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Export, Import, Productivity and Growth:

A theoretical and empirical study of an endogenous relationship

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Abstract

Numerous studies in the international economic literature suggest that foreign trade has a large positive effect on growth. From the theoretical aspects there are several reasons to believe in both export- and import-led productivity growth as well as productivity-led exports. The empirical results have been mixed, with earlier studies indicating a strong relationship and more recent studies that question the exogeneity assumption, finding endogenous results in several directions. In this study I develop a theoretical model, based on Aghion and Howitt's (1990) Schumpeterian framework, which explain important parts of the diverging results by showing that trade can affect the incentives and probabilities for innovations. I also investigate the relevance of the relationship between aggregated exports, imports and TFP in a Johansen approach for cointegration with error correction models and short-run Granger causality tests, for five OECD countries. I conclude that there is some weak support of long-run relations in all except two cases, and that the strong support for trade induced productivity enhancements cannot be found. The results are generally varying between countries and do not get more conclusive when studying the short-run effects. The heterogeneous results from this study therefore tend to question the previous assumption that trade, and especially export, positively affects productivity growth.

Keywords: Export, Import, Productivity, Growth, Cointegration, Johansen approach, ECM, OECD

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1 Introduction

A body of work in international economics suggests that foreign trade has a large positive effect on income[1] and the relationship between export and economic growth has been a subject of great interest in the growth literature. Many studies have emphasised a direct connection between the two and assumed the direction of causality from trade to income. This conclusion is also reached when import is included in general openness measures. However, developments in the econometric methods have enabled researches to question the results and the strong correlation, revealing an endogenous situation, with e.g. both export-led growth (ELG) and growth-led export (GLE). This contradicts the assumption of exports as an exogenous factor and points to the importance of advanced econometric frameworks as well as theoretical developments to be able to explain the contradictions.

Previously, most studies have focused on the relation between export and growth. However, some studies have shown the possible importance of including imports as an endogenous variable. This could otherwise cause an omitted variable problem. The results have supported the additional hypothesis of import-led growth (ILG). Also, other studies have found that the correlation between trade and income might come out of other domestic policy factors that generally have a positive effect on the economy (Rodrik 1994; Frankel - Romer 1999). Hence, the way in which trade affects growth needs to be properly specified not to capture these effects in a too extensive way.

There is a theoretical debate whether trade primarily has an effect on economic growth through an increase in the factor accumulation, as hypostasised by the neoclassical growth theories, or if the effect is an improvement in productivity. There are strong reasons put forth in the endogenous growth theories to assume that trade should have an effect on productivity. Hence, investigating the productivity trade relationship could improve our understanding of how the relationships work. Studies focusing on labour productivity have found an endogenous relation, primarily between export and productivity. However, examples of recent literature that also includes imports in a VAR framework is limited on a macro level, while firm

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[1] See section 2 for previous empirical findings and section 3 and 4 for theoretical arguments.

based studies have found reasons to believe both in a trade caused increase in productivity and productivity-led trade effects.

In this study I formulate an endogenous theoretical framework that incorporates important parts of this complex dynamics of both export and import, and use the Johansen approach of cointegration and vector error correction models, with estimates of the total factor productivity (TFP) level, to empirically investigate the trade-growth relationship for productivity.

1.1 Statement of Purpose

The main purpose of this thesis is to investigate the endogenous relation between exports, imports, productivity and economic growth on a macroeconomic level. The purpose can be further divided into two main aims: (1) to derive an endogenous theoretical growth model, based on Aghion and Howitt (2009), in order to explain the plausible relationships between trade and both GDP as well as productivity growth; and (2) to empirically study the key elements of the theoretical hypothesises through econometric cointegration analysis using the Johansen approach and Vector autoregressions along with Vector error correction models, in order to investigate long-run equilibrium relations and short-run Granger causality.

The questions of issue are:

- How can the relationship between trade, productivity and economic growth theoretically be explained?
- Are there empirical long-run relationships between export, import and productivity?
- If so, does export and/or import positivity affect long-run productivity development?
- or, are there reasons to believe in only short-run effects?
- and, can the relationship be better explained by reversed causality?

By causality I refer to Granger causality, which should not be interpreted as proof of actual causality relations. The empirical study includes five industrialised OECD countries: Canada, France, Japan, the United Kingdom and the United States. For statistical estimations E-views 5 is used.

1.2 Disposition

The structure of the thesis is as follows. In section 2 there is a discussion of the results from previous empirical research. In section 3 is the theoretical foundation formed by discussing the implication on previous theoretical contributions. Section 4 presents the extended theoretical model. Section 5 explains the selection of variables made for the empirical study. Section 6 includes a description of the econometric methodological framework. In section 7 the choice of data material is described and discussed. In section 8 the empirical results from the different tests are presented, both for long- and short-run relations. Finally, section 9 contains the conclusions. For further information about the mathematical derivation of the theoretical model and the results from the empirical study, additional equations and tables are presented in the appendixes.

2 Previous Empirical Research

In this section a description of empirical studies concerning the trade-growth relationship is presented. The content, conclusions and methods of the studies are discussed and constitute a basis for the implications used in both the theoretical model and the empirical study.

2.1 Export and Growth

2.1.1 Cross-Country and Simple Times Series Studies

There are numerous examples of studies that have used regular OLS cross-country procedures to estimate the effects from export. In a survey, studying a great number of reported estimated effects on growth, Lewer and Van den Berg (2003) state that almost all studies of the quantitative relationship between trade and growth specify trade as real export growth and estimates this on GDP growth. One example of this is Balassa (1978) who focus on developing countries and is using pooled estimation of GDP growth and exports. He finds that export contributes more to growth than domestic and foreign capital or labour. Balassa shows this by including exports as an additional factor in the Cobb-Douglas production function. This approach was later used by e.g. Taylor (1981), Kavoussi (1984) and Ram (1985). Taylor (1981), Kavoussi (1984) found that the effect from trade were higher for developed than developing countries.

This approach is criticized by Sheehey (1990) who rather suggests that exports only represent a production factor when is included in addition to capital and labour, and not necessarily represents the effect of trade. Hence, spurious regressions are made if the model is not adjusted for the fact that exports is a component of GDP and could affects all factor inputs in a similar way.

Most of the cross-country studies find positive and significant relationships between export and growth. Lewer and Van den Berg (2003) conclude that there are strong positive quantitative effects from trade and that they are robust to different specifications and estimation types. Their survey does however exclude studies with a dynamic time series structure and endogenous variables, and focuses rather on those that use OLS cross-country or time series to estimate the magnitude of the effects. But, taking unit roots into account seem to lower the estimated effects from trade.

Studies using OLS and cross-sectional data might be problematic due to some important assumptions. First, the order of causality is not investigated. Rather it is assumed that export affects growth and that growth does not affect any of the explanatory variables, i.e. the variables are exogenous. Second, cross-country studies assume that the regression parameters are constant for all countries. Those are, as we will see, both questionable assumptions. The second problem could easily be circumventing by using time series of individual countries, but the first one need a more dynamic estimations procedure. Therefore, also studies using single time series equations, assuming all variables to be exogenous, might not be appropriate. Kwan et al. (1996) use this method for Taiwan and find a positive relationship between GDP growth and export growth. However, they also conclude that exogeneity tests show rejection of strong exogeneity. Hence, cross-country models like those formed in line with Balassa (1978) could be problematic.

In the light of the problematic assumption of early cross-sectional and time series studies the need for more complex estimations emerged. So, in order to establish if there is export-led growth, or just simultaneous correlation, recent studies use more dynamic approaches with cointegration and Granger causality.

2.1.2 Studies of the Dynamic and Endogenous Structure

As mentioned, more recent studies have assumed that growth and trade variables are endogenous[2]. If they are, then the strong exogeneity assumption can be rejected by a Granger causality test. As we will see there are profound reasons to assume the variable of interest not to be strongly exogenous.

In general, the empirical evidence from dynamic studies is mixed. With some giving support to the long-run relationship between export and growth, other rejects the hypothesis. Also, there has been mixed results of the order of causality, as some studies rather support the opposite direction: growth-led export.

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[2] Endogeneity in this case refers to the reversed causality problem, i.e. the possibility that the dependent variable (y) impact the independent variable (x) at the same time as x has an impact on y.

In a study on export-led growth with Granger causality tests, Sharma et al (1991) looks at five industrialized countries in a four-variable vector autoregressive model (VAR) with quarterly data (from 1960.q1 to 1987.q2) and finds that no countries showed similar results. Germany and Japan seemed to experience export-led growth while UK and US showed reverse causality. Also, no relation between growth and factor inputs (capital and labour) were found. But, export did cause both variables to grow. Awokuse (2003) finds the long-run and short-run export-led growth hypothesis to be valid in a similar case study of Canada, from year 1960 to 2000, using a Vector error correction model (VECM)[3] approach. On the contrary, Richards (2001) finds no evidence of export-led growth in Paraguay. Rather, he finds that domestic growth might drain recourses from the exporting sector, hence, lower the growth rate. Oxley (1993:165) has studied Portugal for a long period of time (1865-1985) in an error correction model with real GDP level and real export level, but did not find any support for export-led growth. Instead the Granger causality test showed the reverse causality: growth-led export.

2.2 Import and Growth

As a critique to the single focus of export performance as the trade variable of interest, Esfahani (1991) uses three-stage cross-country equation systems to show the importance of including import. Investigating 31 semi-industrialized countries he finds that exports primary contribution is to finance the import of intermediate products.

In the search for export-led growth Riezman et al. (1996) shows that not adjusting for import growth can give misleading Granger causality test results, as the causal ordering might be spurious. Given this, they only find support for export-led growth in 30 of 126 investigated countries and 25 countries that experience growth-led export. Their analysis does not however directly include the import-led growth hypothesis.

Also, Awokuse (2008) argues that there has been little attention paid to the importance of imports, but that some recent articles have shown that without controlling for import the causality between export and growth might be spurious. In an attempt to study this relation he uses quarterly data for three Latin-American countries from the beginning of the 1990’s to April 2002. The data consists of real GDP growth, real exports, real imports, gross capital formation and labour force. Using a vector error correction approach Awokuse shows that in

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[3] See section 6.2.3 and 6.2.4 for a technical explanation of VAR and VECM.

one case the error correction terms are significant for export and import, signalling a long-run causation for trade to GDP growth. Also, short term Granger effects from imports to growth are found for all countries, while exports only are significant in one case. He therefore draws the conclusion that imports seems to be more important than the exports in explaining economic growth. He states that:

> The exclusion of imports and singular focus of many past studies on just the role of export as the engine of growth may be misleading or at best incomplete (Awokuse 2008:172).

In a similar article Awokuse (2007) studies the situation for Bulgaria, Czech Republic and Poland. He then finds a bi-directional short term export- growth relationship for Bulgaria, export- and import-led growth in Czech Republic and that import-led growth are present in Poland.

Also, in a re-examination of Oxley (1993), Ramos (2001) uses a three variable VECM-model and shows that there might be two long-term equilibriums, where real GDP, export and import are included. Granger causality test also shows a tow-way relationship both between GDP-export growth and GDP-import growth.

In a study of 39 developing countries Krishna et al. (1998:16) point out that both export and import need to be included in the VAR, in order to best explain output growth, and that there are strong reasons also to include investments. The study also finds cointegrating relationships but they are not of significance for the conclusions.

2.3 Trade and Productivity

Difference between countries growth rates can only partly be explained by increases in the employment of the basic factors of production: capital and labour. Instead the differences are mainly due to different rates of increase in productivity of the inputs. And it is possible that long-run income growth rather might be caused by technological progress (Aghion – Howitt 2009:112). Therefore Kunst & Marin (1989) argue that focus should be on productivity rather than income. In their study of labour productivity in Austria, with the use of Granger causality test, they find support for productivity-led export growth.

Just as in the case for GDP studies the studies with productivity mainly includes only export as an explanatory trade variable (Thangavelu - Rajaguru 2004:1084). For example, Awokuse

(2006) finds a bi-directional short term Granger relationship between labour productivity and real export for Japan. He also concludes that terms-of-trade and capital should be a part of the Japanese model. Yamada (1998) also uses labour productivity instead of GDP, concluding that there is only one case (Italy) where export-led growth can be found when looking at six OECD countries[4]. Another study of industrialized countries[5], for the period 1960.q1 to 1987.q2, was made by Marin (1992) who also uses labour productivity, but for the manufacturing sector. By looking at cointegration she finds a long-run relationship between the four variables: productivity, export, terms-of-trade and OECD output, for all countries except the United Kingdom. She also finds Granger causality from export to productivity in all four countries, but with mixed signs of the coefficient sums, and from productivity to export in Japan.

The number of studies using both productivity and import in a dynamic setting is scares. One is a study done by Thangavelu and Rajaguru (2004) where they focus on nine Asian countries[6] using annual data from 1960 to 1996. In this study the authors use labour productivity in the manufacturing sector as the productivity measure and the levels of real total export and real total import. The analysis is done with an error correction model and Granger causality. The results of the study are mixed but support the notion of the importance of import-led growth. A long-run bi-directional import-productivity relationship was found in India, Indonesia, Malaysia and Taiwan. For the Philippines and Singapore import-led productivity growth is exhibited. Also support for export-led productivity growth was found. However, no relationships were found for Hong Kong and only productivity-led export for Japan. This stands in contrast to the results found by Marin (1992) and Awokuse (2006).

2.4 Firm Based Studies on the Productivity Effect from Trade

Besides the study by Thangavelu and Rajaguru (2004) the main examples of imports impact on productivity is based on firm level studies. In a study by Halpnern et al. (2006), based on unique firm data, they study the effect from imported intermediate goods on Hungarian firms. They find support for a large positive effect both in respect to gain from complementarities

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[4] The United States, Canada, the United Kingdom, Italy, France and Japan

[5] The United States, the United Kingdom, Germany and Japan

[6] Hong Kong, India, Indonesia, Japan, Malaysia, Philippines, Singapore, Taiwan and Thailand

and from new varieties. Kasahara and Lapham (2008) show that, in a static exit and entry setting of Chilean firms, the productivity level was higher in plants that were both participating in export and import, followed by those that were just involved in one activity and even lower in firms that were only focused on the domestic market. The studies in the area which includes imports seem to be rather new, with many papers written in the last ten years. Lately Vogel and Wagner (2010) reported results for German firms pointing to similar conclusions as Kasahara and Lapham (2008). However, their results were only in favour for a self-selection process, and not learning-by importing. If this was a general process it would point to productive-led import growth rather than import-led productivity growth, hence, adding to the possible complexity of the problem.

These examples underline the firm level foundation for including import as an important variable also for studies on the macro level.

2.5 Implications from Previous Research

From the discussion in this section it is clear that the empirical search for distinct connections between trade and growth needs to be continued. The diverging results from the dynamic models, finding bi-directional causality in relation to both income and productivity, express the need for both theory and empirics to take endogeneity into account. Therefore I conclude that a VAR framework, used in later macroeconomic studies, is an appropriate method. It is also evident that imports need to be considered as a possible explanatory factor, as it is found to have both long-run and short-run effects on income as well as productivity. This study therefore is an attempt to contribute to the previously few studies testing both trade variables in relation to productivity, with an adjustment for the possible simultaneous policy effect on capital accumulation.

3 Theoretical Aspects

This section discusses the theoretical arguments that have been emphasised in the literature concerning the relationship between economic growth, productivity, export and import. It is intended to give an understanding for the foundations of the derived theoretical model and the following empirical analysis.

3.1 Two Ways of Thinking about the Effects from Trade

The subject of whether trade affects growth has been in focus for a large theoretical debate. On neoclassical grounds trade openness primarily, or only, affects growth through increasing investments and the aggregate capital stock. The argument is developed in a Heckscher-Ohlin context and states that trade affects the capital/labour ratio, so that firms export the production factors that they are relatively abounded in. However, this does not fully correspond to empirical findings and neglect the importance of intra-industry trade. None the less, it does predict that nations should increase their growth by promoting investments, which is seen in cross-country studies - e.g. Levine and Renelt (1992) - according to Baldwin and Seghezza (1996).

In contrast, the new trade and growth theory regard market imperfections and endogenous productivity growth as the main source of growth. In several of these models, innovation and the entrepreneur are of great significance and investments contribute to growth as a mean to enhance the productivity (see: Aghion - Howitt 2009). New theories also incorporate the important results from industrial organization studies about the need of imperfect market to generate growth. As seen previously, recent empirical findings give support to the assumption that trade benefits growth through other means then investments, e.g. by increasing the productivity.

3.2 Export and Growth

A large number of studies have focused on the relation between increased exports and countries economic growth. Generally, it has been assumed that export brings new opportunities and positive effects that cause an increase in the growth rate. Even though, this view does not stand uncriticised, the causal export–led growth hypothesis is theoretically plausible for a number of reasons.

To begin with, export as a response to increasing demand for a country's output is expected to affect growth as a component of aggregated output (Giles - Williams 2000:263). Leaving out a possible effect on productivity, this implies that export enables growth by increasing the relevant market size. This scale effect from opening up a country is, theoretically, a way to increase the country size or the relevant population. In addition, Kremer (1993) argues that historical evidence also shows a consistency between population or labour size and technological change, which is emphasised in theories by Grossman and Helpman (1991b,c) as a increase in the relevant labour force that could positively affects the rate of technological change. Accordingly, scale effects can raise the profitability of the intermediate sector, leading to a higher production and thereby higher growth rate, as is modelled by Barro and Sala-I-Martin (1997) and Connolly (2000). Jones (1995) criticise this on empirical grounds, arguing that there is no proportional relation between the number of researcher and the growth rate. However, he shows the importance of general population growth for the same purpose. Therefore, the importance of a larger general population cannot be fully excluded.

Second, increased production enables better utilization of economies of scale, as well as increasing specialisation (Helpman - Krugman 1985). As a result recourses may transfer from inefficient domestic production to the export sector, causing a productivity increase (Giles - Williams 2000:263).

Third, an increased orientation towards the international markets gives access to foreign technology. It follows that firms might be able to increase their efficiency through the process learning-by-exporting, including both a scale effect from leaning-by-doing and a diffusion effect from e.g. learned management practices (Giles - Williams 2000:263). The effect of technology transfer is shown by Grossman and Helpman (1991b) trough innovation and imitation, where countries using both strategies can experience a positive growth, if the follower is not too far behind technologically, so that innovations can spread between countries. However, research by Clerides et al. (1996) and Bernard (2004) find little evidence of technological diffusion from exporting activities on domestic firms.

Forth, exports may also affect growth through the foreign exchange constraint, and enables greater import flows (Cetintas - Barisik 2009:637) and eliminate controls that result in an overvaluation of the domestic currency (Giles - Williams 2000:263). The increased import, of e.g. intermediate goods, then raises either the capital formation or stimulates productivity growth.

Fifth, export competition with foreign firms could increase the need for domestic innovations and effectiveness, hence lead to increased productivity (Thangavelu - Rajaguru 2004).

Finally, increased international access can provide greater opportunities for entrepreneurial activity as the rewards increases, which in turns increases the rate of innovation and the productivity. It is argued that this is the key to extended growth (Giles – Williams, 2000:263), because there is a need for risk taking in productivity increasing activities. Guerzoni (2010) models the importance of increased demand for the innovation incentives, and concludes that an increased market size will increase the number of innovations, as it improves the maximal profits. He also states that this result is in line with empirical findings. Jones (1995) critique might therefore be valid if one uses labour as an input in the research process but not as much for the argument of increased demand.

It is not concluded if these opportunities imply an export-led growth or if exporters self select into export markets as they become more productive. Several studies have found that efficient firm’s rather self select into the export markets than benefit from foreign knowledge (Thangavelu - Rajaguru 2004). Bernard (2004) point out that this self-selection process also involve reallocation of recourses from less productive to more productive firms. As a result causality from productivity to export could be expected.

This causal relation from export growth to economic growth is denominated as growth-led exports. The outcome is explained by the assumption that productive firms export more, e.g. as a result of lower domestic costs (Thangavelu - Rajaguru 2004), and a general increase in productivity should therefore precede increased exports. The productivity increase can be caused by domestic labour productivity increases or diffusion of foreign technology (Awokuse 2007:390), and not necessarily from increased trade. However, an increased productivity from other aspects of trade than export (e.g. imports) could improve the domestic productivity and cause a growth-led export expansion.

3.3 Imports and Growth

In addition to export-led growth and growth-led export there might be optional gains from trade through imports: import-led growth. Thangavelu and Rajaguru (2004:1084) argue that the theoretical reasons are in favour of possible import- rather the export-led growth.

As mentioned in the last section, increased imports, financed by exports, provides needed factors of production. The imported (intermediate) goods contribute both to the production in the domestic and export (Awokuse 2008:162), leading to both increased growth and export.

New growth models have emphasised the role of imports as a channel for foreign technology and knowledge to enter the domestic economy. For example, Coe and Helpman (1995) demonstrate that foreign R&D investments, as well as domestic, are important for growth, and that the process through which this takes place is either through direct (technological transfers) or indirect (import of productive inputs) effects on the productivity level. They also points to the relation between research expenditure and innovations as the way to increase the productivity of traded and non-traded goods.

In the case a country opens up to trade, it can benefit from the existing range of (intermediate) products from abroad in two ways: trough more varieties (complemetarity mechanism) or products of better quality (Seater 2005). The importance of intermediate inputs is showed by Grossman and Helpman (1991c:50), as innovation improves intermediates quality continuously the aggregated productivity in final production increases. In Rivera-Batiz and Romer (1991), an increased variety of inputs has a level effect directly after opening up to trade, if the prerequisite for the research sector is not affected by foreign intermediates, in their case by access to world knowledge. But, given that inventors can use the whole scope of existing knowledge there will be an increased growth rate, based upon scale effects of knowledge and incentives for innovation, from trade in goods. Their model displays the effects of free spillover, but omits some form of randomness and that also adaptation might requires a cost. Grossman and Helpman (1991a) show that the inclusion of capital accumulation in these types of innovation driven models does not alter the results of the research process.

An increased import of intermediate products might be used for explaining technological diffusion as a scale effect in the use of intermediate products, or through better quality. This relates to the scale effect in Rivera-Batiz and Romer (1991) but now the mean of improvement goes through imports rather than as a direct effect by opening up to world knowledge.

The indirect effect from imports can also come from the possibility to buy e.g. machinery at a lower price, or at the same price but with higher quality (Mazumdar 2001). This emphasis points to the importance of the relative prices and particularly to the importance of the effective price, when the quality effect is taken into account. This model builds on Lee (1995) that stresses the importance of increased capital accumulation by importing cheaper input. I will, however, emphasize that a lower effective price rather increase the incentives to invest in productivity increasing R&D than add to the capital stock.

Furthermore imports can affect the transfer of foreign R&D knowledge (Lawrence - Weinstein 1999; Mazumdar 2001) through learning-by-doing processes. This idea builds on the observation that countries import new goods prior to making them themselves, and only finally exports them (Chuang 1998:700). Thereby one might suggest that it takes some domestic innovative process in order to incorporate the new technology and for it to have an effect on exports. This need for substantial resource costs in assimilating foreign products has been shown by several researchers. For example Teece concludes that:

> The resources required to transfer technology internationally are considerable. Accordingly, it is quite inappropriate to regard existing technology as something that can be made available to all at zero social cost. (Teece 1977:259)

Accordingly, the form of free spillover-effects emphasised by Rivera-Batiz and Romer (1991) do not seem appropriate to use.

It is possible, as Chuang (1998) points out, that adaptation might be able to come out of a single product. Thus, there is no need for increased trade in order to innovate. But the uncertainty of profits, and the fact that trade flows include information about sale possibilities for new products, makes it likely that increased trade flows is relevant for adapting foreign product into domestic markets or firms.

It follows that import-led productivity growth might not only come from intermediate goods, such as machinery, but also from imports of final goods, as adaptation and adjustments of previous foreign innovations might enhance domestic innovation. It is possible that this process accelerates as the domestic labour productivity increases, as knowledge facilitates the understanding of foreign technology (Thangavelu - Rajaguru 2004).

Imports could also increase competition and thereby innovation, especially in previous insulated and concentrated markets (MacDonald 1994). In addition increased competition could also have the negative effect of lowering profits of inefficient firms, hence the resources available for research investment. It could also be that foreign firms are so productive that domestic firms must close down or be bought by foreign capital. This might follow a reallocation process from less to more productive firms similar to that explained by Bernard (2004).

3.4 Different Effects in Different Countries

Some models have provided insight to the fact that trade does not affect all countries in the same way. Traditionally models that state an increase in growth as a result of research and development also include an effect of comparative advantages. Some countries that have an advantage in research will develop this sector and grow faster than their trade partners. In these models growth is not directly affected by trade and comparative advantage, but rather indirectly by research. Hence, if no research occurs then growth will be absent (Seater 2005). This setting builds on a two country or region divided world, which will not be used in the following sections. But, it is notable that research still is the driving force behind economic growth.

Another aspect is that developing countries may lack the technology needed to produce more highly developed products and in addition suffers from a low level of foreign exchange, which hinder imports of this technologies. Exports of labour intensive might therefore improve the stock of foreign exchange and initiate import and growth increases. (Thangavelu - Rajaguru 2004) This is one argument of why a relation between the variables might be less likely to find in an estimation of developed countries. But, given the previous discussion it seems like there are numerous of reasons why the interactions should be valid also in the industrialized world.

3.5 To Sum up the Theoretical Arguments

From the discussion it is notable that there are theoretical reasons to believe in both export- and import-led economic growth. As one aim is to include the results from this discussion into a model, one must realise that not all factors are possible to take into account. However, from the preceding argument it seems likely that the innovation processes is a key part of the growth process and that important effects from trade in the long-run is likely to emerge from productivity enhancement. Notable is that both exports and import may have an increasing effects on the incentives to innovate, as both an larger export market and import of foreign technology, through imported intermediate inputs, could enhance the returns to research. It seems likely that self-selection into exports is more important than learning-by-exporting and that a process where firms invest even harder, if the country is more open to trade, is expected. If this is the case productivity-led export rather than export-led productivity growth would be present, as firms invest in productivity increasing development, in order to sell to a larger market. It is also noted that both innovations and adaptation should include some form of cost and that imports could bring productivity improvements from high quality products.

4 The Theoretical Model

The aim of this section is to explain the extended theoretical model, which includes implications from the previous theoretical discussion. The derivation of the model can be found in appendix A.

4.1 Basic Assumptions

The derivation of the theoretical model has its origin in Aghion and Howitt's (2009:92ff; 1990) Schumpeterian model of a three sector economy with multiple innovating sectors, allowing for productivity improvements in several products simultaneously. However, in this model time is continuous, and production and consumption occurs under an infinitely long lifetime. Also, trade is included in this model since export in the final goods sector is possible, as well as import of intermediate goods in the intermediate sector.

In accordance with Aghion and Howitt (2009) the first of the three sectors is a final goods sector, working under perfect competition. The second is the intermediate sector, producing inputs for the final goods sector as a monopolist. Thirdly, the research sector produces innovations in intermediate products with entrepreneurs as investors. The Schumpeterian aspect is expressed in that growth comes out of "creative destruction" of previous technology. The model incorporates a random draw of productivity increase from technological innovations, which replaces the existing technology as the driving force of economic growth in combination with trade.

The total production of final goods ($Y_{I,t}$) can be used for three things: consumption, investment in research or as input in the intermediate sector. I addition to Aghion and Howitt's (2009) closed economy model, the final goods sector can export to a perfectly competitive international market, with the amount of exports measured in relation to the production for the domestic market. It is also assumed, in contrast to Aghion and Howitt's (2009) model, that the country can import input goods used in the intermediate sector. As the purpose of this study not is to investigate the trade effect on or via the balance of payment, trade is assumed to be balanced and countries are not allowed to lend to each other.

The model will be presented as follows. First, the effects from import of intermediate inputs are presented. Second, the results are adjusted for the effects of exports, and finally the growth rate of consumption and the equilibrium between supply and demand is established.

4.2 The Production of Final Goods

To begin with, the final goods sector is similar to the one used in Aghion and Howitt (2009). The production in the final sector is determined by three factors: the constant amount of labor (L), the amount of intermediate products (x_{it}) and the productivity of the intermediate products (A_{it}). There is a continuum of intermediate products, for both domestic and foreign products, indexed between [0,1]. All intermediate inputs exhibits full capital depreciation, hence capital accumulation in intermediate goods is excluded.

First, leaving out exports, the total output of the final goods market in time t is

$$Y_{I,t} = L^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^{\alpha} di \tag{4.1}$$

with one produced good ($Y_{I,t}$) compounded by a continuum of intermediate products. The final output from using intermediate product x_{it} is:

$$Y_{I,it} = (A_{it}L)^{1-\alpha} x_{it}^{\alpha} \tag{4.2}$$

where x_{it} is the amount of intermediate inputs needed to produce $Y_{I,it}$. A_{it} is the productivity of the intermediate good. The level of A_{it} is, as we will see, determined by the research sector. The final goods sector exhibit constant return to scale in L and x_{it}.

4.3 The Intermediate Sector and Import

Unlike in Aghion and Howitt (2009), the intermediate goods sector produces $\mathrm{x_{it}}$ by either using $\mathrm{Y_{I,it}}$ or by importing inputs from abroad. The price of the intermediate goods is set by the marginal product of $\mathrm{x_{it}}$ in the final goods market.

$$\mathrm{p_{it}} = \frac{\mathrm{dY_{I,it}}}{\mathrm{dx_{it}}} = \alpha(\mathrm{A_{it}L})^{1-\alpha}\mathrm{x_{it}^{\alpha-1}} \tag{4.3}$$

The intermediate firms buy inputs from the final producers at a normalized effective prize[7] of 1 and from the foreign producers at an effective price of 1/B, where the constant B is determined by the relative quality of foreign to domestic products. Only foreign inputs with B>1 is assumed to be imported. The profit in the intermediate goods sector, when the economy is only open to import, can be described as:

$$\Pi_{\mathrm{I,it}} = \mathrm{p_{it}x_{it}} - \mathrm{x_{d,it}} - \frac{1}{\mathrm{B}}\mathrm{x_{f,it}} \tag{4.4}$$

where $\mathrm{x_D}$ and $\mathrm{x_F}$ is domestically respectively foreign produced inputs. Given that the country is not entirely closed, $\mathrm{x_{f,it}} > 0$ is assumed. Production of output in the intermediate sector is categorized by constant return to scale and need to use both types of inputs in the production[8]. The expression can be written as[9]:

$$\mathrm{x_{it}} = \mathrm{x_{d,it}^{\omega}x_{f,it}^{1-\omega}} \tag{4.5}$$

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[7] The effective price is the price adjusted for the quality of the product.

[8] In Aghion and Howitt (2009:86) the production is also be assumed to experience constant return to scale: $x_{it} = x_{d,it}^{1}$. However, the difference in assumptions for the intermediate production makes the two models directly incomparable.

[9] In our case B enables the firm to produce with the same amount of inputs but at a lower price, which is similar to letting B affect to production function (4.5); hence, enable the same production level but with less inputs. The later would however require a formulation of the substitution effect between the inputs.

ω is assumed to be a constant and reflects the relative scale effect in production of x_{it}. This implies that for a given production level x_{it} increased import would decrease the use of domestic inputs. Using this, the (import) profit function can be described as:

$$\Pi_{I,it} = \alpha L^{1-\alpha} A_{it}^{1-\alpha} x_{d,it}^{\omega\alpha} x_{f,it}^{(1-\omega)\alpha} - x_{d,it} - \frac{1}{B} x_{f,it} \tag{4.6}$$

where the first term at the right hand side is total revenue and the second plus third is total costs. The monopolist chooses x_{it} as to maximize profits by letting marginal revenue equal marginal costs. By optimizing $\Pi_{I,it}$ with respect to $x_{d,it}$ and $x_{f,it}$ respectively we get the optimal input levels:

$$x_{d,it}^{*} = \alpha^{\frac{2}{1-\alpha}} \omega^{\frac{1-\alpha+\omega\alpha}{1-\alpha}} (1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}} B^{\frac{(1-\omega)\alpha}{1-\alpha}} A_{it} L \tag{4.7}$$

$$x_{f,it}^{*} = \alpha^{\frac{2}{1-\alpha}} (1-\omega)^{\frac{1-\omega\alpha}{1-\alpha}} \omega^{\frac{\omega\alpha}{1-\alpha}} B^{\frac{1-\omega\alpha}{1-\alpha}} A_{it} L \tag{4.8}$$

Incorporating this in (4.5) gives the optimal level of output: x_{it}^{*}:

$$x_{it}^{*} = \alpha^{\frac{2}{1-\alpha}} \omega^{\frac{\omega}{1-\alpha}} (1-\omega)^{\frac{(1-\omega)}{1-\alpha}} B^{\frac{(1-\omega)}{1-\alpha}} A_{it} L \tag{4.9}$$

Using (4.7), (4.8) and (4.9) in the profit function (4.4) gives the optimal profit of

$$\Pi_{I,it}^{*} = \psi B^{\frac{\alpha(1-\omega)}{1-\alpha}} A_{it} L \tag{4.10}$$

where $\psi = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\omega)^{\frac{\alpha(1-\omega)}{1-\alpha}} \omega^{\frac{\alpha\omega}{1-\alpha}}$. This is the optimal profit for a firm that only imports. As is readily seen in equation (4.10), the profit of the intermediate firm increases in B, hence imports of more productive inputs increases the profit. There is also a proportional effect from the effective labour supply $(A_{it}L)$, as both the demand of x_{it} and the use of inputs increases.

The optimal price (p_{it}^{*}) can be derived by inserting (4.9) into (4.3). This results in a price of:

$$p_{it}^{*} = \frac{1}{\alpha\omega^{\omega}(1-\omega)^{(1-\omega)}B^{(1-\omega)}} \tag{4.11}$$

4.4 Exports of Final Goods

In addition to extending the model with imports, also exports are introduced. Exports have the effect of opening up sales to previously closed markets. In the case of only a domestic market, the total production is equal to Y_t. But, if the country is open to trade, assuming a symmetrical export of N times the production for the domestic market the total production can be expressed as:

$$Y_{trade,t} = Y_{I,t} + \int_0^N Y_{I,t} = (1+N)Y_{I,t} \tag{4.12}$$

where N is the size of the utilized export market in relation to the domestic market[10], giving a total export of $N * Y_{I,t}$. Because the final goods market is characterized by constant return to scale in L and x_{it}, a proportional increase in L and x_{it} must be implied.

$$\lambda f(L, A_{it}x_{it}) = f(\lambda L, \lambda A_{it}x_{it})$$

It follows from an increase of needed intermediate goods, and constant return to scale in its production, that the output and the input in the intermediate sector is increased by the same proportion, (1+N).

$$(1+N)x_{it} = (1+N)\left[x_{d,it}^{\omega}x_{f,it}^{1-\omega}\right] \tag{4.13}$$

This in turn raises the intermediate profit to:

$$\Pi_{Trade,it} = (1+N)\Pi_{I,it} = (1+N)\left[p_{it}x_{it} - x_{d,it} - \frac{1}{B}x_{f,it}\right] \tag{4.14}$$

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[10] It is assumed that there are no productivity differences between the export and domestic sectors, which is a simplification according to empirical findings.

The optimal price remains at (4.11). This implies that the optimal profit increases proportionally to

$$\Pi^*_{\mathrm{trade,it}} = (1+\mathrm{N})\psi \mathrm{B}^{\frac{\alpha(1-\omega)}{1-\alpha}} \mathrm{A}_{\mathrm{it}}\mathrm{L} \tag{4.15}$$

still with $\psi = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\omega)^{\frac{\alpha(1-\omega)}{1-\alpha}}\omega^{\frac{\alpha\omega}{1-\alpha}}$. It can then be seen that the profit of the intermediate sector is increasing both in exports and imports. Further, the profit is still linear in the productivity level A_{it}, which is determined by the research sector. Also, there is a scale effect, represented by L.

4.5 The Research Sector

The technological level A_{it} is determined by the production of innovations in the research sector, as suggested in Aghion and Howitt (2009). In order to gain the monopoly rent an entrepreneur attempts to make an innovation of the intermediate product with the reward of a global patent and monopoly if she succeeds. A successful innovation will also result in a productivity improvement of $\gamma > 0$. The entrepreneur spends the amount R_{it} units of final goods on research and innovates with the probability

$$\mu_{\mathrm{t}} = \phi\left(\frac{\mathrm{R}_{\mathrm{it}}}{\mathrm{A}^*_{\mathrm{it}}}\left(\frac{\mathrm{x}_{\mathrm{ft}}}{\mathrm{A}_{\mathrm{t}}}\right)^{\tau}\right) = \lambda\left(\frac{\mathrm{R}_{\mathrm{it}}}{\mathrm{A}^*_{\mathrm{it}}}\left(\frac{\mathrm{x}_{\mathrm{ft}}}{\mathrm{A}_{\mathrm{t}}}\right)^{\tau}\right)^{\sigma} \tag{4.16}$$

which is increasing[11] in R_{it} and depends inversely on $\mathrm{A}^*_{\mathrm{it}}$. $\mathrm{A}^*_{\mathrm{it}}$ is the productivity level after a successful innovation, making innovating more difficult when the technology become more complex and developed, $\lambda > 0$ is the productivity in the research sector and σ is the elasticity, which lies within [0, 1]. In addition to Aghion and Howitt (2009) the adaptation effect from import is included. It states that the probability of an successful innovation increases if the entrepreneur has access to greater inflow if imports with a high technological level, here measured as x_{ft}. But, this is reduced by the effect from A_{t}, as a generally higher technology

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[11] $\phi'_{R_{it}}\left(\frac{\mathrm{R}_{\mathrm{it}}}{\mathrm{A}^*_{\mathrm{it}}}\left(\frac{\mathrm{x}_{\mathrm{ft}}}{\mathrm{A}_{\mathrm{t}}}\right)^{\tau}\right) = \sigma\lambda\left(\frac{1}{\mathrm{A}^*_{\mathrm{it}}}\left(\frac{(1+\mathrm{N})\mathrm{x}_{\mathrm{ft}}}{\mathrm{A}_{\mathrm{t}}}\right)^{\tau}\right)^{\sigma}/\mathrm{R}_{\mathrm{it}}^{1-\sigma} > 0$

level in the output market makes it more difficult to find suitable information from import, in order to improve upon existing domestic intermediate products. Also, the entrepreneur's ability to derive knowledge from imports ($\tau > 0$) is important. It is assumed that the parameters are of a scale so that $0 < \mu_t < 1$.

As stated, the innovator gains the monopoly profit if the innovation succeeds, but has to pay R_{it} in order to undertake the research. This imply that net present value[12] of the expected profit for the entrepreneur is

$$E[\Pi_{Research,it}] = \phi\left(\frac{R_{it}}{A_{it}^*}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right) * \frac{\Pi_{trade,it}^*}{r} - R_{it} \tag{4.17}$$

The entrepreneur chooses R_{it} in order to maximize the profit, by letting marginal revenue equal marginal cost. The discount rate r is assumed to be constant as the marginal productivity of capital does not change over time, and intermediate goods exhibits full depreciation. This gives us the research arbitrage condition, stating that the market rate of return equals the rate of return to research.

$$\phi'\left(\frac{R_{it}}{A_{it}^*}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right) * \frac{\Pi_{trade,it}^*}{A_{it}^*}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau} = r \tag{4.18}$$

By replacing the profit with equation (4.15) and optimal imports with (4.8) we get

$$\phi'\left(\frac{R_{it}}{A_{it}^*}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right)\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau} = 1 \tag{4.19}$$

where $\vartheta = \alpha^{\frac{2}{1-\alpha}}(1-\omega)^{\frac{1-\omega\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}$ and $A_t = \int_0^1 A_{it}di$. If we solve the expression for R_{it} it is readily seen that the amount of research expenditures, and thereby the amount of patents, are increasing with the profit of the intermediate firm and with an increased amount of import.

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[12] The net present value is taken for the research profit function as the entrepreneur receives a patent over an infinite time. This is an extension of Aghion and Howitt (2009) which will come into use in the general equilibrium derivation.

$$R_{it} = \left(\sigma\lambda \frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}} L}{r} \right)^{\frac{1}{1-\sigma}} \left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}} L \right)^{\frac{\tau\sigma}{1-\sigma}} A_{it}^{*} \tag{4.20}$$

In order to know the probability, μ_t, the first order condition of equation (4.16) is taken with respect to $\left(\frac{R_{it}}{A_{it}^{*}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right)$. Then this is used in equation (4.19) to solve for $\left(\frac{R_{it}}{A_{it}^{*}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right)$. Finally the result is replaced back into equation (4.16). This gives us a constant probability of:

$$\mu = \lambda^{\frac{1}{1-\sigma}} \left[\sigma \frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}} L}{r} \left(\vartheta(1+N)B^{\frac{1-\omega\alpha}{1-\alpha}} L \right)^{\tau} \right]^{\frac{\sigma}{1-\sigma}} \tag{4.21}$$

with $0 < \mu < 1$. In the long-run μ will be the frequency of innovations, implying on average more patents per time period with higher values of μ. It is readily seen that μ increases with both import and export, as larger values of N and B increases both the expected profit and, as given in the second parenthesis, the access to foreign technology.

4.6 Levels of GDP and Final Production

The level of GDP_t and the total production in the final sector, Y_t, is not equal in this model. This is because some of the production in the final goods market is used as input in the intermediate market, either directly as $x_{d,it}$, or as payment for foreign inputs at the effective price $1/B$. In order not to count this production twice it is necessary to subtract the value of inputs in the intermediate market from the resource constraint, so that GDP only reflects the consumption, investment and net export.

$$GDP_{trade,t} = (1+N)\left(Y_t - \int_0^1 x_{d,it} - \int_0^1 \frac{1}{B} x_{f,it} \right) \tag{4.22}$$

This shows one clear relation between GDP and trade. If the effect from trade would be to increase the factor inputs it could result in a positive correlation between export and GDP as well as a negative correlation with imports. Although this does not describe the long-run growth relationship or if the causal interpretation between GDP and trade is (partly) misleading as both are a result of some other factor.

Knowing the relation in eq. (4.22), the levels of GDP per worker and final goods production per worker can be derived. First, combining equation (4.2) and (4.9) results in a description of Y_t/L that is increasing in A_t, N and B.

$$y_{trade,t} = (1+N)\alpha^{\frac{2\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{(1-\alpha)}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_t \tag{4.23}$$

As stated before N, B and L are taken as constants. Hence any equilibrium change in Y_t/L is determined by the productivity measure A_t, the aggregated unweighted average of all the individual productivity parameters

$$A_t = \int_0^1 A_{it}\, di$$

Secondly, using also equation (4.7) and (4.8) in (4.22) gives the GDP level of the economy:

$$gdp_{trade,t} = (1+N)(1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{(1-\alpha)}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_t \tag{4.24}$$

It is readily seen that $gdp_{trade,t}$ is affected by the same variable as $y_{trade,t}$. Thus, the changes in $gdp_{trade,t}$ is determined by the endogenous variable A_t. This imply that the growth rate of the economy is proportional to the growth rate of A_t. Also, the expression reflects that increased export will have a level effect trough N and that it is optimal to import more inputs. Hence, increased production, as an effect of imported foreign technology (B), lowers the effective price of foreign inputs.

4.7 Technological Change and Growth

The aggregated technological level A_t changes according to:

$$\dot{A}_t = \mu\gamma A_t \tag{4.25}$$

On a firm level, a successful innovation lead to an increase in the technological level of γ with the probability μ, and an unsuccessful innovation imply $\dot{A}_{it} = 0$. Hence, the growth for a individual firm is random. However, the aggregate growth is not random but average, as bad luck is offset by good luck. It follows that the growth rate of A_t is

$$g_t = \mu\gamma \tag{4.26}$$

The constant probability of a successful innovation is given by equation (4.21). Hence, replacing μ above gives the average growth rate of the economy

$$g_t = \lambda^{\frac{1}{1-\sigma}} \left[\sigma \frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}} L}{r} \left(\vartheta(1+N)B^{\frac{1-\omega\alpha}{1-\alpha}} L \right)^{\tau} \right]^{\frac{\sigma}{1-\sigma}} \gamma \tag{4.27}$$

The function is increasing in N, B, λ, γ, τ and L but decreasing in r. Therefore, exports (N) increases the growth rate, by supplying funds and increasing the demand for intermediate products. Imports (B) also have a positive effect through the intensive aspect of a lower effective price for imported intermediate inputs. Also a larger productivity increase (γ) from successful innovations and a more productive research sector (λ) increases the growth rate through more and better innovations. And improved adaptation ability (τ) facilitates the innovation process. In addition, there is a scale effect (L), as increased demands raise the profitability for innovations. The interest rate (r) decreases the growth rate as the choice to invest becomes more expensive, lowering the future expected profits of the entrepreneur.

4.8 Consumers

The previous sections of the model have been focusing on examining the supply side of the economy. In order to receive a full picture of the equilibrium conditions the following part looks into the demand side. Primarily, it is assumed that each country consists of a large number of identical households (L). Which, as stated earlier, are supplying the economy with one unit of labour each, for an infinite amount of time. The labour supply does not grow over time, thereby leaving out the effect of growing markets from existing trade partners. Also, all households last forever and experience a utility from consumption in time t of

$$U = \int_0^\infty u(c_t)e^{-\rho t} \tag{4.28}$$

where $\rho > 0$ is the rate of time preference and c_t is consumption in time t, with $c_t \geq 0$. The consumption is for one good, as only one final good is produced according to eq. (4.1).

Assets exist in two forms: either as loans between households or as profits from research investment, and is owned by all identical households. The households use their assets for three things: consumption, investment in research or to buy intermediate inputs. As firms last for an infinite amount of time there will be future profits for the investors. It is assumed that the profits from investments in research is owned by all households, e.g. as shares in the firms. The rate of return of investments and loans is denoted by r, as they are perfect substitutes. Given perfect capital markets, the rate of return is equal for all investments. Hence, profits from the intermediate sector is included in the existing assets per person a_t. Assuming balanced trade and not allowing for international loans imply that all assets not invested are consumed. Therefore the change in assets per capita is described as

$$\dot{a_t} = ra_t + w_t - c_t \tag{4.29}$$

and is dependent on the rate of return (r) on previously earned assets (a_t), the wage rate from one unit of labor (w_t) less the consumption per person (c_t). Consumers maximize their utility from consumption with respect to their assets as the only objective. Hence, the Hamiltonian function for optimization[13] is

$$H = u(c_t)e^{-\rho t} + \mu_t * [ra_t + w_t - c_t] \tag{4.30}$$

Given that we have constant elasticity of substitution the Euler equation describing the growth in consumption is given by:

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[13] See: appendix A for details of the optimization procedure, and derivation of the Euler equation.

$$g_{ct} = \frac{\dot{c}_t}{c_t} = \frac{1}{\theta}(r - \rho) \tag{4.31}$$

where $1/\theta$ is the inverse of the elasticity of substitution. The function is increasing in r, and $g_c > 0$ given that $r > \rho$.

4.9 Equilibrium

In order to find the equilibrium growth rate of the economy we later need to integrate the results from the firms with the Euler equation for private consumption. However, it needs to be shown that for equilibrium to exist, all variables must be able to grow at a constant rate and that the percentage relations between them also are constant.

To begin with, the resource constraint gives that output of the economy equals aggregate consumption C_t, expenditures on intermediates $x_{d,t}$ and $x_{f,t}$, and investment in research R_t.

$$(1+N)Y_{I,t} = C_t + (I + N)x_{d,t} + \frac{(1+N)}{B}x_{f,t} + R_t \tag{4.32}$$

It can be shown that all variables are a linear function of A_t. For the intermediate products equations (4.7) and (4.8) shows that $x_{d,t}$ and $x_{f,t}$ is linear in A_t. Equation (4.23) shows the relation for Y_t. Equation (4.20) gives a function for R_t. It follows that also C_t is a linear in A_t and from equation (4.24) one can also see that GDP_t is a function of A_t. In addition, equation (4.13) show that the same is true for x_t. Therefore the growth rates of Y_t, GDP_t, C_t, $x_{d,t}$, $x_{f,t}$, x_t, and R_t are all equal to the growth rate of A_t.

$$\frac{\dot{C}_t}{C_t} = \frac{\dot{Y}_t}{Y_t} = \frac{G\dot{D}P_t}{GDP_t} = \frac{\dot{x}_{d,t}}{x_{d,t}} = \frac{\dot{x}_{f,t}}{x_{f,t}} = \frac{\dot{x}_t}{x_t} = \frac{\dot{R}_t}{R_t} = g_t \tag{4.33}$$

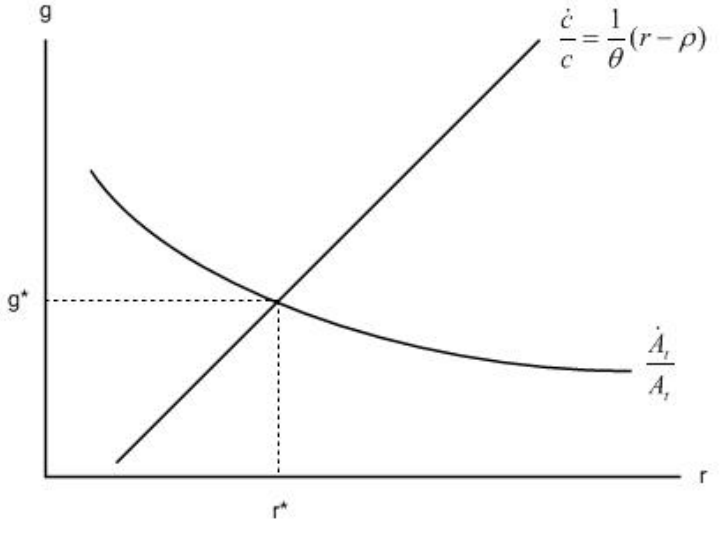
This also implies that, given a constant N, export (NY_t) and production for the domestic market (Y_t) will grow at the same rate. This shows that we should expect a log run relationship between the growth rates of exports, imports and GDP. However, this effect is given by the technological growth as expression (4.33) interprets as the growth of the economy is the growth rate of A_t.

Therefore, the production side of the economy will follow the growth rate in equation (4.27). To find the final solution we need to combine this with the growth rate from the demand side. Previously it has been shown that the growth rate in productivity is decreasing, but that the consumption is increasing, in r_t. Using this, solving for r in eq. (4.31) and replacing into eq. (4.27), a general equilibrium can be found. The equilibrium growth rate can be seen graphically in the intersection between the lines below. Given that $r \neq 0$ this will be the only equilibrium for the economy. The curves are given by equations (4.27) and (4.31). The slope of the consumption growth curve is $1/\theta$ and for the productivity growth it is given by

$$\frac{dg_t}{dr} = -\frac{\sigma}{1-\sigma}\lambda^{\frac{1}{1-\sigma}}\left[\sigma\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L\left(\vartheta(1+N)B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right]^{\frac{\sigma}{1-\sigma}}\gamma\frac{1}{r^{\frac{1}{1-\sigma}}} < 0 \qquad (4.34)$$

Therefore, the productivity growth will be decreasing towards zero in r, at a decreasing rate.

Figure 4.1: General Equilibrium of Theoretical Growth Rates



From equation (4.27), restated below, one can see that a chock in the parameters λ, N, B, L, γ and τ will shift the productivity growth curve upwards.

$$g_t = \lambda^{\frac{1}{1-\sigma}}\left[\sigma\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left(\vartheta(1+N)B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right]^{\frac{\sigma}{1-\sigma}}\gamma$$

One can also see that the effect from the elasticity in the probability function (σ) is of significance as it raises the likelihood of successful innovations.

In order to find the equilibrium effects from changes in the different parameters the Euler equation is solved for r, and replaced in the equation above. It is unfeasible to solve the result for g_t so instead the implicit function theorem is used. The result for B is shown below and one can see that the effect from an increase in B will be strictly positive.

$$\frac{dg_t}{dB} = \left(\left[\frac{\alpha(1-\omega)}{1-\alpha}\right] + \left[\tau\frac{(1-\omega\alpha)}{1-\alpha}\right]\right)\frac{\sigma}{1-\sigma}\frac{1}{\left(\frac{\theta\sigma}{1-\sigma}\frac{1}{(\theta g_t+\rho)} + \frac{1}{g_t}\right)B} > 0 \tag{4.35}$$

As expressed in appendix A similar results can be derived for λ, N, L, τ and γ, while an increase in ρ will have a direct negative effect on growth. Thus, the graphical illustration represents a general result and not a specific case.

4.10 Implications from the Theoretical Model

The basic result from the theoretical model is that countries engaged in trade can experience long-run growth effects both from increased exports and imports and that these are due to improvements in the productivity. It also, shown that there are reasons to be careful about the interpretation of trade-led GDP growth, as growth in these variables might be caused by the change in technology.

There are reasons to believe that both exports and imports can affect the technological growth rate by increasing the incentives for innovations and by improving the access to foreign technology. Hence, the real growth effect comes from the increase in the production of innovations made possible by an increased amount of investment in research, funded by increased income for e.g. exports.

The causal effects with exports might have two directions: First, a shock in exports could increase the possibility to invest in research, resulting in export-led productivity growth. Second, the expectations of an increased out in the intermediate sector, due to improvements in the final good sectors inputs, would pre-increase the investment in research and lead to a productivity improvement, causing increased export possibilities for the final goods sector.

Therefore, the model also predicts a self-selection process aimed at expropriating the additional expected profits from the increased input demand from the final goods sector.

The inclusion of imported intermediate goods in the intermediate sector also contributes to the effect on growth. It can be seen from equation (4.27) that a country that imports inputs with a relatively higher quality will experience a higher growth rate. Also, increases in the import level gives access to foreign technology, which improves the likelihood of innovating, and thereby the average growth rate.

In addition there are links between exports and imports as exports increase the demand for additional imported inputs in the intermediate sector and imports improves the productivity in final goods production, hence increases exports.

The causal effect is not directly treated in the model and the continuous time makes everything occur simultaneously. But, this assumption is naturally not empirically plausible as for examples research and investments requires time to be implemented and successful. Therefore corresponds to the mixed results of previous research and illustrates the complicated web of relationships.

The model states that there are theoretical grounds for trade to improve GDP. But that the economic force behind this is primordially through technological improvements and that investigating the relation between GDP and trade might give misleading results to the importance of trade. Hence, focusing on the relation between export, import and productivity is an area of importance in order to determine whether the export- and/or import-led growth hypothesis is really valid.

5 Choice of Variables for the Empirical Model

As shown in previous sections, the study of trade's effect on growth might be problematic since both export and import might work through different channels. From the theoretical model it is readily seen that an increased amount of exports (and imports) should have a direct level effect on GDP. This is due to the fact that they are bound in via the budget constraint and any direct correlation might therefore risk capturing only this effect. It has also been shown that this relation does not contribute to growth in the long-run, since all equilibrium growth in the model is determined by technological innovations. Therefore, the empirical study is built on the long-run relationship between technological growth and trade.

According to the theoretical model and discussion in previous sections there are a number of different variables that are possible to include in the empirical analysis. Looking at the long-run growth expression in equation (4.27):

$$g_t = \lambda^{\frac{1}{1-\sigma}} \left[\sigma \frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}} L}{r} \left(\vartheta(1+N)B^{\frac{1-\omega\alpha}{1-\alpha}} L \right)^{\tau} \right]^{\frac{\sigma}{1-\sigma}} \gamma$$

the technological growth is determined by the size of exports (N), the quality of imported intermediate inputs (B), both working through an increase in the profits of the entrepreneur and increasing the spillover effect from the level of imported intermediate goods. The growth rate will also increase by the ability to adapt foreign technology (τ) and the productivity in the research sector (λ) and diminish by an increased interest rate (r) - all affecting the probability of innovations, the investments in R&D and the number of patents. Also the general size of an innovation (γ) is of importance.

Although many specification refinements can be considered in a study of the relationship between trade and growth the limited number of observations restricts the empirical study to the use of a simple model that captures the basic relationship of interest. In practice this limitation makes it hard to distinguish between the neoclassical exogenous effects from trade and the endogenous new growth theories. As has been seen in section 2 most studies investigate the direct effect from trade to growth. And, on an aggregated level, the closest to an endogenous study one gets is to look at the effect on productivity instead of GDP.

In this case the number of yearly observations is limited to the period 1960-2007. Hence, only 47 observations are available for each time series. This has econometric implications as the calculations of statistical tests used in the estimations requires an sufficient amount of observations in order both to be reliable and include a satisfactory lag length. The lag length criteria estimations, later described in section 8.2.1, indicates that four variables might give a too extensive model. Therefore, I choose not to include more than four variables in the dynamic estimations. This treatment is in line with the variable selection in studies described in the previous research section.

As an operationalisation of the technological level the Total factor productivity (TFP)[14] is calculated. The derivation builds on a standard output equation of

$$Y_t = A_t^{1-\alpha} L_t^{1-\alpha} K_t^{\alpha} \tag{5.1}$$

which is identical[15] to the final goods equation (4.1) if one assumes the intermediate goods used in the production is financed by capital according to $x_{it} = K_{it}$, and the labour force is allowed to change over time. The productivity level is given by A_t. The calculation of TFP variable is further described in section 7.1.

Following previous research the real level of export (EXPORT) is included. This treatment gives no possibility to distinguish between effects related to domestic innovations from research investments and external spillover embodied in the access to export markets. However, it is reasonable to believe that the size of export signals possibilities to investors. The real level of import (IMPORT) is included to represent the accumulated effects of import. Also in this case the separate effects from quality, number of varieties and the price effects are undistinguishable. Nevertheless, this is in line with previous research, and given the choice of studying the effects related to growth, the time perspective limits the data selection. The choice to include total imports can also be motivated by the thought that also final goods enhance the spillovers of ideas, thus increases the likelihood to adapt foreign technology. This

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[14] A_t is denominated as "labor augmenting productivity" in Aghion and Howitt (2009), I will however use the term TFP as the conclusions from the empirical analysis only differ in the size of the coefficients as; $\ln \mathrm{TFP_t} = (1-\alpha) * \ln \mathrm{A_t}$

[15] This is shown in Aghion and Howitt (2009:114f) for a closed economy.

treatment should be sufficient to investigate if the trade variables are related to productivity, and if these effects are different for each variable. Also, data availability on intermediate inputs are limited for the period of interest[16].

The aggregated investment rate (INVEST) is included for several reasons. Numerous studies has concluded that different sorts of investments has a positive effect on TFP[17] (Stiroh 2000). Although aggregated investments are most commonly used in studies of labour productivity also investments are reasonable to include in studies of TFP. This is because investments might not only increase the capital stock, but can also be embodied with technological change. Also, investments is demanded in order to implement this progress. Aghion and Howitt (2009:116) show that shocks from investments in intermediate goods theoretically should increase the growth rate through increased profits and research. If investments contribute to GDP above adding to the capital stock, one would expect a positive effect on TFP. However, if the major effect is only to increase the factor input, it is possible that investments has a negative effect on TFP. Investment might also be related to trade. As stated in the theoretical background import might primarily affect GDP through capital accumulation. It is also possible that the effect on productivity from trade goes through increased investments. This can be seen in the theoretical model, equation (4.20), as trade increases investment in research and development (R_t), which broadly defined represents the necessary investments for both innovation and adaptation.

Based on the theoretical expectations discussed in the theory sections, the expected signs of the variables included to determine TFP are

$$\mathrm{TFP} = \mathrm{f}(+\mathrm{EXPORT}, +\mathrm{IMPORT}, +\mathrm{INVEST})$$

The hypothesis of productivity-led export and import growth also states that TFP could have a positive effect on both import and export.

As stated in the theoretical model both the number of patents and the research expenditures are two variables closely related to productivity. These are also emphasised in numerous studies as approximations for the domestic level of ideas or innovations. However, empirically there are some concerns with the data which excludes the variable from this study. First,

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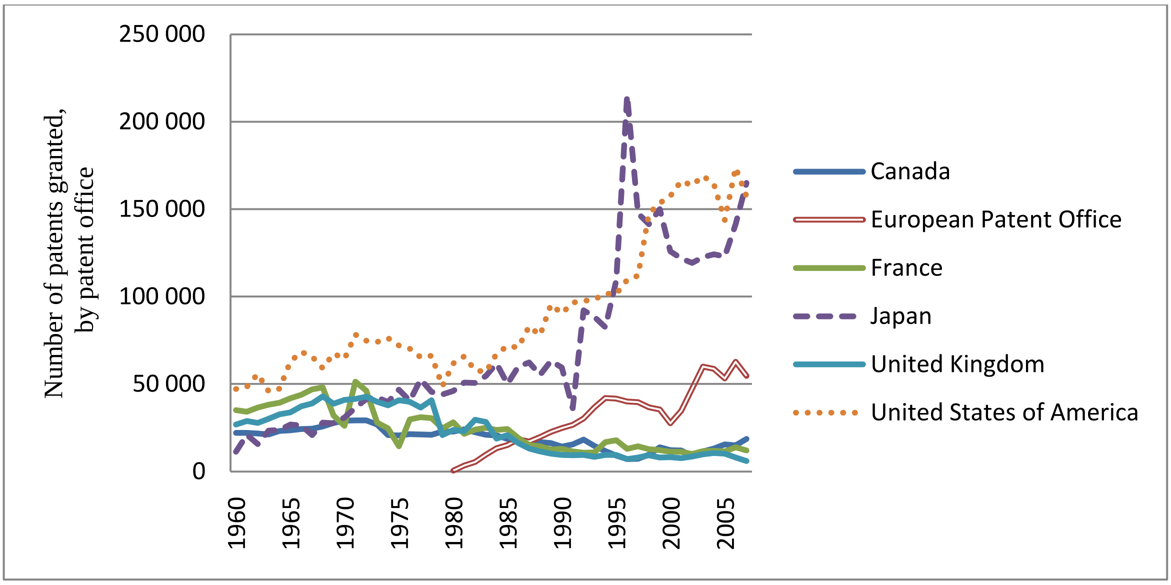
[16] OECD.stat provides data for imported intermediate inputs from 1995 to 2005.

[17] Vecchi and O'Mahony (2003) show this for investments in Information and communication technology; DeLong and Summers (1991) use investments in machinery and Equipment (M&E) and argues that these embodies externalities; finally Bernanke and Gürkaynak (2002) uses the general investment rate.

annual data of research expenditures are not available for the period of interest[18]. Second, the data of patents could be used for long time series, but the consistency of the measure can be questioned. This could be explained by that the importance of patent registration on a national level, for e.g. European countries has decreased over time, as international patents are applied for more frequently at e.g. EU level. At the same time the United States experienced an increase in patent applications. The divergence between the regions, illustrated in figure 5.1, is not likely to reflect on the number of ideas or the technological level. Also, the importance of patents is likely to have changed over time as new technology exhibit significantly different characteristics and an expansion of varieties, rather than new products, might better represent current increases in trade flows.

In addition variables representing terms-of-trade, research productivity, interest rate or adaptation ability is not included, even though there are empirical or theoretical reasons why they could affect the productivity level.

Figure 5.1: Patent Statistics, 1960-2008



Source: WIPO, Patent grants by patent office (1883-2008), *by resident and non-resident*, *http://www.wipo.int/ipstats/en/statistics/patents/*

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[18] WDI delivers data for R&D expenditure as percent of GDP from 1996 for OECD countries, and OECD.stat total R&D expenditures from 1981.

6 Econometric Method

This section describes and discusses the relevant econometric method and theory. It also presents the choices made in relation to this method. In addition, it also includes the delimitations made in the empirical study.

The econometric methodology builds on single country time series estimations. This is in line with the almost all related previous studies using a dynamic approach. It would have been possible to pool the countries into a panel, to gain additional observations. However, the unit root test assume that the countries are not cross-cointegrated (Harris – Sollis 2003:192), which is unlikely since a major part of world trade is undertaken by the included countries. Hence, general conclusions may be misleading. Also, the cointegration analysis would imply a common long-run relationship, which may be problematic since it is possible that the variables are cointegrated differently in different countries, see for example Thangavelu and Rajaguru (2004). A single country approach more clearly reveals if there is heterogeneity between countries. In addition the estimations procedures for estimating a multivariate is not as developed as for single country estimations, e.g. does E-views 5 or 7 not include any per-programmed ability to estimate coefficients for a pooled VECM. I have therefore chosen to use a single country methodology.

6.1 Non-Stationarity

In order to establish the properties of the time series data used in a regression analysis it is of importance to investigate whether the series are stationary or non-stationary. If a time series are stationary then it has time invariant mean, variance and autocovariance. That is, over time, a stationary series tends to return to its mean value and fluctuate around it within a limited range. A non-stationary series does not have a constant mean over time, since it is affected by past disturbance terms, and its variance increases with the sample size (Harris - Sollis 2003:27ff). If a time series are stationary after including a deterministic trend, the series is said to be trend stationary. In order to make a non-stationary series stationary it is possible to differentiate the series. If a series need to be differentiated n times in order to be stationary it is said to be integrated of order n, I(n).

If a series is non-stationary it may not be appropriate to use it in a regression analysis. This is because the use of two or more non-stationary series in one regression might lead to misleading inference. This problem is shown in overestimated R^2- and t-statistics, whereas the Durbin-Watson statistic will be low, signalling autocorrelation in the regression (Westerlund 2005:205). A regression with these problems is called a spurious regression, and it shows a strong relationship between variables that only share correlated time trends. If the series are not trend-stationary it is not possible to de-trend the data in order to solve the problem.

One solution could be to differentiate I(1) series one time in order to make them stationary. However, this transformation implies a loss of information about the long-run relationship (Davidson et al. 1978). Therefore that is not an appropriate choice for this study.

In models that include non-stationary series shocks have long lasting effect, whereas in stationary models shocks can only have a temporary effect (Verbeek 2008:282). In an economic growth framework it is plausible for variables to adjust to an equilibrium level over several years. Thus, a model using non-stationary series is suitable.

6.1.1 Test for unit roots (Non-Stationarity)

There are a number of different unit root tests available for evaluating the presence of a unit root. In this study the Augmented Dickey-Fuller (ADF) test is used[19], and complemented by the Phillip-Perron (PP) test for robustness.

The ADF test builds on the Dickey-fuller test, but expands the possible lag length of the series first difference, in order to get white noise residuals in series that has a higher order of autoregressive process than AR(1). The ADF test assumes a p:th order AR process. The test equations are given by:

$$\Delta y_t = \delta y_{y-1} + \sum_{i=1}^{p-1} \varphi_i \Delta y_{t-i} + u_t \tag{6.1}$$

$$\Delta y_t = \delta y_{y-1} + \sum_{i=1}^{p-1} \varphi_i \Delta y_{t-i} + \mu + u_t \tag{6.2}$$

$$\Delta y_t = \delta y_{y-1} + \sum_{i=1}^{p-1} \varphi_i \Delta y_{t-i} + \mu + \gamma t + u_t \tag{6.3}$$

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[19] This is recommended by Harris and Sollis (2003:42) as a more conservative choice.

where (6.1) leaves out the deterministic components, (6.2) includes a constant and (6.3) includes a constant and a time trend. In order for equation (6.1) to be appropriate the series should be known to have mean value of zero. This is not known in the present case, thus all tests are based on the later two equations.

The ADF null hypothesis is that $\delta = 0$. In that case the series has a unit root and is non-stationary. The alternative hypothesis is that $\delta < 0$, so that the series is stationary (Harris - Sollis 2003:48).

One problem in choosing the appropriate equation is to determine the set of deterministic components. This is done by a combination of graphical evaluations and the significance of the components. Also, the true needed number of lags is unknown. If a too large number is chosen the test will lose power, the risk of not rejecting the null when it is false increases, so that the series looks more non-stationary than is true.

In this study the prime source for determination of lag length is Schwartz Information criteria (SBC). An additional information criterion, the Akaike Information Criterion (AIC), is used for robustness.

As a complement to the ADF test, the Phillip-Perron test uses a non-parametric correction for autocorrelation, and estimates the non-augmented forms of equation 6.1-6.3. The autocorrelation is accounted for by the difference of the variance between the ‘true’ population (σ^2) and from the residuals (σ_{u}^2):

$$\sigma^2 = \lim_{T\to\infty} E(T^{-1}S_T^2), \;\; \sigma_{\mathrm{u}}^2 = \lim_{T\to\infty} \sum_{t-1}^{T} E(\mathrm{u}_{\mathrm{t}}^2) \tag{6.4}$$

This gives consistent estimators of

$$S_u^2 = T^1 \sum_{t=1}^{T}(u_t^2), \;\; S_{Tl}^2 = T^{-1} \sum_{t=1}^{T}(u_t^2) + 2T^{-1} \sum_{t=1}^{l} \sum_{t=j+1}^{T} u_t u_{t-j} \tag{6.5}$$

where l is a lag truncation parameter, capturing the autocorrelation in the residuals. The Phillips Z-test is the given by:

$$Z(\tau_\mu) = (S_u/S_{Tl})\tau_\mu - \frac{1}{2}(S_{Tl}^2 - S_u^2)\left\{S_{Tl}[T^2 \sum_{t=2}^{T} (y_{t-1} - y_{-1})^2\,]^{\frac{1}{2}}\right\}^{-1} \tag{6.6}$$

where τ_μ is the t-statistic from testing H0: $\delta = 0$ in equation 6.2. S_{Tl} is the long-run variance or bandwidth and needs to be selected in advance, e.g. by the use of Newey-West. The same set of critical values, MacKinnon (1996), is used by E-views 5 for both the ADF test and the PP test.

6.2 Cointegration

One possible way to both study the long-run correlation with meaningful inference and use time series that are non-stationary occurs if the series are cointegrated. Two series are said to be cointegrated if they share the same stochastic trend. This trend is the long-run component that relates the two series to a long-term equilibrium situation. Hence, the series might deviate from this equilibrium in the short-run but with time they will return to the common trend.

For non-stationary series to be cointegrated they need to be integrated by the same order, i.e. they need to be differentiated the same amount of times in order to be stationary. Also, the error term from an estimation of the two variables should be stationary, $\varepsilon_t \sim I(0)$. In a simple case of two I(1) variables this can be expressed as:

$$y_t = \beta x_t + \varepsilon_t \tag{6.7}$$

$$y_t - \beta x_t = \varepsilon_t \sim I(0) \tag{6.8}$$

Cointegration then exists if β is such that the residuals are stationary. The parameter β is then called the cointegration parameter. More generally β could be a vector of coefficients and is then called the cointegration vector. If a long-run relationship is found between y_t and x_t then, including an intercept α_0, $z_t = y_t - \beta x_t - \alpha_0$ is the long-run equilibrium error. This measures the extent to which y_t deviates from its “equilibrium value” (Verbeek 2008:329).

6.2.1 Error-Correction Model

It is reasonable to believe that not all economic effects occur instantly. For example might investments, innovations or learning, from increases in trade, not affect the productivity level at once, as there are e.g. costs from implementing new technology. Therefore, the current value of y_t could be affected by past values of x_t. In order to model this dynamic, one can include both lagged values of y_t and x_t, to describe the short-run dynamics.

$$y_t = \alpha_0 + \beta_1 x_t + \beta_2 x_{t-1} + \alpha_1 y_{t-1} + \varepsilon_t \tag{6.9}$$

However, this approach is problematic as the regression could suffer from high levels of multicollinearity. And if the variables are non-stationary it also incorporates the problem mentioned previously.

In order to both adjust for these problems, keeping the dynamics of the model and take into account the long-run relations, it is more suitable to estimate an error-correction model (ECM). A specification of an ECM has been shown by Engel and Granger (1987) to be an equivalent to the existence of a cointegrating relationship. An ECM based on equation (6.9) is expressed as

$$\Delta y_t = \alpha_0 \Delta x_t - (1 - \alpha_1)[y_{t-1} - \beta_0 - \beta_1 x_{t-1}] + u_t \tag{6.10}$$

where $\beta_0 = \frac{\alpha_0}{1-\alpha_1}$ and $\beta_1 = \frac{\alpha_0+\alpha_1}{1-\alpha_1}$. The short term reaction of y_t to a change in x_t if described by α_0 and the long-run effect is captured by β_1. The long-term relationship is given by $[y_{t-1} - \beta_0 - \beta_1 x_{t-1}]$. Hence, if this is equal to zero, then y_t is in its long-run equilibrium. The term $(1 - \alpha_1)$ is known as the speed of adjustment and estimates how much of last periods deviation from the equilibrium that is adjusted in the present period. If $(1 - \alpha_1) \leq 0$, then y_t will not converge back to an equilibrium in case of a deviation, thus there is no meaningful long-run relationship, and therefore no cointegration.

This implies that the ECM only works satisfactory if the variables included in equation (6.9) are cointegrated. Otherwise the regression will be spurious. However, it is possible to extend the model both with stationary variables in the short-run regression and with an augmented number of lags, for a more specific dynamic structure.

The ECM described above has some shortcomings. First, it can only handle two non-stationary variables. Second, it assumes x_t to be treated as given. However, as seen in the theory section, there are strong reasons to believe that some variables might be simultaneously determined. Hence, a multivariate model is needed in order to treat more than one variable as a dependent variable.

6.2.2 Test for Cointegration

If the model only includes two non-stationary variables, that are integrated by the same order, the most common approach to investigate whether the series are cointegrated is to use the Engle-Granger approach (Engle - Granger 1987). This test consists of two steps: first, an OLS-estimation is made in accordance to e.g. equation (6.7); second, the residuals are tested for stationary. If the error terms are stationary then the series are said to be cointegrated. Commonly, the residuals are tested using the ADF-test.

This approach is only useful if the system consists of two cointegrating series. If there are $n > 2$ different variables the Engle-Granger approach is not applicable. In that case it is possible to find up to n-1 cointegration vectors that are I(0). Therefore, it is common to instead use the procedure developed by Johansen (1988).

6.2.3 Vector Autoregressions

In order to understand the Johansen approach it is useful to start by looking at a simple dynamic model with several endogenous variables. A common approach is to consider the Vector autoregressive (VAR) model. Generally this can be expressed as:

$$\mathbf{z}_t = \delta + \mathbf{A}_1 \mathbf{z}_{t-1} + \cdots + \mathbf{A}_k \mathbf{z}_{t-k} + \mathbf{u}_t \qquad \mathbf{u}_t \sim \mathrm{IN}(0, \Sigma) \tag{6.11}$$

where $\mathbf{z}_t$ is an $(n \times 1)$ vector of dependent variables, δ is a $(n \times 1)$ vector of constants, each $\mathbf{A}_j$ is a $(n \times n)$ matrix of coefficients and $\mathbf{u}_t$ is a $(n \times 1)$ vector of error terms, with a zero mean and a covariance matrix Σ. Each individual error term is assumed to be white noise. The VAR includes a number of k lagged values for all variables in all equations (Verbeek 2008:335f).

The numbers of lags of $\mathbf{z}_t$ to include in the VAR can be determined by some form of information criterion, for example SBC or AIC.

By including all endogenous variables in a system of equations, as a function of the lagged values of all endogenous variables in the system, the need for predeterminations of the structure is avoided.

6.2.4 Vector Error Correction Model

Out of the specification of the VAR in eq. (6.11) it is possible to obtain a vector error correction model (VECM) in the same way as in the two-variable case in eq. (6.10).

$$\Delta \mathbf{z}_t = \mathbf{\Gamma}_1 \Delta \mathbf{z}_{t-1} + \cdots + \mathbf{\Gamma}_{k-1} \Delta \mathbf{z}_{t-k+1} + \mathbf{\Pi} \mathbf{z}_{t-1} + \mathbf{u}_t \tag{6.12}$$

Here $\mathbf{\Gamma}_i = -(\mathbf{I} - \mathbf{A}_1 - \cdots - \mathbf{A}_i)$ with $(i = 1, \dots, k-1)$ and $\mathbf{\Pi} = -(\mathbf{I} - \mathbf{A}_1 - \cdots - \mathbf{A}_k)$. In a similar way as in equation (6.10) this specification includes both the short-run and the long-run effect on $\mathbf{z}_t$, by estimations of $\hat{\mathbf{\Gamma}}_i$ and $\hat{\mathbf{\Pi}}$**.** The long-run effect can be seen in $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where $\boldsymbol{\alpha}$ is the speed of adjustment parameter and $\boldsymbol{\beta}'$ consists of long-run coefficients. In an example with $n = 3$ variables and one period lags in first difference this could be expressed as

$$\begin{bmatrix} \Delta y_t \\ \Delta x_t \\ \Delta \epsilon_t \end{bmatrix} = \mathbf{\Gamma}_1 \begin{bmatrix} \Delta y_{t-1} \\ \Delta x_{t-1} \\ \Delta \epsilon_{t-1} \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{21} & \beta_{31} \\ \beta_{12} & \beta_{22} & \beta_{32} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \\ \epsilon_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{xt} \\ u_{\epsilon t} \end{bmatrix} \tag{6.13}$$

In order for $\mathbf{u}_t$ to be white noise and stationary all $\Delta \mathbf{z}_{t-i}$ terms must be stationary and $\mathbf{\Pi} \mathbf{z}_{t-1}$ must also be stationary.

In addition to the model specified in equation (6.12) it is possible to include deterministic components as a part of both the short term VAR and the long-term cointegration relationship.

$$\Delta \mathbf{z}_t = \mathbf{\Gamma}_1 \Delta \mathbf{z}_{t-1} + \cdots + \mathbf{\Gamma}_{k-1} \Delta \mathbf{z}_{t-k+1} + \boldsymbol{\alpha} \begin{bmatrix} \boldsymbol{\beta} \\ \mu_1 \\ \delta_1 \end{bmatrix} \tilde{\mathbf{z}}_{t-1} + \boldsymbol{\alpha}_\perp \mu_2 + \boldsymbol{\alpha}_\perp \delta_2 t + \mathbf{u}_t \tag{6.14}$$

where $\tilde{\mathbf{z}}'_{t-1} = (\mathbf{z}'_{t-1}, 1, t)$. Normally μ_1 is included since the level data seldom has a zero mean. Hence, a model (M1) without any deterministic components is not of interest. There are three other models that are more applicable. First, (M2) if the level data does not have a time trend then $\delta_1 = \delta_2 = u_2 = 0$. The intercept is then only included in the long-run model. Second, (M3) if there is a linear trend in the level data, a intercept is included in the short-run model so that $\delta_1 = \delta_2 = 0$. Third, (M4) if there are some not included long-run growth a linear time trend (δ_1) is included in the cointegration vector. In addition a model (M5) with all components can be used. However, this would imply an ever increasing growth rate and is not theoretically reasonable (Harris – Sollis 2003:132f). Therefore, I only use model M2, M3 and M4.

For a full specification of the VECM it is important to choose both the set of deterministic components and the lag length of the short-run terms. The former is done as a part of the Johansen approach and the latter can be selected by the use of information criterion or by the significance of the different lag orders. Also it is important to include sufficient lags for the VECM residuals to be white noise.

6.2.5 Johansen Approach

The Johansen approach is based on the specification of the VECM in equation (6.14). The test is used to determine the number of cointegration relationships between $\mathrm{n} > 2$ endogenous variables, as there can be up to $(\mathrm{n}-1)$ relationships. This is important in order to find a relation where the residuals $\mathbf{u}_\mathrm{t}$ are white noise.

There are three different situations where $\mathbf{u}_\mathrm{t} \sim \mathrm{I}(0)$ and white noise: First, if there are no cointegration at all, but the variables are I(1), then there might still be short-run relations and a VAR in first difference should be estimated. Second, if all variables are stationary. But this should not be the case in this study. Third, if there exists up to $(\mathrm{n}-1)$ cointegration relationships. This is tested by finding the number of linearly independent columns (r) in the $\mathbf{\Pi}$ matrix, i.e. the number cointegration vectors. Then there must be $\mathrm{r} \leq (\mathrm{n}-1)$ cointegration vectors in $\boldsymbol{\beta}$. If this is true the VECM is said to have reduced rank (Harris - Sollis 2003:111, 122).

To find the rank of $\mathbf{\Pi}$ the Johansen approach is to use two tests: the trace test and the maximum eigenvalue test. The trace test takes the null hypothesis of $\mathrm{H}_0\!: \mathrm{r} \leq \mathrm{r}_0$ against $\mathrm{H}_1\!: \mathrm{r}_0 < \mathrm{r} \leq \mathrm{n}$, where r_0 is tested and r is the true number of existing relationships. The test uses the statistic:

$$\lambda_{\mathrm{trace}}(\mathrm{r}_0) = -\mathrm{T} \sum_{\mathrm{j}=\mathrm{r}_0+1}^{\mathrm{n}} \log\,(1-\hat{\lambda}_\mathrm{j}) \qquad (6.15)$$

where $(\mathrm{r}_0 = 1, \dots, \mathrm{n}-1)$, T is the sample size and $\hat{\lambda}_\mathrm{j}$ are the estimated eigenvalues. The maximum eigenvalue test uses $\mathrm{H}_0\!: \mathrm{r} \leq \mathrm{r}_0$ and $\mathrm{H}_1\!: \mathrm{r} = \mathrm{r}_0 + 1$ and the statistic:

$$\lambda_{\max}(\mathrm{r}_0) = -\,\mathrm{T}\log\big(1-\hat{\lambda}_{r_0+1}\big) \qquad (6.16)$$

6.3 Causality Tests

The approach of using a VECM makes two forms of causality tests possible. First, the matrix **Π** contains information about the long-run relationship. Second, Granger causality tests could be performed by testing the joint significance of the coefficient of lagged variables in first difference. This will give support to conclusions about the short-run structure.

The long-run test is done by looking at the significance of the one period lagged error corrections term (ECT). If it is significant the hypothesis of weak exogeneity can be rejected for that variable (Harris – Sollis 2003:135f). Also, it in interpreted as being possibly caused by any significant variable in the description long-run relationship.

The short-run test uses the χ^2 (Wald) statistics for the joint coefficients of the lagged variables in first difference. The null hypothesis is that variable X does not Granger cause variable Y. An advantage of using Granger-causality tests in a VECM is that the pairwise effect is calculated given past information from all variables. This approach is used by e.g. Thangavelu and Rajaguru (2004) and Awokuse (2008).

7 Data Material

In this section the data material used in the empirical study is explained. A list of data sources can be found in appendix B and a technical description of the capital stock series calculations can be found in appendix E.

All data is collected from either World Development Indicators (WDI) or Penn Worlds Tables 6.3 (PWT). Hence, all series are freely available. A list of measures and sources is located in appendix B. All series are yearly and are estimated from 1960 to 2007. Some data series, underlying the calculations of TFP, are however collected from 1950. It is recognised that long series are needed in time series analysis. However, Hakkio and Rush (1991) and Campbell and Perron (1991) suggest that additional observations from a higher frequency makes little difference. Also, a long time period is of importance when studying long-run growth relations.

All data are in constant dollars for comparability and uses year 2000 as base year. The series from PWT are deflated from year 2005 to year 2000, by the use of the difference between nominal and real GDP per capita series presented in the dataset, before any further calculations were undertaken.

All variables are in levels and natural logarithms. Hence, the coefficients give the elasticity. This follows previous studies, is in accordance with the calculation of TFP levels and is reasonable from looking at the data series, as some series show an exponential development. Also, linear long-run equilibriums in levels are interpreted as finding a common growth rate. The choice to express the series in levels and not growth rates is suitable since the VECM needs the series to be non-stationary. And a use of a VAR setup is needed in order to account for the endogeneity between the variables. The use of growth rates would then limit the VAR to only account for the short-run effect. Because we are interested in the long-run relations, this would limit the analysis fundamentally.

In order to investigate the relation six countries were selected: Canada, France, Italy, Japan, the United Kingdom and the United States. The selection was based on comparability with previous studies. And that there are, to my knowledge, no study investigating the trade growth relationship including both imports, exports and TFP in a VAR framework. The selection includes the major economies in OECD. Germany is left out because of data limitations since PWT's series began in year 1970.

7.1 Total Factor Productivity

As data for calculated TFP levels are not available, all series are calculated from the Penn World Table 6.3 dataset. The dataset contains yearly aggregated data series from 1950 to 2007. TFP is calculated as a residual of a standard Cobb-Douglas production function, in accordance with growth accounting.

$$Y_t = A_t^{1-\alpha} L_t^{1-\alpha} K_t^{\alpha} \tag{7.1}$$

Taking the natural logarithms of the variables and solving for A_t give level series of TFP in ln-form. This requires level series of capita and labour, as well as assumptions about the capital shares (α). I have chosen to use a general standard of 0.32 as the capital share. This is in line with estimates of the OECD average from 1960 to 2007 (Aghion - Howitt 2009:109). The labour force is calculated by the use of Real GDP per capita, Population and Real GDP per worker. The calculations of the capital stock adopt the perpetual inventory method with steady-state initial capital stock and follow a procedure described in Limam and Miller (2004:10ff), who uses the PWT 5.6 dataset. Their estimates are in line with studies of Benhabib and Spiegle (1994) and the reported capital stocks in PTW 5.6. For a technical description of the capital stock calculation see appendix E. The capital stock is assumed to depreciate 7-percent per year and the initial capital stock is calculated for 1950. The significance of the assumptions underlying the initial capital stock is reduced since the first observation used in the analysis is from 1960. This is appropriate since the results otherwise might be considerably affected by these assumptions. The time series from PWT 6.3 used in the calculations are: Real GDP per capita, Investment share of real GDP per capita, Population and Real GDP per capita growth. Also the assumption of the average world GDP per capita growth of 0.04 during the time period was controlled using WDI series for aggregated real GDP growth rates.

7.2 Export and Import

The total levels of export and import were obtained from World Development Indicators[20]. The variables include the value of merchandise, freight, insurance, transport, travel, royalties, license fees, and other services, such as communication, construction, financial, information, business, personal, and government services. They exclude compensation of employees and investment income and transfer payments. The data is expressed in constant $2000.

The choice to use total export and import is based on both theoretical and empirical arguments. As stated in the theoretical equation underlying the analysis both the flow of export and import are theoretically important in determining productivity. Some studies use a less aggregated measure of imports, as it is primarily intermediate goods that are assumed to account for the growth enhancing effects. However, there are no reason to believe that this is the only effect, rather it is possible that also imports of final goods is important as a transmitter of ideas and technology, much in line with the theoretical model. Also, data on imports of intermediate goods are not available for the period of choice. Moreover, this selection of variables follows the general choice of previous studies, for example Thangavelu and Rajaguru (2004).

7.3 Investments

The following underlying series of investments were collected from PWT 6.3: Investment share of real GDP per capita, Real GDP per capita and Population. The investment shares are based on the resource constraint and include all national expenditures on investment. The final series are the level of total gross investments, expressed in $2000 and is the sum of private and public investments.

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[20] OECD.STAT does not provide data for total export or import for the period of interest. However, WDI data is based on OECD national accounts data and World Bank national accounts data.

8 Empiric Results

This section presents the results of the estimations made in the Unit root tests, Johansen cointegration test and the final estimations of the VAR and ECM-models for the long- and short-run relations. Extended tables for results and diagnostic tests are to be found in appendix C.

8.1 Order of Integration

To begin with, all time series need to be tested for the order of integration, as this is essential for determining the time characteristics. However, economic studies must also take theory into consideration in order to make conclusions about the long-run properties. Therefore, I have chosen not to test the series for a quadratic trend, as this would imply an acceptation of ever increasing or decreasing growth in the series. The results from the Augmented Dickey Fuller test and the Phillip-Perron test is presented in table 8.1 for all variables. The results are presented with a constant and trend for the level series and with a constant for the differentiated series. The lag structure (p) in the ADF-test has been determined with a maximum lag order of p=12 and the use of Schwartz Information Criterion. For the Phillip-Perron estimations the Newey-West bandwidth (b) was automatically selected and the Bartlett kernel method was used. In both cases the critical values are MacKinnon (1996) one-sided critical values.

The expectation of the test is that all series should be integrated of order one, I(1). This is since the level series of choice normally are found to be non-stationary, and it is not theoretically likely that any series exhibit an everlasting exponential development. However, since this is only a small sample, graphical illustrations show not only steady increasing trends in the series but also in some cases a tendency for the rate to be decreasing. This could result in a preferred quadratic trend.

Table 8.1: Unit root test results

H0: Variable has a unit root

	Canada						**France**					
	ADF			**PP**			**ADF**			**PP**		
Level	**DF-stat.**	**Prob.**	**p**	**PP-stat.**	**Prob.**	**b**	**DF-stat.**	**Prob.**	**p**	**PP-stat.**	**Prob.**	**b**
TFP	-3.15	0.11	1	-2.38	0.38	1	-3.46	0.06**	1	-3.46	0.06**	2
EXPORT	-2.38	0.39	1	-1.62	0.77	0	-0.94	0.94	0	-1.21	0.90	3
IMPORT	-2.01	0.58	0	-2.16	0.50	1	-2.26	0.45	0	-2.26	0.45	1
INVEST	-2.71	0.24	0	-2.75	0.22	1	-2.85	0.19	0	-2.85	0.19	1
1:st diff												
TFP	-4.88	0.00*	0	-4.91	0.00*	2	-5.07	0.00*	0	-5.03	0.00*	2
EXPORT	-4.63	0.00*	0	-4.56	0.00*	4	-4.69	0.00*	0	-4.75	0.00*	3
IMPORT	-5.79	0.00*	0	-5.76	0.00*	5	-6.22	0.00*	0	-6.26	0.00*	3
INVEST	-7.14	0.00*	0	-7.20	0.00*	4	-5.81	0.00*	0	-5.81	0.00*	0

	Japan						**United Kingdom**					
Level	**DF-stat.**	**Prob.**	**p**	**PP-stat.**	**Prob.**	**b**	**DF-stat.**	**Prob.**	**p**	**PP-stat.**	**Prob.**	**b**
TFP	-2.70	0.24	4	-3.19	0.10**	3	-2.42	0.37	1	-2.10	0.53	2
EXPORT	-2.21	0.47	0	-2.21	0.47	0	-1.91	0.63	0	-2.09	0.54	3
IMPORT	-1.93	0.62	1	-2.47	0.34	0	-1.75	0.71	0	-1.88	0.65	3
INVEST	-2.35	0.40	4	-2.47	0.34	3	-1.88	0.65	0	-2.01	0.58	2
1:st diff												
TFP	-2.73	0.08**	3	-4.38	0.00*	3	-5.45	0.00*	0	-5.34	0.00*	7
EXPORT	-5.35	0.00*	0	-5.58	0.00*	4	-5.95	0.00*	0	-5.91	0.00*	2
IMPORT	-4.06	0.00*	1	-5.57	0.00*	2	-5.40	0.00*	0	-5.25	0.00*	8
INVEST	-1.66	0.44	4	-4.18	0.00*	3	-6.20	0.00*	1	-5.73	0.00*	8

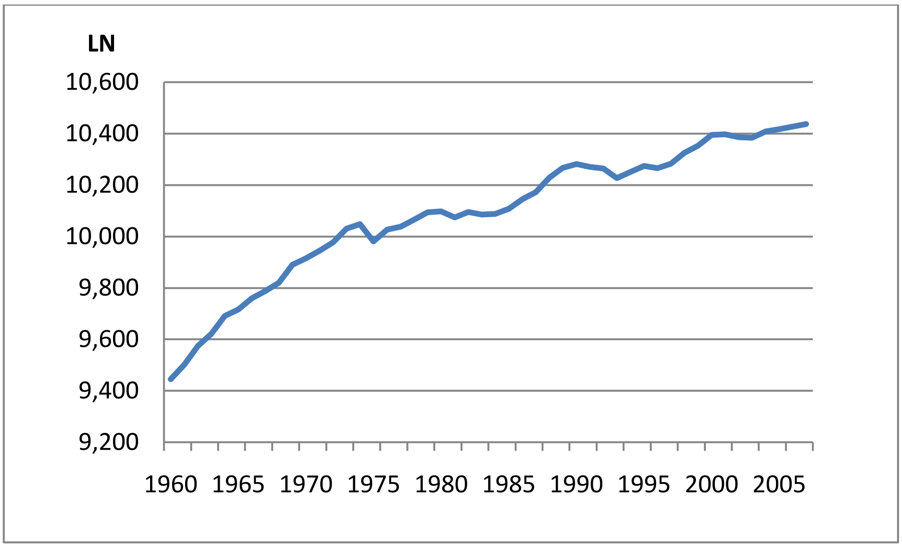
	United States					
Level	**DF-stat.**	**Prob.**	**p**	**PP-stat.**	**Prob.**	**b**
TFP	-3.60	0.04*	1	-2.78	0.21	3
EXPORT	-3.15	0.11	1	-2.08	0.54	0
IMPORT	-2.70	0.24	1	-2.40	0.38	2
INVEST	-4.11	0.01*	1	-3.00	0.14	6
1:st diff						
TFP	-5.42	0.00*	0	-5.28	0.00*	6
EXPORT	-4.80	0.00*	0	-4.61	0.00*	5
IMPORT	-5.63	0.00*	0	-5.61	0.00*	9
INVEST	-6.25	0.00*	0	-6.84	0.00*	2

*significant at 5%, ** significant at 10%

The results of the unit root tests are almost in line with the expectation in all cases. There are however some series that exhibit a different pattern.

For France neither ADF nor the Phillip-Perron test can reject the null hypothesis for TFP in levels at a five percent level. But at 10 percent this is rejected in both cases. The Akaike Information Criterion does however point to an inclusion of three lags, which also makes the series insignificant. Taken together, and noting that it is less harmful to assume the series to be wrongly non-stationary, I assume the series to have a unit root in levels. Also looking at the graphical illustration indicates that there is some sort of non-stationarity. The result might occur because the series is concave in logarithms.

Figure 8.1: Total Factor Productivity in Levels for France



The Japanese TFP series in the Phillip-Perron test also signals stationarity in levels at 10 percent. The ADF-statistic does however clearly show that the null cannot be rejected. Hence, I assume the series to be non-stationary. Also, the ADF can't reject the null in first difference for INVEST. But, with the same reasoning I conclude that the variable is integrated of order I(1).

In two cases does the ADF reject the null in levels for the United States: TFP and INVEST. The Phillip-Perron test does not however support this and I assume the series to be I(1).

The results from the unit-root tests are robust to lag selection by AIC. The recommended lag structure increased in some cases but did not change the conclusions. The result is also robust to the selection of deterministic components. The significance of trends in the level estimations indicates that a constant should be present in the short-run part of the cointegration equation, assuming a linear trend in the data generation process.

To conclude, all series are found to be integrated of order I(1). This enables a straightforward estimation of the number of cointegration relationships, using the Johansen approach.

8.2 Cointegration Structure

In the same way as with the unit root test economic theory has to be applied to the conclusions of the long-run relationships. Also, it is not likely for single lagged years of the independent variables to have a significant impact of the present productivity further back than 12 years. As stated above, the relationships are not expected to follow a long-run concave or convex trend. In addition, it is important for the results to be reliable that the presence of serial correlation and non-normality are dealt with, as the tests are sensitive to these aspects (Johansen & Juselius, 1990). The results are generally also sensitive to the selection of deterministic components.

8.2.1 Selection of the Lag Order

In order to select the number of lags in the VAR different information criteria could be used. The Johansen procedure requires a specification of the common number of first difference lags in order to test for reduced ranks. Therefore, I estimated a VAR in first difference and performed a Lag Order Selection Criteria test for each country. The results are given in table 8.2.

Table 8.2: Lag Order Selection

	FPE	AIC	SBC	HQ
Canada	0	8	0	0
France	0	8	0	0
Japan	1	8	0	8
United Kingdom	0	8	0	8
United States	1	8	1	8

FPE: Final prediction error

AIC: Akaike information criterion

SBC: Schwarz information criterion

HQ: Hannan-Quinn information criterion

The test is limited by the number of observations to include eight lags. This should be enough to capture any expected lag order. The results do not however express any clear recommendation. The large divergence between the criterions points to a potentially too large model, in relation to the number of observations[21]. This motivates the choice not to use any further feasible explanatory variable, and draws attention to a possible limitation of previous studies as well as indicating a conceivable need for a large lag length. The AIC does however not correct for the increase in R^2 achieved simply by adding more lags. Therefore I chose to start searching for an appropriate model by using the number of lags recommended by SBC.

8.3 Estimation Procedure

In order to find a model with white noise residuals I evaluate the residuals from the VAR or VECM. If the residuals show signs of serial correlation or non-normality I re-estimate an equation including an additional lag. Since the procedure is sensitive to the selection of the lag order it is plausible that the rank order and deterministic components, recommended by the Johansen test for reduced rank, change with increased lags. If this happens I prefer to use the new set of lags and deterministic components in the specification of the VECM. Given that an additional lag does not solve the problem, the procedure is repeated.

To know the rank order and the recommended deterministic components, the Johansen approach, described in section 6.2.5, is performed.

8.4 Johansen Test for the Number of Cointegration Relationships

In addition to the lag length, also the set of deterministic components must be chosen in order to test for the number of reduced ranks. This can be done in several ways: by using economic theory, the significance of the coefficients, different information criterion or the Pantula principle[22] recommended by Johansen (1992). The three out of five optional models plausible to theory and practice are described in section 6.2.4. By estimating them and applying the

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[21] See Harris and Sollis (2003:117) for an example with a similar conclusion. Also, limiting the test to fewer lags then eight does give more similar results.

[22] See Harris and Sollis (2003:134f) for an explanation and an applied example of the principle.

Pantula principle as well as studying the cointegration graphs for potential improvements I form the conclusions of the test. A summary of the Johansen test results for each country is given in table 8.3. Tables including the full results can be found in appendix C.

The table shows the first rank where the tests fail to reject the null hypothesis, i.e. the found number of cointegration relationships. The critical values used are the 5 percent level Osterwald-Lenum (1992) critical values, provided by E-views 5.

Following the Pantula principle I base my conclusions on the trace statistic. As can be seen from the table, the test results depend on the number of lags included and differs between the countries. The final model of choice must however take into account the diagnostics of the residuals. Therefore I conducted both a LM autocorrelation test and normality test for each further step in the decision procedure. When possible, given the number of lags, I also performed a heteroskedasticiy test. Diagnostic tables underlying the conclusions can be found in appendix C.

As stated previously, when the Johansen test results in $r = 0$ long-run relationships a VAR in first difference should be used to test for the short-run dynamics. If the test indicates reduced ranks a VECM is instead estimated.

First, for Canada serial correlation is found in the first lag when using the recommended lag length of SBC of zero lags. Following the procedure an inclusion of three lags seems necessary to avoid all serial correlation. At this lag length the tests can not reject the hypothesis of no serial correlation, normality or homoskedaticity. The cointegration test on this level gives a result of no long-run relationships. Therefore I conclude that a VAR in first difference with a constant, including three lags, should be used.

Second, a model including both a constant and a trend in the cointegration vector is recommended at the first step of zero lags for France. But, the residuals suffer from serial correlation and a skewness problem in the first component. Three lags need to be included for not rejecting the null hypothesis of no serial correlation at a 10 percent level and lose the skewness problem. The test result shows problems with the kurtosis but this is overlooked since the Johansen approach is not particularly sensitive to this problem (Johansen - Juselius 1990). So for France a VECM including a constant in both the short- and long-run part (M3) with three lags and one cointegration vector will be estimated.

Table 8.3: Summary of Result from the Johansen Test for Reduced Ranks

Canada					France				
Lag order	**Test statistic**	**M2**	**M3**	**M4**	**Lag order**	**Test statistic**	**M2**	**M3**	**M4**
0	Trace	3	0*	0	0	Trace	3	3	2*
	Max	3	1	1		Max	3	1	1
1	Trace	3	0*	0	1	Trace	2	2	1*
	Max	2	0	0		Max	2	2	1
2	Trace	1	0*	0	2	Trace	1*	1	1
	Max	0	0	0		Max	1	0	1
3	Trace	3	0*	0	3	Trace	3	1*	2
	Max	0	0	0		Max	1	1	0
4	Trace	4	1*	1	4	Trace	2*	2	2
	Max	2	1	1		Max	1	1	1
5	Trace	3*	4	3	5	Trace	2*	2	3
	Max	3	2	3		Max	2	2	3
6	Trace	3*	4	3	6	Trace	4*	4	4
	Max	3	4	3		Max	4	4	4

Japan					United Kingdom				
Lag order	**Test statistic**	**M2**	**M3**	**M4**	**Lag order**	**Test statistic**	**M2**	**M3**	**M4**
0	Trace	3	2*	2	0	Trace	1	0*	0
	Max	3	3	1		Max	1	0	0
1	Trace	3	2*	2	1	Trace	1	0*	0
	Max	3	3	1		Max	1	0	0
2	Trace	1*	1	1	2	Trace	1	0*	0
	Max	1	1	1		Max	1	0	0
3	Trace	2*	2	2	3	Trace	2	1*	1
	Max	0	0	0		Max	1	0	0
4	Trace	1*	1	2	4	Trace	1*	1	1
	Max	1	1	1		Max	1	1	1
5	Trace	2*	2	3	5	Trace	1*	1	2
	Max	1	1	1		Max	1	1	2
6	Trace	4	2*	2	6	Trace	2*	2	3
	Max	2	2	2		Max	2	2	3

United States									
Lag order	**Test statistic**	**M2**	**M3**	**M4**	**Lag order**	**Test statistic**	**M2**	**M3**	**M4**
0	Trace	1	0*	1	4	Trace	3	1*	2
	Max	1	0	1		Max	3	2	2
1	Trace	2	1*	1	5	Trace	3	1*	3
	Max	2	1	1		Max	3	2	3
2	Trace	2	1*	1	6	Trace	3	2*	3
	Max	2	1	1		Max	3	2	3
3	Trace	3	1*	2	Note: Numbers are significant cointegration vectors (r)				
	Max	0	0	1	* Preferred model using the Pantula principle				

Third, for Japan moving to a situation with three lags the Pantula principle indicates that M2 should be used with $r = 2$. By looking at the cointegration graph, it is readily seen that a constant in the short-run relationship should improve the results. This is confirmed by the significance. However, there are signs of serial correlation and using the same procedure for the forth lag solves that problem. As the residuals test indicates white noise I conclude that a VECM using M3 and four lags and $r = 1$ is appropriate. The inclusion of a fourth lag is supported by a lag exclusion test.

Forth, following the estimation procedure six lags are needed for the United Kingdom. The Pantula principle indicates that M2 should be used with $r = 2$, however it was clearly seen from the cointegration graphs that the significance of a constant in the long-run relationships should be tested. This show the M3 is more suitable. Hence, I use a VECM with two cointegration relationships, six lags and a constant in both parts of the equation.

Finally, the SBC in table 8.2 recommend that one lag should be included for the United States. And the the Pantula principle gives us a recommendation of $r = 1$. There are however problems with serial correlation in the first four steps. But with five lags, using the recommended rank of one by the trance statistic, the problems disappear. A lag exclusion test support the inclusion of a fifth lag and changing into two cointegration relationships (because the trace statistic is very close to rejecting one) give worse residuals. Also the inclusion of six lags is not supported by a lag exclusion test. Hence I conclude that a VECM with one long-run relationship, five lags and using M3, is advisable.

8.5 Results for the Long-run Relationships

Cointegration relationships were found for all countries except Canada. Hence, Canada will only be included in the analysis of the short-run effects. For the United Kingdom two cointegration relationships were found, which is theoretically problematic since it implies that there are two different growth paths present. But similar result has been found in previous studies, e.g. Thangavelu and Rajaguru (2004). This implies that the long-run matrix $\mathbf{\Pi} = \boldsymbol{\alpha\beta}'$ is generally expresses for the United Kingdom as:

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \\ \alpha_{41} & \alpha_{42} \end{pmatrix} \boldsymbol{\beta}' = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \\ \beta_{31} & \beta_{32} \\ \beta_{41} & \beta_{42} \end{pmatrix} \tag{8.1}$$

and for the others as:

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \\ \alpha_{31} \\ \alpha_{41} \end{pmatrix} \boldsymbol{\beta}' = \begin{pmatrix} \beta_{11} \\ \beta_{21} \\ \beta_{31} \\ \beta_{41} \end{pmatrix} \tag{8.2}$$

where each row includes the results for a given variable and each column is a separate cointegration relationship. If the variables are structured according to $\boldsymbol{\beta} = (\text{TFP}, \text{EXPORT}, \text{IMPORT}, \text{INVEST})$, the normalized[23] results sets $\beta_{11} = 1$ in the single case, and in the case of the United Kingdom: $\beta_{11} = 1, \beta_{12} = 0, \beta_{21} = 0$ and $\beta_{22} = 1$. The results for the cointegration vectors and the error correction terms (ECT) are as presented below.

The inference is based on a two sided t-test with the approximated five percent significance of 2 and 10 percent significance of 1.67. It can be clearly seen from the table that all coefficients of the one period lagged variable in levels, in the cointegration vector (β), are significant at a five percent level. The long-run cointegration relationship and is given by:

$$\left(\beta_{1j}TFP_{t-1} + \beta_{2j}EXPORT_{t-1} + \beta_{3j}IMPORT_{t-1} + \beta_{4j}INVEST_{t-1}\right) \tag{8.3}$$

Here, β_{ij} shows the effect on the cointegration relationship from a shock in variable i and cointegration vector j, or if the variable deviate from the common growth rate. This is given in the rows marked with β in table 8.4.

For France this implies that an increase in EXPORT and INVEST raises the cointegration relationship above the growth path and IMPORT reduces it. The same is given for the United States. For Japan the opposite is shown, so that increased exports and investments lowers the relationship.

The first relationship of the United Kingdom, normalised for TFP, shows that both IMPORT and INVEST decreases the level. And for the second cointegration relationship IMPORT still decreases the cointegration relationship below the growth path while increased

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[23] Normalizations do not chance the interpretation of the long-run relationships as it does not affect the significance of the variables, it does however affect the magnitude of the coefficients.

investments seems to have an increasing effect. The interactions between the cointegration relationships are unfortunately difficult to determine.

This implies that there is some long-run relation found in the data between exports, imports and productivity for all countries except Canada, and that all variables should be a part of these cointegration relationships. There are however some mixed signs of the coefficients.

The question of whether trade improves productivity is however not revealed in the cointegration description, as all variables are possible dependent variables. The Granger causality can however be seen in the reaction when the economy moves out of equilibrium, and in order for the system to hold there needs to be some dynamics explaining what will occur when the economy is not in steady-state.

Table 8.4: Long-run Results from the VECM-estimates

	France					**Japan**			
	TFP	**EXPORT**	**IMPORT**	**INVEST**		**TFP**	**EXPORT**	**IMPORT**	**INVEST**
β	1	3.8030	-5.8019	2.8044	β	1	-0.3179	0.5546	-0.8185
		(1.0424)	(1.4666)	(0.8329)			(0.0918)	(0.0807)	(0.1454)
		[3.6482]*	[-3.9561]*	[3.3671]*			[-3.4618]*	[6.8712]*	[-5.6306]*
ECT	-0.0261	-0.0512	-0.0812	-0.1971	ECT	0.0563	0.6739	0.3388	-0.0932
	(0.0532)	(0.0801)	(0.1099)	(0.1249)		(0.1849)	(0.3052)	(0.3291)	(0.2763)
	[-0.4905]	[-0.6400]	[-0.7384]	[-1.5781]		[0.3047]	[2.2082]*	[1.0293]	[-0.3373]
	United Kingdom					**United States**			
	TFP	**EXPORT**	**IMPORT**	**INVEST**		**TFP**	**EXPORT**	**IMPORT**	**INVEST**
$\beta1$	1	0	-0.3285	-0.0942	β	1	0.2845	-0.6297	0.2403
			(0.0185)	(0.0345)			(0.0299)	(0.0581)	(0.0844)
			[-17.777]*	[-2.7210]*			[9.5216]*	[-10.846]*	[2.8476]*
$\beta2$	0	1	-1.2692	0.6857	ECT	-0.2687	-1.0805	0.8409	-0.1924
			(0.1528)	(0.2852)		(0.5158)	(0.6137)	(0.7423)	(1.1792)
			[-8.3046]*	[2.4041]*		[-0.5210]	[-1.7607]**	[1.1329]	[-0.1631]
$ECT1$	-4.9154	1.0418	-4.7990	-7.9709					
	(0.9928)	(1.8501)	(2.0301)	(3.5453)					
	[-4.9511]*	[0.5631]	[-2.3640]*	[-2.2483]*					
$ECT2$	-0.6405	-0.4351	-0.6581	-1.0531					
	(0.1542)	(0.2873)	(0.3153)	(0.5506)					
	[-4.1546]*	[-1.5143]	[-2.0873]*	[-1.9127]**					

* Significant at 5 % ** significant at 10%, Numbers in ( ) are standard deviation and numbers in [ ] are t-statistics

By looking at the lagged error correction terms, i.e. the adjustment parameters, it is possible to analyse what happens in case of disequilibrium, i.e. what variables bring the economy back towards the growth path in case of a positive shock above that level, and whether the response is an increase of decrease in the growth of the variable. Therefore, a significant lagged ECT implies that past equilibrium errors affect current outcomes of that variable (Awokuse 2007:391). The ECT_{ij} is given by:

$$a_{ij}\left(\beta_{1j}TFP_{t-1} - \beta_{2j}EXPORT_{t-1} - \beta_{3j}IMPORT_{t-1} - \beta_{4j}INVEST_{t-1}\right) \qquad (8.4)$$

where α_{ij} is the ECT coefficient (or the speed of adjustment) for variable i in cointegration relation j, and the expression in the parenthesis is equal to zero when the economy is in equilibrium.

For France there is no ECT that is significant at a 10 percent level. Therefore, it is difficult to conclude what dynamics are present, and the result indicates the absence of a long-run relationship between trade and productivity. However, INVEST seems to be the only ECT that is close to be significant. Hence, it could be expected that investments adjust to correct any deviation. But, since there is no significance, no conclusions are possible. It is more possible that the method used has trouble detecting the true number of cointegration relationships or that the specification of the deterministic components is wrong. By looking at the cointegration graph D.1 in appendix D it is visibly seen that the calculated relationship, following equation (8.3), starts above the equilibrium growth path and tends to follow a negative trend to finish below the equilibrium level. This could imply that a trend should be included in the short-run model. However, this is not in line with the theoretical expectations, as it would imply an ever decreasing growth rate. The significance of the model (M5) is tested and both the trend and some ECTs are significant. The presence of the trend is probably due to the short observed time period.

In the Japanese case only EXPORT is significant [2.2082] at a five percent level. This indicates that every adjustment required from a deviation from the equilibrium growth path can only be statistically explained by exports. And, as seen in the size of the coefficient, 0.67, the speed of adjustment is rather fast. The interpretation is that a positive shock in TFP (or IMPORT) causes EXPORT to increase as seen in the positive ECT coefficient. But according to equation (8.3) an increase in EXPORT should have a lowering effect on the relationship. This is seen in $\beta_{21} = -0.3179$, and an increased growth of export will adjust the relationship back towards its long-run equilibrium path. The same happens if there is a negative shock in the investments. This supports the hypothesis of productivity-led export growth found in the expectation interpretation of the theoretical model and corresponds well to the findings for Japan in Marin (1992) and Thangavelu and Rajaguru (2004) and partly to the short-term results in Awokuse (2006). The results also indicate the existence of import-led export growth, but this cannot be explained by a productivity increasing effect from imports that in turn increase exports, as the theoretical model implies.

For the United Kingdom, with two cointegration relationships, there are numerous possible interactions, making it difficult to determine any clear relationship. A long-run shock affecting one relationship might not only affect the adjustment parameters in that relation. However, it is notable that the ECT term for EXPORT is not significant in any vector. But, the ECT for both IMPORT and TFP are significant, hence, changes in these variables could be caused by each other or by any of the other variables. Any, conclusive causal effects are however hard to determine as the interaction between the two vectors needs to be considered and the results indicate a complex web of interactions. But the results support the conclusion that there are some cointegration relationships and therefore that there is some form of causation between the variables.

The United States exhibit only significance for the EXPORT ECT. However, in this case a positive shock, from an increase in TFP (or investments), cause export growth to decrease $\alpha_{21} = -1.08$. In this case a decrease in exports do however increases the cointegration relation as seen in $\beta_{21} = 0.2845$. This will bring the economy towards the equilibrium path. The causal effect from productivity is in this case negative, which is not expected from the theoretical discussions. Mixed signs of the coefficients are however found in Marin (1992), and Thangavelu and Rajaguru (2004) found a negative effect from labour productivity to export growth in four cases. The ECT is however only significant at a 10 percent level.

As a complement to the results from residual diagnostic tests, discussed in section 8.4 the cointegration vectors can be examined graphically. Two aspects need to be considered: (1) the macroeconomic series used in the study is expected to adjust slowly to its equilibrium level, and (2) In order for cointegration relationships to be valid the cointegration series should be stationary. The graphs are presented in appendix D and shows that there are some differences between the countries. The graph for France has already been commented above. Graph D.2 for Japan reveals that the cointegration relationship does seem to exhibit some form of non-stationarity. Even though the equilibrium level is crossed, there is no indication of the series to return to equilibrium after 2007. This indicates that the long-run relationship is weak. In Graph D.3 and D.4 the relationships for the United Kingdom are presented. These graphs correspond fairly well to the expectations of the series, as they alter between periods above and below the equilibrium level. The same conclusion is reached by looking at graph D.5 for the United States.

As mentioned, the choice of econometric method was based on the assumption of possible endogeneity between variables. In order for a variable to be weakly exogenous the adjustment parameter should be insignificant, i.e. no long-run Granger causality exists. However, it is readily seen that all variables ECTs are significant in at least one case. Hence, no variable should be generally treated as exogenous.

8.6 Results for the Short-run Relationships

The results from the short-run part of the ECMs, and the VAR used for Canada, are presented in table 8.5. The short-run dynamics are given by the lagged variables in first difference (see equation (6.14) in section 6.2.4). The magnitude of the effect is hard to quantify but the presence of a causal relationship can be established by a Granger causality test of the joint significance of the coefficients. The sign of the joint coefficient is given by the sum of the individual coefficient for each variable, and indicates if an increase in the independent variable causes an increase or decrease in the growth rate of the dependent variable.

The results, presented in table 8.5, differ largely between countries and variables, and generally few relationships are found. Although, in Canada, the United Kingdom and the United States some short-run effects are found at the five and 10 percent significance level. But, for France the null hypothesis of no Granger causality cannot be rejected in any case, hence no short-run relations can be found. Neither in Japan can any causality be found at a 10 percent level. Although, exports positive effect on productivity and investments are both borderline cases. Thus, it weakly supports for the hypothesis of export-led productivity growth.

For Canada there are some clearly significant results. It seems that productivity has a positive effect on both import growth and investment growth. This would support the firm based studies of productivity-led import growth, as it states that firms adjust their production and become more productive before they import. Also, investments have a negative effect on both import and productivity growth. This could indicate overinvestment in non-productivity increasing activities. In the theoretical model this implies a too large R, which reduces the recourses for importing. It could also be that investments tend to decrease before the production in a recession (and increase again before production does).

Table 8.5: Short-run Granger Causality Results

H0: X does not Granger cause Y

		Dependent variables (Y)							
Independent variables (X)		**TFP**		**EXPORT**		**IMPORT**		**INVEST**	
Canada	TFP	-		4.6813	(+)	17.9793	(+)	10.8833	(+)
				(0.1967)		(0.0004)*		(0.0124)*	
	EXPORT	2.9455	(-)	-		5.7002	(-)	3.7616	(-)
		(0.4001)				(0.1271)		(0.2884)	
	IMPORT	1.5678	(+)	1.8864	(+)	-		2.6145	(+)
		(0.6667)		(0.5963)				(0.4549)	
	INVEST	11.4928	(-)	5.3488	(-)	19.0883	(-)	-	
		(0.0093)*		(0.1480)		(0.0003)*			
France	TFP	-		1.8106	(+)	1.6936	(+)	4.6211	(+)
				(0.6126)		(0.6384)		(0.2017)	
	EXPORT	0.3289	(+)	-		2.1175	(+)	1.5366	(+)
		(0.9545)				(0.5484)		(0.6738)	
	IMPORT	0.6282	(-)	0.8256	(+)	-		2.0067	(-)
		(0.8899)		(0.8433)				(0.5710)	
	INVEST	2.1521	(-)	2.4854	(-)	0.8539	(-)	-	
		(0.5414)		(0.4779)		(0.8365)			
Japan	TFP	-		2.5740	(-)	1.0660	(-)	5.3807	(+)
				(0.6314)		(0.8996)		(0.2504)	
	EXPORT	7.6281	(+)	-		1.3565	(-)	7.4768	(+)
		(0.1062)				(0.8517)		(0.1127)	
	IMPORT	4.8592	(-)	3.1553	(-)	-		5.7943	(-)
		(0.3021)		(0.5322)				(0.2150)	
	INVEST	3.1524	(+)	3.8615	(+)	2.8872	(+)	-	
		(0.5326)		(0.4251)		(0.5769)			
United Kingdom	TFP	-		6.1511	(+)	12.6950	(+)	7.7110	(+)
				(0.4065)		(0.0481)*		(0.2601)	
	EXPORT	54.2309	(+)	-		13.4898	(+)	15.5428	(+)
		(0.0000)*				(0.0359)*		(0.0164)*	
	IMPORT	34.8550	(-)	9.7717	(-)	-		11.2017	(-)
		(0.0000)*		(0.1346)				(0.0823)**	
	INVEST	15.7571	(+)	11.6439	(-)	8.3409	(+)	-	
		(0.0151)*		(0.0704)**		(0.2142)			
United States	TFP	-		15.6307	(+)	8.2701	(+)	5.0585	(-)
				(0.0080)*		(0.1420)		(0.4088)	
	EXPORT	0.8940	(+)	-		6.0463	(-)	1.2904	(+)
		(0.9707)				(0.3017)		(0.9359)	
	IMPORT	2.7485	(+)	10.7764	(-)	-		2.9744	(+)
		(0.7387)		(0.0560)**				(0.7039)	
	INVEST	3.8269	(-)	9.8702	(+)	9.1377	(-)	-	
		(0.5746)		(0.0790)**		(0.1037)			

* significant at 5 % ** significant at 10%

Numbers in parenthesis is p-values, others are chi-square statistics

In the United Kingdom productivity does Granger cause import growth, giving additional support the productivity-led import growth hypothesis. Also exports seem to have a positive effect on import growth. Export also contributes to increased productivity as well as increased investments. In contrast imports lower both the productivity growth and investments. Investments appear to have two effects: a positive effect on productivity growth, in line with expectations, and a negative effect on export growth, however only significant at about seven percent level. This indicates that exports improve the productivity growth both directly and by increasing investment growth, indicating export-led productivity growth. These results for exports are clearly in line with the theoretical expectation.

Finally, productivity does have a positive and significant effect on export growth in the United States, giving the only short-run support for productivity-led export growth. Also, investment Granger cause increases in export growth, which is in line with the theoretical expectations. Import does however negatively affect export growth.

9 Conclusions

In this study I have investigate the endogenous relation between export, import, productivity and economic growth. There are theoretical reasons to believe that export and import can affect both long-run productivity and economic growth. The theoretical discussion as well as the derived theoretical model indicates that the long-run relation primarily depends on export and imports effect on productivity development. The theoretical model shows that productivity could then increase as a response to both improved incitement for domestic research or as an influence by technological spillovers from imports: either through imported goods of higher quality or from adaptation of foreign technology. It can also be shown that expectations can explain the plausible causal relation from productivity to export. Thus, there are several ways that trade and productivity can interact.

Also, using the Johansen approach and short-run Granger causality to empirically investigate the separate relations to export and import, does however not support these arguments. The included series are found to be non-stationary, indicating the need for a cointegration analysis for long-run studies. For the OECD countries studied, the Johansen approach finds significant support of long-run relationships in four out of five countries (not for Canada), with two relationships for the United Kingdom. The result for Japan is however weakened by the interpretation of the cointegration graph and for France no ECT was significant. Therefore, I conclude that there are only robust long-run relationships fund in two out of five countries.

Also, the results from the error correction analysis are highly divergent between the countries with no support for a long-run positive effect from export to TFP and neither for import to TFP. Only in Japan is the reversed causality of productivity-led export supported. Taken together the support of a long-run general relationship between export, import and TFP is weak and especially the support for a positive effect from trade to productivity is not found.

The short-run Granger Causality test does give some significant results. However, also in this case is the result varying between the countries and the variables. Therefore, neither for the short-run are any general conclusions possible. Productivity-led exports are only found in the United States and export-led productivity growth in United Kingdom. Productivity-led import is found in two cases, the United Kingdom and Canada, although this is not an aspect generally found in the macroeconomic literature and does not seem to have a profound support in the discussed theory. It is however found in firm based studies, and should therefore not be seen as a problematic result.

The study indicates that the large positive effects on growth from trade found in cross-country studies and expected from theory do not appear when focusing on the relation to productivity. This could indicate that the effect disappear when controlling for production factor input accumulation, which would indicate that trade either is related to GDP by some exogenous factor affecting both income and trade or that trade primarily increase factor accumulation.

Since this study only includes OECD countries, and positive results have been found for emerging economies, it is possible that the effects differ between the countries relative development and that import of e.g. intermediate goods only is of general importance when a country is far from the technological frontier.

To sum up, the study does find theoretical reasons of a relationship between trade and growth, but not empirical results in support of a specific general assumption about the existence of long-run relationships or the order of causality between export, import and productivity. Even though, different hypothesises are supported in different countries. Nevertheless, the results point to some important aspects: the importance of studying each country individually, to not assume the variables included to be generally exogenous and to further investigate the relation between import and growth, as both examples of exports and imports to either Granger cause or to be Granger caused by productivity are found.

10 References

Aghion, Philippe - Howitt, Peter, 1990. "A Model of Growth Through Creative Destruction," *NBER Working Papers 3223*, National Bureau of Economic Research, Inc.

Aghion, Philippe – Howitt, Peter, 2009. *The Economics of Growth*, MIT Press, Cambridge, Massachusetts

Awokuse, Titus O., 2003. "Is the export-led growth hypothesis valid for Canada?". *Canadian Journal of Economics*, Canadian Economics Association, vol. 36(1), pages 126-136, February.

Awokuse, Titus O., 2006. "Export-led growth and the Japanese economy: evidence from VAR and directed acyclic graphs". *Applied Economics*, Taylor and Francis Journals, vol. 38(5), pages 593-602, March.

Awokuse, Titus O., 2007. "Causality between exports, imports, and economic growth: Evidence from transition economies". *Economics Letters*, Elsevier, vol. 94(3), pages 389-395, March.

Awokuse, Titus O., 2008. "Trade openness and economic growth: is growth export-led or import-led?," *Applied Economics*, Taylor and Francis Journals, vol. 40(2), pages 161-173.

Balassa, Bela, 1978. "Exports and economic growth: Further evidence". *Journal of Development Economics*, Elsevier, vol. 5(2), pages 181-189, June.

Baldwin, Richard E. - Seghezza, Elena, 1996. "Trade-Induced Investment-led Growth". *NBER Working Papers 5582*, National Bureau of Economic Research, Inc.

Barro, Robert J. - Sala-i-Martin, Xavier, 1997. ”Technological Diffusion, Convergence, and Growth”. *Journal of Economic Growth*, Vol 2, Issue 1, p. 1-26

Benhabib, Jess - Spiegel, Mark M. 1994. “The Role of Human Capital in economic Development: Evidence from Aggregate Cross-Country Data”. *Journal of Monetary Economics*, vol.34, pages 363-390.

Bernanke, Ben S. – Gürkaynak, Refet S., 2002. "Is Growth Exogenous? Taking Mankiw, Romer, and Weil Seriously," *NBER Chapters, in: NBER Macroeconomics Annual 2001*, Volume 16, pages 11-72 National Bureau of Economic Research, Inc.

Bernard, Andrew B., 2004. "Exporting and Productivity in the USA". *Oxford Review of Economic Policy*, Oxford University Press, vol. 20(3), pages 343-357, Autumn.

Campbell, John Y – Perron, Pierre, 1991. "Pitfalls and Opportunities: What Macroeconomists Should Know About Unit Roots". *NBER Chapters, in: NBER Macroeconomics Annual 1991,* Vol. 6, pages 141-220, National Bureau of Economic Research, Inc.

Cetintas, Hakan - Barisik, Salih, 2009. "Export, Import and Economic Growth: The Case of Transition Economies". *Transition Studies Review*. Springer, vol. 15(4), pages 636-649, February.

Chuang, Yih-Chyi, 1998. "Learning by Doing, the Technology Gap, and Growth". *International Economic Review,* Department of Economics, University of Pennsylvania and Osaka University Institute of Social and Economic Research Association, vol. 39(3), pages 697-721, August.

Clerides, Sofronis – Lach, Saul – Tybout, James, 1996. "Is Learning-by-Exporting Important? Micro-Dynamic Evidence from Colombia, Mexico and Morocco". *NBER Working Papers 5715,* National Bureau of Economic Research, Inc.

Coe, David T. - Helpman, Elhanan, 1995. “International R&D spillovers”. *European Economic Review,* Elsevier, vol. 39(5), pages 859-887, May.

Connolly, M., 2000. “North-South Technological Diffusion: A New Case for Dynamic Gains from Trade”. Duke University Economics Department, working paper #99-08.

DeLong, J. Bradford, - Summers, Lawrence H. 1991. “Equipment Investment and Economic Growth.” *Quarterly Journal of Economics*, Vol.106(2), pages 445-502.

Davidson, J. - Hendry, D. - Srba, F. - Yeo, S., 1978. “Econometric Modeling of the Aggregate Time-Series Relationship between Consumers”. *The Economic Journal,* Vol. 88(352), pages 661-669.

Engle, R.F. - Granger, C.W.J., 1987. “Cointegration and Error-Correction: Representation, Estimation and Testing”. *Econometrica,* Vol. 55(2), pages 251-276.

Esfahani, S. H., 1991 “Exports, imports, and economic growth in semi-industrial countries” *Journal of Development Economics*, Vol.35, pages 93–116.

Frankel, Jeffrey A. – Romer, David, 1999. “Does Trade Cause Growth?”. *American Economic Review,* American Economic Association, vol. 89(3), pages 379-399, June.

Giles, J. A. and Williams, C. L. 2000. “Export-led growth: a survey of the empirical literature and some noncausality results”, Part 1, *Journal of International Trade and Economic Development*, 9, 261–337.

Grossman, G.M. and E. Helpman, 1991a. *Innovation and growth in the global economy*. MIT Press, Cambridge, MA.

Grossman, Gene M & Helpman, Elhanan, 1991b. "Quality Ladders and Product Cycles," *The Quarterly Journal of Economics*, MIT Press, vol. 106(2), pages 557-86, May.

Grossman, Gene M & Helpman, Elhanan, 1991c. "Quality Ladders in the Theory of Growth," *Review of Economic Studies*, Blackwell Publishing, vol. 58(1), pages 43-61, January.

Guerzoni, Marco, 2010. "The impact of market size and users' sophistication on innovation: the patterns of demand". *Economics of Innovation and New Technology*, Taylor and Francis Journals, vol. 19(1), pages 113-126.

Hakkio, Craig S. - Rush, Mark, 1991. "Cointegration: how short is the long run?". *Journal of International Money and Finance*, Elsevier, vol. 10(4), pages 571-581, December.

Halpern, László - Koren, Miklós - Szeidl, Adam, 2006. "Imports and Productivity". *Meeting Papers 796*, Society for Economic Dynamics.

Harris, R. - Sollis, R, 2003. *Applied Time Series Modelling and Forecasting,* Wiley, Chichester, Great Britain.

Helpman, E. - Krugman P., 1985. *Market structure and foreign Trade*. MIT Press, Cambridge, MA.

Heston, Alan - Summers, Robert, 1991. “The Penn World Table (Mark 5.6): An Expanded Set of International Comparisons, 1950-1988”. *The Quarterly Journal of Econom*ics (May): 327-368. (Version 5.6 released January 1995, and updated to 1992).

Heston, Alan – Summers, Robert - Aten, Bettina, 2009. *Penn World Table Version 6.3, Center for International Comparisons of Production*. Income and Prices at the University of Pennsylvania, August.

Johansen, S., 1988. “Statistical Analysis of Cointegrating Vectors”. *Journal of Economic Dynamics and Control*, Vol. 12(2-3), pages 231-254.

Johansen, S. - Juselius, K., 1990. “Maximum Likelihood Estimation and Inference on Cointegration - with Applications to the Demand for Money”. *Oxford Bulletin of Economics and Statistics*, Vol. 52(2), pages 169-210.

Johansen, Soren, 1992. "Determination of Cointegration Rank in the Presence of a Linear Trend". *Oxford Bulletin of Economics and Statistics*, Department of Economics, University of Oxford, vol. 54(3), pages 383-97.

Jones, C. I, 1995. “R&D Based Models of Economic Growth”. *Journal of Political Economy*, 103, 759-84.

Kasahara, Hiroyuki - Lapham, Beverly, 2008. "Productivity and the Decision to Import and Export: Theory and Evidence," *CESifo Working Paper Series 2240*, CESifo Group Munich.

Kavoussi, Rostam M., 1984. "Export expansion and economic growth : Further empirical evidence". *Journal of Development Economics*, Elsevier, vol. 14(1), pages 241-250.

King, Robert J., - Levine, Ross 1994. “Capital Fundamentalism Economic Development and Economic Growth”. Carnegie-Rochester Conference Series on Public Policy 40 (June), pages 259-292.

Kremer, Michael, 1993. “Population Growth and Technological Change: One Million B.C. to 1990”. *The Quarterly Journal of Economics*, MIT Press, vol. 108(3), pages 681-716, August.

Krishna, Kala – Ozyildirim, Ataman - Swanson, Norman R., 1998. "Trade, Investment, and Growth: Nexus, Analysis, and Prognosis," *NBER Working Papers 6861*, National Bureau of Economic Research, Inc.

Kunst, R.M. - Marin, D., 1989. “On exports and productivity: A causal analysis”. *Review of Economics and Statistics*, vol. 71(4), pages 699–703.

Kwan, Andy C C - Cotsomitis, John A., - Kwok, Benjamin, 1996. "Exports, Economic Growth and Exogeneity: Taiwan 1953-88". *Applied Economics*, Taylor and Francis Journals, vol. 28(4), pages 467-71, April.

Lawrence, Robert Z., - Weinstein, David E., 1999. “Trade and Growth: Import-Led or Export-Led? Evidence From Japan and Korea”. *NBER Working Papers 7264*, National Bureau of Economic Research, Inc.

Lee, J. W., 1995. “Capital goods import and long-run growth”. *Journal of development Economics*. Vol. 48(1), pages 91-110.

Levine, R. – Renelt, D., 1992. “A sensitivity analysis of cross-country growth regressions”. *American economic review*, vol 82(4), pages 942-963

Lewer, Joshua J. - Van den Berg, Hendrik, 2003. "How Large Is International Trade's Effect on Economic Growth?". *Journal of Economic Surveys*, Blackwell Publishing, vol. 17(3), pages 363-396.

Limam, Yasmina Reem - Miller, Stephen M, 2004. “Explaining Economic Growth: Factor Accumulation, Total Factor Productivity Growth, and Production Efficiency Improvement”. *Working papers 2004-20*, University of Connecticut, Department of Economics.

MacDonald, J. M., 1994. “Does import competition force efficient production?”. *Review of Economics and Statistics*, vol. 74, 678-688.

MacKinnon, James G., 1996. “Numerical Distribution Functions for Unit Root and Cointegration Tests”. *Journal of Applied Econometrics*, vol.11, pages 601-618.

Marin, Dahlia, 1992. "Is the Export-Led.Growth Hypothesis Valid for Industrialized Countries?". *Review of Economics and Statistics,* The MIT press, vol. 74(4), pages 678-688.

Mazumdar, Joy, 2001. “Imported machinery and growth in LDCs”. *Journal of Development Economics,* Elsevier, vol. 65(1), pages 209-224, June.

Osterwald-Lenum, M., 1992. “A Note with Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank Test Statistics”. *Oxford Bulletin of Economics and Statistics,* Vol 54(3), pages 461–472.

Oxley, Les, 1993. "Cointegration, causality and export-led growth in Portugal, 1865-1985". *Economics Letters,* Elsevier, vol. 43(2), pages 163-166.

Ram, Rati, 1985. "Exports and Economic Growth: Some Additional Evidence". *Economic Development and Cultural Change,* University of Chicago Press, vol. 33(2), pages 415-425, January.

Ramos, Francisco F. Ribeiro, 2001. "Exports, imports, and economic growth in Portugal: evidence from causality and cointegration analysis". *Economic Modelling,* Elsevier, vol. 18(4), pages 613-623, December.

Richards, D. G., 2001. “Exports as a determinant of long-run growth in Paraguay, 1966-96”. *Journal of Development Studies,* vol. 38(1), pages 128–46.

Riezman, Raymond G. - Whiteman, Charles H. - Summers, Peter M, 1996. "The Engine of Growth or Its Handmaiden? A Time-Series Assessment of Export-Led Growth". *Empirical Economics,* Springer, vol. 21(1), pages 77-110.

Rivera-Batiz, Luis A - Romer, Paul M, 1991. "Economic Integration and Endogenous Growth". *The Quarterly Journal of Economics,* MIT Press, vol. 106(2), pages 531-55, May.

Rodrik, Dani, 1994.”Getting Interventions Right: How South Korea and Taiwan Grew Rich”. *NBER Working Papers 4964,* National Bureau of Economic Research, Inc.

Seater, John J., 2005. "Trade, Growth, and Technology Equalization". *DEGIT Conference Papers c010_008,* DEGIT, Dynamics, Economic Growth, and International Trade.

Sharma, S. C. - Norris, N. M. and Chung, D. W., 1991. “Exports and economic growth in industrialized countries”. *Applied Economics,* vol. 23, pages 697–707.

Sheehey, Edmund J., 1990. “Exports and Growth: A Flawed Framework”. *Journal of Development Studies,* vol. 27(1), pages 111-117.

Stiroh, K.J., 2000. “Investment and Productivity Growth - A Survey from the Neoclassical and New Growth Perspectives”. *Papers 24*, Gouvernement du Canada - Industry Canada & Gouvernement du Canada - Industry Canada.

Teece, David J, 1977. "Technology Transfer by Multinational Firms: The Resource Cost of Transferring Technological Know-how". *Economic Journal*, Royal Economic Society, vol. 87(346), pages 242-61, June.

Thangavelu, Shandre Mugan - Rajaguru, Gulasekaran, 2004. "Is there an export or import-led productivity growth in rapidly developing Asian countries? a multivariate VAR analysis". *Applied Economics*, Taylor and Francis Journals, vol. 36(10), pages 1083-1093, June.

Tyler, William G., 1981. "Growth and export expansion in developing countries: Some empirical evidence". *Journal of Development Economics*, Elsevier, vol. 9(1), pages 121-130, August.

Vecchi, Michela - O'Mahony, Mary, 2003. "In Search of An ICT Impact on TFP: Evidence from Industry Panel Data". *Royal Economic Society Annual Conference 2003 210*, Royal Economic Society.

Verbeek, M, 2004. *A Guide to Modern Econometrics*, Wiley, Chichester, England

Vogel, Alexander - Wagner, Joachim, 2010. "Higher productivity in importing German manufacturing firms: self-selection, learning from importing, or both?". *Review of World Economics* (Weltwirtschaftliches Archiv), Springer, vol. 145(4), pages 641-665, January.

WDI, *World Development Indicators online*, The World Bank Group, http://data.worldbank.org/indicator, downloaded: 2010-04-14.

Westerlund, Joakim, 2005. *Introduktion till ekonometri*, Studentlitteratur. Lund.

Yamada, Hiroshi, 1998. "A note on the causality between export and productivity: an empirical re-examination". *Economics Letters*, Elsevier, vol. 61(1), pages 111-114, October.

Appendix A: Derivation of the Theoretical Model

The appendix focuses on the mathematical derivations, for reasoning about the basic assumptions and the interpretations of the results readers are referred to section 4.

The derivation of the theoretical model has its origin in the Schumpeterian model of a three sector economy with multiple innovating sectors (i), constituting a continuum of [0, 1], and allowing for productivity improvements in several products simultaneously (Aghion - Howitt 2009:92-96).

The Production of Final Goods

The final goods market, excluding exports, is described as

$$Y_{I,t} = L^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^{\alpha} di \tag{A.1}$$

with production from intermediate product i equal to

$$Y_{I,it} = (A_{it}L)^{1-\alpha} x_{it}^{\alpha} \tag{A.2}$$

The price of the intermediate good is determined by is marginal product

$$p_{it} = \frac{dY_{I,it}}{dx_{it}} = \alpha (A_{it}L)^{1-\alpha} x_{it}^{\alpha-1} \tag{A.3}$$

The effective price[24] of the intermediate products, as input in the intermediate production, is $P_d = 1$ for the domestically produced products, as they are the numeriare, and $P_f = \frac{1}{B}$ for imported intermediate products. The profit in the intermediate sector is equal to

$$\Pi_{I,it} = p_{it}x_{it} - x_{d,it} - \frac{1}{B}x_{f,it} \tag{A.4}$$

The domestic production of intermediate products x_{it} is described by the production function

$$x_{it} = x_{d,it}^{\omega}x_{f,it}^{1-\omega} \tag{A.5}$$

The profit for an importing firm is given by:

$$\Pi_{I,it} = \alpha L^{1-\alpha}A_{it}^{1-\alpha}x_{it}^{\alpha-1}x_{it} - x_{d,it} - \frac{1}{B}x_{f,it}$$
$$\Pi_{I,it} = \alpha L^{1-\alpha}A_{it}^{1-\alpha}x_{it}^{\alpha} - x_{d,it} - \frac{1}{B}x_{f,it}$$
$$\Pi_{I,it} = \alpha L^{1-\alpha}A_{it}^{1-\alpha}(x_{d,it}^{\omega}x_{f,it}^{1-\omega})^{\alpha} - x_{d,it} - \frac{1}{B}x_{f,it}$$
$$\Pi_{I,it} = \alpha L^{1-a}A_{it}^{1-\alpha}x_{d,it}^{\omega\alpha}x_{f,it}^{(1-\omega)\alpha} - x_{d,it} - \frac{1}{B}x_{f,it} \tag{A.6}$$

In order to find the optimal profit, the optimal levels of x_{it}^{*}, $x_{d,it}^{*}$ and $x_{f,it}^{*}$ are found using the f.o.c. of (A.6):

$$\frac{d\Pi_{I,it}}{dx_{d,it}} = \omega\alpha^{2}L^{1-\alpha}A_{it}^{1-\alpha}x_{d,it}^{\omega\alpha-1}x_{f,it}^{(1-\omega)\alpha} - 1 = 0 \tag{A.7}$$
$$\frac{d\Pi_{I,it}}{dx_{f,it}} = (1-\omega)\alpha^{2}L^{1-\alpha}A_{it}^{1-\alpha}x_{d,it}^{\omega\alpha}x_{f,it}^{(1-\omega)\alpha-1} - \frac{1}{B} = 0 \tag{A.8}$$

Solving for $x_{d,it}$ gives:

$$\omega\alpha^{2}L^{1-\alpha}A_{it}^{1-\alpha}x_{d,it}^{\omega\alpha-1}x_{f,it}^{(1-\omega)\alpha} = 1$$
$$x_{d,it}^{\omega\alpha-1} = x_{f,it}^{-(1-\omega)\alpha}/\omega\alpha^{2}L^{1-\alpha}A_{it}^{1-\alpha}$$

---

[24] The effective price is when the price is adjusted for the quality of the product

$$x_{d,it} = x_f^{\frac{(\omega-1)\alpha}{\omega\alpha-1}} \Big/ (\omega\alpha^2 L^{1-\alpha} A_{it}^{1-\alpha})^{\frac{1}{\omega\alpha-1}}$$

$$x_{d,it} = x_{f,it}^{\frac{(1-\omega)\alpha}{1-\omega\alpha}} (\omega\alpha^2 L^{1-\alpha} A_{it}^{1-\alpha})^{\frac{1}{1-\omega\alpha}} \tag{A.9}$$

Solving for $x_{f,it}^*$ is done by using (A.8) and (A.9):

$$(1-\omega)\alpha^2 L^{1-\alpha} A_{it}^{1-\alpha} \left(x_{f,it}^{\frac{(1-\omega)\alpha}{1-\omega\alpha}} (\omega\alpha^2 L^{1-\alpha} A_{it}^{1-\alpha})^{\frac{1}{1-\omega\alpha}} \right)^{\omega\alpha} x_{f,it}^{(1-\omega)\alpha-1} = \frac{1}{B}$$

$$(1-\omega)\alpha^2 L^{1-\alpha} A_{it}^{1-\alpha} x_{f,it}^{\frac{(1-\omega)\alpha^2\omega}{1-\omega\alpha}} x_{f,it}^{(1-\omega)\alpha-1} (\omega\alpha^2 L^{1-\alpha} A_{it}^{1-\alpha})^{\frac{\omega\alpha}{1-\omega\alpha}} = \frac{1}{B}$$

$$(1-\omega)\alpha^2 L^{1-\alpha} A_{it}^{1-\alpha} x_{f,it}^{\frac{\omega\alpha^2-\omega^2\alpha^2+(a-\omega\alpha-1)(1-\omega\alpha)}{1-\omega\alpha}} (\omega\alpha^2 L^{1-\alpha} A_{it}^{1-\alpha})^{\frac{\omega\alpha}{1-\omega\alpha}} = \frac{1}{B}$$

$$x_{f,it}^{\frac{1-\alpha}{1-\omega\alpha}} = (1-\omega)\alpha^2 L^{1-\alpha} A_{it}^{1-\alpha} B (\omega\alpha^2 L^{1-\alpha} A_{it}^{1-\alpha})^{\frac{\omega\alpha}{1-\omega\alpha}}$$

$$x_{f,it}^* = \left(B(1-\omega)\alpha^2 L^{1-\alpha} A_{it}^{1-\alpha} (\omega\alpha^2 L^{1-\alpha} A_{it}^{1-\alpha})^{\frac{\omega\alpha}{1-\omega\alpha}} \right)^{\frac{1}{\frac{1-\alpha}{1-\omega a}}}$$

$$x_{f,it}^* = \alpha^{\frac{2}{1-\alpha}} (1-\omega)^{\frac{1-\omega\alpha}{1-\alpha}} \omega^{\frac{\omega\alpha}{1-\alpha}} B^{\frac{1-\omega\alpha}{1-\alpha}} A_{it} L \tag{A.10}$$

In order to solve for $x_{d,it}^*$ equation (A.9) and (A.10) is used:

$$x_{d,it}^* = \left(\alpha^{\frac{2}{1-\alpha}} (1-\omega)^{\frac{1-\omega\alpha}{1-\alpha}} \omega^{\frac{\omega\alpha}{1-\alpha}} B^{\frac{1-\omega\alpha}{1-\alpha}} A_{it} L \right)^{\frac{(1-\omega)\alpha}{1-\omega\alpha}} (\alpha^2 \omega L^{1-\alpha} A_{it}^{1-\alpha})^{\frac{1}{1-\omega\alpha}}$$

$$x_{d,it}^* = \left(\alpha^{\frac{2}{1-\alpha} * \frac{(1-\omega)\alpha}{1-\omega\alpha}} (1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}} \omega^{\frac{\omega\alpha}{1-\alpha} * \frac{(1-\omega)\alpha}{1-\omega\alpha}} B^{\frac{(1-\omega)\alpha}{1-\alpha}} (A_{it} L)^{\frac{(1-\omega)\alpha}{1-\omega\alpha}} \right)$$
$$* \left(\alpha^{\frac{2}{1-\omega\alpha}} \omega^{\frac{1}{1-\omega\alpha}} (A_{it} L)^{\frac{1-\alpha}{1-\omega\alpha}} \right)$$

$$x_{d,it}^* = \alpha^{\frac{2}{1-\alpha}} (1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}} B^{\frac{(1-\omega)\alpha}{1-\alpha}} A_{it} L * \omega^{\frac{\omega\alpha}{1-\alpha} * \frac{(1-\omega)\alpha}{1-\omega\alpha} + \frac{1}{1-\omega\alpha}}$$

$$x_{d,it}^* = \alpha^{\frac{2}{1-\alpha}} \omega^{\frac{1+\omega\alpha-\alpha}{1-\alpha}} (1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}} B^{\frac{(1-\omega)\alpha}{1-\alpha}} A_{it} L \tag{A.11}$$

Then (A.5), (A.10) and (A.11) is used to solve for x_{it}^*:

$$x_{it}^* = \left(\alpha^{\frac{2}{1-\alpha}} \omega^{\frac{1+\omega\alpha-\alpha}{1-\alpha}} (1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}} B^{\frac{(1-\omega)\alpha}{1-\alpha}} A_{it} L \right)^{\omega} \left(\alpha^{\frac{2}{1-\alpha}} (1-\omega)^{\frac{1-\omega\alpha}{1-\alpha}} \omega^{\frac{\omega\alpha}{1-\alpha}} B^{\frac{1-\omega\alpha}{1-\alpha}} A_{it} L \right)^{1-\omega}$$

$$\mathrm{x}_{\mathrm{it}}^{*} = \alpha^{\frac{2\omega}{1-\alpha}}\alpha^{\frac{2(1-\omega)}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha\omega}{1-\alpha}}(1-\omega)^{\frac{(1-\omega\alpha)(1-\omega)}{1-\alpha}}$$

$$* \, \omega^{\frac{\omega+\omega^2\alpha-\omega\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha(1-\omega)}{1-\alpha}}\mathrm{B}^{\frac{(1-\omega)\alpha\omega}{1-\alpha}}\mathrm{B}^{\frac{(1-\omega\alpha)(1-\omega)}{1-\alpha}}(\mathrm{A}_{\mathrm{it}}\mathrm{L})^{\omega}(\mathrm{A}_{\mathrm{it}}\mathrm{L})^{(1-\omega)}$$

$$\mathrm{x}_{\mathrm{it}}^{*} = \omega^{\frac{\omega+\omega^2\alpha-\omega\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha(1-\omega)}{1-\alpha}} * \alpha^{\frac{2}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)}{1-\alpha}}\mathrm{B}^{\frac{(1-\omega)}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L}$$

$$\mathrm{x}_{\mathrm{it}}^{*} = \alpha^{\frac{2}{1-\alpha}}\omega^{\frac{\omega}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)}{1-\alpha}}\mathrm{B}^{\frac{(1-\omega)}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L} \qquad (\mathrm{A}.12)$$

Replacing equations (A.3), (A.10), (A.11), and (A.12) into the profit function in (A.4) gives the optimal profit of:

$$\Pi_{\mathrm{I,it}}^{*} = \alpha(\mathrm{A}_{\mathrm{it}}\mathrm{L})^{1-\alpha}\left(\alpha^{\frac{2}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)}{1-\alpha}}\omega^{\frac{\omega}{1-\alpha}}\mathrm{B}^{\frac{(1-\omega)}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L}\right)^{\alpha}$$

$$-\left(\alpha^{\frac{2}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}\omega^{\frac{1+\omega\alpha-\alpha}{1-\alpha}}\mathrm{B}^{\frac{(1-\omega)\alpha}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L}\right)$$

$$-\frac{1}{\mathrm{B}}\left(\alpha^{\frac{2}{1-\alpha}}(1-\omega)^{\frac{1-\omega\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}\mathrm{B}^{\frac{1-\omega\alpha}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L}\right)$$

$$\Pi_{\mathrm{I,it}}^{*} = \varphi - \left[(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}\omega^{\frac{1+\omega\alpha-\alpha}{1-\alpha}} + (1-\omega)^{\frac{1-\omega\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}\right]\alpha^{\frac{2}{1-\alpha}}\mathrm{B}^{\frac{(1-\omega)\alpha}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L}$$

where $\varphi = \alpha(\mathrm{A}_{\mathrm{it}}\mathrm{L})^{1-\alpha}\left(\omega^{\frac{\omega}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)}{1-\alpha}}\mathrm{B}^{\frac{(1-\omega)}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L}\right)^{\alpha}$

$$\Pi_{\mathrm{I,it}}^{*} = \varphi - \left[\omega^{\frac{1+\omega\alpha-\alpha}{1-\alpha}} + (1-\omega)\omega^{\frac{\omega\alpha}{1-\alpha}}\right](1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}\mathrm{B}^{\frac{(1-\omega)\alpha}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L}$$

$$\Pi_{\mathrm{I,it}}^{*} = \varphi - \left[\omega^{\frac{\omega\alpha}{1-\alpha}}\omega + (1-\omega)\omega^{\frac{\omega\alpha}{1-\alpha}}\right](1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}\mathrm{B}^{\frac{(1-\omega)\alpha}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L}$$

$$\Pi_{\mathrm{I,it}}^{*} = \alpha^{\frac{1+\alpha}{1-\alpha}}(1-\omega)^{\frac{\alpha(1-\omega)}{1-\alpha}}\omega^{\frac{\alpha\omega}{1-\alpha}}\mathrm{B}^{\frac{\alpha(1-\omega)}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L} - \alpha^{\frac{2}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}\mathrm{B}^{\frac{(1-\omega)\alpha}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L}$$

$$\Pi_{\mathrm{I,it}}^{*} = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\omega)^{\frac{\alpha(1-\omega)}{1-\alpha}}\omega^{\frac{\alpha\omega}{1-\alpha}}\mathrm{B}^{\frac{\alpha(1-\omega)}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L} \qquad (\mathrm{A}.13)$$

This is the expression for a firm that is open to import of foreign intermediate products. It can also be shown by replacing (A.12) into (A.3) that the optimal price is given by:

$$\mathrm{p}_{\mathrm{it}} = \alpha(\mathrm{A}_{\mathrm{it}}\mathrm{L})^{1-\alpha}\left(\alpha^{\frac{2}{1-\alpha}}\omega^{\frac{\omega}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)}{1-\alpha}}\mathrm{B}^{\frac{(1-\omega)}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L}\right)^{\alpha-1}$$

$$\mathrm{p}_{\mathrm{it}}^{*} = \frac{1}{\alpha\omega^{\omega}(1-\omega)^{(1-\omega)}\mathrm{B}^{(1-\omega)}}$$

However, in order to see the effect of full openness to trade one has to add the effect of exports. Exports are included as an increased market for the final goods sector, with all export as proportional increase to the production for the domestic market.

$$\mathrm{Y_{trade,t}} = \mathrm{Y_{I,t}} + \int_0^N \mathrm{Y_{I,t}}\, \mathrm{di} = (1+\mathrm{N})\mathrm{Y_{I,t}} \tag{A.14}$$

Increased production will also give rise to increased demand for inputs in accordance with:

$$\lambda \mathrm{f}(\mathrm{L}, \mathrm{A_{it}x_{it}}) = \mathrm{f}(\lambda \mathrm{L}, \lambda \mathrm{A_{it}x_{it}})$$

And therefore

$$(1+\mathrm{N})\mathrm{Y_{it}} = (1+\mathrm{N})(\mathrm{A_{it}L})^{1-\alpha}\mathrm{x_{it}}^{\alpha} = \big((1+\mathrm{N})\mathrm{A_{it}L}\big)^{1-\alpha}\big((1+\mathrm{N})\mathrm{x_{it}}\big)^{\alpha}$$

so that

$$(1+\mathrm{N})\mathrm{x_{it}} = (1+\mathrm{N})\left[\mathrm{x_{d,it}^{\omega}x_{f,it}^{1-\omega}}\right]$$

Using these equations it can be shown that the optimal price is given by:

$$\mathrm{P_{it}} = \frac{\mathrm{d}(1+\mathrm{N})\mathrm{Y_{it}}}{\mathrm{d}(1+\mathrm{N})\mathrm{x_{it}}} = \alpha\big((1+\mathrm{N})\mathrm{A_{it}L}\big)^{1-\alpha}\big((1+\mathrm{N})\mathrm{x_{it}}\big)^{\alpha-1}$$

The profit of the intermediate sector change with increased demand for $\mathrm{x_{it}}$, and an increased need of inputs. The new profit, with full trade, is given by:

$$\Pi_{\mathrm{Trade,it}} = \mathrm{p_{it}}(1+\mathrm{N})\mathrm{x_{it}} - (1+\mathrm{N})\mathrm{x_{d,it}} - \frac{1}{\mathrm{B}}(1+\mathrm{N})\mathrm{x_{f,it}}$$

Hence, replacing $\mathrm{p_{it}}$ into the function reveals that this can be written as:

$$\Pi_{\mathrm{Trade,it}} = \alpha\big((1+\mathrm{N})\mathrm{A_{it}L}\big)^{1-\alpha}\big((1+\mathrm{N})\mathrm{x_{it}}\big)^{\alpha-1}(1+\mathrm{N})\mathrm{x_{it}} - (1+\mathrm{N})\mathrm{x_{d,it}} - \frac{1}{\mathrm{B}}(1+\mathrm{N})\mathrm{x_{f,it}}$$

$$\Pi_{\mathrm{trade,it}} = (1+\mathrm{N})\Pi_{\mathrm{I,it}} = (1+\mathrm{N})\left(\alpha \mathrm{L}^{1-\mathrm{a}}\mathrm{A}_{\mathrm{it}}^{1-\alpha}\mathrm{x}_{\mathrm{it}}^{\alpha} - \mathrm{x}_{\mathrm{d,it}} - \frac{1}{\mathrm{B}}\mathrm{x}_{\mathrm{f,it}}\right)$$

$$\Pi_{\mathrm{trade,it}} = (1+\mathrm{N})\Pi_{\mathrm{I,it}} = (1+\mathrm{N})\left(\alpha \mathrm{L}^{1-\mathrm{a}}\mathrm{A}_{\mathrm{it}}^{1-\alpha}\mathrm{x}_{\mathrm{d,it}}^{\omega\alpha}\mathrm{x}_{\mathrm{f,it}}^{(1-\omega)\alpha} - \mathrm{x}_{\mathrm{d,it}} - \frac{1}{\mathrm{B}}\mathrm{x}_{\mathrm{f,it}}\right) \qquad (\mathrm{A}.15)$$

Replacing $\mathrm{x}_{\mathrm{d,it}}$ and $\mathrm{x}_{\mathrm{d,it}}$ with (A.11) and (A.10) respectively and the optimal profit is solved in the same manner as before. This gives us the optimal trade profit for an intermediate firm:

$$\Pi_{\mathrm{trade,it}}^{*} = (1+\mathrm{N})(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\mathrm{B}^{\frac{\alpha(1-\omega)}{1-\alpha}}(1-\omega)^{\frac{\alpha(1-\omega)}{1-\alpha}}\omega^{\frac{\alpha\omega}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L} \qquad (\mathrm{A}.16)$$

In addition, the optimal price can also be shown not to have changed

$$\mathrm{P}_{\mathrm{it}} = \alpha\big((1+\mathrm{N})\mathrm{A}_{\mathrm{it}}\mathrm{L}\big)^{1-\alpha}\left((1+\mathrm{N})\alpha^{\frac{2}{1-\alpha}}\omega^{\frac{\omega}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)}{1-\alpha}}\mathrm{B}^{\frac{(1-\omega)}{1-\alpha}}\mathrm{A}_{\mathrm{it}}\mathrm{L}\right)^{\alpha-1}$$

$$\mathrm{p}_{\mathrm{trade,it}}^{*} = \frac{1}{\alpha\omega^{\omega}(1-\omega)^{(1-\omega)}\mathrm{B}^{(1-\omega)}}$$

The Research Sector

The technological level A_{it} is determined by the production of innovations in the research sector. The innovation will result in a productivity improvement of $\gamma > 0$. The entrepreneur spends the amount R_{it} units of final goods on research and innovates with the probability

$$\mu_{\mathrm{t}} = \phi\left(\frac{\mathrm{R}_{\mathrm{it}}}{\mathrm{A}_{\mathrm{it}}^{*}}\left(\frac{\mathrm{x}_{\mathrm{ft}}}{\mathrm{A}_{\mathrm{t}}}\right)^{\tau}\right) = \lambda\left(\frac{\mathrm{R}_{\mathrm{it}}}{\mathrm{A}_{\mathrm{it}}^{*}}\left(\frac{\mathrm{x}_{\mathrm{ft}}}{\mathrm{A}_{\mathrm{t}}}\right)^{\tau}\right)^{\sigma} \qquad (\mathrm{A}.17)$$

$\lambda > 0$ is the productivity in the research sector, $\mathrm{A}_{\mathrm{it}}^{*}$ is the target productivity if the innovations succeeds and σ, [0, 1], is the elasticity. Also, the entrepreneur's ability to derive knowledge from imports ($\tau > 0$) is important. It is assumed that the parameters are of a scale so that $0 < \mu_{\mathrm{t}} < 1$. This implies that net present value of the expected profit for the entrepreneur is:

$$\mathrm{E}\big[\Pi_{\mathrm{Research,it}}\big] = \phi\left(\frac{\mathrm{R}_{\mathrm{it}}}{\mathrm{A}_{\mathrm{it}}^{*}}\left(\frac{(1+\mathrm{N})\mathrm{x}_{\mathrm{ft}}}{\mathrm{A}_{\mathrm{t}}}\right)^{\tau}\right) * \frac{\Pi_{\mathrm{trade,it}}^{*}}{\mathrm{r}} - \mathrm{R}_{\mathrm{it}} \qquad (\mathrm{A}.18)$$

with an interest rate of r as discount factor. The entrepreneur chooses R_{it} in order to maximize the profit. This gives us the research arbitrage condition:

$$\phi'\left(\frac{R_{it}}{A_{it}^{*}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right) * \frac{\Pi_{trade,it}^{*}}{rA_{it}^{*}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau} - 1 = 0$$

$$\phi'\left(\frac{R_{it}}{A_{it}^{*}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right) * \frac{\Pi_{trade,it}^{*}}{A_{it}^{*}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau} = r \qquad (A.19)$$

Using the profit from (A.16) and optimal imports from (A.10) gives:

$$\phi'\left(\frac{R_{it}}{A_{it}^{*}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right)\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}A_{it}L}{A_{it}^{*}}\left(\frac{(1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}A_t L}{A_t}\right)^{\tau} = r$$

where $\psi = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\omega)^{\frac{\alpha(1-\omega)}{1-\alpha}}\omega^{\frac{\alpha\omega}{1-\alpha}}$, $\vartheta = \alpha^{\frac{2}{1-\alpha}}(1-\omega)^{\frac{1-\omega\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}$ and $A_t = \int_0^1 A_{it}di$

$$\phi'\left(\frac{R_{it}}{A_{it}^{*}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right)\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau} = 1 \qquad (A.20)$$

In order to know the probability, μ_t, the first order condition of equation (A.17) is taken with respect to $\left(\frac{R_{it}}{A_{it}^{*}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right)$.

$$\phi'\left(\frac{R_{it}}{A_{it}^{*}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right) = \sigma\lambda\left(\frac{R_{it}}{A_{it}^{*}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right)^{\sigma-1}$$

Then this is used in equation (A.20) to solve for $\left(\frac{R_{it}}{A_{it}^{*}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right)$.

$$\sigma\lambda\left(\frac{R_{it}}{A_{it}^{*}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right)^{\sigma-1}\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau} = 1$$

$$\frac{R_{it}}{A_{it}^{*}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau} = \left(\sigma\lambda\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right)^{\frac{1}{1-\sigma}} \qquad (A.21)$$

Solving for R_{it}, by using eq. (A.10) again, gives an expression of:

$$R_{it} = \left(\sigma\lambda\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right)^{\frac{1}{1-\sigma}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{-\tau}A_{it}^{*}$$

$$R_{it} = \left(\sigma\lambda\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right)^{\frac{1}{1-\sigma}}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{-\tau}A_{it}^{*}$$

$$R_{it} = \left(\sigma\lambda\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\right)^{\frac{1}{1-\sigma}}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\frac{\tau\sigma}{1-\sigma}}A_{it}^{*} \qquad (A.22)$$

Finally the result in equation (A.21) is replaced back into equation (A.17). This gives a constant probability of:

$$\mu = \lambda\left(\left(\sigma\lambda\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right)^{\frac{1}{1-\sigma}}\right)^{\sigma}$$

$$\mu = \lambda^{\frac{1}{1-\sigma}}\left[\sigma\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left(\vartheta(1+N)B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right]^{\frac{\sigma}{1-\sigma}} \qquad (A.23)$$

Y-level and GDP-level

In order to find the equilibrium levels of y_t and gdp_t, x_{it} in (A.12) is replaced in equation (A.1) and (A.14).

$$Y_{trade,t} = (1+N)Y_{I,t} = (1+N)L^{1-\alpha}\int_0^1 A_{it}^{1-\alpha}\left(\alpha^{\frac{2}{1-\alpha}}\omega^{\frac{\omega}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)}{1-\alpha}}B^{\frac{(1-\omega)}{1-\alpha}}A_{it}L\right)^{\alpha}di$$

$$Y_{trade,t} = (1+N)L\alpha^{\frac{2\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}\int_0^1 A_{it}di$$

$$Y_{trade,t} = (1+N)\alpha^{\frac{2\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_tL \qquad (A.24)$$

Because $A_t = \int_0^1 A_{it}\,di$ is the unweighted numerical average of all the individual productivity parameters. In per capita terms this gives:

$$y_{trade,t} = (1+N)\alpha^{\frac{2\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_t \tag{A.25}$$

$GDP_{trade,t}$ can be expressed as:

$$GDP_{trade,t} = (1+N)\left(Y_{Trade,t} - \int_0^1 x_{d,it}di - \frac{1}{B}\int_0^1 x_{f,it}di\right) \tag{A.26}$$

Replacing $x_{d,it}$ and $x_{f,it}$with equation (A.10) and (A.11) gives:

$$\begin{aligned} GDP_{trade,t} = (1+N)\Bigg(&Y_{Trade,t} - \alpha^{\frac{2}{1-\alpha}}\omega^{\frac{1+\omega\alpha-\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_tL \\ &- \frac{1}{B}\alpha^{\frac{2}{1-\alpha}}(1-\omega)^{\frac{1-\omega\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}B^{\frac{1-\omega\alpha}{1-\alpha}}A_tL\Bigg) \end{aligned}$$

This is solved in the same way as the optimal intermediate profit in equation (A.13), which gives:

$$GDP_{trade,t} = (1+N)\left(Y_{Trade,t} - \alpha^{\frac{2}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_tL\right)$$

Replacing $Y_{Trade,t}$ with equation (A.24) gives the final expression for $gdp_{trade,t}$:

$$\begin{aligned} GDP_{trade,t} = (1+N)\Bigg(&\alpha^{\frac{2\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_tL \\ &- \alpha^{\frac{2}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_tL\Bigg) \end{aligned}$$

$$GDP_{trade,t} = (1+N)\left(\alpha^{\frac{2\alpha}{1-\alpha}} - \alpha^{\frac{2}{1-\alpha}}\right)\omega^{\frac{\omega\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_tL$$

$$GDP_{trade,t} = (1+N)(1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_tL \tag{A.27}$$

$$gdp_{trade,t} = (1+N)(1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_t \tag{A.28}$$

It is easily seen that the only two variables in the expression are $\mathrm{gdp}_{\mathrm{trade},t}$ and A_t. The relation is proportional so that taking the log of the expression and differentiates it by time gives the growth rates of:

$$g_{\mathrm{gdp}_{\mathrm{trade},t}} = g_{A_t} \tag{A.29}$$

Hence, in the long-run, the growth rate of the economy is proportional to the growth rate of A_t.

Growth rate of technology

The change in A_t is described as:

$$\dot{A}_t = \mu\gamma A_t \tag{A.30}$$

A successful innovation lead to an increase in the technological level with the probability μ. It follows that the expected growth rate of A_t is:

$$g_t = \frac{\dot{A}_t}{A_t} = \mu\gamma \tag{A.31}$$

The probability of a successful innovation is given by equation (A.18). Hence, replacing μ above gives the average growth rate of the economy:

$$g_t = \lambda^{\frac{1}{1-\sigma}}\left[\sigma\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left(\vartheta(1+N)B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right]^{\frac{\sigma}{1-\sigma}}\gamma \tag{A.32}$$

The change in g_t as result from a increased interest rate is given by:

$$\frac{dg_t}{dr} = -\frac{\sigma}{1-\sigma}\lambda^{\frac{1}{1-\sigma}}\left[\sigma\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L\left(\vartheta(1+N)B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right]^{\frac{\sigma}{1-\sigma}}\gamma\frac{1}{r^{\frac{1}{1-\sigma}}} < 0$$

Consumers

Each country consists of a large number of identical households, each supplying the economy with one unit of labour for an infinite time. The labour supply (L) in each country is fixed: $\partial L_t/\partial t = 0$. The total consumption for a country is set to C_t, with consumption per capita of $c_t = C_t/L_t$. The inequality constraint states a positive consumption of $c_t \geq 0$. Utility from consumption in time t is given by $u(c_t)$, with $u'(c_t) > 0$, $u''(c_t) < 0$ stating a concave function, and Inada conditions of $\lim_{c\to 0} u'(c_t) = \infty$ and $\lim_{c\to\infty} u'(c_t) = 0$. Aggregated household utility is given by: $U = \int_0^\infty u(c_t)e^{-\rho t}$, where $\rho > 0$ is the rate of time preference.

Each person holds assets of $a_t = \text{Assets}_t/L_t$, where Assets_t is the total assets of the country. The change in total assets is given by:

$$\dot{\text{Assets}}_t = \frac{d(\text{Assets}_t)}{dt} = r_t * \text{Assets}_t + w_t * L_t - C_t \qquad (A.33)$$

The change in assets per person is then described as:

$$\frac{da_t}{dt} = \frac{\frac{d\text{Assets}_t}{dL_t}}{dt} = \frac{\text{Assets}_t}{L_t}\left[\frac{\dot{\text{Assets}}_t}{\text{Assets}_t} - \frac{\dot{L}_t}{L_t}\right] = \frac{\text{Assets}_t}{L_t}\left[\frac{r_t * \text{Assets}_t + w_t * L_t - C_t}{\text{Assets}_t} - 0\right]$$

$$\frac{da_t}{dt} = \frac{r_t * \text{Assets}_t}{L_t} + w_t - \frac{C_t}{L_t}$$

$$\dot{a}_t = r_t a_t + w_t - c_t \qquad (A.34)$$

Also the credit market constraint is equal to:

$$\lim_{t\to\infty} a_t e^{-\int_0^\infty r_v dv} \geq 0$$

This states that the stock of dept per capita of a household cannot grow at a rate greater than r_t. Consumers maximize their utility from consumption with respect to their assets as the only objective. Generally the Hamiltonian function used to solve this problem is:

$$H = \text{felicity function} * \text{dicount factor} + \text{costat variable} * \text{change in state variable}$$

From the assumptions above this gives:

$$H = u(c_t) * e^{-\rho t} + \mu_t * [ra_t + w_t - c_t] \tag{A.35}$$

Taking the first order conditions of the function:

$$\frac{dH}{dc_t} = u'(c_t)e^{-\rho t} - \mu_t = 0 \tag{A.36}$$

$$-\frac{dH}{da_t} = \frac{d\mu_t}{dt} = -\mu_t r_t \tag{A.37}$$

$$\lim_{t\to\infty} \mu_t * a_t = 0 \tag{A.38}$$

Solving (A.36) for μ_t and differentiate with respect to time gives:

$$\mu_t = u'(c_t)e^{-\rho t}$$

$$\frac{d\mu_t}{dt} = u''(c_t) * \frac{dc_t}{dt} * e^{-\rho t} + u'(c_t) * (-\rho)e^{-\rho t} = u'(c_t)e^{-\rho t} * \left[\frac{u''(c_t)}{u'(c_t)} * \dot{c}_t - \rho\right]$$

$$\frac{d\mu_t}{dt} = \mu_t * \left[\frac{u''(c_t)}{u'(c_t)} * \dot{c}_t - \rho\right]$$

Then, replacing this in (A.37) and solve for r_t gives:

$$\mu_t * \left[\frac{u''(c_t)}{u'(c_t)} * \dot{c}_t - \rho\right] = -\mu_t r_t$$

$$r_t = \rho - \frac{u''(c_t)}{u'(c_t)} c_t * \frac{\dot{c}_t}{c_t} \tag{A.39}$$

This can be simplified by making an assumption about the elasticity expression in the equation. If it's assumed that the utility function exhibit constant elasticity of substitution (CES) according to

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1 - \theta}$$

and using $u(c_t)'$ and $u(c_t)''$, it can be shown that the elasticity is constant and given by:

$$\frac{u''(c_t)}{u'(c_t)} * c_t = -\theta$$

This imply that if the elasticity (θ) is high, then the utility will be lower as people are less willing to exchange consumption for time. Replacing this in (A.39) gives the final expression for the interest rate:

$$r_t = \rho + \theta * \frac{\dot{c}_t}{c_t} \tag{A.40}$$

This can be rearranges into the growth rate of consumption, also known as the Euler equation:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\theta}(r_t - \rho) \tag{A.41}$$

Equilibrium

In order to find the equilibrium growth rate of the economy, we need to integrate the results from the production side with the Euler equation for private consumption. The resource constraint gives that output of the economy for domestic production equals aggregate consumption C_t, expenditures on intermediates $x_{d,it}$ and $x_{f,it}$, and investment in research R_t.

$$(1 + N)Y_{I,t} = C_t + (I + N)x_{d,t} + \frac{(1 + N)}{B}x_{f,t} + R_t \tag{A.42}$$

It can be shown that all variables are a linear function of A_t. Equations (A.11) and (A.10) shows that $x_{d,it}$ and $x_{f,it}$ is linear in A_t respectively.

$$x^*_{d,it} = \alpha^{\frac{2}{1-\alpha}}\omega^{\frac{1+\omega\alpha-\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_{it}L$$

$$x^*_{d,t} = \int_0^1 x^*_{d,it}di = \alpha^{\frac{2}{1-\alpha}}\omega^{\frac{1+\omega\alpha-\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}L\int_0^1 A_{it}di$$

$$x^*_{d,t} = \alpha^{\frac{2}{1-\alpha}}\omega^{\frac{1+\omega\alpha-\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_tL \tag{A.43}$$

And the same procedure gives:

$$x^*_{f,t} = \alpha^{\frac{2}{1-\alpha}}(1-\omega)^{\frac{1-\omega\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}B^{\frac{1-\omega\alpha}{1-\alpha}}A_tL \qquad (A.44)$$

Also, equation (A.24) shows the relation for $Y_{trade,t}$:

$$Y_{trade,t} = (1+N)\alpha^{\frac{2\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_tL$$

In order to find a function for R_t the f.o.c of eq. (A.17) is replaced into eq. (A.20) which gives:

$$\sigma\lambda\left(\frac{R_{it}}{A^*_{it}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau}\right)^{\sigma-1}\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau} = 1$$

Still with: $\psi = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}(1-\omega)^{\frac{\alpha(1-\omega)}{1-\alpha}}\omega^{\frac{\alpha\omega}{1-\alpha}}$ and $\vartheta = \alpha^{\frac{2}{1-\alpha}}(1-\omega)^{\frac{1-\omega\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}$ and $A_t = \int_0^1 A_{it}di$

$$\frac{R_{it}}{A^*_{it}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{\tau} = \left(\sigma\lambda\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right)^{\frac{1}{1-\sigma}}$$

$$R_{it} = \left(\sigma\lambda\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right)^{\frac{1}{1-\sigma}}\left(\frac{(1+N)x_{ft}}{A_t}\right)^{-\tau}A^*_{it}$$

$$R_{it} = \left(\sigma\lambda\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right)^{\frac{1}{1-\sigma}}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{-\tau}A^*_{it}$$

$$R_t = \int_0^1 R_{it}di = \left(\sigma\lambda\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right)^{\frac{1}{1-\sigma}}$$

$$* \left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{-\tau}\int_0^1 A^*_{it}di$$

$$R_t = \left(\sigma\lambda\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{r}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right)^{\frac{1}{1-\sigma}}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{-\tau}A^*_t \qquad (A.45)$$

From equation (A.42) it is possible to set C_t alone and replace Y_t, $x_{d,t}$, $x_{f,t}$ and R_t with equations (A.24), (A.43), (A.44) and (A.45). It is easily seen that C_t will be a function of A_t. From equation (A.27) it is also shown that this is true for $GDP_{trade,t}$:

$$GDP_{trade,t} = (1+N)(1-\alpha^2)\alpha^{\frac{2\alpha}{1-\alpha}}\omega^{\frac{\omega\alpha}{1-\alpha}}(1-\omega)^{\frac{(1-\omega)\alpha}{1-\alpha}}B^{\frac{(1-\omega)\alpha}{1-\alpha}}A_tL$$

From equation (A.5) it is easily seen that if $x_{d,t}$ and $x_{f,t}$ exhibit the growth rate g_t then so will x_t. Therefore, the growth rates of Y_t, GDP_t, C_t, , $x_{d,t}$, $x_{f,t}$, x_t, and R_t all equal the growth rate of A_t.

$$\frac{\dot{C}_t}{C_t} = \frac{\dot{Y}_t}{Y_t} = \frac{G\dot{D}P_t}{GDP_t} = \frac{\dot{x}_{d,t}}{x_{d,t}} = \frac{\dot{x}_{f,t}}{x_{f,t}} = \frac{\dot{x}_t}{x_t} = \frac{\dot{R}_t}{R_t} = g_t \qquad (A.46)$$

The production side of the economy will follow the growth rate in eq. (A.32). Yet, to find the final solution we need to combine this with the growth rate from the demand side Euler equation in (A.41). The equilibrium occurs where the curves intersect, and is displayed graphically in section 4.9. This is also given by replacing r in eq. (A.32) with eq. (A.40), and letting $g_{ct} = g_t$.

$$g_t = \lambda^{\frac{1}{1-\sigma}}\left[\sigma\frac{\psi(1+N)B^{\frac{\alpha(1-\omega)}{1-\alpha}}L}{g_t\theta+\rho}\left(\vartheta(1+N)B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}\right]^{\frac{\sigma}{1-\sigma}}\gamma \qquad (A.47)$$

This gives a expression with g_t on both sides. In order to investigate how the growth rate is affected by shocks in different parameters, the implicit function theorem is used. It states that, for a function of $F(g_t, q, \ldots)$, one can find the effect on g_t by any parameter q, by using:

$$\frac{dg_t}{dq} = -\frac{dF/dq}{dF/dg_t}$$

Dividing both sides of eq. (A.47) by g_t and setting the left side to zero gives:

$$0=\lambda^{\frac{1}{1-\sigma}}\left[\left(\sigma(1+N)\psi B^{\frac{\alpha(1-\omega)}{1-\alpha}}L\right)\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\tau}(\theta g_t+\rho)^{-1}\right]^{\frac{\sigma}{1-\sigma}}\gamma*\frac{1}{g_t}-1 \qquad (\mathrm{A}.48)$$

Taking the first derivative of (A.48) with respect to g_t:

$$\begin{aligned}\frac{dF}{dg_t}=&-\frac{\sigma}{1-\sigma}\lambda^{\frac{1}{1-\sigma}}\left(\sigma(1+N)\psi B^{\frac{\alpha(1-\omega)}{1-\alpha}}L\right)^{\frac{\sigma}{1-\sigma}}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\frac{\tau\sigma}{1-\sigma}}\gamma\frac{1}{g_t}(\theta g_t+\rho)^{-\frac{\sigma}{1-\sigma}-1}\theta\\&+(-1)\frac{1}{g_t^2}\lambda^{\frac{1}{1-\sigma}}\left(\sigma(1+N)\psi B^{\frac{\alpha(1-\omega)}{1-\alpha}}L\right)^{\frac{\sigma}{1-\sigma}}\\&*\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\frac{\tau\sigma}{1-\sigma}}(\theta g_t+\rho)^{-\frac{\sigma}{1-\sigma}}\gamma\end{aligned}$$

$$\begin{aligned}\frac{dF}{dg_t}=&-\lambda^{\frac{1}{1-\sigma}}\left(\sigma(1+N)\psi B^{\frac{\alpha(1-\omega)}{1-\alpha}}L\right)^{\frac{\sigma}{1-\sigma}}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\frac{\tau\sigma}{1-\sigma}}\gamma\\&*\left(\frac{\theta\sigma}{1-\sigma}\frac{1}{(\theta g_t+\rho)}+\frac{1}{g_t}\right)\frac{1}{g_t(\theta g_t+\rho)^{\frac{\sigma}{1-\sigma}}}\end{aligned} \qquad (\mathrm{A}.49)$$

Then, for each parameter to investigate, we take the f.o.c. of eq. (A.48) with respect to that parameter and divide the result with minus equation (A.49). For B this gives:

$$\begin{aligned}\frac{dF}{dB}=&\left(\left[\frac{\alpha(1-\omega)}{1-\alpha}\right]+\left[\tau\frac{1-\omega\alpha}{1-\alpha}\right]\right)\lambda^{\frac{1}{1-\sigma}}(\theta g_t+\rho)^{-\frac{\sigma}{1-\sigma}}\frac{\gamma}{g_t}\frac{\sigma}{1-\sigma}\\&*\left(\sigma(1+N)\psi B^{\frac{\alpha(1-\omega)}{1-\alpha}}L\right)^{\frac{\sigma}{1-\sigma}}\left((1+N)\vartheta B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)^{\frac{\tau\sigma}{1-\sigma}}\frac{1}{B}\end{aligned}$$

$$-\frac{\frac{dF}{dB}}{\frac{dF}{dg_t}}=\frac{dg_t}{dB}=\left(\left[\frac{\alpha(1-\omega)}{1-\alpha}\right]+\left[\tau\frac{(1-\omega\alpha)}{1-\alpha}\right]\right)\frac{\sigma}{1-\sigma}*\frac{1}{\left(\frac{\theta\sigma}{1-\sigma}\frac{1}{(\theta g_t+\rho)}+\frac{1}{g_t}\right)B}>0 \quad (\mathrm{A}.50)$$

The result for B shows that the effect from an increase in B will be strictly positive. The results for N,γ,λ,L,τ and ρ are given below, and are strictly positive for all except ρ which is strictly negative.

$$\frac{dg_t}{dN}=(1+\tau)\frac{\sigma}{1-\sigma}\frac{1}{\left(\frac{\sigma}{1-\sigma}\frac{\theta}{(\theta g_t+\rho)}+\frac{1}{g_t}\right)}\frac{1}{(1+N)}>0 \qquad (\mathrm{A}.51)$$

$$\frac{dg_t}{d\gamma} = \frac{1}{\left(\frac{\sigma}{1-\sigma}\frac{\theta}{(\theta g_t + \rho)} + \frac{1}{g_t}\right)\gamma} > 0 \qquad (A.52)$$

$$\frac{dg_t}{d\lambda} = \frac{1}{1-\sigma}\frac{1}{\left(\frac{\theta\sigma}{1-\sigma}\frac{1}{(\theta g_t + \rho)} + \frac{1}{g_t}\right)\lambda} > 0 \qquad (A.53)$$

$$\frac{dg_t}{dL} = (1+\tau)\frac{\sigma}{1-\sigma}\frac{1}{\left(\frac{\sigma}{1-\sigma}\frac{\theta}{(\theta g_t + \rho)} + \frac{1}{g_t}\right)}\frac{1}{L} > 0 \qquad (A.54)$$

$$\frac{dg_t}{d\tau} = \frac{\frac{\sigma}{1-\sigma}\ln\left((1+N)\vartheta\, B^{\frac{1-\omega\alpha}{1-\alpha}}L\right)}{\left(\frac{\theta\sigma}{1-\sigma}\frac{1}{(\theta g_t + \rho)} + \frac{1}{g_t}\right)} > 0 \qquad (A.55)$$

$$\frac{dg_t}{d\rho} = -\frac{\sigma}{1-\sigma}\frac{1}{\left(\frac{\theta\sigma}{1-\sigma} + \left(\theta + \frac{\rho}{g_t}\right)\right)} < 0 \qquad (A.56)$$

Appendix B: Data Sources

Table B.1: Data Sources

Series	Measure	Source
Population	Population (in thousands)	Penn World Tables 6.3; POP
Nominal GDP per capita	Current price, US$, Real Gross Domestic Product per Capita	Penn World Tables 6.3; cgdp
Real GDP per capita	US$2005 Constant price, Real GDP per capita (Constant Prices: Laspeyres), derived from growth rates of c, g, i	Penn World Tables 6.3; rgdpl
Investment share	%, US$2005 constant price, Investment Share of Real GDP per capita US	Penn World Tables 6.3; ki
Real GDP per worker	US$2005 Constant price, Real GDP per worker	Penn World Tables 6.3; rgdpwok
Growth rate of GDP per capita	%, Growth rate of Real GDP per capita	Penn World Tables 6.3; grgdpl2
Exports	US$2000 constant price, Exports of goods and services	World Development Indicators; NE.EXP.GNFS.KD
Imports	US$2000 constant price, Imports of goods and services	World Development Indicators; NE.IMP.GNFS.KD
(World) GDP growth	Annual % growth, $2000 Constant price	World Development Indicators; NY.GDP.MKTP.KD.ZG

Note: When needed, variables used in the empirical study are deflated into $2000 constant price and/or multiplied by 1000 to get total levels. Series are either directly used or used as part of calculations for TFP and total investment levels

Appendix C: Diagnostics of Residuals and Results from Johansen Cointegration Test

Table C.1: Results from LM Autocorrelation Tests

H0: No autocorrelation at lag order p

Test is done using a lag length of 12 lags

Canada	VAR	VAR	VAR	VAR	
Lags	0	1	2	3	
1	0.004**	0.520	0.562	0.609	
2	0.451	0.799	0.265	0.826	
3	0.493	0.141	0.183	0.638	
4	0.716	0.397	0.574	0.260	
5	0.766	0.622	0.077*	0.351	
6	0.791	0.898	0.825	0.520	
7	0.617	0.426	0.253	0.532	
8	0.805	0.871	0.707	0.696	
9	0.401	0.183	0.136	0.172	
10	0.197	0.916	0.620	0.400	
11	0.293	0.4833	0.756	0.554	
12	0.201	0.0549*	0.014**	0.138	
France	M4 r=2	M4 r=1	M2 r=1	M3 r=1	M3 r=1
Lags	0	1	2	2	3
1	0.009**	0.022**	0.911	0.838	0.620
2	0.681	0.916	0.476	0.445	0.511
3	0.762	0.832	0.998	0.994	0.454
4	0.198	0.473	0.269	0.337	0.386
5	0.389	0.346	0.896	0.843	0.959
6	0.695	0.354	0.542	0.669	0.264
7	0.879	0.399	0.511	0.649	0.769
8	0.668	0.926	0.835	0.841	0.995
9	0.967	0.790	0.975	0.950	0.979
10	0.429	0.586	0.363	0.646	0.611
11	0.769	0.331	0.294	0.404	0.691
12	0.864	0.137	0.088*	0.063*	0.104

* Significant at 5 percent ** significant at 10 percent, Values are p-values and based on Chi-square statistic with 16 df.

Japan	M3 r=2	M3 r=2	M3 r=1	M3 r=2	M3 r=1
Lags	0	1	2	3	4
1	0.063*	0.458	0.950	0.267	0.844
2	0.709	0.482	0.306	0.599	0.932
3	0.370	0.611	0.970	0.382	0.320
4	0.453	0.898	0.613	0.716	0.696
5	0.116	0.085*	0.371	0.397	0.835
6	0.390	0.107	0.428	0.932	0.608
7	0.711	0.840	0.866	0.888	0.629
8	0.688	0.577	0.886	0.978	0.814
9	0.546	0.135	0.082*	0.033**	0.154
10	0.799	0.626	0.817	0.878	0.378
11	0.732	0.761	0.435	0.889	0.408
12	0.434	0.319	0.520	0.619	0.897

United Kingdom	VAR	VAR	VAR	M3 r=1	M3 r=1	M2 r=1	M2 r=2	M3 r=2
Lags	0	1	2	3	4	5	6	6
1	0.094*	0.434	0.866	0.361	0.319	0.497	0.130	0.256
2	0.530	0.225	0.413	0.096*	0.351	0.377	0.213	0.236
3	0.866	0.920	0.564	0.391	0.142	0.833	0.748	0.598
4	0.923	0.703	0.586	0.860	0.145	0.530	0.834	0.834
5	0.978	0.981	0.964	0.192	0.139	0.041**	0.303	0.318
6	0.200	0.0645*	0.069*	0.126	0.409	0.625	0.671	0.359
7	0.070*	0.645	0.816	0.532	0.002**	0.233	0.479	0.687
8	0.073*	0.013**	0.069*	0.036**	0.162	0.059*	0.254	0.452
9	0.535	0.262	0.384	0.981	0.493	0.177	0.130	0.301
10	0.530	0.454	0.533	0.429	0.195	0.352	0.587	0.625
11	0.200	0.344	0.189	0.106	0.058*	0.698	0.262	0.219
12	0.191	0.227	0.537	0.867	0.855	0.733	0.498	0.188

United States	M3 r=1	M3 r=1	M3 r=1	M3 r=1	M3 r=1
Lags	1	2	3	4	5
1	0.007**	0.817	0.176	0.065*	0.660
2	0.012**	0.003**	0.438	0.155	0.228
3	0.760	0.086*	0.440	0.049**	0.610
4	0.789	0.214	0.561	0.649	0.723
5	0.103	0.152	0.257	0.563	0.505
6	0.646	0.421	0.507	0.164	0.472
7	0.046**	0.095*	0.001**	0.172	0.162
8	0.924	0.966	0.977	0.839	0.860
9	0.797	0.913	0.978	0.891	0.865
10	0.680	0.344	0.152	0.934	0.842
11	0.387	0.315	0.710	0.803	0.944
12	0.621	0.175	0.581	0.949	0.714

* Significant at 5 percent ** significant at 10 percent, Values are p-values and based on Chi-square statistic with 16 df.

Table C.2: Residual Normality Test

H0: Residuals are multivariate normal

Canada	VAR	VAR	VAR	VAR
Lags	0	1	2	3
Component	Skewness	Skewness	Skewness	Skewness
1	0.074**	0.500	0.141	0.291
2	0.458	0.698	0.938	0.731
3	0.225	0.609	0.896	0.916
4	0.036*	0.260	0.280	0.627
Joint	0.830	0.711	0.499	0.830
Component	Kurtosis	Kurtosis	Kurtosis	Kurtosis
1	0.521	0.590	0.366	0.072**
2	0.600	0.167	0.107	0.032*
3	0.721	0.318	0.129	0.028*
4	0.419	0.495	0.133	0.016*
Joint	0.832	0.453	0.092**	0.001*

France	M4 r=2	M4 r=1	M2 r=1	M3 r=1	M3 r=1
Lags	0	1	2	2	3
Component	Skewness	Skewness	Skewness	Skewness	Skewness
1	0.023*	0.175	0.045*	0.183	0.329
2	0.505	0.496	0.390	0.364	0.385
3	0.823	0.143	0.502	0.488	0.951
4	0.989	0.574	0.497	0.573	0.937
Joint	0.223	0.312	0.224	0.495	0.788
Component	Kurtosis	Kurtosis	Kurtosis	Kurtosis	Kurtosis
1	0.070*	0.985	0.609	0.622	0.162
2	0.998	0.218	0.377	0.281	0.073**
3	0.935	0.951	0.023*	0.017*	0.007*
4	0.295	0.125	0.139	0.103	0.022*
Joint	0.357	0.423	0.078**	0.044*	0.002*

* Significant at 5 percent ** Significant at 10 percent, Number are p-values based on chi-square statistic.

Japan	M3 r=2	M3 r=2	M3 r=1	M3 r=2	M3 r=1
Lags	0	1	2	3	4
Component	Skewness	Skewness	Skewness	Skewness	Skewness
1	0.185	0.077**	0.522	0.511	0.784
2	0.447	0.723	0.481	0.595	0.412
3	0.262	0.099**	0.309	0.572	0.374
4	0.178	0.200	0.379	0.559	0.573
Joint	0.248	0.106	0.606	0.849	0.762
Component	Kurtosis	Kurtosis	Kurtosis	Kurtosis	Kurtosis
1	0.766	0.713	0.039*	0.014*	0.004*
2	0.103	0.081**	0.131	0.034*	0.064**
3	0.594	0.471	0.145	0.024*	0.017*
4	0.740	0.322	0.161	0.027*	0.007*
Joint	0.534	0.322	0.031*	0.000*	0.000*

United Kingdom	VAR	VAR	VAR	M3 r=1	M3 r=1	M2 r=1	M2 r=2	M3 r=2
Lags	0	1	2	3	4	5	6	6
Component	Skewness	Skewness	Skewness	Skewness	Skewness	Skewness	Skewness	Skewness
1	0.188	0.064**	0.501	0.589	0.340	0.889	0.996	0.964
2	0.643	0.963	0.694	0.903	0.729	0.946	0.907	0.893
3	0.599	0.864	0.983	0.426	0.641	0.728	0.857	0.859
4	0.934	0.996	0.668	0.548	0.803	0.903	0.927	0.945
Joint	0.693	0.482	0.939	0.861	0.859	0.997	1.000	1.000
Component	Kurtosis	Kurtosis	Kurtosis	Kurtosis	Kurtosis	Kurtosis	Kurtosis	Kurtosis
1	0.788	0.919	0.309	0.136	0.095**	0.002*	0.001*	0.001*
2	0.551	0.311	0.093**	0.014*	0.004*	0.004*	0.001*	0.001*
3	0.123	0.059**	0.030*	0.017*	0.003*	0.001*	0.001*	0.000*
4	0.742	0.804	0.323	0.101	0.010*	0.002*	0.000*	0.000*
Joint	0.572	0.325	0.049*	0.002*	0.000*	0.000*	0.000*	0.000*

United States	M3 r=1	M3 r=1	M3 r=1	M3 r=1	M3 r=1
Lags	1	2	3	4	5
Component	Skewness	Skewness	Skewness	Skewness	Skewness
1	0.477	0.523	0.876	0.714	0.874
2	0.505	0.770	0.724	0.777	0.731
3	0.500	0.605	0.934	0.956	0.766
4	0.968	0.558	0.726	0.699	0.969
Joint	0.843	0.894	0.991	0.985	0.994
Component	Kurtosis	Kurtosis	Kurtosis	Kurtosis	Kurtosis
1	0.069**	0.016*	0.032*	0.007*	0.001*
2	0.300	0.294	0.036*	0.008*	0.001*
3	0.193	0.018*	0.015*	0.002*	0.002*
4	0.048*	0.119	0.037*	0.025*	0.005*
Joint	0.040*	0.005*	0.001*	0.000*	0.000*

* Significant at 5 percent ** Significant at 10 percent, Number are p-values based on chi-square statistic.

Table C.3: Joint Residual Heteroskedasticity Test
H0: No heteroskedasticity or (no misspecification)

Canada	VAR	VAR	VAR	VAR				
Lags	0	1	2	3				
Joint	-	0.823	0.813	0.643				
France	M4 r=2	M4 r=1	M2 r=1	M3 r=1	M3 r=1			
Lags	0	1	2	2	3			
Joint	0.339	0.393	0.727	0.752	0.361			
Japan	M3 r=2	M3 r=2	M3 r=1	M3 r=2	M3 r=1			
Lags	0	1	2	3	4			
Joint	0.032*	0.112	0.026*	0.253	0.343			
United Kingdom	VAR	VAR	VAR	M3 r=1	M3 r=1	M2 r=1	M2 r=2	M3 r=2
Lags	0	1	2	3	4	5	6	6
Joint	-	0.152	0.586	0.530	0.355	-	-	-
United States	M3 r=1	M3 r=1	M3 r=1	M3 r=1	M3 r=1			
Lags	1	2	3	4	5			
Joint	0.537	0.170	0.336	0.283	-			

Numbers are p-values, *significant at 5 percent

Table C.4: Model 2, Trace and Maximum Eigenvalue Test for Cointegration

	Trace statistic				**Max-Eigenvalue**			
C(%)=5%	53.12	34.91	19.96	9.24	28.14	22.00	15.67	9.24
Lags = 0	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	104.04**	43.67**	15.60	2.31	60.38**	28.07**	13.28	2.31
France	150.79**	62.61**	20.86*	3.24	88.18**	41.75**	17.62*	3.24
Japan	132.93**	51.67**	25.74**	3.81	81.26**	25.93*	21.93**	3.81
UK	100.69**	33.49	18.45	7.16	67.20**	15.04	11.29	7.16
US	103.46**	26.50	7.96	2.42	76.95**	18.54	5.54	2.42
Lags = 1	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	64.21**	34.56	14.19	4.34	29.65*	20.37	9.85	4.34
France	82.56**	38.83*	16.15	2.78	43.73**	22.68*	13.37	2.78
Japan	95.91**	57.42**	20.93*	3.99	38.50**	36.49**	16.93*	3.99
UK	54.52*	25.27	12.17	5.36	29.25*	13.10	6.81	5.36
US	89.75**	41.39**	17.88	3.47	48.36**	23.51*	14.41	3.47
Lags = 2	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	58.88*	32.92	18.18	7.44	25.96	14.74	10.74	7.44
France	63.03**	30.95	15.62	5.62	32.08*	15.34	9.99	5.62
Japan	81.71**	34.51	16.32	4.02	47.20**	18.19	12.30	4.02
UK	61.24**	31.98	15.94	4.86	29.26*	16.04	11.08	4.86
US	70.70**	37.81*	15.59	4.39	32.89*	22.22*	11.20	4.39
Lags = 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	63.95**	37.66*	17.29	7.34	26.30	20.36	9.95	7.34
France	68.75**	38.82*	22.75*	8.89	29.93*	16.07	13.86	8.89
Japan	69.13**	42.26**	16.75	5.10	26.87	25.51*	11.66	5.10
UK	68.87**	39.72*	18.21	6.65	29.14*	21.52	11.56	6.65
US	68.23**	41.31**	21.80*	7.08	26.92	19.51	14.71	7.08
Lags = 4	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	80.31**	47.30**	23.17*	9.87*	33.01*	24.13*	13.30	9.87*
France	85.51**	40.34*	18.15	5.08	45.17**	22.19*	13.07	5.08
Japan	70.88**	29.86	11.65	2.86	41.02**	18.21	8.79	2.86
UK	68.26**	28.11	10.47	4.29	40.14**	17.64	6.18	4.29
US	109.32**	49.16**	21.03*	5.25	60.17**	28.13**	15.78*	5.25
Lags = 5	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	112.25**	51.02**	25.69**	8.94	61.23**	25.33*	16.75*	8.94
France	75.28**	44.51**	19.61	6.50	30.77*	24.90*	13.11	6.50
Japan	71.83**	39.55*	19.43	5.66	32.27*	20.13	13.77	5.66
UK	86.99**	31.74	13.68	3.34	55.25**	18.06	10.34	3.34
US	97.93**	53.01	25.14	5.65	44.92**	27.87**	19.48*	5.65
Lags = 6	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	146.11**	79.61**	37.28**	8.24	66.50**	42.34**	29.03**	8.24
France	146.80**	70.01**	31.66**	9.30*	76.79**	38.36**	22.36**	9.30*
Japan	108.07**	55.94**	22.75*	10.46*	52.14**	33.19**	12.29	10.46*
UK	103.20**	41.04*	13.38	3.10	62.16**	27.67**	10.28	3.10
US	104.11**	50.09**	23.34*	6.62	54.03**	26.75*	16.71*	6.62

*Significant at 5 percent **Significant at 1 percent

Table C.5: Model 3, Trace and Maximum Eigenvalue Test for Cointegration

	Trace statistic				**Max-Eigenvalue**			
C(%)=5%	47.21	29.68	15.41	3.76	27.07	20.97	14.07	3.76
Lags =0	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	45.63	17.50	3.79	1.10	28.13*	13.71	2.69	1.10
France	83.53**	36.86**	17.51*	0.10	46.67**	19.35	17.41*	0.10
Japan	83.60**	37.06**	15.01	0.12	46.55**	22.05*	14.88*	0.12
UK	43.47	19.74	7.26	0.09	23.73	12.48	7.17	0.09
US	38.06	11.23	5.16	0.09	26.82	6.07	5.07	0.09
Lags =1	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	36.84	15.94	6.04	0.82	20.90	9.90	5.22	0.82
France	63.40**	34.43*	11.77	0.03	28.97*	22.66*	11.74	0.03
Japan	73.75**	36.92**	15.00	0.08	36.83**	21.92*	14.92*	0.08
UK	32.46	12.53	5.53	0.00	19.93	6.99	5.53	0.00
US	48.94*	21.84	5.36	0.06	27.10*	16.48	5.30	0.06
Lags =2	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	36.49	21.71	10.73	1.84	14.79	10.98	8.89	1.84
France	49.72*	25.02	9.72	0.41	24.70	15.31	9.31	0.41
Japan	64.86**	28.48	11.93	0.14	36.37**	16.55	11.79	0.14
UK	44.80	19.73	5.32	0.42	25.08	14.40	4.91	0.42
US	54.27*	21.55	5.17	0.73	32.72**	16.38	4.44	0.73
Lags =3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	44.76	23.16	12.07	2.15	21.61	11.09	9.92	2.15
France	58.17**	28.56	13.03	2.27	29.60*	15.53	10.76	2.27
Japan	59.89**	33.15*	11.13	0.01	26.73	22.02*	11.13	0.01
UK	50.14*	28.26	9.66	2.85	21.88	18.60	6.81	2.85
US	51.17*	27.69	9.00	0.37	23.48	18.69	8.63	0.37
Lags =4	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	58.36**	29.31	15.09	4.14*	29.05*	14.23	10.95	4.14*
France	71.63**	31.37*	10.80	3.47	40.25**	20.58	7.33	3.47
Japan	59.01**	26.77	8.71	0.07	32.24*	18.06	8.64	0.07
UK	59.31**	20.63	4.60	0.04	38.69**	16.02	4.56	0.04
US	89.53**	29.39	7.07	0.00	60.13**	22.32*	7.07	0.00
Lags =5	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	96.66**	41.17**	17.76*	6.81**	55.50**	23.40*	10.96	6.81**
France	68.07**	37.38**	13.93	4.73*	30.68*	23.45*	9.20	4.73*
Japan	66.37**	34.34*	14.59	0.84	32.03*	19.76	13.74	0.84
UK	78.40**	24.65	7.59	0.43	53.75**	17.06	7.16	0.43
US	72.24**	29.47	5.77	0.06	42.77**	23.70*	5.70	0.06
Lags =6	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	130.88**	67.03**	29.47**	7.87**	63.85**	37.56**	21.60**	7.87**
France	139.17**	63.05**	25.21**	6.28*	76.12**	37.84**	18.94**	6.28*
Japan	99.27**	47.77**	15.03	3.05	51.50**	32.73**	11.99	3.05
UK	92.85**	31.09*	6.55	2.05	61.76**	24.55*	4.49	2.05
US	72.85**	32.26*	7.76	1.13	40.60**	24.50*	6.62	1.13

*Significant at 5 percent **Significant at 1 percent

Table C.6: Model 4 of Trace and Maximum Eigenvalue test for Cointegration

C(%) =	**Trace statistic**				**Max-Eigen value**			
5%	62.99	42.44	25.32	12.25	31.46	25.54	18.96	12.25
Lags =0	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	57.35	21.64	7.52	2.41	35.71*	14.12	5.11	2.41
France	91.52**	44.41*	23.40	5.47	47.10**	21.02	17.93	5.47
Japan	95.66**	46.36*	24.31	9.15	49.30**	22.05	15.16	9.15
UK	55.26	25.69	12.49	5.03	29.56	13.21	7.46	5.03
US	65.61*	33.46	9.46	3.76	32.15*	24.00	5.70	3.76
Lags =1	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	47.95	22.87	10.09	4.76	25.08	12.78	5.33	4.76
France	83.26**	42.35	18.96	6.87	40.90**	23.39	12.09	6.87
Japan	85.88**	46.87*	22.91	7.95	39.01**	23.97	14.96	7.95
UK	42.06	19.42	9.26	3.24	22.64	10.16	6.01	3.24
US	65.58*	32.41	14.97	3.61	33.17*	17.44	11.36	3.61
Lags =2	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	44.92	28.13	14.76	4.33	16.79	13.37	10.43	4.33
France	73.10**	33.72	16.85	7.00	39.38**	16.87	9.85	7.00
Japan	77.68**	39.16	22.60	6.77	38.52**	16.56	15.83	6.77
UK	53.36	26.32	10.92	4.74	27.04	15.40	6.18	4.74
US	82.58**	39.71	20.66	4.44	42.87**	19.05	16.22	4.44
Lags =3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	54.23	29.15	13.72	3.17	25.08	15.43	10.55	3.17
France	75.28**	45.21*	22.31	10.72	30.07	22.90	11.59	10.72
Japan	73.92**	44.81*	21.95	5.46	29.11	22.86	16.50	5.46
UK	64.32*	35.45	15.42	6.81	28.87	20.02	8.61	6.81
US	90.55**	44.56*	21.76	5.46	45.98**	22.80	16.31	5.46
Lags =4	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	73.63**	41.51	16.60	4.68	32.11*	24.91	11.92	4.68
France	88.99**	48.14*	25.11	7.27	40.86**	23.02	17.84	7.27
Japan	76.46**	44.22*	23.14	8.49	32.24*	21.09	14.65	8.49
UK	70.92**	31.96	12.89	4.52	38.96**	19.07	8.37	4.52
US	139.77**	69.04**	23.91	5.99	70.73**	45.12**	17.92	5.99
Lags =5	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	120.43**	61.60**	27.87*	6.83	58.82**	33.73**	21.04*	6.83
France	94.63**	60.43**	29.76*	7.92	34.20*	30.67**	21.85*	7.92
Japan	85.73**	50.63**	27.58*	9.37	35.11*	23.05	18.21	9.37
UK	104.87**	51.00**	21.55	4.84	53.87**	29.44*	16.72	4.84
US	117.03**	66.32**	25.83*	5.56	50.71**	40.49**	20.27*	5.56
Lags =6	r = 0	r ≤ 1	r ≤ 2	r ≤ 3	r = 0	r ≤ 1	r ≤ 2	r ≤ 3
Canada	138.28**	74.43**	35.77**	12.33*	63.85**	38.66**	23.44*	12.33*
France	190.00**	106.96**	51.52**	14.78*	83.05**	55.43**	36.75**	14.78*
Japan	125.81**	71.17**	19.77	7.31	54.63**	51.41**	12.45	7.31
UK	166.72**	89.51**	28.53*	4.04	77.21**	60.98**	24.49**	4.04
US	147.46**	70.73**	30.85**	6.39	76.73**	39.88**	24.46**	6.39

*Significant at 5 percent **Significant at 1 percent

Appendix D: Graphs of Cointegration Relations

Figure D.1: Cointegration Graph, France

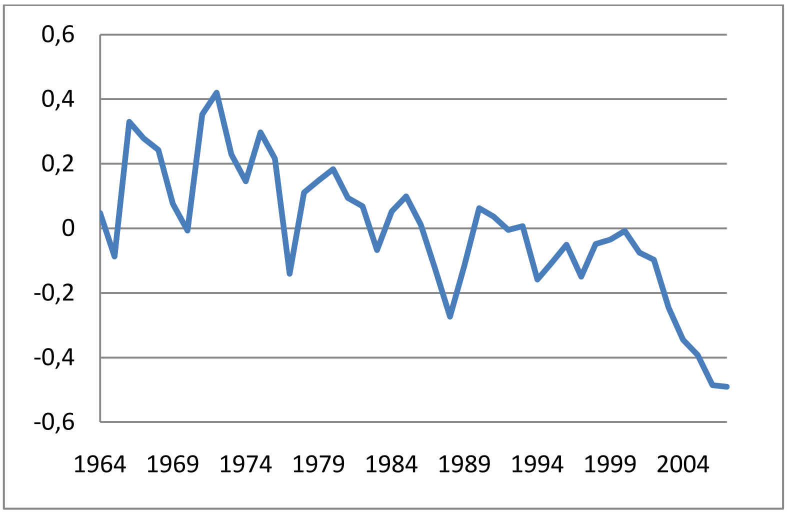


Figure D.2: Cointegration Graph, Japan

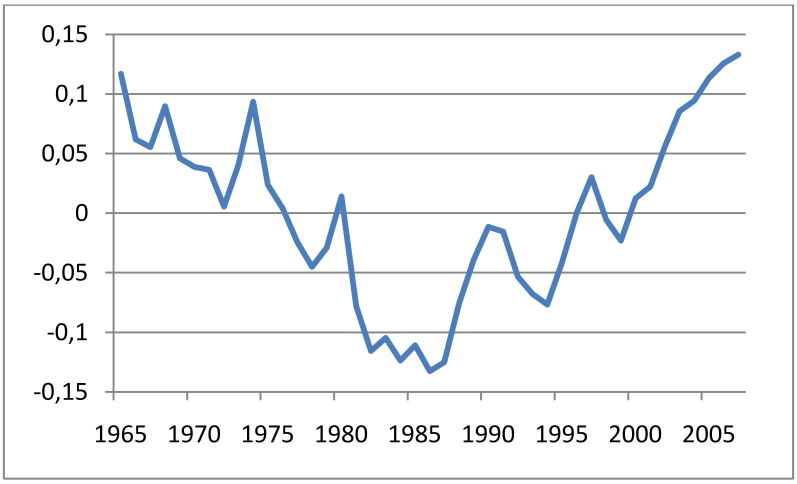


Figure D.3: Cointegration Graph 1, United Kingdom

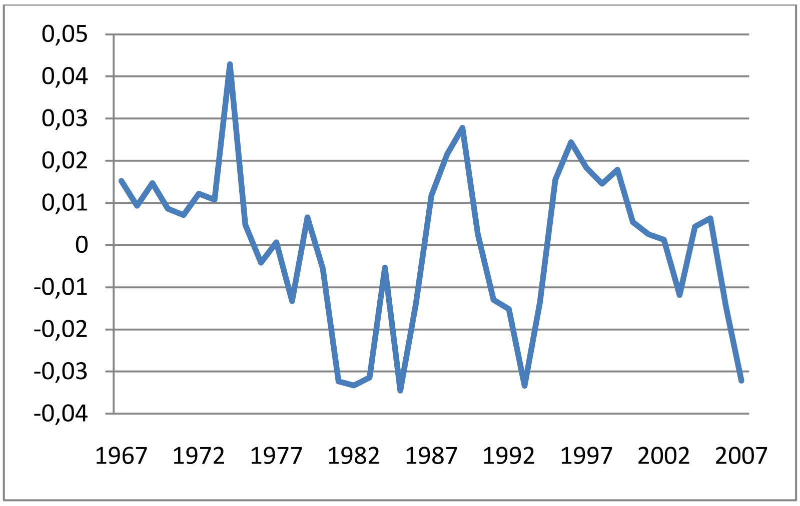


Figure D.4: Cointegration Graph 2, United Kingdom

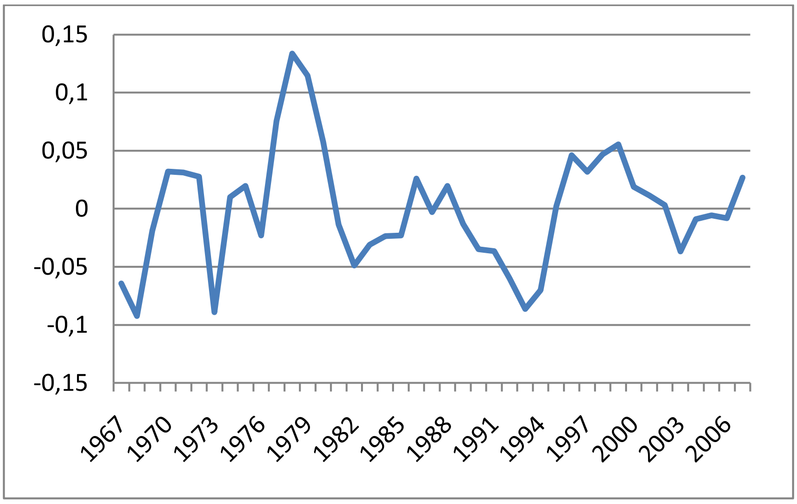
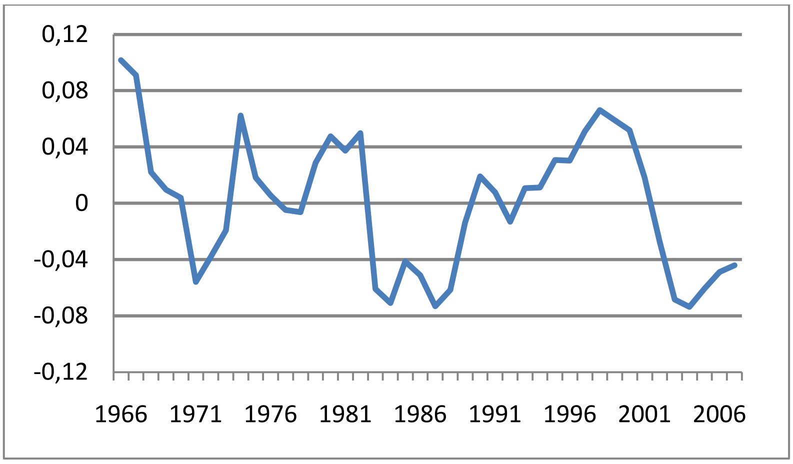


Figure D.5: Cointegration Graph, United States



Appendix E: Calculation of the Capital Stock

This section describes the calculation procedure for the capital stock estimates used in the TFP level calculation. The capital stock was calculated for a period of 1950-2007. I apply a perpetual inventory method with steady-state initial capital stock describe in Limam and Miller (2004) who follows (King – Levine, 1994).

First, I estimated the initial capital stock for year 1950 for each country. Then, based on that starting value I used investment series with the perpetual inventory method to derive the capital stock series. In order to calculate the initial capital stock the steady-state capital-out ratio was derived:

$$\kappa_j^* = i_j^*/(\delta + \gamma_j^*) \tag{E.1}$$

where κ_j^* is the steady-state capital-out ratio, and is assumed to be fixed. i_j^* is the steady state investment rate (share) for country j, which is the average rate for 1950-2007. δ is the depreciation rate of capital and is assumed to be 7 percent. The steady-state growth rate is a weighted average of the country's growth rate (1951-2007) and the word growth rate (1961-2007):

$$\gamma_j^* = \lambda\gamma_j + (1 - \lambda)\gamma_w \tag{E.2}$$

where λ is set to 0.25, and is a measure of mean reversion. $\gamma_w$was calculated to 0.04.

The initial capital stock, $K_{j,50}$, is the derived using: $K_{j,50} = \kappa_j^* Y_{j,50}$, where $Y_{j,50}$ is the real GDP level in 1950. Then, the capital stock estimates for any given following year uses the perpetual inventory method according to:

$$K_{j,t+1} = I_{j,t} + (1 - \delta)K_{j,t} \tag{E.3}$$

The series used in the estimations are then based on the years 1960-2007. All sample series used are from Penn Word Table 6.3 and are deflated into \$2000.