A Numerical Study of Vortex-Induced Vibrations (VIV) in an Elastic Cantilever

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Summary This study treats the subject fluid-structure interaction (FSI) for incompressible flow with small vibrations. The open source packages DEAL.II and OpenFOAM have been used to create a coupling between a finite element formulation for the structure and finite volume formulation for the fluid. A staggered solution algorithm have been implemented in C++ and verified against empirical data of Vortex-Induced Vibration (VIV) frequencies.

Introduction

A cantilever is placed in a domain of a velocity driven fluid. The traction differential acting upon the structure induce a deformation and the movement of the structure affects the fluid as well. This mutual influence referred to as fluid-structure interaction (FSI), is known to cause several interesting phenomena. Among such is vortex-induced vibration[1] (VIV), where the forced movement of a fluid around the structure gives upon point of release from the structure, an angular momentum manifested as a vortex in the fluid with an oscillating transversal force component.

Mathematical and Numerical Description

A physical domain consisting of fluid and structure is described by velocity field ($\mathbf{U}$,$\mathbf{v}$) and pressure p, displacement field ($\mathbf{q}$) and pressure in a continuum model. The equations governing the motion of an incompressible Newtonian fluid and an elastic structure ($D_{ijkl}\epsilon_{kl}$) with damping ($C_{il}v_l$) then takes the following form in reduced variables (*) in the fluid domain and state space formalism for the structure with small strain operator ($\epsilon(\mathbf{q})$),

$$\nabla^* \cdot \mathbf{U}^* = \mathbf{0}, \tag{1}$$

$$\frac{DU_i^*}{Dt^*} = -\partial_i p^* + \frac{1}{Re}\nabla^{*2}U_i^* + b_i^*, \tag{2}$$

$$\dot{q}_i - v_i = 0, \tag{3}$$

$$\partial_t v_i + C_{il}v_l - \partial_j D_{ijkl}\epsilon_{kl}(\mathbf{q}) = f_i. \tag{4}$$

The coupling boundary between fluid and structure is a traction term, i.e. the sum of the pressure force and the viscous force. Both PDE sets have the same character and therefore a monolithic approach is feasible [2]. However, the problem can become too large to handle or unstable, therefore a staggered algorithm is preferred where even the individually domains can further, by divide and conquer, be partitioned [4, 5]. In solving Eqn (2) and (4) it is assumed that the problem can be formulated in two steps, the solution of the physical domain in a steady state formalism, followed by semi-discretization in time.

[1]For an excellent review, C.H.K. Williamson and R. Govardhan, Vortex-induced vibrations. Ann. Rev.Fluid.Mech **36** (2004) 443-455.

Further, by assumption of fixed point solution the fluid domain is solved separately from the structure domain, using FVM respectively FEM.

```
Time Loop
 Staggered Loop
    Solve Fluid State
    Transfer Traction to Solid State Solver
    Solve Solid State
    Exit Staggered Loop if change of deformation < tolerance
    Transfer Deformation to Fluid State Solver
  End Staggered Loop
 End Time Loop
```

The norm for convergence is with respect to the displacement field. However, while combining two solvers the time must be adaptive with respect to the CFL condition in order to meet the convergence criteria. The open source packages used in this study are OpenFOAM[2]. and DEAL.II[3]. The staggered algorithm allows the FSI solver to be run on separate machines/threads. The test case and the staggered algorithm originates from a study using OpenFOAM [5]. The fix-point iteration to locate the quasi-static equilibrium point between the solvers use the Aitkens relaxation method[4] to accelerate the sub cycle loop, the staggered loop.

The Case Study

A cantilever of thickness $D = 0.2$ m and height of $10D$ is placed $5D$ from the inlet, $2.5D$ from the walls and $20D$ from the outlet. The wire frame of the rectangular domain is thus $(26 \times 6 \times 12.5) \cdot D$. The flow is velocity driven with uniform Dirichlet condition at the inlet (mag($\mathbf{U}$)) and Neumann conditions at the outlet. For the pressure a Neumann condition is used at the inlet and a Dirichlet condition at the outlet. At the walls, no-slip conditions are used. The unstructured grid in the fluid domain is created using scaled tetrahedral elements with a structured boundary mesh with size 0.02 m , growth rate 1.1 and 0.1 m as upper limit on cell size, while the structured grid for the structure domain $8 \times 8 \times 64$ cell partition.

Application to VIV

The following empirical expression for the Strouhal number (St) can be used to estimate the frequency of the VIV for a cantilever in an infinite domain,

$$St = \frac{fl}{U} = 0.198(1 - \frac{19.7}{Re}). \tag{5}$$

The result in table 1 presents the frequency of probes placed in respective domain, showing the synchronization between the frequency of the fluid motion (f) and the structure (f_s).

[2] *http://www.opencfd.co.uk/openfoam/*
[3] *http://www.dealii.org.*

It scales within the margin of error with Eqn (5). However, wall effects should also be accounted for. Table 2 gives the observed VIV in the nodamped cases with no fluid probes, note that for $U = 1$ two frequencies appear, where the higher is the first harmonic of the lower and it appears due to discretization error of a sinusoidal function. The VIV is masked

U (ms^{-1})	$f_s(Hz)$	$f(Hz)$	$f_i(Hz)$
1	0.7	1-1.3	1.7
10	8	7-14	11
25	15	17-25	11

Table 1: VIV frequency with damping from section 6.3 in [1].

U (ms^{-1})	$f_s(Hz)$
1	0.79, 0.74, 1.47, 1.53, 1.53, 1.53
10	6.3, 6.6, 5.2
25	17.07, 17.2

Table 2: VIV frequency without damping from table 6.1 in [1].

by the in-line frequency due to release of cantilever and for this reason a Rayleight damping was added with 0.1%. The in-line frequency f_i well match reported elsewhere [3]. Figure 1-2 is the FFT spectra for U=1 in table 1.

Conclusion

This study presents a method to resolve the fluid-structure interaction (FSI) using a fixed-point iterative scheme with a partitioned Gauss-Seidel technique accelerated with Aitkens relaxation method. The validation of the solver involves among others,

- reproduced frequency shift in in-line movement.
- matched frequency in VIV with probes of fluid and structure.
- reproduced VIV frequency with regard to Eqn (5).

The study implicates the need for damping in this model where frequency is obtained in a real time numerical experiment.

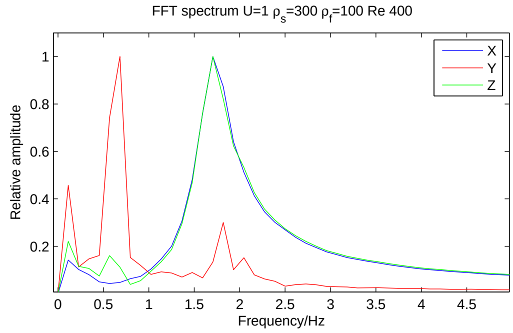

Figure 1: The FFT on marker point at U=1.

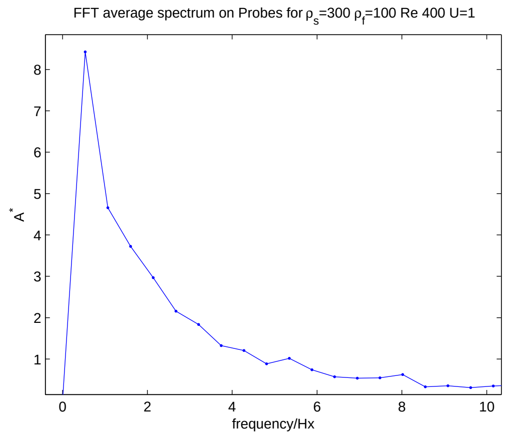

Figure 2: The averaged FFT spectrum for the fluid probes for U=1.

References

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