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Analyzing firm performance in a glass industry: a non-parametric frontier approach

Master thesis within Econometrics

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Lund 20th January 2011

Abstract

A non-parametric data envelopment analysis (DEA) method is applied in this study in order for calculating the technical and scale efficiency of a glass factory by means of a production frontier, despite the common trend among contemporary literatures to use stochastic parametric methods and „estimate" the frontier production function. The available aggregated structure of the firm data paved the way for the authors to rethink and adopt a mathematical (linear programming) approach which would make it possible to conduct the study with a limited number of observations and without the detailed form of the production function. Encouraged by a significant number of comparative studies of parametric and non-parametric methods along with the increasing strength of the updated DEA models; an informed choice was made to determine an input-oriented, variable returns to scale (VRS), multi-stage slack calculation DEA model. The results of the efficiency analysis found four decision making units (DMUs) inefficient in the constant returns to scale (CRS) frontier with the existence of scale efficiencies (SE), the rest of the sixteen DMUs were efficient in both the CRS and the VRS frontiers. In general, the results turned out to be quite in line with prior observation, coherent with the pattern of fluctuating production levels given certain unbalanced amount of labor employed for different decision-making units (DMUs) and the respective level of wastes in production.

Keywords: allocative efficiency, data envelopment analysis, economic efficiency, non-parametric frontier, technical efficiency, scale efficiency, industrial glass factory.

Abbreviations

AE – Allocative Efficiency

CRS – Constant Returns to Scale

DEA – Data Envelopment Analysis

DMU – Decision Making Unit

EE – Economic Efficiency

LP – Linear Program

NIRS – Non-increasing Returns to Scale

OLS – Ordinary Least Squares

SE – Scale Efficiency

TE – Technical Efficiency

TFP – Total Factor Productivity

VRS – Variable returns to Scale

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1 Introduction

Measuring efficiency has always been a crucial part of applied economics. Since Cobb & Douglas (1928), econometricians were estimating „average" production function with the ordinary lest squares (OLS) method based on the mean output rather than the maximum attainable output. After the last three decades of development in economic theories and major advancements in applied methodologies, nowadays, we are able to estimate both the inefficiency of a production unit and any sort of error involved, as well as a frontier of maximum attainable outputs from a given sets of inputs or production technology. Measurement of economic efficiency can be fragmented into technical and allocative efficiency, of which, estimation of technical efficiency is somewhat more complex yet essential. The measurements of such efficiencies are vital for three reasons: First, through the comparison of relevant economic units, a relative measure can be set which indicate either the success or failure of the observed performance. Second, the sources of the variance in efficiency can be identified with further analysis and therefore the causes can be eliminated. Finally, identifying the sources of inefficiency usually bears a great policy implication in terms of helping both public and private institutions to develop an optimal set of policies.

Empirical methodologies of efficiency analysis split into parametric and non-parametric methods. Parametric or statistical methods apply a deterministic, stochastic or time-variant (or invariant) panel data production frontiers and it uses production, cost, profit and revenue functions in explaining the production technology while estimating efficiency (Battese 1991). The non-parametric methods, on the other hand, rely on Data Envelopment Analysis (DEA) purely based on mathematical linear programming to calculate the efficiency scores of a firm in the production frontier (Murillo Zamorano, 2004). DEA has been widely used for evaluating the performance and benchmarking of schools, hospitals, financial institutions, production plants, etc. as a multi-factor productivity analysis model. Thus, it is accounted as an important tool for evaluating and improving the performance of almost every kinds of productive facility that involves an input and output or managing operations research (Charnes et al., 1994).

1.1 Purpose

We are to implement a deterministic non-parametric frontier model to calculate the technical and the scale efficiency scores of an industrial glass factory in Tehran, Iran. In addition, we would identify the sources of inefficiency in its production technology due to any relevant

causes. Moreover, we would be advising on how to achieve an optimal level of production within given constraints.

1.2 Outline

A literature review is conducted in section 2, briefly outlining some of the prominent studies that have been motivating and essential in writing this paper. The theoretical framework presented in section 3, decomposes the concept of economic efficiency. Some major non-parametric methods in practice (which was also offered by the computer program that was implemented in this paper) followed by their advantages and disadvantages are discussed in this section as well. Section 4 describes the glass factory that is analyzed. Section 5 presents, a specification of the factory data and the determination of a non-parametric frontier model. The presentation and analysis of the results of the DEA model served in Section 6. Finally, in Section 7, the discussion concludes with a summary of the main findings, limitations of this paper as well as by providing some advice on increasing efficiency regarding increasing output while minimizing inputs in production.

2 Literature review

There is a vast literature written on frontier production functions and the measurement of technical, allocative efficiency or overall economic efficiency. Most of these literatures can generally be divided into two groups, **a)** parametric methods and **b)** non-parametric methods.

As this paper follows a non-parametric method only a brief account of the development of parametric methods is mentioned. Due to the relatively earlier start of the development of parametric methods along with the undivided attention paid by many prominent researchers, it has achieved an enriched array of methodologies with applications in a broad field. To begin in 1976, Aigner, Lovell, Schimdt, Meeusen and van den Broeck and Battese Corra independently set a mile stone in developing the stochastic frontier production function. By the beginning of 1990"s, a series of major developments led to the three distinct categories of parametric frontier estimation i.e. deterministic, stochastic and panel data model (Battese, 1991).

Although the potential of non-parametric methods was planted parallel in the seminal paper of Farrel (1957) as described in the theoretical background section, it took two more decades for it to be widely discussed by the researchers. A proposal made by Boles (1966) and Afriat (1972) convincingly suggested the use of linear programming to determine a production frontier in measuring efficiency. But it was not enough to get the interest of a larger group of researchers. In 1978, Charnes, Cooper and Rhodes introduced the term *„data envelopment analysis"* and started off the beginning of a series of extensions and developments of the DEA methodology for application. They proposed a model which assumed constant returns to scale (CRS) including input-oriented measures. A subsequent paper in 1984 by Banker, Charnes and Cooper extended the previous model to assume variable returns to scale (VRS). Further extension allowed the calculation of cost efficiency along with revenue maximization or cost minimization, and the use of time-invariant units or panel data model made possible by the pioneering work of Fare, Grosskpof, Lovell et al. (1994), Fare, Grosskpof, Norris and Zhang (1994), Coelli and Perelman (1996). The model that is applied in determining the DEA production frontier in this paper is the one outlined by Fare et al. (1994). This model is implemented using a computer program (Data Envelopment Analysis Program) developed by Tim Coelli (1996) along with some of his own developments in multi-stage slack calculation and measuring super efficiency according to Koopman"s (1951) definition.

On the other hand, a comprehensive survey of the frontier production literature by Murillo-Zamorano (2004) has been an effective and practical guidance to approach this wide field of research in the right direction. This provided some critical details for the use of parametric and non-parametric frontier methods, concluding that none of these is generally preferred to the other. Furthermore, they suggested that one of these methods can be implemented relative to the advantages and disadvantages of each of them by considering the given data and characteristics of the framework. Besides, another survey by Kalirajan and Shand (1999) added some specific perspective to the technical efficiency measurement. In addition, a recent empirical work as Ajibefun (2008) applying both parametric and non-parametric methods in measuring the economic efficiency of a small scale food crop in Nigeria, has been a direct source of motivation. Basically the same conclusion was reached in this paper similar to the majority of literatures that compared parametric and non-parametric methods. That is the efficiency scores are quite close from the parametric and non-parametric methods and a combination of these two methods would be a good approach to policy implications.

In addition, a research conducted by Kazemi (2010), estimated technical efficiency of 36 Electricity Distribution firms in Iran by using yearly data of 2007-2008. In his research, both the parametric deterministic frontier production function and non-parametric Data Envelopment Analysis were applied and an interesting correlation between these two approaches was found. Moreover, the huge applied literature by means of both parametric and non-parametric frontier methods applied in the wide range of fields in Economics ranging from finance-banking to agriculture or environmental economics to public-development economics can be regarded as the single most productive influence in writing this paper.

3 Theoretical background

Measuring economic efficiency has been commonly linked to the use of frontier functions relative to an efficient technology. In the last 50 years, a wide range of research had begun with the seminal paper by Michael J. Farrel. In Farrel‟s (1957) paper, a method was presented for measuring technical efficiency (TE) and allocative efficiency (AE) of a firm as the two components of total economic efficiency (EE)[1]. His work was mostly influenced by the definition provided by Koopman (1951) and the measure of technical efficiency introduced by Debreu (1951). Farrel categorized several ways of a production unit being inefficient, first by obtaining less than the maximum output available from a certain set of inputs, i.e. TE and for a certain level of outputs not using the optimal combination of inputs given their prices and marginal productivities (allocatively inefficient). The original idea had the proposal with the illustration of an input-input space, thereby with a focus on input reduction in terms of producing a particular level of output. The efficiency measurement literature is primarily classified into input-oriented and output-oriented measures.

3.1 Measuring economic efficiency

First to note, the above mentioned efficiency measurements (described elaborately in the following sections) assumed the knowledge of the production function of the efficient firm, though the efficient isoquant must be estimated from the sample data in practice. Secondly, Farrel proposed the construction of either **a)** non-parametric piecewise-linear convex isoquant to ensure no observed point lie on the left or below it, or **b)** a parametric function, as the Cobb-Douglas, that is fitted to the data and ensuring again that no observed point lie to the left or below it as described below in figure 1:

[1] In the original paper, the term *price efficiency* used instead of allocative efficiency and *overall efficiency* used instead of economic efficiency. The latter terminologies used here to be coherent with most of the recent literatures.

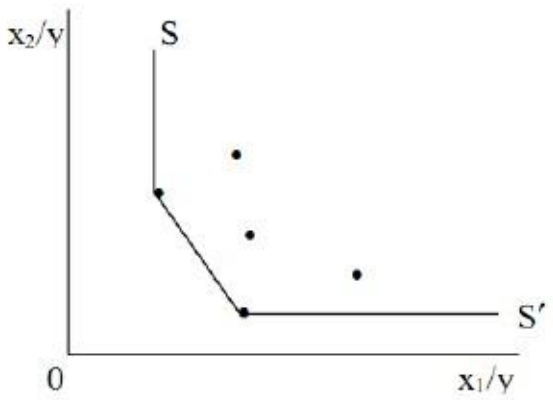

Figure 1. Non-parametric piecewise linear convex isoquant (Farrel 1957).

Throughout the rest of this paper, we shall see that regardless input or output oriented measurement methods, the aforementioned proposal of Farrel will be followed in constructing all the non-parametric piecewise-linear convex isoquants (ensuring no observed point lie on the left or below it).

3.1.1 Input-oriented measures

Technical efficiency: This idea can simply be expressed with an example of a firm using two inputs x_1 and x_2 to produce a single output *y* holding the constant returns to scale (CRS) assumption, which allows representing the technology using a unit isoquant [2]. The production function of a fully efficient firm that is yet to be estimated is represented by this unit isoquant, in figure 2 to measure the technical efficiency. This captures the minimum combination of inputs needed to produce one unit of output. Therefore, any point above or to the right of P would be considered technically inefficient using more inputs than required. The distance QP, along the ray OP represents the amount by which all inputs can be proportionally reduced without decreasing the output. In geometric expression, the technical efficiency level of the package *P* can be expressed by the ratio 1-QP/OP or in the percentage term OQ/OP by which all inputs could be reduced (see figure 2).

$$TE_I = OQ/OP \qquad 0 < TE_I < 1. \tag{1}$$

This therefore gives us an indicator of the degree of technical efficiency. It will take a value between zero and one. A value of one indicates that the firm is fully efficient i.e. technically

[2] However, an extension of this method was discussed by Farrel to accommodate multiple inputs and multiple outputs and the assumption of non-constant returns to scale.

efficient in this case, and any value less than one indicates inefficiency. Thus, the point Q in the figure below is technically efficient as it lies on the technically efficient unit isoquant.

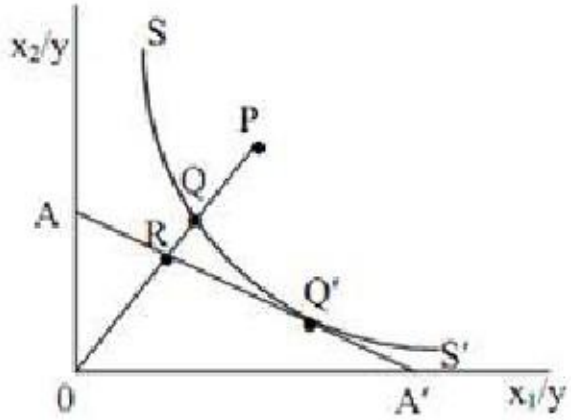

Figure 2. Technical and allocative efficiency measures (Farrel, 1957).

Allocative efficiency: Given the market price information and assuming a particular behavioural objective as cost minimization the slope of the isocost-line reflects the input-price ratio. The distance RP represents the production cost that the producer would be able to reduce by moving from a technically efficient but not allocatively efficient package *Q* to both technically and allocatively efficient package . The allocative efficiency (AE) of the firm producing the package is defined as the ratio:

$$AE_I = OR/OQ \qquad 0 < AE_I < 1. \qquad (2)$$

The measurement of the economic efficiency (EE) is derived from the multiplicative interaction of the two above mentioned efficiency measurements:

$$EE_I = TE_I\, AE_I = (OQ/OP) \qquad (OR/OQ) = OR/OP \quad 0 < EE_I < 1. \qquad (3)$$

3.1.2 Output-oriented measures

In the case of an output-oriented measurement, we could start with a different perspective on increasing the output quantities while holding the level of inputs constant. As Fare and Lovell (1978) explained, the output and input-oriented measures will be equivalent in the presence of constant returns to scale and we will find difference in their measures when either

increasing or decreasing returns to scale are present[3]. This is well described in figure 3 below, where we have decreasing returns to scale (DRTS) technology represented by f(x) involving one input and one output and an inefficient firm operating at the point P.

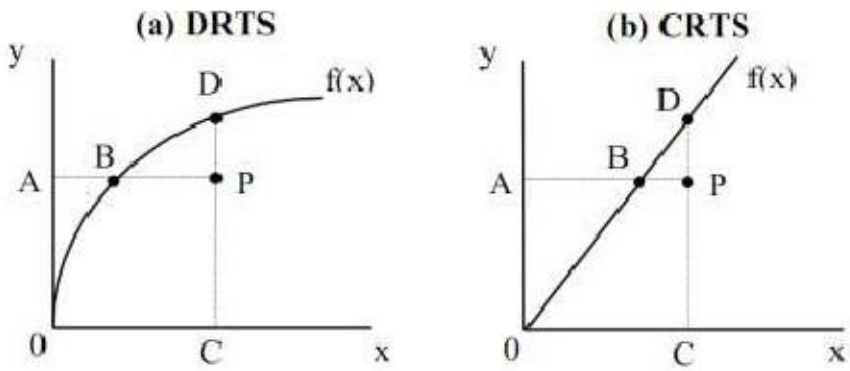

Figure 3. Input and Output-oriented TE measures and returns to scale (Fare and Lovell, 1978).

In Figure 3(a), the Farrel input-oriented measure of TE would be equal to the ratio AB/AP and the output-oriented measure of TE would be CP/CD. As mentioned above these TEs will be unequal when decreasing or increasing returns to scale are present. And in the case of Figure 3(b) for any inefficient point P we would observe AB/AP=CP/CD when constant returns to scale (CRTS) applies.

Technical efficiency measures: Here, the production involves two outputs y_1 and y_2 and a single input x. The technology is represented by a two dimensional unit production possibility curve which is the upper bound of the production possibilities. By assuming CRS, Figure 4 below depicts the Farrel output-oriented TE measure by the distance AB as inefficiency of a firm located at point A, giving us the amount by which the output could be increased without increasing the level of input.

[3] Input and output-oriented technical efficiency measures also shown to be equal to the input, output distant functions by Shephard (1970). This bears more importance in discussing the DEA methods of calculating the Malmquist indices of TFP change in the later section.

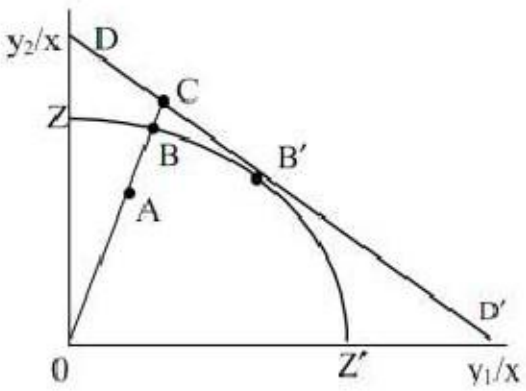

Figure 4. Output-oriented TE and AE measures (Farrel 1957).

By considering figure 4, we get the following output-oriented TE measure

$$TE_0 = OA/OB \qquad 0 < TE_0 < 1. \tag{4}$$

Allocative efficiency measures: With relevant price information the isorevenue line can be drawn. This drawn line tells us an increase in revenue will be gained when moved from TE point B to both TE and AE point .

$$AE_0 = OB/OC \qquad 0 < AE_0 < 1. \tag{5}$$

Again the overall efficiency measurement is derived as a product of these two measures.

$$EE_0 = (OA/OB) \quad (OB/OC) \qquad 0 < EE_0 < 1. \tag{6}$$

Until this part of the discussion, all the measures discussed above were radial (along a line from the origin to the observed production point) in which the relative proportions of inputs or outputs remain constant. This gave an advantage to all the above-mentioned efficiency measures to be units invariant. Thus, the change in the units, i.e. labor hour to labor days will not chance the value of efficiency measure, unlike in the case of a non-radial measure, counting the shortest distance from the production point to the production envelopment surface.

3.2 Non –parametric methods of measuring economic efficiency

This paper implemented Data Envelopment Analysis Program (DEAP) which takes a non-parametric linear convex hull approach to frontier estimation for calculating efficiency scores. Data Envelopment Analysis is a non-parametric mathematical linear programming

method that envelops data to create an efficient surface that gives the possibility to calculate efficiencies of all the DMUs relative to this surface (Coelli, 1996). In micro economic theory, the production function can be said to form the basis of input-output relationships in a farm, efficiency calculation can be made relative to this frontier if it‟s known. This computer program can apply several models but the three principal models which are based on the work of Fare, Grosskopf et al. (1994) will be discussed in the following sections. The three principal models are:

1. Standard CRS and VRS DEA models that involve the calculation of technical and scale efficiencies (Fare, Grosskopf and Lovell, 1994).

2. Extended version of the CRS and VRS models can be accounted for analysing cost and allocative efficiencies, which are outlined in Fare et al. 1994.

3. Malmquist DEA methods applied to panel data which are used for calculating indices of total factor productivity (TFP), can be broken into, technological change, technical efficiency change and scale efficiency change (Fare, Grosskopf, Norris and Zhang, 1994).

3.2.1 Constant Returns to Scale (CRS) model

Let us assume a data on K inputs and M outputs on each of N DMU‟s and for the i-th DMU these are represented respectively by x_i and y_i vectors. The input matrix, X, and the output matrix, Y, represent the data of all *N* DMU‟s. As mentioned earlier, the purpose here is to construct a non-parametric envelopment frontier that captures all observed point below or on it. In the simple case of two inputs used to produce one output, it can be visualized as a number of intersecting planes forming a conical hull over a scatter of points in three-dimensional space, that can also be represented by a unit/isoquant in input/input space (see figure 1).

The ratio form (two forms) is a good way to introduce DEA for expository purposes. For each DMU we would like to measure the ratios of all outputs over all inputs as , where u is an vector of output weights and v is a vector of input weights. To select the optimal weight we specify the mathematical programming problem as follows:

(7)

This programming involves finding values for u and v, so that the efficiency measure of the i-th DMU is maximized holding the constraint that all efficiency measures must be less than or equal to one. One major problem here is when is a solution then another solution is giving an infinite number of solutions. Therefore the constraint , is imposed as a remedy, which gives:

(8)

where the notation change from u and v to μ and v (interpreted as normalised shadow prices) denotes the transformation. This form is known as the multiplier form of the linear programming (LP) problem.

An equivalent envelopment form of this problem can be derived using the duality in linear programming:

(9)

here, θ is a scalar and λ is a vector of constants. The envelopment form has fewer constraints in comparison to the multiplier form *(*$K+M < N+1$*)* and is generally preferred in practice. θ represents efficiency scores for the i-th DMU and the value of θ must be less than or equal to one. A value of θ equal to 1, indicates a technically efficient DMU on the frontier according to Farrel (1957). λ denotes the weights of the peers[4] in determining the efficient

[4]By definition, the *Peers* indicates the linear combination of efficient DMUs that define efficient production for inefficient DMUs. For example, in Figure 5, DMUs C and D are referred as the *peers* of DMU B.

points for the inefficient DMUs. To note, the linear programming problem must be solved for each DMU in the sample (N times), therefore each DMU will obtain its own value of θ.

3.2.1.1 Slacks

In the non-parametric frontier in DEA, the piecewise linear form can cause some difficulties in efficiency measurement. It arises due to the sections of the piecewise linear frontier running parallel to the axes (see figure 1) and do not occur in most parametric functions (see figure 2). The problem is illustrated in the figure below:

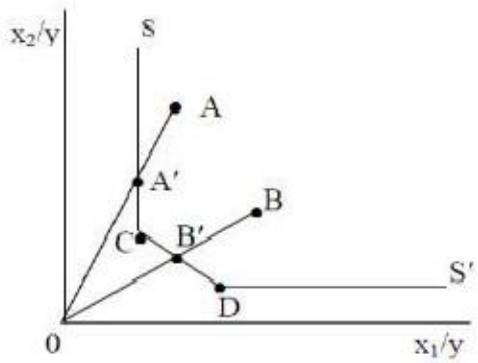

Figure 5. Input Slacks and the measurement of efficiency (Coelli, 1996)

Here, C and D are the two efficient DMU"s which define the frontier, and A and B are the inefficient DMU"s. According to Farrell (1957) measurement, technical efficiency for DMU"s A and *B* are respectively and . It is questionable whether the point is an efficient point as the amount of input can be reduced by the amount , and produce the same output as before. This is known as input slack or input excess in DEA literatures (see Appendix 2, Table 1 for slacks on the horizontal axis). If this simple example extended to a case with more inputs and/or outputs, the diagram would not be that simple any more, output slacks can occur as well. Thus, one can say that both Farrel measure of TE (θ) and any non-zero input or output slacks should be counted to provide an accurate calculation of TE of a DMU in a DEA analysis. For the i-th DMU the output slacks will be equal to zero only if $Y\lambda - y_i = 0$, while the input slacks will be equal to zero only if $\theta x_i - X\lambda = 0$ for the given optimal values of θ and λ.

As shown in figure 5, the input slack for point is from input . But if we have more inputs and outputs, the identification of C as the nearest efficient frontier point and

subsequent calculation of slacks get quite complicated. As a remedy, some studies as Ali and Seiford (1993), suggested a second-stage linear programming problem. This solution proposes an efficient frontier point by moving or maximizing the sum of slacks from an inefficient frontier point such as to a furthest efficient frontier point such as C. This second stage linear programming is defined below:

(10)

OS denotes an vector of output slacks, IS a K vector of input slacks and M1 and *K1* are and *K* respective vector of ones. To note further, in this second stage LP the value of is taken from the first stage results and therefore is not a variable. And this second stage LP must also be solved for each *N* DMU‟s.

There are two major problems regarding this second stage solution. As we know, in an input-oriented method the input level is tried to be minimized while holding the output level unchanged. Unless in the cases of inefficient DMUs where a higher output levels can also be achieved with a lowered input levels for production (as can be seen later in the DEA results analysis section). Thus, the first problem arising from the second stage LP cause it should minimize the sum of slacks not maximise. Therefore, it will identify not the nearest efficient point but the furthest efficient point. Second, this approach is not invariant to the units of measurement. For example, the alteration the units of measurement of heights from centimetres to inches (leaving other units of measurements unchanged) could result in identifying different efficient boundary points and different slacks and lambda measures. However, these problems are important when slacks occurs in two or more dimensions as it does quite often, treatments of these problems can then be essential.

Many studies implement the first stage LP as we have in LP 9, to obtain Farrel radial technical efficiency measures (θ) for each DMU and ignore the slacks completely or they report both the (θ) and the residual slacks, which may be calculated as $OS = -y_i + Y\lambda$ and $IS = \theta x_i - X\lambda$. This approach also comes with a problem that either these residual slacks may not always provide all Koopmans slacks i.e. a number of observations appearing on the vertical

section of the frontier as in Figure 5, and hence may not always identify the nearest (Koopmans) efficient point for each DMU.

Three choices were given for the treatment of slacks in the DEAP software. First, a one-stage DEA, which conducts the LPs (as LP 9) then calculate slacks residually. Second, is a two-stage DEA which conducts the LP 9 and 10. The last and the most crucial one is multi-stage DEA which conducts a sequence of radial LPs to identify the efficient projected point.

To note, this demanding multi-stage method of calculating both input and output slacks which is generally recommended by Coelli (1997) was selected for this paper. As the multi-stage method identifies the projected efficient points of input-output mixes which are as similar to those of the inefficient points and also being invariant to units of measurements. Ferrier and Lovell (1990) had shown that we are over with slacks given an infinite sample size and an alternate frontier construction method involving a smooth function surface also that slacks may be viewed as allocative efficiency (AE).

3.2.2 VRS model and scale efficiencies

In the CRS frontier the DMUs are producing at an optimal level (in the flat portion of the LRAC curve) but it is unlikely in reality at the presence of imperfect competition, various environmental and economic constraints. So we realise firms operating at sub-optimal levels, measuring TEs that are confounded by scale efficiencies (SE) from an extension of the input oriented CRS model (Charnes, Cooper Rhodes 1978) proposed in Banker, Charnes and Cooper (1984). There, assuming VRS, TE in CRS is decomposed into “pure” TE in VRS times SE all in an input oriented measure. As the VRS model calculated both the CRS and the VRS frontiers and get the SE. The convexity constraint is added to the CRS dual LP that derived an equivalent envelopment form in LP 9 to account for VRS:

(11)

Here, is an vectors of ones. This approach forms a convex hull of intersecting planes, enveloping the data points more tightly than the CRS conical hull giving TE scores greater or equal to those obtained from the CRS model.

3.2.2.1 Calculation of scale efficiencies/ratio efficiencies

Total efficiency scores which are obtained from CRS DEA decomposed into two components, one because of scale inefficiency and the second one because of "pure" technical inefficiency. This can be seen by conducting both a CRS and a VRS DEA upon the same data. Thus, a difference in the TE scores for a particular DMU indicating the presence of SE. And that SE can be calculated from the difference between the CRS TE scores and VRS TE scores. Figure 6 below illustrates the calculation of scale economies:

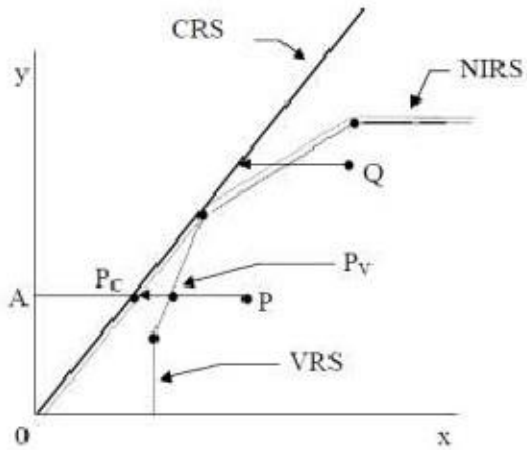

Figure 6. Scale Economies in DEA (Coelli, 1996)

Here we have one input to produce one output and the CRS and VRS DEA frontiers. In CRS, the input-oriented technical inefficiency of the point *P* is the distance but in VRS the technical inefficiency would be . Therefore the difference between these two distances is accounted as scale inefficiency. This can also be expressed as ratio efficiency measures:

All these measures are bounded between zero and one, and the subscript I denoting an input-oriented measure. Also for the reason,

We get the TE score in CRS frontier as,

Therefore the CRS TE measure is split into pure TE and SE. But this measurement of SE cannot identify the area, i.e. whether the DMU is operating in an area of increasing or decreasing returns to scale. This can be determined by running an additional DEA problem imposing a non-increasing returns to scale (NIRS) assumption. Altering the VRS LP 11 and substituting with to get:

(12)

Figure 6 also presents the NIRS DEA frontier. The nature of the scale inefficiency can be determined by seeing if the NIRS TE score is equal to the VRS TE score or not. If the NIRS TE score is equal to the VRS TE score as for the point *Q,* then decreasing return to scale exist but if they are unequal as for point *P* then there exist increasing return to scale.

3.2.3 Cost efficiency

With the availability of price information a behavioural objective as cost minimization or revenue maximization can be considered to measure both TE and AE scores. In the case of VRS cost minimization, the input-oriented DEA model set out in LP 11 is run first to obtain the technical efficiencies (TE). Then the cost minimization DEA that is given below needs to be run:

(13)

Here, is the vector of input prices for the i-th DMU and calculated by the linear programming, is the cost-minimizing vector of input quantities for the i-th DMU with input prices, and the output levels . Therefore the total cost efficiency (CE) or economic efficiency of the i-th DMU would be calculated:

This is the ratio of minimum cost to observed cost in practice. Then allocative efficiency can simply be calculated residually as,

To note, this method includes any slacks into the allocative efficiency measure, which is often justified since slacks reflects an inappropriate input mix (Ferrier and Lovell, 1990).

3.2.4 Panel data model

When panel data is available, linear programming and input/output-oriented Malmquist TFP index, as there is in DEA is typically preferred. One of the routine and basic measurements of this method is to measure productivity change and to identify technical change and technical efficiency change from this productivity change. Fare et al. (1994) specifies an output-based Malmquist productivity change index such as,

(14)

In this model, “d” denotes the four distance functions. Distance function defines multi-output, multi-input production technology without considering concepts of cost minimization and profit maximization (Basti and Akin, 2008). Also the subscript “o” shows that these are

output-oriented measures.[5] Therefore, “ ” denotes the output-oriented distance function which indicates the maximum expansion of output with presumption input .[6] This model depicts the productivity of the production point (relative to the production point ( ). If the value of TFP is greater than one, it shows a positive growth from period t to period t+1. This output based Malmquist TFP index is, the geometric means of two output-based Malmquist TFP indices, where one index uses period t technology and the other uses t+1 technology. To calculate LP 14 above, we need to calculate the four distance functions d"s that involve linear programming problems similar to those for calculating Farrel TE measures.

Initially assuming CRS to start with, further decomposition to find SE is done later in this section. A CRS output-oriented linear programming used to calculate $d_{0t}(x_t, y_t)$, without the convexity restriction from VRS and added time subscripts:

(15)

The rest of three LP problems are simple variant of this:

(16)

[5] There are two distance functions that can be defined by Malmquist index. First one is input distance function. An input distance function explains technology by maximum comparative decreasing of inputs by having presumption outputs. While output-distance function deliberates maximum comparative increasing of the output by presumption input vector (Coelli and Rao, 2005).

[6] In Malmquist TFP index, changes of TFP between two data periods (t, t+1) is calculated by using distances of each data period relative to common technology.

(17)

(18)

In LP 17 and 18, where we compare the production points, to technologies from different time periods, parameter need not be ≥1, as it is a must when calculating Farrel efficiencies (θ). The production point could lie above the feasible production set. This is most likely in LP 17 where a production point from period $t+1$ is compared to technology in period t. If there was a technical progress then a value of Φ 1 is possible. And it is also possible to occur in LP 18 but less likely.

To note, the value of Φ and λ"s are likely to take different values in above four LPs. Moreover, the four LPs above must be calculated for each DMU. For instance, if we have 30 DMUs and 2 time periods, 120 LPs must be calculated. For adding extra time periods, an extra three LP"s for each DMU must be calculated to construct a chained index i.e. with T time periods and N DMUs; LPs must be calculated for each DMU. For instance, if we have 20DMUs and 10 time periods, we will have LPs. We should note that results on each and every DMU for each and every adjacent pair of time periods can be tabulated and summary measures across time and space can be presented.

3.4.5 Calculation of scale efficiency

The approach described above can be extended to fragment (CRS) technical efficiency change into scale efficiency and "pure" (VRS) technical efficiency. This requires calculating two additional LPs when comparing two production points i.e. repeating LP 17 & 18 with the

VRS convexity restriction added to each of them. That means calculating the distance functions relative to a VRS instead of a CRS production technology. Then the CRS and VRS values can be used to calculate the scale efficiency residually i.e. $TE_{CRS}/TE_{VRS} = SE$. Now, for the case of N DMUs and T time periods the number of LPs would increase from to (see Fare et al. 1994, p 75).

3.3 Advantages and disadvantages of non-parametric methods

The principal advantages and disadvantages of non-parametric methods in comparison to parametric methods in measuring efficiency using production frontier is summarized from Ajibefun (2008) in the table below:

Table 1

Methods of Efficiency Analysis		
Methods	Parametric methods	Non-parametric methods
	Stochastic Frontier Production Function (SFPF).	Deterministic Data Envelopment Analysis (DEA).
	Alternatively use production, cost, profit and revenue functions for describing the production technology and estimate technical,allocativeandeconomic efficiency.	Mainlyuselinearprogramming techniques for describing the production frontierandcalculatetechnical, allocative, cost and economic efficiency from it.
Advantages	Parametric methods allow for the test of DEA do not require a particular hypothesis concerning the goodness of fit of functional form for the production the model.technology.	
Disadvantages	It requires specification of the production technology and causes both specification and estimation problems.	It is not possible to estimate parameters for the model and impossible to test hypothesis concerning the performance of the model. And it does not distinguish between statistical noise ε_i and inefficiency u_i.

Developments in non-parametric methods: As mentioned above, the deterministic nature of non-parametric methods has caused the traditional literature to describe them as non-statistical methods. However, a series of recent developments made it possible to determine statistical properties of non-parametric frontier estimators in a statistical model. Nowadays, statistical inference based on non-parametric frontier approaches in measuring the economic efficiency is available basically by using asymptotic sampling distributions or by using bootstrap.

Asymptotic sampling distributions: A prominent work in this field by Grosskopf (1996) provided a thorough survey of statistical inference in non-parametric, deterministic and linear programming frontier models. This paper also had a proper treatment for non-parametric regularity tests, sensitivity analyses, and non-parametric statistics tests. Moreover, the

asymptotic properties of these DEA estimators were analyses based on maximum likelihood. Some major limitations of the method developed by Grosskopf (1996) were outlined in later developments by Kneip, Park and Simar (1998) and Park, Simar and Weiner (2000), who mainly pointed out that Grosskopf‟s results, may be misleading when using small samples. Basically, this method introduces extra noise when estimating unknown parameters in constructing estimates of confidence intervals. In addition, although most applications of the DEA estimator are usually carried out in the multivariate framework, the asymptotic sampling distribution in Grosskopf (1996) is only available for univariate DEA framework.

Bootstrap: Bootstrapping techniques have recently been proposed as a way to deal with the disadvantages of DEA. The bootstrap provides a convenient way to analyze the sensitivity of efficiency scores relative to the sampling variations of the calculated frontier by avoiding the aforementioned drawbacks of asymptotic sampling distribution (Murillo-Zamorano, 2004). A method first developed by Ferrier and Hirschberg (1997), introduced a stochastic element into the TE scores obtained by DEA techniques, which provided means to calculate the empirical distributions for the efficiency measures which were used to derive confidence intervals for the original efficiency levels. However, this methodology was later criticized by Simar and Wilson (1999a, 1999b) who showed that the bootstrap procedure suggested by Ferrier and Hirschberg (1997) is inconsistent. An alternative was proposed by the same critics, Simar and Wilson (1998) showing that it is necessary to define a reasonable data-generating process and propose a reasonable estimator of it to validate the bootstrap. Simar and Wilson (2000a) used this procedure for constructing confidence interval which basically corrects for the bias via simulations. Here again, this bias correction procedure via simulation introduces a further source of noise into the process. This modus operandi‟s weakness is overcome in Simar and Wilson (1999c) with an improved procedure which automatically corrects for the bias without explicitly using a noisy biased estimator. Moreover, the initial methodology introduced in Simar and Wilson (1998) is also extended by Simar and Wilson (2000b).

Finally, some major issues remain that involve the use of asymptotic results and bootstrap, first, the high sensitivity of non-parametric approaches to extreme value and outliers and second, the way to allow stochastic noises in a non-parametric frontiers.

To address the first issue, a non-parametric estimator which is more robust to extreme values, noise or outliers, is proposed by Cazals, Florens and Simar (2002). This does not envelope all

the data points and based on a concept on expected minimum input function. Then to address the latter issue with stochastic noises, Sengupta (2000a) generalized the non-parametric frontier approach in the stochastic case when the input prices and capital adjustment costs vary. Another discussion covered the issue of stochastic noises in Huang and Li (2001) by discussing the relationships of their stochastic DEA models with some conventional DEA models based on utilizing the theory of chance constrained programming.

4 ASA glass factory

4.1 The glass industry and its products

In the modern industrial world the glass industry is an integral part of national economies and economic activities. Glass is used in numerous ranges of products from household glass wares to buildings to air, land, and water vehicles, literally in everything we can think of. Industrial glass can be defined as a packaging which is used for food and beverage, lighting products for homes and businesses, tubes for televisions and etc. But in a specific classification the range of industrial glass wares can be categorized as follow (www.glassindustry.info):

- Automotive glass which includes window, rear view, side view mirrors and wind shields of motor vehicles.
- Hollow glass which includes containers such as bottles and jars.
- Flat glasses which includes window and door installation.
- Crystal glass and tableware.
- Fibre glass product which includes optical cables.
- Optical glass which includes all sorts of lenses used in the optical industry.
- Scientific glass consists microscope and telescope lenses.
- Glass processing that involves toughening, laminating or curving glass, engraving, etching etc.
- Art glass which includes stained glass, milk glass, tiffany and processing tools (www.apprenticeships.org.uk).

The glass wares that are produced by the glass factory (ASA Glass Factory) of this paper‟s interest are also listed below:

- Durable glass which is used for the roof or the top of the oven (noted as roof door in our data).
- Durable glass that is used for the front door of the oven (noted as front door in the data).

- Key frame which is installed on the oven (noted as key frame in the data).
- Greenhouse, the terms of greenhouse is used for the bottom of the ovens specially for preserving the heat within to keep the food warm (noted as greenhouse in the data)
- Double layer glass, meaning the durable two-layer glasses which are useful in several particular kinds of productions for ovens, heaters and buildings and etc.
- Refrigerator glasses which are durable glasses that are used as the shelves inside the refrigerator.

Every glass factory could differ in their production process according to the range of their products and the combination of manual labour and automatic machines. A general description of ASA glass factories production process is given below.

4.2 Steps in producing industrial glass wares in ASA glass

1. **Cutting:** The first step consists of cutting the raw glass into different shapes and sizes according to the orders of the customers. This step can be done in a factory either manually by labourers or by automatic production lines i.e. mechanical hands. In ASA glass factory, it is done by the labourers. In addition, straightening, shaping, drilling and milling are also done in this step. Smoothing the edges to take away the unevenness of the glass is done by „Diamond machines". The amount of labour required in this part of the production process fluctuates according to the demand for the goods.

2. **Cleaning & polishing:** In the second step, the glasses are washed then polished three times until the surfaces of the glasses become demagnetized. First of all they are washed by water then in the second and third steps they are cleaned and polished by chemical (cleaning) agents. Since this part is done by machines, the amount of labourers employed in this area are limited, as each machine would just need one operator standing by.

3. **Drilling and piercing:** this step is immediately followed after the second step. Here, the glasses are drilled and pierced according to the requirement of the customers or the design of the devise they will fit into. These are basically done by machines and the extent of its use is decisive in terms of the speed of the production process.

4. **Printing:** In this step different kinds of templates are used to print or design the glasses according to the itemized order. These templates are put on the glasses, afterward the colours are applied to make the designs or images appear. This is one of the most important steps since it requires creativity in terms of designing the templates and applying them to the products. And variations of attractive designs are significant to achieve more demand for the product in the market. This step has always been quite time consuming. An increasing marginal product can be observed in this step since the production process making the use of machineries an important factor regarding the overall productivity.

5. **Drying:** In this step the printed glasses are dried in special heating cabinets to be prepared for the next step. This is relatively more time consuming in comparison to the others steps since it must be done properly and with patience. Besides, this step is quite sensitive to the indoor temperature. Therefore it is essential that the production facility avoids unfavourable fluctuation of the temperature causing any damage to the glass.

6. **Annealing:** In the sixth step, the glasses are placed into a „furnace" for annealing. The glasses will be heated up to the annealing point then cooled down slowly to the room temperature. This process relieves the internal stress of the glasses to make them more durable. To note, the furnace is always turned on due to the time required to raise the temperature to the desired level and the relevant cost factors. As a result, we can observe the night-shift production units as well.

7. **Coating:** In the last section after annealing, a layer of polyethylene wax is applied via a water based emulsion. This makes the glass surface slippery, protecting it from scratching and preventing the glass wares from sticking together when they are transported. Thus, the invisible combined coating gives a virtually uncatchable, anti-scratch surface to the glass.

A necessary description has been given above get an overall image of the steps and processes in the glass production at ASA glass factory. Based on the production data of the quantity produced per months in two daily shifts of production combined with some input data of labourers, machineries and amount of wastes per types of product a non-parametric DEA frontier model will calculate the efficiency score of this glass factory.

5 DEA model specification and the data

An inductive approach was taken to analyse the empirical production data from the glass factory of our interest. To begin, the choice of method and the selection of model were made in accordance with the availability of the data, its structure and the type of the production process. An input-oriented, VRS (variable returns to scale), DEA model outlined in Fare, Grosskopf and Lovell (1994) was used to calculate technical efficiency (TE) and scale efficiency (SE) from the frontier function. A multi-stage calculation method was selected for slacks calculation counting for invariant units and providing a super efficiency measures (Coelli, 1997). A DEA analysis software named DEAP (Data Envelopment Analysis Program) developed by Tim Coelli (1996) was used for the calculation of the model and the efficiency measurements. In the following sections we will start a discussion by looking at the data table, followed by the model selection process and the analyses of the results from the models with some implications.

5.1 Description of the data

We have the monthly production data of day and night shift production units for ten months from October 2009 until July 2010 from ASA glass factory (see appendix 1, table 2). This gives us twenty DMUs (implying twenty firms in this question) in the DEA computer program implemented for the analysis. These twenty DMUs equals the ten months production data times the two production shifts. DEA allows us such a liberty to consider two shifts of the same factory as two separate DMU"s. Therefore DMU 1 and DMU 2 are respectively the day and night shifts monthly production data of October 2009 and the rest of eighteen DMUs are monthly day and night shifts production data for next nine months.

All the even numbered DMUs represent the night shift units and the odd numbered ones represent the day shift units (see appendix 1, table 2). The numbers of outputs are denoted by y's i.e. y_1 for DMU1 is eight thousand seven hundred thirty two pieces of that particular kind of glassware produced. Here, y_1 to y_6 are representing the following six types of glass wares respectively, Roof Door, Front Door, Key Frame, Green House, Double, Refrigerator. These names are given by the manufacturer for the purpose of their uses.

The numbers of inputs are denoted by x's i.e. the numbers of labourers per shift is denoted by x_1 i.e. 28 labourers for all the day shifts and 14 labourers for all the night shifts. The amount of wastes in producing each six types of outputs are denoted respectively by x_2 to x_7, i.e. x_2 in

DMU 1 is one hundred three pieces of rejected product of y_1 type. The number of machineries used in almost all steps of production processes is denoted by i.e. the furnace, the polishing machine etc. To note, these machineries are switched and shared among different steps of the productions processes. In addition, the night shift DMUs employ half number of labourers than the day shift DMUs i.e. 14 labourers, utilizing same amount of resources and producing 10-40% higher on a regular month to month basis (see appendix 1, table 2 & 3).

5.2 Model Selection

As mentioned above we chose an input oriented, VRS, multi-stage DEA model for the purpose of our frontier analysis. Due to the lack of availability of daily data and for longer period of time, we had to choose DEA instead of Econometric methods to represent a frontier function relative to an efficient technology. Since linear programming can handle simultaneous equations conveniently, a choice of orientation in this case was not much crucial. In addition, as it is already explained in the theoretical background section, Input and output oriented technical measures are equivalent when CRS is present. And input-output oriented model will estimate the same frontier and identify the same set of DMU‟s by definition as being efficient (Coeli, 1996).

However, an input oriented measure was taken observing the highly fluctuating level of production of the firm and considering the number of wastes, though the amount of waste is quite steady around 1-2% of the whole production output every month. Besides, the amount produced is rather higher than the amount ordered every month even after excluding the amount of wastes as informed by the factory manager.

Now, in reality it is likely that the DMUs are producing at sub-optimal levels at the presence of various random causes or errors. Therefore, by assuming VRS in the model, a CRS frontier is also calculated and presented. To find the SE the TE in CRS is decomposed into “pure” TE in VRS times SE all in an input oriented measure. This approach gives VRS TE scores greater or equal to those obtained from the CRS model. Moreover, by having both the CRS and VRS frontiers, this model provides an effective way of checking the robustness of the results to the change in the returns to scale assumptions.

A multi-stage method of calculating both input and output slacks provided in the software DEAP (Coeli, 1996) was chosen, that identifies the projected efficient points of input-output

mixes which are as similar as possible to those of the inefficient points and also being invariant to units of measurements. Thus, it also identifies a firm as technically efficient only when it‟s operating on the production frontier giving a super efficiency measure.

6 Results and analysis

6.1 DEA Results

The results from the previously described DEA model are presented in Table 2 below. As we can see that except DMU 7, 13, 15 and 17 the rest are efficient in both the CRS and VRS frontiers and all of them are efficient in VRS TE measures. It‟s visible that DMU 7, DMU 13, DMU 15 and DMU 17 are inefficient in the CRS assumption and all of them are on the increasing returns to scale (IRS) portion of the VRS frontier (see figure 6, section 3.3.2). Some scale efficiency (SE) exists between the CRS and VRS frontiers regarding the TEs. These results from the CRS and VRS frontiers can be interpreted to be robust to misspecification of the returns to scale assumption which is discussed further in the sensitivity analysis sub-section.

Table 2

DMUs	CRS TE	VRS TE	Scale Efficiency	
1	1.000	1.000	1.000	
2	1.000	1.000	1.000	
3	1.000	1.000	1.000	
4	1.000	1.000	1.000	
5	1.000	1.000	1.000	
6	1.000	1.000	1.000	
7	**0.990**	1.000	**0.990**	**IRS**
8	1.000	1.000	1.000	
9	1.000	1.000	1.000	
10	1.000	1.000	1.000	
11	1.000	1.000	1.000	

12	1.000	1.000	1.000	
13	**0.738**	1.000	**0.738**	**IRS**
14	1.000	1.000	1.000	
15	**0.963**	1.000	**0.963**	**IRS**
16	1.000	1.000	1.000	
17	**0.981**	1.000	**0.981**	**IRS**
18	1.000	1.000	1.000	
19	1.000	1.000	1.000	
20	1.000	1.000	1.000	
Mean	0.984	1.00	0.984	

The four inefficient DMUs are the day shift production unit in the months of January, April, May and June of 2009. Primarily, a distinct pattern of relative inefficiency can be observed amongst the day shift production units (see appendix 1, table 2). In addition, a higher number of waste and lower productivity levels are evident among all day shift production units in comparison to the night shifts in general. Secondly, except DMU 1 and DMU 5, no other odd numbered day shift DMU was considered as a peer to the four inefficient DMUs in terms of defining efficient production point for them. Whereas, the most counted DMUs were DMU 2 and DMU 14 as peers of the inefficient DMUs, both belonging to the night shifts (see appendix 1, table 7). And except DMU 8 and DMU 16, the rest of the eight night shifts DMU have contributed to determine efficient points on the production frontier for all the inefficient DMUs.

6.2 Inefficient units

DMU 7

It has a technical efficiency (TE) score of .990 in CRS frontier, being the least inefficient among the inefficient ones. It‟s located on the increasing returns to scale portion on the VRS frontier. Its efficient point of production has been defined by DMU 1, DMU 2, DMU 18, DMU 14, DMU 10, DMU 6 putting the following weights (λ„s) respectively 0.027, 0.059, 0.364, 0.140, 0.038, 0.372 (see appendix 1, table 7 & 8). As we see, that no day shift DMU has been chosen to define efficient points for DMU 7 except DMU 1.

It"s interesting to identify that the input slacks are basically concerning all the amount of wastes (to) in producing each 6 types of glassware (see appendix 1, table 5). And the next major source of inefficiency in input slack is rooted in the amount of labourers ( ), which is almost double the day shift i.e. 14.162. On the other hand, the number of machineries ( ) employed in the production processes either in assisting the labourers or automatic production turn out to be optimal in service.

DMU 13

It has a technical efficiency (TE) score of .738 in CRS frontier, being the poorest performer of all. Although it"s located on the increasing returns to scale portion on the VRS frontier. Its efficient point of production has been defined by DMU 12, DMU 2, DMU 4, DMU 14 putting the following weights (λ,,s) respectively 0.113, 0.314, 0.179, 0.393 (see appendix 1, table 7 & 8). Again, no day shift DMU has been chosen to define efficient points for DMU 13.

Now, observing the input slacks we find the amount of wastes (to) in producing each 6types of glassware is least among the 4 inefficient units (see appendix 1, table 5). However, the inputs slacks also tell us that this unit can carry out the production by half of its current amount of labourers ( ), equal to the night shift. The number of machineries ( ) employed in the production processes either in helping the labourers or automatic productions are as optimal in service as before.

DMU 15

With a technical efficiency (TE) score of .963 in CRS frontier, DMU 15 is the third most inefficient of all, located on the increasing returns to scale portion on the VRS frontier. Its efficient point of production has been defined by DMU 6, DMU 2, DMU 5, DMU 14, DMU 20 putting the following weights (λ,,s) respectively 0.041, 0.463, 0.131, 0.246, 0.118 (see appendix 1, table 7 & 8). Only one day shift DMU has been chosen to define efficient points for DMU 15 for the second time.

Now, observing the input slacks we find the amount of wastes (x_2 to x_7) are the highest among all four inefficient units in producing each 6types of glassware (see appendix 1, table

5). This time, the inputs slacks indicating that there is not a need for a drastic 50% cut in the amount of labourers ( ) to lead an efficient level of production; a 43% reduction would be enough. The number of machineries ( ) employed in the production processes either in helping the labourers or automatic productions are optimal in service as before.

DMU 17

With a technical efficiency (TE) score of 0.981 in CRS frontier, DMU 15 is the second least efficient of all, situated on the increasing returns to scale portion on the VRS frontier. Its efficient point of production has been defined by DMU 10, DMU 20, DMU 6, DMU 2, DMU 14 putting the following weights (λ,,s) respectively 0.062, 0.060, 0.000, 0.377, 0.501(see appendix 1, table 7 & 8). No day shift DMU has been chosen to define efficient points for DMU 17.

Now, observing the input slacks we find the amount of wastes (to) in producing each 6types of glassware is not as high as DMU 15 but around DMU 7 (see appendix 1, table 5). This time as well, the inputs slacks indicating a similar need for a drastic 50% cut in the amount of labourers ( ) to achieve an efficient level of production. The numbers of machineries ( ) employed in the production processes in helping the labourers or automatic production are optimal in service as before. Figure 7 below illustrates the general overview of the slacks relative to the inefficient DMUs:

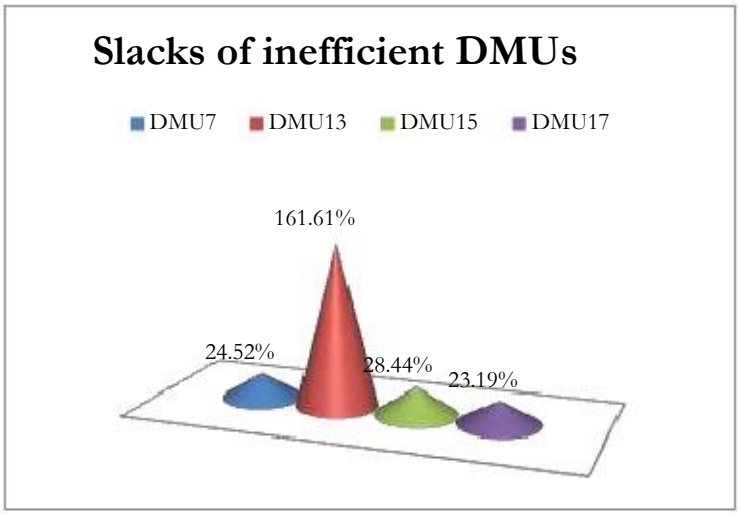

Figure 7

6.3 Average change in inputs and output slacks to reach technical efficiency

Table 3

DMU 7		DMU13		DMU15		DMU17	
x	y	x	y	x	Y	X	y
-48.64%	0.00%	-50.00%	25.81%	-43.43%	4.84%	-50.00%	0.00%
-32.48%	51.37%	-16.64%	101.27%	-43.61%	0.00%	-46.40%	4.08%
-50.69%	0.00%	-24.59%	124.15%	-45.25%	0.00%	-42.20%	0.00%
-32.47%	18.78%	-62.93%	28.42%	-42.66%	0.00%	-28.34%	0.00%
-43.83%	76.97%	-16.64%	677.07%	-42.66%	154.88%	-28.34%	129.94%
-55.21%	0.00%	-16.64%	12.96%	-69.42%	10.94%	-74.73%	5.14%
-32.48%	**+24.52%**	-16.64%	**+161.61%**	-50.49%	**+28.44%**	-35.49%	**+23.19%**
0.00%		0.00%		0.00%		0.00%	
-36.98%		**-25.51%**		**-42.19%**		**-38.19%**	

The average percentage reduction in all inputs resulting the average percentage increase in the outputs (slacks) among the inefficient DMUs 7, 13, 15, 17 converted them into technically efficient DMUs, it is discussed below from table 3 above derived from appendix 1, table 4 & 6.

A 36.98% average reduction in the inputs can lead to a 24.52% increase in the level of output of production for DMU 7 as calculated from the DEA model. Thus, wastes in production process and overcrowding of labor force may be identified as a bigger reason of technical or scale inefficiency and perhaps a cause of lower level of production output.

As for DMU 13 a relatively lower 25.51% average reduction in the inputs can lead to a staggering 161.61% average increase in the level of output of. In this case, the main source of inefficiency can said to be in the decreasing marginal output per labourer and eventual wastes in production leading to a large-scale decrease in the level of production

A rather higher 42.19% average reduction in the inputs can lead to a 28.44% average increase in the level of output of production for DMU 15. Now, the main source of inefficiency can be conveniently identified, primarily for the wastes in production then in the decreasing marginal output per laborer, which are causing a high amount of under production

Finally, a significant 38.19% average reduction in the inputs can lead to a 23.19% average increase in the level of output of production for DMU 17. The sources of inefficiency can said to be primarily caused by the amount of wastes in production accompanied by decreasing marginal output per laborer, causing a poorly low level of under production.

6.4 Efficient Units

Out of the twenty DMUs except the above mentioned four, the rest sixteen DMUs are efficient in both CRS and VRS frontier. Dominance in terms of superior productivity and efficiency in labour management is reflected among the night shift (even numbered) DMUs throughout the sample 10months of our observation from October 2009 to July 2010 (see appendix 1, table 1 & 2). There are instances when the day shift output level for a particular type of glassware was 50% to even 80% lowers than the night shift in the same month (see appendix 1, table 3 & 6). However, 40% of the day shift production units have found to be inefficient and the rest 60% are efficient despite the noticeable discrepancy in comparison to the night shift production units of the same month. Night shift DMU 2 and DMU 4 have the most number of peer count i.e. 4 in determining the efficient points for every inefficient DMUs (see appendix 1, table 7 & 8). A super efficient measure was chosen in the method (multi-stage) of DEA to determine efficient frontier according to the Koopman‟s (1951) rather strict definition of efficiency. In either case, the night shift DMUs did not come out to be inefficient.

6.5 Sensitivity analysis

As mentioned earlier, choosing a VRS model served a critical purpose of testing the robustness of the results in the model to the change in the scale to returns assumptions. This was for a practical reason to check for any misspecification, as the VRS model also calculated the CRS frontier. The results already show that all the twenty DMUs found efficient in the VRS frontier. To deepen this discussion, a comparison made (see appendix 1, table 9) between not only the VRS and CRS frontiers in a VRS model but among three additionally calculated DEA models. Which are *a CRS model that was calculated in 2-stage, a CRS model calculated in multi-stage,* and also another *VRS model containing both VRS and CRS frontiers that was calculated in 2-stage.*

It is interesting to see that the results are identical in all four models for all twenty DMUs. The exact same night shift DMUs i.e. 7, 13, 15, 17 are identified as inefficient in the CRS frontiers in all fours DEA models with identical technical efficiency scores. And they all show the trend of increasing returns to scale in both the VRS models. On the other hand, each and every DMU found to be efficient in the VRS frontiers despite the choice of slack calculation method.

It can therefore be stated that in a perfect world where every environmental and financial factor is at optimal level for the production facility of the given glass factory assuming a CRS frontier; we will have sixteen DMUs operating at optimal level of production and four falling behind at a sub-optimal level of production. On the contrary, we can consider all the twenty DMUs producing optimally in the reality despite numerous ups and downs in the day to day life of the environment they are located in. To conclude, the results of the DEA model is tested for robustness to the changes in the returns to scale assumptions and therefore found free of any misspecification.

7 Conclusions

A non-parametric frontier method was undertaken according to the available data structure of ASA glass factory in Tehran, Iran to determine its technical efficiency (TE) and scale efficiency (SE) scores. We exploited an input-oriented, VRS model with multi-stage slack calculation to ensure the determination of super efficiency measures. Not to our surprise, all the four inefficient DMUs turned out to be from the day shift production units and all of them displaying increasing returns to scale, whereas, the rest of the sixteen DMUs turned out efficient in both the CRS and VRS frontiers. In addition, all ten even numbered night shift DMUs demonstrate a general nature of efficiency.

To focus on the limitations of this non-parametric model of production frontier; had we been provided with the production cost data (e.g. electricity, gas, water price) for both day and night shift production units or particular data on labour wage for the day and the night shifts respectively. We could have discovered some behavioural aspect of the efficiency score fluctuation amongst the day and the night shift units. Or even solve the mystery of almost twice the level of productivity amongst the night shift production units with remarkably low level of wastage, employing half of the day shift amount of labourer and utilizing identical amount of all the other inputs.

The calculated DEA results deliberately indicated output slacks for outputs , , , , , with respective modes of 2 3 1 2 4 3. Meaning for instance, there was two occasions in two inefficient DMUs where the production level of lacked from the efficient level (see appendix 1, table 4). An investigation could be carried out in ASA glass factory regarding the combination of laborer and machineries engaged especially in the production unit of , , and as they lacked behind the efficient level 3, 4 and 3 times respectively. It could be of great interest not only for the employer but also for the labourers to identify the sources of inefficiency. A better managerial approach could be taken along with improved training for the labourers directly involved in producing those particular products.

In addition, a model could be developed in the shadow of the night shifts to cross compare and improve the day shift performance. A vital factor here to consider is either if the labourers are specialized in a particular step of production process or for a particular type of glassware. Also if they are asked to switch places and work anywhere else in the factory in case of emergency. If not so, a lack of specialization can be implied cancelling the inference of increasing returns to scale.

Aggregated production data was an initial downfall of this study, although 10 months production data was provided by ASA glass factory for this analysis. But it was only two observations per month for the night and the day shifts for those 10 months. Implementing a non-parametric method allows to dig deeper even with this kind of aggregated monthly data in the „Malmquist Panel Data Model‟ if provided with few years data. Availability of daily data or weekly data could have been crucial to interpret numerous other causes of inefficiency. For instances, if there was a working day during the holidays and combined with the wage rate, a simple negative or positive linear relation may have been discovered in relation to the efficiency score fluctuation. An alternative hypothesis can be proposed in describing the lower efficiency score of the day shift production units. The likelihood of on duty socialization is greater among the day shifts labourers than the night shifts in considering the total amount of labourers who are employed, as well as all the other labourers in the premises involved with transportation, administration etc. However, a further research is strongly recommended with data on factors of production cost, especially for labour wage rate. A panel data approach is believed to be invaluable for deeper analysis. In addition, use of qualitative data on the labourer‟s demography, working condition, and education can be of versatile use.

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Appendix1. Tables

1. Analysis of all DMU's efficiency scores by a VRS multi-stage DEA model

Table 1

Units	Status	Efficiency
DMU1		1
DMU2		1
DMU3		1
DMU4		1
DMU5		1
DMU6		1
DMU 7		**0.99**
DMU8	**i.r.s**	1
DMU9		1
DMU10		1
DMU11		1
DMU12		1
DMU13		**0.738**
DMU14		1
DMU15	**i.r.s**	**0.963**
DMU16		1
DMU17	**i.r.s**	**0.981**
DMU18		1
DMU19	**i.r.s**	1
DMU20		1
Min.Value		0.738
Max.Value		1
Mean		0.9836
Std.Dev.		0.058532

Table 2

Units	y1	y2	y3	y4	y5	y6	x1	x2	x3	x4	x5	x6	x7	x8
DMU1	8732	7666	7016	4390	2342	7231	28	103	147	51	44	113	190	10
DMU2	8863	9560	2708	5934	8272	13753	14	103	118	19	33	32	101	10
DMU3	7789	6585	4485	6709	4096	5024	28	214	214	145	85	115	72	10
DMU4	10435	11952	4528	2063	3735	2779	14	160	118	143	26	104	39	10
DMU5	8493	7397	7334	6018	1827	3143	28	196	223	249	98	108	72	10
DMU6	9575	11102	6798	5601	3770	5204	14	174	149	229	91	53	45	10
DMU7	**9883**	**6202**	**5732**	**4058**	**2371**	**6939**	**28**	**260**	**286**	**175**	**123**	**109**	**113**	**10**
DMU8	7946	10841	4079	4899	3378	5321	14	208	221	123	103	77	94	10
DMU9	10473	8027	6174	6071	3195	3736	28	273	186	175	156	142	63	10
DMU10	12027	9761	4998	6291	4070	2276	14	239	154	140	152	112	37	10
DMU11	6322	6933	6534	4777	2585	347	28	204	235	198	126	87	31	10
DMU12	9245	9377	3659	4418	2210	2049	14	140	174	110	116	73	6	10
DMU13	**7162**	**4833**	**2318**	**3405**	**576**	**6199**	**28**	**155**	**144**	**174**	**55**	**50**	**77**	**10**
DMU14	8411	8947	7928	4166	2434	4961	14	133	78	52	45	12	63	10
DMU15	**8354**	**9191**	**5143**	**5527**	**2161**	**8034**	**28**	**245**	**243**	**127**	**97**	**131**	**168**	**10**
DMU16	10245	9787	3396	3894	5199	9901	14	201	230	83	67	55	137	10
DMU17	**8847**	**8904**	**5657**	**5086**	**2146**	**7760**	**28**	**248**	**182**	**65**	**71**	**111**	**119**	**10**
DMU18	10790	7869	4266	3975	4805	8839	14	204	159	49	55	50	105	10
DMU19	9511	7453	6724	5591	2341	5859	28	221	256	89	117	122	81	10
DMU20	9085	9585	5886	6191	5762	5859	14	210	201	77	107	50	81	10
Min.Value	6322	4833	2318	2063	576	347	14	103	78	19	26	12	6	10
Max.Value	12027	11952	7928	6709	8272	13753	28	273	286	249	156	142	190	10
Mean	9109.4	8598.6	5268.15	4953.2	3363.75	5760.7	21	194.6	186	123.7	88.35	85.3	847	10
Std.Dev.	1329.97	1771.26	1573.61	1165.53	1710.57	3079.45	7.18	49.33	53.1	63.6	37.95	36.44	45.8	0

2. Analysis of all DMU's efficiency scores (Inefficient DMUs turning efficient by fulfilling the given targets) by a VRS multi-stage DEA model

Table 3

Units	y1	y2	y3	y4	y5	y6	x1	x2	x3	x4	x5	x6	x7	x8
DMU1	8732	7666	7016	4390	2342	7231	28	103	147	51	44	113	190	10
DMU2	8863	9560	2708	5934	8272	13753	14	103	118	19	33	32	101	10
DMU3	7789	6585	4485	6709	4096	5024	28	214	214	145	85	115	72	10
DMU4	10435	11952	4528	2063	3735	2779	14	160	118	143	26	104	39	10
DMU5	8493	7397	7334	6018	1827	3143	28	196	223	249	98	108	72	10
DMU6	9575	11102	6798	5601	3770	5204	14	174	149	229	91	53	45	10
DMU7	**9883**	**9388**	**5732**	**4820**	**4196**	**6939**	**14**	**176**	**141**	**118**	**69**	**49**	**76**	**10**
DMU8	7946	10841	4079	4899	3378	5321	14	208	221	123	103	77	94	10
DMU9	10473	8027	6174	6071	3195	3736	28	273	186	175	156	142	63	10
DMU10	12027	9761	4998	6291	4070	2276	14	239	154	140	152	112	37	10
DMU11	6322	6933	6534	4777	2585	347	28	204	235	198	126	87	31	10
DMU12	9245	9377	3659	4418	2210	2049	14	140	174	110	116	73	6	10
DMU13	**9010**	**9727**	**5196**	**4373**	**4476**	**7002.3**	**14**	**129**	**109**	**64.5**	**46**	**42**	**64**	**10**
DMU14	8411	8947	7928	4166	2434	4961	14	133	78	52	45	12	63	10
DMU15	**8759**	**9191**	**5143**	**5527**	**5507.9**	**8912.9**	**16**	**138**	**133**	**72.8**	**56**	**40**	**83**	**10**
DMU16	10245	9787	3396	3894	5199	9901	14	201	230	83	67	55	137	10
DMU17	**8847**	**9267**	**5657**	**5086**	**4934.5**	**8158.6**	**14**	**133**	**105**	**46.6**	**51**	**28**	**77**	**10**
DMU18	10790	7869	4266	3975	4805	8839	14	204	159	49	55	50	105	10
DMU19	9511	7453	6724	5591	2341	5859	28	221	256	89	117	122	81	10
DMU20	9085	9585	5886	6191	5762	5859	14	210	201	77	107	50	81	10
Min.Value	6322	6585	2708	2063	1827	347	14	103	78	19	26	12	6	10
Max.Value	12027	11952	7928	6709	8272	13753	28	273	256	249	156	142	190	10
Mean	9222	9021	5412	5040	3956.8	5864.7	18	178	168	112	82	73	76	10
Std.Dev	1238	1421	1413	1096	1553.8	3144.2	6.5	46.5	50.8	63.8	39	37	40	0

Table 4

Parameters	Relative Deviation Required for Inefficient DMUs To Achieve Targets of Efficiency			
	DMU7	DMU13	DMU15	DMU17
x1	-48.64%	-50.00%	-43.43%	-50.00%
x2	-32.48%	-16.64%	-43.61%	-46.40%
x3	-50.69%	-24.59%	-45.25%	-42.20%
x4	-32.47%	-62.93%	-42.66%	-28.34%
x5	-43.83%	-16.64%	-42.66%	-28.34%
x6	-55.21%	-16.64%	-69.42%	-74.73%
x7	-32.48%	-16.64%	-50.49%	-35.49%
x8	0.00%	0.00%	0.00%	0.00%
y1	0.00%	+25.81%	+4.84%	0.00%
y2	+51.37%	+101.27%	0.00%	+4.08%
y3	0.00%	+124.15%	0.00%	0.00%
y4	+18.78%	+28.42%	0.00%	0.00%
y5	+76.97%	+677.07%	+154.88%	+129.94%
y6	0.00%	+12.96%	+10.94%	+5.14%

3. Slacks

Table 5

Input Slacks:	x1	x2	x3	x4	x5	x6	x7	x8
DMU 7	13.618	84.438	144.965	56.833	53.911	60.183	36.698	0.000
DMU13	14.000	25.790	35.407	109.487	9.151	8.319	12.812	0.000
DMU15	12.159	106.843	109.953	54.181	41.382	90.937	84.816	0.000
DMU17	14.000	115.063	76.801	18.423	20.124	82.953	42.229	0.000
mean	2.689	16.607	18.356	11.946	6.228	12.120	8.828	0.000

Table 6

Output Slacks:	y1	y2	y3	y4	y5	y6
DMU 7	0.000	3186.320	0.000	762.742	1825.240	0.000
DMU 13	1848.317	4894.184	2877.758	967.659	3899.948	803.273
DMU 15	404.611	0.000	0.000	0.000	3346.868	878.859
DMU 17	0.000	363.138	0.000	0.000	2788.507	398.613
Mean	112.646	422.182	143.888	86.520	593.028	104.037
Mode	2	3	1	2	4	3

4. Peers of Inefficient DMUs

Table 7

DMUs	Peers	Quantity
7	1 2 18 14 10 6	6
13	12 2 4 14	4
15	6 2 5 14 20	5
17	10 20 6 2 14	5

Table 8

DMUs	Peer weights
7	0.027 0.059 0.364 0.140 0.038 0.372
13	0.113 0.314 0.179 0.393
15	0.041 0.463 0.131 0.246 0.118
17	0.062 0.060 0.000 0.377 0.501

5. Sensitivity analysis of the returns to scale assumption from three additional DEA models

Table 9

a. CRS 2-stage

DMUs	CRS TE
1	1
2	1
3	1
4	1
5	1
6	1
7	**0.99**
8	1
9	1
10	1
11	1
12	1
13	**0.738**
14	1
15	**0.963**
16	1
17	**0.981**
18	1
19	1
20	1
Mean	0.984

b. CRS Multi-stage

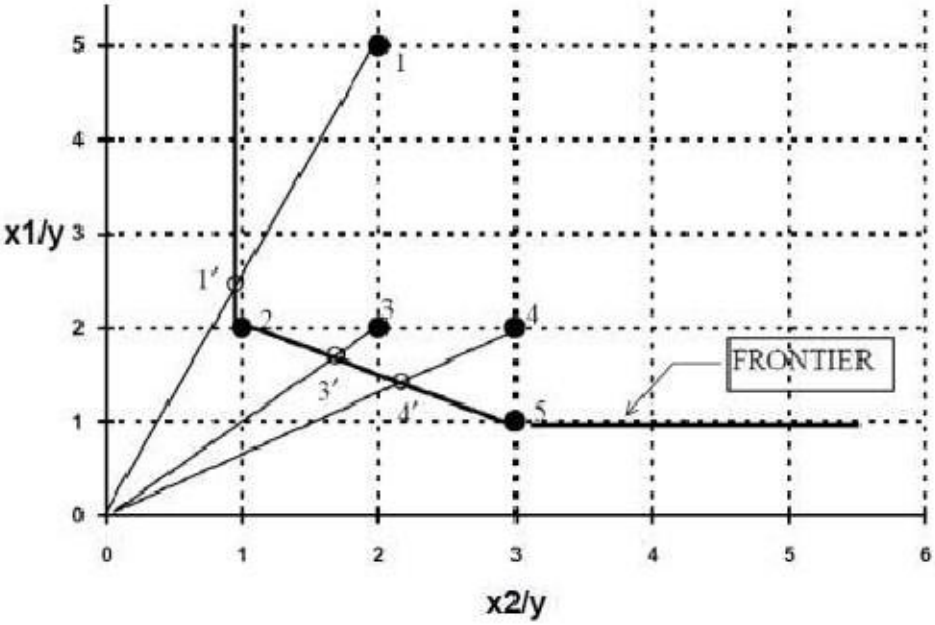
DMUs	CRS TE
1	1
2	1
3	1
4	1
5	1
6	1
7	**0.99**
8	1
9	1
10	1
11	1
12	1
13	**0.738**
14	1
15	**0.963**
16	1
17	**0.981**
18	1
19	1
20	1
Mean	0.984

c. VRS 2-stage

DMUs	CRS TE	VRS TE	SE	Status
1	1	1	1	
2	1	1	1	
3	1	1	1	
4	1	1	1	
5	1	1	1	
6	1	1	1	
7	**0.99**	1	**0.99**	
8	1	1	1	
9	1	1	1	**IRS**
10	1	1	1	
11	1	1	1	
12	1	1	1	
13	**0.738**	1	**0.738**	
14	1	1	1	
15	**0.963**	1	**0.963**	
16	1	1	1	**IRS**
17	**0.981**	1	**0.981**	
18	1	1	1	**IRS**
19	1	1	1	
20	1	1	1	
Mean	0.984	1	0.984	**IRS**

Appendix2. Graphs

Graph1. Slacks occurring in the horizontal part of the isoquant (Coeli, 1996).



Graph2. Frontier performance of all DMUs.