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The Monetary Model – A panel data approach

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Abstract

The purpose of this paper is to determine if effective exchange rate pricing can be based on the (flexible) monetary model. If so, it implies a cointegration relationship with a cointegrating vector between the spot exchange rate and the non-stationary determining variables of the monetary model; relative money supplies and relative national incomes. Cointegration tests for individual countries have been done frequently and without evidence pro the monetary model. But there are some indications that the increased power achieved form a multi-country cross-setting, or panel data, show evidence in favour of the monetary model. In this paper we find evidence of at least one cointegrating relationship of the monetary model in a small panel with post Bretton-Woods observations. We also find supporting evidence related to the significance of modelling the variables determining the long-run real exchange rate equilibrium in the monetary model.

Keywords: Panel cointegration, the monetary model, the real and the nominal exchange rate.

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1 Introduction

After the breakdown of the Bretton Woods-system in 1973, the exchange rates between the major economies of the world were left to be determined by market forces in the foreign exchange market (Forex). With floating exchange rates and the continual development of the financial market, together with daily international trade in goods and services requiring different national currencies, the need of such a market can't be exaggerated. Today, the daily volume of transactions in the foreign exchange market exceeds $5 trillion, making it the largest market in the world and the place of global business for speculators, firms, governments, banks and other financial institutions. The size and importance of the foreign exchange market attract attention from several institutions in our society. Governments are interested in an effective way to perform sound monetary policy. Firms aspire to minimize the loss in trade due to exchange rate volatility. Speculators try their best to invest their money in a currency that is about to gain value. These economic agents all depend on the implications that the exchange rate behaviour have on trade, economic growth and the general functioning of the world economy.

The monetary models are the first most frameworks to analyse exchange rate behaviour. Since their births researchers have had substantial trouble in validating the empirical connection with the theoretical suggestions, and the monetary models have consistently failed to beat the forecasting performance of simple random walks. In the 80's there was great pessimism regarding to its empirical performance. In research by Frankel (1984) and the independent article of Dornbusch (1983) both ended up with insignificant variable estimations and the wrong parameter signs. Meese and Rogoff (1983) confirmed these gloomy results in two famous papers and concluded that none of the monetary models were capable of producing better out of sample forecast than the random walk model. In a later article the duo showed that the monetary models were equally bad when they used economically plausible values instead of the estimated equivalence, in order to correct for small sample and simultaneity bias. It is safe to say that the early empirical work showed that it was virtually impossible to make short-run predictions of the exchange rate and that even the forward exchange rate was of little use in predictions of the spot value. In fact, Meese and Rogoff showed that it wasn't until after 12-months that the monetary model predictions started to outperform the random walk models. The random walk model was beaten in the works of MacDonald and Taylor (1993) with their sophisticated forward looking monetary model of the deutschmark-dollar parity. In their study, which had more of a modern approach, they tested for long-run cointegration and found evidence of such between the exchange rate and its fundamental determinants. But it is difficult to say whether the results were "case-specific" and, in fact, Sarantis (1994) came to the opposite conclusion using the same model and method but on data with a larger time-span. Sarantis concluded that a larger set of time-series data actually produces worse results. Hence, panel data may help in that it includes more cross-sectional observations which should improve the power of cointegration tests. More recent cointegration tests have been done by for example Groen (1998), Islam and Hasan (2006) and Zhang and Lowinger (2005). Groen found very little support for cointegration in individual exchange rates, but some support in his pooled data set. His method was basically a pooled version of the Engle-Granger procedure that suffered from low power since it didn't model cross-country differences in short run dynamics. Islam and Hasan specified a monetary model based on the error-correction model and managed to outperform random walk models. Using the Johansen procedure they identified a cointegrating vector between dollar-yen exchange rates and exchange rate determinants. For similar studies see Moosa (1994) or Francis et al. (2001). The works of Zhang and Lowinger showed similar results but they were sensitive to the countries included in the tests. Rapach and Wohar (2002) constructed individual test for 14 countries and found mixed results; there were some evidence of cointegration for 8 countries but for the remaining 6 such evidence was completely absent. Many studies have shown similiar lack of evidence in favour of monetary model cointegration when looking at individual time-series only, and the overall empirical evidence is mixed at best. These discouraging results may very well reflect low power of the tests and not necessarily the legitimacy if the monetary model. Sarno et al. offer an interesting alternative to the standard procedure of statistical measures of forecast accuracy. Instead, Sarno et al. use a model with economic fundamentals in order to predict exchange rate out of sample and find evidence for their model using three major US dollar exchange rates. They compare the economic value, to a utility maximizing investor, of out of sample exchange rate forecasts using a monetary model based on fundamentals with the value from a random walk (Sarno et al., 2003). But overall there has been little success in tying to match floating exchange rates to macroeconomic fundamentals. Some recent surveys however point on a long term relationship.

Lately, there has been an increase in the number of articles that consider monetary cointegration with the increased power of panel data. Rapach and Wohar (2004) compare individual test of cointegration between the monetary and its macroeconomic fundamentals with the modern panel tests. In contrast to country-by-country tests Rapach and Wohar show that there is support for the monetary model in panels and find evidence of cointegration between US dollar exchange rates and both the relative money supplies variable as well as the relative income variable. In the same paper the authors show that assumptions on homogeneity is very important for the results, and that tests that impose homogeneity in the case of heterogeneity in the true data-generating process, may produce false positive results of cointegration. Groen (2002) uses a rather complicated VEC model that is a mix of the pure time series approach and the panel approach, allowing for cross-sectional correlation. Groen stress the importance of allowing for heterogeneous short-run dynamics in the panel data and that the error-correction coefficient of the cointegration relationship works very slow. Groen also compares his VEC model against the individual time-series Johansen approach and the Engle-Granger pooled method used in Groen (1998), in which cross-section dependence is neglected. Groen finds that the VEC model provides more optimistic result in favor of the monetary model, and some strong evidence in favor of the monetary model for three major European countries. Westerlund and Basher (2009) consider both cross-sectional dependence, heterogenity and the presence of structural breaks, and argue that taking this into consideration is crucial when empirically evaluating the monetary model. Based on their cointegration test they find evidence in favor of the monetary model but fail to repeat their findings in the same data set when ignoring the structural breaks and the cross-sectional dependence that their alternative test incorporates. Another study in the same category is that of Mark & Sul (2001) who uses a bootstrap procedure to control for small sample size distortion as well as cross-sectional dependence. Mark & Sul finds supporting evidence for the monetary model in a small panel and concludes that the results aren’t solely consistent for the US dollar. Mark and Sul’s results also raise an interesting question regarding why monetary fundamentals are so much better in predictions than the PPP, although their close relationship with one another.

Our aim is to re-examine the monetary model using panel-data for a set of five countries and one economic region; the Euro-zone. Above that we will examine whether the results are sensitive to the use of a wider money supply aggregate than the traditional narrow equivalent. We will also relax the rather bold PPP assumption and include real exchange rate variables under the premise that real exchange rate is not constant over time, and when prices are sticky the nominal and real exchange rate move in tandem. Hence, we will conduct cointegration test for an additional specification of the monetary model. We will begin by describing the monetary model theory and the assumptions that are used to derive the model, followed by the econometric theory that is used in the following analysis. After that we will present the additional variables and re-do the cointegration tests.

2 Exchange rate determination theory

2.1 Definitions of the Nominal and the Real exchange rate

In order to compare the price of domestic goods with goods produced abroad, we express them in a common currency using the *nominal* exchange rate. The nominal exchange rate is the price of domestic money in terms of foreign money. Its price is determined at the foreign exchange market and throughout this paper it will be denoted as the number of domestic monetary units in terms of a single foreign money unit, also called "European terms". As reasonably customary we will call the nominal exchange rate *S*. Later we might find cause to also introduce the *real* exchange rate, which is closely connected to the nominal equivalence.

The real exchange rate is an expression of the relative price of foreign *goods* in terms of domestic goods. The only difference from the nominal exchange rate is that the former is adjusted for the domestic and foreign price levels. If the nominal exchange rate is denoted by *S*, the real exchange is:

$$Z = \frac{P}{P^f S} = \frac{P/S}{P^f} \qquad (2.1)$$

Here, P is the domestic price level and P^f the foreign equivalence. $P^f S$ is the domestic price of foreign goods. Hence, we have expressed the real exchange rate with both the domestic and foreign prices in terms of domestic currency as well as in foreign currency.

2.2 Purchasing Power Parity

Purchasing power parity (PPP) is an important exchange rate theory. The concept of PPP is that arbitrage forces will make sure that international prices, expressed in the same currency, will converge into a unified price. Its origins come from the theory of "the law of one price". In a seemingly important article Rogoff (1996) points out that the belief in PPP as a short term mechanism is dismal among economist, but on the other hand most economists adopt the believe that PPP plays at least some role in long-term exchange rate behaviour (Rogoff, 1996).

2.2.1 The law of one price

The law of one price states that identical goods that are sold in different marketplaces will have the same price when expressed in the same currency. If P is the domestic price in domestic currency of a good, P* is the foreign equivalence and S the nominal exchange rate we have that:

$$P_t = SP_t{}^f \tag{2.2}$$

It is based on the idea of perfect goods arbitrage. Arbitrage means that economic agents can exploit differences in prices in order to make risk free profits. Once an arbitrage possibility appears economic agents take full advantage of the opportunity, profoundly changing the balance of the market. This rational behaviour from economic agents together with competitive markets and the absence of transportation costs, and other obstacles to trade, make up the prerequisite for the law of one price (Pilbeam, 2006). The same perquisites make the law of one price unlikely to hold in a strict manner. The law of one price is more likely to coincide for merchandises that are heavily traded[1] and in close geographical proximity (Rogoff, 1996).

When an arbitrage possibility arise agents purchase commodities where they are cheap, only to sell them in a market where they can achieve a higher price. To do so, a domestic agent would have to exchange his home currency into foreign currency, thereby creating a higher relative demand for foreign money[2]. As long as there is a price discrepancy agents will continue to act in the same manner, selling

[1] For example gold.
[2] The example implies that domestic goods are more expensive then foreign goods.

domestic money to buy foreign currency, until the exchange rate appreciate[3] enough to ultimately leaving the two prices identical when expressed in the same currency.

2.2.2 Absolute and relative PPP

The PPP theory is divided into two fields; an absolute version of PPP which applies the law of price in a strict sense, and a relaxed version, relative PPP, which hypothesizes that exchange rates will adjust according to the difference in inflation between two countries. The difference from the "law of one price" is that PPP is primarily designed for identical *bundles* of goods, or rather, aggregate consumer price indexes such as CPI or WPI. In the absolute version of PPP a bundle of goods in one country will have the same price as an identical bundle of goods in a different country once the price is converted into a common currency. The implication on the exchange rate is that a rise in the home price level in relation to the foreign price level will lead to a proportional depreciation of the home currency, in order for the equality relationship below to hold (Pilbeam, 2006). The absolute version of PPP implies:

$$P_t{}^h = S_t P_t^f \qquad (2.3)$$

where P^h is the price level in the "home" country, P^f is the foreign price level and S is the nominal exchange rate. In turn, the relative version of PPP can be modelled as:

$$s_t = p_t{}^h - p_t{}^f \qquad (2.4)$$

Here, lower-case letters denote log variables so that $s_t = \ln(S_t)$. The above relationship is a fundamental building block of the monetary model. If the exchange rate doesn't reflect the relative prices of goods, arbitrage possibilities should induce a massive shift of products from one place to another. This would require currency to facilitate transactions and the demand and supply relationship of the exchange rate should balance the relationship in (2.4). But transportation costs, imperfect information, tariffs and other barriers to trade make the absolute version of PPP unlikely to hold. Such costs are immense and are very likely to play a substantial role in the difference between the real exchange rate and PPP. In the relative version of the PPP theory the

[3] Note that the definition of the exchange rate as "European terms" imply that an appreciation of the exchange rate is equivalent with a loss of value of the "home" currency.

exchange rate will adjust according to the difference of inflation between two countries. Thus, if the Swedish inflation is 10% while the Danish inflation is 5%, we can expect the Swedish krona to depreciate 5%. The law of one price would then be restored (Marrewijk, 2006).

2.2.3 PPP evaluation

Prior to the adoption of floating exchange rates, many proponents of PPP believed that the changes in the exchange rates would develop in close relationship to the relative price levels just as predicted by the theory. Contrary, prices tend to be sticky in the short and medium run, while nominal exchange rates may fluctuate wieldy, where daily changes of approximately ±1% is normal (Burda & Wyplosz, 2001). On the other hand, the PPP theory shows some promising results between similar countries within close proximity were transportation costs are low and barriers to trade are small. But first and foremost evidence pro the PPP theory is found in empirical research of the *long run*. Although, in the studies supporting the PPP theory, the speed of convergence towards equilibrium have been shown to be very slow (Rogoff, 1996). With the increased power of panel data more recent research has sprung some revival to the PPP theory and supplied support for the long run relative PPP. See for example Taylor and Sarno (1998) who argue that the hypothesis of PPP being mean reverting might hold even post Bretton Woods for the G5 countries. Pedroni (2001) on the other hand, rejects the hypothesis of the strong version of PPP for a panel of post Bretton Woods countries using the latest econometric techniques and conclude:

”…the failure of strong PPP appears to be pervasive in the post Bretton Woods period”.

With that being said, nominal exchange rates have been much more volatile than the relative changes in the price levels, and the deviations from the PPP level and the law of one price have been both substantial and prolonged. For many years researchers failed to conclude that there was any long-term relationship at all between PPP and the nominal exchange rate and several modern tests of cointegration between PPP and exchange rates have failed to reject the null hypothesis of a random walk. Some measurements of the return to equilibrium have shown that the deviations are corrected for at a rate of only 15% per year (Rogoff, 1996). Engel and Rogers (1995) found that relative prices are a function of the distance between marketplaces. But, they also conclude that when the placement of one market is found across the boarder there are unexplained differences in the prices (Engel and Rogers, 1995). Taylor (2003) stress the logical importance of a long data *span* rather than numerous observations in order to get high power in unit root and cointegration time series tests. Hence, increasing the observations by changing the data set from yearly observations into quarterly should show very small effects on unit root tests. Above that, Taylor points out that working with panel data, that is, "cross-sectional time-series data", have the same positive effect.

One should bare in mind the difficulties with measuring and evaluating the PPP theory. First, there are problems of comparing identical bundles of goods. A typical bundle of good for the rich Western countries doesn't withhold the same merchandises as a typical bundle from the developing world. There are also measurement problems between the Western countries. Even if the bundles contain the same type of merchandises the quality of these can obviously be different. Furthermore, there are good reasons to believe that separating traded goods from non-traded goods make the PPP more likely to hold since the price in traded goods will tend to be adjusted by international competition while the price of non-traded goods is determined primarily by domestic supply and demand relationships (Taylor, 2003). Above that, numerous difficulties may arise in time-series data; a bundle of goods may change over time, the consumption pattern within and hence between countries may shift, and price indices may divert from country to country.

The failure of short run PPP can be partly be attributed to stickiness in prices. Hence, when various factors affect the nominal exchange rate the real exchange rate is equally affected and the two "variables" move in tandem. In turn, there are many "chocks" that affect the real exchange rate (and consequently also the nominal). Here, the Balassa and Samuelson hypothesis and the net foreign position of a country play an important role. Note that even if traded prices are equalized so that the law of one price holds in the presence of a Balassa and Samuleson productivity effect the real exchange rate will move since the nominal exchange rate doesn't offset the price changes in the non-traded sector. This and other determinants of the real exchange rates is analyzed further in chapter 3. If we chose to instead illustrate the real exchange rate in terms of logged variables we have for example:

$$z_t = s_t + p_t{}^f - p_t \tag{2.5}$$

If PPP were to hold (2.5) above would equal zero! Hence, any movements in the real exchange rate corresponds to a deviation from PPP (Taylor, 2003).

2.3 The monetary model

Since the mid-70's there has been a substantial development of modern models of exchange rate determination. These models are called monetary models since they are primarily based on the supply and demand of money. The exchange rate should be regarded as a forward looking asset price. It's long run steady-state in determined by macroeconomic variables and the adjustment toward equilibrium is determined by rational agents expectations of the long run. While the PPP model primarily focuses its analytical weight on goods arbitrage the monetary models also include international capital movements, an important feature in an ever-growing global financial world.

Since the devastating results by Meese and Rogoff (1983) there has been an enormous evolvement of research that tries to determine its empirical soundness. Still, after almost 30 years (!), the question of whether the monetary model is able to outperform the naive random walk is still elusive. One should also note that there are many different specifications of the monetary model. Although many researchers have claimed success for various versions, the results of these articles aren't consistent (Engel and West, 2005). On the contrary, Cheung et al. (2002) point out with the result of their comprehensive study, that there isn't any dominant strength to any particular model.

2.3.1 The money demand

Since the birth of the monetary models exchange rates have been regarded as the relative price of two monies. Hence, monetary models attempt to model this relative price in terms of supply and demand variables for money. The demand for money is assumed to be determined by real income, the price level and the nominal interest rate (Taylor, 1995).

Money is a means of payment that is generally accepted. We *hold* it because we can use it as an aid of exchange. For the sake of this paper, our interest in money is primarily the currency, i e banknotes and coins[4], and why we hold it[5].

As mentioned above money isn't demanded for its' intrinsic value but rather from what it can buy us. The amount of services and goods that a given unit of money will buy depends on the price level. Thus, the nominal money demand is positively related to the price level. When prices go up, we demand more money to facilitate our transactions. If we take the price level into account we get the real money stock, M/P. Real money is demanded to carry out transactions and thus increases with real GDP (Burda & Wyplosz, 2001). The price and alternative cost to holding money is the nominal interest rate. Money itself holds no nominal interest rate, and even a negative real interest due to inflation. Thus, a higher interest rate discourages the will to hold money. We can write a simple real money demand function as:

$$M/P = \varphi(\underset{+}{Y}, \underset{-}{i}) \tag{2.6}$$

Here, the real stock of money is a function off the nominal interest rate *i* and the real income level *Y*. From the signs it's clear that the money demand increases with the real income level but decreases with the nominal interest rate.

2.3.2 Uncovered interest rate parity [UIP]

If we assume that capital is perfectly mobile an international investor can swift his money from one asset to another at any point in time. Let's say that the investor is valuing his options of either investing in a Swedish bond or in a Danish bond. If the bonds are regarded as equally risky and if they face the same maturity, the investor's choice depends upon their currency of denomination and the belonging interest rate (Pilbeam, 2006). Thus, the expected development of the respective currencies plays a large part on the basis of a decision.

We write the UIP condition as:

$$E\dot{s} = E(s_{t+1}) - s_t = r - r^* \tag{2.7}$$

[4] Formally called M0, M1 or Currency in Circulation depending on the country of interest.
[5] Although, we will also introduce a wider money aggregate for comprisal purpose.

The dot over *Es* denotes the change from one period to the next in the variable. According to the UIP condition the expected depreciation of one currency to another is equal to the difference between the interest rates of the Swedish bond to the Danish bond. Thus, an interest rate of 10% on the Swedish bond and equally 3% on a Danish bond would make the investor expect the SEK to depreciate 7%. This example illustrates the point of the UIP condition well; given perfectly agile capital movements and that the bonds are regarded as equally risky, the expected rate of return on the different bonds are the same.

2.3.3 A flexible-price monetary model

To set up the monetary model we make use of our previously described framework. The monetary model features both the UIP condition and that the PPP condition holds continuously, two rather bold assumptions. Following Francis et al. (2001) we will end up with the following 3 key equations:

$$\begin{aligned} s &= p - p^f \\ p &= m - \eta y + \sigma i \qquad (2.8) \\ p^f &= m^f - \eta y^f + \sigma i^f \end{aligned}$$

They are easily derived from a common real money demand function. We start with a description of domestic and foreign demand for real balances expressed in logarithms:

$$m_t - p_t = y_t - i_t \qquad (2.9)$$

Here, m is the domestic money stock, *p* the domestic price level, *i* the nominal interest rate and *y* the domestic production level. *f* indicates that it is a foreign variable. According to (2.9) the real money supply is dependent of the real level of production and the interest rate.

The foreign equivalence is identical:

$$m_t^f - p_t^f = y_t^f - i_t^f \qquad (2.10)$$

From (2.9) and (2.10) we get the money demand functions of the home and foreign country respectively. With some easy rearranging we get:

$$p_t = m_t - \eta y_t + \sigma i_t \tag{2.11}$$

$$p_t^{\,f} = m_t^{\,f} - \eta y_t^{\,f} + \sigma i_t^{\,f} \tag{2.12}$$

While the money supply has a unit effect on the money demand functions the income elasticity is $\eta > 0$, and the interest semi-elasticity $\sigma > 0$.

Assuming that the PPP holds continuously we have the nominal exchange rate:

$$s_t = p_t - p_t^{\,f} \tag{2.13}$$

Substituting (2.11) and (2.12) into (2.13) yields the monetary model:

$$s_t = m_t - \eta y_t + \sigma i_t - m_t^{\,f} - \eta y_t^{\,f} + \sigma i_t^{\,f} \tag{2.14}$$

Or,

$$s_t = \left(m_t - m_t^f\right) - \eta\left(y_t - y_t^f\right) + \sigma\left(i_t - i_t^{\,f}\right) \tag{2.15}$$

The above equation tells us that relative money supplies, relative levels of national income and relative interest rates determine the spot exchange rate. For the sake of this paper we can exclude the second last component of (2.15), (i_t-i_t^f), since it is assumed to be stationary. Using the UIP condition we can introduce expectations into (2.15). E is the normal expectations operator. Remembering the conclusion in (2.7) we have that

$$UIP = E\left(s_{t+1}\right) - s_t = \left(i_t - i^f{}_t\right) \tag{2.16}$$

Clearly, the above relationship doesn't include a stochastic trend why we conclude that the term is I(0). Substituting (2.16) into (2.15) we get the final model

$$s_t = \left(m_t - m_t^{\,f}\right) - \sigma\left(y_t - y_t^{\,f}\right) + u_t \qquad (2.17)$$

where u_t is equal to $u_t = \lambda\left[E_t\left(s_{t+1} - s_t\right)\right] + \delta_t$

Since both the components of the disturbance term, δ and $[E_t(s_{t+1}\text{-}s_t)]$, are I(0) that implies that also u_t is I(0). We assume that s, m, and y are I(1) and that Δs, Δm, and Δy are stationary. Then, their exist a linear combination of $s - (m - m^f) + \eta(y - y^f)$ that is also stationary given that the disturbance term u is I(0). Hence, the variables in the nominal exchange equation are cointegrated with the cointegration vector
$\beta = (1, -1\eta)'$ based on $(m - m^f)$ and $(y - y^f)$. See for example Taylor (1995), Groen (2002), or Westerlund and Basher (2009).

2.3.4 Interpreting the non-restricted exchange rate equation

First, we see in (2.15) that the relative money supply affect the exchange rate. The impact of the relative money supply on the exchange rate is equal to unity in the equation. That means that a 1% increase (decrease) of the home money supply would lead to an equally large depreciation (appreciation) of the currency. The explanation is that a χ% increase in the domestic money supply induce an equivalent increase in prices, χ%, and since PPP holds continuously this leads to a χ% depreciation of the currency. Secondly, relative levels of income also affect the exchange rates. Although here, the influence of $(y\text{-}y^f)$ isn't equal to unity as in the case of the relative money supply variable, but rather some arbitrary value, here set to η. η is the elasticity of relative levels of income on the exchange rate and measures the percentage influence a change in relative income have on the exchange rate. Again, the PPP comes into play. If there's a rise in the domestic income this would increase the transactions in the economy and equivalently the amount of money demanded. If the interest rate and the money supply are constant the only way to meet the increased demand of money is through lower domestic prices. To maintain PPP the exchange rate has to appreciate to make up for the fall in prices.

The last part of the exchange rate equation is the relative difference in interest rates. Again the impact isn't equal to unity but to some arbitrary value, here σ. As seen from the equation in (2.15) an increase in the domestic interest rate would lead to a depreciation of the domestic currency. The reason is that a rise in the domestic interest rate leads to a fall in the demand for money and that would in turn lead to a depreciation of the domestic currency.

3 Relaxing the PPP assumption

In the monetary model we have imposed a strong assumption that we will look further on now. We've assumed that the PPP condition holds continuously, and as mentioned earlier that is unlikely to be true. As for the *real* exchange rate, the PPP condition implies that z is constant over time. We know that z is constant when the nominal exchange rate fully offsets the relative discrepancy in the domestic and foreign price levels. We also know that in the case of fixed prices the nominal and the real exchange rate move in tandem. Experience and empirics tells us that z isn't constant, at least not in the short run, and that it reacts to several macroeconomic relationships. As mentioned the nominal exchange rate is determined by relative money supplies (Taylor, 1995). In turn, the domestic money supply determines the domestic price levels. If we rearrange the familiar equation of the log real exchange rate from (2.5) we have that the nominal exchange rate is equal to its PPP value plus the real exchange rate:

$$s_t = z + p_t - p_t{}^f \quad \text{or,} \tag{3.1}$$

$$s_t = z + (m_t - m_t{}^f) - \eta(y_t - y_t{}^f) + \sigma\big(i_t - i_t{}^f\big) \tag{3.2}$$

Hence, according to our previous framework the real exchange rate has entered as a fixed constant in our regular model. But as mentioned, it is unlikely that z is constant over time which is why we will also set up a monetary model that incorporates the determinants of the real exchange rate and the variables that are most likely to induce a discrepancy of the real exchange rate from its PPP value. We than have the following model:

$$s_t = c + z_t + (m_t - m_t{}^f) - \eta(y_t - y_t{}^f) + \sigma\big(i_t - i_t{}^f\big) \tag{3.3}$$

3.1 The real exchange rate

The real exchange rate has great importance in models of the open economy where international trade is the point of focus. If we define it with respect to CPI, or another general price level, and express it in terms of log variables we have a rather standard definition:

$$z_t = s_t + p_t{}^f - p_t \tag{3.4}$$

A rise in z means an appreciation of the real exchange rate. There are different ways of analyzing the real exchange rate but in the majority of the literature the importance of tradable goods prices are stressed. If we assume a world with rather liberal trade, there will be little scope for differing prices and the world competition will equalize the prices of traded goods. Non-traded goods, which are produced and consumed locally, are on the other hand not affected by the pressure of world competition. It is fruitful to express the general price level in terms of both traded (normal goods) and non-traded goods (typically services) price:

$$p_t = \alpha_t p_t^{NT} + (1 - \alpha_t) p_t^T \tag{3.5}$$

$$p_t^f = \alpha_t^f p_t^{NT(f)} + (1 - \alpha_t^f) p_t^{T(f)} \tag{3.6}$$

Where *NT* denotes non-tradable goods and *T* traded goods and α is the geometric weights of their shares in the economy. If we insert (3.5) and (3.6) into (3.4) we get a general expression of the long run equilibrium real exchange rate:

$$z_t \equiv (s_t - p_t^T + p_t^{Tf}) + \left[-\alpha (p_t^{NT} - p_t^T) + a^f (p_t^{NT(f)} - p_t^{T(f)}) \right] \tag{3.7}$$

The first term in (3.7) is the price of traded goods which will be equal to zero if PPP for traded goods holds. It's also possible that all goods are tradable but imperfect substitutes in which case the first term equals the equivalence in (3.5). But there are many variables that affect the two terms in (3.7) and it is likely that both can show non-zero characteristics (Chinn, 2006).

Equation (3.7) indicates that there are 3 ways in which the real exchange rate is affected: (i) the relative price of tradable goods is not constant and the traded bundles of goods are not perfect substitutes, (ii) possible differences in the domestic and foreign relative prices of traded to non-traded goods and, (iii) differences in α during different periods of time (MacDonald, 1997).

The real exchange rate also has a special function of balancing the PCA deficits and surpluses (the PCA is rarely balanced) in order to meet the intertemporal budget constraint of the government. If the PCA is needed to change to satisfy the budget constraint, then so must the real exchange rate since the latter constitutes the spur of change. For example, at a consistent negative primary current account, the real exchange rate will have to depreciate to improve the balance of payments. A depreciation would induce the producers to reallocate resources towards traded goods, and consumers to spend more of the income in domestically produced non-traded merchandises. Both changes will improve the primary current account (Burda & Wyplosz, 2001). The fact that the primary current account eventually must meet the budget constraint defines the long run real exchange rate. Since it's unlikely that the economy is at its' steady state equilibrium, it is also unlikely that the real exchange rate is equivalent to the long run equilibrium real exchange rate at any point in time (Lane and Milesi-Ferretti, 2002).

In the works of Lane and Milesi-Ferretti they stretch the fact that one should separate the long-run relation between net foreign assets and the real exchange rate and proposes a decomposition between the net foreign asset position and the trade balance, and in other hand the relation between the trade balance and the real exchange rate. They argue that a country with high production growth that is able to earn more on its' foreign assets than they'll have to pay on their foreign liabilities requires a smaller trade surplus to balance its net foreign asset position. Hence, the real exchange rate depreciation for a country that fulfil the above assumption can be smaller than the required depreciation for a country with less growth and inferior net investment flows (Lane and Milesi-Ferretti, 2002).

In an open-economy model the steady state can be described with the following equations:

$$tb = -r^* nfa \quad (3.8)$$

$$z = -\varphi tb + \lambda X \quad (3.9)$$

where *tb* is the trade balance as share of GDP, *r** the interest on external assets and liabilities, *nfa* the share of net foreign assets in relation to GDP, *z* is real exchange rate, and *X* stands for other deterministics that make up the real exchange rate. From the above equations we learn that a country can run a steady state trade deficit that equals the net investment income on the foreign asset position and that the real exchange rate will be more depreciated the larger the steady state trade surplus (Lane and Milesi-Ferretti, 2002).

Combining (3.8) and (3.9) yields:

$$z = \varphi(r^* nfa) + \lambda X \quad (3.11)$$

where we see that the real exchange rate increases with the net foreign asset position. That is somewhat restrictive according to Lane and Milesi-Ferretti due to differences in rates of return and different growth speeds.

3.2 The determinants of *z*

The first term of (3.7) relate to the relative price of tradable goods. If PPP applied continuously and if foreign and domestic tradable goods were perfect substitutes we would assume that the term is constant (or zero). But that is unlikely since, among other things, the inter-country quality between different goods is likely to divert. Apart from that, the relative price of traded goods has a large impact on the current account and the current account is affected by the determinants of national savings and investment. National savings and investments in turn, are to some extent determined by the national fiscal balance. Hence, we arrive at the conclusion that fiscal policy has an impact on the real exchange rate, but it is unclear whether it is positive or negative[1]. Another component regarding to national savings that affect the real exchange rate is private sector net savings (MacDonald, 1997).

The second term of (3.7) expresses the relative price of non-tradable goods in terms of tradable goods across countries. The phenomenon of changes in the relative price of traded to non-traded goods has been successfully described by the Balassa-Samuelson effect. Balassa and Samuelson observed that western countries have systematically higher prices than developing countries, even when expressed in the same currency. They argued that labour productivity is much higher in developed countries and, more importantly, that the productivity gain is concentrated to the traded goods sector rather than the non-traded goods sector (Pilbeam, 2006). Contrary, the productivity in the non-traded goods sector is assumed to be close to constant and equal over developed and developing countries. A western barber is likely to be as productive as a barber from a developing country, and it is equally likely that the productivity of barbering has been constant over the last centuries!

In the Balassa-Samuelson model wages are assumed to be the same between the traded and the non-traded goods sectors and positively linked to productivity. In turn, prices are linked to wages. It is also assumed that PPP holds for the traded sector only. Hence, the relative price of traded goods will be smaller for a developed country with high growth in the tradable goods sector. That is a direct consequence from the fact that PPP applies between countries in the traded goods sector at the same time as prices in the non-tradable goods sector rise faster in developed countries, due to the link between sector wages, even though the productivity between countries in the non-traded sectors are the same. Of such, a high productivity country or a country with a higher focus of production on traded goods tend to have a higher equilibrium exchange rate (Coudert, 2004).

While the Balassa-Samuelson impact on the real exchange rate is a supply-side effect, the relative price of tradable goods to non-traded goods might also be affected by a demand bias towards non-traded goods. That would occur if the demand income elasticity of non-traded goods is larger than one, meaning that the consumer demand of services is larger than the demand for normal goods (Coudert, 2004). That effect is enhanced in an economy where governments spending it guided towards the non-traded sector and if the size of the government grows over time. That implies wealth redistribution from private spending in the traded sector to the non-traded sector (MacDonald, 1997).

A part from the previously described deterministics the real exchange rate is affected by a number of different things. The structure of the production side of the economy is one of those which in turn is affected by its' resource endowments, comparative advantages and sectoral structures (Burda & Wyplosz 2001). The famous catching-up hypothesis described by Abramovitz may also play a crucial role during the transition period of country where the development of new industries probably shift the production from non-traded goods to traded goods. During this transitional phase it is also not unlikely that consumer preferences change although the net effect is somewhat unclear.

3.3 Modelling *Z*

From the reasoning above we have that the real exchange rate can be described as function of the following variables:

$$z = f(PROD, DEM, FISC, PS, TT)$$

Here, *PROD* is the Balassa-Samuelson productivity differential related to the relative price of tradables to non-tradables, *DEM* is the demand side bias towards non-tradables, *FISC* is the effect of fiscal policy on z, *PS* is private savings and *TT* is the terms of trade. Apart from *FISC* the effect of the variables works positively on the real exchange rate. Theoretically, a change in *FISC* will induce a depreciation according to the Mundell-Fleming two-country model. A tightening of fiscal policy would increase national savings, lower the real interest rate and improve the current account while the real exchange rate depreciates. But the Mundell-Fleming model ignores the effect that stock-flows have on the initial current account position. Models that do capture this effect, such as the Dornbusch model (1984), imply an increase in the net foreign assets and an appreciation of the real exchange rate (MacDonald, 1997).

Finding proxies for the variables that enter the real exchange rate function can be tricky. We will try to capture the essence with the inter-country differences between:

- The relative quota between exports and imports as a share of real GDP

- The relative productivity between countries measured as the relative difference between the labour force to real GDP ratio.

We have shown how the trade balance influence the real exchange rate directly in equation (3.8). But the trade balance also work as a proxy for the relative price of non-tradables. A country that runs persistent trade surpluses will have a negative wealth effect in order to maintain resource utilization below production. That will induce a downward pressure on the demand of non-tradable goods. The negative wealth effect also channels a secondary effect on the non-traded sector; raising the labour supply and lowering the total (wage) costs (Lane and Milesi-Ferretti, 2002).
Lane and Milesi-Ferretti note that with larger trade deficits leads to a real appreciation.

Relative output per worker, or productivity, is used as a proxy for the Balassa-Samuelson productivity effect since we don't have access to sectoral productivity data. It is also possible that a rise in the productivity is align with an increase of demand in the non-tradable sector through the above described taste channel (fortunately the effects move in the same direction!). The proxies are not a complete set of variables but are sufficient to examine whether real variables may have an effect on the nominal exchange rate or not. Later we will look for cointegration in the following model:

$$s_{it} = \alpha + \beta_1 \dot{m}_{it} + \beta_2 \dot{y}_{it} + \beta_3 \dot{tb}_{it} + \beta_4 \dot{pr}_{it} + e_{it} \tag{3.12}$$

where *tb* is the relative trade balance and *pr* is the relative productivity. The small dot over the right hand side variables indicate the difference between the domestic and foreign variables.

4 Econometric methodology & the data

4.1 The Data

The data is ranged from the first quarter of 1973, following the breakdown of the Bretton Woods-system, to the last quarter of 2007. There are five countries and one economic region included: Australia, Canada, the Eurozone, Japan, the United Kingdom and the United States. The UK and The US are the base countries and their respective currencies constitute the currency of denotation in two separate dataset samples. United Kingdom is added as a numeraire country to control for the risk of result misspecification due the high peak of the US dollar in the mid 1980s' which can be explained by a combination of sizable tax cuts and climbing interest rates, but also a strong speculative pressure in dollar assets that exacerbated the appreciation Groen (1998). This chock, as well as a common sharp downturn after 2002, is verified by visual examination of the data. Strangely, there is no trace in the data of the British downturn in the seventies.

The GDP variables are expressed in fixed prices, and the monetary variables are chosen to ensure comparability between regions. We will look at a model with both a narrow definition of money as well as a broader definition, in order to examine the sensitivity of the results from choosing between different money aggregates. Both the real GDP variables and the two different sets of money variables are seasonally adjusted using the U.S. Census bureau's X12 method.

The data is downloaded from Datastream and processed in Eviews and Excel. All variables are in natural logarithms. Real data for the Eurozone was introduced in 1999 when the euro as currency was first launched and the European Central Bank was established. For empirical purposes a dataset with Eurozone aggregates called "the Area Wide Model" has been published containing constructed data from 1970. We have used this constructed AWM-data for the Eurozone[6].

[6] See http://www.eabcn.org/area-wide-model for a thorough explanation of the AWM-data.

When we introduce the "*real*" variables we lose the five first years of data since we were unable to find sufficient observations for certain variables.

4.2 Unit root and cointegration econometrics

The normal characteristics of the OLS-estimator provide that the data series in time-series models are stationary. This is not always the case which is why economists have to be aware of the risk of encountering spurious regressions. But a regression containing one or more unit roots can still be of great economic interest since it may hold a long term relationship, i.e. *cointegration*. When testing for the presence of a unit root in a variable one examines if its' lagged value is equal to 1. If we consider a simple AR(1)-model we have the Dickey-Fuller specification $\Delta y_t = \beta_0 + \gamma y_{t-1} + e_t$. Our hypothesis is to determine whether or not y_i has a unit root. Hence we test:

$$H_0 : \gamma = 0 \Leftrightarrow (\beta_1 = 1) \text{ against}$$
$$H_1 : \gamma < 0 \Leftrightarrow (\beta_1 < 1)$$

Here, rejecting the null hypothesis is indication of the presence of a unit root. The Dickey-Fuller test could easily be extended to consider a higher lag order, AR(*p*) processes[7].

The Engle-Granger two-step approach is another essential and basic econometric framework. If there exists a cointegration relationship between two or more non-stationary variables cointegration means that there is still a linear combination of these series that is stationary. Hence, in a simple two variable model with to I(1) series there exists a cointegration relationship if the error terms, e_t, are I(0). The first step is to estimate your model followed by the second stage where you check whether the error terms contain a unit root or not. The Engle-Granger approach draws with several flaws if there exists more than one cointegration relationship. If so, the test is bound to find cointegration, since the null hypothesis is that of no cointegration, but the resulting vector will be a linear combination of the true cointegration vector. The E-G approach is also sensitive to the normalization of variables (the LHS variable)

[7] See for example Verbeek.

and lacks power due to the fact that a regression on the residuals doesn't capture all of the dynamics in the variables[8].

The Johansen approach is developed for a multivariate setting where more than one cointegration relationship may exist. The test doesn't draw with the same problems as the E-G approach but is instead quite sensitive. For the multivariate case Johansen has developed a maximum likelihood approach that makes it possible to test the number of *cointegrating ranks*. If we have a set of *k* I(1)-variables it is possible that their exists *k*-1 stationary linear relationships. As mentioned above, it implies the problem of identifying the individual cointegration vectors which is why econometricians in the multivariate framework rather refer to the term *cointegration space*. If we consider a first order VAR system for the exchange rate equation we have:

$$S_t = \delta_1 + \theta_{11}S_{t-1} + \theta_{12}M_{t-1} + \theta_{13}Y_{t-1} + \varepsilon_{1t}$$
$$M_t = \delta_2 + \theta_{21}S_{t-1} + \theta_{22}M_{t-1} + \theta_{23}Y_{t-1} + \varepsilon_{2t}$$
$$Y_t = \delta_3 + \theta_{31}S_{t-1} + \theta_{32}M_{t-1} + \theta_{33}Y_{t-1} + \varepsilon_{3t}$$

Which is equivalent to

$$\begin{pmatrix} S_t \\ M_t \\ Y_t \end{pmatrix} = \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{pmatrix} + \begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \\ \theta_{31} & \theta_{32} \end{pmatrix} \begin{pmatrix} S_{t-1} \\ M_{t-1} \\ Y_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix}$$

If we consider a general *p*-dimensional VAR for a *k*-dimensional vector y_t of non-stationary variables instead and use conventional definitions we have:

$$y_t = \delta + \emptyset y_{t-1} + \cdots + \emptyset_p y_{t-p} + \varepsilon_t$$

where δ is a vector of deterministic variables and ε a *k*-dimensional vector of white noise error terms with covariance matrix ∑ since $\varepsilon_{1t}, \ldots, \varepsilon_{kt}$ might be correlated. In this case there might be one or more cointegration vectors β that make Z= $\beta' y_t$ stationary.

[8] See for example Harris & Sollis or Enders.

We can write the VAR model as a vector error correction model (VECM) representation. If we take for example a VAR(3) model we can write it as:

$$
\begin{aligned}
\Delta y_t &= \delta + (\phi_1 - I_k) y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \varepsilon_t \\
&= \delta + (\phi_1 - I_k) y_{t-1} + \phi_2 y_{t-1} - \phi_2 \Delta y_{t-1} + \phi_3 y_{t-3} + \varepsilon_t \\
&= (\phi_1 + \phi_2 + I_k) y_{t-1} - \phi_2 y_{t-1} + \phi_3 y_{t-3} + \varepsilon_t
\end{aligned}
$$

where $\phi_2 \Delta y_{t-1} = \phi_2 (y_{t-1} - y_{t-2})$ and I_k is a *k*-dimensional identity matrix. If we continue to write the VAR(*3*) in the same manner and group similar variables we get

$$
\begin{aligned}
&\Delta y_t = (\phi_1 + \phi_2 + \phi_2 - I_k) y_{t-1} - \phi_2 y_{t-1} +\acute{}-\phi_3 (\Delta y_{t-1} + \Delta y_{t-2}) + \varepsilon_t = \\
&\delta + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + (\phi_1 + \phi_2 + \phi_2 - I_k) y_{t-1} + \varepsilon_t
\end{aligned}
$$

For a general VAR(*p*) model we can write the last coefficient terms as $\prod = (\phi_1 + \ldots + \phi_p - I_k)$ which is a coefficient matrix that determines the properties of y_t in the long run. The VAR(p) specification is then:

$$
\Delta y_t = \delta + \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_{p-1} \Delta y_{t-p+1} + \Pi y_{t-1} + \varepsilon_t
$$

where Πy_{t-1} is stationary since we assume that e_t and Δy_t are stationary. If the variables y_t are non-stationary the elements in Πy_{t-1} are I(0) and represent linear combinations that correspond to cointegration vectors of the order *r*, where the rank *r* of Π is: $0 < r < k$ and *k* is the number of variables in the y_t vector. That is, there are *r* linear stationary combinations in the vector y_t (cointegration relationships). When *r* meets the previously stated condition we say that Π has reduced rank and can write an error correction model of the form:

$$
\Delta y_t = \delta + \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_{p-1} \Delta y_{t-p+1} + \gamma \beta' y_{t-1} + \varepsilon_t
$$

where β is the matrix of coefficients and γ is the speed of adjustment, and the whole term represents the r cointegration relationships. In the error correction model only the cointegration relationships are included or Πy_{t-p} wouldn't be stationary. Hence, there are (k-r) columns of γ that are insignificant. Thus, the testing of cointegration is the same as testing which columns of γ that are zero or finding the r linearly columns in Π.

It is possible to test the null hypothesis with either the trace statistic or the max eigenvalue statistic. The trace statistic tests the null of r cointegration relationships against the alternative hypothesis of k cointegrating relationships. K relationships is equivalent with $r = 0$ and all variables being stationary. The trace statistic is computed as:

$$\lambda_{trace}(r_0) = -T\sum_{j=r+1}^{k}\log\left(1-\hat{\lambda}_j\right)$$

The max eigenvalue statistic is equal to:

$$\lambda_{\max}(r_0) = -T\log\left(1-\hat{\lambda}_{r+1}\right)$$

and tests the null hypothesis of r cointegration relations against the alternative of $r+1$ cointegration relations[9].

4.3 The Panel Case

A panel data set contains several observations over the same cross-section dimension i, stretched like time-series data over a certain period of time t. Panel data allows the researcher to examine multiple source data from heterogeneous individuals simultaneously, be that from different countries, firms or individuals. That allows for more advanced and realistic models than a single cross-section or time-series would. Single Dickey-Fuller tests have lower power in detecting stationary series than the pooled equivalent. The gain comes from the large number of observations that the two dimensions of time-series and cross-section data give, leading to greater efficiency and accuracy when estimating the model parameters. The drawbacks are primarily applicatory; when estimating the same cross-section variables over and over again

[9] See for example Verbeek or Harris & Sollis.

there is likely to be a loss of independence between the observations. One might also encounter the problem of missing observations.

4.3.1 Panel Unit Root

With the increasing knowledge of the advantages of panel data there have been a surge in panel data unit root and cointegration procedures. In contrast to single time-series test, assumptions have to be made about cross-sectional dependence in the error terms. The solution in most procedures is to simply assume that e_{it} is cross-sectionaly independent. That assumption might oppose the empirical insight that many tests perform poorly when the error terms are in fact correlated. Another problem regards heterogeneity or serial correlation in the model parameters across different individuals, such as countries. Failing to recognise heterogeneity may likely produce biased results and poor testing performance. Given the complexity in accurately modelling panel unit root tests there has sprung a wide set of tests. Their main functional difference applies to the testing of either individual unit root processes or tests with a common unit root process.

Levin and Lin (1992) have proposed a test where they assume that the error terms are IID(0, σ_e^2) across countries and time, hence individual processes that are identical over the whole cross-section panel. That implies an alternative hypothesis where all the series have the same stationary parameter, $\rho<1$, for each country/firm etc. Originally they also imposed the strong assumption that the error terms are cross-sectionaly independent. Since their first publication the test has been developed to account for serial correlation. In the new model by Levin, Lin and Chu (2002) they propose the following specification[10]:

$$\Delta y_{it} = \rho * y_{i,t-1} + \sum_{L=1}^{pi} \theta_{iL} \Delta y_{i,t-L} + z'_{it} \gamma + u_{it}$$

Note that $\rho=\rho^*$ and $p=p_i$ (lag specification) for all individuals, i. The hypothesis to be tested are:

$$H_0 : \rho^* = (\rho-1) \Leftrightarrow \rho=0 \text{ (in the I(0) model)}$$

[10] See Harris & Sollis.

$$H_1 : \rho^* < 0 \quad \Leftrightarrow \rho < 1$$

Here, failure to reject the null hypothesis implies that all the series contain a unit root, while the alternative hypothesis implies that all the series are stationary. The procedure consists of individual ADF-tests for each specification *i*. Unlike the original model LLC show that their *t*-statistic for the estimation of ρ is asymptotically normally distributed.

In contrary to the LLC test where homogeneity in ρ was assumed Im, Pesaran and Shin (IPS) proposed a similar test with the important difference that the homogeneity restriction was relaxed allowing ρ_i to vary across the individuals *i*. Hence, the IPS test is a test with individual unit root processes. Other than that, the test is based on basically the same principals. You may, or may not, want to exclude the deterministic time trend, though one has to be consistent in his choice. The IPS test is also based on individual ADF regressions. Just as in the previously described case the lag length, p_i, is allowed to vary between the panel cross sections. It is important to carefully chose the p_i lag orders or the error terms will be serially correlated, making the critical values biased. We have the following ADF specification:

$$\Delta y_{it} = \rho_i * y_{i,t-1} + \sum_{L=1}^{pi} \theta_{iL} \Delta y_{i,t-L} + z'_{it} \gamma + u_{it}$$

where the hypothesis are:

$$H_0 : \rho_i^* = 0$$
$$H_1 : \rho_i^* < 0$$

The null hypothesis is that each series contain a unit root, while the alternative hypothesis is that at least one of the series is stationary. If the null hypothesis is rejected there is no way of telling which series that or non-stationary and which ones are not. The test statistic follows a student t distribution that can be approximated by a normal distribution as T and N → ∞. The IPS test runs an individual ρ for each cross-section *i* and averages the result to get the *t*-statistic:

$$\bar{t} = \frac{1}{N}\sum_{i=1}^{N} t_{\rho^*}$$

Each regression of the ISP specification will yield an individual ρ for each i that is run. $t_{\rho*}$ is the individual t-statistic for each i. This is a rather simple tests based on the premise that if the individual parameter estimations are unbiased then so will the average be.

Another alternative test of individual unit root processes is the Maddala and Wu (1999) Fisher ADF test. In their test they use all the corresponding p-values for *rejecting the null* that is generated from individual unit root tests for each i tested. The combined p-values gives:

$$P = -2\sum_{i=1}^{N} \ln p_i$$

which has a χ^2-distribution with 2N degrees of freedom. The above P-statistic can be used for any unit root test statistic and is easy to compute. Although it is similar to the IPS test which averages the ADF t-statistics from each cross-section individual, averaging the p-values (for rejecting the null) shows better results than both the IPS and the LL test[11].

4.3.2 Panel cointegration

The benefits of panel unit root tests in terms of increased power also apply to panel cointegration tests. There are two fields of cointegration tests; one that tests whether or not there exists a cointegration relationship at all, and one that identify the cointegration vector.

Kao (1999) and Pedroni (1995, 1999) have developed a single equation test with the null hypothesis of no cointegration. McKoskey and Kao (1999) have developed a residual based test where the null hypothesis instead is cointegration. Both the Kao and Pedroni tests are based on the Engle-Granger (1987) cointegration test; if the residuals of a spurious regression from a model with I(1) variables are I(0) the

[11] See for example Baltagi.

variables are cointegrated. If the residuals are I(1) the model is in fact a spurious regression[12].

The Kao (1999) test is a panel data approach very similar to the Engle-Granger two-step procedure. Starting with a simple panel regression model:

$$y_{it} = x'_{it}\beta + z'_{it}\gamma + e_{it}$$

where y and x are assumed to be I(1). As described above the Kao test doesn't allow β to vary over the *i* individuals, thus assuming homogeneity. The error terms are equal to:

$$\hat{e}_{it} = \rho\hat{e}_{i,t-1} + \upsilon_{it}$$

If the error terms are I(0) than there exists a cointegration relationship. Hence, we test the null hypothesis:

$H_0 : \rho = 1$

$H_1 : \rho < 1$ which means that x and y are cointegrated.

There are five versions of the Kao test. All the tests are asymptotically distributed under the standard normal distribution.

The Pedroni test however relaxes the assumption of homogeneity and suggested the following model:

$y_{it} = \alpha_i + \delta_t t + \beta_{1i}x_{1i,t} + \beta_{2i}x_{2i,t} + ... + \beta_{Ki}x_{Ki,t} + e_{it}$ with the residuals

$\hat{e}_{it} = \rho_i\hat{e}_{i,t-1} + \upsilon_{it}$

As we can read in the equation α_i and β_i can vary across *i*, hence the test allows for heterogeneous intercepts and slope coefficients. Here, the null is no cointegration:

[12] See Harris & Sollis.

$H_0 : \rho_i = 1$

$H_1 : \rho_i < 1$ (between-dimension) or $\rho_i = \rho < 1$ (within-dimension)

The alternative hypothesis depends on how the model in (X) is specified in regards to the nature of α_i and $\delta_t t$; either a within-dimension specification that pools ρ across the different individuals so that $\rho_i=\rho$ or a between-dimension specification that allows for heterogeneity that is based on an average of the individual ρ_i's. Pedroni has shown that the less restricting between-dimension estimator perform better in small samples. All together there are seven different tests within the Pedroni framework. The test statistics are normally distributed.

The combined Fisher/Johansen test was developed by Maddala and Wu (1999) from the works by Fisher (1932) on individual significance levels for rejecting the null hypothesis. Much as described above, they propose a test where they combine the individual cross-section cointegration tests to obtain an aggregated test for the entire panel data set. If π_i is the *p*-value from an individual cointegration test for cross-section *i*, the test statistic is:

$$P = -2\sum_{i=1}^{N} \log(\pi_i)$$

which has a χ^2 distribution with 2N degrees of freedom as $T_i \rightarrow \infty$ for finite *N*. To drawback with the combined Fisher test is that to get exact critical values one have to calculate these with Monte Carlo simulations[13].

[13] See for example Baltagi.

4 Econometric evaluation

4.1 Individual tests

We have constructed individual unit root and cointegration tests in order to compare with the panel case. We start with determining the order of integration in the series. In the vast majority of the cases we were unable to reject the null hypothesis of presence of a unit root in the series. There are some conflicting results depending on whether you include a trend or not. In those cases we performed a standard Dickey-Fuller test according to the procedure recommended in Enders (2004)[14]. We concluded that there are four variables that are stationary according to the tests; the narrow money supply of Australia^{US} and Japan^{US}, and the exchange rates of Australia^{UK} and Canada^{UK}. Overall, the evidence of non-stationarity is strong and we went on performing cointegration tests for the full set of countries.

In the table below we show the *t*-values from testing whether or not γ is equal to zero in the following test regression: $\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \sum \beta_i \Delta y_{t-1} + \varepsilon_t$. In some cases the trend was insignificant implying that $a_2 t = 0$. The stars (*) indicate stationarity and *c/ct* indicate the deterministic components included in the model; a model including both a constant and a trend or a model holding only the constant. The second relative money term, $(m - m^f)^w$, constitute the wide money aggregate.

Individual unit root tests. US numeraire.

	s		$(m-m^f)$		$(m-m^f)^w$		$(y-y^f)$	
	c	ct	c	ct	c	ct	c	ct
Australia	-1,437	-0,573	-4,041*	-2,912	-0,803	-2,541	-1,560	0,087
Canada	-0,822	1,247	-0,294	-2,354	-3,162	-2,632	-1,411	-0,326
Eurozone	-1,732	-1,315	-0,710	-0,326	-1,125	0,087	-0,235	-2,894
Japan	-1,491	-1,907	-0,220	-5,506*	-0,903	-3,201	0,111	-3,201
United Kingdom	-2,563	-2,087	-0,521	-2,894	-1,451	-1,295	0,757	-1,295

14 This is only done in the cases where we reject the null hypothesis of a unit root since the model including a trend and a constant is the least restrictive one.

The lag order was automatically chosen according to the Schwarz information criterion. The result for the UK numeraire sample was very similar to the US equivalence. Further, there were only insignificant differences between the narrow and wide money aggregates. The overall indication of non-stationarity was in line with our expectations and we continued with individual cointegration tests.

Individual unit root tests. UK numeraire.

	s		$(m-m^f)$		$(m-m^f)^w$		$(y-y^f)$	
	c	ct	c	ct	c	ct	c	ct
Australia	-2,032	-3,790*	-1,380	-1,729	-1,547	-1,397	-0,959	-3,944
Canada	-3,198*	-3,213	-1,715	-2,965	1,440	-2,702	-2,869	-2,504
Eurozone	-2,567	-2,756	-0,233	-2,616	-1,245	-1,113	-1,597	-2,434
Japan	-2,279	-0,933	-1,878	-1,923	5,497	-0,124	-1,560	-1,016
United States	-2,563	-2,087	-0,521	-2,894	-1,451	-1,295	-1,676	-1,761

4.1.2 Individual cointegration tests

The individual cointegration tests were done with the Johansen test procedure described in chapter 3. The lag order was selected using the Akaike and Schwarz information criterion. We will display the results of two different models of deterministic components in the auxiliary regression. The first is “Model 3” which implies a linear trend in the data, and an intercept but no trend in the cointegrating equation (the error correction term). “Model 4” is specified with a linear trend in the data, and both an intercept and a trend in the cointegrating equation. In the case of conflicting results we have tried to narrow down the most likely model using the so-called Pantula principle; moving “down” from the least restrictive model and continue until the first time the null hypothesis is not rejected[15]. In that case we’ve also looked at Johansen model 2 in which the level data are absent of deterministic trends, while the cointegrating equations have intercepts. We’ve also chosen to include the test results of an alternative model using four lags (in level)[16] instead of the suggested lag order according to the Akaike and Schwarz criterion. The reason is twofold; the Akaike and Schwarz information criteria tend to suggest a fairly short lag length in VAR specifications. At the same time inclusion of too few lags may lead to the wrong conclusion that the model is stationary (Chinn & Moore, 2009). These alternative results are displayed in parenthesis.

[15] See for example Harris & Sollis.
[16] Implying 3 lags in difference as in the case of a Johansen test.

US numeraire

From the table below it is very clear that increasing the number of lags has a devastating effect on the chase of cointegration. Using the Pantula principle we conclude that there is evidence of one cointegrating rank in the case of Japan and Canada. This, clearly, isn't overwhelming results in favor of the monetary model. If we look at the results from the tests with lag order 4 the picture looks even more catastrophic; illustrating only a weak indication of cointegration for Canada and Japan.

Johansen summary of individual cointegration tests. US numeraire.

Lag:*	Model 3		Model 4	
	Trace statistic	Max eigenvalue	Trace statistic	Max eigenvalue
Australia	0 (1)	0 (0)	0 (0)	0 (0)
Canada	1 (1)	1 (0)	1 (1)	2 (1)
Eurozone	0 (0)	0 (0)	0 (0)	0 (0)
Japan	1 (0)	1 (0)	1 (1)	1 (0)
United Kingom	0 (0)	0 (0)	1 (0)	1 (0)

In the case where we include a wider definition of money in the monetary model the results are very conflicting depending on whether you look on the trace statistic or the maximum eigenvalue statistic. The only clear-cut indication is found in the Japanese case where the tests suggest one cointegrating rank. In the case of Canada and the Eurozone, where Model 3 explains the most appropriate deterministic, the Johansen test suggests either 2 or 0 cointegration ranks for Canada, and 1 or 0 for the Eurozone. For Australia and United Kingdom there is no evidence of cointegration.

Johansen summary of individual cointegration tests. US numeraire. MW model.

	Model 3		Model 4	
	Trace statistic	Max eigenvalue	Trace statistic	Max eigenvalue
Australia	0 (0)	0 (0)	0 (0)	0 (0)
Canada	2 (1)	0 (1)	2 (2)	1 (2)
Eurozone	1 (1)	0 (2)	1 (2)	1 (2)
Japan	1 (0)	1 (0)	1 (1)	1 (1)
United Kingom	0 (0)	1 (1)	0 (0)	0 (0)

UK numeraire

The results are worse when we use the British Sterling as the numeraire. From the below table we conclude that there isn't evidence of cointegration in a single case (model 4 holds the relevant deterministics for Japan according to the Pantula principle)!

Johansen summary of individual cointegration ranks. UK numeraire.

Lag:*	Model 3		Model 4	
	Trace statistic	Max eigenvalue	Trace statistic	Max eigenvalue
Australia	0 (0)	0 (0)	0 (0)	0 (0)
Canada	0 (1)	0 (1)	0 (0)	0 (0)
Eurozone	0 (1)	0 (1)	0 (1)	0 (0)
Japan	1 (2)	1 (1)	0 (1)	0 (0)
United States	0 (0)	0 (0)	0 (0)	0 (0)

In the wide money model of the UK sample there is suggested evidence of 1 cointegration rank in the case of Japan. These cases will be further studied in a VECM model.

Johansen summary of individual cointegration tests. UK numeraire. MW model.

	Model 3			
	Trace statistic	Max eigenvalue	Trace statistic	Max eigenvalue
Australia	0 (0)	0 (0)	0 (0)	0 (0)
Canada	0 (1)	0 (1)	0 (1)	0 (0)
Eurozone	0 (0)	0 (0)	0 (0)	0 (1)
Japan	1 (0)	1 (0)	1 (0)	1 (0)
United States	0 (0)	0 (0)	0 (0)	0 (0)

4.1.3 The short-run VECM model

There is a crucial difference between the standard VAR model in first difference and the VEC model in that the latter allows you to study both the long and the short run relationships between a set of variables since the VECM includes the cointegration space among the series. As such, the error correction term measures the speed of adjustment towards the long-term equilibrium; a proportion of the disequilibrium in the current period is adjusted for in the next period, while the "normal" slope coefficients describe the short-run dynamics of the model. Hence, the change in a variable is sensitive to the gap between the variables in the previous period as well as changes in the variables. We showed how the VAR could be transformed into a VEC in chapter 3, but to grasp the dynamics it's easier to illustrate a simpler error-correction model. For that we refer the reader to the Appendix.

4.1.4 VECM output

Turning again to the sample with the US dollar as denominator, estimating the normalized cointegrating vector for Canada and Japan yields the following results:

$$s_t^{JPN} = -21.8 - 3.4\dot{m} + 2.5\dot{y}$$

$$s_t^{CAN} = -8.1 + 2.3\dot{m} - 6.4\dot{y}$$

The first, illustrating the monetary model between Japanese Yen and American dollars, doesn't seem to correspond to an economically interpretable long-run relationship and the coefficients are of the wrong sign! Hence, we assume that our previous conclusion of one cointegration rank is probably false. In the case of Canada however, the signs are correct and the coefficients are significant. Although, the size of the coefficients are disturbingly large, probably due to the large and negative intercept. Using the principle of Pantula again the test actually suggests that the "true" model for Canada is a model without the intercept term in the VAR model but an intercept in the cointegration equation while the most appropriate model in the case of Japan is suggested by the lack of intercept and trends in both the VAR model and in the cointegrating equation. The results are then reversed, indicating no cointegration at all. From the error correction table below we see that the error correction term has the wrong sign in the case of Canada. The money variables are significant in both cases but have the wrong sign in the Canadian regression. The variables in the Japanese VECM is of the correct sign, but the error correction term is insignificant. It's safe to conclude that there isn't any real evidence of cointegration.

Exchange rate estimation results CAN/$ and JPN/$

	[Canada]	[Japan]
Error correction term	**0.04**	-0.03
Lag money	**-0.03**	**0.03**
Lag income	0.001	**-0.001**

Notes: Estimates of VECM model with 1 lag of first differences, not all off which are reported.
Bold style indicates significance at the 10% level.

The VECM representations of the other models that were suggested to have a cointegrating relationship had coefficient far more bizarre why we chose to not display them. We simply conclude that there is very scarce evidence of cointegration in *any* of the individual cases.

4.1 Panel Unit Root Tests

Below we will demonstrate the results from the panel unit root tests for the two different samples. We will display the test statistics for the fixed effects model, where an individual intercept term in included (test statistic C), as well as a model with both an intercept and a time trend (test statistic CT). The lag order was automatically chosen using the Schwarz information criteria.

Here, indication of the presence of a unit root at the 5% significance level is denoted by a star and the corresponding *p*-values are in parenthesis. Notice that there are two terms with (m-m^f) corresponding to the narrow money aggregate and the wider one.

T1.3 Panel Unit Root Tests. US *numeraire*.					
	Type of test	Null hypothesis	Lag order[1]	Test statistic C[23]	Test statistic CT
s	LLC	Common unit root	0	-0,28* (0.39)	4.70 *(1.00)
	Breitung	Common unit root	0	3.02* (0.99)	3.84* (1.00)
	IPS	Individual unit root	0	0.20* (0.42)	3.62* (1.00)
	Fisher ADF	Individual unit root	0	9.13* (0.51)	2.37* (0.99)
	Hadri	Common unit root[4]	Bartlett Kernel	12.8* (0.00)	6.77* (0.00)
$(m\text{-}m^f)^n$	LLC	:	1 to 7	-0.61* (0.27)	-2.72 (0.00)
	Breitung	:	1 to 7	1.24* (0.89)	1.16* (0.88)
	IPS	:	1 to 7	0.80* (0.79)	-1.80 (0.04)
	Fisher ADF	:	1 to 7	13.8* (0.18)	28.98 (0.00)
	Hadri	:	Bartlett Kernel	14.6* (0.00)	10,67* (0.00)
$(m\text{-}m^f)^w$	LLC	:	1 to 3	-1.77 (0.04)	1.25* (0.89)
	Breitung	:	1 to 3	-0.56*(0.29)	1.84* (0.97)
	IPS	:	1 to 3	-2.64 (0.00)	-1.77 (0.04)
	Fisher ADF	:	1 to 3	41.93 (0.00)	43.73 (0.00)
	Hadri	:	1 to 3	17.04* (0.00)	12.35* (0.00)
$(y\text{-}y^f)$	LLC	:	0 to 3	1.93* (0.97)	-0.63* (0.26)
	Breitung	:	0 to 3	-0.33* (0.37)	-0.13* (0.45)
	IPS	:	0 to 3	2.75* (1.00)	-0.70* (0.24)
	Fisher ADF	:	0 to 3	2.72* (0.99)	12.33* (0.26)
	Hadri	:	Barlett Kernel	17.2* (0.00)	11.27* (0.00)

[1]The lag order is based on the Schwarz information criterion and may vary depending on country.

[2]We display test statistics based on the deterministic of both a constant and a constant and trend.

[3]The test statistics: LLC t*-statistic, Breitung t-statistic, IPS avarage t-statistic, F-ADF χ^2-statistic, Hadri Z-statistic.

[4]Hadri null hypothesis is of no unit root.

We can read of the table that the evidence of possible unit roots is strong although there are some deviating values, primarily in the wide money variable which is comforting since we have a (more relevant) control term in the narrow equivalence.

The test of the UK numeraire subsample show similar results. Almost every test of every variable implies the presence of a unit root. Hence, we conclude that the variables are I(1) for both subsamples and that it makes sense to continue with cointegration tests.

	Type of test	Null hypothesis	Lag order	Test statistic C	Test statistic CT
s	LLC	Common unit root	0	-1.91* (0.03)	0.74* (0.77)
	Breitung	Common unit root	0	0.67* (0.75)	0.64* (0.74)
	IPS	Individual unit root	0	-2.60 (0.00)	-1.07 * (0.14)
	Fisher ADF	Individual unit root	0	22.7 (0.01)	16.8* (0.08)
	Hadri	Common unit root[1]	Bartlett Kernel	13.76* (0.00)	9.06* (0.00)
$(m\text{-}m^f)^n$	LLC	:	0 to 1	-0.39* (0.35)	0.20* (0.58)
	Breitung	:	0 to 1	2.66* (0.99)	1.61* (0.95)
	IPS	:	0 to 1	1.00* (0.84)	-0.71* (0.24)
	Fisher ADF	:	0 to 1	5.32* (0.87)	11.53* (0.32)
	Hadri	:	Bartlett Kernel	16.49* (0.00)	5.25* (0.00)
$(m\text{-}m^f)^w$	LLC	:	0 to 2	3.18* (1.00)	0.66* (0.75)
	Breitung	:	0 to 2	-0.50* (0.31)	1.29* (0.90)
	IPS	:	0 to 2	0.54* (1.00)	2.42* (0.99)
	Fisher ADF	:	0 to 2	3.39* (0.97)	3.60* (0.96)
	Hadri	:	Bartlett Kernel	17.72* (0.00)	8,65* (0.00)
$(y\text{-}y^f)$	LLC	:	0 to 1/0 to 2	-2.21 (0.02)	-0.17*(0.43)
	Breitung	:	0 to 1/0 to 2	0.68* (0.75)	0.74* (0.77)
	IPS	:	0 to 1/0 to 2	1.26* (0.90)	-1.01* (0.16)
	Fisher ADF	:	0 to 1/0 to 2	11.67* (0.31)	15.49* (0.12)
	Hadri	:	Barlett Kernel	18.52* (0.00)	11.28* (0.00)

T1.4 Panel Unit Root Tests. UK *numeraire*.

4.2 Panel cointegration tests

We have constructed panel cointegration tests based on both the Pedroni test and the Kao test, as well as the Johansen cointegration test. The results of the Pedroni test is consistent regardless what model one chooses. Again *c* is an individual constant and *ct* involves a model with both an individual constant and a trend. Model 1 constitutes the monetary model with the narrow definition of money and Model 2 is the "wide" equivalence. The lag selection was automatically chosen based on the Schwarz criterion, and was specified as lag order p=0. We display the result of both the within estimator and the between estimator.

T1.3 Panel Cointegration Tests. US *numeraire*.				
	Model 1		Model 2	
Type of test	Within	Between	Within	Between
	c	*c*	*c*	*c*
Pedroni[1]				
v-Statistic	0.496* (0.353)	—	0.602* (0.333)	—
rho-Statistic	-1.095* (0.219)	0.718* (0.308)	0.563* (0.341)	1.381* (0.154)
PP-statistic	-1.635* (0.105)	0.344* (0.376)	0.980* (0.247)	1.929* (0.062)
ADF-Statistic	-1.516* (0.126)	0.817* (0.286)	1.420* (0.146)	2.756 (0.009)
	ct	*ct*	*ct*	*ct*
	0.504* (0.351)	—	0.504* (0.351)	—
	0.235* (0.388)	1.338* (0.163)	-0.010* (0.399)	0.963* (0.251)
	0.480* (0.355)	2.108 (0.043)	0.226* (0.389)	1.512* (0.127)
	1.293* (0.173)	3.012 (0.004)	1.121* (0.213)	2.773 (0.008)
Kao[2]	*c*		*c*	
ADF t-Statistic	-2.206 (0.014)		-1.78* (0.038)	

[12]Null hypothesis of no cointegration.

The Pedroni and Kao null hypothesis are of *no* cointegration. The Kao test only allows specifying a model with individual intercepts and imposes homogeneity; the slope coefficient in the auxiliary model is fixed across the individuals in the panel. This is restrictive and may influence the power of the test. Pedroni has constructed four different models of dynamics in order to correct for serial correlation in the error terms. Above that, Pedroni specifies two alternative hypothesis; a homogenous alternative called the within-dimension test where the auxiliary slope coefficients are fixed, $(p=p_i) < 1$ for all i, and a heterogeneous alternative where $p_i < 1$ for all i, called the between-dimension statistic test. The interpretation of the alternative hypothesis depends on the assumptions of the underlying data generating process. For example, a possible interpretation could be that some but not all individuals of the panel are cointegrated. The between-dimension estimator is less restrictive but suffers from poor size properties. The stars indicate that the null hypothesis isn't rejected and both the Pedroni and the Kao test suggest that the error terms in Model 2 is a non-stationary I(1) process. Hence, we lack empirical support that the variables in Model 2 are cointegrated.

Looking at again the Pedroni test the results are the same in Model 1 but that result is conflicting with the Kao test. This is quite usual and a troublesome fact of modern cointegration tests since there isn't any clear-cut way of knowing which model to rely on.

T1.3 Johansen Cointegration Tests. US *numeraire*.

		Model 1		Model 2	
	Rank/Statistic	Fisher trace	Fisher Max eigenvalue	Fisher trace	Fisher Max eigenvalue
Johansen Model	r = 0	23.72 (0.008)	16.92 (0.076)	25.85 (0.004)	23.39 (0.010)
Model 3	r ? 1	14.51 (0.151)	10.01 (0.440)	11.19 (0.343)	11.80 (0.298)
	r ? 2	21.51 (0.018)	21.51 (0.018)	8.196 (0.610)	8.196 (0.610)
Model 2	r = 0	19.29 (0.037)	18.03 (0.054)	45.04 (0.000)	42.71 (0.000)
	r ? 1	8.670 (0.564)	6.686 (0.755)	15.24 (0.124)	15.09 (0.129)
	r ? 2	9.360 (0.498)	9.360 (0.498)	8.144 (0.615)	8.144 (0.615)

The Johansen test consists of two different tests; the trace test and the maximum eigenvalue test. The trace statistic tests the null of *r* cointegration relationships against the alternative hypothesis of *k* cointegrating relationships, where *k* is the total number of variables. In the maximum eigenvalue test one tests the null hypothesis of *r* cointegration relations against the alternative of *r+1* cointegration relations. The probabilities in parenthesis are computed using an asymptotic Chi-square distribution and may be misguiding due to cross-sectional correlation. The advantage with the Johansen test is that it doesn't only test the presence of a cointegration relationship but also the number of cointegration relationships that exist. In the Johansen test one has to choose between specifying a model according to 5 different assumptions. To specify the correct model is somewhat tricky; we compared the corresponding Akaike and Schwarz information criterion and examined the residuals based on each of the 5 different deterministic assumptions, but it was difficult to derive a clear-cut answer. The residual analysis wasn't perfect and showed some outliers, but it corresponded to neither model specification nor lag length selection. We've chosen to display the results of the Johansen specification Model 2 and Model 3, since we believe that these are likely to describe the underlying data in the most correct manner. Model 3 is specified with a linear trend in the data, and an intercept but no trend in the cointegrating equation (error correction term). Model 2 is specified without linear trends in the data, and an intercept in the cointegrating equation. Again, Model 1 and Model 2 correspond to the wide and narrow money definition models.

In the Johansen test above for the US numeraire data-sample we present the estimated eigenvalues $\lambda_1,\ldots,\lambda_k$ in descending order. An eigenvalue that is different from zero implies a cointegrating vector. The results are clear-cut for both models according to the two Johansen specifications and the corresponding trace and maximum eigenvalue test statistics. The null hypothesis of rank 0 has to be rejected at the 5% significance level against the alternative hypothesis of rank 1 and rank k. The null hypothesis of 0 or 1 cointegrating vectors, rank ≤ 1, is not rejected on behalf of the alternative hypothesis $r = 2$. Thus, the Johansen test indicates the presence of one cointegration vector in the monetary model. The results of the Johansen test are somewhat conflicting with the Pedroni and Kao test. A possible explanation might be that the lag-order is too small which is known to induce a positive bias of rejection of the null of stationarity. Unfortunately it is not possible to determine lag length by looking at univariate auto- and partial correlation functions as in the standard time-series case. One alternative it to chose after the Akaike and Schwarz information criterion or an LR test. We followed that path and the lag order proposed was 2 in level. Another explanation might be that endogenity and co-variance in the residuals affect the estimations. Of the same reason, we will not specify a VEC-model.

The Pedroni and Kao cointegration tests below for the UK numeraire subsample show similar results, there is no clear-cut indication on whether there are a cointegration relationship or not. Although, the Kao test indicates that there do exist a cointegration vector in both Model 1 and Model 2. The Pedroni test shows mixed results. Simulation studies of the properties of the Pedroni test have shown that under our panel characteristics with $t = 144$ and $N = 5$ the panel-ADF test has the best size and size-adjusted power properties. This result was consistent in the presence of strong cross-country correlation which is likely in our sample. The panel-ADF test proved much better than the group-ADF which is less restrictive but have poor size properties (Örzal, 2007). Hence, one should primarily focus on the within ADF-statistic.

T1.3 Panel Cointegration Tests. UK *numeraire*.				
	Model 1		Model 2	
Type of test	Within	Between	Within	Between
	c	*c*	*c*	*c*
Pedroni				
v-Statistic	3.582 (0.001)	—	4.319 (0.000)	—
rho-Statistic	-1.629* (0.106)	-1.186* (0.198)	-2.733 (0.010)	-2.036 (0.050)
PP-statistic	-1.267* (0.179)	-1.298* (0.172)	-2.311 (0.028)	-2.246 (0.032)
ADF-Statistic	-0.883* (0.270)	-0.865* (0.274)	-1.931* (0.062)	-1.786* (0.081)
	ct	*ct*	*ct*	*ct*
	1.593* (0.112)	—	1.987* (0.055)	—
	-1.259* (0.181)	-0.565* (0.340)	'-1.616* (0.108)	-1.011* (0.239)
	-1.276* (0.177)	-0.922* (0.261)	-1.680* (0.097)	-1.379* (0.154)
	-0.742* (0.303)	-0.409* (0.367)	-0.895* (0.267)	-0.549* (0.343)
Kao	*c*		*c*	
ADF t-Statistic	-2.124 (0.017)		-2.300 (0.011)	

In the Johansen test the results are unambiguous. Both the trace and the maximum eigenvalue test indicate the presence of one cointegrating rank, as opposed to the theory of two cointegrating ranks.

T1.3 Johansen Cointegration Tests. UK *numeraire*.					
		Model 1		Model 2	
	Rank/Statistic	Fisher trace	Fisher Max eigenvalue	Fisher trace	Fisher Max eigenvalue
Johansen Model	r = 0	23.53 (0.001)	17.86 (0.057)	18.81 (0.043)	18.15 (0.052)
Model3	r ? 1	13.32 (0.206)	10.32 (0.413)	9.351 (0.499)	9.278 (0.506)
	r ? 2	17.80 (0.058)	17.80 (0.058)	10.08 (0.433)	10.08 (0.433)
Model2	r = 0	20.37 (0.026)	18.37 (0.049)	40.95 (0.000)	26.16 (0.004)
	r ? 1	9.584 (0.477)	8.169 (0.612)	23.27 (0.010)	19.28 (0.037)
	r ? 2	8.780 (0.553)	8.780 (0.553)	12.87 (0.231)	12.87 (0.231)

4.2.1 The two-way error correction model and the Engle-Granger procedure

The cointegration tests previously done are based on the assumption of cross-section independence between the countries in the panel. As pointed out in Westerlund and Basher (2008) this is highly unlikely since a common numeraire country (such as the US and UK in our case) makes it very likely that the remaining countries are correlated. Above that Westerlund et al. also stretch the importance of not ignoring structural breaks since this may induce incorrect inference. Hence, there may be problems with the power of the above tests.

Instead of dealing with the traditional panel data model we will take a look at a two-way error component model. First consider the traditional model:

$y_{it} = \alpha + X'_{it}\beta + u_{it}$ where $u_{it} = \mu_i + v_{it}$.

Here, μ_i is the unobservable individual specific effect and v_{it} is the remaining disturbance. μ_I is time invariant and includes any individual effects or abilities that are not specified in the regression. In the two-way error component model however, we include both an individual effect and a time-effect that is invariant of the individuals. The two-way error disturbances are then: $u_{it} = \mu_i + \lambda_t + v_{it}$ where μ_I again is the individual effect from above and λ_t is the unobservable individual-invariant time effect. v_{it} is the remaining normal disturbance. Our hope is that the time effect capture all the time-specific chocks to the system, making the residuals independent.

Using the two-way error fixed effects model (two-way within-estimator) we proceed with pooled Engle-Granger cointegration tests, which should prove more robust since we are now able to estimate the time effect. The Engle-Granger test is appropriate since all previous testing indicates that there is presence of maximum one cointegration relationship. In the first step we will run the following specification of our panel data:

$$s_{it} = \mu_i + \lambda_t + \beta_1(m_{it} - m_{it}^f) + \beta_2(y_{it} - y_{it}^f) + v_{it}$$

We were not able to include dummy variables to control for structural breaks since the combined presence of break-dummy variables and cross-section and period dummy variables would cause perfect multicollinearity. The parameters of the monetary deterministics, relative real incomes and relative money supply, are identical across the individuals. Hence, we have to impose homogeneity across the individuals which is a bold but in this case necessary assumption.

In the second step we test whether the error-terms, v_{it}, are stationary or not. If so, the residuals can be interpreted as long-run equilibrium errors implicating that there exists a linear combination of the non-stationary variables that is stationary; a cointegrating relationship. We apply the Dickey Fuller test on the residuals:

$$\Delta\hat{v}_{it} = \alpha\hat{v}_{i,t-1} + \sum_{j=1}^{p}\psi_{ij}\Delta\hat{v}_{i,t-j} + \varepsilon_{it}$$

where we have set the lag length p of the lagged first difference terms to 1, based on the significance of the lags. This seems sufficient to deal with the serial correlation in the disturbance terms. One could also estimate () with feasible generalized least squares that is based on different user specified transformations of the variables that eliminates cross-section in ε_{it}. The results of the Engle-Granger procedure is displayed below. The result is very clear and we find evidence of cointegration in all our panels.

Engle-Granger two-step procedure

	t-value	Cointegration
Panel US	0.0001	YES
Panel US_mw	0.0009	YES
Panel UK	0.0001	YES
Panel UK_mw	0.0007	YES

If we construct a panel cointegration test from individual unit root tests we get the same result. When using the Fisher ADF test proposed by Maddala and Wu one combines the individual p-values corresponding to the null hypothesis of unit root. Working with individual time-series means that we can use different lag lengths in the individual ADF tests, include dummy variables that correct for structural breaks and let the slope coefficents vary over the individual countries. Hence, we specify the following model for our different countries:

$$s_t = \alpha + \varphi t + \Psi D + \beta_1(m_t - m_t^f) + \beta_2(y_t - y_t^f) + e_t$$

With an ADF-test we examine the 5 different sets of residuals for the presence of a unit root:

$$\Delta\hat{e}_t = \alpha\hat{e}_{t-1} + \sum_{j=1}^{p}\psi_i\Delta\hat{v}_{t-j} + \varepsilon_t$$

The combined p-values give:

$$P = -2\sum_{i=1}^{N}\ln p_i$$

which has a χ^2 distribution with 2N degrees of freedom as $T_i \rightarrow \infty$ for finite N. The result of the combined-*p* test is equally clear regardless whether a dummy is included or not and we conclude that there is one cointegrating relationship in the monetary model.

The combining *p*-value test

	P	χ2-critical value	Cointegration?
US numeraire	44,44	18,3	YES
US numeraire (ΨD_i=0)	38,14	18,3	YES
UK numeraire	49,61	18,3	YES
UK numeraire (ΨDi=0)	50,51	18,3	YES
US Wide money	41,09	18,3	YES
UK Wide money	46,41	18,3	YES

4.3 Results from relaxing the PPP assumption

We ran unit root test for the new variables and concluded that they were I(0). Below we have summarized the results from the cointegration tests. The first table are the results from estimating only the nominal exchange rate and the new variables, as they are described in chapter 3. All tests but the Pedroni test indicates the presence of one cointegrating rank. At the 10% significance level all tests suggests the presence of cointegration in the case of the UK panel.

Summary of cointegration tests

	Johansen Model 2		Pedroni Panel-t (p-value)	Pooled Engle-Granger
	Trace statistic	Max eigenvalue		
US Panel	1	1	0	1
UK Panel	2	0[1]	0[2]	1

[1,2] The test suggests 1 cointegrating relationship at the 10% significance level

When we include all variables that enter equation (3.12) the Johansen test indicates the presence of 3 cointegrating ranks! This is somewhat surprising and even if the test is likely to be affected to some extent of the assumptions made, there is strong indication that the new variables play an important role in the monetary model.

Summary of cointegration tests

	Johansen Model 2		Pedroni Panel-t (p-value)
	Trace statistic	Max eigenvalue	
US Panel	3	2	0,206
UK Panel	3	3	0,107

In the VECM model all of the variables were significant but money supply and the relative trade balance were of the wrong sign. The error correction terms were of the correct sign and the adjusted R^2 improved particularly with the relative trade balance variable.

5 Conclusion

We have examined two panels of exchange rate data in order to validate the long-run properties of the monetary model. In doing so we arrive at four main conclusions. First, there is an obvious need to develop more reliable and consistent panel unit root and cointegration tests. As of today, the practical researcher is likely to stumble on inconsistent results and at the same time have little or no reason to choose between either of the tests. Not knowing which results to rely on is obviously a problem when conducting statistical based research.

Given the limitations of the tests and the possible reduced power due to cross-section correlation, homogeneity and structural breaks we found evidence of one cointegrating rank in both the UK and US panel according to all but the Pedroni test. These results were consistent for the panels where a wider money aggregate was used and there was no clear-cut difference between the two sub-samples.

The results from the individual time-series were basically in line with our expectations and didn't give any real evidence of cointegration. In fact, the results brought our expectations to the limit as we didn't find evidence of cointegration in a single case which was somewhat surprising, although there were some indications primarily in the case of Japan. Hence, the more promising results in the panels show of either the increased strength and power of including several cross-sections or the sensitivity of the tests as regard to the restrictive assumptions sometimes made.

Relaxing the PPP assumption showed very promising results and it is likely that at least one of the introduced variables have a long-term cointegration relationship with the nominal exchange rate.

Further development and research of panel cointegration tests are welcomed, and a necessity for the soundness of economical research. Some progress have been made by for example Westerlund and Groen, but their tests require dire econometrical knowledge and isn't an integrated part of any statistical platform, such as Eviews.

Another future interesting point of focus is to investigate the other variables that enter the real exchange function and their possible impact in the monetary model.

Because of lack of data, we only included two of several possible variables and the promising result motivates further research.

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Appendix A. The ECM.

If we assume that the true monetary model can be described by money supply alone we have that:

$$S = K\dot{M} \tag{1}$$

where time indexes are left out for the sake of simplicity, a “dot” over a variable indicates the inter-country difference, and K is the true slope coefficient for money supply. In log we have

$$s = k + m \tag{2}$$

If we model a general representation of the data with a wide set of dynamics we might have:

$$s = \beta_0 + \beta_1 m + \beta_2 m_{-1} + \alpha_1 s_{-1} + u \tag{3}$$

Now, equation (3) will only be consistent with the long-run equilibrium equation in (1) when the factors that could cause a departure from equilibrium are cancelled out. Hence, when we set u = 0 and the variables equal to their long run equilibrium value so that

$$s^* = \beta_0 + \beta_1 m^* + \beta_2 m^* + \alpha s^* = \tag{4}$$

$$(1-\alpha_1)s^* = \beta_0 + (\beta_1 + \beta_2)m^* =$$

$$s^* = \frac{\beta_0}{(1-\alpha_1)} + \frac{(\beta_1 + \beta_2)}{(1-\alpha_1)} m^*$$

Here, if the above equation shall correspond to the long run equilibrium equation in (2) we must have that:

$$\frac{\beta_0}{1-\alpha_1} = k$$

$$\frac{\beta_0 + \beta_0}{1-\alpha_1} = 1$$

This imply that $\beta_1+\beta_2 = (1-\alpha)$. If we let Π describe the common value of these terms we have that $\beta_2 = \Pi-\beta_1$, and $\alpha = 1-\Pi$. Inserting this into (3) and taking the change of *s*, we can express the equation in error correction form:

$$s = \beta_0 + \beta_1 m + (\Pi - \beta_1)m_{-1} + (1-\Pi)s_{-1} + u$$

$$s = \beta_0 + \beta_1 m - \beta_1 m_{-1} + \Pi m_{-1} + s_{-1} - \Pi s_{-1} + u$$

$$\Delta s = \beta_0 + \beta_1(m - m_{-1}) + \Pi(m_{-1} - s_{-1}) + u$$

$$\rightarrow \Delta s = \beta_0 + \beta_1 \Delta m - \beta_3 \Delta y + \prod(m_{-1} - s_{-1}) + u$$

This is our ECM model. In equilibrium we have that $S = K\dot{M}$. But in the short run the nominal exchange rate react to two chocks as can be seen in the ECM model that we derived above: the movements of m according to k and in response to disequilibrium in the previous period, i.e. the error correction term. If we put the relationship in log once again we have that: $s = k + \dot{m}$ so that $s - \dot{m} = k$. If the value of "*s-m*" was high in period 1, that is, above *k*, than the exchange rate should be corrected downwards in the next period, and vice versa. Hence, the expected sign of Π should be negative. Since we will work with more than two variables a general description of our model may look as follows:

$\Delta s_t = \Gamma \Delta X_{t-1} + \rho_1 \Delta s_{t-1} + \Pi(s_{t-1} - X_{t-1}) + u_t$ where X is a vector of monetary fundamentals.