



Master's Thesis

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SUSTAINABILITY IN PUBLIC FINANCE: The Case of Lithuania 1999-2008

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Abstract

This essay discusses the long-run fiscal sustainability issues for one of the EU member states – Lithuania. Investigating properties of the public debt time series for the period 1999:1-2008:2, it is concluded that past fiscal policy in the country has been sustainable. A set of econometrical procedures was employed to analyse past fiscal time series, leading to the main conclusion that a sustainable public debt development is achieved through the total budget balance management policy. In the long-run, the control over the total budget deficits outcomes is maintained by adjusting public spending to the movements in revenue. Such a fiscal policy reaction function thus allowed the government to keep the debt-to-GDP ratio on the sustainable path.

Keywords: Budget deficit; Public debt; Sustainability; Fiscal policy; Lithuania

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1. Introduction

1.1. Background

Fiscal policy formation in the short- and long-run is an important issue in analyzing the overall macroeconomic outlook of an economy. Fiscal misbalances disturb the overall macroeconomic outlook by among other things pressing up inflation, giving a rise to current account deficits and crowding out private sector investments. Qualitative fiscal policy-making on the other hand has stabilizing effects on the macroeconomic imbalances, it reduces inflationary pressure helping to maintain a favourable macroeconomic environment and promoting economic growth in the long-run. ([Giammarioli et al. 2007, p.5](#))

The concept of the qualitative fiscal policy-making in the short-run is summarised in terms of fiscal stability. Stability concept comprises government's ability to carry out its responsibilities, which are associated with the commitments made, in the short-run. This ability is necessary to secure the continuity of the economic activity without disruption. In some periods liquidity problems for the government might arise when financing its responsibilities and to solve these problems the government might be required to borrow funds on the financial market. The direct linkage between the short-run and the long-run might be afterwards established through the behaviour of the participants on the financial market – the government and its creditors. The creditors are willing to finance the short-run liquidity problems of the economy in case these arise, as long as they are sure that the government is capable to fulfil its obligation to its creditors in the long-run. Thus, it is expected that in the long-run the economy will be able to carry out its obligations by covering its liabilities to the investors. The quality of public finance in the long-run might thus be summarised by the concept of sustainability in public finance which suggests that the government is supposed to conduct fiscal policy in a way that the economy is capable to cover its obligations to creditors through its assets in the long-run. Evaluation of the soundness of fiscal policy in the long-run might be set up by evaluating whether an economy satisfies its intertemporal public budget constraint. ([Giammarioli et al. 2007, p.5-6](#))

1.2. Purpose

Hence, the purpose of this essay is to analyse sustainability in public finance in the long-run for the one of the Baltic States – Lithuania – which is also a part of the EU-27. The interest in this Baltic State arises out of the fact that not many research papers on the fiscal policy issues might be found in the empirical literature for this particular country. The analysis is performed by applying the most conventional measures for evaluating whether public finance has been sustainable. These measures are found in the empirical fiscal policy literature which usually focuses on the

investigation of the properties of the fiscal time series also taking into consideration the aspects of economic growth and level of the interest rate in the economy.

Analysis starts by presenting the theoretical and mathematical concept of sustainability. Focusing on the basic model which characterizes how the relevant fiscal variables are expected to develop to sustain soundness of the fiscal policy in the long-run, econometrically tested models found in the empirical literature are presented. On this background, the general fiscal outlook of Lithuania is presented and available fiscal time series are evaluated in order to answer the questions of:

1. Has fiscal policy of Lithuania been sustainable in the past?
2. Can any direct fiscal channels be identified pointing towards sustainable fiscal outlook in the country?

1.3. Disposition

The essay is organized as follows: section 2 presents the concept of sustainability in public finance in the long-run while in section 3 the mathematical model of the economy's budget constraint is developed. Section 4 summarises empirical literature - econometric models to test for soundness of the fiscal policy. In the section 5 the overall macroeconomic outlook of Lithuania is in short presented, emphasising the development of public finance since independence in 1990. Section 6 presents fiscal data and econometrical methodologies used to analyse sustainability issues and section 7 concludes.

2. Theoretical concept of sustainability

The capability of the fiscal authorities to fulfil their obligations in the long-run is a result of qualitative fiscal policy-making which is summarized by the concept of sustainability in public finance. Therefore, soundness of the fiscal policy is explained and further considered in the theory of economy's intertemporal budget constraint.

In the early fiscal policy literature it was common to raise two questions regarding public finance issues. The first question was whether it was reasonable for the governments to run fiscal deficits indefinitely. Consequences of such activities are considered to cause misbalances in economic activity since it might among other things press up inflation, give rise to the increased current account deficits and crowd out private sector investments. The second question was whether it was possible at all to run permanent deficits. (Hamilton & Flavin 1986, p.808) Fiscal policy literature on this background has developed the theoretical intertemporal budget constraint framework in order to answer the latter question and has built mathematical models in accordance to the theory.

2.1. Households' budget constraint

The theoretical framework develops an approach that any household faces budget constraints throughout its lifetime. This in general implies that a private household is expected to consume and spend no more than it earns throughout its existence period. Throughout its lifetime, however, deviations from the balanced budget are desirable in a sense that, taking the utility maximization problem into consideration, in some periods the household is expected to consume and spend more than it earns or has saved. In these periods the household is consequently borrowing. In the other periods the same household is on the contrary expected to spend less than it earns thus being able to repay in the previous periods borrowed funds and possibly in addition to save.

It might be considered that permanent borrowing is still possible by continuously rolling over these loans by borrowing additional funds to consume and to repay previously borrowed amounts. However it is also obvious that not every financial institution or private entity would be willing to continue lending to such a household indefinitely. Therefore it is typical to assume that any household is subjected to its budget constraint in present value terms in the long-run as conventionally it is expected to earn at least as much as it spends throughout the lifetime. (Hamilton & Flavin 1986, p.808) These issues are the background to the economic theory, which suggests that a household faces the intertemporal budget constraint which is also referred to the balanced budget throughout its lifetime implying that household's consumption in present value terms net of interest payments is expected to be at least equal to household's initial wealth and discounted income over the lifetime. (Romer 2006, p.560)

2.2. Economy's budget constraint

In the same way as households, governments might be also considered to be subjected to the budget constraint in the long-run, which in fiscal policy literature is usually referred to the economy's intertemporal budget constraint. Generally it is assumed that unlimited borrowing is not possible and thus governments are obliged to repay borrowed funds. Behaviour of the unlimited borrowing is not consistent with the expectations of the lenders and these would be willing to continue crediting the economy as long as these might expect the government to fulfil the obligations to them in the long-run. (Romer 2006, p.560) For the dynamically efficient economy ability carrying out obligations becomes therefore possible by sustaining the budgetary position where government spending net of interest payments in a present value term together with the initial stock of the debt are not exceeding discounted public revenue over the existence path of economy. (Romer 2006, p.560, Greiner, Koeller & Semmler 2006, p. 130)

Different fiscal positions over time however are feasible and balanced, in-surplus and in-deficits budgetary positions might be observed. This implies that in some periods the government is required to borrow on the financial markets whereas in other periods the economy is saving.

Differences in the budgetary positions might be considered to arise for instance due to the general cyclical position of the economy as well as discretionary stabilization policy measures taking into account the stabilizing effects of the fiscal policy-making. Borrowing is thus necessary from the fiscal stability perspective in order to secure that no disruption in economic activity occurs in the short-run and therefore it is of importance to establish linkages between budgetary positions that are consistent with the concept of intertemporal budget constraint.

2.3. Theoretical concept of sustainability in the public finance

Maintaining, consequently, budgetary positions over time which allow an economy to satisfy its intertemporal budget constraint is referred to the sustainable public finance. Soundness of fiscal policy might be analyzed by establishing the direct linkage between the dynamic of the economy's budgetary positions and public debt. Since the necessity for governmental borrowing emerges when economy is running a public budget deficit arising out of excessive public spending compared to tax income, in national accounts this borrowing appears as an accumulation of the public debt. As a rule, the deficit is financed by issuing governmental securities. Meanwhile budgetary surpluses in general might be considered to be used for repurchasing governmental securities issued which in national accounts would appear as a decumulation of the public debt.

Under usual circumstances economies are therefore exposed for the fluctuations in the public debt level and one of the main institutional concerns is to keep the development of the debt under control. Keeping development of the public debt on the sustainable path thus characterizes proper fiscal policy making. (Romer 2006, p.560)

2.4. Implication of the sustainability in the public finance

Extending further the concept of sustainability, which suggests that fiscal institutions of an economy should keep the development of the public debt under control, has a direct economic implication since uncontrollable increases in public deficits and debt level might lead to serious economic problems. If the debt level of an economy is increasing rapidly or is already relatively high the stability in the macroeconomic environment of a country is violated.

Generally it is considered that a rapid increase in the public debt level might be the result of a rise in the public spending thus putting pressure on the domestic demand. Expansionary fiscal policy in the economy leads to the increase in the demand for domestic funds and interest rates in the economy are supposed to rise. Increase in the interest rates resulting from an increase in public spending crowds out private sector investments and threatens economic growth in the long-run. On the other hand, if the central bank interferes in the money market by supplying an additional monetary base, loose fiscal policy induces inflationary pressure which also misbalances the overall macroeconomic environment. (Makin 2005, p.286)

Increase in the public debt level is usually assumed to increase a probability of the debt default, resulting rise in the risk premium on the governmental securities required by the debt financiers. Unbalanced increase in public spending therefore leads to a rise in the debt servicing costs through two channels – an increase in the interest rates required by investors and continuous growth in interest obligation as a consequence of the general increase in the debt level. This in turn again causes additional burden to the public budget through a rise in debt servicing costs whereas under other circumstances these funds could be reallocated for more productive activities of the public sector. (Makin 2005, p.286)

Additional debt servicing costs might further call for the incentive for the debt default or monetarisation of it. Monetarization of the debt occurring through issuance of the monetary base, presses up inflation while debt default makes further debt financing with reasonable interest rates complicated since reliance of the debt investors to the fiscal institutions is violated. Debt default or monetarisation of it as a rule causes the outflow of the capital, threatening the stability of the financial system in the economy. (Makin 2005, p.286)

Capital outflow usually occurs even before the actual monetarization or the debt default if investors might expect the government of taking such actions. Therefore keeping the development of the public debt under control is an important aspect for a disciplined government to secure a stable macroeconomic outlook of an economy. (Makin 2005, p.286) Efficient governments would take corrective actions through efficient public deficit management policies in order to stabilize or maintain public debt development on the sustainable path in order so secure favourable macroeconomic conditions.

3. Mathematical approach

Soundness of the public finance is highlighted in the proper public deficit and debt management policies. Developing the mathematical model of intertemporal budget constraint allows seeing what debt and deficit properties should be maintained in order to define the sustainable path of the fiscal policy-making in the long-run. Interrelationship between public debt and deficits is thus developed by introducing one-period model connecting these two variables, afterwards establishing the general representation which satisfies economy's intertemporal budget constraint.

3.1. One-period budget identity

One-period interrelationship between public debt and budget deficit might be established as follows. Denoting the gross stock of the debt at period t as \bar{B}_t , nominal budgetary revenue and expenditure, excluding interest payments on the debt as \bar{T}_t and \bar{G}_t respectively and nominal

interest rate on the debt service as i_t , the general representation of the stock of the debt in nominal terms at period t is given by:

$$\bar{B}_t = (1 + i_t)\bar{B}_{t-1} + \bar{G}_t - \bar{T}_t \quad (1)$$

The notation in (1) suggests¹ that the stock of the debt at time t is a sum of the public debt at the previous period \bar{B}_{t-1} with its servicing costs $i_t\bar{B}_{t-1}$ and the difference between primary public expenditure and tax recipients at the end of period t . This latter term $\bar{G}_t - \bar{T}_t$ yields primary or non-interest budget deficits figure of the period t .

The dynamics of the stock of debt over time is given by subtracting the term \bar{B}_{t-1} from both sides of equation (1), yielding the one-period adjustment of the public debt in discrete time, which in turn suggests that the change in the stock of the debt between periods t and $t - 1$ equals the amount of the interest paid to finance the outstanding debt $i_t\bar{B}_{t-1}$ and a primary budget deficits $\bar{G}_t - \bar{T}_t$ at the end of the period t :

$$\Delta\bar{B}_t \equiv \bar{B}_t - \bar{B}_{t-1} = i_t\bar{B}_{t-1} + \bar{G}_t - \bar{T}_t \quad (2)$$

In accordance to the notation in (2), the stock of the debt grows at the rate equal to the interest rate i_t throughout time whenever a primary budget balance equals zero, i.e. $\bar{G}_t - \bar{T}_t = 0$. If an economy is running primary deficits, i.e. $\bar{G}_t - \bar{T}_t > 0$, the stock of the debt is growing at the higher rate than the interest rate i_t . Running primary surpluses, which is the case of $\bar{G}_t - \bar{T}_t < 0$, the stock of the debt is growing at the rate lower than the interest rate i_t . If a primary surplus is high enough to cover also debt servicing costs, i.e. $i_t\bar{B}_{t-1} + \bar{G}_t - \bar{T}_t < 0$, then reduction in the debt level is observed throughout time. (Naime 2004, p.60)

In notation (2), a change in the stock of the debt between periods t and $t-1$ denotes thus an accumulation or decumulation of debt and the term $\Delta\bar{B}_t$ equals the budget balance including interest payments on the debt – total public budget balance. (Bravo & Silvestre 2002, p.519) The

¹ This equation might be considered as the most general representation of the development of the stock of debt. In empirical literature and applied macroeconomics it is more usual to consider the model of $\bar{B}_t = (1 + i_t)\bar{B}_{t-1} + \bar{G}_t - \bar{T}_t \pm \bar{D}_t$ instead where the term \bar{D}_t denotes “the rest terms”. The rest term denoted by \bar{D}_t might for instance include excessive capital gain on holding governmental bonds, measurement errors (Hamilton & Flavin 1986, p.811), monetarization, i.e. seigniorage or privatization revenue and revaluations of financial assets. (Bravo & Silvestre 2002, p.519) These terms are usually included in the mathematical formulation of the debt dynamic in the empirical literature. In applied macroeconomics this term might be referred to the debt-deficit adjustment (or stock-flow adjustment). This term regularly appears in the EU fiscal policy framework when the development of the gross public debt of the general government is analyzed. See further section 6.1.1

expression in (2) is also known as the most general representation of the accounting identity of the public debt.

In the economic literature it is however more usual to consider relevant variables in real terms or as their ratios-to-GDP². In this sense, rearrangement of the nominal change in the public debt to the real one and its ratio-to-GDP is more advisable to induct the macroeconomic sense into the variables. To obtain variables in the real terms or as their proportion to GDP, all the relevant variables of the accounting identity should be divided by the price level in economy or by nominal GDP respectively. These rearrangements yield the representation of one-period dynamics of the public debt in discrete time in the real terms:

$$\Delta B_t \equiv B_t - B_{t-1} = r_t B_{t-1} + G_t - T_t \quad (3)$$

identifying real interest rate paid to the debt holders as r_t (Bravo & Silvestre 2002, p.519) as well the dynamic of the debt-to-GDP ratio³ where the term $r_t - h_t$ corresponds to the difference between the real return rate on the public debt and the growth rate of the real GDP:

$$\Delta b_t \equiv b_t - b_{t-1} = (r_t - h_t) b_{t-1} + g_t - t_t \quad (4)$$

The notation in (4) suggests that not only development of the primary balance-to-GDP ratio and level of the interest rate, but also the growth rate of the economy, should be considered in the dynamics of the debt-to-GDP ratio.

In a fast growing economy with a low interest rate on the debt service, debt-to-GDP ratio might decumulate. In a very general case of the zero primary balance, i.e. $g_t - t_t = 0$ and high growth rate in the economy, i.e. $r_t < h_t$, the stock of the debt accumulates at the constant rate equal to the real interest rate on the debt service, but since at the same time GDP grows at a higher rate, it leads to the total effect of the decumulation of the debt-to-GDP ratio. In the same sense high interest rates on the debt service combined with low GDP growth rates might lead to the accumulation of the debt-to-GDP ratio.

This might also be expressed in a way that if $r_t < h_t$ then an economy could still run a primary deficits without accumulating debt-to-GDP ratio while if $r_t > h_t$ then primary surplus might be not

² For example, in the EU fiscal policy framework, one of the measures defining the long-run sustainability in public finance is the debt-to-GDP ratio which under usual circumstances should not exceed 60 percent. In emerging market economies the optimal debt-to-GDP ratio might considered not to exceed 25% percent. (Makin 2005, p. 290)

³ See appendix A.1 where the initial model in nominal terms is rearranged to the ones in real terms as well as the ratios to GDP. The other representations such as relevant series adjusted to population might also be found in the empirical literature, see for example Hakkio & Rush (1991)

enough for decumulation of the debt-to-GDP ratio. Equivalently, running primary budgetary surpluses is necessary, for the case where $r_t > h_t$, in order to keep the debt-to-GDP ratio constant whereas for the case where $r_t < h_t$, permanent primary deficit is feasible to be able to keep the debt ratio on the constant level. (Makin 2005, p.288)

3.2. Intertemporal budget constraint model in real terms

One-period public budget identity might be extended further to see what conditions the economy should fulfil to satisfy its life-time budget constraint. Therefore assuming that accounting identity holds each period, it should also be incorporated into the model. Expanding the procedure to the infinite future would yield the general representation of the intertemporal budget constraint in present value terms.

Mathematical derivation of intertemporal budget constraint model in real terms starts by expanding the terms in the accounting identity (3) to the infinite future, yielding the equation of:

$$B_t = \rho^n \left(\sum_{n=1}^{\infty} T_{t+n} - \sum_{n=1}^{\infty} G_{t+n} \right) + \rho^n B_{t+n} \quad (5)$$

where $\rho^n = \frac{1}{(1+r)^n}$ is a discount factor⁴.

Equation (5) suggests that the initial stock of public debt B_t should be equal to the sum of the discounted primary future surpluses $\rho^n \left(\sum_{n=1}^{\infty} T_{t+n} - \sum_{n=1}^{\infty} G_{t+n} \right)$ and the present value of the future stock of the debt $\rho^n B_{t+n}$. However, the latter term should fulfil some specific properties. The last term in the right-hand side of equation (5) thus should converge towards zero throughout time, i.e. when $n \rightarrow \infty$

$$\lim_{n \rightarrow \infty} \rho^n B_{t+n} = 0 \quad (6)$$

The restriction in (6) is also known as the transversality or the No-Ponzi-Game condition. (Hamilton & Flavin 1986, p. 811)

⁴ See Appendix A.2 for the derivation of this model under assumption of the constant and positive real interest rate $r_{t+n} = r > 0$. In addition the other assumptions regarding the real interest development are considered also in Appendix A.2 with their implications on the outlook of the equation (5).

3.2.1. No-Ponzi-Game condition

Thus, imposing additional condition for the last term in equation (5) and mathematically restricting future debt development as in (6), the following economic intuition behind this constraint might be explained as follows.

A Ponzi-Game refers to a position when an economy borrows funds continuously by issuing a new debt. In this way the economy is rolling over it indefinitely without eventually retiring it. It happens when an economy is spending more than it is earning and public spending thus permanently exceeds tax revenue. (Romer 2006, p.52)

Such an economy therefore rolls its debt over since it borrows continuously to finance public deficits and it is borrowing more funds than it is eventually capable to repay in the future. This would be a violation of the initially established condition associated with the behaviour of the debt holders – these would not be willing to finance such an economy. Investors in the governmental securities expect the total reimbursement of their funds invested causing entire debt to decumulate in the infinite future. This is why the restriction in the last term in the right-hand side in equation (5), $\rho^n B_{t+n}$, should be imposed. (Hamilton & Flavin 1986, p. 811) Assuming therefore that any economy has to cover all the debt issued, the term $\rho^n B_{t+n}$ must approach zero throughout time, i.e. $\lim_{n \rightarrow \infty} \rho^n B_{t+n} = 0$. In such a case an economy totally reimburses the amounts borrowed resulting total decumulation of the stock of debt when $n \rightarrow \infty$. (Hamilton & Flavin 1986, p. 811) On the other hand a Ponzi-Game might mathematically be expressed by $\lim_{n \rightarrow \infty} \rho^n B_{t+n} > 0$, which noticeably would not be considered as appropriate by the investors to the governmental securities. (Romer 2006, p.52; Neaime 2004, p. 53-54)

3.2.2. Intertemporal budget constraint model

Equations (5) and (6) comprise the theory that economies are subjected to the intertemporal budget constraint where No-Ponzi-Game condition of $\lim_{n \rightarrow \infty} \rho^n B_{t+n} = 0$ eliminates the possibility of rolling over debt forever restricting the economy to repay totally all the debt issued. (Romer 2006, p.50)

Therefore assuming that equality in (6) holds, equation in (5) might be reduced to

$$B_t = \rho^n \left(\sum_{n=1}^{\infty} T_{t+n} - \sum_{n=1}^{\infty} G_{t+n} \right) \quad (7)$$

suggesting that an economy is satisfying its intertemporal budget constraint if discounted future debt converges towards zero which is equivalent the implication that the outstanding debt is equal

to the sum of future primary surpluses in the present value terms. If one of these properties hold, then the other also should hold. (Hakkio & Rush 1991a, p. 431; Neaime 2004, p. 54)

For the term B_{t+j} in equation (6) to converge towards zero a necessary condition is that the denominator of the discount factor is lower than the growth rate of the term B_{t+j} . Economic interpretation of this implication is that the real the debt should on average grow no faster than the level of the real interest rate. Although the debt is continually growing, but the growth rate of it on average is lower that the level of the real interest rate to finance the outstanding debt service, the convergence of the future discounted debt towards zero is observed throughout time. (Hamilton & Flavin 1986, p. 811)

3.3. Intertemporal budget constraint model for variables as their ratios to GDP

Thus, when the case for the real economy is considered, the intertemporal budget constraint suggests that outstanding stock of debt should be equal to the discounted future primary surpluses, implying that the real debt on average should grow no faster than the level of the real interest rate. (Hamilton & Flavin 1986, p. 811) When the intertemporal budget constraint is considered for the growing economy with interest rate exceeding the GDP growth rate, a constant debt-to-GDP ratio over time is in line with the definition of the sustainable fiscal policy. (Bohn 1995, p. 263)

For the variables as their ratios to GDP, the intertemporal budget constraint might be derived by firstly establishing expression

$$b_t = \rho^n \left(\sum_{n=1}^{\infty} t_{t+n} - \sum_{n=1}^{\infty} g_{t+n} \right) + \rho^n b_{t+n} \quad (8)$$

where $\rho^n = \frac{(1+h)^n}{(1+r)^n}$ is a discount factor⁵.

In the same way as for the real economies, the No-Ponzi-Game condition should be imposed for the growing ones suggesting that the future discounted debt-to-GDP ratio should converge towards zero at the limit. For this to happen, the constant debt-to-GDP ratio over time is consistent with the No-Ponzi-Game restriction. It might be expressed by equation

$$\lim_{n \rightarrow \infty} \rho^n b_{t+n} = c_1 \lim_{n \rightarrow \infty} \rho^n = 0 \quad (9)$$

⁵ See Appendix A.3 for the derivation of this model under assumption of the constant interest rate and GDP growth rate. The general representation of the model is also considered in Appendix A.3.

suggesting that for the constant term c_1 , which denotes a constant fraction of debt-to-GDP ratio over time, the debt-to-GDP ratio converges towards zero when $n \rightarrow \infty$, whenever $r > h$. (Giammarioli et al. 2007, p.5-7, Greiner, Koeller & Semmler 2006, p. 130)

The intertemporal budget constraint model might be thus established by reducing equation (8) to

$$b_t = \rho^n \left(\sum_{n=1}^{\infty} t_{t+n} - \sum_{n=1}^{\infty} g_{t+n} \right) \quad (10)$$

limiting sound fiscal policy-making to the sum of the discounted future primary surpluses to GDP being equal to the outstanding debt-to-GDP ratio. (Giammarioli et al. 2007, p.7-8)

3.4. Additional implications

The intertemporal budget constraint approach imposes a No-Ponzi-Game restriction which should hold for dynamically efficient economies. This restriction states that for the real and growing economies the discounted future debt should converge towards zero at the limit. For real economies this term approaches zero throughout time if the stock of the debt grows at a lower rate than the real interest rate to finance the outstanding debt. For growing economies this term approaches zero throughout time if the debt-to-GDP ratio is maintained constant over time.

However, it is not obvious that the No-Ponzi-Game condition should be satisfied for every economy. This condition is rather a general requirement for sustainability. Under circumstances when GDP growth rate exceeds the interest rate it is still possible to run a Ponzi-scheme. (Romer 2006, p.564)

Thus for the case when GDP growth rate exceeds interest rate on the debt service, i.e. $r < h$, the following aspects should be considered. Restricting the growing economy to keep the constant debt-to-GDP ratio, the case of $b_t = b_{t+n}$, and normalizing the discount factor to $\rho_t = 1$, equation (8) might be rewritten to

$$(1 - \rho^n) b_t = \rho^n \left(\sum_{n=1}^{\infty} t_{t+n} - \sum_{n=1}^{\infty} g_{t+n} \right) \quad (11)$$

suggesting that for the GDP growth rate exceeding the interest rate, the left hand-side of the notation in (11) is less than zero. This implies that, for the equality in (11) to hold, the government might permanently run primary deficits still keeping the debt-to-GDP ratio constant over time. The only one restriction in this case arises only for the magnitude of the primary deficit-to-GDP ratio.

(Giammarioli et al. 2007, p.8) It is however argued that under such circumstances when primary deficit is persistently accumulated, the government is playing a Ponzi-Game and the intertemporal budget constraint in a strict sense is violated. Such economies are considered being dynamically inefficient in a view of the fact that they are playing a Ponzi-Game. (Greiner, Koeller & Semmler 2006, p. 130) Theoretically such policies thus imply violation of the intertemporal budget constraint, although feasibility for such a policy-making on the permanent basis arises only in very rare cases. (Romer 2006, p.564)

4. Empirical literature overview

Empirical literature has, on the background of the mathematical model, developed an econometrical framework to test whether real economies satisfy the No-Ponzi-Game thus recognizing or rejecting the long-run sustainability in the public finance. Assessment of whether an economy is satisfying its intertemporal budget constraint focuses on evaluating of what properties the fiscal time series should exhibit to make the discounted future debt to approach zero. Testing procedure usually employs univariate econometrical techniques afterwards focusing on the interrelationship between the development of the revenue and expenditure items. Also the required features on the public deficit or debt time series are defined which are consistent with the sustainable fiscal outlook. Empirical literature also shows that the assumptions regarding the real interest rate development are crucial for establishing empirically tested models.

Recent fiscal policy literature, on the background of previously developed methodologies, expands further empirically tested models to conclude whether sustainability in the public finance is maintained or not. New approaches which have lately become more important in the empirical literature, however criticize commonly used univariate test techniques and develop complementary approaches to test for the solvency in the public finance. These methodologies in general focus on growing economies and suggest that fiscal policy reaction functions should be considered for concluding whether sustainability of fiscal policy is maintained.

4.1. Empirical models for real economies

Fiscal policy literature on the topic of the intertemporal budget constraint for real economies suggests that the sum of the discounted future primary budget balances should be equal to the outstanding stock of debt. (Hamilton & Flavin 1986, p.811) Therefore econometrical analysis on the fiscal sustainability issues starts investigating the development of the primary budget deficit and public debt time series. Later studies derive additional approaches focusing on the total budget deficit series instead; while in the latest empirical research both cases are widely considered. A review of the most relevant empirical insights is outlined below.

4.1.1. Hamilton & Flavin (1986)

One of the first empirically tested models for the real economy was proposed by [Hamilton & Flavin \(1986\)](#) whose aim was to examine whether the intertemporal budget constraint restrictions are fulfilled. The authors firstly incorporate inconsistency in mathematical No-Ponzi-Game restriction as summarised in (6), assuming that discounted future primary expenditure is expected to be somewhat higher than revenue and thus a part of the public debt is expected never be repaid. It implies that $E_t \left(\lim_{n \rightarrow \infty} \rho^n B_{t+n} \right) = A_0 > 0$ and an additional positive term in equation (7) appears so

that $B_t = \rho^n E_t \left(\sum_{n=1}^{\infty} T_{t+n} - \sum_{n=1}^{\infty} G_{t+n} \right) + A_0$. The term A_0 in this equation expresses the amount of

expenditure which is going to be financed entirely by the debt holders, not by the state, violating the No-Ponzi-Game restriction. Constructing linear regression of

$B_t = A_0 + \rho^n \left(\sum_{n=1}^{\infty} T_{t+n} - \sum_{n=1}^{\infty} G_{t+n} \right) + \varepsilon_t$ where ε_t is an error term, allows for empirical testing of the

hypothesis whether the debt holders might expect the economy to satisfy its intertemporal budget constraint.

In general, for the null hypothesis of a constant $A_0 = 0$, a balanced primary budget throughout economy's life time is observed. This in turn is assumed to be true if and only if both primary balance $\rho^n \left(\sum_{n=1}^{\infty} T_{t+n} - \sum_{n=1}^{\infty} G_{t+n} \right)$ and debt series B_t follows a stationary stochastic process.

Technically intertemporal budget constraint is thus satisfied if $\rho^n \left(\sum_{n=1}^{\infty} T_{t+n} - \sum_{n=1}^{\infty} G_{t+n} \right) \sim I(0)$ and

$B_t \sim I(0)$ implying that $E_t \left(\lim_{n \rightarrow \infty} \rho^n B_{t+n} \right) = A_0 = 0$. The main intuition behind this test is that

stationary primary balance series imply fluctuations around some mean in the long-run. For $A_0 = 0$, stationary primary balance fluctuates around a zero mean leading to the balanced primary budget in the long-run and stationary public debt time series in accordance to the established empirically tested model. For any other outcomes, the alternative hypothesis of $A_0 > 0$ is valid and fulfilment

of the country's intertemporal budget constraint is violated. Non-stationary primary budget deficit series or/and non-stationary public debt time series implies the existence of some positive term of $A_0 > 0$ and since $E_t \left(\lim_{n \rightarrow \infty} \rho^n B_{t+n} \right) = A_0 > 0$ the discounted future debt does not converge towards

zero at the limit. ([Hamilton & Flavin 1986, p.815](#))

After deriving the empirically tested model, [Hamilton & Flavin \(1986\)](#) conduct unit root tests and test for the stationarity in both – debt time series B_t and primary deficit time series $T_t - G_t$ in real

terms on the U.S. annual data. As developed in the model, both series are discounted by the discount factor $\rho^n = \frac{1}{(1+r)^n}$ where r is assumed being a constant ex post real interest rate⁶ obtained by averaging real rate for the period of estimation. (Hamilton & Flavin 1986, p.815-816) In general, rejection of the unit root in the tests for the relevant time series allows authors to conclude sustainability in the U.S. public finance for the chosen sample. (Hamilton & Flavin 1986, p.818)

4.1.2. Trehan & Walsh (1988)

Trehan & Walsh (1988) continue the work of Hamilton & Flavin (1986) suggesting that the total deficit instead of the primary budget balance should be of economic interest in order to conclude whether governments conduct their fiscal policies in a way that the discounted future debt approaches zero through the sample period.

Deriving econometrical implications which satisfy the No-Ponzi Game restriction, properties on the debt development are evaluated further. Derivations show that consistency with the intertemporal budget constraint might be achieved if stationarity in the first difference of the public debt might be proved so that $\Delta B_t \sim I(0)$, or, expressed in the levels, public debt time series should be integrated of the first order $B_t \sim I(1)$. This implication is derived under assumption of the constant and positive real interest rate $r_{t+n} = r > 0$. From the accounting identity in (3) it might eventually be seen that the total deficit series should follow a stationary stochastic process, i.e. $(rB_{t-1} + G_t - T_t) \sim I(0)$ to satisfy intertemporal budget constraint. Alternatively the authors suggest that a stationary stochastic process in the total deficit time series might be verified by applying co-integration techniques. Assuming that total expenditure and revenue are stationary in their first differences, i.e. $(rB_{t-1} + G_t) \sim I(1)$ and $T_t \sim I(1)$, stationarity in total deficit might be proved by seeking co-integration relationship between these series with the co-integration vector of $(1, -1)$. (Trehan & Walsh 1988, p.432) This approach with additional insights has been further explored by Hakkio & Rush (1991a). The work of Trehan & Walsh (1988) also points towards additional conclusion that stationarity in the primary balance which has been considered by Hamilton & Flavin (1986) is not sufficient to conclude whether fiscal policy is conducted in a proper way. (Trehan & Walsh 1988, p.432)

4.1.3. Wilcox (1989)

Wilcox (1989) develops further the work of Hamilton & Flavin (1986) and raises a question of what might happen to the development of the real debt time series B_t if the term A_o follows a

⁶ Technically $r_{t+n} = r > 0$, i.e. the real interest rate is assumed to be constant and positive.

stochastic process instead. [Wilcox \(1989\)](#) questions stationarity restrictions imposed by [Hamilton & Flavin \(1986\)](#) on the public debt time series as a necessary condition for the intertemporal budget constraint to be satisfied.

In accordance to [Wilcox \(1989\)](#), the method proposed by [Hamilton & Flavin \(1986\)](#) does not allow for the deviations of debt from the discounted future primary surpluses any period for stationary public debt time series $B_t \sim I(0)$ to be valid. The author suggests that it is possible for the public debt time series to follow a non-stationary stochastic process still satisfying intertemporal budget constraint. Non-stationary debt series might just arise out of the fact that deviations of the public debt from the future surpluses persists longer periods. Thus, unit root in B_t might be still consistent with government behaviour in terms of the proper fiscal policy-making. ([Wilcox 1989, p.295-296](#))

4.1.4. *Hakkio & Rush (1991)*

[Hakkio & Rush \(1991a\)](#) extend on the other hand the work of [Trehan & Walsh \(1988\)](#) which was based on the approach which seeks the co-integration relationship between public revenue and spending. [Hakkio & Rush \(1991a\)](#) derive a model under the assumption that the real interest rate follows any stationary stochastic process instead⁷. It is also suggested that the same model is applicable when the relevant time series are analyzed as their ratios to GDP, but not when nominal data set is considered. ([Hakkio & Rush 1991a, p.435](#)) Derivation of empirically tested model is based on the co-integration techniques with some additional implications.

The authors firstly make some mathematical rearrangements in (7) and derive the model testable within the co-integration framework. Some mathematical rearrangements yields the equation

$$\Delta B_t \equiv G_t + r_t B_{t-1} - T_t = \rho^n E_t \left(\sum_{n=1}^{\infty} \Delta T_{t+n} - \sum_{n=1}^{\infty} \Delta S_{t+n} \right) + E_t \left(\lim_{n \rightarrow \infty} \rho^n \Delta B_{t+n} \right) \quad (12)$$

where $\rho^n = \frac{1}{(1+r)^n}$ is a discount factor and $S_t = G_t + (r_t - r)B_{t-1}$ is defined as total spending

with interest rate following any stationary stochastic process⁸. Equation (12) also suggests that total deficit time series should be under consideration. ([Hakkio & Rush 1991a, p.432](#))

Further the authors assume that the series T_t and S_t are non-stationary with a drift, i.e. both are random walks and both have a non-zero mean. This might be expressed by providing the expected

⁷ Instead of $r_{t+n} = r > 0$ as in [Trehan & Walsh \(1988\)](#), it is assumed that interest rate follows a stochastic stationary process around the mean of r , i.e. $r_{t+n} \sim (r, \sigma^2)$, $r > 0$

⁸ Equation (12) is obtained using the difference operator on equation (7) under assumption that the interest rate follows a stochastic stationary process. Since the same model was used by [Quintos \(1995\)](#) which is consider in later section, see appendix A.3. for extended version of this equation.

development of them in the univariate equation form of $T_{t+n} = \alpha_1 + \phi_1 T_{t+n-1} + \varepsilon_{t,1}$ and $S_{t+n} = \alpha_2 + \phi_2 S_{t+n-1} + \varepsilon_{t,2}$. The first differences ΔT_{t+n} and ΔS_{t+n} are assumed to be stationary, i.e. $T_{t+n} \sim I(1)$ and $S_{t+n} \sim I(1)$, so that $\Delta T_{t+n} \equiv T_{t+n} - T_{t+n-1} = \alpha_1 + \varepsilon_{t,1}$ and $\Delta S_{t+n} \equiv S_{t+n} - S_{t+n-1} = \alpha_2 + \varepsilon_{t,2}$ as $\phi_1 = \phi_2 = 1$. Under these processes the equation in (12) might be rewritten to $G_t + r_t B_{t-1} = \alpha + T_t + \rho^n \lim_{n \rightarrow \infty} \Delta B_{t+n} + \varepsilon_t$ where the constant is equal to $\alpha \equiv \sum_{n=1}^{\infty} \rho^n (\alpha_1 - \alpha_2)$ and the error term $\varepsilon_t \equiv \sum_{n=1}^{\infty} \rho^n (\varepsilon_{t,1} - \varepsilon_{t,2})$. Since No-Ponzi-Game restriction is assumed to hold, the final regression to test for sustainability is obtained by setting $E_t \left(\lim_{n \rightarrow \infty} \rho^n \Delta B_{t+n} \right) = 0$ and takes form of

$$T_t = \alpha + \beta(G_t + r_t B_{t-1}) + \varepsilon_t \quad (13)$$

which is a conventional equation of the Engle-Granger methodology to test for co-integration relationship between the variables. The economic intuition behind this type of methodology is that even if public revenue and public spending are random walk series, but these series follow each other in some pattern in the long-run, then it is enough to conclude that No-Ponzi-condition is satisfied. It is because the government reacts to the increase in the public debt by adjusting budget revenue and/or cutting public spending in the upcoming periods. (Hakkio & Rush 1991a, p.432-433)

In general evaluation procedure in Hakkio & Rush (1991a) start by applying Augmented Dickey-Fuller type tests on the variables, testing first for the null hypothesis of $\phi_1 = 1$ and $\phi_2 = 1$ in regressions $T_t = \alpha_1 + \phi_1 T_{t-1} + \varepsilon_{t,1}$ and $(G_t + r_t B_{t-1}) = \alpha_2 + \phi_2 (G_{t-1} + r_{t-1} B_{t-2}) + \varepsilon_{t,2}$ respectively. Non-rejection of the nulls allows to evaluate further the properties of the error term ε_t in the regression (13) seeking for $\varepsilon_t \sim I(0)$, i.e. co-integration between $(G_t + r_t B_{t-1})$ and T_t . For the valid null hypotheses of $\delta = 1$ in $\varepsilon_t = \delta \varepsilon_{t-1} + \eta_t$ here $\eta_t \sim NIID(0, \sigma^2)$, the unit root in the error term ε_t also implies no co-integration between total spending and revenue.

Further Hakkio & Rush (1991a) impose some restrictions on the regressor $\hat{\beta}$ in the equation (13) suggesting that the co-integration vector between those time series should have some specific properties. With $\hat{\beta} = 1$ implying that the vector of $(1, -1)$ is obtained, economic intuition allows considering the situation where the government conducts its fiscal policy in a way that budget revenue growth in the long-run is exactly follows the past development in total spending. Thus, having $T_t \sim I(1)$ and $(G_t + r_t B_{t-1}) \sim I(1)$ co-integrated with the co-integration vector of $(1, -1)$

implies from equation (13) that $T_t - \alpha - (G_t + r_t B_{t-1}) = \varepsilon_t \sim I(0)$ and thus $\Delta B_t = r_t B_{t-1} + G_t - T_t = \alpha + \varepsilon_t \sim I(0)$ meaning stationary total budget deficit and stationarity in the change of the public debt time series. (Hakkio & Rush 1991a, p.433-434)

However, mathematically it is derived that even if total expenditure and revenue are co-integrated and the estimated regressor $\hat{\beta}$ lies within the interval of $0 < \hat{\beta} < 1$ the No-Ponzi-Game condition of $E_t \left(\lim_{n \rightarrow \infty} \rho^n \Delta B_{t+n} \right) = 0$ is still satisfied. (Hakkio & Rush 1991a, p.433-434) Economically this corresponds to the situation when the government reacts to the increase in the public debt to some extent by adjusting the revenue growth to the total spending development in the long-run, but this correction does not equally and completely correspond to the growth in the public expenditure. In this case growing budget deficit and non-stationary change in the public debt time series are observed. Hence, obtaining $T_t \sim I(1)$, $(G_t + r_t B_{t-1}) \sim I(1)$, $\varepsilon_t \sim I(0)$ and some co-integration vector of $(1, -\beta)$ still implies that $E_t \left(\lim_{n \rightarrow \infty} \rho^n \Delta B_{t+n} \right) = 0$. Thus, these derivations suggest that the restriction on the estimated regressor being equal to $\hat{\beta} = 1$ is doubted as being necessary for the No-Ponzi-Game restriction to hold. It is nevertheless suggested that even if the condition of $0 < \hat{\beta} < 1$ mathematically complies with the No-Ponzi-Game restriction of $E_t \left(\rho^n \lim_{n \rightarrow \infty} \Delta B_{t+n} \right) = 0$, it is highly questioned and rejected as being economically appropriate within the intertemporal budget constraint approach. (Hakkio & Rush 1991a, p.433-434) These findings later were developed by Quintos (1995).

4.1.5. Trehan & Walsh (1991)

Trehan & Walsh (1991) explore further intertemporal budget constraint approach primarily developed by Hamilton & Flavin (1986) and establish empirically tested implications to examine whether No-Ponzi-Game condition holds. Within the public finance framework again a primary budget balance is considered as the main indicator of the sustainability in the public finance. The authors derive the implication that sufficient and necessary conditions for intertemporal budget constraint to be satisfied are co-integration relationship between primary deficit and debt time series and stationarity in the quasi-difference of the primary deficit⁹. This implies that if the real interest rate follows a process of $E_t(r_{t+n}) = r > 0$, primary budget balance and debt are both non-stationary but a co-integration relationship between B_{t-1} and $(G_t - T_t)$ exists so that $(G_t - T_t) - \beta B_{t-1} = \varepsilon_t \sim I(0)$ and $(G_t - T_t) - \lambda(G_{t-1} - T_{t-1}) \sim I(0)$ with zero mean is valid for some $0 \leq \lambda < (1+r)$, implying that the primary deficit grow at the lower rate than the real interest rate,

⁹ Technically it is assumed that expected interest rate is positive and constant, i.e. $E_t(r_{t+n}) = r > 0$

then $E_t \left(\lim_{n \rightarrow \infty} \rho^n B_{t+n} \right) = 0$. Alternatively the intertemporal budget constraint is satisfied also if both the debt and the primary deficit time series are stationary, which is the case when $B_t \sim I(0)$ and $(G_t - T_t) \sim I(0)$. (Trehan & Walsh 1991, p.207-211)

4.1.6. Quintos (1995)

Quintos (1995) introduces the concept of “strong” and “weak” sustainability. Accordingly “strong” sustainability corresponds to the Hamilton & Flavin’s (1986) findings suggesting that the debt time series should be stationary in their levels, i.e. $B_t \sim I(0)$, to confirm sustainability of the fiscal policy. “Strong” sustainability also corresponds to the Hakkio & Rush (1991a) findings referring to the first order integration of the total expenditure and revenue items and also co-integration between those time series with the co-integration vector of $(1, -1)$, leading to stationarity in the total deficit and thus in the first difference of the public debt time series.

“Weak” sustainability on the other hand does not require co-integration with vector of $(1, -1)$ in a strict sense between revenue and total expenditure, it is enough that these series are stationary in their first differences and co-integrated with some vector of $(1, -\beta)$ so that $T_t - \alpha - \beta(G_t + r_t B_{t-1}) = \varepsilon_t \sim I(0)$ in the equation (13)¹⁰. The restriction on $\hat{\beta}$ is that it lies within the interval of $0 < \hat{\beta} < 1$. This condition leads to non-stationarity in levels but stationarity in first difference of the total deficit time series. The debt series are then integrated of the second order, which is the case when $B_t \sim I(2)$ since $\Delta B_t = r_t B_{t-1} + G_t - T_t = (1 - \hat{\beta})(G_t + r_t B_{t-1}) + \alpha + \varepsilon_t \sim I(1)$. The conclusion that $B_t \sim I(2)$ supports the “weak” form of sustainability in the public finance is derived from the implication that the term $E_t \left(\lim_{n \rightarrow \infty} \rho^n \Delta B_{t+n} \right) = 0$ when restriction on $0 < \hat{\beta} < 1$ is imposed. In Hakkio & Rush (1991a) the existence of $0 < \hat{\beta} < 1$ was relaxed as being economically inappropriate as a gap between revenue collection and total expenditure is a permanent feature in the economy. (Quintos 1995, p.411)

The intuition behind the introduction of a “weak” sustainability approach is that existence of a co-integration relationship between revenue and expenditure still might allow public debt to grow slower than the level of the real interest rate is which is consistent with a conventional intertemporal budget constraint approach for real economies. “Weak” sustainability therefore refers to behaviour of a debt time series which allows the discounted future debt as defined in the No-

¹⁰ Under assumption that interest rate follows a stochastic stationary process around the mean of r , i.e. $r_{t+n} \sim (r, \sigma^2)$, $r > 0$

Ponzi game restriction to converge towards zero slower than under “strong” sustainability condition. However, in this case the deficit process is somewhat explosive implying that economy is permanently running total deficit. Whereas this policy is feasible and refers to the sustainable fiscal policy, such deficit management might have serious economic implications in the long-run with an increased risk for debt default. (Quintos 1995, p.410-411)

4.2. Empirical models for the growing economies

On the background of these univariate time series approaches, fiscal policy literature develops further alternative methods for sustainability testing. The analysis of the soundness of the fiscal policy for the growing economies in general investigates whether the governments are reacting to the increase in the debt-to-GDP ratio by adjusting primary balance the same periods and other periods onwards. (Bohn 1998, p.950-951) These new approaches are developed on the earlier established empirically tested models, and in general, question the correctness of them in concluding whether sustainability in public finance is maintained or not. The summary of the relevant empirical insights is therefore outlined in the sections below.

4.2.1. Bohn (1998)

Bohn (1998) explores somewhat different methodological approach to define whether public finance is sustainable or not. Firstly, the commonly accepted econometric tests based on the univariate time series methodologies, which are usually based on the stationarity tests in the public debt time series are questioned. It is suggested that these tests might not be the most appropriate for determining soundness of the fiscal policy. It is due to the properties of the real debt or debt-to-GDP the time series which even decreasing might lead to the non-rejection of the hypothesis of the unit root in those thus initially pointing towards unsustainable public finance. (Bohn 1998, p.949-950)

Difficulties in rejecting a unit root in samples finds its origin in the properties of debt-to-GDP time series as those encompass much more additional information than only dynamics of the public budget balance. Thus, debt-to-GDP ratio might increase or decrease not only as a result of the fiscal policy design, but also due to the additional factors that influence debt dynamics. For instance variations in the interest rate or the GDP growth rate, to a large extent, are outside government control, but still influence debt development. These additional factors normally influence the dynamics of the public debt time series while the definition of sustainable public finance however should account only for the fiscal policy design. Therefore even normal cyclical fluctuations in the budget balance series combined with interest or GDP shocks might make debt time series difficult to mean-revert and the rejection of the non-stationarity in the tested samples becomes difficult. (Bohn 1998, p.949-950)

An alternative approach is therefore developed to test for the solvency in the public finance. Instead, it is suggested to decompose debt time series and focus only the systematic primary budget balance response to the dynamic of the public debt. This in turn shows only fiscal policy design – if any government is reacting to the increase or decrease in the public debt by adjusting primary budget balance this and next periods onwards. (Bohn 1998, p.950-951)

The general regression proposed to test for sustainability takes form of

$$(t_t - g_t) = \beta b_t + AZ_t + \eta_t \quad (14)$$

where Z_t is a vector of other components than debt that might affect the development of the primary surplus and $\eta_t \sim NIID(0, \sigma^2)$. (Bohn 1998, p.951) These components might for example include cyclical component of the output and temporary expenditure measures as in Bohn (1998) or cyclical component of the output or expenditure as in Haber & Neck (2006). Inclusion of other known country-specific variables might be found in other empirical works and in some cases replacement of b_t with its lagged value b_{t-1} might be necessary since some governments plan budget in advance as considered in Greiner, Koeller & Semmler (2007).

Two additional implications regarding the properties of the relevant variables in this regression (14) should be considered. Rejecting a unit root in the primary surplus and the debt time series, i.e. finding that $(t_t - g_t) \sim I(0)$ and $b_t \sim I(0)$ calls for inclusion of the additional variables in the regression (14), i.e. modelling explicitly Z_t , otherwise the model would suffer from the omitted variables. The regressor $\hat{\beta}$ should be positive and significant indicating that the government is increasing the primary balance ratio as a response to the increase in the debt-to-GDP ratio in order to maintain a sound fiscal stance. Finding these two series non-stationary, both integrated of order one and co-integrated, i.e. $(t_t - g_t) \sim I(1)$, $b_t \sim I(1)$ and $\varepsilon_t \sim I(0)$ there is no need to model Z_t explicitly as the joint process of $AZ_t + \eta_t = \varepsilon_t \sim I(0)$ (Bohn 1998, p. 951) Co-integration vector properties should however be restricted to $(1, -\beta)$ pointing towards a positive primary surplus reaction to the increase in the public debt.

This alternative approach is considered to capture only the reaction of the fiscal institutions to the increase or decrease in the public debt through fiscal policy design by modelling the primary budget balance and all the other factors affecting dynamics of the public debt series separately. Limiting the analysis of the dynamic of the debt only on the fiscal policy-making is in turn is consistent with the definition of the sustainability in the public finance. (Bohn 1998, p.950)

Summarising all the aspects considered in [Bohn \(1998\)](#) together, it is suggested that soundness of the fiscal policy should not be limited to evaluating stationarity of the debt-to-GDP ratio. Fiscal policy reaction functions should be established for assessing sustainability issues. These should focus on the investigation on whether the governments are reacting to the fluctuations in the debt-to-GDP ratio by adjusting primary balances in the onwards periods. The main reason to this extension is thus that univariate time series approaches might in some cases be misleading in sustainability assessment. The concrete examples of this would be decreasing non-stationary debt-to-GDP ratio which would point to the unsustainable public finance as considered in [Bohn \(1998\)](#) or alternatively trend stationary increasing debt-to-GDP ratio which would initially point towards the sound fiscal policy-making.

4.2.2. *Bohn (2007)*

[Bohn \(2007\)](#) summarises all the previous research techniques developed by for example [Hamilton & Flavin \(1986\)](#), [Trehan & Walsh \(1988\)](#), [Quintos \(1995\)](#) that seeks ways to test whether the No-Ponzi game restriction is satisfied. These techniques in general derive empirically tested models which are based on the tests of stationarity in the levels or differences of the public debt time series under certain assumptions about the interest rate development. Alternatively these techniques test if total expenditure and revenue time series are properly co-integrated. These implications are essential to prove that the discounted future debt approaches zero when $n \rightarrow \infty$ in No-Ponzi-Game restriction in notation (6) and suggests therefore that a certain order of integration of the relevant time series leads to $E_t \left(\lim_{n \rightarrow \infty} \rho^n B_{t+n} \right) = 0$. In [Hamilton & Flavin \(1986\)](#) it was shown that zero order of integration in the debt time series leads to sustainable public finance whereas in [Trehan & Walsh \(1988\)](#) the first order of integration in those was considered for the same conclusion. In [Quintos \(1995\)](#) on the other hand the concept of “weak” sustainability in the public finance was introduced suggesting that second order of integration in the debt time series is also consistent with fulfilment of the intertemporal budget constraint. ([Bohn 2007, p.1838-1841](#))

[Bohn \(2007\)](#) proves however that for any finite order of integration of the debt time series, the discounted future debt approaches zero when $n \rightarrow \infty$ and thus the No-Ponzi game restriction is satisfied independently of whether it is low- or high-order of integration in the debt time series. Technically, it is proved that if $B_t \sim I(m)$ for any finite $m \geq 0$, then $E_t \left(\lim_{n \rightarrow \infty} \rho^n B_{t+n} \right) = 0$. ([Bohn 2007, p.1840](#)) [Bohn \(2007\)](#) also explains that in [Trehan & Walsh \(1988\)](#) as well as in [Quintos \(1995\)](#) the mathematical derivations to test for whether the No-Ponzi-Game restriction is satisfied are correct, but the possibility of the higher order of integration in the relevant variables is not considered in the proofs. Consequently [Bohn \(2007\)](#) also shows that the co-integration relationship between revenue and total expenditure time series is not a necessary condition for the No-Ponzi-Game restriction being satisfied. Thus, any finite order of integration of these series separately,

leads also to the fulfilment of the intertemporal budget constraint. Technically, it implies that if $T_t \sim I(m_T)$, $(G_t + r_t B_{t-1}) \sim I(m_G)$ for any finite $m_T \geq 0$ and $m_G \geq 0$ where possibly $m_T \neq m_G$, then $B_t \sim I(m)$ and $E_t \left(\lim_{n \rightarrow \infty} \rho^n B_{t+n} \right) = 0$. (Bohn 2007, p.1842)

All these insights might be summarised by saying that the No-Ponzi game condition where discounted future debt should approach zero, i.e. $E_t \left(\lim_{n \rightarrow \infty} \rho^n B_{t+n} \right) = 0$ will be satisfied as long as $B_t \sim I(m)$. It might be only of interest to distinguish the strength of sustainability – “strong” with $B_t \sim I(1)$, “weak” with $B_t \sim I(2)$ or even “absurdly weak” with $B_t \sim I(m)$. These findings in any case suggest that unit root and co-integration techniques are not appropriate for concluding whether sustainability in public finance is maintained or not as these would always point towards conclusion that the debt series are integrated of any finite order which implies fulfilment of the No-Ponzi game restriction. (Bohn 2007, p.1842)

Bohn (2007) therefore proposes to pay more attention to the fiscal reaction functions which might be much more appropriate for analysing sustainability of fiscal policy. (Bohn 2007, p.1846) One of these was developed in Bohn (1998) seeking for the systematic response of the primary budget balance to the dynamics of the public debt. Positive reaction of the primary surplus to the increase in the public debt ratio was referred to as a sustainable public finance. Also distinction of the order of integration in debt time series might have economic interest. Distinguishing “strong” and “weak” forms of sustainability might be of importance as it sets some bounds of how the debt dynamics is expected to look like under certain economic conditions. Order of integration of debt or deficit time series still is informative for showing how the government is succeeding managing its public finance since higher order of integration is associated with macroeconomic risks in the long-run. (Bohn 2007, p.1846)

4.3. Summary over empirical literature

With all the empirical insights presented in the previous section a short review over the propositions for concluding a sound fiscal stance might be summarised as follows:

1. Hamilton & Flavin (1986) suggest that sustainability in public finance is achieved if both primary deficit and debt time series are integrated of order zero, the case of $T_t - G_t \sim I(0)$ and $B_t \sim I(0)$.
2. Trehan & Walsh (1988) propose the model suggesting that if total deficit time series are integrated of order zero while public debt of order one, the case of $(rB_{t-1} + G_t - T_t) \sim I(0)$ and $B_t \sim I(1)$, the sustainable fiscal outlook is maintained.

3. [Hakkio & Rush \(1991a\)](#) derive the model suggesting that if revenue and total expenditure are stationary in the first differences, the case of $(rB_{t-1} + G_t) \sim I(1)$ and $T_t \sim I(1)$, and co-integrated with the co-integrating vector $(1, -1)$, then sustainability in the public finance might be concluded. It also leads to the implication that total deficit time series are stationary in levels, i.e. $(rB_{t-1} + G_t - T_t) \sim I(0)$ and debt time series are stationary in their first differences, i.e. $B_t \sim I(1)$.

4. [Quintos \(1995\)](#) on the other hand suggests that second order of integration in the public debt time series and total deficit time series integrated of order one, the case of $(rB_{t-1} + G_t - T_t) \sim I(1)$ and $B_t \sim I(2)$, are enough to conclude sustainable fiscal outlook.

5. [Bohn \(1998, 2007\)](#) proposes to pay more attention on the fiscal policy reaction functions implying that governments should react to the increase in the public debt ratio by adjusting primary budget balance next periods onwards. If the primary deficit and debt time series are integrated of order zero, the case when $T_t - G_t \sim I(0)$ and $B_t \sim I(0)$, one should seek for the reaction of the primary budget balance to the development in the public debt time series by employing OLS estimation methods. If the primary deficit and debt time series are on the other hand integrated of order one, the case when $T_t - G_t \sim I(1)$ and $B_t \sim I(1)$, the co-integrating relationship with a co-integration vector $(1, -\beta)$ would point towards soundness of the fiscal policy.

4.4. Recently applicable methods

Recent fiscal policy literature on the topic of sustainable public finance shows that combinations of all the previously defined methods are used in applied works. The method proposed by [Bohn \(1998\)](#) focusing on fiscal reaction functions with possible extensions is applicable; the methods derived on the univariate time series analysis basis proposed by [Hakkio & Rush \(1991a\)](#) and [Quintos \(1995\)](#) are also widely used. Usually these two alternatives complement each other since the methods used also depend in many cases on the availability of data, quality of the data sets and properties of the relevant time series.

Some specific examples might be considered as follows. [Haber & Neck \(2006\)](#) evaluates Austrian while [Greiner, Koeller & Semmler \(2006\)](#) German fiscal time series using only the method proposed by [Bohn \(1998\)](#). Since a unit root in both the public debt and primary surplus time series is rejected, so that $(t_t - g_t) \sim I(0)$ and $b_t \sim I(0)$, the OLS estimation techniques are employed.

[Baharumshah & Lau \(2007\)](#) for instance apply only the methods derived by [Hakkio & Rush \(1991a\)](#) and [Quintos \(1995\)](#) seeking for a co-integration relationship between revenue and total spending for some East Asian countries. The conclusions on sustainability are made by evaluating

the properties of the co-integration vector as well as evaluating order of integration of the debt time series.

In the other works such as for example [Gurbuz, Jobert & Tuncer \(2007\)](#) who investigate Turkish public debt development, the authors combine both alternatives. Univariate methods are used to investigate the order of integration of the public debt, also suggesting that stationarity in its levels leads to sustainable public finance. Co-integration between revenue and total spending is also evaluated where the properties of a co-integration vector is assumed permit conclusions of whether fiscal policy is sustainable or not. These methods correspond to [Hamilton & Flavin \(1986\)](#), [Hakkio & Rush \(1991a\)](#) and [Quintos \(1995\)](#) derived models to detect sustainability. The method proposed by [Bohn \(1998\)](#) is also considered suggesting that a proper co-integration relationship between primary budget balance and public debt time series, if these both are $(t_t - g_t) \sim I(1)$ and $b_t \sim I(1)$, points towards sustainability.

5. Macroeconomic and fiscal outlook of Lithuania

On the background of the mathematical and empirical fiscal policy literature presented above, some facts about the general macroeconomic outlook of Lithuania are further presented emphasising fiscal development of the country which are important for assessing fiscal sustainability issues in the economy.

Thus, since independence in 1990 Lithuania's economy has experienced a period of a robust economic growth and a period of economic downturn¹¹. The transitional period of early 1990's was characterized by extensive structural changes in the whole economy. The general fiscal outlook of the country was that it historically started with a zero public debt and sweeping public sector reforms took place at that time. Public budget was reorganized – new taxes were introduced, other was abolished, tax rates changed and the compositional effects on expenditure were observed. Regular discretionary fiscal policy measures with high ratios of the public deficits were the main feature of these years.

The recession of late 1990's was to a large extent caused by the general transition period and the Russian financial crisis. High inflation, external imbalances, high rates of unemployment and at times negative real GDP growth characterises these years. The period of early 2000 was one of robust economic growth, moderate inflation, enhanced external competition and improved current account balance as well as declining level of unemployment. At the same time further changes in the public finance management were observed with a steadily decreasing fiscal deficit ratio. Decline in public deficits ratio was however to some extent driven by the fact that Lithuania started

¹¹ See Appendix B with the main macroeconomic indicators of Lithuania for 1999-2007. The national currency of Lithuania is Litas which is fixed to euro at the rate of 3.4528 LTL/EUR.

the official negotiations to enter the European Union (EU) in 2000 since EU fiscal policy rules applicable also to the candidate member states require public deficit be no greater than 3 percent of GDP on the annual basis. Since 2002 this requirement has never been violated and the country has never been subjected to the excessive deficit procedure.

After 2004 when the country entered the EU, economic activity has showed exceptional growth rates. The main driving factor behind it was boost in domestic demand. Favourable external crediting conditions combined with simplified access to the EU structural support funds contributed to the robust growth in private consumption and investment. Fiscal contribution to the strong GDP growth was also considerable since relatively strong growth rates in government consumption were observed. General fiscal outlook nonetheless remained well-favoured as economic expansion contributed positively to boosting public revenue, relatively low deficit and steadily decreasing debt ratio. Permanent fiscal deficits on the annual basis however remained even under very favourable macroeconomic conditions.

Lately development of the economic activity has been worrisome – the economy was considered to show signs of overheating with increasing current account deficits and raising inflation. Such a macroeconomic outlook might also to some extent be attributable to the pro-cyclical fiscal policy-making in the last years when revenue windfalls were used to finance additional expenditure instead of reducing fiscal deficits. Since a part of cyclical revenue was used to permanently increase current public spending, cooling down of the economic activity made general fiscal outlook and development of the public deficits in the medium-run highly uncertain.

Concern about fiscal development has recently resulted in difficulties in financing public deficits and continuity of economic activity was disrupted when financial institutions refused funding public sector. Therefore past fiscal policy-making might probably be questioned even though public debt ratio has never been of high concern of the fiscal institutions before. On the other hand difficulties to finance public activities could also be associated with the uncertainties regarding future fiscal policy-making. Extensive public sector reform was therefore enforced and changes in the tax system adopted with the budget of 2009 reduced pressure and distrust of the financial institutions to the fiscal authorities. On this background, the quality of public finance of Lithuania also in the past can be questioned and the low debt ratio might not necessarily imply sustainable fiscal policy-making in the long-run. Therefore these theoretical insights call further for assessing past sustainability issues in a quantitative way, in addition focusing shortly on the future fiscal development.

6. Data and methods

In the following section, by presenting data and methods, the fiscal sustainability issues of Lithuania are further analysed to see whether past fiscal policy of the country has been problematic or not with additional considerations regarding expected future development of the public debt.

6.1. Data

6.1.1. Data properties

Time series used for this analysis includes the series of a so called “Maastricht” debt and “Maastricht” or general government deficit. Covering these series implies that one-period accounting identity for the debt-to-GDP ratio dynamic is given by $\Delta b_t \equiv b_t - b_{t-1} = (r_t - h_t)b_{t-1} + g_t - t_t \pm d_t$ and intertemporal budget constraint is summarised by the model of $b_t = \rho^n \left(\sum_{n=1}^{\infty} t_{t+n} - \sum_{n=1}^{\infty} g_{t+n} \right) \pm \rho^n \sum_{n=1}^{\infty} d_{t+n} + \rho^n b_{t+n}$ where the No-Ponzi-Game restriction should hold so that $\lim_{n \rightarrow \infty} \rho^n b_{t+n} = 0$. The additional term d_t denotes the debt-deficit adjustment.

Positive debt-deficit adjustment implies that the debt increases more than accounted by the total deficit or the debt decreases less than accounted by surplus. This term captures for instance differences in the methodology to account for general government deficit and gross debt. The main reasons for this term to arise might be timing of recording fiscal transactions. As deficit figures are calculated on accrued while public debt on cash basis meaning that the government makes the commitment to finance some expenditure postponing payments, this fact leads to the differences in recording figures. Sometimes governments might also choose to use surpluses in the other ways than repaying existing debt, such as for instance invest these extra funds to the financial assets which also appear in the debt-deficit adjustment figure with a positive sign. Besides additional debt might be issued and also re-invested to the financial assets if deficits does not need to be financed. These statistical discrepancies, different timing of recording and financial asset transactions causes accounting identity of the public debt to deviate, sometimes significantly on temporary basis, by the debt-deficit adjustment term. Even if debt-deficit adjustment term usually tends to be quite volatile, it does not on average influence the public debt ratio significantly for the EU economies. (EC 2004, p.1-7)

However this term might differ also significantly across countries, especially when a particular time series sample is considered. (EC 2004, p.1-7) In case of Lithuania, throughout the period of 1998:1 -2008:2 the debt-deficit adjustment-to-GDP ratio averages to approximately -0,17 percent of GDP through the sample period. This implies that debt-deficit adjustment is on average negligible for the

development of the public debt and therefore fiscal policy design together with GDP growth is a driving force in accumulation or decumulation of the public debt ratio.

In empirical literature, when data gathered is usually restricted only by some relevant variables, to obtain a full accounting identity, this difference is referred to the seigniorage revenue and accounted in the total revenue figure. For Lithuania, as the origin of the debt-deficit adjustment is well-known in the accounting identity, this term is considered separately, but not analyzed due to its relative average insignificance for the debt-to-GDP dynamics.

6.1.2. Sample size

The sample size covers period of 1999:1-2008:2 since data availability is restricted to this period. Quarterly data in nominal levels is taken from the Department of Statistics database. Variables available account for total revenue, total expenditure, interest paid on the debt service, total deficit and gross consolidated debt. A debt-deficit adjustment figure in the data sets is calculated as a remainder in the accounting identity. For the year 1999 where only yearly data is available for the debt time series, quarterly data is extrapolated from yearly data by assuming constant proportion of the debt-deficit adjustment each quarter since no clear seasonal pattern might be observed in this term. Primary budget balance is calculated by summing up total budget balance with interest paid. Afterwards relevant nominal variables are recalculated to their ratios to GDP. Transformation of all the variables, except public debt, to the ratios of GDP is done by dividing nominal variables accumulated as four-quarter moving sums with four-quarter moving sum of the nominal GDP time series. Since debt by itself is an accumulated process, quarterly debt time series are divided with four-quarter moving sum of the nominal GDP. Series analyzed are seasonally adjusted with Census X12 and data cut is of November 3, 2008. EViews program package is used for estimation. Precautious treatment of the results followed further is required since data sets analysed are restricted by the small sample size.

6.1.3. Defining long-run

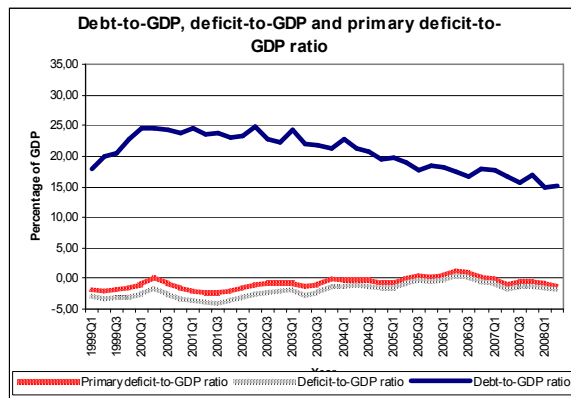
To be able to perform analysis on the small sample, it is of importance to distinguish the properties of data which are consistent with a definition of the long-run for the particular economic issue being analysed. Since intertemporal budget constraint develops the approach of the sustainability in the public finance in the long-run and fiscal data in Lithuania is restricted by ten years, an additional implication should be of consideration.

Definition of the long-run in general is addressed to each specific economic problem being analyzed. (Hakkio & Rush 1991b, p.578) Three fiscal years are the most conventional measure defining medium-run within the public finance approach. For the Lithuanian economy analysed here, the long-run is assumed to coincide with the sample size of almost ten years.

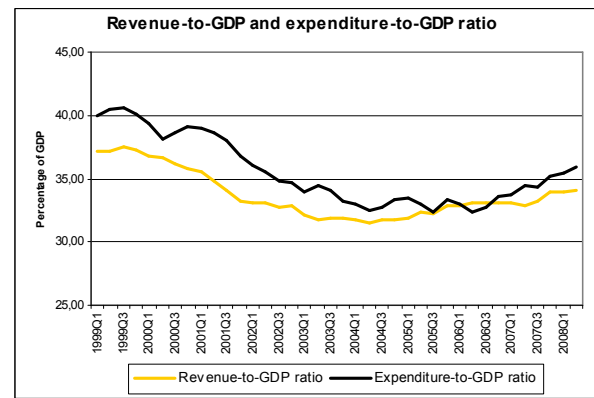
6.1.4. Initial remarks on data

Plotting further initial time series of the public debt, deficits, primary deficits, revenue and expenditure ratios in the graphs 1 and 2 for the sample in question, some initial brief insights might already be summed up. These will be of importance in the later sections when past fiscal policy issues will be quantified.

Graph 1. Debt-to-GDP, deficit-to-GDP and primary deficit-to-GDP ratio



Graph 2. Revenue-to-GDP and expenditure-to-GDP ratio



Accordingly the development of the debt ratio, which showed increasing pattern and afterwards started to decline, indicates that in early 2000's the change in the time trend occurred. Deficit time series on the other hand suggests that economy stayed in permanent deficits although in decreasing pattern up to 2006, several quarters in 2006 budgetary surpluses were observed on the four-quarter accumulated basis and increasingly in-deficits fiscal position since 2007 was recorded again. This is also noticeable when revenue and expenditure time series are considered where decreasing deviations between revenue and expenditure might be observed up to 2007. Besides it is also evident that these series clearly follow some mutual pattern which is also accounted in the development of the total the deficit time series.

Whether primary deficit is responding to the fluctuations in the debt ratio is on the other hand not that obvious. In empirical policy literature this is an important policy issue to conclude whether fiscal policy is sustainable or not. A primary deficit, in line with the total budget balance, was improving from the early 2000's, when the debt ratio started to decline. However from 2007, still declining debt ratio was followed by the increase in the primary deficits. Therefore, from the initially plotted series, no clear pattern of how and whether the primary deficits reacts to the changes in the public debt series at all in the long-run can be identified. In order to identify whether eventually primary deficits is responding to the development in the public debt, a set of the econometrical tests is employed and quantitative testing procedure starts by presenting the methods used to assess fiscal sustainability issues.

6.2. Methods

The testing procedure starts with the evaluation of the order of integration in the relevant time series by applying several types of the unit root tests. Unit root tests are performed in order to evaluate stationarity in revenue, total expenditure, primary budget balance, total deficit and public debt ratios. In case of the unit root in levels-to-GDP, the order of integration in those series is evaluated.

This procedure is employed with reference to the theoretical and applied fiscal policy literature on the topic, where the methods to test for sustainability primarily depend on the data availability and stationarity test results. Therefore in accordance to the unit-root test results the following modelling procedure is applied:

1. Finding public debt and primary surplus time series sets stationary, the model proposed by [Bohn \(1998\)](#) should be applied. In this method the general regression takes form of $t_t - g_t = \beta b_t + A\mathbf{Z}_t + \eta_t$ with \mathbf{Z}_t being as a vector of the other components than debt that might affect the development of the primary surplus. In accordance to this approach, finding $\hat{\beta}$ being positive and sufficiently large, proper fiscal policy-making might be concluded¹².

2. Alternatively, a proper fiscal policy-making is concluded if public debt and primary surplus time series are integrated of order one and co-integrated¹³ with a co-integration vector attaining some specific properties. For the co-integration vector being $(1, -\beta)$ with sufficiently large β , sustainability in public finance should be concluded. This technique corresponds to the method proposed by [Bohn \(1998\)](#).

These two approaches focus thus on the direct primary balance reaction to the development of the public debt. However, in order to establish such policy reaction functions, the same order of integration is required for the relevant time series. Only if regressor $\hat{\beta}$ or co-integrating vector $(1, -\beta)$ exhibits proper signs and magnitude, one may conclude that fiscal policy-making is sustainable.

Different order of integration on the other hand points towards lack of direct reaction of the primary surplus to the increase in the public debt and in this case univariate time series techniques might still be employed. If the government performs some other kind of the public debt management policy, which is still consistent with the concept of sound fiscal-policy making, a more complex set based on the univariate techniques might be applied.

¹² See for example [Haber & Neck \(2006\)](#), [Greiner, Koeller & Semmler \(2006\)](#)

¹³ See for example [Gurbuz, Jobert & Tuncer \(2007\)](#)

3. According to the empirical fiscal policy literature, stationary public debt time series alone should support the concept of the sustainable public finance¹⁴. This method partially corresponds to the method developed by [Hamilton & Flavin's \(1986\)](#) and is in general consistent with the mathematically derived intertemporal budget constraint restriction. This restriction suggests that a constant debt-to-GDP ratio over time, which econometrically is captured by the mean-stationary debt time series, is enough to conclude sustainable public finance. The additional implication would arise in case of trend stationary debt-to-GDP ratio where the pattern of how the series are trending is important issue for concluding sustainability. Thus, mean-stationary or trend stationary decreasing debt series are consistent with the conventional intertemporal budget approach. Application of this method however would call for establishment of the fiscal reaction functions as it of interest to define a source of the proper debt management policy.

If none of the previously described methodologies points towards sustainability, other fiscal policy reaction functions might also be investigated as existence of them would suggest that some kind of the debt management policy is still evident.

4. It might be considered that total deficit time series should be of economic interest as finding total deficit time series stationary points towards sound fiscal policy-making. Stationary figures would eventually imply that either both total spending and revenue series are stationary, or that these are integrated of order one and co-integrated with a vector of $(1,-1)$. This method of testing was basically developed by [Hakkio & Rush \(1991a\)](#) and it is still considered to point towards “strong” version of the sustainability in the applied fiscal policy literature¹⁵. Nevertheless, having a full set of data, first order of integration in the debt time series would also be necessary to obtain a complete accounting identity. Besides the causality functions between revenue and expenditure might in this case be of interest in order to define a reaction function in the debt management policy.

5. Finally, higher order of integration of the total deficit and debt time series as well as co-integration relationship between revenue and spending with some vector of $(1,-\beta)$ might be assumed to allow distinguishing “weaker” form of sustainability¹⁶, but if this approach is valid, then economical implications as developed in [Quintos \(1995\)](#) should also be of consideration for concluding whether public finance is sustainable or not.

The summary on the latter two univariate time series techniques points towards a suggestion that the conclusion whether a sustainable fiscal outlook is maintained should be carried in a more

¹⁴ See for example [Gurbuz, Jobert & Tuncer \(2007\)](#)

¹⁵ See for example [Baharumshah & Lau \(2007\)](#), [Gurbuz, Jobert & Tuncer \(2007\)](#)

¹⁶ See for example [Baharumshah & Lau \(2007\)](#), [Gurbuz, Jobert & Tuncer \(2007\)](#)

complex way depending on the set of implications carried by the unit-root and co-integration testing and results should be evaluated and interpreted jointly.

6.3. Unit-root tests

Evaluation of the order of integration of the relevant time series is made by shortly describing three types of the standard unit-root test procedures – Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP) test and Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test, afterwards focusing on the interpretation of the test results. In addition a supplementary method called Perron’s unit root test with trend break, which is based on the unit root testing around the broken deterministic components is constructed since one of the time series data sets points towards the necessity of it.

6.3.1. ADF test

In the generalized ADF test equation, denoting the relevant variable by y_t , where $t = \{1, 2, \dots, k\}$, the regression of

$$\Delta y_t = \mu + \beta \cdot t + \gamma y_{t-1} + \varphi_k \sum_{i=1}^k \Delta y_{t-i} + \varepsilon_t$$

is estimated with t as a deterministic time trend, lagged differences of Δy_{t-k} to account for autocorrelation in residuals and $\varepsilon_t \sim IID(0, \sigma^2)$. (Dickey & Fuller 1979, p.430). Under the null hypothesis of the unit root it is tested whether $\gamma = 0$, while under the alternative - $\gamma < 0$. Critical values are calculated and provided in Dickey & Fuller (1981).

In case of a unit root in the levels, the order of integration is evaluated by running regression of

$$\Delta^2 y_t = \mu + \beta \cdot t + \gamma \Delta y_{t-1} + \varphi_k \sum_{i=1}^k \Delta y_{t-i} + \varepsilon_t$$

and applying the same hypothesis testing.

6.3.2. PP test

Phillips & Perron (1988) shows however that ADF type tests do not have correct statistics when autocorrelation and heterogeneity, arising out of a wide range of reasons, is present in the error term. Phillips & Perron (1988) has on this background developed a corrected test statistics for a unit root testing in the non-augmented DF type regressions allowing the possibility for autocorrelation and heteroskedasticity in the residuals.

Therefore, the PP test for the unit root regression takes the same form as non-augmented DF

$$\Delta y_t = \mu + \beta \cdot t + \gamma y_{t-1} + \varepsilon_t$$

where the possibility of the autocorrelation in ε_t and non-constant variance, i.e. $\sigma_\varepsilon^2 \neq \sigma^2$, is allowed. (Phillips & Perron 1988, p.338-340) Under the null hypothesis of the non-stationarity, it is tested whether $\gamma = 1$, while under the alternative - $\gamma < 1$. Test statistics for γ is thus corrected to these particular properties of the error term. Critical values are the same as given in Dickey & Fuller (1981).

6.3.3. KPSS test

Both ADF and PP tests has suffers however from a low power when γ is close to unity especially in the small samples. Due to the limitations in data when time series are not enough informative, unit root tests might fail to rejects a unit root when it actually exists. KPSS test is supposed to distinguish between and detect a unit root in series that fluctuate around a deterministic trend or accept a random walk in time series. KPSS test assumes that a variable in question is a sum of a linear time trend, a random walk variable and $\varepsilon_t \sim IID(0, \sigma^2)$. The regression takes form of

$$y_t = \beta t + r_t + \varepsilon_t \text{ where } r_t = r_{t-1} + u_t \text{ with } u_t \sim N(0, \sigma_u^2)$$

testing whether the variance of u_t equals zero, thus pointing towards trend stationary series y_t . This arises out of the fact that as the initial value of r in this regression is assumed to be fixed, then having $\sigma_u^2 = 0$ means that r_t serves as a simple constant in the regression implying that y_t fluctuates around a deterministic trend. Thus under the null hypothesis of the trend stationary series, it is tested if $\sigma_u^2 = 0$. Meanwhile under alternative hypothesis of the unit root in y_t , this variable has a stochastic trend which is accounted by r_t . (Kwiatkowski, Phillips, Schmidt & Shin 1991, p.160-168)

6.3.4. Perron's unit root test with trend break

As noticed above, development of the debt ratio clearly points towards trend break in the time series and therefore an additional unit root test is constructed to account for it. In the debt time series it might be suspected that change in the deterministic trend occurred the third quarter of 2000.

Standard unit root tests often fail to reject the unit root when the trend break in data is present although time series are trend stationary around a deterministic trend with a break in it. Therefore an additional unit root test allowing one-time break in the deterministic trend is constructed in order to account for it, inducing precision in the estimates.

[Perron \(1989\)](#) constructed a unit root test which initially was referred to the model of the “changing growth”. This expression in turn refers to situation when change in the slope of the time trend occurs without any drastic change in time series, but rather the time trend changes in its slope at the date of the break. The model takes form of $y_t = \mu_1 + y_{t-1} + (\mu_1 - \mu_2)DU_t + \varepsilon_t$ under the null hypothesis of the unit-root around the broken deterministic trend while $y_t = \mu_1 + \beta t + (\beta_2 - \beta_1)DT_t^* + \varepsilon_t$ under the alternative of the stationarity in series with a deterministic trend where one-time change in the slope of it occurs. Dummy variables are

$$DU_t = \begin{cases} 1 & \text{if } t > T_B \\ 0 & \text{otherwise} \end{cases}, \quad DT_t^* = \begin{cases} t - T_B & \text{if } t > T_B \\ 0 & \text{otherwise} \end{cases} \quad \text{where } T_B \text{ is a time of the trend break}$$

and $\varepsilon_t \sim IID(0, \sigma^2)$. ([Perron 1989, p.1363-1364](#)) Using the difference operator, the first model might be re-defined to $\Delta y_t = \mu_1 + \gamma DU_t + \varepsilon_t$ under the null, while the second one to $\Delta y_t = \mu_1 + \beta t + \gamma DT_t^* + \delta y_{t-1} + \varepsilon_t$ under the alternative. Thus the latter regression is serving as the test regression, suggesting that for $\delta = 0$ the series are non-stationary, while for $\delta < 0$ the series are stationary and fluctuates around the broken deterministic trend. The test is performed by applying OLS and testing for $\delta = 0$. Critical values are calculated and provided in [Perron \(1989\)](#).

6.3.5. Starting conclusions based on the unit-root test results

Standard unit root test results indicate that all series in question, except debt, are non-stationary in their levels and stationary in their first differences. Public debt time series on the other hand are stationary around the broken deterministic where such conclusion is made by employing the Perron’s unit root with trend break test. All the unit-root test results are listed below in table 1.

The first conclusion which might be deduced from the unit-root test results is that debt development in the past has been on the sustainable path. This conclusion is supported by the fact that the debt-to-GDP series are stationary around the broken decreasing deterministic trend paying a special attention to the fact that the trend break occurred in the beginning of the period in question. This implication is thus based essentially on the univariate time series approach¹⁷ since the different order of integration of the debt and the primary deficit time series, such that $b_t \sim I(0)$ while $(t_t - g_t) \sim I(1)$, does not allow establishing and testing for the fiscal reaction function of whether the primary budget balance responses to the fluctuations in the public debt

¹⁷ This method corresponds to No.3 in section 6.2

directly as initially considered in the methods section¹⁸. Different order of integration in these series in general implies that primary balance as such is most likely targeted independently on the development of the public debt.

However other fiscal policy reaction functions should be considered as they are of interest to define a source of the proper debt management policy. Therefore in accordance to the unit root test results which indicated that and revenue items and total spending are integrated of order one, such that $t_t \sim I(1)$ and $(g_t + r_t b_{t-1}) \sim I(1)$, the mutual relationship between these might be established. Co-integration between them in this particular case would point towards fiscal reaction function through the total budget balance instead, which is insightful since primary budget balance in a strict sense is a part of the total balance. Fiscal function through the total budget balance going from the particular revenue and expenditure management policy is likely to make the debt-to-GDP ratio to trend-revert. Establishment of such function also would allow confirming that some debt management policy is feasible that makes debt-to-GDP ratio to remain on the sustainable path.

¹⁸ These methods correspond to No.1 alternatively No.2 in section 6.2

Table 1. Unit-root tests results, sample 1999:1 – 2008:2¹⁹

Test statistics	ADF	PP	KPSS	Perron's unit root test with trend break ²⁰
In levels:				
Revenue/GDP (T/Y)	0.2362(0)	0.0248 (2)	0.1923 (5)*	
Expenditure/GDP ($G+rB$)/ Y	0.7261(0)	0.9689(4)	0.1879(5)*	
Total deficit/GDP ($T-G-rB$)/ Y	-1.0894(0)	-1.1140(1)	0.5529(5)*	
Primary deficit/GDP ($T-G$)/ Y	-1.6603(0)	-1.6603(0)	0.5128(4)*	
Debt/GDP (B/Y)				-4.1363 (0)*
In first differences:				
Revenue/GDP $\Delta(T/Y)$	-4.8229(0)*	-4.7751(1)*	0.1111(2)	
Expenditure/GDP $\Delta(G+rB)/Y$	-5.2081(0)*	-5.2031(3)*	0.1376(2)	
Total deficit/GDP $\Delta(T-G-rB)/Y$	-5.1438 (0)*	-5.1612(1)*	0.1081(1)	
Primary deficit/GDP $\Delta(T-G)/Y$	-5.0177(0)*	-5.0444(1)*	0.1225(2)	

Notes:

- Numbers denotes statistics for ADF, PP, KPSS tests as well as Perron's unit root test with trend break statistics.
- (*) Denotes rejection of the null hypothesis at the 5% significance level.
- Numbers in the parenthesis denotes number of lagged differences included in the regressions.
- In ADF and PP tests revenue and expenditure in levels and first differences are tested with both time trend and constant; total and primary deficit are tested without any deterministic components. In KPSS test revenue and spending both in levels and first differences are tested with constant and deterministic trend. Other variables are tested only on the constant.
- For the debt time series Perron's unit root test with trend break is constructed. The test regression takes form $\Delta y_t = \mu + \beta t + \gamma DT_t^* + \delta y_{t-1} + \varepsilon_t$ where DT_t^* is constructed as $DT_t^* = \begin{cases} t - T_B & \text{if } t > T_B \\ 0 & \text{otherwise} \end{cases}$ with T_B being the seventh observation of the sample since the trend break is assumed to occur in 2000:3. The estimated OLS regression is given by $\Delta y_t = 13.7414 + 0.5194 t - 0.7386 DT_t^* - 0.6652 y_{t-1}$ with standard errors in parenthesis. Estimated residuals of the regression are $\varepsilon_t \sim NIID(0, \sigma^2)$. t-value for $\hat{\delta}$ equals $t_{\hat{\delta}} = -4.1363$ while critical values for $\lambda = T_B / t = 0.2$ are taken from Perron (1989) table V.B. at page 1377. The critical value for the test equals -3.80 at 5% significance level. See also Gurbuz, Jobert & Tuncer (2007) equation B at page 347 as an applied example of this model.

¹⁹ Table 1 reports unit root test results on seasonally adjusted time series. To account for seasonal roots in data, it is considered that these tests should also be performed on the unadjusted time series. See for example Schreiber & Wolters (2007) for applications. However, since the test results for the unadjusted data sets lead to the same conclusions, these are not reported separately.

²⁰ The trend break is assumed to occur in the third quarter of 2000.

6.4. Co-integration tests

Thus, as unit-root tests indicate that revenue and expenditure-to-GDP are both integrated of order one, the co-integration analysis is further employed in order to evaluate whether some particular debt management policy is apparent. Co-integrated time series per definition would eventually point towards a clear fiscal reaction function in the long-run. To see this, some theoretical insights firstly should be of consideration.

A co-integration relationship between time series implies that adjustment between revenue and total spending exist in the long-run and therefore a sustainable public debt development is achieved through the total budget balance management policy. Finding a co-integration relationship between revenue and spending implies that these series are related together and some different cases of linkage, or causality, between them might be considered.

The first case is when fiscal authorities determine the level of expected tax receipts and then plan expenditure. Then the one-way causality goes from revenue to expenditure as spending is adjusted to the revenue movements. The second case is an opposite case – one-way causality where the government first decides the level of expenditure and then adjusts its income to the expected spending. The last two possible outcomes might be either bidirectional causality or a case of no-causality. Bidirectional causality presumes the situation where the government plans both tax receipts and spending at the same time whereas the latter case assumes the situation when revenue and expenditure are planned not depending on each other. In this case it would be usual to consider a case of not co-integrated time series and in general problematic fiscal policy-making. (Baharumshah & Lau 2007, p.885)

Therefore, finding the series not co-integrated implies that no causal relationship exists between them. If one finds series co-integrated, the following aspect is then of interest. In the latter case it becomes possible to establish a causality function between revenue and spending in order to see how public deficit is managed and consequently the development of the public debt is kept under control.

6.4.1. Johansen's co-integration test

Therefore to test for co-integration between revenue and spending, Johansen's co-integration test is employed. The test focuses on the determining the number of co-integrating relationships between the variables within the vector autoregressive (VAR) approach. Having a vector of non-stationary variables \mathbf{X}_t , that are integrated of the first order, the estimation procedure starts by running the VAR model of

$$\Delta\mathbf{X}_t = \mu + \Gamma_1\Delta\mathbf{X}_{t-1} + \dots + \Gamma_{k-1}\Delta\mathbf{X}_{t-k+1} + \Pi\mathbf{X}_{t-k} + \boldsymbol{\varepsilon}_t$$

where $\Gamma_i = -I + \Pi_1 + \dots + \Pi_i$ and $\Pi = I - \Pi_1 - \dots - \Pi_k$ are coefficient matrixes and the vector of the error term is $\varepsilon_t \sim NIID(0, \Sigma^2)$. In the coefficient matrix Π the long-run relationship estimates between the variables in vector \mathbf{X}_t are included. When $p \times p$ matrix Π has a zero rank, i.e. $r = 0$, all the combinations between the variables in a vector \mathbf{X}_t are non-stationary and co-integration between those does not exist. When $p \times p$ matrix Π has a full-rank, i.e. $r = p$, then all combinations among the variables in the vector \mathbf{X}_t are stationary and thus co-integrated. For the case when $r < p$, the number of co-integration relationships among the variables in \mathbf{X}_t equals to r . (Payne & Sahu 2004, p.203-204)

Maximum likelihood procedure is employed to test for the number of co-integration relationships in the system by applying λ_{\max} maximum eigenvalue and λ_{trace} trace tests where the null hypothesis of no co-integration, i.e. the case when $r = 0$, is in general tested against the alternative one in several stages. λ_{\max} test sequentially tests that a number of co-integrating vectors is r under the null and $r + 1$ under the alternative, starting from the first hypothesis of no co-integration, the case when $r = 0$. λ_{trace} test consecutively tests the case when a number of co-integrating vectors is equal to r under the alternative of at least $r + 1$ co-integrating vectors also starting from the first hypothesis of no co-integration. (Payne & Sahu 2004, p.203-204)

6.4.2. Pre-testing and co-integration test results

Co-integration test results based on this methodology depends on the number of lags included in VAR model as well as on the number of deterministic components included in the system and therefore statistical pre-testing combined with some economic intuition is first necessary.

Taking into account a fact that quarterly fiscal time series are considered and one of the variables, namely expenditure, does not depend on the market forces rather than is exogenously determined, budgetary planning process is meaningful in order to determine appropriate lag length in VAR. Since budget planning process in Lithuania ends approximately one quarter before the next budgetary year, thus planning revenue and spending only for the coming year, the economic intuition suggests that no longer than five lags in VAR model should be considered. Elsewhere the appropriate lag length is chosen by applying econometrical techniques.

Therefore in order to determine the number of the lags included in VAR process, VAR (5) is estimated and some pre-testing based on the minimization of Akaike information criterion (AIC), Final prediction error (FPE), Schwarz information criterion (SIC) and Hannan-Quinn information criterion (HQ) information criteria is conducted. The pre-test results are presented in table 2.

Table 2. VAR lag structure, sample 1999:1 – 2008:2

VAR order	Constant	FPE	AIC	SIC	HQ
A					
1	with constant	0.021479*	1.834073*	2.1061*	1.9256*
2	with constant	0.023128	1.904351	2.3578	2.0569
3	with constant	0.027714	2.076839	2.7117	2.2904
4	with constant	0.032035	2.206500	3.0227	2.4811
5	with constant	0.033852	2.237045	3.2347	2.5727
B					
1	no constant	0.022828*	1.895699*	2.0770*	1.9567*
2	no constant	0.024234	1.953365	2.3161	2.0754
3	no constant	0.029177	2.133202	2.6773	2.3163
4	no constant	0.034892	2.300559	3.0261	2.5446
5	no constant	0.037343	2.348841	3.2558	2.6540

From the table 2 it might be seen that all the criterions, except SIC, points towards the appropriate VAR model being VAR (1) with a constant and on this background it is chosen to deal with the representation chosen by the majority of information criteria. Therefore VAR(1) with a constant term is estimated²¹ and further pre-testing procedure is based on the assessment of whether the deterministic components should be included in the system of equations. Sensitivity of the co-integration test results therefore calls for testing the presence of deterministic components in vector error-correction model (VECM) and statistical results of the test are presented below in table 3.

Table 3. Test for deterministic components in VECM (1) for sample 1999:1 – 2008:2

Model	VAR order	Number of co-integration equations	Trend assumptions in data	Trend and intercept assumptions in co-integration equation (CE)	AIC	SIC
1	1	0	No trend	No trend, no intercept	2.1861	2.3621
		1			1.9337*	2.2856*
2	1	0	No trend	No trend, intercept	2.1861	2.3621
		1			1.9771	2.3730
3	1	0	Linear trend	No trend, intercept	2.2766	2.5405
		1			2.0306	2.4704
4	1	0	Linear trend	Trend, intercept	2.2766	2.5405
		1			2.0451	2.5289
5	1	0	Quadratic trend	Trend, intercept	2.0802	2.4320
		1			1.9936	2.5215

Consequently, in table 3, both AIC and SIC statistic points towards statistical assessment that model of type 1 should be considered when testing for co-integration²². In line with these results, VECM (1) from the type 1 model is estimated yielding co-integration test results presented in table 4 and VECM (1) coefficients presented in the table 5.

²¹ In addition, dealing with this model, it is checked that estimated VAR (1) with constant satisfies stability conditions.

²² It is usually considered that type 1 models are rarely used especially if series seem to have a constant deterministic component. Plotting relevant time series a constant term was evident. However, in VECM(1) representation with a constant term, it turned to be insignificant at 5% significance level and thus excluded from VECM turning back to the model 1.

Table 4. Co-integration test results between revenue and expenditure to GDP for sample 1999:1 – 2008:2 – model 1

	Lags	Null hypothesis	Alternative hypothesis	Statistics	Critical values at 5%
λ_{\max}	1	$r = 0$	$r = 1$	17.0855*	11.2248
		$r = 1$	$r = 2$	0.0887	4.1299
λ_{trace}	1	$r = 0$	$r = 1$	17.1743*	12.3209
		$r \leq 1$	$r = 2$	0.0887	4.1299

- (*) Denotes rejection of the null hypothesis at 5% significance level.

Table 5. VECM (1) estimates for the sample 1999:1 – 2008:2 – model 1²³

Variable	Coefficient estimates	Standard errors	Coefficient estimates	Standard errors
	Dependent variable $\hat{\Delta}t_t$		Dependent variable $\hat{\Delta}(g_t + r_t b_{t-1})$	
$t_{t-1} - 0.9596 \cdot (g_t + r_t b_{t-1})$	0.1100	0.0577	0.3408*	0.0800
Δt_{t-1}	0.1538	0.1941	-0.3957	0.2693
$\Delta(g_{t-1} + r_{t-1} b_{t-2})$	0.1509	0.1018	0.2988*	0.1412

- (*) Denotes significant coefficients at the 5% significance level.

The tests results thus support the co-integration relationship between revenue and expenditure at 5% significance level where the normalized co-integrating coefficient equals (1.0, -0.95). It is rather close to the theoretical value of (1.0, -1.0) which is the requirement for stationary total deficit time series, but statistical properties of the co-integration vector however suggests that this relationship is weaker than (1.0, -1.0)²⁴.

Therefore conclusions which might be drawn from the co-integration test results are summarised as follows. Existence of the co-integration relationship between revenue and expenditure supports previously made conclusion that fiscal policy in Lithuania has been sustainable in the past since the source of fiscal adjustment is evident in the long-run. Proper debt management flows by controlling

²³ The extended VECM(1) system takes representation of:

$$\begin{cases} \hat{\Delta}t_t = 0.1100 \cdot (t_{t-1} - 0.9596 \cdot (g_t + r_t b_{t-1})) + 0.1538 \cdot \Delta t_{t-1} + 0.1509 \cdot \Delta(g_{t-1} + r_{t-1} b_{t-2}) \\ \hat{\Delta}(g_t + r_t b_{t-1}) = 0.3408 \cdot (t_{t-1} - 0.9596 \cdot (g_t + r_t b_{t-1})) - 0.3957 \cdot \Delta t_{t-1} + 0.2988 \cdot \Delta(g_{t-1} + r_{t-1} b_{t-2}) \end{cases}$$

Estimated VECM passes residual normality test at 5% significance level (orthogonalization method: Cholesky of covariance matrix) and no-serial autocorrelation is present up to 12th lag (LM test). The coefficient in front of $(t_{t-1} - 0.9596(g_t + r_t b_{t-1}))$ measures the speed of adjustment of the variables t_t and $(g_t + r_t b_{t-1})$ towards long-run equilibrium; the coefficients in parenthesis denotes co-integration relationship of (1.0, -0.95) between variables while other coefficients in front of lagged differences captures short-run dynamics.

²⁴ Restricting the co-integration vector being (1.0, -1.0) and testing validity of this restriction, the test yields results allowing rejecting the null hypothesis of binding restriction of (1.0, -1.0) at 5% significance level. LR statistics for this test equals to 9.7763.

the development of the total public deficits instead. Even though a gap between public spending and tax collection is a permanent feature in the long-run which is captured by the statistical properties of the co-integrating vector and non-stationary total deficit series, it is in line with the conventional intertemporal budget constraint approach. The reason for this implication also might be drawn from the theoretical point of view suggesting that for permanent deficits including interest payments the intertemporal budget constraint is still satisfied, however with possible additional macroeconomic considerations²⁵.

6.5. Impulse-response functions

Unit-root tests results thus support first order of integration between revenue and spending items allowing testing for existence of co-integration. Meanwhile co-integration test results support the linkage between these series and shows that the source of the debt adjustment goes through deficit management policies, allowing further establishing fiscal policy reaction functions. These fiscal reaction functions will show whether bidirectional causal relationship between these series exist or only one-way causality might be established depending on the properties of the time series. Economically this is insightful since it gives better understanding of the deficits and debt management processes in the country. Therefore a special type of causality test is further considered where the VECM (1) estimates serves establishing reaction functions between revenue and expenditure in the graphical form.

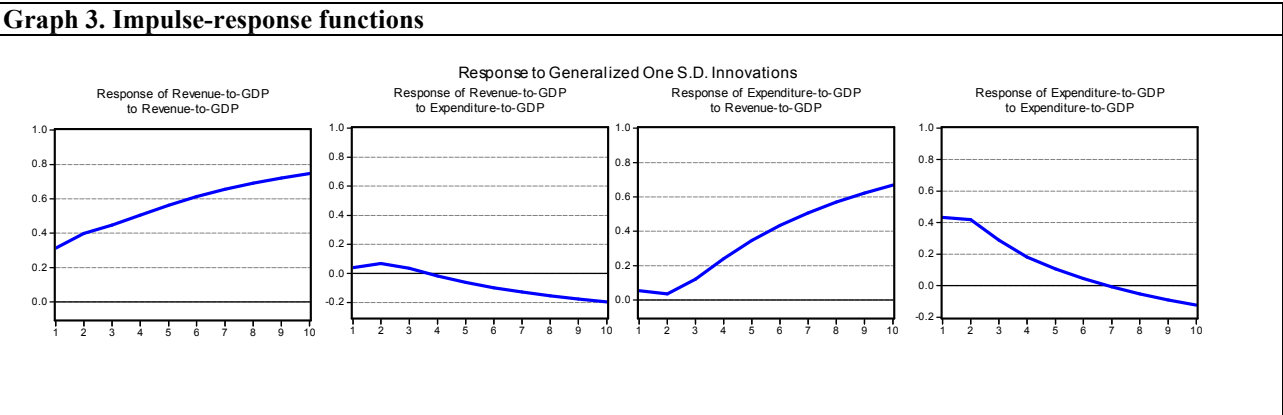
This specific type of the test is called impulse-response analysis and variance decomposition and it is employed to see what causality between revenue and spending might be observable. Impulse-response and variance decomposition functions show how, how long and in what magnitude public expenditure reacts to the one-time positive or negative shock in tax revenue. The opposite function might also be established linking the reaction of the public receipts to the shock in public spending. In addition, these functions show the reaction of these both variables to the shocks of their own.

The impulse-response function and variance decomposition is further applied to analyze endogeneity and thus causality of the revenue and expenditure in their system of equations. Impulse-response function in general measures how the one-time shock in revenue influences its own movements in the future as well as movements in expenditure and persistency of this shock in the system. Meanwhile, decomposing error variance, variance decomposition function measures to what extent variation in revenue might be explained by its own movements as well as variability in expenditure items and vice versa.

²⁵ The methods No.4 and No.5 considered in section 6.2 suggest that non-stationary total deficit figures should eventually lead to the second order of integration of the debt time series. This relationship was established through accounting identity under assumptions of a certain properties of the discount factor. It should be therefore pointed out that a non-stationary deficit and stationary debt series raises some controversy to the fourth and fifth method considered in the relevant section since a full accounting identity imposed by these methods can not be obtained. This possibly points towards an implication that properties of the debt-to-GDP ratio might to some extent be influenced by the dynamic of the debt-deficit adjustment term. Also assumptions regarding the interest rate development used to derive these models might be not compatible assessing fiscal sustainability issues for Lithuania's economy.

Since the system of non-stationary variables in this particular case is considered, it is sometimes suggested that impulse-response analysis should be performed on the stationary differences in VAR²⁶. In the other empirical works on the other hand the estimated VECM representation is used for creating impulse-response functions²⁷. In this case, it is chosen to deal with the latter method and the procedure starts by using the estimated VECM model as a baseline for establishing impulse-response functions. It is chosen to deal with the generalized impulse-response functions due to its independence to the ordering of the variables in VECM. This method is applicable to stationary variables in VAR system as well as integrated and possibly co-integrated processes in VECM. In VAR system with variables $I(0)$ the impact of shocks dies throughout time whereas permanent effects of the shocks in VECM system with variables $I(1)$ persist. (Pesaran & Shin 1997, p.21-24)

Generalized impulse-response functions shows how revenue and spending responds to a shock set to one-standard-deviation at time $t = 0$ in the mutual system of the revenue-expenditure items. Impulse-response functions are thus plotted in the graph 3 below over ten quarter's horizon where the horizontal scale reports time horizons and the vertical - standard-deviation units.

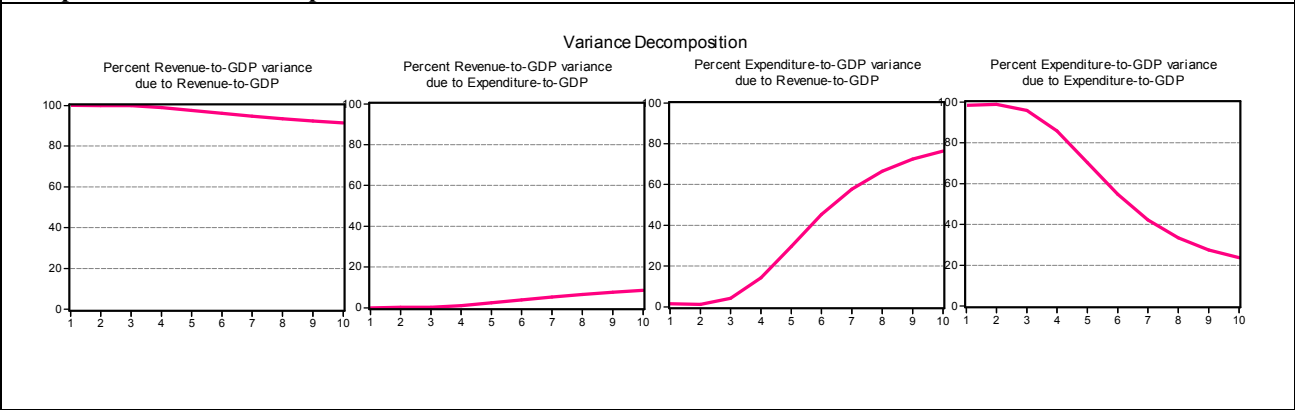


Impulse-response functions thus suggest that the one-time shock in revenue measured in one standard deviation is influential to the movements in expenditure. It does not however influence public spending immediately whereas the positive effect of a shock starts affecting expenditure movements some periods thereafter. This is due to the budgeting process since it takes time for the government to react to the movements in revenue by adjusting public spending. Revenue items on the other hand are not to a large extent responding to the movements in spending showing that the one-way causality function might be established flowing entirely from revenue to expenditure.

²⁶ See for example Payne & Sahu (2004)
²⁷ See for example Giorgioni & Holden (2003), Schreiber & Wolters (2007).

Additional insights might be obtained from the variance decomposition function where the standard Cholesky decomposition is used²⁸. It shows how much of the variations in expenditure might be explained by the movements in revenue and own items in percent and vice versa. The variance decomposition functions are therefore plotted in the graph 4 below.

Graph 4. Variance decomposition functions



The one-time shock in revenue at time $t = 0$ influences public spending and explains considerable proportion of the movements them, however some period thereafter. The influence of the shock is thus lagging and co-movements of expenditure might be observable only after some periods. On the other hand variations in revenue to a large extent depends only on its own variations supporting previously made conclusion of exogeneity of the tax receipts in the system to the shocks in expenditure.

Economic interpretation of the results obtained by plotting impulse response functions might be summarised and interpreted by observing that, in general, the one-way causality goes from revenue to expenditure since throughout time spending is adjusting to the movements in tax receipts. This also shows that development of public debt and deficit are controlled by adjusting governmental spending to the growth in the budgetary revenue.

The existence of such a causality function is highly plausible, but leads to some additional economic considerations. A shortcoming of such a policy reaction function is that in the short-run, in the case of for instance unexpected positive shock to the economy, there is an incentive to use revenue windfalls to finance additional current expenditure leading to an expansionary pro-cyclical fiscal policy. Also in case of an unexpected negative shock to the economy, if revenue was for some reason overestimated, the decrease in the growth rates of revenue also creates an incentive to cut public spending thus deepening the economic downturn. One of the possible solutions to avoid pro-cyclicality would be expenditure ceiling policies leading to a more predictable fiscal policy thus strengthening the quality of public finance in the short- and medium-run.

²⁸ Using this technique however might yield different results depending on the ordering of the variables in the system (Pesaran & Shin 1997, p.21-24) and therefore re-ordering of variables is made in order to perform sensitivity test. This test yields the same results as considered in the main text.

6.6. Future policy implications

The analysis above suggests that fiscal policy in Lithuania has been sustainable. However, these conclusions are based entirely on the statistical properties of the past fiscal time series and are limited only to the past fiscal policy-making disregarding expected future developments of the public deficits and debt. Even though it is sometimes argued that past fiscal developments should receive less attention and one should focus on the future fiscal sustainability issues instead, proper fiscal policy-making in the past might be of importance even for future fiscal policies since it shows that the country in general is willing to maintain a sound fiscal stance.

Also paying special attention to the difficulties which Lithuanian economy copes with at present, it should be noted that some problems in fiscal policy-making still persist. The current economic downturn²⁹ is challenging public finance showing that future attempts to improve the quality of public finance are necessary. These should primarily aim to avoid pro-cyclicality thus maintaining favourable macroeconomic outlook in the short- and medium-run. In the long-run, fiscal challenges are less severe. Even though debt-to-GDP ratio is likely to start increasing³⁰, the relatively low debt ratio and the pension reform undertaken in the past, aiming to reduce the fiscal burden associated with the ageing population, makes such a country as Lithuania a low-risk country in relation to the future long-run sustainability issues.

7. Concluding remarks

This essay has analyzed the topic of an intertemporal budget constraint in the long-run from the public finance perspective for Lithuania. A set of econometrical tests allowed concluding that past fiscal policy in the country for the period 1999:1-2008:2 has been sustainable through a proper public debt management. Properties of the fiscal time series suggested that the primary budget balance, those proper adjustments to the debt development considered being an important feature for maintaining soundness of fiscal policy, was targeted independently on the development of the public debt. Nevertheless other fiscal policy reaction functions were clearly apparent in order to keep the debt-to-GDP ratio on the sustainable path. Soundness of fiscal policy has been maintained instead through the total budget deficit management policy. Since the government is adjusting public spending to the revenue movements, which points towards one-way causality function from revenue to public expenditure, the control over budgetary outcomes is achieved leading to proper public debt management policies. The small sample size requires however highly precautionary treatment of the estimates and conclusion drawn from the tests results.

²⁹ See for example medium-run forecasts of the Bank of Lithuania at www.lb.lt or forecasts presented by the the Ministry of Finance of Lithuania presented in Convergence Program 2008.

³⁰ See for example medium-run forecasts of the Ministry of Finance of Lithuania presented in the Convergence Program 2008.

References

Ahmad Zubaidi Baharumshar & Evan Lau (2007), "Regime Changes and the Sustainability of Fiscal Imbalance in East Asian Countries". *Economic Modelling* 24 (2007), pp. 878-894.

Henning Bohn (1995), "The Sustainability of Budget Deficits in a Stochastic Economy". *The Journal of Money, Credit and Banking*, Vol.27, No.1, pp. 257-271.

Henning Bohn (1998), "The Behavior of U.S. Public Debt and Deficits". *The Quarterly Journal of Economics*, Vol.113, No.3, pp. 949-963.

Henning Bohn (2007), "Are Stationarity and Cointegration Restrictions Really Necessary for the Intertemporal Budget Constraint?". *Journal of Monetary Economics*, Vol.54, pp. 1837-1847.

Ane Bela Santos Bravo & Antonio Luis Silvestre (2002), "Intertemporal Sustainability in Fiscal Policies: Some Tests for European Countries". *European Journal Of Political Economy* Vol.18 (2002), pp. 517-528.

Burda & Wyplosz (2005), "*Macroeconomics*". Web Appendix 15. The Algebra of Budget Deficits. Fourth edition.

European Commission (2004), "The Stock-Flow Adjustment in the EU Member States". ECFIN/B(2004)REP/50035.

David A. Dickey & Wayne A. Fuller (1981), "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root". *Econometrica*, Vol.49, No.4, pp. 1057-1072.

LR Finansų Ministerija (2008), *Lietuvos Konvergencijos Programa (LKP)*, Patvirtinta LR Vyriausybės 2009 m. sausio 21 d. nutarimu Nr. 30. (2009-02-01) www.finmin.lt

Alfred Greiner, Uwe Koeller & Willi Semmler (2006), "Testing the Sustainability of German Fiscal Policy: Evidence for the Period 1960-2003". *Empirica*, 33, pp. 127-140.

Yeim Gurbuz, Thomas Jobert & Ruhi Tuncer (2007) "Public Debt in Turkey: Evaluation and Perspectives". *Empirica*, 33, pp. 127-140

Gottfried Haber & Reinhard Neck (2006), "Sustainability of Austran Public Debt: a Political Economy Perspective". *Applied Economics*, 39:3, pp. 343-359.

Craig S. Hakkio & Mark Rush (1991a), "Is the Budget deficit "Too Large?". *Economic Inquiry*, Vol.XXIX, pp. 429-445.

Craig S. Hakkio & Mark Rush (1991b), "Cointegration: How Short is the Long Run?". *Journal of International Money and Finance*, 10, pp. 571-581.

Nicola Giammarioli, Christiane Nickel, Phillipp Rother & Jan-Pierre Vidal (2007) "Assessing Fiscal Soundness. Theory and Practice." ECB Occasional Paper Series. No.56/March 2007.

James D. Hamilton & Marjorie A. Flavin (1986), "On the Limitations of Government Borrowings: A Framework for Empirical Testing". *The American Economic Review*, Vol. 76, No.4, pp. 808-819.

Denis Kwiatkowski, Peter C.B. Phillips, Peter Schmidt and Yongcheol Shin (1991), "Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root". *Journal of Econometrics*, Vol.54, pp. 159-178.

Anthony J. Makin (2005), "Public Debt Sustainability and Its Macroeconomic Implications in ASEAN-4". *ASEAN Economic Bulletin*, Vol.22, No.3, pp. 284-296.

Simon Neaime (2004), "Sustainability of Budget Deficits and Public Debt in Lebanon: a Stationarity and Co-integration Analysis". *Review of Middle East Economics and Finance*, Vol.2, No.1, pp.43-61.

Carmela E. Quintos (1995), "Sustainability of the Deficit Process with Structural Shifts". *Journal of Business & Economic Statistics*, Vol.13, No.4, pp. 409-417.

James E. Payne & Anandi P. Sahu (2004), "Random Walks, Cointegration, and the Transmission of Shocks across Global Real Estate and Equity Markets". *Journal of Economics and Finance*, Vol.28, No.2, pp. 198-210.

Peter C.B. Phillips & Pierre Perron (1988), "Testing for a Unit Root in Time Series Regression". *Biometrika*, Vol.75, No.2, pp. 335-346.

Pierre Perron (1989), "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis". *Econometrica*, Vol.57, No.6, pp. 1361-1401.

H. Hashem Pesaran & Yongcheol Shin (1998), "Generalized Impulse Response Analysis in Linear Multivariate Models". *Economics Letters*, Vol.58, pp. 17-29.

David Romer (2006), "*Advanced Macroeconomics*". Third edition. McGraw-Hill/Irwin.

Sven Schreiber & Jurgen Wolters (2007), "The Long-run Phillips Curve Revisited: Is the NAIRU Framework Data-Consistent?". *Journal of Macroeconomics*, Vol.29, pp. 355-367.

Bharat Trehan & Carl E. Walsh (1988), "Common Trends, the Government's Budget Constraint, and Revenue Smoothing". *Journal of Economic Dynamics and Control*, Vol.12, pp. 425-444.

Bharat Trehan & Carl E. Walsh (1991), "Testing Intertemporal Budget Constraints: Theory and applications on U.S. Federal Budget and Current Account Deficits". *Journal of Money, Credit, and Banking*, Vol.23, No.2, pp. 206-223.

David W. Wilcox (1989), "The Sustainability of Government Deficits: Implications of the Present-Value Borrowing Constraint". *Journal of Money, Credit, and Banking*, Vol.21, No.3, pp. 291-306.

Appendix A

A.1. Dynamic of the real debt and debt-to-GDP ratio

Consider the accounting identity of the public debt identity in nominal terms as in (3)

$$\Delta \bar{B} = i\bar{B} + \bar{G} - \bar{T} \quad (\text{A.1.1})$$

where the primary deficit is denoted by $\bar{G} - \bar{T}$ and the term $i\bar{B}$ represents nominal interest payments on the outstanding public debt. The sum of these two terms in turn represents the nominal budget deficit. The right-hand term in equation defines how the total deficit is financed – in this case by issuing new debt $\Delta \bar{B}$. (Burda & Wyplosz 2005, p.1-3)

A.1.1 Accounting identity of the public debt as its ratio-to-GDP

Dividing all the relevant variables by nominal GDP, equation in A.1.1 might be redefined to

$$\frac{\Delta \bar{B}}{Y} = i \frac{\bar{B}}{Y} + \frac{\bar{G}}{Y} - \frac{\bar{T}}{Y} \quad (\text{A.1.2})$$

And applying the derivation rules of two functions³¹ and inserting results obtained to (A.1.2) yields the following results:

$$\Delta \left(\frac{\bar{B}}{Y} \right) + \frac{\bar{B}}{Y} \cdot \frac{\Delta Y}{Y} = i \frac{\bar{B}}{Y} + \frac{\bar{G}}{Y} - \frac{\bar{T}}{Y} \quad (\text{A.1.3})$$

Defining public spending, tax recipients and public debt as their ratios to the nominal GDP as $g = \frac{\bar{G}}{Y}$,

$t = \frac{\bar{T}}{Y}$, $b = \frac{\bar{B}}{Y}$ and nominal GDP growth rate to $H = \frac{\Delta Y}{Y}$, equation (A.1.3) might be redefined to:

$$\Delta b + H \cdot b = ib + g - t \quad (\text{A.1.4})$$

Defining further the growth rate of the real GDP as $h = \frac{\Delta \left(\frac{Y}{P} \right)}{\frac{Y}{P}}$, inflation rate as $\pi = \frac{\Delta P}{P}$ and applying

again the mathematical rules of derivation, the growth of the real GDP is equal to:

³¹ $\Delta \left(\frac{\bar{B}}{Y} \right) = \frac{Y\Delta \bar{B} - \bar{B}\Delta Y}{Y^2} = \frac{\Delta \bar{B}}{Y} - \frac{\bar{B}}{Y} \cdot \frac{\Delta Y}{Y}$

$$h = \frac{\Delta\left(\frac{Y}{P}\right)}{\frac{Y}{P}} = \frac{\frac{P\Delta Y - Y\Delta P}{P^2}}{\frac{Y}{P}} = \frac{\frac{\Delta Y}{P} - \frac{Y\Delta P}{P^2}}{\frac{Y}{P}} = \left(\frac{\Delta Y}{P} - \frac{Y\Delta P}{P^2}\right) \cdot \frac{P}{Y} = \frac{\Delta Y}{Y} - \frac{\Delta P}{P} = H - \pi$$

and therefore $H = h + \pi$ (A.1.5)

Defining nominal interest rate as the sum of the real interest rate and inflation in economy $i = r + \pi$ and with help of equation (A.1.5) rearranging equation (A.1.4), the change in ratio of public debt-to-GDP is equal to:

$$\Delta b = (r + \pi)b + g - t - (h + \pi)b = g - t + (r - h)b$$
 (A.1.6)

where the term $r - h$ corresponds to the difference between the real return rate on the public debt and the growth rate of the real GDP. (Burda & Wyplosz 2005, p.1-3)

A.1.2. Accounting identity of the public debt in real terms

To obtain variables in the real terms all the variables in A.1.1 are divided by the price level in economy which yields:

$$\frac{\Delta\bar{B}}{P} = i \frac{\bar{B}}{P} + \frac{\bar{G}}{P} - \frac{\bar{T}}{P}$$
 (A.1.2)

Applying the derivation rules of two functions³² and inserting results obtained to (A.1.2) yields the following results:

$$\Delta\left(\frac{\bar{B}}{P}\right) + \frac{\Delta P}{P} \cdot \frac{\bar{B}}{P} = i \frac{\bar{B}}{P} + \frac{\bar{G}}{P} - \frac{\bar{T}}{P}$$
 (A.1.3)

Defining public spending, tax recipients and public debt in real terms as $G = \frac{\bar{G}}{P}$, $T = \frac{\bar{T}}{P}$, $B = \frac{\bar{B}}{P}$,

inflation rate to $\pi = \frac{\Delta P}{P}$ and the nominal interest rate as a sum of the real interest rate and inflation in economy $i = r + \pi$, equation (A.1.3) is redefined to

$$\Delta B + \pi \cdot B = (r + \pi)B + G - T$$
 (A.1.4)

³² $\Delta\left(\frac{\bar{B}}{P}\right) = \frac{P\Delta\bar{B} - \bar{B}\Delta P}{P^2} = \frac{\Delta\bar{B}}{P} - \frac{\bar{B}}{P} \cdot \frac{\Delta P}{P}$

and one-period dynamic of the real debt in discrete time equals to

$$\Delta B = rB + G - T \quad (\text{A.1.5})$$

identifying real interest rate paid to the debt holders as r . (Burda & Wyplosz 2005, p.1-3)

A.2. Deriving the intertemporal budget constraint model in real terms

Deriving the intertemporal budget constraint model in real terms, assumptions regarding the development of the real interest rate should be made. Under three most common assumptions on the interest rate the first stage of the intertemporal budget constraint model is derived in the following way.

The representation of one-period dynamic of the public debt in discrete time in the real terms is given as defined in notation (3):

$$\Delta B_t \equiv B_t - B_{t-1} = r_t B_{t-1} + G_t - T_t$$

Assuming that this identity holds for each period, so that

$$\Delta B_{t+1} \equiv B_{t+1} - B_t = r_{t+1} B_t + G_{t+1} - T_{t+1}$$

and solving this equation for B_t , the real stock of debt at t is equal to:

$$B_t = \frac{B_{t+1}}{1 + r_{t+1}} - \frac{G_{t+1}}{1 + r_{t+1}} + \frac{T_{t+1}}{1 + r_{t+1}}$$

Furthermore defining the real stock of the debt for the finite $n-1$ periods forward yields representations of

$$B_{t+1} = \frac{B_{t+2}}{1 + r_{t+2}} - \frac{G_{t+2}}{1 + r_{t+2}} + \frac{T_{t+2}}{1 + r_{t+2}}$$

$$B_{t+2} = \frac{B_{t+3}}{1 + r_{t+3}} - \frac{G_{t+3}}{1 + r_{t+3}} + \frac{T_{t+3}}{1 + r_{t+3}}$$

...

$$B_{t+n-2} = \frac{B_{t+n-1}}{1 + r_{t+n-1}} - \frac{G_{t+n-1}}{1 + r_{t+n-1}} + \frac{T_{t+n-1}}{1 + r_{t+n-1}}$$

$$B_{t+n-1} = \frac{B_{t+n}}{1+r_{t+n}} - \frac{G_{t+n}}{1+r_{t+n}} + \frac{T_{t+n}}{1+r_{t+n}}$$

Recursively substituting the term B_{t+n-1} into B_{t+n-2} , B_{t+n-2} afterwards into B_{t+n-3} , ..., B_{t+2} into B_{t+1} and B_{t+1} into B_t , the latter term might be defined as

$$B_t = \frac{B_{t+1}}{1+r_{t+1}} - \frac{G_{t+1}}{1+r_{t+1}} + \frac{T_{t+1}}{1+r_{t+1}} = \frac{B_{t+n}}{(1+r_{t+n}) \cdot (1+r_{t+n-1}) \cdot \dots \cdot (1+r_{t+2}) \cdot (1+r_{t+1})} - \frac{G_{t+n}}{(1+r_{t+n}) \cdot (1+r_{t+n-1}) \cdot \dots \cdot (1+r_{t+2}) \cdot (1+r_{t+1})} + \frac{T_{t+n}}{(1+r_{t+n}) \cdot (1+r_{t+n-1}) \cdot \dots \cdot (1+r_{t+2}) \cdot (1+r_{t+1})} - \dots - \frac{G_{t+2}}{(1+r_{t+2}) \cdot (1+r_{t+1})} + \frac{T_{t+2}}{(1+r_{t+2}) \cdot (1+r_{t+1})} - \frac{G_{t+1}}{1+r_{t+1}} + \frac{T_{t+1}}{1+r_{t+1}}$$

A.2.1. Constant and positive interest rate

Assuming that the real interest rate is constant and positive over time, i.e. $r_{t+n} = r > 0$, summing up the terms in the last equation for infinite periods forward, the general representation of the stock of the debt B_t is equal to present value of the future stock debt and the sum of the discounted primary budget deficit. (Bohn 2007, p.1839)

$$B_t = \frac{B_{t+n}}{(1+r)^n} - \sum_{n=1}^{\infty} \frac{G_{t+n}}{(1+r)^n} + \sum_{n=1}^{\infty} \frac{T_{t+n}}{(1+r)^n}$$

A.2.2. Positive and constant expected interest rate

Assuming that the expected real interest rate is positive and constant, i.e. $E_t(r_{t+n}) = r > 0$ for all periods, yields the same general representation of the dynamic of the stock of the debt B_t for infinite periods forward. (Bohn 2007, p.1839)

$$B_t = \frac{B_{t+n}}{(1+r)^n} - \sum_{n=1}^{\infty} \frac{G_{t+n}}{(1+r)^n} + \sum_{n=1}^{\infty} \frac{T_{t+n}}{(1+r)^n}$$

A.2.3. Stochastic stationary process in the interest rate with positive mean

Considering the general representation of the stock of the public debt at $t+1$ as in (1) equivalently for the one in the real terms

$$B_{t+1} = (1+r_{t+1})B_t + G_{t+1} - T_{t+1}$$

and assuming that the real interest rate follows a stochastic stationary process around the mean of r , i.e. $r_{t+n} \sim (r, \sigma^2)$, $r > 0$ the equation can be rewritten by subtracting the term of rB_t from both sides, such that

$$B_{t+1} - (1+r)B_t = (r_{t+1} - r)B_t + G_{t+1} - T_{t+1}$$

where the term $G_{t+1} + (r_{t+1} - r)B_t$ is equal to $G_{t+1} + r_{t+1}B_t$ if the real interest rate is assumed to fluctuate around the zero mean. (Quintos 1995, p.410) Solving the equation for B_t , the real stock of debt at t is equal to:

$$B_t = \frac{B_{t+1}}{1+r} - \frac{G_{t+1}}{1+r} - \frac{(r_{t+1} - r)B_t}{1+r} + \frac{T_{t+1}}{1+r}$$

Assuming further that this identity holds for each period forward, recursive substitution yields the general representation of B_t

$$B_t = \frac{B_{t+n}}{(1+r)^n} - \sum_{n=1}^{\infty} \frac{G_{t+n}}{(1+r)^n} - \sum_{n=1}^{\infty} \frac{(r_{t+n} - r)B_{t+n-1}}{(1+r)^n} + \sum_{n=1}^{\infty} \frac{T_{t+n}}{(1+r)^n}$$

where the term $G_{t+n} + (r_{t+n} - r)B_{t+n-1}$ is total spending with interest rate following a stationary stochastic process around a mean of r , $r > 0$. (Quintos 1995, p.410)

A.2. Deriving the intertemporal budget constraint model in ratios to GDP

Starting with the accounting identity in real terms as in (3) one can derive the intertemporal budget constraint model when variables are considered as their ratios to GDP. Assuming that this identity holds each period, so that $\Delta B_{t+1} \equiv B_{t+1} - B_t = r_{t+1}B_t + G_{t+1} - T_{t+1}$ and GDP grows at the rate of $Y_{t+1} = Y_t(1+h_{t+1})$, division of all the variables to GDP, yields:

$$\frac{B_{t+1}}{Y_{t+1}} = (1+r_{t+1})\frac{B_t}{Y_{t+1}} + \frac{G_{t+1}}{Y_{t+1}} - \frac{T_{t+1}}{Y_{t+1}} = (1+r_{t+1})\frac{B_t}{Y_t(1+h_{t+1})} + \frac{G_{t+1}}{Y_{t+1}} - \frac{T_{t+1}}{Y_{t+1}} = \frac{(1+r_{t+1})}{(1+h_{t+1})}\frac{B_t}{Y_t} + \frac{G_{t+1}}{Y_{t+1}} - \frac{T_{t+1}}{Y_{t+1}}$$

This implies that the ratio of $\frac{B_t}{Y_t}$ is given by:

$$\frac{B_t}{Y_t} = \frac{(1+h_{t+1})}{(1+r_{t+1})}\frac{B_{t+1}}{Y_{t+1}} - \frac{(1+h_{t+1})}{(1+r_{t+1})}\frac{G_{t+1}}{Y_{t+1}} + \frac{(1+h_{t+1})}{(1+r_{t+1})}\frac{T_{t+1}}{Y_{t+1}}$$

Assuming further that this identity holds each period, such that:

$$\frac{B_{t+1}}{Y_{t+1}} = \frac{(1+h_{t+2})}{(1+r_{t+2})} \frac{B_{t+2}}{Y_{t+2}} - \frac{(1+h_{t+2})}{(1+r_{t+2})} \frac{G_{t+2}}{Y_{t+2}} + \frac{(1+h_{t+2})}{(1+r_{t+2})} \frac{T_{t+2}}{Y_{t+2}}$$

$$\frac{B_{t+2}}{Y_{t+2}} = \frac{(1+h_{t+3})}{(1+r_{t+3})} \frac{B_{t+3}}{Y_{t+3}} - \frac{(1+h_{t+3})}{(1+r_{t+3})} \frac{G_{t+3}}{Y_{t+3}} + \frac{(1+h_{t+3})}{(1+r_{t+3})} \frac{T_{t+3}}{Y_{t+3}}$$

...

$$\frac{B_{t+n-2}}{Y_{t+n-2}} = \frac{(1+h_{t+n-1})}{(1+r_{t+n-1})} \frac{B_{t+n-1}}{Y_{t+n-1}} - \frac{(1+h_{t+n-1})}{(1+r_{t+n-1})} \frac{G_{t+n-1}}{Y_{t+n-1}} + \frac{(1+h_{t+n-1})}{(1+r_{t+n-1})} \frac{T_{t+n-1}}{Y_{t+n-1}}$$

$$\frac{B_{t+n-1}}{Y_{t+n-1}} = \frac{(1+h_{t+n})}{(1+r_{t+n})} \frac{B_{t+n}}{Y_{t+n}} - \frac{(1+h_{t+n})}{(1+r_{t+n})} \frac{G_{t+n}}{Y_{t+n}} + \frac{(1+h_{t+n})}{(1+r_{t+n})} \frac{T_{t+n}}{Y_{t+n}}$$

and recursively substituting the term $\frac{B_{t+n-1}}{Y_{t+n-1}}$ backwards into $\frac{B_{t+n-2}}{Y_{t+n-2}}$, $\frac{B_{t+n-2}}{Y_{t+n-2}}$ afterwards

into $\frac{B_{t+n-3}}{Y_{t+n-3}}$, ..., $\frac{B_{t+2}}{Y_{t+2}}$ into $\frac{B_{t+1}}{Y_{t+1}}$ and $\frac{B_{t+1}}{Y_{t+1}}$ into $\frac{B_t}{Y_t}$, the latter term might be defined as

$$\begin{aligned} \frac{B_t}{Y_t} &= \frac{(1+h_{t+1})}{(1+r_{t+1})} \frac{B_{t+1}}{Y_{t+1}} - \frac{(1+h_{t+1})}{(1+r_{t+1})} \frac{G_{t+1}}{Y_{t+1}} + \frac{(1+h_{t+1})}{(1+r_{t+1})} \frac{T_{t+1}}{Y_{t+1}} = \frac{(1+h_{t+n}) \cdots (1+h_{t+1})}{(1+r_{t+n}) \cdots (1+r_{t+1})} \frac{B_{t+n}}{Y_{t+n}} + \frac{(1+h_{t+n}) \cdots (1+h_{t+1})}{(1+r_{t+n}) \cdots (1+r_{t+1})} \left(\frac{T_{t+n}}{Y_{t+n}} - \frac{G_{t+n}}{Y_{t+n}} \right) + \dots \\ &+ \frac{(1+h_{t+2}) \cdots (1+h_{t+1})}{(1+r_{t+2}) \cdots (1+r_{t+1})} \left(\frac{T_{t+2}}{Y_{t+2}} - \frac{G_{t+2}}{Y_{t+2}} \right) + \frac{1+h_{t+1}}{1+r_{t+1}} \left(\frac{T_{t+1}}{Y_{t+1}} - \frac{G_{t+1}}{Y_{t+1}} \right) \end{aligned}$$

which is a general representation of the debt-to-GDP ratio at time t .

Assuming positive and constant interest rate and GDP growth rates, and summing up the terms in the equation for the infinite periods forward, the general representation of the debt-to-GDP ratio $\frac{B_t}{Y_t}$ at period t is equal to:

$$\frac{B_t}{Y_t} = \frac{(1+h)^n}{(1+r)^n} \frac{B_{t+n}}{Y_{t+n}} - \sum_{n=1}^{\infty} \frac{(1+h)^n}{(1+r)^n} \frac{G_{t+n}}{Y_{t+n}} + \sum_{n=1}^{\infty} \frac{(1+h)^n}{(1+r)^n} \frac{T_{t+n}}{Y_{t+n}}$$

suggesting that debt-to-GDP ratio is equal to present value of the future stock debt and the sum of the discounted primary budget deficit adjusted to the GDP growth. (Giammarioli et al. 2007, p.5-7)

A.3. Deriving the intertemporal budget constraint model in real terms in the empirical co-integration framework

In [Hakkio & Rush \(1991a\)](#) and [Quintos \(1995\)](#) the intertemporal budget constraint model within co-integration approach in the extensive version for equation (12) might be derived by using the difference operator. Defining the stock of the public debt at $t + 1$ in real terms as in (1) and assuming that the real interest rate follows a stochastic stationary process around the mean of r yields the equation of:

$$B_{t+1} - (1+r)B_t = (r_{t+1} - r)B_t + G_{t+1} - T_{t+1}$$

Solving this equation for B_t , the real stock of debt at t is equal to:

$$B_t = \frac{B_{t+1}}{1+r} - \frac{G_{t+1}}{1+r} - \frac{(r_{t+1} - r)B_t}{1+r} + \frac{T_{t+1}}{1+r}$$

Subtracting from this equation the representation defining the dynamic of the stock of the debt one period backwards yields the equation of:

$$\Delta B_t \equiv B_t - B_{t-1} = \frac{\Delta B_{t+1}}{1+r} + \frac{\Delta T_{t+1}}{1+r} - \frac{\Delta G_{t+1}}{1+r} + \frac{r\Delta B_t}{1+r} - \frac{\Delta r_{t+1}B_t}{1+r}$$

Furthermore defining the change of the real stock of the debt infinite periods forward and recursively substituting these terms, ΔB_t is given by:

$$\Delta B_t = \frac{\Delta B_{t+n}}{(1+r)^n} + \sum_{n=1}^{\infty} \frac{\Delta T_{t+n}}{(1+r)^n} - \sum_{n=1}^{\infty} \frac{\Delta G_{t+n}}{(1+r)^n} + \sum_{n=1}^{\infty} \frac{r\Delta B_{t+n-1}}{(1+r)^n} - \sum_{n=1}^{\infty} \frac{\Delta r_{t+n}B_{t+n-1}}{(1+r)^n}$$

where the sum of the last three terms in [Hakkio & Rush \(1991a\)](#) and [Quintos \(1995\)](#) are defined as the change in total spending when the interest rate follows a stationary stochastic process around a positive mean of r in equation (12) obtaining the term of ΔS_{t+n} . ([Quintos 1995, p.410](#))

Appendix B

B.1. Main macroeconomic indicators of Lithuania 1999-2007

	1999	2000	2001	2002	2003	2004	2005	2006	2007
Real GDP annual growth (%)	-1,5	4,1	6,7	6,9	10,2	7,4	7,8	7,8	8,9
Final consumption expenditure annual growth (%)	0,7	N.A.	3,7	4,8	8,9	11,0	10,2	9,1	10,3
of which: Households' consumption annual growth (%)	4,1	N.A.	4,2	5,9	10,4	11,9	12,3	10,6	12,3
of which: Government's consumption annual growth (%)	-8,2	N.A.	2,6	1,1	4,1	8,2	3,5	3,7	3,3
Gross capital formation annual growth (%)	-6,3	N.A.	15,6	14,7	24,8	19,9	-1,3	9,4	21,9
Exports of goods and services annual growth (%)	-16,4	N.A.	21,2	19,4	6,9	4,4	17,7	12,0	4,3
Imports of goods and services annual growth (%)	-12,4	N.A.	17,6	17,7	10,4	14,9	16,4	13,7	11,6
Nominal GDP (milliard Lt)	43666,7	45736,8	48636,9	52070,0	56959,4	62697,8	72060,4	82792,8	98138,7
Final consumption expenditure (milliard Lt)	38243,8	39967,6	41941,1	44271,8	47816,9	52901,9	59958,3	69415,4	81354,2
of which: Households' consumption (milliard Lt)	28449,0	29447,5	31424,3	33230,6	36357,5	40562,4	46312,0	53268,6	63237,8
of which: Government's consumption (milliard Lt)	9731,5	10413,1	10425,0	10893,8	11308,5	12158,3	13502,7	15966,2	17884,7
Gross capital formation (milliard Lt)	9835,1	8639,4	9379,6	10774,5	12461,4	14234,2	17228,3	21803,6	29950,0
Exports of goods and services (milliard Lt)	16973,0	20465,9	24213,9	27444,0	29137,4	32635,5	41457,9	48917,1	53371,8
Imports of goods and services (milliard Lt)	21385,1	23336,1	26897,6	30420,4	32456,3	37073,7	46584,1	57343,3	66537,3
Inflation (average) (%)	0,7	1,0	1,4	0,3	-1,1	1,2	2,7	3,7	5,7
Unemployment rate (%)	14,6	16,4	17,4	13,8	12,4	11,4	8,3	5,6	4,3
Foreign trade balance (% of GDP)	-17,2	-14,6	-14,6	-16,1	-14,4	-13,7	-14,4	-17,4	-18,7
Current account deficit (-) or surplus (+) (% of GDP)	-10,9	-5,9	-4,7	-5,1	-6,8	-7,7	-7,1	-10,6	-14,6
General government revenue (% of GDP)	37,3	35,8	33,2	32,9	31,9	31,8	32,8	33,1	33,8
General government expenditure (% of GDP)	40,1	39,1	36,7	34,7	33,2	33,3	33,3	33,5	35,1
General government deficit (-) or surplus (+) (% of GDP)	-2,8	-3,2	-3,6	-1,9	-1,3	-1,5	-0,5	-0,5	-1,2
Gross consolidated debt (% of GDP)	22,8	23,7	23,1	22,3	21,1	19,4	18,4	18,0	17,0

External sources:

Department of Statistics <http://www.stat.gov.lt/lt> and Bank of Lithuania at <http://www.lb.lt>. The last observation is for November 3, 2008.