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# **Portfolio Optimization**

*-The Mean-Variance and CVaR approach*

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# Abstract

**Title:** Portfolio Optimization -*The Mean-Variance and CVaR approach*

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**Abstract:** The recent economic turmoil has increased volatility on the Swedish stock market and made investors more exposed to risk in an uncertain environment. This research will investigate if the quantitative portfolio optimization models Mean-Variance and CVaR can produce risk-adjusted returns for investors acting in the Swedish stock market. From the classic Mean-Variance model different investment strategies with restrictions on short-selling are applied and the CVaR approach is applied on confidence levels of 95% and 99% respectively. The optimized portfolios are constructed using 3 different input periods of 1, 2 and 3 years that are rebalanced on a monthly basis. To be able to grasp the results a benchmark index and an equal weight strategy are included.

We show, by using the Sharpe-ratio as an evaluation method, that the equal weight strategy produces the most efficient risk/reward during the time-period 2005-2009. The optimization models Mean-Variance and CVaR, with their applied strategies, turned out to underperform both the benchmark index and the equal weight strategy in the risk/reward universe. Finally, when analyzing the 3 different lengths of input periods it is found that no length is superior to be used in the investment strategies.

**Keywords:** Mean-Variance, CVaR, Portfolio Optimization, Volatility risk, Sharpe-ratio.

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# 1. Introduction

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*The introductory chapter begins with the background and a discussion of the problem. Next, the formulation of the research question and the purpose are presented for the reader. The chapter ends by stating some limitations and a short plan.*

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## 1.1 Background

The problem of constructing optimal portfolios of assets is attractive for all risk averse market participants. Thus the aim of this paper is to compose optimized portfolios using different models and strategies. By rebalancing the portfolio holdings monthly, according to different underlying algorithms, it can be assumed that investors better can exploit new information available on the market. The optimized portfolios will be evaluated by using a risk/reward approach according to the Sharpe-ratio to conclude if any model and strategy is superior to the others.

First approach is the Markowitz's (1952) classical Mean-Variance, M-V, model. M-V is the pioneer work within the portfolio selection problem and is also known as the modern portfolio theory. Among researchers and practitioners the model has both been criticized and celebrated during the years. Karacabey (2007) claims in his paper that even though the model's theoretical reputation is strong, it has not been used extensively in practise. Frankfurter et al. (1971) found some limitations of the Markowitz model in their study. The authors claim that with a quite realistic simulation approach, M-V portfolios are not able to generate more efficient portfolios than randomly selected or equally weighted portfolios. However, on the other hand Markowitz won the Nobel Prize 1990 for his work.

The classical M-V model is then followed up by forming different investment strategies by restricting some of the assumptions, hence imposing a no short-selling restriction and a limit for the short-selling. Jagannathan & Ma (2003) conclude in their research that imposing restrictions will lead to better out-of-sample results for the optimized portfolios. On the other hand the recent paper from Fan et al. (2008) show that the no short-selling portfolio is not diversified enough and can be improved by allowing some short positions. Also Chang et al. (2009) found that adding constraints for short-selling and over-weighting of an international investment portfolio reduce, but do not completely eliminate, the diversification benefits.

Another model that will be optimized is the more recently developed model, CVaR, Conditional Value-at-Risk. The model has its roots in the more well known, VaR, Value-at-Risk approach. Compared to VaR, the model has beneficial properties of being a coherent and solid risk measure (Krokhmal et al., 2001). Based on this, former research proposes that one should optimize using CVaR rather than VaR (Alexander & Baptista, 2004). Even though CVaR is preferable in many aspects it has not become a standard in the finance industry. The founder is positive to the CVaR model and expresses that: "...it is likely to play a major role as it currently does in the insurance industry" (Uryasev, 2000).

## **1.2 Problem discussion**

A problem in finance, when it comes to investing in the stock market, is to combine weights of individual stocks optimally. Rational investors seek to create a well diversified portfolio based on different risk measures, thus in that way keep control of the diversifiable risk. Where risk is often measured as volatility, a measure that both the M-V and CVaR approach utilize. It is then interesting to compare whether the results are consistent with each other or if one of the models is preferable.

In this empirical research, the performance of the different models and strategies will be evaluated in order to highlight the differences between the portfolio returns in relation to risk. Since the models applied has shown to be sensitive to the input (Best & Grauer, 1991). The research will also evaluate how different time periods, as input, affect the stock allocation and thereby the risk/reward.

Former empirical studies have shown that traditional active portfolio management is underperforming benchmark indices over time (SPIVA Report Year-End, 2008). There may then be an issue as to whether the related quantitative active management approach is able to succeed or not. According to economic theories such as CAPM, APT and the efficient market theory it is almost a premise that successful active management is not possible (Grinold & Kahn, 1995). However, investing subsequent to the quantitative approach is a way to reduce human biases according to the theory of Behavioural finance (Chincarini & Kim, 2006). Instead of complex and subjective investments, applying portfolio optimizations techniques leads to a systematic and formal decision making for an investor (Sharpe et al., 2000).

Further, according to Fama (1970) the only way investors are able to beat the market over the long run is by taking greater risk.

Finally, another common problem in financial data is that the distribution tend to behave non-normal, hence financial data has shown to be leptokurtotic and skewed (Osborne, 1959; Fama, 1965; Rachev & Mittnik, 2000). However, according to the Central limit theorem the distribution goes towards normality when the observations increase to infinity (Rice, 1995). Additionally, Fama (1976) found that monthly stock returns can be well described by a normal distribution approach.

### **1.3 Formulation of question**

*Which portfolio optimization model and strategy produce the most efficient risk/reward return evaluated by the Sharpe-ratio?*

### **1.4 Purpose**

The purpose of this research is to test if different portfolio optimization models and strategies can help investors to produce a more risk-adjusted return. To be able to grasp the results, this research will also include a benchmark index and an equally weighted portfolio.

### **1.5 Limitations**

To limit the optimization problems, this research will include the 30 most traded companies on the Swedish stock market, OMXS30 index, over a specified period.

### **1.6 Plan**

The outline of the thesis is as follows: chapter 2 explains the theory behind the applied portfolio optimization techniques, chapter 3 presents a summary of some previous scientific studies and describes the handling of the data, chapter 4 contains the analysis of the empirical results and concluding remarks are in chapter 5.

## 2. Theory

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*In this section the theories behind the two optimization approaches Mean-Variance and CVaR, are described. Then, the different optimization problems are outlined with the corresponding restrictions. Finally the theory behind the Sharpe-ratio is described.*

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### 2.1 Mean-Variance

#### 2.1.1 Classical approach

The classical Mean-Variance approach in portfolio theory was introduced by Markowitz in 1952. The objective is that investors want to maximize expected returns, for every level of risk, and expected return is considered as a desirable variable. The financial risk is measured as variance or standard deviation of returns and is considered as an undesirable variable among investors. Markowitz expresses the portfolio mean and the variance as follows:

$$E(r) = \sum_{i=1}^N X_i \mu_i$$
$$\sigma^2 = \sum_{i=1}^N \sum_{j=1}^N \sigma_{ij} X_i X_j$$

Where  $E(r)$  is the expected mean return of some portfolio,  $X_i$  is the vector of weights of asset  $i$  and  $\mu_i$  is the vector of asset  $i$ 's returns. In the portfolio variance,  $\sigma^2$ , the  $\sigma_{ij}$  denotes the covariance between two assets. Expressed as follows:

$$\sigma_{ij} = E \{ [R_i - E(R_i)][R_j - E(R_j)] \}$$

or

$$\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$$

Where  $\rho_{ij}$  is the correlation,  $\sigma_i$  and  $\sigma_j$  are the standard deviation of asset  $i$  and  $j$  respectively.

Investors who want to maximize returns would place all wealth in the security with the highest expected return regardless of the volatility. However, risk averse investors seek

diversification, since portfolio diversification will lower variance and thereby the level of risk through the covariance between assets. Thus, rational investors desire to maximize expected return and minimize variance by investing in several different assets.

One is now able to find the efficient portfolios of the portfolio frontier, the ones located above the Mvp. These are the set of portfolios that have the best Mean-Variance profile, where each efficient portfolio presents the minimum variance given the expected return. Depending on investors' risk aversion they chose portfolios along the efficient part of the portfolio frontier. (Markowitz, 1952)

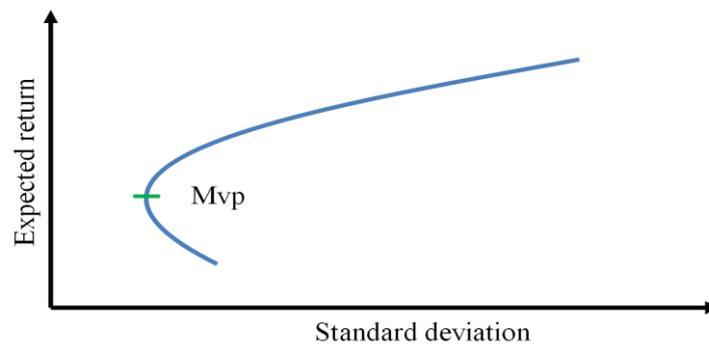


Figure 1. Mvp in risk/reward universe

The efficient portfolios mean and variance stated in matrix formulation:

$$\mu_p = \mu'w$$

$$\sigma_p^2 = w'Vw$$

Where  $\mu'$  is a vector of expected return of each asset and  $w$  is a vector of each assets weight in the portfolio. Multiplying these two vectors is equal to the expected return of a portfolio,  $\mu_p$ .

The Variance-covariance matrix is denoted as  $V$ :

$$V \equiv \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1N} \\ \vdots & \ddots & \vdots \\ \sigma_{N1} & \cdots & \sigma_{NN} \end{bmatrix}$$

The method used for optimizing the M-V portfolios is the indirect utility function within the parametric quadratic programming method, PQP, which is a reformulation of the traditional

Mean-Variance optimization framework. Below, the optimization problem to the classical approach with no restrictions is presented.

$$\max \quad \mathbb{U}_{(\mu_p, \sigma_p^2)} = t * \mu'w - \frac{1}{2}w'Vw$$

$$\text{subject to } w'I = 1$$

Where  $t$  is the risk tolerance and the weights,  $w'I$ , sum to one, thus all wealth is invested in the risky assets.

To estimate individuals risk tolerance, Sharpe et al. (2000) propose that one should set the slope of the investor's indifference curve,  $1/t$ , equal to the slope of the efficient frontier at the point the investor chose his or her portfolio. This will reflect the investor's tradeoff between expected return and variance. Hence, the following formula is used to calculate the risk tolerance:

$$t = \frac{2[\bar{r}_c - r_f]\sigma_S^2}{(\bar{r}_S - r_f)^2}$$

However, in this research the risk tolerance is set equal to zero, which would make it analogous to the most risk averse investor in the market. Since Mvp-portfolios are constructed the only risk investors are exposed to is the unavoidable risk.

### 2.1.2 Short-selling restriction

This strategy implies no short-selling in the portfolios. According to empirical findings this will reduce the number of selected assets (Fan et al., 2008). Thus, the new efficient frontier will always be to the right of the classical efficient frontier.

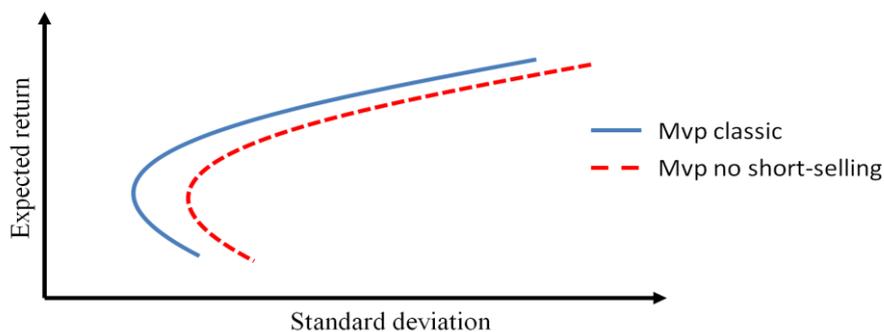


Figure 2. Short-selling restriction

The formula being optimized is:

$$\begin{aligned} \max \quad & \mathbb{U}_{(\mu_p, \sigma_p^2)} = t * \mu'w - \frac{1}{2} w'Vw \\ \text{subject to} \quad & w'I = 1 \quad \text{where } w \geq 0 \end{aligned}$$

Total weights still sum to one, but the weights are now restricted to be non-negative.

### 2.1.3 Limits of short-selling restriction

In this strategy a limit of short-selling positions is set to -10%, which is presupposed to be a “normal” level of short-selling boundary. This will reduce the drawback of the classical Mean-Variance portfolio where weights often take on extreme values, both positive and negative (Merton, 1980).

The formula being optimized is:

$$\begin{aligned} \max \quad & \mathbb{U}_{(\mu_p, \sigma_p^2)} = t * w'\mu - \frac{1}{2} w'Vw \\ \text{subject to} \quad & w'I = 1 \quad \text{where } w \geq -0,1 \end{aligned}$$

Where  $w \geq -0,1$  implies that maximum amount of short-selling is -10%.

## 2.2 Conditional Value-at-Risk

The CVaR is closely related to risk measures such as Expected shortfall, PVaR and tail-VaR among others. However, they are all extensions developed from the original VaR approach. VaR has a bias toward optimism, since it only gives information that a loss can be expected to be higher than VaR itself. The probability of a loss not exceeding the threshold  $\alpha$ -tail is therefore the quartile:

$$\Psi(x, \alpha) = \int_{f(x,y) \leq \alpha} p(\mathbf{y}) d(\mathbf{y})$$

CVaR on the other hand, gives the investor information of how big loss one actually should expect over some chosen time period and level of confidence. Thus, being an average of high losses beyond the  $\alpha$ -tail of the loss distribution. Moreover, CVaR is capable to capture non-normal distributions both in the form of kurtosis and skewness. (Rockafellar & Uryasev,

2000). Mathematical expression for CVaR is the one below and an extended derivation is found in Appendix A.

$$\Phi_{\beta}(x) = (1 - \alpha)^{-1} \int_{f(x,y) \geq \alpha_{\beta}(x)} f(x,y)p(y)dy$$

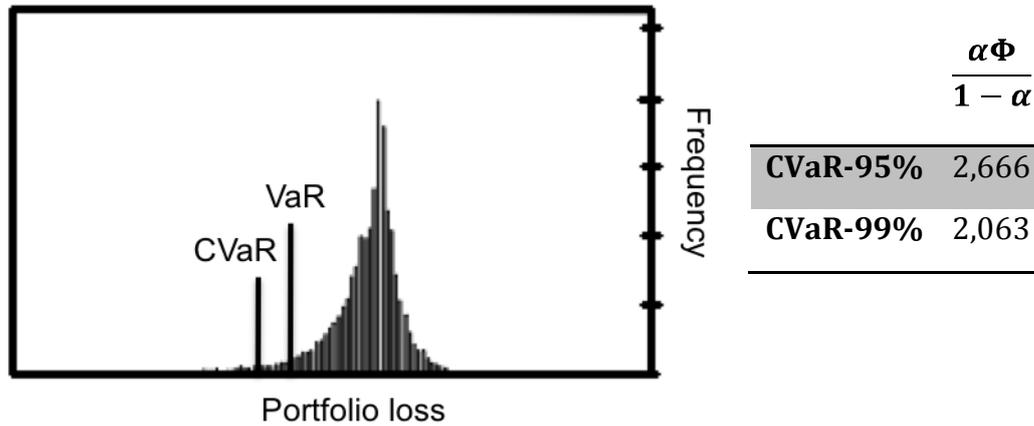


Figure 3. CVaR

### 2.2.1 Coherent risk measure

Artzner et al. (1997) proposed conditions for a risk measure to be coherent. The conditions that must be satisfied are postulated as a set of axioms, which are composed as follows:

- 
- (i) *Monotonicity:*  $Y \geq X \Rightarrow \rho(Y) \leq \rho(X)$
  - (ii) *Sub-additivity:*  $\rho(X + Y) \leq \rho(X) + \rho(Y)$
  - (iii) *Positive homogeneity:*  $\rho(hX) = h\rho(X)$  for  $h > 0$
  - (iv) *Translation invariance:*  $\rho(X + n) = \rho(X) - n$
- 

Pflug (2000) proved that CVaR satisfies the axioms above and therefore is a coherent risk measure. Further, the sub-additivity of the CVaR ensures that the portfolio risk surface is convex which implies that the optimization problem is linear. Moreover, the convexity has the properties of a unique optimum which eliminates the possibility of a local minimum being different from a global. (Uryasev, 1999)

Being a coherent risk measure is not true for the VaR approach, due to the lack of fulfilling the sub-additivity condition (Dowd, 2005). The VaR of a portfolio may therefore be larger than the sum of the individual assets VaR (Alexander & Baptista, 2004). Moreover, the optimization of VaR therefore becomes very complicated, due to being non-convex and the possibility of several local minima (Uryasev et al., 1999). An exception is when return-loss distributions are normal distributed, the measures in that case provide the same optimal portfolios (Uryasev, 2000).

However, if a non-normal distribution should be applied for CVaR there are significant computational challenges. This amounts to precise knowledge of the underlying distribution, a multi-dimensional integral and computationally beyond the fourth dimension (Goh et al., 2009). Therefore, in line with several previous studies, this research will assume normal distributed returns (Krokhmal et al., 1999).

### 2.2.2 Confidence level

The confidence levels of 99% and 95% that are applied are the typical levels chosen in estimating and taking control of the risk through the model (Rockafellar & Uryuasev, 2000). The confidence level is connected to the investors risk aversion, where a high confidence level refers to a more risk averse investor compared to a low confidence level (Alexander, 2008).

### 2.2.3 Optimization problem

The formula being optimized is:

$$\begin{aligned} \min \quad CVaR &= -\mu_p + \frac{\alpha\Phi}{1-\alpha}\sigma_p \\ \text{subject to} \quad w'I &= 1 \quad \text{where } w \geq 0 \end{aligned}$$

This optimization problem constructs portfolios that minimize the exposure to expected losses where  $\alpha$  is the confidence levels of 95% and 99% respectively and  $\alpha\Phi$  follows the standard normal density function:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

## 2.3 Sharpe-ratio

The Sharpe-ratio as a risk measure was introduced by William F. Sharpe 1966 in his famous article “Mutual Fund Performance”. It estimates a risk-adjusted return, by setting the return in relation to the standard deviation of the return. A Sharpe-ratio of 1 indicates that the return is proportional to the risk taken. The higher ratio the better, since it indicates a higher return given the risk alternative a lower risk given return. (Sharpe, 1994)

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$$\frac{R_p - R_F}{\sigma_p}$$

$R_p$  = Return of portfolio

$R_F$  = Return of risk free asset

$\sigma_p$  = Standard deviation of portfolio returns

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(Eiteman et al., 2007)

The numerator is the excess return, while the denominator is the standard deviation of portfolio returns over the same period (Eiteman et al., 2007). The best portfolio according to the Sharpe-ratio is therefore the one that exhibits the highest excess return over the risk free rate relative portfolio risk (Saunders & Allen, 2002).

The Sharpe-ratio, as a risk measure, has some shortcomings. To begin with, it assumes that the only risk that affects the portfolio risk is the standard deviation. Moreover, the fact that diversifiable risk can be diversified away is nothing that the Sharpe-ratio takes into account (Saunders & Allen, 2002).

A further shortcoming is the misleading interpretation of negative values. In the stated example, it can be seen that the Sharpe-ratio becomes higher when the risk increases even though return and risk free rate is the same in both cases.

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Both portfolios:  $R_p$  3%,  $R_f$  4%

Different risk:  $\sigma_{p1} = 5\%$  and  $\sigma_{p2} = 10\%$

$$\text{Sharpe - ratio1} : \frac{0,03 - 0,04}{0,05} = -0,2$$

$$\text{Sharpe - ratio2} : \frac{0,03 - 0,04}{0,1} = -0,1$$

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(Own production)

## 3. Method

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*This chapter first presents a summary of some previous scientific studies in the field. This is followed by a motivation to methods employed in this thesis and goes on to describe the handling of the data. The chapter ends with a graphical presentation of the different models and strategies that will be included in the research.*

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### 3.1 Previous scientific studies

Mulvey & Simsek (2005) study the problem of multi-stage portfolio optimization, where the assets in the study consist of four categories; S&P 500, global stocks, real estates and commodities during the period of 1972-1997. The mean return, standard deviation and covariances are calculated from historical data. They study a 10 year investment horizon and assume quarterly rebalancing. Their goal is to maximize the risk/reward for total asset wealth at the end of the period by using a simulation approach. They find that investment performance of a portfolio can be improved by means of dynamically rebalancing compared with the single-period Mean-Variance model.

Uryasev & Rockafellar (1999) propose the new approach CVaR, an extension of VaR, that is able to capture kurtosis and skewness in returns. They show mathematically how to optimize portfolios by minimizing CVaR using linear programming techniques.

The paper by Krokmal et al. (2001) compares the performance of different optimized portfolios of hedge funds using the following models: CVaR, Conditional Drawdown-at-Risk (CDaR), Mean-Absolute Deviation and Maximum Loss. The data consists of monthly returns for 500 hedge funds over the period 1995-12-01 to 2001-05-31. They show, among other results, that imposing risk constraints for in-sample data may degrade portfolios expected returns whereas the control of risk is improved. Further they find from the out-of-sample test that tightening of risk constraints significantly increases portfolio return and at the same time decreases losses. CVaR and CDaR demonstrate the most solid performance among the risk measures in out-of sample tests.

Krokmal et al. (1999) extends the CVaR optimization problem to be applicable to finance by maximizing the returns under the CVaR constraint. A case study is performed on the stocks from the S&P 100 index with historical end-of-the-day prices over the period 1997-07-14 to

1999-07-28. No short-selling is allowed but a limitation of that one single asset cannot contribute to more than 20 percent of the total portfolio is applied. The case study show that the approach may lead to efficient investment strategies.

## **3.2 The choice of methods**

The M-V approach is, as described in the background, the pioneer work within the field of portfolio optimization. The classical approach and two additional strategies are therefore included in this research. CVaR minimizes the value of the expected losses while its main competitor VaR optimization minimizes the lower bound losses. From a risk management point of view, the CVaR is preferable since it is neutral or conservative rather than optimistic.

This research contributes with information of how the different optimization models and strategies performed on the Swedish stock market in a risk/reward approach, evaluated by the Sharpe-ratio with different length of input periods. This will be of interest to investment companies, mutual funds and other more theoretical practitioners.

## **3.3 Handling of data**

### **3.3.1 Selection of data and time periods**

The assets to invest in are the stocks listed on the OMXS30 index, as of the last observation 2010-01-09, Appendix D. The index is constructed of the thirty stocks with the highest turnover and is updated two times per year (NASDAQ OMX). However, this research disregards the changes of stocks included in the index. The restricting simplification is believed not to influence the results since the purpose of this study is to evaluate and compare portfolio optimization models, not to create mimicking portfolios of the index. Further, by selecting the stocks included at the end of the period the problem of survivorship bias is eluded (Sharpe et al., 2000).

The stock prices are collected from Datastream and consist of the monthly closing prices for the period 2001-12-09 to 2010-01-09. All thirty stocks are included in the sample input with one exception, Alfa Laval. Due to insufficient data for Alfa Laval, for the first six months in 2001, it constrains the portfolios of three year input to include only 29 stocks over this period. The risk-free rates, in the form of the 12 month Treasury-bill, are collected from the Swedish Central Bank and are used as an input in the Sharpe ratio.

### 3.3.2 Approach

The input data for the models consist of one, two and three years respectively with a one month rolling window, figure 1. An assumption of that future values of mean, variance and covariances are monotonically related to historical estimates is made. The historical returns are calculated as follows:

$$r_t = \ln \left( \frac{S_t}{S_{t-1}} \right)$$

Where  $r_t$  represents the return for the period, while  $S_t$  and  $S_{t-1}$  are the closing price of the stocks at time t and t-1. From the returns the arithmetic mean can be calculated.

$$\hat{\mu} = \frac{1}{M} \sum_{t=1}^M r_t, \text{ where } M \text{ is the number of time periods}$$

From every sample input the different portfolio optimization strategies are applied on a one month out-of-sample forecast. Further, every portfolio is fully invested in the stock market each month.

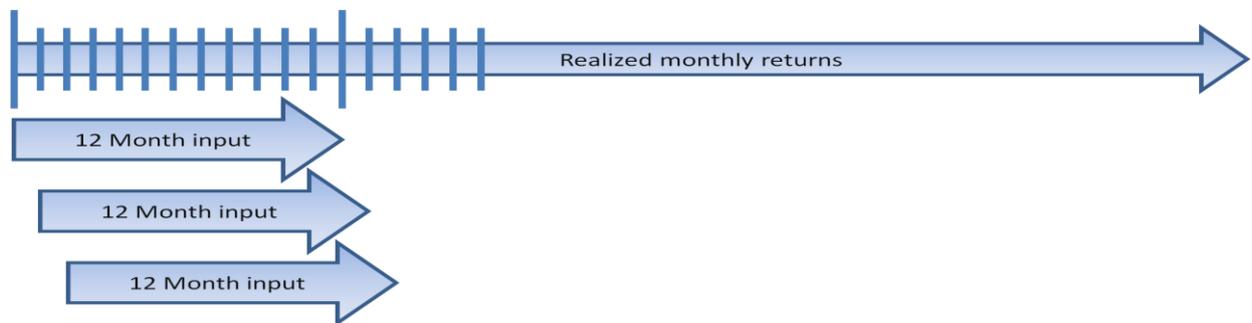


Figure 4. Rolling window

This gives a total output of sixty monthly return observations which will be evaluated and compared in a risk/reward approach by applying the Sharpe-ratio on a yearly basis, over the period of 2005 - 2009. The return is the continuous compounded return over the year and the risk is the standard deviation of the returns over the year.

$$R = (1 + r_{jan}) * (1 + r_{feb}) \dots * (1 + r_{dec})$$

$$\sigma = \left( \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2} \right) * \sqrt{12}$$

In the graph below it can be seen that the time period covers a business cycle and consists of different market trends.

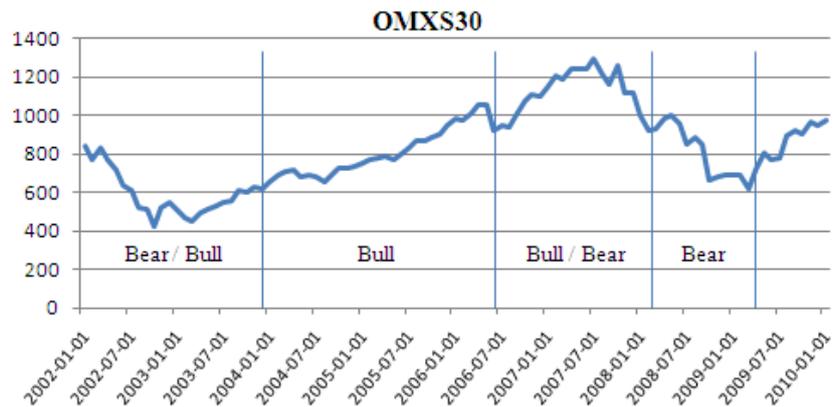


Figure 5. OMXS30 chart

### 3.3.3 Graphical approach

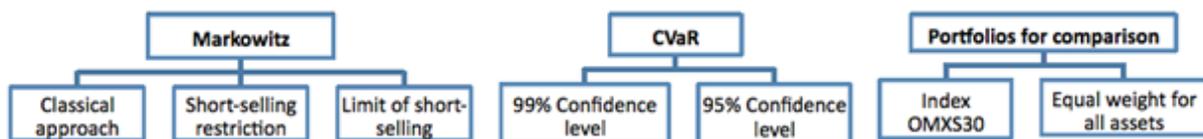


Figure 6. Graphical approach strategies

### 3.4 Critic on method and relevance

Transaction costs are ignored in this research. Even though the results become more theoretical the research still satisfies the main purpose of evaluating different models and strategies.

Also, dividends are not taken into account due to the short timeframe and complicated data handling. This will result in negative returns for some months in the spring, since this is generally the time when dividends are distributed to shareholders. Moreover, all strategies are exposed to the same shortcoming and therefore it is assumed not to influence the final result.

## 4. Results

*In this section the results from the different strategies are presented on an annual basis. The numerical results are followed up with an analysis of the most important results. The main focus is on the Sharpe-ratio but also other factors are analyzed.*

### 4.1 Yearly evaluation

2005	CVaR-95%	CVaR-99%	Mvp no short-selling	Mvp classic	Mvp-10%	Equal weight	OMXS30
Sharpe-ratio 1Y input	1,09	2,63	2,82	2,42	2,14	4,18	3,88
Sharpe-ratio 2Y input	3,13	3,04	2,57	-0,06	1,50	4,18	3,88
Sharpe-ratio 3Y input	3,54	3,67	3,74	1,87	2,43	4,18	3,88
Return 1Y input	24,96%	34,12%	32,57%	30,48%	28,43%	42,62%	30,65%
Return 2Y input	38,15%	36,75%	30,03%	0,51%	28,81%	42,62%	30,65%
Return 3Y input	57,23%	56,44%	43,22%	49,79%	42,17%	42,62%	30,65%
Std returns 1Y input	21,00%	12,14%	10,81%	11,71%	12,27%	9,70%	7,35%
Std returns 2Y input	11,49%	11,39%	10,84%	26,77%	17,83%	9,70%	7,35%
Std returns 3Y input	15,54%	14,79%	10,99%	25,51%	16,46%	9,70%	7,35%
Risk free rate	2,14%						

**Table 1.** Results 2005

All strategies and input periods produce a risk-adjusted return in the form of positive Sharpe-ratios with the exception of the Mvp classical approach with 2 year input. Even though positive values, in most cases above 1 which implies that the return is more than proportional to the risk taken, the results are not very impressive if compared to the index. This implies that the market conditions for the period have been very favorable with a relatively low risk free rate, low volatility and increasing stock returns.

Overall, the highest Sharpe-ratios are produced using 3 year input due to very high returns where the risk increase, but not proportional to the returns produced. One can also observe that the strategies that are restricted to not allow short-selling outperforms the strategies which are allowed. The single highest Sharpe-ratio over the year is produced by the equal weight strategy. The equal weight strategy outperforms the other strategies since it produces a high return with relatively low risk.

2006	CvaR-95%	CvaR-99%	Mvp no short-selling	Mvp classic	Mvp-10%	Equal weight	OMXS30
Sharpe-ratio 1Y input	0,30	0,41	0,31	0,04	0,12	0,84	0,63
Sharpe-ratio 2Y input	0,65	0,72	0,68	0,73	0,85	0,84	0,63
Sharpe-ratio 3Y input	0,47	5,22	0,37	-0,09	0,62	0,84	0,63
Return 1Y input	10,48%	12,85%	8,43%	2,99%	4,57%	20,31%	14,24%
Return 2Y input	14,50%	15,26%	12,92%	21,88%	15,62%	20,31%	14,24%
Return 3Y input	10,93%	41,18%	8,23%	0,49%	13,05%	20,31%	14,24%
Std returns 1Y input	27,33%	26,09%	19,74%	17,82%	19,20%	21,49%	18,95%
Std returns 2Y input	18,94%	18,06%	15,69%	27,08%	15,71%	21,49%	18,95%
Std returns 3Y input	18,45%	7,46%	15,96%	20,52%	17,56%	21,49%	18,95%
Risk free rate	2,25%						

**Table 2.** Results 2006

Compared to previous year, 2006 was a period with significantly higher risk and lower returns for the strategies and index. Nevertheless, CVaR-99% produces the research highest Sharpe-ratio over the same year using 3 year input. When analyzing this result further the high Sharpe-ratio was produced by positive stable monthly returns. Since they were stable and positive, eleven months out of twelve, the risk became very low.

Once again, only one negative Sharpe-ratio is produced and this was from the same strategy as previous year, the Mvp classical approach. The results are mixed concerning if respectively strategy outperformed the index or not. Moreover, the equal weight strategy outperformed the index again, since it had the same properties as previous year.

2007	CvaR-95%	CvaR-99%	Mvp no short-selling	Mvp classic	Mvp-10%	Equal weight	OMXS30
Sharpe-ratio 1Y input	-0,26	-0,36	-0,57	-1,06	-0,60	-0,56	-0,81
Sharpe-ratio 2Y input	0,70	0,82	1,15	-1,65	1,05	-0,56	-0,81
Sharpe-ratio 3Y input	-0,03	-0,11	-0,52	-0,59	0,90	-0,56	-0,81
Return 1Y input	0,56%	-0,33%	-3,38%	-17,98%	-6,15%	-9,12%	-14,27%
Return 2Y input	11,05%	11,96%	14,37%	-28,81%	21,46%	-9,12%	-14,27%
Return 3Y input	3,15%	2,06%	-2,52%	-23,69%	19,77%	-9,12%	-14,27%
Std returns 1Y input	10,97%	10,52%	11,94%	20,29%	15,96%	22,32%	22,02%
Std returns 2Y input	10,77%	10,32%	9,52%	19,57%	17,19%	22,32%	22,02%
Std returns 3Y input	12,50%	12,23%	11,50%	45,90%	18,08%	22,32%	22,02%
Risk free rate	3,46%						

**Table 3.** Results 2007

Due to the break-out of the financial crisis, the stock market faced both a bull and bear trend, see figure 2, where the index ended with a decrease of -14,27% for the period. Nevertheless, all strategies of two year input manage to generate positive Sharpe-ratios, with the exception

of the Mvp classic. Noteworthy is that the interest rate has increased from previous year and that this leads to some deteriorating of the Sharpe-ratios.

The strategies that are expected to perform best in a market downturn are the ones which allow short-selling. Remarkable is that the Mvp classic is the only strategy that underperforms the index, due to low returns with high volatility. As described in the theory chapter 2, negative Sharpe-ratios cannot be interpreted correctly. Still, one can interpret the return and risk individually. By doing this it can be seen that the index produces high negative return with high risk compared to the optimization strategies, excluding Mvp classic. Especially we want to emphasize the Mvp-10% strategy that managed to generate high positive returns in a declining market.

2008	CvaR-95%	CvaR-99%	Mvp no short-selling	Mvp classic	Mvp-10%	Equal weight	OMXS30
Sharpe-ratio 1Y input	-1,62	-1,65	-1,75	-1,17	-1,39	-1,57	-1,31
Sharpe-ratio 2Y input	-1,75	-1,75	-1,66	-0,53	-1,11	-1,57	-1,31
Sharpe-ratio 3Y input	-1,53	-1,55	-1,54	-1,43	-1,34	-1,57	-1,31
Return 1Y input	-40,10%	-40,37%	-40,54%	-29,60%	-35,67%	-40,88%	-35,29%
Return 2Y input	-43,38%	-43,01%	-40,46%	-34,18%	-40,68%	-40,88%	-35,29%
Return 3Y input	-39,28%	-39,38%	-38,51%	-58,05%	-43,91%	-40,88%	-35,29%
Std returns 1Y input	27,21%	26,91%	25,56%	28,86%	28,53%	28,55%	29,96%
Std returns 2Y input	27,06%	26,94%	26,82%	72,30%	40,42%	28,55%	29,96%
Std returns 3Y input	28,26%	28,10%	27,63%	43,52%	35,80%	28,55%	29,96%
Risk free rate	4,08%						

**Table 4.** Results 2008

In this “financial lost year”, none of the strategies distinguish in a positively manner of performance. The underlying assumption of normal distributed returns is not a good approximation when extreme events occur, also known as Black swans. Although, the one year input strategies are expected to be the ones that fastest capture and accommodate to the change in market condition, they still underperform over the year in relation to the benchmark index.

It seems to be a disadvantage for quant funds, in comparison of traditional mutual managed funds, because they generally become slow to adjust to changes in the economic environment.

2009	CvaR-95%	CvaR-99%	Mvp no short-selling	Mvp classic	Mvp-10%	Equal weight	OMXS30
Sharpe-ratio 1Y input	1,13	1,10	1,05	1,14	1,38	1,33	1,33
Sharpe-ratio 2Y input	0,72	0,75	0,86	2,08	1,42	1,33	1,33
Sharpe-ratio 3Y input	0,82	0,85	0,84	-1,11	0,49	1,33	1,33
Return 1Y input	24,96%	24,50%	38,34%	29,62%	58,07%	44,62%	36,89%
Return 2Y input	14,85%	16,23%	24,11%	83,42%	42,89%	44,62%	36,89%
Return 3Y input	15,06%	16,13%	17,94%	-49,10%	9,00%	44,62%	36,89%
Std returns 1Y input	21,00%	21,23%	35,45%	24,92%	41,10%	32,60%	26,80%
Std returns 2Y input	18,93%	19,98%	26,58%	39,48%	29,35%	32,60%	26,80%
Std returns 3Y input	16,89%	17,44%	19,81%	45,54%	15,76%	32,60%	26,80%
Risk free rate	1,25%						

**Table 5.** Results 2009

Once more the Mvp classic strategy proved to be unreliable for different input periods, since it produces extreme both positive and negative returns, along with that the highest and lowest Sharpe-ratio. The strategy's extreme high return, for 2 year input, can be explained by 5 monthly returns above 10% over the year. Taking a closer look at the remarkable loss, for 3 year input, it can be explained by one single large position.

Additionally, the Mvp-10% strategies are, like in 2007, able to take advantage of the short positions in a bull market and generate high Sharpe-ratios for one and two year input data. Note that the interest rate has decreased sharply by 283 basis points to 1,25% which, everything held constant, improves the Sharpe-ratios.

Finally we want to stress the result of that the equal weight strategy and the index produced equal Sharpe-ratio, but with different characteristics. The equal weight strategy is exposed to a higher risk than the index, but the additional risk is exactly compensated by a higher return.

## 4.2 Yearly evaluation summarized

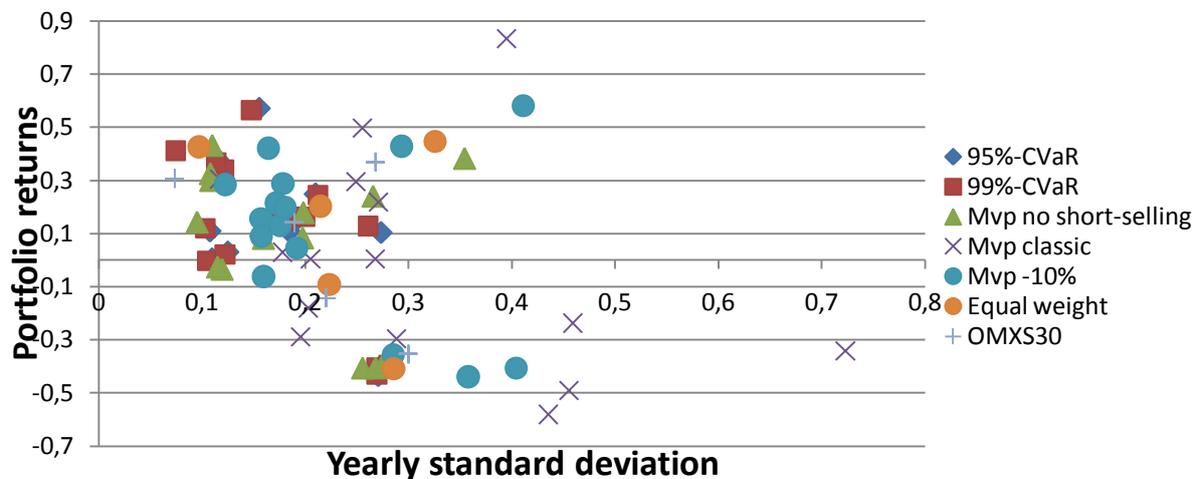


Figure 7. Results risk/reward universe

During the evaluation period between 2005-2009, average portfolio return was approximately 7% with a risk level of 22%. This approximation was calculated from an average of all observations and is presented in figure 7. Although the Mvp classic constructs portfolios that optimize Sharpe-ratios in-sample, it is not dependable for out-of-sample performance. Recalling from previous results, the Mvp classic strategies is characterized by large fluctuations as can be seen in the risk/reward universe figure 7. The large fluctuations can be explained by the drawback of the classical approach which takes on extreme stock weights, a finding which is in line with what Merton (1980) also found. Furthermore, since this strategy has the highest weight turnover it will also contribute with the highest transaction costs. As can be seen in figure 7, a more sound approach to short-selling is the strategy that limits the short-selling to -10%. This strategy showed to be more solid and still have the advantage of short-selling while at the same time reduce the problem of taking on extreme weights.

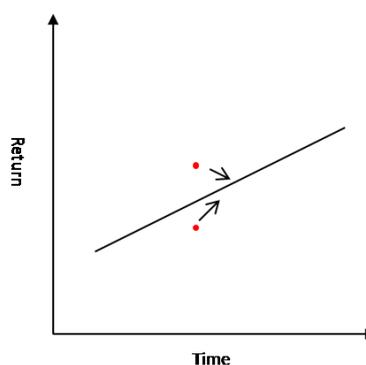
In addition, we find that CVaR-99% produce higher Sharpe-ratios than CVaR-95% in several periods. This is because it systematically generates lower risk, with the exception of 2009, to a return that is equal or greater than CVaR-95%. The difference is in many cases not substantial but still we propose that the CVaR-99% is the most prudent CVaR strategy. Thus, the additional risk which CVaR-95% is exposed to is not compensated proportionally.

When analyzing the lengths of input periods, a remarkable result is that there is no clear pattern that one length of input period is superior to any other in the out-of-sample tests.

Further, there is no intuition that the 1 year input strategies accommodate faster to major changes in market conditions if comparing to 2 and 3 year input. However, one should remember that the estimation risk problem is reduced for the Mvp strategies, since we apply a zero risk tolerance, thus the optimization problem does not depend on mean return. Appendix B shows the five year performance of each input period over time by tracking the compound growth of each strategy. It can be seen, as mentioned before, that the Mvp classic strategy is characterized by extreme fluctuations.

While the Mvp classic strategy was characterized by extreme fluctuations the equal weight strategy follows a much smoother and predictable path. The equal weight strategy can be seen as the reversal of a momentum fund. In practise, the stocks that have risen in value are sold and the money is re-invested in the shares that have fallen in value to such an extent that every asset has an equal weight for the coming period.

An intuitive explanation to why this strategy outperforms the market could be that there exists a “trend equilibrium” for the stock returns. If one individual stock return is higher than the others one month, it will be lower the next to converge with the others and vice versa for lower returns. This is exactly what the equal weight strategy in this research capture on a one month basis. The graph below gives an illustrative explanation where the solid line is the “trend equilibrium” and the red dots are individual stocks that outperform and underperform the portfolio respectively.



**Figure 8.** *Trend equilibrium*

When conducting a Jarque-Bera test of the sample data we find, in line with recent research (Osborne, 1959; Fama, 1965; Rachev & Mittnik, 2000), that the distribution of returns behaves indeed non-normal, as shown in Appendix A. Thus the return distribution has properties of negative skewness and excess kurtosis that exist in financial data. This implies

that there are periods with extreme events that the applied strategies fail to capture and take into consideration.

### 4.3 M-V versus CVaR

As described in the problem discussion both the M-V and the CVaR approach utilize the volatility as a risk measure. From the results we can observe that the models produce mixed out-of-sample results where the models sometimes are consistent and sometimes not. It is therefore interesting to analyze where the different models choose their in-sample optimal portfolios in the risk/reward universe.

For reliability the CVaR-99%, which in this case consider the most risk averse investor, and the Mvp no short-selling strategy are analyzed. In Appendix C, the charts of the efficient frontier for the Mvp no short-selling are constructed combined with dots for the optimal portfolio selection for the two CVaR strategies. To be able to find the presented cases when the models are consistent or not, the following formula is applied:

$$\sqrt{\frac{\sum_{i=1}^N (W_{iMvp} - W_{iCVaR-99\%})}{N}}$$

The formula captures the in-sample consistency between the models by summing up the difference in weights for every individual asset.

Intuitively, no point from the CVaR strategies can be outside of the efficient frontier produced by the Mvp no short-selling strategy since it minimizes risk for every return. Noteworthy is that the CVaR strategies choose portfolios on, or very close to, the efficient frontier. Hence near optimal portfolios in a M-V approach. What makes the models more or less consistent is dependent upon the expected return where the Mvp approach with risk tolerance equal to zero ignores the expected return while it is a component of the CVaR approach. Hence, if expected return is close to zero the models becomes more consistent and vice versa. In a quite unrealistic case if expected returns are exactly equal to zero both approaches construct optimal portfolios by only minimizing the portfolios standard deviation, thereby constructing the same optimal portfolios.

## 5. Conclusion

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In this final chapter conclusions based on the main results are stressed. Moreover, it includes feedback on the thesis purpose and the question is answered. The chapter ends with some suggestions for future research.

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When reviewing the results to be able to answer the question, “*Which portfolio optimization model and strategy produce the most efficient risk/reward return evaluated by the Sharpe-ratio?*”, it could be concluded that the simplest strategy is also the strategy that produced the most efficient Sharpe-ratios. Neither of the two optimization models, M-V or CVaR, and their applied strategies managed to outperform the strategy that invested an equal weight in every stock each month.

The equal weight strategy was described to be seen as the reversal of a momentum fund. We proposed an intuitive explanation to how the strategy was able to capture that there may exist a “trend equilibrium” for stock returns which the strategy was described to take advantage of. The returns for the strategy followed a smooth and predictable path and it produced solid Sharpe-ratios that were more efficient than the other strategies including the index on an overall perspective.

While the equal weight strategy showed to be stable, the Mvp classical approach showed to be very unstable and characterized by extreme fluctuations in both returns and Sharpe-ratios. The fact that Mvp classic produced the most efficient Sharpe-ratios in-sample proved to be meaningless for out-of-sample performance. An explanation to why this occurred was found to be that the Mvp classic takes on extreme weights. Even though not considered in the research, we could also conclude that the Mvp classic had the highest weight turnover and would therefore also contribute with the highest transaction costs. Furthermore, we found that a more sound approach to short-selling would be to have limits of short-selling, which is in line with previous research (Krokhmal et al., 2001; Krokhmal et al., 1999).

In addition, we found that CVaR-99% produced higher Sharpe-ratios than CVaR-95% for several periods. CVaR-95% was in many cases exposed to a slightly higher risk but did not get compensated for it. We therefore proposed that CVaR-99% was a more prudent strategy than CVaR-95%.

When analyzed the in-sample consistency between the two models, M-V and CVaR, we found that the CVaR strategies selected portfolios on, or very close, to the Mvp classic efficient frontier. It could be concluded that the expected return is what makes the models more or less consistent since it is a component in the CVaR approach and is excluded in the Mvp approach.

Due to the random results in the Sharpe-ratios for the different length of input periods we could conclude that no length was superior. Remarkable was that there were no intuition of that the 1 year input strategies accommodated faster to different market conditions.

Finally, we made a concluding remark when showed that the data sample was non-normal distributed. That would imply that the strategies did not capture the extreme events that occurred, which affected the construction of optimal portfolios and caused unreliable estimates of risk.

As suggestions for future research, we would find it interesting to extend the equal weight strategy, since it produced the most efficient Sharpe-ratios. By imposing an additional algorithm to also short-sell the “outperformers” and re-invest in the “underperformers” the extended strategy possibly becomes more effective. Due to the impact of transaction costs, one could also investigate if any excess adjusted return is sufficient to compensate for the costs of rebalancing.

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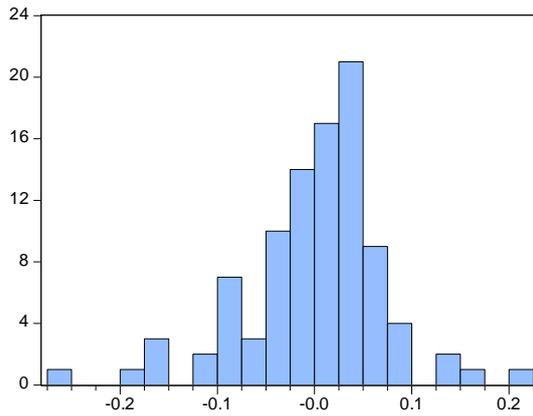
[http://www2.standardandpoors.com/spf/pdf/index/SPIVA\\_2009\\_Midyear.pdf](http://www2.standardandpoors.com/spf/pdf/index/SPIVA_2009_Midyear.pdf)

# Appendix A

## CVaR derivation

$$\begin{aligned}
 CVaR &= (1 - \alpha)^{-1} \int_{f(x,y) \geq VaR} f(x,y)p(y)dy \\
 &= \frac{1}{1 - \alpha} \int_{VaR}^{+\infty} \frac{t}{\sqrt{2\pi}\sigma_p} \exp\left\{-\frac{[t + E(r_p)]^2}{2\sigma_p^2}\right\} dt \\
 &= \frac{1}{1 - \alpha} \int_{VaR}^{+\infty} \frac{t + E(r_p) - E(r_p)}{\sqrt{2\pi}\sigma_p} \exp\left\{-\frac{[t + E(r_p)]^2}{2\sigma_p^2}\right\} dt \\
 &= \frac{\sigma_p}{(1 - \alpha)\sqrt{2\pi}} \exp\left(\frac{-[VaR + E(r_p)]^2}{2\sigma_p^2}\right) \\
 &\quad - \frac{1}{1 - \alpha} \int_{VaR}^{+\infty} \frac{E(r_p)}{\sqrt{2\pi}\sigma_p} \exp\left\{-\frac{[t + E(r_p)]^2}{2\sigma_p^2}\right\} dt \\
 &= \frac{\Phi[\Phi^{-1}(\alpha)]}{1 - \alpha} \sigma_p - E(r_p) = \frac{\alpha\Phi}{1 - \alpha} \sigma_p - \mu_p
 \end{aligned}$$

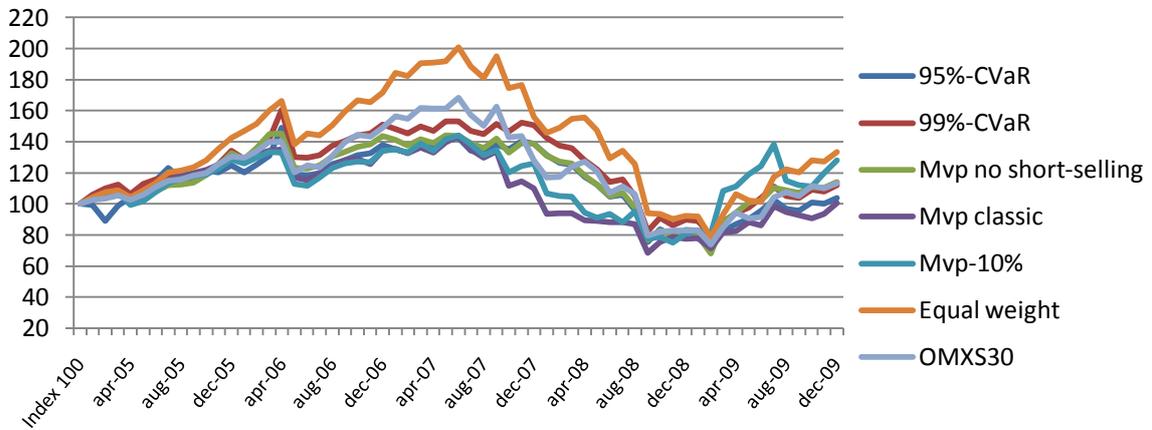
## Data distribution



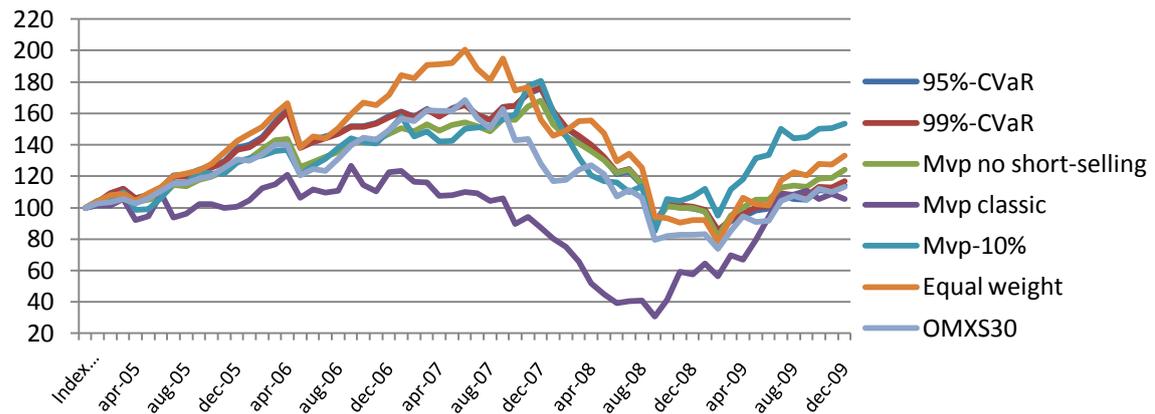
Series: LOG_RETURNS_MONTHLY	
Sample 2002M01 2010M01	
Observations 96	
Mean	0.001493
Median	0.013282
Maximum	0.201713
Minimum	-0.254397
Std. Dev.	0.071843
Skewness	-0.742801
Kurtosis	4.790361
Jarque-Bera	21.64962
Probability	0.000020

# Appendix B

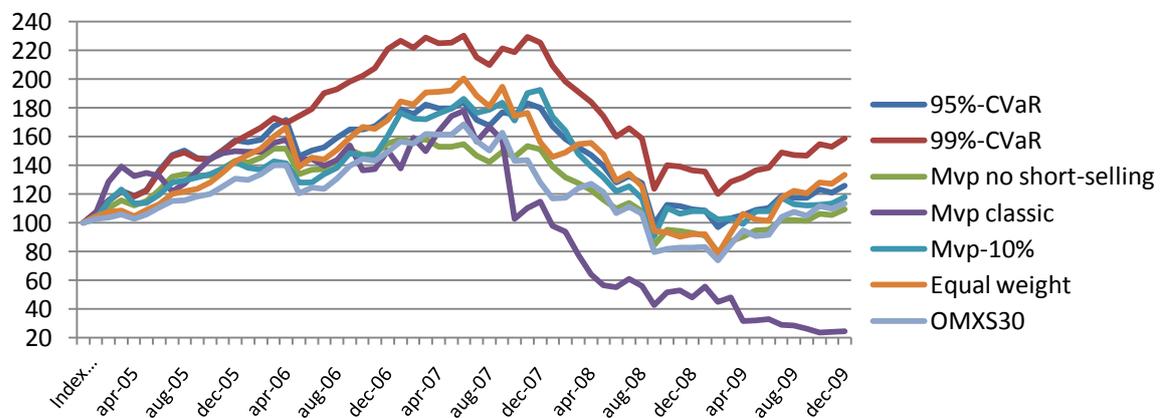
## Portfolio returns 1 year input



## Portfolio returns 2 year input

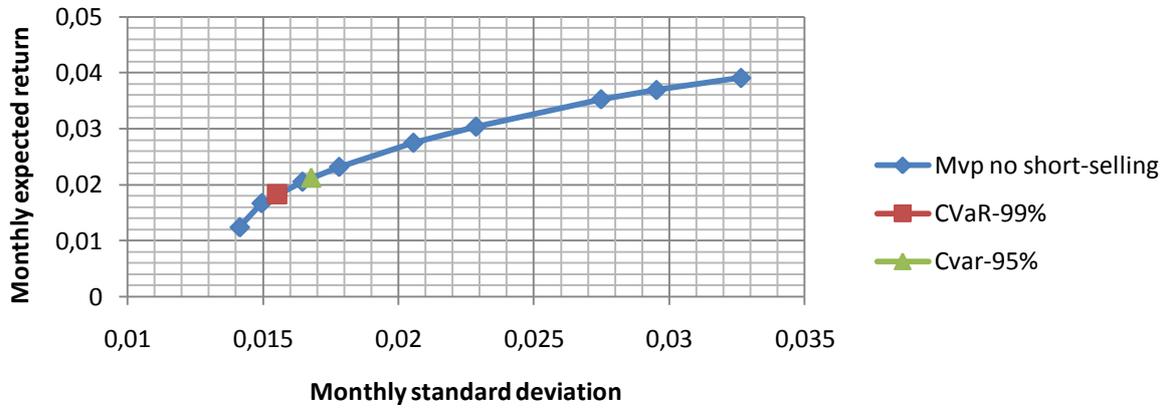


## Portfolio returns 3 year input

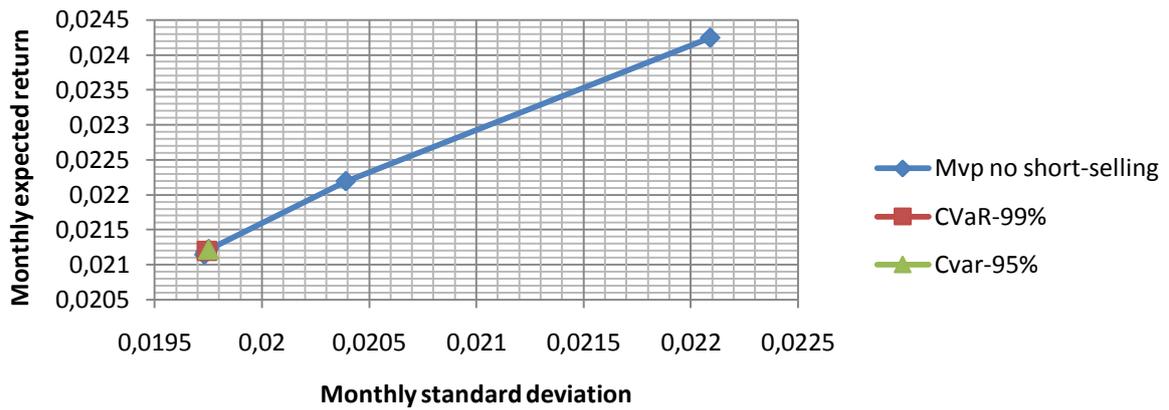


# Appendix C

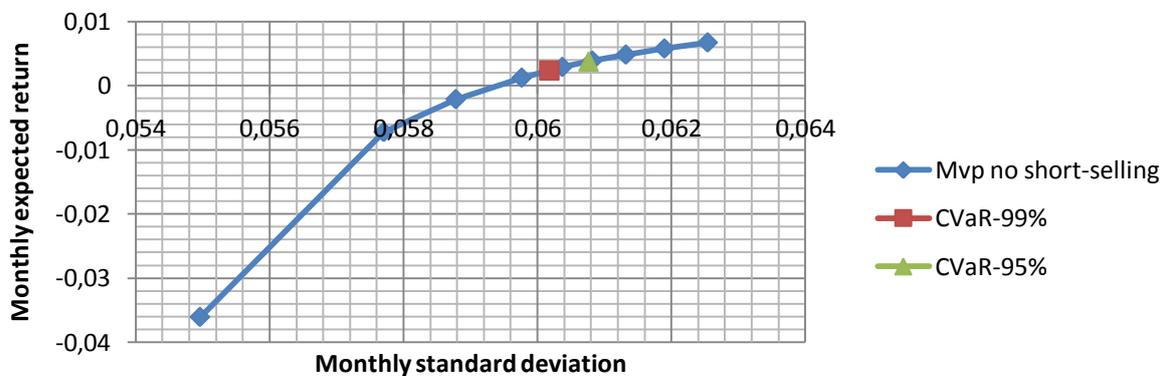
## Portfolio selection average difference



## Portfolio selection small difference



## Portfolio selection big difference



## **Appendix D**

### **Stocks included in OMXS30 as of 2010-01-09**

ABB

ALFA LAVAL

ASSA ABLOY B

ASTRA ZENECA

ATLAS COPCO A

ATLAS COPCO B

BOLIDEN

ELECTROLUX B

ERICSSON B

GETINGE

HENNES&MAURITZ B

INVESTOR B

LUNDIN PETROLIUM

MTG B

NOKIA

NORDEA

SANDVIK

SCA B

SCANIA B

SEB A

SECURITAS B

SKANSKA B

SKF B

SSAB B

SVENSKA HANDELSBANKEN A

SWEDBANK A

SWEDISH MATCH

TELE 2 B

TELIASONERA

VOLVO B