have described the concept of ideas and the structure of the market underlying the endogenous growth model. This section moves away from the microeconomic perspective and focuses on the aggregate economy, the engine of growth and constraints arising from discrimination, in other words, the macroeconomic modelling of the gender-based endogenous growth model.

The thesis modifies the Romer model in connection to labour. The labour force is divided in two groups of males and females and the concept of labour, as interpreted in the original

Romer model, is extended with labour equations that consists of different average productivity of males and females depending on the degree of discrimination in the labour market. The first subsection concentrates on the framework of the Romer model developed by Jones (2002) while the presentation of the model extensions and the solution to the gender-based endogenous growth model in steady state are introduced in the remaining parts of the subsection.

3.3.1 The Romer Model

Two types of output are produced in the aggregate economy - final-goods and technology. Final-goods are produced by firms in the final-goods sector and are either consumed or saved by firms and households. Furthermore, technology is created by ideas which, in turn, are produced in the research sector. Firms take the level of new ideas as given, both in the final-goods sector and the research sector, and each sector consists of many small firms with negligible influence on the aggregate economy.

The basic elements of the Romer model can be divided into two parts, a production function of final output and a number of equations that describe how components of the production function develop in the long-run. The production function of the final-goods sector is a Cobb-Douglas type which is widely used in the modelling of growth models in general. Inputs to the production function are the capital stock, K , the stock of technology, A , and the share of the labour force employed in the final-goods sector, L_y . The share of capital (versus labour and technology) to total output is determined by a fractional exponent, α .

$$Y = K^\alpha (AL_y)^{1-\alpha} \quad 0 \leq \alpha \leq 1 \quad (9)$$

When technology is given, the combination of capital and labour results in a production function which has constant return to scale. However, since the stock of technology is continuously expanding in line with the creation of new ideas resulting from an idea-production that uses productive inputs, the production function exhibits increasing returns to scale. Hence idea-production, imperfect competition and increasing returns are related - recall the discussion held in section 3.1. This implies that the production function generates sustained economic growth in the long-run.

The capital accumulation function describes how the capital stock develops over time. The stock of capital increases when individuals save a fraction of their income, s , and decreases with an exogenous depreciation rate, d . The fraction of the output that is saved is constant.

$$\dot{K} = sY - dK \quad (10)$$

As aforementioned, the key input to the production function of final output is the production function of ideas. The technology accumulation function, i.e. the rate at which new ideas are discovered over time, \dot{A} , depends on the stock of ideas already invented and the amount of the labour force working in the research sector, L_A :

$$\dot{A} = A^\phi L_A^\lambda \quad \phi < 1 \quad 0 < \lambda < 1 \quad (11)$$

The output elasticity, ϕ , determines how the stock of ideas, A , affects the rate at which new ideas are discovered. The output elasticity is positive if old ideas aid the creation of new ideas by making researchers more productive, which is referred to as the standing-on-shoulders effect. If the output elasticity is equal to zero, the discovering rate of new ideas is independent of the existing stock of ideas. The labour input of the research sector is raised by an externality, λ , that measures the stepping-on-toes effect. If λ is less than one, the externality takes into account that not all ideas discovered by individual researchers are new. Hence λ reflects decreasing returns to scale with regard to labour. If λ is equal to one, ideas invented are proportional to the labour input of the research sector and all ideas invented increase the stock of technology. The fact that new ideas sometimes displace old ideas is a process called creative destruction.

The assumptions made in the thesis in connection to the parameters of the production function of ideas ($\phi < 1$ and $0 < \lambda < 1$) are those promoted by Jones (2002) and differ from the original assumptions made by Romer (1990). Romer assumes that $\phi = \lambda = 1$ which results in an economy where economic growth is determined by the size and the productivity of the research workforce. Hence these assumptions give a completely different solution to what determines economic growth in the long-run, as will be shown in section 3.3.3. The assumptions made by Jones are more plausible in the sense that there may exist a so-called fishing-out effect related to ϕ . The fishing-out effect reflects the fact that the easiest ideas are discovered first and that it

therefore becomes more and more difficult to invent new ideas, hence that $\phi < 1$. In addition, the stepping-on-toes effect presented above implies that $\lambda < 1$. For more information about the assumptions made by Romer and economic growth in Romer's original model, see Romer (1990). Furthermore, readers who are familiar with the Romer model may have noticed that the parameter that measure researchers' productivity is excluded from the technology accumulation function. The productivity term is modified and included in the labour equations introduced in later parts of the thesis.

The labour force is equal to the total population of the economy and grows at a constant and exogenous rate, n .

$$\frac{\dot{L}}{L} = n \quad (12)$$

The labour force is employed in either the final-goods sector or the research sector. The share of the labour force employed in each sector is constant, s_R , see equation 13. In the original work of Romer and in more advanced models, the share of the labour force employed in the production of idea-based technology is determined endogenously.

$$\frac{L_A}{L} = s_R \quad (13)$$

3.3.2 Labour, Productivity and Discrimination

The extensions of the Romer model are made with regard to labour. In the original model, the amount of labour is given. In the gender-based endogenous growth model, the concept of labour is extended. Labour is instead given by labour equations that consist of the amount of labour employed in each sector as well as the average productivity of that particular set of labour. Productivity is defined as individuals' talent for producing final-goods and ideas, i.e. how many units of goods individuals produce in a given set of time. Productivity takes a value between 1 and 2, where the least productive individual has a productivity of 1 and the most productive individual has a productivity of 2. Furthermore, productivity is distributed uniformly between individuals of the economy.

An important assumption of the model is that productivity is relatively more important in the research sector than in the production of final-goods. Production of idea-based technology requires labour with high educational qualifications, e.g. advanced academic degrees in fields related to science and technology. More productive individuals acquire more education than less productive individuals and educational attainment in general aids productivity. Employees earn a wage that is equal to their marginal product of producing either final-goods or ideas, e.g. researchers earn a wage based on the value of the ideas they discover. Less talented individuals are therefore better off working with final-goods production and more talented individuals are better off being researchers. Hence the most productive individuals of each sex are employed in the research sector and the remaining part in the production of final-goods. Worth mentioning again is that productivity is distributed uniformly within the population, hence also between males and females.

In order to create a model that takes into account differences in the treatment of men and women in the labour market, the labour force is divided into two groups of equal size - males and females. Male and female workers are denoted with the exponents M and F respectively:

$$L = L^M + L^F, \quad L^M = L^F \quad (14)$$

3.3.2.1 *The Redistribution and Productivity Effect of Discrimination*

Discrimination, D , enters the model through the labour equations of the final-goods sector and the research sector. Discrimination measures *discrimination of females in the research sector*. For simplifying purposes it is assumed that there is no discrimination in the production of final-goods. Individuals receive employment in the final-goods sector regardless of whether they are males or females. The level of discrimination is exogenous and takes a value between 0 and 1, where 0 means no discrimination and 1 means complete discrimination. Discrimination is interpreted as a type of employer discrimination where women are restricted to enter the labour market of R&D due to prejudice against their capacity as researchers. Think of the economy as existing of many small firms producing either final-goods or ideas. Prejudice employers believe that females are less productive than males in the research sector and hire thereby only male workers. When the level of discrimination is complete, all firms in the research sector hire male workers, while when discrimination is less than complete, some firms are prejudiced and some are not. The share of firms being prejudice depends on the level of discrimination in the

society. Since productivity is distributed uniformly between males and females, the thesis defines an economy free of discrimination (i.e. when there is no employer discrimination in the research sector) when there is a fifty-fifty share of males and females in the research sector.

Discrimination affects the model in two ways. Firstly, discrimination affects the fraction of males and females working in the research sector and thereby indirectly the fraction of males and females working with production of final output. Discrimination is expressed as follows.

$$D = 1 - \frac{s_R^F}{s_R} \quad (15)$$

s_R^M and s_R^F denote the share of male and female researchers, respectively, and s_R denotes the total share of the labour force employed in the research sector. s_R^M and s_R^F are shown to be equal to the total share of researchers in the economy multiplied with a term which level depends on the level of discrimination, see equation 16. For proof, see appendix B.

$$s_R^M = s_R(1 + D) \quad (16)$$

$$s_R^F = s_R(1 - D) \quad (17)$$

The total share of the labour force working in the research sector is an exogenous and constant fraction and there is no unemployment in the economy. This implies that when discrimination prevails, male and female workers are redistributed between the final-goods sector and the research sector. Female workers are replaced by less productive male workers in the research sector and the opposite occurs in the final goods sector where male workers are replaced by more productive female workers. This leads to the second way discrimination affects the model. Discrimination affects the average productivity of workers of both sectors and of both sexes. (Note that productivity at individual level is given, which implies that males and females acquire the same amount of schooling compatible with their level of productivity when discrimination prevails than when discrimination is absent.) It follows from the assumption that productivity is distributed uniformly between males and females, that the average productivity function of males and females employed in the final-goods sector are given by equation 18 and 19:

$$P_y^M(D) = \frac{3 - s_R(1 + D)}{2} \quad (18)$$

$$P_y^F(D) = \frac{3 - s_R(1 - D)}{2} \quad (19)$$

The average productivity function of males and females in the R&D sector are given by equation 20 and 21:

$$P_A^M(D) = 2 - \frac{1}{2}s_R(1 + D) \quad (20)$$

$$P_A^F(D) = 2 - \frac{1}{2}s_R(1 - D) \quad (21)$$

Recall that the average productivity functions measure the average productivity of the labour force working in each sector divided into males and females. Separately, in the case of discrimination, it is still the case that the most talented individuals of each group are employed in the research sector, but in unequal proportions. Hence individuals do not recognise that the average productivity increases or decreases due to discrimination.

The redistribution of labour results in that the average productivity of males is a decreasing function of discrimination while the average productivity of females is an increasing function of discrimination. This *redistribution- and productivity effect* of discrimination is explained as follows. The share of male workers employed in the research sector increases with labour of less productivity from the final-goods sector. Hence average productivity of male researchers *decreases*. The less productive male workers of the final-goods sector, which have received employment in the research sector, are the most productive male workers of the final-goods sector. This implies that the group of males' of the final-goods sector have lost their most productive workers to the research sector. Hence the average productivity of male final-goods workers *decreases*. The reasoning with regard to female workers is analogous. The share of female workers employed in the research sector decreases with the least productive labour of the research sector. Hence the average productivity of female researchers *increases*. The less productive female workers of the research sector, which receive employment in the final-goods sector, are more productive than female final-goods workers in general. This implies that the group of females' of the final-goods sector have gained more productive workers from the

research sector. Hence the average productivity of female final-goods workers *increases*. The positive and negative effects of discrimination on the average level of productivity of males and females are summarized in the equations below. (Thorough derivations of the average productivity functions as well as the derivations of the productivity functions with respect to discrimination are found in appendix C.)

$$\text{Males:} \quad \frac{\delta P_A^M(D)}{\delta D} < 0 \quad \frac{\delta P_y^M(D)}{\delta D} < 0 \quad (22)$$

$$\text{Females:} \quad \frac{\delta P_A^F(D)}{\delta D} > 0 \quad \frac{\delta P_y^F(D)}{\delta D} > 0 \quad (23)$$

3.3.2.2 The Labour Equations

In the remaining part of the subsection, the labour equations of the final-goods sector and the research sector are introduced. The labour equations replace the expressions of the labour force working in the final-goods sector, L_y , and the research sector, L_A , in the production function of final output and the technology accumulation function of the original Romer model. See equation 9 and 11.

The labour equation of the final-goods sector, L_y' , consists of two separate equations of males and females where each equation reflects the average productivity of the final-goods workers and the number of males and females of the total population engaged in the production of final output. The latter is found from multiplying the share of males and females employed in the final-goods sector with the total population. Recall that both the average productivity and the share of males and females in the final-goods sector differ in degree and size depending on the level of discrimination in the research sector:

$$L_y' = p_y^M(D)(1 - s_R(1 + D))L^M + p_y^F(D)(1 - s_R(1 - D))L^F \quad (24)$$

$$p_y^M \neq p_A^M \quad \text{when } D > 0$$

The average productivity of males and females are identical when discrimination is absent since it is assumed that productivity is distributed uniformly between males and females and that the same type of labour in terms of their productivity level chooses to work with final-goods production or research production.

The labour equation of the research sector, L_A' , is similar to the final-goods labour equation. It reflects the average productivity and the amount of males and females working in the research sector, which differ depending on the degree of discrimination in the society. The amount of males and females employed in the research sector are found from multiplying the share of males and females of the research sector with the total amount of males and females in the labour force.

$$L_A' = p_A^M (D)^\gamma s_R (1 + D) L^M + p_A^F (D)^\gamma s_R (1 - D) L^F \quad (25)$$

$$p_Y^F \neq p_A^F \quad \text{when } D > 0, \gamma > 1$$

The average productivity functions of the research labour force are raised with an elasticity that measures the fact that productivity is relatively more important in the research sector than in the final-goods sector, captured by $\gamma > 1$. The fact that the elasticity is strictly greater than 1 is important since otherwise, if γ is equal to 1, productivity is equally important in the production of ideas as in the production of final output. Finally, when discrimination is absent the average productivity of males and females are identical.

3.3.2.3 The Complete Model

The model of gender-based endogenous growth is summarized by equation 26 to 31. The production function of final output is modified with the labour equation of the final-goods sector and the technology accumulation function is modified with the labour equation of the research sector. The capital accumulation function as well as the growth rate of the labour force are identical to the original model.

$$Y = K^\alpha (AL_y)'^{1-\alpha} \quad (26)$$

$$\dot{A} = A^\phi L_A'^\lambda \quad (27)$$

$$\dot{K} = sY - dK \quad (28)$$

$$\dot{L} = nL \quad (29)$$

$$L_y' = p_y^M (D)(1 - s_R(1 + D))L^M + p_y^F (D)(1 - s_R(1 - D))L^F \quad (30)$$

$$L_A' = p_A^M (D)^\gamma s_R (1 + D) L^M + p_A^F (D)^\gamma s_R (1 - D) L^F \quad (31)$$

The model reflects a dynamic system of research-based growth which explains endogenous growth of gross domestic product, GDP (Y). The system consists of variables (K , A and L) and technical and behavioural parameters (α , ϕ , λ , γ , s , n , d , D , s_R , s_R^M , s_R^F , P_A^M , P_A^F , P_y^M and P_y^F). Greek numbers represent technical parameters.

3.3.3 Steady State Solution

The concept of *steady state* is referring to a state of the economy where the variables of the model grow at a constant rate over time. When the steady state rate of growth is positive, the variables grow along a balanced growth path. In the short-run, shocks move the economy away from or change the steady state. The steady state concept is therefore only useful for making predictions with regard to the economy's level of development and its progress over time in the long-run.

The first part of the subsection examines the relationship between economic growth and technological progress and the determinants of technological progress in the long run. The second part of the subsection explores how features of the economy affect the level of GDP per capita in steady state. Finally, the question whether gender-based discrimination affects GDP per capita and economic growth in the long-run is examined.

3.3.3.1 Growth of the Model

It can be shown, which corresponds with the result of the original Romer model, that the rate of growth of GDP per capita is the same as the rate of growth of the capital-labour ratio and rate of growth of the stock of ideas in the long run. For proof, see appendix E.

$$g_y = g_k = g_A \quad g > 0 \quad (30)$$

From the above relationship it follows that the growth rate of GDP per capita is determined by the growth rate of the stock of technology. The determinants of the growth rate of technology in steady state are found from rewriting the technology accumulation function. For proof, see appendix E.

$$g_A = \frac{\lambda n}{1 - \phi} \quad (31)$$

The technology growth rate is equal to the population growth rate, n , and parameters of the technology accumulation function. The latter determines to what extent the stock of technology affects the production of new ideas (i.e. the size of the standing-on-shoulders effect, ϕ) and whether there are decreasing returns to labour in the production of ideas (i.e. the stepping-on-toes effect, λ). The technology growth rate reflects the fact that the number of researchers increases with the population growth rate and that the more researchers employed, the more ideas invented that are beneficial to everyone in an economy regardless of its size. This also implies that the variables grow along a balanced growth path, hence $g > 0$.

It can be concluded that in the long-run the economy grows with the same rate as the growth rate of its population. Hence gender-based discrimination does not affect the growth rate of the economy. Another important implication is that standard measures to increase economic growth (e.g. through government policies) are ineffective in creating sustainable economic growth in the long run. These conclusions are related to the assumptions of the parameters of the technology accumulation function (hence $\phi < 1$ and $0 < \lambda < 1$ from equation 11) emphasized by Jones and is not a prediction made by Romer.

When the economy is outside its steady state, which is oftentimes the case in the short run, the variables adjust to the steady state level of growth over time. This process is called transitory growth. During the adjustment period, the economy exhibits periods of growth above or below its steady state which result in the economy reaching a higher or lower level of GDP per capita in steady state. Related to economic growth is also the concept of exponential growth. This means that the larger an economy gets, the faster it grows. Hence even small changes in the annual growth rate generate large effects in the long-run. The income level per capita and the presence of transitory growth are thus both aspects of great importance in the analysis of economic prosperity in the long run.

3.3.3.2 Solving for the Steady State

In order to determine the features of the economy that affect the level of GDP per capita in the long-run, the solution to GDP per capita in steady state is solved in terms of the parameters of the model. For proof, see appendix D.

$$y^* = \left[\frac{s}{d + g_A + n} \right]^{\frac{\alpha}{1-\alpha}} \left(\frac{P_y^M(D) s_R (1 - (1 + D)) L^M + P_y^F(D) (1 - (1 + D)) L^F}{L} \right) \quad (30)$$

$$\left(\frac{(P_A^M(D)^\gamma s_R (1 + D) L^M + P_A^F(D)^\gamma s_R (1 - D) L^F)^\lambda}{g_A} \right)^{\frac{1}{1-\phi}}$$

It can be concluded that the results of the above equation correspond with the results of the original Romer model. Hence the savings rate, s , affects the steady state level of GDP per capita in a positive manner. Countries that invest more are richer because investments increase the stock of capital in the economy. Analogously, the depreciation rate, d , affects the steady state level of GDP per capita in a negative manner because it reduces the stock of capital in the economy. The population growth rate, n , has a negative impact on GDP per capita in steady state. Since the steady state solution is expressed in terms of per capita, individuals of the economy need to invest more in order for the growth rate of GDP per capita to stay constant.

Another determinant that influences GDP per capita negatively in steady state is the technology growth rate, g_A . The technology growth rate, which will be examined in more detail later, reflects the rate at which the stock of technology changes over time and enters the expression in two ways. Firstly, the negativity of the technology growth rate arises because individuals need to invest more in order to take advantage of all new inventions created and secondly, because researchers need to maintain the rate of the production of ideas in order for the steady state rate to stay constant over time. It is however the case that the negative effect of the growth rate of technology is to be viewed as a technicality.

The size of the labour force affects the growth rate of GDP per capita both positively and negatively. The positive impact arises from the fact that the larger the economy, the larger the amount of labour employed in the research sector. The negative impact arises because individuals working in the research sector cannot at the same time produce final-goods. The impacts of the labour force are reflected on both the numerator and denominator of the two parenthesis of the equation. Notice that the presence of the total population in the equation implies that the model exhibits a scale effect in levels.

Productivity enters the steady state solution as a numerator. Hence productivity affects the level of GDP per capita positively. Although, since productivity is given within an interval of 1 to 2 and distributed uniformly between the population in an economy where there is no unemployment, average productivity *per se* does not affect the level of GDP per capita. The same reasoning is applied for the shares of males and females in the final-goods sector and the research sector. Hence the size of the shares of males and females in the two sectors does not affect the level of GDP per capita *per se* since the total amount of employees in the economy is constant.

3.3.3.3 Effects of Discrimination

How does gender-based discrimination affect the level of GDP per capita in steady state? As aforementioned, discrimination generates a redistribution- and productivity effect of male and female workers in the final-goods sector and the research sector and there is no unemployment in the economy. These features of the economy suggest that the contribution of the final-goods sector to final output increases while the contribution of the research sector to production of ideas decreases. Hence discrimination affects the level of GDP per capita in steady state both positively and negatively. In addition, if the positive effect exactly outweighs the negative effect of discrimination, the total effect of discrimination vanishes.

Hence does discrimination affect GDP per capita in steady state positively, negatively or not at all? From an analytical point of view, if productivity is equally important in the final-goods sector as it is in the production of ideas, the contribution of the final-goods sector outweighs the contribution of the research sector and the effect of discrimination vanishes. Productivity is however more important in the research sector, captured by $\gamma > 0$, which implies that the negative impact of discrimination exceeds the positive impact. In order to investigate whether this is true, the solution of the gender-based endogenous growth model in steady state is derived with respect to discrimination:

$$\frac{\delta y^*}{\delta D} = \left[\frac{s}{d + g_A + n} \right]^{1-\alpha} \frac{1}{g_A^{\frac{1}{1-\phi}} L} \left(P_A^M(D)^\gamma s_R(1+D)L^M + P_A^F(D)^\gamma s_R(1-D)L^F \right)^{\frac{\lambda}{1-\phi}}$$

$$\left[L^{M,F} \left(P_y^M(D)(1-s_R(1+D)) - P_y^M(D)s_R + P_y^F(D)(1-s_R(1-D)) + P_y^F(D)s_R \right) + \frac{\lambda}{1-\phi} \left(\frac{P_y^M(D)(1-s_R(1+D)) + P_y^F(D)(1-s_R(1-D))}{P_A^M(D)^\gamma s_R(1+D) + P_A^F(D)^\gamma s_R(1-D)} \right) \right]$$

$$\left[L^{M,F} \left(\gamma P_A^M(D)^{\gamma-1} P_A^M(D)s_R(1+D) + P_A^M(D)^\gamma s_R + \gamma P_A^F(D)^{\gamma-1} P_A^F(D)s_R(1-D) - P_A^F(D)^\gamma s_R \right) \right]$$

From analyzing the derivative of the growth model, it becomes apparent that it is too cumbersome to determine how discrimination affects GDP per capita in steady state without inserting numerical values into the model. Hence to investigate how discrimination affects GDP per capita in the long-run and the magnitude of this effect, the gender-based endogenous growth model needs to be simulated.

4 Simulations

This section undertakes simulations of the gender-based endogenous growth model. The simulations are predictions of the dynamic system of research-based growth developed in the thesis and are made in order to investigate whether and to what extent discrimination affects GDP per capita in the long-run. Hence simulations are imitations of how the economy develops over time and are computed from the production function of final output, the capital accumulation function and the technology accumulation function. In order to examine how gender-based discrimination affects the economy in the long run, it is assumed that the economy is in steady state when the simulations begin. The simulations would otherwise not be able to isolate the effect of gender-based discrimination from other features of the economy that determine how the economy develops over time. Furthermore, the simulations are undertaken over a time period of 50 years and start at year 2000.

The first subsection introduces the values chosen for the parameter and variables of the endogenous growth model, i.e. the so-called *default values*. The results from the simulations are presented and analyzed in the remaining part of the section.

4.1 Default Values

The model consists of both technical and behavioural *parameters*, where Greek numbers represent technical parameters, which are, due to technical considerations, constant over time. Furthermore, the parameters are divided into static and dynamic parameters depending on whether their value change with regard to the level of discrimination in the economy. The static parameters are unchanged by the level of discrimination and vice versa.

The parameters of the model are given numerical values that are widely used in economics and in simulations of simple economic growth models in general, see for example Jones (2002), and which are based on empirical observations. In addition, the choice of parameter values is also based on the parameters ability to generate plausible simulations, i.e. simulations that behave realistically. The static parameters of the model are summarized in table 1.

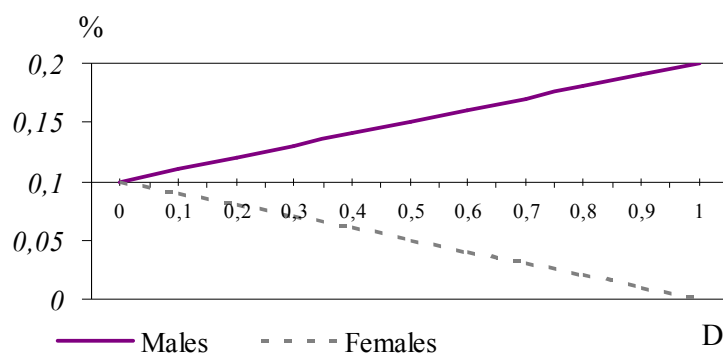
Tab 1: Static parameters

s	0,1
d	0,05
n	0,02
α	0,3
λ	0,5
ϕ	0,5
s_R	0,1
γ	1
	4
	8

The gross national savings rate as a percentage of disposable income is 10 percent, the depreciation rate of the capital stock is 5 percent, the annual population growth rate is 2 percent and capital's share of total output is 0,3.²¹ There is a stepping-on-toes effect of the labour force employed in the research sector, reflected in λ equal to 0,5 and ϕ equal to 0,5 implies that there is a standing-on-shoulders effect with regard to the stock of technology. The share of the labour force employed in the research sector is 10 percent and the relative importance of talent in the research sector, captured by γ , is given three different values – 1, 4 and 8 – during the simulations.

The dynamic parameters of the model are D , s_R^M , s_R^F , P_A^M , P_A^F , P_y^M and P_y^F . Discrimination is a value between 0 and 1, where 0 means no discrimination and 1 means complete discrimination. Figure 6 illustrates the fraction of male and female employees in the research sector at different degrees of discrimination.

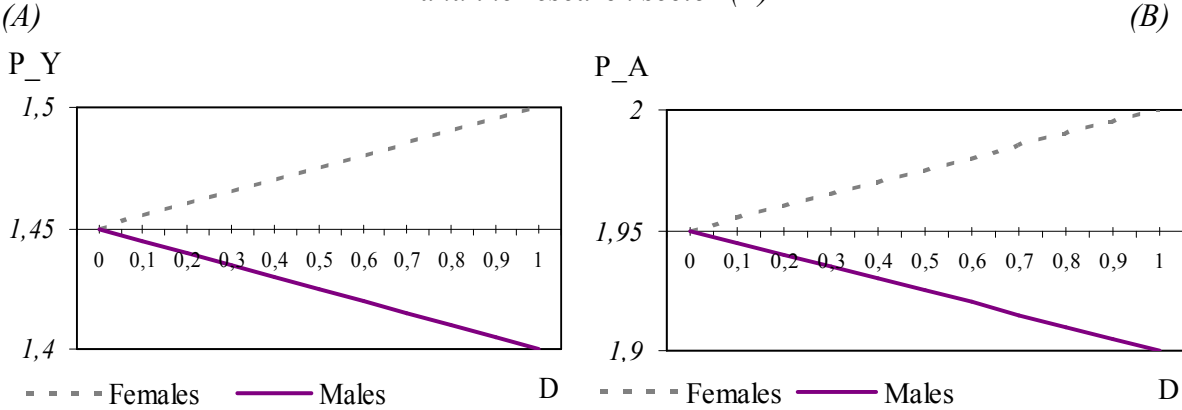
Fig 6: The fraction of males and females employed in the research sector



²¹ Jones, 2002: 20 - 22

The share of males in the research sector increases with discrimination and the share of females in the research sector decreases with discrimination. In the case of complete discrimination, the share of female researchers vanishes. The productivity of individuals is given within an interval of 1 to 2, where individuals with a productivity close to 2 are the most productive and vice versa. The average productivity of males and females are identical in the absence of discrimination – 1,95 in the research sector and 1,45 in the final-goods sector. When the level of discrimination increases, males’ average productivity decreases and females’ average productivity increases. This redistribution- and productivity effect of discrimination is illustrated in figure 7.

Fig 7: The average productivity of males and females in the final-goods sector (A) and the research sector (B)



The numerical values of the variables A_0 , K_0 and L_0 are chosen so that the economy is close to its steady state when the simulations begin. These values are found from solving the equations presented below. For derivations, see appendix F.

The value of L_0 is an arbitrary number and has been given the value 10. This number may be interpreted as in million inhabitants in the economy.

The initial value of GDP per capita is found from inserting the parameter values of the model into the solution of the model in steady state (For more information about the solution in steady state, see section 3.3.3.2):

$$y^* = \left[\frac{s}{d + g_A + n} \right]^{\frac{\alpha}{1-\alpha}} \left(\frac{P_y^M(D) s_R (1 - (1 + D)) L^M + P_y^F(D) (1 - (1 + D)) L^F}{L} \right) \quad (34)$$

$$\left(\frac{(P_A^M(D)^\gamma s_R (1 + D) L^M + P_A^F(D)^\gamma s_R (1 - D) L^F)^\lambda}{g_A} \right)^{\frac{1}{1-\phi}}$$

The initial endowment of the stock of capital in the economy is found by inserting the solutions to y^* and relevant parameters into equation 35. The stock of technology is a theoretical construction and is to be considered an index value.

$$A = \frac{y^*}{\left[\frac{s}{d + g_A + n} \right]^{\frac{\alpha}{1-\alpha}} \left(\frac{L_y}{L} \right)} \quad (35)$$

By inserting relevant variables and parameters into equation 36, the default value of the stock of capital is determined. Recall that y^* , A and K are endogenous and that their values change depending on the choice of parameters of the model.

$$K = \frac{y^{\frac{1}{\alpha}}}{A^{\frac{1-\alpha}{\alpha}} \left(\frac{L_y}{L} \right)^{\frac{1-\alpha}{\alpha}}} L \quad (36)$$

4.2 The Simulations

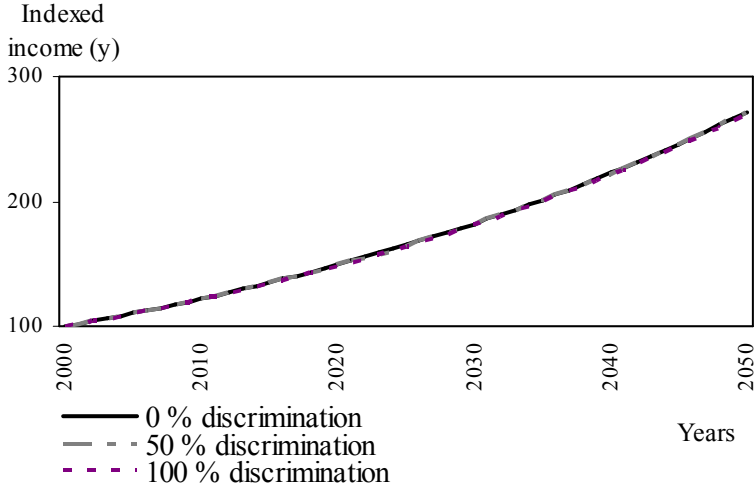
This subsection presents and analyzes the results from the simulations of GDP per capita and the annual growth rate in steady state. Hence this section answers the question posed in the introduction of the thesis. It was shown in the previous section that discrimination affects economic growth both positively and negatively and the total effect of discrimination may be related to the relative importance of productivity in the research sector. For this reason, the simulations are undertaken three times and each time with a different assumption with regard to $\gamma - \gamma$ equal to 1, 4 and 8 – is used. When the relative importance of productivity is equal to 1, productivity is equally important in the research sector and in the production of final output.

When the relative importance of productivity is $\gamma > 1$, productivity is more important in the research sector. The larger the value of γ , the more important is productivity in the production of idea-based technology. The simulations are in all other aspects beside γ identical.

Each simulation consists of three separate simulations based on different scenarios of the level of discrimination in the society – 0 percent discrimination, 50 percent discrimination and 100 percent discrimination. When the simulations of the three scenarios differ, discrimination affects the aggregate level of GDP per capita in steady state. Hence regardless whether discrimination has an impact on the aggregate level of the economy or not, workers at the micro level of the economy are still affected by discrimination. The relative size of the difference between the discrimination scenarios determines to what extent GDP per capita is influenced by discrimination. The simulations are illustrated both graphically and in table form and have for comparable purposes been indexed over time. Hence depending on the assumption of γ , the value of GDP per capita as well as the growth rate of GDP per capita differ. It is thus the relative difference between the discrimination scenarios that is of interest and not the actual level of GDP per capita. Finally, in order to examine how the variables develop in the long-run, the simulations are undertaken over a time period of 50 years.

The first simulation illustrates the development of GDP per capita over time where productivity is equally important in both sectors of the economy, captured by $\gamma = 1$, see figure 8 and table 2 below.

Fig 8: Indexed value of GDP per capita in steady state, $\gamma = 1$



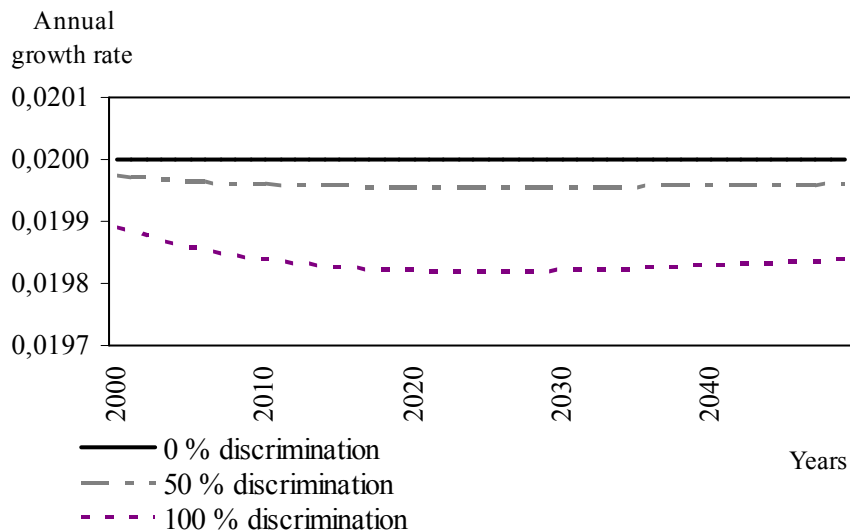
Tab 2: Indexed value of GDP per capita in steady state, $\gamma = 1$

Years	0 % discrimination	50 % discrimination	100 % discrimination
2000	100	100	100
2010	122	122	122
2020	149	149	149
2030	182	182	181
2040	223	222	221
2050	272	271	270

The simulation shows that when females are not discriminated in the research sector, the economy grows by 172 percent over a time period of 50 years. However, when comparing the simulations of the different discrimination scenarios, it is revealed that if the economy is characterized by complete discrimination, it grows by 170 percent, which is less than in the case of no discrimination. Hence it can be concluded that discrimination affects the economy negatively in the long run and that the effects of discrimination are visible even though productivity is equally important in the production of final output than in the production of idea-based technology. The total effect of discrimination is however small, which implies that the positive and the negative effect of discrimination to a large extent outweigh each other.

The annual growth rate of GDP per capita in steady state when γ is equal to 1 is displayed in figure 9.

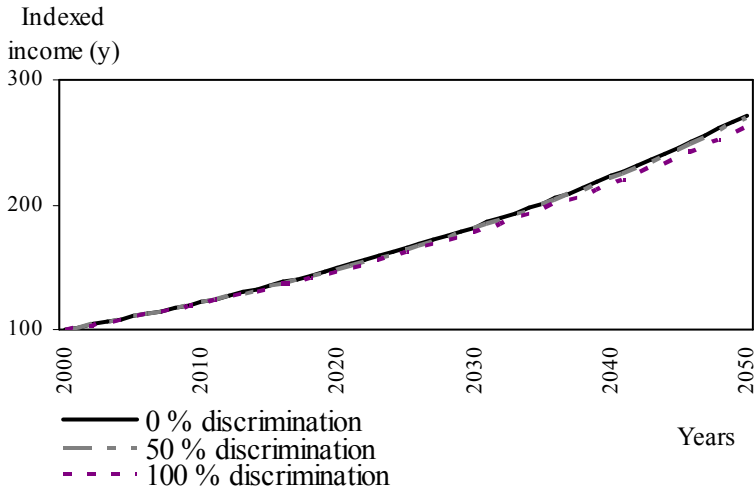
Fig 9: The annual growth rate of GDP per capita (g_A) in steady state, $\gamma = 1$



It is reflected in the figure that when discrimination is absent, the annual growth rate of the economy is 2 percent. This result confirms the result from the previous section of the thesis. Hence that the annual growth rate of GDP per capita is determined by the growth rate of the stock of ideas, which, in turn, is determined by the growth rate of the population. This implies that discrimination does not affect economic growth in the long run. It is however revealed that discrimination influences the growth rate of GDP per capita in a negative manner and that the higher the level of discrimination in the society, the lower is the level of economic growth. The fact that the growth rate deviates from the population growth rate at 2 percent when discrimination is prevalent implies that discrimination gives rise to growth rates outside the steady state level of growth. This means that the different growth rates displayed in the figure reflect transitory growth and that there is a tendency of the growth rates to adjust to the long run growth rate over time. Furthermore, since the differences between the discrimination scenarios persist out of scope of the simulations, it can be concluded that the speed of the adjustment process is slow. This implies that, in terms of transitory growth, discrimination affects the growth rate of GDP per capita during a long period of time.

The thesis does however take stand point in the belief that productivity is more important in the research sector than in the final-goods sector, captured by $\gamma > 1$. The second simulation of the model is portrayed in figure 10 and table 3 and illustrates GDP per capita in steady state when γ is equal to 4.

Fig 10: Indexed value of GDP per capita in steady state, $\gamma = 4$



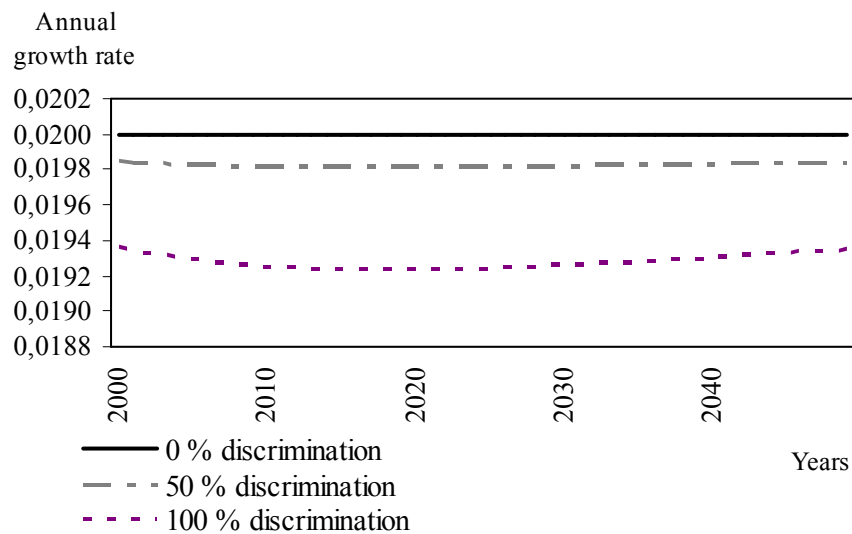
Tab 3: Indexed value of GDP per capita in steady state, $\gamma = 4$

Years	0 % discrimination	50 % discrimination	100 % discrimination
2000	100	100	100
2010	122	122	121
2020	149	149	147
2030	182	181	178
2040	223	221	216
2050	272	269	262

It can be concluded that in an economy where there is no gender-based discrimination, the level of GDP per capita increases by 172 percent, which is equal to the increase of the previous simulation. Furthermore, it is revealed that, when productivity is more important in the research sector than in the final-goods sector, discrimination affects the level of GDP per capita more negatively than when productivity is equally important in both sectors of the economy. This is illustrated by the fact that an economy characterized by complete discrimination of females in the research sector increased by 170 percent when productivity was equally important in both sectors, as was showed earlier, compared to the current simulation, where an economy characterized by complete discrimination increases by 162 percent during the same period of time.

Figure 11 illustrates the annual growth rate of GDP per capita in steady state when $\gamma = 4$.

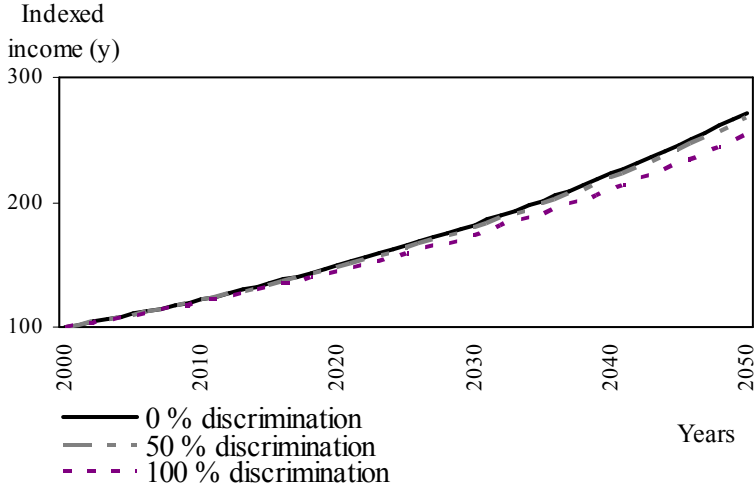
Fig 11: The annual growth rate of GDP per capita (g_A) in steady state, $\gamma = 4$



It is thus discovered that the growth rate in steady state is unaffected by the assumption that productivity is more important in the production of ideas than in the production of final output. Hence the long run growth rate is equal to the growth rate of the population. It can however also be concluded that when there is discrimination of females in the research sector, the growth rate of the economy is outside its steady state and that discrimination influences the level of transitory growth negatively. These conclusions confirm the conclusions from the previous simulation. In addition, it is showed that when productivity is more important in the research sector, the affect of discrimination on transitory growth is strengthen. Hence the effect of discrimination is more negative when γ is equal to 4 than when γ is equal to 1.

The third and final scenario carried out in the thesis is γ equal to 8 and is depicted in figure 12 and table 4. Analogously, this simulation confirms the results from the previous simulations. Hence that discrimination affects GDP per capita negatively and that the effect of discrimination is strengthen the more important productivity is in the production of ideas compared to the production of final output. The level of GDP per capita of an economy with no prejudiced employers increases by 172 percent compared to an economy with prejudiced employers which increases by 153 percent.

Fig 12: Indexed value of GDP per capita in steady state, $\gamma = 8$

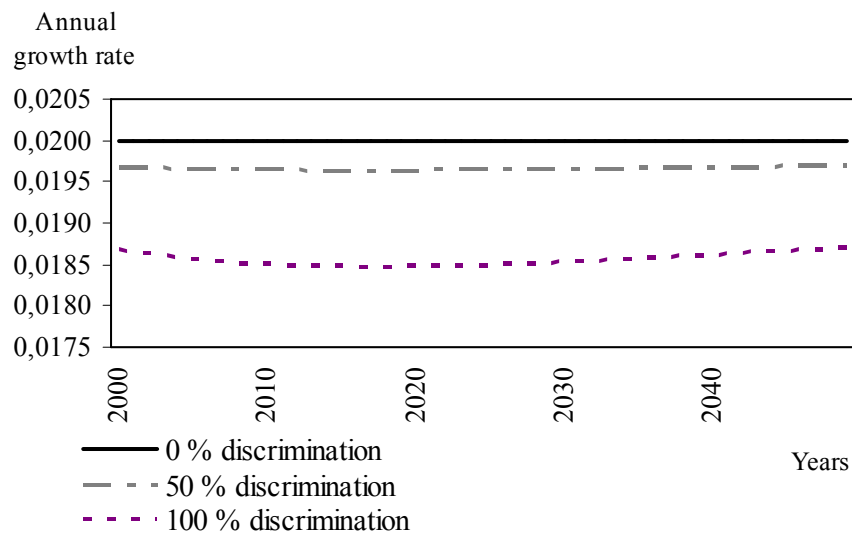


Tab 4: Indexed value of GDP per capita in steady state, $\gamma = 8$

Years	0 % discrimination	50 % discrimination	100 % discrimination
2000	100	100	100
2010	122	122	120
2020	149	148	145
2030	182	180	174
2040	223	219	210
2050	272	267	253

Finally, figure 13 illustrates the annual growth rate of GDP per capita when $\gamma = 8$. The growth rate in steady state is unaffected and equal to the growth rate of the population. The economy is outside its steady state when discrimination prevails, hence the growth rates of the discrimination scenarios reflect transitory growth, and the higher the level of discrimination, the lower the rate of growth of the economy.

Fig 13: The annual growth rate of GDP per capita (g_A) in steady state, $\gamma = 8$



To conclude, discrimination influences the level of GDP per capita negatively in the long run regardless whether productivity is relatively more important in the research sector than in the production of final output. The negative effect of discrimination is however small when productivity is equally important in both sectors. Hence the larger the value of γ , the more harmful is the effect of discrimination on GDP per capita in the long run. Furthermore, the differences of the level of GDP per capita between the discrimination scenarios are

strengthened over time due to the existence of exponential growth. This means that the difference between 0 percent discrimination and 100 percent discrimination is larger than proportional to the difference between 0 percent discrimination and 50 percent discrimination, as is shown in the simulations. Since the value of GDP per capita increases with a constant rate – the larger the economy gets, the faster it grows. Discrimination is thus an inefficient practice in the economy which generates negative effects that fortify in the long-run.

In addition, the annual growth rate is determined by the growth rate of technology, which, in turn, is determined by the population growth rate. It can thereby be concluded that economic growth in steady state is unaffected by the level of gender-based discrimination in the economy, as was concluded in the theoretical part of the thesis. However, discrimination has been found to affect the rate of transitory growth of the economy, i.e. when the economy adjusts to a new level of steady state. Furthermore, the transfer period seems to adjust slowly and persists out of scope of the simulations. This implies that discrimination in fact generates long-run growth effects in terms of long-term periods of transitory growth and that the larger the value of the importance of productivity in the research sector, the more harmful is discrimination on transitory growth. Hence, measures to decrease discrimination in the society may be motivated from a growth perspective as well.

A final remark is that it is possible to imagine that the extent of which discrimination affects GDP per capita in steady state differs depending on the choice of the numerical values of the remaining parameters of the model. For this reason, another set of simulations were made with different assumptions with regard to the population growth rate, the share of the labour force employed in the research sector, the savings rate, the depreciation rate, the stepping-on-toes effect and the standing-on-shoulders effect. During the simulations, the relative importance of productivity in the research sector was constant at $\gamma = 4$. As expected, the parameters that affects the extent to which discrimination affects GDP per capita are those related to the technology accumulation function and/or the growth rate of technology. Hence changes in the savings rate of disposable income and the depreciation rate of the capital stock leave the effect of discrimination unchanged. The degree of the effect of the parameters that affects the extent to which gender-based discriminations affects GDP per capita, is related to the concept of exponential growth. The results from these additional simulations will therefore not be introduced further.

5 Conclusions

The main objectives behind the construction of the gender-based endogenous growth model developed in the thesis were twofold. At the same time as it is of greatest importance to find additional insights to the concept of economic growth, the reason was also to investigate whether one could find economic arguments, applicable to Western economies, of reducing gender-based discrimination in society. The relation between discrimination and labour market differentials has been proven significant by several studies. Few studies have however tried to connect gender-based discrimination with macroeconomic analysis. Hence the aim of the thesis was to formalize a relationship between gender-based discrimination and GDP per capita in order to investigate how discrimination affects the level of GDP per capita and economic growth in the long run.

The result of the thesis is that gender-based discrimination affects the level of GDP per capita negatively in the long run. Hence, economies which are characterized by low levels of gender-based discrimination have a higher level of GDP per capita in steady state and vice versa. The extent to which gender-based discrimination affects the level of GDP per capita is determined by how much more important productivity is in the production of idea-based technology than in the production of final output. Hence the larger the relative importance of productivity in the research sector, the larger the effect of gender-based discrimination.

In terms of economic growth, it was found that gender-based discrimination does not affect the growth rate of the economy in the long-run. Hence the growth rate is, which corresponds with the version of the Romer model used in the thesis, determined by the growth rate of the population. It was however found that gender-based discrimination affects transitory growth, i.e. the adjustment process for when the economy transfers to a higher or lower level of GDP per capita in steady state. The rate of transitory growth was found to be lower the higher the level of discrimination in the society. When discrimination is absent, the growth rate is equal to the steady state rate of growth. In addition, the transfer period was found to prevail out of scope of the simulations. Hence discrimination does, in this perspective, affect economic growth during a very long period of time. This result confirms the result found by Sedgley and Elmslie (2005) who conclude that GDP per capita increases if the level of discrimination decreases for high skill levels relative to low skill levels. The growth effect of discrimination found in the

study by Sedgley and Elmslie may therefore reflect transitory growth in accordance with the result of the thesis.

An important question is whether government subsidies to decrease discrimination affect GDP per capita in the long-run. The answer is that policies that reduce the level of discrimination in the society, for example through affirmative action that aids women in the research sector, result in a higher level of GDP per capita in the long run. Furthermore, due to the slow and long-term transfer period of transitory growth, policies directed against gender-based discrimination may be motivated from a growth perspective as well. In addition, there is evidence in the literature of a gap with regard to social and private returns to research which suggests that the society can expect large gains from encouraging researching activities²². Policy makers have much to win from encouraging women in scientific careers, hence “unleashing the potential” of women.

Furthermore, there are several reasons to believe that the results of thesis are underestimated and that the features of the gender-based endogenous growth model are such that they give the hypothesis of the thesis relatively limited effects.

Firstly, the productivity of individuals are assumed to be uniformly distributed. It is however more likely that the productivity of individuals are distributed according to the normal distribution. Hence that there are more individuals with an average skill level than there are individuals of high or low skill levels. If the number of individuals at the top range of the productivity scale is small, the effect of discrimination is likely to increase. Secondly, the thesis does, for simplifying purposes, not take into account that, in a society characterized by gender-based discrimination, discrimination prevails in the final-goods sector as well. Hence if one models discrimination of females in both sectors of the economy, the effect of discrimination of GDP per capita is likely to increase. One may model discrimination of females in both sectors by dividing each sector into subsectors and model the redistribution and productivity effect of discrimination in relation to horizontal and vertical segregation of women. Thirdly, if women have limited access to employment in the research sector, women does probably not, which is assumed in the thesis, acquire the same amount of schooling as they would have if discrimination was absent. This implies that the average productivity of individuals employed

²² Jones, 2002: 121

in the final-goods sector should decrease more than proportional to the level of discrimination in the society, hence the effect of discrimination increases. Fourthly, the fact that not all groups in the society who are subject to discrimination, e.g. due to ethnicity, sexual orientation or looks, are included in the model suggests that the total effect of discrimination results in larger effects of discrimination of GDP per capita in the long-run. Fifthly, since the model does not take into account the existence of unemployment in the economy, the effect of discrimination is likely to be underestimated. The reason is that females that are dislocated in the labour market, i.e. in terms of their productivity level, may be unemployed instead of receiving employment in the final-goods sector. If labour are put outside the labour market, the total contribution of the labour force to final output production and research activities decreases which result in a lower level of GDP per capita in the long-run. Finally, if one uses the set of assumptions promoted by Romer (1990) instead of the ones promoted by Jones (1995, 2002), it has been concluded in earlier parts of the thesis that the growth rate of GDP per capita is determined by the size and the productivity of the research workforce. This implies that discrimination would affect the growth rate of GDP per capita in steady state as well as the level of transitory growth. All of the above aspects of discrimination and its connection to GDP per capita and economic growth are examples of topics interesting for future research.

Another suggestion for future research is related to the key assumption of the model, i.e. that productivity is relatively more important in the research sector than in the production of final output. One may thus examine whether the assumption is true and the size of γ . Individuals are key input into the research sector, hence their ability to produce ideas determines the overall productivity of the sector. It is therefore reasonable to assume that productivity is highly important in the research sector. It is also plausible that γ differs with regard to the size of the research sector, in different sub-sectors and depending on the distribution of talent among the population etc., which suggests that the size of γ is large. It is however also fair to assume that productivity affects the final-goods sector as well and in innumerable ways, suggesting that the size of γ is smaller than expected.

A final remark of the thesis is that, besides economic arguments of non-discrimination, non-discrimination is a goal in itself and that the integrity and quality of research²³ are related to the fact that all individuals of a society are of concern and participate in research.

²³ OECD, 2006 (2): 11

References

Literature

Albrecht, J.W., Björklund, A. and Vroman, S.B. (2003), "Is There a Glass Ceiling in Sweden?", *Journal of Labor Economics*, vol. 21, no. , p. 145-177.

Borjas, George J. (2005), "Labour Economics" 3d ed. The McGraw-Hill Companies, New York

Esteve-Volvart, B. (2000), "Sex Discrimination and Growth". IMF Working Paper No. 84, International Monetary Fund

Fregert, K. and Jonung, L., (2003), "Makroekonomi – Teori, Politik & Institutioner". Studentlitteratur. Lund

Garcia-Minguez, P. and Sanchez-Losada, F. (2002), "Statistical discrimination and growth : Should we subsidize discriminated against workers?" *Economic Letters* 79, 255 – 261

Goldin, C. and Rouse, C. (2000), "Orchestrating Impartiality": The Impact of 'Blind' Auditions on Female Musicians", *American Economic Review*, no. 4, pp. 715-741

Hausmann, R. Tyson, L. D., and Zahidi, S. (1997), "The Global Gender Gap Report 2007", World Economic Forum, Geneva

Johansson, M. and Katz, K. (2007), "Wage Differences between Women and Men in Sweden – the Impact of Skill Mismatch", IFAU Working Paper, 2007: 13

Jones, Charles I. (2002), "Introduction to Economic Growth" 2d ed. W.W. Norton & Company, New York

Jones, Charles I. (1995), "R&D-Based Models of Economic Growth" *Journal of Political Economy* 103: 759 – 84.

Leahey, Erin. (2006), "Gender Differences in Productivity: Research Specialization as a Missing Link" Gender Society 2006; 20; 754 - 780

Lundberg, S. and Rose, E. (2000), "Parenthood and the earnings of married men and women", Labour Economics" no. 6, p. 689-710

Romer, P. (1990). "Endogenous technological change". The Journal of Political Economy 98. S71-S102

Sedgley, N. and Elmslie, B. (2005). "Discrimination and growth: The distribution across skill matters". Economic Letters 90, 194 – 199

Sørensen, P and Whitta-Jacobsen, H. (2005), "Introducing Advanced Macroeconomics: Growth & Business Cycles". The McGraw-Hill Companies, Berkshire

Electronic sources

OECD Employment Outlook (2008:5), www.sourceoecd.com

OECD: Main Science and Technology Indicators database (2008:1),
<http://dx.doi.org/10.1787/451463482377>

OECD: Science, Technology and Industry Outlook (2008:1), www.sourceoecd.com

OECD: Main Science and Technology Indicators database (2006),
<http://dx.doi.org/10.1787/235006333017>

OECD(1): Science, Technology and Industry Outlook (2006), www.sourceoecd.com

OECD(2): Women in Scientific Careers: Unleashing the potential(2006) www.sourceoecd.com

Statistics Sweden: "Research and experimental development in Sweden 2005. An overview". (2007),
http://www.scb.se/statistik/UF/UF0301/2005A01/UF0301_2005A01_SM_UF16SM0701.pdf

APPENDIX A: Sectorwise Derivations

(i) The final-goods sector

Final goods firms compete on a perfectly competitive market and combine labour and capital in the production of a homogenous output, denoted Y . The production function for final goods is:

$$Y = L_Y^{1-\alpha} \sum_{j=1}^A x_j^\alpha \quad (1)$$

L_y is the labour working in the final goods sector and x_j are different intermediate goods. There are more than one capital good in the economy, the number denoted with an A . Firms in the final goods sector take the number of intermediate goods as given. It can be shown, see Jones (2002), that the summation sign can be replaced by an integral, where the integral measures the number of capital goods available in the final good sector.

$$Y = L_Y^{1-\alpha} \int_0^A x_j^\alpha dj \quad (2)$$

The profit-maximisation problem is:

$$\max L_Y^{1-\alpha} \int_0^A x_j^\alpha dj - wL_Y - \int_0^A p_j x_j dj \quad (3)$$

The first order conditions are²⁴:

$$w = (1 - \alpha) \frac{Y}{L_Y} \quad (4)$$

$$p_j = \alpha L_Y^{1-\alpha} x_j^{\alpha-1}, \quad (5)$$

(ii) The intermediate goods sector

Each intermediate firm buys the design of a capital good from the research sector at a fixed cost. The designs are protected by patents which make the intermediate goods firms monopolists in their respective markets. The profit maximisation problem is:

²⁴ Jones, 2002; 110 – 113

$$\max \pi_j = p_j(x_j)x_j - rx_j \quad (6)$$

Finding and rewriting the first-order condition (without subscript j) gives the following equations:

$$p'(x)x + p(x) - r = 0 \quad (7)$$

$$p = \frac{1}{1 + \frac{p'(x)x}{p}} r \quad (8)$$

$\frac{p'(x)x}{p}$ is an elasticity which is equal to $\alpha - 1$. Rewriting the equation with the knowledge of the elasticity gives a solution where price is a mark-up over marginal cost, r :

$$p = \frac{1}{\alpha} r \quad (9)$$

Since the above solution as well as the demand curve facing the intermediate goods firms is the same for each firm, firms in the final goods sector utilise the capital goods produced by the monopolists by the same amount and each intermediate goods firm earns the same profit. The final goods sector's demand for intermediate capital goods from the intermediate goods sector is equal to the total stock of capital in the economy.

$$\int_0^A x_j dj = K \quad (10)$$

Since capital goods are used in equal proportions by final goods firms, one can ignore subscript j . The number of capital goods in the economy can thereby be illustrated by the following equation:

$$x = \frac{K}{A} \quad (11)$$

Considering the fact that $x_j = x$, the final goods production function can be rewritten as:

$$Y = AL_Y^{1-\alpha} x^\alpha \quad (12)$$

When substituting the two equations we get the economy's aggregate production function²⁵:

$$Y = AL_y^{1-\alpha} A^{-\alpha} K^\alpha = K^\alpha (AL_Y)^{1-\alpha} \quad (13)$$

(iii) The research sector

The price of a patent is determined by the present discounted value of the profits earned by the intermediate-goods firm buying a design, denoted P_A . The present discounted value changes over time in accordance to the so-called arbitrage condition. The arbitrage condition reflects that the return from investing in a unit of capital and earning the interest rate r is equal to investing in a patent and earning profit from it.

$$rP_A = \pi + \dot{P}_A \quad (14)$$

Rewriting the equation gives:

$$r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A} \quad (15)$$

When the economy grows along a balanced growth path, r is constant. For r to be a constant, the quotient $\frac{\pi}{P_A}$ must also be constant. The quotient is constant only if π and P_A grow at the same rate. If denoting the rate with an n and rewriting the equation, we get the price of a patent along a balanced growth path.²⁶

$$P_A = \frac{\pi}{r - n} \quad (16)$$

²⁵ Jones, 2002; 113 – 114

²⁶ Jones, 2002; 115 – 116

APPENDIX B: Discrimination

Discrimination is measured as shares of men and women in the labour force and is found from the following steps of calculations:

$$\begin{aligned}
 D = 1 - \frac{s_R^F}{s_R} &\Leftrightarrow D - 1 = \frac{s_R^F}{s_R} \Leftrightarrow s_R(D - 1) = -s_R^F \Leftrightarrow s_R^F = s_R(1 - D) \\
 s_R = \frac{s_R^M + s_R^F}{2} = \frac{s_R^M + s_R(1 - D)}{2} &\Leftrightarrow 2s_R = s_R^M + s_R(1 - D) \Leftrightarrow 2s_R - s_R(1 - D) = s_R^M \Leftrightarrow \\
 \Leftrightarrow s_R(1 + D) = s_R^M &
 \end{aligned} \tag{17}$$

The resulting equations are representing the fraction of the labour force engaged in the research sector of males and females, respectively:

$$s_R^M = s_R(1 + D) \tag{18}$$

$$s_R^F = s_R(1 - D) \tag{19}$$

APPENDIX C: The Average Productivity Functions

The probability that an individual has a productivity level of x is $f(x)$. Assume further that the probability that an individual has a productivity level that lies between 1 and k is an integral of the following kind:

$$F(k) = \int_1^k f(x) dx \tag{20}$$

This implies that, if $F(k) = 0.9$, there is a 90 percents probability that the individual has a productivity level that lies between 1 and k . It is assumed that the share of the labour force working with research is exogenous and constant at 10 percent. In order to calculate the average productivity of the 10 percent most talented individuals we have to find what k results in $F(k) = 0.9$. Hence the 10 percent with the highest productivity will have a productivity level

that lies between k and 2. The average productivity of the most talented individuals can then be calculated as follows:

$$\frac{\int_k^2 f(x)xdx}{\int_k^2 f(x)dx} = \frac{\int_k^2 f(x)xdx}{1 - \int_1^k f(x)dx} = \frac{\int_k^2 f(x)xdx}{1 - 0,9} = \frac{\int_k^2 f(x)xdx}{0,1} \quad (21)$$

Note that the denominator is equal to the share of the individuals for whom the average is calculated. Even though the aggregate share of the labour force is constant at 10 percent, the shares of males and females working with R&D are going to differ depending on the level of discrimination. This result in an equation of k that changes with the level of discrimination, which is equal to $k = 2 - s_R(1 + D)$ for men and $k = 2 - s_R(1 - D)$ for women. It is now possible to calculate the average productivity levels of males and females in the research sector and the final-goods sector, respectively. Starting with the research sector gives:

$$P_A^{M,F} = \frac{\int_k^2 f(x)xdx}{\int_k^2 f(x)dx} = \frac{\int_k^2 f(x)xdx}{1 - \int_1^k f(x)dx} \quad (22)$$

The function above is rewritten with the expression for k and solved for according to the steps of calculations below:

$$P_A^M = \frac{\int_{2-s_R(1+D)}^2 f(x)xdx}{\int_{2-s_R(1+D)}^2 f(x)dx} \quad (23)$$

$$\begin{aligned} P_A^M &= \frac{\left[\frac{x^2}{2} \right]_{2-s_R(1+D)}^2}{\left[x \right]_{2-s_R(1+D)}^2} = \frac{\frac{2^2}{2} - \frac{(1-s_R(1+D))^2}{2}}{2 - (2-s_R(1+D))} = \frac{\frac{1}{2}(4 - (4 + s_R^2(1+D) - 4s_R(1+D)))}{s_R(1+D)} \\ &= \frac{1}{2} \left(\frac{-s_R^2(1+D)^2 + 4s_R(1+D)}{s_R(1+D)} \right) = \frac{1}{2} (s_R(1+D) + 4) = \frac{4}{2} - \frac{s_R(1+D)}{2} = 2 - \frac{1}{2}s_R(1+D) \end{aligned}$$

$$\Rightarrow P_A^M(D) = 2 - \frac{1}{2}s_R(1+D) \quad (24)$$

The function above represents the average productivity of male researchers. The derivative of the function with respect to discrimination is negative:

$$P_A^{M'}(D) = -\frac{1}{2}s_R \quad (25)$$

The average productivity of females in the research sector is calculated as follows:

$$P_A^F = \frac{\int_{2-s_R(1-D)}^2 f(x)xdx}{\int_{2-s_R(1-D)}^2 f(x)dx} \quad (26)$$

$$\begin{aligned} P_A^F &= \frac{\left[\frac{x^2}{2} \right]_{2-s_R(1-D)}^2}{\left[x \right]_{2-s_R(1-D)}^2} = \frac{\frac{2^2}{2} - \frac{(1-s_R(1-D))^2}{2}}{2 - (2-s_R(1-D))} = \frac{1(4 - (4 + s_R^2(1-D) - 4s_R(1-D)))}{s_R(1-D)} \\ &= \frac{1}{2} \left(\frac{-s_R^2(1-D)^2 + 4s_R(1-D)}{s_R(1-D)} \right) = \frac{1}{2}(s_R(1-D) + 4) = \frac{4}{2} - \frac{s_R(1-D)}{2} = 2 - \frac{1}{2}s_R(1-D) \end{aligned}$$

$$\Rightarrow P_A^F(D) = 2 - \frac{1}{2}s_R(1-D) \quad (27)$$

The derivative of the average productivity function of female researchers with respect to discrimination is positive.

$$P_A^{F'}(D) = \frac{1}{2}s_R \quad (28)$$

Turning attention to the final-goods sector, the average productivity of males and females is instead:

$$P_y^{M,F} = \frac{\int_1^k f(x)xdx}{\int_1^k f(x)dx} \quad (29)$$

The average of male final-goods workers is found from the following calculations:

$$P_y^M = \frac{\int_1^{2-s_R(1+D)} f(x)xdx}{\int_1^{2-s_R(1+D)} f(x)dx}$$

$$P_y^M = \frac{\left[\frac{x^2}{2} \right]_1^{2-s_R(1+D)}}{\left[x \right]_1^{2-s_R(1+D)}} = \frac{\frac{(2-s_R(1+D))^2}{2} - \frac{1^2}{2}}{2-s_R(1+D)-1} = \frac{3+s_R^2(1+D)^2-4s_R(1+D)}{2(1-s_R(1+D))}$$

$$= \frac{3-4s_R(1+D)+s_R^2(1+D)^2}{2-2s_R(1+D)} = \frac{3-s_R(1+D)}{2}$$

$$\Rightarrow P_y^M(D) = \frac{3-s_R(1+D)}{2} \quad (30)$$

The sign of the derivative of the average productivity function of male final-goods workers is negative:

$$P_y^{M'}(D) = -\frac{1}{2}s_R \quad (31)$$

Finally, the average productivity of female final-goods workers is found from the following calculations:

$$P_y^F = \frac{\int_1^{2-s_R(1-D)} f(x)xdx}{\int_1^{2-s_R(1-D)} f(x)dx}$$

$$\begin{aligned}
P_y^F &= \frac{\left[\frac{x^2}{2} \right]_1^{2-s_R(1-D)}}{\left[x \right]_1^{2-s_R(1-D)}} = \frac{\frac{(2-s_R(1-D))^2}{2} - \frac{1^2}{2}}{2-s_R(1-D)-1} = \frac{3+s_R^2(1-D)^2-4s_R(1-D)}{2} \\
&= \frac{3-4s_R(1-D)+s_R^2(1-D)^2}{2-2s_R(1-D)} = \frac{3-s_R(1-D)}{2} \\
\Rightarrow P_y^F(D) &= \frac{3-s_R(1-D)}{2} \tag{32}
\end{aligned}$$

In opposite to the derivative of males in the final-goods sector, the sign of the derivative of the average productivity of female final-goods workers with respect to discrimination is positive:

$$P_y^{F'}(D) = \frac{1}{2} s_R \tag{33}$$

APPENDIX D: GDP per capita in Steady State

The derivation of GDP per capita in steady state is taking its starting-point in the aggregate production function. Note that variables raised with an asterisk denote variables in steady state. Rewriting the equation to denote the level of GDP in per capita and technology units gives:

$$\begin{aligned}
\frac{Y}{AL} &= \frac{K^\alpha (AL_y)^{1-\alpha}}{AL} \\
\Rightarrow \tilde{y} &= \tilde{k}^\alpha \left(\frac{L_y}{L} \right)^{1-\alpha} \tag{34}
\end{aligned}$$

In equilibrium $\dot{\tilde{k}} = 0$, which is equal to:

$$\dot{\tilde{k}} = \frac{\dot{K}}{AL} = \frac{K}{AL} \left(\frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L} \right) = 0$$

$$\dot{\tilde{k}} \left(\frac{sy - dK}{K} - g_A - n \right) = 0$$

$$\Rightarrow s\tilde{y}^* = \tilde{k}^* (d + g_A + n) \quad (35)$$

\tilde{y} is replaced with its function and the level of the capital stock in steady state and per capita and technology units is obtained:

$$s\tilde{k}^\alpha \left(\frac{L_y}{L} \right)^{1-\alpha} = \tilde{k} (d + g_A + n)$$

$$\Rightarrow \tilde{k}^* = \left[\frac{s}{d + g_A + n} \right]^{\frac{1}{1-\alpha}} \left(\frac{L_y}{L} \right) \quad (36)$$

By inserting the equation above into the equation for GDP per capita and technology units, one can solve for GDP per capita in steady state:

$$\tilde{y} = \left[\frac{s}{d + g_A + n} \right]^{\frac{\alpha}{1-\alpha}} \left(\frac{L_y}{L} \right)^\alpha \left(\frac{L_y}{L} \right)^{1-\alpha}$$

$$\tilde{y} = \left[\frac{s}{d + g_A + n} \right]^{\frac{\alpha}{1-\alpha}} \left(\frac{L_y}{L} \right)$$

$$\Rightarrow y^* = \left[\frac{s}{d + g_A + n} \right]^{\frac{\alpha}{1-\alpha}} \left(\frac{L_y}{L} \right) A \quad (37)$$

The equation describing the growth rate of technology in steady state g_A (see appendix E) rewritten for A is:

$$A = \left(\frac{L^\lambda}{g_A} \right)^{\frac{1}{1-\phi}} \quad (38)$$

Replace A in the equation for GDP per capita with the technology equation. The replacement gives:

$$y^* = \left[\frac{s}{d + g_A + n} \right]^{\frac{\alpha}{1-\alpha}} \left(\frac{L_y}{L} \right) \left(\frac{L_A^\lambda}{g_A} \right)^{\frac{1}{1-\phi}} \quad (39)$$

Before the final equation is obtained, specified versions of the labour equations are inserted into the above equation, the result being the solution to GDP per capita in steady state:

$$\begin{aligned} y^* &= \left[\frac{s}{d + g_A + n} \right]^{\frac{\alpha}{1-\alpha}} \left(\frac{P_y^M(D)(1-s_R(1+D))L^M + P_y^F(D)(1-s_R(1-D))L^F}{L} \right) \\ \Rightarrow &\left(\frac{(P_A^M(D)^\gamma s_R(1+D))L^M + P_A^F(D)^\gamma s_R(1-D)L^F}{g_A} \right)^{\frac{1}{1-\phi}} \end{aligned} \quad (40)$$

APPENDIX E: Growth in Steady State

To determine the growth rate of variables in the model along a balanced growth path, the starting-point is the aggregate production function and the accumulation function of capital. Lower-case letters denote per capita variables while g_x denotes the growth rate of variables along the balanced growth path.

$$Y = K^\alpha (AL_y)^{1-\alpha} \quad (41)$$

$$\dot{K} = sY - dK \quad (42)$$

The growth rate of the capital accumulation function is equal to:

$$g_k = \frac{\dot{K}}{K} = s \frac{Y}{K} - d \quad (43)$$

In order for the growth rate of the capital stock to be constant, as defined along a balanced growth path, the quotient $\frac{Y}{K}$ must also be constant. This implies that Y and K grow at the same rate:

$$g_y = g_k \quad (44)$$

Rewriting the aggregate production function in per capita form and taking the logs and derivative of the per capita equation gives:

$$\begin{aligned} \frac{Y}{L} &= \frac{K^\alpha (AL_y)^{1-\alpha}}{L} \\ \Rightarrow y &= k^\alpha A^{1-\alpha} \left(\frac{L_y}{L} \right)^{1-\alpha} \end{aligned} \quad (45)$$

$$\Rightarrow \frac{d \ln y}{dt} = \alpha \frac{d \ln k}{dt} + (1-\alpha) \frac{d \ln A}{dt} + (1-\alpha) \frac{d \ln \left(\frac{L_y}{L} \right)}{dt} \quad (46)$$

Rewriting the equation using the fact that $g_{\frac{L_y}{L}} = 0$ and $g_y = g_k$, the growth rates obtained for GDP per capita, the capital-labour output and the level of technology are equal along a balanced growth path:

$$g_y = \alpha g_k + (1-\alpha) g_A$$

$$g_y = \alpha g_y + (1-\alpha) g_A$$

$$g_y = g_A$$

$$\Rightarrow g_y = g_k = g_A \quad (47)$$

A derivation of the growth rate of technology in steady state takes its starting-point in the accumulation function of technology:

$$\dot{A} = A^\phi L_A^\lambda$$

$$\frac{\dot{A}}{A} = \frac{A^\phi L_A^\lambda}{A}$$

$$\Rightarrow g_A = A^{\phi-1} L_A^\lambda \quad (48)$$

Calculating the growth rate of the growth rate in technology gives the final solution for what determines the growth of technology in steady state:

$$\frac{\dot{g}_A}{g_A} = (\phi-1) \frac{\dot{A}}{A} + \lambda \frac{\dot{L}_A}{L_A}$$

$$0 = (\phi-1)g_A + \lambda n$$

$$\Rightarrow g_A = \frac{\lambda n}{1-\phi} \quad (49)$$

APPENDIX F: Default Values

The default value of GDP per capita is found from solving the GDP per capita equation in steady state derived in appendix D, that is:

$$y^* = \left[\frac{s}{d + g_A + n} \right]^{\frac{\alpha}{1-\alpha}} \left(\frac{P_y^M(D) s_R (1-(1+D)) L^M + P_y^F(D) (1-(1+D)) L^F}{L} \right)$$

$$\Rightarrow \left(\frac{(P_A^M(D)^\gamma s_R (1+D) L^M + P_A^F(D)^\gamma s_R (1-D) L^F)^\lambda}{g_A} \right)^{\frac{1}{1-\phi}} \quad (50)$$

The appropriate starting value of the stock of ideas is found from solving the following equation:

$$y^* = \left[\frac{s}{d + g_A + n} \right]^{1-\alpha} \left(\frac{L_y}{L} \right) A$$

$$\Rightarrow A = \frac{y^*}{\left[\frac{s}{d + g_A + n} \right]^{1-\alpha} \left(\frac{L_y}{L} \right)} \quad (51)$$

Finally, the default value of the capital labour ratio is given by:

$$\tilde{y} = \tilde{k}^\alpha \left(\frac{L_y}{L} \right)^{1-\alpha}$$

$$k = \frac{K}{L} = \frac{y^\alpha}{A^{\frac{1-\alpha}{\alpha}} \left(\frac{L_y}{L} \right)^{\frac{1-\alpha}{\alpha}}}$$

$$\Rightarrow K = \frac{y^\alpha}{A^{\frac{1-\alpha}{\alpha}} \left(\frac{L_y}{L} \right)^{\frac{1-\alpha}{\alpha}}} L \quad (52)$$