Department of Statistics

Comparison of Multivariate GARCH Models with Application to Zero-Coupon Bond Volatility

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Abstract

The purpose of this thesis is to investigate different formulations of multivariate GARCH models and to apply two of the popular ones – the BEKK- GARCH model and the DCC- GARCH model – in evaluating the volatility of a portfolio of zero-coupon bonds. Multivariate GARCH models are considered as one of the most useful tools for analyzing and forecasting the volatility of time series when volatility fluctuates over time. This feature demonstrates its availability in modeling the co-movement of multivariate time series with varying conditional covariance matrix. From this point of view, firstly we focus on understanding the model specifications of several widely used multivariate GARCH models so as to select appropriate models; and then construct the BEKK form and the DCC form separately by employing the financial data obtained from the website of the European Central Bank. The next work is dedicated to diagnose the goodness of fit of the established models even though there are comparatively few tests specific to multivariate models according to previous literatures. On top of those, we compare the fitting performance of these two forms and predict the future dynamics of our data on the ground of these two models.

1. Introduction

With the increase in the complexity of the instruments in the risk management field, huge demands for the various models which can simulate and reflect the characteristics of the financial time series have expanded. One of the significant features of financial data that has won much attention is the volatility; because it is a numerical measure of the risk faced by individual investors and financial institutions. It is well known that the volatility of financial data often varies over time and tends to cluster in periods, i.e., high volatility is usually followed by high volatility, and low volatility by low volatility. This phenomenon corresponds to the fluctuating volatility. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model and its extensions have been proved to be able to capture the volatility clustering and predict volatilities in the future.

Specifically, when analyzing the co-movements of financial returns, it is always essential to estimate, construct, evaluate, and forecast the co-volatility dynamics of asset returns in a portfolio. This task can be fulfilled by multivariate GARCH (MGARCH) models. The development of MGARCH models could be thought as a great breakthrough against the curse of dimensionality in the financial modeling. Many different formulations have been constructed parsimoniously and still remain necessary flexibility. MGARCH models can be applied to asset pricing, portfolio theory, VaR estimation and risk management or diversification, which require the volatilities and co-volatilities of several markets [Bauwens *et al*., 2006].

In this thesis, MGARCH models are estimated for volatility and co-volatility of three zero coupon bond prices with different maturities. The data is provided by the website of the European Central Bank (ECB) which is the institution of the European Union tasked with administrating the monetary policy of the 16 EU member states taking part in the Eurozone.

A zero coupon bond is a non-coupon-bearing bond that pays face value at the time of maturity even though it is bought at a price lower than its face value. It has no reinvestment risk and is more sensitive to interest rate change than coupon-bearing bonds. Due to these features, zero-coupon bonds can be easily used to create any type of cash flow stream and thus match asset cash flows with liability cash flows (e.g. to provide for college expenses, house-purchase down payment, or other liability funding.), and are used by pension funds and insurance companies to offset, or immunize the interest rate risk of these firms' long-term liabilities.

Moreover, the return of zero coupon bond, referred to as zero rate, is a fundamental element in the field of fix-income pricing and risk evaluation. By using cash-flow-mapping method [Hull, 2005], any fixed cash flow can be mapped to a portfolio consisting of a few zero coupon bonds, which match the cash flow’s return and volatility. This viewpoint exemplifies how to generalize the specific zero coupon bond volatilities into a general case. It also motivates our study to model volatility and co-volatility of three zero-coupon bonds with different maturities of 6 month, 1 year and 2 year. In later section, we estimated two MGARCH models based on the BEKK form and DCC form by the quasi - maximum likelihood method and also tested the goodness of fit of these models.

The reminder of this thesis is organized as follows. Section 2 reviews MGARCH models, including its different forms, diagnostics and the forecasting. In section 3 we present the BEKK and DCC MGARCH models of volatility and co-volatility of ECB zero coupon bond data set. Section 4 provides conclusions and further work.

2. Model Specification and Estimation Methodology

In order to accurately capture the characteristic heteroskedasticity of many financial data, which refers to the fact that the market volatility varies and tends to cluster in periods of high volatility as well as periods of low volatility, the ARCH model was introduced by Engle (1982).

Even though this model captures the varying volatility of financial time series in contrast with the constant volatility in previous research, there was still need for a better model to measure risk which is reflected as the volatility. This section mainly concerns a more generalized model of the ARCH model from the univariate case to multivariate cases.

2.1 ARCH models

The mean process of ARCH models can be expressed by

$$r_t = \mu + \varepsilon_t., \qquad t = 1,\ldots\ldots,T \tag{1}$$

Here, μ is the mean of the time series r_t and ε_t denotes its residual. T is the number of observations.

Regarding the residuals' variance process of ARCH models, assume $\varepsilon_t=\sigma_t z_t$, where $z_t \sim N(0,1)$ and the series σ_t^2 are modeled by

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2, \tag{2}$$

where $\alpha_0 > 0$ and $\alpha_i \geq 0$, $i > 0$.

It specifies a stochastic process for the residuals and predicts the average size of the residuals.

However, it has its own drawbacks in that the assumption that positive and negative shocks have the same effects on volatility goes contrary to the reality. It is very common that the price of a financial asset responds differently to positive and negative shocks [Paul, 2007]. In addition, it is always the case that ARCH models require the estimation of a large number of parameters as a high order of ARCH terms has to be selected for the purpose of catching the dynamic of the conditional variance.

2.2 GARCH models

The following subsections introduce the general formulation of a univariate GARCH model, the most widely used GARCH form – GARCH (1, 1) and some extensions.

2.2.1 General form of GARCH models

In view of the ARCH model's limitations, Bollerslev (1986) proposed the Generalized ARCH model (GARCH), in which the conditional variance satisfies the following form.

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_{t-p}^2 \quad (3)$$

where $\alpha_i > 0$ and $\beta_i > 0$.

In GARCH models, residuals' lags can be replaced by a limited number of lags of conditional variances, which simplifies the lag structure and as well the estimation process of coefficients.

2.2.2 GARCH (1, 1) models

The most frequently used GARCH model is the GARCH (1, 1) model. In GARCH (1, 1), the conditional variance matrix is calculated from a long-run average variance rate, V_L, and also from the lag terms σ_{n-1} and ε_{n-1}. The equation of the conditional variance for GARCH (1, 1) is

$$\sigma_n^2 = \gamma V_L + \alpha \varepsilon_{n-1}^2 + \beta \sigma_{n-1}^2 \quad (4)$$

where γ is the weight assigned to V_L, α is the weight assigned to ε_{n-1}^2, and β is the weight assigned to σ_{n-1}^2. In addition, the weights sum to one, that is,

$$\gamma + \alpha + \beta = 1 \tag{5}$$

The GARCH (1, 1) models specifies that σ_n^2 is based on the most recent observation of ε_n^2 and the most recent variance rate σ_{n-1}^2.

Setting $\omega = \gamma V_L$, the GARCH (1, 1) model can be rewritten as

$$\sigma_n^2 = \omega + \alpha \varepsilon_{n-1}^2 + \beta \sigma_{n-1}^2 \tag{6}$$

This is the form that is usually used for the estimation of parameters in the univariate case.

2.2.3 Extensions of the GARCH models

There are many extensions of the standard GARCH models[1]. Nonlinear GARCH (NGARCH) was proposed by Engle and Ng in 1993. The conditional covariance equation is in the form $\sigma_t^2 = \omega + \alpha(\varepsilon_{t-1} - \theta\sigma_{t-1})^2 + \beta\sigma_{t-1}^2$, where $\alpha, \beta, \omega > 0$. The integrated GARCH (IGARCH) is a restricted version of the GARCH model, where the sum of all the parameters sum up to one. The exponential GARCH (EGARCH) introduced by Nelson (1991) is to model the logarithm of the variance rather than the level. The GARCH-in-mean (GARCH-M) model adds a heteroskedasticity term into the mean equation. The quadratic GARCH (QGARCH) model can handle asymmetric effects of positive and negative shocks. The Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model (1993) can also model asymmetry in the GARCH process. The threshold GARCH (TGARCH) model is similar to GJR-GARCH with the specification on conditional standard deviation instead of conditional variance. Family GARCH (FGARCH) by Hentschel (1995) is an omnibus model that is a mix of other symmetric

[1] http://en.wikipedia.org/wiki/Autoregressive_conditional_heteroskedasticity

or asymmetric GARCH models.

2.3 Multivariate GARCH models

The basic idea to extend univariate GARCH models to multivariate GARCH models is that it is significant to predict the dependence in the comovements of asset returns in a portfolio. To recognize this feature through a multivariate model would generate a more reliable model than separate univariate models.

In the first place, one should consider what specification of an MGARCH model should be imposed. On the one hand, it should be flexible enough to state the dynamics of the conditional variances and covariances. On the other hand, as the number of parameters in an MGARCH model increases rapidly along with the dimension of the model, the specification should be parsimonious to simplify the model estimation and also reach the purpose of easy interpretation of the model parameters. However, parsimony may reduce the number of parameters, in which situation the relevant dynamics in the covariance matrix cannot be captured. So it is important to get a balance between the parsimony and the flexibility when designing the multivariate GARCH model specifications. Another feature that multivariate GARCH models must satisfy is that the covariance matrix should be positive definite.

2.3.1 Formulations of Multivariate GARCH models

This section emphasizes on giving a brief introduction to several different multivariate GARCH models.

- VEC/DVEC-GARCH models

The first MGARCH model was introduced by Bollerslev, Engle and Wooldridge in 1988, which is called VEC model. It is much general compared to the subsequent formulations. In the VEC model, every conditional variance and covariance is a function of all lagged conditional variances and covariances, as well as lagged squared returns and cross-products of returns. The model can be expressed below:

$$vech(H_t) = c + \sum_{j=1}^{q} A_j vech(\varepsilon_{t-j}\varepsilon'_{t-j}) + \sum_{j=1}^{p} B_j vech(H_{t-j}), \tag{7}$$

where *vech* (·) is an operator that stacks the columns of the lower triangular part of its argument square matrix, H_t is the covariance matrix of the residuals, N presents the number of variables, t is the index of the tth observation, c is an $N(N+1)/2 \times 1$ vector, A_j and B_j are $N(N+1)/2 \times$ N(N+1)/2 parameter matrices and ε is an $N \times 1$ vector.

The condition for H_t to be positive definite for all t is not restrictive. In addition, the number of parameters equals $(p+q)\times(N(N+1)/2)^2+N(N+1)/2$, which is large. Furthermore, it demands a large quantity of computation.

The DVEC model, the restricted version of VEC, was also proposed by Bollerslev, *et al* (1988). It assumes the A_j and B_j in equation (7) are diagonal matrices, which makes it possible for H_t to be positive definite for all t. Also, the estimation process proceeds much smoothly compared to the complete VEC model. However, the DVEC model with $(p+q+1)\times N\times(N+1)/2$ parameters is too restrictive since it does not take into account the interaction between different conditional variances and covariances.

• BEKK-GARCH models

To ensure positive definiteness, a new parameterization of the conditional variance matrix H_t was defined by Baba, Engle, Kraft and Kroner (1990) and became known as the BEKK model, which is viewed as another restricted version of the VEC model. It achieves the positive definiteness of the conditional covariance by formulating the model in a way that this property is implied by the model structure.

The form of the BEKK model is as follows

$$H_t = CC' + \sum_{j=1}^{q}\sum_{k=1}^{K} A'_{kj}\varepsilon_{t-j}\varepsilon'_{t-j}A_{kj} + \sum_{j=1}^{p}\sum_{k=1}^{K} B'_{kj}H_{t-j}B_{kj} \tag{8}$$

where A_{kj}, B_{kj}, and C are $N\times N$ parameter matrices, and C is a lower triangular matrix.

The purpose of decomposing the constant term into a product of two triangular matrices is to guarantee the positive semi-definiteness of H_t. Whenever $K > 1$ an identification problem would be generated for the reason that there are not only a single parameterization that can obtain the same representation of the model.

The first-order BEKK model is

$$H_t = CC' + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B. \tag{9}$$

The BEKK model also has its diagonal form by assuming A_{kj}, B_{kj} matrices are diagonal. It is a restricted version of the DVEC model. The most restricted version of the diagonal BEKK model is the scalar BEKK one with $A = aI$ and $B = bI$ where a and b are scalars.

Estimation of a BEKK model still bears large computations due to several matrix transpositions. The number of parameters of the complete BEKK model is $(p+q)KN^2+N(N+1)/2$. Even in the diagonal one, the number of parameters soon reduces to $(p+q)\ K\times N+N\times(N+1)/2$, but it is still large. The BEKK form is not linear in parameters, which makes the convergence of the model difficult. However, the strong point lies in that the model structure automatically guarantees the positive definiteness of H_t. Under the overall consideration, it is typically assumed that $p = q = K = 1$ in BEKK form's application.

•Constant Conditional Correlations (CCC) models

The Constant Conditional Correlation model was introduced by Bollerslev in 1990 to primarily model the conditional covariance matrix indirectly by estimating the conditional correlation matrix. The conditional correlation is assumed to be constant while the conditional variances are varying. Obviously, this assumption is impractical for real financial time series. Then certain modifications were made grounded on this form [Annastiina and Timo, 2008].

•Dynamic Conditional Correlations (DCC) models

The Dynamic Conditional Correlation model was proposed by Engle in 2002. It is a nonlinear combination of univariate GARCH models and it is also a generalized version of the CCC model. The form of Engle's DCC model is as follows:

$$H_t = D_t R_t D_t \tag{10}$$

where

$$D_t = diag(h_{11t}^{1/2}, \cdots, h_{NNt}^{1/2})$$

and each h_{iit} is described by a univariate GARCH model. Further,

$$R_t = diag(q_{11t}^{1/2}, \cdots, q_{NNt}^{1/2}) Q_t diag(q_{11t}^{1/2}, \cdots, q_{NNt}^{1/2}),$$

where $Q_t = (q_{ijt})$ is the $N \times N$ symmetric positive definite matrix which has the form:

$$Q_t = (1 - \alpha - \beta)\overline{Q} + \alpha u_{t-1} u'_{t-1} + \beta Q_{t-1}. \tag{11}$$

Here, $u_{it} = \varepsilon_{it} / \sqrt{h_{iit}}$, α and β are non-negative scalars that $\alpha + \beta < 1$, $\overline{Q}$ is the $N \times N$ unconditional variance matrix of u_t.

The shortcoming of the model is that all conditional correlations follow the same dynamic structure.

The number of parameters to be estimated is $(N+1) \times (N+4)/2$, which is relatively smaller than the complete BEKK form with the same dimension when N is small. When N is large, the estimation of the DCC model can be performed by a two-step procedure which decreases the complexity of the estimation process. In brief, in the first place, the conditional variance is estimated via univariate GARCH model for each variable. The next step is to estimate the parameters for the conditional correlation. The DCC model can make the covariance matrix positive definite at any point in time.

•Other multivariate forms

To overcome the difficulty of large number of parameters, the O-GARCH model was proposed by Alexander in 2000. It tries to express a multivariate GARCH in terms of univariate ones. The advantage of this model is that the fluctuating volatility can be explained by a few principle components. One of the disadvantages is that it is usually uncertain whether the unconditional variances have the coherent scaling. Another multivariate GARCH model GO-GARCH model is proposed by Bauwens *et al*. in 2006.

2.3.2 Estimation of MGARCH models

The most usual way to estimate the conditional covariance matrix in the MGARCH model is by the quasi maximum likelihood method.

Let $H_t(\theta)$ be a positive definite $N\times N$ conditional covariance matrix of some $N\times 1$ residual vector ε_t, parameterized by the vector θ. Denoting the available information at time t by $\mathcal{F}_t$, we have

$$E_{t-1}[\varepsilon_t \,|\, \mathcal{F}_{t-1}] = 0; \tag{12}$$

$$E_{t-1}[\varepsilon_t \varepsilon_t' \,|\, \mathcal{F}_{t-1}] = H_t(\theta). \tag{13}$$

Generally the conditional covariance matrix $H_t(\theta)$ is well specified based on a certain MGARCH model. Suppose there is an underlying parameter vector θ_0 which one wants to estimate using a given sample of T observations. The quasi maximum likelihood (QML) approach estimates θ_0 by maximizing the Gaussian log likelihood function

$$\log L_T(\theta) = -\frac{N\cdot T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log|H_t| - \frac{1}{2}\sum_{t=1}^{T}\varepsilon_t' H_t^{-1}\varepsilon_t \ . \tag{14}$$

One needs to notice its assumption that the time series treated should be stationary and the distribution of its residual is pre-defined as a conditional Gaussian distribution. The latter assumption can meanwhile give us hints on how to check the adequacy of the established MGARCH model.

2.3.3 Diagnostics of MGARCH models

The check of the adequacy of MGARCH models is essential in identifying whether a well specified MGARCH model can attain reliable estimates and inference.

Graphical diagnostics for MGARCH models can be fulfilled by examining plots of the sample autocorrelation (ACF) and the sample cross correlation functions (XCF). To ensure the inference from the estimated parameters in the MGARCH model is enough valid, the residuals should be exhibited as a set of white noise with features like expected zero mean vector, no autocorrelations, constant variance, and normal distribution of the residuals.

The autocorrelation and cross correlation functions for the squared process are shown to be useful in identifying and checking time series behavior in the conditional variance equation of the GARCH form.

In the literature, several tests have been developed to test the autocorrelation no matter in univariate or multivariate form. Box and Pierce derived a goodness-of-fit test, called the portmanteau test. It may be the most popular one among all the diagnostics for conditional heteroscedasticity models. The test statistic may be expressed as a function of the covariances between the residuals of the fitted model [Hosking, 1980].

A multivariate version is given by

$$HM(M) = T^2 \sum_{j=1}^{M} (T-j)^{-1} tr\left\{ C_Y^{-1}(0) C_Y(j) C_Y^{-1}(0) C_Y{}'(j) \right\}, \tag{15}$$

where T is the number of observations, $C_Y(j)$ is the sample autocovariance matrix of order j and $Y_t = vech(y_t y_t')$.

The distribution of $HM(M)$ is the asymptotical $\chi^2(K^2 M)$ under the null hypothesis that there is no MGARCH effects.

But still, the fact is that very few tests are adaptable to multivariate models even though there are many diagnostic tests dealing with univariate models.

To summarize, once the model is assumed to catch the dynamics of the time series, the standardized residual $\hat{z}_t = \hat{H}_t^{-1/2} \hat{\varepsilon}_t$ should satisfy the following conditions [Bauwens *et al*., 2006]:

1) $E(\hat{z}_t \hat{z}_t') = I_N$; (16)

2) $Cov(\hat{z}_{it}^2, \hat{z}_{jt}^2) = 0$, for all pairs of the variable index $i \neq j$; (17)

3) $Cov(\hat{z}_{it}^2, \hat{z}_{j,t-k}^2) = 0$, for $k > 0$. (18)

Testing 1) would find the misspecification in the conditional mean; testing 2) is to verify whether the conditional distribution is Gaussian; the purpose of testing 3) is to check the adequacy of the dynamic specification of H_t even without knowing the validity of the assumption on the distribution of z_t.

Concerning the comparison of the BEKK-GARCH model and the DCC-GARCH model, the mean absolute error (MAE) is used to evaluate the fitting performance of both models.

2.3.4 Forecasting

In the class of multivariate ARCH/GARCH models and their extensions, the covariance matrix is no longer constant over time. After such model has been estimated, it is always meaningful to get to understand the mechanism that how the future series can be generated and whether they fit well with the real series.

•Forecasting by the BEKK-GARCH model

In the conditional covariance equation of the BEKK-GARCH model

$$H_t = CC' + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'H_{t-1}B, \tag{19}$$

H_t is a function of the past information, i.e., H_{t-1} and ε_{t-1}. For this reason, the parameter estimation of MGARCH models can be used to predict the future covariance matrix.

•Forecasting by the DCC-GARCH model

The forecast of the covariance matrix of the DCC model is implemented in a two-step procedure. The prediction of the diagonal matrix of the time-varying standard variation through the univariate GARCH models and the forecast of the conditional correlation matrix of the standardized residuals are dealt with separately.

Under the assumption that the volatility at time t is known, what is its forecast value at time $t+k$? In a three-variable case, the answer when $k = 1$ is given below,

$$h_{ii,t+1} = \omega + \alpha\varepsilon_{i,t}^2 + \beta h_{ii,t} \tag{20}$$

where $i = 1, 2, 3$.

To obtain the forecast $h_{ii,t+k}$ at time $t+k$, one just need to repeat the substitution successively.

Cited from the definition formula of the DCC-GARCH model, the structure of the conditional correlation matrix is the equation (11).

Under the assumption that $\overline{R} = \overline{Q}$ and $R_{t+i} = Q_{t+i}$ for $i = 1, \ldots, k$, a successive calculation as before can be performed to derive R_{t+k}.

MGARCH models can be used for forecasting. However, by analyzing the relative forecasting accuracy of the two formulations BEKK and DCC, it can be deduced that the forecasting performance of the MGARCH models is not always satisfactory. Many studies, e.g. see Andersen and Bollerslev (1998), reveals that the apparent poor forecasting effect of the MGARCH models is due to using the squared shocks as an approximate value for the true conditional volatility.

3. Construction of Multivariate GARCH Models

3.1 Data Description

The original data is provided by the European Central Bank (ECB) website[2]. It contains daily zero rates of AAA-rated euro area central government bonds, from 01/01/2007 to 30/04/2010. The following table gives a fraction of the data, for example, on 2-Jan-07, the zero rate with maturity 6 month is 3.61% in continuous compounding.

Table 3.1 the Zero Rate Data from ECB

	6m	1y	2y
2-Jan-07	3.611032	3.749662	3.790767
3-Jan-07	3.611704	3.74577	3.782817
4-Jan-07	3.618205	3.754924	3.792305
5-Jan-07	3.626887	3.776559	3.823927
8-Jan-07	3.625807	3.76995	3.815602
9-Jan-07	3.636011	3.777952	3.822553
10-Jan-07	3.65271	3.798284	3.843286

With given ZR_{it} , the zero rate at time t, and maturity T, the zero coupon bond price p_{it} is calculated as

$$p_{it} = S \times e^{-ZR_{it} \cdot T},\ i = 1, 2, 3. \tag{21}$$

where S is the par value, in our case taking the value 100. The daily log return r_t is calculated as follows:

$$r_{i,t} = \ln\left(\frac{p_{i,t}}{p_{i,t-1}}\right),\ i = 1, 2, 3. \tag{22}$$

Their associated line graphs are plotted in the following Figure 3.1. Three variables (*var*1/*var*2/*var*3) correspond to three daily returns with different maturities (6*m*/1*y*/2*y*). One may see that during the second half of year 2008, the daily returns exhibits high

[2] http://www.ecb.int/stats/money/yc/html/index.en.html

volatility, reflecting a financial crisis. Besides, their descriptive statistics are given in Table 3.2. Moreover, the result of ARCH effect [Walter, 2009] test proposed by Engle of each return series is given in Table 3.3, where "*H*" being 1 indicates rejecting of null hypothesis that there is no ARCH effect. One may see that each variable/return has significant ARCH effect.

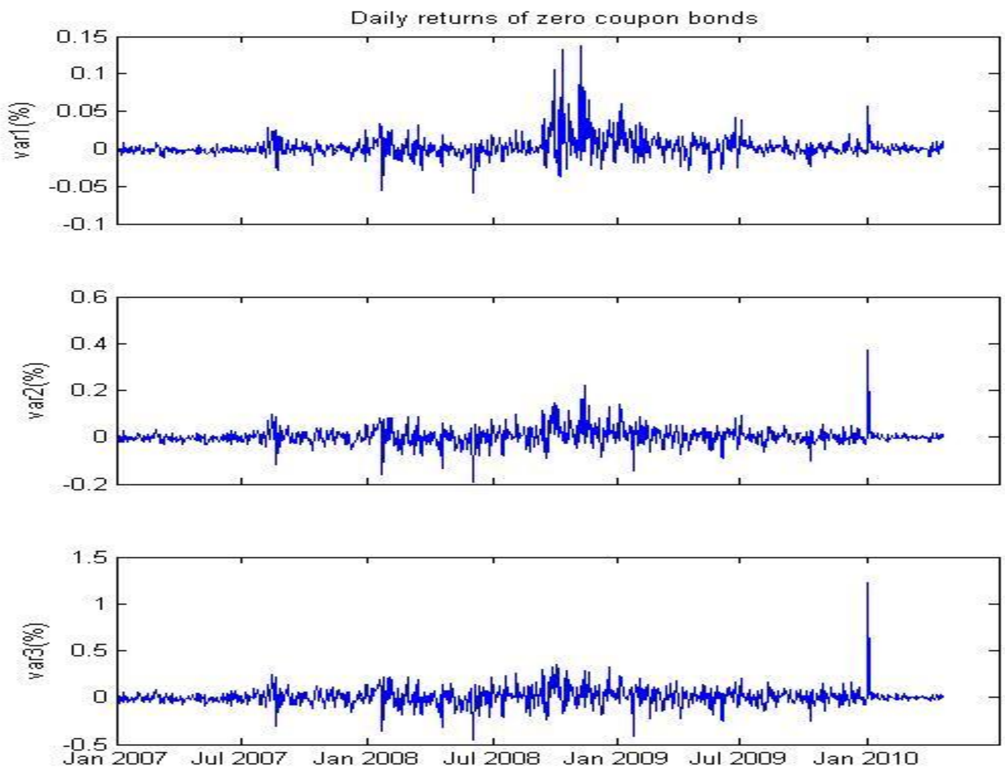


Figure 3.1 Daily Log-Return of Bonds with Different Maturities - 6m, 1y and 2y from top to bottom

Table 3.2 Descriptive Statistics of Return Series with different maturities (6*m*/1*y*/2*y*)

	Mean	Median	Max	Min	Std.Dev.	Skewness	Kurtosis	Jarque-Bera	Prob
*var*1	0.002325	-0.000084	0.137364	-0.058140	0.016573	2.532435	19.69058	8049.379	0.0000
*var*2	0.004493	0.001949	0.218942	-0.194397	0.040345	0.305750	6.771629	386.2683	0.0000
*var*3	0.008162	0.006838	0.353296	-0.452186	0.097917	-0.139231	5.129021	121.9801	0.0000

Table 3.3 GARCH Effect Testing of Return Series

	*var*1		*var*2		*var*3	
Lag	H	pValue	H	pValue	H	pValue
1	1	0.0357	1	0.1887×10^{-5}	1	0.2385×10^{-5}
2	1	0.0121	1	0.4053×10^{-5}	1	0.1165×10^{-5}
3	1	0.0000	1	0.0157×10^{-5}	1	0.0279×10^{-5}
4	1	0.0000	1	0.0002×10^{-5}	1	0.0108×10^{-5}
5	1	0.0000	1	0.0006×10^{-5}	1	0.0086×10^{-5}

One can detect from Table 3.2 or Figure 3.1 that the bonds with the longer maturity are much more volatile than those with a shorter maturity.

Additionally, the financial data here exhibits features like:

• Volatility clustering – Volatility does not keep constant. It is quite common that high returns tend to be followed by high returns and low returns tend to be close with low returns.

• Leptokurtosis effect – By viewing the value of kurtosis, one can conclude that the return series can show the feature of fat tails relative to the normal distribution as high kurtosis indicates a larger possibility of extreme movements.

• Leverage effect – Volatility increases more after low returns than after high returns. A simple explanation for this is that negative returns imply a larger proportion of debt which leads to a high volatility after smaller changes.

• Skewness – All of three variables show evidence of some degree of skewness. The effect of skewness may be positive or negative, which describes their departure from symmetry.

• Long-run memory effect – The existence of this effect reflects persistence temporal dependence even between distant observations.

In addition, the Jarque-Bera statistics reject the null hypothesis that the log return series are normally distributed as the probability of BJ test are all equal to zero.

3.2 Multivariate-GARCH modeling

The data of 2007, 2008 and first half of 2009, totally 635 observations, is used to estimate MGARCH models, and the rest data, i.e., from Jul/2009 is used to evaluate model forecasting. As the BEKK-GARCH and DCC-GARCH models are the two most widely used multivariate GARCH models, we will restrict to model the volatility and co-volatility of the three variables by using BEKK and DCC forms.

Next we present the estimated model, and their diagnostics and forecasting are provided in following subsections.

3.2.1 Model Estimation

As stated before, MGARCH models are estimated by maximum likelihood techniques. In our case, the process was performed by the econometrics software package RATS 7.0 (Regression Analysis of Time Series) which is used worldwide for analyzing time series, developing or estimating econometric models and forecasting. Because of the flexible maximum likelihood estimation capabilities of RATS [Estima, 2007a; 2007b], it has advantages over many other software packages on estimating standard multivariate-ARCH and multivariate-GARCH models.

RATS supports different forms of MGARCH models, including general MGARCH, BEKK, diagonal, VECH, CCC (Constant Conditional Correlation), DCC (Dynamic Conditional Correlations), and EWMA (Exponentially Weighted Moving Average) models. In this thesis, only two widely used MGARCH forms, BEKK form and DCC form are estimated.

The optimization algorithm used for the maximum likelihood estimation is BFGS proposed independently by Broyden (1970), Fletcher (1970), Goldfarb (1970) and Shanno (1970). As a numerical optimization algorithm, it uses iteration routines to obtain the coefficient estimation.

Convergence is assumed to occur if the change in the coefficients to be estimated, i.e. $\min(|\beta_2 - \beta_1|/|\beta_1|, |\beta_2 - \beta_1|)$, is less than the convergence criterion option *cvcrit* specified. The convergence criterion option *cvcrit* used in this thesis was chosen as the default value 0.00001.

3.2.2 BEKK models

As illustrated before, the BEKK form [Engle and Kroner, 1995] of MGARCH takes the following form:

$$H_t = CC' + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B \tag{23}$$

Note that an advantage of BEKK form over VECH form is that positive-definiteness is automatically ensured. Parameter estimation of BEKK form is provided in Table 3.4.

Table 3.4 the Estimation of BEKK-GARCH Model Parameters

Variable	Coeff	Std Error	T-Stat	Signif
Mean(1)	-0.0012	0.0003	-4.3372	0.0000
Mean(2)	-0.0021	0.0010	-2.1793	0.0293
Mean(3)	-0.0039	0.0026	-1.4932	0.1354
C(1,1)	0.0007	0.0003	2.3633	0.0181
C(2,1)	0.0035	0.0009	3.9886	0.0001
C(2,2)	-0.0000	0.0016	0.0004	0.9998
C(3,1)	0.0077	0.0023	3.2883	0.0010
C(3,2)	-0.0000	0.0048	-0.0003	0.9998
C(3,3)	0.0000	0.0008	0.0002	0.9999
A(1,1)	0.3400	0.0711	4.7793	0.0000
A(1,2)	0.1881	0.1658	1.1345	0.2566
A(1,3)	0.6015	0.3843	1.5651	0.1175
A(2,1)	-0.0415	0.0670	-0.6197	0.5354
A(2,2)	-0.1082	0.1356	-0.7985	0.4246
A(2,3)	-0.5756	0.3350	-1.7179	0.0858
A(3,1)	0.0245	0.0216	1.1343	0.2567
A(3,2)	0.1713	0.0424	4.0429	0.0001
A(3,3)	0.5036	0.1129	4.4603	0.0000
B(1,1)	1.2262	0.0172	71.3272	0.0000
B(1,2)	0.3803	0.0392	9.6977	0.0000
B(1,3)	-0.1912	0.1171	-1.6328	0.1025
B(2,1)	-0.3316	0.0169	-19.6146	0.0000
B(2,2)	0.6331	0.0140	45.0757	0.0000
B(2,3)	0.1752	0.0183	9.5760	0.0000
B(3,1)	0.0997	0.0061	16.3942	0.0000
B(3,2)	0.0802	0.0098	8.1893	0.0000
B(3,3)	0.9035	0.0085	48.2091	0.0000

We can see from Table 3.4 that most of variables estimated here are statistically significant.

The estimated BEKK-GARCH model can be obtained by substituting the following matrices into equation (23).

$$A = \begin{pmatrix} 0.340011859 & 0.188091926 & 0.601514375 \\ -0.041537184 & -0.108241876 & 0.575566895 \\ 0.024501622 & 0.171321966 & 0.503572680 \end{pmatrix}$$

$$B = \begin{pmatrix} 1.226159448 & 0.380299285 & -0.191173524 \\ -0.331628418 & 0.633147678 & 0.175165485 \\ 0.099658347 & 0.080216739 & 0.903467974 \end{pmatrix}$$

$$C = \begin{pmatrix} 0.000653422 & 0 & 0 \\ 0.003544914 & -0.000000480 & 0 \\ 0.007689214 & -0.000001374 & 0.000000121 \end{pmatrix}$$

3.2.3 DCC models

As reviewed in previous chapter, the DCC model has the following form:

$$H_t = D_t R_t D_t ,$$

where $D_t = diag(h_{11t}^{1/2}, \cdots, h_{NNt}^{1/2})$, each h_{iit} is a univariate GARCH model, and

$$R_t = diag(q_{11t}^{1/2}, \cdots, q_{NNt}^{1/2}) Q_t diag(q_{11t}^{1/2}, \cdots, q_{NNt}^{1/2}) .$$

The matrix $Q_t = (q_{ijt})$ is the $N \times N$ symmetric positive definite matrix updated by the following:

$$Q_t = (1 - \alpha - \beta)\overline{Q} + \alpha u_{t-1} u'_{t-1} + \beta Q_{t-1} .$$

where $u_{it} = \varepsilon_{it} / \sqrt{h_{iit}}$.

Parameter estimation of DCC model from RATS is provided in Table 3.5.

Table 3.5 the Estimation of DCC-GARCH Model Parameters

Variable	Coeff	Std Error	T-Stat	Signif
Mean(1)	-9.0478e-04	2.8310e-04	-3.1959	0.0014
Mean(2)	-2.0104e-03	8.6466e-04	-2.3250	0.0201
Mean(3)	-4.2393e-03	2.2654e-03	-1.8713	0.0613
C(1)	1.2142e-06	4.3198e-07	2.8109	0.0049
C(2)	4.6008e-07	2.4914e-06	0.1847	0.8535
C(3)	-5.1693e-06	1.7046e-05	-0.3033	0.7617
A(1)	0.2145	0.0250	8.5835	0.0000
A(2)	0.1692	0.0185	9.1365	0.0000
A(3)	0.1502	0.0179	8.3777	0.0000
B(1)	0.8259	0.0161	51.2268	0.0000
B(2)	0.8566	0.0124	69.2042	0.0000
B(3)	0.8713	0.0130	67.1575	0.0000
DCC(1)	0.0934	0.0110	8.4944	0.0000
DCC(2)	0.8971	0.0119	75.6349	0.0000

Except for the constant terms, all the other estimated variables are statistically significant.

Then the estimated DCC model is as following, where $\overline{Q}$ is the 3×3 unconditional covariance matrix of u_t:

$$
\begin{aligned}
h_{11t} &= 1.2142 \times 10^{-6} + 0.2145\varepsilon_{1,t-1}^2 + 0.8259 h_{11,t-1} \\
h_{22t} &= 4.6008 \times 10^{-7} + 0.1692\varepsilon_{2,t-1}^2 + 0.8566 h_{22,t-1} \\
h_{33t} &= -5.1693 \times 10^{-6} + 0.1502\varepsilon_{3,t-1}^2 + 0.8713 h_{33,t-1} \\
Q_t &= (1 - 0.0934 - 0.8971)\overline{Q} + 0.0934 u_{t-1} u'_{t-1} + 0.8971 Q_{t-1} \\
R_t &= diag(q_{11t}^{1/2}, q_{22t}^{1/2}, q_{33t}^{1/2}) Q_t diag(q_{11t}^{1/2}, q_{22t}^{1/2}, q_{33t}^{1/2}) \\
\overline{Q} &= \begin{pmatrix} 0.9934 & 0.8301 & 0.6979 \\ 0.8301 & 1.0124 & 0.9683 \\ 0.6979 & 0.9683 & 1.0034 \end{pmatrix}
\end{aligned}
$$

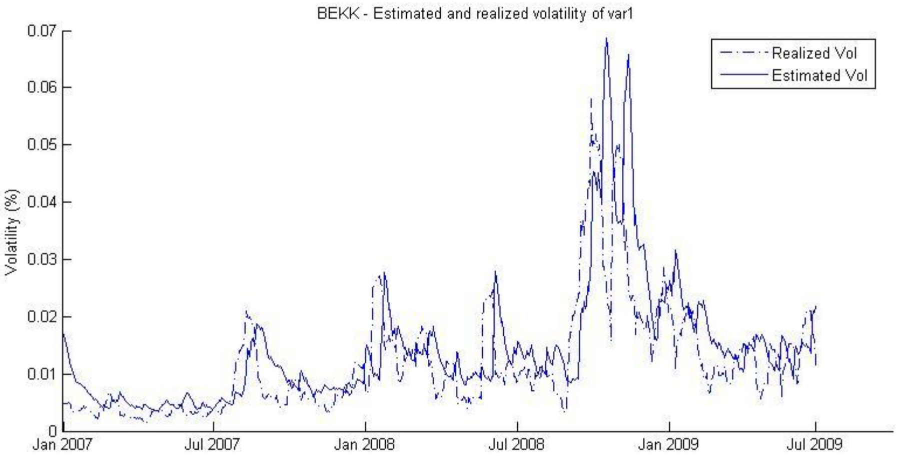
where $u_{it} = \varepsilon_{it} / \sqrt{h_{iit}}$.

3.3 Model Diagnostics

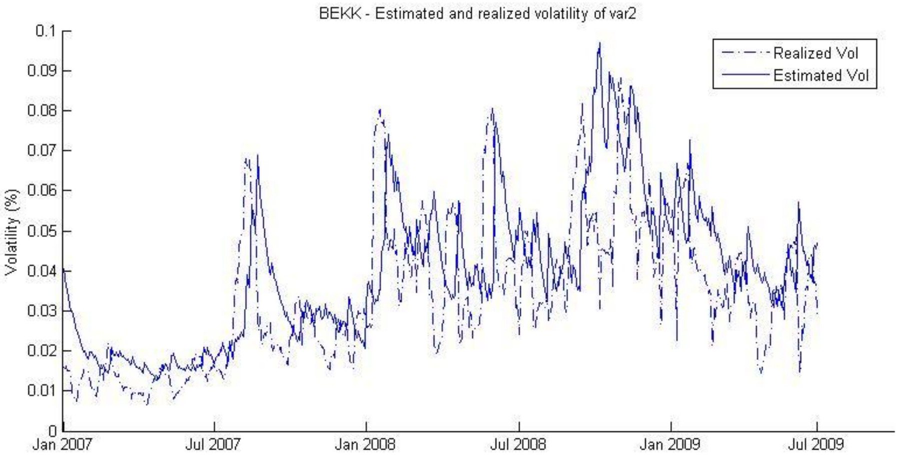
3.3.1 Diagnostics of BEKK models

The empirical measure of logarithmic daily return variability is called the realized volatility. It is computed from high-frequency logarithmic returns [Hull, 2005].

It is calculated using the subsequent 10 observations on the log-returns in our case. In contrast realized volatility constructed from high-frequency returns with the restrictive parametric multivariate GARCH models, links between realized volatility and the diagonal elements of the conditional covariance matrix have been established [Andersen *et al.*, 2003].



(a)



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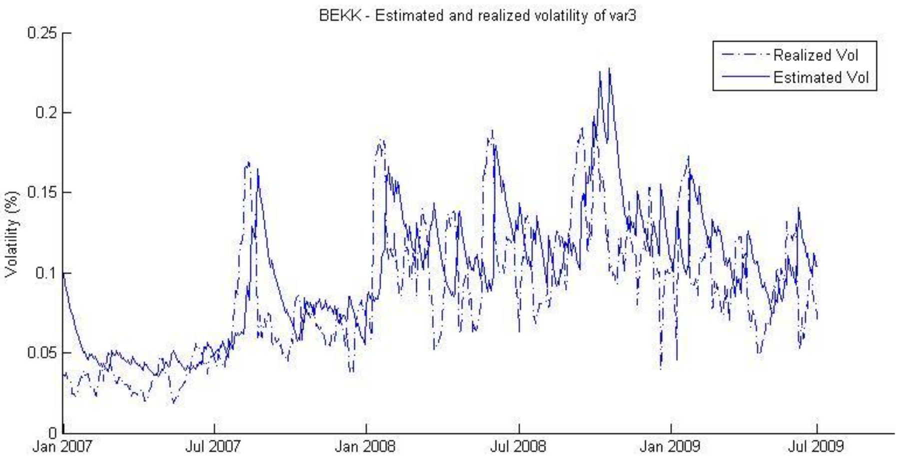


(c)

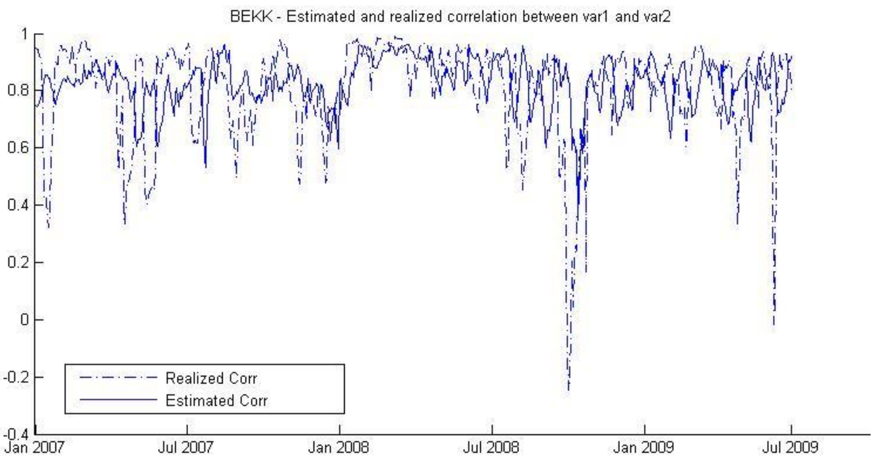
Figure 3.2 Estimated and Realized Volatility of the BEKK Model

Basically, the estimated volatility follows the dynamic of the realized volatility. And the graph reveals two of the financial data's features, the volatility clustering and the relation between maturity and volatility, that is, longer maturity corresponds to higher volatility as indicated in Fig. 3.2 (c). On average, there exists a horizontal lag between these two lines for the reason that we calculated the realized volatility by using the next ten observations.

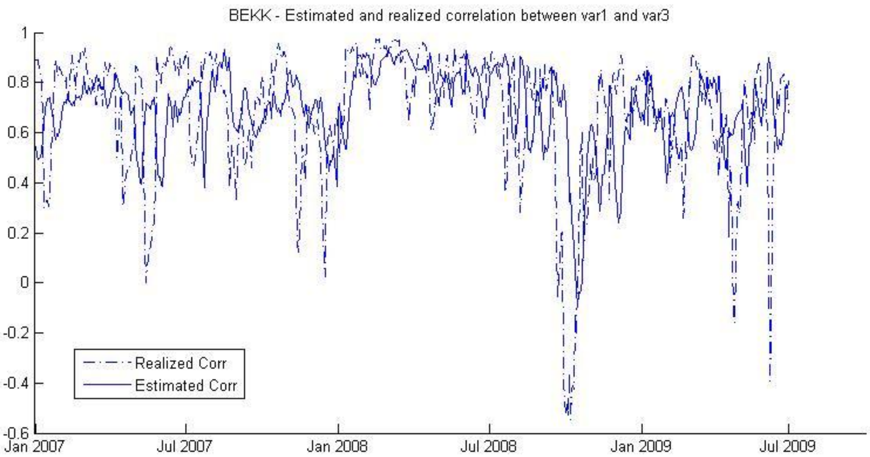
The realized correlation over a horizon of *T* days is approximated by a consistent, empirical estimate. In our case, the realized correlation between the log daily return of var *i* and var *j* at time t over a *T*-day horizon is calculated as

$$\rho(\operatorname{var} i, \operatorname{var} j)_{t,T} = \frac{\sum_{k=1}^{T}(\operatorname{var} i_{t+k} - \mu_i)(\operatorname{var} j_{t+k} - \mu_j)}{\sqrt{\sum_{k=1}^{T}(\operatorname{var} i_{t+k} - \mu_i)^2}\sqrt{\sum_{k=1}^{T}(\operatorname{var} j_{t+k} - \mu_j)^2}} \tag{24}$$

where μ_i and μ_j are the corresponding sample means over the *T*-day period.



(a)



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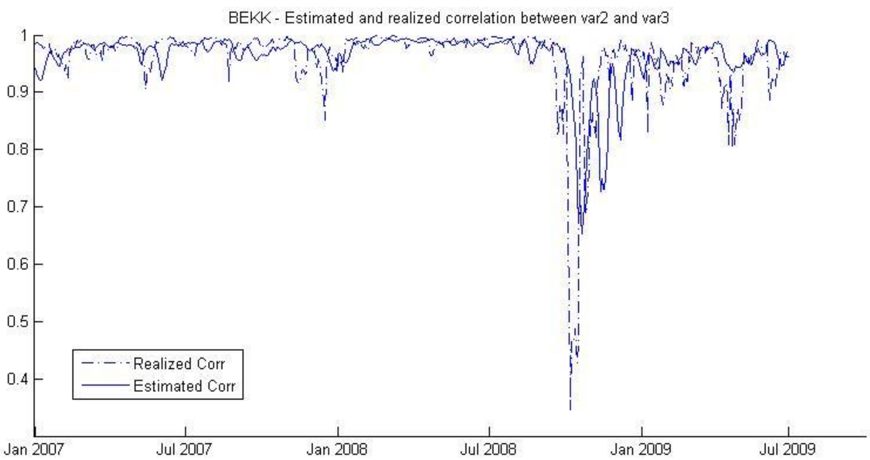


(c)

Figure 3.3 Estimated and Realized Correlation of the BEKK Model

The comparison between the estimated and realized correlation is shown above. It can be seen that there is a huge decline on estimated and realized correlation during the second half of year 2008. With regard to other time periods, the value of correlation between *var*1 and *var*2/*var*3 is around 0.8 and the value of correlation between *var*1 and *var*3 is even above 0.9. As for the performance of fit, the estimated correlation more or less follows the dynamics of the realized correlation except there is also a horizontal lag between them.

The model estimation employed here is the Gaussian quasi MLE method. One of its assumptions is that the residuals have a Gaussian distribution. Hence, to test whether the estimations of the model parameters are robust, we can check whether the residuals of the estimated process are white noise.

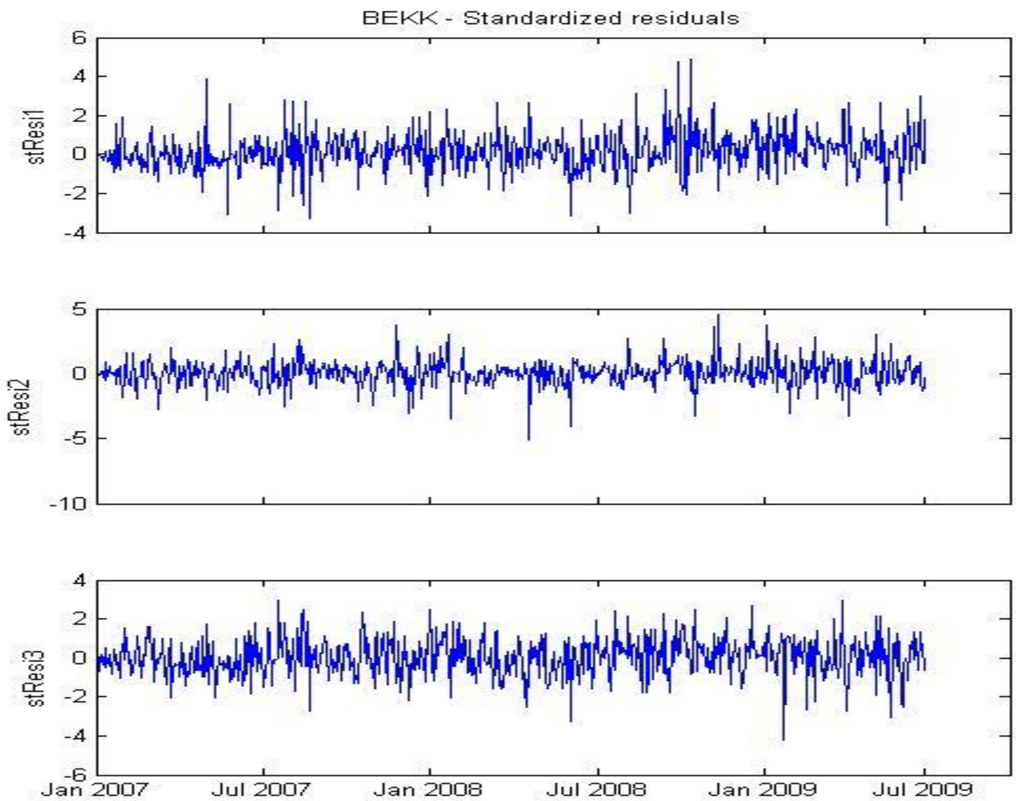


Figure 3.4 Standardized Residuals of the BEKK Models

Calculated by the formula $\hat{z}_t = H_t^{-1/2}\hat{\varepsilon}_t$, the standardized residuals are shown in Fig

3.4 for the three variables. It indicates that no distinct difference exists among the distributions of the three residuals. They all look like white noise on a certain degree.

Table 3.6 shows the testing result of GARCH effect on the standardized residuals of the BEKK model. H = 0 represents the acceptance of the null hypothesis that no GARCH effects exist. In contrast with Table 3.3, we can conclude that GARCH effect has eliminated quite a lot. The Ljung-Box test based on the autocorrelation plot tests the randomness at each distinct lag. H = 0 means that we tend to accept the null hypothesis that the series is random.

Table 3.6 GARCH Effect Testing of each Standardized Residuals (BEKK)

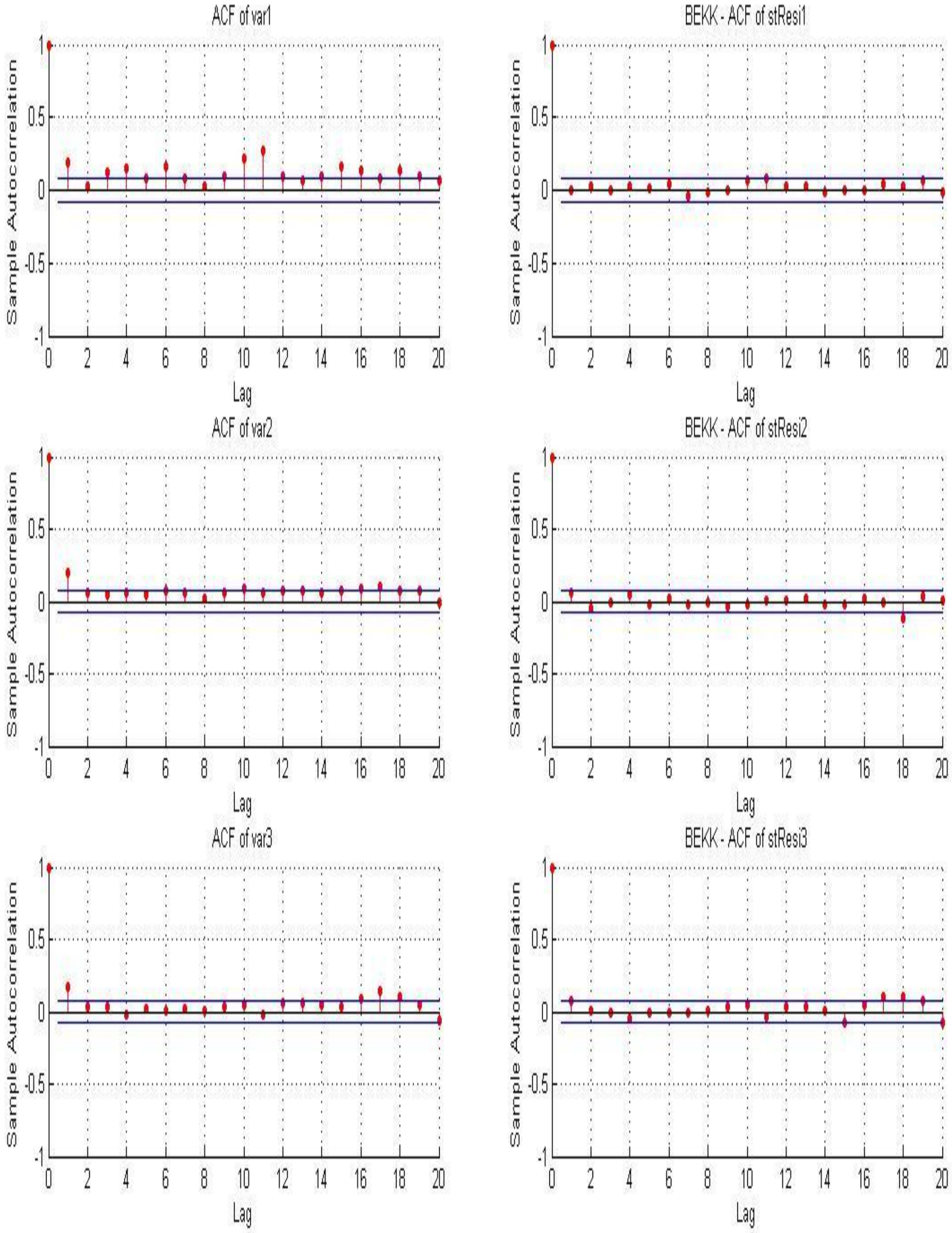
	*var*1		*var*2		*var*3	
Lag	H	pValue	H	pValue	H	pValue
1	0	0.2117	0	0.1537	0	0.8706
2	0	0.3649	0	0.3154	0	0.8924
3	0	0.4380	0	0.5108	0	0.9291
4	1	0.0309	0	0.3234	0	0.6651
5	1	0.0163	0	0.3108	0	0.6329

Table 3.7 LBQ Test of each Standardized Residuals of the BEKK Model

	*var*1		*var*2		*var*3	
Lag	H	pValue	H	pValue	H	pValue
1	0	0.9488	0	0.0935	0	0.0507
2	0	0.8372	0	0.1144	0	0.1412
3	0	0.9459	0	0.2220	0	0.2655
4	0	0.9164	0	0.2025	0	0.2158
5	0	0.9477	0	0.2984	0	0.3276

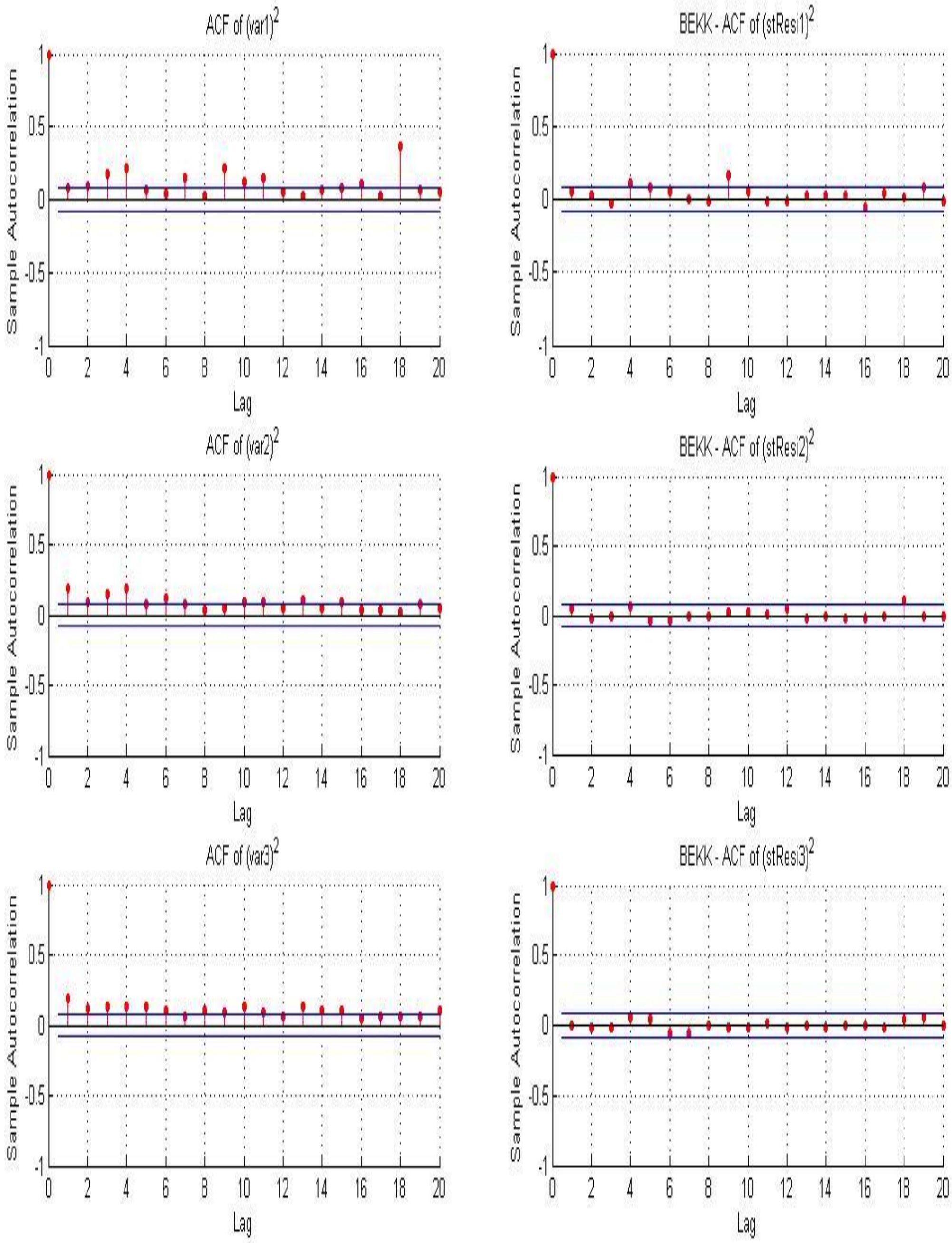
Results of the sample autocorrelation and the sample cross-correlation of the standardized residuals and the squared standardized residuals are presented here to examine the adequacy of the MGARCH model.

Figure 3.5 shows the sample autocorrelation function of the standardized residual of the BEKK model (b) and compare it to the sample autocorrelation of the returns ahead of modeling (a). For most of lags, the sample ACFs and XCFs are within the distance between positive and negative 2 times standard deviation lines at 95% confidence level. A comparison between the ACFs of the premodel data and the standardized residual indicates that GARCH effect has been removed quite a lot. In Fig 3.8, the contrast between the XCFs of the corresponding squared terms before and after the BEKK model also proves that the GARCH effect has been diminished a lot.



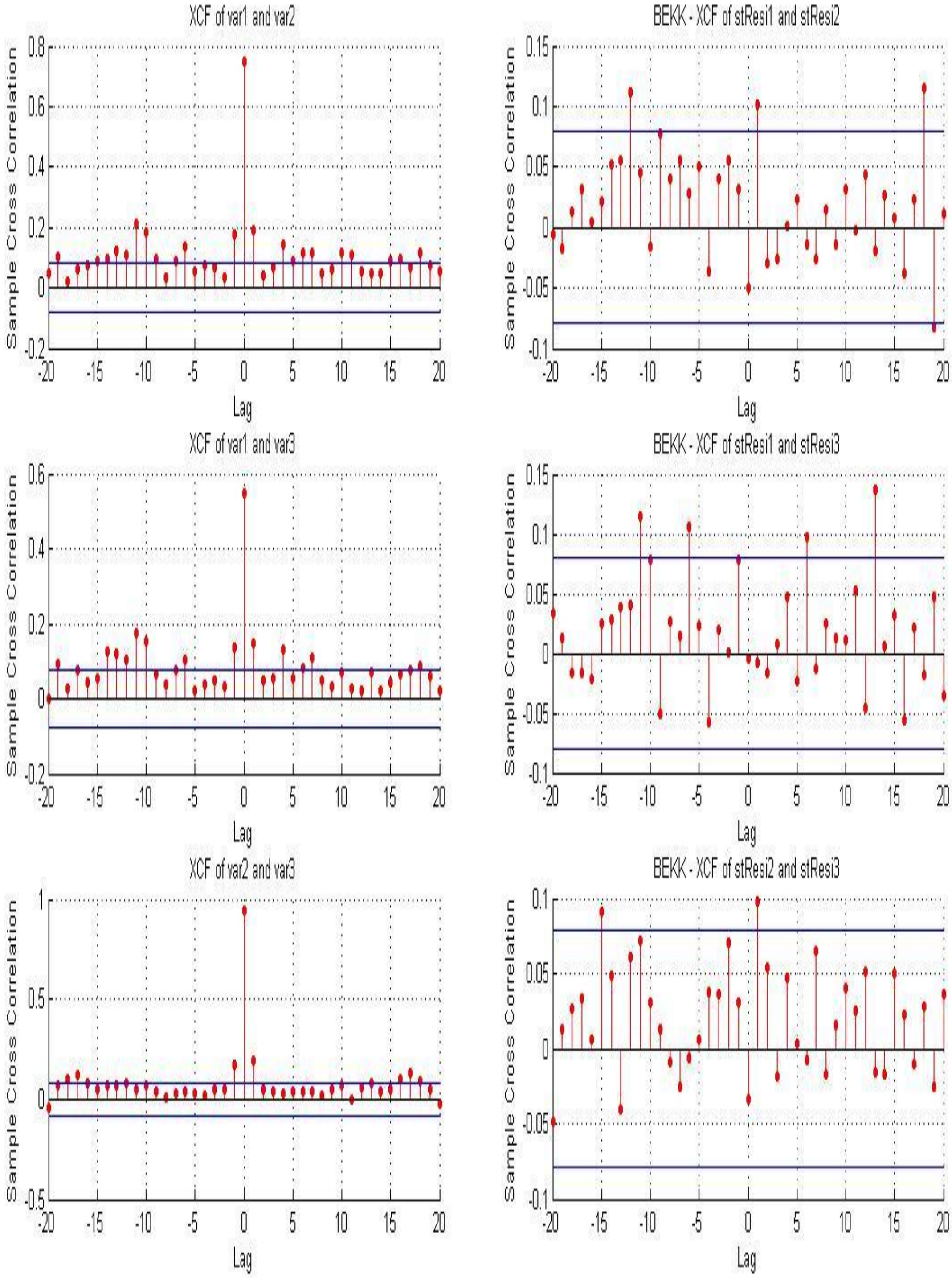
(a) (b)

Figure 3.5 ACFs of Premodel Data and Standardized Residual of the BEKK Model



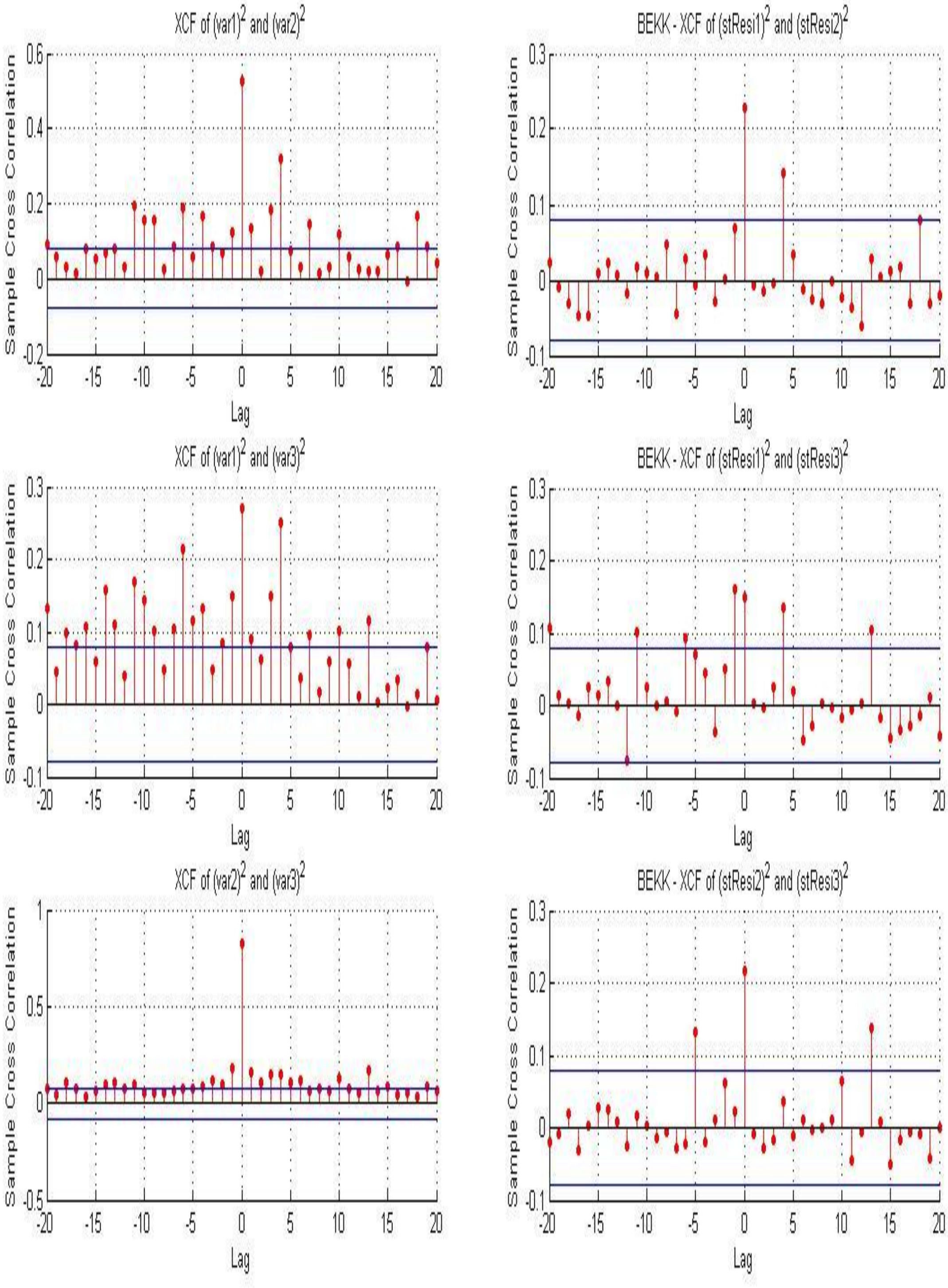
(a) (b)

Figure 3.6 ACFs of the Squared Premodel Data and the Squared Standardized Residual of the BEKK Model



(a) (b)

Figure 3.7 XCFs of Premodel Data and Standardized Residuals of the BEKK Model

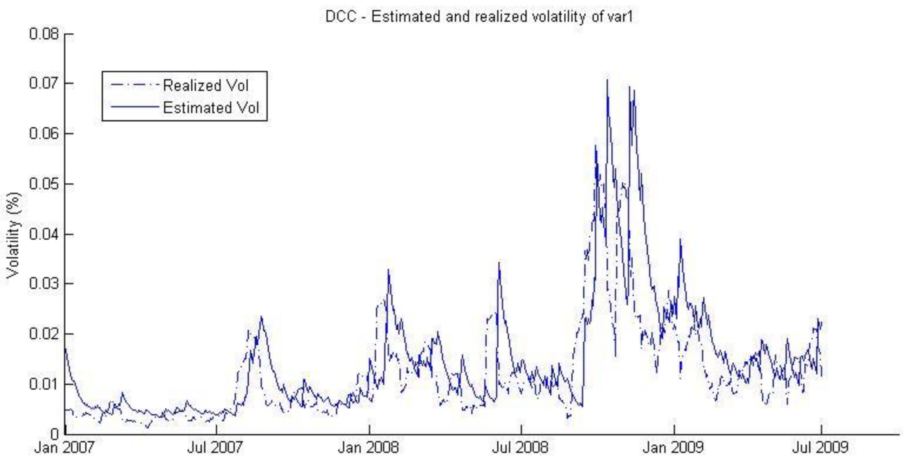


(a) (b)

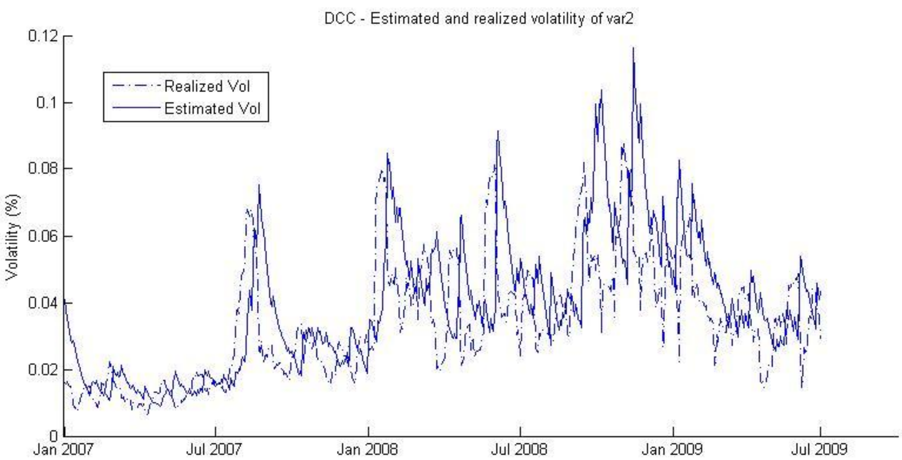
Figure 3.8 XCFs of the Squared Premodel Data and the Squared Standardized Residuals of the BEKK Model

3.3.2 Diagnostics of DCC models

Volatility clustering is also presented in Fig 3.9. The estimated volatility on the whole changes along with the realized volatility. Also, there exists a horizontal lag between these two lines for the same reason explained before. The fitting performance of the DCC model is shown in Fig 3.9 and Fig 3.10 not such satisfying as that of the BEKK model shown in Fig 3.2 and Fig 3.3.



(a)



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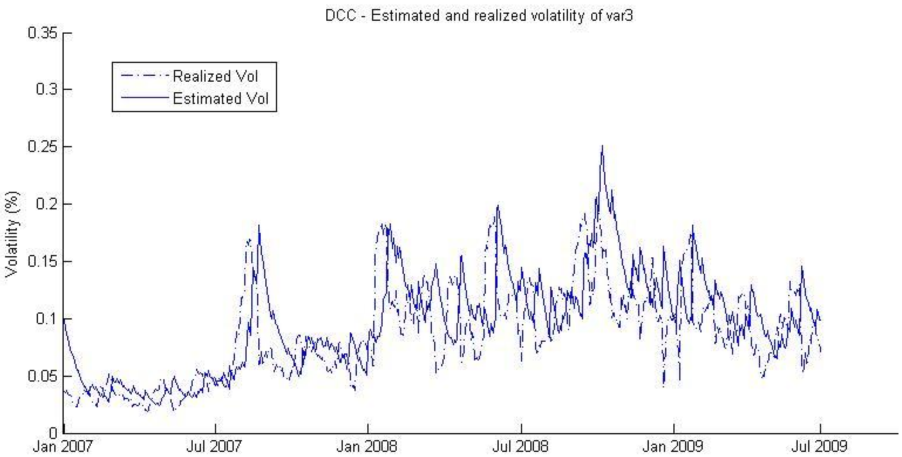
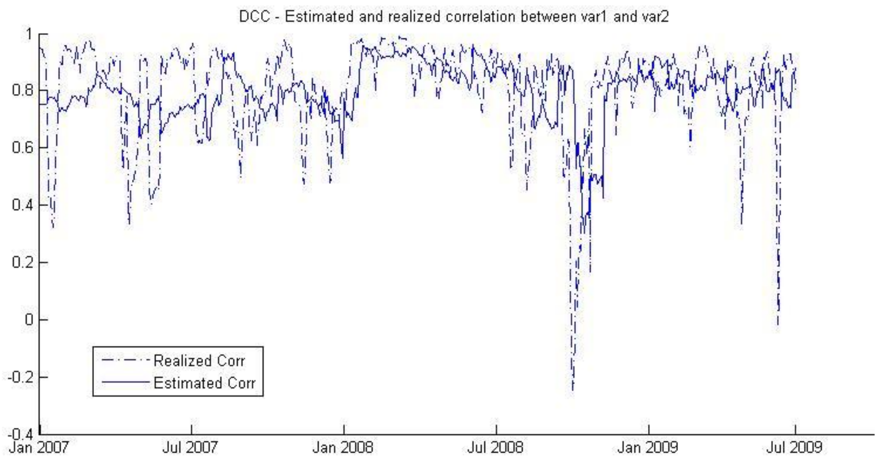
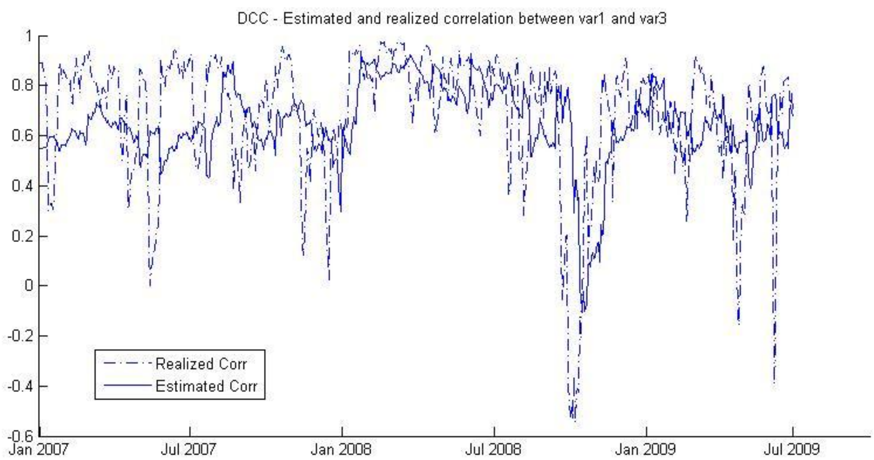


(c)

Figure 3.9 Estimated and Realized Volatility of the DCC Model



(a)



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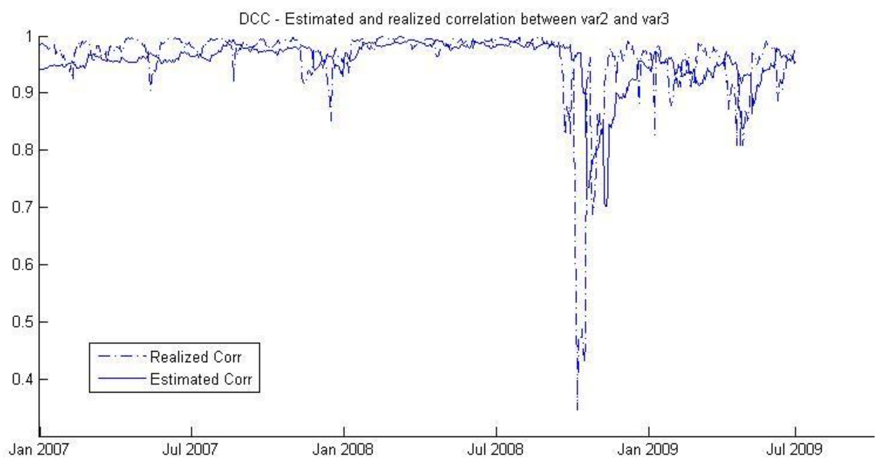


(c)

Figure 3.10 Estimated and Realized Correlation of the DCC Model

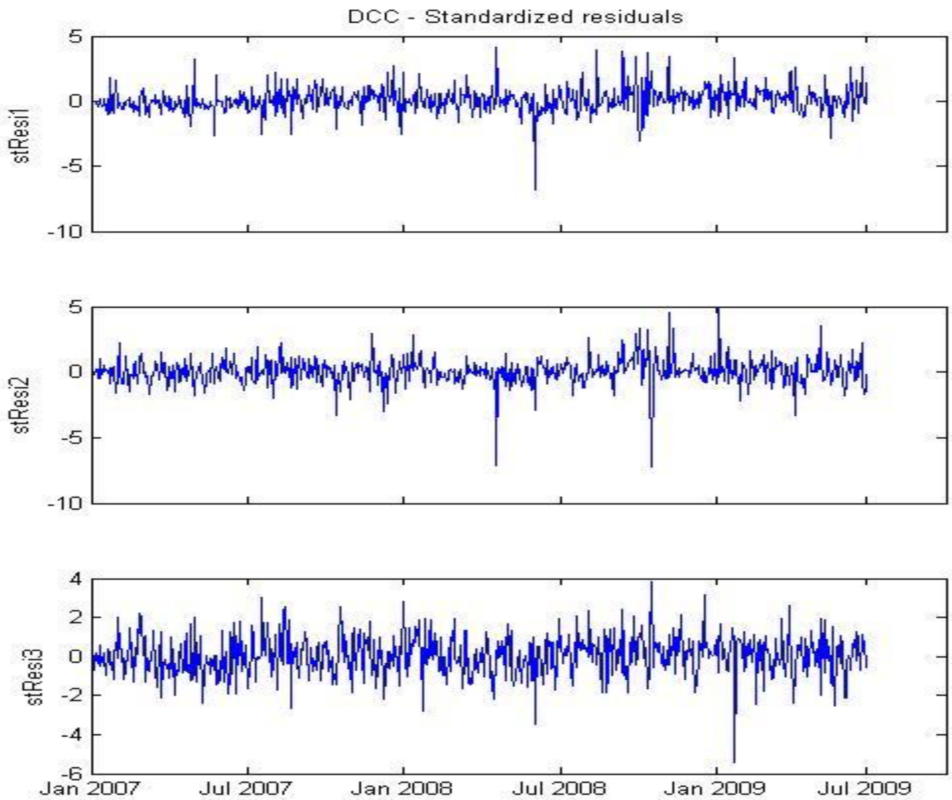


Figure 3.11 Standardized Residuals of the DCC Models

Table 3.8 shows the testing result of GARCH effect on the standardized residuals of the DCC model. In contrast with Table 3.3, we can also conclude that GARCH effect has eliminated quite a lot. The Ljung-Box test based on the autocorrelation plot tests the randomness at each distinct lag.

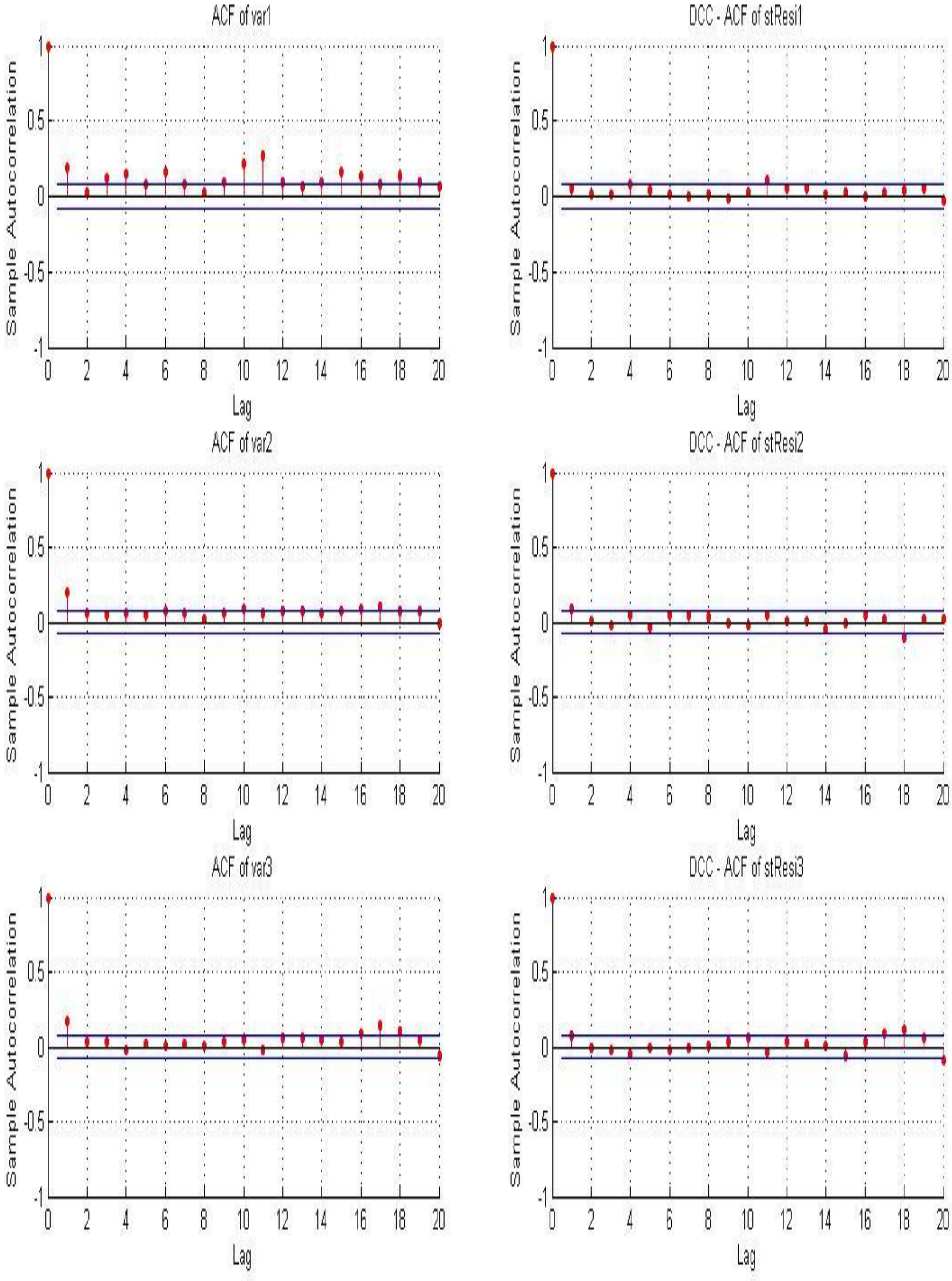
Table 3.8 GARCH Effect Testing of each Standardized Residuals (DCC)

	*var*1		*var*2		*var*3	
Lag	H	pValue	H	pValue	H	pValue
1	0	0.0500	0	0.4501	0	0.5339
2	0	0.1458	0	0.5963	0	0.6506
3	0	0.2669	0	0.1113	0	0.7538
4	0	0.3477	0	0.1851	0	0.8506
5	0	0.4898	0	0.2481	0	0.9222

Table 3.9 LBQ Test of each Standardized Residuals of the DCC Model

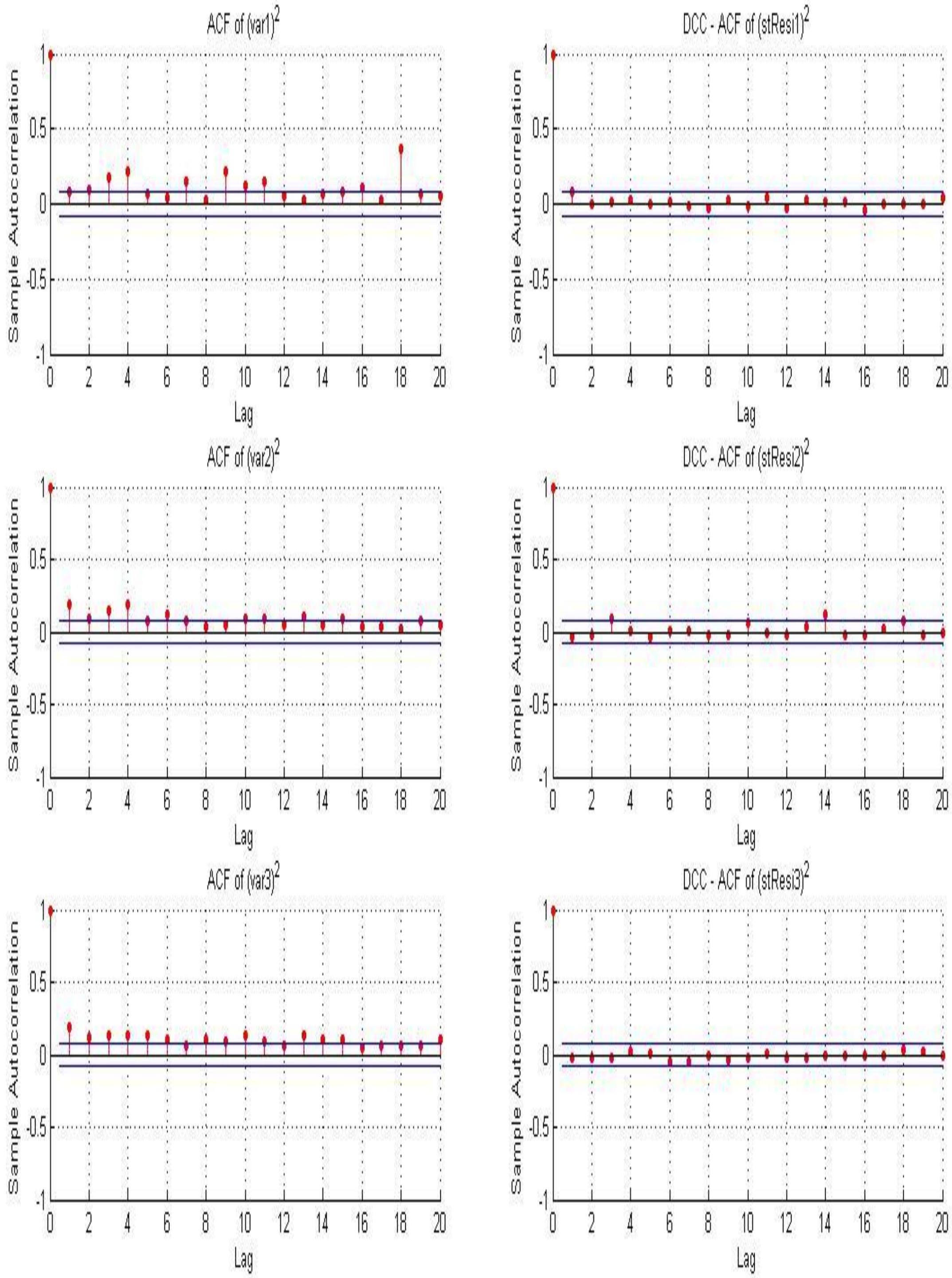
	*var*1		*var*2		*var*3	
Lag	H	pValue	H	pValue	H	pValue
1	0	0.1951	0	0.0292	0	0.0830
2	0	0.4152	0	0.0928	0	0.2226
3	0	0.5695	0	0.1779	0	0.3635
4	0	0.2261	0	0.1891	0	0.2889
5	0	0.2339	0	0.2262	0	0.4050

A comparison, see in fig. 3.12, between the ACF of the premodel data and the standardized residual indicates that GARCH effect has been erased much. For most of lags, the sample ACFs and XCFs are within the distance between positive and negative 2 times standard deviation lines at 95% confidence level. ACFs of the squared data before and after the modeling show that they are serially uncorrelated. The cross-correlations of the squared pre-model data and the squared standardized residuals of the DCC model also reflects that less GARCH effect exists in the squared standardized residuals after modeling.



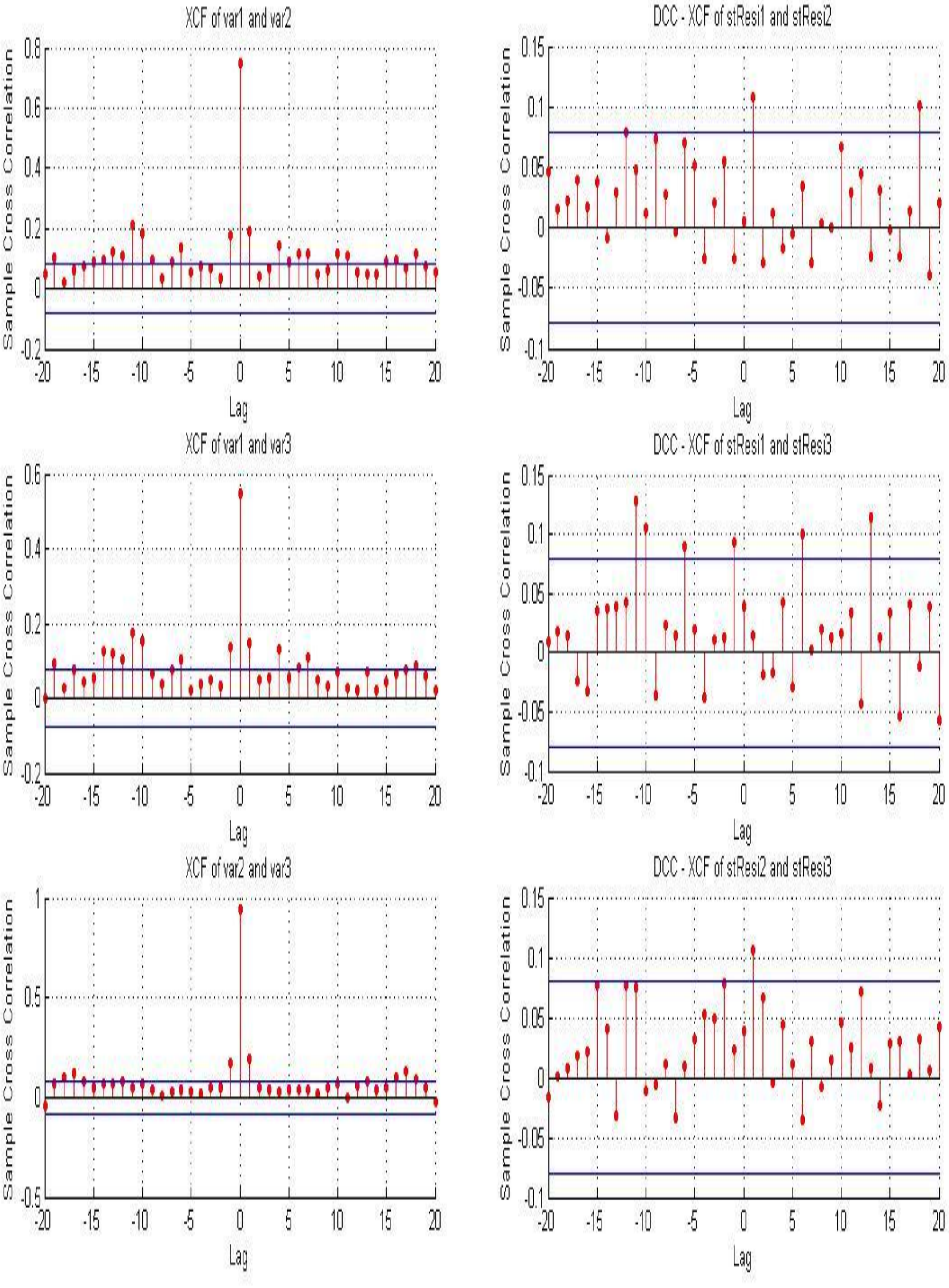
(a) (b)

Figure 3.12 ACFs of Premodel Data and Standardized Residual of the DCC Model



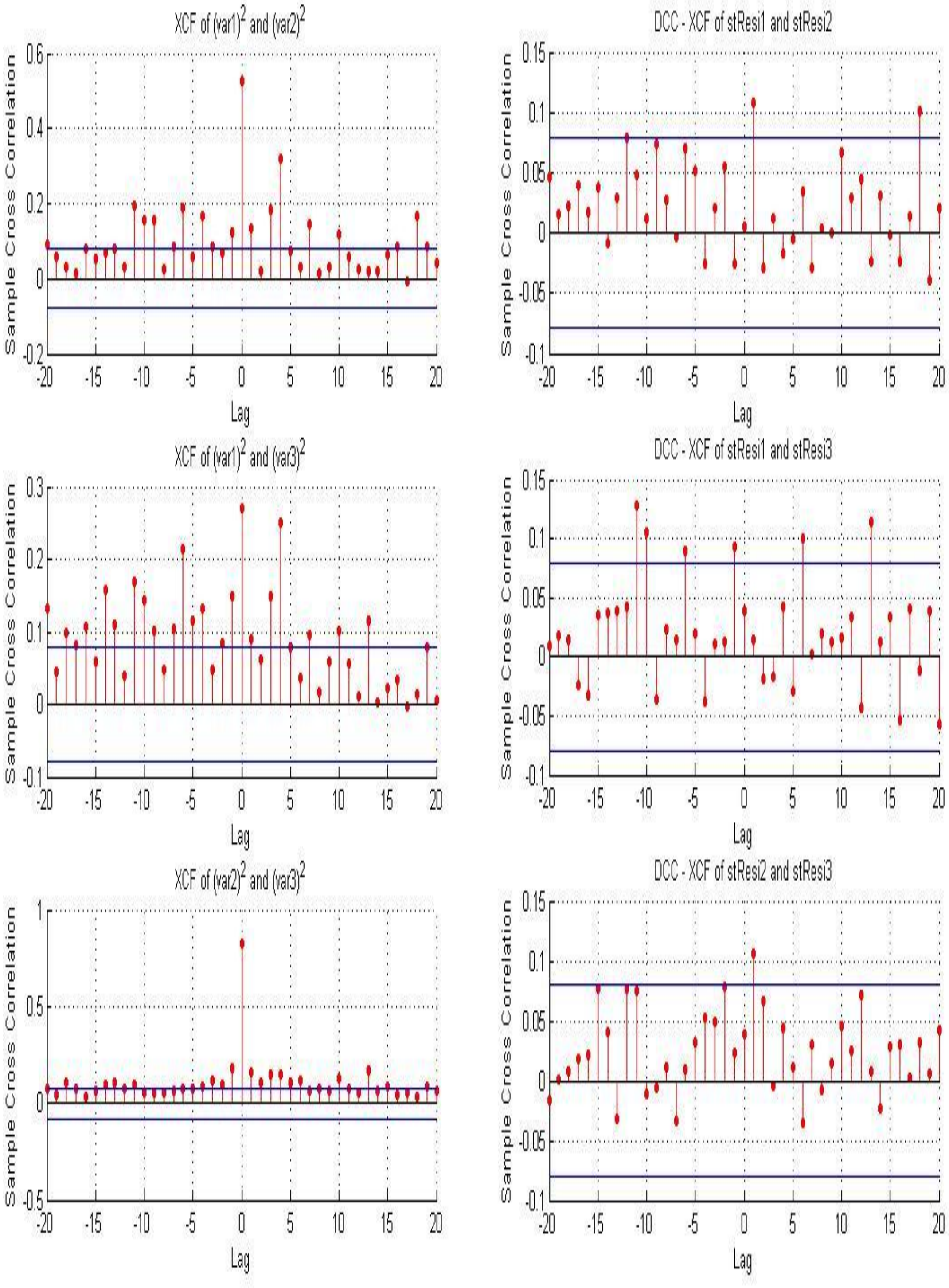
(a) (b)

Figure 3.13 ACFs of the Squared Premodel Data and the Squared Standardized Residual of the DCC model



(a) (b)

Figure 3.14 XCFs of Premodel Data and Standardized Residuals of the DCC Model



(a) (b)

Figure 3.15 XCFs of the Squared Premodel Data and the Squared Standardized Residuals of the DCC model

3.3.3 Comparison of BEKK and DCC models

The mean absolute error (MAE) [Engle, 2000] can measure how close the estimated variables are to the realized values. It is also called the mean average error. In our case MAE is calculated by

$$MAEvi = \frac{1}{n}\sum_{k=1}^{n}\left|\sigma_{ik} - \hat{\sigma}_{ik}\right| \tag{25}$$

for volatility where n is the total number of observations or

$$MAEij = \frac{1}{n}\sum_{k=1}^{n}\left|\rho_{ijk} - \hat{\rho}_{ijk}\right| \tag{26}$$

for correlation where i, j = 1, 2, 3.

Table 3.10 MAE in correlation and volatility of the BEKK model

Average error in correlation		Average error in volatility	
*MAE*12	0.1317	*MAEv*1	0.0056
*MAE*13	0.1990	*MAEv*2	0.0132
*MAE*23	0.0324	*MAEv*3	0.0306

Table 3.11 MAE in correlation and volatility of the DCC model

Average error in correlation		Average error in volatility	
*MAE*12	0.1354	*MAEv*1	0.0062
*MAE*13	0.2027	*MAEv*2	0.0142
*MAE*23	0.0352	*MAEv*3	0.0309

The values of the measure absolute error between these models suggest that the parameter estimation of the BEKK model is more accurate than that given by the DCC model even through the magnitude of the difference between their corresponding MAEs is not enough.

3.4 Forecasting

We split our sample into two parts, 2.5-year estimation period and the subsequent half-year forecast periods. The dynamic characteristics of the logarithmic daily returns of the zero-coupon bonds have been simulated during the estimation period by the multivariate GARCH models. After the parameters of the model are estimated, the determination of the prediction on the conditional covariance matrix H_{t+k} at time t+k can be attained.

As for BEKK-GARCH models, the iteration formula for the purpose of forecast is

$$H_{t+1} = C'C + A'\varepsilon_{t+1}\varepsilon'_{t+1}A + B'H_t B.$$

With regards to DCC-GARCH models, the iteration formula for the purpose of forecast is

$$H_{t+k} = D_{t+k} R_{t+k} D_{t+k}$$

where D_{t+k} and R_{t+k} can be computed separately and

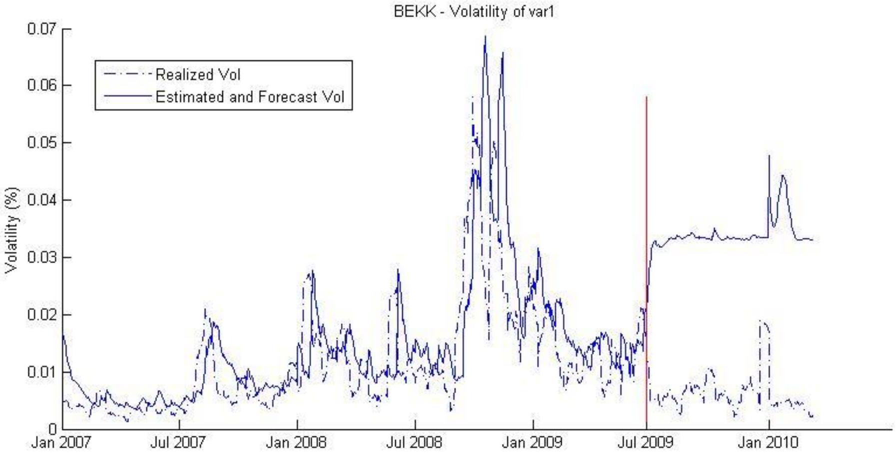
$D_{t+k} = diag(h_{11,t+k}^{1/2}, \cdots, h_{33,t+k}^{1/2})$, each $h_{ii,t+k}$ is a univariate GARCH model, and

$$R_{t+k} = diag(q_{11,t+k}^{1/2}, \cdots, q_{33,t+k}^{1/2}) Q_{t+k} diag(q_{11,t+k}^{1/2}, \cdots, q_{33,t+k}^{1/2}).$$

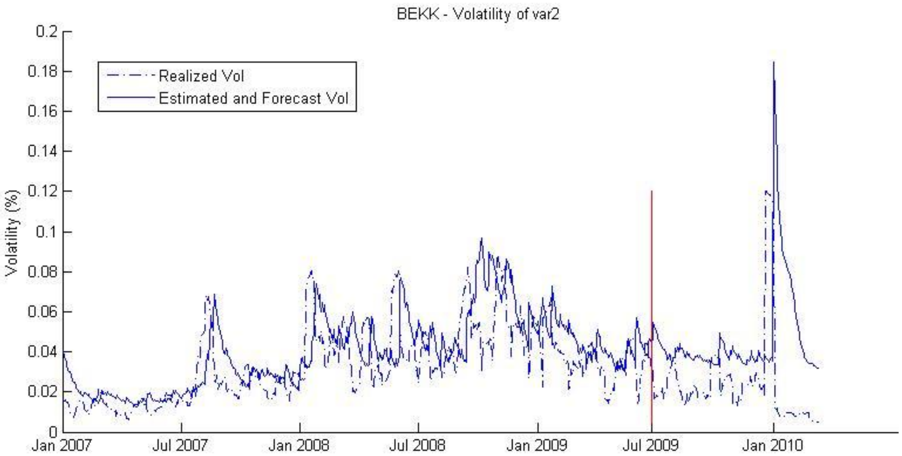
The matrix $Q_{t+k} = (q_{ij,t+k})$ is the 3×3 symmetric positive definite matrix updated by following:

$$Q_{t+k} = (1-\alpha-\beta)\overline{Q} + \alpha u_{t+k-1}u'_{t+k-1} + \beta Q_{t+k-1}.$$

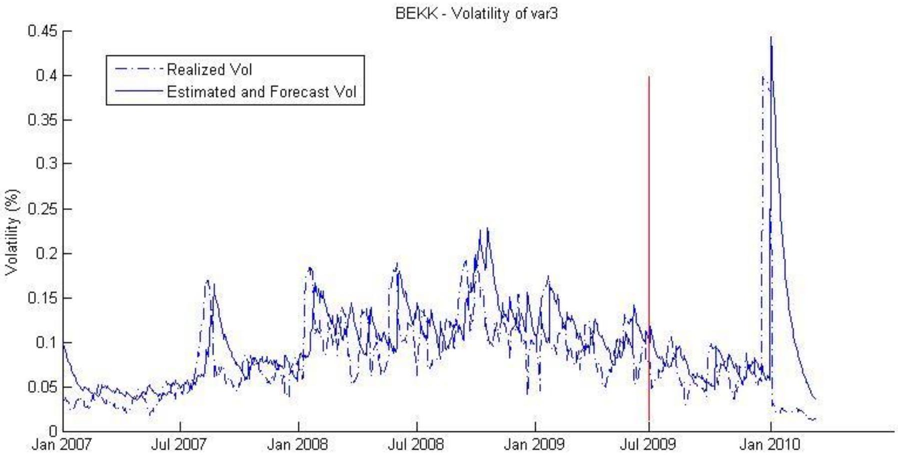
where $u_{i,t+k} = \varepsilon_{i,t+k} / \sqrt{h_{ii,t=k}}$.



(a)



(b)



(c)

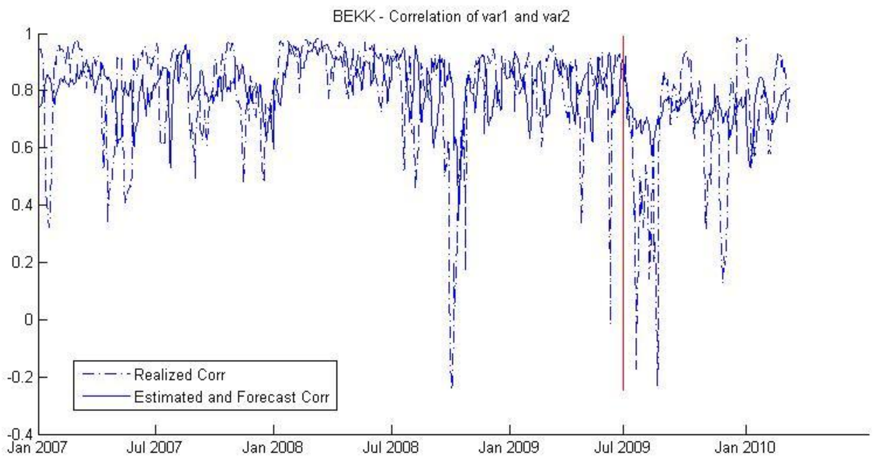
Figure 3.16 the Estimated and Forecasted Volatilities-BEKK

Fig. 3.16 shows the performance of the prediction is better as the maturity gets longer. Especially in the third plot (c), a horizontal lag is presented clearly. What is needed to pay attention is that a very sparse observation appears at the very beginning of year 2010. That is why a peak suddenly emerges. But the tendency is that the forecast volatility then goes back to its normal dynamics exponentially.

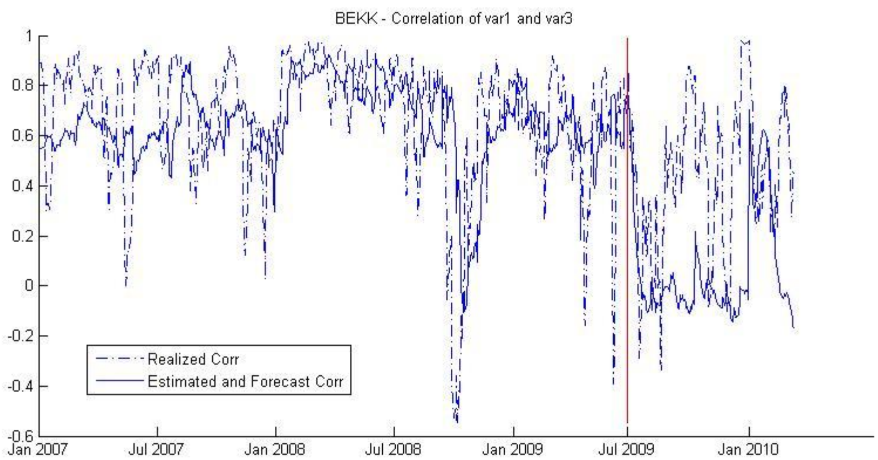
Fig 3.17 presents the poor performance of the BEKK-GARCH model on forecasting through the comparison of the realized correlations and the forecast correlations. On the left of the vertical line in Fig 3.17 presents the comparison between the realized correlation and the estimated correlation by the BEKK form. On the right side of the vertical line, it shows the poorer performance on forecasting the correlation among each pair of variables in the subsequent half year.

The forecasting performance of DCC-GARCH models looks better than that of the BEKK-GARCH model. The forecast volatility generally follows the dynamics of the realized volatility. For the same reason that there is a very sparse observation, a peak also appears in the following figure.

The forecasted correlations by the DCC form in Fig 3.19 also fit better with the realized ones.



(a)



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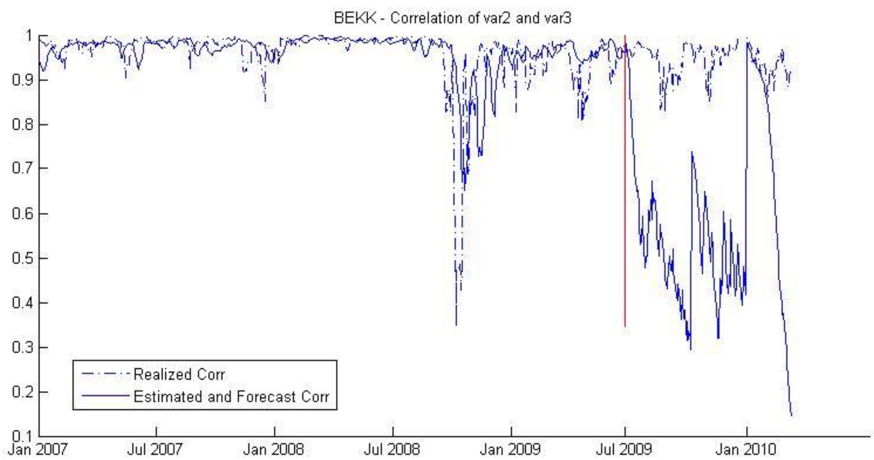
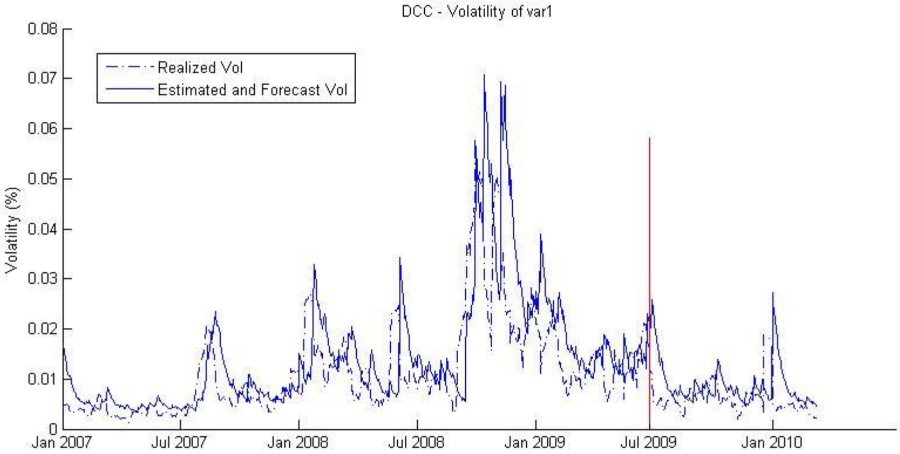
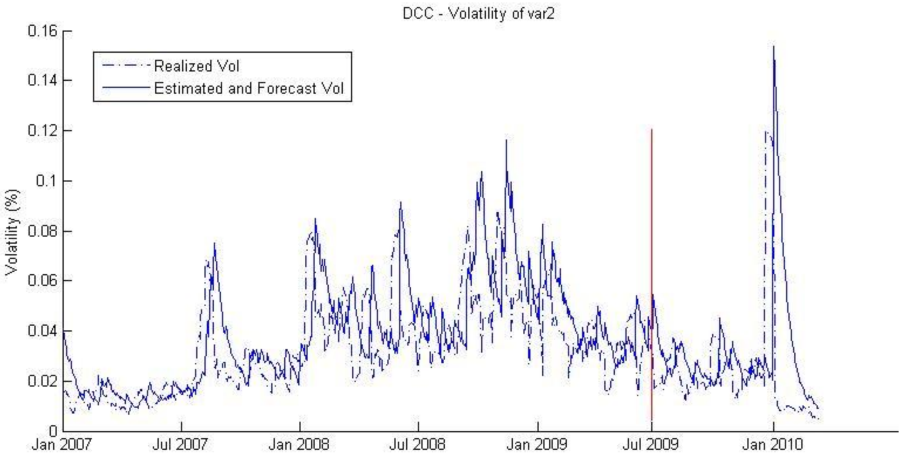


(c)

Figure 3.17 the Estimated and Forecasted Correlations-BEKK



(a)



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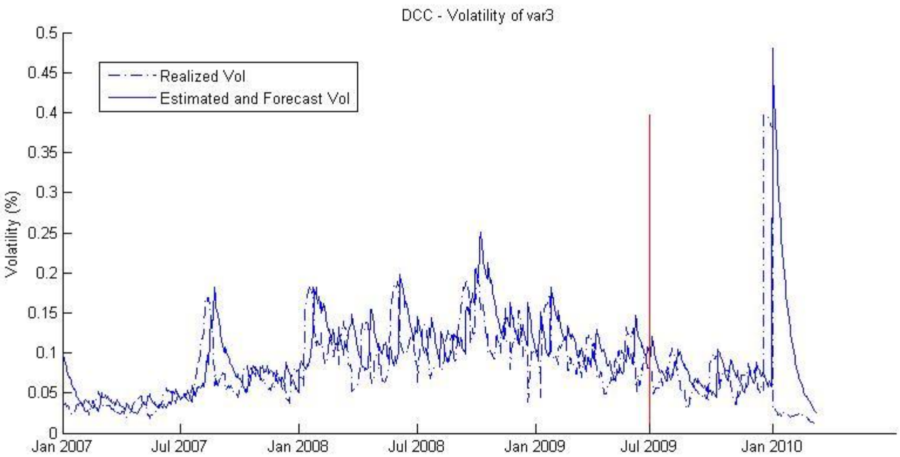
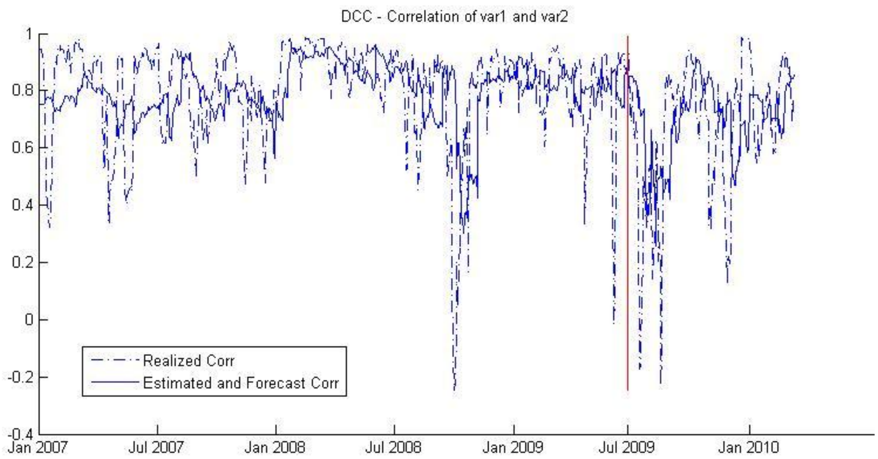
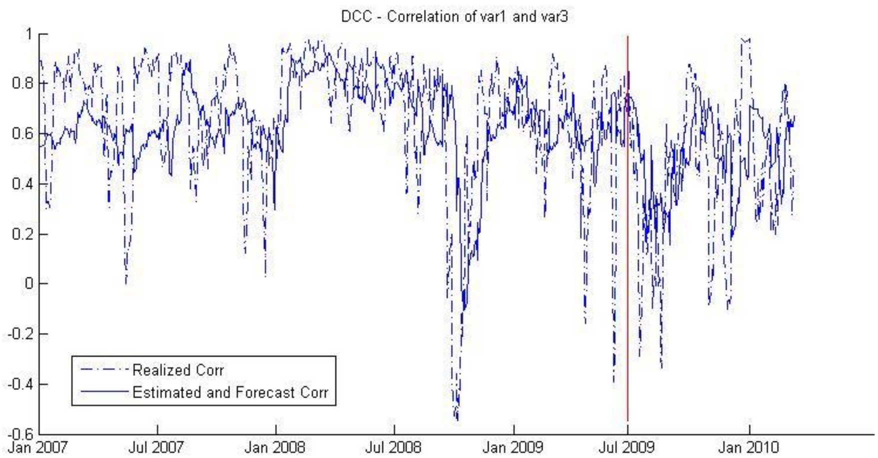


(c)

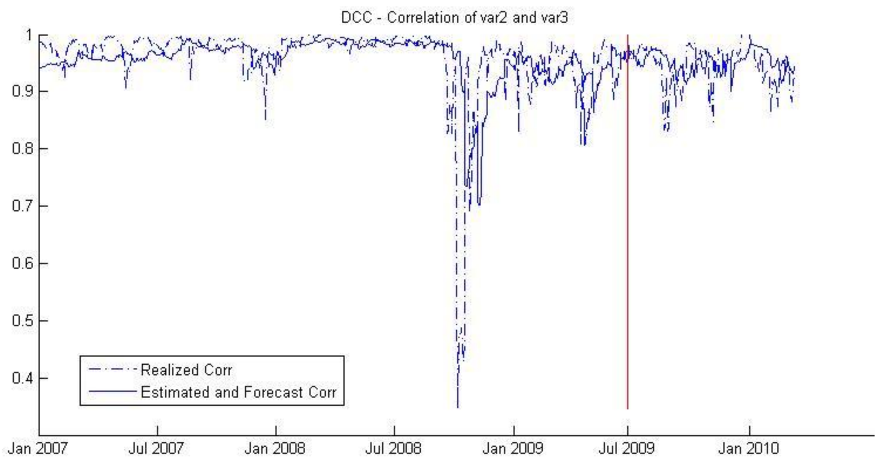
Figure 3.18 the Estimated and Forecasted Volatilities-DCC



(a)



(b)



(c)

Figure 3.19 the Estimated and Forecasted Correlations-DCC

The relatively better prediction performance of DCC-GARCH models can also be presented in the comparison of the estimated and forecast correlations. One of the reasons for this distinction is that the number of parameters estimated in the BEKK-GARCH models is more than that of DCC-GARCH models so that the summation of the error accumulated by each parameter of the BEKK-GARCH models tends to be larger than that of the DCC-GARCH models.

4. Conclusions and Future Work

This thesis focuses on the construction and the diagnostics of two formulations of multivariate GARCH models – the BEKK and DCC forms. The estimation process is fulfilled in the software package RATS 7.0 through the maximum likelihood method. After the parameters of these models are estimated, the forecast of the conditional covariance matrix is conducted by the iteration process. All our implementations are realized under the assumption that the residual terms are followed by a Gaussian distribution. Therefore, the diagnostics in evaluating the adequacy of modeling are operated by checking whether such assumption is credible enough.

By comparing the goodness of fit through the mean absolute error, we find that the fitting performance of the BEKK – GARCH form is better than DCC – GARCH form in our case. This difference may due to the number of parameters of the BEKK – GARCH model is comparatively more; so that BEKK – GARCH model has a better capability in explaining the information hidden in the history data. In the opposite, the DCC – GARCH model has an advantage over the BEKK – GARCH model in the area of forecasting as the DCC – GARCH model is more parsimonious than the BEKK – GARCH model. In this sense, it is crucially important to balance parsimony and flexibility when modeling multivariate GARCH models.

Regarding the diagnostic tests applied to multivariate GARCH models, our work is inadequate because of the fact that few tests are applicable to multivariate cases and also due to the difficulty in implementing those extended forms of the tests for detecting the univariate GARCH effect.

References

Alexander C. 2000. A primer on the orthogonal GARCH model. Unpublished manuscript. ISMA Centre, University of Reading, UK.

Andersen T., Bollerslev T., Diebold F.X. and Labys P. 2003. Modeling and forecasting realized volatility. *Econometrica* 71: 529-626.

Andersen T. and Bollerslev T. 1998. Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International Economic Review* 39: 885-905.

Annastiina S. and Timo T. 2008. Multivariate GARCH models. *SSE/EFI Working Paper Series in Economics and Finance No. 669.*

Baba Y., Engle R.F., Kraft D. and Kroner K. 1990. Multivariate simultaneous generalized ARCH, unpublished manuscript, University of California, San Diego.

Bauwens L., Laurent S., and Rombouts J.V.K. 2006. Multivariate GARCH models: A survey. *Journal of Applied Econometrics* 21: 79-109.

Bollerslev T. 1986. Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics* 31: 307-327.

Bollerslev T. 1990. Modeling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. *Review of Economics and Statistics* 72: 498-505.

Bollerslev T., Engle R.F., Wooldridge J.M. 1988. A capital asset pricing model with time varying covariances. *Journal of Political Economy* 96: 116-131.

Box G.E.P., Pierce D.A. 1970. Distribution of the autocorrelations in autoregressive moving average time series models. *Journal of American Statistical Association* 65: 1509-1526.

Broyden C.G. 1970. The convergence of a class of double-rank minimization algorithms. *Journal of the Institute of Mathematics and Its Applications* 6: 76-90.

Engle R.F. 1982. Autoregressive conditional heteroskedasticity with estimates of the variance of the United Kingdom inflation. *Econometrica* 50: 987-1007.

Engle R.F. 2002. Dynamic conditional correlation – A simple class of multivariate GARCH models. *Journal of Business and Economic Statistics* 20(3): 339-350.

Engle R., Kroner F.K. 1995. Multivariate simultaneous generalized ARCH. *Econometric*

Theory 11: 122-150.

Engle R.F., Ng V.K. 1993. Measuring and testing the impact of news on volatility. *Journal of Finance* 48(5): 1749-1778.

Estima, 2007a. *RATS User's Guide*. United States of America.

Estima, 2007b. *RATS Reference Manual.* United States of America.

Fletcher R. 1970. A new approach to variable metric algorithms. *Computer Journal* 13: 317- 322.

Glosten L., Jagannathan R. and Runkle D. 1993. Relationship between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance* 48: 1779-1801.

Goldfarb D. 1970. A family of variable metric updates derived by variational means. *Mathematics of Computation* 24: 23-26.

Hentschel, Ludger, 1995. All in the family nesting symmetric and asymmetric GARCH models. *Journal of Financial Economics* 39(1): 71-104.

Hosking J.R.M. 1980. The multivariate portmanteau statistic. *Journal of American Statistical Association* 75: 602-608.

Hull J. C. 2005. *Options, futures and other derivatives*. Prentice Hall: New York.

Nelson D.B. 1991. Conditional heteroskedasticity in asset returns: a new approach. *Econometrica* 59: 347-370.

Paul R.K. 2007. Autoregressive conditional heteroscedastic (ARCH) family of models for describing volatility. Seminar Paper. Indian Agricultural Statistics Research Institute.

Shanno D.F. 1970. Conditioning of quasi-newton methods for function minimization. *Mathematics of Computation* 24: 647-656.

Walter E. 2009. Applied econometric time series. *John Wiley & Sons, Inc*. The third edition.