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Monte Carlo based collateralized loan obligation (CLO) valuation under different copulas

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Sammanfattning

Titel: Monte Carlo baserad collateralized loan obligation (CLO) värdering under olika copulas

Seminariedatum: 2010-06-03

Kurs: NEKA31 – Uppsats kandidatnivå, 15hp

Författare: Karl Johan Sundin

Handledare: Professor Hossein Ashgarian

Nyckelord: CLO, collateralized loan obligation, Monte Carlo, copula

Syfte: Att utveckla och implementera en Monte Carlo baserad kassaflödesmodell för värdering av en godtycklig CLO och sedan undersöka prisinverkan av olika beroendestrukturer bland dem underliggande krediterna.

Teori: Teorin för förlustmodellering hos en kreditpool baserar sig på tidigare arbeten inom prissättning av syntetiska collateralized debt obligations (CDOs).

Metod: En deterministisk kassaflödesmodell för värdering av en godtycklig CLO utvecklas och implementeras. Därefter utvecklades och implementerades en modell för förlustmodellering hos en kreditpool. Till sist länkades dessa modeller tillsammans till den kompletta modellen.

Slutsatser: Givet magnituden av andra osäkerhetskällor så är effekten av olika beroendestrukturer bland dem underliggande krediterna inte tillräckligt stor för att rättfärdiga tidsödande arbete för att finna den rätta beroendestrukturen.

Abstract

Title: Monte Carlo based collateralized loan obligation (CLO) valuation under different

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Keywords: CLO, collateralized loan obligation, Monte Carlo, copula

Purpose: To implement a Monte Carlo based cash flow model able to value an arbitrary collateralized loan obligation (CLO). The valuation is done under three different copulas in order to see how the default dependence structure among obligors influences the valuation.

Theory: The theory for loss distribution modeling in a credit pool is based on earlier works in synthetic collateralized debt obligations (CDOs) pricing.

Methodology: A deterministic cash flow model for valuing an arbitrary collateralized loan obligation (CLO) is implemented. After that a credit pool loss distribution model is developed and implemented. At last the two models are linked together into the complete model.

Conclusions: The impact from the credit dependency structure is not great enough to justify time consuming efforts to find the right dependence structure since there are a lot of other potentially larger error sources in the valuation.

Summary

In this thesis a model for valuing an arbitrary collateralized loan obligation (CLO) is implemented. A CLO is a structured credit product whose performance relies on a pool of corporate loans. Valuation in the context of this thesis is the price on a CLO security and the distribution for the price. The major driver behind the performance of the loan pool is the default distribution among obligors. Along with individual default probabilities the default dependence structure is the main factor influencing the default distribution. The dependence structure is hard to observe in the market, it is therefore of interest to see how various dependence structures affect the CLO valuation and thus how much effort should be put into finding the most suitable dependence model. The dependence structure for the defaults in the pool is modeled using three different copulas namely; Gaussian, Student t and Clayton. The computation is done using Monte Carlo simulations. I my results I find that the dependence structure affects the valuation of different CLO securities differently. My conclusion is that the impact is not great enough to justify time consuming efforts to find the right dependence structure since there are a lot of other potentially larger error sources in the valuation.

The first chapter holds the background and the purpose of this thesis. In the second chapter an introduction to CLOs is presented, the presentation is rather detailed. Chapter three then presents the most common CLO valuation techniques used by market participants. In the fourth chapter one of the valuation techniques is selected and implemented, the technique selected is the discounted cash flow analysis. In chapter five the implemented cash flow model is tested and compared to a commercial cash flow tool use by market participants. In chapter six a model for loss distribution modeling is implemented and linked to the cash flow model. As base for the loss distribution model techniques from synthetic CDO pricing is used. In the seventh chapter the complete model (cash flow + loss distribution) is tested. The eight, and last chapter holds my conclusions.

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1 Introduction

1.1 Background

Originally the business model of most banks was to, at some margin, redeploy funds from depositors and investors into households and businesses by means of lending. Banks are then, by regulation, forced to hold risk-based capital equal to some percentage of the complete loan portfolio in order to protect the depositors. To optimize the balance sheet banks started to securitize loans on the balance sheet. By bundling together loans and repacking them into tradable capital market instruments the credit risk could be removed from the balance sheet and placed with various investors. One instruments which has shown popular for balance sheet management is the collateralized loan obligation (CLO). In a CLO the investors acquire a structural claim on the interest proceeds stemming from a portfolio of bank loans in exchange for absorbing a certain degree of defaults on the collateral. In the wake of the increased interest in CLOs a balance sheet tool, CLOs also developed into a popular arbitrage vehicle. Investment banks and asset manager realized that CLOs was suitable for exploiting the excess spread between the sum of the separate loans and the sum of the loans bundled together. Instead of moving loans from the issuers balance sheet the arbitrage CLO issuer acquires loans in the in the secondary market or by directly by participating in loan syndications. Between 1999 and 2006 the CLO market grew by almost 500% to a peak in issuance of 97 billion $, before crashing during the credit crunch 2007, S&P Leveraged commentary and Data [1]. Along with the increased market size the need for good valuation models also grew. The theoretically fair price under various economical scenarios is interesting both for secondary market trades but also for regular balance sheet valuation of CLO positions where mark-to-market is not possible.

1.2 Purpose

The purpose of this thesis is to implement a Monte Carlo based cash flow model able to value an arbitrary collateralized loan obligation (CLO). Valuation in this context means computing the price and the distribution for the price on a CLO tranche. The valuation is done under three different copulas in order to see how the default dependence structure among obligors influences the valuation. The implementation of the model is done in Visual Basic for Applications (VBA). In order to enhance usability the relevant input parameters must be easy to access and the model should be easy to understand. The model should also be computationally efficient so that computation time is kept reasonably low on a regular PC.

The implementation of the model can be divided into two parts:

- The first part focuses on the development of a discounted cash flow (DCF) model able to value an arbitrary CLO.
- The second part involves the modeling of the expected loss distribution using concepts from synthetic collateralized debt obligation pricing. The DCF model is linked to the loss distribution model and the valuation is done using Monte Carlo simulations.

Since the aim of this thesis is to implement a model using available techniques the thesis will be more biased towards application then theory. A lot of time of this thesis has been devoted to the design and implementation of model.

2 Collateralized debt obligations (CDOs)

2.1 Basic concept of CDOs

A collateralized debt obligation (CDO) is a type of asset-backed security backed by a diversified pool of debt securities or bank loans. Depending on the underlying asset type the CDO has difference reference names e.g. a CDO backed by bonds is known as a collateralized bond obligation (CBO). Later in this chapter a deeper survey will go through the most common types of CDOs.

When setting up a CDO a bankruptcy-remote entity, also referred to as special purpose vehicle (SPV), is created and a collateral manager is assigned. The SPVs is used for isolating the financial risk involved and the manager is responsible for managing the pool of debt obligations. In order to fund purchase of the underlying assets, also known as collateral assets, the SPV issues different types of debt securities (tranches). This process is known as securitization. The investors buying the tranches are entitled to the proceeds from the collateral portfolio. The issued tranches are divided into different classes depending on their senior claim on the collateral portfolio. This so-called "tranching" enables a customization of the risk/reward profile on the different tranches and can thereby attract various types of investors. The tranches are in a broad sense divided into the following classes:

- Senior tranche
- Mezzanine tranche
- Subordinate/equity tranche

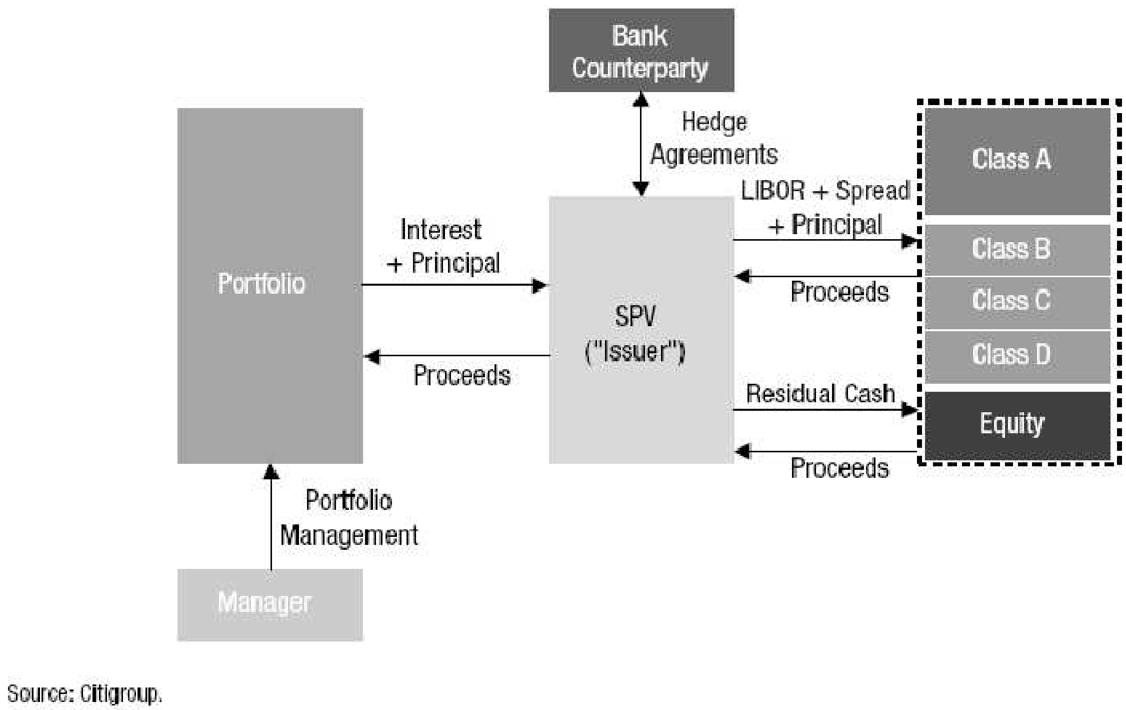


Figure 2-1: Schematic CDO structure

Each tranche, except from the subordinate/equity tranche, is assigned a credit rating from at least one of the major rating agencies (S&P, Moody and Fitch). Usually the *senior* tranches (Class A in figure 2-1) are rated at least A (S&P), the *mezzanine* tranches (Class B,C,D in figure 2-1) are rated between BBB and B (S&P), Goodman and Fabozzi [2]. For the *subordinate/equity* tranche no rating is assigned since it is only entitled to the residual proceeds. The subordinate/equity tranche offers a leveraged investment in the collateral since it has a higher expected return and volatility on return then the underlying assets in the CDO.

The distribution of principal and interest payments to the various tranches, referred to as "Distribution of payments", is made according to specifications in the prospectus. The prospectus contains all relevant information about the CDO and is originally distributed to all initial investors. In general the payments to the different tranches are made sequentially, such that payments are first made to the most senior tranches and then to the other classes in order of their subordination. To ensure that these payments can be done throughout the life of the CDO, and thereby keep the original rating on the tranche, it is important to keep sufficiently and a good quality on the underlying assets. Therefore certain tests (cash flow coverage tests) are imposed which must be fulfilled before any payments can be done to the following subordinate tranche. The payments which should have been used to pay junior tranches are instead used to redeem rated tranches or to reinvest in more assets. In severely distressed deals, broken tests can also lead to a liquidation of the CDO. It should be said that the complexity of the distribution of payments change dramatically between various CDO types.

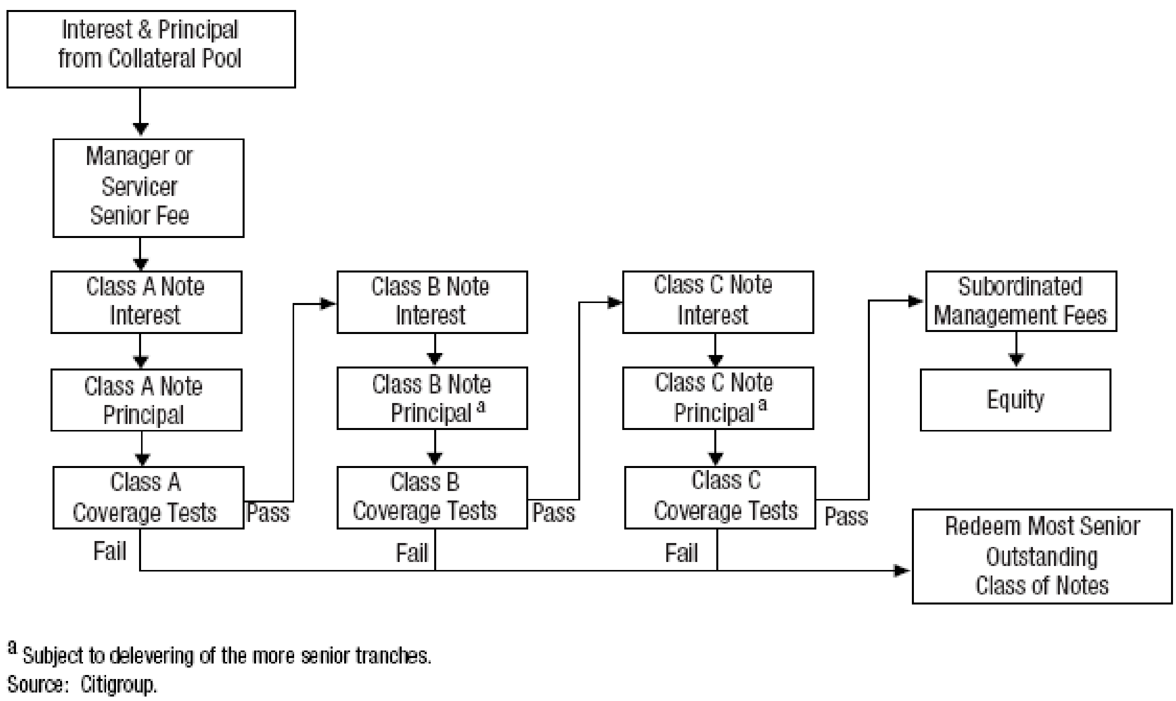


Figure 2-2: Distribution of payments example

Beside cash flow coverage tests there are also so called asset quality tests which should protect the tranche holders. Those tests must be fulfilled in order to maintain the original rating on the tranches. The tests usually monitor:

- Minimum rating on the collateral assets.
- Industry/obligor limits among the collateral assets
- Minimum coupon or spread among collateral assets
- Maturity profile

There exists three important periods in the life of a CDO, here stated in chronological order;

- Ramp-up period
- Reinvestment period
- Redemption period

The *ramp-up period* is the period following the closing date of the CDO. After the closing date no new investors are able to buy-in to the transaction. At the same time the CDO manager starts to build up the collateral by buying assets with the funds from the existing investors. This period usually lasts less than one year. After the end of the ramp-up period the *reinvestment period* starts. During this period all principal payments from the underlying assets is reinvested in new assets in order to keep the collateral amount somewhat constant. This period usually lasts for at least five years. The last period is called *redemption* period and it starts when the reinvestment period ends and continues until the maturity date of the CDO. In this period principal payments are not reinvested, instead they are used to redeem the tranches. Usually the CDO is retired before the period ends as the manager liquidates the CDO when the collateral falls below some arbitrary threshold. This is explained by the fact that as the sum of the collateral assets (principal balance) decrease the fixed costs relative to the proceeds flowing into the CLO increase and thus the structure becomes economically inefficient.

2.2 CDO Asset classification

As mentioned in the beginning of chapter 2.1, CDOs are often classified on behalf of their underlying assets. Over time, CDO collateral has expanded the types of assets that can be secured. Traditionally assets such as *bank loans*, *high yield bonds* and *emerging market debt* have been used. On the later years products such as *residential mortgage backed securities* (RMBS), *commercial mortgage backed securities* (CMBS), *credit default swaps* (CDS) and other CDOs have been used for securitization. Some of the most common CDOs are listed below, The Bond Market Association [3]:

- Collateralized loan obligation (CLO): In a CLO the underlying assets of the CDO are mainly investment grade or leveraged loans.
- Collateralized bond obligation (CBO): In a CBO the underlying assets of the CDO are mainly corporate bonds and emerging market debt.
- CDO of RMBSs: In this type of structure the underlying assets of the CDO are mainly tranches in different RMBSs. A RMBS is structured in the same way as a CDO with the only difference that the underlying assets are residential mortgage loans.
- CDO of CMBSs: In this type of structure the underlying assets of the CDO are mainly tranches in different CMBSs. A CMBS is structured in the same way as a CDO with the only difference that the underlying assets are commercial mortgage loans.
- CDO of CDSs, known as “synthetic CDO”: In this type of structure the underlying assets of the CDO are mainly single name credit default swaps (CDSs). A CDS can in a simple sense be described as insurance on a loan. The buyer of the CDS is protected from losses stemming from an underlying loan, meanwhile the seller of the CDS will pay any losses on the underlying loan to the CDS buyer. In chapter 2.3.2 there is a deeper survey on synthetic CDOs.
- CDO of CDOs, known as “CDO^2”: In this type of structure the underlying assets of the CDO are mainly tranches in different CDOs.

2.3 Conventional versus Synthetic

It is possible to distinguish between two broad categories of CDO structures: Cash structures and synthetic structures.

2.3.1 Cash structures

Cash structures are the most “straight-forward” way of securitizing a pool of assets. The SPV issues liabilities for cash, which is then used for buying assets. From a balance sheet point of view the assets are transferred from the seller’s balance sheet to the assets side of the SPVs balance sheet. This process of selling assets such as bank loans however brings two main problems.

The funding issue: Since banks often are low-cost funders the problem of transferring high grade assets to higher cost funders occurs. The reason is that it is simply not profitable for a high-cost funder to hold high grade assets. Consider for example a high-cost funder borrowing at LIBOR+30 basis points (bps), while the low-cost funder receives the same funding at LIBOR-5 bps. Also consider an AAA rated loan paying LIBOR+35 bps, with the cost of offsetting the credit risk of 20 bps. In this case the high-cost funder can only finance this asset as loss of 15 bps, while the low-cost funder can hold the same asset and earn 20 bps. This is especially important since CLO financing is relatively expensive with a AAA tranche cost of LIBOR+(35-45 bps), [2]. It should be said though that the CLO finance itself via term funding which, in opposite to regular funding, guarantees availability of funding and mitigates the risk of increased spreads.

The confidentiality issue: If a bank sells a customer loan, and thereby removes it from its own balance sheet a notification to the borrower is always needed. In some cases the bank might even require a consent regarding the loan sale from the borrower. This is because the terms and conditions of the loan might include confidential information about the borrower. For banks, this is particularly important, since selling loans into a SPV is by many considered as compromising the client relationship.

2.3.2 Synthetic structures

In order to avoid two highlighted problems with cash structures a synthetic structure is often chosen. A synthetic structure uses credit derivatives to transfer the credit risk of some reference assets from the “sellers” balance sheet to the SPVs balance sheet, without actually selling the assets. In this way the assets remain on the balance sheet of the “seller” while the credit risk is offset. The most common way of creating synthetic structures is to use credit default swaps (CDSs).

A credit default swap can be seen as an insurance against credit events on a single reference asset or a reference pool of assets. The protection buyer pays a periodic fee in return for a contingent payment from the protections seller in case of a credit event on the reference asset. This contingent payment is done in order to compensate the buyer for the losses stemming from the credit event. Usually the payment from the buyer is done annually and is denoted in basis points on the notional amount on the swap. The most common reference assets are loans, bonds or derivatives. It is important to define which credit events will trigger a loss in the reference asset. Often a definition from International Swaps and Derivatives Association [4] is used. Accordingly the credit events applicable to credit default swaps include:

- Bankruptcy
- Failure to pay

- Obligation acceleration
- Repudiation/moratorium
- Restructuring

Depending on the funding level in synthetic CDOs two main types can be distinguished: fully funded and partially funded synthetic CDOs. In both types the confidentiality issue is solved since the assets remain on the banks balance sheet and no notification to the borrowers is needed for using their loans as reference assets in a synthetic structure. The purpose of a regulatory capital relief for the bank is still intact since the credit risk is transferred.

In a *fully funded* synthetic CDO, the sum of the issued liabilities is approximately equal to the asset value of the reference pool. The proceeds from the liability issuance are then used to invest in high grade assets, such as government securities and AAA rated asset-backed securities. Those assets are then used as collateral in the CDO. Simultaneously the SPV enters into one or several credit default swap contracts by selling protection on a reference asset or pool of assets. The periodically income to the CDO is then the sum of the interest proceeds from the high grade assets and swap premium earned from the buyer of the CDSs. In case of credit events among the reference assets a part of the high grade assets is liquidated in order pay the protection buyers. However, a fully funded structure does not solve the funding issue.

In a partially funded synthetic CDO the reference pool of assets is divided into a super senior unfunded piece and a junior funded piece. The super senior piece is called so since it is considered to carry less risk then a AAA (S&P) security, thus the cost for buying protection on this part of the collateral structure is relative low. The distribution among them is usually 85-90% super senior and 10-15% junior. The SPV then issues liabilities equal to the value of the junior piece and uses the proceeds to buy high grade assets, which are in turn used as collateral. At the same time the SPV sells protection on the complete reference pool of assets and buys protection on the super senior piece. The SPV is now left with only the credit risk from the junior piece and thereby absorbs the first part of the losses (10-15%) in the reference pool of assets. In this structure the confidentiality issue is still solved and now the funding issue is also solved since only a part of the deal is funded (10-15%). To show the difference in funding, consider a partially funded (90% super senior/10% junior) CDO with an AAA tranche cost of LIBOR+50 BP and a CDS protection cost of 10 BP, also assume a yield of LIBOR+5 BP on the high grade assets. Then the CDO manager saves 35 BP ((50-5)-10) on the complete amount it does not have to fund), [2].

2.4 Economics of CDOs

The development of CDOs is closely linked to two capital market imperfections. First, banks and certain other financial institutions have regulatory capital requirements making it costly for them to hold risk on their balance sheets. Second, individual assets such as bonds and loans might be highly illiquid, leading to a reduction in their market values, Duffie and Singleton [5]. With securitization it is possible to mitigate or make use of these two market imperfections. On the back of these issues CDOs are categorized with respect to the motivation of the issuer of the CDO.

2.4.1 Balance sheet CDOs

In a balance sheet CDO the purpose of the issuer is to remove assets from its balance sheet. The reason for it is that regulators require banks and other financial institutions to hold risk-based capital equal to some percentage of the assets, in order to protect depositors and thereby the financial system. Balance sheet CDOs are often referred to as "bank balance sheet CLOs" since the securitization is primarily based on commercial and industrial loans originating from the balance sheet of highly rated banks. By lowering the required risk-based capital it is possible for the bank to redeploy more money and thus increase its return on capital (ROC) and return on risk assets. The great majority of balance sheet transactions use a synthetic structure to transfer risk, [2].

2.4.2 Arbitrage CDOs

The driver behind an arbitrage CDO is the issuers desire to earn the excess spread between the largely speculative-grade assets and the rated liabilities that have been issued. A great spread induces a potential high leveraged yield on the equity tranche. By securitization the issuer tries benefit from mispricing in imperfect capital markets. When pooling together the assets it is possible to increase the total value of the CDO structure relative to the total market value of the individual assets. The issuer mainly earns cash by holding a portion of the equity tranche and from performance fees which are usually linked to the performance of the equity tranche. The key question when issuing an arbitrage deal is therefore whether or not the structure can offer a competitive yield on the equity tranche, and at the same time sell the debt tranches in the capital markets. Arbitrage CDOs are mainly issued by asset managers and investment banks. Since the assets used as collateral in arbitrage CDOs are, in opposite to balance sheet CDOs, bought in the primary and secondary market arbitrage deals have in recent years been seen as a driver behind issuance of speculative-grade loans and high-yield bonds.

2.5 Cash flow versus Market value

Depending on where the primary source of the proceeds from the assets backing to CDO originates from, it is possible to differentiate between two types of CDO transactions. If the proceeds used for paying the tranche holders largely depend on the total return generated by active management of the asset pool by the collateral manager the transaction is known as a *market value CDO*. If instead the primary source is the interest and maturing principal from the underlying assets the transaction is referred to as a *cash flow CDO*.

2.5.1 Market value CDOs

Similar to hedge fund managers, the asset manager of a market value CDO invest in pool of assets that, in the manager's estimation, have a favorable risk/reward ratio. Naturally this brings about increased volatility on the tranches compared to cash flow structures. A market value structure is often deployed when issuer, to a large extent, intends to invest in assets with non predictable cash flows or maturity beyond the life of the CDO, e.g. distressed debt and Treasury bonds. Usually a mark-to-market is performed on a daily basis. The manager possess a great trading flexibility, usually there exists no trading restrictions as long as the asset test requirements are met. Finally it should be said that market value CDOs are exclusively set up as arbitrage structures. The reason for it is that the issuer's motivation behind a market value structure is to take advantage of mispricing in the capital markets and not to remove assets from the balance sheet.

2.5.2 Cash flow CDOs

In a cash flow CDO the objective of the asset manager is to generate sufficient cash flows to pay interest and principal to the senior and mezzanine tranches without active trading of the underlying assets. Since the structure is designed to secure the payments to the rated tranches restrictive covenants are imposed on the asset manager. Already in the ramp-up period of a cash flow CDO the rating agencies rating the tranches demand that certain requirements are meet. On the back of those restrictions the asset manager is very limited in the buying and selling of underlying assets. Instead the manager focuses on controlling defaults and recoveries among the assets.

2.6 Stylized facts on CLOs

Since we are mainly interested in the characteristics of CLOs it is necessary to provide a deeper survey on this particular CDO class.

The underlying asset in most CLOs is, as mentioned earlier in this chapter, primarily leveraged loans. Chapter 2.7 will give a brief introduction to this asset class. The liability side of the CLO usually holds 5-7 rated tranches and then 1 unrated (equity) tranche. The average collateral pool size is usually between €300 million and €700 million in par value spread across 50-150 distinct obligors in 20-30 different industries. Often the CLO holds more than one credit from each obligor. Investor in the CLO usually receives an investor report on a monthly basis containing:

- Information on defaulted obligors in the portfolio
- Recent purchases and sales of credits
- All credits with information such as spread, maturity, rating, etc.
- Cash flow coverage and asset quality test results
- Account statements

Note that all reports do not provide the same information and it is usually hard to extract much data from the reports due to the formatting. The information provided will show immensely important in later modeling as it provides us with information on which CLO related information that is easy to access.

2.7 Stylized facts on leveraged loans

There exists no definition for leveraged loans, but in general a corporate loan is called leveraged if the loan is rated BB+ or lower (S&P) or if the loan carries as spread of at least LIBOR+125 bps. Leveraged loans are mainly issued in the context of leveraged buy-out (LBO) and merger and acquisition (M&A) activity. Leveraged loans are in most cases secured, meaning that they have a claim on the issuing company's assets in case of a default. A leveraged loan is often issued in different tranches carrying different features and thus appeals various investors. CLO managers invest mainly in, Leveraged loans as an asset class [6]:

- Term loan Bs/Cs (TLB and TLC). TLB/TLC represents the largest part of the total leveraged loan and is the most commonly traded in the secondary market. Its popularity among non-bank investors is mainly due to its simple construction. A TLB/TLC can be described as a straight loan paying a fix or floating annual interest without any amortization schedule for the principal payments. The principal is repaid as a bullet payment at the maturity date for the loan. The maturity for a TLB/TLC is usually around 6 years. The loans are in most cases callable at par without penalty, meaning that the loan can be repaid in part or in full at any time without penalty. TLB/TLC are the most senior in the capital structure in terms of claim on collateral in case of default (first lien).
- Second lien loans are junior to term loans in terms of asset claim in case of default. They therefore carry higher spreads and mostly some kind of call protection.

- Mezzanine loans are ranked behind second lien loans in the capital structure. The increased risk is again compensated by higher spreads and some form of call protection. In addition mezzanine loans usually carry a Payable-in-kind (PIK) feature. In the context of mezzanine loans this means that some portion of the coupon at each payment date is capitalized and thus added to the principal. Consider for example a mezzanine loan with a principal balance of 100, a coupon of 5% and a PIK rate of 5%. At the first payment date the coupon payment is calculated as 100*0.05=5 and the new principal is calculated as 100*1.05=105. The following payment date the coupon is the 105*0.05=5.25 and the new principal amount is calculated as 105*1.05=110.25. In this way it is possible for the lender to take part in the up-side of successful issuer.

There exist two indices tracking the bid/offer prices for secondary market leveraged loans, namely European Leveraged loan index (ELLI) tracking European leveraged loans and Leveraged Loan index (LLI) tracking US leveraged loans.

3 CLO valuation techniques

CDOs are more or less complex structures depending on the assets backing the CDO and its specific features. The main problem in valuation is thus the lack of standardization in the CDO market. As a result there is no standard way of valuating CDOs. As mentioned in the *Purpose* of this thesis, the aim is to implement a CLO valuation model. Therefore the 3rd chapter will be devoted to, briefly, go through the problems and methodologies involved when valuating CLOs, [3].

Cash flow CLOs rely on the performance of the underlying assets to generate sufficient cash flows to satisfy the rated tranche holders and provide a competitive return on the equity tranche. The most basic level of evaluation of a CLO tranche therefore involves an analysis of the underlying portfolio of assets and the structure of the CLO. It is important to consider how the CLO has performed so far with respect to cash flow coverage and quality tests as well as expected losses stemming from the portfolio. Most important factors influencing the number of defaults and the resulting losses in the portfolio are: credit quality, diversity among obligors and recovery rates. A slightly more sophisticated, still straight-forward, approach for valuing CLOs and other asset-backed securities is based on the present value of projected cash flows. This technique is known as discounted cash flow (DCF) valuation and it attempts to simulate the cash flow characteristics of the tranche. In situations where a liquidation of the CLO is expected, e.g. when the equity holders exercise their possibility to call the CLO or when the collateral manager is forced to liquidate due to broken test triggers. The call possibility is sometimes exercised when the deal performs very badly and equity holders believe that the CLO is worth more liquidated (rare). In these cases a net asset value (NAV) approach for valuing the CLO might be more suitable. NAV technique tries to estimate the value of the CLO to the different tranche holders if it was liquidated at time. Depending on other investor needs yet a third way of valuing CLOs can be used. The thought behind it is to separate the principal and interest components from the CLO tranche and to value them on interest only (IO) and principal only (PO) basis. In most cases investors use more than one of the mentioned techniques when analyzing a CLO tranche.

3.1 Discounted cash flow analysis

The discounted cash flow method (DCF) is one of the most common ways of valuating a CLO tranche. As in regular discounted cash flow modeling the future cash flow streams are estimated and discounted to present value using an appropriate discount rate. In a CLO tranche the cash flows largely depend on the performance of the underlying assets. In a 'pure' cash flow CLO the assets performance is the determining factor, whereas market value CLOs which are actively managed put yet another dimension to consider when evaluating the deal. When doing a DCF analysis a number of scenario-related assumptions are applied to the model. Usually investor runs a set of different assumptions to stress the deal and thereby determine how sensitive the value is to changes in assumptions. Another key assumption that is usually applied is that the deal is not called. Most of the model inputs are deal specific, whereas the assumptions mainly are market specific. Below some of the key assumptions are given, a deeper explanation will be given in chapter 4.

Deal specific:

- Current capitalization of the CLO.
- Priority of payments.
- Characteristics of the underlying assets.

Market specific:

- Yield curve.
- Expected default and recovery rate.
- Expected prepayments.

The discount rate used is supposed to reflect the risk in the future cash flows; if the risk is high the discount rate should also be high and vice versa. This naturally brings that different tranches with different risk also use different discount rates. The discount rate can either be given as a spread above a reference rate, usually LIBOR or EURIBOR, or as a yield. Usually the discount rates are implied from currently traded prices on similar CLO tranches. For investors this often means that the appropriate discount rates are obtained from a CLO trading desk. Since CLOs often are highly illiquid in the secondary market the implied rates can be highly volatile.

3.2 Net Asset Value

The most straight forward approach for valuating CLO tranches is the net asset value (NAV) technique. The technique assumes a liquidation of the current asset pool and the resulting distribution of the proceeds to the different tranches in accordance with the distribution of payments. NAV is usually used as a complement to DCF valuation, particularly in the case of:

- Market value CLOs, since it is extremely hard to model the trading behavior of the collateral manager.
- Distressed CLOs in which liquidation is an expected scenario.
- CLOs where an optional call scenario of the deal is expected.
- Comparing relative value of a CLO in which NAV can be seen as an indication of for example the quality if the underlying assets.

Due to the illiquid nature of most underlying assets NAV suffers from mark-to-market volatility, which may not been taken account in the original pricing of the CLO. Generally the pricing of the CLO tranche (calculating a fair spread on the different tranches) are done under the assumption that the underlying assets are held until maturity. This means that non-credit-related price volatility of the underlying assets have a significant impact on the result of a NAV valuation. When estimating the proceeds earned from liquidating the CLO all sources of are taken into account, including:

- Current market value of the underlying assets including accrued interest.
- Balance of interest and principal collection accounts.
- Currency and interest hedge agreements.

3.3 Interest only and principal only

Similar to strip bonds it is possible to decompose a CLO tranche into an interest only (IO) and a principal only (PO) component. By considering the components individually it is possible to use different discounting factors and thus make the valuation more accurate in some cases. The two components are considered as having different ability to fulfill their debt obligations depending on the distribution of payments. This is the case in certain distressed CLOs, where back-pay senior and mezzanine tranches continue to receive interest while repayment of the principal is unlikely.

3.3.1 IO valuation

The two most important factors to consider when valuing the interest only component of a CLO tranche is the distribution of payments and the definitions of "events of default" of the deal. In case any definitions in "events of default" are breached the senior note holders can decide to put the deal to default, usually leading to liquidation of the deal. Another important factor to determine is whether the interest on the tranche is deferrable. In tranches were the interest payment is not deferrable the interest payments can be senior to principal payments in all other classes. Whereas in tranches were the interest is deferrable the interest payment can be subordinate to principal payments in senior classes. As a result the interest components on a tranche with non-deferrable interest holds relatively low risk and thus it is possible to use a tighter discounting factor then the risk on the full tranche implies. Selecting an appropriate discounting factor for tranches with deferrable interest is more complex. It is here necessary to distinguish whether the tranche is performing or distressed. In the case of a distressed tranche the IO should be heavily discounted since the interest will only be paid after the cash flow tests are cured. If instead, the tranche is performing it is important to evaluate the amount of losses the CLO can experience before cash flow tests are breached and cash are redirected. It is also necessary to estimate the cumulated expected loss rate in the CLO and under which conditions any deferred interest is paid. Once those factors are analyzed it is possible to select an appropriate discounting factor.

3.3.2 PO valuation

The first step when valuing the PO component is to distinguish between three different groups of tranches. Tranches with a:

- Low likelihood of receiving principal payments.
- Low level of credit enhancement (e.g. excess spread or amount of subordination).
- High level of credit enhancement.

PO components with low likelihood of receiving principal payments can be compared to out-of-the-money options. The sum of the expected principal payments is zero, but depending on the volatility of the projected loss rate and the priority of payments it is still possible for receive principal payments. In PO components with a low level of credit enhancement it is most important to calculate the expected loss rate. Using the expected loss rate calculations of the expected principal payments can be made, and thus a value on the component. The value can then be seen as a premium on an option. In PO components with a high level of credit enhancement the most important factor to consider is the exact amount of credit enhancement and thereby the amount of losses in the collateral the component can withstand.

4 Implementation of discounted cash flow model

4.1 Why focus DCF?

The purpose of the implemented model is to value cash flow CLO tranches. For a number of reasons the model will therefore be based on discounted cash flow analysis:

- Since the aim is to value cash flow CLOs and not market value CLOs it is advantageous to use a valuation methodology that uses projected cash flows as basis for the valuation. This is explained by the significantly lower and somewhat predictable trading activity in a cash flow CLO. The possibility to predict the trading patterns is due to trading restrictions imposed on the collateral manager. This relatively low and predictable trading activity combined with a number of other assumptions makes it possible project the cash flows stemming from the underlying assets.
- Possibilities to compare model results with Intex Solutions, a commercial DCF tool currently in use by many market participants.

4.2 Model features

In the first step of this thesis the aim is to develop and implement a deterministic cash flow model which uses a set of different parameters as input and returns a price as percentage of par on a selected tranche. The model and its implementation carry two important features:

Flexible: The model should be able to value a multi name portfolio consisting of CLO tranches, with each CLO having an individual collateral profile and distribution of payments structure. This lack in homogeneity therefore induces a strong need for flexibility and simplicity in the implementation of the model. The user of the implemented model should be able to calibrate the model to any CLO and to have easy access to all input parameters.

Fast and robust: Since the model, in a next stage, is to be used as a base for Monte Carlo simulations the implemented model has to be computationally efficient and robust. This is done by optimizing the programming code. In VBA this mainly means minimizing:

- The number of loops performed when running the code
- Communication with the Excel worksheet

4.3 Model implementation

It is possible to separate the model into two parts. The first considers the assets backing the CLO and the related interest and principal payments coming into the CLO. The second part involves the distribution of the projected asset cash flows to the different tranche holders. When putting the two parts together the model should therefore at each pay date estimate the cash flows that are paid into the CLO since last pay date. The total cash flows stemming from the underlying assets are then put through the distribution of payments of the CLO, and thus distributed to the different tranche holders. Since both underlying assets and the tranches usually carry floating rate coupons (reference rate, e.g. LIBOR 6 month, plus a fixed spread) it is necessary to construct a yield curve for the reference rate. The yield curve will be used later for discounting purposes as well.

4.3.1 Part one: Yield curve

In order to be able to forecast the reference rate for floating rate securities and later for discounting purposes a yield curve is needed. Once the yield curve is computed is possible to us a simple bootstrapping method to compute the reference rate at any point in time. For the construction of the yield curve natural splines are used. Natural splines are chosen since they are easy to implement and performs at least as satisfactorily as other methods, Beim [7].

Consider a set of observed interest rates $[T_i, r_i]$ for $i = 1,2, \dots, n$ where T_i is the maturity and r_i the interest rate. It is then possible to construct a cubic spline $S(T)$ with $n-1$ piecewise cubic polynomials $S(T)_i$ for $i = 0,1, \dots, n-1$ between each observed rate:

$$S(T) = \begin{cases} S(T)_0, & T \epsilon [T_0, T_1] \\ & \dots \\ S(T)_{n-1}, & T \epsilon [T_{n-1}, T_n] \end{cases}$$

Where:

$$S(T)_i = a_i(T - T_i)^3 + b_i(T - T_i)^2 + c_i(T - T_i) + d_i\,, \qquad T \epsilon [T_i, T_{i+1}]$$

Under the conditions:

$$S(T_i)_i = r_i, \qquad S(T_{i+1})_i = r_{i+1}$$

$$S'(T_i)_{i-1} = S'(T_i)_i, \;\; S''(T_i)_{i-1} = S''(T_i)_i$$

For natural splines:

$$S''(T_0)_0 = 0, \qquad S''(T_n)_{n-1} = 0$$

Using the yield curve and a simple bootstrapping method it is possible to construct an arbitrary interest rate contract at time t with a maturity m. Consider two observed market rates $F_{0,T}$ and $F_{0,T+m}$ at $t = 0$ with at maturity of T, $T + m$. Assuming that there exists no arbitrage it is possible to set up the following relationship using monthly compounding:

$$\left(1 + F_{0,T}\right)^{\frac{T}{12}}\left(1 + F_{T,m}\right)^{\frac{m}{12}} = \left(1 + F_{0,T+m}\right)^{\frac{T+m}{12}}$$

An interest rate at $t = T$ with a maturity m can then be expressed as:

$$F_{T,m} = \left(\frac{\left(1 + F_{0,T+m}\right)^{\frac{T+m}{12}}}{\left(1 + F_{0,T}\right)^{\frac{T}{12}}}\right)^{\frac{12}{m}} - 1$$

Figure 4-1 and 4-2 shows the yield curve and the bootstrapped 6 month EURIBOR using the interest rates in table 4-1.

Maturity [months]	1	3	6	24	36	60	120	360
Euribor rate	0.918%	1.344%	1.543%	1.849%	2.196%	2.748%	3.483%	3.787%

Table 4-1: Euribor rates, as of 2009-05-05

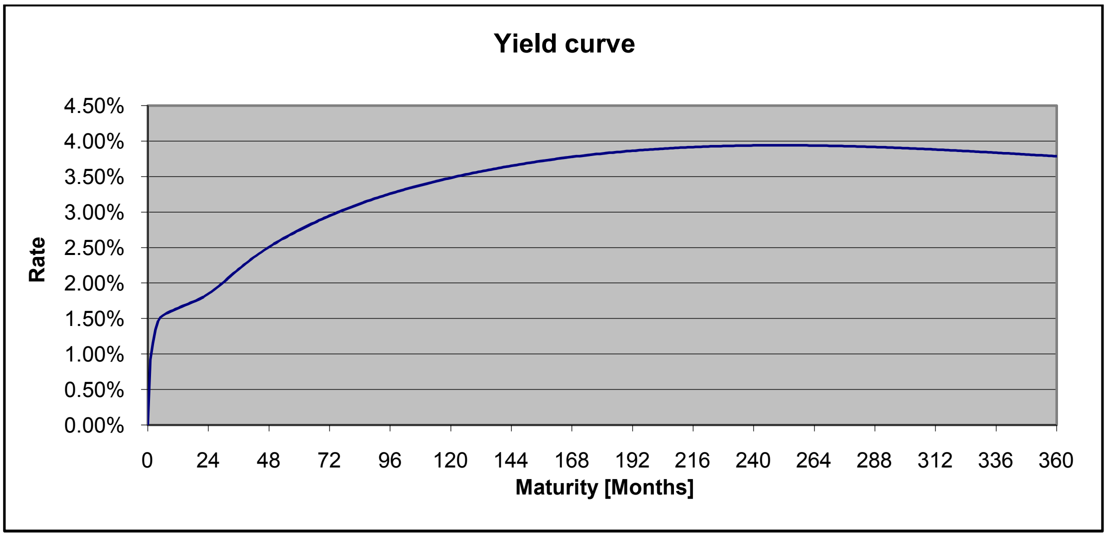


Figure 4-1: Yield curve, computed using natural splines and rates from table 4-1

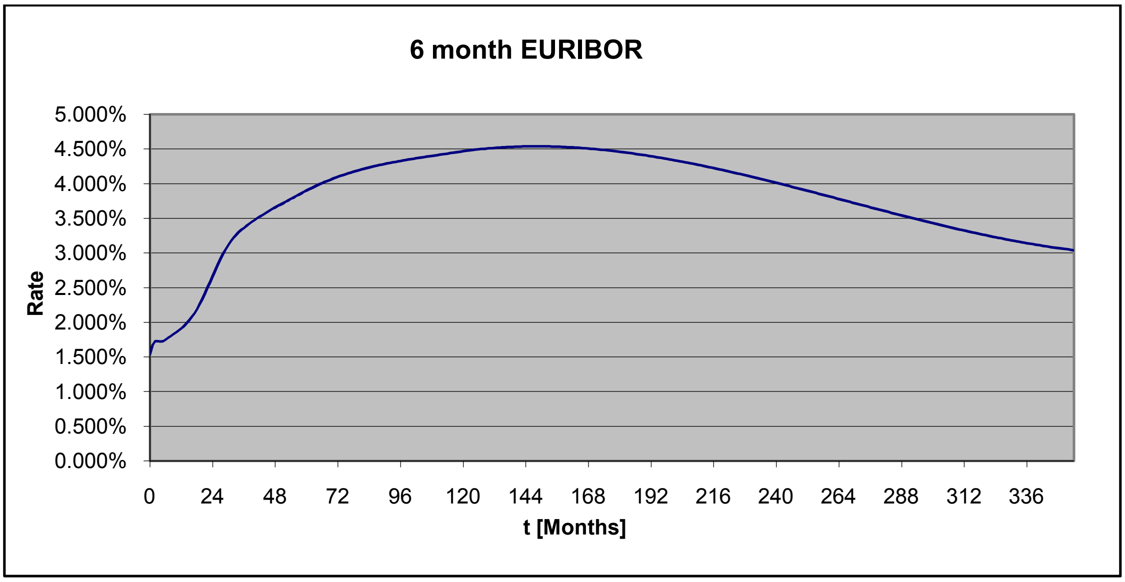


Figure 4-2: 6 month EURIBOR, computed using a the yield curve from figure 4-1 and a bootstrapping method

4.3.2 Part two: Project assets cash flows

The aim of this part of the model is to estimate the principal and interest cash flows streaming into the CLO during the current period. A period is defined as the time between the current payment date and the previous payment date. The number of annual periods is known as the payment cycle and varies from CLO to CLO. Most common is to make payments to the tranches semi annually or quarterly. It is important to separate the principal payments from the interest payments since they are differently treated in the distribution of payments. The cash flows during the period are estimated under a given scenario.

DCF model inputs

In order to be able to project the cash flows stemming from the assets a set of input parameters are needed. The amount of information needed as input is a trade-off between accessibility and precision in the projections. If too many and to high precision in the input parameters were demanded the implemented model would suffer from a lack in usability and vice versa. Since it is hard to handle all underlying assets the idée is to keep the sum of all assets and their corresponding weight instead of giving in all assets in the model.

Collateral specific inputs: It is necessary to define N different underlying asset types; if only one asset type is defined the portfolio is considered homogenous. The common collateral parameters needed are then:

- Original principal balance of all assets.
- Principal balance of all performing assets.
- Principal balance of all non-performing (defaulted) assets.
- Discount definition for assets. An asset is defined as discounted of it was purchased for less than the discount definition.
- Principal balance of all discounted assets.
- Market value of all discounted assets.
- Average maturity of all assets.

Asset specific inputs: For the N defined assets some key features are needed. These are:

- Principal weight $w_{t_0,j}^{Perf}$ of asset j of the principal balance of all performing assets at last payment date t_0 so that:

$$1 = \sum_{j=1}^{N} w_{t_0,j}^{Perf}$$

- Coupon type; fixed or floating.
- Annual coupon.
- Reinvestment maturity. This is the maturity if cash is reinvested in this type of asset.
- Reinvestment coupon. This is the coupon received if cash is reinvested in this type of asset.
- Pay-in-kind (PIK) coupon. Assumed to be the same when reinvesting in the asset.
- Reinvestment price. That is the price paid when reinvesting in this asset type, given as percentage of par.
- Reinvestment weight $v_{t_i,j}$ in asset n in period i of the total amount reinvested so that $1 = \sum_{j=1}^{N} v_{t_i,j}$

Scenario specific inputs: The asset cash flows in each period over the life of a CLO is estimated under a given scenario. Therefore a set of parameters describing the current and forecasted economic climate are defined. These are:

- Conditional default rate CPR_{t_i} in period i. CDR represents an assumed default rate in each period, expressed as a per annum percentage of the total principal balance of all performing loans in the portfolio. Banks usually calculate the CDR for analytical purposes only as it is not a correct indicator for actual defaults since these are likely to differ in timing and amounts from the assumed end-of-period loss at constant default rate, Jobst [8]. Even with these drawback it is market standard since it is considered a tractable way of inducing defaults to the asset pool.

- Conditional prepayment rate CPR_{t_i} in period i. CPR is analogues to CDR and therefore suffers from the same advantages and drawbacks.
- Recovery rate Rec_{t_i} in period i. The recovery rate describes the percentage of defaulted nominal which is expected to be recovered.
- Default and prepayment timing within a period; start, middle or end. The timing of these parameters can have a significant impact on the interest cash flows.
- Recovery lag. This is the time it takes from that a loan defaults until the recovery amount is received.

CLO structure specific inputs: Except from collateral, asset and scenario related information some general information regarding the CLO under valuation is needed. These are:

- Payment cycle for the CLO.
- The date for the end of the reinvestment period. This date coincide with a payment date.
- The maturity date for the CLO. This date also coincide with a payment date.
- First payment date for the CLO minus one period.
- Cash amount in the CLO principal account or any corresponding account.
- Call threshold. The CLO is liquidated as soon as the principal balance of performing assets plus non-performing assets fall below the defined threshold.

DCF model assumptions

To make it possible to estimate the cash flows using the input parameters and in order to limit the complexity of the model it is necessary to make a set of assumptions. To get higher precision in the projected cash flows it is possible to relax some of the assumptions. On the other hand this would probably increase the number of input parameters needed and surely increase the complexity. The assumptions made follow below:

- Defaults and prepayments occur at the same time in a period.
- In each period an asset can either: survive, mature, prepay or default.
- The asset pool exhibits first maturity payments, second prepayments and third defaults. This is similar to the Intex solutions approach.
- The reference rate for the floating rate assets and the tranches is fixed at the payment date of each period.
- All the underlying assets carry a “bullet payment” for the principal.
- Asset coupon payments occur at the payment date of each period.
- Each defined asset is assumed to carry a maturity profile. The maturity profile is evenly distributed as average maturity rounded to closest year$\pm$ 2 years. Meaning that for an asset with an average maturity of 6 years a semi annually pay cycle the maturity profile holds (2*2)*2+1=9 different maturities.
- Each asset type is assumed to consist of infinity many small assets. This is necessary in order to be able to apply scenario parameters such as CDR and CPR.

- CDR and CPR are recalculated to periodic CDR and CPR by dividing them with the pay cycle.
- The collateral manager is only allowed to reinvest principal proceeds in new assets if all coverage tests are in line.
- The CLO is only liquidated when the call threshold is breached.

Calculation of asset interest and principal proceeds

With all input parameters and assumptions on hand it is possible to compute the interest and principal cash flows in each period of the CLO life. Below all the steps involved are described:

1. Time slicing. Start by computing the number K of future payment dates from previous payment date to the CLO maturity date (last payment date). It is then possible to define a discrete time grid $t_0 < t_1 < t_i \dots < t_{K-1} < t_K$ where t_0 represent the previous CLO payment date. In most cases the CLO is retired before the last payment date. As a result it is, in advance, only possible to calculate the upper boundary of payment dates. The K periods associated with each payment date is then defined as the continuous time t: $t_{i-1} < t \leq t_i$.
2. Apply maturity payments to the total performing assets $Nom_{t_{i-1}}^{Perf}$ in the start of the period. For that we consider the N defined assets with an asset weight $w_{t_{i-1},j}^{Perf}$, each with M_j different parts, each with the asset part weight of $w_{j,k}^{Perf}$ and an asset part maturity $T_{j,k}^{Mat}$. Also consider a counting process $F_{j,k}(t) = 1_{\{T_{j,k}^{Mat} < t\}}$ which takes the value 1 if the asset part has matured before t and 0 otherwise. The nominal of the maturing assets is the computed as:

$$Nom_{t_i}^{Matur} = Nom_{t_{i-1}}^{Perf} \sum_{j=1}^{N} \sum_{k=1}^{M_j} w_{t_{i-1},j}^{Perf} w_{t_{i-1},j,k}^{Perf} F_{j,k}(t_i)$$

3. If $Nom_{t_i}^{Matur} < Nom_{t_{i-1}}^{Perf}$ apply prepayments to the performing assets $Nom_{t_{i-1}}^{Perf}$ in the start of the period:

$$Nom_{t_i}^{Prepay} = Nom_{t_{i-1}}^{Perf} CPR_{t_i}$$

4. If $Nom_{t_i}^{Matur} + Nom_{t_i}^{Prepay} < Nom_{t_{i-1}}^{Perf}$ apply defaults to the performing assets $Nom_{t_{i-1}}^{Perf}$ in the start of the period:

$$Nom_{t_i}^{Default} = Nom_{t_{i-1}}^{Perf} CDR_{t_i}$$

5. Consider the total nominal of non-performing assets $Nom_{t_{j-1}}^{NonPerf}$, consisting of B individual parts each with a weights $w_j^{NonPerf}$ and a time-to-recovery T_j^{Rec}. The recovery rate Rec_{t_i} is equal for all assets. Also consider a counting process $G_j(t) = 1_{\{T_j^{Rec} < t\}}$ which takes the value 1 if the part has been recovered before t and 0 otherwise. The nominal of the recovery payments is then computed as:

$$Nom_{t_i}^{Recovery} = Nom_{t_{i-1}}^{NonPerf} \sum_{j=1}^{B} w_{t_{i-1},j}^{NonPerf} G_j(t_i) Rec_{t_i}$$

6. Defining the time $t_{Reinvest}$ as the last day in the reinvestment period, the amount of cash redirected for reinvestment at last payment date due to breached coverage tests $Nom_{t_{i-1}}^{Redirect}$, $v_{t_i,j}$ as the percentage cash reinvested in asset j in period i and p_j the price of asset j. It is then possible to computed the performing principal amount in the end of the period $Nom_{t_i}^{Perf}$:

$$Nom_{t_i}^{Perf} = \begin{cases} Nom_{t_{i-1}}^{Perf} - Nom_{t_i}^{Matur} - Nom_{t_i}^{Prepay} - Nom_{t_i}^{Default} + \\ \left(Nom_{t_i}^{Recovery} + Nom_{t_i}^{Matur} + Nom_{t_i}^{Prepay} + Nom_{t_{i-1}}^{Redirect}\right) \sum_{j=1}^{N} \frac{v_{t_i,j}}{p_j}, if\ t_i < t_{Reinvest} \\ Nom_{t_{i-1}}^{Perf} - Nom_{t_i}^{Matur} - Nom_{t_i}^{Prepay} - Nom_{t_i}^{Default}, if\ t_i \geq t_{Reinvest} \end{cases}$$

7. As the nominal $Nom_{t_j}^{Perf}$ for the end of the period is computed it is possible to compute the interest and principal cash flows using the timing of defaults and prepayments and a counting process counting process $E(t) = 1_{\{OC,IC\ or\ IR\ breached\ at\ t_{i-1}\}}$ which takes the value 1 if all coverage tests are in line at the start of the period and 0 otherwise:

$$CF^{Interest} = \begin{cases} Nom_{t_i}^{Perf} \sum_{j=1}^{N} \sum_{k=1}^{M} w_{t_i j} w_{t_i j,k} Coupon_{j,k}, if\ timing = start \\ \frac{1}{2}\left(Nom_{t_i}^{Perf} \sum_{j=1}^{N} \sum_{k=1}^{M} w_{t_i,j} w_{t_i,j,k} Coupon_{j,k} + Nom_{t_{j-1}}^{Perf} \sum_{j=1}^{N} \sum_{k=1}^{M} w_{t_{j-1},j} w_{t_{j-1},j,k} Coupon_{j,k}\right), timing = mid \\ Nom_{t_{i-1}}^{Perf} \sum_{j=1}^{N} \sum_{k=1}^{M} w_{t_{j-1},j} w_{t_{j-1},j,k} Coupon_{j,k}, if\ timing = end \end{cases}$$

$$CF^{Principal} = \begin{cases} 0, if\ t_i < t_{Reinvest}\ and\ E(t) = 0 \\ Nom_{t_i}^{Recovery} + Nom_{t_i}^{Matur} + Nom_{t_i}^{Prepay} + Nom_{t_{i-1}}^{Redirect}, if\ t_i \geq t_{Reinvest}\ or\ E(t) = 1 \end{cases}$$

4.3.3 Part three: Distribute assets cash flows

Once the asset cash flows for the current payment date are computed it is time to distribute the cash flows the different tranche holders. Here one major problem when implementing a model for valuing an arbitrary CLO occurs – No CLOs are equally structured when it comes to the distribution of payments. This presents at delicate programming problem since implemented model has to be flexible enough to value any CLO. The idea is therefore to implement a set of distribution modules enabling the user to build up the distribution of payments for a particular CLO using the predefined modules. This approach also makes it possible to gradually add new modules when it is needed. The cash flows from the underlying assets are assumed to be paid to an interest respective a principal account. For each module in the distribution of payments any of those to accounts can be charged depending on where the cash should be paid from.

Calculation of ACB

Before defining any modules it is necessary to define what here is called aggregated collateral balance ACB, which is a reduced collateral amount frequently used in the distribution of payments. In order to compute the ACB some input parameters are needed:

- The percentage CCC rated obligations as part of the performing collateral balance at time t_i, CCC_{t_i}.
- The percentage CCC rated obligations counted on the performing collateral balance eligible to be counted into the ACB at par, also known as CCC-haircut.
- Average market value of the CCC rated assets MV^{CCC}.
- The discounted obligation definition $\mathrm{Discount}^{\mathrm{Def}}$. Obligations purchased at a price above this threshold is eligible to be counted into the ACB at par.

The ACB at time t_i, can then be computed as:

$$\mathrm{CCC}_{\mathrm{t_i}}^{\mathrm{Excess}} = Nom_{t_i}^{Perf} \max\big(CCC_{t_i} - CCC^{Haircut}, 0\big)\, MV^{CCC}$$

$$H\big(p, \mathrm{Discount}^{\mathrm{Def}}\big) = \begin{cases} p, & p \leq \mathrm{Discount}^{\mathrm{Def}} \\ 1, & p > \mathrm{Discount}^{\mathrm{Def}} \end{cases}$$

$$ACB_{t_i} = Nom_{t_i}^{Perf} \sum_{j=1}^{N} \sum_{k=1}^{M_j} w_{t_i,j}^{Perf} w_{t_i,j,k}^{Perf} H\big(p_{j,k}, \mathrm{Discount}^{\mathrm{Def}}\big) - \mathrm{CCC}_{\mathrm{t_i}}^{\mathrm{Excess}}$$
$$+ Nom_{t_i,j}^{NonPerf} \sum_{j=1}^{B} w_{t_i,j}^{NonPerf}\, Rec_{t_i}$$

Predefined modules

In this section all predefined modules with a short description are shown. Due to the large number of modules the detailed descriptions and computations will not be displayed here, instead details are found in the Appendix A-1. It is possible to distinguish between three types of modules depending on their main function:

Fees, administrative and hedging expenses. These types of distribution of payment features are the most changing in different CLOs. The defined modules are:

- *Pay any fix cost*. This module pays any type of fixed cost in the CLO, e.g. fix trustee fee.
- *Pay any deferred fix cost*. This module pays any type of deferred fixed cost in the CLO.
- *Pay variable cost on original asset balance*. This module pays any type of variable cost based on the in original principal balance of all assets in the CLO.
- *Pay deferred variable cost on original asset balance*. This module pays any deferred type of variable cost based on the in original principal balance of all assets in the CLO.
- *Pay variable cost on collateral balance*. This module pays any type of variable cost based on the average sum of performing and non-performing assets in the period.

- *Pay deferred variable cost on collateral balance*. This module pays any type of deferred variable cost based on the average sum of performing and non-performing assets in the period.
- *Pay variable senior management fee on ACB*. Module that pays the so called senior management fee based on the average ACB in the period.
- *Pay deferred variable senior management fee on ACB*. Module that pays deferred senior management fee based on the average ACB in the period.
- *Pay variable senior management fee on collateral balance*. Module that pays the so called senior management fee based on the average sum of performing and non-performing assets in the period.
- *Pay deferred variable senior management fee on collateral balance*. Module that pays the deferred senior management fee based on the average sum of performing and non-performing assets in the period.
- *Pay variable subordinate management fee on ACB*. Module that pays the so called senior management fee based on the average ACB in the period.
- *Pay deferred variable subordinate management fee on ACB*. Module that pays deferred senior management fee based on the average ACB in the period.
- *Pay variable subordinate management fee on collateral balance*. Module that pays the so called senior management fee based on the average sum of performing and non-performing assets in the period.
- *Pay deferred variable subordinate management fee on collateral balance.* Module that pays the deferred senior management fee based on the average sum of performing and non-performing assets in the period.
- *Pay incentive management fee.* This module pays the simplest form of incentive management fee. It is usually paid as soon as the internal rate of return (IRR) on the equity tranche has reached above a certain threshold.

Cash flow test and the mechanisms for curing failing tests are usually identical in different CLOs. This of course presents a lot of convenience when modeling them. The defined modules are:

- *Over collateralization (OC) test*. This module performs the OC test on selected tranche, usually all debt tranches carry an OC test. It measures the ratio between the ACB and the tranche nominal plus the nominal of any senior tranches. The test is considered breached if the ration falls below predefined tranche specific OC trigger.
- *Interest coverage (IC) test*. This module performs the IC test on a selected tranche, like OC tests all debt tranches usually carry an IC test. It measures the ratio between the interest cash flows minus senior expenses and the interest supposed to be paid on the tranche plus interest supposed to be paid on any senior tranche. The test is considered breached if the ration falls below predefined tranche specific IC trigger.
- *Cure OC test by redemption*. This module cures an OC test by redeeming a selected tranche. OC tests can either be cured by decreasing the denominator (redemption of tranches) or by increasing the numerator (investing in more assets) in the OC test ratio.
- *Cure OC test by pari passau redemption.* This module cures the OC tests by redeeming two or more selected equally senior (pari passau) tranches with regard to their nominal.

- *Cure OC test by reinvesting in new assets*. This module cures an OC by investing in new assets.
- *Cure IC test by redemption*. This module cures an IC test by redeeming a selected tranche. IC tests can be cured either by reducing the denominator (reducing the interest to be paid by redemption of tranches) or by increasing the numerator (increasing the interest from underlying assets by investing in more assets) in the IC test ratio.
- *Cure IC test by pari passau redemption*. This module cures the IC tests by redeeming two or more selected pari passau tranches with regard to their nominal.
- *Cure IC test by reinvesting in new assets*. This module cures an IC by investing in new assets.
- *Interest reflection (IR) test*. This module performs the IR tests. Most CLOs carry an IR tests. It measures the ratio between the ACB and the sum of all debt tranche nominals. The test is considered breached if the ration falls below predefined IR trigger.
- *Cure IR test by redemption*. This module cures the IR test by redeeming a selected tranche. The IR test can either be cured by decreasing the denominator (redemption of tranches) or by increasing the numerator (investing in more assets) in the IR test ratio.
- *Cure IR test by reinvesting in new assets.* This module cures an IR by investing in new assets.

Pay tranches. This module type carries the most common distribution of payments features, the principal and interest payments to the different tranches. The defined modules are:

- Pay tranche interest. This module pays interest on a selected tranche.
- Pay pari passau tranche interest. This module pays interest on two or more selected pari passau tranches with regard to the interest they should have received.
- Pay deferred tranche interest. This module pays any deferred interest on a selected deferrable interest tranche.
- Pay pari passau deferred tranche interest. This module pays deferred interest on two or more selected pari passau tranches with regard to the interest they should have received.
- Pay tranche principal. This module redeems a selected tranche.
- Pay pari passau tranche principal. This module redeems two or more selected pari passau tranches with regard to their nominals.

In order to show how distribution of payments is modularized a real together with the corresponding module distribution of payments is shown in the Appendix A-2.

4.3.4 Part four: Use the tranche cash flows to compute the tranche value

Once the interest and principal cash flows for all tranches and periods in the life of the CLO are projected it is possible to compute the value of a particular tranche by discounting the associated cash flows. The discounting factor for each period can either be fixed or floating with a spread. Using monthly compounding the price is computed as:

$$TrancheValue = \frac{\sum_{i=1}^{K} \frac{CF_{t_i}}{\left(1 + \left(F_{0,t} + spread\right)\right)^{\frac{t_i}{12}}}}{TrancheNominal}$$

$$TrancheValue = \frac{\sum_{i=1}^{K} \frac{CF_{t_i}}{(1 + Yield)^{\frac{t_i}{12}}}}{TrancheNominal}$$

5 Performance of cash flow model

In order to ensure that the cash flow model provide correct cash flows under various scenarios it is of course necessary to test it. Most important is to see wheatear the model reacts appropriately when changing key parameters such as CDR, CPR and recovery rate. The results will be compared to Intex Solutions.

5.1 A few words about Intex

Intex Solutions, Inc. [9] is the world's leading provider of structured fixed-income cash flow models. According to themselves most major investment banks, brokers and investment managers use their tool. They provide a large number of RMBS, ABS, CMBS and CDO deal models, created and maintained. The library contains 20,000 modeled deals. Each deal is updated with regard to underlying assets, account statements, test performance, etc. using monthly or quarterly investor reports obtained from trustees, servicers and issuers. One main advantage in using Intex is that they keep all of the underlying assets with the respective spread, maturity and etc.

5.2 Test scenarios

Three different scenarios with varying constant periodic CDR have been chosen as reference scenarios. There are a lot of other parameters influencing the cash flows and how they are distributed to the different tranches making it hard to calibrate Intex to my implemented DCF Tool. The problem with Intex is that their computations are not fully transparent. For example Intex does not allow the user to choose when the defaults and prepayments occur and it is hard to find out what assumption they use. In order to make the comparison fair it is important the assumptions in use are as equal as possible. Table 5-1 shows the scenarios used.

	“Good”	“Average”	“Bad”
CDR	4%	8%	16%
CPR	10%	10%	10%
Recovery	50%	50%	50%
Recovery lag	0.5 years	0.5 years	0.5 years
CPR/CDR timing	Start	Start	Start

Table 5-1: Test scenarios

Next stage is to select at least one arbitrary reference CLO to apply the scenarios on. I choose to use two reference CLOs to make the test more valid. The selected CLOs are two real CLOs, they will be called RealCLO1 and RealCLO2. Some of their respective key characteristics are shown in table 5-2.

	RealCLO1	RealCLO2
Principal account balance	620,087.18	0
Average spread	2.72%	2.96%
Reinvestment spread	3.66%	3.66%
Average Maturity	68 months	84 months
Performing assets	393,232,110.36	306,570,537.27
Non-performing assets	1,390,066.92	18,996,382.55
Term loans	100%	98%
Mezzanine	0%	2%
Currency	EURO	EURO

Table 5-2: RealCLO1 and RealCLO2 characteristics as of 2009-05-29

5.3 Results

The first to look at when comparing the results is the size and distribution of the cash flows coming into the CLO during each period. Figure 5-1 and 5-2 shows the cumulative cash flows for the test CLOs. First thing to observe is that the projected cash flows are most similar for the "Good" scenario and then rather unlike for the "Average" and "Bad" scenarios. It is also seen that worse scenario in terms of defaults lead to larger cash flows early. The explanation for this is that high default rates cause coverage (OC and IC) tests to breach. As a result the principal proceeds which should have been deployed in new assets during the reinvestment period are not allowed to be reinvested and thus stay in the principal account and are then paid in accordance with the distribution of payments at the payment date. Therefore it can make a significant difference whether a coverage test was fully cured or not at the last payment date. Small differences in cash flows between Intex and the implemented DCF Tool can therefore be amplified and propagate. It would be advantageous to make comparisons with principal and interest separated, but unfortunately Intex does not provide the interest cash flows from the underlying assets explicitly.

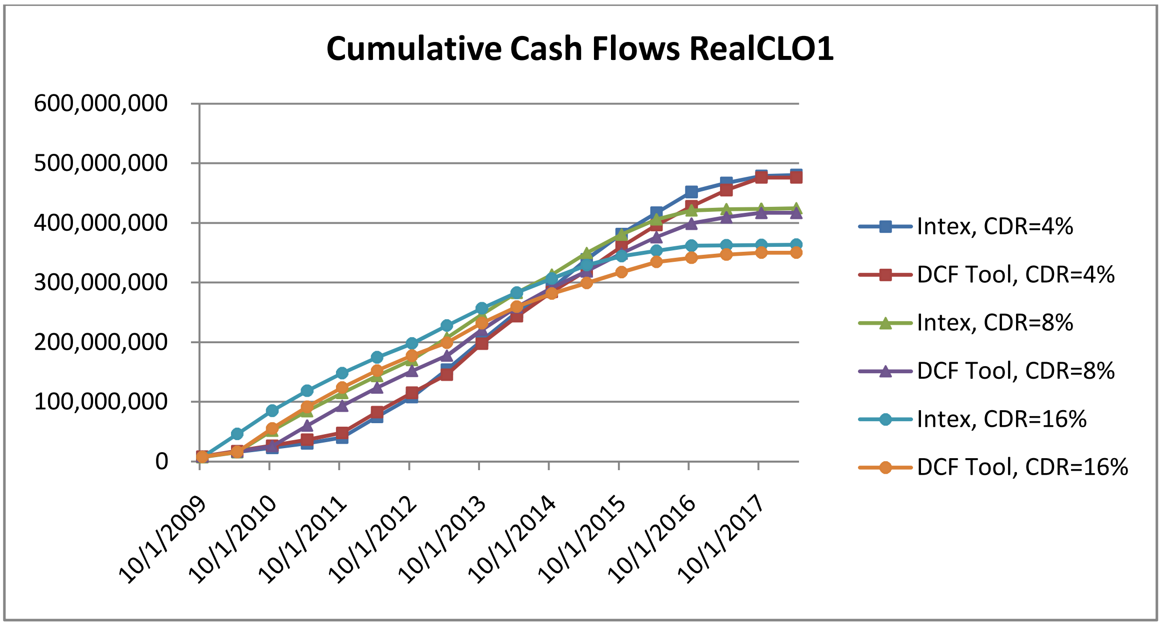


Figure 5-1: Cumulative cash flows RealCLO1

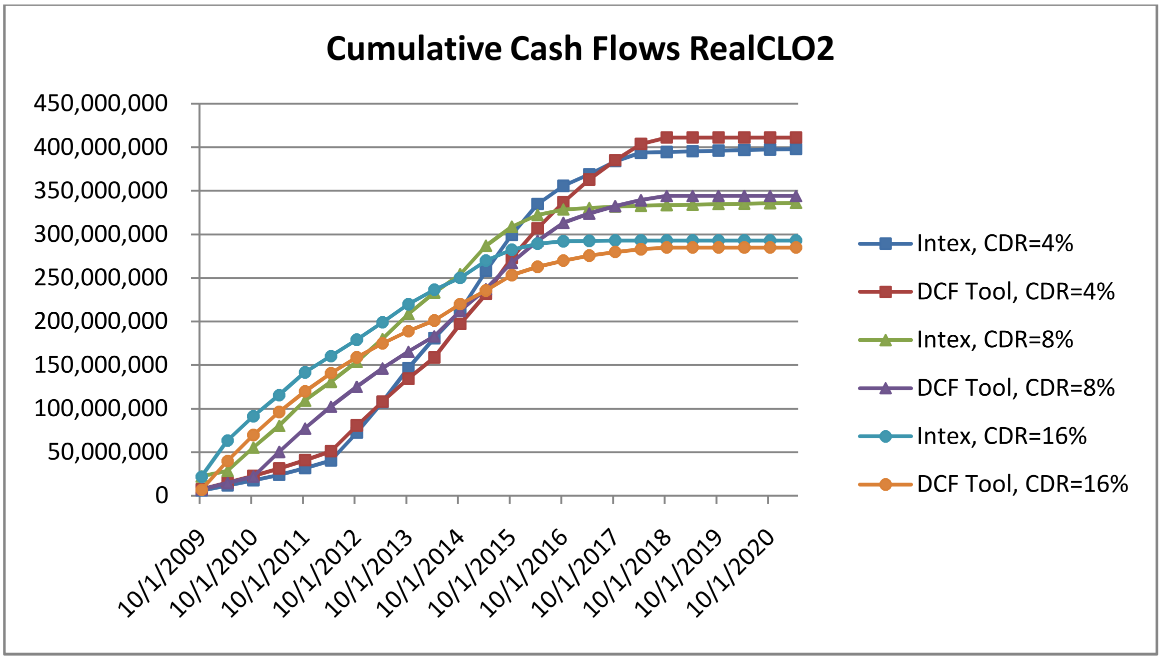


Figure 5-2: Cumulative cash flows RealCLO2

The cash flows projected at each payment date are then distributed via the distribution payments for the respective CLO. I decided to only show the cash flows for the most senior tranche and the equity tranche in each deal. Starting by studying figure 5-3 it is shown that the projected cash flows are relatively concordant for all scenarios. The difference seen in the "Average" and "Bad" scenario is explained by one early large cash flow which shifts the curves upwards. Looking at figure 5-4 the distribution of the cash flows deviate more between Intex and the implemented DCF Tool. It should be noticed that in the "Average" and "Bad" scenario Intex distributes large cash flows at the first payment date. These cash flows are principal proceeds which have not been reinvested. In the implemented DCF Tool this would mean that any of the coverage tests were breached at the previous payment date, a thing that I do not keep tracks on. Turning to the equity tranche cash flows one should be aware that these are the hardest to project correctly. Due to the high leverage on the equity tranche any difference in the cash flows coming in to the CLO or in the distribution of them will be amplified in the equity cash flows. The large differences seen in figure 5-5 and 5-6 are therefore expected. Although it seems that the implemented DCF Tool underestimates the late cash flows relative to Intex. As an example Appendix A-3 shows all the cash flows for RealCLO1 computed using the DCF Tool.

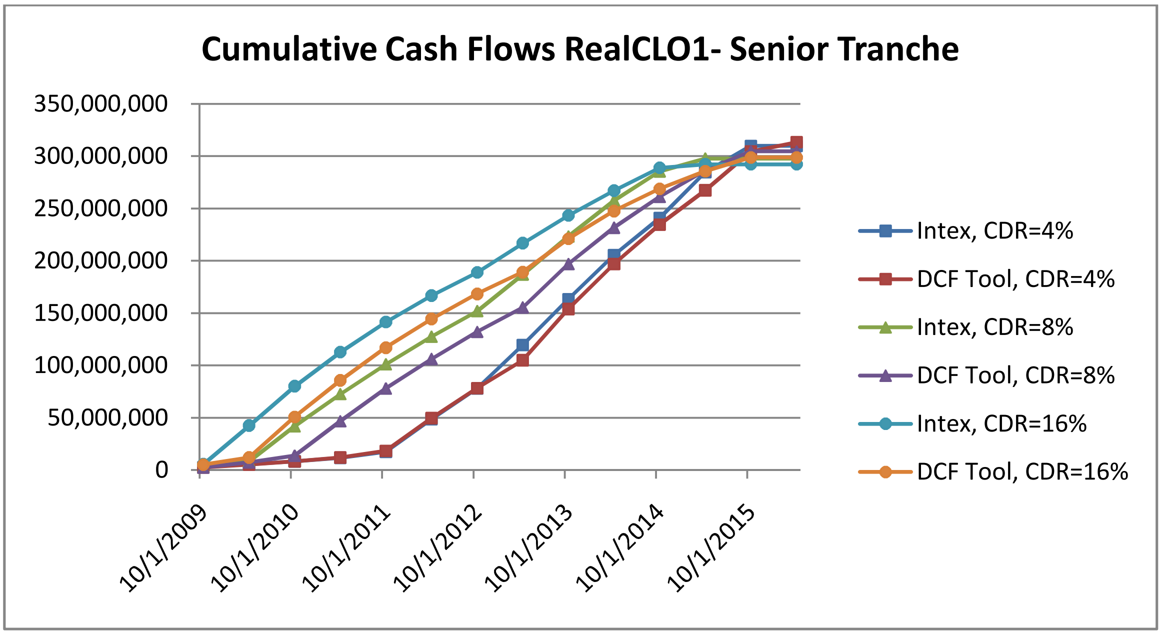


Figure 5-3: Cumulative cash flows RealCLO1 - Senior Tranche

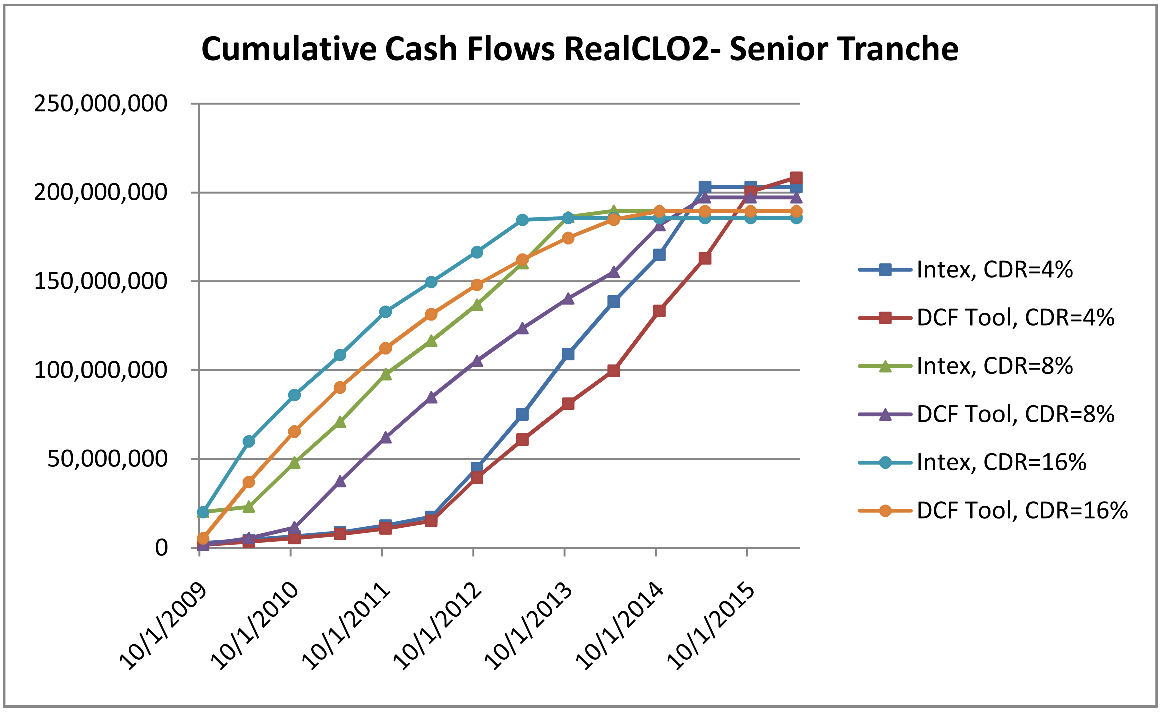


Figure 5-4: Cumulative cash flows RealCLO2 - Senior Tranche

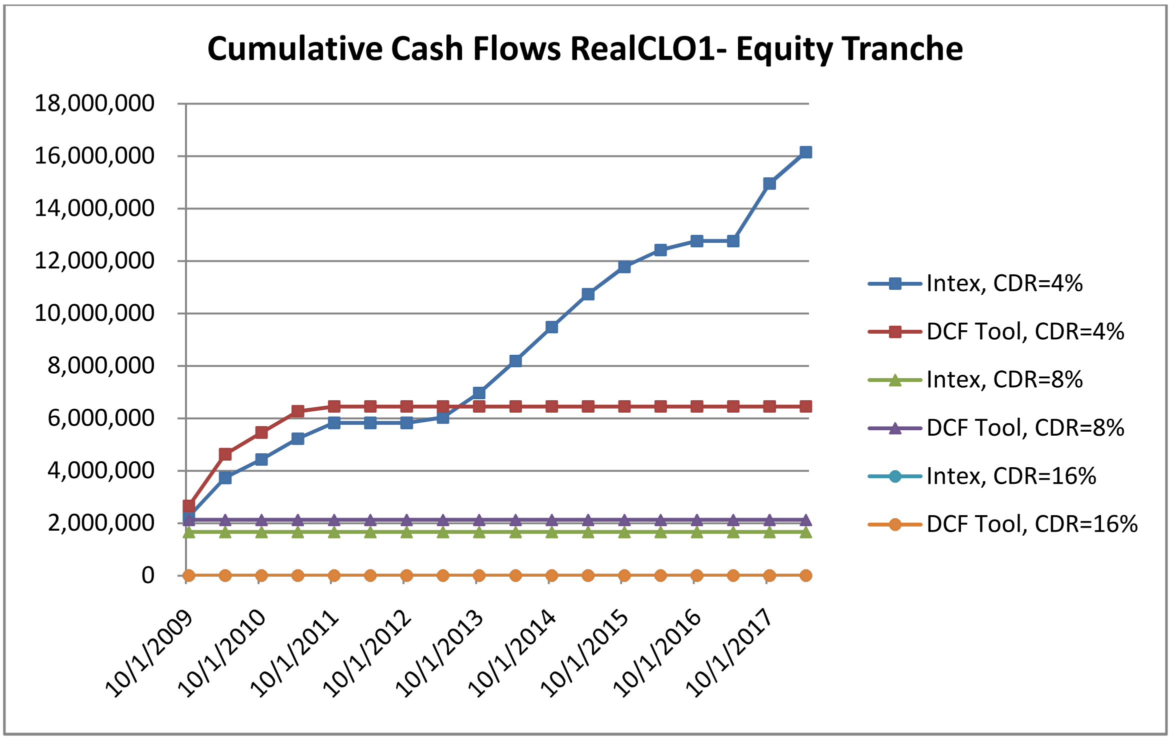


Figure 5-5: Cumulative cash flows RealCLO1 - Equity Tranche

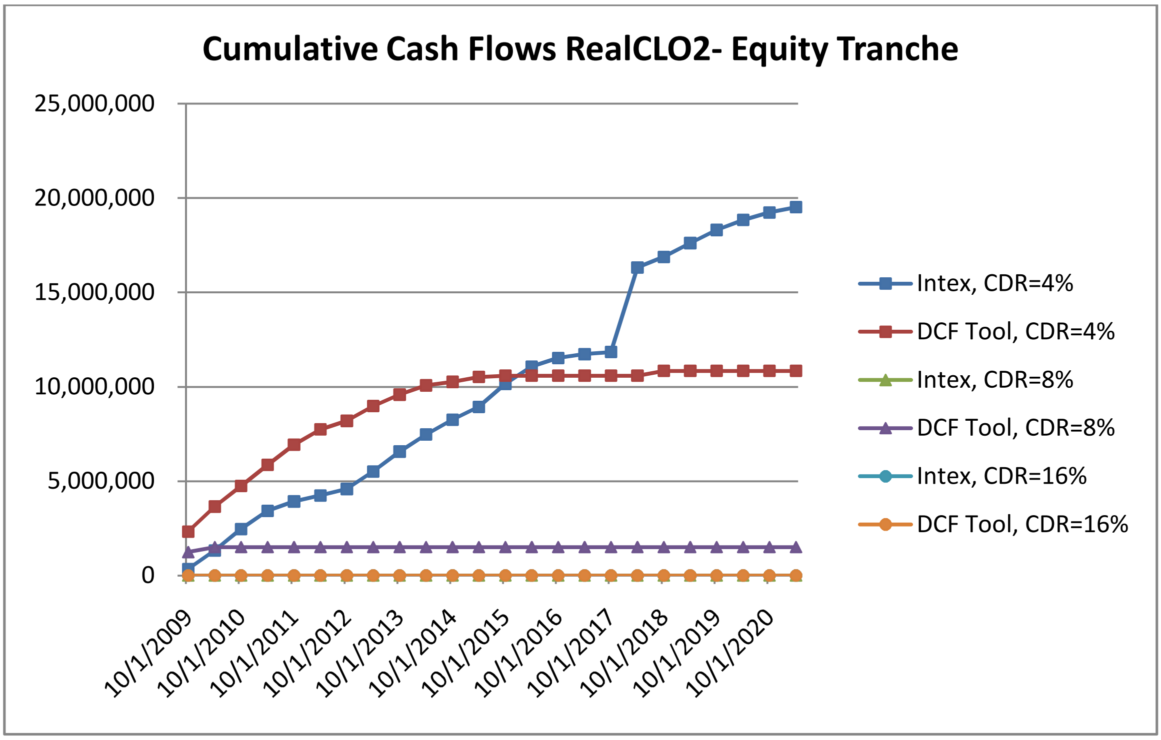


Figure 5-6: Cumulative cash flows RealCLO2 - Equity Tranche

Using the cash flows it is straight-forward to compute the price on each tranche. The determining factor in computing the price is then what discounting factor is used. As earlier mentioned the discounting factor used should reflect the uncertainty in the cash flows. On the back of that, and with some guidance from recent market discounting factors the spreads are selected. Comparing the prices computed in table 5-3 and 5-4 it is seen that the price on the senior tranches only deviate a few percentage in the different scenarios for both deals. For the equity tranche in RealCLO1 the prices deviate less than expected when looking at the cash flows, especially in the "God" scenario, where the cash flows differ a lot. This is explained by the heavy discounting factor which punishes back-loaded cash flows. Turing to the equity tranche in RealCLO2 the prices deviate significantly and this is simply explained by large differences in early cash flows.

	RealCLO1			
Tranche	Senior	Senior	Equity	Equity
Discount spread	700 bps	700 bps	6500 bps	6500 bps
Price	Intex	DCF tool	Intex	DCF tool
"Good"	0.7543	0.7560	0.1043	0.1060
"Average"	0.8115	0.7901	0.0374	0.0440
"Bad"	0.8439	0.8160	0.0000	0.0000

Table 5-3: Results RealCLO1

	RealCLO2			
Tranche	Senior	Senior	Equity	Equity
Discount spread	700 bps	700 bps	6500 bps	6500 bps
Price	Intex	DCF tool	Intex	DCF tool
"Good"	0.7481	0.7415	0.0677	0.1296
"Average"	0.8488	0.8047	0.0000	0.0341
"Bad"	0.8864	0.8603	0.0000	0.0000

Table 5-4: Results RealCLO2

6 Loss distribution modeling in CLOs

As mentioned earlier, one of the key issues when valuing CLOs is to estimate the amount of losses among the underlying assets. As of today a lot of research has be conducted in the area of pricing CDOs, the vast majority though have been focused on the pricing of synthetic CDOs. The reason for this is that synthetic CDOs can be rather standardized, simply structured and information on the underlying assets is usually easy to access. As a result it is possible to isolate the problem of pricing synthetic CDOs only to the loss distribution modeling.

This chapter will start by going through the standard approach for pricing synthetic CDOs. After I will adapt and implement the presented synthetic CDO pricing techniques to value CLOs.

6.1 Synthetic CDOs and credit-indices

Until the credit crunch 2007 the market for standardized CDSs swelled. As a result the credit derivatives market developed two indices to track CDS spreads. In Europe the Dow Jones iTraxx EUR evolved containing the average CDS spread of the 125 most liquid European investment-grade companies. The corresponding US index is called Dow Jones CDX NAIG and contains the average CDS spread of the 125 most liquid US investment-grade companies. The indices greatly increased the liquidity, transparency and cost efficiency in the credit derivatives market. The portfolio underlying indexes could now be used to create standardized synthetic CDO index tranches. The object of CDO pricing is then to calculate a fair spread for each tranche.

Dow Jones Itraxx Europe	
Tranche Name	Default Exposure
Equity	0-3%
Junior Mezzanine	3-6%
Senior Mezzanine	6-9%
Senior	9-12%
Super Senior	12-22%

Table 6-1: Tradable tranches on Dow Jones Itraxx Europe

Consider for example an investor selling protection on the equity tranche of Dow Jones Itraxx Europe. Assuming recovery rate of 0 percentages the investor is then willing to bear the loss of the first 3% of defaults among the 125 companies.

6.2 Synthetic CDO pricing

This chapter will contain a brief introduction to standardized synthetic CDO pricing from a theoretical perspective. It is important to bear in mind that this pricing technique is only possible since standardized synthetic CDOs have no distribution of payments and instead of writing down the assets when defaults occur the tranches are written down as soon as they are hit with losses.

In order to compute the loss in a portfolio of credits consider a portfolio consisting of N credits, each with a nominal Nom_j , *a* recovery rate Rec_j and a default time τ_j , for $j = 1,2, \dots, N.$ Also consider counting process $I_j(t) = 1_{\{\tau_j < t\}}$ which takes the value 1 if credit j has defaulted before t and 0 otherwise. The expression for the portfolio loss is then:

$$L(t) = \sum_{j=1}^{N} Nom_j \left(1 - Rec_j\right) I_j(t)$$

Since the intention is to value a tranche it is necessary to derive an expression transforming the portfolio loss into loss exhibited by a particular tranche. The determining factor for how much losses a tranche will exhibit is which interval of losses it is exposed to. Therefore suppose that the tranche under consideration covers losses on the portfolio between K_L and K_U. Where K_L is known as the attachment point and K_U as the detachment point. In the case of the equity investor in chapter 6.1 $K_L = 0\%$ and $K_U = 3\%$. For an arbitrary tranche l with the attachment point K_{L_l} and the detachment point K_{U_l} the tranche loss can be expressed as:

$$TL^{K_{L_l}, K_{U_l}}(L) = \frac{min\left(L, K_{U_l}\right) - min\left(L, K_{L_l}\right)}{K_{U_l} - K_{L_l}}$$

The expected tranche loss at each point in time can then be computed by multiplying the portfolio loss with the corresponding probability:

$$E\left(TL^{K_{L_l}, K_{U_l}}(L)\right) = \sum_{L=0}^{N} TL^{K_{L_l}, K_{U_l}}(L)\, Pr(Loss = L)$$

$$= \frac{1}{K_{U_j} - K_{L_j}} \int_0^1 min\left(x, K_{U_j}\right) - min\left(x, K_{L_j}\right) dF(x)$$

Finding the fair spread of a tranche can then be done using a similar approach to pricing single names CDSs. The CDO is said to consist of two legs: a default leg and a premium leg. The premium leg represents the payments the investor receives for selling protection, whereas the default leg represents the payments the investor pay to the protection buyer.

Consider K payment dates, $t_1 < t_i \dots < t_{K-1} < t_K$. Where the pay frequency is equal to $\eta = t_{i-1} - t_i$, the current discounting factor at time t_i is denoted $B(0, t_i)$ and S is the spread paid on the tranche. The premium leg can then be expressed as:

$$PL = \sum_{i=1}^{K} \eta S B(0, t_i) \left[1 - E\left(TL^{K_{L_j}, K_{U_j}}\left(L_{t_{i-1}}\right)\right)\right]$$

Assuming that defaults occur in the middle of the period the default leg can be expressed as:

$$DL = \sum_{i=1}^{K} B(t_0, t_i) \left(E\left(TL^{K_{L_j}, K_{U_j}}\left(L_{t_i}\right)\right) - E\left(TL^{K_{L_j}, K_{U_j}}\left(L_{t_{i-1}}\right)\right)\right)$$

Setting the premium leg equal to the default leg yields:

$$S = \frac{\sum_{i=1}^{K} B(t_0, t_i)\left(E\left(TL^{K_{L_j}, K_{U_j}}(L_{t_i})\right) - E\left(TL^{K_{L_j}, K_{U_j}}(L_{t_{i-1}})\right)\right)}{\sum_{i=1}^{K} \eta SB(t_0, t_i)\left[1 - E\left(TL^{K_{L_j}, K_{U_j}}(L_{t_{i-1}})\right)\right]}$$

It can be seen that as the expected loss on the tranche is computed it is straight forward to compute the fair spread. In the computation of the expected tranche loss it is necessary to derive the loss distribution $F(x)$. This is often quite cumbersome since it is necessary to model the dependence structure among the credits in the portfolio.

6.3 Loss distribution modeling

6.3.1 Default time distribution function

A cornerstone in loss distribution modeling is the individual default probabilities of the obligors in the portfolio. The introduction of hazard rates is analogue to Li [10]. Consider a security A with an associated default-time τ_A. Let τ_A be a continuous random variable which measures the time from now until the time when the default occurs also let $Q(t)$ represent the distribution function of τ_A under a risk-neutral or physical probability measure. Then:

$$Q(t) = Pr(\tau_A < t)$$

$Q(t)$ is then the cumulative default probability, measuring the probability for the obligor to default by time t. The corresponding cumulative survival probability is then $V(t)$=1- $Q(t)$.
The probability of default between t and $t + \Delta t$ conditional on survival until t:

$$Pr(t < \tau_A \leq t + \Delta t \,|\tau_A > t) = \frac{V(t) - V(t + \Delta t)}{V(t)} = \lambda(t)\Delta t$$

Where $\lambda(t)$ denotes the so called hazard rate, the expression can be rewritten to:

$$\frac{V(t) - V(t + \Delta t)}{\Delta t} = \lambda(t)V(t)$$

Taking the limits yields:

$$\frac{dV(t)}{\mathrm{d}t} = \lambda(t)V(t)$$

Solving for $V(t)$:

$$V(t) = e^{-\int_0^t \lambda(s)ds}$$

$$Q(t) = 1 - e^{-\int_0^t \lambda(s)ds}$$

By assuming that $\lambda(s)$ is constant between 0 and t:

$$V(t) = e^{-\lambda t}$$

$$Q(t) = 1 - e^{-\lambda t}$$

6.3.2 Structural model for credit risk

There exist two main suggestions for default processes in credit risk literature: reduced form and structural models, Hull [10] Reduced form models assume that the hazard rate for a company follows an exogenously given stochastic process. The parameters governing the hazard rate process are usually implied from the market data. Reduced form models are therefore said to consider the relationship between the financial situation and default for a company in an explicit way. Structural models, in opposite, tries to model the value of the assets and debt for a company in order to decide the time for default. This approach was originally proposed by Merton [12]. The basic idea is that, if the assets of a company are below its debt at the time for servicing the debt the company will default. The basic Merton model has been extended in numerous ways. For example Black and Cox [13] suggested a modified version which assume that at company defaults whenever its assets fall below a certain threshold, thus the company could default at any time and not only when debt is to be serviced. The structural model can be used in conjunction with either a risk-neutral or physical probability measure.

Merton's structural model assumes that the value of the assets A_j for company j follow a geometric Brownian motion:

$$\frac{dA_j}{A_j} = \mu_j dt + \sigma_j dW_j$$

Where μ_j and σ_j are the instantaneous expected rate of return and volatility of the assets of the company respectively, dW_j is a standard Wiener process where $W_j \sim N(0,t)$. Under the assumption that the capital structure of the company consists of equity, assets and debt, the equity can be seen as a call option on the company's assets with a strike price equal to the debt value. Conditional on the current assets value $A_{0,j}$, the probability of default for the company at time T when the debt is to be serviced, is then expressed as:

$$Pr\left(A_{T,j} < D_j \middle| A_{0,j}\right)$$

Using Ito's lemma it is possible to derive an expression for the value of the assets at the maturity for the debt $A_{T,i}$ based on the current assets value $A_{0,i}$:

$$A_{T,j} = A_{0,j} e^{\left\{\left(\mu_j - \frac{\sigma_j^2}{2}\right)T + \sigma_j \sqrt{T} X_j\right\}}$$

Where $X_j \sim N(0,1)$. It is now possible express the default of company j conditional on X_j:

$$Pr\left(X_j < K_j\right) = \Phi\left(K_j\right)$$

Where:

$$K_{T,j} = \frac{ln \frac{D_j}{A_{0,j}} - \left(\mu_j - \frac{\sigma_j^2}{2}\right)T}{\sigma_j \sqrt{T}}$$

It is here clear that the default process for company j is driven by the latent risk variable X_j.

Assume that the company can default at any time and will eventually do so. The link between X_j and the default time τ_j is then given via the distribution of the default time:

$$Q(t)_j = Pr\left(X_j < K_j\right)$$

$$\tau_j = Q_j^{-1}\left(\Phi\left(X_j\right)\right)$$

6.3.3 Calibration of default thresholds

Under a structural framework the default thresholds are defined as:

$$K_j = F_K^{-1}\left(Q(t)_j\right)$$

Under the assumption that the default probabilities are exponentially distributed and that the hazard rate is constant:

$$K_i = F_K^{-1}\left(1 - e^{-\lambda_j t}\right)$$

The hazard rates λ_j can be calibrated using either observed market data (risk-neutral probabilities) or historical default data (physical probabilities). In the case of synthetic CDOs it is common to use the observed spreads on the CDSs on the underlying assets. If the

6.3.4 Introduction to copulas

The major problem when computing the excepted loss in a credit portfolio is the modeling of the joint default times. As mentioned in chapter 6.3.3, using information on the underlying credits it is possible to extract the individual credit curves or, in a joint distribution, the marginal distributions. Under the assumption that the time-to-default for the credits in the portfolio is independent it would be straight forward to compute the loss. However, the independence assumption does not hold since it is observed that the default rate tends to be high in a booming economic environment and vice versa. This implies that the credits are all affected by the same set of macroeconomic variables and that there exists a mutual dependence among them [9]. By introducing copulas it is possible to model the individual margins and the dependence structure separately before linking them together to their full multivariate distribution.

A copula functions is function that links N arbitrary marginal distributions to their full multivariate distribution. Consider N univariate distribution functions $F_1(x_1), F_2(x_2), \ldots, F_N(x_N)$ the joint distribution function can then be written as:

$$Pr(X_1 \leq x_1, X_2 \leq x_2, \ldots, X_N \leq x_N) = C\big(Pr(X_1 \leq x_1), Pr(X_2 \leq x_2), \ldots, Pr(X_N \leq x_N)\big)$$

Which is equal to:

$$F(x_1, x_2, \ldots, x_N) = C(F_1(x_1), F_2(x_2), \ldots, F_N(x_N))$$

Were C is the copula function. In the following section some of the basic properties of copula functions are summarized. It is done without proofs, for a more formal representation I refer to Nelsen [14].

Definition: A 2-dimensional copula is a function $C : [0,1]^2 \rightarrow [0,1]$ with the following properties:

1. For every uniform random variable $u \in [0,1]$

$$C(0,u) = C(u,0) = 0$$

$$C(1,u) = C(u,1) = u$$

2. For every uniform random variable $u_1, u_2, v_1, v_2 \in [0,1]$ and $u_1 \leq u_2, v_1 \leq v_2$

$$C(u_1,u_2) - C(u_2,v_1) - C(u_1,v_2) + C(v_1,v_2) \geq 0$$

3. For every uniform random variable $u, v \in [0,1]$ C is considered symmetric if:

$$C(u,v) = C(v,u)$$

Sklar's theorem: Consider a joint distribution function F with margins F_1 and F_2. Then there exists a copula C such that:

$$F(x_1, x_2) = C\big(F_1(x_1), F_2(x_2)\big)$$

And vice versa:

$$C(u_1, u_2) = F\left(F_1^{-1}(u_1), F_2^{-1}(u_2)\right)$$

For $x_1, x_2 \in [-\infty, \infty]$. If F_1 and F_2 are continuous then, then C is unique. Otherwise, C is uniquely determined on $Range\ F_1 \times Range\ F_2$. Using Sklar's theorem it is possible to separate the structure of dependence into two parts: The first part is the specification of the marginal distribution function of the random variables. The second part is to select a suitable copula that captures the dependence structure between the random variables.

6.3.5 Copulas under consideration

6.3.5.1 Gaussian

A Gaussian or Normal copula with the correlation matrix Σ is defined as:

$$C_\Sigma^{Ga}(u_1, \dots, u_N) = \Phi_\Sigma\big(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_N)\big)$$

In order to show how Merton's structural model adopts a Gaussian copula consider a portfolio consisting of N companies. The default of each company j is triggered by a latent normal random risk variable X_j. The joint distribution of the latent variables can then be written as:

$$F(X_1, \dots, X_N) = Pr\big(F_1(X_1), \dots, F_N(X_N)\big)$$

Using the copula terminology it can be rewritten as:

$$F(X_1, \dots, X_N) = C\big(F_1(X_1), \dots, F_N(X_N)\big)$$

Since the latent variables X_j are standard normal distributed in the classic Merton model:

$$C(u_1, \dots, u_N) = \Phi_\Sigma\big(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_N)\big)$$

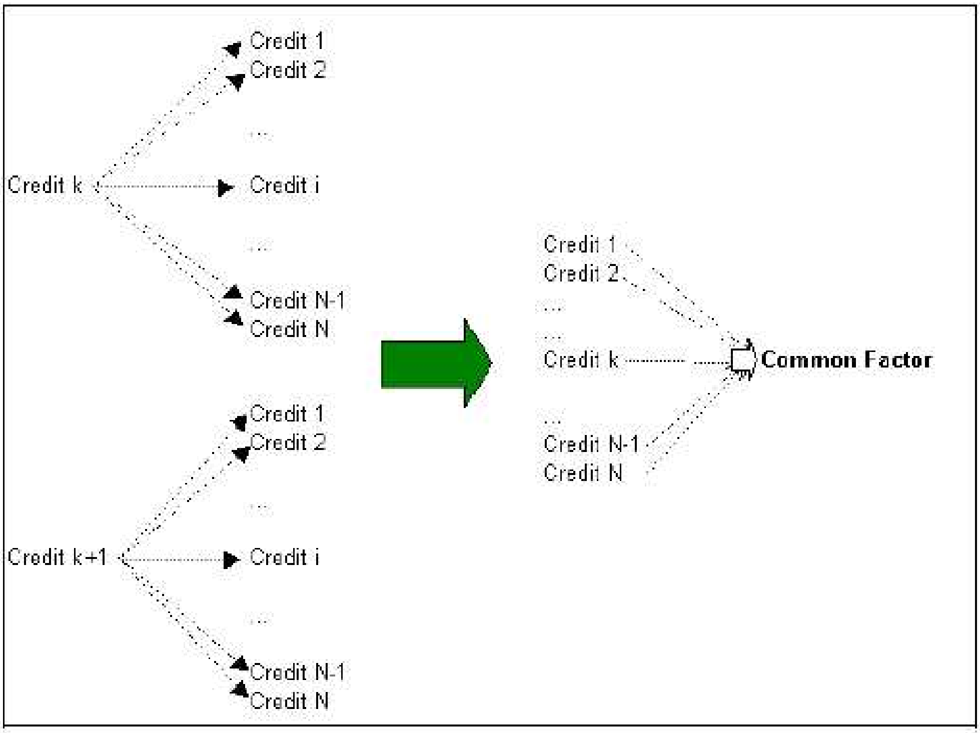
The Gaussian copula is the most commonly used copula in finance. Its popularity can be explained two main reasons:

- The marginal distributions are normal
- The dependence structure can be fully described by the correlation matrix and the marginal distributions

Implementation of a Gaussian copula involves the generation of vector X with correlated Gaussian variables under the correlations matrix Σ. Apart from the problem of calibrating such matrix, the use of a correlation matrix in a Monte Carlo framework suffers from two main drawbacks, Galiani [15] :

- In order to obtain a decent level of accuracy in the loss distribution a large number of simulations are needed, and is therefore time consuming.
- It is necessary to compute $\frac{N(N-1)}{2}$ pair wise correlation, were N is the number of obligors in the portfolio

In order to avoid the inconvenience associated with the use of a correlation matrix it is common market standard to introduce a factor based correlations structure. Instead of considering an obligor-vs.-obligor dependency the focus is on obligor-vs.-common factor dependency. There is no upper limit for the number of common factors that can be introduced, but the efficiency in computation time diminishes as more factors are introduced. In a multi factor dependence structure one could for example argue that an obligor is affected by a macroeconomic factor affecting all obligors and a sector factor affecting only the obligors in this particular sector. If we assume 10 different sectors and one general macroeconomic environment we would need to adopt 11 common factors. In a 100 obligor portfolio this would be necessary to estimate 1100 correlations instead of 4950 when using a correlation matrix. It has become market standard to use a one factor approach.



Source: Merrill Lynch

Figure 6-1: From pair wise to common factor dependency

Under a one factor Gaussian copula model the latent X_j driving the risk in each company in the portfolio is defined as:

$$X_j = \beta_j Z + w_j \varepsilon_j$$

Where Z is the common (systematic) risk factor which could represent the general macroeconomic environment. ε_i is the idiosyncratic, or firm specific risk factor which could be interpreted as the risk factors such as management, capital structure and etc. β_j is the sensitivity to the common risk factor and w_j is the idiosyncratic weight. Z and ε_j are independent standard random variables. Since the sum of these two is stable under convolution X_j also follows a standard normal distribution. The variance can therefore be expressed as:

$$Var[X_j] = \beta_j^2 Var[Z] + w_j^2 Var[\varepsilon_j] + 2\beta_j w_j Cov[Z, \varepsilon_j] = \beta_j^2 + w_j^2 = 1$$

$$w = \sqrt{1 - \beta_j^2} = \sqrt{1 - \rho_j} \, with \, \rho_j = \beta_j^2$$

$$Corr[X_j, X_l] = \frac{Cov[X_j, X_l]}{\sqrt{Var(X_j)}\sqrt{Var(X_l)}} = \sqrt{\rho_j}\sqrt{\rho_l} E[Z^2] = \sqrt{\rho_j \rho_l}$$

The latent variable X_j can thus be rewritten to:

$$X_j = \sqrt{\rho_j} Z + \sqrt{1 - \rho_j} \, \varepsilon_j$$

It can be seen that ρ_j drives the relative contribution of common risk and idiosyncratic risk into the company. The parameter ρ_j can also be seen as the correlation to the common risk source. Under the assumption that the correlation $Corr[X_j, X_l]$ is equal between all obligors X_j can be simplified to:

$$X_j = \sqrt{\rho} Z + \sqrt{1 - \rho} \, \varepsilon_j$$

A part from the regular linear correlation ρ two others dependence measures are usually also considered in a copula context. The first one is Kendall's tau τ_K, and is defined as:

$$\tau_k = 4 \iint C(u, v) dC(u, v) - 1$$

The second alternative dependence measure is the tail dependence. According to Joe [16] two random variables X_1 and X_2 , with the distribution functions $F_1$and F_2 and the associated copula C is said to have upper tail dependence if:

$$\lambda_U = \lim_{u \to 1} Pr\left(X_1 > F_1^{-1}(u) \middle| X_2 > F_2^{-1}(u)\right) = \lim_{u \to 1} \frac{C(u,u) + 1 - 2u}{1-u} > 0$$

Correspondingly it is said to have lower tail dependence if:

$$\lambda_L = \lim_{u \to 0} Pr\left(X_1 \le F_1^{-1}(u) \middle| X_2 \le F_2^{-1}(u)\right) = \lim_{u \to 0} \frac{C(u,u)}{u} > 0$$

Thus the tail dependence can be interpreted as the probability that one margin exceeds a certain threshold; conditional on the other margin has already exceeded that threshold.

For the Gaussian copula it can be shown that:

$$\tau_k = \frac{2}{\pi} arcsin\, \rho^2$$

$$\lambda_U = \lambda_L = \begin{cases} 0, & \rho < 1 \\ 1, & \rho = 1 \end{cases}$$

The following algorithm generate correlated uniform variables $\left(u_{1,}, u_2, \dots, u_N\right)$ from a Gaussian copula using a one factor approach and pair wise equal correlation ρ.

1. Simulate a standard normal variable Z;
2. Simulate N independent standard normal variables ε =$(\varepsilon_i, \varepsilon_2, \dots, \varepsilon_N)$
3. Set $X = \sqrt{\rho} Z + \sqrt{1-\rho}\, \varepsilon$
4. Set X back to a N-dimensional vector u of uniform variables by computing $u = \Phi(\mathrm{X})$

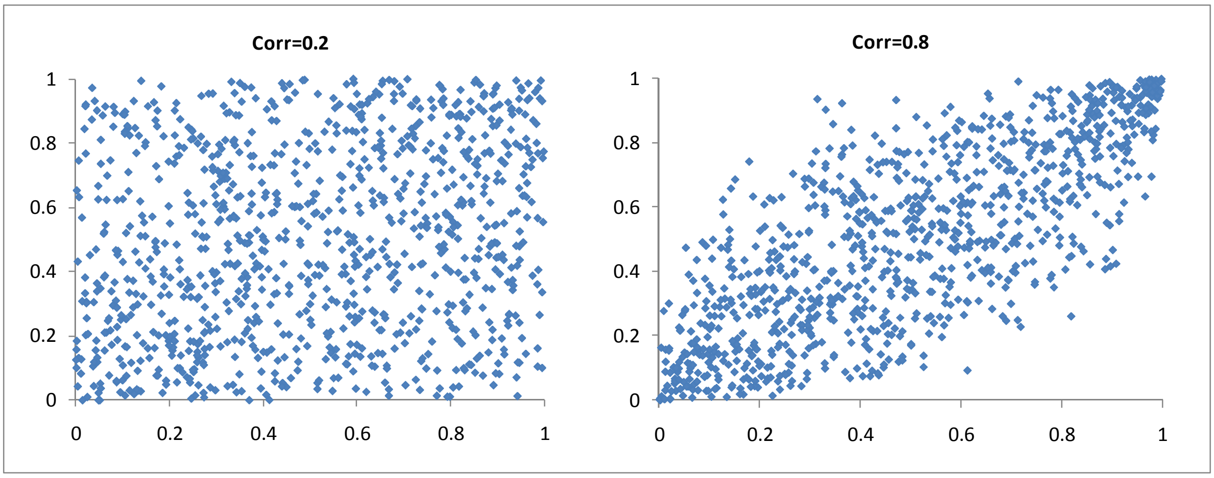


Figure 6-2: 1000 simulated standard uniform random variables using a Gaussian copula

6.3.5.2 Student t copula

A Student t copula with v degrees of freedom and the correlation matrix Σ is defined as:

$$C_{v,\Sigma}^{t}(u_1, \dots, u_N) = t_{v,\Sigma}\left(t_v^{-1}(u_1), \dots, t_v^{-1}(u_N)\right)$$

The student t copula is a natural extension to the Gaussian copula. In the Gaussian copula the underlying assumption of a normal distributed risk factor is preserved and thus assigns a low weight to extreme events. Although it is observed in the financial markets that extreme events takes place more often than suggested by a normal distribution, Abid and Naifar [17]. In order to obtain fatter tails it is natural to turn to the student t distribution. The student t distribution has the same bell shaped non-skewed characteristics as the normal distribution but it has fatter tails. Like the Gaussian copula the student copula belongs to the family of elliptical copulas.

Under a one factor Student t copula with equal pair wise correlation ρ and v degrees of freedom the latent variable X_j driving the risk in each company in the portfolio is defined as:

$$X_j = \sqrt{\frac{v}{s}}\left(\sqrt{\rho}Z + \sqrt{1-\rho}\,\varepsilon_j\right)$$

Where Z is the common (systematic) risk factor, ε_j is the idiosyncratic risk factor and s is a χ_v^2 distributed random variable with v degrees of freedom. When v is sufficiently large the Student t copula converges to the Gaussian copula.

In terms of alternative dependence measures it is possible to show that for the student t copula:

$$\tau_k = \frac{2}{\pi} arcsin\, \rho^2$$

$$\lambda_U = \lambda_L = 2t_{v+1}\left(-\sqrt{v+1}\sqrt{\frac{1-\rho^2}{1+\rho^2}}\right)$$

The following algorithm generate correlated uniform variables $\left(u_{1,}, u_2, \dots, u_N\right)$ from a student t copula with v degrees of freedom using a one factor approach and pair wise equal correlation ρ:

1. Simulate an independent random variable Z from standard normal distribution;
2. Simulate N independent random variables ε =$(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N)$ from standard normal distribution;
3. Simulate an independent random variable s from a χ_v^2 distribution with v degrees of freedom;
4. Set X= $\sqrt{\frac{v}{s}}\left(\sqrt{\rho}Z + \sqrt{1-\rho}\,\varepsilon\right)$;
5. Set X back to a N-dimensional vector u of uniform variables by computing $u = \mathrm{t_v}(\mathrm{X})$. Where $\mathrm{t_v}$ is a student t distribution with v degrees of freedom.

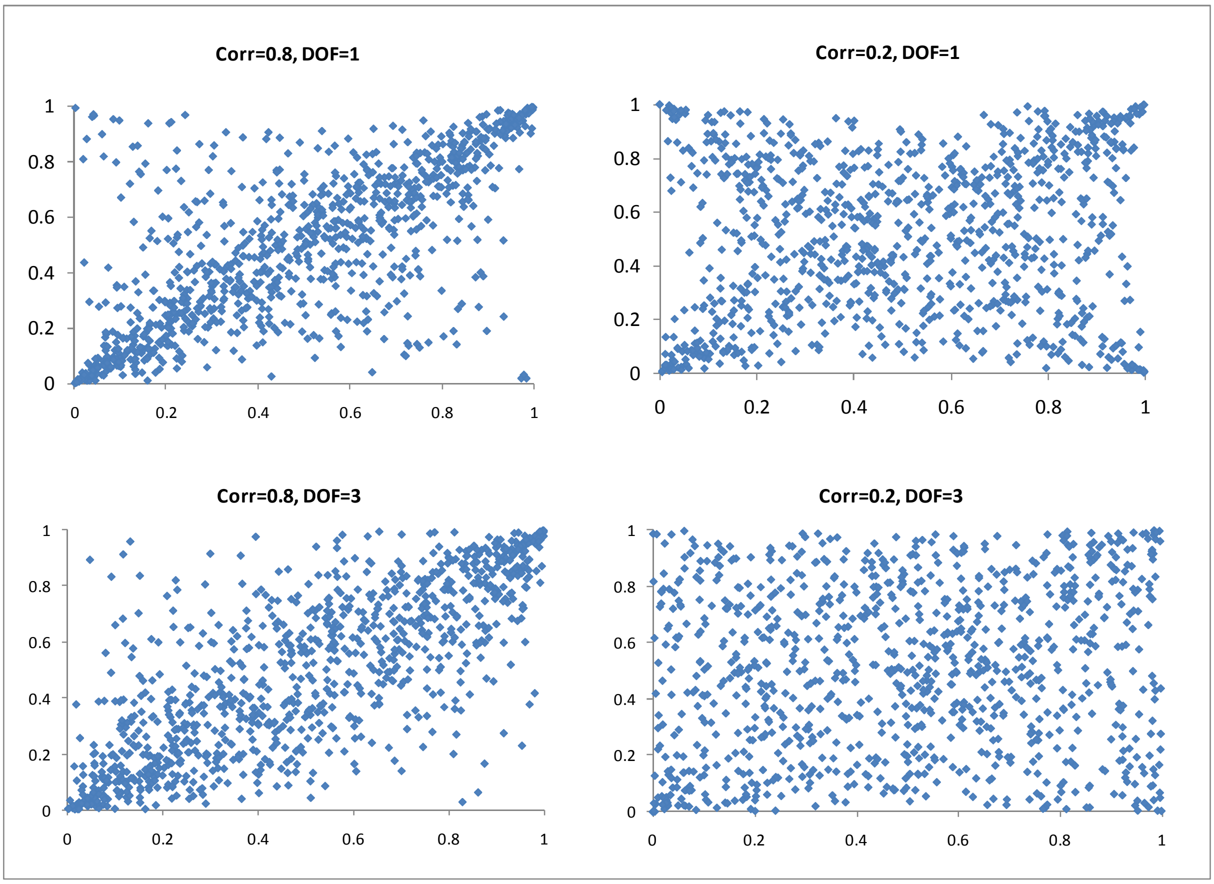


Figure 6-3: 1000 simulated standard uniform random variables using a Student t copula

6.3.5.3 Clayton copula

The Clayton copula belongs to the family of Archimedean copulas. Any Archimedean copula can be defined as:

$$C^{Arch}(u_1, \dots, u_m) = \phi^{-1}\big(\phi(u_1) + \dots + \phi(u_m)\big)$$

Where ϕ is as a decreasing function known as the generator of the copula and correspondingly ϕ^{-1} denotes the inverse of the generator, Frees and Valdes [18]. In the case of the Clayton copula ϕ and ϕ^{-1} are defined as, for $\theta > 0$:

$$\phi(t) = \left(t^{-\theta} - 1\right)$$

$$\phi^{-1}(s) = (1 + s)^{-\frac{1}{\theta}}$$

In terms of alternative dependence measures it is possible to show that for the Clayton copula:

$$\tau_k = \frac{\theta}{\theta + 2}$$

$$\lambda_L = \begin{cases} 0, & \theta = 0 \\ 2^{\frac{-1}{\theta}}, & \theta > 0 \end{cases}$$

$$\lambda_U = \begin{cases} 0, & \theta = 0 \\ 0, & \theta > 0 \end{cases}$$

The following algorithm generate correlated uniform variables $(u_{1,}, u_2, \ldots, u_N)$ from a Clayton copula with pair wise equal correlation ρ:

1. Compute $\tau_k = \frac{2}{\pi}\sin^{-1}\rho^2$;
2. Compute $\theta = \frac{2\tau_k}{1-\tau_k}$;
3. Simulate an independent random variable g from a Gamma distribution with the shape parameter $\frac{1}{\theta}$;
4. Simulate N independent random variables $u = (u_1, u_2, \ldots, u_N)$ from a uniform distribution;
5. Set $Y = \frac{ln(u)}{g}$;
6. Generate a N-dimensional vector u of uniform variables by computing $u = (1+Y)^{\frac{-1}{\theta}}$.

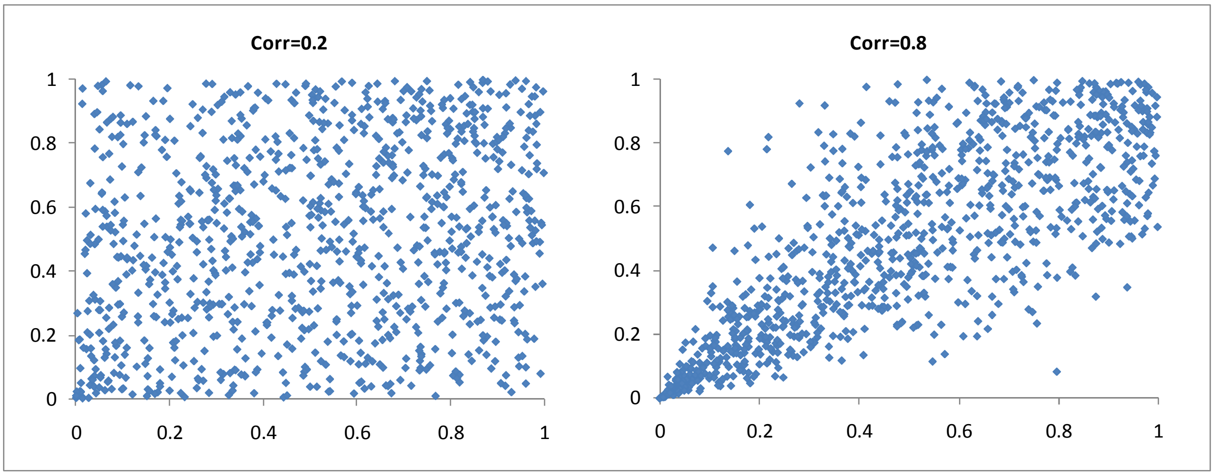


Figure 6-4: 1000 simulated standard uniform random variables using a Clayton copula

6.4 Implied correlation

Analogous to computing the implied volatility on options using Black and Schools formula and observed market quotes it is possible to compute the implied correlation on any "traded" tranche using spreads quoted on Itraxx or CDX. The implied correlation is compute by solving the one factor Gaussian copula model for the correlation under an observed spread (market standard). The correlation has different influence on different tranches. If for example the market perceived correlation were up this would reduce the spread on the equity tranche whereas the spread on the most senior tranche would increase. This can be explained by an illustrative example by JPMorgan. Consider a cat walking blindfold through a room with mousetraps. If the cat had only one life (equity tranche) it would prefer the mousetraps to be positioned in clustered (high correlation) because it will lose its life no matter if it stepped on one trap or on a cluster of traps. If the cat instead had nine lives (senior tranche) it would prefer to have the traps evenly distributed in the room since it could afford to step on nine traps but not on a cluster containing more than nine traps.

When pricing tranches under the one factor Gaussian copula the same correlation applies for all tranches, but when implying the correlation on different tranches on the same index various correlations are obtained. This is known as the correlation smile and the major conclusion from it is that either the dependence structure obtained by a one factor Gaussian copula is not correct or the market is pricing is inconsistent. A major share of all research on CDOs is devoted to finding a loss distribution model which solves the problem of the correlation smile.

6.5 CLO valuation using concepts from synthetic CDO pricing

In this chapter I will present the concepts for applying synthetic CDO pricing techniques on CLOs. The loss distribution will be done using Monte Carlo simulations. In terms of loss distribution modeling, this means simulating dependent default times for the different obligors in a portfolio. Although there are many obvious similarities between CLOs and standardized synthetic CDOs there exists some small differences which makes it far more problematic to adopt loss distribution models. A summary of the problems are given below:

Complex distribution of payments

In opposite to standardized synthetic CDOs, CLOs have a highly complex distribution of payments structure and thus a cash flow model able to capture the projected cash flows from the assets and the related distribution of these proceeds is needed.

Calibration of credit curves

The problem with calibration lies in the difference in underlying assets in a CLO compared to a standardized synthetic CDO. The CDS contracts acting as collateral in a standardized synthetic CDO are liquid products making it possible to get market quotes for the spreads. Using the CDS spreads for different maturities written on the same obligor it is possible to imply the market perceived hazard rate and the associated term structure. This is done using an assumed recovery rate and a simple bootstrapping method. If desired, it is also possible to construct a full correlation matrix for the CDSs using the market quotes. Moving on to CLOs where the underlying assets are mainly leveraged loans used for buy-outs and M&A activity there exist market quotes for a great part of the loan universe, but a vast part of the quotes is considered unreliable due to the low liquidity. For the term structure of the hazard rate it is also problematic since it is hard to find several similar loans with different maturities. With market quotes it is possible to compute of the implied hazard rate. This can be done using a set of assumptions regarding prepayments and recoveries together with a similar risk-free asset.

Term structure

This is major problem in loss distribution modeling in a CLO. In standardized synthetic CDOs the maturity of the tranche is equal to the maturity of the underlying CDS contracts. Hence, we can simulate one default time for each of the underlying obligors since we know that the credit risk of the company will stay in the CDO until maturity or fall out before maturity. In case any obligor defaults we will only write down the value of the affected tranche. If we analogues simulate a default time for each of the underlying obligors in a CLO we run into problems. Imagine that company A has a simulated default time of 10 years, the loan issued by the company that we have in the CLO portfolio has a maturity of 5 years. As the loan matures before the company defaults the CLO does not suffer from it. If we are still in the reinvestment period the money are instead reinvested in a new loan with a different risk and default time. We will also experience a related problem when we try to imply the hazard rate from a loan since the maturity of a loan is usually 6 years whereas the maturity of a CLO is 15 years. This means that we can only imply the 6 years average hazard rate even though we would at least need the 15 years average to make a fair assumption.

6.6 Model implementation of synthetic CDO loss modeling

In order to easily integrate the loss distribution model in the cash flow model the output should be in the form of CDR in each period. The correlation is assumed to be pair-wise equal and exogenous given by the user of the tool.

Complex distribution of payments

Loosely spoken you can say that this problem is solved by replacing the equation for pricing standardized synthetic CDOs derived in chapter 6.4 with the DCF model implemented in chapter 4.

Calibration of credit curves

Due to the lack of information with regard to the underlying assets I think the best way to imply the hazard rate is to use historical default rates. The historical default rates are given annually by for example Moody's. A positive aspect with the use this kind of data is that the default rates are usually given per rating; hence it is possible to use different hazard rates for various rated loans in the portfolio. Luckily this works out well since CLO investor reports usually provide the distribution of ratings among the underlying loans. The hazard rate is assumed to be constant over the life of the CLO. The average hazard over T years is then calculated using the distribution for the default-time from chapter 6.3.2:

$$\lambda = -\frac{ln\big(1 - Q(T)\big)}{T}$$

To make the model flexible the user of the implemented model has to choose the time T. Theoretically the most suitable time to use is the time-to-maturity for the CLO.

Term(years)/Rating	1	2	3	4	5	7	10	15	20
Aaa	0.00%	0.00%	0.00%	0.03%	0.10%	0.25%	0.52%	0.99%	1.19%
Aa	0.01%	0.02%	0.04%	0.11%	0.18%	0.34%	0.52%	1.11%	1.93%
A	0.02%	0.10%	0.22%	0.34%	0.47%	0.76%	1.29%	2.36%	4.24%
Baa	0.18%	0.51%	0.93%	1.43%	1.94%	2.96%	4.64%	8.24%	11.36%
ba	1.21%	3.22%	5.57%	7.96%	10.22%	14.01%	19.12%	28.38%	35.09%
B	5.24%	11.30%	17.04%	22.05%	26.79%	34.77%	43.34%	52.18%	54.42%
Caa-C	19.48%	30.49%	39.72%	46.90%	52.62%	59.94%	69.18%	70.87%	70.87%

Table 6-2: Moody's average cumulative default rates 1970-2006

An alternative approach to implement the hazard rate is to use the price on a credit. Using the assumption that the credit quality of the LLI or ELLI reflects the CLO portfolio I also implement a model for computing the implied hazard rate in the index. The obtained hazard rate is the average hazard rate for the average maturity, in years, of the assets in the CLO. The intention is therefore not to provide the user with a correct hazard rate, but to give the user an indicative and comparable measure of it.

Consider a loan with a nominal Nom, an annual floating coupon C with a reference rate at $D(0, t_i)$, *a* constant recovery rate Rec, a constant prepayment rate CPR and a discount factor $B(0, t_i)$. Also consider K payment dates, $t_1 < t_2 < t_i \dots < t_{K-1} < t_K$ where the pay frequency is equal to $\eta = t_{i-1} - t_i$. Assuming that prepayments occur immediately after coupon payments the risk-free price $P^{Riskfree}$ can be expressed as:

$$P^{Riskfree} = \sum_{i=1}^{K} \eta(CPR + C + D(0, t_i))\mathrm{B}(0, \mathrm{t_i})Nom(1 - \eta CPR)^{i-1} + Nom(1 - \eta CPR)^{K}\mathrm{B}(0, \mathrm{t_K})$$

Assuming that default can occur immediately before payments the present value of the expected loss EL can be expressed as:

$$EL = \sum_{i=1}^{K} \left(\sum_{j=1}^{K-i} \eta \left(CPR + C + D(t_i, t_j) \right) \mathrm{B}(\mathrm{t_i}, \mathrm{t_j}) Nom(1 - \eta CPR)^{j-1} + Nom(1 - \eta CPR)^{K-(i-1)}\mathrm{B}(t_i, t_K) - NomRec \right) \mathrm{B}(0, \mathrm{t_i})$$

The constant CDR can then be compute as:

$$CDR = \frac{P^{Riskfree} - P^{Market}}{EL\eta}$$

Using the CDR the expression for the hazard rat h is:

$$h = -ln(1 - CDR)$$

Term structure

I assume that if a loan prepays or matures, the principal is reinvested in a loan issued by the same company. I further assume that recovery payments from defaulted companies are reinvested equally among the remaining performing companies. This is a very strong assumption to make but it enables us to simulate only one default-time for each company.

As mentioned in the beginning of the chapter a simulated default scenario should be induced to the DCF model via periodic CDR in order to easily link the DCF model to the loss distribution model. Under the assumption that the nominal of all obligors are equal the periodic CDR can be computed by considering a portfolio consisting of *N* credits, each with a default-time τ_j for $j = 1,2, \dots, N.$ Also consider counting process $I_j(t) = 1_{\{\tau_j < t\}}$ which takes the value 1 if credit j has defaulted before t and 0 otherwise. The expression for the annual CDR in the portfolio is then:

$$CDR_{t_i} = \frac{\sum_{j=1}^{N} I_j(t_{i-1}) - \sum_{j=1}^{N} I_j(t_i)}{\sum_{j=1}^{N} I_j(t_{i-1})} \frac{12}{\eta}$$

Where the pay frequency is equal to $\eta = t_i - t_{i-1}$

The following algorithm generates correlated default times $\left(\tau_{1,}, \tau_2, \ldots, \tau_N\right)$:

- Simulate N independent random variables $U = (u_1, u_2, \ldots, u_N)$ under a desired copula
- Generate the N -dimensional vector τ of default times by computing $\tau = -\frac{ln(U)}{h}$

Where h is the hazard rate vector.

7 Performance of loss distribution model

7.1 Test scenarios

In order to see how the implemented tool performs and compare how prices are affected by changing dependence structure two scenarios are defined. The two scenarios are equal expect from varying linear correlation. The degrees of freedom for the Student t copula is assumed to be DOF=1 and DOF=3 (same as chapter 6.3.5.2). The only idea behind the selection of degrees of freedom is that they should show a clear difference in the dependence structure; studying figure I 6-3 think that this is the case with DOF=1 and DOF=3. The two scenarios are presented in table 7-1.

	"Low"	"High"
Correlation	0.20	0.80
CPR	10%	10%
Recovery	50%	50%
Recovery lag	0.5 years	0.5 years
CPR/CDR timing	Start	Start

Table 7-1: Test scenarios loss distribution model

The CLO under price consideration is RealCLO1 from chapter 4. All parameters are the same as before, but in order to compute the credit curves for the different rated obligors in the CLO the rating distribution and the number of obligors is also needed. In RealCLO1 there are 65 different obligors.

Rating (Moody's)	10 year average hazard rate	Portfolio weight RealCLO1
Aaa	0.000522362	0%
Aa	0.000523367	0%
A	0.001295354	0%
Baa	0.004747952	1%
Ba	0.021217888	22%
B	0.056815464	65%
Caa-C	0.117694147	12%

Table 7-2: Rating distribution RealCLO1, as of 2009-05-29

After a few test I experienced that 30000 simulations is sufficient to return identical distributions. Therefore this is the number if simulations that will be used in the test.

7.2 Results

We start analyzing the results by looking at the price on the senior tranche under the different copulas and correlation. The first observation to make is how homogenous the prices and the standard deviations are for both correlations. This is not unexpected since CLOs are structured to protect the senior tranches. High default rates early will trigger coverage tests to break and thus stop reinvesting principal proceeds and redirect interest cash flows to redeem the senior tranche. This will increase the price on the senior tranche since large cash flows will be received early. If the CLO exhibits extremely high default rates early it can affect the senior tranche negatively, but under the assumption of a recovery rate of 50% extreme events are damped. Looking at figure 7-1 and 7-2 we can also observe that the distribution for the price looks very similar for the different copulas.

Looking at the results for the mezzanine tranche we see that the prices and standard deviations are slightly more homogenous for the higher correlation. The prices are also lower for the higher correlation. This can be explained by the increased risk of high default rates which cuts all cash flows to the tranche. This can also be seen on the left tail on the distribution for all copulas on figure 7-4. For the lower correlation on figure 7-5 the distributions look very similar. At last turning to the equity tranche, here we observe that the prices and standard deviation for lower correlation differ much whereas the higher correlation is more homogenous except from the Clayton copula. It can also be seen that low correlation is, as expected, positive for the equity tranche.

	Senior		Mezzanine (Class C)		Equity	
	Average Price	σ	Average Price	σ	Average Price	σ
Gauss, Corr=0.2	0.7719	0.0255	0.2540	0.0510	0.1091	0.0689
Student, DOF=1, Corr =0.2	0.7706	0.0279	0.2334	0.0784	0.1427	0.0716
Student, DOF=3, Corr =0.2	0.7724	0.0280	0.2447	0.0651	0.1254	0.0719
Clayton, Corr =0.2	0.7716	0.0218	0.2625	0.0312	0.0969	0.0593
Gauss, Corr =0.8	0.7604	0.0221	0.2116	0.1010	0.1640	0.0752
Student, DOF=1, Corr =0.8	0.7560	0.0213	0.2112	0.1005	0.1710	0.0690
Student, DOF=3, Corr =0.8	0.7582	0.0213	0.2122	0.1005	0.1676	0.0720
Clayton, Corr =0.8	0.7694	0.0285	0.2184	0.0951	0.1389	0.0854

Table 7-3: Prices on RealCLO1, senior, mezzanine, equity tranches with Corr=0.2 and Corr=0.8. Number of simulations is 30000.

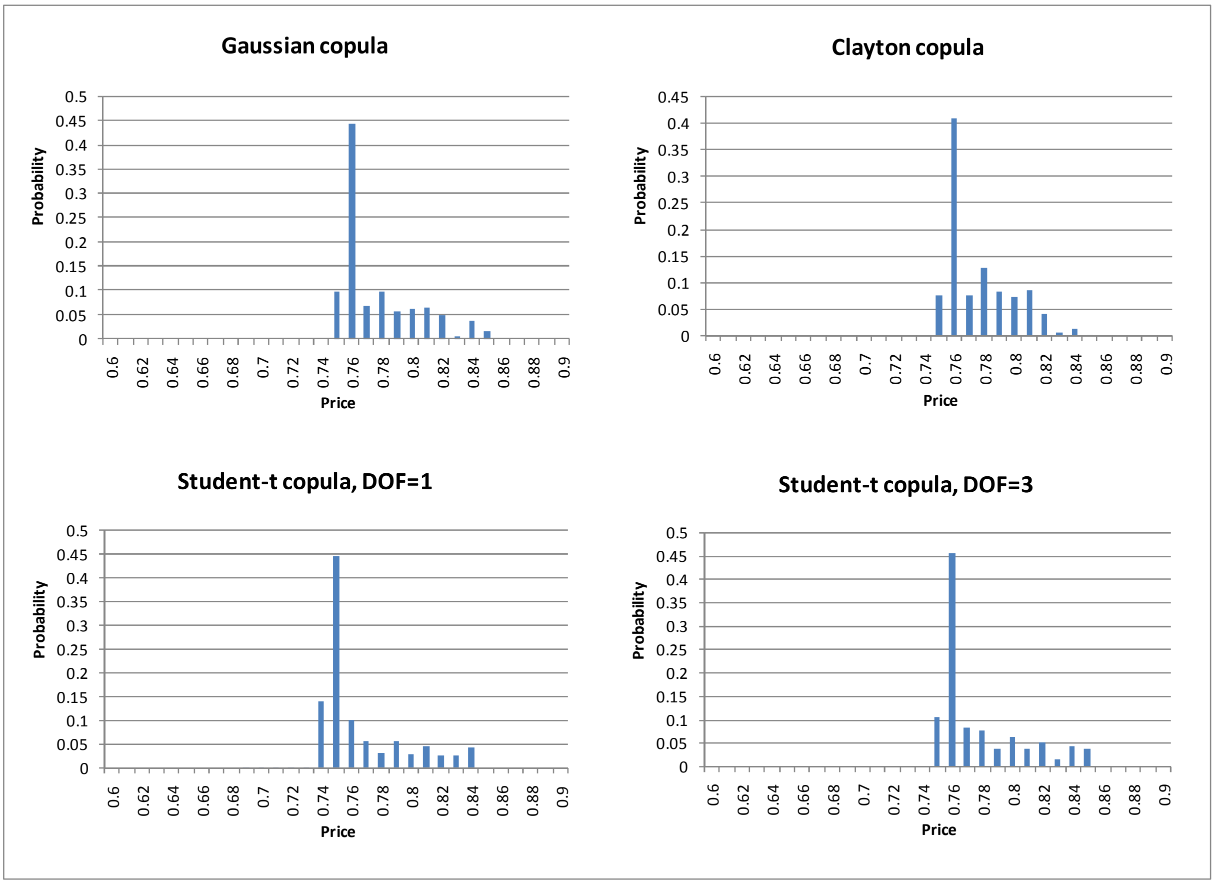


Figure 7-1: Price distribution RealCLO1 senior tranche, correlation=0.2, 30000 simulations

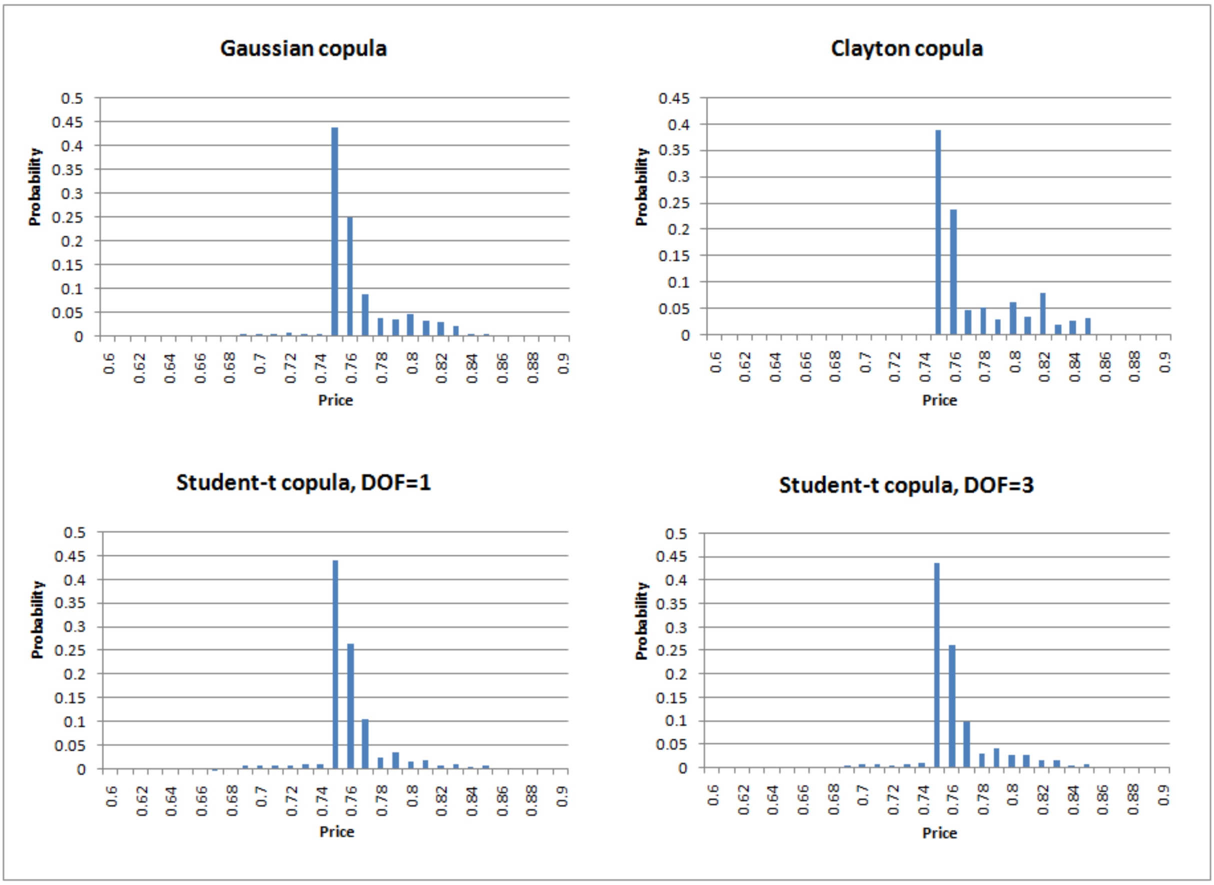


Figure 7-2: Price distribution RealCLO1 senior tranche, correlation=0.8, 30000 simulations

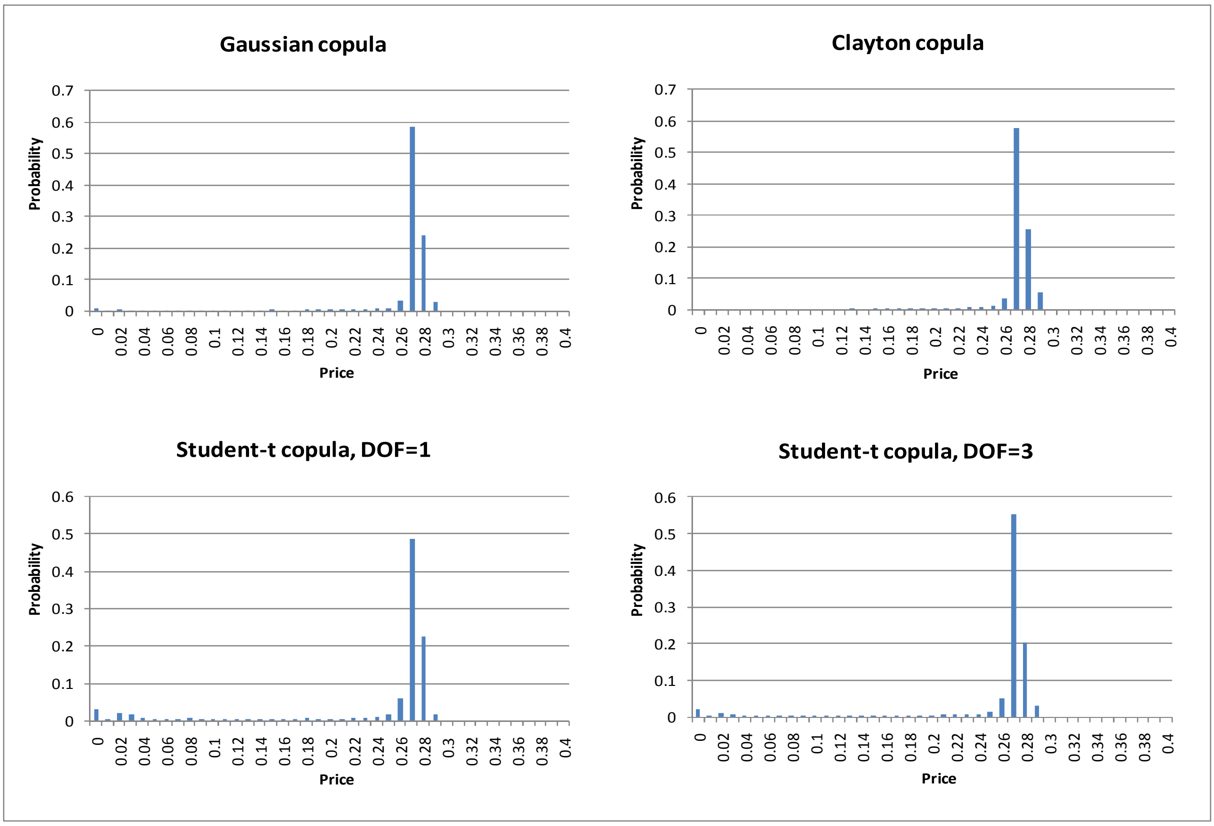


Figure 7-3: Price distribution RealCLO1 mezzanine tranche, correlation=0.2, 30000 simulations

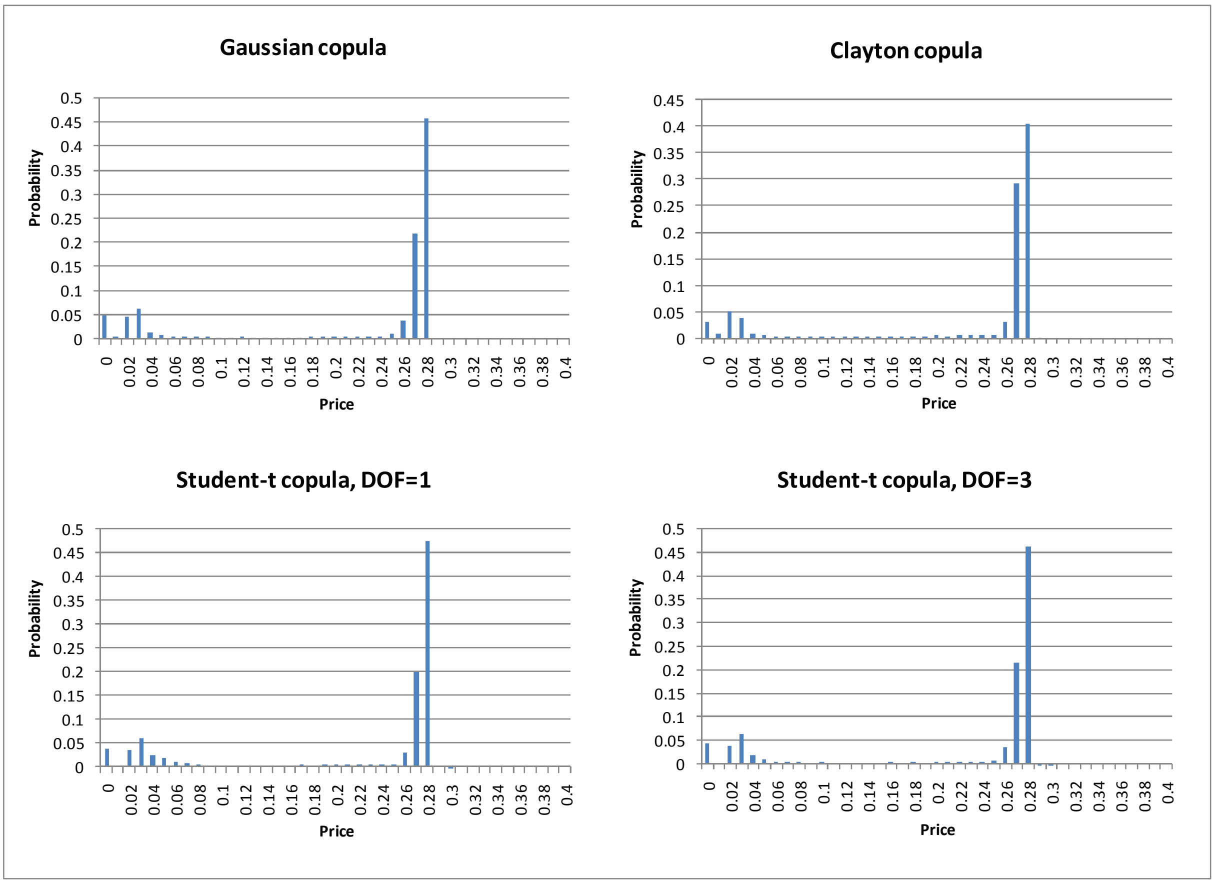


Figure 7-4: Price distribution RealCLO1 mezzanine tranche, correlation=0.8, 30000 simulations

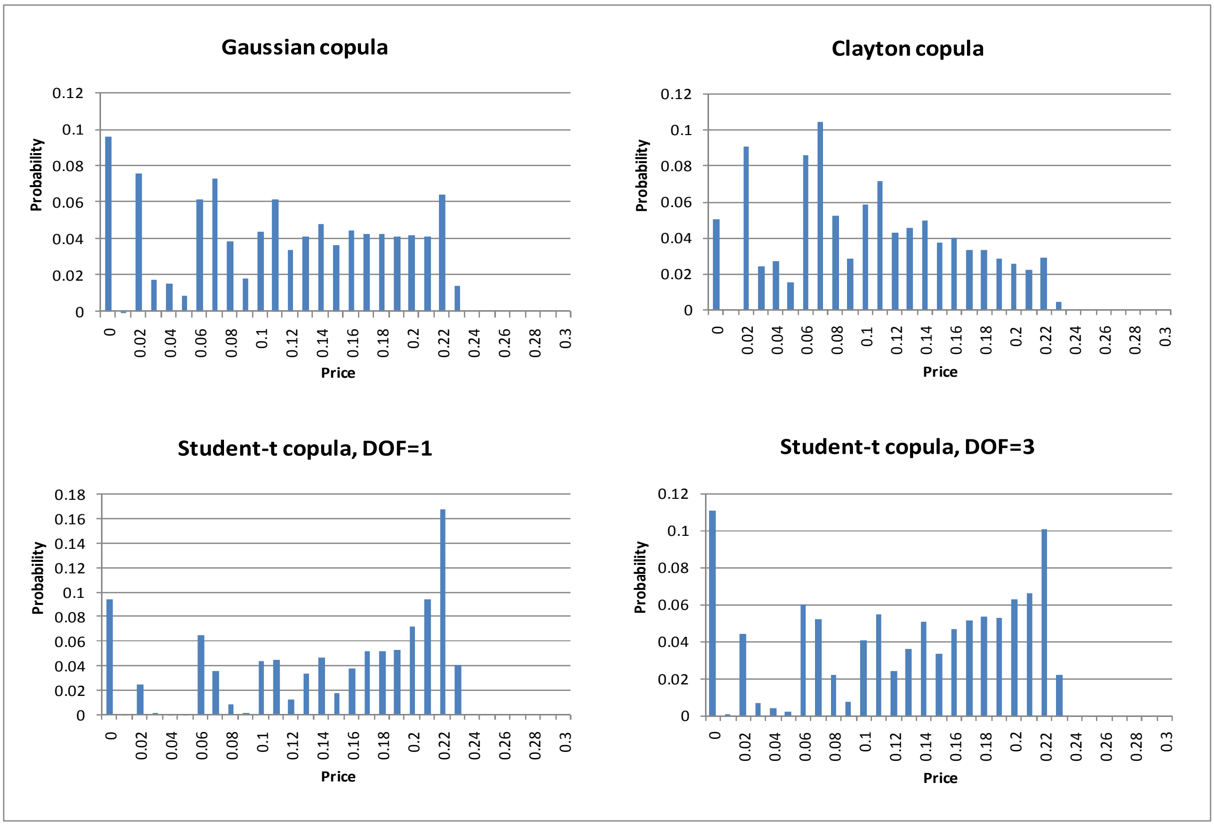


Figure 7-5: Price distribution RealCLO1 equity tranche, correlation=0.2, 30000 simulations

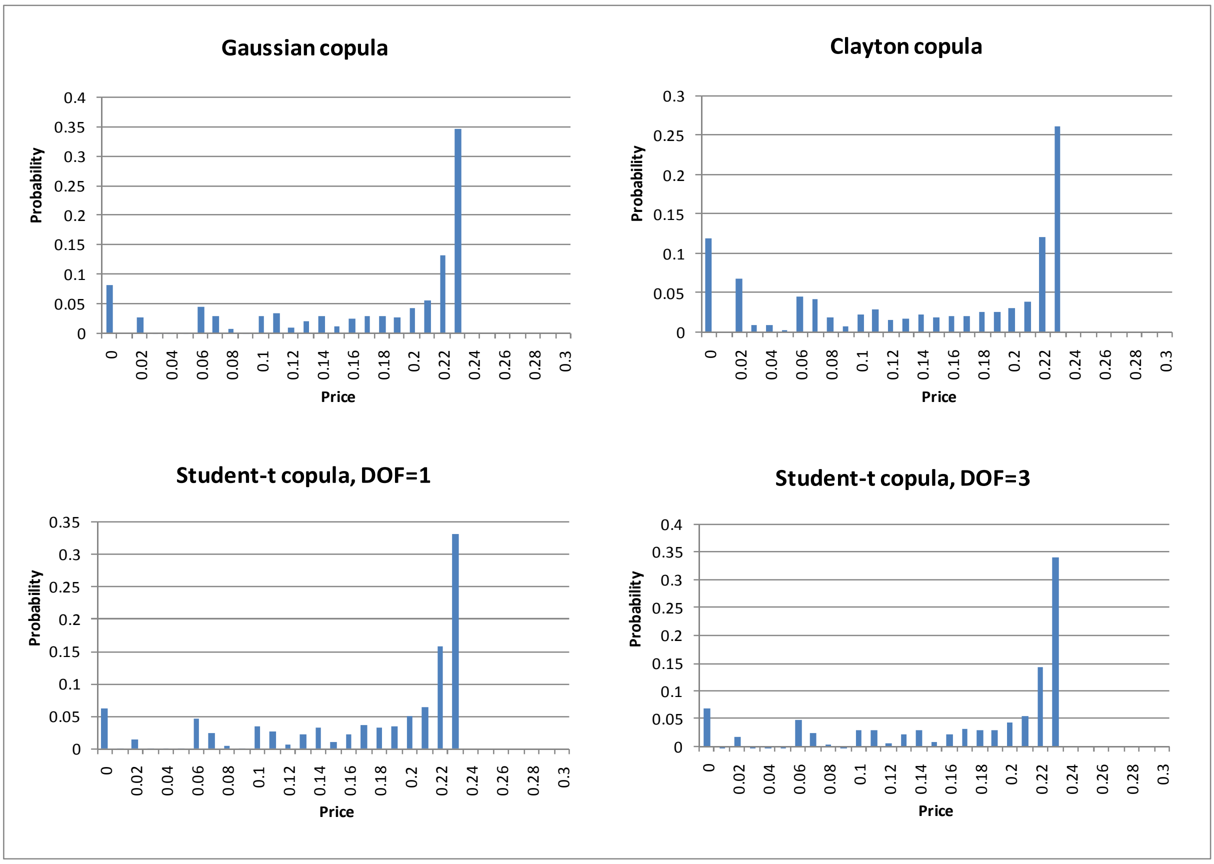


Figure 7-6: Price distribution RealCLO1 equity tranche, correlation=0.8, 30000 simulations

8 Conclusions

The aim for this thesis is to implement a model for valuing arbitrary CLOs. Therefore the first part of this thesis presents techniques for implementing a discounted cash flow (DCF) model for pricing arbitrary CLOs. The model is compared to Intex a commercial DCF tool in used for pricing CLOs. The implemented DCF tool produces similar cash flows to Intex. As expected the projected cash flows differs more for junior tranches. Junior tranches are more sensitive to the projected cash flows from the underlying assets due to their position in the distribution of payments. This is especially true for the equity tranche due to the high leverage.

The second part of the thesis then involves the implementation of a loss distribution model for the underlying assets in the CLO. In the DCF model the annual default rate was exogenously given. In this part I adopt concepts from standardized synthetic CDO pricing to compute the periodic CDR. Three different copulas are implemented. The exogenously given variables are now the pair-wise equal linear correlation and the historically cumulative default probabilities. Tests were undertaken to see how the different tranches react to various correlations and how much the choice of copula (dependence structure) influence the price on different tranches. It was possible to see that the senior tranche is very robust and is least influenced by changes in correlation and choice of copula. And then, as expected, more junior tranches are more sensitive as the price difference and standard deviation for the price increase for changes in correlation and copula. For all price distributions it can be seen that the Gaussian and student t copula with 3 degrees of freedom perform most similar, which I also expected as the student t copula converges to the Gaussian as the degrees of freedom grew large. Another interesting feature from a risk perspective is the fat lower tails for the mezzanine and equity tranche under the high correlation. The aim of keeping the computation time down succeeded well as 30000 simulations with 65 obligors (RealCLO1) take approximately 20 minutes on a regular PC. 30000 simulations might seem low, but after trying some different I find this number to be a good trade-off between computation time and precision.

The main problem in the valuation of CLOs is the amount of parameters affecting the price. Apart from all scenario related assumptions such as prepayments and recovery rates, it is also necessary to make assumptions regarding the reinvestment profile. It is also a necessity to have an accurate cash flow model, otherwise good assumptions are worthless. Based on the result for the price distributions my opinion is that, under the current model, the gain in finding the correct dependence structure for the default correlation is limited as other error sources are potentially larger.

There are a number of ways to further develop the model. From one side one could improve the DCF model to take into features such as potential liquidation scenarios of the CLO. As mentioned in chapter 1 the equity investors can liquidate the CLO in case it performs bad early and the senior investors can choose to liquidate the CLO in case of an “event of default”. Another way to improve the DCF model would be to avoid assumptions that recovery payments is reinvested in the same company. One could also implement some kind of model for the recovery and prepayments. Since recovery rates and prepayment rates are negatively correlated with the default rate, if might be possible to model those as a function of the default rate.

Bibliography

[1] S&P Leveraged Commentary and Data, www.lcdcomps.com

[2] Goodman B., *CDOs structure and analysis*, 2004

[3] The Bond Market Association, *CDO Primer*, 2004

[4] International Swaps and Derivatives Association, www.isda.org

[5] Duffie D., Singleton K. J., *Credit Risk Pricing, Measurement and Management,* 2003

[6] Morgan Stanley, *Leveraged loans as an asset class*, 2008

[7] Beim D., *Term Structure and the Non-Cash Value in Bonds*, 1992

[8] Jobst A., *Loan Securitization: and Default term structure and asset pricing based on loss prioritization*, 2005

[9] Intex Solutions, www.intex.com

[10] Li D. X., *On Default Correlation: A Copula Function Approach*, 1999

[11] Hull J. C., *Options, Futures and Derivatives 7th edition*, 2009

[12] Merton R., *On the Pricing of Corporate Debt: The Risk Structure of Interest Rates*, 1974

[13] Black F., Cox J. C., *Valuing Corporate Securities: Some Effects of Bond Indenture Provisions*, 1976

[14] Nelsen R. B., *A Introduction to Copulas*, 1999

[15] Merill Lynch, *Factor Models for Synthetic CDO valuation,* 2004

[16] Joe, H., *Multivariate models and dependence concepts*, 1997

[17] Abid F., Naifar N, *Copula based simulation procedures for pricing basket Credit Derivatives*, 2007

[18] Frees E. W., E. A. Valdez, *Understanding relationships using copulas*, 1998

APPENDIX

A-1 Predefined distribution of payments modules

This part of the appendix consist of the computation behind the different distribution of payments modules. In each module the inputs and computations are displayed. Inside the brackets after the name for each module the sub routine name is also displayed to make easier to later track the modules programming code:

Pay any fix cost (“PayFixAdmin”)

Main input parameters for this module:

- Total fixed cost for the CLO FC
- Account to pay from; interest or principal

The transferred amount TA is then calculated as:

$$TA = FC$$

Pay variable cost on original asset balance (“FlAdminOrgTotAssets”)

Module that pays any type of variable cost based on the original collateral balance. Main Input parameters for this module:

- Variable cost VC
- Original collateral balance Nom^{Org}
- Account to pay from; interest or principal

The transferred amount TA is then calculated as:

$$TA = VC \times Nom^{Org}$$

Pay variable cost on collateral balance (“FlAdminCurrTotAssets”)

Module that pays any type of variable cost based on the current collateral balance. It is calculated as the average collateral balance in the current period. Main input parameters for this module:

- Variable cost VC
- Nominal of performing assets in the start of the period $Nom_{t_{i-1}}^{Perf}$ and in the end of the period $Nom_{t_i}^{Perf}$
- Nominal of non-performing assets in the start of the period $Nom_{t_{i-1}}^{NonPerf}$ and in the end of the period $Nom_{t_i}^{NonPerf}$
- Account to pay from; interest or principal

The transferred amount TA is then calculated as:

$$TA = VC\frac{1}{2}\left(Nom_{t_i}^{Perf} + Nom_{t_i}^{NonPerf} + Nom_{t_{i-1}}^{Perf} + Nom_{t_{i-1}}^{NonPerf}\right)$$

Pay variable senior management fee on ACB (“FlSenManFeeCurrACB”)

Module that pays the fee denoted “Senior management fee” in most CLOs. This fee is called senior management fee because it is senior to any tranche payments. It is calculated on the average aggregate collateral balance (ACB) in the period. Main input parameters for this module:

- Senior Fee amount Fee^{Senior}
- ACB in the start of the period $ACB_{t_{i-1}}$ and in the end of the period ACB_{t_i}
- Account to pay from; interest or principal

The transferred amount TA is then calculated as:

$$TA = Fee^{Senior}\frac{1}{2}\left(ACB_{t_i} + ACB_{t_{i-1}}\right)$$

Pay variable senior management fee on collateral balance (“FlSenManFeeCurrACB”)

Module that pays the “Senior management fee” based on the average collateral balance in the current period. Main input parameters for this module:

- Senior Fee amount Fee^{Senior}
- Nominal of performing assets in the start of the period $Nom_{t_{i-1}}^{Perf}$ and in the end of the period $Nom_{t_i}^{Perf}$
- Nominal of non-performing assets in the start of the period $Nom_{t_{i-1}}^{NonPerf}$ and in the end of the period $Nom_{t_i}^{NonPerf}$
- Account to pay from; interest or principal

The transferred amount TA is then calculated as:

$$TA = Fee^{Senior}\frac{1}{2}\left(Nom_{t_i}^{Perf} + Nom_{t_i}^{NonPerf} + Nom_{t_{i-1}}^{Perf} + Nom_{t_{i-1}}^{NonPerf}\right)$$

Pay variable subordinate management fee on collateral balance (“FlSubManFeeCurrACB”)

Module that pays the fee denoted “Subordinate management fee” in most CLOs. This fee is called subordinate management fee because it is usually not paid until all debt tranches have been paid. It is calculated on the average ACB in the period. Main input parameters for this module:

- Subordinate fee amount $Fee^{Subordinate}$
- ACB in the start of the period $ACB_{t_{i-1}}$ and in the end of the period ACB_{t_i}
- Account to pay from; interest or principal

The transferred amount TA is then calculated as:

$$TA = Fee^{Subordinate}\frac{1}{2}\left(ACB_{t_i} + ACB_{t_{i-1}}\right)$$

Pay variable subordinate management fee on ACB ("FlSubManFeeTotAssets")

Module that pays the "Subordinate management fee" based on the average collateral balance in the current period. Main input parameters for this module:

- Subordinate fee amount $Fee^{Subordinate}$
- Nominal of performing assets in the start of the period $Nom_{t_{i-1}}^{Perf}$ and in the end of the period $Nom_{t_i}^{Perf}$
- Nominal of non-performing assets in the start of the period $Nom_{t_{i-1}}^{NonPerf}$ and in the end of the period $Nom_{t_i}^{NonPerf}$
- Account to pay from; interest or principal

The transferred amount TA is then calculated as:

$$TA = Fee^{Subordinate} \frac{1}{2} \left(Nom_{t_i}^{Perf} + Nom_{t_i}^{NonPerf} + Nom_{t_{i-1}}^{Perf} + Nom_{t_{i-1}}^{NonPerf} \right)$$

Pay type 1 incentive management fee ("FlIncent1")

Module that pays the simplest form of "Incentive management fee" or "performance fee". As the name insinuate it is a fee which is closely linked to the performance of the manager. Often this is measured in the ability to provide a good yield on the equity tranche. The main problem is that there exist a lot of different structures for this fee. Therefore I decided only to define the simplest type. Main Input parameters for this module:

- Internal rate of return (IRR) on the equity tranche in the end of the period $IRR_{t_i}^{Equity}$.
- IRR hurdle IRR^{Hurdle}. As soon as the IRR on the equity tranche exceeds this threshold the performance fee starts paying.
- Incentive fee amount $Fee^{Incentive}$.
- Account to pay from; interest or principal

The transferred amount TA is then calculated as:

$$IRR^{Excess} = \begin{cases} 0, & IRR^{Equity} < IRR^{Hurdle} \\ IRR^{Excess}, & IRR^{Equity} \geq IRR^{Hurdle} \end{cases}$$

Where IRR^{Excess} is the amount left when the IRR^{Hurdle} is fulfilled. The transferred amount TA is then computed as:

$$TA = IRR^{Excess} Fee^{Incentive}$$

Pay deferred fees, administrative and hedging expenses ("DeferredFee")

Module that pays any deferred fee, administrative or hedging expense, assuming that it is deferrable. The incentive management fee is for example not deferrable. Main input parameters for this module:

- Which fee, administrative or hedging expense to pay
- Deferred amount DA
- Account to pay from; interest or principal

The transferred amount TA is then calculated as:

$$TA = DA$$

Over collateralization (OC) test ("CheckOC")

Module that performs an OC test on a specified tranche. The output from this module is two values. The first value is the redemption amount needed to cure the test and the second value is the amount of cash needed in order to cure the test by investing in new assets. Main input parameters for this module:

- Nominal the specified tranche all nominals on any senior tranches
- OC Trigger on specified tranche $OCTrigger$
- Account balance on principal $Cash^{PrincipalAccount}$ account

Consider m tranches $A, B, ..j \dots, m$, each a nominal Nom_j. The OC test can then be then calculated as:

$$OC_j = \frac{ACB + Cash^{PrincipalAccount}}{\sum_k^{A,B,\dots,j} Nom_k}$$

The nominal which is needed to cure the test can then calculated as:

$$Cure_j^{Redeem} = \frac{ACB + Cash^{PrincipalAccount}}{OCTrigger_j} - \sum_k^{A,B,\dots,j} Nom_k$$

$$Cure_j^{Reinvest} = OCTrigger_j \sum_k^{A,B,\dots,j} Nom_k - \left(ACB + Cash^{PrincipalAccount} + Cash^{ReinvestAccount}\right)$$

This implies that if $Cure_j^{Redeem} < 0$ or $Cure_j^{Reinvest} > 0$ the test is breached.

Interest coverage (IC) test ("CheckIC")

Module that performs a specified IC test. The output from this module is two values. The first value is the redemption amount needed to cure the test and the second value is the amount of cash needed in order to cure the test by investing in new assets. Main input parameters for this module:

- Nominal and coupon on the specified tranche and on any senior tranches.
- Total interest cash flow in the period $CF^{Interest}$
- IC Trigger on specified tranche $ICTrigger$.
- Sum of senior costs $SenCosts$

Consider m tranches $A, B, ..j \dots, m$, each a nominal Nom_j and a current coupon (incl. any reference rate) $Coupon_i$. The OC test can then be then calculated as:

$$IC_j = \frac{CF^{Interest} - SenCosts}{\sum_k^{A,B,\dots,j} Nom_k\, Coupon_k}$$

The interest needed to cure the test can then be calculated as:

$$CureInt_j^{Redeem} = \frac{CF^{Interest} - SenCosts}{ICTrigger_j} - \sum_{k}^{A,B,\dots,j} Nom_k\, Coupon_k$$

$$CureInt^{Reinvest} = ICTrigger_j \sum_{k}^{A,B,\dots,j} Nom_k\, Coupon_k - (CF^{Interest} - SenCosts)$$

This implies that if $CureInt_j^{Redeem} < 0$ or $CureInt^{Reinvest} > 0$ the test is breached.

<u>Cure OC test by redemption ("CureOCRedeem")</u>

Module that cures an OC test by redeeming a specified tranche using redirected interest or principal cash flows. It is necessary to perform an OC test before this module in the distribution of payments. Main input parameters for this module:

- Amount needed to cure the OC test $Cure^{Redeem}$
- Tranche nominal of tranche to redeem Nom
- Account to pay from; interest or principal

The new nominal Nom_{New} is then calculated as:

$$Nom_{\ New} = \begin{cases} Nom - Cure^{Redeem}, & Cure^{Redeem} < 0 \\ Nom, & Cure^{Redeem} \geq 0 \end{cases}$$

<u>Cure OC test by pari passau redemption ("CureOCPPRedeem")</u>

Module that cures an OC test by redeeming specified pari passau tranches using redirected interest or principal cash flows. It is necessary to perform an OC test before this module in the distribution of payments. Main input parameters for this module:

- Amount needed to cure the OC test $Cure^{Redeem}$
- Tranche nominal of m pari passau tranches $Nom_1, Nom_2, \dots, Nom_j, \dots, Nom_m$.
- Account to pay from; interest or principal

The new nominal on tranche j $Nom_{j,New}$ is then calculated as:

$$Nom_{j,New} = \begin{cases} Nom_j + Cure^{Redeem} \frac{Nom_j}{\sum_{k=1}^{m} Nom_k}, & Cure^{Redeem} < 0 \\ Nom_j, & Cure^{Redeem} \geq 0 \end{cases}$$

<u>Cure OC test by reinvesting in new assets ("CureOCReinvest")</u>

Module that cures an OC test by reinvesting in more assets using redirected interest or principal cash flows. It is necessary to perform an OC test before this module in the distribution of payments. Main input parameters for this module:

- Amount needed to cure the OC test $Cure^{Reinvest}$
- Account to pay from; interest or principal

The transferred amount TA is the calculated as:

$$TA = \begin{cases} 0, & Cure^{Reinvest} < 0 \\ Cure^{Reinvest}, & Cure^{Reinvest} \geq 0 \end{cases}$$

Cure IC test by redemption ("CureICRedeem")

Module that cures an IC test by redeeming a specified tranche using redirected interest or principal cash flows. It is necessary to perform an IC test before this module in the distribution of payments. Main input parameters for this module:

- Interest amount needed to cure the IC test $CureInt^{Redeem}$
- Tranche nominal Nom and $Coupon$ coupon of tranche to redeem
- Account to pay from; interest or principal

The new nominal Nom_{New} is then calculated as:

$$Nom_{\ New} = \begin{cases} Nom - \dfrac{CureInt^{Redeem}}{Coupon}, & CureInt^{Redeem} < 0 \\ Nom, & CureInt^{Redeem} \geq 0 \end{cases}$$

Cure IC test by pari passau redemption ("CureICPPRedeem")

Module that cures an IC test by redeeming specified pari passau tranches using redirected interest or principal cash flows. It is necessary to perform an IC test before this module in the distribution of payments. Main input parameters for this module:

- Interest amount needed to cure the IC test $CureInt^{Redeem}$
- Tranche nominal of m pari passau tranches $Nom_1, Nom_2, \dots, Nom_j, \dots, Nom_m$ with the coupons $Coupon_1, Coupon_2, \dots, Coupon_j, \dots, Coupon_m$
- Account to pay from; interest or principal

The new nominal on tranche j $Nom_{j,New}$ is then calculated as:

$$Nom_{j,New} = \begin{cases} Nom_j + \dfrac{CureInt^{Redeem} \dfrac{Nom_j}{\sum_{k=1}^{m} Nom_k}}{Coupon_j}, & CureInt^{Redeem} < 0 \\ Nom_j, & CureInt^{Redeem} \geq 0 \end{cases}$$

Cure IC test by reinvesting in new assets ("CureICReinvest")

Module that cures an IC test by reinvesting in more assets using redirected interest or principal cash flows. It is necessary to perform an IC test before this module in the distribution of payments. Main input parameters for this module:

- Amount needed to cure the IC test $CureInt^{Reinvest}$
- Current performing collateral profile consisting of N defined assets with an asset weight w_j^{Perf}, each with M_j different parts with the asset part weight of $w_{j,k}^{Perf}$ and an asset part coupon $Coupon_{j,k}$

- Account to pay from; interest or principal

The transferred amount TA is the calculated as:

$$TA = \begin{cases} 0, & CureInt^{Reinvest} < 0 \\ \dfrac{Cure^{Reinvest}}{\sum_{j=1}^{N}\sum_{k=1}^{M_j} w_{j,k}^{Perf}\, w_j^{Perf}\, Coupon_{j,k}}, & CureInt^{Reinvest} \geq 0 \end{cases}$$

Interest reflection (IR) test ("CheckIR")

Module that performs the IR test. The output from this module is two values. The first value is the redemption amount needed to cure the test and the second value is the amount of cash needed in order to cure the test by investing in new assets. Main input parameters for this module:

- Nominal of all debt tranches, assuming m debt tranches $Nom_A, Nom_B, \dots, Nom_j, \dots, Nom_m$
- IR trigger $IRTrigger$.
- Account balance on principal $Cash^{PrincipalAccount}$ account.

The IR test is calculated as:

$$IR = \frac{ACB + Cash^{PrincipalAccount}}{\sum_{j=1}^{A,B,\dots,m} Nom_j}$$

The nominal which is needed to cure the test can then calculated as:

$$Cure^{Redeem} = \frac{ACB + Cash^{PrincipalAccount}}{IRTrigger} - \sum_{j=1}^{A,B,\dots,m} Nom_j$$

$$Cure^{Reinvest} = IRTrigger \sum_{j=1}^{A,B,\dots,m} Nom_j - \left(ACB + Cash^{PrincipalAccount}\right)$$

This implies that if $Cure^{Redeem} < 0$ or $Cure_j^{Reinvest} > 0$ the test is breached.

Cure IR test by redemption ("CureIRRedeem")

Module that cures the IR test by redeeming a specified tranche using redirected interest or principal cash flows. It is necessary to perform an IR test before this module in the distribution of payments. Main input parameters for this module:

- Amount needed to cure the IR test $Cure^{Redeem}$
- Tranche nominal of tranche to redeem Nom
- Max redirect amount for curing IR test $IRRedirectMax$
- Account to pay from; interest or principal

The new nominal Nom_{New} is then calculated as:

$$Nom_{\,New} = \begin{cases} Nom - Cure^{Redeem} IRRedirectMax, & Cure^{Redeem} < 0 \\ Nom, & Cure^{Redeem} \geq 0 \end{cases}$$

Cure IR test by pari passau redemption ("CureIRPPRedeem")

Module that cures the IR test by redeeming specified pari passau tranches using redirected interest or principal cash flows. It is necessary to perform an IR test before this module in the distribution of payments. Main input parameters for this module:

- Amount needed to cure the IR test $Cure^{Redeem}$
- Tranche nominal of m pari passau tranches $Nom_1, Nom_2, \dots, Nom_j, \dots, Nom_m$.
- Max redirect amount for curing IR test $IRRedirectMax$
- Account to pay from; interest or principal

The new nominal on tranche j $Nom_{j,New}$ is then calculated as:

$$Nom_{j,New} = \begin{cases} Nom_j + Cure^{Redeem} \dfrac{Nom_j}{\sum_{k=1}^{m} Nom_k} IRRedirectMax, & Cure^{Redeem} < 0 \\ Nom_j, & Cure^{Redeem} \geq 0 \end{cases}$$

Cure IR test by reinvesting in new assets ("CureIRReinvest")

Module that cures the IR test by reinvesting in more assets using redirected interest or principal cash flows. It is necessary to perform an OC test before this module in the distribution of payments. Main input parameters for this module:

- Amount needed to cure the IR test $Cure^{Reinvest}$
- Max redirect amount for curing IR test $IRRedirectMax$
- Account to pay from; interest or principal

The transferred amount TA is the calculated as:

$$TA = \begin{cases} 0, & Cure^{Reinvest} < 0 \\ Cure^{Reinvest} IRRedirectMax, & Cure^{Reinvest} \geq 0 \end{cases}$$

Pay tranche interest ("TrancheInt")

Module that pays tranche interest on any tranche (debt or equity). Main input parameters for this module:

- Nominal Nom and coupon $Coupon$ on tranche to pay
- Account to pay from; interest or principal

The transferred amount TA is then calculated as:

$$TA = CouponNom$$

Pay pari passau tranche interest ("TranchePPInt")

Module that pays tranche interest on any pari passau tranches (debt or equity). The pari passau payment is only different to regular payment if there is not enough available funds to pay the tranches in full. Main input parameters for this module:

- Tranche nominal of m pari passau tranches $Nom_1, Nom_2, \dots, Nom_j, \dots, Nom_m$ with the coupons $Coupon_1, Coupon_2, \dots, Coupon_j, \dots, Coupon_m$
- Available interest $AvailFunds$
- Account to pay from; interest or principal

The transferred amount TA_j is then calculated as:

$$TA_j = \begin{cases} Coupon_j Nom_j, & \sum_{j=1}^{m} Coupon_j Nom_j < AvailFunds \\ \frac{Coupon_j Nom_j}{\sum_{j=1}^{m} Coupon_j Nom_j} AvailFunds, & \sum_{j=1}^{m} Coupon_j Nom_j \geq AvailFunds \end{cases}$$

Pay deferred tranche interest ("DeferredTrancheInt ")

Module that pays deferred tranche interest on any tranche (debt or equity). Main input parameters for this module:

- Deferred interest $DefInt$
- Account to pay from; interest or principal

The transferred amount TA is then calculated as:

$$TA = DefInt$$

Pay pari passau deferred tranche interest ("DeferredTranchePPInt ")

Module that pays tranche deferred interest on any pari passau tranches (debt or equity). The pari passau payment is only different to regular payment if there is not enough available funds to pay the deferred interest on the tranches in full. Main input parameters for this module:

- Tranche nominal of m pari passau tranches $Nom_1, Nom_2, \dots, Nom_j, \dots, Nom_m$ with the deferred interest $DefInt_1, DefInt_2, \dots, DefInt_j, \dots, DefInt_m$
- Available funds $AvailFunds$
- Account to pay from; interest or principal

The transferred amount TA_j is then calculated as:

$$TA_j = \begin{cases} DefInt, & \sum_{j=1}^{m} DefInt_j < AvailFunds \\ \frac{Nom_j}{\sum_{j=1}^{m} Nom_j} AvailFunds, & \sum_{j=1}^{m} DefInt_j \geq AvailFunds \end{cases}$$

Pay tranche principal ("TranchePrinc")

Module that pays tranche principal on any tranche (debt or equity). Main input parameters for this module:

- Nominal Nom on tranche to pay
- Account to pay from; interest or principal

The transferred amount TA is then calculated as:

$$TA = Nom$$

Pay pari passau tranche principal ("TranchePPPrinc")

Module that pays tranche principal on any pari passau tranches (debt or equity). The pari passau payment is only different to regular payment if there is not enough available funds to pay the tranches in full. Main input parameters for this module:

- Tranche nominal of m pari passau tranches $Nom_1, Nom_2, \ldots, Nom_j, \ldots, Nom_m$
- Available principal $AvailFunds$
- Account to pay from; interest or principal

The transferred amount TA_j is then calculated as:

$$TA_j = \begin{cases} Coupon_j Nom_j, & \sum_{j=1}^{m} Nom_j < AvailFunds \\ \frac{Nom_j}{\sum_{j=1}^{m} Nom_j} AvailFunds, & \sum_{j=1}^{m} Nom_j \geq AvailFunds \end{cases}$$

A-2 Real vs. modularized distribution of payments

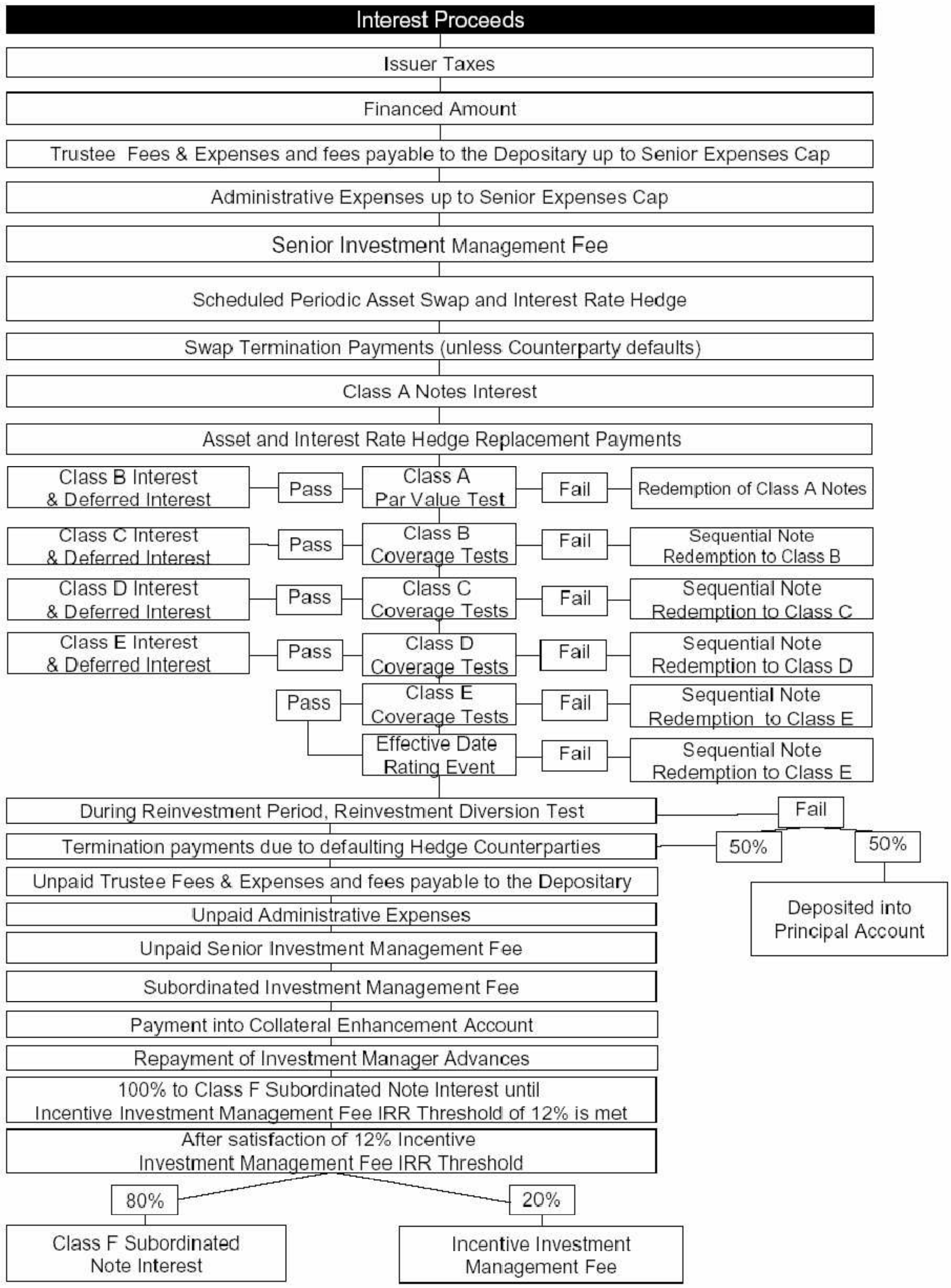


Figure A-1: Original distribution of interest payments RealCLO1

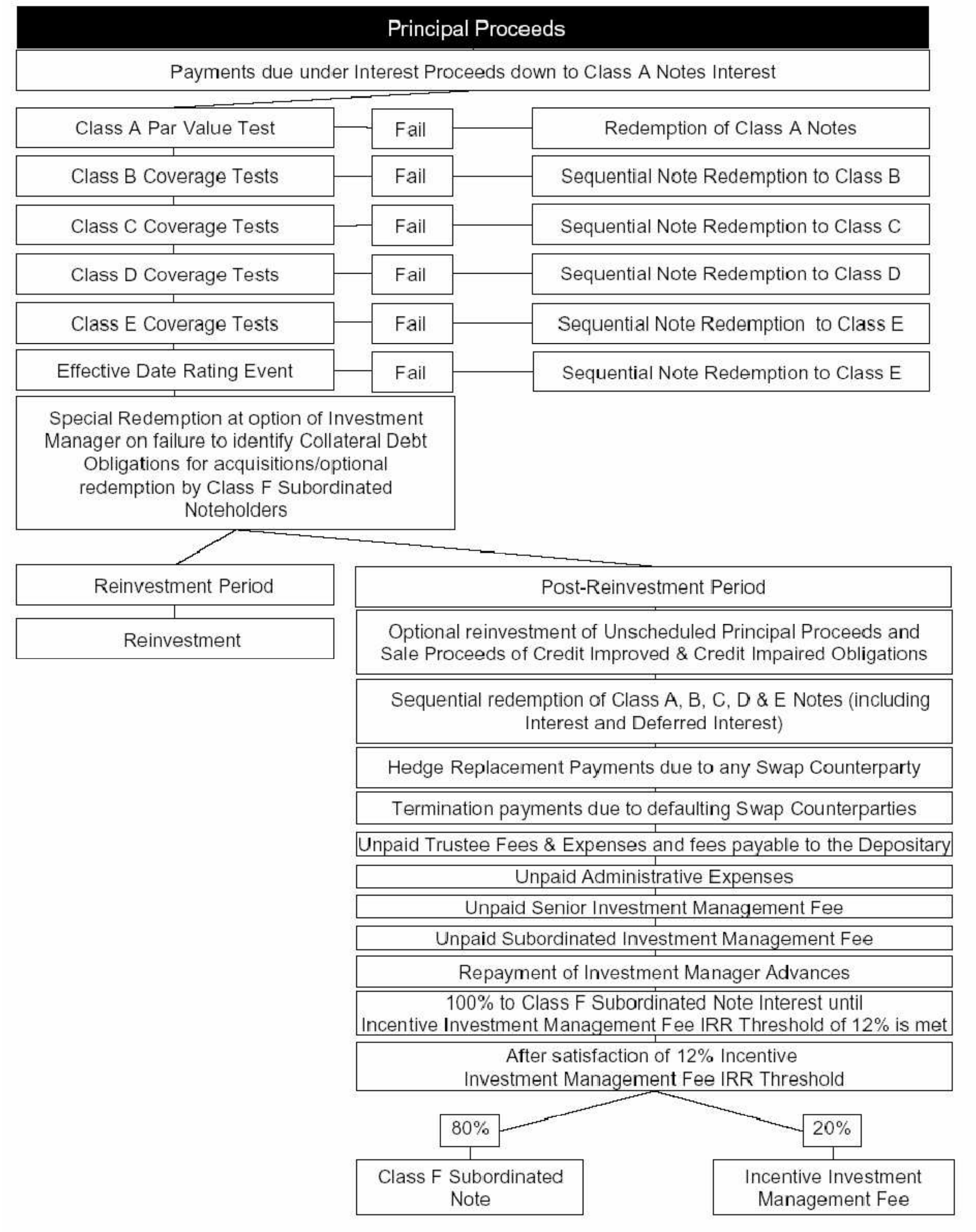


Figure A-2: Original distribution of principal payments RealCLO1

Position	Type	Pay to/Reference	Pay from
1	Fee	FixAdmin	Interest
2	Fee	FIAdminCurrTotAssets	Interest
3	Fee	FISenManFeeTotAssets	Interest
4	TrancheInt	Class A	Interest
5	CheckOC	Class A	
6	CureOCRedeem	Class A	Interest
7	TrancheInt	Class B	Interest
8	CheckOC	Class B	
9	CureOCRedeem	Class A	Interest
10	CureOCRedeem	Class B	Interest
11	CheckIC	Class B	
12	CureICRedeem	Class A	Interest
13	CureICRedeem	Class B	Interest
14	TrancheInt	Class C	Interest
15	CheckOC	Class C	
16	CureOCRedeem	Class A	Interest
17	CureOCRedeem	Class B	Interest
18	CureOCRedeem	Class C	Interest
19	CheckIC	Class C	
20	CureICRedeem	Class A	Interest
21	CureICRedeem	Class B	Interest
22	CureICRedeem	Class C	Interest
23	TranchePPInt	Class D1	Interest
24	TranchePPInt	Class D2	Interest
25	CheckOC	Class D2	
26	CureOCRedeem	Class A	Interest
27	CureOCRedeem	Class B	Interest
28	CureOCRedeem	Class C	Interest
29	CureOCPPRedeem	Class D1	Interest
30	CureOCPPRedeem	Class D2	Interest
31	CheckIC	Class D2	
32	CureICRedeem	Class A	Interest
33	CureICRedeem	Class B	Interest
34	CureICRedeem	Class C	Interest
35	CureICPPRedeem	Class D1	Interest
36	CureICPPRedeem	Class D2	Interest
37	TrancheInt	Class E	Interest
38	CheckOC	Class E	
39	CureOCRedeem	Class A	Interest
40	CureOCRedeem	Class B	Interest
41	CureOCRedeem	Class C	Interest
42	CureOCPPRedeem	Class D1	Interest
43	CureOCPPRedeem	Class D2	Interest
44	CureOCRedeem	Class E	Interest
45	CheckIC	Class E	
46	CureICRedeem	Class A	Interest
47	CureICRedeem	Class B	Interest
48	CureICRedeem	Class C	Interest
49	CureICPPRedeem	Class D1	Interest
50	CureICPPRedeem	Class D2	Interest
51	CureICRedeem	Class E	Interest
52	CheckIR	Class E	
53	CureIRReinvest		Interest
54	DeferredFee	FixAdmin	Interest
55	DeferredFee	FIAdminCurrTotAssets	Interest
56	DeferredFee	FISenManFeeTotAssets	Interest
57	Fee	FISubManFeeTotAssets	Interest
58	Fee	FIIncent1	Interest
59	TrancheInt	Class F	Interest
60	CheckOC	Class A	
61	CureOCRedeem	Class A	Principal
62	CheckOC	Class B	
63	CureOCRedeem	Class A	Principal
64	CureOCRedeem	Class B	Principal
65	CheckIC	Class B	
66	CureICRedeem	Class A	Principal
67	CureICRedeem	Class B	Principal
68	CheckOC	Class C	
69	CureOCRedeem	Class A	Principal
70	CureOCRedeem	Class B	Principal
71	CureOCRedeem	Class C	Principal
72	CheckIC	Class C	
73	CureICRedeem	Class A	Principal
74	CureICRedeem	Class B	Principal
75	CureICRedeem	Class C	Principal
76	CheckOC	Class D2	
77	CureOCRedeem	Class A	Principal
78	CureOCRedeem	Class B	Principal
79	CureOCRedeem	Class C	Principal
80	CureOCPPRedeem	Class D1	Principal
81	CureOCPPRedeem	Class D2	Principal
82	CheckIC	Class D2	
83	CureICRedeem	Class A	Principal
84	CureICRedeem	Class B	Principal
85	CureICRedeem	Class C	Principal
86	CureICPPRedeem	Class D1	Principal
87	CureICPPRedeem	Class D2	Principal
88	CheckOC	Class E	
89	CureOCRedeem	Class A	Principal
90	CureOCRedeem	Class B	Principal
91	CureOCRedeem	Class C	Principal
92	CureOCPPRedeem	Class D1	Principal
93	CureOCPPRedeem	Class D2	Principal
94	CureOCRedeem	Class E	Principal
95	CheckIC	Class E	
96	CureICRedeem	Class A	Principal
97	CureICRedeem	Class B	Principal
98	CureICRedeem	Class C	Principal
99	CureICPPRedeem	Class D1	Principal
100	CureICPPRedeem	Class D2	Principal
101	CureICRedeem	Class E	Principal
102	DeferredTrancheInt	Class A	Principal
103	TranchePrinc	Class A	Principal
104	DeferredTrancheInt	Class B	Principal
105	TranchePrinc	Class B	Principal
106	DeferredTrancheInt	Class C	Principal
107	TranchePrinc	Class C	Principal
108	DeferredTranchePPInt	Class D1	Principal
109	DeferredTranchePPInt	Class D2	Principal
110	TranchePPPrinc	Class D1	Principal
111	TranchePPPrinc	Class D2	Principal
112	DeferredTrancheInt	Class E	Principal
113	TranchePrinc	Class E	Principal
114	DeferredFee	FixAdmin	Principal
115	DeferredFee	FIAdminCurrTotAssets	Principal
116	DeferredFee	FISenManFeeTotAssets	Principal
117	DeferredFee	FISubManFeeTotAssets	Principal
118	Fee	FIIncent1	Principal
119	TranchePrinc	Class F	Principal

Figure A-3: Modularized distribution of payments RealCLO1

A-3 Cash flows RealCLO1

Approx. date	Period	CDR	Recovery	CPR	Total Cashflow Principal	Interest	FixAdmin	FlAdminCurrTotAssets	FlSenManFeeTotAssets	FlSubManFeeTotAssets	FlIncent1
10/15/2009	1	0.04	0.5	0.1	-	8,641,186	150,000	98,724	394,895	888,513	176,162
4/15/2010	2	0.04	0.5	0.1	-	8,869,759	150,000	98,144	392,578	883,300	491,329
10/15/2010	3	0.04	0.5	0.1	-	9,140,551	150,000	97,223	388,891	875,004	206,041
4/15/2011	4	0.04	0.5	0.1	-	10,045,022	150,000	96,553	386,211	868,974	201,930
10/15/2011	5	0.04	0.5	0.1	-	11,364,226	150,000	96,073	384,291	864,654	45,244
4/15/2012	6	0.04	0.5	0.1	23,656,424	11,333,080	150,000	92,540	370,161	338,492	-
10/15/2012	7	0.04	0.5	0.1	21,570,020	10,837,690	150,000	86,124	344,496	310,144	-
4/15/2013	8	0.04	0.5	0.1	20,055,465	10,276,237	150,000	80,095	320,381	255,967	-
10/15/2013	9	0.04	0.5	0.1	43,107,174	8,902,323	150,000	71,428	285,710	-	-
4/15/2014	10	0.04	0.5	0.1	38,476,101	7,663,345	150,000	60,478	241,910	-	-
10/15/2014	11	0.04	0.5	0.1	34,253,650	6,536,605	150,000	50,688	202,751	-	-
4/15/2015	12	0.04	0.5	0.1	30,428,882	5,494,945	150,000	42,012	168,046	-	-
10/15/2015	13	0.04	0.5	0.1	35,860,404	4,183,627	150,000	33,230	132,922	-	-
4/15/2016	14	0.04	0.5	0.1	33,265,726	2,938,845	150,000	24,179	96,718	-	-
10/15/2016	15	0.04	0.5	0.1	29,545,630	1,815,451	150,000	16,004	64,016	-	-
4/15/2017	16	0.04	0.5	0.1	26,358,033	800,591	150,000	8,781	35,123	-	-
10/15/2017	17	0.04	0.5	0.1	21,086,457	-	-	-	-	-	-

Class A Principal	Interest	Class B Principal	Interest	Class C Principal	Interest	Class D1 Principal	Interest	Class D2 Principal	Interest	Class E Principal	Interest	Class F Principal	Interest
-	2,632,467	-	314,996	-	392,696	-	149,624	-	322,000	-	451,498	-	2,669,611
-	2,780,394	-	331,433	-	410,776	-	154,829	-	322,000	-	459,169	-	1,965,315
-	2,958,691	-	351,243	-	432,568	-	161,102	-	322,000	-	468,414	-	824,164
156,499	3,567,983	-	418,943	-	507,037	-	182,540	-	322,000	-	500,007	-	807,721
1,842,680	4,484,462	-	521,063	-	619,369	-	214,878	-	322,000	-	547,663	-	180,976
26,245,665	5,039,994	-	586,677	-	691,544	-	235,656	-	322,000	-	578,282	-	-
23,945,323	4,768,029	-	614,177	-	721,794	-	244,364	-	322,000	-	591,116	-	-
22,281,553	4,437,858	-	633,748	-	743,322	-	250,562	-	322,000	-	600,249	-	-
44,804,666	4,097,532	-	651,186	-	762,505	-	256,084	-	322,000	-	608,387	-	-
39,774,489	3,256,322	-	670,638	-	783,902	-	262,244	-	322,000	-	617,464	-	-
35,200,481	2,471,788	-	690,857	-	806,143	-	268,646	-	322,000	-	626,900	-	-
31,067,849	1,731,365	-	708,201	-	825,221	-	274,139	-	322,000	-	634,994	-	-
35,878,084	1,043,049	-	722,834	-	841,317	-	278,772	-	322,000	-	641,823	-	-
8,802,711	209,100	24,463,014	735,121	-	854,833	-	282,663	-	322,000	-	264,231	-	-
-	-	5,536,986	137,580	24,008,645	866,165	-	273,585	-	308,102	-	-	-	-
-	-	-	-	8,991,355	238,603	6,911,615	301,392	10,185,538	336,218	-	-	-	-
-	-	-	-	-	-	2,588,385	79,294	3,814,462	87,733	12,014,408	2,502,176	-	-

Figure A-4: Cash flows RealCLO1