



**LUND UNIVERSITY**  
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# **Additional Information in Higher Order Derivatives of the Black-Scholes formula**

**Department of Economics**  
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## **Abstract**

Volatility smile arising from the Black-Scholes model is a long studied subject in option pricing theory. By analysing higher order derivatives of the model, I hope to put another perspective to the problem. According to a study on the American S&P 500 future options, about 90% of the risks of holding an option in five days can be removed by using higher order derivatives of Black-Scholes. The objective of this thesis is to investigate whether or not the price changes of Swedish OMX-index options follow the Taylor expansion of Black-Scholes. The method used here is to compare the observed option price changes after five holding days with the price changes predicted by first, second and third order Taylor series.

The findings and conclusions of this study can be summarised as follows: 1. The mean of prediction errors increases as higher order derivatives are used to predict the price changes, suggesting that the OMX-index options do not follow Taylor expansion of Black-Scholes. 2. As the standard deviation of the prediction errors is decreasing when orders of derivatives in use increase, I conclude that there is additional information in the second and third order derivatives of Black-Scholes. 3. Increasing maturity leads to better predictions, suggesting that the Black-Scholes differential equation is better at describing the movement of long maturity options.

**Keywords:** *Greeks, Black-Scholes, Higher order derivatives, Taylor expansion.*

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## **I. Background**

The Black-Scholes model is the most famous model for option pricing. The elegance of the model lies in that it only uses a few parameters, the calculation is simple, and it creates a perfect hedge to the underlying asset. However, it is also the most debated model. In spite of its elegance it also has deficiencies. One of the variables in the model, volatility, which is assumed to be constant for all strikes and maturities, appears to be different when observing the market. The market prices implies higher volatility for low strikes and lower volatility for high strikes. It forms a pattern known as the volatility smile. Numerous research have studied this phenomenon, some researchers conclude that the volatility smile is merely a post crash fear caused by hedging pressure, which is not seen before 1987. While others have proposed a modified Black-Scholes model with jump diffusion or stochastic volatility incorporated, or both, in order to fit the market better. An analysis of higher order derivatives of the Black-Scholes will put another perspective to this contradiction. The objective of this thesis is to investigate whether or not the price change of the Swedish OMX index option will follow a Taylor expansion of the Black-Scholes model.

### **Basics about options**

Option is a contract between a buyer and a seller in which the buyer has the right, but not the obligation, to buy or sell a certain asset (called the underlying asset) to a predetermined price (called strike price), in a predetermined time in the future (time-to-maturity). If the buyer (long position) chooses to exercise the right, the seller (short position) is obliged to sell or buy the asset at the agreed price. Call options gives the right to buy whereas put option gives the right to sell.

The concept of option is known to be used since ancient time. However the first well-known option market was established in Holland in the early 17<sup>th</sup> century, trading tulips. The purpose of options served as security for producers and merchandisers at the beginning, but soon the

market had attracted many speculators. This incidence, known as the tulip mania, is the first recorded economic bubble in history<sup>[1]</sup>.

Today it is possible to trade options on all kinds of assets, stocks, foreign exchange, commodities, energy etc. The variety of option styles is even more substantial. There are standardised options like European option, American option and Asian option. A standardised option can be traded in a market to a bid-ask spread and supposedly free from arbitrage. Options that are not standardised are exotic options, barrier options, shout options, rainbow option, binary option, power options to name a few. Exotic options can usually not be found in a market but are tailor made and traded over the counter. Usually there are no standardised models for pricing different types of exotic options. Depending on the contract, pricing exotic options can be very complex.

The option style discussed here is European options. At the expiration day if the price of the underlying asset exceeds the strike price of a call option, the buyer receives the difference. However if the price of the underlying asset is below the strike price, the buyer loses the premium he paid for the option. It is the same principle for an European put option.

### **The Black-Scholes model – theory and practice**

The Black-Scholes model was first published in 1973 and its impact on option pricing is evolutionary. It is believed that the Black-Scholes model laid the foundation for the rapid growth of markets for derivatives since publishing<sup>[2]</sup>. The beginning of today's modern option pricing theory dates back to the early 20<sup>th</sup> century. A French mathematician, Louis Bachelier, offered the earliest known analytic valuation for options using stochastic process, now known as the Brownian motion. This is presented in his Ph.d. dissertation, “The Theory of Speculation”<sup>[3][4]</sup>. Since Bachelier, the model has been developed and improved by later researchers. One of them is James Boness, who derived a formula which laid the foundation for today's Black-Scholes. All predecessors of Black-Scholes suffered from one shortcoming, the risk premium were not dealt with in a proper way. To estimate the risk premium, one had

to know or make assumptions on the investor's risk preferences. But Black and Scholes found a way to derive so that the risk premium was removed in the formula. This does not mean that the risk premium has disappeared, in fact, it is already included in the stock price<sup>[5]</sup>. This improvement earned Fisher Black, Myron Scholes and Robert Merton the Economy price in honour of Nobel 1997.

The central idea of Black-Scholes (here after BS) is dynamic delta hedging. By rebalancing a portfolio consisting of bonds and the underlying asset, it is possible to replicate the value of the option. The proportion of the underlying asset in the portfolio is depended on the delta of the option. A portfolio that is delta neutral is relatively insensitive to change in the price of the underlying asset.

While the academics praise the BS-models, it is not as appreciated or maybe even used by the practitioners<sup>\*[6]</sup>. To be able to remove the risks related to the underlying asset, continuous hedging (rebalancing portfolio with infinitely small time intervals) is required. As this is not implementable in reality, static hedging (rebalancing the portfolio once in a while) will creat risks. If the time intervall between each rebalancing is large, then movement of the underlying asset creates risks. In contrast, if the time intervall between each rebalancing is small, transaction fees quickly adds up. Further more it is evident that some of the assumptions made by the BS do not conform to reality. The assumptions used to derive the Black-Scholes-Merton differential equation<sup>[7]</sup>:

1. The stock price follows a geometric Brownien Motion with constant drift and volatility.
2. There are no restrictions on short selling.
3. There are no transaction costs or taxes. All securities are perfectly divisible.
4. There are no dividends during the life of the derivative.
5. The are no riskless arbitrage opportunities.
6. The riskless rate is constant during the life of the option.

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\* Practitioners here refers to traders operating on option markets.

The first assumption is sometimes formulated as stock price has log normal properties. Many studies have tried to recover the empirical stock price distribution. Often the empirical distribution is more skewed to the left than log normal distribution<sup>[8]</sup>. Also, the volatility of the stock is not constant over the option's lifetime. As new informations arriving to the market continuously, the market's expectation on the asset's volatility will change. This risk is not included in the BS-model.

The third assumption is conditions of an ideal world. While in the real world there are transaction costs and taxes, and securities are not perfectly divisible. These are the technicalities that prevent dynamic replication.

The last assumption, riskfree rate is a stochastic variable and not constant. It is possible to hedge the riskless rate though. There are other option pricing models that relaxis this assumption.

### **The volatility smile**

All parameters in the BS-formula can be observed, all except for one, the stock price volatility. The implied volatility can be obtained by substituting all parameters, stock price, strike price, riskfree interest rate, time to maturity, and the market price of the option into the BS formula. This is the volatility estimated by the market. When plotting implied volatility of the options against strikes and maturities, it appears not to be constant, as the BS-model assumes. For equity options, the implied volatility is higher for low strikes and diminishes as the strike compounds<sup>[9][10]</sup>. It is often refered to as the volatility smirk in the literature. For foreign exchange options, the implied volatility forms a smile pattern when plotted against the strikes. Term structure of volatility refers to that the smile is more pronounced closer to the maturity. Over the years the research about the volatility smile has been extensive but so far it is a debate without an solution. Some researchers claim that the smile found in stock options is caused by hedging pressure, crash-o-phobia after the stock market crash in 1987<sup>[11]</sup>. While some others argue that BS underestimates extreme events which the market have priced.

## Criticism and justification

If the BS-model is correct, one could theoretically profit from the market abnormality. Literature testing market inefficiencies are extensive, for example Black and Scholes(1972), Galai(1977), Chiras and Manaster(1978) and Ederington and Guan(2002). The general conclusion is without transacting fee, there are substantial winnings. But once transaction fee is included, there are no arbitrage opportunities. Ederington and Guan (2002) even found that the true smile is some what flatter than the observed smile but it is not completely flat as assumed in the BS\*. In another paper Ederington and Guan (2007: *Higher Order Greeks*) showed that the 2:nd and 3:rd order derivatives of BS are quite good in explaining price changes of the S&P 500 future options, 90% of the risks are removed when higher order derivatives are considered. This justifies the BS-model. Bakshi, Cao and Chen (1995) compared the empirical performance of BS and other alternative models that allowed stochastic volatility, or jump diffusion, or stochastic interest rate. They tested the models' performance based on three yardsticks, internal consistency of implied parameters (implied volatility), the out of sample data, and the hedging ability. Overall, BS had the worst performance in all three categories whereas stochastic volatility model had the best performance. Jump-diffusion added to stochastic volatility improved the fit of the short maturity options. They also found that delta hedging BS consistently resulted in overhedging. According to Haug and Taleb (2008), Nelson (1904) pointed out that delta hedging worked better in theory than in practice. Delta hedging is not a new idea coming with BS, it wasn't used because it wasn't considered as a good hedging strategy.

Analysing Taylor expansion of BS is one way to evaluate the model. Based on the idea of Ederington and Guan (2007), I will do a study on Swedish OMX-index options. The thesis have the following disposition; in section two the issues arising from implemeting the study is discussed. Followed by an analysis of data in section three. In section four the mathematics

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\* The authors meant that the true smile is the part of the smile one can not profit from even before transaction costs. The true smile should be due to model deficiencies. Whereas a part of the observed smile is due to market inefficiency and one can profit from it before transaction costs.

and formulae are described. In the next section programming process is described in detail. In the last section the results are analysed, followed by a conclusion and a discussion.

## II. Problem

One way to find out whether or not BS is an adequate model for options pricing is to analyse its power to predict option's price changes. Traditionally only the four first order derivatives and a second order derivative, gamma, are mentioned in the literature. Ederington and Guan are the first and the only ones to publish a study of all second and third order derivatives of BS. Could the second and third order derivatives of the BS-model contain useful information as well? Does the importance of the additional information from the derivatives depend on the option's strike or maturity? To be able to answer these questions I will compare the observed option price change with the price changes predicted by first, second and third order Taylor expansion of BS.

### Issues and former study

In *Higher Order Greeks* the authors concluded that the option price changes follow the Taylor expansion (here after T-E) of BS. This conclusion could be wrongly interpreted since their result suggests, based on a study of a prespecified holding period, to which they set to five days, most of the option price changes can be explained by combining first, second and third order derivatives of BS. However, nothing is known about when the holding period is less than five days nor when it is more than five days. To verify that option price changes follow T-E of BS, studying different holding days is necessary. If the prediction of option price changes is more precise when holding period is short and less precise as holding period extends, then we can conclude that option prices follow T-E of BS and BS could be an adequate option pricing model. But if the prediction for shorter holding period is less accurate or equally precise as predictions for longer holding period, in both cases the price changes do not follow a T-E of BS. Since nothing is known about the prediction ability for other holding days, we cannot come to any conclusion. However, the author's motivation for setting holding period to five days is that it is long enough to eliminate bid-ask spread and short enough for

the derivatives to remain relatively unchanged. Doing a study on different holding days is preferred, but since the thesis will become too extensive, I limited this thesis to study only five days.

The second issue is the choice of volatility. Since the constant volatility property is violated, the choice of volatility is not obvious. The authors of *Higher Order Greeks* have chosen to use an average of five implied volatilities, volatilities from the strike above and below the option concerned, and the opposite option type with the three strikes closest to the option. For example if we are calculating an averaged volatility for a put option with strike 800, we take an average of implied volatilities from put with strikes 810 and 790, calls with strikes 810, 800 and 790. Motivation for this method is that the observed volatility increase or decrease arising from bid-ask spread will be eliminated by averaging the implied volatilities. This is a good idea, but, if the authors use implied volatility, which is already a deviation from the BS-model, whatever result they will get will not justify the BS-model but a modified BS-model. Second, if using an average implied volatility which most likely will differ from the implied volatility of the option concerned, we will derive a new call or put price when putting back the average implied volatility into the BS-model. When calculating the observed price change of the option, should we use the observed prices or the option prices derived from the averaged implied volatility? This study will focus on how well the BS model fits the market. Since the bid-ask spread is a reality, I will not attempt to eliminate or compromise it.

The last issue concerns framing the subsamples, i.e. dividing the data into maturity and strike subsamples. Referring to *Higher Order Greeks* again, their data was divided into two maturity samples and each maturity sample had five strike samples. The division of maturity samples, one short maturity sample with options within 4-12 weeks to maturity, and one long maturity sample, with options within 13-26 weeks to maturity, is due to the S&P 500 futures having maturity four times a year. Option from the long maturity sample uses an S&P future different from options of the short maturity sample. The strike samples are respectively one ITM sample, one ATM sample and three OTM samples. There are three OTM-samples and only

one ITM-sample because trading in OTMs is much heavier than trading in ITMs. The samples here will be framed in a similar way as in *Higher Order Greeks*. The motivation for the framing needs data analysis and is therefore discussed in the next section instead.

### III. Data

The options chosen to be examined here are options on index of Swedish exchange, OMXS30. The analysing period is 2001-2006. The reason for the choice of the underlying asset is firstly, I want to evaluate BS-model for equity. Secondly, the options traded most frequently on the exchange are preferred. I have collected the closing prices for all options on OMXS30 during the selected period. Closing quotes of the index and daily rate of the STIBOR (riskfree rate) are collected from the homepage of OMX<sup>[12]</sup> respectively from the homepage of the Swedish Central Bank<sup>[13]</sup>.

#### Issues of data

Closing quotations are not the ideal data to use since they can give misleading information. By analysing the daily highs and lows of OMXS30, I found that during the period of 2001-2006, the range of OMXS30 could be up to 12% a day. For example if OMX is 1000kr some day and the range that day is 12%, then at least 6 strikes could be ATM that day. When options are categorised according to strikes, an ITM-option traded earlier that day could be treated as an OTM-option according to closing quote information. If tick data is used instead, it is then possible to match the option's moneyness with the OMX quote at the time the option is executed. But since neither the exchange nor other public sources provide tick data, I can only use closing quotes here. Since the mismatching problem will only trouble options that are thinly traded, I will only use option strikes that are with certainty traded until closing.

Table 1. *Maximum and minimum daily range of OMX from 2000 to 2006. As we can see the difference is huge, lowest range is 0.34% of the OMX and highest range is up to 12% of OMX.*

	min	max	mean	std
<b>2001</b>	0.95%	12.16%	2.70%	1.31%
<b>2002</b>	0.73%	8.86%	2.60%	1.26%
<b>2003</b>	0.71%	7.30%	1.80%	0.78%
<b>2004</b>	0.38%	3.32%	1.20%	0.49%
<b>2005</b>	0.37%	8.23%	0.98%	0.62%
<b>2006</b>	0.34%	5.67%	1.34%	0.79%

By analysing the volume distribution of OMX options over maturity, I found that at an average, 97% of all options traded have less than three months to maturity. The rest of the 3% of the options are between three months to one year to maturity. Since trading in long maturity options is light, the prices are considered to be not efficient, these options are not included in this study.

**Table 2.** *This table shows the trading volume distributed over maturity. The Trading volume has increased since 2001; especially the percentage volume of longer maturity options has increased.*

	<i>One Month</i>	<i>Two Month</i>	<i>Three Month</i>	<i>Volume</i>
<b>Calls 01</b>	80.8%	96.0%	98.2%	1864198
<b>Puts 01</b>	75.3%	94.3%	97.3%	1800397
<b>Calls 02</b>	82.1%	95.3%	98.7%	2023676
<b>Puts 02</b>	77.5%	93.4%	98.0%	2011138
<b>Calls 03</b>	80.5%	95.9%	98.2%	2327175
<b>Puts 03</b>	81.2%	96.0%	98.4%	1885156
<b>Calls 04</b>	75.5%	95.5%	99.2%	2485458
<b>Puts 04</b>	66.0%	89.5%	96.8%	2445902
<b>Calls 05</b>	73.5%	94.7%	98.9%	3957714
<b>Puts 05</b>	66.8%	89.9%	96.1%	3725646
<b>Calls 06</b>	64.3%	89.2%	96.4%	5305543
<b>Puts 06</b>	59.0%	83.5%	94.7%	5233856

### Sample framing

The data will be divided into four strike samples each for calls and puts. I separate the call options from the put options because generally, the implied volatility is high for OTM puts and ITM calls, while it is lower for OTM calls and ITM puts. The ITM sample includes calls with moneyness\* between 0.94 and 0.98 and puts between 1.02 and 1.06. The ATM sample includes calls and puts within moneyness 0.98-1.02. There are two OTM samples, OTM1 includes calls with moneyness between 1.02 and 1.06 and puts between 0.94 and 0.98. In OTM2 sample we find calls with moneyness of 1.06-1.10 and puts with moneyness of

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\* Moneyness is calculated by dividing strike price with stock price.

0.90-0.94. Other data will be excluded from this study, since the options outside this range are considered thinly traded. The moneyness calculated from options that are thinly traded are not always reliable.

Options with maturities longer than 13 weeks are not included in this study. The maturity-samples will be divided on a weekly bases. For example the sample that has 13 weeks to maturity includes options that have 61 to 65 business days to maturity. (which means it uses option prices from same the week as the option expires. Usually it is not recommended to use this data since the options prices can be manipulated. But I have used it in my study anyway and as we can see the data looks a bit strange.)

## IV. Mathematics and formulae

### Black-Scholes

After deriving the Black-Scholes-Merton differential equation, we obtain the following formulae for call and put<sup>[14]</sup>. The notations used here are the same as in J.C. Hull (2006).  $S$  is the stock price,  $K$  is the exercise price,  $r$  is the riskfree interest rate,  $T$  is time-to-maturity and  $\sigma$  is the volatility.  $c$  is the European call price and  $p$  is European put price.  $N(x)$  is the standard cumulative normal-distribution function.

$$c = SN(d_1) - Ke^{-rT}N(d_2)$$
$$p = Ke^{-rT}N(-d_2) - SN(-d_1)$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} = d_2 + \sigma\sqrt{T}$$
$$d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

$$N(x) = \Phi_{(0,1)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$$

### The five Greeks

First order derivatives of the BS-model are often mentioned and analysed in literature. They are referred to as the greeks, with delta being the most important. “Delta hedging” refers to hedging this derivative. Delta measures the change of the option's value relative to the change of underlying asset's price. The delta is different for call and put options.

*Delta for calls :*

$$\Delta = N(d_1)$$

*Delta for puts :*

$$\Delta = N(d_1) - 1$$

The second most important greek is probably vega. Hedging vega is wise since the volatility is not constant over an option's life time. It measures the option's sensitivity to change of the

underlying asset's volatility.  $n(x)$  is the standardised probability distribution function.

*Vega :*

$$v = S\sqrt{T}n(d_1)$$

$$n(X) = N'(X)$$

Theta is sometimes referred to as the time decay of the option. It measures the option's value sensitivity to time. The change of time is a predictable parameter. As the time-to-maturity is always decreasing, the textbooks usually display the negative theta. The thetas here are positive.

*Theta for calls :*

$$\theta = \frac{\sigma Sn(d_1)}{2\sqrt{T}} + rKe^{-rT}N(d_2)$$

*Theta for puts :*

$$\theta = \frac{\sigma Sn(d_1)}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$

Rho measures the option's sensitivity to change of riskfree interest rate. Interest rate is not constant over an option's life time as one of the BS-model's assumptions but changes in interest rate often have a small impact on the option's value.

*Rho for calls :*

$$\rho = TKe^{-rT}n(d_2)$$

*Rho for puts :*

$$\rho = -TKe^{-rT}n(-d_2)$$

Gamma is actually a second order derivative of the BS-model but since gamma is a useful indicator, it is regarded as one of the greeks. Gamma measures the sensitivity of the delta, how much the option's price will change relative to delta changes.

*Gamma :*

$$\Gamma = \frac{n(d_1)}{S\sigma\sqrt{T}}$$

### **Higher order derivatives**

Some of the second and third order derivatives have names, probably because they are useful or interesting in one way or another. For example, “vanna” is the derivative of both stock

price and volatility, and “color” is the derivative of gamma and time. There are fifteen second and third order derivatives in total, excluding gamma and all derivatives of rho. The derivatives of rho are not included here due to their impact on option price changes are very small<sup>[15]</sup>. The formulae are shown in Appendix Formulae. Since not all of the derivatives have names I will present the derivatives with symbols.

**Taylor expansion**

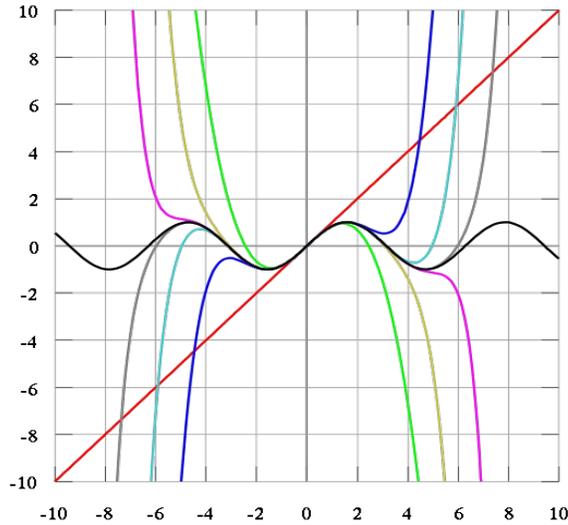
Taylor expansion is used for approximating either an unknown or known function  $f(x)$  at an arbitrary fixed point  $a$  by a polynomial  $p(x)$ <sup>[16]</sup>.  $r_n(x)$  is the remainder term of the  $n$ :th order Taylor expansion of  $f(x)$ , if  $f(x)$  is continuous around  $a$ . The BS is continuous in any point before expiration.

$$p(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \dots + \frac{f^{(n)}(a)(x-a)^n}{n!} + r_n(x)$$

As  $n$  goes to infinity,  $r_n(x)$  goes to zero and  $p(x) = f(x)$ . As  $n$  is finite, we will have a remainder term, an “approximation error”, the difference between the real value calculated from  $f(x)$ , and the approximated value calculated from  $p(x)$ . We want to minimise the approximation error as much as possible, of course. The magnitude of the remainder term depends on two things, one, how far  $x$  is from  $a$ . The closer to  $a$  the better is the approximation (see figure 1). Two, the size of  $n$ . The larger is the  $n$  the better is the approximation.

The function in figure 1 is  $\sin(x)$  (black line) and point  $a$  is set to zero. If we do Taylor expansion of first order (red line), we can approximate function  $\sin(x)$  quite well for  $x = a \pm 1$ . If we try to approximate  $x = 4$ , the first order expansion will give you a completely wrong idea of what  $\sin(4)$  is. The further away  $x$  is from  $a$  the larger is the approximation error  $r(x)$ . But if we do Taylor expansion of sixth degree (pink line), the polynomial will approximate  $x=4$  quite well. This thesis will examine up to the 3:rd order Taylor expansion of BS. Mathematically speaking, the “approximation error” can get quite large doing only a third

**Figure 1.** This figure provides the idea of Taylor expansion.



order expansion. Also, the holding days are set to 5,  $x - a = 5$ , this could also generate a sizable remainder term. This means even if BS is the right model, this study could show other result.

First order expansion of BS,  $n = 1$ :

$$\delta \hat{O}_1 = \Delta \cdot \delta s + v \cdot \delta \sigma + \theta \cdot \delta t + \rho \cdot \delta r$$

$\delta \hat{O}_1$  is the predicted option price change after 5 holding days. Delta is calculated from the option at time  $t$  and  $\delta s$  is the difference in OMX-price between time  $t$  and  $t+5$ . Using the term “predicting option price change” is actually a bit misleading since ex post data is used to “predict” future price change. The other variables are calculated in the same way. The second and third order Taylor expansion are shown in Appendix Formulae.

### The testmodel

To test whether or not option price changes will follow a T-E of BS, one can analyse the

absolute errors or the mean squared errors. The absolute error is calculated by taking the absolute difference between the observed data and the data predicted by the model. The mean squared errors are the square of the absolute errors. The absolute errors are analysed here because they can be interpreted as monetary units. For example, if the error between an observed price change and the change predicted by model is 1.5, then the model has mispriced the change with 1.5 Swedish Kronor.

The test model:

$$\text{Absolute Forecast Error 1} = |\delta O - \delta \hat{O}_1| \quad (4)$$

$$\text{Absolute Forecast Error 2} = |\delta O - \delta \hat{O}_1 - \delta \hat{O}_2| \quad (5)$$

$$\text{Absolute Forecast Error 3} = |\delta O - \delta \hat{O}_1 - \delta \hat{O}_2 - \delta \hat{O}_3| \quad (6)$$

$\delta O$  is the change in option price between time  $t$  and  $t+5$ .  $\delta \hat{O}_1, \delta \hat{O}_2, \delta \hat{O}_3$  is calculated according to equations (1) (2) (3) in Appendix Formulae.

## **V. Procedure**

To be able to handle the massive option data and make computations efficient, programming is inevitable. I choose to use MatLab because it is powerful for calculations and matrix operations. The programming is carried out in 4 steps, the first step is to handle the collected data, to read it into Matlab, sorting the data and make them easy to manage. The second step is to perform calculations of implied volatilities, moneyness and all derivatives of the options. The third step is to find data with specific attributes, for example, to find all ATM call options with 10-15 days to maturity. After, the observed option price change and the AFEs (absolute forecast errors from equations (4)(5)(6)) are calculated for all samples. At the fourth and final step the means and standard deviations of observed price change and AFEs are calculated and put together in the Appendix Tables. The rest of the programmes are made to help analysing the results.

### **Data management**

The option data collected from OMX are in text format and one file contains data from one month. All options that were listed on OMX each day. Program *extractdata* reads the text files into Matlab and sorts the options that are not OMX options or were not traded. The read-in data is then processed in the next program *sortera*, where the option data is sorted by their names. For example an option is named OMXS305A700, the program finds all options with this name in the monthly data and save the date it was traded and the closing prices. In october 2004 the OMX options changed names from OMX/year/month/type/strike to OMXS30/year/month/type/strike, hence the treatment of option data from 2004-2005 required special care.

### **Calculations**

To proceed with calculations of derivatives we must first find implied volatilities for all

options. I use a built-in Matlab function, *blsimpvol*, to obtain implied volatilities. The built-in function was able to solve most but not all implied volatilities. The reason why some implied volatilities could not be obtained is unknown. The problem could be the data or the built-in function. After, moneyness is computed for all options in program *moneyness*. Before proceeding to calculations of derivatives, options with invalid implied volatility had to be deleted first in *invimpvol*. I choose not to use options with implied volatility higher than 45% because high volatilities are more likely to be caused by bid-ask spread or non-synchronised data rather than estimated by the market. These data are also deleted in *invimpvol*.

### **Making subsamples**

The main program at this step is *run*, it is mainly for getting datas and sending variables between the functions *sampcol* and *forecaster*. The parameters are set in *run*, the strikes, maturity and holding period. *Sampcol* finds the options that fit the parameters, and if it finds the very same option traded 5 days later, it saves the option's name and traded dates in a table. For example we are looking for all ATM calls that expires in March 2003 and have 3 weeks to maturity. The program first looks for the date of expiration, then it finds the dates between 15-11 days before expiration. The matlab class *fints* is used to find whether or not an option is traded in those specified days. The programme *forecaster* starts with converting all option information (including implied volatilities and all derivatives) from class *struct* to class *fints*. By using *fints*, it is easy to calculate  $\delta s$ ,  $\delta\sigma$ ,  $\delta t$ ,  $\delta r$ , which are needed for calculating  $\delta O$ ,  $\delta\hat{O}_1$ ,  $\delta\hat{O}_2$  and  $\delta\hat{O}_3$ .

### **Analysis**

At the first attempt, I intend to analyse the data annually, considering the fact that market's up and down trends or the variation in volatility could have effect on prediction abilities. However I realised it would be too much data to be analyse. Instead, I put together the data of six years to be one sample. Even by doing this way, a total of 16 tables were resulted, let alone

how many tables I would have had if the data are analysed annually. The programme *gruppering* begins with putting together the data of six years, following by calculating the AFEs. *Analys* calculates the mean and standard deviation for all AFEs. After the first observation, *analys* was extended to carry out some linear regressions. *Analys* put the data together in a table while *linreg* is the programme actually performing the linear regression<sup>[17]</sup>. The programme *acrosstrikes* tests the relation between moneyiness and mean of AFEs and the relation between moneyiness and standard deviation of AFEs. Programme *lilliefors* makes lilliefors test for all samples and puts the result in tables 17-18. More details about the test and why it is used is explained in the next section. I used the built-in Matlab function, *lillietest* when testing for normality. When testing for log normality, I take the logarithm of the AFEs before doing the normal lilliefors test. After, a hypothesis test was made for AFEs that showed normality or log normality in program *diffm*. The program *ny* puts the result in table 19.

## VI. Analysis

Results from the test model are put together in tables 1-16 in Appendix Tables. First, we want to analyse the result by strike categories. The first four tables are the result of the four strikes of the calls. The following four tables are result of the four strikes of puts. In continuing, the result will be analysed by categorising the AFEs. For example, table 9 contains the result of all call AFEs using first order T-E. The a-tables (of tables 1-16) are the means of  $\delta O$  (observed option price change) or any of the AFEs. The b-tables are the standard deviations of  $\delta O$  or any of the AFEs. I have made the following observations:

**I.** The mean of AFEs is increasing as more derivatives are used to explain price changes. This seems to be true for most strikes and maturities. The standard deviation of AFEs on the other hand, seems to be decreasing as more derivatives are considered.

**II.** Both mean and standard deviation of the AFEs decreases with increasing maturity. While for the observed price change, the mean is rather random and standard deviation is more less constant.

**III.** Mean and standard deviation of AFEs decreases as options go from ITM to OTM2. In contrast, for the observed option price change, the mean increases as the options go from ITM to OTM2. Standard deviation decreases quite dramatically with increasing/decreasing moneyness for calls/put.

**IV.** The difference between mean of AFE2 and AFE3 is only marginal.

Because the observations should be varified in a more scientific way, simple linear regressions will be used to test the relations. To verify a positive relation, i.e. increasing  $x$  yields increasing  $y$ , I set a constraint for beta from the regressions, slope of the line, must be greater than zero, and  $R^2$  \* must be greater than 0.5. To varify a negative relation, i.e. increasing  $x$  yields decreasing  $y$ , beta estimated from the regression must be less than zero and  $R^2$  must be

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\*  $R^2$  measures how much of the data can be explained by the regression.

greater than 0.5. In the following text each observation will be tested by the method mentioned above.

**I.** *Mean of AFEs increases as orders of derivatives in use increase. Standard deviation of AFEs decreases as orders of derivatives in use increase.*

Here the variable  $x$  is the number of orders of derivatives in use, so  $x = [1,2,3]$ . Variable  $y$  is the respective calculated mean of AFEs. Note that  $\delta O$  is not included in the regression. Tables 1.c – 8.c show results from the regressions. I found that only 4 of 96 regressions did not show a positive relation. Tables 1.d – 8.d summarise the regression result of the second statement. Here, 36 of 96 regressions did not show negative relation. The regressions revealed that the mean of AFEs increases as orders of derivatives in use increase. As the expected or desirable result is the other way around, the mean should go towards zero. The Taylor approximation is less exact when more derivatives are considered, the option price changes does not follow a T-E of BS. The standard deviation on the other hand is often decreasing when higher order derivatives are considered. This is true for about 2/3 of the data. Interpretation of decreasing standard deviation is that BS always predicts price changes with about the same error.

**II.** *The mean of AFEs decreases as time-to-maturity increases. Standard deviation of AFEs also decreases as time-to-maturity increases.*

Tables 1.e – 8.e show regression results from relation between mean of AFEs and time to maturity. Tables 1.f-8.f shows regression result from relation between standard deviation of AFEs and time to maturity. All regressions show downward trend for increasing maturity except for ITM puts. Regressions are also made for observing the relation between  $\delta O$  and maturity. The regressions show no relation between observed option price and maturity. This result suggests that while the observed option price changes are rather random and it's standard deviation is rather constant, the models prediction is more precise as the maturity increases.

**III.** *Mean of AFEs decreases as moneyness goes from ITM to OTM2. Standard deviation of*

*AFEs also decreases as moneyness goes from ITM to OTM2.*

Tables 9.c – 14.c summarise the regression results of the first statement. Only 3 out of 72 regression failed to show positive relation. Tables 9.d – 14.d show how standard deviation vary with moneyness. Out of 72 regressions, 28 did not show a negative relation. Same regression was made for observed option price changes for comparison. I found that the mean of  $\delta O$  increases as moneyness increases/decreases for call/put. As for standard deviation, the regression betas decreases drastically as strikes go from ITM to OTM2\*. For the observed price changes, the difference between the standard deviation estimated from the ITM options and from the OTM2 options is large. But once the higher order derivatives are considered, the difference gets smaller. If standard deviation is considered as risk, the BS derivatives can eliminate some risk for high risk options, i.e. ITMs and ATMs. Where as for options with less risk, hedging with BS derivatives will not have much effect.

**IV.** *The difference of AFE from second and third derivatives is only marginal.*

To test the difference between the means of two samples, I will do a two side hypothesis test<sup>[18]</sup>. This test takes the assumption that the samples are normally distributed. First I have to perform a normality test on all samples. I use Lilliefors test<sup>[19]</sup> to test for normality because it is a non-parametric tes and the mean and standard deviation of the samples does not have to be known. The mean and standard deviation estimated from the samples are not to be mixed up with the “real” mean and standard deviation, that is the reason why it is best to use a non-parametric test. Result from Lilliefors test for all samples are shown in table 17. Most of the samples are not normally distributed. Normality appears mostly in ITM samples for both calls and puts, and almost non-existent in OTM-samples. By looking at distributions from randomly picked samples, I observed that the distributions are often skewed to the left and leaves a long tail to the right. Since we are analysing absolute errors, all values are greater than zero. One possible distribution for left side skewness and positive numbers is log normal

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\* The betas estimated from  $\delta O$  has a range from  $-1.47$  to  $-5.41$ , compared to betas estimated from AFEs, which are almost never less than  $-1$

distribution. If AFEs are log normally distributed, then the logarithm of AFEs are normally distributed. The samples are tested for log normality and the result is shown in table 18. I found that log normality appear mostly in ITMs and OTM2s and often in maturities longer than 5 weeks. The mean comparisons will only be made for samples that are normal or log normal. The result is shown in 19a. Also I made mean comparisons between AFE1 and AFE2. The result is shown in 19b. As we can see, statistically speaking, we cannot show that there is difference between means of AFE2 and AFE3. Whereas we can find significant difference between means of AFE1 and AFE2. This also supports that option prices do not follow T-E of BS.

## **Conclusion**

I have come to the following conclusions:

1. Higher order derivatives of BS failed to explain option price changes after 5 holding days. The mean of AFEs does not decrease as more derivatives are used, which is the result we expected. This is a parallel to delta vs. gamma hedging. In theory the hedging performance should be improved by making both delta and gamma neutral, but in practise it is quite the opposite. Also we cannot find statistic significance in the difference between the means of AFE2 and AFE3. Should options price follow T-E of BS the mean of AFE 3 should be smaller than AFE2.
2. Is there additional information in 2:nd and 3:rd order derivatives? Yes, considering the standard deviation of AFEs are substantially smaller once higher order derivatives are considered. BS fails to predict the price changes, on the other hand we can predict the prediction error within a small interval.
3. BS is much better at predicting the longer maturity samples than the short maturity samples. One explanation could be that the differential equation of BS can describe the movement of long maturity options quite well, while short maturity options are more sensitive to new information arrived in the market and cannot be described by the differential equation.

## Discussion

*Why is the result in contradiction with “Higher order greeks”?* There are some main differences in the two studies. First of all, we use different underlying asset and thus we actually tested two different models. Second, as we use different measurement for volatility, one can wonder how much will the result differ by this fact. Also, this study did not include options with maturities longer than three month, whereas in the other study the options with shorter than four weeks maturity are not included.

*What lacked in this study and what can be suggested for further studies?* Using tick data would significantly strengthen the result. But such data will be extensive and harder to find. If one is interested in using the derivatives in practice, for example to estimate risks of option portfolios, one should study the distributions and extreme values of the AFEs. Also the practitioner would find it interesting to study the forecast errors, not the absolute forecast errors. I have looked at some data where I calculated the errors instead of absolute errors, the result was very little in difference. This could mean that BS tend to overhedging, the same result as in Baksi, Cao and Chen (1995). Also, it would be interesting to investigate whether the ups and downs of the market trend will affect the predicting ability of the derivatives. This could be interesting both from a theoretical point of view and practical point of view. Theoretically market trends shouldn't affect the prediction. But if it does one can exploit this result in practice.

*Is BS an adequate model for pricing options?* Actually, there is nothing wrong with the theory. Black and Scholes solved a differential equation and it was genius. Besides the obvious unconformities, the problem is perhaps that the differential equation does not describe the real option's movement so well. Maybe the real option's movement should be described more as a discontinuous function with jumps, because of the gap of the bid-ask spread. My general opinion is that BS is an insufficient model, though it seems that the BS is fairly good at pricing long maturity options.

## Endnotes

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- [3] [http://en.wikipedia.org/wiki/Louis\\_Bachelier](http://en.wikipedia.org/wiki/Louis_Bachelier)
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## Appendix:

### Tables

Tables 1-4 show the result of the four call strike samples. Table 5-8 show the result of the four put strike samples. Tables 9-16 show the same result as tables 1-8 but are organised after AFE (absolute forecast error). For the tables 1-16, the a-tables show the means of the samples, whereas b-tables show the standard deviations. C-tables display the regression results from testing the relation between mean and AFE. Alfa is where the linear function intersect with the y-axis. Beta is the slope of the line and  $R^2$  is how much of the data can be explained by the function. D-tables are regression result from testing the relation between standard deviation and AFE. E-tables are the results from testing the relation between mean and maturity. F-tables are the results from testing the relation between standard deviation and maturity. The numbers that are bold mark the regressions that violate against the assumed relations. Table 17 displays the test result from testing normality. Table 18 shows test result from testing log normality. Table 19.a1 and b1 show the test result from testing whether there or not there is difference between the mean of AFE2 and AFE3. Table 19.a2 and b2 show the test result from testing the difference between the mean of AFE1 and AFE2. Table 19.a test the samples that showed normality, based on result from table 17. Table 19.b test the sample that showed log normality, based on result from table 18.

## ITM Calls

**Table 1.a**

weeks to maturi	dO	AFE 1	AFE 2	AFE 3
2	-9.09	7.73	8.61	8.62
3	-8.95	5.51	6.00	6.55
4	-4.94	4.32	5.64	5.58
5	-7.17	4.37	5.03	5.32
6	-6.61	3.76	4.51	4.52
7	-6.43	3.52	3.93	3.96
8	-8.91	3.45	3.74	3.92
9	-8.33	2.52	4.12	4.19
10	-7.78	3.30	3.99	3.73
11	-4.06	2.89	3.78	4.02
12	-3.54	1.65	3.17	3.39
13	-5.89	1.92	3.17	3.18

**Table 1.b**

dO	AFE 1	AFE 2	AFE 3
16.44	5.66	5.55	9.3
16.80	4.59	2.93	3.73
17.49	3.27	5.06	3.13
15.14	4.54	2.19	2.95
12.31	3.20	2.75	3.95
14.14	5.14	2.74	2.65
18.03	3.41	1.33	1.57
14.76	1.76	1.22	1.27
12.93	1.53	1.26	1.32
13.28	2.19	1.43	3.88
12.99	1.86	0.84	0.82
10.93	1.18	0.61	0.81

**Table 1.c**

weeks to maturi	alfa	beta	R <sup>2</sup>
2	7.43	0.44	0.76
3	4.98	0.52	1.00
4	3.93	0.63	0.71
5	3.96	0.47	0.95
6	3.50	0.38	0.77
7	3.36	0.22	0.81
8	3.23	0.24	0.98
9	1.94	0.84	0.78
10	3.24	0.21	<b>0.38</b>
11	2.43	0.57	0.90
12	1.00	0.87	0.84
13	1.49	0.63	0.75

**Table 1.d**

alfa	beta	R <sup>2</sup>
3.19	<b>1.82</b>	0.73
4.61	-0.43	<b>0.27</b>
3.96	-0.07	<b>0.00</b>
4.82	-0.80	<b>0.44</b>
2.56	<b>0.37</b>	<b>0.38</b>
6.00	-1.24	0.78
3.94	-0.92	0.65
1.90	-0.24	0.67
1.58	-0.10	0.54
0.81	<b>0.84</b>	<b>0.45</b>
2.22	-0.52	0.76
1.24	-0.19	<b>0.41</b>

**Table 1.e**

	alfa	beta	R <sup>2</sup>
dO	-8.89	0.28	0.27
AFE 1	6.84	-0.41	0.81
AFE 2	7.39	-0.37	0.74
AFE 3	7.58	-0.38	0.76

**Table 1.f**

alfa	beta	R <sup>2</sup>
17.86	-0.43	0.49
5.88	-0.36	0.73
5.19	-0.38	0.75
6.4	-0.46	0.51

## ATM Calls

**Table 2.a**

weeks to maturi	dO	AFE 1	AFE 2	AFE 3
2	-3.24	6.60	8.72	8.55
3	-2.35	4.67	6.44	6.54
4	0.14	3.78	5.40	5.46
5	-0.96	3.53	4.75	4.76
6	-2.83	3.17	4.15	4.17
7	-2.58	2.78	3.73	3.64
8	-1.20	2.35	3.39	3.33
9	0.76	2.24	3.08	3.05
10	-2.74	2.31	2.93	2.92
11	-0.36	1.89	2.62	2.54
12	0.48	1.92	2.49	2.43
13	-3.00	2.42	3.21	3.11

**Table 2.b**

dO	AFE 1	AFE 2	AFE 3
11.33	4.80	4.77	4.41
10.33	3.18	3.36	3.09
11.45	2.43	2.73	2.28
10.52	2.48	2.08	2.02
9.18	2.23	1.83	1.74
9.48	1.83	1.59	1.52
10.79	1.69	1.60	1.29
9.27	1.40	1.33	1.33
9.12	1.69	1.30	1.26
9.77	1.01	1.25	1.28
10.2	1.30	1.09	1.20
7.06	1.37	1.06	1.21

**Table 2.c**

weeks to maturi	alfa	beta	R <sup>2</sup>
2	6.00	0.98	0.69
3	4.02	0.93	0.79
4	3.20	0.84	0.78
5	3.11	0.62	0.76
6	2.83	0.50	0.77
7	2.52	0.43	0.67
8	2.05	0.49	0.70
9	1.98	0.41	0.72
10	2.10	0.31	0.74
11	1.70	0.33	0.66
12	1.77	0.26	0.67
13	2.22	0.35	0.64

**Table 2.d**

alfa	beta	R <sup>2</sup>
5.05	-0.19	0.80
3.30	-0.04	<b>0.11</b>
2.63	-0.08	<b>0.12</b>
2.65	-0.23	0.85
2.42	-0.24	0.88
1.95	-0.15	0.92
1.93	-0.20	0.90
1.43	-0.04	0.81
1.85	-0.22	0.82
0.91	<b>0.13</b>	0.84
1.30	-0.05	<b>0.26</b>
1.37	-0.08	<b>0.26</b>

**Table 2.e**

	alfa	beta	R <sup>2</sup>
dO	-2.22	0.1	0.06
AFE 1	5.58	-0.33	0.74
AFE 2	7.61	-0.45	0.77
AFE 3	7.62	-0.46	0.79

**Table 2.f**

alfa	beta	R <sup>2</sup>
11.59	-0.23	0.48
3.98	-0.25	0.74
4.02	-0.27	0.76
3.58	-0.23	0.69

## OTM1 Calls

**Table 3.a**

weeks to maturi	dO	AFE 1	AFE 2	AFE 3
2	-1.14	4.91	5.71	5.56
3	-0.48	3.42	4.82	4.69
4	0.66	2.97	4.06	4.10
5	0.17	2.76	3.79	3.81
6	-1.70	2.66	3.65	3.62
7	-0.62	2.30	3.23	3.17
8	0.47	2.11	2.87	2.85
9	2.02	1.88	2.45	2.55
10	-0.06	1.74	2.22	2.28
11	0.17	1.73	2.24	2.21
12	1.05	1.71	2.07	2.09
13	1.81	1.54	2.11	2.22

**Table 3.b**

dO	AFE 1	AFE 2	AFE 3
5.67	3.67	4.27	4.06
6.49	2.58	3.27	3.02
7.36	2.30	2.59	2.37
7.49	2.01	2.24	2.10
6.88	1.81	1.96	1.88
7.58	1.57	1.76	1.62
7.00	1.58	1.58	1.34
6.72	1.35	1.19	1.17
5.98	1.00	1.03	1.00
6.26	1.12	1.12	1.01
5.96	1.28	1.04	1.02
6.52	0.89	0.82	0.94

**Table 3.c**

weeks to maturi	alfa	beta	R <sup>2</sup>
2	4.74	0.33	0.59
3	3.03	0.64	0.68
4	2.58	0.57	0.78
5	2.40	0.53	0.77
6	2.35	0.48	0.73
7	2.03	0.44	0.70
8	1.87	0.37	0.73
9	1.62	0.34	0.86
10	1.55	0.27	0.82
11	1.59	0.24	0.70
12	1.58	0.19	0.79
13	1.29	0.34	0.86

**Table 3.d**

alfa	beta	R <sup>2</sup>
3.61	<b>0.19</b>	<b>0.41</b>
2.52	<b>0.22</b>	<b>0.39</b>
2.36	<b>0.03</b>	<b>0.04</b>
2.03	<b>0.04</b>	<b>0.16</b>
1.82	<b>0.03</b>	<b>0.23</b>
1.59	<b>0.03</b>	<b>0.08</b>
1.74	-0.12	0.76
1.41	-0.09	0.82
1.01	<b>0.00</b>	<b>0.00</b>
1.19	-0.05	0.73
1.37	-0.13	0.79
0.84	<b>0.02</b>	<b>0.15</b>

**Table 3.e**

	alfa	beta	R <sup>2</sup>
dO	-1.22	0.19	0.38
AFE 1	4.28	-0.24	0.80
AFE 2	5.60	-0.31	0.91
AFE 3	5.48	-0.30	0.91

**Table 3.f**

alfa	beta	R <sup>2</sup>
6.97	-0.04	0.06
3.26	-0.20	0.82
3.92	-0.27	0.86
3.62	-0.24	0.83

## OTM2 Calls

**Table 4.a**

weeks to maturi	dO	AFE 1	AFE 2	AFE 3
2	-0.68	4.08	3.65	3.44
3	0.36	3.36	3.58	3.59
4	0.62	2.47	2.99	3.03
5	-0.25	2.35	3.14	3.22
6	-1.40	2.39	3.34	3.27
7	0.93	2.29	2.99	3.00
8	0.57	2.09	2.77	2.83
9	1.74	1.66	2.23	2.35
10	1.12	1.66	2.41	2.36
11	0.62	1.57	2.02	2.01
12	2.25	1.33	1.99	2.05
13	3.51	1.26	1.81	1.99

**Table 4.b**

dO	AFE 1	AFE 2	AFE 3
3.21	2.78	2.79	2.76
5.27	2.47	2.32	2.32
4.96	1.91	1.95	1.91
4.93	1.64	1.82	1.81
5.07	1.63	1.80	1.74
7.21	1.52	1.53	1.44
5.57	1.42	1.52	1.48
4.74	1.30	1.25	1.23
7.48	1.21	1.26	1.11
4.38	1.08	0.87	0.89
7.40	0.71	0.80	0.88
5.17	0.84	1.06	1.15

**Table 4.c**

weeks to maturi	alfa	beta	R <sup>2</sup>
2	4.37	<b>-0.32</b>	0.97
3	3.28	0.11	0.78
4	2.26	0.28	0.81
5	2.04	0.43	0.82
6	2.13	0.44	0.69
7	2.04	0.36	0.77
8	1.82	0.37	0.81
9	1.39	0.34	0.88
10	1.44	0.35	0.70
11	1.42	0.22	0.75
12	1.08	0.36	0.81
13	0.97	0.36	0.92

**Table 4.d**

alfa	beta	R <sup>2</sup>
2.8	-0.01	0.66
2.52	-0.07	0.78
1.92	<b>0.00</b>	<b>0.03</b>
1.59	<b>0.08</b>	0.73
1.61	<b>0.06</b>	<b>0.42</b>
1.58	-0.04	0.69
1.41	<b>0.03</b>	<b>0.35</b>
1.33	-0.04	0.97
1.30	-0.05	<b>0.41</b>
1.14	-0.1	0.69
0.63	<b>0.08</b>	<b>1.00</b>
0.70	<b>0.16</b>	<b>0.95</b>

**Table 4.e**

	alfa	beta	R <sup>2</sup>
dO	-1.29	0.28	0.58
AFE 1	3.82	-0.21	0.86
AFE 2	4.01	-0.17	0.92
AFE 3	3.92	-0.15	0.91

**Table 4.f**

alfa	beta	R <sup>2</sup>
4.22	0.16	0.21
2.75	-0.16	0.90
2.75	-0.16	0.91
2.68	-0.15	0.88

## ITM Puts

**Table 5.a**

weeks to maturi	dO	AFE 1	AFE 2	AFE 3
2	-1.82	9.60	10.64	13.52
3	-7.51	4.88	5.73	5.97
4	-8.06	3.52	4.41	4.66
5	-6.96	3.10	3.84	3.98
6	-0.32	2.77	3.70	3.78
7	-4.30	2.50	3.36	3.40
8	-4.18	2.06	2.95	2.93
9	-8.15	1.65	2.61	2.70
10	7.76	3.54	3.76	3.77
11	-6.79	1.70	2.53	2.74
12	-17.56	1.79	3.29	2.97
13	-7.00	2.12	3.29	3.38

**Table 5.b**

dO	AFE 1	AFE 2	AFE 3
23.15	7.59	11.47	46.67
18.01	3.10	3.22	4.51
17.71	2.38	2.41	2.36
15.81	2.12	1.98	1.98
16.99	1.99	1.81	1.84
15.29	1.82	1.54	1.67
16.53	1.43	1.35	1.19
14.72	1.40	1.00	1.09
18.25	3.26	1.49	2.30
15.09	0.61	0.65	0.73
7.34	1.42	0.45	0.85
18.47	1.26	0.89	0.87

**Table 5.c**

weeks to maturi	alfa	beta	R
2	7.34	1.96	0.93
3	4.44	0.55	0.91
4	3.05	0.57	0.90
5	2.76	0.44	0.87
6	2.41	0.50	0.81
7	2.19	0.45	0.78
8	1.78	0.44	0.73
9	1.27	0.52	0.82
10	3.47	0.11	0.79
11	1.29	0.52	0.90
12	1.51	0.59	0.56
13	1.67	0.63	0.80

**Table 5.d**

alfa	beta	R
-17.17	<b>19.54</b>	0.82
2.20	<b>0.71</b>	0.81
2.40	-0.01	<b>0.10</b>
2.16	-0.07	0.73
2.03	-0.08	0.59
1.83	-0.08	<b>0.29</b>
1.56	-0.12	0.97
1.47	-0.15	0.53
3.31	-0.48	<b>0.29</b>
0.54	<b>0.06</b>	0.98
1.47	-0.28	<b>0.33</b>
1.40	-0.20	0.80

**Table 5.e**

	alfa	beta	R
dO	-3.09	-0.31	0.04
AFE 1	6.53	-0.44	<b>0.50</b>
AFE 2	7.32	-0.42	<b>0.47</b>
AFE 3	8.59	-0.55	<b>0.44</b>

**Table 5.f**

alfa	beta	R
20.62	-0.56	0.30
4.82	-0.33	<b>0.42</b>
6.52	-0.55	<b>0.45</b>
19.91	-1.92	<b>0.28</b>

## ATM Puts

**Table 6.a**

weeks to mat	dO	AFE 1	AFE 2	AFE 3
2	-0.83	7.08	8.97	9.10
3	-0.78	4.30	5.95	6.06
4	-2.50	3.38	4.86	4.97
5	-2.19	3.06	4.17	4.25
6	-0.39	2.76	3.67	3.72
7	-0.72	2.44	3.17	3.19
8	-2.66	1.95	2.75	2.80
9	-4.16	1.69	2.61	2.62
10	-0.33	2.03	2.54	2.60
11	1.07	1.77	2.65	2.66
12	-2.47	1.85	2.45	2.46
13	-2.28	1.55	2.30	2.32

**Table 6.b**

dO	AFE 1	AFE 2	AFE 3
13.43	5.95	6.48	9.61
14.01	2.84	3.52	4.24
11.99	2.25	2.21	2.11
11.79	1.93	1.82	1.82
11.24	1.87	1.64	1.60
12.79	1.60	1.54	1.36
9.08	1.28	1.06	1.02
8.73	1.13	0.91	0.95
11.93	1.88	0.86	0.89
13.99	1.06	0.94	0.88
7.63	1.06	0.95	0.97
7.93	0.98	0.83	0.85

**Table 6.c**

weeks to mat	alfa	beta	R
2	6.37	1.01	0.80
3	3.68	0.88	0.80
4	2.81	0.80	0.80
5	2.64	0.59	0.80
6	2.42	0.48	0.79
7	2.18	0.38	0.77
8	1.65	0.43	0.79
9	1.37	0.47	0.76
10	1.83	0.28	0.82
11	1.47	0.44	0.76
12	1.64	0.31	0.76
13	1.28	0.38	0.77

**Table 6.d**

alfa	beta	R
3.69	<b>1.83</b>	0.86
2.14	<b>0.70</b>	1.00
2.33	-0.07	0.94
1.97	-0.05	0.75
1.97	-0.14	0.88
1.74	-0.12	0.92
1.38	-0.13	0.85
1.18	-0.09	0.59
2.19	-0.49	0.72
1.13	-0.09	0.97
1.08	-0.05	0.53
1.01	-0.06	0.62

**Table 6.e**

	alfa	beta	R
dO	-1.25	-0.04	0.01
AFE 1	5.53	-0.36	0.68
AFE 2	7.30	-0.46	0.72
AFE 3	7.44	-0.47	0.72

**Table 6.f**

alfa	beta	R
14.21	-0.4	0.39
4.13	-0.29	0.57
4.56	-0.36	0.61
5.90	-0.49	0.50

## OTM1 Puts

**Table 7.a**

weeks to mat	dO	AFE 1	AFE 2	AFE 3
2	0.80	5.55	6.44	6.13
3	0.70	3.85	4.82	4.82
4	-0.50	3.03	4.26	4.26
5	-0.30	2.83	3.76	3.84
6	0.25	2.67	3.38	3.42
7	0.26	2.23	2.96	2.96
8	-0.32	1.97	2.74	2.72
9	-1.23	1.69	2.46	2.46
10	0.22	1.70	2.22	2.26
11	0.58	1.82	2.17	2.12
12	-1.18	1.31	1.92	1.93
13	-2.51	1.12	1.92	1.91

**Table 7.b**

dO	AFE 1	AFE 2	AFE 3
9.57	4.28	6.27	6.87
10.14	3.12	3.75	2.90
8.64	2.31	2.27	2.10
8.79	2.23	1.83	1.85
8.41	2.18	1.63	1.57
9.14	1.46	1.44	1.30
9.32	1.49	1.39	1.13
6.65	1.05	0.92	0.92
8.29	0.96	0.89	0.83
7.60	2.64	1.36	0.91
6.15	0.77	0.72	0.72
6.44	0.71	0.55	0.54

**Table 7.c**

weeks to mat	alfa	beta	R
2	5.46	0.29	<b>0.41</b>
3	3.52	0.49	0.75
4	2.62	0.62	0.75
5	2.46	0.50	0.81
6	2.40	0.38	0.79
7	1.99	0.37	0.76
8	1.73	0.37	0.73
9	1.43	0.38	0.75
10	1.49	0.28	0.81
11	1.74	0.15	0.62
12	1.09	0.31	0.77
13	0.85	0.40	0.74

**Table 7.d**

alfa	beta	R
3.21	<b>1.30</b>	0.91
3.48	-0.11	<b>0.06</b>
2.44	-0.11	0.89
2.34	-0.19	0.70
2.41	-0.31	0.83
1.55	-0.08	0.80
1.69	-0.18	0.94
1.10	-0.07	0.74
1.02	-0.06	1.00
3.36	-0.86	0.93
0.79	-0.03	0.74
0.77	-0.09	0.78

**Table 7.e**

	alfa	beta	R
dO	0.99	-0.17	0.39
AFE 1	4.81	-0.31	0.81
AFE 2	5.91	-0.35	0.86
AFE 3	5.82	-0.34	0.89

**Table 7.f**

alfa	beta	R
10.50	-0.30	0.69
3.71	-0.24	0.63
4.63	-0.36	0.65
4.55	-0.37	0.58

## OTM2 Puts

**Table 8.a**

weeks to mat	dO	AFE 1	AFE 2	AFE 3
2	0.43	4.93	4.35	3.95
3	0.87	3.34	3.39	3.46
4	-0.29	2.66	3.26	3.28
5	0.18	2.54	3.04	3.12
6	0.57	2.37	2.82	2.83
7	1.45	2.11	2.48	2.51
8	0.03	1.84	2.40	2.44
9	-1.04	1.64	2.16	2.15
10	0.58	1.57	1.89	1.89
11	-0.22	1.44	1.63	1.59
12	-0.94	1.13	1.50	1.49
13	-0.93	1.14	1.62	1.61

**Table 8.b**

dO	AFE 1	AFE 2	AFE 3
6.42	3.71	3.99	3.15
7.19	3.60	2.48	2.46
5.15	1.81	1.96	1.92
6.74	2.09	1.85	1.85
6.09	2.02	1.59	1.49
9.42	2.28	1.55	1.35
6.26	1.24	1.12	1.16
4.09	1.00	0.94	0.94
5.38	1.29	0.78	0.73
4.17	1.33	0.64	0.57
2.95	0.53	0.52	0.52
4.36	0.70	0.62	0.66

**Table 8.c**

weeks to mat	alfa	beta	R
2	5.39	<b>-0.49</b>	0.99
3	3.28	0.06	0.99
4	2.44	0.31	0.78
5	2.33	0.29	0.85
6	2.21	0.23	0.77
7	1.96	0.20	0.81
8	1.62	0.30	0.80
9	1.47	0.25	0.73
10	1.46	0.16	0.75
11	1.40	0.07	0.56
12	1.01	0.18	0.74
13	0.99	0.23	0.73

**Table 8.d**

alfa	beta	R
4.17	-0.28	<b>0.43</b>
3.99	-0.57	0.76
1.78	0.06	0.52
2.17	-0.12	0.73
2.23	-0.27	0.88
2.66	-0.46	0.90
1.26	-0.04	<b>0.44</b>
1.02	-0.03	0.72
1.49	-0.28	0.81
1.61	-0.38	0.82
0.54	-0.01	0.75
0.70	-0.02	<b>0.27</b>

**Table 8.e**

	alfa	beta	R
dO	0.98	-0.12	0.32
AFE 1	4.26	-0.27	0.82
AFE 2	4.28	-0.23	0.94
AFE 3	4.19	-0.22	0.97

**Table 8.f**

alfa	beta	R
7.88	-0.29	0.37
3.69	-0.25	0.79
3.38	-0.25	0.82
3.02	-0.22	0.90

## AFE1

**Table 9.a**

weeks to mat	ITM	ATM	OTM1	OTM2
2	7.73	6.60	4.91	4.08
3	5.51	4.67	3.42	3.36
4	4.32	3.78	2.97	2.47
5	4.37	3.53	2.76	2.35
6	3.76	3.17	2.66	2.39
7	3.52	2.78	2.30	2.29
8	3.45	2.35	2.11	2.09
9	2.52	2.24	1.88	1.66
10	3.30	2.31	1.74	1.66
11	2.89	1.89	1.73	1.57
12	1.65	1.92	1.71	1.33
13	1.92	2.42	1.54	1.26

**Table 9.b**

ITM	ATM	OTM1	OTM2
5.66	4.80	3.67	2.78
4.59	3.18	2.58	2.47
3.27	2.43	2.30	1.91
4.54	2.48	2.01	1.64
3.20	2.23	1.81	1.63
5.14	1.83	1.57	1.52
3.41	1.69	1.58	1.42
1.76	1.40	1.35	1.30
1.53	1.69	1.00	1.21
2.19	1.01	1.12	1.08
1.86	1.30	1.28	0.71
1.18	1.37	0.89	0.84

**Table 9.c**

weeks to mat	alfa	beta	R
2	8.99	-1.26	0.98
3	6.16	-0.77	0.91
4	4.98	-0.64	0.99
5	4.95	-0.68	0.98
6	4.15	-0.46	0.97
7	3.77	-0.42	0.87
8	3.58	-0.43	0.75
9	2.81	-0.29	0.99
10	3.63	-0.55	0.88
11	3.05	-0.41	0.80
12	1.94	-0.12	<b>0.37</b>
13	2.49	-0.28	0.54

**Table 9.d**

alfa	beta	R
6.67	-0.97	1.00
4.95	-0.70	0.85
3.53	-0.42	0.90
4.96	-0.91	0.84
3.5	-0.51	0.89
5.29	-1.11	0.67
3.54	-0.61	0.71
1.81	-0.14	0.79
1.77	-0.17	<b>0.47</b>
2.16	-0.32	0.55
2.16	-0.35	0.91
1.45	-0.15	0.60

AFE2

Table 10.a

weeks to mat	ITM	ATM	OTM1	OTM2
2	8.61	8.72	5.71	3.65
3	6.00	6.44	4.82	3.58
4	5.64	5.40	4.06	2.99
5	5.03	4.75	3.79	3.14
6	4.51	4.15	3.65	3.34
7	3.93	3.73	3.23	2.99
8	3.74	3.39	2.87	2.77
9	4.12	3.08	2.45	2.23
10	3.99	2.93	2.22	2.41
11	3.78	2.62	2.24	2.02
12	3.17	2.49	2.07	1.99
13	3.17	3.21	2.11	1.81

Table 10.b

ITM	ATM	OTM1	OTM2
5.55	4.77	4.27	2.79
2.93	3.36	3.27	2.32
5.06	2.73	2.59	1.95
2.19	2.08	2.24	1.82
2.75	1.83	1.96	1.80
2.74	1.59	1.76	1.53
1.33	1.60	1.58	1.52
1.22	1.33	1.19	1.25
1.26	1.30	1.03	1.26
1.43	1.25	1.12	0.87
0.84	1.09	1.04	0.80
0.61	1.06	0.82	1.06

Table 10.c

weeks to mat	alfa	beta	R
2	11.14	-1.79	0.89
3	7.43	-0.89	0.80
4	6.85	-0.93	0.94
5	5.84	-0.67	0.96
6	4.91	-0.40	0.99
7	4.3	-0.33	0.97
8	4.05	-0.34	0.95
9	4.54	-0.63	0.92
10	4.25	-0.55	0.79
11	4.08	-0.57	0.87
12	3.42	-0.40	0.89
13	3.87	-0.52	0.86

Table 10.d

alfa	beta	R
6.54	-0.88	0.95
3.44	-0.19	<b>0.27</b>
5.45	-0.95	0.81
2.32	-0.10	<b>0.45</b>
2.76	-0.27	0.61
2.77	-0.35	0.63
1.38	<b>0.05</b>	<b>0.31</b>
1.26	<b>0.00</b>	<b>0.01</b>
1.28	-0.03	<b>0.07</b>
1.62	-0.18	0.98
0.99	-0.02	<b>0.02</b>
0.61	<b>0.11</b>	<b>0.45</b>

AFE3

Table 11.a

weeks to mat	ITM	ATM	OTM1	OTM2
2	8.62	8.55	5.56	3.44
3	6.55	6.54	4.69	3.59
4	5.58	5.46	4.1	3.03
5	5.32	4.76	3.81	3.22
6	4.52	4.17	3.62	3.27
7	3.96	3.64	3.17	3.00
8	3.92	3.33	2.85	2.83
9	4.19	3.05	2.55	2.35
10	3.73	2.92	2.28	2.36
11	4.02	2.54	2.21	2.01
12	3.39	2.43	2.09	2.05
13	3.18	3.11	2.22	1.99

Table 11.b

ITM	ATM	OTM1	OTM2
9.3	4.41	4.06	2.76
3.73	3.09	3.02	2.32
3.13	2.28	2.37	1.91
2.95	2.02	2.10	1.81
3.95	1.74	1.88	1.74
2.65	1.52	1.62	1.44
1.57	1.29	1.34	1.48
1.27	1.33	1.17	1.23
1.32	1.26	1.00	1.11
3.88	1.28	1.01	0.89
0.82	1.20	1.02	0.88
0.81	1.21	0.94	1.15

Table 11.c

weeks to mat	alfa	beta	R
2	11.18	-1.85	0.91
3	8.02	-1.07	0.90
4	6.80	-0.90	0.92
5	6.09	-0.72	0.99
6	4.98	-0.43	0.99
7	4.28	-0.33	0.97
8	4.17	-0.38	0.89
9	4.54	-0.60	0.89
10	4.01	-0.48	0.84
11	4.28	-0.64	0.81
12	3.59	-0.44	0.82
13	3.74	-0.45	0.90

Table 11.d

alfa	beta	R
10.13	-2.00	0.81
4.12	-0.43	0.93
3.31	-0.35	0.81
3.05	-0.33	0.74
3.95	-0.65	0.60
2.68	-0.35	0.65
1.48	-0.02	<b>0.05</b>
1.32	-0.03	<b>0.33</b>
1.40	-0.09	0.64
4.07	-0.92	0.71
0.98	<b>0.00</b>	<b>0.00</b>
0.84	<b>0.08</b>	<b>0.28</b>

PAFE1

Table 12.a

weeks to mat	ITM	ATM	OTM1	OTM2
2	9.60	7.08	5.55	4.93
3	4.88	4.30	3.85	3.34
4	3.52	3.38	3.03	2.66
5	3.10	3.06	2.83	2.54
6	2.77	2.76	2.67	2.37
7	2.50	2.44	2.23	2.11
8	2.06	1.95	1.97	1.84
9	1.65	1.69	1.69	1.64
10	3.54	2.03	1.70	1.57
11	1.70	1.77	1.82	1.44
12	1.79	1.85	1.31	1.13
13	2.12	1.55	1.12	1.14

Table 12.b

ITM	ATM	OTM1	OTM2
7.59	5.95	4.28	3.71
3.10	2.84	3.12	3.60
2.38	2.25	2.31	1.81
2.12	1.93	2.23	2.09
1.99	1.87	2.18	2.02
1.82	1.60	1.46	2.28
1.43	1.28	1.49	1.24
1.40	1.13	1.05	1.00
3.26	1.88	0.96	1.29
0.61	1.06	2.64	1.33
1.42	1.06	0.77	0.53
1.26	0.98	0.71	0.70

Table 12.c

weeks to mat	alfa	beta	R
2	10.68	-1.56	0.93
3	5.36	-0.51	1.00
4	3.88	-0.29	0.97
5	3.36	-0.19	0.92
6	2.97	-0.13	0.80
7	2.67	-0.14	0.97
8	2.11	-0.06	0.82
9	1.68	<b>0.00</b>	<b>0.04</b>
10	3.78	-0.63	0.79
11	1.87	-0.08	<b>0.32</b>
12	2.15	-0.25	0.84
13	2.32	-0.34	0.86

Table 12.d

alfa	beta	R
8.71	-1.33	0.96
2.72	<b>0.18</b>	0.53
2.60	-0.17	0.68
2.04	<b>0.02</b>	<b>0.05</b>
1.91	<b>0.04</b>	<b>0.17</b>
1.48	<b>0.12</b>	<b>0.19</b>
1.44	-0.03	<b>0.14</b>
1.46	-0.13	0.87
3.55	-0.68	0.75
0.47	<b>0.37</b>	<b>0.31</b>
1.68	-0.29	0.99
1.40	-0.20	0.90

PAFE2

Table 13.a

weeks to mat	ITM	ATM	OTM1	OTM2
2	10.64	8.97	6.44	4.35
3	5.73	5.95	4.82	3.39
4	4.41	4.86	4.26	3.26
5	3.84	4.17	3.76	3.04
6	3.70	3.67	3.38	2.82
7	3.36	3.17	2.96	2.48
8	2.95	2.75	2.74	2.40
9	2.61	2.61	2.46	2.16
10	3.76	2.54	2.22	1.89
11	2.53	2.65	2.17	1.63
12	3.29	2.45	1.92	1.50
13	3.29	2.30	1.92	1.62

Table 13.b

ITM	ATM	OTM1	OTM2
11.47	6.48	6.27	3.99
3.22	3.52	3.75	2.48
2.41	2.21	2.27	1.96
1.98	1.82	1.83	1.85
1.81	1.64	1.63	1.59
1.54	1.54	1.44	1.55
1.35	1.06	1.39	1.12
1.00	0.91	0.92	0.94
1.49	0.86	0.89	0.78
0.65	0.94	1.36	0.64
0.45	0.95	0.72	0.52
0.89	0.83	0.55	0.62

Table 13.c

weeks to mat	alfa	beta	R
2	12.94	-2.14	0.99
3	7.01	-0.81	0.82
4	5.21	-0.41	0.61
5	4.4	-0.28	0.58
6	4.13	-0.29	0.85
7	3.71	-0.29	0.94
8	3.12	-0.17	0.88
9	2.83	-0.15	0.83
10	4.08	-0.59	0.88
11	3.04	-0.32	0.80
12	3.77	-0.59	0.97
13	3.63	-0.54	0.92

Table 13.d

alfa	beta	R
12.71	-2.26	0.86
3.74	-0.20	<b>0.21</b>
2.54	-0.13	0.79
1.97	-0.04	<b>0.48</b>
1.84	-0.07	0.81
1.54	-0.01	<b>0.04</b>
1.31	-0.03	<b>0.07</b>
0.98	-0.02	<b>0.28</b>
1.53	-0.21	0.68
0.81	<b>0.04</b>	<b>0.02</b>
0.66	<b>0.00</b>	<b>0.00</b>
1	-0.11	0.74

PAFE3

**Table 14.a**

weeks to mat	ITM	ATM	OTM1	OTM2
2	13.52	9.10	6.13	3.95
3	5.97	6.06	4.82	3.46
4	4.66	4.97	4.26	3.28
5	3.98	4.25	3.84	3.12
6	3.78	3.72	3.42	2.83
7	3.40	3.19	2.96	2.51
8	2.93	2.80	2.72	2.44
9	2.70	2.62	2.46	2.15
10	3.77	2.60	2.26	1.89
11	2.74	2.66	2.12	1.59
12	2.97	2.46	1.93	1.49
13	3.38	2.32	1.91	1.61

**Table 14.b**

ITM	ATM	OTM1	OTM2
46.67	9.61	6.87	3.15
4.51	4.24	2.90	2.46
2.36	2.11	2.10	1.92
1.98	1.82	1.85	1.85
1.84	1.60	1.57	1.49
1.67	1.36	1.30	1.35
1.19	1.02	1.13	1.16
1.09	0.95	0.92	0.94
2.30	0.89	0.83	0.73
0.73	0.88	0.91	0.57
0.85	0.97	0.72	0.52
0.87	0.85	0.54	0.66

**Table 14.c**

weeks to mat	alfa	beta	R <sup>2</sup>
2	16.09	-3.17	0.97
3	7.27	-0.88	0.87
4	5.51	-0.49	0.73
5	4.55	-0.30	0.64
6	4.22	-0.31	0.87
7	3.74	-0.29	0.96
8	3.11	-0.15	0.93
9	2.94	-0.18	0.92
10	4.12	-0.60	0.90
11	3.28	-0.40	0.93
12	3.45	-0.50	1.00
13	3.73	-0.57	0.91

**Table 14.d**

alfa	beta	R
49.9	-13.33	0.72
5.4	-0.75	0.93
2.45	-0.13	0.90
1.97	-0.04	<b>0.43</b>
1.89	-0.11	0.84
1.68	-0.10	0.60
1.12	<b>0.00</b>	<b>0.00</b>
1.09	-0.05	0.63
2.38	-0.48	0.68
0.89	-0.05	<b>0.14</b>
1.08	-0.13	0.69
0.97	-0.09	0.59

dO

**Table 15.a**

weeks to mat	ITM	ATM	OTM1	OTM2
2	-9.09	-3.24	-1.14	-0.68
3	-8.95	-2.35	-0.48	0.36
4	-4.94	0.14	0.66	0.62
5	-7.17	-0.96	0.17	-0.25
6	-6.61	-2.83	-1.7	-1.4
7	-6.43	-2.58	-0.62	0.93
8	-8.91	-1.2	0.47	0.57
9	-8.33	0.76	2.02	1.74
10	-7.78	-2.74	-0.06	1.12
11	-4.06	-0.36	0.17	0.62
12	-3.54	0.48	1.05	2.25
13	-5.89	-3	1.81	3.51

**Table 15.b**

ITM	ATM	OTM1	OTM2
16.44	11.33	5.67	3.21
16.8	10.33	6.49	5.27
17.49	11.45	7.36	4.96
15.14	10.52	7.49	4.93
12.31	9.18	6.88	5.07
14.14	9.48	7.58	7.21
18.03	10.79	7	5.57
14.76	9.27	6.72	4.74
12.93	9.12	5.98	7.48
13.28	9.77	6.26	4.38
12.99	10.2	5.96	7.4
10.93	7.06	6.52	5.17

**Table 15.c**

weeks to mat	alfa	beta	R
2	-10.37	2.73	0.83
3	-10.3	2.98	0.83
4	-5.18	1.72	0.67
5	-7.52	2.19	0.67
6	-7.32	1.67	0.82
7	-8.18	2.40	0.95
8	-9.8	3.01	0.75
9	-8.82	3.15	0.67
10	-9.71	2.94	0.92
11	-4.55	1.46	0.77
12	-4.43	1.80	0.85
13	-9.14	3.30	0.97

**Table 15.d**

alfa	beta	R
20.5	-4.53	0.98
19.33	-3.84	0.91
20.73	-4.17	0.96
17.93	-3.37	0.98
14.37	-2.40	0.98
15.27	-2.27	0.85
20.64	-4.12	0.91
17.03	-3.26	0.94
13.75	-1.95	0.71
15.97	-3.02	0.98
14.38	-2.10	0.76
11.87	-1.78	0.87

PdO

**Table 16.a**

weeks to mat	ITM	ATM	OTM1	OTM2
2	-1.82	-0.83	0.8	0.43
3	-7.51	-0.78	0.7	0.87
4	-8.06	-2.5	-0.5	-0.29
5	-6.96	-2.19	-0.3	0.18
6	-0.32	-0.39	0.25	0.57
7	-4.3	-0.72	0.26	1.45
8	-4.18	-2.66	-0.32	0.03
9	-8.15	-4.16	-1.23	-1.04
10	7.76	-0.33	0.22	0.58
11	-6.79	1.07	0.58	-0.22
12	-17.56	-2.47	-1.18	-0.94
13	-7	-2.28	-2.51	-0.93

**Table 16.b**

ITM	ATM	OTM1	OTM2
23.15	13.43	9.57	6.42
18.01	14.01	10.14	7.19
17.71	11.99	8.64	5.15
15.81	11.79	8.79	6.74
16.99	11.24	8.41	6.09
15.29	12.79	9.14	9.42
16.53	9.08	9.32	6.26
14.72	8.73	6.65	4.09
18.25	11.93	8.29	5.38
15.09	13.99	7.6	4.17
7.34	7.63	6.15	2.95
18.47	7.93	6.44	4.36

**Table 16.c**

weeks to mat	alfa	beta	R
2	-2.45	0.84	0.81
3	-8.34	2.67	0.76
4	-9.16	2.53	0.81
5	-8.14	2.33	0.85
6	-0.8	0.33	0.86
7	-5.39	1.82	0.90
8	-5.53	1.50	0.94
9	-9.71	2.42	0.89
10	7.31	-2.10	0.50
11	-6.14	1.92	0.46
12	-18.33	5.12	0.67
13	-7.68	1.80	0.77

**Table 16.d**

alfa	beta	R
26.65	-5.41	0.92
21.43	-3.64	1.00
21.13	-4.10	0.98
18.34	-3.02	0.98
19.56	-3.55	0.95
16.98	-2.13	0.88
17.94	-3.06	0.81
17.04	-3.40	0.94
21.52	-4.22	0.97
20.00	-3.92	0.94
9.68	-1.47	0.78
20.26	-4.38	0.81

**normality**

**Table 17**

$H_0$ : the sample is log normally distributed

$H_1$ : reject the null hypothesis.

Null accepts  $H_0$  at 5% significance level. One rejects  $H_0$ .

**Calls**

ITM	dO	AFE1	AFE2	AFE3
2	1	1	1	1
3	1	1	1	1
4	1	1	1	1
5	0	1	1	1
6	0	1	1	1
7	0	1	1	1
8	1	1	1	1
9	0	0	0	0
10	0	0	0	0
11	0	0	1	1
12	0	0	0	0
13	0	0	0	1

**ATM**

2	1	1	1	1
3	1	1	1	1
4	1	1	1	1
5	1	1	1	1
6	1	1	1	1
7	1	1	1	1
8	1	1	1	1
9	0	1	1	1
10	1	1	1	1
11	1	1	1	1
12	1	0	1	1
13	0	0	0	1

OTM1 non

OTM2 2, at week 12

**Puts**

ITM	dO	AFE1	AFE2	AFE3
2	1	1	1	1
3	1	1	1	1
4	1	1	1	1
5	1	1	1	1
6	1	1	0	0
7	1	1	1	1
8	1	0	0	0
9	1	1	0	0
10	0	1	1	1
11	0	0	0	0
12	1	0	1	0
13	0	0	0	0

**ATM**

2	1	1	1	1
3	1	1	1	1
4	1	1	1	1
5	1	1	1	1
6	1	1	1	1
7	1	1	1	1
8	1	1	1	1
9	1	1	1	1
10	1	1	1	1
11	1	0	0	0
12	0	0	1	0
13	0	1	1	1

OTM1 1, at week 13

OTM2 non

## lognormality

**Table 18**

$H_0$ : the sample is log normally distributed

$H_1$ : reject the null hypothesis.

Null accepts  $H_0$  at 5% significance level. One rejects  $H_0$ .

**Calls**

ITM

weeks	AFE1	AFE2	AFE3
2	1	1	1
3	1	1	1
4	1	1	1
5	1	1	0
6	1	1	0
7	1	1	1
8	0	0	1
9	1	0	0
10	0	0	0
11	0	0	1
12	0	0	0
13	0	0	0

ATM 6 samples

OTM1 4 samples

OTM2

2	1	1	1
3	1	1	1
4	1	1	1
5	1	1	1
6	1	1	1
7	1	1	1
8	1	0	0
9	1	1	0
10	1	0	0
11	1	0	0
12	0	0	0
13	0	1	1

**Puts**

ITM

weeks	AFE1	AFE2	AFE3
2	1	1	1
3	1	1	1
4	1	1	1
5	1	1	1
6	1	1	1
7	1	1	0
8	1	1	1
9	0	1	0
10	1	1	0
11	0	0	0
12	0	0	0
13	0	0	0

ATM 6 samples

OTM1 1 sample

OTM2

2	0	1	1
3	1	1	1
4	1	1	1
5	1	1	1
6	1	1	1
7	1	0	1
8	1	1	1
9	1	0	0
10	1	0	0
11	1	0	1
12	0	1	1
13	1	1	1

diffm

Table 19

a1.

option type/week	$m_{AFE3} - m_{AFE2}$	H-test	T-cdf
C ITM			
9	0.07	0	0.87
10	-0.26	0	0.13
12	0.22	0	0.93
P ITM			
6	0.08	1	0.98
8	-0.02	0	0.35
9	0.10	0	0.96
11	0.21	0	0.96
13	0.09	0	0.87
P ATM			
11	0.01	0	0.55

b1.

option type/week	$m_{AFE3} - m_{AFE2}$	H-test	T-cdf
C ITM			
9	0.02	0	0.86
10	-0.08	0	0.13
12	0.07	0	0.93
13	-0.01	0	0.44
C OTM2			
8	0.04	1	1.00
10	0.00	0	0.55
11	0.00	0	0.44
12	0.02	0	0.88
P ITM			
11	0.08	0	0.95
12	-0.14	0	0.23
13	0.03	0	0.89
P OTM2			
9	0.00	0	0.23
10	0.01	0	0.91

a2.

option type/week	$m_{AFE3} - m_{AFE2}$	H-test	T-cdf
C ITM			
9	1.60	1	1.00
10	0.69	1	1.00
12	1.52	1	0.99
13	1.26	1	0.99
C ATM			
13	0.79	1	1.00
P ITM			
8	0.89	1	1.00
11	0.83	1	1.00
13	1.17	1	1.00
P ATM			
11	0.88	1	1.00

b2.

option type/week	$m_{AFE3} - m_{AFE2}$	H-test	T-cdf
C ITM			
8	0.3	1	1.00
10	0.29	1	0.99
11	0.41	1	0.98
12	1.72	0	0.96
13	0.73	1	0.98
C OTM2			
12	0.51	1	1.00
P ITM			
11	0.42	1	1.00
12	0.96	0	0.91
13	0.56	1	0.99

Appendix:

Formulae

## Black-Scholes

$$c = SN(d_1) - Ke^{-rT} N(d_2)$$

$$p = Ke^{-rT} N(-d_2) - SN(-d_1)$$

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} = d_2 + \sigma\sqrt{T}$$

$$d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

## Normal distribution function

$$N(x) = \Phi_{(0,1)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$$

$$n(x) = \frac{\partial}{\partial x} N(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$n'(x) = \frac{\partial}{\partial x} n(x) = -\frac{x}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} = -xn(x)$$

## Derivatives of $d_1$ and $d_2$

$$\frac{\partial d_1}{\partial S} = \frac{\ln(S)}{\sigma\sqrt{T}} = \frac{1}{S\sigma\sqrt{T}} = \frac{\partial d_2}{\partial S}$$

$$\frac{\partial d_1}{\partial \sigma} = \frac{\sigma^2 T^{3/2} - \sqrt{T}(\ln(S/K) + (r + \frac{\sigma^2}{2})T)}{\sigma^2 T} = \sqrt{T} - \frac{d_1}{\sigma} = -\frac{d_2}{\sigma} \quad \frac{\partial d_2}{\partial \sigma} = -\sqrt{T} - \frac{d_2}{\sigma} = -\frac{d_1}{\sigma}$$

$$\frac{\partial d_1}{\partial T} = \frac{\sigma\sqrt{T}(r + \frac{\sigma^2}{2}) - \frac{\sigma}{2\sqrt{T}}(\ln(S/K) + (r + \frac{\sigma^2}{2})T)}{\sigma^2 T} = \frac{2(r + \frac{\sigma^2}{2})T}{2T\sigma\sqrt{T}} - \frac{\ln(S/K) + (r + \frac{\sigma^2}{2})T}{2T\sigma\sqrt{T}} = \frac{\hat{d}_1}{2T}$$

$$\frac{\partial d_2}{\partial T} = \frac{\hat{d}_2}{2T}$$

$$\hat{d}_1 = \frac{-\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \quad \hat{d}_2 = \frac{-\ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$\frac{\partial \hat{d}_1}{\partial T} = \frac{d_1}{2T} \quad \frac{\partial \hat{d}_2}{\partial T} = \frac{d_2}{2T}$$

$$\frac{\partial \hat{d}_1}{\partial S} = -\frac{\partial d_1}{\partial S} = -\frac{1}{S\sigma\sqrt{T}} = \frac{\partial \hat{d}_2}{\partial S}$$

$$\frac{\partial \hat{d}_1}{\partial \sigma} = \sqrt{T} - \frac{\hat{d}_1}{\sigma} = -\frac{\hat{d}_2}{\sigma} \quad \frac{\partial \hat{d}_2}{\partial \sigma} = \sqrt{T} - \frac{\hat{d}_2}{\sigma} = -\frac{\hat{d}_1}{\sigma}$$

## Derivatives of stock price

$$\frac{\partial c}{\partial S} = N(d_1)$$

$$\frac{\partial p}{\partial S} = N(d_1) - 1$$

$$\frac{\partial^2 o}{\partial S^2} = n(d_1) \cdot \frac{\partial d_1}{\partial S} = \frac{n(d_1)}{S\sigma\sqrt{T}}$$

$$\frac{\partial^3 o}{\partial S^3} = \frac{n'(d_1) \frac{\partial d_1}{\partial S} \cdot S\sigma\sqrt{T} - \sigma\sqrt{T}n(d_1)}{(S\sigma\sqrt{T})^2} = \frac{-d_1n(d_1) - \sigma\sqrt{T}n(d_1)}{(S\sigma\sqrt{T})^2} = -\frac{n(d_1)}{S^2\sigma\sqrt{T}} \left( \frac{d_1}{\sigma\sqrt{T}} + 1 \right)$$

## Derivatives of volatility

$$\frac{\partial o}{\partial \sigma} = S\sqrt{T}n(d_1)$$

$$\frac{\partial^2 o}{\partial \sigma^2} = S\sqrt{T} \cdot n'(d_1) \cdot \frac{\partial d_1}{\partial \sigma} = -S\sqrt{T}n(d_1)d_1 \frac{-d_2}{\sigma} = \frac{\partial o}{\partial \sigma} d_1 \frac{d_2}{\sigma}$$

$$\frac{\partial^3 o}{\partial \sigma^3} = \frac{\partial^2 o}{\partial \sigma^2} \cdot d_1 \frac{d_2}{\sigma} + \frac{\partial o}{\partial \sigma} \cdot \frac{\partial}{\partial \sigma} \left( d_1 \frac{d_2}{\sigma} \right)$$

$$\frac{\partial}{\partial \sigma} \left( d_1 \frac{d_2}{\sigma} \right) = \frac{\partial d_1}{\partial \sigma} \cdot \frac{d_2}{\sigma} + d_1 \cdot \frac{\sigma \frac{\partial d_2}{\partial \sigma} - d_2}{\sigma^2} = -\left(\frac{d_2}{\sigma}\right)^2 - \frac{d_1^2 - d_1 d_2}{\sigma^2} = -\frac{d_1^2 + d_2^2 + d_1 d_2}{\sigma^2}$$

$$\begin{aligned} \frac{\partial^3 o}{\partial \sigma^3} &= \frac{\partial^2 o}{\partial \sigma^2} \cdot d_1 \frac{d_2}{\sigma} + \frac{\partial o}{\partial \sigma} \cdot \frac{\partial}{\partial \sigma} \left( d_1 \frac{d_2}{\sigma} \right) = \frac{\partial o}{\partial \sigma} \left( \frac{d_1 d_2}{\sigma} \right)^2 - \frac{\partial o}{\partial \sigma} \cdot \frac{d_1^2 + d_2^2 + d_1 d_2}{\sigma^2} \\ &= \frac{\partial o}{\partial \sigma} \cdot \frac{1}{\sigma^2} (d_1^2 d_2^2 - d_1^2 - d_2^2 - d_1 d_2) \end{aligned}$$

## Derivatives of time

$$\frac{\partial o}{\partial T} = \sigma \frac{Sn(d_1)}{2\sqrt{T}} \pm rKe^{-rT} N(d_2) = \sigma st \pm th N(d_2)$$

$$\begin{aligned} \frac{\partial}{\partial T} \sigma st &= S\sigma \left( \frac{2\sqrt{T}n'(d_1) \frac{\partial d_1}{\partial T} - \frac{2}{2\sqrt{T}}n(d_1)}{4T} \right) = \frac{S\sigma}{4T} (-2\sqrt{T}d_1n(d_1) \frac{\hat{d}_1}{2T} - \frac{n(d_1)}{\sqrt{T}}) \\ &= -\sigma st \frac{(d_1 \hat{d}_1 + 1)}{2T} \end{aligned}$$

$$\frac{\partial}{\partial T} th N(d_2) = rK (-r e^{-rT} N(d_2) + e^{-rT} n(d_2) \frac{\partial d_2}{\partial T}) = th (-rN(d_2) + n(d_2) \frac{\hat{d}_2}{2T})$$

$$\frac{\partial^2 o}{\partial T^2} = \sigma st \frac{(d_1 \hat{d}_1 + 1)}{2T} \pm th(-rN(d_2) + n(d_2)) \frac{\hat{d}_2}{2T}$$

### Partial derivatives of stock price and volatility

$$\frac{\partial c}{\partial S} = N(d_1)$$

$$\frac{\partial^2 o}{\partial S \partial \sigma} = n(d_1) \frac{\partial d_1}{\partial \sigma} = -n(d_1) \frac{d_2}{\sigma}$$

$$\frac{\partial^2 o}{\partial S^2 \partial \sigma} = -n'(d_1) \frac{\partial d_1}{\partial S} \frac{d_2}{\sigma} - \frac{n(d_1)}{\sigma} \frac{\partial d_2}{\partial S} = \frac{n(d_1)}{S \sigma^2 \sqrt{T}} (d_1 d_2 - 1)$$

$$\begin{aligned} \frac{\partial^2 o}{\partial S \partial \sigma^2} &= -n'(d_1) \frac{\partial d_1}{\partial \sigma} \frac{d_2}{\sigma} - n(d_1) \left( \frac{\sigma \frac{\partial d_2}{\partial \sigma} - d_2}{\sigma^2} \right) = -d_1 n(d_1) \left( \frac{d_2}{\sigma} \right)^2 - n(d_1) \frac{(-d_1 - d_2)}{\sigma^2} \\ &= \frac{n(d_1)}{\sigma^2} (-d_1 d_2^2 + d_1 + d_2) \end{aligned}$$

### Partial derivatives of stock price and time

$$\frac{\partial c}{\partial S} = N(d_1)$$

$$\frac{\partial^2 o}{\partial S \partial T} = n(d_1) \frac{\partial d_1}{\partial T} = n(d_1) \frac{\hat{d}_1}{2T}$$

$$\frac{\partial^3 o}{\partial S^2 \partial T} = \frac{1}{2T} (n'(d_1) \frac{\partial d_1}{\partial S} \hat{d}_1 + n(d_1) \frac{\partial \hat{d}_1}{\partial S}) = \frac{1}{2T} \left( \frac{-d_1 n(d_1)}{S \sigma \sqrt{T}} \hat{d}_1 - \frac{n(d_1)}{S \sigma \sqrt{T}} \right) = -\frac{n(d_1)}{2TS \sigma \sqrt{T}} (d_1 \hat{d}_1 + 1)$$

$$\begin{aligned} \frac{\partial^3 o}{\partial S \partial T^2} &= n'(d_1) \frac{\partial d_1}{\partial T} \cdot \frac{\hat{d}_1}{2T} + n(d_1) \left( \frac{2T \frac{\partial \hat{d}_1}{\partial T} - 2\hat{d}_1}{4T^2} \right) = -d_1 n(d_1) \left( \frac{\hat{d}_1}{2T} \right)^2 + \frac{n(d_1)}{4T^2} (d_1 - 2\hat{d}_1) \\ &= \frac{n(d_1)}{4T^2} (-d_1 \hat{d}_1^2 + d_1 - 2\hat{d}_1) \end{aligned}$$

### Partial derivatives of volatility and time

$$\frac{\partial o}{\partial \sigma} = S \sqrt{T} n(d_1)$$

$$\frac{\partial^2 o}{\partial \sigma \partial T} = S \left( \frac{1}{2\sqrt{T}} n(d_1) + \sqrt{T} n'(d_1) \frac{\partial d_1}{\partial T} \right) = S \left( \frac{1}{2\sqrt{T}} n(d_1) - \sqrt{T} d_1 n(d_1) \frac{\hat{d}_1}{2T} \right) = st(1 - d_1 \hat{d}_1)$$

$$\begin{aligned}\frac{\partial^3 o}{\partial \sigma^2 \partial T} &= \frac{Sn'(d_1) \frac{\partial d_1}{\partial \sigma}}{2\sqrt{T}} (1 - d_1 \hat{d}_1) - \frac{Sn(d_1)}{2\sqrt{T}} \left( \frac{\partial d_1}{\partial \sigma} \hat{d}_1 + \frac{\partial \hat{d}_1}{\partial \sigma} d_1 \right) \\ &= st \cdot d_1 \frac{d_2}{\sigma} (1 - d_1 \hat{d}_1) - st \left( \frac{-d_2}{\sigma} \hat{d}_1 - \frac{\hat{d}_2}{\sigma} d_1 \right) = \frac{st}{\sigma} (d_1 d_2 - d_1^2 d_2 \hat{d}_1 + d_2 \hat{d}_1 + \hat{d}_2 d_1)\end{aligned}$$

$$\frac{\partial}{\partial T} st = \frac{\partial}{\partial T} \sigma st \cdot \frac{1}{\sigma} = -\frac{st}{2T} (d_1 \hat{d}_1 + 1)$$

$$\frac{\partial}{\partial T} (d_1 \hat{d}_1) = \frac{\hat{d}_1}{2T} d_1 + \frac{d_1}{2T} \hat{d}_1 = \frac{1}{2T} (d_1^2 + \hat{d}_1^2)$$

$$\frac{\partial^3 o}{\partial \sigma \partial T^2} = -\frac{st}{2T} (d_1 \hat{d}_1 + 1)(1 - d_1 \hat{d}_1) - \frac{st}{2T} (d_1^2 + \hat{d}_1^2) = -\frac{st}{2T} (-d_1^2 \hat{d}_1^2 + 1 + d_1^2 + \hat{d}_1^2)$$

### Derivative of stock price, volatility and time

$$\frac{\partial^2 o}{\partial S \partial T} = n(d_1) \frac{\hat{d}_1}{2T}$$

$$\frac{\partial^3 o}{\partial S \partial \sigma \partial T} = \frac{n(d_1)}{2T \sigma} (d_1 d_2 \hat{d}_1 - \hat{d}_2)$$

### Other notations

$$st = \frac{Sn(d_1)}{2\sqrt{T}}$$

$$theta_2 = rKe^{-rT}$$

## Taylor expansion of Black-Scholes

$$(1) \quad \partial \hat{O}_1 = \frac{\delta O}{\delta S} \cdot \delta s + \frac{\delta O}{\delta \sigma} \cdot \delta \sigma + \frac{\delta O}{\delta T} \cdot \delta t + \frac{\delta O}{\delta r} \cdot \delta r$$

(2)

$$\partial \hat{O}_2 = \frac{1}{2} (\Gamma \cdot (\delta s)^2 + \frac{\partial O^2}{\partial^2 \sigma} \cdot (\delta \sigma)^2 + \frac{\partial O^2}{\partial^2 T} \cdot (\delta t)^2 + 2 \frac{\partial O^2}{\partial S \partial \sigma} \cdot \delta s \delta \sigma + 2 \frac{\partial O^2}{\partial S \partial T} \cdot \delta s \delta t + 2 \frac{\partial O^2}{\partial \sigma \partial T} \cdot \delta \sigma \delta t)$$

(3)

$$\begin{aligned} \partial \hat{O}_3 = & \frac{1}{6} \frac{\partial O^3}{\partial^3 S} \cdot (\delta s)^3 + \frac{\partial O^3}{\partial^3 \sigma} \cdot (\delta \sigma)^3 + \frac{\partial O^3}{\partial^3 T} \cdot (\delta t)^3 + 3 \frac{\partial O^3}{\partial S^2 \partial \sigma} \cdot (\delta s)^2 \delta \sigma + 3 \frac{\partial O^3}{\partial S \partial \sigma^2} \cdot \delta s (\delta \sigma)^2 + 3 \frac{\partial O^3}{\partial S^2 \partial T} \cdot (\delta s)^2 \delta t + 3 \frac{\partial O^3}{\partial S \partial T^2} \cdot \delta s (\delta t)^2 + 3 \frac{\partial O^3}{\partial \sigma^2 \partial T} \cdot (\delta \sigma)^2 \delta t + 3 \frac{\partial O^3}{\partial \sigma \partial T^2} \cdot \delta \sigma (\delta t)^2 + 6 \frac{\partial O^3}{\partial S \partial \sigma \partial T} \delta s \delta \sigma \delta t \end{aligned}$$

$$\frac{\delta O}{\delta S} = \Delta$$

$$\frac{\delta O}{\delta \sigma} = \nu$$

$$\frac{\delta O}{\delta T} = \theta$$

$$\frac{\delta O}{\delta r} = \rho$$

Appendix:  
Programmes

## Programmes

### 1. Data management

#### extractdata

```
global Ommyy
global option
global data
Y = input('vilket år?');
for rep = 1:12
A = ['200' Y '/I200' Y int2str(rep) '.txt']
fid = fopen(A,'r')
Ommyy = textscan(fid,'%s %s %*f %*f %*f %*f %*f %*f', 'headerLines', 1)
fclose(fid)
```

```
Ommyy{2} = char(Ommyy{2});
c1 = size(Ommyy{2},2)
c = 0;
for h = 1:length(Ommyy{1})
    Strike = str2num(Ommyy{2}(h,6:c1));
    if Ommyy{3}(h) == 0 || isempty(Strike)
        c = c+1;
        crap(c)= h;
    end
end
disp(c)
Ommyy{1}(crap)= [];
Ommyy{2}(crap,:)= [];
Ommyy{3}(crap)= [];
```

```
clear h Strike fid c crap
sortera(rep, c1, Y);
end
k=1;
for t = 1:length(option)
    if str2num(option(t).name(4)) ~= str2num(Y)+1
        bort(k) = t;
        k=k+1;
    end
end
end
```

#### function sortera(b, c1, year)

```
global Ommyy
global option
global data

Ommyy{1} = char(Ommyy{1});
[date] = day([Ommyy{1}]);

h = 1;
for k = 1:length(Ommyy{3})
    m = 1;
    while m <= length(option)
        if strcmp(Ommyy{2}(k,:), option(m).name, c1-1)
            n = size(option(m).date,1);
            option(m).date(n+1,:)= date(k,:);
            option(m).price(n+1,1)= Ommyy{3}(k);
            h = h+1;
            break
        end
        m = m+1;
    end

    if k == h
        a = length(option);
        option(a+1).name = Ommyy{2}(k,:);
        option(a+1).date = date(k,:);
        option(a+1).price = Ommyy{3}(k);
        option(a+1).strike = str2num(option(a+1).name(6:c1));
        h = h+1;
    end
end
```

```

    end
end

a = size(option)
clear s h k m n

s1 = [year,char(64+b)]
s2 = [year,char(76+b)]
h = 1;
g = 1;
k = 1;
for i =1:length(option)
    if option(i).name(4:5) == s1
        data.(genvarname(s1))(h) = option(i);
        del(k) = i;
        k = k+1;
        h = h+1;
    elseif option(i).name(4:5) == s2
        data.(genvarname(s2))(g) = option(i);
        del(k) = i;
        k = k+1;
        g = g+1;
    end
end

option(del) = [];
b = size(option)

```

## 2. Calculations

```

impvol
[TS] = TScreaterer(y);
for month = 1:24
    code = ['x' int2str(y),char(month+64)];
    if month<13, exp = expiration(month,year,TS); else exp = expiration(month-12,year,TS); end
    for t = 1:length(data.(code))
        omxprice = fts2mat(TS(data.(code)(t).date).OMXS30);
        ld = size(omxprice);
        strike = data.(code)(t).strike*ones(ld);
        time = (fts2mat(TS(exp).busday)-fts2mat(TS(data.(code)(t).date).busday))/250;
        value = data.(code)(t).price;
        if month<13, type = ones(ld); else type = zeros(ld); end
        for m = 1:ld(1)
            if time(m) <= 1/6
                interest(m) = fts2mat(TS(data.(code)(t).date(m)).rf1m);
            elseif time(m) <= 1/4
                interest(m) = fts2mat(TS(data.(code)(t).date(m)).rf2m);
            elseif time(m) <= 1/2
                interest(m) = fts2mat(TS(data.(code)(t).date(m)).rf3m);
            elseif time(m) <= 3/4
                interest(m) = fts2mat(TS(data.(code)(t).date(m)).rf6m);
            elseif time(m) <= 1
                interest(m) = fts2mat(TS(data.(code)(t).date(m)).rf9m);
            else
                interest(m) = fts2mat(TS(data.(code)(t).date(m)).rf12m);
            end
        end
        interest = interest';
        data.(code)(t).impvol = blsimpv(omxprice,strike,interest,time,value,[],[],[],type);
        clear interest
        disp(ld)
    end
end

moneyness
b = input('year?')
a = b-1;
s = ['OMX/OMX200' int2str(a) '.txt']
fid = fopen(s,'r');
omx = textscan(fid, '%s %*f %*f %*f', 'headerLines',1);
fclose(fid)

```

```

s = ['OMX/OMX200' int2str(b) '.txt']
fid = fopen(s,'r');
omx2 = textscan(fid, '%s %*f %*f %*f', 'headerLines',1);
fclose(fid)
TS = fints([omx{1};omx2{1}], [omx{2};omx2{2}], 'OMXS30');

for g = 65:88
    s = ['x' int2str(b),char(g)]
    for i = 1:length(data.(s))
        data.(s)(i).moneyness = data.(s)(i).strike./fts2mat(TS(data.(s)(i).date).OMXS30);
    end
end

inimpvol
for h=65:88
    code=['x1' char(h)]
    for i=1:length(data.(code))
        mu=0;
        for j=1:length(data.(code)(i).impvol)
            if isnan(data.(code)(i).impvol(j)) || data.(code)(i).impvol(j)>45
                mu=mu+1;
                elim(mu)=j;
            end
        end
        if mu >=1
            data.(code)(i).date(elim)=[];
            data.(code)(i).price(elim)=[];
            data.(code)(i).moneyness(elim)=[];
            data.(code)(i).impvol(elim)=[];
            elim = [];
        end
    end
end

calculations
function calculations(year)
global data

TS = TScreator([year,year-1],['1m '; '2m '; '3m '; '6m '; '9m '; '12m']);
y = num2str(year);

for month = 1:24
    s = ['x' y, char(month+64)]
    if month<13
        exp = expiration(month,year,TS);
    else
        exp = expiration(month-12,year,TS);
    end

    for k = 1:length(data.(s))
        disp(k)
        OTS = fints(data.(s)(k).date,data.(s)(k).impvol, 'iv');
        omxprice = fts2mat(TS(data.(s)(k).date).OMXS30);
        ld = size(omxprice,1);
        strike = data.(s)(k).strike*ones(ld,1);
        time = (fts2mat(TS(exp).busday)-fts2mat(TS(data.(s)(k).date).busday))/250;
        volatility = fts2mat(OTS.iv);
        for m = 1:ld
            if time(m) <= 1/6
                interest(m) = fts2mat(TS(data.(s)(k).date(m)).rf1m);
            elseif time(m) <= 1/4
                interest(m) = fts2mat(TS(data.(s)(k).date(m)).rf2m);
            elseif time(m) <= 1/2
                interest(m) = fts2mat(TS(data.(s)(k).date(m)).rf3m);
            elseif time(m) <= 3/4
                interest(m) = fts2mat(TS(data.(s)(k).date(m)).rf6m);
            elseif time(m) <= 1
                interest(m) = fts2mat(TS(data.(s)(k).date(m)).rf9m);
            else
                interest(m) = fts2mat(TS(data.(s)(k).date(m)).rf12m);
            end
        end
    end
end
end

```

```

interest = interest';
t1 = log(omxprice./strike);
t2 = interest.*time;
t3 = volatility.^2./2.*time;
n = volatility.*sqrt(time);

d1 = (t1+t2+t3)./n;
d2 = (t1+t2-t3)./n;
d1star = (-t1+t2+t3)./n;
d2star = (-t1+t2-t3)./n;
st = FORM1(omxprice,d1,time);
th = FORM2(interest,strike,time);

clear m t1 t2 t3 n

if month<13, data.(s)(k).delta = S(d1); else data.(s)(k).delta = S(d1)-1; end
data.(s)(k).vega = V(omxprice,time,d1);
if month<13, [data.(s)(k).theta,U] = blstheta(omxprice,strike,interest,time,volatility);
else [U,data.(s)(k).theta] = blstheta(omxprice,strike,interest,time,volatility); end
if month<13, [data.(s)(k).rho,U] = blsrho(omxprice,strike,interest,time,volatility);
else [U,data.(s)(k).rho] = blsrho(omxprice,strike,interest,time,volatility); end

data.(s)(k).gamma = blsgamma(omxprice,strike,interest,time,volatility);
data.(s)(k).v2 = V2(volatility,d1,d2,data.(s)(k).vega);
if month<13, data.(s)(k).t2 = T2(interest,time,volatility,d1,d2,d1star,d2star,th,st,'c');
else data.(s)(k).t2 = T2(interest,time,volatility,d1,d2,d1star,d2star,th,st,'p'); end
data.(s)(k).sv = SV(volatility,d1,d2);
data.(s)(k).st = ST(time,d1,d1star);
data.(s)(k).vt = VT(d1,d1star,st);

data.(s)(k).s3 = S3(omxprice,time,volatility,d1,data.(s)(k).gamma);
data.(s)(k).v3 = V3(volatility,d1,d2,data.(s)(k).vega);
% data.(s)(k).t3 = T3(volatility,d1,d2,data.(s)(k).theta);
data.(s)(k).s2v = S2V(volatility,d1,d2,data.(s)(k).gamma);
data.(s)(k).s2t = S2T(time,d1,d1star,data.(s)(k).gamma);
data.(s)(k).sv2 = SV2(volatility,d1,d2);
data.(s)(k).st2 = ST2(time,d1,d1star);
data.(s)(k).v2t = V2T(volatility,d1,d2,d1star,d2star,st);
data.(s)(k).vt2 = VT2(time,d1,d1star,st);
data.(s)(k).svt = SVT(time,volatility,d1,d2,d1star,d2star);
clear interest
end
end

function st = FORM1(P,d1,T)
st=P.*normpdf(d1)/(2*sqrt(T));

function th = FORM2(R,S,T)
th=R.*S.*exp(-R.*T);

function delta = S(d1)
delta = normcdf(d1);

function dS2dT = S2T(T,d1,d1star,gamma)
dS2dT = -gamma./(2*T).*(d1.*d1star+1);

function dS2dV = S2V(V,d1,d2,gamma)
dS2dV = gamma./V.*(d1.*d2-1);

function dS3 = S3(P,T,V,d1,gamma)
dS3 = (-gamma./P).*(d1./(V.*sqrt(T))+1);

function dSdT = ST(T,d1,d1star)
dSdT = normpdf(d1).*d1star./(2*T);

function dSdT2 = ST2(T,d1,d1star)
dSdT2 = normpdf(d1)/(4*T.^2).*(-d1.*d1star.^2+d1-2*d1star);

function dSdV = SV(V,d1,d2)
dSdV = -normpdf(d1).*d2./V;

function dSdV2 = SV2(V,d1,d2)

```

```

dSdV2 = normpdf(d1)./V.^2.*(-d1.*d2.^2+d1+d2);

function dSdVdT = SVT(T,V,d1,d2,d1star,d2star)
dSdVdT=normpdf(d1)./(2*T.*V).*(d1.*d2.*d1star-d2star);

function dT2 = T2(R,T,V,d1,d2,d1star,d2star,th,st,type)
if type=='c'
    dT2 = V.*st.*(d1.*d1star+1)./(2*T)+th.*(-R.*normcdf(d2)+normpdf(d2).*d2star./(2*T));
elseif type=='p'
    dT2 = V.*st.*(d1.*d1star+1)./(2*T)-th.*(-R.*normcdf(d2)+normpdf(d2).*d2star./(2*T));
end

function vega = V(P,T,d1)
vega = P.*sqrt(T).*normpdf(d1);

function dV2 = V2(V,d1,d2,vega)
dV2 = vega.*d1.*d2./V;

function dV2dT = V2T(V,d1,d2,d1star,d2star,st)
dV2dT = st./V.*(d1.*d2-d1.^2.*d2.*d1star+d2.*d1star+d2star.*d1);

function dV3 = V3(V,d1,d2,vega)
dV3=vega./V.^2.*(d1.^2.*d2.^2-d1.^2-d2.^2-d1.*d2);

function dVdT = VT(d1,d1star,st)
dVdT=st.*(1-d1.*d1star);

function dVdT2 = VT2(T,d1,d1star,st)
dVdT2=-st./(2*T).*(-d1.^2.*d1star.^2+1+d1.^2+d1star.^2);

```

### 3. Forming samples

```

run
global TS
for y=1:6
    TScreater([y,y-1],['1m '; '2m '; '3m '; '6m '; '9m '; '12m']);
    eval(['load P0' int2str(y) '.mat'])

    for vecka = 2:13
        boundary = [1.02,1.06];
        holding = 5;

        eval(['C=sampcol(y,vecka,holding,boundary,P0' int2str(y) ')'])

        if isempty(C)==0
            eval(['T=forecaster(C,y,vecka,P0' int2str(y) ')'])

            h = length(T);
            L = []; for i=1:h, if isempty(T{i})==0, for k=1:size(T{i},1),L(length(L)+1:length(L)+length(T{i})
{k,1}),1)=T{i}{k,1};end,end,end
            L1 = []; for i=1:h, if isempty(T{i})==0, for k=1:size(T{i},1),L1(length(L1)+1:length(L1)+length(T{i})
{k,2}),1)=T{i}{k,2};end,end,end
            L2 = []; for i=1:h, if isempty(T{i})==0, for k=1:size(T{i},1),L2(length(L2)+1:length(L2)+length(T{i})
{k,3}),1)=T{i}{k,3};end,end,end
            L3 = []; for i=1:h, if isempty(T{i})==0, for k=1:size(T{i},1),L3(length(L3)+1:length(L3)+length(T{i})
{k,4}),1)=T{i}{k,4};end,end,end

            eval(['X' int2str(y),'IV' int2str(vecka) '=[L,L1,L2,L3];'])
        end
    end
clear global TS
eval(['clear P0' int2str(y)])
end

sampcol
function C=reldates(year,vecka,holding,boundary,data)
global TS

C={};
y = num2str(year);
next=1;

```

```

for month = 1:12
s = ['x' y, char(month+76)];
exp = expiration(month, year);
b = fts2mat(TS(exp).busday)-vecka*5;
c = b+4;
st = 0;
bla = {};
for k = 1:length(data.(s))
count=0;
for loop = b:c
OTS = fints(data.(s)(k).date, data.(s)(k).moneyness, 'moneyness');
starting = TS.dates(loop);
ending = TS.dates(loop+holding);
if OTS.dates(1) <= starting && OTS.dates(length(OTS)) >= ending && isempty(OTS(datestr(starting)))==0
mnn = fts2mat(OTS(datestr(starting)).moneyness);
if mnn >= min(boundary) && mnn < max(boundary) && isempty(OTS(datestr(ending)))==0
count = count+1;
sdate(count, :) = datestr(starting);
edate(count, :) = datestr(ending);
end
end
end
if count >=1
st = st+1;
bla(st, :) = {k, sdate, edate};
clear sdate edate
end
end
if isempty(bla)==0
C{next,1}=s;
C{next,2}=bla;
next=next+1;
end
end

forecaster
function T=statbuild(C, year, vecka, data)
global TS

if vecka/52 < 1/12
RF = 'rf1m'
elseif vecka/52 < 1/6
RF = 'rf2m'
elseif vecka/52 < 1/4
RF = 'rf3m'
elseif vecka/52 < 1/2
RF = 'rf6m'
end

for i = 1:size(C,1)
exp = expiration(i, year);
A = C{i,1};
if isempty(C{i,2})==0
for k = 1:size(C{i,2},1)
B = C{i,2}{k,1};
OTS = fints(data.(A)(B).date, [data.(A)(B).price, data.(A)(B).impvol], {'price', 'iv'});
OTS.delta = data.(A)(B).delta;
OTS.vega = data.(A)(B).vega;
OTS.theta = data.(A)(B).theta;
OTS.rho = data.(A)(B).rho;

OTS.gamma = data.(A)(B).gamma;
OTS.v2 = data.(A)(B).v2;
OTS.t2 = data.(A)(B).t2;
OTS.sv = data.(A)(B).sv;
OTS.st = data.(A)(B).st;
OTS.vt = data.(A)(B).vt;

OTS.s3 = data.(A)(B).s3;
OTS.v3 = data.(A)(B).v3;
OTS.s2v = data.(A)(B).s2v;
OTS.s2t = data.(A)(B).s2t;

```

```

OTS.sv2 = data.(A)(B).sv2;
OTS.st2 = data.(A)(B).st2;
OTS.v2t = data.(A)(B).v2t;
OTS.vt2 = data.(A)(B).vt2;
OTS.svt = data.(A)(B).svt;

ds = fts2mat(TS(C{i,2}{k,3}).OMXS30)-fts2mat(TS(C{i,2}{k,2}).OMXS30);
dv = fts2mat(OTS(C{i,2}{k,3}).iv)-fts2mat(OTS(C{i,2}{k,2}).iv);
dt = 1/250.*(fts2mat(TS(C{i,2}{k,2}).busday)-fts2mat(TS(C{i,2}{k,3}).busday));
dr = fts2mat(TS(C{i,2}{k,3}).(RF))-fts2mat(TS(C{i,2}{k,2}).(RF));

d0 = fts2mat(OTS(C{i,2}{k,3}).price)-fts2mat(OTS(C{i,2}{k,2}).price);
d01 = fts2mat(OTS(C{i,2}{k,2}).delta).*ds+fts2mat(OTS(C{i,2}{k,2}).vega).*dv+fts2mat(OTS(C{i,2}
{k,2}).theta).*dt+fts2mat(OTS(C{i,2}{k,2}).rho).*dr;
d02 = 1/2*(fts2mat(OTS(C{i,2}{k,2}).gamma).*ds.^2+fts2mat(OTS(C{i,2}{k,2}).v2).*dv.^2+fts2mat(OTS(C{i,2}
{k,2}).t2).*dt.^2+2*fts2mat(OTS(C{i,2}{k,2}).sv).*ds.*dv+2*fts2mat(OTS(C{i,2}
{k,2}).st).*ds.*dt+2*fts2mat(OTS(C{i,2}{k,2}).vt).*dv.*dt);
d03 = 1/6*(fts2mat(OTS(C{i,2}{k,2}).s3).*ds.^3+fts2mat(OTS(C{i,2}{k,2}).v3).*dv.^3+3*fts2mat(OTS(C{i,2}
{k,2}).s2v).*ds.^2.*dv+3*fts2mat(OTS(C{i,2}{k,2}).sv2).*ds.*dv.^2+3*fts2mat(OTS(C{i,2}
{k,2}).s2t).*ds.^2.*dt+3*fts2mat(OTS(C{i,2}{k,2}).st2).*ds.*dt.^2+3*fts2mat(OTS(C{i,2}
{k,2}).v2t).*dv.^2.*dt+3*fts2mat(OTS(C{i,2}{k,2}).vt2).*dv.*dt.^2+6*fts2mat(OTS(C{i,2}{k,2}).svt).*ds.*dv.*dt);

T{i}{k,1}=d0;
T{i}{k,2}=d01;
T{i}{k,3}=d02;
T{i}{k,4}=d03;

end
end
end
T=T';

```

#### 4. Analysis

```

gruppering
s=input('strike?');
eval(['load P' s 'tmAll.mat'])
for j=2:13
    num=int2str(j)
    for k=1:6
        eval(['exist X' int2str(k) s 'V' num ';' ])
        if ans==0
            eval(['X' int2str(k) s 'V' num '=[];' ])
        end
    end
end

eval(['ny=[X1' s 'V' num ';' X2' s 'V' num ';' X3' s 'V'...
num ';' X4' s 'V' num ';' X5' s 'V' num ';' X6' s 'V' num ';' ');'])
Ny1=abs(ny(:,1)-ny(:,2));
Ny2=abs(ny(:,1)-ny(:,2)-ny(:,3));
Ny3=abs(ny(:,1)-ny(:,2)-ny(:,3)-ny(:,4));
eval(['P' s 'V' num '= [ny(:,1),Ny1,Ny2,Ny3]; '])
end

analys
for i=2:13
    for j=1:4
        eval(['it=P' s 'V' int2str(i) '(:, ' int2str(j) ');'])
        M(i,j)=mean(it);
        S(i,j)=std(it);
    end
    ss(i)=length(it);
    [a,b,c]=linreg([1:3],M(i,2:4));
    regDM(i,:)= [a,b,c];
    [a,b,c]=linreg([1:3],S(i,2:4));
    regDS(i,:)= [a,b,c];
end
for j=1:4
    x=[2:13]';
    [a,b,c]=linreg(x,M(2:13,j));
    regTM(j,:)= [a,b,c];
end

```

```

    [a,b,c]=linreg(x,S(2:13,j));
    regTS(j,:)=a,b,c];
end

```

### Linreg

```

function [a,b,r]=linreg(x,y)
if length(x)~=length(y)
    disp('error')
    a=[];b=[];r=[];
else
    n=length(x);
    Sxx=sum(x.^2)-1/n*sum(x)^2;
    Sxy=sum(x.*y)-1/n*sum(x)*sum(y);
    Syy=sum(y.^2)-1/n*sum(y)^2;

    b=Sxy/Sxx;
    a=sum(y)/n-b/n*sum(x);
    r=Sxy^2/Sxx/Syy;
end

```

### Acrosstrikes

```

j=4;
for i=2:13
    a=['PIV' int2str(i)];
    b=['PAV' int2str(i)];
    c=['PO1V' int2str(i)];
    d=['PO2V' int2str(i)];

    col=['(:, ' int2str(j) ')'];
    eval(['SM(i,:)=mean(' a col '),mean(' b col '),mean(' c col '),mean(' d col ');'])
    eval(['SS(i,:)=std(' a col '),std(' b col '),std(' c col '),std(' d col ');'])
    [a,b,c]=linreg([1:4],SM(i,:));
    regSM(i,:)=a,b,c];
    [a,b,c]=linreg([1:4],SS(i,:));
    regSS(i,:)=a,b,c];
end

```

### lilliefors

```

s=input('strike');
eval(['load ' s 'klar.mat'])
for i=2:13
    eval(['it=' s 'V' int2str(i) ');'])
    for j=2:4
        eval(['ny=it(:, ' int2str(j) ');'])
        lt(i,j)=lillietest(log(ny));
    end
end

```

### ny

```

dessa = [8,10,11,12];
for i=1:length(dessa)
    eval(['[m,h,t]=diffm(log(O2V' int2str(dessa(i)) ');'])
    tabell(i,:)=m,h,t];
end

```

### diffm

```

function [m,h,t]=diffm(samp)
x=samp(:,4)-samp(:,3);
n=length(x);
m=mean(x);
s=std(x);
d=s/sqrt(n);
u=m/d;
t=tcdf(u,n-1);
if t>0.975
    h=1;
else
    h=0;
end

```