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Mean-Earnings-Variance Based Portfolio Selection

Abstract:

The objective of the thesis is to provide a portfolio selection methodology which will take into account the earnings of the companies whose shares are under consideration, though a look into the problem of estimation of the efficient frontier for a basket of stocks. Following an overview of standard portfolio selection theory, a simple risk-return based model will be expanded to a return-earnings-risk model and the portfolio frontier will be projected as a surface in a third dimensional portfolio space. An inquiry into the optimal estimation method will seek to provide the methodology that will lead to the combination of stocks with the greatest Sharpe ratio.

The expanded model is then back tested against the standard one employing data for Dow Jones components for the period between 1997 and 2008. Looking closer into these results, it will be shown how the inclusion of earnings lead to better portfolio risk weighted performance, as measured by the Sharpe ratio.

Keywords: Portfolio Theory, Earnings, Mean - Variance, Estimation Problem, Portfolio Frontier

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1. Introduction

The question of how can it be possible to maximize the long term growth of one's wealth, far predates the economics science. As long as an individual can store wealth and demand for the use of the wealth-in-storage exists, the question of what is the optimal allocation will be dominant. Investors will thus need a method to rank their alternatives in terms of maximization of their utility.

Even though various approaches to this end have been proposed in the past, a recurring theme is the concept of a trade off between the expected return from an investment and the risk associated with it. The rationale behind the trade off rests on the premise that should all assets yield the same return, it would be natural for the risk averse investor to prefer the asset with the lowest risk. Seeing return as a premium that compensates for the undertaken risk can help explain the behaviour of investor when allocating wealth among assets. However, even though the practice of charging higher premiums for riskier ventures remains present from shipping insurance in ancient Athens to today's equity and venture capital markets, the representations of risk and its premium have varied over time.

Even when it comes to the simplest investment, for example a loan it is not clear how one should evaluate whether the risk is justified. To illustrate this, one might drift away from the explicit scope of the thesis and consider for instance an investor who may only achieve return by lending his wealth to other agents, each of whom faces a different probability of default while promising a different but constant interest payment. If the investor lends his entire wealth to the most creditworthy agent, in the event of default he would be bankrupt. Thus there could be a way for an individual to protect himself by lending not one but many, as creditworthy as possible, borrowers, thus decreasing his chances of large losses due to diversification.

This line of thinking can be extended into assets that represent investments in companies, the most common of which would be a stock, purchase of which would represent ownership of a fraction of a company, and which might or might not pay a dividend. If the investor can not only lend money at a fixed rate of interest but also participate in investments whose return varies according to the success of the underlying business, then the potential returns will obviously increase, further complicating the problem. In this case one will need to worry about the possibility of default as well as the possibility that the investment will have a smaller return than anticipated, or even a negative one, in which case one might lose part but not all of the funds that have been invested. Furthermore, if the potential payoff can be traced down by examining past returns, one also needs a measure for risk. When it comes to investing in shares of companies, the volatility of their returns is the most common and obvious parameter that can be used to account for risk. However, as in the problem of the investor in loans, the investor in shares must take into account that low volatility is of little use when losses in various investments occur simultaneously. In that respect it might make more sense to try and pick a basket of shares whose returns and losses seem unrelated with each other. In essence the ultimate purpose of the investor will be to maximize the return on his wealth while minimizing the volatility of the entire portfolio rather than that of individual shares.

Extensive amount of literature exists on portfolio selection and a great proportion of it is based on the Mean-Variance portfolio frontier, pioneered by the research of Markowitz in the early 50s. The model produced by Markowitz (1959) produces an optimal combination of securities which can be levered in such way that a single optimal portfolio exists for every level of risk tolerance. This in some cases implies the existence of a risk free rate of interest. In this respect, one assumes the existence of an omnipotent economic agent, such as a government, that only lends and borrows but may never default.

The inherent implication of the above is that an individual or mutual fund selecting their portfolio as proposed by the mean variance criterion will be expected to outperform the returns of an alternate strategy consisting of buying and holding an index in which all assets have equal weights. In effect, the investor utilizing this approach seeks to benefit by attaining the optimal diversification among investments (Markowitz, 1959). Essentially, if this is true, then an investor can achieve better returns for any given level of risk tolerance when weighting investments through Mean-Variance, rather than assigning equal weights in each stock in the index or following some other arbitrary methodology to assign weights.

The Mean-Variance theory requires in essence the selection of a measure for risk and a measure for return. The most straightforward way would be to use the volatility of portfolio returns to account for risk, while the past returns of the portfolio account for reward. This approach however has some significant shortcomings, the most notable of which is the backward looking nature of the methodology and its tendency to overvalue securities showing high returns, regardless of whether these are generated by an increase in the company's ability to generate earnings or are due to irrational behaviour of market participants generated by cognitive biases on their behalf, whose excess demands pushes securities prices higher thus forming a stock market bubble.

In light of this, the investor, or the individual responsible for investment decisions on behalf of others might consider a number of alternatives to account for reward. Should one be able to select a more effective way to account for returns, it would be possible to achieve a better trade off with respect to the corresponding risk. In fact, the selection of portfolio weights based on historical returns is itself a source of risk, which is not related to the market; the estimation risk, which cannot be accounted for through volatility. There exists thus a very distinct additional problem, which concerns the estimation method employed by the investor in stocks.

The objective of this thesis will be to construct a model which incorporates the companies' earnings in its dynamics, thus allowing a more efficient model for investments, in order to provide the optimal solution to the portfolio selection problem, in conjunction with the estimation problem.

The importance of earnings have been inquired into by in the work of Shiller (2005), who makes a very compelling case that earnings matter. His methodology consists in a model which has the objective of optimizing allocation with respect to the earnings of a passive investment at the time that one chooses to invest in the market. Notable research includes that done by Sanjoy Basu (1968) and Francis Nicolson (1977), who by employing earnings to provide evidence against the Efficient Market Hypothesis (the interested reader is referred to its outline as done by Fama, 1970 and Samuelson, 1965), show a synergy of attractive earnings and returns. This thesis differs from the work of the aforementioned authors in that it does not seek to ratify or disprove the efficient market hypothesis but rather than provide a model that functions regardless of its existence, by seeing the earnings issue as part of a larger solution to the estimation problems associated with the standard portfolio theory.

The Thesis is outlined as follows. First, the groundwork will be laid through an overview of the standard model that Markowitz proposed, along with a discussion of its properties, advantages and disadvantages, paying particular attention to unwanted properties met when its estimation is done employing historical returns. Then the model will be modified and extended in an attempt to sustain the advantages while discarding part of the drawbacks. Finally the new models will be tested on the terms of the original theory.

2. Standard Portfolio Theory Overview

i. The theoretical motivation of portfolio selection and its estimation through historical returns.

Markowitz (1959) proposes the basic model of portfolio selection for the individual that aims in utility maximization subject to his preferences regarding return and risk. In Markowitz's theory as well as for the purpose of this Thesis, investment choices are constrained to stocks an asset which is assumed to be risk free, such as short term government bonds. Inherent in this proposition is the postulate that while all individuals will prefer the maximum possible return they have different levels of risk tolerance, thus as long as different stocks entail different risks, a portfolio selection model must not only produce the most risk efficient combination of assets, but also must allow for individuals to fine tune their investments to allow for different attitudes towards risk.

In this respect, the most obvious estimation method of Markowitz's model consists in utilizing historical returns of various stocks to model future expected returns and their volatility and correlations to model portfolio risk. The use of volatility is rather straightforward to justify; if uncertainty for a stock is high, substantial volume of unexpected information concerning the stock will reach the market, leading to consecutive selling or buying. On the other hand if uncertainty for a stock is low most news associated with the stock will be anticipated by the market participants, thus investors will tend to stick with their strategy and its price will follow a more or less stable path. In the standard portfolio theory context, volatility of a portfolio can also be seen as a value at risk measure, should one accept that stock returns are distributed normally, or follow some other kind of PDF which remains constant over time. If one accepts this property of volatility, one can measure the probability of any level of loss associated with a stock or stock portfolio, by plotting the standard deviation of the portfolio into the PDF in order to calculate probabilities for each level of loss.

Nevertheless, justifying the use of past returns as a proxy for future ones is not so simple, as a sceptical reader might not find it easy to accept that past returns of a stock can say much about the future. However, should one accept the efficient market hypothesis, as discussed in section iii, such justification becomes much easier. One can then state that past stock returns represent the premium that their buyers were demanding in order to take on the risk associated with the corresponding stock. In this respect, the utilization of past returns can be seen as a way to sort those securities which have offered the most attractive premia in the past and not as some mystic mechanism which predicts future returns.

Consequently, one must choose a way to model the investor's risk tolerance into the model. This is done by incorporating a tolerance constant in the utility function, which is meant to account for each investor's view on the trade-off between risk and return. The more risk an investor is willing to take, the more loading of risky but lucrative investments will be warranted.

Finally a special case can sometimes be assumed, where the market is under CAPM equilibrium (Sharpe 1964, Sharpe 1970). In this case all agents have picked the optimal mean variance investment choice and the and there exists a tangency portfolio, investment in which will be optimal regardless of the individual's risk preferences. In this case, individuals will be attaining the optimal Sharpe ratio when investing in the tangency portfolio while using a positive, negative or zero investment in the risk free rate to account for their risk tolerance. In this case, the investor who attains higher risk weighted return than the tangency portfolio will have a positive alpha, which is the CAPM measure of what part of an investor's returns can be attributed to skill rather than loading of risky assets. In this context, how risky an asset is can be measured by inquiring into the degree to which its moves are related to market moves and is represented by the constant beta.

ii. Expressing the Mean-Variance model mathematically

The above can be expressed mathematically through the utility function (Hansson 2009)

$$U\left(\mu_p, \sigma_p^2\right) = t\,\mu - \sigma^2/2 \tag{2.1}$$

Where mu denotes return, sigma denotes volatility and t is the risk tolerance constant which is distinct for each individual investor. It follows that the utility is maximized by allocating wealth as follows:

$$\max_w \left(t\, w' \mu - \frac{w' V w}{2} \mid w' I = 1 \right) \tag{2.2}$$

Where w is the vector which contains the weights under which wealth is allocated to each asset and V is the covariance matrix of the assets. As individuals have finite wealth, the $w' I = 1$ constraint allows a finite solution to the maximization problem. In equation 2.2, the investor can load more risk into the portfolio by increasing the value of t (which is the investor specific measure of how much risk one is willing to take). Projection of expected returns on portfolio variance for increasing values of t produces the "efficient frontier". This represents the hyperbola of asset combinations that produce the highest return for a given level of risk and investors simply need to pick a point on it in order to achieve the highest returns for their given level of risk tolerance. A further implicit assumption of no transaction costs must also be noted. Finally, one must assume that markets trade in continuous time, i.e. that transactions do not have breaks but are taking place in a marketplace that is open at all times.

Now, should one consider a universe of possible investments and the presence of a risk free rate of interest there will be a tangency point on the efficient frontier where it meets the straight line representing the interest rate. That point is the market portfolio and any move beside it will reduce Mean-Variance efficiency. Mathematically the market portfolio weights can be shown to be[i]:

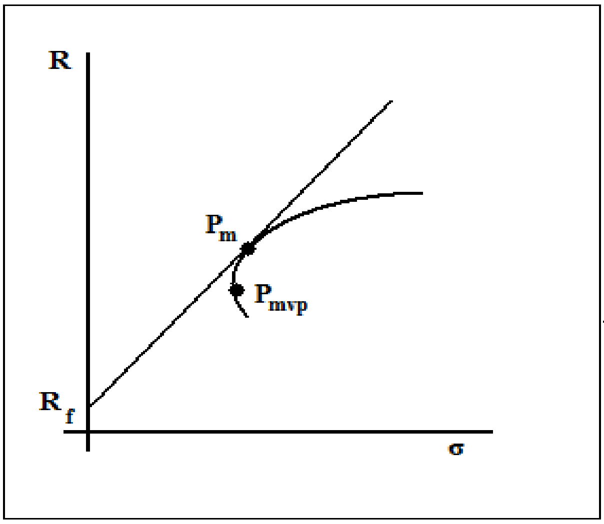
$$W_{MV} = \frac{(\mu - r_f)[V^{-1}(\mu - r_f I)]}{(\mu' - r_f I')V^{-1}(\mu - r_f I)} \tag{2.3}$$

Where r_f denotes the risk free rate of interest and W_{MV} the vector of asset weights. It follows that investors will instead adjust their return target and portfolio variance tolerance by the appropriate combination of investment in the risk free asset and the market portfolio. This is mathematically equivalent to stating that the market portfolio has the maximum Sharpe ratio (defined as the ratio of return in excess of the risk free rate to the standard deviation of the portfolio), thus the Sharpe ratio will be used as the main benchmark for the evaluation of portfolios in back testing.

Note that solving equation 2.2 for zero risk tolerance yields the portfolio which is characterized by minimum variance. If the return of this portfolio is lower than the risk free rate, the market portfolio will lie outside the efficient frontier on the lower half of the hyperbola and investors will want to short sell it and invest the proceeds as well as their initial wealth in the risk free rate. A special case exists when the return of the minimum variance portfolio is equal to the risk free rate, in which case the tangency portfolio does not exist and investors will employ a combination of the risk free asset and a zero investment portfolio.

Finaly, Chart 2.1 provides a brief graphical depiction of the standard portfolio space under Markowitz's' Mean-Variance model with the market in CAPM equilibrium, with P_m denoting the market portfolio at the point where the efficient frontier crosses the line whose intercept is the risk free rate, R_f.

Chart 2.1: The efficient frontier



iii. Shortcomings of the Mean-Variance model and unwanted properties of its estimation through historical returns.

Despite the advantages of the model discussed above, there are some serious shortcomings associated with it, not only regarding its theoretical aspects but also concerning the frequent occasions in which it has failed to function as expected in practice. The performance of the model during the dotcom bubble as well as during the credit crunch will be discussed in more detail later on, however it can be said that in most bear markets the model tends to underestimate downside risks for stock prices (Mandelbrot and Hudson 2006).

One explanation about this can be given if one rejects or even relaxes the efficient market hypothesis. The very strong version of the hypothesis implies that the returns of a stock or a model will reasonably resemble a statistical distribution. This line of thinking is further amplified by the assumption that information unexpected events cause price movements and these events, once priced in a stock, cannot under any circumstances provide further guidance for a company's future. If movements in stock prices are a result of unexpected news reaching a market that has priced in all information available until then, one can reasonably expect that a time series of stock returns will not be serially correlated, which is equivalent to stating that there is no possibility to predict the future movements of the stocks based on their past. If this is the case then volatility not only accounts for risk but also provides a value at risk based measure of risk. However in practice, even before the credit crisis, historical stock returns have generally not followed a constant distribution in the long run and have shown that the case for serial correlation can be made, in such way that a random walk based model would not be consistent with observation (Mandelbrot and Taylor 1967). Thus one may conclude that either stock returns are not identically and independently distributed, or that the market does not efficiently price available information.

Additionally, there are further concerns about the foundations of the estimation of the mean variance model through historical data. Should one relax the efficient market hypothesis, the use of past returns as a proxy for future ones is no longer so simple to justify, as one could then state that the returns of a market that cannot efficiently price in information, is under no way a good indication for the future as premiums calculated by the market in the past might be misleading.

Finally, the most striking shortcoming of the mean-variance model when estimated through historical returns is not associated with the efficient market hypothesis. It rather follows from the implicit recommendation of the model towards the overweighting of stocks that have had solid returns in the past. This estimation method brings the model dangerously close to resembling a bubble-built up mechanism. Indeed, as under the model investors would tend to allocate wealth into assets that have given high returns in the past, the rise in their demand will lead to even higher prices which will in turn be translated into even better returns, leading to a vicious circle of further investment in these stocks. Then at some point the intrinsic value of the stocks associated with the ability of their respective companies to generate earnings will no longer be able to catch up with the stock price, leading to the inevitable burst of the bubble. This is the essence of the estimation problem one encounters when seeking to yield the optimal mean variance solution using historical stock returns.

3. Expansion of the standard model

i. Earnings as a factor in portfolio selection.

From the analysis of the mean variance model in the previous sector, it follows that one needs to address the fundamental problems associated with it, even in the expense of simplicity. In this respect, a new variation of the model can be proposed, which will incorporate the earnings of the companies in which one may invest. The rationale behind the use of earnings is that if a company can produce enough profit to pay its investors with dividends or load the balance sheet with cash and equivalent assets, they will be very reluctant to sell and allocate their wealth elsewhere. Assume that two similar stocks have had exactly identical returns and volatility and that one of the companies is making a profit while the other one is making a loss. In such case it would be reasonable to conclude that one would rather sell the loss making company rather than the profit making one. Now if we expand this line of thinking it becomes reasonable to conclude that the earnings of the companies should one way or the other incorporated in a model which seeks to propose a forward looking allocation of wealth into assets.

The above remarks concerning earnings can be expressed from a different view point. As discussed in section iii. Of the previous chapter, there are issues concerning the estimation of the mean variance model through historical returns, as this may lead one to invest in stocks exhibiting bubble like properties. However, using the earnings per share or some equivalent measure in the estimation process can be a very effective solution. This happens because as a stock's price would rise, the earnings/price ratio of a stock would decrease, thus representing lower return and guiding the investor away from bubble-like assets.

It follows that one must determine what variable is the best to be used as a proxy for earnings. The simplest way is to simply use the companies' earnings as reported in their quarterly statements, i.e. their PE ratios. Another alternative is to use the analyst forecast consensus for the stocks or the earnings guidance of the company, which have the obvious advantage of being forward looking, but are subject to the risk related to the abilities of the analysts.

Another question arises regarding the way that should be used to incorporate earnings into the asset allocation model. If one completely rejects the use of past returns as a proxy for future returns, one can simply postulate that earnings should be used instead. This produces a model that is mathematically identical to the mean-variance one, where mu is simply replaced with a variable to account for earnings. However, a more elegant solution, albeit a more complicated one might be to introduce earnings in addition to returns, along with a constant to account for the importance an individual places in protecting himself against the emergence of a bubble.

ii. The Mean-Earnings-Variance model.

Assuming that one has a measure for the earnings of the companies, one can begin by modifying the standard mean variance model as follows:

$$\max_w (\kappa w' \varepsilon - \frac{w' V w}{2} \mid w' I = 1) \tag{3.1}$$

Where epsilon is the vector of earnings over price and kappa is a factor representing the weight that an individual investor assigns to earnings. Note that the above relation is mathematically equivalent to (2.2), and its implicit statement is simply “earnings forecast future returns better than historical returns”, something that can be tested in a straightforward way as will be discussed in chapter 4.

In the presence of a risk free rate and following the exact Mean-Variance methodology for solving the utility maximization problem, the weights of the market portfolio will be

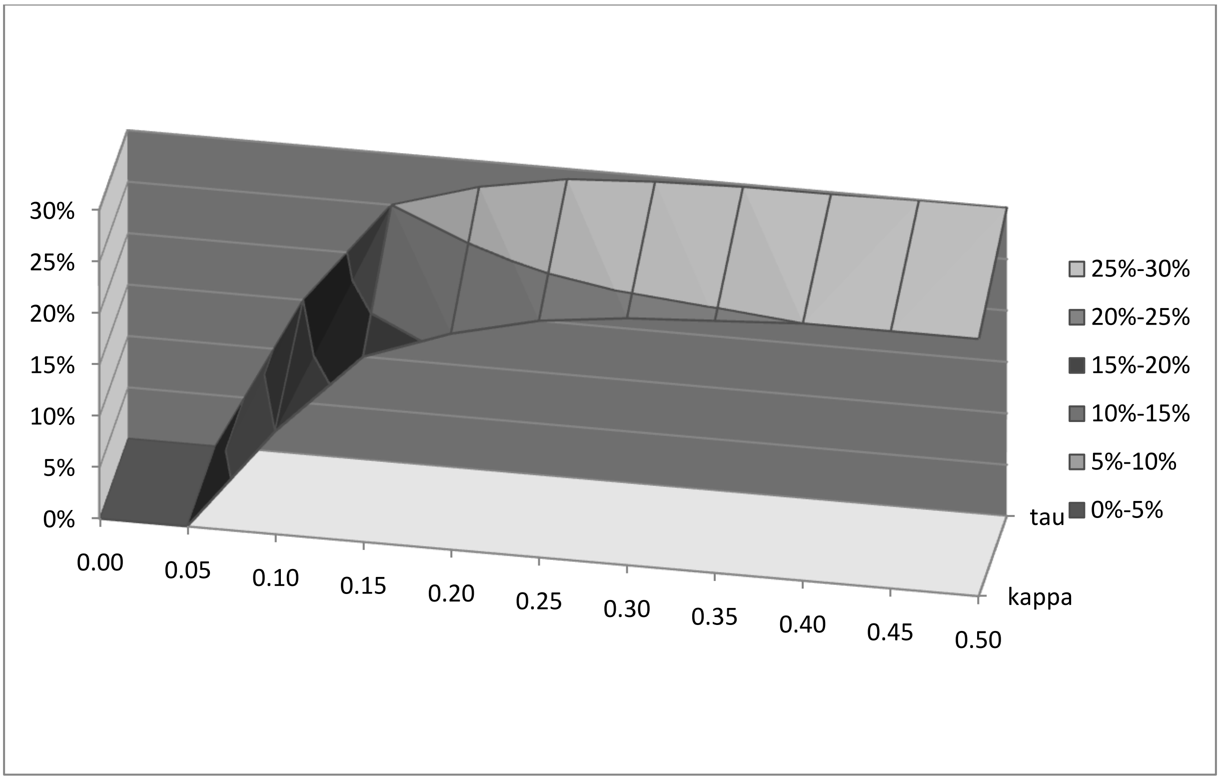
$$W_{EV} = \frac{(\varepsilon - r_f)[V^{-1}(\varepsilon - r_f I)]}{(\varepsilon' - r_f I')V^{-1}(\varepsilon - r_f I)} \tag{3.2}$$

Now the question one needs to ask is whether historical returns should be completely discarded from the model, as in (3.1). As discussed above, one can postulate that the more confidently one adopts the efficient market hypothesis, the more importance one has to place on past earnings. A fundamental dilemma on this issue can be traced here, as while the mean-variance model and the efficient market hypothesis seem pretty robust within normal economic environments, the opposite can be said about downturns where even a relatively risk averse investor can be lead to overloading of stocks. A further discussion of the hypothesis being beyond the scope of this thesis, one can suffice to admit that some weak version of it holds and needs to be incorporated in the model. Following this line of thought, the investor needs to decide in addition to the risk level (t) the level of importance one wants to place in the companies’ earnings (κ). If one further assumes CAPM equilibrium and that investors allocate their entire wealth on a combination of stocks, these weights will sum to one, as any other number would imply investment in either a combination of stocks and the risk free asset, or an overinvestment in stocks through the borrowing at the risk free rate. Note that even if one does not accept this assumption, the Sharpe ratio of the portfolio is not affected. Mathematically this corresponds to:

$$\max_w (t\, w' \mu + \kappa w' \varepsilon - \frac{w' V w}{2} \mid w' I = 1, (t + \kappa) = 1) \tag{3.3}$$

The above relation can be seen as an expansion of the traditional portfolio frontier to include a third dimension that accounts for earnings. The frontier instead of a hyperbola plotted on a two dimensional mean-variance space, will now correspond to a portfolio surface plotted on a three dimensional mean-earnings-variance space.

Chart 3.1: Representation of the portfolio surface



In chart 3.1, simulated data of stock market returns are used to give an illustrative impression of such a frontier surface for all increasing combinations of t and κ. The chart is simplified as instead of placing the tau and kappa in separate axis, the surface is folded and they are shown one behind the other.

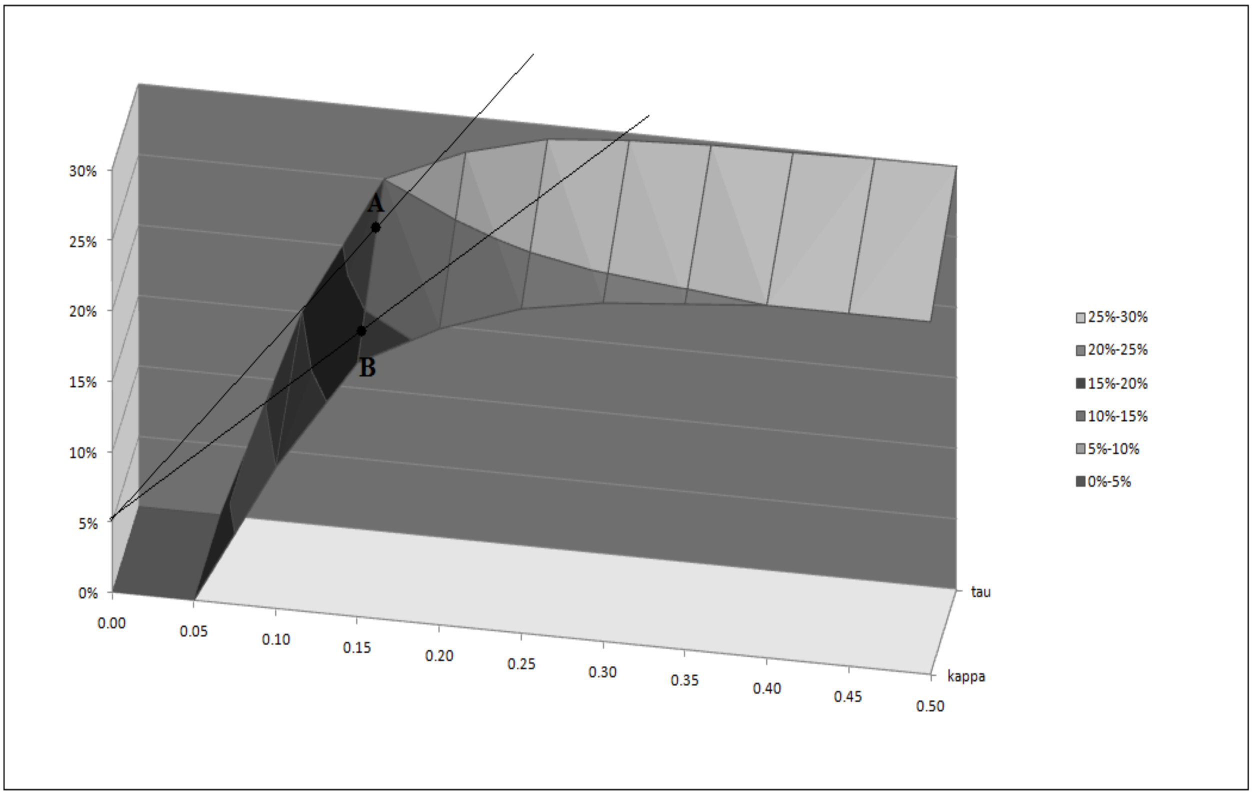
It must also be noted that the minimum variance portfolio of all aforementioned models is identical and can be computed by solving 2.2, 3.1 and 3.3 with zero weight assigned to the risk tolerance constants tau and kappa. Assuming that the return of the minimum variance portfolio is lower than that of all efficient portfolios, it follows that there will be a plethora of portfolios tangent to the portfolio surface, each corresponding to a combination of values of κ and t. In the presence of a risk free rate, the investor will have to decide on the values of the constants depending on how much weight he assigns to past performance versus present earnings. The weights of the portfolios can be assumed to be

$$W_{MEV} = tW_{MV} + \kappa W_{EV} \,|(t+\kappa) = 1. \tag{3.4}$$

The above implies that the investor has to pick among two portfolios. The first one (MV) is constructed through the past returns of the assets and is expected to yield the highest return should these assets continue to behave in the future as they had in the past, it is thus obviously prone to investing in a potential bubble if the earnings of these companies cannot catch up with the increases in the share prices that are a natural outcome of buyer initiated transactions. The second one (EV) is a portfolio where instead of past returns; the various companies are evaluated in their ability to generate cash. The implied expected return might be lower, but this portfolio can be expected to perform better when the market enters a "reality check" mode and investors dump the most spuriously inflated assets.

The possible combinations of tau and kappa graphically correspond to a choice among portfolios tangent to the portfolio surface as in chart 3.2. Assuming that there exists a risk free rate of 5% at which investors borrow and lend, there will be infinity of combinations of efficient portfolios that result from the combination of W_{MV} and W_{EV}. All of them will lie where the line whose slope is the risk free rate is tangent to the surface formed by the two portfolios. Recall that the chart shows a simplified representation where the surface is folded, so what appears to be a straight line is the curve formed by the continuity of tangent points. The investor who wants to assign more weight to the earnings factor in equation 3.4 will prefer portfolio B, as opposed to portfolio A. This would not be efficient in a mean-variance context, as the investor in B would achieve lower return for the same risk. However, in the Mean-Earnings-Variance context the efficiency is achieved through equation 3.4 and even though the risk of A and B appear equal, in practice the investor who mistrusts the robustness of the Mean-Variance model seeks to benefit by diversifying not among portfolios but among investment strategies. After deciding on a combination of kappa and tau, the investor can commence borrowing or lending at the risk free rate can be used to adjust for the investor's risk tolerance.

Chart 3.2: Portfolio choice in the Mean-Variance space



4. Application of the Mean-Earnings-Variance model

i. Data Overview – Dow Jones in the 90s and 00s.

Before proceeding to benchmarking the Mean-Earnings-Variance and Mean-Variance models, a brief discussion of the data set is in order. As the rationale behind the Mean-Earnings-Variance model is to outperform the Mean-Variance one in the long term, the benchmarks must be run in a variety of market and macroeconomic conditions. It must also be done through selection among stocks representing companies that are active in different industries.

In this light, the most obvious solution is the Dow Jones industrial average index. As the components of the index are not constant, a basket of the 29 companies that comprised the index as of December 2009 will be used, dropping the stock of Kraft due to earnings data availability issues. This basket is characterized by large capitalization companies, spanning a wide range of industries and sectors. It should also be noted that since all companies are large Multinational Corporations, their price will be very sensitive to global macroeconomic developments. Furthermore, where applicable, the yield of the one month T- bill will be used as the risk free rate.

The data series starts in 1994 while the back testing period will begin in January 1997 and end in November 2010. This period is very appealing for the purpose of evaluation of asset allocation models, as it contains two very rigid bull markets in the early 90s and mid 00s as well as the two remarkable bear markets of the last decade. The data source is the Reuter's market news international data base, except for the T-Bill yields which come from the data base of the New York Fed.

Table 4.1: Overview of the companies in the data series

Company	Expected Return	Volatility	Average PE Ratio	Industry
3M	9.41%	21.78%	21	Industrials
Alcoa	3.45%	38.90%	50	Basic Materials
American Express	12.32%	33.24%	19	Financials
AT&T	5.12%	25.54%	19	Telecoms
Bank Of America	4.65%	42.40%	13	Financials
Boeing	6.75%	30.47%	33	Industrials
Caterpillar	11.53%	32.63%	15	Industrials
Chevron	10.87%	20.06%	21	Energy & Petroleum
Cisco	15.41%	41.38%	49	Technology

Coca Cola	7.90%	23.04%	33	Non Cyclical Consumer Goods
E. I. Du Pont De Nemours	4.01%	25.83%	35	Basic Materials
Exxon	11.89%	16.85%	17	Energy & Petroleum
General Electric	5.59%	27.33%	24	Industrials
Hewlett Packard	12.26%	37.08%	23	Technology
Home Depot	7.82%	28.58%	29	Retail
Intel	10.69%	42.18%	29	Technology
IBM	14.61%	29.59%	19	Technology
Johnson & Johnson	12.80%	20.07%	23	Healthcare
JP Morgan	10.49%	34.52%	27	Financials
McDonalds	9.83%	23.78%	22	Non Cyclical Consumer Goods
Merck	6.76%	28.86%	21	Healthcare
Microsoft	16.15%	34.92%	35	Technology
Pfizer	9.81%	23.89%	36	Healthcare
Procter & Gamble	10.72%	22.21%	26	Non Cyclical Consumer Goods
Travellers	7.92%	27.75%	19	Financials
United Technologies	14.43%	25.72%	19	Technology
Verizon	5.05%	24.57%	30	Telecoms
Wal-Mart	9.35%	24.14%	26	Retail
Disney	4.46%	27.13%	40	Media
AVERAGE	9.38%	28.77%	27	

Table 4.1 shows the basic properties of the companies that will be used to test the models. The technology sector is the one most heavily present (7 companies), followed by Industrials (4 companies), Financials (4 companies), Healthcare (3 companies), Non Cyclical Consumer Goods (3 companies), Telecoms (2 companies), Retailers (2 companies), Energy (2 companies) and Media (1 company). On first glance, volatility is in line with one what would expect based on the uncertainty in the industry. Shares of companies that are considered defensive such as those in the Healthcare and Non Cyclical Consumer Goods Sectors are characterized by lower volatilities relatives to those in traditionally riskier sectors such as Financials and Technologies.

ii. Construction of Model Portfolios

a. Estimation Windows

In an effort to reproduce the behaviour of a fund which allocates wealth into combinations of positions in stocks and the risk free asset as accurately as possible, it will be assumed that when the allocation takes place, the decision making process is influenced only by the data which are available at the time. Then the accuracy of the models can be asserted by examination of the degree to which the portfolios achieve the desired goal of risk-return efficiency.

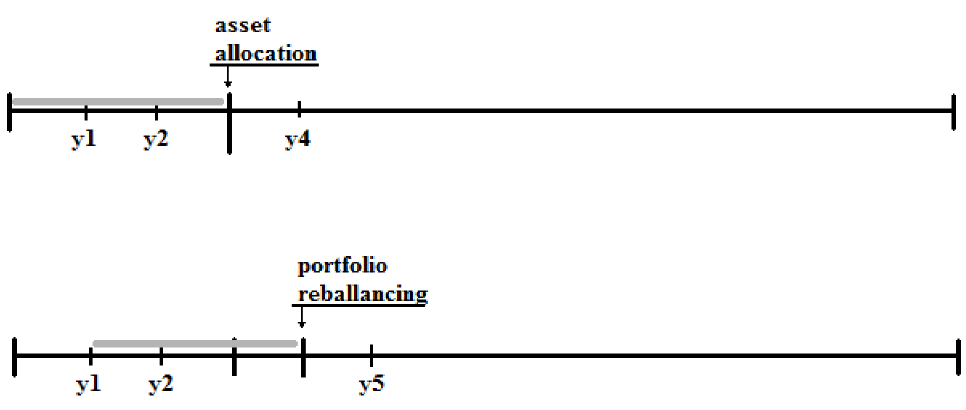
The exact methodology that will be followed is rather simple:

- At the beginning of each year, the fund manager gathers the information on the historical behaviour of the assets for the last three years.
- The first working day of the month, the wealth of the fund is allocated on a combination of stocks and the risk free rate, as dictated by the utilized model.
- At the end of the year the fund closes all open positions and settles the incurred profit or loss.

This process is repeated thirteen times, from 1997 to 2009. Then one can assess the long term performance of each model based on the risk/return achieved when utilizing only data for the last three years as well as the asset allocation model. The main ranking tool will be the attained Sharpe ratios.

The process is out lined in chart 4.1. The fund manager at the start of year 3 allocates the wealth into assets based on the data for the last three years (marked in grey). At the end of the year he re-estimates the model he follows based on the latest three year data and rebalances the portfolio. This process is followed for 13 years.

Chart 4.1: The asset allocation process



b. Returns and Earnings on Individual Stocks

In order to accurately reproduce the returns from buying and holding a specific stock, one needs to incorporate the effects of dividend payouts on wealth. Thus the returns for holding a stock over a period from t-1 to t will be calculated as follows:

$$R_t = Ln((P_t + div_t)/P_{t-1}) \tag{4.1}$$

Where R_t represents the return for the period, div_t represents the dividend paid for holding one share in the period, while P_t and P_{t-1} are the closing price of the stock at points t and t-1. In practice, P_t and P_{t-1} incorporate the effects of asset price change on wealth, while div_t accounts for the effects of dividend payouts.

Earnings, represented by ε in equation 3.1 are of course of particular importance. The estimation of monthly earnings ratios, which show how much profit a company generates based on its share price, will be done by dividing the annualized earnings per share as reported in the company's quarterly balance sheet by the closing price of the company's stock for the month. Thus for month t it will be:

$$E_t = \frac{Annualized\ \ Earnings\ \ Per\ Share\ for\ the\ last\ quarter}{Share\ Closing\ \ Price\ \ for\ \ the\ Month} \tag{4.1}$$

It must be noted that when companies make a loss, earnings per share will be assumed to be zero

c. Model Construction Overview

In sections 2 and 3, the basic models are outlined with the analysis being focused on their theoretical aspects. An overview of the models focused on the more practical estimation issues is thus in order.

- *The DOW* Model.* As the stocks in the Dow Jones Industrial Average have been changing, the values of the index will not be used. Instead, DOW* portfolio will be utilized defined as the portfolio consisting of investing in all 29 shares with equal weights.
- *The Minimum Variance Portfolio (MVP).* It can be calculated by solving any of the utility maximization relations (2.1, 3.1,3.3) assigning zero weights to the risk tolerance constants tau and kappa.
- *The Mean-Variance (MV) model.* It is the market portfolio under the traditional Mean Variance model, estimated by equation 2.3.
- *The Earnings-Variance (EV) model.* It is identical to the Mean Variance model, using earnings as a proxy for return. It is estimated by equation 3.2
- *The Mean-Earnings-Variance (MEV) model.* It is the portfolio resulting from the linear combination of MV and EV portfolios that correspond to varying values of tau and kappa. Regarding the analysis of model portfolios performance, an equal

weighting of 0.5 in each will initially be assumed. Consequently the case of varying weights will be examined.

Note that depending on the degree to which one accepts the CAPM equilibrium assumption, MV portfolio or the DOW* portfolio can be seen as a passive investment. In the context of the analysis, the DOW* will be considered as the passive investment, and alpha and beta measures will be computed with respect to it.

d. Model Performance Indicators

The most reasonable assumption for the objectives underlying asset allocation decisions is that the ultimate goal is to achieve the most efficient combination of return and risk, given risk tolerance. Consequently, in order to assess and rank the models, one needs to take into account a number of indicators related to the risk and returns imposed and whether the trade-off offered by each model is attractive in comparison to the others. The following will be considered:

- Total Return. The compounded return of a $1 investment over 13 years (note that for 2009 the 9-month return is annualized), which an investor achieves when following the strategy dictated by the one of the specified models.
- Expected Return. Calculated by dividing Total Return over the number of Years
- Sigma. It is the standard deviation of the expected returns observed when following an specific investment strategy.
- Sharpe Ratio. The efficiency maximizing ratio of expected return in excess of the risk free rate over the standard deviation of returns. It will be used to evaluate whether the return offered by a specific model strategy justifies the risk. As higher Sharpe Ratio implies greater efficiency with respect to risk, it will be used to rank the performance of each strategy.
- Beta. The correlation of a model strategy with the DOW* portfolio.
- Alpha. A measure of excess returns that cannot be explained by beta and may thus be attributed to the performance of the model assuming a naturally distributed error element.

iv. Models Back Testing and Comparison.

This sector splits the analysis of the problem in two environments, one where short selling is allowed and one where it is not allowed. This allows for a more spherical approach to the issue of inclusion of the earnings in the asset allocation process.

It might be useful to note, that the analysis might become more interesting if all model portfolios are to be seen as funds. In that sense the problem can be limited to the question of picking the optimal fund or combination thereof, each of which follows an explicit investing methodology that can be replicated with ease, should one assume insignificant effect of commissions and non continuous trading time, restrictions bound to be present in a real investing environment.

a. Short Selling not Allowed

One may begin analysing the performance of the model in the case that short selling is not allowed. In this sense, the five portfolios can be seen as different mutual funds allocating 100% of their capital to long positions in the 29 stocks. Then the individual investor may invest in one of the funds any amount by borrowing and lending at the risk free rate of interest.

Recall from section iii. Of chapter 3, that a 100% investment in the tangency portfolio implies CAPM equilibrium. This is expressed by tau and kappa summing to one in equation 3.3. Should one reject this assumption, attention to alpha and beta should be diminished. However, the conclusions concerning the Sharpe ratio are unaffected by the acceptance or non acceptance of CAPM equilibrium.

Table 4.2: Model performance when short selling is not permitted

	DOW*	MV	EV	MVP	MEV
1997	28.81%	34.20%	28.36%	28.34%	31.28%
1998	25.64%	62.51%	-6.44%	20.14%	28.04%
1999	21.96%	54.22%	15.85%	22.22%	35.03%
2000	-6.10%	-33.30%	12.28%	-5.40%	-10.51%
2001	-9.50%	-47.07%	1.43%	-8.82%	-22.82%
2002	-21.08%	1.11%	-20.90%	-19.41%	-9.90%
2003	26.63%	17.99%	12.59%	19.70%	15.29%
2004	5.20%	19.42%	15.06%	15.31%	17.24%
2005	2.35%	11.73%	7.97%	3.63%	9.85%
2006	19.17%	10.27%	25.43%	15.01%	17.85%
2007	8.93%	1.53%	6.17%	6.95%	3.85%
2008	-38.94%	-32.25%	-31.85%	-25.93%	-32.05%
2009 **	14.44%	-3.90%	16.52%	-4.86%	7.96%
Total Return	78%	96.45%	82.47%	66.87%	91.11%
Average Return	6%	7.42%	6.34%	5.14%	7.01%
Standard Deviation	20%	32%	17%	17%	21%
Sharpe Ratio	0.29	0.13	0.20	0.11	0.20
Beta vs. DOW'*	NA	1.06	0.61	0.70	0.84
Alpha	NA	0.007	0.014	-0.004	0.012

The MEV portfolio returns are calculated assuming 0.5 weights in each of the risk constants in eq. 3.4. Calculations are based on market and dividend data from Reuters Finance and T-bill rates data from NY Fed.

As seen in table 4.2, the earnings based approaches outperform all others, as seen in their Sharpe Ratios and their respective values of Alpha. The Mean-Earnings-Variance portfolio when assuming equal weights of 0.5 in the equations 3.3 and 3.4, yields a higher rate of return without substantially increasing the risk, it should thus be noted as a relatively successful trade off.

The Mean-Variance portfolio yields the highest return, however this is not sufficient to compensate the degree of risk which is implicit in the investment. One can illustrate this further by considering a fund of funds that has the choice to invest in any combination of the above portfolios and may leverage the investment through borrowing at the risk free rate. Assume that when the equity of the fund of fund reaches zero the fund is in default with a recovery rate of 0. Should the investment take place only in MV, the fund of funds will default for any degree of leverage higher than 1.25/1. However, investments in the EV may be levered up to 3.14 times while investments in the MEV fund 3.12 times before facing default. In practice, a fund which is characterized by a high degree of leverage, would most likely respond to losses by a rapid sale of assets at a depressed price in order to "stay in business", so should one pick an arbitrary leverage ratio of 2/1, it is reasonable to conclude that the MV fund would be led to either default or at best case low returns due to sales of assets in depressed prices, while the EM and MEV funds would succeed to multiply their returns through external financing.

Table 4.3: Portfolio risks when short selling is not permitted

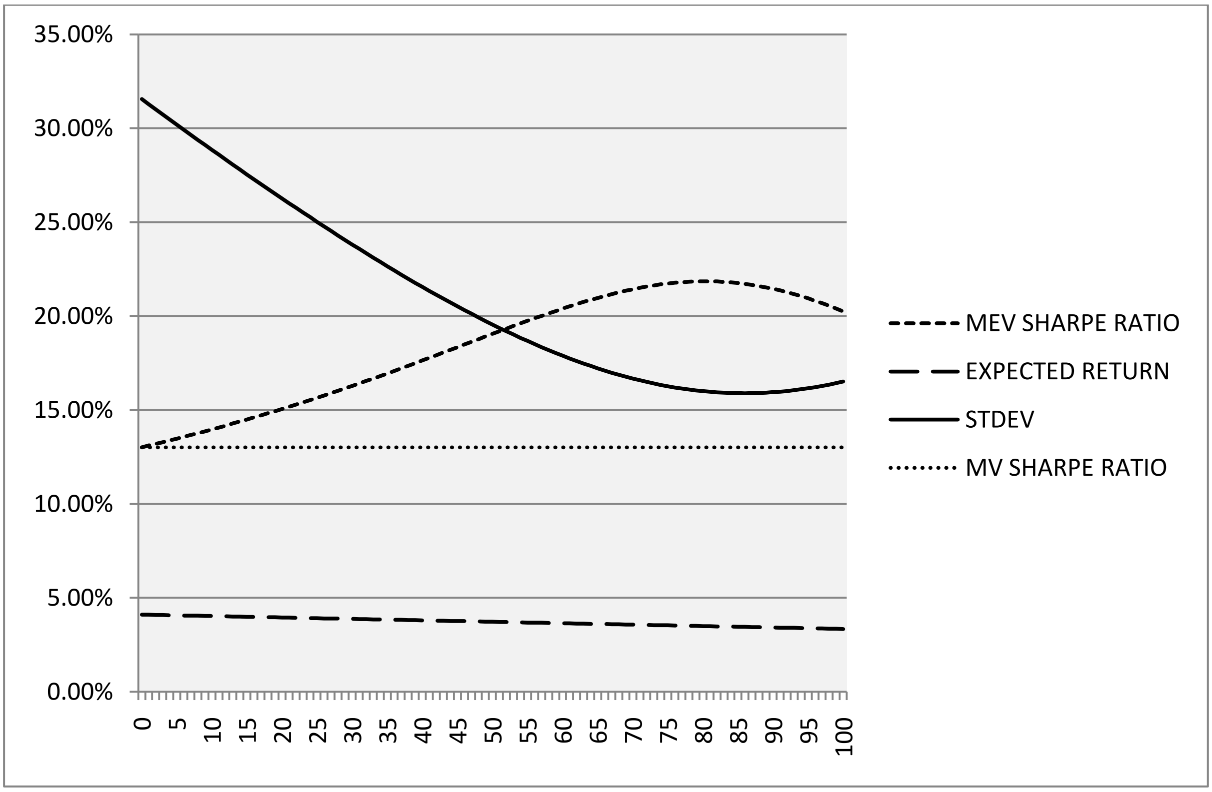
	DOW*	MV	EV	MVP	MEV
Sigma	20.51%	32.44%	17.28%	16.95%	17.09%
Sharpe Ratio	16.11%	13.01%	20.25%	11.50%	23.09%
Beta	NA	106.32%	61.12%	69.86%	70.01%
Largest Yearly Loss	-38.94% *(2008)*	-47.07% *(2001)*	-31.85% *(2008)*	-25.93% *(2008)*	-31.93% *(2008)*
Largest Consecutive Yearly Loss	-38.94% *(2008)*	-80.37% *(2001-2002)*	-31.85% *(2008)*	-30.79% *(2000-2002)*	-31.93% *(2008)*
Largest Monthly Loss	-16.19% (Oct08)	-33.29% (Feb01)	-21.63% (Feb00)	-13.65% (Feb01)	-19.16% (Feb01)
Largest Consecutive Monthly Loss	-28.30% (Sep08-Nov08)	-59.01% (Feb01-Mar01)	-30.90% *(Jun01-Aug01)*	-41.88% (Oct08-Feb09)	-31.74% (Feb01-Mar01)

The MEV portfolio returns are calculated assuming 0.5 weights in each of the risk constants in eq. 3.4. The periods in which the worst losses were incurred are noted in parentheses.

Table 4.3 can be very useful in further illustrating the implicit risk associated with each portfolio. The earnings based portfolios and the minimum variance portfolio (recall that it is identical for all models) are characterized by the lowest standard deviations and correlations with DOW*. Additionally, their worst months are remarkably milder than those of the DOW* and Mean-Variance models. Particularly concerning the latter, the higher expected return comes at a stiff price, as in addition to the relatively low Sharpe Ratio, there are cases where very large losses are incurred.

Up to this point, the estimation of the MEV portfolio is done without particular consideration as to what weights should the investor assign to kappa and tau in equations 3.2 and 3.4, where equal weights of 0.5 are assumed for simplicity. It is thus natural to wonder what weights are optimal and how an investor should decide upon this. This is a rather tough issue as the investor would need to go many more years in the past to draw some conclusions about the weights, but the further back in history one looks the less relevant their analysis becomes, in the context of the asset allocation problem. While there is no direct closed form solution to this problem, the importance of picking such an exact solution remains questionable. As Chart 4.2 shows (the weight of the Earnings-Variance portfolio is represented on the horizontal axis, ranging from 0 percent to 100 percent), assigning practically any weight other than zero to kappa, leads to increase in the Sharpe ratio. Thus, recalling that weights kappa and tau must sum to one, it can be stated that the analysis of the model portfolios back tests shows that nonzero values of kappa lead the portfolio to better performance. This improvement is noticeable in most obvious arbitrary combinations such as 0.25 and 0.75.

Chart 4.2: The effects of increasing weight of kappa on the Sharpe ratio



It would be noteworthy to answer the question of the optimal tau and kappa weights with the benefit of hindsight, i.e. by assuming that the investor knows the outcomes of each investment strategy with certainty but for some reason cannot base the investment decision on this knowledge, but only apply the knowledge in order to assign the weights that will produce the highest Sharpe Ratio. Then differentiating the Sharpe Ratio equation with respect to kappa and tau and solving the system of first order conditions for the value that sets the derivative equal to zero, one can calculate that the highest achievable return is accomplished by assigning a value of 0.8156 to kappa and 0.1844 to tau. Then the maximum Sharpe Ratio of 0.23 is achieved. Even though this has no bearing on the actual asset allocation problem it is important that the Sharpe Ratio maximizing weight values are to a reasonable degree distant from the extreme values of 0 and 1. Should such extreme values be met, one could simply invest everything in the portfolio whose weight constant is one, but the above values make it rather clear that it is the combination of the extreme portfolios that bring the improvement in the Sharpe Ratio. In other words the investor should combine the two portfolios not because of his ignorance as to which one should exclusively be picked, but in the context of inferring that combining the portfolios is by itself sufficient to improve performance.

Finally, chart 4.3 shows the performance of each portfolio over time, by tracking the compound growth of an investment in each of the portfolios. The fluctuations of the values of the investments over time can be seen providing a visual representation of the risk associated with them. It can be seen that the Mean-Variance portfolio is characterized by wild jumps and slumps in value, while the earnings based portfolios follow a much smoother and predictable path.

Chart 4.3: Growth of investments through time

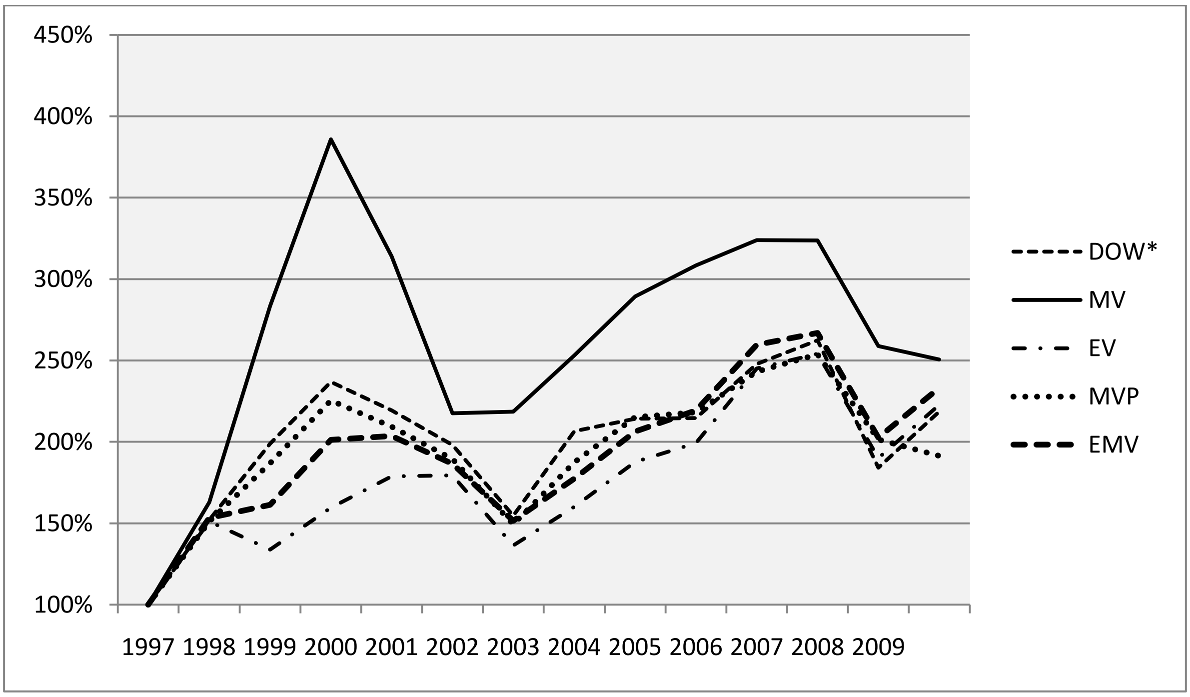
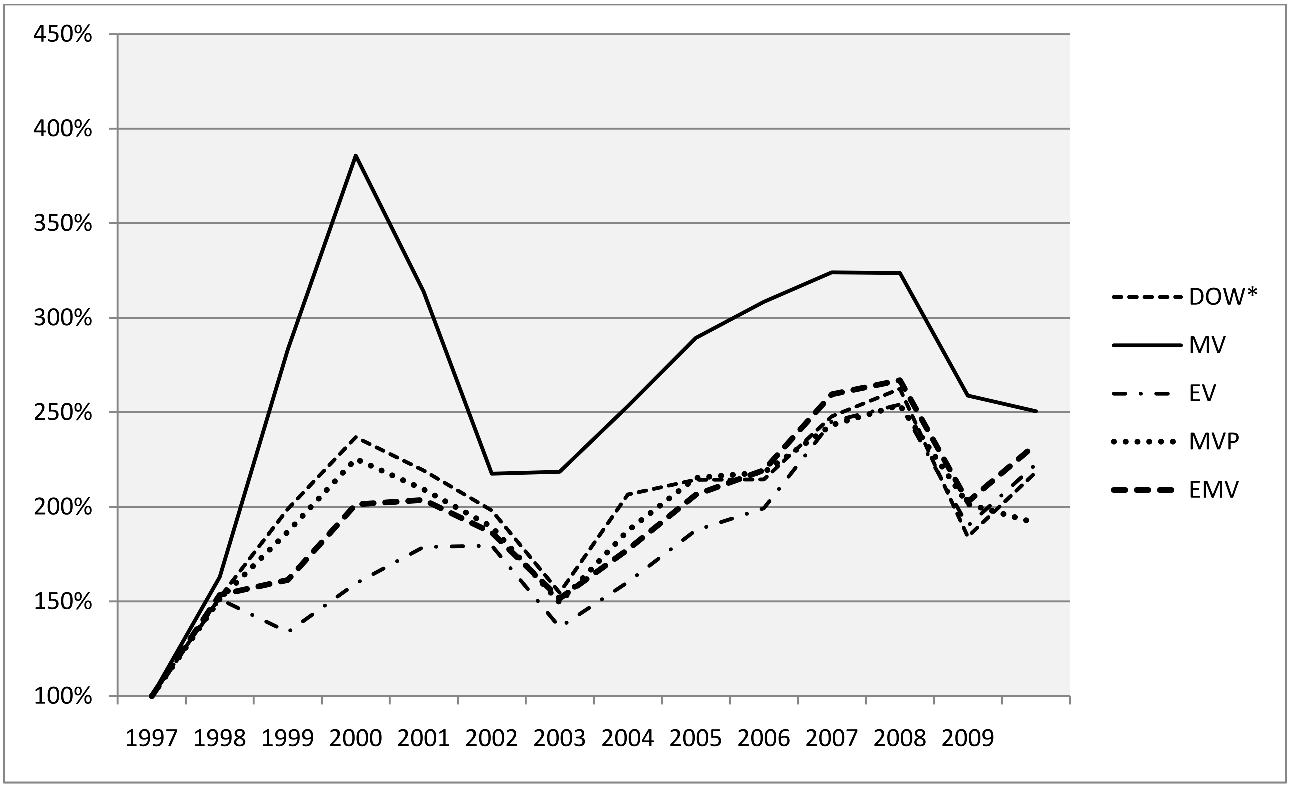


Chart 4.4 takes the gives another perspective by looking at the growth of an investment levered by 2/1. In this case the investor each year receives twice the return of the portfolio while paying the risk free rate.

Chart 4.4: Growth of a 2/1 levered investment



b. Short Selling Allowed

One may now proceed to analyze the behaviour and performance of the model portfolios in a slightly more complex environment, where the investors are allowed to short sell the assets. The mechanics of this are assumed to be as simple and straight forward as possible: The investors may borrow the shares and sell them as if they owned them. In doing so, they seek to profit in two ways. First by investing the short sales proceeds in stocks that are expected to yield higher returns, thus selling them at a higher price; and second by buying back the sold shares at a lower price.

Recall that we assume markets to be frictionless and continuous. In a real investing environment, these effects would be much more severe than those of the previous section where short selling is prohibited. This is because of the unpredictable costs of borrowing a share, which can range from practically insignificant, to substantially high, depending on the availability of each share.

Table 4.4: Model performance when short selling is permitted

Year/Portfolio	DOW*	MV	EV	MVP	EMV
1997	28.81%	8.10%	47.17%	60.44%	28%
1998	25.64%	-94.61%	-96.10%	-11.50%	-95%
1999	21.96%	160.62%	63.95%	35.13%	112%
2000	-6.10%	-70.95%	1768.42%	2.99%	849%
2001	-9.50%	-2968.96%	-235.42%	5.95%	-1602%
2002	-21.08%	-82.90%	-53.43%	-38.86%	-68%
2003	26.63%	7.28%	-35.03%	-5.69%	-14%
2004	5.20%	7.45%	18.71%	11.22%	13%
2005	2.35%	2.56%	-6.44%	-8.66%	-2%
2006	19.17%	-5.13%	-3.25%	12.79%	-4%
2007	8.93%	0.42%	0.39%	2.13%	0%
2008	-38.94%	7.51%	-68.83%	-59.58%	-31%
2009 **	14.44%	-36.19%	-8.59%	-20.37%	-22%
SUM	78%	-3065%	1392%	-14%	-837%
AVERAGE	6%	-236%	107%	-1%	-64%
STDEV	20%	824%	505%	30%	521%
SHARPE RATIO	0.29	-0.29	0.21	-0.04	-12%
BETA	NA	9.12	-2.55	0.97	329%
ALPHA	NA	-2.90	1.22	-0.07	-84%

The MEV portfolio returns are calculated assuming 0.5 weights in each of the risk constants in eq. 3.4. Calculations are based on market and dividend data from Reuters Finance and T-bill rates data from NY Fed.

Table 4.2 shows the attained returns of each model portfolio when short selling is allowed. The first thing that one notices is the upside jump in the volatility of all model portfolios. In essence, the risk free rate here allows for much more aggressive risk taking, which in turn leads the portfolios towards attaining very high profits and losses for most years. Note that as the DOW* portfolio is constructed without the use of leverage, there is therefore no possibility to short sell, its performance and volatility are exactly the same as in the case where short selling is not allowed. This provides a useful benchmark in comparing portfolios in the two settings.

Proceeding to the performance of each individual model, one may start by focusing on the Mean-Variance model. The drawbacks of the methodology are very clearly outlined, as the portfolio accumulates a return of -3065% over the benchmark period, which translates into a 236% yearly loss. In other words, should a mutual fund that utilizes the Mean-Variance methodology as described above be allowed to trade without any obligations to deposit collateral, its lender would have incurred an accumulated loss equal to 30 times the fund's capital. On more notable property is that the greatest loss is incurred close to the dot com crash period, rather than the recent credit crisis.

The roots of the problem are rather clear. The enormous loss of the portfolio in 2001 is incurred in large part because of the tendency towards overloading portfolio weights in inflated assets.

Table 4.5: 2001 Model portfolio weights.

	1998-2000 Exp Return	2001 Return	PE	DOW*	MV	EV	MVP
MMM	15.41%	0.21%	3.97%	3%	1692%	382%	16%
AA	23.04%	7.55%	5.46%	3%	-558%	-162%	3%
AXP	21.20%	-42.19%	3.77%	3%	1155%	-66%	-52%
T	10.89%	-17.43%	5.08%	3%	281%	202%	13%
BAC	-6.17%	35.60%	10.53%	3%	-1065%	-352%	-13%
BA	11.28%	-51.82%	4.02%	3%	-657%	-38%	21%
CAT	1.87%	12.78%	6.21%	3%	-1578%	-304%	-21%
CVX	6.08%	8.91%	8.20%	3%	-2368%	-259%	24%
CSCO	47.17%	-74.77%	1.20%	3%	1300%	49%	-6%
KO	-1.95%	-24.47%	1.25%	3%	-2375%	-73%	14%
DD	-4.75%	-9.61%	1.35%	3%	547%	456%	21%
XOM	13.95%	-7.84%	4.85%	3%	4607%	14%	0%
GE	23.67%	-16.28%	2.58%	3%	-447%	335%	36%
HPQ	9.54%	-41.54%	5.71%	3%	-38%	-271%	-15%
HD	28.45%	11.39%	2.54%	3%	-1127%	-136%	17%
INTC	18.15%	4.79%	5.18%	3%	-308%	-28%	8%
IBM	16.72%	35.80%	4.98%	3%	-9%	254%	16%
JNJ	16.85%	13.13%	3.14%	3%	782%	387%	13%
JPM	9.53%	-19.29%	8.47%	3%	1685%	97%	1%
MCD	12.15%	-24.95%	4.46%	3%	-447%	135%	13%
MRK	20.64%	-44.36%	3.07%	3%	2343%	114%	-4%
MSFT	9.81%	42.36%	4.17%	3%	-861%	28%	-1%
PFE	21.34%	-13.31%	1.43%	3%	-773%	-238%	1%
PG	0.94%	3.01%	3.98%	3%	-1780%	-196%	11%
TRV	12.22%	-18.63%	7.25%	3%	469%	-107%	-8%
UTX	27.04%	-18.34%	3.80%	3%	1299%	173%	-16%
VZ	6.20%	-2.47%	7.52%	3%	-1001%	-442%	-15%
WMT	33.49%	8.55%	2.62%	3%	1146%	-135%	-6%
DIS	-3.61%	-32.38%	1.97%	3%	-1816%	280%	30%

Table 4.5 shows this in detail. Recall from the asset allocation methodology that the fund manager in the beginning of 2001 will gather data on earnings, correlations, variances and expected returns from the three year period of 1998-2000, which coincides with the market topping process in the dot com era. The expected returns in this period are presented in the second column, while the returns that were actually achieved are presented in the third column. The Mean-Variance investing fund, mislead by the past returns of technology names such as Cisco and United Tech, or financials such as JP Morgan and American express, is lead to overloading their stocks, while shorting relatively "safer" companies such as Procter & Gamble, Disney and Coca Cola. This coincides with long positions in companies that have a high PE ratio (which is equivalent to a low Earnings/Price ratio as presented in the fourth column) and short positions in the companies that show more solid earnings record. In essence, the Mean-Variance earnings fund loads assets that have been preferred by the market in the last three years, while ignoring their earnings record. The result is disastrous, as in 2001 it becomes clear that many asset prices are artificially inflated and their share values plunge as the market comes back into contact with reality. It could be said that the positive return achieved by this methodology when short selling is not allowed, is not an inherent property of the model, but rather a result of the leverage cap that the no short selling condition implies. When short selling is allowed, the Mean-Variance investor may finance long positions both by borrowing at the risk free rate and by using the short sales proceeds. Removal of the second option obviously implies a cap in the ability to lever the long positions and it is not completely clear whether the positive returns are a result of the methodology or the cap.

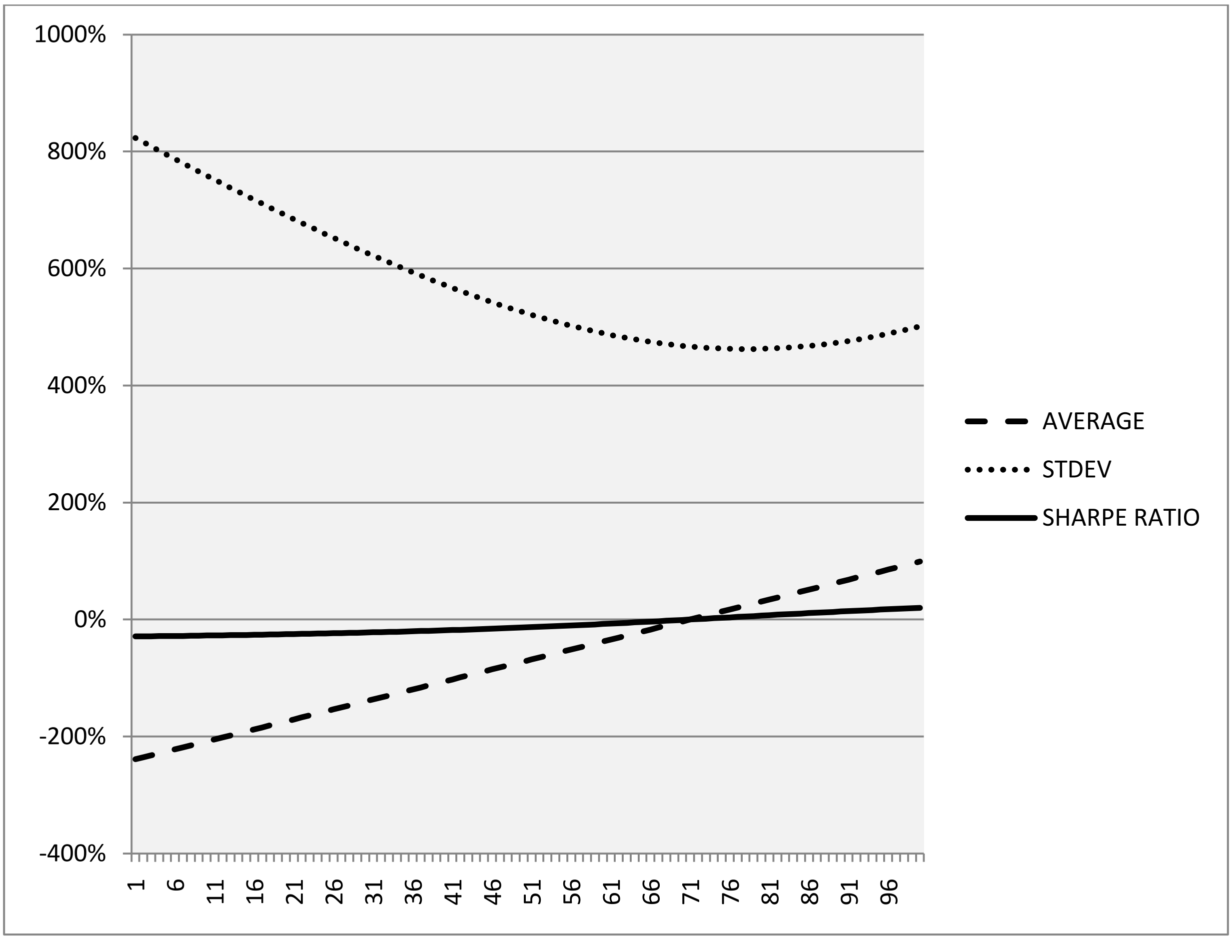
One may also note that the model proposes a long-Exxon short-Chevron allocation, again loading an asset that exhibited robust past returns at the expense of an asset that has a solid earnings record. As these companies are rather similar, it is very interesting to watch how the model proposes the exact opposite of what a rational investor would most likely decide, again with extremely poor results, as in 2001 Exxon drops by 8% while Chevron achieves a healthy 9% return.

So if the investment based on the Mean-Variance methodology fails, it is natural to inquire whether the Earnings-Variance methodology performs better. The answer is an overwhelming yes, as this strategy leads to achieving an accumulated 1392% return, or 107% year over year. In 2001, loading into assets that have the property of solid past earnings protects the portfolio against excessive drops, so even in the face of a bubble busting market environment, the Earning-Variance fund suffers a 235% loss which means that only a fraction of the return it achieved during the bull market is lost when the tides change.

Furthermore, the Earnings-Variance model portfolio vastly outperforms the Mean-Variance model and the DOW* benchmark portfolio in every risk weighted benchmark, with much lower volatility, higher Sharpe ratio and higher alpha. A very remarkable property of the Earnings-Variance portfolio's risk is the negative value of beta, which in combination with the returns record of the portfolio, leads to the conclusion that while always keeping a net long position towards the market, the Earnings-Variance investment performs better when Dow* is dropping. Recall from the early chapters of this thesis, how finding a combination of assets that “half flip heads when the other half flips tails” is nothing short of a holy grail in portfolio theory. While such a combination of stocks is not achievable in any obvious and consistent way, the beta value of the Earnings-Variance model portfolio in the back testing period achieves just that. Both portfolios are net long in equities and have a positive net return, but in the 1997-2009 period, while one flipped heads, the other one was tending towards flipping tails indeed.

Proceeding to the case of a 50% loading on each of the Mean-Variance and Earnings Variance portfolio (corresponding to the EMV column in table 4.4), one may draw the conclusion that the inclusion of earnings does not provide sufficient cushion to the investor. In this case a negative return is which might be smaller than the loss incurred in the Mean-Variance investment but it’s still enough to wipe out more than eight times the initial wealth. In order to profit from the combination one would actually need to short sell the Mean-Variance portfolio instead, or at least assign as little as possible weight to it. Indeed, as chart 4.5 shows (Again the horizontal axis is used to plot for the weight assigned to the EV, ranging from 0 percent to 100 percent), loading the Earnings-Variance portfolio improves almost every risk property of the portfolio, as well as the Average yearly return which is denoted by the dashed line.

Chart 4.5: Effects of EV portfolio loading in the EMV model



Finally, it is noteworthy how the Minimum Variance Portfolio underperforms in just about every aspect, except for the achievement of the smallest variance. It seems that while a relatively low variance can be achieved, this is no guarantee of uniform or even positive returns.

5. Conclusions

The objective of this thesis has been to overview how the classical Mean Variance allocation theory performs in a realistic investing environment, in comparison to a different proposed estimation strategy in which companies' earnings rather than past returns are used in part or in whole in order to determine the portfolio weights.

Initially, the investment problem is investigated. The main question is how an investor can quantify and manipulate the risk – reward trade off, in order to maximize his long term returns. As the future is uncertain, the problem is in large part focused in the proxies that one may use to identify risk and return. Historically, a number of approaches has been utilized, but nothing better than rules-of-the-thumb resembling methods had ever been proposed, until the introduction of Markowitz's Mean-Variance portfolio theory.

The Mean-Variance portfolio theory, rather than focusing on the volatility of each individual security, utilizes the variance of the entire portfolio as a proxy for risk. In doing so, the investor using this methodology attempts to minimize risk by combining securities that seem to rise and fall at different periods. However, a number of problems remain. First, there is no guarantee that the shares future behaviour will resemble the past. Then the historical returns of the various shares are used as a proxy for the risk premium that the investor attains and the optimal trade off is proposed by following relatively simple mathematical manipulations.

However, a number of drawbacks in this methodology are identified. The most obvious is that there is no guarantee or indication that the share prices will behave in the future as they did in the past. The capital markets are characterized by uncertainty and as the risk sources vary in time, not only the returns but also the volatility of investments can be expected to be subject to substantial divergence from what history would suggest.

An even more striking drawback is related to the model's estimation problem and consists in the tendency of the model to load onto securities that have exhibited solid returns in the past. As these returns might not be a result of the companies' profitability but rather result of speculation about the future growth of their share prices, in the event that a stock market bubble is formed, the Mean-Variance portfolio can be very misleading, as it would tend to propose higher loading of risky assets. Additionally, the ability to lever long position in these assets through borrowing at the risk free rate, or short selling other assets, amplifies the potential losses.

In light of this, the Earnings-Variance estimation methodology, which is based on the companies' earnings rather than their past returns, is proposed. This model uses the same mathematical framework as the Mean-Variance portfolio, while assuming that earnings have a better forecasting value in bear markets. Then the investors have the choice to load their wealth or part there off into a portfolio constructed on the basis of earnings.

The various methodologies are then tested in a realistic investing environment. Four alternative funds are considered, each using its own methodology for asset allocation. Different models are utilized and asset weights are proposed based on historical data regarding the companies' volatilities, past earnings, past returns and correlations. It is shown that the Earnings-Variance based approaches are indeed effective in achieving a more efficient risk/reward trade off.

In an environment where short selling is not allowed, the earnings based approaches produce slightly lower accumulated and average returns, but this is compensated by their much more appealing risk properties. As judged by the measures of Sharpe ratio, as well as each model portfolios Alpha, the earnings based approaches do indeed achieve the optimal allocation, when comparing them to the traditional Mean-Variance based investment approach.

When short selling is allowed the results are even more striking. The Mean-Variance based approach not only fails to produce returns, but is lead to striking losses amounting to many times the initial investment. On the other hand the Earnings based approaches provide robust returns in the long run, while still being subject to downside risks due to the ability of short selling.

Combinations of the two strategies are also investigated. When short selling is not allowed, any combination of the Mean-Variance and Earnings-Variance approach produces a better Sharpe ratio than that of the classical Mean Variance portfolio. When short selling is allowed, the cushion of the earnings based approach is deemed insufficient to compensate the vast losses that are incurred in the Mean-Variance based investment. Thus, in the short selling environment one may conclude that the best strategy would be to abandon the latter altogether and invest solely on what is proposed by the Earnings-Variance based methodology.

Finally, one may make a brief remark on the efficient market hypothesis, which states that market price changes reflect the effect of unexpected events that affect the companies and as a result, share prices develop in a random walk process, or in the case where normal distribution of returns is assumed, a wiener process. If this is the case, then the historical returns would correspond accurately to the risk premium associated with an investment. Furthermore, under efficient markets and in the presence of a risk free rate, the market Mean Variance, portfolio can be expected to be characterized by the highest Sharpe ratio in comparison to that of all other portfolios, if not in each period then at least in the long run. However, looking at the performance of the Mean Variance portfolio in the back testing period, it is clear that this is not always the case. When the market's plunge, the premiums are far distant than what Mean-Variance portfolio theory would suggest and the market portfolio loses the optimal Sharpe ratio property.

The most striking conclusion regarding the efficient market hypothesis can be reached by recalling that the investor in the EV portfolio employs some information that has been already priced in the stock and yet attains a better Sharpe ratio than the passive investor. Recall that the efficient market hypothesis states that once information has arrived in the market (in the case of earnings this happens when companies announce their quarterly results) and is priced in, it is impossible to improve one's portfolio efficiency on the basis of the priced-in information, thus all future moves in a stock's price will be due to unexpected news rather than news that reached the market days or weeks ago. The very fact that by investing in the EV model portfolio one can achieve a better Sharpe ratio than the passive investment (be it in the MV or DOW* portfolio), using past information such as Earnings, is by itself inconsistent with the efficient market hypothesis.

Inquiring into the back testing results; one could say that there are large periods in which markets tend to behave efficiently. In the absence of bear markets, the efficient market hypothesis and the Mean-Variance portfolio theory provide a rather accurate and satisfactory description of share price changes. However, when the markets experience steep declines, such as those incurred in the crises at the beginning and the end of the last decade, the efficiency seems to break down and share prices no longer resemble what the risk-premium line of thinking would suggest.

It could be that some varying degree of efficiency exists in the share prices. However this efficiency seems to be very sensitive to large downward price movements. When the efficiency brakes down, a larger return/risk premium will not necessarily imply a large risk and vice versa. And since the prediction of market plunges is impossible, the most prudent strategy seems to be the result of the combination of the Earnings-Variance and Mean-Variance approach, together with caps on leverage and the ability to short sell a specific instrument.

6. References

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