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Robbing Peter to pay Paul? A Box Jenkins analysis of the future of the

UK pensions system

Abstract

By isolating and forecasting the trend in two key variables this paper analyses the future plausibility of the current UK pensions system. Box Jenkins methods (1970, 1976, 1994) are applied to UK post-war government pensioner expenditure and population over 65. Conclusions are made with particular reference to the recent reports of the pensions commission who recommended a drastic reorganization of pensions in the UK. The demographic forecasts are shown to be very close to the predictions made by the pensions commission which both demonstrates the applicability of Box Jenkins methods to pension modelling problems and provides support for the conclusions reached by the commission, including a steady increase in retirement age. The generated forecasts are uniformly higher than the population predictions made by the government actuary's department which provides further support for a wide range of academic work demonstrating governmental underestimation of UK population. Analysis of government pension expenditure provides forecasts that do not contradict with the modelling carried out by Turner. Therefore the Box Jenkins analysis of two key variables – pensioner population and government pensioner expenditure – has corroborated the forecasting work carried out by the pensions commission and provided support for their wide ranging conclusions.

Keywords: UK pensions system; pension reform; Box Jenkins methods; mathematical forecasting

Contents

List of figures and tables

1. Preamble

Box and Jenkins (1970, 1976, 1994) methods will be applied to two time series data sets: government pension expenditure and the number of people over 65. It is hoped that the forecasting of these two variables will provide insights into the plausibility of the current UK pensions system. Conclusions will be made with reference to the recent reports of the pensions commission who recommended a drastic reform of the UK pensions framework. Analysis of the trend in government expenditure will allow evaluation of the viability of the current pensions regime. The extent of population ageing will be evaluated using the generated population forecasts. These projections can then be used to evaluate the commission's recommendation of an increase in retirement age.

The analysis will begin with an introduction to Box Jenkins methods, followed by a brief overview of the current UK pensions system and the reports by the pensions commission. The next section will review a selection of relevant academic literature for this exercise. This will be followed by the methodology section which aims to explain the statistical grounding of the methods to be employed in this paper. The penultimate section will implement the techniques described in the methodology, producing a set of forecasts for both variables. Finally, a discussion of the implications and relevance of the generated forecasts will be presented, with particular reference to the future state of the UK pensions system.

2. Overview

In this section the basic definitions surrounding Box Jenkins ARIMA modelling will be elucidated, followed by a brief overview of the UK pensions system. Particular concentration will be placed on the recent reform proposals forwarded by the pensions commission. Discussion of important reform drivers, including population ageing and poor private pension fund performance will also be provided.

2.1 Box Jenkins methods – an introduction

The Box Jenkins model has gained great popularity since the publication of their book in 1970. Box Jenkins techniques are based on the idea that a time series in which successive values are highly dependent can be regarded as being generated from a series of independent shocks. Modelling such series leads to the class of autoregressive, integrated, moving average (ARIMA) models. An autoregressive process is essentially a regression equation where a variable is related to its own past values instead of to a set of independent variables (Nelson, 1973, p.38):

$$y_t = \mu + \zeta_1 y_{t-1} + \zeta_2 y_{t-2} + ... + \zeta_p y_{t-p} + u_t$$

where $y_t, y_{t-1}, y_{t-2}.....y_{t-p}$ is a time series of observations; μ is a constant and u_t a set of independent and identically distributed random variables with $E(u_t) = 0$ and $\text{var}(u_t) = \sigma^2$ known as zero mean white noise. A moving average process relates the current value of a variable to a linear combination of zero mean white noise shocks (Nelson, 1973, p.33).

$$y_t = \mu + \theta_1 u_{t-1} + \theta_2 u_{t-2} + ... + \theta_q u_{t-q} + u_t$$

A model's level of integration depends on how many differences[1] are required to induce stationarity – a stationary series has constant mean and variance over time - more detail on stationarity will be given in the methodology section 4.1.

The Box Jenkins methodology can be viewed as a four step iterative procedure (Box, Jenkins and Reinsel, 1994, p.181-2). In the *identification* stage stationary data are used to identify an appropriate model by observing the behaviour of the

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[1] First differences can be calculated as $\nabla y_t = y_t - y_{t-1}$. The second differences are the differences of the first differences $\nabla^2 y_t = (y_t - y_{t-1}) - (y_{t-1} - y_{t-2}) = y_t - 2y_{t-1} + y_{t-2}$ etc.

autocorrelation function and partial autocorrelation function[2]. The second stage is *estimation* where historical data are used to estimate the parameters of the identified model. The penultimate step is *diagnostic checking* where various methods are used to check the adequacy of the model and an improved model can be suggested if necessary. *Forecasting* is the final stage where future values of the time series are predicted using the specified model. The methodology section of this paper will be arranged according to these four steps.

Box Jenkins techniques are theoretically simple and can be applied to a wide range of time series data sets, for example Edlund & Karlsson (1993) modelled the Swedish unemployment rate[3], Harris & Liu (1993) forecasted monthly electricity consumption and du Preez & Witt (2003) analysed monthly tourist demand. It is important to note that Box Jenkins methods forecast using only historical data. A data set of at least 50 observations is required for Box Jenkins methods to produce reasonable results (Box and Jenkins, 1976, p.18). Data sets over 50 observations are available for both variables in this exercise therefore use of Box Jenkins forecasting is justified. Applying Box Jenkins techniques to such drastically different data sets demonstrates how applicable they are. It is hoped that the theoretical simplicity of Box Jenkins methods will make this paper accessible to practitioners and academics from a wide range of disciplines.

2.2 A short introduction to the UK pensions system[4]

The UK has a complex pension system, which mixes defined benefit and defined contribution formulae and public and private provision. The public scheme has two tiers, but most workers contract out of its second tier into private pensions. Pension age, currently 60 for women and 65 for men, will be equalized from 2010. The first tier of the system is the basic state pension which pays a flat rate to all people of pensionable age who meet the contribution condition. Occupational schemes are mainly defined benefit, but there has been recent rapid growth in defined contribution schemes. Defined benefit plans provide a pension usually related to

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[2] Definitions of these concepts will be provided in the methodology section 4.2 of this paper.

[3] The authors used a VARIMA which is a multivariate generalization of the Univariate ARIMA model, an outline of the theory underlying VARIMA models is provided by Riise & Tjostheim (1984).

[4] A concise history of pensions and pension funding is provided by Shapiro (2005).

years of membership in the scheme and some measure of final salary when covered by the plan. In 1988 the government introduced the option of contracting out of Serps (the second state pension at the time) into a personal scheme. Personal pensions are individual retirement savings accounts mainly sold by life insurance companies and banks. These schemes are defined contribution: the ultimate value depends on contributions made, the investment returns earned and the level of annuity rates when the member retires. As will be discussed later in this paper, the multifaceted nature of the UK pensions system makes analysing it very difficult.

2.3 Pension reform in the UK

Population ageing and other drivers

A major driver of the demand for pension reform are the rapidly increasing old age dependency ratios in the large western European countries. This trend is illustrated in Whiteford and Whitehouse (2006) who note that average pension spending in the OECD in 2001 was 7.4% of GDP, while the figure for France was 10.4% and Germany 10.8%. Observed population ageing is caused by a declining fertility rates coupled with increasing life expectancy. The UK has particularly severe ageing problems with the government actuary's department predicting that life expectancy for both sexes will rise by three years over the next decade (Banks and Blundell, 2005)[5]. Population ageing has and will continue to have a significant effect on the plausibility of the UK pensions system – analysis of ageing trends is a key driver of pension reform. This paper will attempt to model population ageing by forecasting the UK dependent population.

Problems with private pension funds

Another driver of pension reform includes a series of high profile problems in private pensions such as Allied Steel and Wire, coupled with a lack of confidence in private money generated by the scandals such as Enron and Parmalat. Besley and Prat (2005) define a series of requirements for "credible pensions" arguing that credibility problems can be mitigated by shifting to a mix of means tested or flat rate state pensions and individual defined contribution plans. The UK also has its own specific problems because of its high dependency on private pension funds –

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[5] When using actuarial projections we should be aware that, historically, public sector actuaries projections have systematically underestimated life expectancy improvements (Booth *et al.*, 2006).

today many pension funds face large deficits because of recent drops in the stock market and the removal of fund tax advantages in 1997 (Barr, 2006). In response to stock market problems many companies redirected their pension funds towards bonds, increasing the demand for long-dated gilts and reducing yields considerably. These combined factors have had a harsh impact on UK company pension funds which have, in turn, greatly jeopardized the whole UK pension system.

The pension commission reports

The UK government commissioned an independent report into the future plausibility of the current pensions system in December 2002: the findings of the commission were published in three separate stages over the last three years (Turner 2004, 2005, 2006). Using the starting point of population ageing as outlined above, the commission recommended to abolish the state pension, second state pension and employer-backed funds and to form a universal scheme, called the National Pensions Savings Scheme (NPSS). Employees would be automatically enrolled into the NPSS or good quality existing employer schemes. People would have the right to opt out and/or make additional contributions above the automatic minimum. The NPSS would be underpinned by a flat rate pension which is less means tested than the current state pension (Hills, 2006a). In order to enact these changes the commission recommend that both public spending and state pension age increase (Hills, 2006b). However the commissions recommendations - particularly the proposed increase in retirement age - were met with public, industrial[6] and academic criticism, see Congdon (2005). The objective of this paper is to evaluate the conclusions of the pensions commission using Box Jenkins forecasting, particularly analysing if the population ageing trend is as severe as the commission project and if the amount of public spending on pensions needs to drastically increase.

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[6] "Turner defends his universal pension system against industry criticism" *Pensions Management* 1 May 2006

3. Literature review

This section will examine academic literature relevant to the forecasting objective of this paper. Evaluations of the use of Box Jenkins techniques in the realm of macroeconomic and demographic forecasting will be provided, followed by an analysis of more general pension modelling efforts. It is hoped that the techniques discussed in this section will provide a problem solving framework for the methodology section of this paper.

3.1 Box Jenkins methods in macroeconomic forecasting

Box Jenkins models have been applied to macroeconomic time series consistently since the forecasting boom of the 1980s[7]. An early analysis comes from Oller (1985) who used quarterly Finnish macroeconomic data over the period 1970-80, applying a VARIMA[8] specification to six variables: foreign demand, exports, price of oil, domestic unemployment, wages and investment. He found that VARIMA worked well in projecting developments in the Finnish economy for the early 1980s. VARIMA models have been used extensively in econometrics, particularly the special case of vector autoregressive (VAR) models which allow an unrestricted approximation to the reduced form of a wide variety of dynamic econometric models. VAR models are extremely flexible (Funke, 1990), but suffer from overfitting problems with too many free insignificant parameters. As a result, these models can provide good within sample fitting but poor out of sample forecasts[9]. This is a major problem and therefore the analysis in this paper will not use a VAR-type model.

The macroeconomic forecasting in this paper will mainly follow the work of Wong *et al.* (2005) who used univariate ARIMA models to predict five key indicators in the construction labour market of Hong Kong. The authors are rigorous in their application and testing of Box Jenkins methods while providing detailed analysis of

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[7] Box Jenkins methods have also been applied to theoretical macroeconomic problems: Dhrymes & Peristiani (1988) fitted a structural ARIMA-type specification to examine the standard open economy macroeconomic paradigm, finding that a structural multiscale ARMA(1,1) was an appropriate model for analysis of an open economy system.

[8] c.f. footnote 3.

[9] The mechanics of forecasting will be discussed in the methodology section 4.5.

forecast accuracy. Wong *et al.* note that a multivariate approach (e.g. VARIMA) would provide more accurate forecasts for the construction market, however this paper will follow their univariate method because of the problem of variable inclusion. It is difficult to find theoretical motivations for variable selection and consequently subjectivity will bias the resulting forecasts. It is clear that there is plenty of academic literature supporting the use of univariate ARIMA models for macroeconomic variables, this exercise places particular importance on the paper by Wong *et al.* (2005).

3.2 Box Jenkins methods in population forecasting

There is a small body of forecasting literature supporting the application of ARIMA models to demographic data (Gooijer and Hyndman, 2006). The population of Sweden[10] is analysed by Saboia (1974) who uses an extremely large data set (1780-1960), with a ten year out of sample period. Saboia's results compare favourably with the demographic approximations of Keyfitz (1968, 1972)[11] which form one of main keystones of demographic analysis. The similarity of Box Jenkins forecasts with the predictions by Keyfitz illustrates the utility of using Box Jenkins techniques on demographic data. Another early paper comes from Kashyap & Rao (1976) who compare different time series methods for forecasting the U.S. population using observations from 1900 to 1971, concluding that an AR(1) process of the first logarithmic differences is a suitable model. McDonald (1981) used an ARIMA(0,1,1) model to analyse an Australian births series, first nuptial confinements, which are the first pregnancy of the current marriage resulting in a live birth. Although his work is theoretically rigorous, McDonald acknowledges that his long run point forecasts lack precision, the problem of forecast accuracy will be addressed in the methodology section. An extensive review of early population forecasting attempts is provided by Land (1986), who describes some of the problems related to forecast accuracy (Land, 1986, section v). This exercise will adopt the framework of a paper by Pflaumer (1992) who forecasted the population of the U.S. up to 2080 using an ARIMA(2,2,0). The author concludes by noting that

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[10] Sweden is acknowledged as the country with the best historical demographic data and is commonly used in population estimating exercises.

[11] The majority of Keyfitz's work in the late 1960s was devoted to converting observed death rates into probabilistic measures that could be used to construct life tables. The later and more sophisticated models used a Taylor approximation to obtain survival probabilities.

Box Jenkins techniques should be used to reconcile more complex demographic techniques, this aim fits well with the objectives of this paper which aims to qualify the demographic projections made by the pensions commission.

3.3 Other pension modelling methods

Reform and macroeconomics[12]

Macroeconomic analysis of pension systems has mainly concentrated on the influence of pension reform on macroeconomic variables, see for instance Adema *et al.* (2005)[13]. A recent paper by Borsch-Supan *et al.* (2006) models the effect of population ageing and pension reform on international capital markets using a complex simulation method called first order tatonnement iteration. Their main conclusion is that countries most affected by ageing, such as those in the European Union, will initially be capital exporters, while countries less affected by ageing, such as the United States, will import capital. They also conclude that pension reforms with the higher degrees of pre-funding are likely to induce the highest amounts of capital exports. The iterative methods employed by Borsch-Supan *et al.* could be used to quantify the effect of the pension commission's reform proposals on the UK economy, however their model would need to be extended because currently it only deals with the simplistic pay as you go (PAYG) continental European systems. As mentioned previously UK pension arrangements are complex (see Whitehouse, 2002) and multifaceted which makes their explicit modelling, particularly the application of iterative simulations, extremely difficult.

Reform and population ageing

There has been a recent explosion in literature recommending various modes of pension reform with particular reference to the problems of rapidly ageing western populations. Lindbeck and Persson (2003) outline the basic reform models and compare developments in a wide array of countries. A paper by Oksanen (2005) explicitly analyses ageing and benefit structure by building a simple three period

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[12] The economic theoretical context of pension reform is discussed by Barr (2002). There is a growing literature on the "economics of pensions" examining the policy implications of pension strategy with particular reference to conventional economic theory e.g. Barr and Diamond (2002); Cesaratto (2002).

[13] The authors analyse the international spillover effects of pension reform by examining how pension reforms in countries with pay as you go schemes affect countries with funded systems.

model that examines the relation between population ageing and public pension rules on national saving[14]. His definition of "actuarial neutrality across generations" provides a framework for analysing the behaviour of pension systems where it is important to avoid any unintended increase in the pension burden. This paper will attempt to model population ageing by forecasting the dependent population of the UK; the resulting forecasts will then be used to make a qualitative assessment of the impact of ageing on the pension system. Although simplistic, this method will traverse the traditional boundary between demography and economics, a feat which none of the aforementioned publications achieve.

Pension plan design

Modelling the structure of the UK pension system is of particular interest considering the plethora of reform proposals being suggested by academics. Pries (2007) examines the welfare effects of a switch from a defined benefit (DB) pension system to a defined contribution (DC) system of personal retirement accounts with reference to current U.S. pension arrangements. He solves equations approximating an agent's social security and personal retirement accounts using dynamic programming (treating them as Bellman equations[15]). Pries concludes that earners at the bottom of the wage scale are better off under DB plans than under DC personal retirement accounts with constant contribution rates. Such analysis is relevant for the UK because many companies are closing their DB funds in favour of DC often with much lower employer contributions being made into them (Hills, 2006a). Applying the model suggested by Pries to UK firm pension schemes would provide some indication of how the current DB to DC trend will affect the welfare of pensioners. Such an investigation would provide useful information about contracted out or firm pension funds but would not give a holistic assessment of the UK pensions system.

Conclusions

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[14] Oksanen (2002) constructs a simulation model to deal with pension reform and transition more explicitly, which is built on by Oksanen (2004).

[15] The recursive Bellman equation can be used to find a maximum of a dynamic programming problem. Definitions of Bellman equations are given by Bouten et al. (2005).

The extreme difficulties of pension modelling are elucidated by Herce (2003)[16], who concludes that analysis "should be parsimonious using different methodologies for different purposes". The UK benefits framework is inherently complex, therefore modelling and subsequently forecasting UK pensions using dynamic linear programming or iterative computation will require extreme simplifying assumptions and consequently biased forecasts. This explains the lack of academic attempts to explicitly model or forecast the UK pensions system (with the exception of Blake and Mayhew, 2006). It therefore appears that the use of Box Jenkins forecasting is the most appropriate for this exercise.

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[16] "A comprehensive pension model firmly based on theoretical foundations and capturing the sophisticated detail of current pension arrangements is a very expensive and time consuming endeavour."

4. Methodology

All of the methods and statistical techniques employed in this paper will be explained in this section. Mathematical proofs will be provided where relevant. Much of the methodology is constructed with reference to computer implementation: the computer package Econometric Views (EViews) 4.0 will be used. A review of an earlier version of EViews is presented by Sparks (1997).

4.1 Stationarity

Dickey Fuller test

An ARIMA model is designed for stationary time series data, for which the process can be modelled via an equation with fixed coefficients that can be estimated from past data. A stationary series has constant mean and variance over time. The early work on testing for stationarity came from Dickey and Fuller (1979), who conceptualised the method as "testing for a unit root". They investigate if $\phi = 1$ in the following three types of model:

$$\text{(i)} \quad y_t = \phi y_{t-1} + u_t$$

$$\text{(ii)} \quad y_t = \mu + \phi y_{t-1} + u_t$$

$$\text{(iii)} \quad y_t = \mu + \beta t + \phi y_{t-1} + u_t$$

where μ is a constant; βt a deterministic trend term; $u_t \sim N(0, \sigma^2)$ as before. If there is no evidence to reject the null hypothesis $H_0 : \phi = 1$ then there is a unit root and the process is non-stationary. Equation (iii) can be rearranged to simplify the test procedure:

$$\Delta y_t = \mu + \beta t + \psi y_{t-1} + u_t$$

in this case the null hypothesis is $H_0 : \psi = 0$.

Augmented Dickey Fuller test

The above tests assume no autocorrelation in the dependent variable Δy_t, however in practice it is likely that there will be some autocorrelation in data. Therefore an

adjustment needs to be made to the traditional test, producing the augmented Dickey Fuller[17].

$$\Delta y_t = \psi y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-1} + u_t$$

The new term in the regression takes account of any autocorrelation existing in the dependent variable. The augmented test will be used in this paper. In order to test the above hypotheses we use a t-ratio with the following test statistic.

$$\hat{\psi} \Big/ S\hat{E}(\hat{\psi})$$

This statistic does not follow a standard t-distribution because the null hypothesis is non-stationarity. Therefore critical values need to be found by simulation methods: Fuller (1976, p.371,373) uses Monte Carlo simulation to find the set of critical values.

Table 4.1.1 Critical values for the augmented Dickey Fuller test

Significance level	5%	1%
Constant	-2.86	-3.43
Constant & Trend	-3.41	-3.96

The same values can be used for both DF and ADF[18]. A major criticism of unit root tests is that they do not work well if the process is stationary but with a root close to the non-stationarity boundary. This problem will be taken into account when applying the ADF in this paper.

Order of differencing

The literature review has revealed that many population time series may require more than one level of differencing to induce stationarity. In general, econometric time series do not require more than one level of differencing which means that there are few academic papers studying the tests for higher orders of differencing.

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[17] This version of the test is presented without a trend term; a trend term can be added to the specification without loss of generality.

[18] It should be noted that these tests do not take account of the significance of the constant and/or trend terms – a modified set of critical values presented by Dickey and Fuller (1981) takes full account of the significance of the constant and trend terms.

One exemption is Dickey and Pantula (1987) who develop a formal statistical test for determining the appropriate level of differencing in a model. Various empirical papers have evaluated the impact of different levels of differencing on forecasting performance, the majority conclude that determining the correct order of differencing is not essential for forecasting (Tiao and Tsay, 1983). Therefore, this paper will investigate higher orders of differencing using ADF statistics rather than implementing the Dickey and Pantula (1987) method.

4.2 Identification

Graphical identification

(i) The ACF and PACF. Box and Jenkins (1970) recommend using plots of the autocorrelation function (ACF) and partial autocorrelation (PACF) to determine the order of model to be used. Some basic definitions are required before ACF and PACF can be explained. For a stationary process autocovariance at lag k is $\gamma_k = \text{cov}(y_t, y_{t-k})$, and consequently $\gamma_0 = \text{var}(y_t)$. Then autocorrelation is defined by the following relation.

$$\rho_k = \frac{\gamma_k}{\gamma_0}$$

The nxn autocorrelation matrix of a stationary process with autocorrelation sequence ρ_k is shown below.

$$P_n = \begin{pmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{n-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{n-2} \\ \rho_2 & \rho_1 & 1 & \cdots & \rho_{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{n-1} & \rho_{n-2} & \rho_{n-3} & \cdots & 1 \end{pmatrix}$$

The partial autocorrelation (PAC) at lag k of a stationary process is given by the ratio of determinants:

$$\phi_k = \frac{\left| P_k^* \right|}{\left| P_k \right|}$$

where P_k is the autocorrelation matrix and P_k^* is the matrix obtained from P_k by replacing the last column by $\begin{pmatrix} \rho_1 & \rho_2 & \cdots & \rho_K \end{pmatrix}^T$, where T denotes transpose[19].

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[19] The variable T will be used to denote sample size in the rest of this paper.

Hence the PAC examines whether the correlation between two lags of a time series is solely due to both these random variables being correlated with intermediate random variables of the process.

(ii) Estimation of the ACF and PACF. In Practice the ACF and PACF need to be estimated. The following equation can be used to estimate the AC coefficient for the series k periods apart:

$$\rho_k = \frac{\sum_{t=k+1}^{T}(y_t - \bar{y}_t)(y_{t-k} - \bar{y}_{t-k})}{\sum_{t=1}^{T}(y_t - \bar{y})^2}$$

where $\bar{y}_t$ is the sample mean of the whole series, while $\bar{y}_{t-k}$ is the sample mean up to lag k. Eviews uses a slightly different estimator for computational simplicity:

$$\rho_k = \frac{\sum_{t=k+1}^{T}\frac{(y_t - \bar{y})(y_{t-k} - \bar{y})}{T-k}}{\sum_{t=1}^{T}\frac{(y_t - \bar{y})^2}{T}}$$

where the overall sample mean has been used as the mean of both y_t and y_{t-k}. This simplification is still a consistent estimator. To estimate the PAC at lag k EViews uses recursive methods as recommended by Box and Jenkins (1974, part v).

$$\phi_k = \begin{cases} \rho_1 & k = 1 \\ \dfrac{\rho_k - \sum_{j=1}^{k-1}\phi_{k-1,j}\rho_{k-j}}{1 - \sum_{j=1}^{k-1}\phi_{k-1,j}\rho_{k-j}} & k \succ 1 \end{cases}$$

Having explained how EViews calculates the ACF and PACF the remainder of this subsection will elucidate how these plots can be used to identify suitable ARIMA models.

(iii) Applying the ACF and PACF to ARIMA models. For identification purposes it is of interest to derive the autocorrelation sequences for moving average and autoregressive processes. Starting with the MA(q) process without constant term (defined previously):

14

$$y_t = \theta_1 u_{t-1} + \ldots + \theta_q u_{t-q} + u_t$$

where $u_t \sim N(0, \sigma^2)$. We then have:

$$\text{cov}(y_t, y_{t-k}) = \text{cov}\left(\sum_{i=0}^{q} \theta_i u_{t-i}, \sum_{j=0}^{q} \theta_j u_{t+k-j}\right)$$

because an autoregressive process of finite order can be expressed as a

$$= \sum_{i=0}^{q} \sum_{j=o}^{q} \theta_i \theta_j \text{cov}(u_{t-i}, u_{t+k-j}) \text{ by the definition of covariance}$$

$$= \sum_{i=0}^{q} \sum_{j=o}^{q} \theta_i \theta_{l+k} \text{cov}(u_{t-i}, u_{t-l}) \text{ where } l = j - k$$

By independence:

$$\gamma_k = \text{cov}(u_t, u_{t-l}) = \begin{cases} \sigma^2 & l = i \\ 0 & l \neq i \end{cases}$$

$$\therefore \gamma_k = \text{cov}(y_t, y_{t+k}) = \begin{cases} 0 & k > q \\ \sigma^2 \sum_{i=0}^{q-k} \theta_i \theta_{i-k} & k = 0,1,2\ldots q \end{cases}$$

$$\therefore \rho_k = \frac{\gamma_k}{\gamma_0} = \begin{cases} 0 & k > q \\ \dfrac{\sum_{i=0}^{q-k} \theta_i \theta_{i-k}}{\sum_{i=0}^{q} \theta_i^2} & k = 0,1,2\ldots q \end{cases}$$

This proof shows that the ACF cuts off after lag q. Similar work can be carried out to prove the following (see Box, Jenkins and Reinsel, 1994, p.187).

Table 4.2.1 Behaviour of the ACF and PACF for ARMA processes

	ACF	PACF
White noise	All zero	All zero
MA(q)	Drop off after lag q	Geometric decay
AR(p)	Geometric decay	Drop off after lag p
ARMA(p,q)	Geometric decay	Geometric decay

Box Jenkins recommended using the above information to identify the appropriate model for the time series.

Information criteria

Sometimes it is difficult to interpret the ACF and PACF plots, so tests known as information criteria have been developed. They use two factors: a term which is a function of the residual sum of squares from the chosen model[20] and some penalty for the loss of degrees of freedom from adding extra parameters (Tsay, 2005, p.41). The most popular criteria are Akaike's (1974) information criterion (AK) and Schwarz's (1978) Bayesian information criterion (SB). There are many different formulations of these statistics, Chatfield (2001, p.224) uses:

$$AK = \frac{-2(\log likelihood)}{T} + 2\frac{v}{T}$$

where $\hat{\sigma}^2$ is the residual sum of squares divided by the degrees of freedom; v is the number of parameters; T is sample size. For a Gaussian AR(p) this equation transforms into the specification shown below.

$$AK = \ln(\hat{\sigma}^2) + \frac{2v}{T}$$

The Schwarz criterion places harsher penalty on extra parameters.

$$SB = \ln(\hat{\sigma}^2) + \frac{v}{T}\ln T$$

When constructing a model the aim is to minimize the value of these information criteria. There are many other information criteria,[21] but only the AK and SB information criteria will be used in this paper because these are provided in the EViews regression output[22]. These information criteria will be used in addition to the ACF and PACF plots for model identification .

4.3 Estimation

Least squares estimators

There are many different ways to estimate the parameters of an ARIMA model, with the two most commonly used being maximum likelihood and least squares regression. Least squares will be used in this paper, following the methods set out by Box and Jenkins (1976, pp.212-24). The premise of least squares is that a

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[20] The sum of squared differences between a fitted line and the set of observations.

[21] For instance the Hannan-Quinn (1979) criterion: $HQ = \ln(\hat{\sigma}^2) + \frac{2v}{T}\ln(\ln T)$.

[22] EViews uses a slightly modified version of the AC and SB which rely on a log likelihood function based on maximum likelihood estimation (see Brooks, 2002, p.265).

straight line be fitted through a given set of points so that the sum of the squares of the distances of those points from the straight line is a minimum, where the distance is measured in vertical direction (Kreyszig, 1970, p.288). The least squares estimates and their standard errors are easy to derive and proofs will not be discussed in this paper (for derivations see Brooks (2002, pp.127-9). The general model:

$$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + ... + \beta_v x_{vt} + u_t$$

can be written in terms of matrices:

$$y_{Tx1} = X_{Txv}\beta_{vx1} + u_{Tx1}$$

where the subscript is the dimension of the matrix. Using this formulation the coefficient estimates are given by:

$$\widehat{\beta} = \begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_v \end{pmatrix} = (X'X)^{-1}X'y$$

while the corresponding coefficient standard errors are:

$$s^2 = \frac{\hat{u}'\hat{u}}{T-2}$$

where v is the number of regressors including a constant. EViews regression output provides a t-ratio test of whether a given coefficient is zero against a two-sided alternative, the statistic used is given below.

$$\frac{\hat{\beta}_i - \beta_i^*}{SE(\hat{\beta}_i)} = \frac{\widehat{\beta}_i}{SE(\hat{\beta}_i)} \sim t_{t-v}$$

The above test can be used to determine if a coefficient should be included in the model and thus is useful for determining the order of the ARIMA structure.

Goodness of fit statistics

Evaluation of total model performance can be made with reference to goodness of fit statistics, the most commonly used being the adjusted R^2. Unadjusted R^2 can be found from the following,

$$R^2 = \frac{ESS}{TSS} = \frac{TSS - RSS}{TSS} = 1 - \frac{RSS}{TSS}$$

where TSS is the total sum of squares $\sum_t (y_t - \bar{y})^2$, ESS the explained sum of squares $\sum_t (\hat{y}_t - \bar{y})^2$ and RSS the residual sum of squares $\sum_t \hat{r}_t^2$. An important property of R^2 is $0 \le R^2 \le 1$, if $R^2 = 1$ then the model has explained all of the variability of the dependent variable about its mean i.e. all observations would lie on the fitted line. A major problem with unadjusted R^2 is that it never decreases if more parameters are added to a model. This problem led to the modification:

$$R_a^2 = 1 - \left[\frac{T-1}{T-v}(1-R^2) \right]$$

where v is the number of parameters in the model. This expression takes into account the loss of degrees of freedom associated with adding extra variables. Adjusted R^2 is a useful measure of the quality of a fitted model. These basic building blocks allow estimation of the coefficients for any ARIMA process.

4.4 Diagnostic checking

Box and Jenkins (1970) suggest two forms of diagnostic checking: overfitting and residual diagnostics. The analysis here will concentrate on residual diagnostics because they are of greater statistical interest. Residual tests examine if the covariance between error terms over time will be zero – if a model is a good explanation of a set of data then errors will be uncorrelated. One of the most important residual tests is the portmanteau lack of fit test developed by Box and Pierce (1970). It was originally defined as:

$$Q = T \sum_{t=0}^{\varsigma} r_{t-\varsigma}^2 \sim \chi^2(\varsigma - p - q)$$

where $r_t, r_{t-1}, r_{t-2}, \ldots, r_{t-\varsigma}$ is a series of lagged residuals from the fitted OLS, ς is the maximum lag length, while p and q are the number of estimated parameters. However, later work by Ljung and Box (1978) uses Monte Carlo simulation to find a more accurate approximation to the chi-square distribution,

$$Q = T(T+2) \sum_{t=0}^{\varsigma} \frac{r_{t-\varsigma}^2}{T-\varsigma} \sim \chi^2(\varsigma - p - q).$$

If the model being tested is an adequate fit then the statistic will belong to the relevant chi-squared distribution[23], whereas if the model is not appropriate then the statistic will be inflated. EViews has a Q-statistic function which will be used to analyse and compare models in this paper.

In order to consolidate the Ljung Box test this paper will consider another residual diagnostic check – the Breusch (1978), Godfrey (1978) test. Assume again that there are a time series of residuals $r_t, r_{t-1}, r_{t-2}....r_{t-\varsigma}$ obtained from OLS regression of the ARIMA(p,q) model. The first step of Breush Godfrey is to regress the residuals from the most recent OLS on the regressors used in the ARIMA model plus the series $r_t, r_{t-1}, r_{t-2}....r_{t-\varsigma}$ i.e. a regression of the form:

$$r_t = \omega_1 + \omega_2 x_{2t} + \omega_3 x_{3t} + ... + \omega_v x_{vt} + \eta_1 r_{t-1} + \eta_2 r_{t-2} + \eta_3 r_{t-3} + ... + \eta_k r_{t-\varsigma} + u_t$$

where u_t is a zero mean white noise error term. Then calculate R^2 for this regression. The test statistic is:

$$(T - \varsigma)R^2 \sim \chi_\varsigma^2$$

where the null hypothesis is that there is no autocorrelation in residuals. The main problem in implementing this test is deciding the number of lagged residuals ς to include in the analysis. There is no clear answer to this problem other than the use of trial and error. The Ljung Box and Breusch Godfrey residual tests will be used to carry out the diagnostic checking stage of the Box Jenkins process.

4.5 Forecasting

Basic definitions

The final, and most important stage of the Box Jenkins process is forecasting. There are two broad types of forecast: one step ahead forecasts are generated for the next observation only whereas multi-step ahead forecasts are generated for 1,2,3,…..,s steps ahead. Using EViews it is possible to create a sequence of one step ahead forecasts using a fixed set of observations: "static" forecasts. This type of forecasting will be applied in this paper. It should be noted that EViews also allows for the computation of "dynamic" multi-step ahead forecasts using a rolling window. In order to evaluate forecast accuracy it is common to construct a holdout sample

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[23] The joint null hypothesis is that all $\varsigma + 1$ of the residuals are zero.

from the data set which can be used to create out of sample forecasts. Once the out of sample period has been defined the remainder of the data (the in-sample period) would be used to estimate the parameters of the model. Generating forecasts for the out of sample period and comparing them with the actual observations would then provide a useful measure of forecast performance. The mechanics of forecasting with ARIMA models is straightforward: if $f_{t,s}$ is a forecast made at time t for s steps in the future then the forecast function is of the form:

$$f_{t,s} = \sum_{i=1}^{p} a_i f_{t,s-i} + \sum_{j=1}^{q} b_j u_{t+s-j}$$

where a_i is the autoregressive and b_i the moving average parameter; $f_{t,s} = y_{t+s}$ when $s \leq 0$. Series of point forecasts will be generated for each model in section 5.6 of this paper.

Measuring forecast accuracy

Quantifying forecast accuracy is an important part of ARIMA modelling. There are many simple measures of forecast accuracy, for instance the mean squared error (MSE), mean absolute error (MAE) and mean squared deviation (MSD). However the most appropriate simple error measure for this exercise is the mean absolute percentage error (MAPE).

$$MAPE = \frac{1}{T} \sum_{i=1}^{T} \frac{|y_t - f_t|}{y_t}$$

This statistic can be used to compare the accuracy of forecasts based on two entirely different series (Hanke and Wichern, 2005, p.80) because it is defined in relative not absolute terms. The time series to be analysed in this paper are very different, but use of MAPE allows comparison of forecast accuracy between them. Another useful measure of forecast accuracy is Theil's "U" inequality coefficient statistic (1966, p.28) which compares the forecast error of the proposed model with the error of a benchmark model which is typically a simple model like random walk (Armstrong and Collopy, 1992). There are many variations of the U statistic, Wong *et. al* (2005) make the simplifying assumption that the benchmark forecasts are zero ($fb_t = 0$) giving:

$$U = \sqrt{\frac{\frac{1}{T}\sum_{t=1}^{T}(y_t - f_t)^2}{\frac{1}{T}\sum_{t=1}^{T}(y_t - fb_t)^2}} = \sqrt{\frac{\frac{1}{T}\sum_{t=1}^{T}(y_t - f_t)^2}{\frac{1}{T}\sum_{t=1}^{T}(y_t)^2}}$$

this coefficient lies between zero and one. If U=0 the forecast error is zero for all t and the model is a perfect fit; if U=1 the predictive performance of the model totally fails. This statistic is computed by EViews.

EViews also provides a decomposition of the mean squared error (MSE) which allows further assessment of forecast accuracy:

$$\text{MSE} = \frac{1}{T}\sum_{i=1}^{T}\frac{(y_t - f_t)^2}{y_t} = \left(\frac{\sum f_t}{T} - \bar{y}\right)^2 + (\sigma_{\bar{y}} - \sigma_y) + 2(1-\upsilon)\sigma_{\bar{y}}\sigma_y$$

where σ denotes standard deviation and υ is the correlation between f_t and y_t. The right hand side of the expression can be used to analyse the three forces driving MSE. The bias proportion describes how far the mean of the forecast is from the mean of the series,

$$bias = \frac{\left(\dfrac{\sum f_t}{T} - \bar{y}\right)^2}{\dfrac{1}{T}\sum_{i=1}^{T}\dfrac{(y_t - f_t)^2}{y_t}}$$

while the variance proportion compares the variance of the forecast and the series,

$$\text{var} = \frac{(\sigma_{\bar{y}} - \sigma_y)}{\dfrac{1}{T}\sum_{i=1}^{T}\dfrac{(y_t - f_t)^2}{y_t}}$$

and the covariance proportion measures the remaining unsystematic errors,

$$\text{cov} = \frac{2(1-\upsilon)\sigma_{\bar{y}}\sigma_y}{\dfrac{1}{T}\sum_{i=1}^{T}\dfrac{(y_t - f_t)^2}{y_t}} .$$

Using these measures it is possible to examine what factors are contributing to the MSE. Empirical work has shown that accurate forecasts would be unbiased and would also have a small variance proportion, so that most of the forecast error should come from the covariance (Brooks, 2002, p.293). In this paper MAPE, Theil's

inequality and a decomposition of MSE will be used to assess the quality of generated forecasts.

5. Forecasting

In this section the techniques described in the methodology section will be applied to the government pension expenditure and population over 65 data sets. Analysis will start with a short discussion of the data sources, followed by examination of data stationarity. The remainder of the section will deal with the implementation and examination of Box Jenkins techniques.

5.1 Data

Two sets of data have been acquired for this exercise: UK population over 65 years of age from 1943-2004 and UK government pension expenditure for 1948-2005. The population data set are mid-year estimates obtained from the *Annual Abstract of Statistics* 1945-2006 published by the Office for National Statistics (previously the Central Statistics Office). This data set quantifies all persons over 65 years of age (despite women's pension age currently being 60) because pension age will be equalised from 2010. The data look fairly smooth, but with an obvious trend, implying that stationarity may be a problem. The issue of stationarity will be dealt with in the next subsection.

Figure 5.1.1 Time series of UK population over 65 for 1943-2005
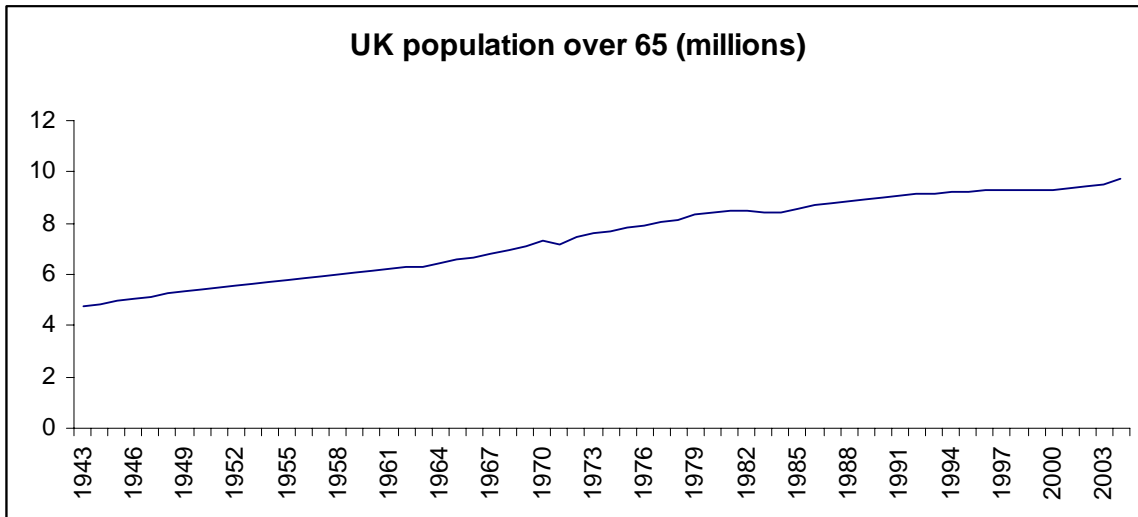
The data for government pension expenditure, expressed in nominal terms, were downloaded from the Office for National Statistics (ONS) website[24]. Modelling of this time series will provide assessment of future government pension burden. It is also hoped that the analysis of this data set will provide some insight into the poor performance of private pension funds because this atrophy will cause more employees to enroll in the state second pension (S2P) instead of private employment pensions. Therefore a rise in government pension expenditure caused by an increase in S2P outlay will reflect a downturn in private pension fund performance. A plot of the expenditure data set expressed in nominal terms is given below.

Figure 5.1.2 Time series of UK government expenditure (£million) for 1948-2006
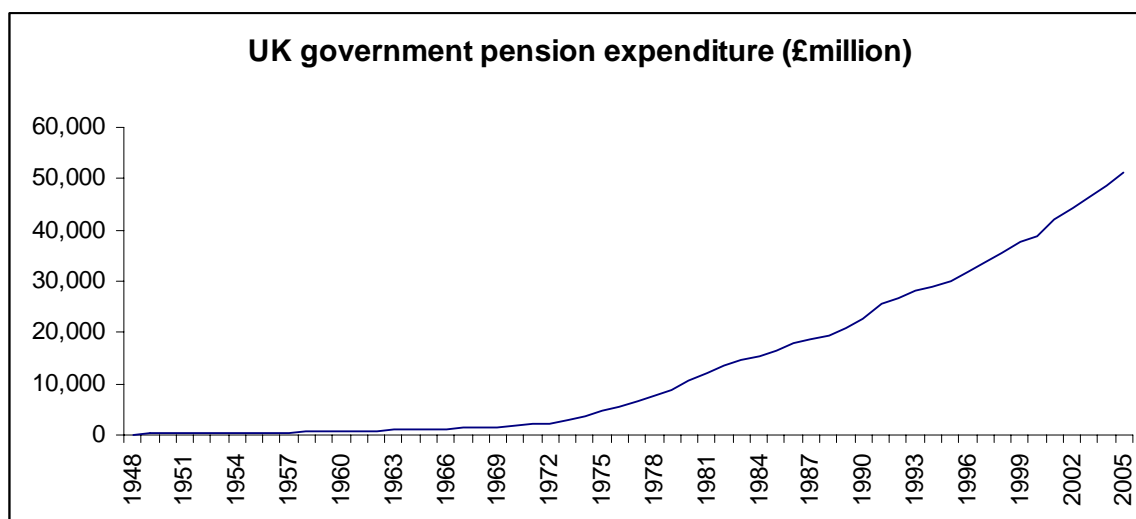
As the data were in nominal terms they were deflated using a time series of consumer price index (CPI) levels[25]. The resulting series was thus expressed in relative rather than absolute terms.

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[24] The ONS provides a wide variety of economic, demographic and social security statistics that are available for download in Microsoft Excel form: www.statistics.gov.uk.

[25] This time series was also obtained from the ONS website.

Figure 5.1.3 Deflated time series of UK government pension expenditure 1948-2006

UK goverment pension expenditure (deflated)
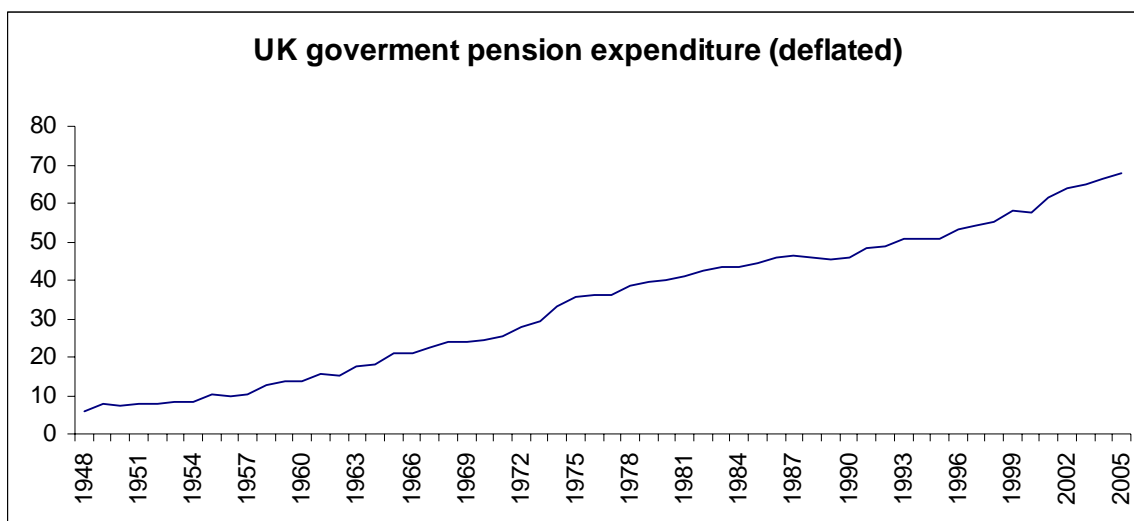
As elucidated earlier both data sets have over 50 observations which makes them suitable for Box Jenkins modelling. The remainder of this section will deal with fitting ARIMA models to these data sets.

5.2 Stationarity analysis

The plots provided in the previous subsection imply that both data sets may be non-stationary. However, graphical methods are not the best way to decide on stationarity, instead the augmented Dickey Fuller (ADF) test should be used. Carrying out the ADF test in EViews using six lags of the dependent variable (a reasonable assumption because the data appear to be highly correlated) produced the statistics given below. The corresponding critical values as computed by Fuller (1976) are given in section 4.1 of this paper.

Table 5.2.1 ADF test statistics for the differenced demographic series

	No differences	First differences	Second differences
Intercept	-0.98	-2.07	**-3.88**
trend and intercept	-1.58	-2.26	**-3.84**

The above statistics provide reason to reject the ADF null hypothesis (which is non stationarity) for the second differences of the demographic series. Therefore second differences should be taken in order to induce stationarity. The government pension data gives the following statistics.

Table 5.2.2 ADF test statistics for the differenced macroeconomic series

	No differences	First differences	Second differences
Intercept	0.26	-2.18	**-4.16**
trend and intercept	-2.35	-2.23	**-4.11**

Analysis of the pension expenditure data set reveals that second differences will induce stationarity. Now that transformations have been found to make the data stationary the remainder of this section will deal with fitting the Box Jenkins model.

<u>5.3 Model identification</u>

Graphical identification. The first step of model identification is to find the ACF and PACF of the stationary data. Visual methods can then be used to determine model orders. For the population data the plots (over twelve lags) look as follows.

Figure 5.3.1 ACF and PACF (on twelve lags) for the population series
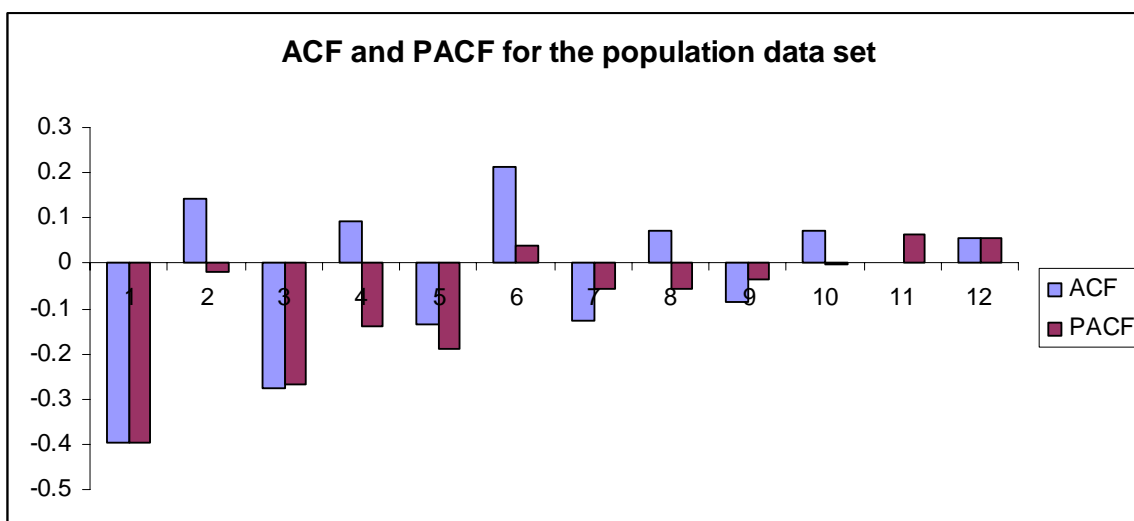
It is difficult to draw conclusive results from the above plots. Both functions show signs of decay after a significant term at a lag one. The work of Box, Jenkins and Reinsel (1994, p.187) discussed in the methodology section, would then imply that the model is of order ARMA(1,1). This is a tentative result which will need to be confirmed using information criteria. Using EViews to find the ACF and PACF of the expenditure data produced the plot shown below.

Figure 5.3.2 ACF and PACF (on twelve lags) for the pension expenditure series
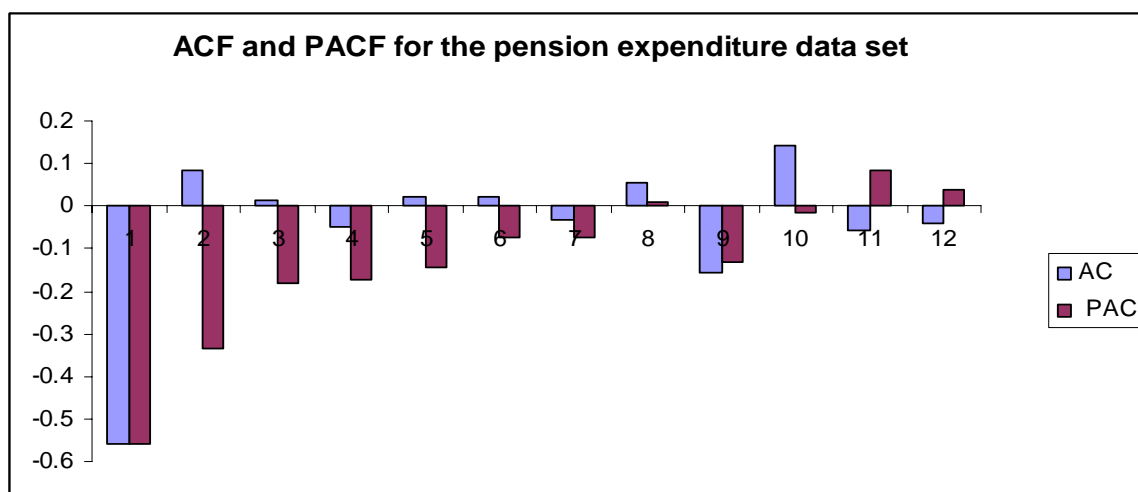
The above diagram shows a decaying PACF coupled with an ACF that cuts off after lag one. Using the Box Jenkins identification criterion this would imply that the series is of the form MA(1), giving an ARIMA model of order (0,2,1) for the macroeconomic data.

Information criteria

It is possible to use information criteria to provide more insight into model performance. Once ARIMA models have been estimated via regression (see next sub-section) it is possible to compute these criteria. EViews provides criteria values in regression output.

Table 5.3.3 Akaike's criterion for a selection of demographic ARMA models

AR(p) / MA(q)	0	1	2
0	-2.93	-3.12	-2.91
1	-3.05	**-3.17**	-3.08
2	-2.89	-3.07	-2.98

Table 5.3.4 Schwarz's criterion for a selection of demographic ARMA models

AR(p) / MA(q)	0	1	2
0	-2.89	-3.05	-2.84
1	-2.98	**-3.17**	-2.97
2	-2.82	-2.97	-2.87

Use of criteria therefore implies that an ARMA(1,1) model is the most suitable for the over 65 population time series. This supports the conclusion from the graphical analysis. Therefore the ARIMA(1,2,1) will be applied to the population data in section 5.6 of this paper. The corresponding information criteria for the expenditure data set are tabulated below.

Table 5.3.5 Akaike's criterion for a selection of macroeconomic ARMA models

AR(p) / MA(q)	0	1	2	3
0	3.75	2.99	3.78	3.78
1	3.37	3.02	**2.96**	3.40
2	3.77	3.04	3.81	3.81

Table 5.3.6 Schwarz's criterion for a selection of macroeconomic ARMA models

AR(p) / MA(q)	0	1	2	3
0	3.78	**3.06**	3.85	3.86
1	3.44	3.13	3.07	3.51
2	3.85	3.15	3.92	3.92

Therefore the Akaike and Schwarz criteria recommend different models – Akaike chooses an ARMA(1,2) while Schwarz favours an ARMA(0,1), the same model as suggested by graphical identification. This difference is caused by the Schwarz criterion placing a heavier penalty on adding extra parameters to a model (see the definitions in section 4.2). Given that estimating extra parameters reduces model accuracy this paper will proceed under the assumption that the expenditure data is of ARIMA(0,2,1) form.

5.4 Model estimation

Least squares regression will be used in EViews to estimate models for both data sets. In order to test forecast accuracy each data set will be truncated, creating a holdout period used to create out of sample forecasts. Therefore the population data running from 1943-1994 and the macroeconomic series from 1948-1995 will be used for model estimation. The previous section identified the demographic model as:

$$\nabla^2(y_t) = \mu + \theta u_{t-1} + \zeta \nabla^2(y_{t-1}) + u_t$$

where ∇ is the differencing operator, μ a constant and u_t a white noise error term. Using the least squares regression function in EViews allows estimation of model parameters.

Table 5.4.1 Regression output from demographic model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000480	0.000897	-0.534863	0.5953
AR(1)	0.309106	0.146021	2.116856	0.0397
MA(1)	-0.962586	0.031402	-30.65321	0.0000

The t-statistic of the constant provides no evidence to reject the null hypothesis that it is zero (see the methodology section 4.3 for a formulaic explanation). Therefore the constant should be removed from the model.

$$\nabla^2(y_t) = \theta u_{t-1} + \zeta \nabla^2(y_{t-1}) + u_t$$

Running this model in EViews gives the coefficients $\theta = -0.96$ and $\zeta = 0.31$. Therefore the estimated model for the population time series looks as follows.

$$\nabla^2(y_t) = -0.96u_{t-1} + 0.31\nabla^2(y_{t-1}) + u_t$$

Diagnostic checks will be used to test the accuracy of this model in section 5.5.

For the macroeconomic pension data the identification stage suggested a model of the following form.

$$\nabla^2 y_t = \mu + \theta u_{t-1} + u_t$$

Using EViews to estimate this model using least squares produces the output shown below.

Table 5.4.2 Regression output from macroeconomic model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002548	0.012555	0.202951	0.8401
MA(1)	-0.971239	0.023650	-41.06781	0.0000

Again the constant is not a significant parameter. The final model is therefore:

$$\nabla^2 y_t = -0.97u_{t-1} + u_t$$

with an adjusted R^2 of 0.52. Now that the parameters in both models have been estimated the next stage of the Box Jenkins process involves checking the adequacy of fit using various diagnostics.

5.5 Model diagnostic checking

In this section tests of the correlation between residuals will be carried out. If a model is an adequate fit to a data series then the residuals formed from its estimation will be uncorrelated. Two tests will be used. The first is the Ljung Box analysis which is described in detail in section 4.4. Applying this test with ten residual lags (reasonable considering the sample size minus holdout period is only 48) to the demographic fitted model in EViews gives the following figures.

Table 5.5.1 Output from Ljung Box residual test of the demographic model

	AC	PAC	Q-Stat	Prob
1	0.001	0.001	7.E-05	
2	-0.023	-0.023	0.0329	
3	-0.082	-0.082	0.4585	0.498
4	0.006	0.006	0.4611	0.794
5	-0.020	-0.024	0.4884	0.921
6	0.067	0.061	0.7922	0.939
7	-0.059	-0.060	1.0314	0.960
8	0.034	0.034	1.1117	0.981
9	-0.002	0.005	1.1120	0.993
10	-0.003	-0.012	1.1129	0.997

It is evident that there is no reason to reject the null hypothesis that the residuals are uncorrelated.

For the macroeconomic model EViews finds the following.

Table 5.5.2 Output from Ljung Box residual test of the macroeconomic model

	AC	PAC	Q-Stat	Prob
1	-0.037	-0.037	0.0665	
2	0.111	0.110	0.6884	0.407
3	-0.048	-0.041	0.8069	0.668

|				
4	0.060	0.045	0.9933	0.803
5	-0.021	-0.009	1.0180	0.907
6	0.005	-0.009	1.0195	0.961
7	0.136	0.146	2.0647	0.914
8	-0.001	0.003	2.0647	0.956
9	-0.030	-0.060	2.1185	0.977
10	0.108	0.123	2.8287	0.971

This result also gives no reason to reject the null hypothesis. Therefore the Ljung Box test cannot find any evidence of autocorrelation in the residuals of the fitted models which implies that they are an adequate fit.

Application of the Breush Godfrey test (described in section 4.4) to the population model gives the test statistic 5.99 which corresponds to a p-value of 0.82, giving no cause to reject the null hypothesis that there is no autocorrelation in residuals. The macroeconomic data yields statistic 3.16 with p-value 0.98, again giving no reason to reject the null hypothesis. Therefore the Breusch Godfrey test has shown that both models are a good fit for their data. All diagnostic checks have given positive results meaning that both models can be used for forecasting.

5.6 Final forecasts

Before implementing forecasts it is important to test predictive power using out of sample forecasts based on data not used in model estimation. Due to the relatively small sample sizes out of sample forecasts will be based on ten observations. Static one step ahead forecasts will be used. EViews gives the following outputs for the out of sample period.
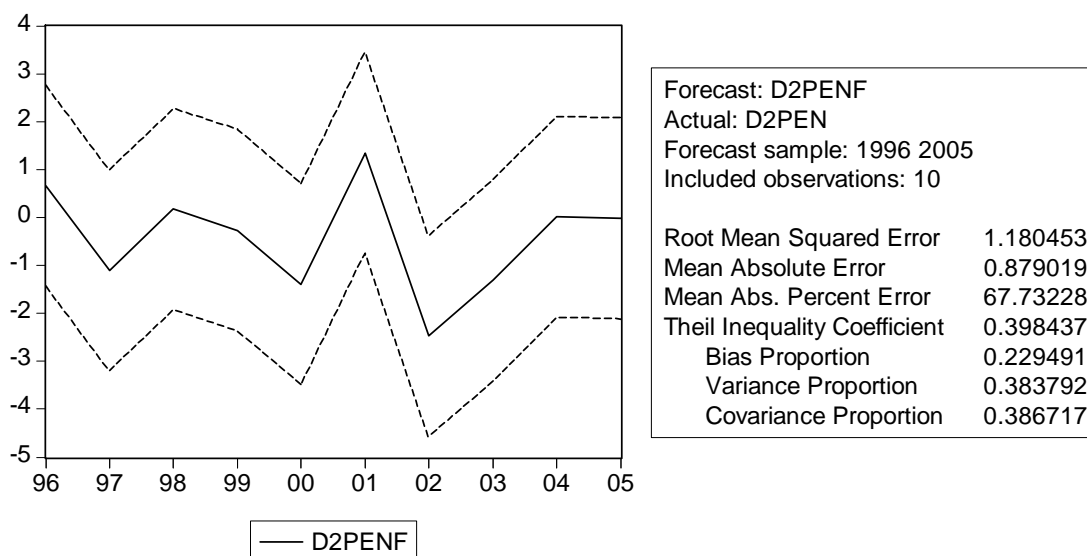
Figure 5.6.1 Out of sample forecast results for the population model
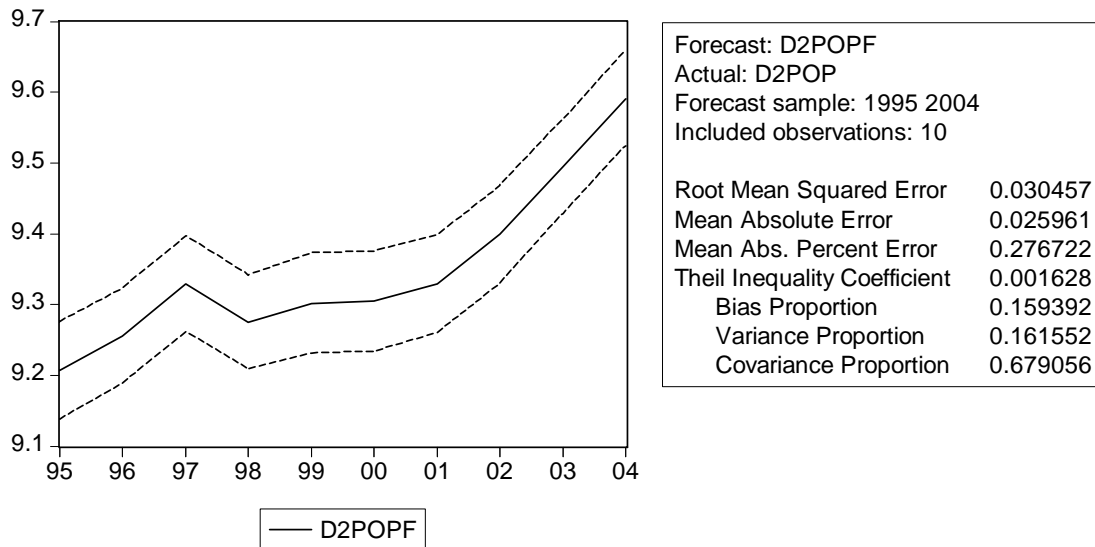
The variable name "D2POPF" represents the second differences of the UK population over 65. Theil's inequality measure is close to zero which implies that the model is a good fit (definitions of the accuracy measures are given in section 4.5). The MAPE is 27% which is reasonable. The covariance portion of the MSE is the larger than the bias or variance which shows that the model works well. Therefore the summary of accuracy statistics give good out of sample results for the demographic model.

The out of sample results for the macroeconomic government expenditure model are presented below.

Figure 5.6.2 Out of sample forecast results for the macroeconomic model


```
 4
 3
 2
 1
 0
-1
-2
-3
-4
-5
   96   97   98   99   00   01   02   03   04   05
```

| Forecast: D2PENF |
| Actual: D2PEN |
| Forecast sample: 1996 2005 |
| Included observations: 10 |

Root Mean Squared Error	1.180453
Mean Absolute Error	0.879019
Mean Abs. Percent Error	67.73228
Theil Inequality Coefficient	0.398437
Bias Proportion	0.229491
Variance Proportion	0.383792
Covariance Proportion	0.386717

——— D2PENF

The variable "D2PEN" is the second differences of the UK government pension expenditure time series. MAPE for this model is 68% which is higher than the corresponding value for the demographic series. The Theil coefficient is 0.40 which shows that the model is a reasonable fit. From the MSE breakdown the covariance portion is slightly higher than the variance portion although the results are not as conclusive as for the population model. Therefore the accuracy measures for the macroeconomic model appear reasonable, although not quite as convincing as those for the demographic specification.

Choice of forecast horizon

Before computing the forecasts for each time series it is important to decide on an appropriate forecast horizon. Firstly with regard to demographic forecasting, following the Pflaumer (1992) paper described in the literature review section it is possible to forecast up to 90 years in the future using ARIMA modelling. The reports of the pension commission (Turner, 2004, figures 1.5-1.10) use demographic projections up to 2050. Therefore forecasts will be made for the time period up to the year 2050. The macroeconomic forecasts of the current pension system made by Turner (2004, chapter 4) were generally made to 2050 or 2060. In this paper the forecast horizon of 2050 will also be applied to the government pension expenditure data set.

Now that the forecast horizons have been decided on it is possible to generate point forecasts for the two data series. Some mathematics will be required to derive these forecasts. The process starts with the demographic model found by the Box Jenkins method.

$$\nabla^2(y_t) = -0.96u_{t-1} + 0.31\nabla^2(y_{t-1}) + u_t.$$

Removing the difference operator from the l.h.s. gives:

$$\nabla^2 y_t = (1-B)^2 y_t = (1-2B+B^2)y_t = y_t - 2y_{t-1} + y_{t-2}$$

where B is the backward shift operator. Substituting and rearranging produces:

$$y_t = 2y_{t-1} - y_{t-2} - 0.96u_{t-1} + 0.31(y_{t-1} - 2y_{t-2} + y_{t-3}) + u_t$$

$$\therefore y_t = 2.31y_{t-1} - 1.62y_{t-2} + 0.31y_{t-3} - 0.96u_{t-1} + u_t$$

assuming parameter stability it is possible to write:

$$y_{t+1} = 2.31y_t - 1.62y_{t-1} + 0.31y_{t-2} - 0.96u_t + u_{t+1}$$

$$y_{t+2} = 2.31y_{t+1} - 1.62y_t + 0.31y_{t-1} - 0.96u_{t+1} + u_{t+2}$$

$$y_{t+3} = 2.31y_{t+2} - 1.62y_{t+1} + 0.31y_t - 0.96u_{t+2} + u_{t+3}$$

$$y_{t+4} = 2.31y_{t+3} - 1.62y_{t+2} + 0.31y_{t+1} - 0.96u_{t+3} + u_{t+4}\text{ etc.}$$

Applying conditional expectations (assuming we have full information for up to and including time t) leads to:

$$E(y_{t+1/t}) = 2.31E(y_t) - 1.62E(y_{t-1}) + 0.31E(y_{t-2}) - 0.96E(u_t) + E(u_{t+1})$$

$$f_{t,1} = E(y_{t+1/t}) = 2.31y_t - 1.62y_{t-1} + 0.31y_{t-2} - 0.96u_t + 0$$

where $f_{t,1}$ represents the one step ahead forecast from time t. A value for u_t can be obtained from the EViews out of sample forecasts[26]. The expression $E(u_{t+1})$ is set to its unconditional mean of zero because it has not yet been observed. The two step ahead forecast can be found from:

$$E(y_{t+2/t}) = 2.31E(y_{t+1}) - 1.62E(y_t) + 0.31E(y_{t-1}) - 0.96E(u_{t+1}) + E(u_{t+2})$$

$$\therefore f_{t,2} = E(y_{t+2/t}) = 2.31f_{t,1} - 1.62y_t + 0.31y_{t-1}$$

in this expression both $E(u_{t+1})$ and $E(u_{t+2})$ have not been observed and are thus set to their unconditional expectation which is zero. The best information available

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[26] Using the out of sample forecast function in EViews to find $u_t = f_t - y_t$

about $E(y_{t+1})$ is the one step ahead forecast $f_{t,1}$. The three step ahead forecast is therefore:

$$f_{t,3} = E(y_{t+3/t}) = 2.31f_{t,2} - 1.62f_{t,1} + 0.31y_t$$

while the four step ahead forecast is:

$$f_{t,4} = E(y_{t+4/t}) = 2.31f_{t,3} - 1.62f_{t,2} + 0.31f_{t,1}$$

It is therefore possible to construct forecasts up to the forecast horizon using this framework. The resulting series of forecasts appear in the figure below.

Figure 5.6.3 Projections of UK population over 65 from 2005 to 2050
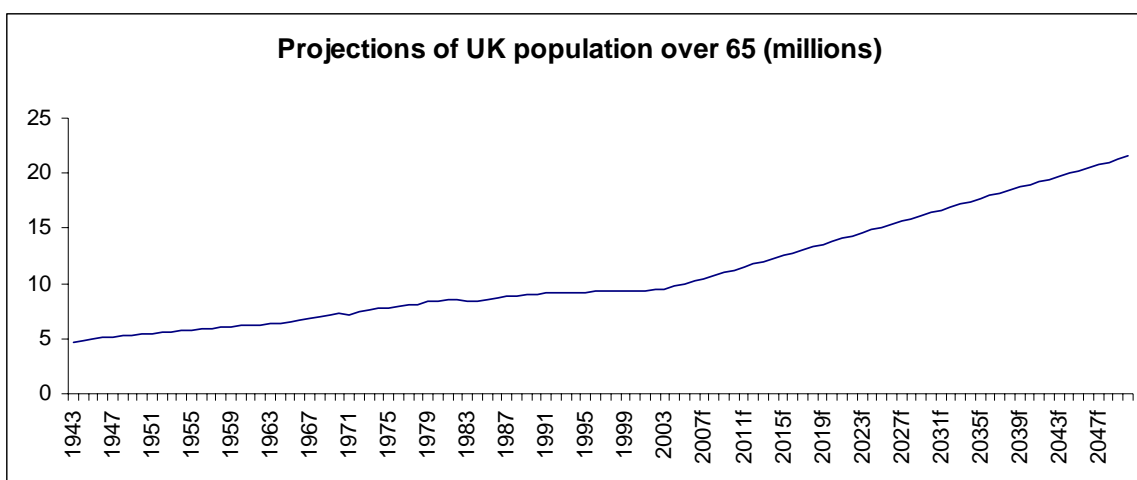
Remember that the final macroeconomic pension model was:

$$\nabla^2 y_t = -0.97u_{t-1} + u_t$$

Removal of the difference operator gives the following.

$$\nabla^2 y_t = (1-B)^2 y_t = (1-2B+B^2)y_t = y_t - 2y_{t-1} + y_{t-2}$$

Substituting and rearranging produces:

$$y_t = 2y_{t-1} - y_{t-2} - 0.97u_{t-1} + u_t$$

assuming parameter stability:

$$y_{t+1} = 2y_t - y_{t-1} - 0.97u_t + u_{t+1}$$

$$y_{t+2} = 2y_{t+1} - y_t - 0.97u_{t+1} + u_{t+2}$$

$$y_{t+3} = 2y_{t+2} - y_{t+1} - 0.97u_{t+2} + u_{t+3}$$

$$y_{t+4} = 2y_{t+3} - y_{t+2} - 0.97u_{t+3} + u_{t+4} \text{ etc.}$$

Applying conditional expectations:

$$E(y_{t+1/t}) = 2E(y_t) - E(y_{t-1}) - 0.97E(u_t) + E(u_{t+1})$$

$$\therefore f_{t,1} = 2y_t - y_{t-1} - 0.97u_t$$

where $f_{t,1}$ represents the one step ahead forecast from time t as before. The value of u_t is obtained from the EViews out of sample forecasts. The two step ahead forecast can be found from the equations below.

$$E(y_{t+2/t}) = 2E(y_{t+1}) - E(y_t) - 0.97E(u_{t+1}) + E(u_{t+2})$$

$$\therefore f_{t,2} = E(y_{t+2/t}) = 2f_{t,1} - y_t$$

For the three step ahead forecast use:

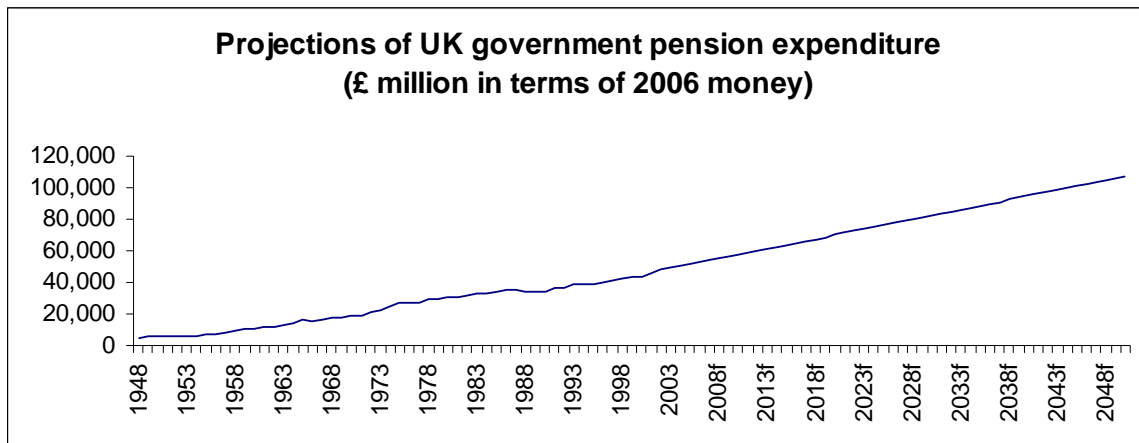$$E(y_{t+3/t}) = 2E(y_{t+2}) - E(y_{t+1}) - 0.97E(u_{t+2}) + E(u_{t+3})$$

$$\therefore f_{t,3} = 2f_{t,2} + f_{t,1}$$

while the four step ahead forecast is defined as follows.

$$f_{t,4} = 2f_{t,3} - f_{t,2}$$

Forecasts up to the forecast horizon can be constructed using this method. The resulting forecast series is shown below.

Figure 5.6.4 Projections of government pension expenditure from 2006 to 2050

A full discussion of the implications of this set of forecasts will be provided in section 6 of this paper. Now that the final forecast models have been generated the remainder of this paper will be devoted to discussions of their implications for the future of the UK pensions system.

6. Discussion

In this section final forecasts will be compared to the projections made by the pensions commission and government actuary's department. This will be followed by a detailed analysis of the implication of the final forecasts and an evaluation of their conclusions for the future state of the UK pensions system. The chapter will end with some suggestions for further research in the realm of pension modelling.

6.1 Evaluation of model projections

It is possible to reconcile the population forecasts made in this paper using population projections obtained from the government actuary's website[27].

Figure 6.1.1 Government actuary's projections for UK population over 65 for 2004 to 2051

Government actuary's projections for UK population over 65 (millions)
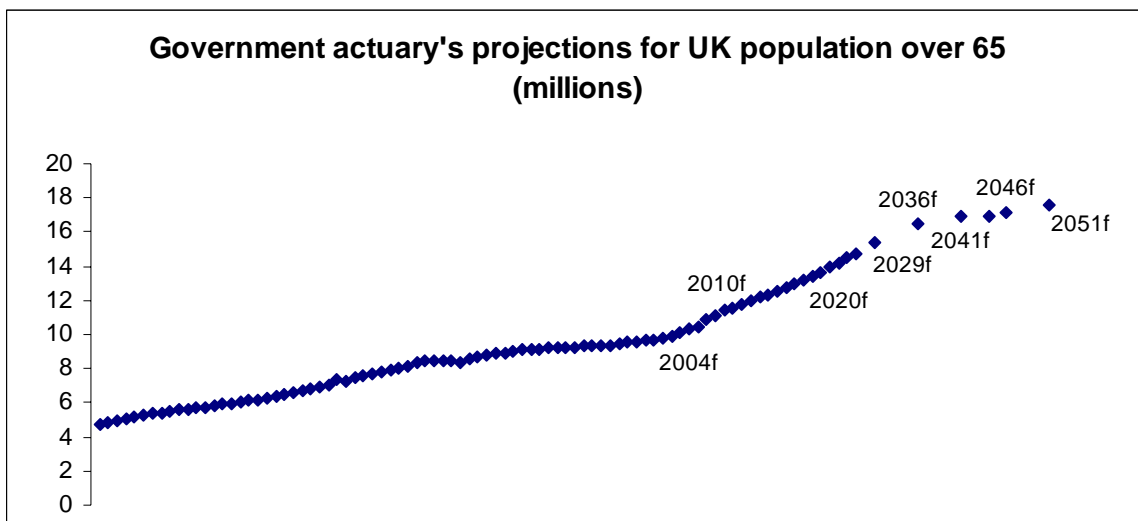
The government actuary's model shows an increase in the population trend post 2004, this rise can also be seen in the model developed in this paper. A selection of point forecasts from the government actuary's department are tabulated below.

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[27] See www.gad.org.uk/population, the time series was based on 2004 projections.

Table 6.1.2 Government actuary and model population point forecasts

	Model's prediction	Government actuary's prediction
2005f	9.9628	9.6562
2010f	11.2440	10.2931
2015f	12.5317	11.5932
2020f	13.8215	12.5436
2025f	15.1119	13.6600
2031f	16.4025	15.3395
3036f	17.6932	16.4571
2041f	18.9838	16.8938
2044f	20.2745	16.9653
2051f	21.8035	17.6050

Therefore the point forecast of the population model is higher than the government actuary forecast in every period. This is a satisfying result because government actuaries have consistently underestimated population in the past (Booth *et al.,* 2006). The comparison with government actuary projections demonstrates the usefulness of the new model. Now it is important to consider what demographic projections were made by the pensions commission. Although they did not explicitly model the UK population over 65, the pensions commission projected the old age dependency ratio (Turner, 2004, figure 1.6) which is the ratio of 65+ year olds to 20-65 year olds. The time series plot is provided below.

Figure 6.1.3 Pensions commission projections of old age dependency ratio from 2003 to 2050

38
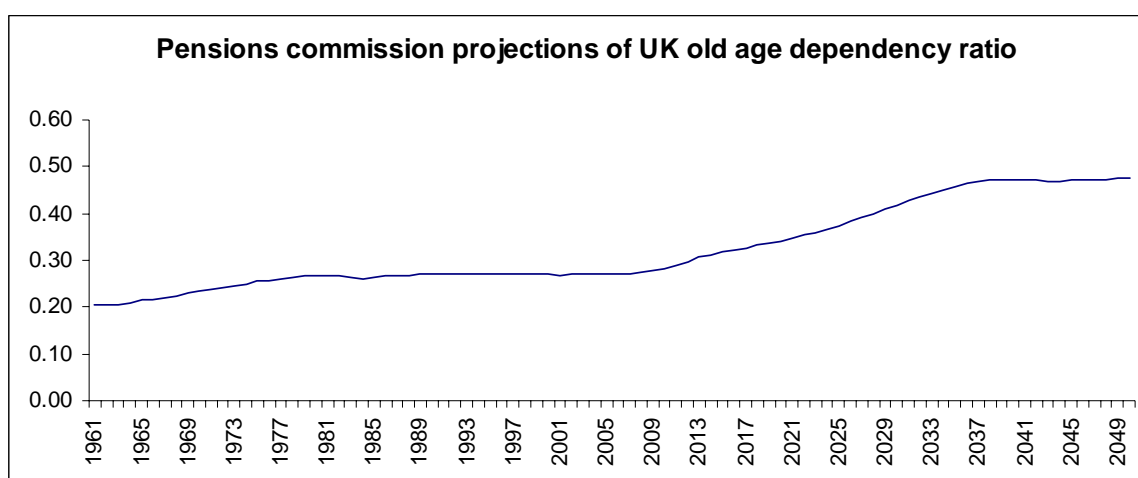
The figure above shows a marked increase in the dependency ratio between 2005 and 2039 is interesting to note. This conclusion does not contradict the results posited by this paper. The prediction of 48% dependency at 2050 appears rather high, however the dependency ratio implied by the model predictions in this paper coupled with the government actuary's forecast of the population aged 20-65 for 2050 (adjusted upwards) yields a dependency ratio of 47%[28]. Therefore the model constructed in this paper supports the demographic model presented by the pensions commission.

The pensions commission modelled government pensioner expenditure as a percentage of GDP (Turner, 2005, figure 1.19).

Figure 6.1.4 Pensions commission projections of government pensioner expenditure as a percentage of GDP
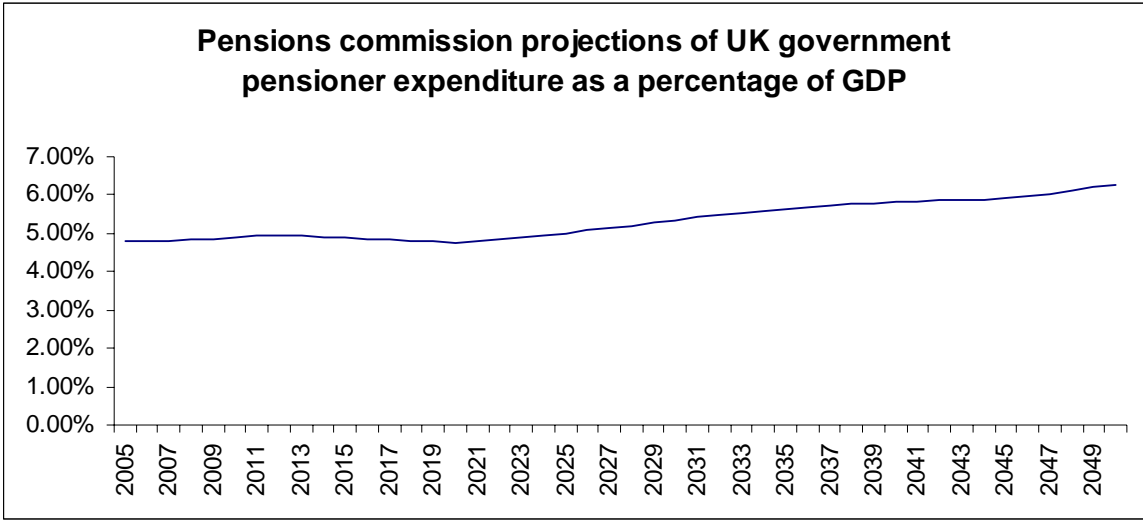

The above graph shows that pension expenditure will increase to around 6% of GDP by 2051. The model developed in this paper can be compared to the above figure by taking current GDP and assuming a constant real GDP growth rate up to 2051.

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[28] $\dfrac{21.8035}{37.507 * \dfrac{21.8035}{17.6050}} = 0.47$ where 21.8035 is the model point forecast for 2051; 37.507 is the

projected population forecast for 20 to 65 year olds from the government actuary's department adjusted upward.

Using growth rates of 2%, 2.5% and 3% to find the proportion of GDP absorbed by pension expenditure provides the following plot.

Figure 6.1.5 Box Jenkins Projections of UK government pension expenditure as a percentage of GDP
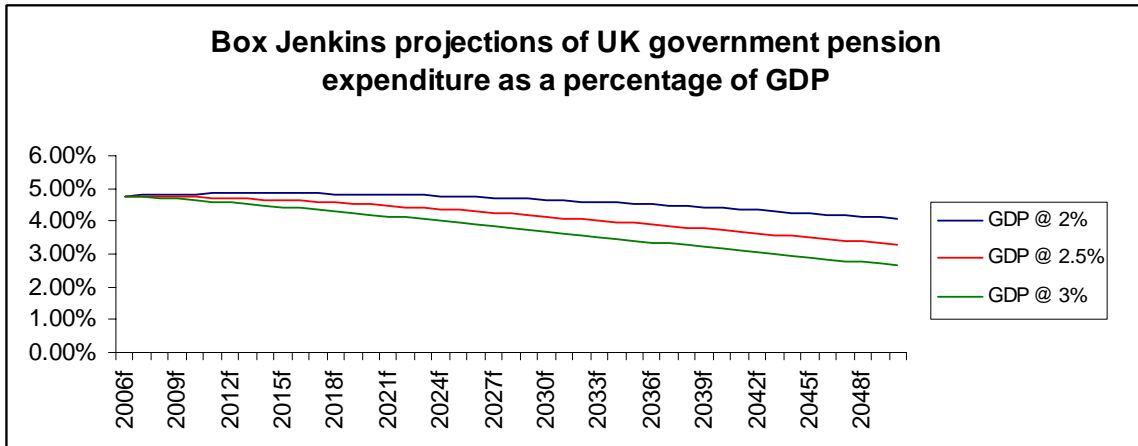
It is therefore clear that the forecasts in this paper predict a decrease in the percentage of GDP paid out in pensions (assuming that the current system remains unchanged). These results implies that the current pension system may not be as unsustainable as the pensions commission concluded. However this conclusion does not take into account mortality decreases and the related problems of an increasing dependent population. The forecasts above should therefore be interpreted with caution – they depend on the future state of the UK economy and the extent of population ageing. If upward adjustments were made to the above forecasts to account for a larger dependent population then it is likely that the predictions would lie close to those made by Turner. Therefore the results here have shown a major drawback of Box Jenkins methods – their use of historical data means that they cannot take account of future macroenvironmental changes like increases in net migration. In summation, the macroeconomic modelling in this paper has provided no reason to criticise the predictions made by the pensions commission while the demographic forecasting has strongly corroborated the commission's work.

6.2 Policy implications and beyond

The modelling in this paper has provided many insights into the future of the UK pensions system and the validity of reform proposals from the pensions commission

(Turner, 2005, 2006). Firstly, the macroeconomic forecasting model has allowed insights into the future governmental pension burden. The forecasts show that if the UK pension framework remains unchanged then it is likely that the proportion of GDP spent on pensions will decrease. However these forecasts should be adjusted upward to reflect future life expectancy increases meaning that the end result will be close to the model developed by the pensions commission.

The demographic model developed in this paper has been shown to support the much derided population projections made by the pensions commission. Many analysts were critical of Turner's warning of a "demographic timebomb" and the increases in the national retirement ages that he proposed. The forecasts generated for this exercise were very close to the projections made by the commission when evaluated in terms of the dependency ratio. Therefore the work in this paper can be seen as a validation of Turner's forecasting work and the conclusions which resulted from his projections. Comparison of the population estimates made here with those proposed by the government actuary's department provide yet more evidence of the undervaluation of demographic forecasts by national and government organizations (Booth *et al.*, 2006). Therefore this paper has affirmed the projections made by the pensions commission and provided some statistical support for their wide ranging conclusions.

Having evaluated the pensions commission modelling it is now relevant to question the role that politicians have and will play in pension reform in the UK. The formation of the pensions commission and the concept of private sector involvement in governmental decisions is an idea championed by the Blairite New Labour movement. Some academics have examined the drivers behind Labour's move toward reformation of the pension system. For instance, Mann (2005) examines the political and ideological motivations behind the Labour government's proposed reforms of the UK benefits system with reference to the Turner report and its implications. He concludes that "[c]hoice, flexibility and simplicity are the velvet glove puppets but compulsion and regulation the claws they conceal". The author is obviously cynical about the commissions proposal for a more comprehensive and universal state pension. It is likely that a move toward stronger state control of the pension system will lead to a right-wing backlash in the UK media which already has a strong cynicism about the workings of UK pensions. It is therefore extremely

important that the manifesto published by the pensions commission has a rigorous modelling foundation. This paper has corroborated some of the mathematical work carried out by the commission. However the story is far from over. When the recommendations of the pensions commission are implemented, which will happen gradually over the next decades, the political and ideological motivations underlying reform are likely to be questioned in more detail.

6.2 Modelling method

Box Jenkins techniques are an unconventional technique for pension modelling and their successful application in this paper is of great interest. Previous academic work on pension modeling and forecasting has generally involved simulation methods based on broad macroeconomic assumptions (see for instance Borsch-Supan *et al.* (2006) outlined in the literature review section of this paper). The forecasting methods proposed by Box Jenkins forecast variables using only historical data and require no macroeconomic assumptions. This means that subjectivity will not obstruct the modelling process; the same cannot be said for the simulation techniques applied in contemporary pension modelling research. A drawback of Box Jenkins is that it cannot account for future changes in mortality, future migration etc. because of its implicit historical basis. The inherent simplicity and applicability of Box Jenkins forecasting make it an extremely useful way of modelling pension systems and it has many advantages over the simulation techniques which are currently fashionable amongst researchers.

6.3 Suggestions for further research

There are many opportunities for the use of Box Jenkins methods for the purpose of modelling pension systems: Box Jenkins techniques are relatively easy to apply as long as suitably large data sets are available. A useful addition to this paper would be the creation of a model to analyse the future performance of firm pension funds. There are many potential time series that could be analysed including employee participation numbers, pension fund assets as a % of GDP (Turner, 2004, figure 3.4), the price of contracted out rebates (Turner, 2004, figure 3.18) etc. Using a private pension fund model in conjunction with an analysis of public pension spending like the one provided here would give a holistic assessment of the future health of the pensions system. The application of multivariate ARIMA models could be used to model complex problems like the economic impact of pension reform or

the welfare effects of a switch from a defined benefit to defined contribution system. Box Jenkins methods, although used extensively in macroeconomics and demography, have been used very rarely for pension modelling problems: it is hoped that this paper has demonstrated the relevance of Box Jenkins techniques for this area of research and that they will be applied in the future.

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