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Department of Economics

# MARKET MODELS VS. ACCOUNTING MODELS

## - DEFAULT PREDICTION DURING THE FINANCIAL TURMOIL

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#### PART I. INTRODUCTION

#### 1.1 Background

During the past few years we have experienced an extraordinary turbulence in the financial markets. Stock markets in freefall, countless of bankruptcies and government interventions to save huge financial institutions have been regular events. During these times the focal point has been risk management. Poor risk management has been one of the main reasons for the experienced crisis. Credit risk in particular has been a widely discussed topic, which has divided both academics and professionals. Credit rating agencies' assessments have been subject to immense criticism and many are questioning their methods and the accuracy of credit risk models in general. Credit risk is the risk you are exposed to when lending money to someone, that the counterparty fails to meet its obligation with agreed terms. When the borrower cannot payback the loan, he defaults on the loan. The lender looses out on the money and is required to write off the claim from its balance sheet. This may in some cases lead to the lender experiencing financial distress and a bankruptcy threat. The reason for managing credit risk correctly is the lender's wish to maximise the return by maintaining a sound exposure to credit risk<sup>1</sup>.

### 1.2 Problem specification

To be able to manage the credit risk, the lender need to have a good idea on how likely the borrower is to default. There are a number of different models available to estimate the likelihood of a borrower defaulting. We have examined two models using different sort of input when predicting default: the famous z-score by Altman's from 1968 uses date collected from companies' financial statements and a modified Merton model, where the input are obtained from the financial markets. The modified Merton, created by Byström (2005) model is built on Merton's (1974) model.

<sup>&</sup>lt;sup>1</sup> PRINCIPLES FOR THE MANAGEMENT OF CREDIT RISK, Consultative paper issued by the Basel Committee on Banking Supervision (1999).

#### 1.3 Purpose

The purpose of the study is to assess the relative performance of one accounting and one market based model in a particularly challenging climate with suffering economies and turbulent markets.

#### 1.4 Disposition

The first part of the paper is available to give the reader an adequate background to the reason for this study, while Part II provides a comprehensive description of the theory behind the models used in the analysis. The focus on the analysis procedure can be found in Part III, where an explanation on how the study has been conducted is included. Part IV runs through the result and an interpretation of the models' test score is given. Part V stresses the importance of using appropriate comparison methods and briefly focus on the theory behind the particular process carried out in this study. Results from the comparison are also presented in this section followed by an analysis of the two models' default predicting ability. The final part, Part VI is a concluding section, where the study and its findings are summed up and an ultimate statement is made.

#### PART II. THEORY

Altman's z-score model and the Merton model are widely used among professionals handling credit risk and have therefore been chosen to represent the two different model categories that are the cause of this paper. However, the modified Merton is a modification of the original Merton (1974) and contains a few new critical assumptions in order to simplify the model. The result of the comparison will be interpreted as an indication to which model category is the most accurate. Will the modified Merton model, due to the fact that it uses market information, give a better forecast of companies in distress or will the accounting based z-score model be more accurate when predicting bankruptcies.

The important and also the appealing matter of this paper is that it examines the accuracy of the models during recent year. During years when the markets have been extremely volatile and many companies have experienced financial distress due to tremendous uncertainty and turbulence in the markets. We should therefore expect a market-based model, such as the modified Merton, to perform better than the z-score model. The market based model should be quicker to react to the changes of the firm's financial position. Whereas the accounting based model only presents information of the past, which may not be sufficient in predicting the future. The modified Merton model however, also uses historical share/asset prices to compute the volatility but is more forward looking in its approach to predicting bankruptcies, according to Saunders and Allen (2002). Since the price of the stock is representative for the future prospects of the firm. In an efficient market, the share prices should reflect the books of the company. Other information (not found in the accounting statement of the company, e.g. future cash flows) might have an impact on the likelihood of default and is also reflected in the valuation of the company. Another weakness of the z-score is the possibility that the firm's accounting people have manipulated the input data and the financial reports does not represent the actual state of the firm. This would obviously take away the truthfulness of the model. The accuracy of the default predicting models should (theoretically) not be depended on time or sample used in the model. Mensah (1984) found sample dependence to occur in accounting based models. Mensah (1984) suggested the models to be redeveloped and variables changed to fit the specific sample. The Merton model's major weakness is the set of assumptions that may not represent reality. For example is the assumption that stock returns are normally distributed, something that is not always reflected in the real world. (Assumptions explained in detail below). Previous comparing research has had contradicting result and a final decision on which model is superior has not been able to have been taken. For example did Campbell et al (2006) find that the Merton model was not accurate enough to predict default, but Kealhofer and Kurbat (2001) found that the Merton approach outperforms various accounting ratios and argues that the Merton model already contain all the information in accounting ratios, and more.

#### 2.1 Previous Research

Many studies highlighting the accuracy of default prediction models have been carried out before, for both academics and professionals have relied on the models. The marketbased Merton model has been under scrutiny from a vast number of academics and many modifications in attempts to improve the model have been conducted. However, there are few published studies where accounting based models and market based models have been compared on the basis of their power. Reisz and Perlich (2004) found that the accounting based z-score outperform the structural models on a one year time horizon, but looses its power when the time horizon is further away from failure. Agarwal and Taffler (2008) tests a UK adapted z-score model and two versions of the Merton model. The two versions of Merton models came from Hillegeis et al (2004) and Bharath and Shumway (2004), where slightly different methods of computing the asset value and volatility are used. Agarwal and Taffler (2008) set of data is taken from before the financial crisis. Agarwal and Taffler (2008) find little difference between the accounting based and the market based model. Hillegeis et al (2004) reaches the same conclusion when they test the models. They argue that due to the model's assumptions, it is not surprising that the market based models do not perform better. The need to back out asset value and volatility is another reason for the inaccuracy according to Hillegeis et al (2004).

#### 2.2 Models

Two models have been chosen to represent one of the model categories each. The zscore model was created by Edward I. Altman in the 1960's and uses accounting data as it's main components. The market based category is represented by a modified Merton model. The z-score model and the Merton model has been around for decades and have been used by many credit risk managers and analysed in many studies. They are two of the most frequently used models. The models will be explained in detail below, first the z-score model and then the original Merton model followed by the modified version.

The test in this study is made on a random sample of companies. Some of them manufacturing firms and some of them non-manufacturing firms. They are/were all

publicly held companies. We therefore decided to use the original z-score model from 1968, as it would be the most appropriate in our study due to our general sample. A model that is applicable on many different types companies is more useful and as a reason the original z-score model from 1968 is representing the accounting based category in this study.

The reason for using the modified Merton model in the paper is its simplicity's moderate impact on the result Byström could extract from his study. It would be interesting to see by representing the market based models, if this simplified model could outperform a classic model like z-score model.

#### 2.3 Z-score model

Altman builds his work on Beaver's (1966) article on financial ratios as a tool for predicting failures.<sup>2</sup> Beaver conducted a univariate analysis and found that a number of ratios could distinguish between failed companies and non-failed companies up to five years prior to failure. His work implied that by using more than only one ratio at the time (a multivariate approach) the analysis could be more successful. Altman (1968) assesses the analytical quality of ratio analysis by combing a set of financial ratios in a multi discriminant analysis approach on corporate bankruptcy prediction. A multi discriminant analysis is a method used to separate the observations into defined groups. In Altman's case he referrers to the two classes as bankrupt and non-bankrupt. The discriminant function is

$$\mathbf{Z} = \mathbf{V}_1 \mathbf{X}_1 + \mathbf{V}_2 \mathbf{X}_2 + \dots + \mathbf{V}_n \mathbf{X}_n$$

Where:

 $V_1X_1, \dots, V_n$  = discriminant coefficient

and

 $\mathbf{V}_2 \mathbf{X}_2, \dots, \mathbf{X}_n$  = independent variables.

<sup>&</sup>lt;sup>2</sup> Beaves defined failure as the inability of a firm to pay its financial obligations as they mature. A firm is said to have failed when any of the following events have occured: bankruptcy, bond default, an overdrawn bank account, or nonpayment of a prefered stock dividends.

The analysis results in a z-value that then can distinguish the bankrupt and non-bankrupt companies from each other. Altman divides the resulting z-values into three categories, (see table 1)

Z-Value	Category
- 1.80	Bankrupt
1.81 - 2.99	Zone of Ignorance
3.00 -	Non-Bankrupt

**Table 1.** Altman defines the z-scores in to three classes. A score up to 1.80 describes a firm that will go bankrupt. If the company score 3 or above, it should have no problems with bankruptcy. 1.81 – 2.99 is a zone of ignorance, where the future of the firm is unclear.

A z-value below 1.81 indicates a distressed company likely to file for bankruptcy, while a z-value above 2.99 show financial strength and are unlikely to go bankrupt. Altman defines a zone of ignorance, a grey area, when the model produces a z-value 1.81 - 2.99, where the model is unable to distinguish a bankrupt from a non-bankrupt firm.

Altman produced five appropriate ratio categories for the model. He tested a total of 22, variables and found that five variables were doing a better job predicting failure than the others. He tested the variables' statistical significance and for inter-correlation between them. To test the individual discriminant ability of the variables, Altman conducted a F-test. The test stressed the difference between the average values of the ratios to the values of the ratios within each group. He found  $X_1 - X_4$  to be significant at the 0.001 level, while  $X_5$  did not show a significant difference between the classes. Altman still chose to include all five of the variables since on a strictly univariate level, they all (including  $X_5$ ) produced a lower value for bankrupt firms then for non-bankrupt firms. His sample contained of 66 observations, which half of them were bankrupt companies and the other half non-bankrupt companies. The firms in his sample all had total assets between \$1 - \$25 million. In his study, Altman finds his approach to be accurate in 95% of the cases. He finds that his model can predict bankruptcies accurately up to two years prior to the companies were actually failing. The model's accuracy is declining quickly the further away from the two years prior to bankrupt the model is predicting.

His model is:

#### $\mathbf{Z} = 0.012 \mathbf{X}_1 + 0.014 \mathbf{X}_2 + 0.033 \mathbf{X}_3 + 0.006 \mathbf{X}_4 + 0.999 \mathbf{X}_5$

Where

#### $X_1 = Working capital/Total asset$

Working capital is the difference between current assets and current liabilities. Altman considered two other liquidity ratios but found working capital/total assets to be the most suitable. It is a measure that compares the size of the net liquidity to the total capitalisation.

#### $X_2$ = Retained Earnings/Total assets

This measurement refers to the cumulative profitability over time; the gains and losses over the firm's lifetime in relation to its total assets. A young firm is likely to have a lower ratio and is more likely to be classified bankrupt. The measurement can therefore be considered to be discriminating towards young companies. Altman claims that this is in fact how reality is. Young companies are more likely to find themselves in financial distress than more mature companies. Paid dividends to shareholders can affect the ratio and should be adjusted for.

#### $X_3$ = Earnings before interest and tax/Total assets

Is a measurement of a how productive a firm is using its total assets, not considering tax and interest aspects. Since a company's long-term survival requires profitability, this is a relevant measure especially when you look closer at the companies that are performing poor.

#### $X_4$ = Market value equity/Book value of total debt

Refers to the total market value of the firm (i.e. total value of all shares) in relation to the total debt, including both current and long-term liabilities. The measurement indicates

how much the stock price can decrease before the liabilities exceeds the assets and the firm acquire insolvency problems.

#### $X_5 = Sales/Total assets$

This measure is the least important ratio on an individual basis (as explained above), based on the statistical significant measure, but Sales/Total assets is important due to its relationship with the other ratios. The measurement shows the value of the sales the firm is able to generate in relation to the total assets and could be according to Atman, interpreted as management's ability to handle competitive conditions.

Altman's model from 1968 is specifically developed to predict bankruptcies for publicly held, manufacturing companies in the US. His tests therefore only included publicly held, manufacturing firms where financial data were available. The z-score model was revisited by Altman (2000), where he found by conducting tests on samples from three time periods that the bankruptcy predicting accuracy was still strong, between 82 – 94 per cent, depending if he used the traditional 1.81 cut off threshold or a newer, more accurate z-score of 2.675. We will be using the original threshold of 1.81 as the distinguishing value in this study. Altman et al (1977) put together a newer model, the **ZETA**<sup>®</sup> model, based on the z-score model from 1968. The **ZETA**<sup>®</sup> model was updated to better predict bankruptcies in the US in a more modern time. They found that by adding two more variables the model gave an improved result predicting bankruptcies.

 $\mathbf{X}_1$  = Return on Asset,  $\mathbf{X}_2$  = Stability of earnings,  $\mathbf{X}_3$  = Debt service

 $\mathbf{X}_4$  = Cumulative profitability,  $\mathbf{X}_5$  = Liquidity,  $\mathbf{X}_6$  = Capitalization,  $\mathbf{X}_7$  = Size

Altman (2000) implies that the **ZETA<sup>®</sup>** model improves failure accuracy over the z-score model and is based on date more relevant to today's conditions. Since the **ZETA<sup>®</sup>** model is a proprietary effort, the parameters are not available to the public. That rules out an attempt to include the model in our analysis.

Altman et al (1977) also adapted the z-score model for private firms and nonmanufacturing firms to be satisfactory by substituting the book value of the net worth for the market value equity.

#### 2.4 Merton 1974

While the likelihood of failure in the z-score model is calculated from accounting data, the Default Probability (PD) in Merton (1974) is modelled with market information. The PD depends on the value of the firm's assets at the beginning in relation to its outside debt, and the market value of the firm's assets. The Merton model is often referred to as a structural model since it only depends upon the capital structure of the firm (debt and equity). The model from 1974 is based on Black-Scholes (1973) general equilibrium theory of option pricing framework, where the equity of the firm can be referred to as call option on the underlying value of the firm with the strike price equal to the face value of firm's debt.<sup>3</sup> The Merton model can produce immediate updates of the default probability by backing out asset values and asset volatilities from observable equity values<sup>4</sup> and balance sheet reports. The model assumes that a company has a certain amount of zero-coupon debt that will become due in the future. The company defaults if it the size of the amount it needs to payback is greater then the value of the company's assets.

To work out the default probability using the Merton model we need to calculate Distance to Default (DD). To calculate the DD, three pieces of information are needed.

- 1. Asset Market Value
- 2. Asset Value Risk (Asset Value Volatility)
- 3. The Book Value of the Firm's Debt (Liabilities)

Distance to Default = <u>Asset Value Risk</u> - Liabilities <u>Asset Market Value</u>

 $<sup>^{3}</sup>$  Equity holders have the right, but not the obligation, to pay back the debt of the firm to the lenders and in that way take over the firm.

<sup>&</sup>lt;sup>4</sup> Observed equity values and volatility are taken from the quoted stock prices.

As mentioned above, the asset market value and asset value risk are not observable, but by using the Black-Scholes option pricing model, the Merton framework backs out the these values from equity value and equity volatility by assuming the firm's underlying assets follow the stochastic process:

 $\mathrm{d}V_{\mathrm{A}} = \mu_{A}V_{A}\mathrm{d}t + \sigma_{A}V_{A}\mathrm{d}z$ 

Where

 $V_A$ ,  $dV_A$  = the firm's asset value and asset value change  $\mu_A$ ,  $\sigma_A$  = the firm's asset value drift and volatility dz = Wiener process

In the Black-Scholes framework there are only two types of liabilities, a single class debt and a single class of equity. The market value at time T can be expressed by the following option function equation if the book value of debt is denoted X:

$$V_E = V_A \cdot N(d_1) - e^{-rT} \cdot X \cdot N(d_2)$$

 $N(\cdot)$  is a cumulative Normal distribution function and

$$d_{1} = \frac{\ln\left(\frac{V_{A}}{X}\right) + \left(r + \frac{\sigma_{A}^{2}}{2}\right) \cdot T}{\sigma_{A}\sqrt{T}} \quad \text{and} \quad d_{2} = d_{1} - \sigma_{A} \cdot \sqrt{T}$$

Where r is the risk free interest rate.

The value of the option is referred to as the total value of the firm's equity, which is easily observed as the market value of the total shares of stock. The asset value however, has to be estimated. The same method is used when obtaining the volatility, the equity volatility can be observed but the asset volatility has to be approximated.

Under Merton's assumptions it could be shown that asset volatility  $\sigma_A$  and equity volatility  $\sigma_E$  are related to each other as:

$$\sigma_E = N(d_1) \cdot \sigma_A \cdot \frac{V_A}{V_E}$$

All the variables needed are obtained and distance to default can be calculated

,

Distance to Default = 
$$\frac{\ln\left(\frac{V_A}{X}\right) + \left(r - \frac{\sigma_A^2}{2}\right)}{\sigma_A \sqrt{T}}$$

The DD result can be interpreted as the number of standard deviations the company are away from default. The larger the value of DD, the further the firm is from defaulting i.e. the lower the probability of default. The Merton model assumes that the returns are Normally distributed and uses the number of standard deviations to produces a default probability (PD).

#### Modified Merton 2.5

Since asset value and volatility are not observable and need to be backed out from equity value and volatility, Byström (2005) created a simplified model. Drawing on three critical assumptions.

1. The drift term 
$$\left(r - \frac{\sigma_A^2}{2}\right) \cdot T$$
 is "small".

- 2.  $N(d_1)$  is close to one.
- 3. The book value of debt (X) is used to calculate the leverage ratio (debt-equity ratio)  $\frac{X}{V_A}$

The Modified Merton model only uses observable parameters. Byström justifies the first assumption by claiming that the drift term in most practical situations are small in comparisons to the first term.

It reduces Merton's model to

$$\mathrm{DD}_{Modified} = \frac{\ln\left(\frac{V_A}{X}\right)}{\sigma_A}$$

The second assumption can be explained by observations suggesting that in most cases  $N(d_1)$  is close to one. Only in extreme cases, where the option is almost at-the-money, (i.e. the asset volatility is very high and the asset value is close to the book value of debt), there is a  $N(d_1)$  is significantly different to one. This means that Byström (2006) replaces

$$\sigma_{A}$$
 with  $\frac{\sigma_{E} \cdot V_{E}}{V_{A}}$  , and gets

$$DD_{Modified} = \frac{\ln\left(\frac{V_A}{X}\right)}{\frac{\sigma_E \cdot V_E}{V_A}}$$

Byström argues that since it is the book value (not the market value) of debt that needs to be paid back at maturity, the third assumption is plausible. Theory suggest that when calculating the total value of the firm, market values should be used. Since only equity has a quoted market value and not debt, an error is generated when adding the value of equity to the book value of debt.

The leverage ratio can be defined by

$$L = \frac{X}{V_A}$$

By using a debt-equity ratio, Byström (2006) claims that the model actually gain strength by not relaxing the assumption from the original Merton model, that the debt is constant over time. Empirical studies suggest that companies adjust their debt according to the value of the company and keep the leverage ratio rather constant, see for example March (1982) and Auerbach (1985). After the third assumption the distance to default in the simplified Merton model finally looks like

$$\mathrm{DD}_{Modified} = \frac{\ln\left(\frac{1}{L}\right)}{\sigma_{E}(1-L)} = \frac{\ln(L)}{(L-1)} \cdot \frac{1}{\sigma_{E}}$$

One more assumption is made in the modified model. The book value of debt can be used to calculate the leverage ratio

$$L = \frac{X}{V_E + X}$$

After these assumptions the distance to default can be calculated by only including observables, all needed is the book value of debt, equity value and equity volatility.

#### PART III. EMPERICAL STUDY

The idea with the study is to compare two different types of credit risk measures. The zscore model, based on ratios collected from companies' accounting data and the modified Merton model, where the value of the company and its volatility are gathered from the market. To be able to find the superior default predicting model, it is important to look not only on the accuracy of predicting failing firms but also the accuracy of predicting non-failing firms. We therefore include both bankrupt companies and companies' still operating in the sample.

#### 3.1 Data

A total of thirty-four firms have taken part in the study. Twenty-four of the companies are non-bankrupt companies. Ten of the thirty-four companies were chosen for the reason that they filed for bankruptcy<sup>5</sup> between the second half of 2007 and the first six months of 2009.<sup>6</sup> All companies included in the paper are or were publicly traded on an American stock exchange. The non-bankrupt companies have been randomly chosen with the restrictions of still being active and trading and have sufficient information available. The amount of companies included has been subject to time and availability of information (data) about firms. However, thirty-four analysed firms are a reasonable amount of firms and should give a clear view of the accuracy of the models<sup>7</sup>.

To be able to conduct the study a range of company data has been required. The accounting data has been collected from companies' 10-K reports<sup>8</sup>. A 10-K report is a thorough report of a company's performance that must be submitted annually to the Securities and Exchange Commission (SEC). Typically, the 10-K reports contain much more details than other annual reports. The reports have been chosen to represent the actual accounting statement one year prior to the bankruptcy and therefore the closest statement to the analysed data was used. Financial statements released during 2008 have been the source for the non-bankrupt companies. All the critical ratios included in the zscore model, except the market value of equity have been derived from these reports. Market value of equity has been calculated by multiplying the number of outstanding shares at that time with the share price. The information required for the z-score model was fairly straightforward and did not involve any major complications. The only assumption made was that Sales was taken from Total revenue in the report. The original z-score model was only tested on manufacturing firms and therefore sales were used as good measurement. However, in this study the industry has not been restricted to manufacturing firms and total revenue would be more appropriate measure and to a greater extent, more precisely represent the firm's ability to compete in the industry. The term working capital is the difference between Current assets and Current liabilities. The

 $<sup>^{5}</sup>$  The term bankruptcy is referring to companies filing for chapter 7 or 11 under United States bankruptcy laws.

<sup>&</sup>lt;sup>6</sup> The reason for the time period in which the sample is stretching is that many people hold the situation with the run on Northern Rock in September 2007 as the starting point of the crisis and a decision was made to consider it (or at least the worst part) to be over by the middle of 2009.

<sup>&</sup>lt;sup>7</sup> The statement is made based on similar studies carried out in the past.

<sup>&</sup>lt;sup>8</sup> 10-K reports are available at the U.S. Securities and Exchange Commission. http://www.sec.gov/

Book value of total debt used in the z-score model (denoted X in the modified Merton model) is the total liabilities the company has at certain time. The value of the z-score was computed following Altman's model from 1968.

### $Z = 0.012 X_1 + 0.014 X_2 + 0.033 X_3 + 0.006 X_4 + 0.999 X_5$

Historical share prices, gathered from DataStream and Yahoo Finance have been the sources behind finding the default probability in the modified Merton model. A time period of one-year prior to failure was used; consequently the share prices were collected 12 months prior to estimation. The data in modified Merton model needed a transformation into practical input before it could be used. Share prices multiplied with the total number of outstanding shares in the company at that specific time provided us with the market capitalisation, where market capitalisation is the value referred to as equity value. To be able to provide the equity volatility the logarithmic return  $(R_i)$  is needed.

$$R = \ln \left( \frac{Share \ price_{t+1}}{Share \ price_{t}} \right)$$

Equity volatility is then simply the standard deviations of the daily change of market capitalisation over a continuing period. To fit the model, the equity volatility should be represented annually; therefore we need to convert it from daily values to an annual value by multiplying it with the square root of the number of days in a year.

The number of days is assumed to be 250, since most stocks are traded during 250 days per year on average.

$$\sigma_{annual} = \sigma_{daily} \cdot \sqrt{250}$$

The leverage ratio is calculated by using the Book value of the debt and the Equity value

$$L = \frac{X}{V_E + X}$$

and we get Distance to default from

$$DD_{Modified} = \frac{\ln\left(\frac{1}{L}\right)}{\sigma_E(1-L)} = \frac{\ln(L)}{(L-1)} \cdot \frac{1}{\sigma_E}$$

A Standard Normal Distribution  $[N \sim (0,1)]$  is used to obtain the probability of default.

#### PART IV. TEST RESULTS

#### 4.1 Type I and Type II Errors

The models analysed in this paper can make two types of errors. They can either predict non-failure (ex ante) for a company that in reality defaults (ex post). This error is labelled a Type I error. A type I error is very costly for the lender, since a default results in the lender losing out on a large part or the whole amount of the claim. Financial institutions experiencing huge credit losses are partly what caused the recent financial instability. The other kind of error that is subject to inaccuracy in a credit risk model is a Type II error, predicting a default when in fact the company should have been ranked very unlikely to default, see figure 1. Unfortunately it is not possible to have type I and II errors equal to zero since there is a trade off between the two. One can argue that that the more vital of the two errors when evaluating a credit risk model is the type I error. The lenders would rather overestimate the probability that the borrower will default than underestimate it. Even so, there are costs of both types of errors. A high type II results in a lower (than optimal) risk exposure and results in less profit. The costs can be explained in loss of alternative income, loss in interest income and origination fees. Also to be required to sell at disadvantages times is a consequence Keenan et al (2000) gives as a possible cost of a high type II error.

Actual (ex post)

		Low credit quality	High credit quality
	Low credit quality	Correct prediction	Type II error
Model (ex ante)	High credit quality	Type I error	Correct prediction

Figure 1. Type I and Type II errors, based on a Keenan et al (2000) figure.

#### 4.1 Z-score

The computed values by the two analysed models are by completely different nature and therefore require different interpretations. The z-score model is quite clear in theory, distinguishing between failing and non-failing firms by certain thresholds. A score above 2.99 indicates a healthy firm, not experiencing any financial distress, while a score below 1.81 on the other hand is a sign of a troubled firm that will be filing for bankruptcy in the near future. Any score in between is treated as inconclusive and Altman's model is not able to predict the future of the firm. The results from the test of bankrupt companies can be seen in table 2 and the test for non-bankrupt companies in table 3.

	Bankrupt companies	Z-score	Modified Merton (PD)
1	Bally Total Fitness Holding Corp.	1.121	5.744%
2	Tweeter Home Entertainment Group, Inc.	3.268	8.460%
3	Circuit City Stores, Inc.	4.276	0.466%
4	Tribune Company	0.962	0.000%
5	Pope & Talbot	2.213	77.705%
6	Aventine Renewable Energy Holdings	3.640	4.802%
7	Finlay Enterprises Inc.	0.905	53.157%
8	Pilgrim's Pride Corp.	2.332	0.012%
9	Midway Games	3.973	0.536%
10	Fleetwood Enterprises	27.003	5.886%

**Table 2.** The z-score and PD results from test of the bankrupt companies listed.

When testing the thirty-four firms we found only twenty scores to be accurate. Particularly weak was the prediction of failing firms, where the model only classified three out of teen possible firms into correct categories. Five firms were predicted not to fail by the model, while in fact they did fail within a year from the date the estimation took place. Two landed in the grey area and should arguably be considered as correct, (or at least not incorrect). However, the fact that the z-score model is not able to describe the future of companies in the zone of ignorance is its major flaw and discussed widely among academics. A value in the grey zone is to no help for anyone using the z-score model to evaluate the credit risk. It can only provide the person with a warning: the firm

is not strong enough (according to the model) to be classified as a non-failing firm. The model was considerably stronger predicting non-failures. Seventeen out of twenty-four received a score above 2.99 and one ended up in the grey zone. We can therefore conclude the model to have committed more type I errors than type II errors since its ability to predict non-failing firms were more accurate that its ability to predict failing firms.

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	Non-bankrupt companies	Z-score	Modified Merton (PD)
1	Blue Coat System Inc.	1.790	0.098%
2	Inspire Pharmaceutical Inc.	-2.855	4.432%
3	Itron Inc.	1.660	2.079%
4	Deckers Outdoor Corp.	10.118	0.044%
5	Digi International Inc.	4.943	0.024%
6	Deswell Industries Inc.	12.080	0.000%
7	Friedman Industires Inc.	4.887	0.018%
8	National Coal Corp.	-0.392	40.084%
9	NeuroMetrix Inc.	-3.564	30.606%
10	Nobility Homes Inc.	20.685	0.002%
11	Napco Security Technologies Inc.	4.740	0.003%
12	Universal Forest Products Inc.	4.883	1.816%
13	Alkermes Inc.	4.682	0.000%
14	Cal-Maine Foods Inc.	6.407	0.527%
15	Casella Waste Systems Inc.	0.807	1.218%
16	Cherokee Inc.	17.293	0.000%
17	MKS Instruments Inc.	6.142	0.001%
18	Macy's Inc.	1.709	0.103%
19	Bitstream Inc.	7.259	0.000%
20	Buffalo Wild Wings Inc.	6.216	0.315%
21	GigaMedia Ltd.	3.072	2.464%
22	Hansen Natural Corp.	9.127	0.040%
23	Raven Industries Inc.	15.379	0.000%
24	Royal Gold Inc.	11.049	0.000%

 Table 3. The z-score and PD results from test of the non-bankrupt companies listed.

#### 4.2 Modified Merton

To express the size of type I and type II errors regarding the modified Merton model is more difficult than for the z-score model. The test of a firm using the modified Merton model results in a probability of default between 0% - 100%. It is more difficult to the PD only tells us how likely a default is. Let's say a firm receives a PD of 10%. It would indicate that one out of ten companies, all with PDs equal to 10% is expected to default within a year. The PD does not explain if the firm will or will not fail. It does only tell us that if we test enough companies, the number of the PD (e.g. ten) will fail in that particular subset of PDs. Therefore, we cannot determine the size of the type I and type II errors. Still, we can look at the accuracy of the modified Merton model by comparing the PDs of the bankrupt companies versus the non-bankrupt companies. Our results show that eleven of the twenty-four non-bankrupt firms received PDs lower than 0.01%, which would represent one in ten thousand firms are expected to go bankrupt. A total of sixteen firms have a default probability lower than 1% (i.e. less than one in a hundred firms will fail on average). Two non-failing firms have very high PDs: 30% and 40%. In these cases the model cannot be considered reliable. The model's test results of the bankrupt companies should ideally result in high default probability. In reality they show four out of ten companies producing a PD lower than 1%. However, the six remaining firms produce values above 4%, with two of them showing PDs above 53%. We can then express an instinctive reasoning over the size of the type I and type II errors. The bankrupt companies seem to have a majority of rather high PDs and a couple of them are very high. Thus there is a minority of low PDs and that would imply a low type I error. On the other hand, four out of ten values are below one percent, and that would imply a large type I error. The default probabilities regarding the non-bankrupt companies have a majority of low values, which is consistent with the theory of the model and therefore indicate a low type II error. But the non-bankruptcy list below also consists of a few high PDs, which would be the result of a large type II error. Remember this is merely an intuitive expression and by no means justified by a statistical test, but the plausible reasoning hopefully provides the reader with a satisfaction. It is also interesting to look at the average default probability the model draw out of the tested sample and it might help us to get a clearer view. Table 4 show the average defaults predicted by the model in both percentage and amount of firms. The modified Merton model predicted 2.4 firms to fail on average, while in fact 10.0 firms failed in reality. It is

obvious that the Merton model underestimated the default probability in the test. A possible explanation could be that the important assumption of normal distribution does not hold at turbulent times. The underestimation of risk would imply that the distribution has fatter tails than in the normal distribution and the model therefore does not recognise all the firms in distress.

	Percentage	Firms
Average defaults model	7.1%	2.4
Average defaults actual	29.4%	10.0

**Table 4.** Show the average PD predicted by the Merton model and the actual defaulting firms. The model underestimated the PD of the sample.

The results so far have not been able to give us an answer to which model has been superior in predicting failures over the past few years and the analysis to this point has been unsatisfactory. A different approach will now be taken, where we hopefully will be able to produce a clear answer to the introductorily problem.

#### PART V. COMPARING THE MODELS

Since the two analysed models produce different credit risk measures, evaluating and comparing the accuracy of the credit risk models can be difficult. As described above, the z-score generates a continuous number, where certain thresholds determine the supposed future of the company. According to Keenan and Sobehart (1999), the many frequently used diagnostics, like F-statistics and Akaike Information Criteria are not sufficient comparing default predicting and risk scoring models. The Merton framework on the other hand produces a probability of default and is explanatory to the degree that it gives you a probability of a company going bankrupt. To be able to conduct an objective and detailed evaluation and comparison of the models, specific measures needs to be taken. In this paper the Cumulative Accuracy Profile<sup>9</sup> is the particular method used to evaluate and compare the models.

<sup>&</sup>lt;sup>9</sup> Also referred to as Gini Curves or Gini Coefficients.

#### 5.1 Cumulative Accuracy Profile (CAP)

CAP is a commonly used technique when measuring the discriminating power between default predicting models. It does so by give a graphical illustration of the model's ability to distinguish between failing and non-failing firms. Another feature of CAP is that it can evaluate and compare the predicting ability of quantitative measures or qualitative grading systems to each other. CAP curves do also uncover information about model's prediction power over its entire range of risk scores over a particular time period. By ordering the analysed firms from riskiest to safest the CAP curve can be plotted. Keenan et al (2000) explains that for a given fraction x% of the total number of companies ordered by estimated risk, the type I curve can be created by calculating the percentage y(x) of the defaulters whose risk score is equal to or lower than the one for fraction x. It represents the cumulative fraction of default events for different percentiles of the rating scale. Similarly, the CAP curve for type II errors represent its compliment and is constructed by using the z(x) of non-defaulters. The perfect credit-scoring model (capturing 100% of the defaulting firms and 0% of the non-defaulting firms) would assign the lowest scores (for the z-score model) and the highest PD (for the Merton model) to the defaulting firms and the CAP curve would increase linearly until it reaches one and then stay constant at one. For an uninformative model with very poor discriminating power (i.e. randomly distinguishing between defaulting and non-defaulting firms), the fraction of x of all borrowers with the lowest rating will contain x percent of all defaulters and CAP would graphically appear as a straight 45-degree line. A superior model also produce a low type II error by generate low risk levels for the surviving firms. The CAP curve for this model would be slowly increasing in the beginning. A model, which CAP curve is the closest to the ideal model's CAP curve is the most accurate default predicting curve.

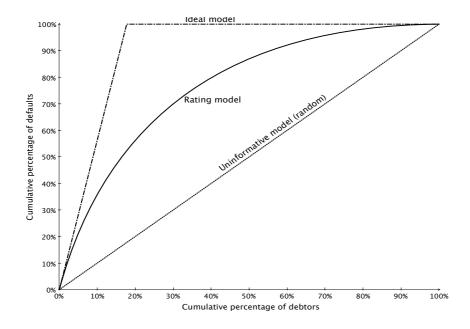
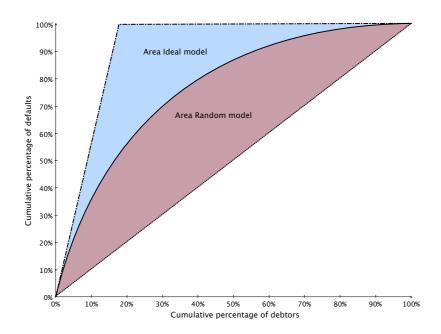


Figure 2. Cumulative accuracy ratio curves. The figure is based on Van der Burght (2008)

## 5.2 Accuracy Ratio (AR)

A way to measure the quality of the rating models is by using the accuracy ratio. The accuracy ratio is closely related to CAP as it uses the rating model's CAP curve to derive its value. It is the ratio of the area between the uninformative model (random) and rating model, and the area between the ideal model and the uninformative model. The AR summarises the predicting ability of type I and type II errors by allocating a number between zero and one, where a number closer to one is superior to a number close to zero. AR is evaluating which model is the closest to the ideal model's curve, in other words, which model is the best to discriminate between failing and non-failing firms. By comparing the accuracy ratio of the models being analysed, the answer to which model has been more accurate in predicting defaults should be easily observable.

 $AR = \frac{\text{Area Random Model}}{\text{Area Ideal Model} + \text{Area Random Model}}$ 



**Figure 3.** Accuracy ratio is the ratio between the area of the model's CAP curve and the random curve, and the area between the ideal model and the random model. The figure is based on Keenan et al (2000).

The AR function can also be defined as

$$AR = \frac{2\int_{0}^{1} y(x)dx - 1}{1 - f}$$

where y(x) is the CAP profile function and *f* is the actual observed fraction of defaults. Since the area in the CAP framework are restricted by the model's CAP curve and the CAP curve of a random model, y(x) = x, the numerator is 2. The denominator is twice the area enclosed from below by the same boundary, and from above by the same ideal CAP curves.

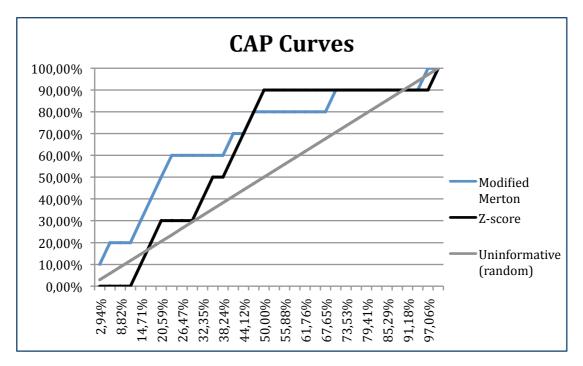
## 5.3 Computing the CAP and AR (empirically)

To be able to draw the rating curves, the companies have to be listed by riskiness, from riskiest to safest. In the modified Merton's case that means that the highest PD and for the z-score, the lower score will be listed at the top, as the riskiest. We then list the fraction of defaults of each of those PDs and z-scores. The two lists combined create the

CAP curve for each model. Recall that the AR is the fraction of the area between the model's CAP curve and the uninformative curve, and the difference between the ideal curve and the uninformative curve. Therefore, both the ideal curve and the uninformative curve needs to be constructed. We draw the random curve with a 45-degree angle, according to theory. The ideal curve is computed by matching the correct amount of failures with actual amount of failures at the specific subset. We can calculate the difference between the models' and the uninformative curve at each of the subsets and then sum up the differences to obtain the area for each model. We conduct the same procedure to find the area between the ideal curve and the uninformative curve. By dividing the area of the rating model and uninformative curve with the area of the ideal model and uninformative curve we find the accuracy ratio.

#### 5.4 Results

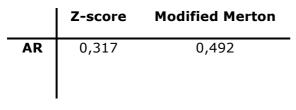
Figure 4 shows the CAP curves for the modified Merton model and the z-score model. We can see that both models give a better prediction of firms in financial distress than a random model, which was expected since both types of models have been exercised frequently in the past. The model that is furthest away from the uninformative model is the better credit risk model.



**Figure 4.** The CAP curves of the z-score and modified Merton model visualised. The modified Merton's curve seems to be farther away from the Uninformative curve, which would imply that the modified Merton model has performed better.

Unfortunately, the graph does not present an obvious answer to this. However, it does seem like the modified Merton model's gap is greater than the gap between the z-score and the random curve and would therefore imply that the modified Merton model has been performing better.

To clarify our suspicion we look at the result from our accuracy ratio calculations. Table 5 show that the z-score produced an AR of 0.317 and the AR for evaluating the modified Merton model turned out to be 0.492 percent. Previous studies suggests that the accuracy ration in our tests are low and indicates that the models are not very powerful in predicting defaults on our sample of firms. For example do Keenan et al (2000) find higher accuracy ratios for both the z-score model and a version of the Merton model, 0.48 for the z-score and 0.67 for the Merton model. Keenan and Sobehart (1999) discover AR equal to 0.56 for the z-score model and 0.67 for the version they use as the Merton model. Why their studies have resulted in overall higher ARs is difficult to determine. One reason can be the turbulent time and that it is more difficult to predict corporate default in financial turmoil.



**Table 5.** The result from the Accuracy ratio tests. The modified Merton model has a higher AR than the z-score model and can be distinguished as the better model in this test.

Since theory suggests that the closer the AR is to one, the better the model, we can conclude the modified Merton model to have performed better during the past few years of financial tension, represented by our sample firms. Our result of a stronger predicting modified Merton model should establish market information to be more accurate during a time of high volatility on stock markets. A possible reason can be the fast changes in share prices representing the current condition of the company and its expected future. Consistent with earlier arguments made in PART I and PART II.

Our test shows the modified Merton model to be more accurate when predicting default for the sample firms. This is coherent with Mensah (1984) and his suggestion of redeveloping the accounting based model depending on the sample and circumstances. This is another major flaw with the z-score model; by always having to amend an existing model weakens the need of the model. To consider if it is economically justified to spend time and money to improve a model for it to fit the sample is what should be behind the decision. A model that is created and based on a sample instead of being created to fit all samples looses some of its cause and therefore the demand for its results. However, the z-score model has a strong advantage, the fact that it is easy to interpret in comparison to the Merton model. A firm's z-score reveals the future of that particular firm, while the PD derived from the Merton model only show an average bankruptcy value. Together with the earlier revealed Achilles' heel of the z-score model; the inability to classify firms in the zone of ignorance, these are the two most important reasons for choosing a different default predicting model. As another vital motivation for using the modified Merton model instead of the z-score model, is the result of the accuracy ratio test. It is easy to see that the Merton model clearly performed better (see table 3) than the z-score model.

Factors like time and effort should, since they are costly, be evaluated before choosing default prediction models. In general, we are interested in the future of a firm. It can therefore be a substantial difference of the workload required to collect data for the accounting based model. Normally the market data is easier to compile since the financial reports needed for the z-score model might be unavailable at first. When back testing to find the accuracy of models (as we have in this study), data gathering have required rather similar amount of time and labour for both models. However, we propose that the two factors are not overlooked when using the models as a predictive tool. The easy way to gather appropriate data for the Merton model, gives the user a possibility to do immediate test updates, while someone using the z-score needs to wait for the next financial statement to be released.

#### PART VI. CONCLUSION

Throughout paper the tests have been carried out with the objective to distinguish the default predicting accuracy between market based and accounting based models. Two particular models were selected to represent one category each. The Altman (1968) z-score model denoted the model using information collected from companies' financial statements and a modified Merton model created by Byström (2006), which is a simplified version of Merton (1974), represented a market based approach. The important result derived from the analysis is that both models do predict corporate bankruptcies better than a random model. The test gave an AR of 0.317 for the z-score and 0.420 for the modified Merton model. These are lower than previous test done on similar accounting and market based models. This inconsistency, regarding the Merton model can be due to the fact that the modified Merton model is not the same version as the ones used in other research. The fact that the modified Merton model underestimates the default probabilities is most likely due to its assumption of normality does not hold in times when economies suffer from major turbulence.

The reason for the z-score model achieve a worse AR than tests carried out in the past could be related to Mensah (1984) reasoning about the need to modify the z-score model to appropriately suit the sample and circumstances around the test. Our sample might be even further away from the orignal sample Altman used in 1968. In this particular case, a

version of the z-score, specifically modified to fit companies during a financial crisis would probably prove to be more accurate and would make for interesting future research. It would be fascinating to find out if a model, accounting based or market based that could be adapted to suit these circumstances. Another reason for the weak z-score predication could be the fact that Altman used only manufacturing, medium sized firms with assets in the range \$1 - \$25 million and we have in this paper randomly selected firms of different sizes. Nevertheless, our test recognized the modified Merton model to be more accurate than the original z-score model and should therefore preferable be used in an economy with highly volatile and uncertain financial markets.

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#### Appendix A.

Shows the input variables used when computing the PD in the modified Merton model and the ratios used when calculating the z-score. Where Market cap is the Equity value and Vola. Market cap is the Equity volatility. DD is the Distance to default. X is the Book value of debt and L is the leverage ratio. X1 – X5 are the ratios used in the z-score model. The highlighted columns are the final PDs (in decimals) and the final z-score values. See PART II and PART III for more details.

	Non-Bankrupt Companies	Market cap	Vola. market cap	×	_	QQ	D	X1	X2	X3	X4	X5	Z-score
	1 Blue Coat System Inc.	810175900	0,686651	136310000	0,144017	3,296956	0,000977	0,318642	-2,281945	0,078732	5,926782 (	0,787683	1,790429
	2 Inspire Pharmaceutical Inc.	204012000	0,912038	69837000	0,255020	2,011057	0,044319	0,459728	-3,155817	-0,411621	2,921260	0,617191	-2,855489
. '	3 Itron Inc.	2198392600	0,621791	1,836E+09	0,460456	2,311702	0,020794	0,102151	0,017516	0,039289	1,197497	0,665089	1,659679
1	4 Deckers Outdoor Corp.	1045498300	0,761767	99469000	0,086875	3,512546	0,000444	0,656897	0,555103	0,249115	10,510795	1,425295	10,117846
-,	5 Digi International Inc.	254307600	0,631283	39482000	0,134389	3,672858	0,000240	0,413517	0,289684	0,070331	5,213307	0,681817	4,942990
2	6 Deswell Industries Inc.	312000000	0,471218	19150000	0,057829	6,419989	0,000000	0,389945	0,426553	0,065460	16,292428	1,024208	12,079766
	7 Friedman Industires Inc.	34884000	0,412861	22002000	0,386774	3,751983	0,000175	0,517324	0,200932	0,101347	1,638033	2,670107	4,886796
~	8 National Coal Corp.	43408600	1,355751	142318000	0,766277	0,840128	0,400837	-0,040857	-0,739360	-0,117286	0,305011	0,897435	-0,391632
	9 NeuroMetrix Inc.	12058200	1,469164	8546000	0,414770	1,023531	0,306057	0,689400	-2,861750	-0,673370	1,410976	0,991810	-3,563887
Ť	10 Nobility Homes Inc.	51943000	0,826758	1736000	0,032340	4,289189	0,000018	0,470475	0,929915	0,060579	29,921083	0,666172	20,684518
H	11 Napco Security Technologies Inc.	86477700	0,469093	23181000	0,211392	4,200898	0,000027	0,538209	0,593460	0,040783	3,730542 (	0,891089	4,739801
Ħ	12 Universal Forest Products Inc.	472668400	0,670287	274136000	0,367079	2,362298	0,018162	0,282234	0,481989	0,023571	1,763904	2,735713	4,882568
H	13 Alkermes Inc.	1134302400	0,381114	350997000	0,236314	4,956487	0,000001	1,750132	-1,300772	0,539036	3,231658	0,685809	4,682015
Ĥ	14 Cal-Maine Foods Inc.	740064000	0,680077	225556000	0,233587	2,789996	0,005271	0,242501	0,533912	0,477027	3,281065	1,827361	6,406839
Ĥ	15 Casella Waste Systems Inc.	271297000	0,466794	711405000	0,723927	2,506920	0,012179	-0,020738	-0,178443	0,048545	0,381354 (	0,693130	0,806742
ī	16 Cherokee Inc.	293284900	0,381082	13257000	0,043247	8,614419	0,000000	0,425073	0,310681	0,637446	22,123022	0,971635	17,293089
H	17 MKS Instruments Inc.	728851200	0,553639	98241000	0,118779	4,366853	0,000013	0,459717	0,245120	0,042266	7,419012	0,656887	6,141942
ĩ	18 Macy's Inc.	1,1763E+10	0,388065	1,788E+10	0,603209	3,282822	0,001028	0,034690	0,072259	0,068336	0,657801	0,946885	1,708920
Ť	19 Bitstream Inc.	44109900	0,501562	4409000	0,090872	5,259621	0,000000	0,716396	-0,870610	0,124416	10,004513	1,206372	7,259268
7	20 Buffalo Wild Wings Inc.	409681000	0,756123	72225000	0,149874	2,952649	0,003151	0,083530	0,349749	0,149144	5,672288	1,732509	6,216209
2	21 GigaMedia Ltd.	306103100	0,858108	88337000	0,223955	2,246944	0,024644	0,166026	-0,143008	0,119179	3,465174 (	0,600926	3,071739
22	2 Hansen Natural Corp.	3028764900	0,729317	325521000	0,097046	3,542031	0,000397	0,490906	0,606009	0,227709	9,304361	1,356957	9,127155
2	23 Raven Industries Inc.	540660200	0,458634	29586000	0,051883	6,804279	0,000000	0,532669	0,894211	0,285565	18,274190	1,582277	15,378674
Ň	24 Royal Gold Inc.	1048097700	0,429666	62633000	0,056389	7,092292	0,000000	0,370144	0,035684	0,072775	16,988565	0,121456	11,048760
	Bankrupt Companies	Market cap	Vola. market cap	×	-	00	Qd	X1	X2	ХЗ	X4	X5	<mark>Z-score</mark>
	1 Bally Total Fitness Holding Corp.	179880572	0,551825	1,797E+09	0,909017	1,899975	0,057436	-2,226637	0,108544	0,287607	0,042240	2,669174	1,120948
	2 Tweeter Home Entertainment Group, Inc.	167027210	0,781460	190417000	0,532718	1,724612	0,084597	0,152177	-0,092996	-0,044187	0,610420	2,998248	3,268101
. /	3 Circuit City Stores, Inc.	1156062600	0,432725	2,216E+09	0,657169	2,829861	0,004657	0,222650	0,261914	0,094075	0,332788	3,135054	4,275898
۷	4 Tribune Company	3866086948	0,190942	1,666E+10	0,811684	5,802549	0,000000	-0,061214	0,264212	0,047054	0,208880	0,385026	0,961686
-1	5 Pope & Talbot	41120	3,531606	541594000	0,999924	0,283168	0,777048	0,235401	0,068456	0,141381	0,165288	1,270566	2,213344
2	6 Aventine Renewable Energy Holdings	226645182	0,623136	418314000	0,648590	1,977175	0,048022	0,398036	0,085196	0,065026	1,280268	2,061976	3,639579
	7 Finlay Enterprises Inc.	23052500	1,630869	560710000	0,960510	0,625605	0,531574	0,292844	-0,108694	-0,188442	0,000417	1,328870	0,905175
۵ د	8 Pilgrim's Pride Corp.	2002554400	0,340798	2,602E+09	0,565095	3,850911	0,000118	0,104885	0,182230	-0,278963	1,024573	2,258763	2,331653
	9 Midway Games	640095000	0,692922	184280000	0,223539	2,784561	0,005360	0,323944	2,251930	-0,342982	1,379401	0,736756	3,973253
Ä	10 Fleetwood Enterprises	272229500	0,645036	539303000	0,664025	1,889271	0,058855	0,172156	-0,761236	0,035635	41,822983	2,653544	27,003132