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Model-Free Implied Volatility,

Its Time-Series Behavior And Forecasting Ability

Supervisor:
Hans Byström

Author:
Olena Mickolson

Summary

Title: Model-Free Implied Volatility, Its Time-Series Behavior And Forecasting Ability

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Author: Olena Mickolson

Supervisor: Hans Byström

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Purpose: The aim of this thesis is to research various dependencies, trends and facts known about the volatility by looking at the time-series behavior of the implicit and historical volatility measures of the SMI and EURO STOXX 50 over the period covering January 1999 and December 2009

Theoretical framework: The theoretical framework consists mainly of the option theory.

Method: The quantitative method is used.

Empirical platform: The implied volatility indices used in this analysis, VSMI and STOXX, created by SIX Swiss Exchange Ltd and STOXX Ltd respectively, while the historical volatility indices used in this analysis, HVSMI and HVSTOXX, are calculated by taking the standard deviation of actual daily returns on the SMI and EURO STOXX 50.

Conclusion: Several widely acknowledged stylized statistical facts about the performance of volatility were observed during the analysis. First, the volatility exhibits mean-reversion towards its long-run mean. Second, certain skewness is present in the volatility response to changing market circumstances – falling markets raise volatility more substantially, than rising markets diminish the expected volatility. Results of this study signify that even though there exists a strong relationship between both volatility measures for SMI and EURO STOXX 50, the implied volatility values lay above the historical volatility values most of the time, whereas the latter is more scattered and volatile. Finally, we conclude based on the OLS results, that model-free implied volatility has no significant relationship with the future realized volatility and thus cannot be used as a market forecast of future volatility index.

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1 Introduction

Expected volatility as a measure of risk involved in economic decision making is a crucial ingredient in modern financial theory. The reason is that volatility indicators provide traders with an estimate of how much movement a stock can be expected to make over a given time span. The knowledge of the degree of fluctuations in asset returns is fundamental for determining whether an option is likely to expire in or out of the money as well as understanding whether an option is cheap or expensive relative to the historical facts of the underlying instrument.
The importance of volatility measures has been increasing rapidly as a result of the breakthroughs in the financial theory and the cascade of financial innovations of the past decades. Markowitz (1952) prominent work on the modern portfolio theory introduced the idea of a tradeoff between the risk and the expected return of a portfolio. The development of the portfolio selection theory marked the beginning of an intense focus on the "quantification" of the risk as measured by variances and covariances of and between asset returns. In 1963 William F. Sharpe with the publication of his dissertation "A Simplified Model of Portfolio Analysis" introduced the world to the Capital Asset Pricing Model (CAPM), which has become an essential part of the investment theory and its wide-spread application strengthened the significance of volatility measures even further. One of the later boosts to the importance of volatility measures originated Black-Scholes model or Black-Scholes-Merton model formulated in 1973, the underlying assumption of which is that price volatility of the underlying instruments follow a predictable pattern, thus making volatility the most vital input to the model. Since then numerous option-pricing models have been developed and almost all of them rely on estimates for the volatility to derive their forecasts.

This paper is structured as follows: after outlining the significant distinction between historical and implicit volatility we recall the theoretical derivation of the Black-Scholes formula. Further, a careful analysis of the two volatility measures is carried out in a sampling period from January 2000 to January 2010 with 120 months volatility data for the S&P 500, DAX 30 and DJ EuroStoxx 50. Finally, the results of a series of tests comparing performance of implied and historical volatility in predicting future realized volatilities and stock market returns will be presented along with the analysis of the nature of relationship between the implied volatility and the returns of the underlying asset.

2 Theoretical Framework

2.1 Historical Volatility vs Implicit Volatility

In this section concepts of the two volatility measures and their distinctive features will be concisely introduced. It will also introduce the most prevalent historical volatility measure following with the examination on how the implicit volatility can be estimated.

The sequence of asset returns over time embodies their empirical probability distribution, which can be interpreted as a sequence of random variables with a particular realization or outcome at each point in time. Thus, all that can be seen in any empirical data set is just one particular realization of the stochastic process.

The remark above is, however, not important for the backward-looking concept of historical, or explicit, volatility, which is merely a degree or level of up and down movement in a value over time. The historical volatility is usually computed by taking the standard deviation of the price returns over a given number of sessions, multiplied by a factor (e.g. the square root of the annual number of trading days) to produce an annualized volatility level. There are a number of mathematical formulas one could use to calculate historical volatility. The most widely used is the following:

$$\sigma = \sqrt{\frac{1}{(n-1)} \sum_{y=1}^{n} \left(\ln \frac{y_i}{y_{i-1}} - \mu \right)^2} \tag{1}$$

$$\mu = \frac{1}{n} \sum_{y=1}^{n} \left(\ln \frac{y_i}{y_{i-1}} \right) \tag{2}$$

where N is the number of time periods being considered; μ - mean of a sample of measurements and y_i is the price of asset at time i.

The historical volatility as stated in the equation (1) does not require any assumptions about the probability distribution of returns. However, if the explicit volatility measure is used for the statistical inference about the future volatility, assumptions about returns' probability distribution become inescapable. The subject of modeling the volatility gained a lot of attention due to uncovering one of its most prominent features, namely the fact that the volatilities of asset returns fluctuate over time. As a result a number of stylized statistical facts prevailing among most of financial assets has been identified:

1. *Leptokurtosis* and *skewness* - asset returns follow a distribution which is far from normal assumed in most option-pricing models, in particular it exhibits a substantial degree of excess kurtosis and is slightly skewed rather than properly bell-shaped.

2. *Volatility clustering* - different measures of volatility display a positive autocorrelation over several days. This quantifies the fact that high-volatility events tend to cluster in time, confirming that the volatility is neither constant over time, nor independent from past volatilities.
3. *Mean-reversion* – the volatility tends to revert to some long-run average.
4. *Leverage effect* – asset volatility and asset returns are negatively correlated.

To account for these stylized facts, several seminal models have been developed. They include the widely used Autoregressive Conditional Heteroscedasticity (ARCH) model by Robert Engle in Engle (1982), its generalization, the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model by Tim Bollerslev in Bollerslev (1986) and the models that use combinations between (G)ARCH and a moving average, so-called ARMA models. The problem of the volatility being inconstant over time is of a great importance, since it contradicts the assumptions of the Black-Scholes formula, discussed in section 3.

There are two distinct categories of measures for the implicit volatility: measures based on option-pricing models and "model-free" measures, both of which are deduced from option prices. This guides us to another vital difference between the two volatility concepts: the historical volatility is calculated using observed asset prices, whereas the implicit volatility is calculated from observed option prices.
The most fundamental reason that explains the paramount popularity of the implicit volatility is that a volatility concept implied from the option prices possesses a noteworthy property: as options are bets on the future development of the underlying asset, the key advantage of this option implied volatility is the fact that it is a forward-looking variable by nature. Thus, unlike volatility measures based on the historical data, it should reveal market expectations on the volatility over the remaining life time of the option.
Consequently, the information content of the implicit volatility and its capability of being a predictor for future asset price volatility has been of primary concern in the literature from the early studies up to now.

2.2 Implicit Volatility Measures

Keeping in mind the crucial distinction between the two volatility concepts, we now proceed to outlining the two wide categories of implicit volatility measures. First, we will briefly recall the derivation of the Black-Sckoles formula and show how the volatility can be inferred from option-pricing models and section 3.2 will outline two examples of model-free volatility measures.

2.3 Notation

Throughout the paper the following notation is used:

c - option premium if the option is a call option

p - option premium if the option is a put option

X - strike or exercise price of the option

S_t - price of the asset underlying the option in period t

r - risk-free rate of return

$\tau = T - t$ - remaining time from today, t until maturity in period T

σ - stock return standard deviation

2.4 Volatility Implied From Black-Scholes Model

The option pricing model developed by Black and Scholes (1973) and further extended by Merton (1973) is a benchmark in the financial theory. The B-S model provided the framework for thinking about option pricing and despite its rather restrictive assumptions, it remains an anchor of financial model building.

The assumptions involved in the derivation of the Black-Scholes equation:

1. Markets are efficient - are price-continuous, liquid and provide all players with equal access to the available information. Hence, transaction costs are assumed to be zero in the Black-Scholes analysis.
2. Constant and known interest rate - there exists a risk-free security which generates €1 at time T when € $e^{-r(T-t)}$ is invested at the time t.
3. Continuous trading
4. The no-arbitrage principle is satisfied
5. The underlying asset is perfectly divisible and short selling is allowed.
6. The price of the underlying asset follows the geometric Brownian process of the form $dS = (\phi S + \sigma S W)dt$ where W is white noise.

The Black-Scholes formula can be derived as the limit of the binomial pricing formula as the time between trades diminishes, or directly in the continuous time model using an arbitrage argument. The option value is a function of the stock price and time, and the local movement in the stock price can be computed using a result called Ito's lemma[1], which is an extension of the chain rule from calculus.

For a smooth function $F(S,t)$, the normal Taylor series expansion goes as

$$dF(S,t) = \frac{\partial F}{\partial S}dS + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(dS)^2 + \frac{\partial^2 F}{\partial S \partial t}dSdt + \frac{1}{2}\frac{\partial^2 F}{\partial t^2}(dt)^2 + \cdots \qquad (3)$$

[1] John C. Hull, Options, Futures, and Other Derivatives, 7th Edition, *Prentice Hall: 237-251,* 2008

The Ito's lemma states that if a variable S follows a stochastic process of the form

$$dS = \mu(x,t)dt + \sigma(x,t)dW \quad (4)$$

where W is a Wiener process with a property $(dW)^2 = dt$

$$(dS)^2 = \mu^2(x,t)(dt)^2 + \sigma^2(x,t)(dW)^2 + 2\mu(x,t)\sigma(x,t)\,dtdW = \sigma^2(x,t)\,dt \quad (5)$$

Substituting properties from the equation (5) into the Taylor series expansion generates the Ito's lemma:

$$dF(S,t) = \frac{\partial F}{\partial S}(\mu(x,t)dt + \sigma(x,t)dW) + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(dS)^2 =$$

$$= \frac{\partial F}{\partial S}(\mu(x,t)dt + \sigma(x,t)dW) + \frac{\partial F}{\partial t}dt + \frac{1}{2}\sigma^2(x,t)\frac{\partial^2 F}{\partial S^2}dt =$$

$$= \left(\mu(x,t)\frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2(x,t)\frac{\partial^2 F}{\partial S^2}\right)dt + \sigma(x,t)\frac{\partial F}{\partial S}dW \quad (6)$$

Generalized expression for the Ito's lemma is:

$$dF(S_1 \dots S_n, t) = \sum_{i=1}^{n}\frac{\partial F}{\partial S_i}dS_i + \frac{\partial F}{\partial t}dt + \frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial^2 F}{\partial S_i \partial S_j}\sigma_i\sigma_j dt \quad (7)$$

The Black-Scholes equation can now be derived. Consider a derivative F whose value is a function of the value of the underlying asset S, which assumed to follow the stochastic process:

$$dS = \phi S dt + \sigma S W dt \quad (8)$$

where the average growth rate of the underlying security (ϕ) and the volatility (σ) are constants. Employing the Ito's lemma gains:

$$dF = \frac{\partial F}{\partial S}dS + \left(\frac{\partial F}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 F}{\partial S^2}\right)dt = \left(\phi S\frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 F}{\partial S^2}\right)dt + \sigma S W\frac{\partial F}{\partial S}dt \quad (9)$$

To eliminate the stochastic term in equation (8) consider the portfolio $\Pi = F - \frac{\partial F}{\partial S}S$. Following the no-arbitrage condition (since there is no stochastic term, Π is a risk-free investment and must therefore generate the equivalent return as any other risk-free investment)

$$d\Pi = dF - \frac{\partial F}{\partial S}dS = \left(\frac{\partial F}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 F}{\partial S^2}\right)dt = r\Pi dt = r\left(F - \frac{\partial F}{\partial S}S\right)dt \quad (10)$$

Simplifying equation (9) gives the Black-Scholes formula

$$\frac{\partial F}{\partial t} + r\frac{\partial F}{\partial S} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 F}{\partial S^2} = rF \tag{11}$$

It is clear that the principle of a risk-neutral valuation is satisfied in this case since the Black-Scholes equation is independent of the expected rate of growth of the underlying security price (ϕ).

The portfolio Π represents a self-financing, replicating, hedging strategy: it replicates a risk-free investment and it is hedged due to the absence of the stochastic component.

There are several ways of solving the Black-Scholes equation. Here the solution of the Black-Scholes equation using the principle of risk-neutral valuation is presented. It involves analyzing the assumed process for the stock prices using the Ito's lemma and applying the principle of the risk-neutral valuation to the result. Applying the Ito's lemma to the equation (8) gives

$$d(\ln S) = \left(\phi - \frac{\sigma^2}{2}\right)dt + \sigma W \tag{12}$$

The time integral of the white noise W gains a random walk whose distribution is known to be normal. From equation (12) it can be easily seen that

$$\ln S - \ln S_0 \sim N\left[\left(\phi - \frac{\sigma^2}{2}\right)\left(T - t, \sigma\sqrt{T-t}\right)\right] \tag{13}$$

where S and S_0 are the prices of the underlying security at time T and t respectively. Rearranging expression (13) shows that S follows a lognormal distribution:

$$\ln S \sim N\left[\ln S_0 + \left(\phi - \frac{\sigma^2}{2}\right)\left(T - t, \sigma\sqrt{T-t}\right)\right] \tag{14}$$

The risk-neutral valuation principle indicates that the present value of the option is the expected final value $E[\max(S-K,0)]$ of the option discounted at the risk-free interest rate (Strikant (2000)).

$$c = e^{-r(T-t)}E[\max(S-K,0)] = e^{-r(T-t)}\int_K^\infty (S-K)g(S)dS \tag{15}$$

where $g(S)$, the probability density function of S can be written as

$$g(S) = \frac{1}{\sigma S\sqrt{2\pi(T-t)}}\exp\left(-\frac{\left(\ln\left(\frac{S}{S_0}\right)-\left(r-\frac{\sigma^2}{2}\right)(T-t)\right)^2}{2\sigma^2(T-t)}\right) \tag{16}$$

The value of the integral in (15) is

$$c = SN(d_1) - Ke^{-r(T-t)}N(d_2) \tag{17}$$

where

$$d_1 = \frac{ln\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}},\ d_2 = \frac{ln\left(\frac{S}{K}\right)+\left(r-\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t} \tag{18}$$

and $N(x)$ is the cumulative standard normal distribution.
Intuitive interpretation of the result can be obtained by rearranging the expression (17)

$$c = e^{-r(T-t)}\left[e^{r(T-t)}SN(d_1) - KN(d_2)\right] \tag{19}$$

In a risk-neutral world $e^{r(T-t)}SN(d_1)$ is the expected value of a variable that equals S if $S > K$ and 0 otherwise while $N(d_2)$is the probability that the final stock price will be above the exercise price so that $KN(d_2)$ is the strike price times the probability that the strike price will be paid. Consequently $e^{r(T-t)}SN(d_1) - KN(d_2)$ is merely an expression for the expected value of the option at maturity.

2.4.1 Phenomenon of the Volatility Smile

The Black-Scholes formula was originally developed to compute option premiums. However, for options where a reliable market price exists, the formula can be used to predict the volatility. This is done by solving the Black-Scholes formula for σ, using an iteration method and plugging in the observed values for the other variables, including the observed option premium.

The problems with the volatilities implied by option-pricing models become visible when plotting the implied volatility values of options with the same time to maturity across various strike prices. The emerging pattern is known as the "volatility smile" and is the most widely-noted phenomenon testifying to the limitations of the classical Black-Scholes model.

Even though there is no simple explanation to the existence of this phenomenon, it seems to be strongly related to the stock market crash of 1987, when the stock market fell in two days by 23 %. Due to the lacking supply of put options, market participants were unable to protect themselves against the market fall fast enough. After the crash, in order to have downside protection, traders were willing to pay premium for the put index options. This theory of supply and demand is therefore one of the reasons for the existing smile.

Another reason for the existence of the smile phenomenon can be found in the fact that traders might not have homogeneous beliefs about distribution of the underlying asset. These differing views result in the violation of the Black-Scholes model's constant volatility assumption and lead therefore to the non-normality in the return distribution, thus causing the volatility smile.

The existence of the "volatility smile" phenomenon raised much uncertainty about the volatility estimates obtained from the Black-Scholes model and in response to its rather unrealistic assumptions "model-free" measures of the implicit volatility emerged.

2.5 Model-Free volatility measures

The greatest distinction between model-free measures of the implicit volatility and the volatility derived from the option-pricing models is that the former rely on much less restrictive assumptions. Even though, assumptions of frictionless markets with no arbitrage opportunities and continuous return distribution of the underlying asset are also present in the model-free implicit volatility measures, no assumptions about the process generating the asset returns are made. The examples of such model-free indexes include VIX and VDAX-new.

2.5.1 VSTOXX

The VSTOXX index is a measure of future short-term market volatility conveyed by stock index option prices with the EURO STOXX 50 as the underlying asset. VSTOXX calculates the implied volatility using a large sample of call and put options on the EURO STOXX 50 with time to maturity (τ) 30 days.

The VSTOXX is calculated using the two nearest expiration months of EURO STOXX 50 options. A rollover to the next expiration occurs eight calendar days prior to the expiry of the nearby option. The value of the index is derived from the prices of out-of-the-money and at-the-money puts and calls. The closer the option's strike price to the at-the-money value, the higher the weight its price receives in the calculation. Hence, the methodology of calculating VSTOXX is independent of an option-pricing model. VSTOXX is calculated directly from option prices rather than solving it out of an option-pricing formula, which considerably relieves the problems of measurement errors and model misspecification that arise when option-pricing models are employed.

The implied volatility for the VSTOXX index is calculated as follows:

$$\sigma = \sqrt{\frac{2}{\tau}\left[\sum_{i=1}^{N-1}\frac{\Delta X_i}{X_i^2}e^{r\tau}Q(X_i,\tau) + \frac{X_2-X_1}{2X_1^2}e^{r\tau}Q(X_1,\tau) + \frac{X_N-X_{N-1}}{2X_N^2}e^{r\tau}Q(X_N,\tau)\right] - \frac{1}{\tau}\left[\frac{F}{X_0}-1\right]^2}$$

where F is the forward index level derived from option prices; X_i is the strike price of the i^{th} out-of-the-money option (put if $X_t < F$ and call if $X_t > F$); ΔX_i is the interval between strike prices; X_0 is the first strike below F and $Q(X_i, \tau)$ is the midpoint of the bid-ask spread for each option with strike X_i. Put and call options are included up to the point where there exist two consecutive strike prices with a bid price equal to zero.

The VSTOXX index value itself is calculated as

$$VSTOXX = 100 \cdot \sigma \tag{20}$$

2.5.2 VSMI

The principle of calculating VSMI is exactly the same as for VSTOXX with the only difference being the underlying asset. While for the VSTOXX it is the EURO STOXX 50 index, for the VSMI it is the Switzerland's most important stock index SMI. The VSMI applies implicit variances to all Eurex-traded SMI options of the same duration. VSMI – as the duration-independent main index – is determined on the basis of a fixed residual term of 30 days.

3 Review of Financial Studies

The expected volatility of financial markets is a key variable in financial investment decisions. Asset allocation decisions are frequently reduced to a two-dimensional decision problem by focusing purely on the risk and the expected return of an asset or portfolio, with the risk being related to the volatility of the returns. The volatility of the returns plays also a central role in the valuation of financial derivatives such as options, which "can be used either as part of a dynamic hedging strategy, to protect a portfolio against adverse price movements, or as a speculative asset which gains from expected price changes. To assess the fair value of an option or to hedge market risk, an investor needs to specify his or her expectations regarding future volatility."[2]

As mentioned earlier, there are essentially two approaches to generate volatility forecasts: the information about the variance of future returns can be derived from their history or from observed option prices by eliciting market expectations. Given informationally efficient markets the volatility, implied by options, will reflect all information contained in past returns. Hence, prognostications based on the past returns should not outperform prognostications based on the implied volatilities.

Early studies find, however, that the implied volatility is a biased forecast of the future volatility and contains little incremental information beyond the historical volatility. For instance, Canina and Figlewski (1993) concluded that the implied volatility from the Standard

[2] H. Claessen & S. Mittnik, Forecasting Stock Market Volatility and the Informational Efficiency of the DAX index Options Market, *Center for Financial Studies*, No 4, 2002

& Poor's (S & P) 100 index options is a poor forecast for the subsequent realized volatility of the underlying index. Their analysis relied on an encompassing regression analysis based on which they concluded that the implicit volatility has essentially no correlation with the future realized volatility and thus does not subsume the information conveyed by the historical volatility. In contrast, Day and Lewis (1992), Lamoureux and Lastrapes (1993) and Jorion (1995) found that the historical volatility outperformed the implied volatility, since it contained more information about the future realized volatility

Christensen and Prabhala(1998) point out some econometric problems associated with the early research that arise from maturity mismatch and usage of the overlapping data. In their study Christensen and Prabhala(1998) avoid these problems by using the non-overlapping data. They adopt the instrumental variables estimation to resolve the errors-in-variable problem associated with the estimated implied volatility and find that the implied volatility outperforms the past volatility in forecasting the future volatility and contains the information content of the past volatility in their specifications. Fleming (1998) also finds that the implied volatility from the S&P 100 index options outperforms the historical volatility in terms of the ex ante forecasting power.
More recent research attempts, therefore, to correct various data and methodological problems in earlier studies. The later studies use instrumental variables (IVs) to correct for the errors-in-variable (EIV) problem in the implied volatility, consider longer time series to take into account possible regime shift around the October 1987 crash and adopt high-frequency asset returns to provide a more accurate estimate for the realized volatility. Jointly, these studies present evidence that the implied volatility is a more efficient forecast for the future volatility than the historical volatility.

Most of the earlier research on the information content of implied volatility concentrates on the Black-Scholes implied volatility from the at-the-money options. Even though at-the-money options seem to be a good starting point, granted they are generally more actively traded than other options, by concentrating merely on them, the information contained in other options is not incorporated. Furthermore, the tests based on the Black-Scholes implied volatility are joint tests of the market efficiency and the Black-Scholes model, leading to a possible model misspecification error. Using the model-free implied volatility should eliminate this problem as it is derived entirely from no-arbitrage conditions and isn't based on any specific option pricing model.

More recently, for the S&P100 index and VIX implied volatility index, Blair, Poon, and Taylor (2001) show that historical prices (even intraday prices) do not provide much incremental information compared to the information given by the VIX index of the implied volatility; moreover, the VIX index provides the best out-of-sample forecasts of the realized volatility (their forecast horizon ranges from 1 to 20 trading days).

Jiang and Tian (2005) use model-free implicit volatility to test the informational efficiency of the option market. Their results from the Standard & Poor's 500 index (SPX) options advocate that the model-free implied volatility subsumes all information conveyed by both historical volatility and the volatility implied by the Black-Scholes model, thus making model-free volatility a better forecast for the future realized volatility.

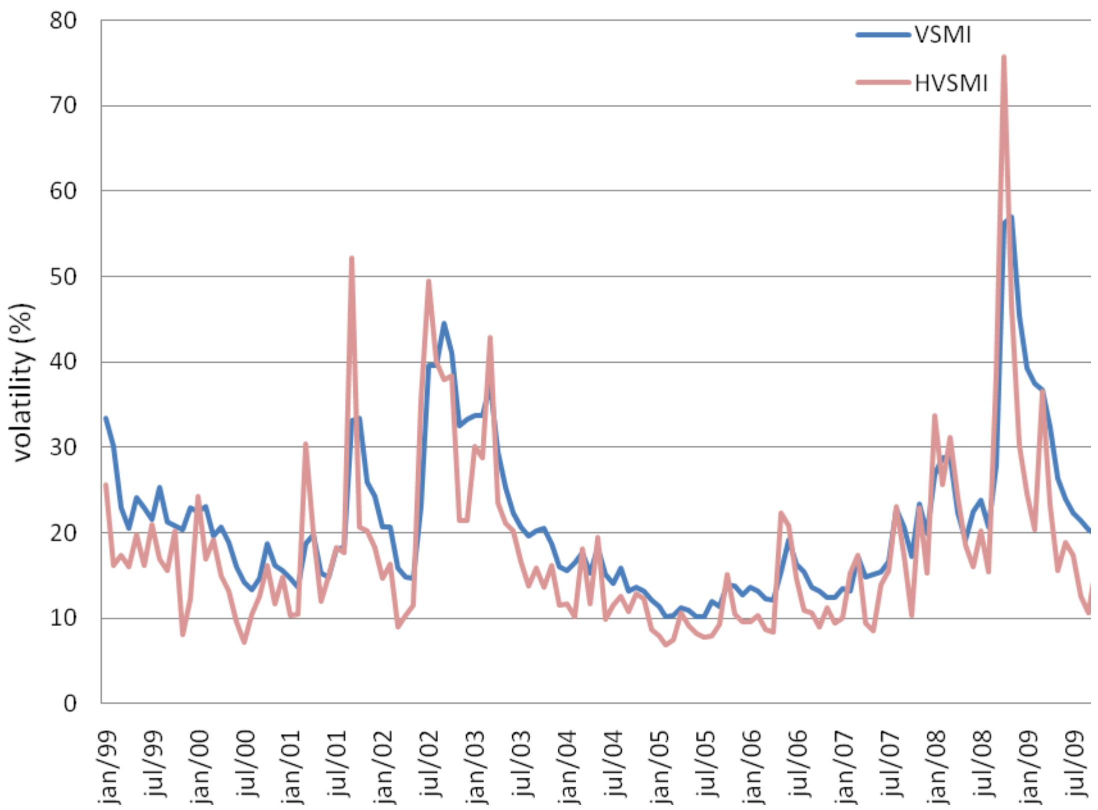
Most of these studies focus on the link between the implied volatility and the future realized volatility. However, some of them investigate possible relationships between the implied volatility and the future stock returns. Interests in the type of relationship between the two stem from the belief that financial markets are efficient. Hence, no relevant information about the stock prices movement can be provided by the implied volatility. This is in contrast with the opinion of non-academic market participants for whom very large implied volatility levels are usually seen as signaling attractive entry levels for long traders. The explanation is that very high implied volatility levels indicate periods of financial turbulence during which investors are believed to be overreacting and selling their financial assets to lessen their losses or boost cash. The empirical research confirms that there exists a relationship between extremely high levels of the implied volatility and a ‘market bottom’. For example, in an article “Fixated on the VIX: soaring volatility means fear - and opportunity”, K. Tan wrote in the July 29, 2002 issue of Barron’s that “A big VIX spike indicates the kind of extreme fear contrarians associate with market bottoms”.

4 Data and Methodology Description

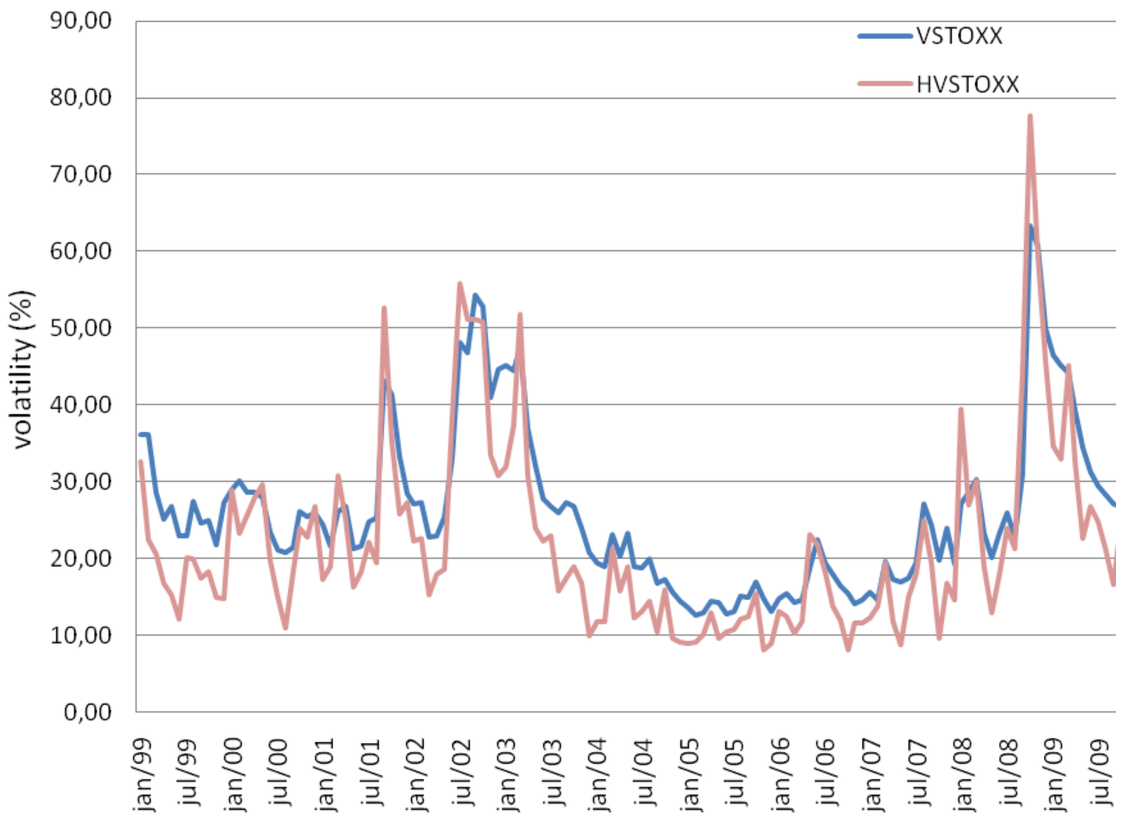
The data set used in this study consists of daily observations of SMI and EURO STOXX 50 covering ten years, from 01.01.1999 to 30.12.2009. Public holidays that fall on weekdays, when Eurex is closed, were omitted from the data set.
The historical monthly volatility for SMI and EUROSOXX 50 is determined by multiplying the standard deviation of the daily returns by the square root of the number of trading days per year.
The implied monthly volatility is calculated taking the average of daily closing VSMI and VSTOXX prices over each month period.
The monthly closing prices of the SMI and EURO STOXX 50 are determined as the average of daily closing prices within one month. Data on SMI and EURO STOXX 50 is taken from SIX Swiss Exchange Ltd and STOXX Ltd respectively.

4.1 Historical volatility vs. model-free implicit volatility

Since option prices reflect market participants' expectations of future movements of the underlying asset, the volatility implied from the option prices should be informationally superior to the historical volatility of the underlying asset. Assuming informational efficiency of the option markets and correct option pricing model, the implicit volatility should subsume the information contained in other variables in explaining the future volatility. Following the work of Chiu(2002) we start out by analyzing the kind of relationship that exists between implied and historic volatility for the SMI (Figure 1) and EURO STOXX 50 (Figure 2) during the sample period January 1999 through December 2009.



<u>Figure 1.</u> Monthly levels of Implied Volatilities (VSMI) and historical Volatilities (HVSMI) for SMI



<u>Figure 2.</u> Monthly levels of Implied Volatilities (VSTOXX) and historical Volatilities (HVSTOXX) for EURO STOXX 50

The notable feature of the figures above is that there exists a substantial relationship between both volatility measures for SMI and EURO STOXX 50, however the implied volatility graph lies above the historical volatility graph most of the time.
The summary statistics reported in Table 1 is compatible with Figures 1 and 2, confirming our earlier observation of implicit volatility being somewhat higher than the corresponding historical volatility.
Table 1 reports the summary statistics for monthly levels of the implied and historic volatilities for the SMI and EURO STOXX 50 portfolios over the sampling period. The mean statistics is consistent with figures 1 and 2, confirming that the implied volatility is, on average, higher than the historic volatility. The mean level for VSMI is 3.16% higher than the HVSMI level of mean; the difference between the equivalent values for EURO STOXX 50 is equal to 4.11%.

	Mean(μ)	**Std.dev(σ)**	**Range**	**Median**	μ/σ	$\mu - \sigma$	$\mu + \sigma$
VSMI	21.08	9.01	46.78	19.35	2.34	12.07	30.09
HVSMI	17.92	10.43	68.85	15.60	1.72	7.49	28.35
VSTOXX	26.07	10.47	50.63	24.51	2.49	15.60	36.54
HVSTOXX	21.96	12.15	69.70	18.90	1.81	9.82	34.11

Table 1. Summary Statistics

Another noteworthy feature that can be extracted from the summary statistics is the distinction in the range of fluctuation of the volatility measures. It is substantially higher for both HVSMI and HVSTOXX when compared to VSMI and VSTOXX, indicating that the former are more volatile. This observation is confirmed by the lower mean-to-standard deviation ratio of 1.72 for HVSMI as opposed to VSMI ratio of 2.34. The same is true for HVSTOXX and VSTOXX.
Adjusting the mean by one standard deviation indicates that the historical volatility is indeed more scattered. For instance, the HVSMI ranges from 7.49% to 28.35% (a range of 20.86%), whereas the VSMI ranges from 12.07% to 30.09% (a range of 18.02%); and the HSTOXX ranges from 9.82% to 34.11% (a range of 24.29%), while the VSTOXX ranges from 15.60% to 36.54% (a range of 20.95%).

The next step is to compare the monthly percentage changes in the volatility measures for the SMI and the EURO STOXX 50.

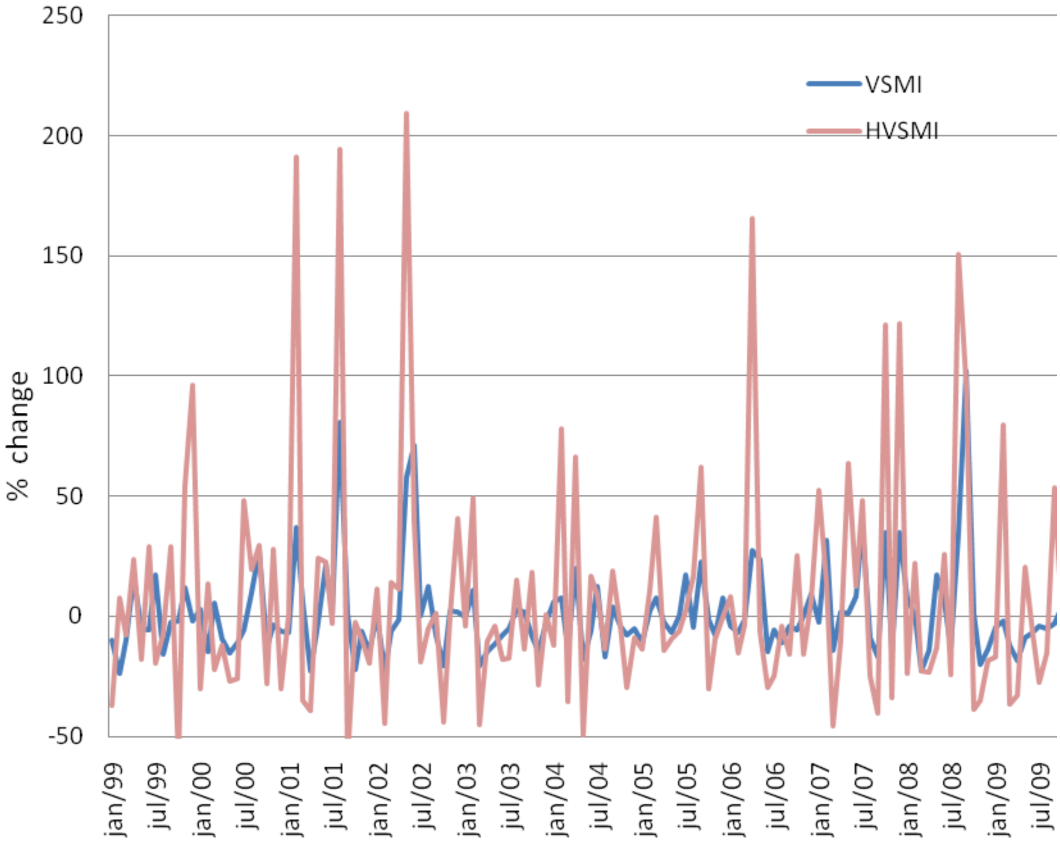


Figure 3. Monthly percentage change in Implied Volatilities (VSMI) and historical Volatilities (HVSMI) for SMI.

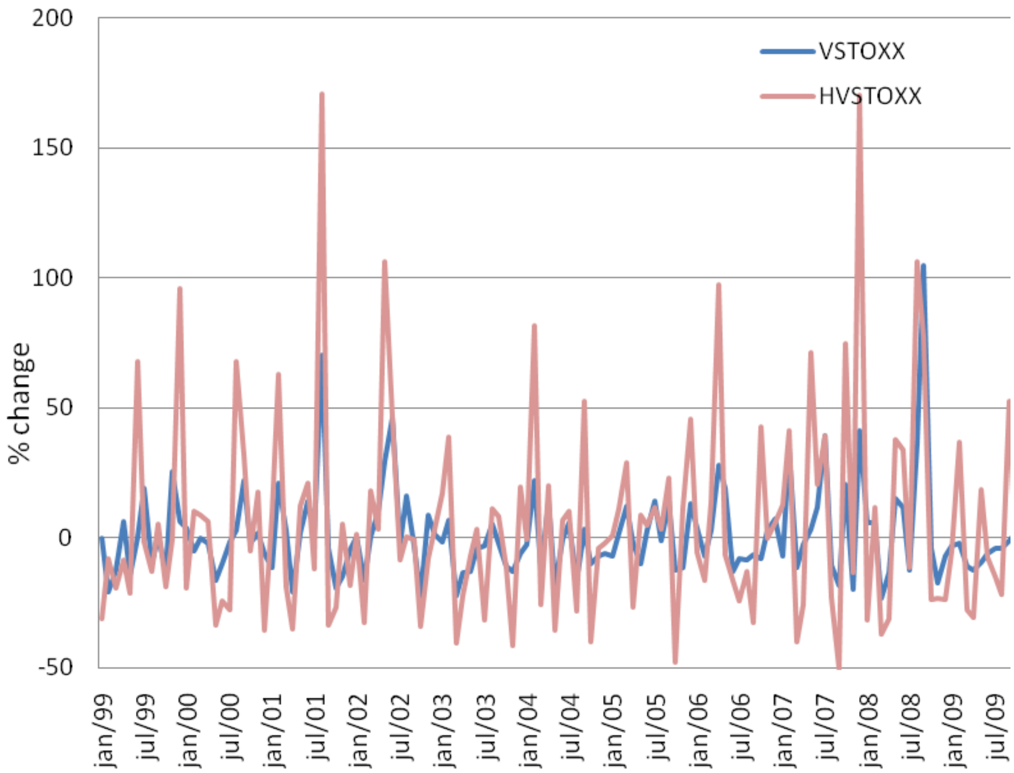


Figure 4. Monthly percentage change in Implied Volatilities (VSTOXX) and historical Volatilities (HVSTOXX) for EURO STOXX 50.

Figures 3 and 4 clearly indicate that the hypothesis stated earlier is indeed true for both indexes – the historical volatility measures are more volatile and scattered than the equivalent implicit volatility measures . The fact that the historic volatility is more volatile accords with the notion that the implied volatility is a smoothed expectation of the future realized volatility, defined as the historic volatility at time *t+1*.

	Mean (μ)	Std.dev (σ)	Range	Median	μ/σ	$\mu - \sigma$	$\mu + \sigma$
% Δ in VSMI	1.05	19.38	125.84	-3.20	0.05	-18.32	20.43
% Δ in HVSMI	7.74	49.15	269.81	-5.32	0.16	-41.41	56.89
% Δ in VSTOXX	0.98	17.61	128,13	-2.12	0.06	-16.63	18.59
% Δ in HVSTOXX	5.39	39.32	220.83	-0.80	0.14	-33.93	44.71

Table 2. Summary Statistics

Summary Statistics presented in Table 2 demonstrates that over the considered period, the average percentage change for the HVSMI is 7.74%, and only 1.05% for the VSMI; for the HVSTOXX and VSTOXX the corresponding numbers are 5.39% and 0.98% respectively.

Keeping in mind statistical differences between the two volatility measures, we next examine whether there exists a significant relationship between them. The results of the regression analysis are presented in Table 3.

	β_0	β_1	R^2	t-stat $(\beta_1 = 0)$	1% sign. level lower	1% sign. level upper
VSMI vs. HVSMI	7.9141	0.7348	0.7348	18.4450	-2.6142	2.6142
VSTOXX vs. HVSTOXX	8.6903	0.7913	0.8421	26.3352		

Table 3. Regression Data

The two-tailed T-test is adopted to analyze whether a significant relationship exists between the VSMI and the HVSMI. The null hypothesis of the slope coefficient being equal to zero $(H_0: \beta_1 = 0)$ is tested against the alternative of it differing from zero $(H_1: \beta_1 \neq 0)$. As shown in Table 3, t-statistic equals 18.4450 which considerably exceeds the upper bound of 1% significance level. The null hypothesis can therefore be rejected in favor of the alternative hypothesis. The conclusion hence is that there exists a significant positive relationship between the VSMI and the HVSMI.

The same testing procedure is carried out for the VSTOXX and the HVSTOXX. This gains the t-statistic value equal to 26.3352, which also exceeds the upper bound of 1% level of significance. Hence, there is a sufficient evidence to reject the null hypothesis and conclude that there exists the significant positive relationship between the VSMI and the HVSMI.

The extent to which the two volatility measures move together is illustrated in Table 4.

	VSMI	*HVSMI*	*VSTOXX*	*HVSTOXX*
VSMI	1			
HVSMI	0.8506	1		
VSTOXX	0.9661	0.8224	1	
HVSTOXX	0.8998	0.9334	0.9177	1

Table 4. Correlation Matrix

The correlation coefficients presented in Table 4 indicate that despite the statistical differences between the historical and implicit volatility measures, a strong correlation exists between them. VSMI and HVSMI share a correlation 85.06% while for VSTOXX and HVSTOXX the corresponding value is 91.77%

4.2 Relationship between the implied volatility and stock market returns

In order to check the efficiency of the implied volatility in predicting the future market returns, we next examine the type of the relationship that exists between the stock market returns and the implied volatility. It is a known phenomenon that the implied volatility of an index tends to decline when its rate of return is positive. This negative relationship between the returns and the volatility has been actively studied and analyzed by economists like Whaley (2000) and Black (1976). The evidence from Whaley (2000) advocates that the implied volatility index indicates the extent of investor stress and fear, thereby revealing useful information about market sentiment. The intuition behind this conclusion is that the demand for option protection declines as the stock price rises, suggesting that investors indirectly set the level of the implied volatility. The investors' demand for stock's call and put options sets the prices, and these prices, in turn, are used to determine the level of corresponding implied volatilities. Another possible explanation of the negative phenomenon that price movements are negatively correlated with the volatility is generally referred to as the "leverage effect". First suggested by Black (1976) it postulates that when a leveraged firm's stock price declines, its debt-to-equity ratio raises, which, in turn, increases the firm's financial leverage, resulting in an increase in the volatility of the firm's equity as it becomes more risky.

Regardless of what the explanation for the phenomenon of this negative relationship is, it is safe to assume that all studies agree on the fact that such phenomenon exists. Figures 5 and 6 lead us to the same conclusion, namely, that declining volatility is often followed by rising markets, whereas the opposite is true for the raising volatility.

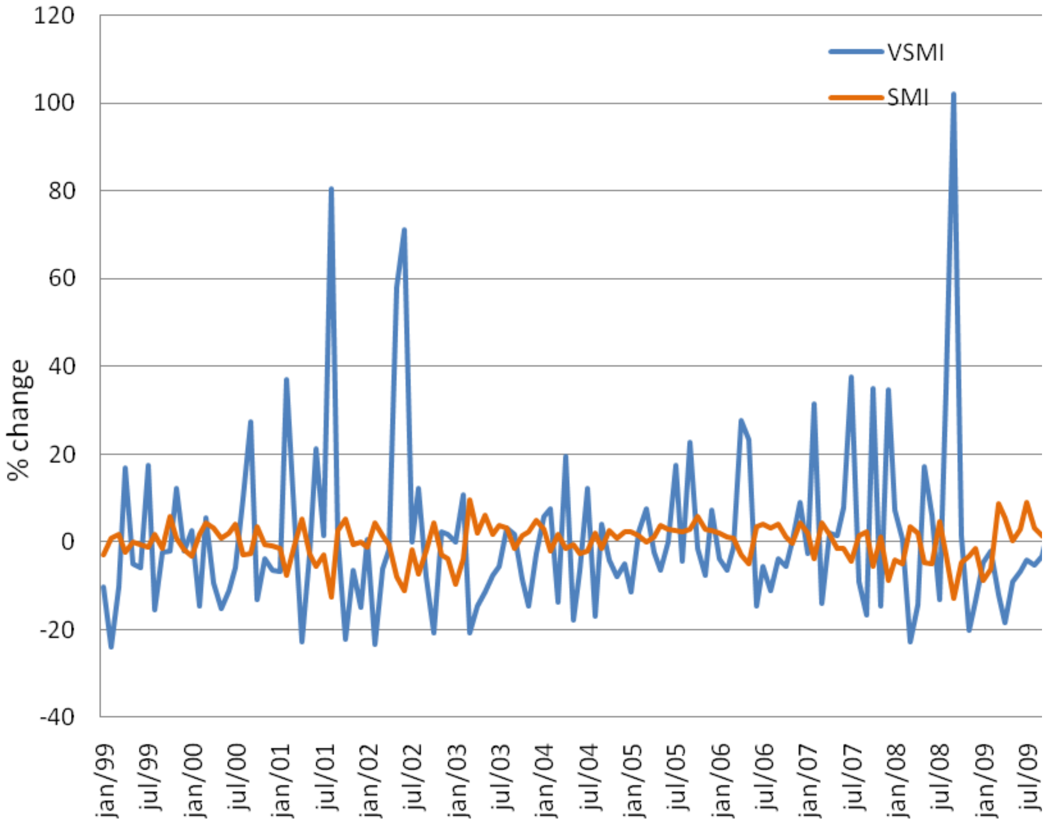


Figure 5. *Monthly percentage change in Implied Volatilities (VSMI) and SMI.*

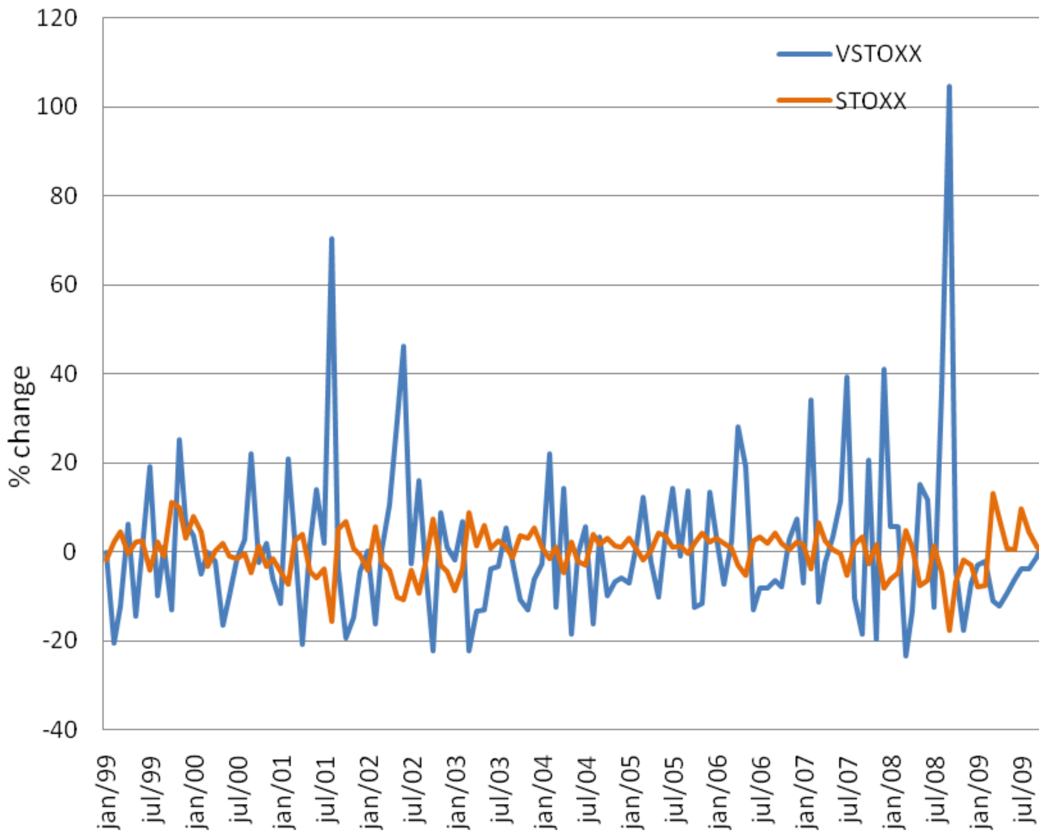


Figure 6. *Monthly percentage change in Implied Volatilities (VSTOXX) and EURO STOXX 50.*

Figures 5 and 6 demonstrate an inverse movement in the monthly percentage changes in the VSMI and VSTOXX and monthly percentage changes in the SMI and EURO STOXX 50 respectively. Another noteworthy feature of the figures above is the recurring at regular intervals spikes which are present in both VSMI and VSTOXX. The large percentage declines in the VSMI and VSTOXX correspond to smaller percentage increases in the underlying SMI and EURO STOXX 50 indices respectively. The two most significant spices took place in September 2001 following the World Trade Center terrorist attack, and from late August 2008 following the financial crisis throughout the world. In the recent financial crisis, especially in September 2008, the VSMI experienced a raise of 102.02% making a -12.96% drop in the SMI, while the VSTOXX experienced a raise of 104.67% making a -17.74% drop in the EURO STOXX 50.

In his study Whaley (2000) used regression analysis and concluded that there exists a significant negative relationship between stock market returns and changes in the implied volatility. The results of regressing the VSMI and VSTOXX on the SMI and EURO STOXX 50 respectively are presented in Table 5.

	β_0	β_1	R^2	t-stat $(\beta_1 = 0)$	1% sign. level lower	1% sign. level upper
% Δ VSMI vs. % Δ SMI	0.1524	-0.1470	0.4766	-10.8378	-2.6142	2.6142
% Δ VSTOXX vs. % ΔSTOXX	0.1647	-0.1904	0.4650	-10.5888		

Table 5. Regression Data

The two-tailed T-test is adopted to check whether there is a significant relationship between the percentage change in the VSMI and SMI. The null hypothesis of the slope coefficient being equal to zero $(H_0: \beta_1 = 0)$ is tested against the alternative of it being other than zero $(H_1: \beta_1 \neq 0)$. As shown in Table 5, t-statistic equals -10.8378 and is far below the lower bound of 1% level of significance. The null hypothesis can therefore be rejected in favor of the alternative hypothesis and we can conclude that there exists a significant negative relationship between the percentage change in the VSMI and the percentage change in the SMI.

The same testing procedure is carried out for the VSTOXX and the HVSTOXX gains the t-statistic value equal to -10.5888, which is also far below the lower bound of 1% level of significance. Even in this case, the null hypothesis is rejected and we conclude that there exists a significant negative relationship between the percentage change in the VSTOXX and the percentage change in the EURO STOXX 50.

	%Δ VSMI	*%Δ HVSMI*	*%Δ SMI*	*%Δ VSTOXX*	*%Δ HVSTOXX*	*%Δ STOXX*
%Δ VSMI	1					
%Δ HVSMI	0.6977	1				
%Δ SMI	-0.6904	-0.5624	1			
%Δ VSTOXX	0.9461	0.6851	-0.6884	1		
%Δ HVSTOXX	0.6990	0.8516	-0.5910	0.7241	1	

%Δ STOXX	-0.6841	-0.5340	0.8708	-0.6819	-0.5389	1

Table 6. Correlation Matrix of percentage changes

The negative relationship between changes in the implied volatility and the rate of return on the underlying asset that we observed in figures 5 and 6 is confirmed by the correlation coefficients in table 6. The percentage changes in the SMI and VSMI share the correlation coefficient of -0.6904 and the percentage changes in the EUROSTOXX 50 and VSTOXX share the correlation coefficient of -0.6819.
This result has an important implication: since index returns cannot be predicted completely accurately, if a change in the implicit volatility can be forecasted, than the high degree of the negative correlation between the two series can be used to create a mean-variance portfolio.

Having established that the significant negative relationship exists between changes in the implied volatility and the rate of return on the underlying asset we now proceed to examining whether the implied volatility is, in fact, a good predictor of the actual realized volatility. The following OLS regression is run for both VSTOXX and EURO STOXX 50:

$$rv_t = \beta_0 + \beta_1 iv_{t-1} + \beta_2 hv_{t-1} + \varepsilon_t \tag{21}$$

where rv_t denotes the natural logarithm of realized volatility of the index, iv_{t-1} and hv_{t-1} denote the natural logarithm of the model-free implied and historical volatility during the period before time t.

	β_0	β_1	β_2	$Adj\ R^2$	t-stat (β_1)	t-stat (β_2)	1% critical values
SMI	5.4848	0.1595	0.5034	0.3825	1.0500	3.8319	$\pm$ 2.6150
EURO STOXX 50	4.3074	0.1176	0,6614	0.5676	0.6968	4.5439	

Table 7. Regression Data

The results of the final regression analysis presented in Table 6 lead us to the conclusion that the model-free implied volatility has no significant relationship with the future realized volatility and thus cannot be used as a market forecast of the future volatility index. On the other hand, the coefficient in front of the lagged historical volatility value is significant at the 1% level, indicating that the historical volatility can be used to forecast the future realized volatility.
There are a number of possible explanations for this result. The most obvious one is that the time-lag of one month is too long to predict the future stock index development. Perhaps a lag of one to five days would generate better results. Second, as mentioned previously, the results of the OLS regression might be distorted by an error-in-variables problem – this problem caused faulty results in the early studies within this field. Third, high correlation between the explanatory variables might lead to a multicollinearity problem. As shown in Table 6 the historical and the implied volatility share the correlation coefficient of 0.6977 for

the SMI index and 0.7241 for EURO STOXX 50. Finally, an explanation to why changes in the lagged implicit volatility values might not serve as a good forecasting tool lies in the nature of the implicit volatility, which is essentially the market's assessment of the underlying asset's volatility over the option's life. It is safe to assume that most traders are unable to make an accurate prediction of what volatility is going to be during the life of an option and are therefore forced to make a guess, when buying or selling the derivative. As a result, the current implicit volatility might not resemble the actual statistical volatility which will disclose itself later during the option's life. Consequently, the faulty forecasted volatility results in the implicit volatility being a poor predictor of the future realized volatility and, hence, unable to forecast future returns of the underlying asset.

Deeper analysis of the time-series behavior is required to answer which of the above reasons, if any, applies to our data. Given the results of the OLS regression presented in Table 7 it would be reasonable to investigate whether the errors-in-variable problem is significant for the implied volatility series and if so, it should be incorporated into the analysis by means of instrumental variables. However, such analysis is beyond the scope of this thesis.

5 Conclusion

In this thesis by adopting the technique suggested by Chiu(2002) we have researched various dependencies, trends and facts known about the volatility by looking at the time-series behavior of the implicit and historical volatility measures of the SMI and EURO STOXX 50 over the period covering January 1999 and December 2009.

Results of this study signify that even though there exists a strong relationship between both volatility measures for SMI and EURO STOXX 50, the implied volatility values lay above the historical volatility values most of the time.
This pattern can be associated with the fact that the negative market reaction to a rising implicit volatility is of a greater magnitude than its positive reaction towards the diminished implicit volatility. This phenomenon can be explained by agents' risk aversion, and hence may illustrate that declining markets result in an increased options demand, which, in turn, results in an elevated option price. An overpriced option implies a volatility that is too high, compared to the historic or actual volatility, since the implicit volatility is derived from an observed market price of an option.

When comparing the percentage changes in the implied and historic volatility for the SMI and EURO STOXX 50, it is observed that although the implied volatility values lay above the historical volatility values most of the time, the historical volatility is more volatile and scattered than the implied volatility for both portfolios.

In this analysis, we also observe several widely acknowledged stylized statistical facts related to the time series behavior of the volatility. First, the volatility exhibits mean-reversion towards its long-run mean. Second, certain skewness is present in the volatility response to changing market circumstances – falling markets raise volatility more substantially, than rising markets diminish the expected volatility.

The importance of the relationship between the implicit and historic volatilities lies in the fact that it often is the basis of forming trading strategies in the options market. The volatility implied in an option price is likely to be a good prognosticator of the future volatility if the market is efficient and the option-pricing model is correct. If the implied volatility does contain the information about forecasting the future realized volatility, then the implied volatility may be useful in forecasting future market movements.
The results of two distinguished works suggest that the implicit volatility is an efficient forecast of the future realized volatility. Christensen and Prabhala (1998) study the relation between the implied and historic volatility using the S&P 100 index option data, and find

that the implied volatility outperforms the historic volatility in forecasting the future volatility. Fleming (1998), who examines the quality of market volatility forecasts implied by the S&P 100 index option price, confirms Christensen and Prabhala's (1998) findings. He finds that the implied volatility from the S&P 100 index options outperforms the historical volatility in terms of the exante forecasting ability.

The results of the OLS regression carried out in this thesis lead us to the conclusion that the model-free implied volatility has no significant relationship with the future realized volatility and thus cannot be used as a market forecast of the future volatility index.

There are a number of possible explanations of this result. The most obvious one is that the time-lag of one month is too long to predict the future stock index development and a lag of one to five days should be used instead. Second, the results of the OLS regression might be distorted by an error-in-variables problem; third, a high correlation between the explanatory variables might lead to a multicollinearity problem and finally, the implicit volatility might not serve as a good forecasting tool due to the fact that it is merely a market's assessment of the underlying asset's volatility over the option's life. If the market's view turns out to be inaccurate, the current implicit volatility might not resemble the actual statistical volatility which will reveal itself later during the option's life. Consequently, an inaccurately predicted volatility results in the implicit volatility being a poor predictor of the future realized volatility and, hence, is unable to forecast future returns of the underlying asset.

This paper has focused on analyzing the predictive power of the model-free implied volatility. One possible extension of this study is to examine the relationship between the implied and the realized volatility using the volatility implied by one of many other well-known option pricing models, such as the deterministic volatility function approach by Dumas et al. (1998), the stochastic volatility models of Heston (1993) and the jump model of Bates (1996).

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