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MASTER ESSAY

Can the use of leverage adjustment techniques give reliable estimates of beta risk?

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Abstract

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Key words: financial leverage, unlevered beta, Hamada's leverage adjustment

Purpose: Using the sample of American companies within the five industries for the period 1994-2009 the following hypothesis shall be tested: does the use of leverage adjustment techniques over-penalize the cost of equity for a high level of debt to equity ratio.

Methodology: To analyze how well the leverage adjustment works under the assumption of constant risk classes, the cross-sectional linear regression is used. With the purpose of relaxing the assumption of constant business risk within the industry to a constant business risk for a given firm, the time series methodology is employed.

Conclusions: In line with the previous studies it is found that the leverage adjustment over-penalizes the beta risk for the high level of debt to equity ratio. The biggest difference between the theoretically implied betas and their empirical counterparts is observed for the cases with high leverage estimated by the model, which does not account for the tax savings from debt.Thus, one should be aware of the problems connected with the over-penalization of beta risk while dealing with the industries which a) include large number of the companies with different operational characteristics, and b) have a high level of debt to equity ratio. Therefore, I can suggest that it is possible to use leverage adjustment of beta, which accounts for tax savings by only averaging the unlevered beta for a small number of companies within homogeneous activities and low leverage level.

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1. Introduction

1.1 Background and Specification of the Problem

The process of estimating a project's cost of capital and the cost of equity in particular is heavily based on the Capital Asset Pricing Model (CAPM), despite the fact that the use of CAPM itself is questioned due to the following reasons. Starting from the most famous Fama and French (1992, 1993) many studies do not support CAPM empirically. For instance Iqbal and Brooks (2007) argue that the relationship between risk and return is not found to be linear, Spyrou and Kassimatis (2009) find that the beta of individual securities is not stationary over time. Moreover, if historical information on realized financial returns is not available, the use of beta as a measure of systematic risk is not feasible. The weaknesses of CAPM have led to considerable amount of beta decomposition research, which potentially could improve the estimation of beta. In particular, to overcome the problem of absence of historical information on the company's returns (e.g. the company is not publically traded), rather than simply averaging the betas from peer companies, Hamada's (1969,1972)/ Bowman's (1980) leverage adjustment technique[1] is used. In addition, this adjustment is sometimes used for the traded companies to reduce statistical noise in estimating individual company's beta, hence increase its precision. Thus to calculate the systematic risk for the particular security, the average industrial unlevered beta and the debt to equity ratios of the firms are used.

This procedure is based on the theory that estimates of equity beta consist of business (operating) risk and financial risk, and the firms in one industry face the same operating risk. However, if the constant risk assumption does not hold in practice, then using the average industry betas and financial leverage of a company to calculate the systematic risk of equity can lead to unreliable estimates of the cost of equity, hence the cost of capital. For instance, it can over-penalize the cost of capital for financial leverage.

In general, the use of Hamada's technique is questioned not only because of the constant operating risk assumption, but as mentioned by Paulo (2010) "Hamada's equation is subject to a number of non-trivial deficiencies, each of which is of sufficient importance to nullify its intended purpose, render impossible its function, and epistemologically contradict its functioning" (Paulo, 2010, p.61). Such strong criticism of Paulo (2010) is connected with the fact that the equation is based on

[1] Further in text I will call it just Hamada's technique

the empirically invalid CAPM, which derived on a set of strong assumptions. In addition, to the restrictions of CAPM, Hamada based the derivation of his equation on supposing a market with perfect competition. Thus, if one considers the impact of all these theoretical hypotheses on commercial activity, then there would be no market since there would be no need to transact when information is fully available to all, there would be no transaction costs, no taxes. Borrowing and lending are realized on the same risk-free rate, all transactions are made at the same point of time, everyone has the same expectations and identical estimations of risk and return. Hamada himself warns that the equation developed by him is not operational, in particular he mentions that “a word of caution is necessary in conclusion. We opened the analytical part of this paper with an enumeration of the assumptions. The results presented here are conditional on these assumptions not grossly violating reality” (Hamada, 1969, p. 30). Nevertheless, today this technique is widely used in practice to show the impact of leverage on the cost of capital and value of the firm. Some studies, in particular Martson and Perry (1996), Faff et al (2002) have demonstrated that one has to be careful when applying the traditional approach for delivering beta risk because it tends to over-penalize beta, especially when a high level of financial leverage is employed. The data examined by them is dated at latest to 1994, but since this technique has not become less popular today, I find it important to revisit this topic for the most recent time period.

1.2 *Hypothesis and Purpose*

The initial hypothesis which I make in this master essay is that the use of leverage adjustment techniques over-penalizes the cost of equity for a high level debt to equity ratio. Consequently, the purpose of the study is to test this hypothesis using the sample of American companies in five industries for the period of 1994-2009.

1.3 *Outline of the Paper*

The paper is structured as the following: section one introduces the reader to the background of the problem and the purpose of the essay. In section two a review of previous theoretical and empirical research in the field is presented. Section three sets forth the description of the data and methodology while empirical results are presented in the fourth section. Section five makes final conclusions and outlines further research in the field.

2. Review of Research in the Field

2.1. THEORETICAL FRAMEWORK

2.1.1. The Concept of Systematic and Unsystematic risk

The total risk of firm's equity can be subdivided into two components: systematic risk, which is the measure of how an asset covariates with the economy, and unsystematic risk, the risk independent of risk in the economy as a whole. As the latter can be diversified through portfolio formation, financial research is mostly focused on systematic, undiversifiable risk. As described, for example, in Copeland et al (2005), the return on any asset is a linear function of market return and a random error term ε_j, which is independent of the market:

$$R_J = a_j + b_j R_m + \varepsilon_j \qquad (2.1)$$

where R_J is the return on asset *j*, a_j - constant term, $b_j R_m$ - constant times a random variable (market return) and ε_j is a random variable, which has zero covariance with R_m. The variance of this relationship can be written as

$$\sigma_j^2 = b_j^2 \sigma_m^2 + \sigma_\varepsilon^2 \qquad (2.2)$$

σ_j^2 represents the total risk, which is portioned into systematic risk, $b_j^2 \sigma_m^2$, and unsystematic risk, σ_ε^2. And the slope coefficient b_j is equal:

$$b_j = COV\ (R_j, R_m)/VAR(R_m) \qquad (2.3)$$

This is equivalent to the beta of the capital asset pricing model, which shows that the rate of return on risky assets is a function of its covariance with the market portfolio. And beta today is the most popular measure of systematic risk of equities. At the same time as it was mentioned above, starting from the well-known Fama and French (1992), there are plenty of discussions, which question the use of CAPM for the estimation of cost of equity. Nevertheless, as long as an equally simple and intuitive alternative for the estimation of cost of equity is not proposed in modern financial theory, CAPM still remains popular in many applications together with the numerous adjustments to it. In particular, the estimation of the cost of capital is closely connected with the leverage adjustment technique, based on the work of Hamada (1969, 1972) and Bowman (1980), which relates the debt to equity ratio and beta. The details of this adjustment are presented below.

2.1.2. Financial Leverage and Equity Required Returns

The theoretical foundation of the relationship between financial leverage and required return is based on the work of Modigliani and Miller (1958,1963). Based on a set of assumptions, including the "equivalent risk" classes, which is similar to the concept of industry, they show that required returns of equity linearly increase with the firm's debt to equity ratio. According to the Proposition II of Modigliani and Miller, the rate of return R of any company j within the kth class looks as follows:

$$R_j = \rho_k + (\rho_k - r)\frac{D_j}{S_j} \qquad (2.4)$$

Meaning that "the expected yield of a share of stock is equal to the appropriate capitalization rate ρ_k for a pure equity stream in the class, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between ρ_k and *r*" [where *r* is a risk free rate]. (Modigliani and Miller (1958), p.271)

Hamada (1969) first links the corporate finance issue described above and the portfolio theoretical framework through the effect of firms' leverage on the systematic risk of their common stocks. He argues that both the Modigliani and Miller proposition and CAPM state that borrowing from any source, while maintaining a fixed amount of equity, increases the risk to the investor, hence "in the mean-standard deviation version of the capital asset pricing model, the covariance of the asset's rate of return with the market's portfolio rate of return (which measures the nondiversifiable risk of the asset – the proxy β will be used to measure this) should be greater for the stock of a firm with a higher debt-equity ratio than for the stock of another firm in the same risk-class with a low debt-equity ratio." (Hamada (1972), p. 435). Assuming that the perfect competition and that the Modigliani and Miller proposition holds from the outset, the differences between the observed systematic risk, B^β and the non-leveraged systematic risk measure A^β are only due to leverage. He derives the following relationship between the observed systematic risk, B^β and the adjusted rate of return time-series A^β :

$$A^\beta = \left(\frac{S_B}{S_A}\right)_{t-1} B^\beta \qquad (2.5)$$

where S_B is the market value of the common stock and S_A is the market value of the firm, which has no debt and preferred stock. Since most of the firms have either debt or preferred stock or both of them, S_{At-1} is not observable directly and Hamada proposes to estimate it using the Modigliani and Miller theory, in particular:

$$S_{At-1} = (V - \tau D)_{t-1} \qquad (2.6)$$

That is, if the tax subsidy for financing debt D (market value of debt) is subtracted from the observed value of the firm V_{t-1} (which is the sum of S_B, D and the observed market value of the preferred stocks). S_B is estimated with an OLS regression between the stock's and market's portfolio historical rates of return.

Using Hamada's (1969) work and implying the paradigm of unlevered firm U (firm without a debt in capital structure), which then issues debt, reduces its common equity and becomes the levered firm L, Bowman (1980) derives the relationship, which now is the most commonly used in financial textbooks:

$$\beta_L = \left(1 + \frac{D_L}{S_L}\right) \beta_U \qquad (2.7)$$

where D_L, S_L are the market values of debt and equity respectively and systematic risk of the levered firm (β_L) equals systematic risk of the unlevered firm, β_U, times one plus debt to equity ratio. If one assumed that Modigliani and Miller (1963) tax subsidy mentioned in (2.6) is correct, the relationship in (2.7) will look as follows:

$$\beta_L = \beta_U + \beta_U (1 - T) \frac{D_L}{S_L} \qquad (2.8)$$

where T is a corporate tax rate. To proxy the corporate tax rate in empirical analysis, one can use a statutory income tax, marginal or effective tax rates. However, as Armitage (2005) notes, a corporation tax rate that a firm faces should be thought as an effective tax rate rather than a statutory tax rate. The effective tax rate is the actual tax rate, which takes into account all other types of taxes paid by the firm. Hence, differences in the tax-related expenses across industries and across types of companies can create differences between the effective tax rate and a fixed statutory tax rate. Moreover, if a company for example faces a loss during the reporting period and pays no tax during that period, then it offsets the loss against the profits in the subsequent periods, thus using the statutory tax rate for the analysis in these periods will give the biased estimate of the tax-related expenses, hence can give inappropriate estimates of the cost of equity. It is also worth noticing that in addition to the corporate tax, the personal taxes of investors can also influence the cost of equity and the cost of capital. "Personal taxes affect the cash flows that providers of capital receive from the projects, and project cash flows are always measured before personal tax. Therefore the cost of capital is always expressed before personal tax, and personal taxes affect the cost of capital" (ibid, p.181). However, it is hard to find a single, market-wide rate of personal tax that applies to any financial asset. And this complication often leaves the personal tax rate behind the estimation of the cost of capital.

As first pointed by Modigliani and Miller (1963) one effect of taxes is to make the risk of levered equity less than it would be without taxes. The reason of the risk reduction is that there are tax savings arising from interest expenses, and these savings are often assumed to be free of risk. However, the assumption of risk-free savings from taxes is not easy to justify and it is often made just for simplification. Because it can be uncertain as the effective rate of corporation tax can change over time, the project's level of leverage can also be changed in the future and if the company reduces the percentages of debt financing, then the amount of tax savings also decreases. Conine (1980) shows that allowing for the risky debt, the relationship between the beta and financial leverage becomes:

$$\beta_L = \beta_U + (\beta_U - \beta_D)(1-T)\frac{D_L}{S_L} \qquad (2.9)$$

where β_D is beta on the risky debt. And the equation (2.9) implies that if the debt is risky, then part of financial risk is borne by the lenders, hence part of the risk which bear the shareholders decreases and the relationship between the equity beta and leverage is less steep. To summarize, three types of leverage adjustments can be specified: a) the leverage adjustment without taxes and risky debt (equation 2.7); b) leverage adjustment with taxes (equation 2.8) and c) leverage adjustment with taxes and risky debt (equation 2.9).

Today all three types of leverage adjustments are widely used in academic and practical applications. Nevertheless, as it was mentioned in the introduction, they are subject to a severe criticism because:

1. They are based on the empirically invalid CAPM.
2. Hamada's equation is derived from the assumption of perfect competition, but in practice the market is ineffective and not perfectly competitive in most cases.
3. The model is based on the assumption of constant operating risk within the risk-class, but it is very hard to proxy the latter because companies gathered under modern industrial classifications (e.g. SIC codes in the USA, SEIC codes in UK etc) often perform very diverse activities, finance their expenses from different sources. Hence they could face quite different business risk.

The extensive empirical research is made to test if the use of leverage adjustment techniques can reflect the real relationship between the cost of capital and financial leverage. Some of these studies are described in the following subsection.

2.2 EMPIRICAL STUDIES

The empirical research, evaluating the validity of leverage adjustment techniques can be divided into two groups: 1) those that are directly testing the equivalent-risk class hypothesis and 2) those that analyze the impact of financial leverage on beta risk. In this subsection, I consequently present the review of the studies from both groups.

2.2.1 Review of the Studies Directly Testing the Equivalent-Risk Class Hypothesis

One of the first attempts to test the equivalent- risk class hypothesis is made by Gonedes (1969). He examines a random sample of ten U.S. firms from each of eight randomly selected industries. To proxy the business risk, the author employs "the relative deviation of firm's annual rate of growth in net operating income from the given firm' s compound rate of growth in annual net operating income with respect to the period 1958-1967." (Gonedes ,1969, p.164). To test the null hypothesis that there is no significant difference between the companies within an industry he implements the Kruskal-Wallis (KW) nonparametric test, which allows to identify if k independent samples are drawn from the same population. He finds that only two of eight industries are consistent with the null hypothesis of homogeneity, hence a selection of firms from a single industry does not ensure the assumption of constant business risk holds and it is not an appropriate procedure for examining the relationship between the cost of capital and financial risk. Sudarsanam and Taffler (1985) examine the homogeneity of Stock Exchange Industrial Classifications (SEIC) in terms of their fundamental economic characteristics. They use multiple discriminant analysis (MDA) to analyze 18 financial ratios relating to economic, financial and trade structure of the industry (among others, economies of scale, degree of mechanization). Their analysis shows that there are considerable differences among 14 examined SEIC industries (in total the sample included 263 companies for the period 1973-1977), but several of them are lack of homogeneity with respect to their economic and structural characteristics. Hence, Sudarsanam and Taffler (1985) suggest that a higher level of aggregation than SEIC can be more appropriate. Another interesting paper related to the topic is written by Lord (1996). He empirically investigates a theoretical model relating operational characteristics (including the degree of operational and financial leverage) of the firm to the total, systematic and unsystematic risk of equity. Lord (1996) examines a pooled cross-section of 35 American companies in the automotive, electric utility and airline industries for the period of 1963-1988 using seemingly unrelated regression to relate four independent variables. His main findings are that the degree of financial leverage is positively correlated with total and unsystematic risk, but not with systematic risk. No evidence is found of any interaction between the degree of financial and operating leverage.

2.2.2. Review of the Studies Analyzing the Impact of Financial Leverage on Beta Risk

Martson and Perry (1996) examine the relationship between beta and financial leverage for two- and four-digit industry codes for the period of 1974-1988, dividing it into three subperiods. They initially estimate beta from a regression of returns on equal-weighted and value-weighted market indices (using individual monthly returns and the corresponding return on the equal-weighted index of NYSE and AMEX securities). For the debt to equity ratio the five-years average is used, more specifically they employ two ratios: the book value of debt to market value of equity *(D/E$_M$)* and the book value of debt to total book equity *(D/E$_B$)*. However, they only report the results for the former because they are identical. Then using average industry betas (β_{ui}) estimated from time-series regression and five –years average of *D/E$_M$*, the leveraged adjusted beta (β_{ei}) for each of the industry *i* is calculated:

$$\beta_{ei} = \beta_{ui} + \beta_{ui}\frac{D}{E} \qquad (2.10)$$

Then the results of calculations from (2.10) are compared to the results of the OLS regression of equation (2.11), where *j* denotes the firm and *i* denotes the industry:

$$\widehat{\beta}\, e_{ji} = \gamma_0 + \gamma_1\frac{D}{E_j} + \widehat{\varepsilon_{ji}} \qquad (2.11)$$

The version of the model accounting for the corporate tax rate and risky debt is also analyzed. Consequently, authors conclude that two-digit SIC codes industry classifications should be avoided in practical and academic applications of this model. For the four-digit SIC level a much stronger relationship between financial leverage and beta is demonstrated, but in most cases “the penalty for financial leverage continues to fall below that posited by both the no tax and corporate tax versions of the Hamada/Bowman models.” (Martson and Perry (1996), p.93). In addition, they test a constant penalty for financial leverage across all levels of *D/E* against the dual penalty system as advocated by tradition theory. Traditionalists argue that the market does not require higher return for more highly levered firms until some critical level is achieved. Thus using switching regression Martson and Perry (1996), analyze the high and low leverage regimes to test if the relationship between required return and financial leverage becomes steeper after an unspecified critical leverage point. The authors do not find the conclusive support for either Hamada/Bowman or the traditional position, but they doubt any theory based on constant risk classes.

A time-series approach for the investigation of the relationship between financial leverage and beta is employed by Faff et al (2002). According to the authors, this approach is superior to the cross-sectional approach employed by Martson and Perry (1996) because it allows to avoid the strong assumption of constant systematic business risk within an industry. Instead, only the assumption of constant operating risk for a given firm is made. Moreover, the time-series approach allows for time variation in the *D/E* ratio which causes time variation in beta risk and total risk. Faff et al (2002) using the sample of 348 U.S. stocks over the period from 1979 to 1994, substituting the time-series version of the Hamada/Bowman equation into standard market model resulting in the following restricted model (the notation below is similar to the notation introduced earlier in this essay):

$$R_{jt} = \alpha_j + \beta_{uj} R_{mt} + \beta_{uj} \left(\frac{D}{E_{jt}}\right) R_{mt} + \varepsilon_{jt} \qquad (2.12)$$

which then compared to the unrestricted model:

$$R_{jt} = \alpha_j + b_j R_{mt} + \gamma_j \left(\frac{D}{E_{jt}}\right) R_{mt} + \varepsilon_{jt} \qquad (2.13)$$

And the tax-adjusted version of the model looks as following:

$$R_{jt} = \alpha_j + \beta_{uj} R_{mt} + \beta_{uj} \left((1 - t_c)\left(\frac{D}{E_{jt}}\right) R_{mt}\right) + \varepsilon_{jt} \qquad (2.14)$$

Which statistically compared to unrestricted model (2.15):

$$R_{jt} = \alpha_j + b_j R_{mt} + \gamma_i \left((1 - t_c)\left(\frac{D}{E_{jt}}\right) R_{mt}\right) + \varepsilon_{jt} \qquad (2.15)$$

The leverage hypothesis is thus verified by testing the restriction: $\gamma_j = b_j$. Overall, the authors find that leverage adjustments are justified, but only for relatively low *D/E* the leverage adjustments of beta risk seem well specified. Hence, they actually confirm "that traditionally applied leverage adjustments tend to over-penalize beta, particularly when high levels of financial leverage are being employed" (Faff et al (2002), p.18). Moreover, they commend to use tax-adjusted leverage technique, which can give better estimation results.

Summary of the previous research in the table 2.1 allows to motivate the choice of the methodology for this master essay. The methodology is chosen in a way that it can be can be applied to test the hypothesis stated above and it can be successfully implemented in a period of time given for this essay. Some methods require the use of variables which are not widely available (e.g. the economies of scale, degree of mechanization) as used by Sudarsanam and Taffler (1985). That is why in this essay I mostly use the methodology employed by Martson and Perry (1996) and Faff et al. because all necessary data is available for this type of analysis. So, to analyze how well the leverage adjustment works under the assumption of constant risk classes, I first use the cross-sectional linear regressions. With the purpose of relaxing the assumption of constant business risk within the industry to a constant business risk for a given firm, the time series methodology is employed then. The details of the methodology and data used are presented in Section three.

Table 2.1. Summary of previous research in the field

Author, year	Method	Data	Results
Gonedes (1969)	Kruskal-Wallis (KW) nonparametric test, which allows to identify if k independent samples are drawn from the same population.	A random sample of ten U.S. firms from eight randomly selected industries for the period of 1958-1967	Selection of firms from a single industry does not ensure the assumption of constant business risk, hence it is not an appropriate procedure for examining the relationship between the cost of capital and financial risk.
Sudarsanam and Taffler (1985)	Multiple Discriminant Analysis (MDA) for analyzing 18 financial ratios, relating to economic, financial, and trade structure of the industry.	263 UK companies in 14 SEIC for the period of 1973-1977	Several industries lack homogeneity with respect to their economic and structural characteristics, higher level of aggregation than SEIC can be more appropriate.
Lord (1996)	Seemingly Unrelated regression to relate total, systematic, and unsystematic risk with four independent variables	35 American companies in three industries for the period of 1963-1988	The degree of financial leverage is positively correlated with total and unsystematic risk, but not with systematic risk
Martson and Perry (1996)	OLS for cross-sectional data, switching regime regression	Sample of American companies in two- and four-digit industry codes for period of 1974-1988	Four-digit SIC level demonstrates much a stronger relationship between financial leverage and beta; beta is over-penalized for financial leverage.
Faff et al (2002)	OLS for time-series data	Sample of 348 U.S. stocks over the period from 1979 to 1994	Leverage adjustments tend to over-penalize beta, particularly when high levels of financial leverage are employed.

3. Methodology and Data

To test the hypothesis if the use of leverage adjustment over-penalizes the cost of equity for a high level of leverage, the definition of risk classes is first needed. Thus, to proxy the identical risk classes I use the Standard Industrial Classification (SIC) –the United States government system of industry classification. It is a system with hierarchical structure which allows to classify the industries with the codes up to the four-digits. As it was mentioned above, Martson and Perry (1996) compare the two-digit and four digit levels of SIC classification and find that two-digit SIC codes should be avoided to proxy the operating risk in practical and academic applications. Thus, in this master essay I examine five industries with four-digit codes: 2834 - Pharmaceutical Preparations, 3579 - Office Machines, 3577 – Computer Peripherals Equipment, 3533 - Oil and Gas Field Machinery and Equipment, 3674 – Semiconductors and Related Devices. In the cross-sectional analysis 369 stocks are analyzed for the period of 1998-2009 and for time-series analysis I use data for 53 companies for the period 1994-2009. The choice of industries is based on the criteria of availability of the data. The list of companies for each of the SIC industries is taken from the web page [2]of Professor of Finance and David Margolis Teaching Fellow at the Stern School of Business at New York University - Aswath Damodaran The data on stock pricing, debt to equity ratios, earnings and sales are downloaded from Thompson Reuters Datastream.

The taxation rate which is theoretically recommended to use in the estimation of the cost of equity is the effective tax rate. However, to calculate the effective tax rate, financial statements of each of the company have to be examined and this can be very time consuming for the sample of 369 companies. Moreover, it is hard to find a single rate as a proxy because the corporate tax rates in the USA differ by the level (state and local level). In addition, the income tax is levied according to a progressive scale, depending on how much the income is as well as the tax rate can vary over time. Hence, the best proxy for the corporate tax rate could be the marginal tax rate which accounts for the dynamic and specific features of the taxes paid. Graham (1996) shows that simulated marginal tax rates can be superior to the alternative proxies of the tax rate including statutory marginal tax rates. Simulated marginal tax rates, which account for deferred taxes, the progressivity of the statutory tax schedule, certain tax credits, and other important particulars which can change the value of the marginal tax rate considerably can be downloaded from Graham's web-page. However, it was impossible to use them in this paper because the data for 10000 companies is based on Compustat CIC codes, to which I do not have access. Thus, alternatively I use the tax rate downloaded from the web-page of Professor Damodaran mentioned above in this section. The tax rate is calculated as the ratio of tax paid to taxable income as reported to the shareholders. This ratio

[2] (http://pages.stern.nyu.edu/~adamodar/New_Home_Page/data.html).

is not available for all companies in the sample. Thus, I average the available tax rates for the given SIC codes for each year. For the years 1998-1999 I assume a constant tax rates equal to the next year's (2000) because more detailed data for this period is not available. Consequently, the corporate tax rate used for the cross-sectional analysis is equal to 0.306. The calculation of tax rates is available in Appendix 1.

Ultimately, the analysis below can be divided into two parts. In the first part I implement a simple cross-sectional regression of raw time series betas (betas calculated as the slope coefficient of log returns and the returns on market portfolio) on the average debt to equity ratio of the company. Then the comparison of raw betas and betas computed using Hamada's adjustment with and without taxes is made. In the second part, the time-series analysis of stated hypothesis is presented.

3.1 Cross -Sectional Approach for Testing the Hypothesis

The cross-sectional analysis is implemented in the following order:

1. Estimate the time-series betas - $\hat{B}_{eji}$ for the monthly log returns for all of the companies in each of the five industries. As the benchmark for beta, the value-weighted S&P 500 composite index is used.
2. Average the annual book value of debt to market value of equity ($\frac{\bar{D}}{E}$) for the total period and calculate $\beta_{uji} = \hat{B}_{eji}/(1+\frac{\bar{D}}{E})$. Theoretically, one should use the market value of debt, but it can be quite cumbersome to calculate it for each of the companies because the debt of the company can have different structure and then it could be necessary to estimate the cost of debt using the yield to maturity of the company's long-term bonds, Moody's or Standard and Poor's credit ratings and match it with the company's cash flow duration. However, Bowman (1980) shows that the ratio of book value of debt to market value of equity (market capitalisation) can have superior explanatory power in relating beta to leverage in comparison to market or pure accounting measures.
3. Estimate the equation (2.11) using OLS regression.
4. Assuming a Modigliani and Miller world without taxes, γ_1 and γ_0 are assumed to be equal to $\bar{\beta}_{ui}$ (the average calculated from equation (2.10)), hence should be equal. Therefore the leverage hypothesis is tested as null hypothesis of H_0: $\gamma_0 = \gamma_1$ against H_1: $\gamma_0 \neq \gamma_1$
5. Compare industry averages calculated in part 1 with respective averages computed by Hamada's adjustment:

$$\beta_{eji} = \beta_{ui} + \beta_{ui}\frac{D_{ji}}{E_{ji}} \qquad (3.1)$$

for each of the companies j in industry i and Hamada adjustment with taxes:

$$\beta_{eji} = \beta_{ui} + \beta_{ui}(1-T)\frac{D_{ji}}{E_{ji}} \qquad (3.2)$$

Where T is a proxy for a corporate tax rate.

It is also possible to test the version of the Hamada's adjustment assuming the risky debt, however finding the suitable proxy for the risky debt needs complex calculations which can be the subject of the separate essay.

3.2 Time-Series Approach for Testing the Hypothesis

To see how Hamada's model performs under the relaxed assumption of constant operating risk within the industry, I additionally implement the time-series analysis. As it has been stated before, it has some advantages compared to the cross-sectional approach. In particular, it allows for time-variation in the level of leverage, which could cause the variation in beta risk, provides stronger control for operating risk. Here I use the companies from the initial sample, but in order to ensure the feasibility of the OLS estimation it is necessary to increase the number observations over time. Thus I take only 53 companies for which the quarterly data for the period of 1994-2009 are available. So for each of the companies I have 65 observations. Similar to the previous subsection I use the log returns, on the composite S&P500 index as a benchmark for the calculation of the equity beta. The time series approach also allows for time variation in tax rates. The proxies for tax rates for the period of 1998-2009 were discussed above and they are presented in Appendix 1. For the period 1994-1997 I assume the same constant tax rate as for the cross sectional analysis (0.306). It is also important to take into account potential unit roots problems. However, the types of the variables used (log returns, ratio of debt to equity) preclude this problem from outset, moreover the series of variables look mean-reverting on the graph as well as the standard unit roots tests confirm the stationarity of the series.

The analysis implemented is similar to the one in Faff et al (2002). Thus, first I calculate the theoretically implied unlevered beta:

$$\beta_{uj} = \beta_{ei}/(1 + \frac{D_j}{E_j}) \tag{3.3}$$

and then compare it with empirically estimated unlevered beta, which equals to the slope coefficient of the restricted time-series model:

$$R_{jt} = \alpha_j + \beta_{uj}\left[R_{mt} + \left(\frac{D}{E_{jt}}\right)R_{mt}\right] + \varepsilon_{jt} \tag{3.4}$$

The empirical estimates for the tax-adjusted unlevered beta are produced from the following model:

$$R_{jt} = \alpha_j + \beta_{uj}\left[R_{mt} + \left((1 - t_c)\left(\frac{D}{E_{jt}}\right)R_{mt}\right)\right] + \varepsilon_{jt} \tag{3.5}$$

In addition I estimate models (2.13) and (2.15)[3] and check the leverage hypothesis: $H0\colon \gamma_j = b_j$ with statistical F test.

3.3 Exploring Potential Industry Effect

With the purpose of exploring the potential industry effect, 53 companies used for the time series analyses are disaggregated under their respective SIC codes. As a result, there are 11 companies in the Pharmaceutical Preparations industry (2834), 20 companies in Oil and Gas Field Machinery and Equipment (3533) and 22 in Semiconductors and Related Devices (3674) industry. The correlation coefficients for the key variables are first calculated and compared between the industries. Then I test the constant risk class hypothesis. Thus to proxy the business risk of the industry the Degree of Operating Leverage (DOL) is used. It is calculated as:

$$DOL = \frac{\%\Delta EBIT}{\%\Delta SALES} \tag{3.6}$$

where, $\%\Delta EBIT$ is a percentage change in the company's Earnings Before Interest and Tax and $\%\Delta SALES$ is a percentage change in companies sales (revenues) in period *t* relative to period *t-1.* This ratio is used because the business (operating) risk of a company is a risk which connected to the competitive position of a firm, changes in consumer preferences or their purchasing power. Doff (2008) makes good survey on defining and measuring the business risk, among others he cites the following definition: "Business risk is the risk that operating income is lower than expected because of lower than expected revenues" (Doff (2008), p. 318 (definition developed by ABN Amro)) . Thus the proxy of operating risk must reflect the changes in the earnings relative to the changes in the revenues. Moreover, the empirical studies in the area of total equity risk (Darrat and Mukherjee (1995), Lord (1996)), show that the interactions between the firm's degree of operating and financial leverage determines the firm's level of systematic risk.

To analyze how homogeneous are the industries with respect to their operating risk, similar to Gonedes (1969) the non-parametric Kruskal-Wallis (KW) one-way analysis of variance by ranks is used. I choose this method because first of all it allows to test the hypothesis of interest. In particular, it allows identifying if k independent samples are drawn from the same population. And secondly, as the non-parametric test it does not make any strong assumptions, for example the assumption of normal distribution, which often does not hold in practice. The null hypothesis of KW test is that k samples are drawn from the same population or from identical populations with respect to average. In the computation of KW test each of the *N* observations is replaced by ranks. The smallest score is replaced by rank 1 and the largest by rank *N*. If the k samples are from the

[3] Note that Faff et al (2002) in estimation of the similar models use only the market return for R_{mt}. However, theoretically it is more correct to use the market risk premium: $R_{mt} - r_f$. Thus I deduct the risk free rate, r_f from the market return to estimate the equation. To proxy the risk free rate I use market yield on U.S. Treasury securities at 10-year constant maturity.

same population then H – statistics of the KW test exhibits chi-squared distribution with k-1 degrees of freedom. The statistics is calculated as follows:

$$H = \left(\frac{12}{N(N+1)}\right)\left(\sum_{j=1}^{k}\frac{R_j^2}{n_j}\right) - 3(N+1) \qquad (3.7)$$

where n_j is the number of observations in the j-th sample, *N*- the number of observations in all samples combined, k is the number of samples and R_j is the sum of ranks in the j-th sample.

4. Results

4.1. Cross-Sectional Approach

In Table 4.1 the results of the cross-sectional analysis are summarized. The very low value of R^2 (it does not exceed 7 %) indicates that debt to equity ratio explains only a low percentage of variation in equity beta. Moreover the coefficient, $\gamma_1$estimated with OLS regression from equation (2.11) is insignificant for all of the industries, except of 3577 - Computer Peripherals Equipment. However, its negative sign contradicts the theory, which predicts a positive linear relationship between the systematic risk of the leverage and the equity. The insignificance and unexpected signs of the coefficients are not surprising because many previous studies discussed above already questioned the influence of leverage on the value of systematic risk of a company. However, Matson and Perry (1996) find most of the slope coefficients on debt to equity significant. For instance, for the Pharmaceutical Preparations industry (2834) the slope coefficient is significant on a 5% confidence level in their case, but it is not significant in the present sample. This can be explained by the fact that different companies with different debt structure are included in the samples. It is also worth noticing that the error terms in simple linear regression here are tested for heteroskedasticity and autocorrelation and both are not supported by the corresponding tests.

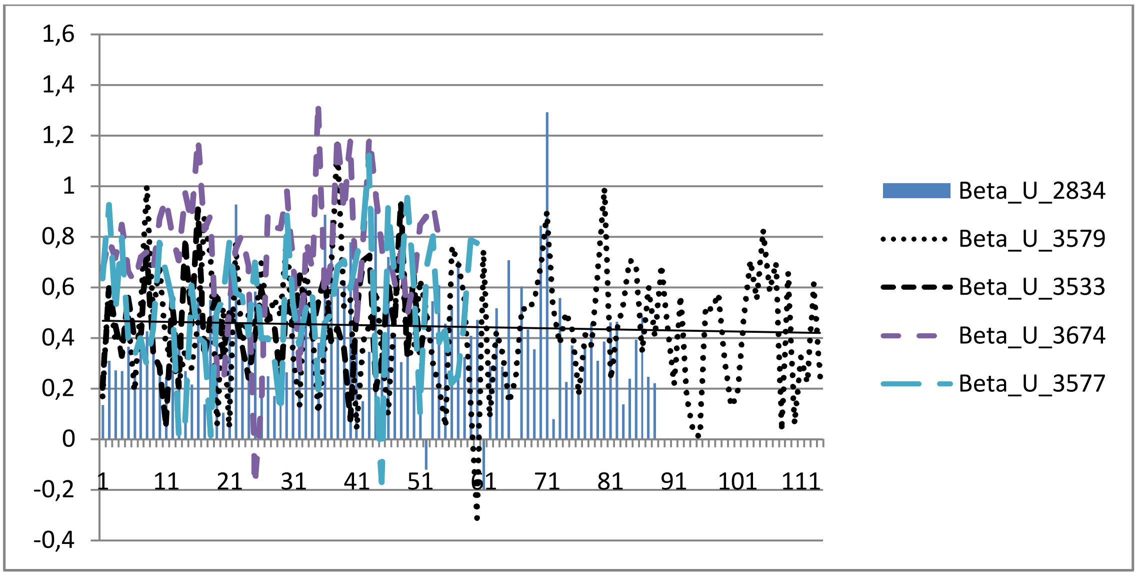
The null hypothesis of equality γ_1 ,γ_0 and consequently $\bar{\beta}_{ui}$ is strictly rejected by the statistical F-test as well as by direct comparison of the averages. So, high values of F- statistics combined with corresponding low p-values statistically reject the leverage hypothesis H_0: $\gamma_0 = \gamma_1$. Also it is obvious that the direct comparison of the average theoretical values and empirical estimates allows rejecting the leverage hypothesis because the disparity between the values is too large. In three cases of five it exceeds 100 %. The lowest difference (85.31 %) is found for the Pharmaceutical Preparations industry (2834) and this lowest difference corresponds to the smallest debt to equity ratio among all the industries. The variation in the value of unlevered beta within the industry may also suggest that companies gathered under one SIC code do not share the same operating risk because they experience significant differences in their operational characteristics, in D/E in particular. In Figure 1, the series of unlevered betas for each of the firm within the industry are plotted. The figure illustrates that the values for unlevered beta deviate significantly from its mean value. Hence, the use of Standard Industrial Classification as a good proxy of the constant operating risk could be questioned and the level of the company's leverage does most likely not have significant influence on the systematic risk of a company, according to the results of cross-sectional analysis. However, the measurement error can arise because of some weaknesses of the approach, in particular because of averaging the debt to equity ratio over twelve years, despite the fact that beta risk can vary together with the variation in the leverage. Therefore, the use of the time series approach for the estimation of unlevered beta proposed by Faff et al (2002) might be more appropriate.

Table 4.1 The outcomes of cross-sectional analysis of leverage hypothesis

SIC code	2834	3674	3533	3579	3577
D/E ratio:					
Mean	0.24	0.31	0.76	0.85	0.90
Standard deviation	0.52	0.64	2.36	2.66	4.36
Beta unlevered, β_{ui}					
Mean	0.41	0.75	0.46	0.44	0.52
Standard deviation	0.24	0.27	0.19	0.25	0.26
γ_0	0.46***	0.89***	0.61***	0.55***	0.63***
t-statistics	14.40	21.80	23.62	21.27	17.83
p-value	0.00	0.00	0.00	0.00	0.00
γ_1	0.06	-0.05	0.01	0.00	-0.02**
t-statistics	1.07	-0.88	1.09	-0.20	-2.09
p-value	0.29	0.38	0.28	0.84	0.04
F-statistics	28.24	125.75	379.48	338.00	292.98
p-value	0.00	0.00	0.00	0.00	0.00
Difference, %	85.31	106.80	97.56	100.41	103.24
R^2	0.01	0.01	0.02	0.00	0.07
N	88	55	52	114	60

This table reports the following figures: mean and standard deviation of debt to equity ratio; mean and standard deviation of theoretically implied unlevered beta: $\beta_{uji} = \hat{B}_{eji}/(1+\frac{\bar{D}}{E})$; the slope, γ_0 and the intercept, γ_1 estimated by OLS from the equation $\widehat{\beta}\, e_{ji} = \gamma_0 + \gamma_1 \frac{D}{E_j} + \widehat{\varepsilon_{ji}}$ and their corresponding t-statistics and p-values. The coefficients marked with*** and ** are significant on the 1% and 5% significance levels correspondingly. F-test represents the F-statistics and respective p-values for the for testing the null hypothesis: H_0: $\gamma_0 = \gamma_1$; R^2 represents the goodness of fit measure for the model; N – is the number of firms in each of the industries.

Figure 1 Plot of series of unlevered beta



In this figure the series of unlevered betas for each of the firm within the industry are plotted.

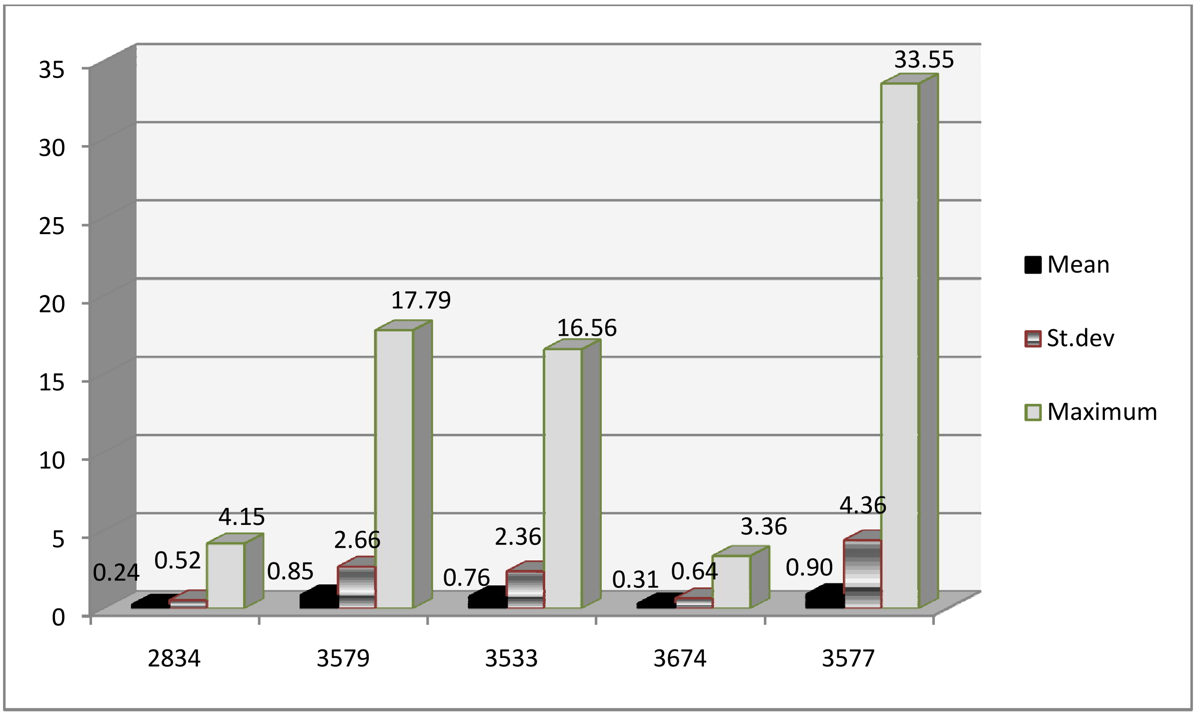
Finally, in Table 4.2 the estimates of the equity betas computed using Hamada's adjustment technique with and without taxes are compared. Computed values illustrate that equity betas calculated with leverage adjustment are higher than raw betas calculated as the slope coefficient of the variance of log returns with the market portfolio. It is also notable that the type of adjustment which allows for tax savings from the debt over-penalizes the systematic risk of equity to a lesser extent than the adjustment without taxes. This is quite expectable because by introducing tax savings to equation (3.2), we are subtracting the positive term from the right-hand side of the equation, hence decreasing the value of the left-hand side. It is also in line with the theory, which implies that tax savings are reducing the risk of levered equity if the tax savings are assumed to be free of risk. Matson and Perry (1996) come to similar conclusions for their sample. However, another point to be emphasized is that for two of the industries, namely Pharmaceutical Preparations industry (2834) and Semiconductors and Related Devices (3674) the differences between the betas are smallest. In the case of the Pharmaceutical Preparations industry (2834) raw equity beta and leverage adjusted beta with taxes are almost equal, the latter is even smaller than the former. And overall the results suggest that the smallest value of the debt to equity ratio for each of the industries corresponds to the smallest difference between the adjusted and unadjusted equity betas. This can be further illustrated by the data presented in the Figure 2. The histogram shows that Pharmaceutical Preparations (2834) and Semiconductors and Related Devices (3674) industries have the lowest deviation from mean (0.24 for 2834 and 0.31 for 3674), so possibly the companies gathered under this SIC codes are more homogeneous, hence share more similar operating risk than companies in other industries. Moreover, the maximum value of debt to equity ratio (4.15 for 2834 and 3.36 for 3674) is much smaller here than for example for the Computer Peripherals Equipment (3577), where maximum value reaches 33.55. These results are similar to Faff et al (2002), who conclude that the approach of unlevering beta over-penalizes the equity risk especially in a situation with high leverage. In general, the results suggest that the leveraged adjustment technique is sensitive to homogeneity of the firms in the same risk class and to the level of leverage. Thus, in the next sub-section I present the results of the time-series analysis, which allow to relax the assumption of constant operating risk within an industry and only assumes the constant operating risk within a firm. Moreover, it allows the debt to equity ratio to vary over time, which potentially can improve the precision of estimates.

Table 4.2 Raw equity betas and betas calculated using Hamada's adjustment averaged by industry

SIC code	Raw equity beta	Leverage adjusted equity beta	Difference between columns II and III, %	Leverage adjusted equity beta with taxes	Difference between columns II and V,%	D/E ratio, industry's mean
2834	0.48	0.50	5.43	0.47	-0.48	0.24
3674	0.88	0.98	10.07	0.91	3.06	0.31
3533	0.62	0.82	24.55	0.71	13.06	0.76
3579	0.54	0.82	33.48	0.70	22.64	0.85
3577	0.62	0.98	37.26	0.84	26.60	0.90

In this table the following figures are presented: raw equity beta is calculated as slope coefficient of monthly log returns of the individual companies and corresponding returns of value-weighted S&P 500 composite index; the values of leverage adjusted equity betas are computed from equation $\beta_{eji} = \beta_{ui} + \beta_{ui}\frac{D_{ji}}{E_{ji}}$; the values of leverage adjusted equity betas with taxes are computed form equation $\beta_{eji} = \beta_{ui} + \beta_{ui}(1-T)\frac{D_{ji}}{E_{ji}}$; debt to equity ratios are averaged by industry.

Figure 2 Descriptive statistics of the sample's D/E ratio



In this Figure the mean, standard deviation and the maximum values of debt to equity ratios for each of the SIC codes are presented.

4.2 TIME-SERIES APPROACH

In table 4.3 I present the results of analysis for the whole sample and for the sample divided by groups. In particular, in the low leverage group the stocks, which debt to equity ratio does not exceed 20%, are included. In the medium leverage and the high leverage groups there are the stocks which leverage level is greater than 20%, but lower than 40% and greater than 40 % correspondingly. In the table 4.3 the descriptive statistics of the debt to equity ratio for each group is presented first. It can be noticed that the sample is not characterized by extremely high levels of debt to equity ratio (the minimum equals 0.07 and the maximum is 6.05). This can be possibly explained by the fact that industries included in the sample are not from the financial sector, which could have high levels of debt to equity ratios. Moreover, the companies operating on the market for the period of 15 years can potentially have good operational characteristics and optimal structure of financing. Next I consider the results for the all stocks in the sample. They are consistent with the findings of the previous sub-section as well as with results of Faff et al. (2002). In particular theoretically implied unlevered beta is almost 14 % greater than its empirical counterpart in the no-taxes case and more than 15 % greater in the case of tax-adjusted model. Moreover, p-value for the t-test is zero, which suggests that we reject the null hypothesis that estimated and theoretically implied betas are likely to come from the populations with the same mean. Roughly speaking, the statistical test confirms that series of theoretically implied and empirically estimated betas are not equal.

The average value for the unlevered beta in the no-taxes case is equal to 0.98 and 1.10 for the tax-adjusted version of the model, which is consistent with the results of Faff et al. (2002) who also confirms that unlevered beta is understated (and equity beta is overstated) more when we do not subtract the tax savings from the debt. However, in contrast to the article mentioned above the sample data used in current essay allows to reject H0: $\gamma_j = b_j$ in most of the cases, statistically rejecting the leverage hypothesis. It is also worth noticing that in no-tax (tax adjusted) setting coefficient γ_j is significant only in 3 (9) out of 53 cases, in contrast to coefficient b_j, which is significant in 36 (47) regressions and R^2 varies from 13% to around 30%. This may suggest that part of variation in return on equity is explained by variation in the market risk premium, but there is no evidence on direct linear relationship between the return on equity and the leverage of the firm. Similar results are achieved in the cross-sectional regression presented above, where the coefficients on D/E ratio are either insignificant, or have the signs which contradict the theory.

As to the results for different leverage groups, the most important to notice here is the fact that in line with previous subsection, the difference between the theoretically implied unlevered beta and its empirical counterpart increases with the debt to equity ratio, implying the higher penalty for the firms with the high level of leverage in comparison to the firms with the low level of leverage. Thus for no-taxes (tax adjusted) model the difference between the betas is only around 7% (5.6%), but for the high leverage group the difference is 24.7% (29%). The t-test also strictly rejects the hypothesis of equality between the series of theoretical and empirical betas.

Overall results of time-series analysis confirm the findings of the previous subsection. Specifically, the estimates of the beta risk with the Hamada's adjustment are closer to the estimates of the beta from the market model (raw equity beta) for the companies with non-high leverage. For the companies with the high level of leverage the estimates of the beta risk are over-penalized for the level of debt to equity ratio. It is consistent with the previous research in the field and with the theoretical predictions. For instance, if the non-zero cost of debt was assumed here, then the value of this cost could be smaller for the companies with the low leverage, than with the high leverage. Hence, assuming zero cost of debt these type of estimations produce less biased estimates for the low leverage group in comparison to the high leverage group.

If we consider the comparison of cross-sectional and time-series analysis, the leverage hypothesis is strictly rejected in both of the cases. However, the differences between the theoretical and empirical values of unlevered beta are more than 100 % in all examined industries in cross-sectional analysis. While in time-series analysis the differences between the series are only around 15 %. Hence, Hamada's model performs better without assumption of constant business risk. And in the next sub-section I further examine this issue by exploring the potential industry effect on the performance of the model.

Table 4.3 The outcomes of time-series analysis of leverage hypothesis

	All stocks	Low leverage	Medium leverage	High leverage
Number of stocks	53	17	16	20
Raw equity beta	1.60	1.14	1.96	1.70
D/E, Average	0.62	0.15	0.26	1.30
Standard Deviation	1.11	0.04	0.05	1.59
Minimum	0.07	0.07	0.21	0.41
Maximum	6.05	0.20	0.38	6.05
No-taxes model				
Theoretical β_{uj}, Average	1.14	0.99	1.55	0.93
Standard Deviation	0.55	0.44	0.53	0.48
Minimum	0.11	0.26	0.99	0.11
Maximum	2.76	2.12	2.76	1.88
Empirically estimated β_{uj}				
Average	0.98	0.93	1.39	0.70
Standard Deviation	0.54	0.43	0.48	0.50
Minimum	0.09	0.38	0.93	0.09
Maximum	2.63	1.96	2.63	1.75
H0: ($\gamma_j = b_j$) not rejected[a]	7	0	1	6
p value for T test[b]	0.00	0.00	0.00	0.00
Difference, %[c]	13.96	6.92	10.71	24.67
Tax-adjusted model				
Theoretical β_{uj}, Average	1.29	1.041	1.68	1.20
Standard Deviation	0.54	0.46	0.48	0.50
Minimum	0.09	0.27	0.93	0.47
Maximum	2.63	2.22	2.63	2.09
Empirically estimated β_{uj}				
Average	1.10	0.99	1.52	0.85
Standard Deviation	0.58	0.44	0.52	0.56
Minimum	0.14	0.40	1.02	0.14
Maximum	2.83	2.12	2.83	2.03
Difference, %	15.34	5.58	9.40	29.19
H0 ($\gamma_j = b_j$) not rejected[a]	4	0	0	4
p value for T test [b]	0.00	0.00	0.00	0.00
Difference, %[d]	15.34	5.58	9.40	29.19

This table reports the descriptive statistics and the summary of time-series analysis for the whole sample and the three leverage groups. For each of the sub-samples the following figures are presented: raw equity beta (calculated as the slope coefficient of the quarterly log returns and the corresponding returns of value-weighted S&P 500 index); average, standard deviation and maximum and minimum values of debt to equity ratio, theoretically implied and empirically estimated unlevered beta for the model with taxes and without taxes. Theoretically implied unlevered beta, β_{uj}, without taxes computed as $\beta_{ujt} = \beta_{ejt}/(1 + \frac{D_{jt}}{E_{jt}})$ and for the tax-adjusted case: $\beta_{ujt} = \beta_{ejt}/(1 + (1 - T_t)\frac{D_{jt}}{E_{jt}})$. The empirical estimates for the model without taxes are the slope coefficient from regression $R_{jt} = \alpha_j + \beta_{uj}\left[R_{mt} + \left(\frac{D}{E_{jt}}\right)R_{mt}\right] + \varepsilon_{jt}$ and for the tax-adjusted model: $R_{jt} = \alpha_j + \beta_{uj}\left[R_{mt} + \left((1 - t_c)\left(\frac{D}{E_{jt}}\right)R_{mt}\right)\right] + \varepsilon_{jt}$. ;

[a] – the number of cases when H0: ($\gamma_j = b_j$) is not rejected. Where γ_j and b_j are estimated from equations (2.13) and (2.12) for no-taxes and tax-adjusted cases respectively; [b] - the p-values of the t-tests, which examine if the series for empirically estimated betas and theoretically implied betas are likely to come from the populations with the same mean; [c] – the percentage difference between the theoretical unlevered betas and empirical estimates for no-taxes case; [d] - the percentage difference between the theoretical unlevered betas and empirical estimates for model with taxes.

4.3. EXPLORING POTENTIAL INDUSTRY EFFECT

With the purpose of exploring the potential industry effect, 53 companies used for the time series analysis are gathered under their respective SIC codes. In table 4.4 the cross-sectional correlations for the key variables for three industries are presented. If we assume that these industrial classifications (namely Pharmaceutical Preparations (2834), Oil and Gas Field Machinery and Equipment (3533), Semiconductors and Related Devices (3674)) represent the good proxy of a constant business risk, then we should expect a) strong positive correlation between the D/E ratio and raw market model beta and b) strong negative correlation between the D/E and unlevered beta. As to the former, the correlation between the variables is positive indeed, however, it can be named to be strong only for the Pharmaceutical Preparations industry (2834), where the correlation coefficient equals 0.79 in contrast to other groups where correlation varies from 0.23 to 0.28. This can be explained by the fact that only 11 companies are analyzed under the 2834 code, which may suggest that they share more similar in operating risk in comparison to other SIC codes. Another point that this industry has the lowest level of leverage (0.36) with the lowest standard deviation (0.22) among other industries. This confirms the previous assumptions about the best performance of Hamada's adjustment in the situation of homogenous companies and low debt to equity ratios.

However, if we consider the correlation between the D/E and unlevered beta for the model without taxes (with taxes) it is strong and negative for all the groups (varying from -0.45 (-0.43) for the whole sample to -0.73 (-0.75) for the Oil and Gas Field Machinery and Equipment (3533)). And only Pharmaceutical Preparations industry exhibits strong positive correlation (0.62 for no taxes model and 0.68 for the model with taxes) of leverage and unlevered beta, and this contradiction could possibly be explained by the fact that despite the lowest leverage level, the value of raw equity beta here is not the lowest among other industries. Nevertheless, the correlation between the raw beta and empirically estimated unlevered beta for both taxes and no-taxes cases is positive and strong for all the groups, and in the case of Pharmaceutical Preparations it is almost equal to one (0.94 for no taxes and 0.97 for the model with taxes). The latter observation confirms one more time that Hamada's adjustment can give the priciest estimates for the companies which do not have a high level of leverage. However, if we examine the differences between the theoretically estimated betas and its empirical counterpart, the smallest penalty for the low leverage group is observed only for the model with taxes. For the no-taxes model, the relationship between the high leverage group and high penalty for leverage is not so pronounced. Nevertheless, it is interesting to notice that in line with the results of cross-sectional analysis, the least disparity between theoretical values and its empirical estimates are produced for the Pharmaceutical Preparations (2834) and Semiconductors and Related Devices (3674) industries. This may suggest that companies in each of two industries are more homogenous, and hence exhibit similar operating risk in comparison to other industries.

To test this assumption empirically, in Appendix 2 I present the results of Kruskal-Wallis one-way analysis test of variance by ranks for 20 randomly selected companies in each of three industries. And controversially to the suppositions above, the test statistics strictly reject the null hypothesis of homogeneity of the firms in the Semiconductors and Related Devices (3674) industry, but do not reject on the 1% significance level the hypothesis for Pharmaceutical Preparations (2834), Oil and Gas Field Machinery and Equipment (3533). It is worth noticing that results are sensitive to the choice of the proxy to business risk, but I reckon that they emphasize one more time how important for this type of analysis to find the suitable proxy for the business risk class. Therefore the use of different industrial classifications is not the best solution for the proxy of constant risk classes. It also can be explained on the intuitive level. Consider, for example the Semiconductors and Related Devices industry. In this industry the large set of companies producing various products (from microprocessors to solar cells) are included. Thus it is hard to justify that they could share similar business risk because the demand for their products depends on different factors as well as they face different types of risk connected with production process. Consequently, I can suggest that it is possible to use leverage adjustment of beta, preferably the adjustment which accounts for tax savings by only averaging the unlevered beta for a small number of companies within homogeneous activities and low leverage level.

Table 4.4 The outcomes of analysis for standard industrial classification and all stocks

	All stocks	2834	3533	3674
Descriptive statistics				
D/E, mean	0.62	0.36	0.45	0.64
D/E, stand. dev.	1.11	0.22	0.60	1.13
Raw equity beta	1.60	1.36	1.25	2.06
Theoretical β_{uj} (no taxes)	1.14	0.96	0.92	1.50
Theoretical β_{uj} (with taxes)	1.29	1.08	1.02	1.67
Empirically estimated β_{uj} (no taxes)	0.98	0.96	0.75	1.29
Empirically estimated β_{uj}(with taxes)	1.10	0.96	0.83	1.44
Difference, % no taxes	13.96	13.93	18.84	13.69
Difference, % taxes	15.34	11.22	19.25	13.87
Correlation coefficients				
D/E, raw beta	0.27	0.79	0.28	0.23
D/E, empirical β_{uj} (no taxes)	-0.45	0.62	-0.75	-0.58
D/E, empirical β_{uj} (with taxes)	-0.43	0.68	-0.73	-0.55
Raw beta, empirical β_{uj} (no taxes)	0.71	0.94	0.25	0.61
Raw beta, empirical β_{uj} (with taxes)	0.76	0.97	0.33	0.66

This table reports the descriptive statistics and correlation coefficients for the whole sample and the sub-samples grouped by the SIC codes. The key variables presented in descriptive statistics are defined in the notes to table 4.3. The correlation coefficients are computed between a) D/E and raw beta; b) D/E and empirically estimated unlevered beta with taxes and without taxes; c) between raw beta and empirically estimated unlevered beta with taxes and without taxes. The differences represent the percentage disparity between theoretical betas and its empirical counterparts for the model without taxes and model adjusted for taxes.

5.Conclusions

Despite its numerous drawbacks, the capital asset pricing model today still remains one of the popular tools for estimation of the cost of equity. To improve the outcomes of estimation and overcome some of the limitations of CAPM, numerous adjustments exist in financial theory. In particular, to overcome the problem of absence of historical data on stock returns and to improve the estimation of an individual company's beta, Hamada's leverage adjustment technique is used. It assumed that companies within one industry share the same operating risk and to reduce statistical noise, average unlevered industry beta and debt to equity ratios of individual firms are used to calculate the beta risk of the company. However, some empirical studies (Martson and Perry (1996), Faff et al. (2002)) find that this leverage adjustment can over-penalize the calculated systematic risk of the company for the high debt to equity ratio. Thus, given the frequent use of this technique in financial applications and potential problems connected with overestimation of systematic risk of the company (and hence underestimation of its value), I find it reasonable to test this technique empirically for the most recent period of time. Cross-sectional and time-series approaches are implemented for the analysis of the main hypothesis of the study: the use of leverage adjustment techniques over-penalizes the cost of equity for a high level of leverage (debt to equity ratio). For the cross-sectional analysis I use the sample of 369 American companies for the period 1998-2009. To proxy identical risk classes, five industries of standard industrial classification are utilized. For the time-series analysis 53 companies with quarterly data for the period 1994-2009 are used. The cross-sectional analysis is used first to see how the model performs under the assumption of constant risk classes. Secondly, the time series approach in-turn allows the relaxing of the strong assumption of a constant operating risk for a particular company.

Results of cross-sectional regressions indicate that variation in the leverage level explains only a low percentage of variation in equity beta. Moreover, the coefficients on debt to equity ratio are either insignificant or have signs contradictory to the theory, as well as the fact that the leverage hypothesis is strictly rejected by the statistical test of mean differences. The lower difference between the theoretical values and respective empirical estimates is observed for the Pharmaceutical Preparations industry (2834) and this lowest difference corresponds to the smallest debt to equity ratio among all the industries. Overall, the smallest value of the debt to equity ratio for each of the industries conforms to the smallest difference between the leverage adjusted betas and raw equity betas calculated using the ordinary market model. The value of unlevered beta varies considerably within the industry also, which may suggest that companies gathered under one SIC code do not share the same operating risk because they experience significant differences in their operational characteristics, in D/E in particular. As to the cross-sectional performance of Hamada's model with and without taxes, it is found that the type of adjustment which allows for tax savings from the debt over-penalizes the systematic risk of equity to a lesser extent than the adjustment without taxes. It is in line with the theory, which implies that tax savings are reducing the risk of levered equity (if the tax savings are assumed to be free of risk). Matson and Perry (1996) draw similar conclusions for their sample.

The time-series analysis of three leverage groups and the whole sample confirms the findings of cross-sectional analysis. The regression results also indicate insignificant coefficients on the debt to equity ratio and the leverage hypothesis is rejected statistically for the majority of the companies in the sample. Moreover, similar to Faff et al. (2002), the average value of unlevered beta in the no-taxes case is equal to 0.9801 and 1.0958 for the tax-adjusted version of the model, meaning that unlevered beta is understated (and equity beta is overstated) more when we do not subtract the tax savings from the debt. The higher penalty is also implied for the firms with the high level of leverage in comparison to the firms with the low level of leverage. The results for subsamples disaggregated by the SIC codes do not produce such a pronounced relationship between the high leverage level and over-penalization of beta risk. However, the interesting observation is that the lowest discrepancy between the theoretically implied unlevered beta and its empirical counterpart is found for the Pharmaceutical Preparations (2834) and Semiconductors and Related Devices (3674) industries. In spite of this, the companies gathered under the latter SIC codes in the time-series analysis do not have low leverage levels (in contrast to the samples in cross-sectional analysis). Hence, it is possible that the companies in these two industries share more similar operating risk in comparison to others. However, the Kruskal-Wallis test of homogeneity of industries, with respect to their degree of operating leverage, does not support this supposition. Thus further analysis is needed to explore the suitable proxy of business risk as well as the suitable approach to its analysis.

Ultimately, it can be concluded that the leverage adjustment technique is sensitive to homogeneity of the firms in the same risk class as well as to the level of leverage. Thus, in line with the previous studies it is found that the leverage adjustment over-penalizes the beta risk for the high level of debt to equity ratio. The biggest difference between the theoretically implied betas and their empirical counterparts is observed for the cases with high leverage estimated by the model, which does not account for the tax savings from debt.Thus, one should be aware of the problems connected with the over-penalization of beta risk while dealing with the industries which a) include large number of the companies with different operational characteristics, and b) have a high level of debt to equity ratio. Therefore, I can suggest that it is possible to use leverage adjustment of beta, which accounts for tax savings by only averaging the unlevered beta for a small number of companies within homogeneous activities and low leverage level.

Further research in this field could use Granger causality test instead of using the simple analysis of correlation coefficients because the latter does not imply the interpretation of causation. And the analysis of causality could give an answer to whether variation in leverage causes variation in equities risk. Moreover, it is also possible to analyze how the degree of operating and financial leverage influences the total risk of the company using seemingly unrelated regressions.

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Appendixes

Appendix 1 Corporate tax rates

	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	1998-2009
2834													
Mean	0.23	0.23	0.23	0.49	0.31	0.33	0.30	0.31	0.31	0.30	0.27	0.27	0.30
N	71	71	71	59	61	64	68	63	65	74	74	69	
3579													
Mean	0.35	0.35	0.35	0.35	0.33	0.45	0.29	0.27	0.28	0.29	0.29	0.28	0.32
N	193	193	193	147	109	128	146	151	155	138	140	121	
3533													
Mean	0.37	0.37	0.37	0.38	0.37	0.35	0.33	0.34	0.30	0.32	0.30	0.32	0.34
N	35	35	35	52	55	52	52	60	78	72	82	79	
3674													
Mean	0.27	0.27	0.27	0.29	0.31	0.31	0.29	0.24	0.25	0.23	0.23	0.32	0.27
N	75	75	75	77	33	32	44	50	51	71	47	79	
3577													
Mean	0.31	0.31	0.31	0.30	0.31	0.30	0.28	0.26	0.25	0.27	0.30	0.28	0.29
N	61	61	61	54	49	45	53	46	46	45	36	40	
Average	0.31	0.31	0.31	0.36	0.33	0.35	0.30	0.29	0.28	0.28	0.28	0.29	0.31

This table reports the tax rates for each of the SIC codes for each of the years, which is calculated as companies average ratio of tax paid to taxable income (as reported to shareholders). For the years 1998-1999 a constant tax rate is assumed.

Appendix 2 Kruskal-Wallis one-way analysis test of variance by ranks

Pharmaceutical Preparations (2834)

Test for Equality of Medians Between Series
Included observations: 12

Method	df	Value	Probability
Kruskal-Wallis	19	29.02	0.07
Kruskal-Wallis (tie-adj.)*	19	29.02	0.07

Oil and Gas Field Machinery and Equipment (3533)

Test for Equality of Medians Between Series
Included observations: 12

Method	df	Value	Probability
Kruskal-Wallis	19	14.55	0.75
Kruskal-Wallis (tie-adj.)*	19	14.55	0.75

Semiconductors and Related Devices (3674)

Test for Equality of Medians Between Series
Included observations: 12

Method	df	Value	Probability
Kruskal-Wallis	19	130.96	0.00
Kruskal-Wallis (tie-adj.)*	19	130.96	0.00

Tables report the results of Kruskal-Wallis one-way analysis test of variance by ranks for each of the industries. The null hypothesis: k samples are drawn from the same population or from identical populations with respect to average.

*- the statistic corrected for the tied observation