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**Model-based Analysis of Individual Decision Making  
in the DNB Household Survey**

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## **Abstract**

Human financial decisions are known to deviate from 'rational', particularly under uncertainty and if time delays are involved. In the Nobel-winning prospect theory, Tversky and Kahneman showed that deviations from rational behavior were not only robust but also quite predictable, if considering average behavior in a population. They found that people consider gains and losses not in absolute terms, but relative to a reference point. People react stronger to losses than gains of the same magnitude, and they tend to overweight small probabilities and underweight large ones. It has also been shown that future outcomes are usually discounted using a hyperbolic discount function. In this thesis, I applied prospect theory to model risk attitudes and inter-temporal choices based on three different types of questions from the Dutch DNB household survey: probability equivalences, risky choices, and inter-temporal choices. Individual decision-making behavior was analyzed based on model parameters, such as loss and risk sensitivities, probability weighting, reference points, and the discounting rates. As different types of empirical measures (delays, probabilities and choices) were combined in my model, I estimated individually best-fitting parameters by minimizing the  $\chi^2$ -error of goodness-of-fit (Press et al. 1992). The parameter estimation results indicated slightly concave utility functions, strong loss aversion, relatively small discounting and minor influence of reference points, consistent with other model-based studies that addressed some of these aspects. Correlations between the estimated parameters indicated that individuals with more concave utility functions also discounted future less and were less affected by framing (i.e. had lower reference points). This result suggests that these different aspects of valuation may share common cognitive mechanisms, which could be investigated in future behavioral economics studies.

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## CHAPTER 1. INTRODUCTION

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Decision making under risk and uncertainty has been one of the most important topics in economics, psychology, and statistics for many centuries. Numerous theoretical and empirical studies addressed this topic, one of which was the well-known expected utility theory. Expected utility theory measures decision making process by comparing individual values of expected utility. In recent years, a number of empirical studies found that people's actual decision making behavior significantly violates the basic assumptions of expected utility theory. To address such violations, the prospect theory was proposed by Amos Tversky and Daniel Kahneman (1979). People's perception of gains and losses were found to be not solely dependent on the final wealth of the given options, but rather relative to a reference point, which is often the status quo. People's attitudes towards perceived gains and losses are also different: they react stronger to losses than to gains of the same magnitude. Furthermore, people tend to overweight small probabilities and underweight large ones. The main contribution of prospect theory was that it systematized all of these empirical observations into a single model.

In this thesis, I apply Tversky and Kahneman's prospect theory to model risk attitudes (choosing between options with different risk) and inter-temporal choices (deciding between options at different points in time). I use questions from the Dutch DNB household survey, which include three probability equivalence questions, five risky choice questions and eight pairs of inter-temporal choice questions. The part of my model that deals with the inter-temporal choice questions follows Dimmock & Kouwenberg (2010) and Tu et al. (2004), whereas the part addressing probability and risk related questions follows Donkers et al. (2001). I analyze individual decision-making behavior based on a number of model parameters, such as loss and risk sensitivities, probability weighting, references, and the discounting rate. As different types of empirical measures (delays, probabilities and choices) are combined in my model, I estimate individually best-fitting parameters by minimizing the  $\chi^2$ -error of goodness-of-fit (Press et al. 1992). The parameter estimation results largely

correspond to literature, but provide some interesting insights. In addition, novel correlations between the estimated parameters are observed, which could be useful for understanding more fundamental mechanisms of human decision making.

The remainder of this thesis is organized as follows. Chapter 2 outlines the theories and briefly introduces the models of the empirical studies. Chapter 3 describes the empirical data, detailed models and procedures for the parameter estimation, evaluation and analysis. Chapter 4 contains parameter estimation results: evaluation of the goodness-of-fit and the analysis and interpretation of estimated parameter values. Finally, chapter 5 discusses the results in a broader perspective and suggests possible improvements and applications of the study.

## CHAPTER 2. THEORY

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Decision making is a process that an individual (or agent) makes a choice among several alternatives. The process can be very difficult under the condition of risk and uncertainty, as it is hard to predict the outcomes in the future states. Prospect theory is one of the most popular theories in economics to deal with decision making in the presence of risk or uncertainty. It shows how an agent formulates the choices and makes a decision based on his evaluation of the choices. This chapter will be organized as the following. Firstly, it will provide a background of how prospect theory was developed. Then the prospect theory itself will be discussed. Lastly, several concrete applications of models based on prospect theory will be presented to make linkages with the further work.

### **Expected Utility Theory**

#### Bernoulli's utility function

The study on decision making under risk and uncertainty started with the principle of maximizing the expected monetary value of choices. The expected value is equal to the sum of the payoffs in different states times their respective probabilities. It was soon realized that decision-making was guided not by the monetary value of choices but rather by what these choices mean to a decision maker, i.e. their subjective values. Daniel Bernoulli was the first person to propose skepticism on expected value method in 1738. Bernoulli demonstrated the limitation of expected value through a coin-toss game called St. Petersburg Paradox. In fact, he showed that people's willingness to pay for playing this game was much smaller than the expected monetary value of the game. He suggested that decision-making was affected by other factors such as people's perception of the outcomes and the likelihood of winning, rather than solely dependent on the expected monetary value. Based on his study, Bernoulli

proposed the expected utility model where people tended to maximize their subjective value of outcomes (utility), and not the monetary value. Bernoulli's utility function is concave in nature, reflecting the notion of diminishing marginal utility. It means that the differences further away from the origin are less significant for people than those that are closer to it. For example, \$1 compared to nothing seems to be more important for people than \$1001 compared to \$1000. Bernoulli suggested that people would choose a sure outcome, such as \$100, rather than a gamble that has a 50-50 chance to pay \$200 or nothing. Thus, in case of probabilistic choice, the concavity of the utility function can lead to *risk aversion*.

### Von Neumann and Morgenstern Theorem

One of the most important influences in the development of expected utility theory was the theorem by von Neumann and Morgenstern (VNM) in 1944. They developed four axiomatizations according to which people were assumed to behave under uncertain options. Von Neumann and Morgenstern used an opposite approach from Bernoulli's theory: in Bernoulli's theory, utility function was used to define preference, as individuals were assumed to prefer a choice that had the highest utility; in the VNM theorem, it is possible to have *individual* utility function for a particular decision maker and therefore different individuals could have different preference orderings (McDermott, 1998).

The VNM's axioms of expected utility are *completeness*, *transitivity*, *continuity*, and *independence*. I will not elaborate on the first two axioms, because they have little to do with the context of risk and uncertainty. Continuity says that if N is strictly preferred to M, which is strictly preferred to L, a suitable mixture of N and L is strictly preferred to M, and also M will be strictly preferred to another suitable mixture of N and L (Mongin, 1997). The independence axiom says that if N is preferred to M, then the combination of {N, p; L, 1-p} is preferred to the combination of {M, p; L 1-p}, where p is a probability. In other words, the preference of the two combinations should only depend on the preference of option N and M, regardless their common consequence L.

The axioms seem to make a lot sense and have been very influential in modeling

decision-making, yet systematic violations of the axioms have been found in more recent empirical studies. French economist Maurice Allais (1953) questioned the bases of VNM theorem by using a questionnaire that contained two situations. In situation 1, people were asked to choose between (a) *receiving \$1 million for sure*, and (b) *a lottery offering a 10% chance of \$5 million, and 89% chance of \$1 million and a 1% chance of \$0*. In situation 2, people were asked to choose between (c) *an 11% chance of \$1 million*, and (d) *a 10% chance of \$5 million*. Allais found that people who preferred option (a) in situation 1 also preferred option (d) in situation 2. The result shows inconsistent pattern of behavior, because preferring (a) over (b) implies  $0.11u(\$1\text{mil}) > 0.10u(\$5\text{mil})^i$ , whereas preferring (d) over (c) implies  $0.11u(\$1\text{mil}) < 0.10u(\$5\text{mil})$ . Such preference orderings should not co-exist according to VNM axioms. Economist Daniel Ellsberg confirmed Allais's notice on irrational behavior. In his article "Risk, Ambiguity and the Savage Axioms" (1961), Daniel Ellsberg pointed out that a large number of the contemporary studies found violations of the VNM axioms, and he wrote:

*"...others sadly but persistently, having looked into their hearts, found conflicts with the axioms and decided, in Samuelson's phrase, to satisfy their preferences and let the axioms satisfy themselves."*

Almost two decades later, a Nobel Prize winning model that put much more insight into the nature of risk assessment and decision making emerged in the field – the prospect theory developed by Amos Tversky and Daniel Kahneman (1979). The major contribution of this model is that it provides systematic and flexible way to both explaining and predicting behavior under risk and uncertainty. The next section will go more in depth about the prospect theory.

## **Prospect Theory**

Unlike the expected utility model, the prospect theory is an empirically supported descriptive theory, i.e. the one that more accurately describes people's actual behavior under risk and

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<sup>i</sup> If  $u(\$1\text{mil}) > 0.1 u(\$5\text{mil}) + 0.89 u(\$1\text{mil})$ , then  $(1-0.89) u(\$1\text{mil}) > 0.1 u(\$5\text{mil})$ , which is  $0.11 u(\$1\text{mil}) > 0.10 u(\$5\text{mil})$

uncertainty. In VNM theory, there is no clear distinction between normative and descriptive aspects, as people's behavior is assumed to follow the axioms. In prospect theory, Tversky and Kahneman (1979) show that it is fairly difficult to integrate the normative and descriptive aspects into a satisfactory model, because in reality people's behavior tends to violate the most basic assumptions of the normative models, such as how individual utility is assessed. The following example may help to illustrate this point. Assume that government wants to estimate how much tax should be set for increasing industrial pollution by a variable that calculates the additional deaths per year caused by cancer. For this purpose, the first thing to do is to let people evaluate their lives; e.g. by asking them "how much do you think your life is worth?" In most cases people would find it hard to answer because it is uneasy to translate value of life into monetary terms. Now assume that the question is rephrased in a more illustrative way, "Did you drive your car to work? Do you know that you risk your life by 1 in 50,000 per year by driving to work? Then how much do you think your life is worth to you?" (McDermott, 1998) Such formulation may give some clue as to how people would start to think; yet others might feel disturbed hearing such a question and may not be willing to cooperate. Even such a simple example shows how sophisticated it is for people to assess utility. It would not be surprising if researchers could not correctly predict behavior by using normative models in many other complicated situations, because such models were not built on a sufficiently strong and behaviorally realistic foundation. Thus, the very first change to be done is to redefine the foundation of the models, or the utility function.

Prospect theory has put a lot of more insights into its value function (utility function). In prospect theory, the value function includes a domain of gain and a domain of loss, relative to a reference point (illustrated in the later section). The reference point is often the current steady state, or status quo. (Tversky & Kahneman, 1979). Tversky and Kahneman found that people behave differently when they perceive "gain" or "loss", not the type of gain or loss that is based on the final results but rather a psychological sensation of it. In addition, people tend to require more compensation for every additional "loss" they perceive than they value an additional "gain". The value function of prospect theory shows concavity in the domain of gain and convexity in the domain of loss and it is steeper in the domain of loss than in the

domain of gain.

According to prospect theory, decision-making goes through two phases: the editing phase and the evaluation phase. The editing phase consists of representation of the acts, outcomes, and contingencies that are associated with a particular choice (McDermott, 1998). During this phase, decision makers can show inconsistencies in their behavior if different options are phrased differently. This is known as the framing effect. The evaluation phase is when the choices are already edited and decision maker needs to evaluate them to make a decision. In this phase, two important things that influence people's behavior is the subjective value function and the perceptual likelihood. All the above-mentioned aspects are important basics of the prospect theory and are discussed separately in the following sections.

### Framing Effect

The framing effect shows that people's perception of choices can be affected by the way these choices are formatted. Some people may argue that some seemingly obvious formulations of a question, such as the order of the choices, should not substantially affect behavior. However, experiments show that people tend to be sensitive towards questions that contain emotionally extreme content, such as life and death. For example, one group of decision makers was asked to choose between two medical plans that would affect 600 patients' lives. Plan A will save 200 patients for sure. Plan B has one-third chance to save all 600 and two-third chance that nobody will be saved. In this case, 72 percent of the participants chose *plan A*. Then, another group was asked exactly the same question formulated in a different way. Plan A in this case will cause 400 patients to die. Plan B has one-third chance that nobody will die, and two-third chance that all 600 will die. Now 78 percent of the participants chose *plan B*. Clearly, what altered people's behavior is not the question itself, but rather the formulation of the question, as in both cases the expected values are equal.

Why is framing effect happening and why is it so important? Tversky and Kahneman suggest that,

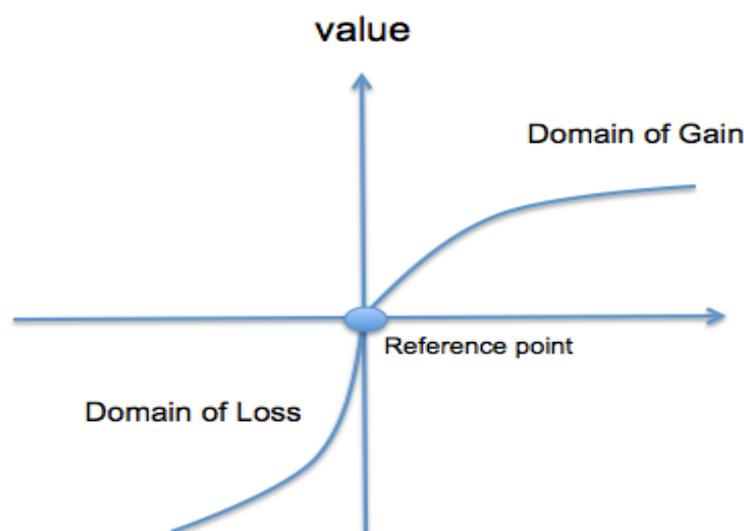
*“Framing is controlled by the manner in which the choice problem is presented as well*

as by the norms, habits, and experiences of the decision maker.” (Tversky & Kahneman, 1986) And more importantly, they wrote, “decision makers are not normally aware of the potential effects of the different decision frames on their preferences, and would wish their preferences to be independent of frames, but are often uncertain how to resolve the undetected inconsistencies.” (Tversky & Kahneman, 1981)

Thus, although people wish that frames did not affect their behavior, most people are unconsciously influenced by them. Framing effect is important not only because it affects the presented choices, but also because it influences the evaluation of the choices in the next phrase through the value function and the perceptual likelihood function. (McDermott, 1998)

### Value Function

In the evaluation phrase, two important functions are the value function and the probability weighting function. An example of the value function of prospect theory is illustrated in the following figure.



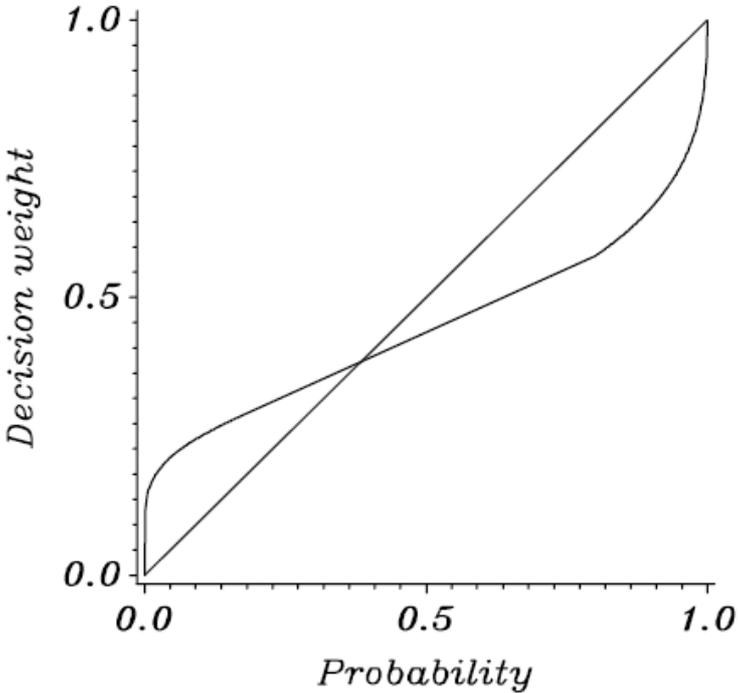
**Figure 1.** Value Function of Prospect Theory

As mentioned earlier, value function of prospect theory has a domain of gain and a domain of loss, relative to a reference point. This is different from the expected utility theory, which considers final value as the measurement of gains and losses. The function is concave in the domain of gain and convex in the domain of loss. The slope of curve represents sensitivity to

change. This means that the changes further away from the starting point are less influential than those that are closer to it. Another feature of the value curve is that it is asymmetrical along the axis. The curve is steeper in the domain of loss than in the domain of gain. This feature implies loss aversion – loss gives worse feeling than a comparable gain pleases. Study also shows that people easily adapt to status quo. They are often satisfied with the current steady state and are not willing to be worse off from it. Therefore, people may value what they currently have to a greater degree than a similarly attractive option that however would require some change to obtain. (McDermott, 1998).

### Probability Weighting Function

In evaluation process, people multiply subjective value of the outcome by its respective probability in order to make a decision. This probability in prospect theory is called the decision weight. The following figure illustrates the probability weighting function.



**Figure 2.** A typical probability weighting function in Prospect Theory (Donkers et al., 2001).

Probability weighting function is different from the subjective probability of expected utility theory in a few aspects. Firstly, decision weight does not follow any standard distribution, as

was in expected utility theory. This means that decision weight does not solely represent the perceived likelihood of outcome, but rather comes from the empirical evidence of actual decision-making. Secondly, the weighting function has different properties near the ends of the [0, 1] range compared to the middle. Typically people tend to overweight low probabilities and underweight high probabilities. Such way of perceiving probabilities certainly affects people's choice behavior. Evidence can often be found in lotteries and insurance examples. In lotteries, people are willing to take a sure loss in order to obtain an almost nonexistent chance of a huge gain (McDermott, 1998). In insurance, people are willing to give up a part of their present wealth for protecting a very small chance of future loss. As Tversky and Kahneman write,

*“...Consequently, people are often risk seeking in dealing with improbable gains and risk averse in dealing with unlikely losses...”*

Such behavior certainly cannot be addressed by expected utility theory, which only includes normal distributions. Prospect theory takes into account the perceptual likelihood of the extreme events that is largely supported by empirical evidence, and therefore is able to provide a more accurate estimation result.

### **Cumulative Prospect Theory**

Cumulative Prospect Theory (CPT) is the more advanced version of prospect theory, which could be more widely applied in real life situations (Tversky & Kahneman 1992). The difference between CPT and the original theory is that in CPT the probability weighting uses cumulative probability distribution function, rather than the probabilities of discrete outcomes. Such formalization is more complicated but more practically relevant because real life situations more often than not deal with continuous distributions of probabilities and values.

Since its establishment, Prospect Theory has been applied to or combined with many existing studies in economics, such the ones regarding inter-temporal choices. Such studies involve choosing between two or more options at different points in time. A number of empirical

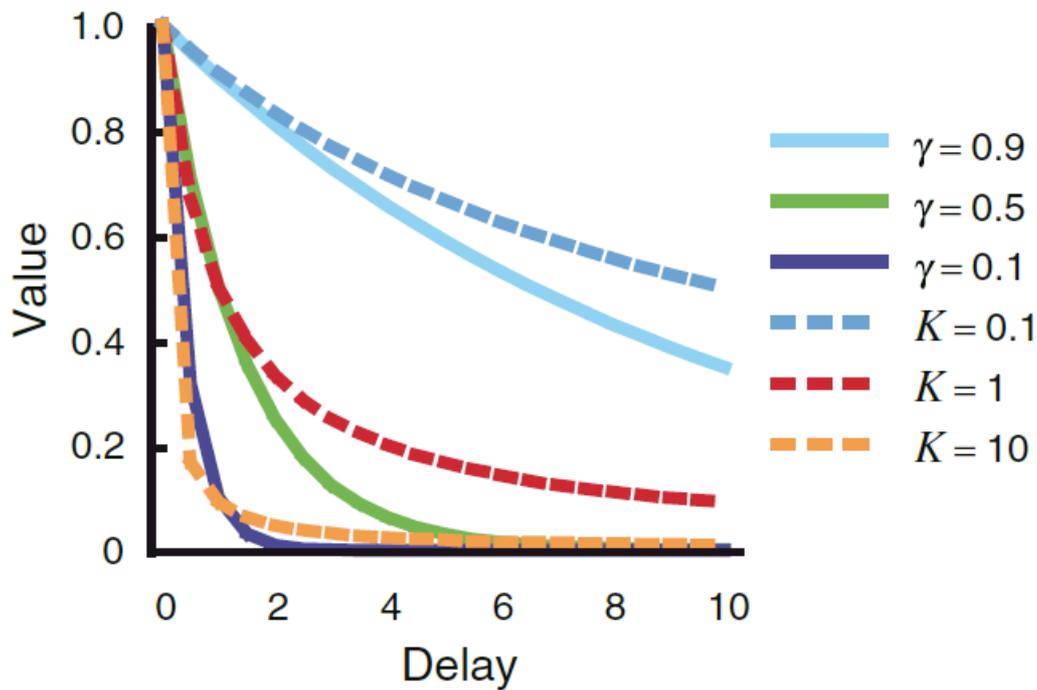
studies found inconsistencies in people's choice behavior from what was defined in the normative models on inter-temporal choice. Eventually such inconsistencies could be better explained by phenomena in Prospect Theory, such as framing effects. The next section will briefly illustrate the recent studies and their main findings in inter-temporal choice behavior.

### **Inter-temporal Choice and Its Anomalies**

There are a number of anomalies in inter-temporal preferences reported by the empirical studies. First of all, Loewenstein and Prelec (1992) found that risk-averse individuals have different discount rate for perceived gains and losses with respect to a reference point. In particular, they found that losses are discounted at a lower rate than gains are. Similarly, Thaler (1981) reported an even stronger result, showing that gains displayed nearly three times higher discount rate than losses did. Secondly, Loewenstein and Prelec (1992) also found asymmetry in preference concerning speed-up and delay of consumption. Particularly, people required much more compensation for delaying a reward to a future time than the money they could forfeit to speedup a future payment.

Regarding the model for calculating the discount factor, most conventional studies used *exponential discounting* (Fisher, 1930; Fishburn & Rubinstein, 1982), while it has been found that the exponential discounting model could not explain some of inconsistent preference patterns in empirical studies. Thaler (1981) used a simple example to illustrate such inconsistency: people who preferred 1 apple today to 2 apples tomorrow were found to prefer 2 apples in 101 days to 1 apple in 100 days. Using exponential discounting, this cannot be valid because exponential discounting has constant discounting rate. In other words, if the value functions of the first case can be written as  $u(1) > u(2) \cdot \gamma^1$ , where  $\gamma$  is the discount rate, then for the latter case the same preference order must be held:  $u(1) \cdot \gamma^{100} > u(2) \cdot \gamma^{101}$ . It is because  $\gamma^{101} = \gamma^1 \cdot \gamma^{100}$ , thus  $\gamma^{100}$  cancels out on both sides of the second inequality. John Horowitz (1991) confirmed the result of Thaler (1982) through experiments with real money

outcomes. Therefore, to solve this problem, some studies suggested that *hyperbolic discounting factor* could better explain such inconsistent preference patterns (Loewenstein, 1988; Loewenstein & Prelec, 1992). The hyperbolic discount function is written as  $f(t) = \frac{1}{1+k \cdot t}$ , where t is the delay in time and k is the discount rate. Figure 3 illustrates the hyperbolic function with different values of k and compares it with the exponential function (with  $\gamma$  corresponding to  $\frac{1}{1+k}$ ).



**Figure 3.** Discounting using the exponential function (with discount rate  $\gamma$ ) and hyperbolic function (with discount rate K) (Doya, 2008).

As shown in the figure, discount rate in the exponential function remains fixed ( $\gamma$  per time unit), whereas in the hyperbolic function it decreases with time, therefore differences between the neighboring time points diminish as they are moved further to the future. For instance, if

the ratio between discounted values on days 0 and 1 is  $\frac{f(0)}{f(1)} = \frac{1+k \cdot 1}{1+k \cdot 0} = 1+k$ , the ratio

between days 100 and 101 becomes only  $\frac{f(100)}{f(101)} = \frac{1+k \cdot 101}{1+k \cdot 100} = 1 + \frac{k}{1+100k}$ , which is

considerably smaller than  $1+k$  for non-zero k values. This explains why people preferring one apple immediately over 2 apples tomorrow may change their preference in 100 days.

## Applications of Prospect Theory and Inter-temporal Choice Models

### Loss Aversion, References, and Inter-temporal Choice (Dimmock & Kouwenberg 2007)

Based on the work of Loewenstein (1988) and Thaler (1981) in studying the inter-temporal choice, Dimmock and Kouwenberg (2007) examines how loss-aversion and reference points affect household portfolio choice, using the Dutch DNB household survey during 1997-2002. In particular, they used 16 questions of the DNB survey that ask households to choose between how much they are willing to pay to speed up a gain or delay a loss, and how much compensation they require to delay a gain or speed up a loss. For each individual household, they estimated four parameters: loss-aversion, discount factor, and two reference points (for speed-up and delay separately). Then they compared the estimated parameters with equity allocation of the households and found that individuals with high loss aversion and high reference point adjustment are less likely to invest in equity.

To simplify the analysis, Dimmock and Kouwenberg used linear value function:  $v(x) = x$  if  $x \geq 0$ , and  $v(x) = \lambda x$  if  $x < 0$ , where  $\lambda > 1$  implies loss-aversion. They also considered ratios of the extra payment for speed-up / delay  $P$  and reference point  $R$  over the status quo  $X$ , i.e.  $p = P / X$  and  $r = R / X$ . In short, they estimated the loss-aversion parameter, the discount rate, and two reference points for each individual from his/her specified payment for speed-up and delay of gains and losses. The method that Dimmock and Kouwenberg applied to estimate the parameters was Generalized Method of Moments (GMM).

Their estimated result of loss-aversion was on average 2.18 that is consistent as in Tversky and Kahneman (1992). The average discount factor is 0.933, which means that the average discount rate is 7% per year. The average reference point  $r$  for delay is 0.52 with a median of 0.34, while for speed-up it is 0.3 with a median of 0.05.

Other similar studies (Dimmock & Kouwenberg, 2010; Tu, 2004) use slightly different formalisms. They do not estimate the values of reference points  $r$  but only of loss aversion and discounting. Instead they try three different reference point values (0.25, 0.5, 1) and see which of them results in the best fit. Using the reference point formalization of Tu 2004, they report that the model with the smallest reference point value fits the best, and the mean values of loss aversion and discounting are respectively 1.64 and 3% per year.

### Measuring Risk Attitude using Lottery Questions (Donkers et al., 2001)

In their paper, Donkers, Melenberg, and Soest (2001) aimed to estimate people's risk attitude by analyzing eight lottery questions of the same Dutch household survey. There are two types of probability related questions in the survey: five questions ask people to choose between lotteries that lead to two different outcomes with certain probabilities; three remaining questions ask people to indicate a minimum probability of winning a large prize, which would make this risky option equally good to receiving a smaller amount of money for sure. For the first five questions, Donkers et al. (2001) did not apply models based on prospect theory, thus they will not be discussed here. The last three probability equivalence questions were modeled using the cumulative prospect theory (CPT).

Donkers et al. (2001) did not estimate individual parameters but rather built their estimation based on individual characteristics, such as age, gender, income, and education. Therefore, overall they have much fewer parameters in comparison to the estimation at the individual level. This makes their model much less flexible but still suitable for analyzing how the model parameters relate to their chosen individual characteristics.

They define the value function as  $V_i = w_i^+(p) \cdot v_i(x)$ , where  $w_i^+(p)$  is the probability weighting function for positive outcomes and  $v_i(x)$  is the value function for monetary outcomes. Using the model specification in Prelec (1998), they define the weighting function as  $w_i^+(p) = \exp(-(-\ln p)^{\gamma_i})$ , with  $\gamma_i$  being a linear function of the individual characteristics. Their value function  $v_i(x) = x^{\alpha_i}$  follows the approach of Tversky and Kahneman (1992),

with  $\alpha_i$  being a quadratic function of the individual characteristics. Their results show that decision weights are significantly different from true probabilities, especially for older people and females, while income plays a negative effect on the difference. In addition, they found substantial relationship between risk attitudes and individual characteristics.

## CHAPTER 3. METHODS

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### Description of the Data

In this study I use data from the DNB Household Survey that is conducted in the Netherlands over the number of years, starting from 1993. It contains diverse information concerning each Dutch household's financial situation and economic preferences, including a set of questions regarding choices in hypothetical situations with risky and/or delayed losses and gains, similar to those typically used in behavioural economics experiments. There is a certain variability of such questions over different years in the survey and unfortunately only in the first two years (1993 and 1994) the survey contains questions regarding *both* risky decisions (lotteries) and inter-temporal choices (delaying or speeding up payments). As we want to include both types into our model, we use questions only for these years.

### The Lottery Questions

The lottery questions (see Appendix) are of two types: probability equivalence questions (*lot1-3*) and choice questions (*wed1-5*). In three probability equivalence questions participants are given a certain sum of money (200, 1000 or 5000 guilders<sup>ii</sup>) for sure and they are asked what should be the smallest acceptable probability of winning a larger prize (20000 guilders) that they would take it instead of the certain smaller sum of money they are given. One example (**lot 1**) is given as the following:

“Imagine you have won f.200 in a game. You can now choose between keeping that f.200, or having a lottery ticket with a certain chance to win a prize of f.20,000. How high would that chance to win f.20,000 need to be such that you prefer the lottery ticket to keeping the f.200 that you had already won?”

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<sup>ii</sup> Dutch currency at the time of survey

- I would prefer the lottery ticket if the chance to win the first prize would be at least.....% (specify a number between 1-100)
- I don't know"

In five choice questions participants are asked to choose between two different options, involving gains and sometimes losses, at least one of which is risky (with clearly defined probabilities of different outcomes). One example (**wed1**) is the following:

“We toss the coin once, and you may choose one of the following two options:

- You receive f.1000 with either heads or tails
- With heads you receive f.2000, and with tail you don't receive anything at all”

### The Inter-temporal Choice Questions

The inter-temporal choice questions (see Appendix) measure choices along 4 different dimensions: (1) gains and losses, (2) different amounts of money involved (1000 vs. 100000 guilders), (3) delaying or speeding up, and (4) different time periods (3 or 12 months) by which a gain or a loss can be delayed or speeded up. Unlike in the later years, where the survey contains all possible options along these 4 dimensions (i.e.  $2^4 = 16$  sets of questions), the first two years have only 9 out of these 16 possible situations. Each situation is represented by a pair of questions (*tijd1-2*, *tijd 3-4*, ..., *tijd17-18*), out of which the first one asks whether the subject would agree with the delay/speed-up by paying or receiving a certain sum of money, and the second one asks what exactly that sum of money should be. It should be noted that the wording is not perfect, because in questions *tijd1-6* that deal with delaying payments by receiving extra money, participants aren't clearly given an option to choose an immediate payment, but rather delaying it without receiving any money. In the other questions (*tijd7-18*) the choice between original payment and delayed/speeded up option is explicit. One example of the inter-temporal choice questions (**tijd2**) can be found below:

“How much EXTRA money (in guilders) do you want to receive AT LEAST, in addition to the f.1000, to compensate for the waiting term of 3 months?

- 1- 2000 guilders
- I don't know”

## Data pre-processing

In real life data sets there are often some missing data points that can potentially make analysis difficult or impossible. For this reason I checked if answers to all or most of the questions were available in the records in all participants. Furthermore, as probability weighting parameter can be reliably estimated only from probability weighting questions (**lot1-3**) and time discounting parameter only from inter-temporal choice questions (**tijd1-18**), I had to exclude the records in which *all* answers to *either type* of questions were missing, „don't know“ or zero<sup>iii</sup>.

Unfortunately for year 1994 the vast majority (around 80%) of records were empty with regard to these questions. Because this may create a significant bias in the data, I decided to only use answers from year 1993 in the analysis. For this year, data pre-processing reduced the number of data records from nearly 4000 to 3135.

## A Model for Determining Value of a Certain Option

In some of the lottery questions where participants have to choose one of the 2 possible options (*wed1-5*), I assume that they determine the value of each option and choose the one whose value they think is higher. In other questions participants either have to estimate a probability  $p_i$  (*lot1-3*) or an amount of extra/less money  $\Delta x_i$  (*tijd1-18*) for one of the options that would make it equivalent to the second option. It is notable that the first type of questions

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<sup>iii</sup> Zero is a valid option in intertemporal choice questions, but if responses to all questions were either zero or missing that indicates that the participant did not understand questions or take them seriously (Dimmock et al., 2009)

(*wed1-5*) are less informative for estimation of individual decision parameters, because the same choice could be produced by a range of underlying parameter values, whereas the second type of questions (*lot 1-3* and *tijd 1-18*) can be used for accurate extraction of parameters.

## The Value Function

If a choice  $x$  consists of a number of possible monetary outcomes  $x_i$  ( $x_1 < x_m < R < x_{m+1} < x_N$ ) that occur with corresponding probabilities  $p_i$  and delays  $T_i$  from the decision moment, then according to the cumulative prospect theory (Tversky & Kahneman, 1992; Donkers et al. 2001) the value of  $x$  is the following:

$$V(x) = \sum_{i=1}^m \left( w \left( \sum_{j=1}^i p_j \right) - w \left( \sum_{j=1}^{i-1} p_j \right) \right) v(x_i - R) \delta(T_i) + \sum_{i=m+1}^N \left( w \left( \sum_{j=i}^N p_j \right) - w \left( \sum_{j=i+1}^N p_j \right) \right) v(x_i - R) \delta(T_i) \quad (1)$$

where  $w(\cdot)$  is the probability weighting function,  $\delta(\cdot)$  is the temporal discounting function,  $v(\cdot)$  is the value / utility function for monetary outcomes, and  $R$  is the reference point that determines whether monetary outcomes are considered as relative gains or losses.

Considering  $R$ , one extreme would be to assume that people value each option separately, therefore  $R$  should be 0. Another extreme would be to assume that people integrate gains/losses with their current wealth, in which case  $R$  should be equal to  $[-\text{wealth}]$ . Although individual's wealth may influence valuation and other financial decision making parameters, empirical evidence suggests that the influence of background financial information is minimal and actual choices are considered mostly based on their relative merits (Dimmock et al., 2009; Loewenstein & Prelec, 1992). In particular,  $R$  is strongly affected by the given certain outcomes. For this reason, following Tu's (2004) approach, if any certain outcomes  $x$  are available, I consider  $R = r \cdot x$  (with  $0 < r < 1$ ) for that moment of time and the value function  $v(x_i - R)$  can be written as  $v(x_i (1 - r))$ ; if not, I assume that  $R = 0$ .

## Each Component of the Value Function

Based on empirical evidence of probability weighting (Tversky & Kahneman, 1992; Donkers et al., 2001), the weighting function  $w(\cdot)$  can be written as follows:

$$w(p_i) = \exp\left(-(-\ln p_i)^\gamma\right) \quad (2)$$

where individual parameter  $\gamma$  ( $0 < \gamma < 1$ ) determines the extent to which low probabilities are overrated and high probabilities underrated. In theory  $w(\cdot)$  may differ between positive and negative outcomes, but for simplicity I use only one parameter value  $\gamma$  for both gains and losses.

For the temporal discounting  $\delta(\cdot)$  I consider the following hyperbolic discounting function, because it has been observed to be more consistent with human behaviour than exponential discounting (Loewenstein & Prelec, 1992; Thaler, 1981):

$$\delta(T_i) = \frac{1}{1 + kT_i} \quad (3)$$

where  $k$  is the discount rate ( $k > 0$ ) and  $T_i$  is delay (in years).

For value / utility function  $v(\cdot)$  I use the following form (Donkers et al., 2001):

$$v(x_i) = x_i^\alpha, \text{ if } x_i \geq 0 \quad (4)$$

and  $v(x_i) = -\lambda(-x_i)^\alpha$ , if  $x_i < 0$ ,

where  $\alpha$  is the risk sensitivity parameter (for risk aversion  $0 < \alpha < 1$ , for risk preference  $\alpha > 1$ ) and  $\lambda$  is the loss sensitivity parameter (for loss aversion  $\lambda > 1$ , for loss preference  $0 < \lambda < 1$ ). Since it is known that most individuals are loss and risk averse, I consider that  $\lambda > 1$  and  $0 < \alpha < 1$ . It is possible that the risk sensitivity parameter  $\alpha$  is different for gains and losses but again for simplicity I use the same value.

Although this model allows for probabilistic delayed gains or losses of different magnitude, it can obviously be used as well, if options are deterministic ( $p_i = 1$ ), if there is no delay ( $T_i = 0$ ), or if I deal with only gains or only losses of one magnitude. However, in order to estimate different parameters, different types of data is necessary: for  $\alpha$  and R options of different monetary value should be available, for  $\lambda$  both losses and gains should be present, for  $\gamma$  at least one option should be probabilistic, and for k one option should be delayed. Although none of the 17 questions<sup>iv</sup> have variability necessary for estimating all parameters at once, for each parameter there are at least several informative questions.

### **Estimating Individual Parameters from Data**

Given the described model, my goal is to estimate individual model parameters that could best describe that individual's choices in 8 lottery questions and 9 pairs of intertemporal choice questions. Although the model with given parameters determines the value of a particular option, such value cannot be observed and therefore directly compared to experimental data. Instead I try to determine what would be a subject's response if his or her behaviour were following the model with that particular set of parameters. Once that is known, the best fitting parameter set can be estimated for each subject that would produce responses as close as those observed in the questionnaire.

#### **Models for Questions lot1-3**

More specifically, for probability equivalence questions (*lot1-3*), using equation 1 the value of the certain option is

$$v(x_i(1-r)) = (x_i(1-r))^\alpha$$

with  $x_1 = 200$ ,  $x_2 = 1000$ , and  $x_3 = 5000$ ,

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<sup>iv</sup> 8 lottery questions and 9 pairs of inter-temporal choice questions

whereas the value of the probabilistic option (the uncertain option) is

$$w(p_i)v(20000 - r \cdot x_i) + w(1 - p_i)v(-r \cdot x_i) = \exp\left(-(-\ln p_i)^\gamma\right)(20000 - r \cdot x_i)^\alpha - \lambda \exp\left(-(-\ln(1 - p_i))^\gamma\right)(r \cdot x_i)^\alpha$$

Therefore, I would need to find such probability  $p_i$  that the value of the certain option equals the value of the uncertain option. Thus, the following equation should hold for a given set of parameters:

$$(x_i(1 - r))^\alpha = \exp\left(-(-\ln p_i)^\gamma\right)(20000 - r \cdot x_i)^\alpha - \lambda \exp\left(-(-\ln(1 - p_i))^\gamma\right)(r \cdot x_i)^\alpha$$

Since the  $p_i$  term in the equation is non-linear and quite complicated, it is solved numerically (Press et al., 1992).

### Models for Questions wed1-5

For 2-choice lottery questions (*wed1-5*) it would be sufficient to determine, which of the two options under each question have higher value given a certain set of parameters.

For *wed1* need to determine if  $(1000 \cdot (1 - r))^\alpha > \exp\left(-(-\ln 0.5)^\gamma\right) \left( (1000 \cdot (2 - r))^\alpha - \lambda (1000r)^\alpha \right)$

For *wed2*, determine if  $\exp\left(-(-\ln 0.8)^\gamma\right) (45 - 30r)^\alpha - \lambda \exp\left(-(-\ln 0.2)^\gamma\right) (30r)^\alpha > (30 \cdot (1 - r))^\alpha$

For *wed3*, determine if  $100^\alpha \exp\left(-(-\ln 0.25)^\gamma\right) > 130^\alpha \exp\left(-(-\ln 0.2)^\gamma\right)$

For *wed4*, determine if  $3000^\alpha \exp\left(-(-\ln 0.02)^\gamma\right) > 6000^\alpha \exp\left(-(-\ln 0.01)^\gamma\right)$

For *wed5*, determine if  $1500^\alpha - 1000^\alpha \lambda > 0$  (here probabilities together with their weights cancel out because they are both equal to 0.5 and the right side is 0).

Then I check whether the actual choice of the first or the second option ( $c_i = 1$  or  $2$ ) is the same as the predicted choice of the model given a certain set of parameters.

## Models for Questions tijd1-18

In the inter-temporal choice questions (*tijd1-18*) participants are asked to determine how much money  $\Delta x_i$  they are willing to pay in order to delay a loss or speed up a gain or how much money  $\Delta x_i$  they ask for in return for delaying a gain or speeding up a loss. I calculate the value of both options given a set of parameters and  $\Delta x_i$ . Since at the chosen value of  $\Delta x_i$  the 2 options should be indifferent, their values should be the same. Thus,

$$\text{for } \textit{tijd1-2}: \frac{(1000 + \Delta x_1)^\alpha}{1 + 0.25k} - \lambda(1000r)^\alpha = (1000 \cdot (1 - r))^\alpha$$

(here 0.25 means a delay of 3 months = 0.25 year,  $\Delta x_1$  – expected additional gain for agreeing a 3 month delay; reference  $r$  is counted only at time = 0 because that is when the gain of 1000 is initially expected).

$$\text{for } \textit{tijd3-4}: \frac{(100000 + \Delta x_2)^\alpha}{1 + 0.25k} - \lambda(100000r)^\alpha = (100000 \cdot (1 - r))^\alpha$$

$$\text{for } \textit{tijd5-6}: \frac{(100000 + \Delta x_3)^\alpha}{1 + k} - \lambda(100000r)^\alpha = (100000 \cdot (1 - r))^\alpha$$

$$\text{for } \textit{tijd7-8}: (1000r)^\alpha - \lambda \frac{(1000 + \Delta x_4)^\alpha}{1 + 0.25k} = -\lambda(1000 \cdot (1 - r))^\alpha$$

$$\text{for } \textit{tijd9-10}: (1000r)^\alpha - \lambda \frac{(1000 + \Delta x_5)^\alpha}{1 + k} = -\lambda(1000 \cdot (1 - r))^\alpha$$

$$\text{for } \textit{tijd11-12}: \frac{(1000 \cdot (1 - r))^\alpha}{1 + 0.25k} = (1000 - \Delta x_6)^\alpha - \lambda \frac{(1000r)^\alpha}{1 + 0.25k}$$

$$\text{for } \textit{tijd13-14}: \frac{(1000 \cdot (1 - r))^\alpha}{1 + k} = (1000 - \Delta x_7)^\alpha - \lambda \frac{(1000r)^\alpha}{1 + k}$$

$$\text{for } \textit{tijd15-16}: \frac{(100000 \cdot (1 - r))^\alpha}{1 + 0.25k} = (100000 - \Delta x_8)^\alpha - \lambda \frac{(100000r)^\alpha}{1 + 0.25k}$$

and for *tijd17-18*: 
$$\frac{(100000 \cdot (1-r))^\alpha}{1+k} = (100000 - \Delta x_9)^\alpha - \lambda \frac{(100000r)^\alpha}{1+k}$$

When participants decline the delayed or speeded up alternative in odd questions (*tijd1*, 3, 5, ..., 17) this happens most likely because the corresponding  $\Delta x_i \leq 0$ . Since  $\Delta x_i$  terms appear only once in these equations, they can be calculated in a straightforward way given parameter values. If the estimated  $\Delta x_i$  is negative and the corresponding participant declined the alternative in the respective question, I consider that there is zero error of fit.

So, for each candidate set of parameters  $((\alpha, \gamma, \lambda, r, k))$ , I do the following:

1. Calculate the corresponding observable variables in all above equations
  - a. In probability equivalence questions the probabilities  $p_i$
  - b. In lotteries the resulting choices ( $c_i = 1$  or  $2$ )
  - c. In inter-temporal choice questions the money differences  $\Delta x_i$
2. Based on them I calculate the  $\chi^2$ -error of goodness-of-fit (Press et al., 1992 – see Chapter 15: Modeling of data) for that parameter set:

$$\chi^2 = \sum_{i=1}^3 \frac{(p_{i\_estimated} - p_{i\_observed})^2}{\sigma^2(p_{i\_observed})} + \sum_{i=1}^5 \frac{(c_{i\_estimated} - c_{i\_observed})^2}{\sigma^2(c_{i\_observed})} + \sum_{i=1}^9 \frac{(\Delta x_{i\_estimated} - \Delta x_{i\_observed})^2}{\sigma^2(\Delta x_{i\_observed})}$$

This is the objective function which I want to minimize, i.e. for each individual the goal is to find such a parameter set  $(\alpha, \gamma, \lambda, r, k)$  that the resulting error would be minimal.

Variances  $\sigma^2$  in the above equation reflect how much each of 17 observable variables varies in the population (as there is only one measurement per variable for each individual, I can only obtain variance of that variable from the population). Dividing by variance is necessary for 2 reasons: firstly, it allows incorporating 3 different types of measures with different units (probabilities, choices, monetary quantities) into one equation that they could be used comparatively; secondly, it takes into account the fact that if a certain variable has high empirical variance then the same absolute errors (e.g.  $(p_{i\_estimated} - p_{i\_observed})^2$ ) should be penalized much less compared to the case if a variable has a much narrower distribution, i.e.

if empirical data itself seems not very reliable, it is unreasonable to ask the model to fit it very closely<sup>v</sup>.

I will perform this optimization based on Monte-Carlo search followed by several gradient-descent runs from the best Monte-Carlo points.

Afterwards I will check using the  $\chi^2$  test of goodness of fit (with  $17-5=12$  degrees of freedom) how good the resulting fit is. At  $p > 0.01$  level this would require  $\chi^2 < 26.22$ . For the participants who did not answer some of the questions the correspondingly lower degrees of freedom will have to be used.

## Evaluation and Analysis of the Estimated Parameters

The main goal of this model is to predict a variety of individual economic behaviors that are observed as answers to economic choices in the DNB household survey. In addition to the  $\chi^2$  goodness-of-fit measure, I also look at the correlations (across individuals) between the observed answers and those predicted by the model. If the model is sufficiently flexible to fit different individuals, I expect these correlations to be high. In addition, these correlations also show whether answers to some of the questions fit the model better than others.

Secondly, for estimated parameters to be of any use, their values have to be reliable, i.e. if the estimation is repeated several times, the resulting parameters should be very close to each other. As there are 5 parameters to be estimated (that can interact with each other) and a heuristic search algorithm is used (which is based on random numbers), this is no trivial issue. To evaluate reliability, I run the whole estimation 2 times and measure correlations between the best-fitting parameters in each run. Very high correlation values and low variance among the best-fitting parameter sets would indicate that the estimation is reliable.

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<sup>v</sup> e.g. if in a choice question each option is chosen 50% of time making an error should be less costly than if one option is chosen 95% of time – in the latter case a trivial model which *always* chooses the first option would have fitting error in only 5% of the data points (which may sound OK), but it does not reproduce a correct qualitative pattern of responses in the population

If the model fits experimental data well enough and estimated parameters are reliable, I will analyze the statistics of these parameters in the population: their mean and median values as well as distributions. I will check whether the results support observations from the Prospect Theory and other studies, such as loss aversion ( $\lambda > 1$ ), concave utility function ( $\alpha < 1$ ) and others. Finally, I will measure correlations between different parameters to detect possible interactions.

## CHAPTER 4. RESULTS

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### Preparing the Estimation of Model Parameters

The model simulation and parameter estimation were done using the MatLab 7.8.0 software. The simulation function *runPTmodel.m* takes 5 model parameters and 17 individual's empirical measurements (questionnaire answers) as input. It calculates the corresponding 17 measures that are produced by the model with given parameters. Measures for probability equivalence questions (*lot1-3*) are calculated using standard MatLab function *fzero* that numerically solves the equation, whereas other measures are calculated directly using the equations in the Methods chapter. Subsequently the simulated measures are compared to empirical ones and the  $\chi^2$ -error of goodness-of-fit is calculated both for each measure separately and in total.

In the parameter estimation procedure *par\_est.m*, firstly 2000 random parameter sets are generated for each individual. After running the model simulation for all of them, 50 best ones (i.e. with the lowest  $\chi^2$ -error) are selected for hill-climbing. In hill-climbing steps are performed for each parameter by adding and subtracting 5% of its range from the current value. If any of these steps leads to improvement in error, it is accepted and search continues from there until no further improvement is observed. Finally, to determine the best-fitting parameter set for each individual, 8 best sets are taken from the hill-climbing and averages of all their combinations ( $2^8-1$ ) are simulated for the refinement and the very best one selected.

An important issue was finding appropriate ranges for the parameters. At first, I started with wider ranges, e.g. loss sensitivity  $\lambda$  and risk sensitivity  $\alpha$  varying between 0.1 and 10, probability weighting parameter  $\gamma$  varying between 0.02 and 2, reference  $r$  between 0 and 1, and discounting rate  $k$  between 0 and 10. Then after first preliminary estimations I looked at the resulting parameter distributions to refine the ranges to smaller that they would cover the vast majority of observed values, but not more. The first noticeable observation was that for

all parameters there were many relatively small values and long tails on the right. Therefore I decided to use log distribution for the generation of initial random parameters and step sizes. For parameter  $\gamma$  there were many values at the preliminary upper bound, therefore I increased this bound to higher. Eventually the following parameter ranges were used:

- risk sensitivity  $\alpha$  in [0.4, 2.5] with geometric mean = 1;
- loss sensitivity  $\lambda$  in [0.6, 9.6] with geometric mean = 2.4;
- probability weighting parameter  $\gamma$  in [0.02, 8] with geometric mean = 0.4;
- reference parameter  $r$  in [0.02, 1] with geometric mean = 0.14;
- discounting rate  $k$  in [0.02, 2] with geometric mean = 0.2.

### **Goodness-of-fit and Reliability of Estimated Parameters**

After performing the parameter estimation procedure, the resulting average  $\chi^2$ -error of goodness-of-fit was 11.9 and its median value was 7.4. The goodness-of-fit test passed in 91.5% of individuals at the  $p > 0.01$  level ( $\chi^2 < 26.22$  for 17 measures) and in 82.7% of individuals at the  $p > 0.10$  level ( $\chi^2 < 18.55$ ), meaning that for the vast majority of individuals the difference between model performance and the actual data was insignificant compared to empirical behavioural variability.

To assess reliability of the estimated parameters I looked at the correlations between 2 independent runs of the estimation procedure. As log distributions were used in the estimation, I used log values of the parameters in correlations. The resulting correlation coefficients  $R$  (do not mix up with the reference point) were the following:

- for risk sensitivity  $\alpha$ :  $R = 0.802$ ;
- for loss sensitivity  $\lambda$ :  $R = 0.803$ ;
- for probability weighting parameter  $\gamma$ :  $R = 0.819$ ;

- for reference parameter r:  $R = 0.829$ ;
- for discounting rate k:  $R = 0.962$ .

These results indicate that the reliability of estimated values was good for the first 4 parameters and excellent for the discounting rate.

Although overall the model with estimated parameters could fit empirical data quite well, it could be that the partial fits varied for different measures, some of them fitting very well, others worse. To analyze this, I looked at the partial goodness-of-fit statistics for each measure as well as correlations between simulated and observed measures (Table 1).

**Table 1.** Mean and median partial goodness-of-fit values as well as correlation between simulations and observed data for each of the 17 variables.

Variable	lot 1	lot 2	lot 3	wed 1	wed 2	wed 3	wed 4	wed 5	tijd 1-2	tijd 3-4	tijd 5-6	tijd 7-8	tijd 9-10	tijd 11-12	tijd 13-14	tijd 15-16	tijd 17-18
Mean partial g.o.f.	1.5	1.9	1.7	0.8	1.3	0.6	0.2	0.1	0.5	0.4	0.3	0.3	0.3	0.6	0.5	0.6	0.5
Median partial g.o.f.	0.5	0.7	0.8	0	0	0	0	0	0.0	0.1	0.0	0.0	0.1	0	0.0	0	0.0
Model-data correlation R	0.4	0.4	0.6	0.7	0.5	0.7	0.9	1.0	0.5	0.5	0.6	0.7	0.8	0.8	0.6	0.7	0.5

One notable observation from Table 1 is that partial fits for the probability equivalence questions (*lot1-3*) are considerably worse than those of the other variables. Their correlations are among the worst, mean goodness-of-fit errors are the highest and median g.o.f. errors are considerable, whereas for the other variables they are very small. To analyze the situation, I looked at the average simulated and empirical values for these variables. Mean simulated values for *lot1-3* probabilities were 0.09, 0.13 and 0.30, whereas mean empirically observed probabilities were 0.35, 0.45 and 0.61. Thus, bad goodness-of-fit is primarily because the model simulated probabilities are much smaller than observed ones. Considering that these questions ask to choose which probability of 20000 guilders is equivalent to surely receiving

200, 1000 or 5000 guilders, the model simulated probabilities seem much more reasonable as they are just slightly larger than the „rational“ values of 0.01, 0.05 and 0.25. Producing the experimentally observed values with the Prospect Theory model would be nearly impossible with realistic parameter settings. For instance, in case of *lot 1*, if we use mean observed value of 0.35, the following equation should hold:

$$w(0.35)v(20000 - 200r) + w(0.65)v(-200r) = v(200 - 200r)$$

The first term  $w(0.35)v(20000 - 200r)$  would normally be much larger than the other two, unless the value function is so concave that  $v(20000-200r)$  is not much higher than  $v(200-200r)$  or reference parameter and loss aversion are so high that  $v(-200r)$  becomes of similar absolute value to  $v(20000-200r)$ . Neither of these possibilities sound realistic based on empirical observations of these parameters in literature. The last possibility for this equation to hold would be if  $w(0.35)$  were very small. However, if that is the case (i.e. small probabilities are underweighted rather than overweighted), then the opposite would apply for larger probabilities, thus it would become impossible to fit mean answers to *lot 2* (0.45) and *lot 3* (0.61).

Another reason of poor fit in these questions may be that while the simulated values are always consistent (i.e.  $p(\text{lot } 1) \leq p(\text{lot } 2) \leq p(\text{lot } 3)$ ), this is not the case in nearly 9% of the individuals, suggesting that these participants possibly were not concentrated and careful with their answers to probability equivalence questions.

To evaluate the results for the choice questions (*wed1-5*), it may also be relevant to look at the percentages of individuals for which the best-fitting model chooses differently from them in each question (Table 2)

**Table 2.** Empirical and model choices in questions *wed1-5*

Question	Both choose option 1	Model chooses option 1, individual option 2	Model chooses option 2, individual option 1	Both choose option 2
<i>wed1</i>	63.9%	0.2%	13.8%	22.2%
<i>wed2</i>	43.7%	31.4%	0%	24.9%
<i>wed3</i>	40.9%	6.6%	8.4%	44.1%
<i>wed4</i>	39.7%	1.5%	2.7%	56.2%

<i>wed5</i>	11.5%	0.7%	0.1%	87.7%
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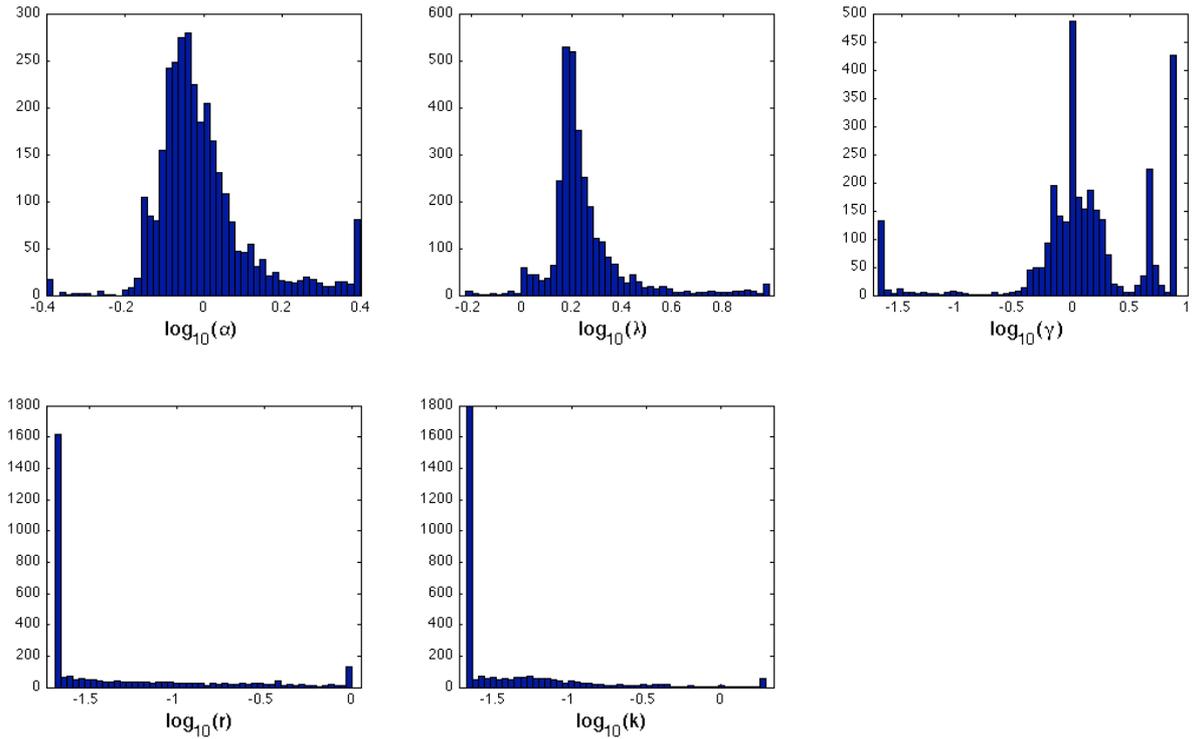
The results show that while for *wed4-5* model responses match the actual behaviour very well, there is a significant problem with *wed2*. The fact that wrongly predicted choices are on one side suggest that the parameters that are appropriate for the answers to other questions can not properly suit responses to *wed2*. In particular, in this question the majority of participants choose sure 30 guilders more often than 80% chance of 45 guilders (expected value = 36), whereas the model more often takes the risky choice. The risk-averse responses in this question could be achieved by higher references combined with high loss sensitivity and/or strong underweighting of high probabilities, but it is apparent from the later section that generally small reference values fit other questions better, whereas probability weighting is sometimes reverse due to high responses to probability equivalence questions *lot1-3*.

### **Analysis of Estimated Parameters**

The resulting means and medians of the estimated parameters are the following:

- for risk sensitivity  $\alpha$ : mean = 1.04, median = 0.93;
- for loss sensitivity  $\lambda$ : mean = 1.94, median = 1.62;
- for probability weighting parameter  $\gamma$ : mean = 2.38, median = 1.14;
- for reference parameter  $r$ : mean = 0.13, median = 0.02;
- for discounting rate  $k$ : mean = 0.11, median = 0.02.

The histograms for log-distributions of these parameters are provided in Figure 4.



**Figure 4.** Log-histograms of estimated model parameters: risk sensitivity ( $\alpha$ ), loss sensitivity ( $\lambda$ ), probability weighting parameter ( $\gamma$ ), reference parameter ( $r$ ), and discounting rate ( $k$ ).

Although literature suggests that most people have concave positive utility functions, in our sample only 61.9% of individuals have risk sensitivity  $\alpha < 1$ . The apparently convex positive utility function for others may be because they deal with hypothetical questions rather than real choices or because small amounts of money could be considered insignificant for some.

Considering the loss sensitivity,  $\lambda$  is higher than 1 in 98.8% individuals, which is remarkable given that not so few individuals give inconsistent answers to some questions. For most individuals  $\lambda$  is much higher than 1, indicating substantial loss aversion, as known in the literature.

Results for the probability weighting parameter  $\gamma$  are a bit unexpected:  $\gamma < 1$  holds for only 38.0% of individuals and  $\gamma = 1$  for further 5.2%. For the majority, the probability weighting curve is opposite from the one that is usual in prospect theory: these participants underweight small probabilities and overweight large ones. However, a substantial population on both lower and upper bounds of the range as well as problems with fit for probability equivalence

questions suggest that my chosen expression of the probability weighting function may be too constrained and not capable to fit different varieties of probability weighting (when participants answer questions consistently).

For both reference parameter  $r$  and discounting rate  $k$ , around half of individuals are at their lower range, suggesting that they do not discount future significantly and are not affected by framing. The other half of values for these parameters are close-to-uniformly distributed along their ranges, although the discounting rate  $k$  has more smaller values, whereas the reference parameter  $r = 1$  in 2.6% of people.

Finally, the estimated parameters can show if some of their value combinations occur more often in individuals, whereas other combinations are more rare. The simplest method to analyze this is to calculate the correlations between the 5 parameters (Table 3). As before I use log values of the parameters for the correlations, but (also as before) that does not change the results much.

**Table 3.** Correlation coefficients  $R$  between the 5 estimated parameters.

vs.	$\alpha$	$\lambda$	$\gamma$	$r$	$k$
Risk sensitivity $\alpha$	-	0.40	0.26	0.61	0.69
Loss sensitivity $\lambda$	0.40	-	-0.04	0.01	0.20
Probability weighting parameter $\gamma$	0.26	-0.04	-	0.28	0.18
Reference parameter $r$	0.61	0.01	0.28	-	0.64
Discounting rate $k$	0.69	0.20	0.18	0.64	-

Firstly, it is remarkable to see so many positive correlations. However, many of them are relatively weak, some even not significant. The strongest positive correlations are between risk sensitivity  $\alpha$ , reference parameter  $r$  and discounting rate  $k$  (all  $R$ s above 0.6), indicating that individuals with concave utility functions are less likely to be affected by framing and do not discount future much, whereas people with convex utility functions are more likely to be affected by framing and discount more. The moderate correlation ( $R = 0.4$ ) between risk and loss sensitivities is also notable: it shows that people with concave utility functions tend to be less loss averse than those with convex utility functions.

## CHAPTER 5. DISCUSSION

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In this thesis I presented a model that is motivated by prospect theory, but which includes various aspects of decision making, ranging from risk and loss sensitivity and subjective probability weighting to discounting and reference points. In addition, I used three different types of economic decisions (probability equivalence, risky choices and inter-temporal choices) which could be reproduced by the same model. Considering that other studies in this field often single out only 2 or 3 aspects of decision making and study only one type of economic decisions, making such a comprehensive model work can be considered a success.

The analysis of goodness-of-fit for different empirical variables revealed certain problems that still remain to be addressed. Most importantly, individual answers to probability equivalence questions could not be reproduced well enough, mainly because the observed values of these probabilities were too high. It is worth noting that despite some oddities in empirical data, model fits to probability equivalence data could have been better if the parameters were not constrained by 14 other variables that also had to be reproduced (and did so more successfully). Another possible reason for these mismatches is the weighting function, which was taken from (Donkers et al., 2001) and (Prelec, 1998). This function was constrained by the fact that the probability weighting curve was monotonically increasing and it intersected the straight line between (0,0) and (1,1) at the predefined point  $p = 1/e = 0.368$ . As a result, irrespectively of whether different probabilities were underweighted or overweighted, those smaller than 0.368 could never become larger than this value, whereas those larger than 0.368 could not become smaller than it. A different and more flexible formulation of probability weighting might be more successful in reproducing the empirically observed data.

The estimated model parameter values suggest that the vast majority of individuals are loss averse and that most people (but not all) have concave positive utility function (i.e. diminishing marginal utility). Nearly half of people have smallest possible values of reference points and discount factors, suggesting that they are not significantly affected by framing and

that they do not discount future significantly. The observed mean and median values for these parameters were largely consistent with previous literature, particularly with (Tu, 2004), who used a similar formalism regarding reference points. The results of the probability weighting parameter, which suggest that more than half of individuals underweight small probabilities and overweight large ones, could not be reliable due to poor fit in probability equivalence questions, but they give the typical probability weighting function some doubt.

Finally, the observed correlations between the estimated parameters suggest that individuals that have more convex utility functions (thus could be considered risk-seeking) also discount more and are more strongly affected by framing, whereas individuals with concave positive utilities are less prone to significant discounting and framing biases. Since variability in different decision making parameters might be caused by some common psychological and neural mechanisms (Doya, 2008), this result suggests that these two groups of people may be predominantly using different strategies. Those that have convex utility function, substantial discounting rate and are affected by framing may predominantly use simpler emotional strategies, whereas people with concave utility functions, less discounting and less framing influence may use more sophisticated cognitive strategies. Performing clustering or principal component analyses may bring further insights to such basic mechanisms of decision making.

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Quest. #	Variables	Question according to questionnaire, answering options and codes	Frequency	Routing
<b>Appendix: choice questions</b>				
		<i>You are probably familiar with games shown on television, where people win prizes and can choose between several options. For example, they can choose to keep a certain prize, or they can choose to take a chance to get a much bigger prize, at the risk of losing the prize altogether.</i>		
		<i>The following questions present similar choices, concerning amounts of money. Some of the amounts are certain for you to have, others you can win in a lottery.</i>		
		<i>We would like to know which choice you would make. There are no right or wrong answers with these questions.</i>		
2087.	wed1	We toss a coin once. You may choose one of the following two options: You receive $f$ 1000 with either heads or tails. . . . . 1 With heads you receive $f$ 2000, with tails you don't receive anything at all. . . . . 2	3105 848	} → 2088
2088.	wed2	Which of the following two options would you choose:  You draw a lottery ticket with an 80% chance to win $f$ 45 (if you lose, you don't get anything at all). . . . . 1 You receive $f$ 30, no matter which ticket is drawn. . . 2	1655 2298	} → 2089
2089.	wed3	Which of the following two options would you choose:  You draw a lottery ticket with a 25% chance to win $f$ 100 (if you lose, you don't get anything at all) . . . . 1 You draw a lottery ticket with a 20% chance to win $f$ 130 (if you lose, you don't get anything at all) . . . . 2	2050 1903	} → 2090
2090.	wed4	Which of the following two options would you choose:  You draw a lottery ticket with a 2% chance to win $f$ 3,000 (if you lose, you don't get anything at all). . . . . 1 You draw a lottery ticket with a 1% chance to win $f$ 6,000 (if you lose, you don't get anything at all). . . . . 2	1771 2182	} → 2091
2091.	wed5	We toss a coin once. Would you accept the following agreement:  Heads, you win $f$ 1,500 Tails, you lose $f$ 1,000  Yes . . . . . 1 No . . . . . 2	473 3480	} → 2092

Quest. #	Variables	Question according to questionnaire, answering options and codes	Frequency	Routing
<b>Appendix: probability equivalence questions</b>				
2092.	<b>lot1</b>	<p><i>Imagine you have won f 200 in a game. You can now choose between keeping that f 200, or having a lottery ticket with a certain chance to win a prize of f 20,000.</i></p> <p>How high would that chance to win f 20,000 need to be such that you would prefer the lottery ticket to keeping the f 200 that you had already won ?</p> <p>I would prefer the lottery ticket if the chance to win the first prize would be at least ... %</p> <p>1-100 %.....</p> <p>Don't know..... -9</p>	3111 842	} → 2093
2093.	<b>lot2</b>	<p><i>Now imagine you have won f 1,000 in a game. You can now choose between keeping that f 1,000, or having a lottery ticket with a certain chance to win a prize of f 20,000. How high would that chance to win f 20,000 need to be such that you would prefer the lottery ticket to keeping the f 1,000 that you had already won ?</i></p> <p>I would prefer the lottery ticket if the chance to win the first prize would be at least ... %</p> <p>1-100 %.....</p> <p>Don't know..... -9</p>	3150 803	} → 2094
2094.	<b>lot3</b>	<p><i>Now imagine you have won f 5,000 in a game. You can now choose between keeping that f 5,000, or having a lottery ticket with a certain chance to win a prize of f 20,000. How high would that chance to win f 20,000 need to be such that you would prefer the lottery ticket to keeping the f 5,000 that you had already won ?</i></p> <p>I would prefer the lottery ticket if the chance to win the first prize would be at least ... %</p> <p>1-100 %.....</p> <p>Don't know..... -9</p>	3124 829	} → 2095
2095.		<p><i>Interviewer:</i> Has the respondent answered the third part of the questionnaire "Economic and Psychological Concepts" (questions <b>tiener1</b> through <b>sprknd2</b>) and is the respondent head of the household or partner of the head of the household?</p> <p><i>Formal description:</i> <math>tiener1^{[2095]} \geq -9</math></p> <p>Yes.....</p> <p>No.....</p>	2958 1138	} → 2096 } → 2109

Quest. #	Variables	Question according to questionnaire, answering options and codes	Frequency	Routing
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**Appendix: inter-temporal choice questions**

2004.	<b>tijd1</b>	Imagine you win a cash prize in a lottery. The prize is worth $f$ 1,000 and can be paid out AT ONCE. Imagine the lottery, which is a financially trustworthy organization, asks if you are prepared to wait 3 months before you get the prize. Would you agree on that proposal, or would you ask for more money if you had to wait for 3 months ? I would agree on the waiting term of 3 months without the need to receive extra money for that. So, after 3 months I receive $f$ 1,000. .... 1 I would agree on the waiting term of 3 months, but I want to receive extra money for that. .... 2	1280 2771	} → 2006 } → 2005
2005.	<b>tijd2</b>	How much EXTRA money (in guilders) do you want to receive AT LEAST, in addition to the $f$ 1,000, to compensate for the waiting term of 3 months ? 1-2000 guilders ..... Don't know ..... -9	2738 33	} → 2006
2006.	<b>tijd3</b>	Imagine the cash prize that you win in the lottery is worth $f$ 100,000. Again, the prize can be paid out AT ONCE, or after 3 months. What would you prefer I would agree on the waiting term of 3 months without the need to receive extra money for that. So, after 3 months I receive $f$ 100,000. .... 1 I would agree on the waiting term of 3 months, but I want to receive extra money for that. .... 2	806 3245	} → 2008 } → 2007
2007.	<b>tijd4</b>	How much EXTRA money (in guilders) do you want to receive AT LEAST, in addition to the $f$ 100,000, to compensate for the waiting term of 3 months ? 1-200000 guilders ..... Don't know ..... -9	3219 26	} → 2008
2008.	<b>tijd5</b>	Imagine the lottery asks if you are prepared to wait a year before you get the prize of $f$ 100,000. What would you prefer ? I would agree on the waiting term of a year without the need to receive extra money for that. So, after a year I receive $f$ 100,000. .... 1 I would agree on the waiting term of a year, but I want to receive extra money for that..... 2	358 3693	} → 2010 } → 2009

368 Variable documentation VSB panel 1993

Quest. #	Variables	Question according to questionnaire, answering options and codes	Frequency	Routing
2009.	<b>tijd6</b>	How much EXTRA money (in guilders) do you want to receive AT LEAST <sub>i</sub> in addition to the <i>f</i> 100,000 <sub>i</sub> to compensate for the waiting term of a year ? 1-200000 guilders..... Don't know..... -9	3663 30	} → 2010
2010.	<b>tijd7</b>	Imagine you receive an assessment for tax arrears. To settle the payment <sub>i</sub> you have two options. One option is paying <i>f</i> 1,000 NOW. The other option is paying LATER <sub>i</sub> but in that case you have to pay MORE. What would you prefer ? I would pay <i>f</i> 1,000 now..... 1 I pay 3 months later <sub>i</sub> and I am prepared to pay more for that..... 2	3752 299	} → 2012 } → 2011
2011.	<b>tijd8</b>	How much EXTRA money (in guilders) would you be prepared to pay AT MOST <sub>i</sub> in addition to the <i>f</i> 1,000 <sub>i</sub> to get the extension of payment of 3 months ? 1-2000 guilders..... Don't know..... -9	282 17	} → 2012
2012.	<b>tijd9</b>	Imagine you could wait a YEAR with settling the tax assessment of <i>f</i> 1,000. What would you prefer <sub>i</sub> I would pay <i>f</i> 1,000 now..... 1 I would pay a year later <sub>i</sub> and I am prepared to pay more for that..... 2	3549 502	} → 2014 } → 2013
2013.	<b>tijd10</b>	How much EXTRA money would you be prepared to pay AT MOST <sub>i</sub> in addition to the <i>f</i> 1,000 <sub>i</sub> to get the extension of payment of a YEAR ? 1-1300 guilders..... Don't know..... -9	469 33	} → 2014
<p><i>The following questions are similar to the questions above<sub>i</sub> but now the question is how much LESS you would be prepared to receive<sub>i</sub> if you receive something at once compared to receiving it later.</i></p>				
2014.	<b>tijd11</b>	Imagine the cash prize that you win in the lottery is worth <i>f</i> 1,000 <sub>i</sub> but is paid out only after 3 MONTHS. The lottery <sub>i</sub> however <sub>i</sub> offers to pay out at once <sub>i</sub> but in that case you will receive less. What would you prefer <sub>i</sub> I would wait 3 months <sub>i</sub> and receive <i>f</i> 1,000..... 1 I would like to have the money now <sub>i</sub> and receive less..... 2	3414 637	} → 2016 } → 2015

Quest. #	Vari- ables	Question according to questionnaire, answering options and codes	Frequen- cy	Rou- ting
2015.	<b>tijd12</b>	How much LESS money (in guilders) would you be prepared to receive AT MOST, if you would get the money at once instead of <i>f</i> 1,000 after 3 MONTHS ? 1-1000 guilders..... Don't know..... -9	564 73	} → 2016
2016.	<b>tijd13</b>	Now imagine the prize is paid out only after A YEAR. What would you prefer? I would wait a year, and receive <i>f</i> 1,000..... 1 I would like to have the money now, and receive less ..... 2	2485 1566	} → 2018 } → 2017
2017.	<b>tijd14</b>	How much LESS money (in guilders) would you be prepared to receive AT MOST, if you would get the money at once instead of <i>f</i> 1,000 after A YEAR ? 1-1000 guilders..... Don't know..... -9	1476 90	} → 2018
2018.	<b>tijd15</b>	Imagine the prize is worth <i>f</i> 100,000, but is only paid out after 3 MONTHS. What would you prefer ? I would wait 3 months, and receive <i>f</i> 100,000,- ..... 1 I would like to have the money now, and receive less ..... 2	3051 1000	} → 2020 } → 2019
2019.	<b>tijd16</b>	How much LESS money (in guilders) would you be prepared to receive AT MOST, if you would get the money at once instead of <i>f</i> 100,000 after 3 MONTHS ? 1-100000 guilders..... Don't know..... -9	927 73	} → 2020
2020.	<b>tijd17</b>	Imagine the prize is worth <i>f</i> 100,000, but is only paid out after A YEAR. What would you prefer ? I would wait a year, and receive <i>f</i> 100,000 ..... 1 I would like to have the money now, and receive less ..... 2	2142 1909	} → 2023 } → 2021
2021.	<b>tijd18</b>	How much LESS money (in guilders) would you be prepared to receive AT MOST, if you would get the money at once instead of <i>f</i> 100,000 after A YEAR ? 1-100000 guilders..... Don't know..... -9	1829 80	} → 2022