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Modeling North Sea oil production

A contemporary evaluation of an empirical model focusing on Norwegian production

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Abstract

In this thesis an empirical model for optimal production of North Sea oil is implemented for the purpose of evaluation in a contemporary context. The model was presented by Pesaran in 1990 and is one of dynamic optimization where a price-taking producer faces a maximization problem subject to constraints describing production conditions. The model along with some background research is initially presented in a quite extensive form. Later the generalized method of moments estimator is used to estimate the equations describing optimal production and exploration of oil. The model proofs to be relatively unsuccessful for the period 1989Q1-2008Q4 and the reason for the failure of it is argued to be implicit assumptions about the stationarity of real oil prices and the lack of cointegration between variables included.

Key terminology: Oil Production, Resource Scarcity, Norway, Dynamic Optimization, GMM.

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1 Introduction

World consumption and production of oil has since the industrial revolution been steadily increasing. The economy of the industrialized countries has been and still is heavily dependent on oil. The recent increase in global economic and population growth ensures that the demand for energy will increase further on into the future. Higher demand for energy will most certainly lead to higher oil prices and depletion of fields where oil is cheap and easy to extract. These factors are likely to move the extraction of oil to locations where the commodity is more inaccessible due to for example extreme weather and complex geological conditions. Locations as such can often be found at sea where the oil is located deep below the surface in seabed wells or on land in shales and sands where the oil requires a lot of processing before it can be sold. Parallel to this process technologies that can provide alternative energy sources are being developed and are likely to continue to do so as the world faces higher oil prices.

Many researchers have attempted to model optimal extraction of exhaustible natural resources by using a dynamic optimization framework, a work that started with Harold Hotelling in 1931. M. Hashem Pesaran is another such researcher who in 1990 developed a model for optimal extraction and exploration of oil in the UK continental shelf for the period 1978-1986. The model is one of a price taking producer of North Sea oil who optimizes the expected future profits by controlling the production and exploration of the commodity. The framework accounts for the fact that discoveries decline over time and that reserves deplete over time. Also it combines a consistent theory with equations that are possible to estimate both for optimal production and exploration over time.

The aim of this thesis is to use the model developed by Pesaran and apply it to a more contemporary context and for a different producer, Norway, who are producing oil under circumstances which are very similar to the UK. The Norwegian oil production began in the early 1970's and was steadily increasing until around the year 2000 when production started to decline. Similarly to the UK the Norwegian oil production has until recently been focused mainly at extracting the commodity from wells located deep below the bottom of the North Sea.

The purpose of replicating Pesarans research is that it has potential to prove a valuable tool for policymakers when forecasting future oil production and making investment decisions about alternative energy technologies. It can also serve as a tool for oil producers providing information about for example what level the price of oil needs to be at for extraction of less accessible oil to be profitable. Moreover the model too has potential to serve as a tool for developers of alternative technology when figuring out what the price of their technology should be for it to be able to compete with oil in the future.

The basic results of this thesis are poor: no convergence is attained when the Generalized Method of Moments estimator is used to estimate the model and the model is subsequently argued to be generally miss-specified.

The outline of this thesis is that section 2 gives an overview of the previous research with the topic of oil and economics, section 3 sets up the theoretical framework from which the main model stems and presents the main model of oil production. Thereafter section 4 presents the econometric method of the Generalized Method of Moments estimator, section 5 presents the specific equations that are to be estimated and section 6 offers a thorough presentation of the data. The results are presented under section 7 and these are analyzed under section 8. Finally the conclusions that draw on the previously conducted analysis are presented under section 9.

2 Previous research

Hotelling (1931) addresses the issue of extracting a natural resource over time. He states that static general equilibrium models are inadequate for determining how producers of non-renewable resources should plan their production. He finds that as long as a non-renewable resource is not a common property competitive extraction is socially optimal. Gilbert (1978) introduces uncertainty about the size of the stock of the non-renewable resource. He finds that it is always optimal for a producer to invest in exploring more stock if storage costs are low and the firms internal valuation of the resource is high. Pindyck (1978) investigates what optimal production and exploration will be if the producer of a natural resource must simultaneously decide on how much to produce and how much effort to put into exploring. He finds that under such circumstances the price profile for the resource might be U-shaped meaning that prices initially decrease from a high level due to low levels of explored reserves and that the price later again will increase due to exploration and depletion of reserves.

Nordhaus (1980) investigates the linkages between shortages of sharp rises in the price of oil and macro economic performance. By analyzing the pre-1973 aspects of the oil market he develops a model for the contemporary 1980 industrialized economy and tests it on US data. He concludes that there is a strong connection between high oil prices and macro economic performance. The author suggests a set of policy strategies implying an introduction of tariffs on oil not produced in the US to reduce dependence on foreign oil.

Rasche and Tatom (1981) investigate the relationship between increases in the oil price and changes in price level, productivity, investments and output for the US, Canada, Japan, Germany, France and the UK. The authors conclude that the evidence supports a strong connection between the price of oil and the macroeconomic variables mentioned above. They also conclude that the relationship is similar for all investigated nations. Hamilton 1983 investigates the occurrence of post World War II recessions in the US economy and the preceding occurrence of sharp rises in the price of crude oil. He concludes that this is not a result of coincidence nor is there a third underlying variable explaining both oil price and the drop in GDP. He does however conclude that there is a theoretical and statistical connection between the variables and that the US economy had developed in a profoundly different way had there not been sharp rises in the oil price.

The relationship between oil and the macro economy is also investigated by Gisser and Goodwin (1986). They find that crude oil prices have a profound impact on a broad range of US macroeconomic indicators. The effect is of such magnitude that it often exceeds the effects of monetary policy and always exceeds those of fiscal policy. Keane and Prasad 1996 investigate the relationship between changes in the oil price and changes in employment and real wages. They find that an increase in the price of oil has a negative effect on wages across all sectors but that it has a positive effect on relative wages for skilled workers. Also, the effect on employment is found to be negative in the short run but positive in the long run.

The relationship between oil and the stock market is analyzed by Jones and Kaul (1996). They look into whether changes in the oil price leads to changes in the price of financial assets in accordance to changes in real future cash flows and expected returns. They find that the US and Canadian stock market react rationally to changes in oil price but that the Japanese and British markets tend to over-react.

Nygreen et. al. (1998) evaluate Norwegian petroleum production and transportation using a model which at the time had been used by Norwegian oil producers over the preceding 15 years. The models objective function either maximizes total net present value of future cash flows or minimizes deviations from an initially set target. This is combined with both economical and engineering constraints such as pipeline and production capacity. The authors find that the model has had an important impact on historical oil production and planning, they do however also conclude that most of the important decisions have been made on a political basis and therefore not always according to the models implications of optimal production.

Berg et. al. (2002) investigate oil exploration under climate treaties for non-OPEC countries that take the world market price as given. Their model is one of global equilibrium for the fossil fuel markets with an objective function that maximizes the value of all future discounted net revenues accounting for carbon taxes introduced gradually. The authors find that non OPEC-oil producers will delay exploration under the presence of a carbon tax and that the incentive to delay will be stronger when taxes are expected to be as high in the future as they are in the beginning. If the carbon tax is gradually increasing over time the authors conclude that production will be moved forward in time.

Ulstein et. al. (2005) present a model for tactical planning of Norwegian petroleum production which takes into account production and pipeline constraints, regulation of production from wells and the splitting of the flows of natural gas and oil as well as consumer demand. The model is then evaluated against market requirements of oil and natural gas. The model results in an optimal extraction path of petroleum products where the most valuable stock is extracted first and less valuable petroleum will be used later. Another important result is that the revenues where very sensitive to changes in quality requirements.

3 Theoretical framework

The theoretical framework that will be explored in this section is based on a mathematical technique called optimal control theory. It was developed in the 1950's by a group of Russian mathematicians lead by Pontryagin for the purpose of space exploration. The technique draws on the works by Euler and Lagrange in the 18th century on the classical calculus of variation (Sydsæter et. al. 2008:305). In this section the theory will be presented in the context of optimal extraction of exhaustible resources. The basic problem is that of profit maximization. Consider an economic agent (e.g. a firm or a state) that owns a stock of an exhaustible natural resource. The agent can extract the resource at a cost and sell it on the market rendering revenue, once extracted the resource cannot be restored so the agent has to be very careful when planning his production. (Hanley et. al. 2007:214).

3.1 Optimal resource extraction under perfect competition

Hotelling (1931) stated that the problem for a profit maximizing firm that extracts an exhaustible natural resource under perfect competition is:

$$max_q \int_0^T \{pq - c(R,q)\} e^{-rt} dt \qquad (1)$$

Subject to:

$$R(0) = R_0 \qquad (2)$$

$$\dot{R} = -q \qquad (3)$$

Here p is the price of oil, q is the extraction of oil, R is the level of oil reserves and R_0 is the initial amount of reserves. Equation (3) implies that the stock of the natural resource changes over time only by how much of it the producer chooses to extract. The function $c(R,q)$ is the cost function for production which depends both on the amount extracted at each point in time and the level of reserves at that point. The reason for reserves being included in the cost function is that lower levels of reserves implies lower reservoir pressure which in turn implies higher costs of extraction. Also, perfect competition implies that the producer has a market share sufficiently small for him not to be able to impact market price. Therefore the price variable is not described as a function of any underlying factors such as for example supply. The current value Hamiltonian is:

$$H(q, R, \mu) = pq - c(R, q) - \mu q \quad (4)$$

With first order condition:

$$H_q = p - c_q - \mu \quad (5)$$

And co-state condition:

$$\dot{\mu} = r\mu + c_R \quad (6)$$

Taking the time derivative of the first order condition and equating with the co-state condition gives Hotelling's rule:

$$r = \frac{\frac{\partial}{\partial t}(p - c_q)}{p - c_q} - \frac{c_R}{p - c_q} \quad (7)$$

This expression states that when the marginal cost of keeping the resource in the ground is zero then the price minus the marginal cost of producing should evolve in the same manner as the discount factor. If there however exists a non-zero marginal cost of keeping stock in the ground then price minus cost plus the offsetting effect of reducing the stock kept in the ground on costs should evolve like the discount factor. This is due to the fact that c_R is assumed to be negative since it describes the cost reducing effect of keeping stock in the ground. In order to find the optimal path of extraction one must specify the functional form of the cost function (Hanley et. Al. 2007:229).

3.2 Optimal resource extraction with exploration costs

An extension of Hotelling's model was introduced in 1978 by Robert Pindyck. His model accounts for the fact that a producer of a natural resource initially does not know how much of the resource exists in his territory. Therefore the restriction of the time derivative on the stock variable is extended by a term that accounts for discovery of new reserves. Also, the objective function is extended to account for the costs of discovery (Pindyck 1978:844).

The problem is:

$$max_{q,x} \int_0^T \{pq - c_1(R, q) - c_2(x)\} e^{-rt} dt \quad (8)$$

s.t.

$$\dot{R} = \dot{X} - q \tag{9}$$

$$\dot{X} = f(X, x) \tag{10}$$

$$X(0) = X_0 \tag{11}$$

Where x is discovery effort and f is the rate of discoveries which in turn is an increasing function of the discovery effort x and also a decreasing function of accumulated discoveries X. The present value Hamiltonian is:

$$H = \{pq - c_1(R)q - c_2(x)\}e^{-rt} + \lambda_1(f(X, x) - q) + \lambda_2 f(X, x) \tag{12}$$

Here λ_1 is the internal valuation of keeping stock in the ground and λ_2 is the shadow price of cumulative reserve discoveries. The price equation of motion can be solved from the time derivatives of the respective internal valuations λ_1 and λ_2:

$$\dot{\lambda_1} = c_1' q e^{-rt} \tag{13}$$

$$\dot{\lambda_2} = -(\lambda_1 + \lambda_2) f_X \tag{14}$$

Together with the market clearing expression:

$$pe^{-rt} - c_1(R)e^{-rt} - \lambda_1 = 0 \tag{15}$$

And equation (9) to be:

$$\dot{p} = rp - rc_1(R) + c_1' f(x, X) \tag{16}$$

In the case of zero marginal cost of reserves the expression breaks down to equation (7), Hotelling's rule. Through this expression it becomes clear that exploration effort will effect the price movement positively. The cumulative exploration will however have a negative effect on the price. This combination of effects reflects the fact that marginal cost increases when more effort is put into exploration. At the same time it contributes to cumulative exploration which adds to reserves which in turn reduces costs (Pindyck 1978:845). When finding the optimal level of exploration a combination of equations (9), (10), (15) and (16) is used while putting $\frac{\partial H}{\partial x} = 0$. This gives the following solution for the equation of motion of exploratory effort:

$$\dot{x} = \frac{c_2'(x)\left[\left(\frac{f_x X}{f_x}\right) * f - f_x + r\right] + c_1'(R) q f_x}{c_2''(x) - c_2'(x)\frac{f_{xx}}{f_x}} \tag{17}$$

This expression states that the effort put into exploring new resources will increase with production but as $t \rightarrow T$ production will go towards zero due to depletion of reserves. Simultaneously the cost of exploring will go towards infinity and the exploratory effort will therefore be equal to zero. The solution for optimal extraction and discovery is also dependent on the specification of the cost function. It is possible to get a solution which implies a U-shaped price curve since at first reserves are low and not much has been discovered, as time passes by more is discovered and produced until reserves once again are scarce and the cost of discovery is high. This is due to the fact that it becomes harder and harder to find new reserves and therefore the profitability of exploiting such reserves is dependent on high prices (Pindyck 1978:846).

3.3 Empirical model of optimal resource extraction under uncertainty

Pesaran 1990 presents an econometric model that is consistent with the models presented above. It also gives attention to the way that firms form expectations and the physical characteristics of the problem (Hanley et. al. 2007:261). The model assumes a risk-neutral producer who makes decisions based upon the information set Ω in the previous period. The general problem is:

$$\underset{x_t, x_{t+1}, \dots}{\text{Max}_{q_t, q_{t+1}, \dots}} \ \text{E}\{\textstyle\sum_{\tau=0}^{\infty} \beta^{\tau} \Pi_{t+\tau} | \Omega_{t-1}\} \tag{18}$$

Where β is the discount factor defined as $\beta = 1/(1+r)$ and Π is the profit function defined as

$\Pi_t = p_t q_t - C(q_t, R_{t-1}) - w_t x_t$. The variables in the profit function are p_t which is the price of crude oil, q_t which is the amount of oil produced at t, $C(q_t, R_{t-1})$ which is the cost function as a function of oil production and reserves as of the previous period and finally, w_t which is the unit cost of exploration (Pesaran 1990:369). On a further note the profit function is assumed to be a convex function and vary positively with the rate of extraction and negatively with the previous periods of reserves. The reason for the profit being decreasing in reserves is that the less oil there is in the reservoir the higher is the costs of extraction due to less pressure in the reservoir which implies that the activity of exploration can be viewed as one of keeping marginal costs low. The constraints facing the producer are:

$$R_{t+\tau} - R_{t+\tau-1} = d_{t+\tau} + e_{t+\tau} - q_{t+\tau}, \quad \tau = 0,1,2,\ldots \qquad (19)$$

Here d is new discoveries, e is extensions to old discoveries and q as before is the extraction rate. The variable e is assumed to be i.i.d. which is a simplifying assumption since the most obvious way to think about it is that it represents a function of past discoveries and effort of development and appraisal wells. The discovery rate is a function of cumulative exploratory effort X_{t-1}, exploration rate x_{t-1} and an error term v_{t-1}:

$$d_t = F(x_t, X_{t-1})v_t \qquad (20)$$

Cumulative discovery efforts are defined as:

$$X_t = X_{t-1} + x_t \qquad (21)$$

The error term v_t is assumed to be orthogonal to the information set which in turn contains information about all current and past values of reserves, prices, exploration rates and exploration costs. The discovery function $F(x_t, X_{t-1})$ is assumed to be increasing and concave in x_t, further it is assumed to be decreasing in X_{t-1} since when cumulative exploration effort has reached a certain point m the effect of the exploration effort is dominated by the effect of exhaustion of reserves so that even if the exploratory effort is high the discovery rate will be negative. Another feature of F is that when cumulative exploration goes to infinity F goes towards zero, simply meaning that when an infinite amount already has been explored it is not likely at all that there is more to discover (Pesaran 1990:370). In order to complete the environment in which the firm maximizes profit Pesaran states that price and cost expectations are formed according to:

$$p_{t+\tau}^{e} = E(p_{t+\tau}^{e}|\Omega_{t-1}) \quad (22)$$

And

$$w_{t+\tau}^{e} = E(w_{t+\tau}^{e}|\Omega_{t-1}) \quad (22)$$

One must of course also define an initial value of reserves R_{t-1}. The optimization framework is now complete and the optimization problem can be presented in its lagranian form:

$$L = E(\sum_{\tau=0}^{\infty} \beta^{\tau} G_{t+\tau}|\Omega_{t-1}) \quad (24)$$

Which is to be maximized with respect to: $q_{t+\tau}, x_{t+\tau}, R_{t+\tau}, X_{t+\tau}, \tau = 1,2,3, ...$

And where

$$G_t = \Pi_t + \lambda_t(d_t + e_t - q_t - R_t + R_{t-1}) + \mu_t(X_t - X_{t-1} - x_t) \quad (25)$$

The auxiliary variables λ and μ can here, as in the previous models, be viewed as the shadow price of keeping stock in the ground and the net value of the marginal product of exploration (Pesaran 1990:371). The corresponding Euler equations form a system which in this case does not have a closed form solution due to the fact that they are non-linear and stochastic. Pesaran deals with this fact in a clever way by assuming that the Euler equations have an interior solution with values of x_t and q_t being strictly positive and focuses on current decision variables x_t and q_t by letting τ=0. The system of Euler equations can then be written as:

$$\mathrm{E}_{t-1}(\lambda_t) = \mathrm{E}_{t-1}(p_t) - \mathrm{E}_{t-1}\left(\frac{\partial C_t}{\partial q_t}\right) \tag{26}$$

$$\mathrm{E}_{t-1}(\lambda_t) = \beta\left[\mathrm{E}_{t-1}(\lambda_{t+1}) - \mathrm{E}_{t-1}\left(\frac{\partial C_{t+1}}{\partial R_t}\right)\right] \tag{27}$$

$$\mathrm{E}_{t-1}(\mu_t) = \mathrm{E}_{t-1}\left(\lambda_t \frac{\partial d_t}{\partial x_t}\right) - \mathrm{E}_{t-1}(\mathrm{w}_{t-1}) \tag{28}$$

$$\mathrm{E}_{t-1}(\mu_t) = \beta\left[\mathrm{E}_{t-1}(\mu_{t+1}) - \mathrm{E}_{t-1}\left(\lambda_t \frac{\partial d_t}{\partial X_t}\right)\right] \tag{29}$$

The first of these equations states that at optimum the internal valuation of the stock kept in the ground is equal to the expected profit of extraction of the stock and selling it on the market. The second equation is the link between time periods since it states that the expected value of future internal valuations is equal to the present value of the expected internal value in the future minus the impact of reduced size of the stock on the marginal cost of extraction. Analogously the third equation is the expected internal valuation of exploration at optimum as the expected marginal value of exploration minus the marginal cost of exploration. The fourth equation can thus be interpreted as the linkage of the internal valuation of exploration over time as the present value of expected future internal valuations of exploration minus the expected impact of cumulative exploration in the future. Note also that in the case when $\mathrm{E}_{t-1}\left(\frac{\partial C_{t+1}}{\partial R_t}\right) = 0$ equation two reduces to the simple Hotelling rule (Pesaran 1990:372).

3.3.1 Finding the optimal time path of extraction

In order to find the optimal extraction over time the cost function must be specified. Pesaran chooses it to be of the form:

$$C(q_t, R_{t-1}) = \delta_0 + \delta_1 q_t + \frac{1}{2}\left(\delta_2 + \frac{\delta_3}{R_{t-1}}\right) q_t^2 + \epsilon_t q_t \tag{30}$$

This equation states that costs are increasing in extraction and decreasing in reserves which means that when the level of reserves go down costs increase, the expected sign of δ_3 is therefore positive. This function is also concave in q_t which implies decreasing marginal cost of extraction. The term ϵ_t is a random variable which is to be viewed as unexpected changes in the cost function (Pesaran 1990:373). Except for defining a cost function another aspect needs to be weighed in before a solution can be attained. This is due to the fact that producing according to the optimal solution invokes costs of extracting that may or may not be proportional to the cost of deviating from optimal extraction. If the cost of producing optimally, which might arise from injecting gas into the reservoir to maintain pressure, is smaller than the loss of profit due to deviation from the optimal path of extraction then the firm might choose to deviate. The relationship between actual rate of extraction and optimal rate of extraction is suggested to be of the form:

$$q_t - q_{t-1} = \phi(q_t^* - q_{t-1}),\ \ 0 < \phi \leq 1 \tag{31}$$

At this point the one thing that remains to be defined before an estimatable equation of extraction can be reached is the price expectations. Pesaran specifies two different price expectations hypothesis who stem from two different ideas about oil price expectations. The first is the rational expectations hypothesis (REH) which implies that expectations are formed according to the following expression:

$$p_t - E_{t-1}(p_t) = \xi_{t1} \tag{32}$$

and

$$p_{t+1} - E_{t-1}(p_{t+1}) = \xi_{t2} \tag{33}$$

Here the ξ_{ti} are white noise processes with expected value zero. The REH therefore implies that the difference between the expected price and the actual price one or two time periods back is zero, in other words the producer is on average correct about the future price of oil. The second approach to the formation of price expectations is the adaptive expectations hypothesis (AEH) which is defined as:

$$E_{t-1}(p_{t+1}) = E_{t-1}(p_t) = (1-\theta)\sum_{i=1}^{\infty}\theta^{i-1}p_{t-i} \tag{34}$$

This implies that price expectations are formed according to a weighted sum of past prices. Under the AEH the supply curve is upwards sloping but the effect of oil price on oil supply is declining in proven reserves (Pesaran 1990:374). The two estimateable equations for optimal extraction can now be defined as:

3.3.1.1 Optimal production under the REH

$$q_t = (1-\phi)q_{t-1} + \alpha_0 z_{t-1} + \alpha_1 z_{t-1}(p_t - \beta p_{t+1}) + \alpha_2 z_{t-1} q_{t+1} + \alpha_3 z_{t-1} h_{t+1} + u_t \quad (35),$$

$$u_t = z_{t-1}(\alpha_1 \xi_{tp} - \alpha_2 \xi_{tq} - \alpha_3 \xi_{th}) \quad (36)$$

3.3.1.2 Optimal production under the AEH

$$q_t = (1-\phi)q_{t-1} + \alpha_0 z_{t-1} + \alpha_1 z_{t-1}(1-\beta)\tilde{p_t}(\theta) + \alpha_2 z_{t-1} q_{t+1} + \alpha_3 z_{t-1} h_{t+1} + v_t \quad (37)$$

$$v_t = -z_{t-1}(\alpha_2 \xi_{tq} + \alpha_3 \xi_{th}) \quad (38)$$

Where the composite error terms are assumed to be martingale difference processes that are also assumed to be serially uncorrelated and satisfy the orthogonality conditions (Pesaran 1990:375):

$$E(u_t|\Omega_{t-1}) = E(v_t|\Omega_{t-1}) = 0 \quad (39)$$

These equations for optimal extraction are expected to render estimates that imply positive marginal effects of production with respect to reserves through the offsetting effect on costs and also with respect to price through the implied increase in expected revenues.

3.3.2 Finding the optimal time path of exploration effort

In order to find the optimal rate of extraction the discovery function needs to be defined. Pesaran refers to Uhler when doing this and states that the discovery function is of the form:

$$F(x_t, X_{t-1}) = A x_t^{\rho} exp(b_1 X_{t-1} - b_2 X_{t-1}^2) \quad (40)$$

This function satisfies all properties stated under section 3.3 for positive values of A, b_1 and b_2. For this specification the threshold value which decides when cumulative discoveries have reached the point where extraction is so large that the function becomes decreasing even in exploratory effort is $X_m = b_1/2b_2$ (Pesaran 1990:375). The problem is that even under the specification of the discovery function the expression is highly complex. In order to find a relatively simple estimatable expression Pesaran lets β be relatively small and ignores the terms involving expectations about future explorations efforts. Also a partial adjustment feature ψ analogous to the production function is included. The exploration function is then defined:

$$logx_t = (1-\psi)logx_{t-1} + \psi a_0 + \psi a_1 X_{t-1} + \psi a_2 X_{t-1}^2 - \psi a_3 log\left(\frac{E_{t-1}(\lambda_t)}{E_{t-1}(w_t)}\right) \quad (41)$$

Where the expectations about the future shadow price of oil links the exploration effort to the optimal rate of extraction and is solved from equation (26) to be:

$$E_{t-1}(\lambda_t) = E_{t-1}(p_t) - \delta_1 - \delta_2(E_{t-1}(q_t)/z_{t-1}) \quad (42)$$

Where $E_{t-1}(q_t) = E(q_t|\Omega)$. This implies that the estimatable equation for the rate of exploration also will be estimated for the REH and AEH through the price expectations term in the expression for the expected shadow price (Pesaran 1990:376). The estimated parameter values are expected to imply positive marginal effects for exploration in price and production and negative marginal effects in accumulative exploration and unit cost of exploration.

4 Method

The model presented above poses two problems that make the use of OLS invalid. The first one is endogeneity which is the problem that arises when the error term and the explanatory variables are not contemporaneously uncorrelated or more formally:

$$E\{\varepsilon_t x_t\} \neq 0 \qquad (43)$$

In this case the use of OLS is not valid since the estimator is no longer unbiased or consistent and one must pursuit different estimation methods (Verbeek 2005:133). The other problem is when the model specification is non-linear in its parameters which obviously renders misspecification problems if the standard OLS is used (Verbeek 2005:65). Both of the problems described above are sufficient for an alternative estimation method to be pursued. The endogeneity problem itself is strong enough for OLS not to be valid and the solution to such a problem in the absence of parameter non-linearity is usually to use the instrumental variables estimator (Verbeek 2005:140). The parameter nonlinearity problem can in absence of the endogeneity problem be solved using the non-linear OLS estimator. When both problems arise simultaneously the non-linear OLS is no longer unbiased or consistent (Verbeek 2005:65). Also the instrumental variables estimator is miss-specified in its functional form (Verbeek 2005:155). In such a situation it is therefore more appropriate to turn to the generalized method of moments (GMM) which is a method of estimation that can handle both endogeneity and parameter non-linearity (Verbeek 2005:159).

The general idea of the method is that a model with a set of moment conditions that is characterized by:

$$E\{f(w_t, z_t, \theta)\} = 0 \qquad (44)$$

Here w_t is the set of endogenous and exogenous variables, z_t is a set of instruments and θ is the set of parameters. This model has the sample equivalent:

$$g_t(\theta) = \frac{1}{T}\sum_{t=1}^{T} f(w_t, z_t, \theta) = 0 \qquad (45)$$

When the model is non-linear in its parameters it is not certain that there is an analytical solution for θ, also, when the number of moment conditions is less than the number of parameters the vector of parameters is not identified (Verbeek 2005:159). Therefore the problem is described in its quadratic form as:

$$min_{\theta} Q_T(\theta) = min_{\theta} g_T(\theta)' W_T g_T(\theta) \quad (46)$$

This expression gives the generalized method of moments estimator. The key assumption above is that of sample averages converging to population means in their probability limits. The properties of the weighing matrix W are:

$$W = (E\{f(w_t, z_t, \theta)\} E\{f(w_t, z_t, \theta)'\})^{-1} \quad (47)$$

This expression is dependent on the parameter values θ which is a bit problematic since it is initially unknown. In order to get around this problem a sub optimal choice of W that is not dependant on θ is used (usually the identity matrix) and the resulting parameter values $\theta_{[1]}$ are then used in the optimal weighing matrix:

$$W_T^{opt} = \left(\frac{1}{T}\sum_{t=1}^{T} f\left(w_t, z_t, \theta_{[1]}\right) f\left(w_t, z_t, \theta_{[1]}\right)'\right)^{-1} \quad (48)$$

In the next step W_T^{opt} is used to minimize the quadratic form and the asymptotically efficient GMM estimator θ_{GMM} is attained (Verbeek 2005:160). The process of finding the optimal weighing matrix may also be continued after this point in order for the purpose of further refinement. The process is then repeated until convergence is attained. This is also known as the iterative GMM and is proven to have good small sample performance. The variance of the GMM estimator is given by:

$$V = (DW^{opt}D')^{-1} \quad (49)$$

Where D is the matrix of the derivatives of f with respect to the paramaters θ. If the elements in D are large it implies a quick and accurate estimation of θ (Verbeek 2005:161). The distribution of the GMM estimates is given by:

$$\sqrt{T}\,(\theta_{GMM} - \theta) \xrightarrow{plim} N(0, V) \qquad (50)$$

Which implies that in the probability limit the estimated parameters θ_{GMM} is equal to the true value θ implying that the estimator is asymptotically unbiased (Verbeek 2005:160). The strength of the GMM estimator other that the fact that it can handle models with non-linear parameters and endogeneity is that it does not rely on any assumptions about the distribution of the error term. It is also resistant towards heteroskedasticity of unknown form.

5 Model

The equations that will be estimated are the output equations (35), (36) and the exploration equation (37). Since these equations are non-linear and are expected render endogeneity problems through their error terms the method that will be deployed in this thesis is GMM. Pesaran has used a method that he refers to as the Non-linear two-stage least squares (TSNLS) which has properties that are similar to GMM in the same way as the normal two-stage least squares. Therefore the use of GMM in this context is valid (Verbeek 2005:159). For convenience the equations to be estimated are re-stated below:

5.1 Output equation

5.1.1 REH

$$q_t = (1-\phi)q_{t-1} + \alpha_0 z_{t-1} + \alpha_1 z_{t-1}(p_t - \beta p_{t+1}) + \alpha_2 z_{t-1} q_{t+1} + \alpha_3 z_{t-1} h_{t+1} + u_t \quad (51)$$

With:

$$u_t = z_{t-1}(\alpha_1 \xi_{tp} - \alpha_2 \xi_{tq} - \alpha_3 \xi_{th}) \quad (52)$$

5.1.2 AEH

$$q_t = (1-\phi)q_{t-1} + \alpha_0 z_{t-1} + \alpha_1 z_{t-1}(1-\beta)\tilde{p_t}(\theta) + \alpha_2 z_{t-1} q_{t+1} + \alpha_3 z_{t-1} h_{t+1} + v_t \quad (53)$$

With:

$$v_t = -z_{t-1}(\alpha_2 \xi_{tq} + \alpha_3 \xi_{th}) \quad (54)$$

And:

$$\tilde{p_t}(\theta) = \theta^t(p_0 - p^*) + p^* \quad (55)$$

With common terms:

$$h_t = (q_t/R_{t-1}) - \frac{1}{2}(q_t/R_{t-1})^2 \text{ and } z_t = \frac{\delta_2 R_t}{\delta_2 R_t + \delta_3} = \frac{R_t}{R_t + \gamma} \text{ where } \gamma = \delta_3/\delta_2 \quad (56)\text{-}(58)$$

Also $\alpha_0 = -\frac{\phi(1-\beta)\delta_1}{\delta_2}, \alpha_1 = \frac{\phi}{\delta_2},\ \alpha_2 = \phi\beta,\ \alpha_3 = \phi\beta\gamma$ (59)-(62)

The presence of parameters in the error terms through z_t, α_1, α_2 and α_3 is the main factor indicating that endogeneity is present the model. In addition to the specification above the output equations will include seasonal dummies in order to capture the effects of possible seasonal variations of oil production in the north sea and will be of the type (dq1-dq4), (dq2-dq4) and (dq3-dq4). In order to get around the endogeneity problem lags and cross products of lags are used as instruments. For the output equation the instruments will be: p_{t-1}, p_{t-2}, q_{t-1}, q_{t-2}, h_{t-1}, h_{t-2}, R_{t-1}, R_{t-2}, $R_{t-1}h_{t-1}$, R_{t-1}^2 and the dummy variables. The parameters to be estimated are: the price weight θ, the discount factor β,the partial adjustment parameter ϕ, the cost parameters δ_1, δ_2 , δ_3 and the dummy coefficients.

5.2 Exploration equation

$$logx_t = \psi a_0 + (1-\psi)logx_{t-1} + \psi a_1 X_{t-1} + \psi a_2 X_{t-1}^2 + \psi a_3 \log\left(\frac{\lambda_t^{\sim e}}{w_t}\right) + \xi_t \quad (63)$$

Where:

$$\lambda_t^{\sim e} = E_{t-1}(p_t) - \delta_2\left(\frac{q_t}{z_{t-1}}\right) \quad (64)$$

And

$$a_0 = (1-\rho)^{-1}\log(A\rho);\ \ a_1 = (1-\rho)^{-1}b_1 \geq 0;\ \ a_2 = -(1-\rho)^{-1}b_2 < 0;\ \ a_3 = (1-\rho)^{-1} > 0$$

(65)-(69)

Where z_t is defined in the same way as for the output equation. For the exploration equation the instruments will be: I, log x_{t-1}, X_{t-1}, X_{t-1}^2, log p_{t-1}, log p_{t-2}, logw_{t-1}, q_{t-1}, and q_{t-2}. The parameters to estimated are: the price weight θ, the cost parameter δ_2 and the partial adjustment parameter ψ along with the a_i terms.

For all equations above the GMM estimator will be used through numerical minimization of the BFGS (Broyden, Fletcher, Goldfarb, Shanno) algorithm. The moment conditions will be that the error terms are orthogonal to the instruments. The initial weighing matrix will be defined as $W = I_m * (Z'Z)^{-1}$ where Z is the matrix of instruments and I_m is the identity matrix of dimension m which is equal to the number of instruments.

6 Data

The data series used for estimating the output and exploration equations where observations on the quarterly real price of oil which was found at the U.S. Energy Information Agency (EIA), the quarterly production of oil, the number of exploration bores drilled per year and the total quarterly expenditure on exploration drilling which were all found at the Norwegian petroleum directorate (NPD). The data on the end of year proven reserves was taken from the statistical review of world energy published by British Petroleum (BP). For the calculation of the real unit cost of exploration a price deflator was used based on a price index for petroleum exporters in Norway, this series was found at Statistics Norway.

6.1 Data treatment

The data that was taken from the Norwegian Petroleum Directorate was found in databases whose structure demanded the use of computer programming in combination with the recording of macros in Microsoft Excel and VBA for the purpose of extracting the data in question. The database where the monthly production was found was for example divided by oil field/platform over time and therefore required summing over all fields/platforms for each year and month. For this to be done efficiently, a macro was recorded where Excels database functions where used in combination with a loop that ran the macro until the end of the time period. For reasons of data harmonizing the production data then needed to be presented in barrels not in standard cubic meters which was the case in the database, also, the data needed to be converted to quarterly observations which was done using a combination of macros and VBA coding.

The process presented above was similar for all variables collected from the Norwegian petroleum directorate. The data on reserves where in the form of end of year observations and therefore needed to be converted to quarterly observations which was done consistently with Pesaran by using exponential interpolation in such a way that quarter four for each year equals the end of that years observation. For this purpose VBA-coding in combination with recorded macros where used to generate the series. In addition to the series presented in the previous section a few more series needed to be generated: the unit cost of exploration, w_t, was generated by 1) assuming uniform distribution of the number of exploration wells started in each quarter of each year in question 2) dividing the quarterly cost of exploration by that number 3) converting the corresponding result into real costs using a price index for oil exporters of petroleum products found at the Norwegian statistics agency. Also the accumulated exploration effort series was generated by simply summing all exploratory wellbores started up to each point in time. Below follows a compilation of the variables used directly in the model:

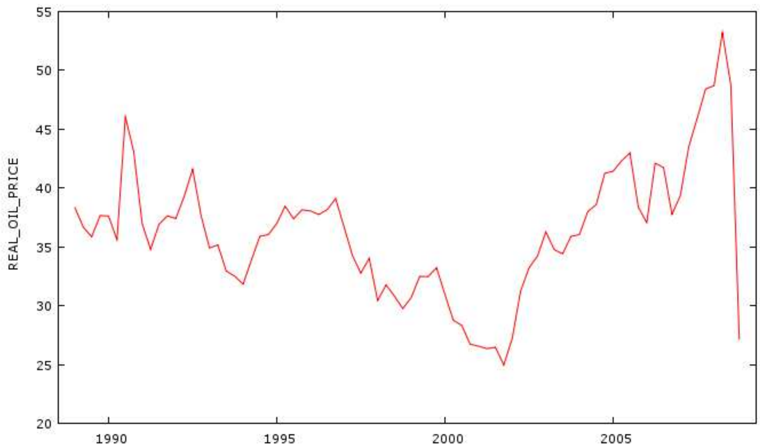
Table 1. Compilation of variables

Variable	Name	Source
$\boldsymbol{p_t}$	Real Price of crude oil in USD	EIA
$\boldsymbol{q_t}$	Quarterly oil production	NPD
$\boldsymbol{x_t}$	End of quarter exploration wellbores started	NPD
$\boldsymbol{EXP_t}$	Quarterly expenditure on exploration wellbores in NOK	NPD
$\boldsymbol{R_t}$	End of quarter proven reserves	BP
$\boldsymbol{PI_t}$	Price index for oil exporters in Norway	Statistics Norway
$\boldsymbol{s_t}$	NOK/USD exchange rate	Norwegian central bank
$\boldsymbol{w_t = \frac{EXP_t}{x_t} * \frac{PI_t}{PI_t^{100}} * s_t}$	Real unit cost of exploration in USD	
$\boldsymbol{X_t = \sum_{s=1}^{t} x_s}$	Accumulated exploratory effort	
$\boldsymbol{h_t = (q_t/R_{t-1}) - \frac{1}{2}(q_t/R_{t-1})^2}$	Composite term	

6.2 Graphical presentation of the data

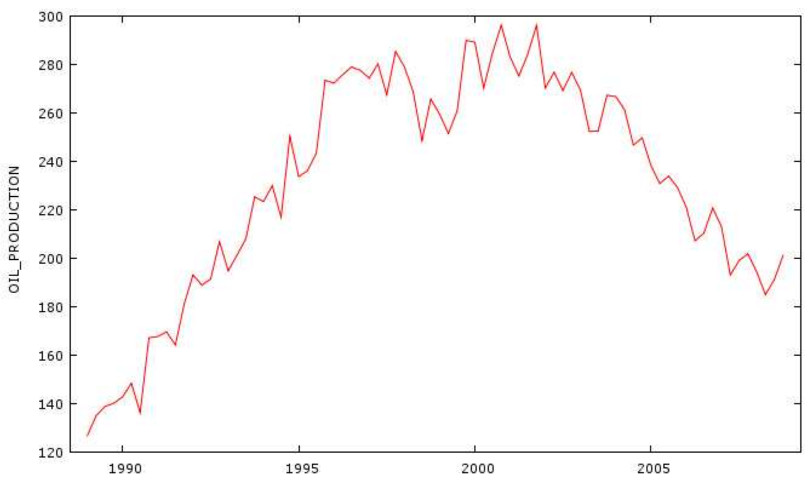
Below follows a graphical presentation of the variables directly used when the model is to be estimated.

Figure 1. Real oil price, $/barrel 1989Q1-2008Q4



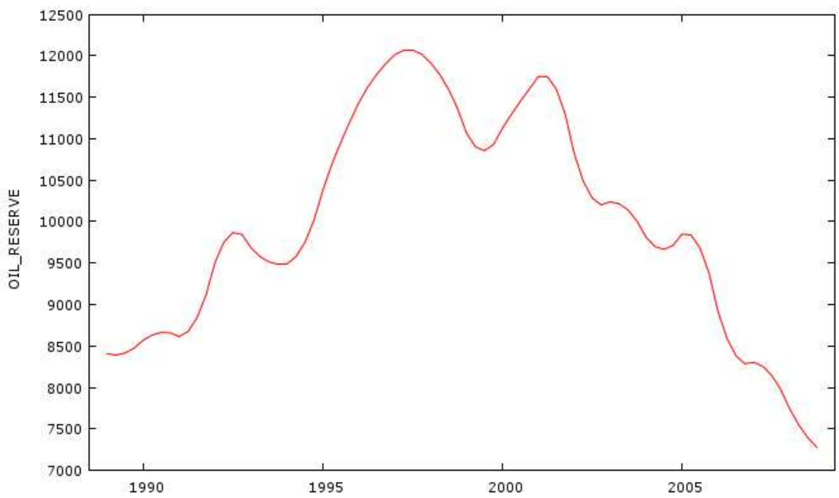
The figure shows that the oil price fluctuated around 30-45 dollars per barrel during the period 1989-2000. During the period 2000-2008 the real price of oil moved from around 25 $/barrel to 53 $/barrel and then at the end of the series the price plummeted to around 27 $/barrel. This is not a surprise since the timing (fall 2008) coincides with the global financial crisis.

Figure 2. Oil production, million barrels, 1989Q1-2008Q4



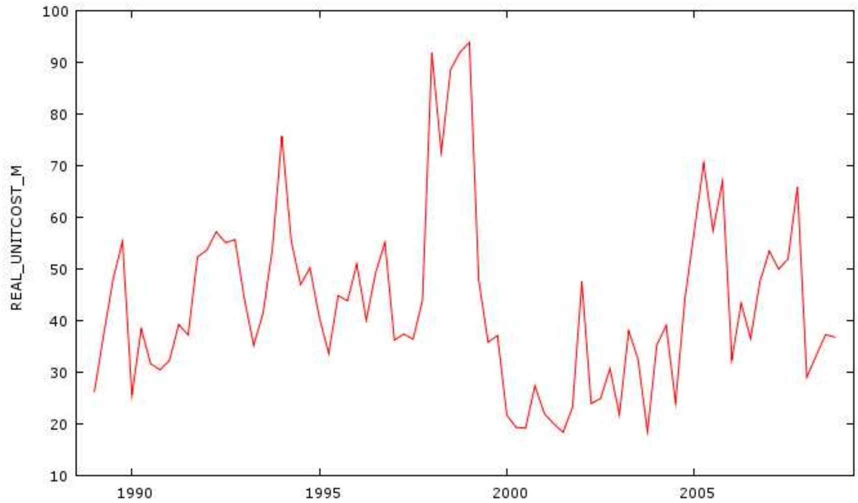
The figure shows how the quarterly oil production was steadily increasing until around 1997 where a temporary slump eventually led to the peak production of 296,06 million barrels around the year 2000. At this point in time oil the oil price was close to its periodic low. After this the production has decreased steadily to 201 million barrels in 2008Q4.

Figure 3. Oil reserve, million barrels, 1989Q1-2008Q4



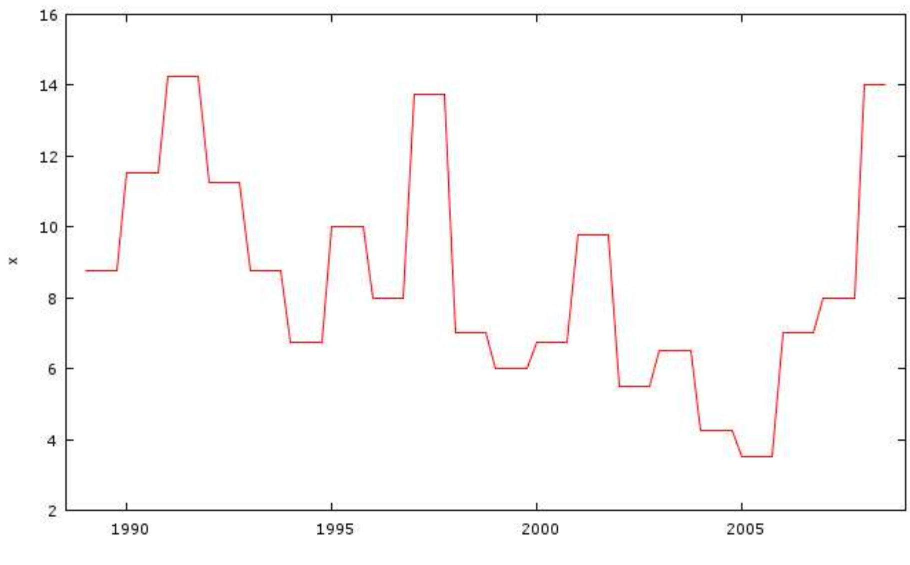
The proven oil reserves in Norway increased until reaching its maximum in 1997 at 12068 million barrels. After a temporary slump and following upturn the reserves have since diminished and are at the end of the series at 7490,8 thousand million barrels. Notable here is that the reserves seem to develop similarly to production. This is not surprising since production is assumed to be a function of reserves (see equations (35)-(38)).

Figure 4. Real unit cost of exploration, million USD, 1989Q1-2008Q4



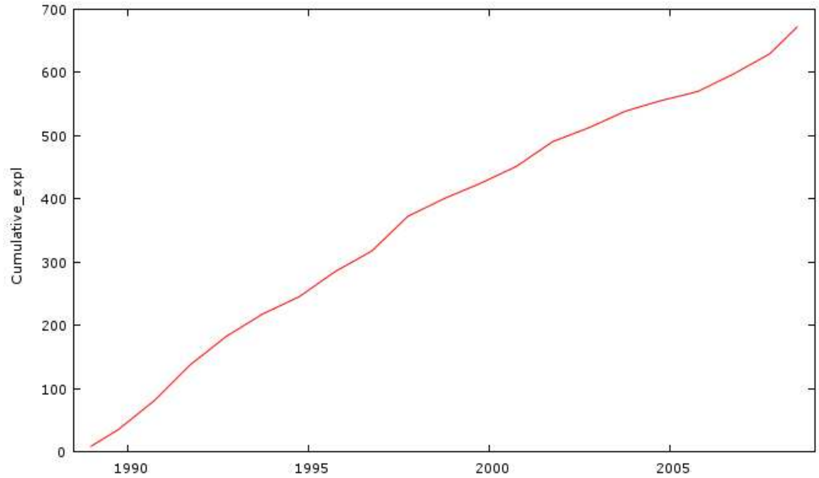
The unit cost of exploration has been varying in the interval 18,399 - 93,739 million USD throughout the period. No clear trend seems to be present, there is however a large peak in unit costs in the end of the 1990's.

Figure 5. Exploration effort, 1989Q1-2008Q4



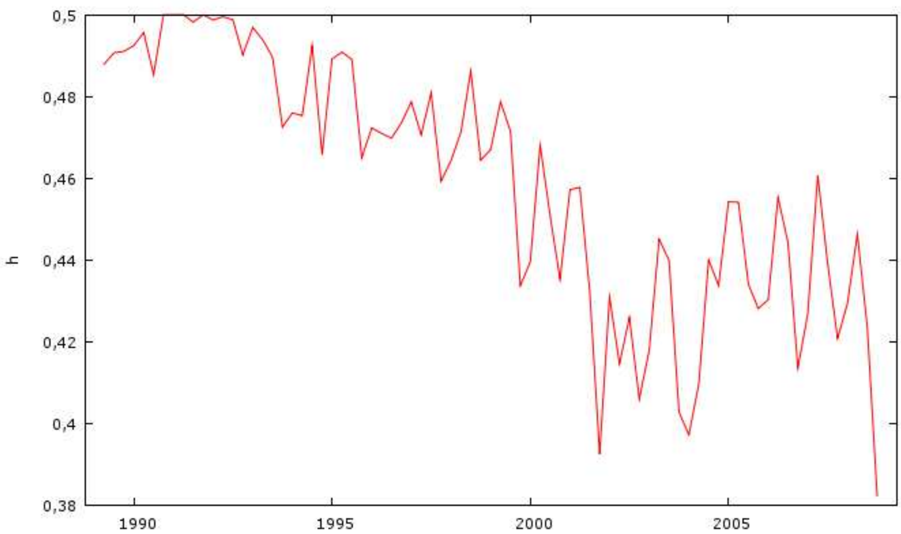
The exploration effort described as the number of exploration wellbores started is here a stepwise series since it has been assumed that the number of exploration bores drilled are identically distributed over the year. Every five years it seems that the exploration effort increases suddenly and temporarily which may be due to static planning of exploration. From 1989 to 2005 the amount of wellbores started seems to decrease on average. From 2005 onwards however the amount increased dramatically.

Figure 6. Cumulative exploration effort, 1989Q1-2008Q4



As expected the cumulative exploration effort is steadily increasing throughout the period, the initial number of exploration wellbores where 8,7500 and by the end of the period 671,00 had been drilled. The mean for this series is 369,41 and the standard deviation is 187,20.

Figure 7. Composite term , 1989Q1-2008Q4



The composite term has decreased steadily during the period, from an initial value of around

0,5 to an end value around 0,4. The mean for this series is 0,45922 and the standard deviation is 0,030705.

6.3 Stationarity analysis

After a graphical inspection of the variables it seems that most of them are non-stationary. In order get more information about this the Augmented Dickey-Fuller (ADF) for existence of a unit root was conducted on all series. The ADF-test is conducted by running the following regression on each variable:

$$\Delta y_t = \mu + \beta t + \gamma^* y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta y_{t-j} + \varepsilon_t \qquad (70)$$

Where μ is a constant, t is a time trend and γ^* and ϕ_j are parameters that concern the lagged values of y_t. The next step is to test the null hypothesis of a unit root that is $\beta = \gamma^* = 0$ by the use of the test statistic:

$$\tau = \frac{\hat{\gamma}-1}{\widehat{\sigma_\gamma}} \qquad (71)$$

Where $\hat{\gamma}$ is the estimated parameter-value of past observations up to the maximum lag order p and $\widehat{\sigma_\gamma}$ is its variance. The basic intuition behind it is to test whether lagged values and of the series in question are significant or not by accounting for time trends of different form (Greene 2002:643). To determine the adequate number of lags included a test down procedure was conducted from a maximum lag order which in turn was approximated from the autocorrelation function. The ADF test was then conducted for level variance with neither intercept or trend, intercept only, both intercept and trend and intercept, trend and quadratic trend. The reason for including the quadratic trend is that both production and reserves show signs of a quadratic component in their graphical representation. A compilation of the resulting test statistics can be viewed below:

Table 2. ADF-test statistics§

Variable	Neither intercept or trend	Intercept	Intercept and trend	Intercept, trend and quadratic trend
p_t	0,0674777	-1,48364	-1,64662	-2,52985
q_t	-0,381144	-1,76556	-0,678295	-3,52493
x_t	-0,363445	-1,97506	-1,83313	-2,06531
R_t	-1,13241	-1,34706	-1,64599	-2,00414
w_t	-0,680855	-4,0202***	-4,02853**	-3,99196**
X_t	0,85489	-1,23888	-2,00629*	-1,72966
h_t	-1,72316*	-0,950853	-3,61807**	-3,47641

§Inference for the no-trend or intercept, and intercept is based on Mackinnon (1996). For the intercept and trend and the intercept, trend and quadratic trend inference is based on Elliott, Rothenberg and Stock (1996).

* Significance on 10%-level

** Significance on 5%-level

*** Significance on 1%-level

From these tests it becomes clear that for most series and test constructions the null hypothesis for existence of a unit root (non-stationarity) cannot be rejected[1]. The exception is the real unit cost of exploration which rejects the null on a 1% level for the ADF with a constant and on the 5% level with a constant and a trend and with constant, trend and quadratic trend. This result is not entirely surprising since it is not obvious from the graphical representation of the variable that it should be non-stationary. Also accumulated production $X_t$is stationary on the 10% level for intercept and trend. For the composite series h_t the null is rejected on the 10% level for the model specification with neither intercept or trend. For this series the test specification of both trend and intercept gives a rejection of the null at the 5% level.

[1] This might prove to be a major problem when the model is to be estimated. Especially the non-stationarity of the real oil price is a big potential threat to the model.

7 Results

The results attained from running the GMM estimator of the model rendered no estimates of any parameters in any of the equations due to lack of convergence. This may due to three things: bad data, poor instruments or bad parameter initialization. In order to establish if the first of these alternatives was the cause of the lack of convergence the data was re-checked and re-calculated and the model run again with no results. Point two was examined by re-running the model with different instruments, in this case using the same variables but with bigger lag length. Once again this rendered no estimates. Another possible source of the lack of convergence could also have been that the initial values that were set to the parameters where too far off their true value and therefore caused the estimation to diverge. To test whether this was the case the parameters to be estimated were set to the exact same value as the results that Pesaran attained in his article. This is valid since one may assume that if the model is good then the parameter values for the current data set should be somewhat similar making it easier for the algorithm to find convergence. This approach however was unsuccessful.

8 Analysis

In general terms the reason for the lack of convergence in the full model can be an effect of non-stationarity in the variables that are included (see table 2). Since the model clinches on its assumption about how price expectations are formed one root of the problem could be that these assumptions do not hold for more contemporary prices.

8.1 Price formation assumptions

The REH is a very strong assumption that directly imposes stationarity of p_t since the expected difference in price between periods is equal to zero. Such a series can obviously not be non-stationary and the lack of convergence under the REH could very well be a direct effect of this. The AEH does not directly impose stationarity of the oil price under equation (34) but that form is not a closed solution implying that it must be transformed before it can be estimated. Pesaran suggests a reduced form for recursive construction that is used for estimation:

$$p_t^{\sim}(\theta) = \theta p_{t-1}^{\sim}(\theta) + (1-\theta)p_{t-1} \quad (72)$$

This is a difference equation of two variables. To find a solution we let: $\theta = a$ and $(1-\theta) = b$:

$$p_t^{\sim}(\theta) = a p_{t-1}^{\sim}(\theta) + b p_{t-1} \quad (73)$$

Starting at $t = 1$:

$$p_1^{\sim}(\theta) = a p_0^{\sim}(\theta) + b p_0 \quad (74)$$

$$p_2^{\sim}(\theta) = a(a p_0^{\sim}(\theta) + b p_0) + b p_1 = a^2 p_0^{\sim}(\theta) + ab p_0 + b p_1 \quad (75)$$

$$p_3^{\sim}(\theta) = a(a^2 p_0^{\sim}(\theta) + ab p_0 + b p_1) + b p_2 = a^3 p_0^{\sim}(\theta) + a^2 b p_0 + ab p_1 + b p_2 \quad (76)$$

And so on which gives us the solution:

$$p_t^{\sim}(\theta) = a^t p_0^{\sim}(\theta) + b \sum_{k=1}^{t} a^{t-k} p_{k-1} \quad (77)$$

The problem is that this is not a closed form solution and is therefore impossible to estimate. To get around this we can treat p_t as a constant which can only be done if one assumes that p_t is a level stationary process, that is: $p_t = p^* + \varepsilon$ where ε is a white noise process with mean zero. By taking expectations we now get:

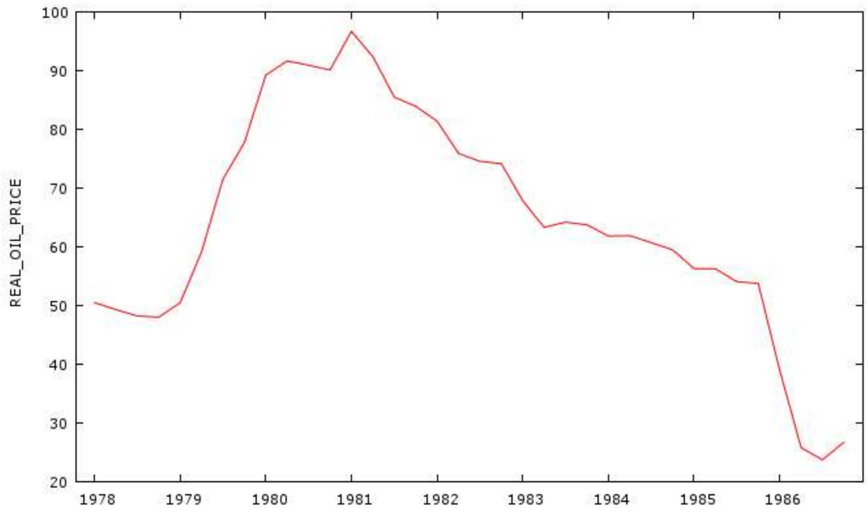
$$E(p_t^{\sim}(\theta)) = E[a^t p_0^{\sim}(\theta) + b \sum_{k=1}^{t} a^{t-k} p_{k-1}] = a^t p_0^{\sim}(\theta) + b \sum_{k=1}^{t} a^{t-k} E(p_{k-1}) = a^t p_0^{\sim}(\theta) + b \sum_{k=1}^{t} a^{t-k} p^* \quad (78)$$

Now the equation can be solved as a linear difference equation of one variable by the rule of the sum of an infinite series and by letting $p_0^{\sim}(\theta) = p_0$:

$$p_t^{\sim}(\theta) = a^t p_0^{\sim}(\theta) + b \frac{1-a^t}{1-a} p^* = \theta^t p_0 + (1-\theta) \frac{1-\theta^t}{1-\theta} p^* = \theta^t (p_0 - p^*) + p^* \quad (79)$$

The resulting equation above is the one used when the model was estimated. Therefore it can be argued that the adaptive expectations hypothesis is estimated under the assumption of stationary prices. Since the real oil price for the period 1989-2008 was found to be non-stationary it can be concluded that this fact probably was a cause of the lack of convergence during estimation. By establishing whether real oil prices where stationary during the 1978-1986 period we can gain more information about this statement.

Figure 8. Real oil price, $/barrel 1978Q1-1986Q4



From an initial value of 50 dollars/barrel the real oil price almost doubled in the course of the following three years reaching a period high at 96 dollars in 1981:1. After this point the price steadily decreased until around 1985:4 when a sharp drop occurred. The price reached its period minimum in 1986:3 at 23,8 dollars. After a graphical examination of the variable it is difficult to determine whether we might expect it to be stationary or not, however, it is not by any means given that it will be stationary since it does seem to trend quite heavily. In order to get more information the ADF test was conducted analogously to section 6:

Table 3. ADF-test on the real oil price 1978-1986

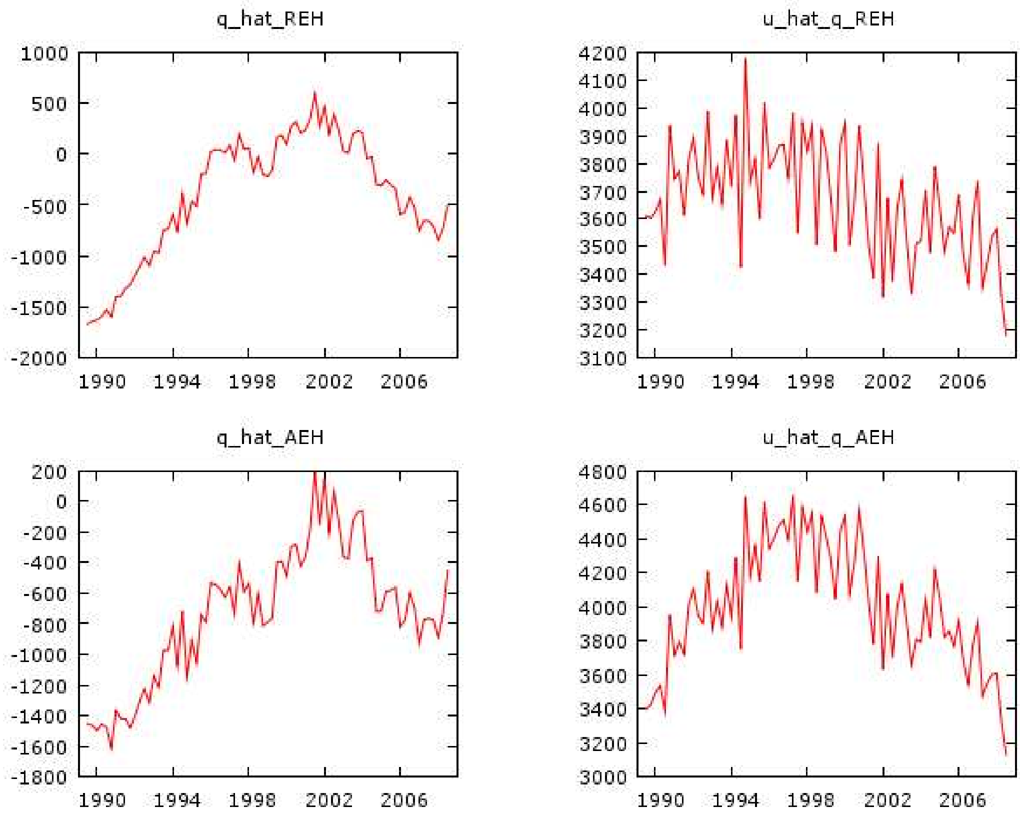
Neither intercept or trend	Intercept	Intercept and trend	Intercept, trend and quadratic trend
-0,594926	-1,34838	-2,10741	-3,69564*

The results indicate that the series is non-stationary for all test specifications except intercept, trend and quadratic trend where the null hypothesis of an existing unit root is rejected at the 10% level. This shows that the model was constructed for a time period that exhibited relatively non-stationary prices thus indicating that the model was miss-specified also in its original context.

8.2 Simulation

In order to get more information about the quality of the estimates that Pesaran attains the parameter values found by him where used to simulate the full model using the Norwegian data. This exercise is on forehand expected to render quite reasonable results since the production and price conditions under which the UK and Norway operate are similar.

Figure 9. Simulated production and residuals using UK parameters and Norwegian data 1989-2008



The simulated production under both the REH and the AEH results in negative production for large part of the time period which is clearly unreasonable since it implies that the producer is buying oil on the market. The shape of the production curves are however quite similar to actual production (see figure 3). The residuals seem to be trending which suggests that autocorrelation might be present[2].

[2] A simulation was carried out for the exploration equation but the Norwegian data was not possible to combine with the estimates attained by Pesaran. The generated series for exploration left empty data points where the function was not defined due to negative values in the logarithmic term (see equation 64).

8.3 Cointegration analysis

The trending residual in the simulated model opens the possibility that the model is more generally miss-specified since it might be the case that the variables used are not cointegrated. A cointegrated relationship can briefly be described as a set of variables that are non-stationary but have some sort of equilibrium relationship usually referred to as the long run relationship. This relationship is of such nature that even though the variables are non-stationary the residual or equilibrium error will be stationary (Greene 2002:650).

A good way to examine whether this is the case is to use the Engle-Granger test for cointegration in which the first step is to examine whether all variables used in the model are integrated of the same order that is if they are stationary after the same number of differencing (Greene 2002:655). By studying the stationarity tests conducted in table 2 under section 6 it becomes clear that the variables included in the exploration equation do not have the same integration order. This is due to the fact that the real unit cost of exploration is concluded to be stationary which implies integration of order zero, the rest of the variables however are not integrated of order zero since they are not stationary. This concludes that the exploration equation is miss-specified. For the case of the output equation it is necessary to test the stationarity of the variables in their first differences to get more information.

Table 4. ADF-test statistics of first differences

Variable	With constant	With constant and trend
Δq_t	-1,89961	-3,54831**
Δp_t	-5,4469***	-5,26993***
Δh_t	-2,63168	-2,62778
Δz_t	-0,24957	-2,78147
ΔR_t	-0,639616	-1,71724

* Significance on 10%-level,

** Significance on 5%-level,

*** Significance on 1%-level

From the results presented in the table above it becomes clear that the variables do not have the same integration order. The only variable that shows signs of being stationary in its first difference is the price variable on the 1 %-level and the production variable on the 5 %-level. For all other variables one can draw the conclusion of non-stationarity and that they therefore must have an integration order larger that one. These results point towards the direction of no cointegration and it may therefore be argued to be the main reason for the lack of convergence when the model was estimated both for the production function and the exploration function. The optimal next step would be to conduct a similar cointergration analysis on the variables for the period 1978-1986 since that could provide information about whether the model was miss-specified with respect to cointegration in its original form. This however proved to be too big a task for this thesis since the relevant data was not available.

9 Conclusions

In this thesis an empirical model of dynamic optimal exploration and production of oil presented by Pesaran in 1990 was evaluated using Norwegian data for the time period 1989-2008. The purpose of this was to investigate the models general applicability and validity. The results from running the full model are poor; no convergence is attained using the GMM estimator for any of the four equations of the full model. It was subsequently concluded that the lack of convergence neither was an effect of calculative mistakes during data treatment nor was it an effect of poor instruments or far off target value assignments of parameters.

On the basis of these results an analysis was carried out which indicated that the model was miss-specified in its original form through the assumption of stationary real oil prices. Also Pesarans estimated parameter-values were used to simulate the full model which resulted in production values that for a large part where negative. A visual inspection of the residual from the simulated model suggested a trending residual which led to a cointegration analysis. It was then found that the variables included in the model were not cointegrated and that the lack of cointegration was the most probable reason for the failure of the model. One of the most striking conclusions that can be drawn from the analysis is that even though one assumes that a cointegrated relationship between the variables exists the model is still miss-specified through the implicit assumption of stationary real oil prices.

From the analysis conducted it becomes clear that the model evaluated in this paper is not a good general model of production and exploration. The use of it in a contemporary context is not valid for three reasons in particular: The first is the miss-specification which is present even in the models original context. It has in this thesis been shown that the specifications of the price expectations hypothesis in the model both assume a level stationary price which is a bad assumption. The second reason is that the variables included in the model are not cointegrated further contributing to the miss-specification of the model. The third argument for not using the model is that the estimation method used is a very sensitive one which in combination with highly complex expressions for optimal production and exploration makes the models general reliability and applicability low.

A suggestion for making the expressions for optimal behavior less complex would be to simplify the cost function so that it for example exhibits constant marginal cost in at least one of the variables. Such a modification can on the other hand push the model further away from reality and its implementation is beyond the scope of this thesis. Ideally a model that can be used for more contemporary oil prices would account for non-stationary of these and use a more robust estimation method than GMM or TSNLS. It should also in a correct way account for the cost structure of production as well as produce relatively simple and intuitive expressions for optimal behavior. Furthermore it should correctly describe the long run relationship that exists between variables. This might be a bit too much to ask of a model developed in a dynamic optimization framework and therefore other methods of finding optimal behavior could be better suited for the task.

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Appendix

A.1 Corresponding Euler equations

The Euler equations that correspond to equation (24) are:

$$\mathrm{E}_{t\text{-}1}\left(p_{t+\tau}-\frac{\partial C_{t+\tau}}{\partial q_{t+\tau}}-\lambda_{t+\tau}\right)=0 \tag{80}$$

$$\mathrm{E}_{t\text{-}1}\left(\beta\lambda_{t+\tau+1}-\lambda_{t+\tau}-\beta\frac{\partial C_{t+\tau+1}}{\partial R_{t+\tau}}\right)=0 \tag{81}$$

$$E_{t-1}\left(\mu_{t+\tau}+w_{t+\tau}-\lambda_{t+\tau}\frac{\partial d_{t+\tau}}{\partial x_{t+\tau}}\right)=0 \tag{82}$$

$$\mathrm{E}_{t-1}\left(\mu_{t+\tau}-\beta\mu_{t+\tau+1}+\beta\lambda_{t+\tau+1}\frac{\partial d_{t+\tau+1}}{\partial X_{t+\tau}}\right)=0 \tag{83}$$

$$\mathrm{E}_{t-1}(R_{t+\tau}-R_{t+\tau-1}-d_{t+\tau}-e_{t+\tau}+q_{t+\tau})=0 \tag{84}$$

$$\mathrm{E}_{t-1}(X_{t+\tau}-X_{t+\tau-1}-x_{t+\tau})=0 \tag{85}$$

By letting $\tau = 0$ the above system can be re-written and give (26)-(29). Since the internal valuation of the stock and the net value of exploration are unobservable they need to be eliminated from (26) in order to find an estimatable equation of optimal extraction. This is done by first noting that:

$$E_{t-1}[E_t(\lambda_{t+1})]=E_{t-1}\left[\mathrm{E}_t(p_{t+1})-\mathrm{E}_t\left(\frac{\partial C_{t+1}}{\partial q_{t+1}}\right)\right] \tag{86}$$

Which, assuming expectations form consistently, yields:

$$E_{t-1}(\lambda_{t+1})=\mathrm{E}_{t-1}(p_{t+1})-\mathrm{E}_{t-1}\left(\frac{\partial C_{t+1}}{\partial q_{t+1}}\right) \tag{87}$$

This result can now be substituted into (27) and be simplified to give:

$$E_t\left(\frac{\partial C_t}{\partial q_t}\right)=E_t(p_t-\beta p_{t+1})+\beta E_{t-1}\left(\frac{\partial C_{t+1}}{\partial q_{t+1}}+\frac{\partial C_{t+1}}{\partial R_t}\right) \tag{88}$$

This equation is neither depending on λ or μ.

A.2 Finding optimal extraction

By using the cost function a solution for the optimal extraction rate over time is attained by substituting equation (30) into equation (86):

$$q_t^* = [-(1-\beta)\delta_1/\delta_2]z_{t-1} + \delta_2^{-1} z_{t-1} E_{t-1}(p_t - \beta p_{t+1}) + \beta z_{t-1} E_{t-1}(q_{t+1}) + \beta\gamma z_{t-1} E_{t-1}(h_{t+1}) \tag{89}$$

By using equation (31) the above solution can be re-written as:

$$q_t = (1-\phi)q_{t-1} + \alpha_0 z_{t-1} + \alpha_1 z_{t-1} E_{t-1}(p_t - \beta p_{t-1}) + \alpha_2 z_{t-1} E_{t-1}(q_{t+1}) + \alpha_3 z_{t-1} E_{t-1}(h_{t+1}) \tag{90}$$

The price expectations term in the equation above can now be replaced by the following term to find the output equation under the REH:

$$E_{t-1}(p_t - \beta p_{t+1}) = p_t - \beta p_{t+1} + \xi_{tp} \tag{91}$$

Where ξ_{tp} fulfills the orthogonality property: $E\left(\xi_{tp} \middle| \Omega_{t-1}\right) = E(\beta(\xi_{t1} - \xi_{t1})|\Omega_{t-1}) = 0$.

The price expectation terms in (90) can now also be replaced by the following expression to find the output equation under the AEH:

$$E_{t-1}(p_t - \beta p_{t+1}) = (1-\beta)(1-\theta)\sum_{i=1}^{\infty} \theta^{i-1} p_{t-i} \tag{92}$$

From REH one can also draw conclusions about the terms in equation (90) that are expectational concerning extraction. Using the same framework as for price we now have that $q_{t+1} - E_{t-1}(q_{t+1}) = \xi_{tq}$ and $h_{t+1} - E_{t-1}(h_{t+1}) = \xi_{th}$ where the ξ_{ti}'s has the same properties as for the price.

A.3 Finding optimal exploration

Solving for the optimal rate of exploration from equations (26)-(29) is much more complex than for optimal extraction. First we can eliminate μ_t from equations (28) and (29) to get:

$$E_{t-1}\left(\lambda_t \frac{\partial d_t}{\partial x_t}\right) = E_{t-1}(w_t) - \beta E_{t-1}\left[w_{t+1} - \lambda_{t+1}\left(\frac{\partial d_{t+1}}{\partial x_{t+1}} - \frac{\partial d_{t+1}}{\partial X_t}\right)\right] \quad (93)$$

The next step is to eliminate λ_t from the expression above and to specify a functional form for the discovery function according to equation (40). The optimal rate of exploration can now be defined:

$$log x_t^* = a_0 + a_1 X_{t-1} + a_2 X_{t-1}^2 - a_3 log\left(\frac{E_{t-1}(\lambda_t)}{E_{t-1}(w_t)}\right) \quad (94)$$

Where

$a_0 = (1-\rho)^{-1}\log(A\rho)$; $a_1 = (1-\rho)^{-1}b_1 \geq 0$; $a_2 = -(1-\rho)^{-1}b_2 < 0$;

$a_3 = (1-\rho)^{-1} > 0$ (95)-(98)