Efficiency and Price in Simultaneous vs. Overlapping Competing Auctions

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Abstract

This essay investigates differences between simultaneous and overlapping competing online auctions with respect to efficiency and price. Recent theoretical findings suggest that overlapping auctions are more efficient and result in a higher average price than simultaneous auctions. These predictions are tested using a data set consisting of 23 935 train ticket auctions submitted by Swedish train operator Statens Järnvägar to the eBay owned auction site Tradera. The results show that for price the data exhibit all expected characteristics and the prediction can thus be confirmed. For efficiency however, the prediction cannot be confirmed, on the contrary simultaneous auctions are found to be significantly more efficient than overlapping auctions.

Keywords: Auction theory, competing online auctions, overlapping auctions, simultaneous auctions, cross-bidding

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1 Introduction

The auction is an ingenious institution for economic transactions that has been around since ancient times[1]. Having the property of information revealer, it is commonly implemented in such dispersed areas as the markets for housing, fresh fish, flowers, financial instruments and for allocating frequency spectra to name only a few. With the advent and growth of the Internet during the last couple of decades, auctions has found yet another frontier – one that is rapidly expanding. More and more individuals and companies use auction sites such as eBay and it's Swedish subsidiary Tradera. Among the two million members at Tradera (Tradera 2011*b*) we find the Swedish train operator Statens Järnvägar (SJ) which has been offering surplus tickets on Tradera since 2008 (SJ 2010). It is precisely that activity that is investigated in this essay. In focus is a change in auction design, a rare event that provides a unique opportunity to compare the effects of two different design features – the conditions for comparison being particularly favorable since we will be dealing with a homogeneous good that has only one seller.

The change in design took place on November 15, 2010. Up until then SJ was offering their surplus train tickets through simultaneous auctions at Tradera, a setup in which auctions with identical tickets, i.e. tickets valid for the same route and departure time, all have the *same ending time*. On the day of the change a press release was published, informing that from that day and onwards train tickets were to be sold in overlapping auctions, a setup which features auctions with identical tickets that *ends a few minutes apart* (SJ 2010). Research on these two types of auctions, jointly referred to as competing auctions, is on the frontier of auction research and the existing theoretical work is rather sparse, but there have been important contributions. Peters & Severinov (2006) pioneered the subject of simultaneous auctions and predicted that it is possible to reach an efficient Bayesian Nash equilibrium if all participators adopt a cross-bidding strategy, i.e. switch between the different auctions of identical objects in order to always bid on the one with the lowest current standing price. Their theoretical model has been tested empirically by Anwar, McMillan & Zheng (2006) and Andersson, Andersson & Andersson (2009), both finding some support for the predictions made. They also find considerable inefficiencies however, and there are a few scholars that have investigated further through both construction of theoretical models (e.g. Stryszowska 2006, Huang, Chen, Chen & Chou 2007) and experimental designs (Hoppe 2008). Their shared theoretical prediction is that it is more efficient to use a design with overlapping rather than simultaneous auctions when selling two or more identical items. As for price, the prediction is that in overlapping auctions prices will on average be higher than in simultaneous auctions, the main reason for this being a decrease in the cross-bidding behavior inducing a higher bidding activity and more bids per auction.

In this essay the two design features are compared with respect to efficiency and price

[1] A famous example is the auction held in A.D. 193, in which the entire Roman Empire was sold (Krishna 2010).

using data on SJ train ticket auctions from approximately two months prior to and one month after the change in auction design. The results show that the theoretical prediction that overlapping auctions are more efficient than simultaneous auctions cannot be confirmed. This is most likely due to the overlapping auctions being a new design which bidders might not yet be familiar with, and possibly also due to sorting issues that arose as an effect of the change.

For price on the other hand, the theoretical prediction that prices are higher in overlapping auctions is confirmed and this is mainly attributed to the effect of the overlap of auctions, the increase in bidding activity and the decrease in the share of cross-bids.

1.1 Research Questions

- Are overlapping auctions more efficient than simultaneous auctions when selling multiple identical objects?
- What happens to price in overlapping competing auctions as compared to simultaneous auctions?

1.2 Outline

The remainder of the essay is laid out as follows. Section 2 holds the theoretical framework including a discussion on efficiency, an introduction to auction design and strategic behavior, and detailed discussions on simultaneous and overlapping auctions. In section 3 the institutional features of the Tradera auction site are accounted for. Section 4 describes the data and the methodology used. The results are presented in section 5 and analyzed in section 6. Section 7 concludes the essay.

2 Theoretical Framework

This section provides the necessary theoretical base for the essay. It starts off with a discussion on efficiency which is followed by a short introduction to auction design and strategic behavior. We then discuss online competing auctions in general and simultaneous and overlapping auctions in particular, including some important definitions. Switching focus, the next issue is price and how that is affected by a change in auction design. We wrap up the section by establishing four hypotheses to be tested.

2.1 What is Efficiency?

The general economic understanding of the concept of efficiency is that an outcome is efficient if it cannot be improved upon other than by increasing the input factors. When resources are given and when comparing different systems that affects the utility of individuals, it is common to use the notion *Pareto efficient* as a benchmark for efficiency. It refers to the state where no one can be made better off without at the same time making someone else worse off. Any other state is inefficient and any move towards efficiency is referred to as a *Pareto improvement*. In the context of competing auctions a group of competing auctions is efficient if there are no Pareto improvements to be made regarding the outcome, i.e. all participators are content with the outcome and would not have it any other way should they be given the opportunity to negotiate between themselves after the closing of the auctions. Clearly, this stipulates that the participators winning the auctions must be the ones with the highest willingness to pay as Pareto improvements would otherwise be possible through post-auction trade (see example 1) (Cowell 2006, p. 232f).

Example 1 *Suppose there are six auctions with identical train tickets and there are ten people bidding, each interested in one ticket each. If the six people with the highest (revealed) willingness to pay end up winning one auction each the outcome will be efficient. We consider revealed willingness to pay as a proxy for valuation, which means that no one of the losing bidders will want to buy a ticket from any of the winners since the price is higher than their valuation. Similarly, no one of the winning bidders will be willing to sell their ticket because they will not want to sell it for a price below their valuation. If however only five of the six bidders with highest willingness to pay end up winning, a Pareto improvement is possible if the participator with low willingness to pay who managed to win offer to sell the ticket to the participator with high willingness to pay who did not win.*

Note that since it is impossible to know the bidders' real willingness to pay for a given train ticket in the data set used, we will make use of their revealed willingness to pay which is simply the highest bid that a bidder submits in a group of auctions with identical goods (i.e. auctions with train tickets for the same route and departure time) (cf. Andersson et al. 2009). Pareto efficiency, as explained above, will be the measure used when evaluating efficiency among competing auctions. How the evaluation is done is explained in further detail in section 4.2.

2.1.1 Absolute and Relative Efficiency

Efficiency can be thought of in absolute and relative terms. The concept of Pareto efficiency as described above originates in absolute efficiency since an outcome is either *efficient*, if no Pareto improvement can be made, or *inefficient*, if Pareto improvements are possible. In other words, it is a binary measure capable of taking only two values and as such it is not very flexible. As Andersson et al. (2009) note, absolute efficiency does not distinguish from an outcome where 9 out of 10 auctions are won by the highest bidders and an outcome where this happens in only 1 out of 10 auctions – both outcomes are equally inefficient even though one may argue that the former in reason is more efficient. To deal with this problem the authors introduce the concept of relative efficiency which is simply defined as the fraction of auctions being efficient. For the example used here this would result in relative efficiency measures of 0.9 and 0.1, which clearly gives a hint of the degree of efficiency. Considering this more revealing property, the measure of relative efficiency will be used in conjunction with absolute efficiency.

2.2 Auction Design and Strategic Behavior

Auctions are commonly constructed according to one of the following standard designs; ascending-bid (English), descending-bid (Dutch), first-price sealed bid or second-price sealed bid – all of which function in a slightly different manner regarding basic rules and what strategy participators ought to adopt. In the descending-bid design the winner is determined by starting at a high price which is continuously lowered until some participant accepts, making the final price of the good equal to the highest, and only, bid. This outcome, where the highest bidder wins and pays a price equal to his or her bid, is the same for the first-price sealed bid auction where participators submit only one bid each which – as the name suggests – is unknown to the other bidders. For these two types of auctions the optimal bid for a given bidder depends not only on the valuation of that bidder but also on his or her beliefs about the valuations of the other bidders involved. Although these designs are used at some online auction sites (at least the descending-bid design[2]) we will not be concerned with them here[3]. Instead, we will focus on the two remaining types; ascending-bid and second price sealed bid auctions (see e.g. Krishna 2010).

In an ascending-bid auction, which is perhaps the most familiar design, the price starts at zero (or at a reservation price set by the seller) and participants then take turns to submit increasing bids until there is only one bidder left in the auction who then becomes the winner.

[2] Bapna, Chang, Goes & Gupta (2009) investigates an online auction house called MegaClub that offers descending-bid auctions. Another example available at the time this essay was written is www.pricefalls.com.

[3] For a more detailed discussion on auction design, see e.g. Klemperer (1999) or Krishna (2010).

Bids are often placed according to standardized minimum increments and the final price of the good will therefore be equal to the second highest bid, where the last losing bidder dropped out, plus this increment. The second-price sealed bid design, also known as a Vickrey auction[4], works pretty much the same way, except that only one bid per participant is submitted in a sealed envelope or in an equivalently secret manner. The striking feature of these two types of auctions is that it is always optimal to bid up to, or place a sealed bid that is equal to, ones true valuation of the good being auctioned. This is because of the second-price feature which guarantees that one will either win the auction and pay a price that is only as high as needed to outbid all the other bidders or that one will not win the auction because someone else has a higher valuation and is therefore willing to pay more. Given that all participants bid according to their true valuations, this means that both of these auction designs guarantees an efficient outcome, i.e. that the bidder with the highest valuation wins (ibid).

In the next section we will focus on online auctions and how we can use the theory on ascending-bid and Vickrey auctions when describing auction design and bidding behavior on sites such as eBay and Tradera.

2.3 Online Auctions

If we assume that the format of the online auctions that we will be dealing with is equivalent to the ascending-bid or Vickrey auction, it would be logical to assume that the optimal bid should be equal to ones valuation of the good being auctioned. However, as Huang et al. (2007) note, online auctions are in many ways different from traditional 'brick-and-mortar' auctions and therefore require a different or at least a modified theoretical approach. One important difference is that traditional auctions usually offer a limited amount of goods for sale while online auction sites such as eBay mediates huge volumes of goods every day and where new auctions start practically every second (eBay 2011). Another difference that has important implications is that online auctions often have a fixed ending time whereas traditional auctions keep going until no more bids are submitted[5]. A third difference, which is crucial to this essay, is that online auction houses often sell identical goods in auctions that progress concurrently. All of these divergences between online and offline auctions affects the behavior of bidders, for example some scholars believe that the phenomenon of last-minute

[4]This design was originally proposed by Nobel laureate William Vickrey in 1961 and although it was first thought to be useful only in theoretical applications it has been acknowledged as particularly useful in online auctions by several scholars, including Anwar et al. (2006), Andersson et al. (2009), Kayhan, McCart & Bhattacherjee (2009), Stryszowska (2006) and Hoppe (2008). This view has however been contested by e.g. Zeithammer & Adams (2010) who find that is more correct to model online auctions using the ascending-bid design. According to the revenue equivalence theorem the two designs yield the same expected revenue and results in the same expected payments for bidders. The prediction that participators have a dominant strategy in bidding their true valuation rests on the assumptions that the goods being auctioned are identical, participators have unit demand and zero search costs, and finally that there is no collusion (Vickrey 1961). See also Milgrom (1989).

[5]As noted by Bajari & Hortaçsu (2004) there exist online auction houses that implement 'soft-ending times', e.g. Amazon where the ending time is automatically extended if a new bid is submitted when the auction is about to close.

bidding, also known as sniping, is in some way related to the fixed ending times of online auctions (Bajari & Hortaçsu 2004). We will not explicitly deal with the issue of sniping here other than acknowledging that it is a phenomenon that has been observed empirically and for which there exist several possible theoretical explanations.

Section 3 will cover how the auctions we deal with here work in practice, however in order to continue we need to cover some important features and assumptions. First, all train tickets offered by SJ are sold individually in separate auctions at the starting price of 1 SEK. This means that we do not need to consider the issue of reservation prices set by the seller. Second, bidders are assumed to have private values, i.e. the valuation of one bidder does not depend on the valuation of other bidders. Third, since all tickets are sold by SJ the feedback or reputation mechanism offered by Tradera is assumed to be irrelevant (cf. Krishna 2010, Anwar et al. 2006, Bajari & Hortaçsu 2004).

Before continuing to discuss concurrent auctions, we need to consider a slight semantic confusion.

2.3.1 Concepts and Definitions

With the increase in interest for Internet auctions, and the surge in research activity that have followed, new theoretical considerations have been introduced and with it has come new terminology which is somewhat non-standardized in the literature. Most important here is the ambiguity over what constitutes simultaneous, overlapping and sequential auctions. All of these terms involve some level of interdepence between auctions which we may commonly refer to as competing auctions[6] and which should be contrasted to independent auctions which does not in any way depend on the properties of another auction. We will use the following definitions (see figure 1[7] for a schematic representation).

Definition 1 *A set of simultaneous auctions consist of two or more auctions of a homogeneous good where every auction has exactly the same starting and ending time*[8].

Definition 2 *A set of overlapping auctions consist of two or more auctions of a homogeneous good where each individual auction to some extent, ε , overlaps with the previous and/or following auction(s)*[9].

[6]This umbrella term is considered to be coined by Anwar et al. (2006) (Bapna et al. 2009).

[7]Adapted from figures in Huang et al. (2007), Hoppe (2008), and Bapna et al. (2009) as well as the theoretical descriptions in Peters & Severinov (2006) and Oren & Rothkopf (1975).

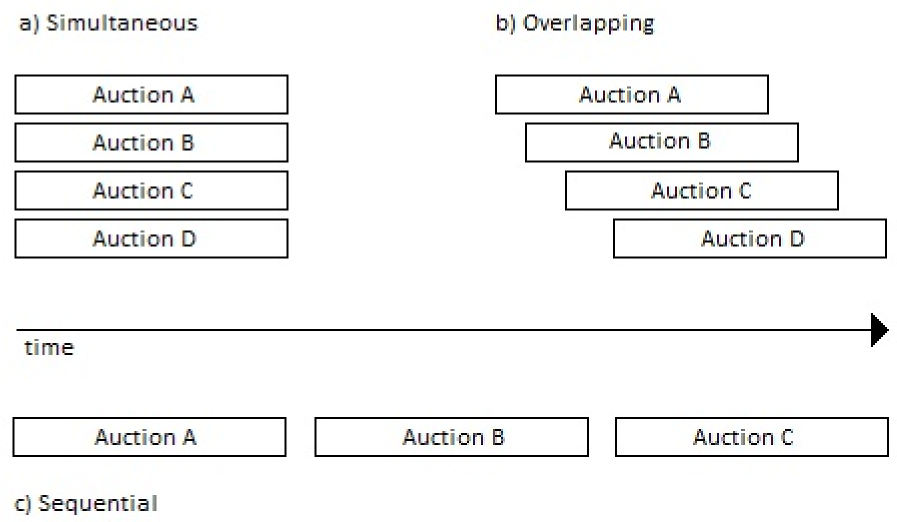
[8]The term simultaneous, defined by Oxford dictionary as 'occurring, operating, or done at the same time', is adapted from Peters & Severinov (2006) and Stryszowska (2006) who use the term *simultaneous auctions* in the way defined here. Anwar et al. (2006) misuse this term since their data contains auctions that are not truly simultaneous but rather overlapping.

[9]The term overlapping, defined by Oxford dictionary as 'extend over so as to cover partly', is adapted from Stryszowska (2006) and Bapna et al. (2009). Huang et al. (2007) use the term parallel to refer to this type of auction design.

Definition 3 *A set of sequential auctions consist of two or more auctions of a homogeneous good where there are some time interval, λ , between each individual auction*[10].

When other research is referenced, above definitions will be used and the concept given in the source will be pointed out in parenthesis or in a footnote.

Figure 1: Auction designs



2.3.2 Simultaneous Auctions

It is useful to begin this section by discussing Peters & Severinov (2006) which is the seminal work within the area of competing online auctions. Using an ascending-bid design they construct a model with multiple simultaneous auctions with many sellers and buyers. Although this normally specifies bidding one's true valuation (as discussed in section 2.2), this is no longer a dominant strategy when faced with multiple auctions of identical objects. Because there are several concurrent auctions ending at the same time, placing a bid equal to one's valuation in only one of the auctions involves a risk of paying too much or of losing the good to someone with a lower valuation. To see this, consider the following example (see table 1 for the example data).

Example 2 *For train number 401 on the route Stockholm – Göteborg departuring at 06.00 on September 15, 2010 two tickets were made available on Tradera.com. Three people showed interest in buying the tickets and placed a total of nine bids. The first bid was placed by*

[10]The term sequential is derived from sequence, defined in Oxford dictionary as 'a set of related [. . .] items that follow each other in a particular order'. It is adapted from Oren & Rothkopf (1975).

the user beartraveller on September 14 at 16.05 making the standing price in that auction 1 SEK[11]*. The second bid was placed by Eldrene in the auction without any bids, making the standing price in that auction 1 SEK. Finally, annika_t entered the bidding process, placed her first bid in the second auction and then engaged in a bidding war with beartraveller which she lost when dropping out at 50 SEK. As can be seen in table 1, beartraveller and annika_t have the highest (revealed) willingness to pay, yet it was beartraveller and Eldrene who won the two auctions.*

Table 1: Example of an inefficient GCA

Auction No.	**Bidder alias**	**Bid (SEK)**	**Submitted**
1	beartraveller	51	2010-09-14 16:05
	annika_t	50	2010-09-14 21:59
	annika_t	20	2010-09-14 21:59
	annika_t	10	2010-09-14 21:59
	annika_t	5	2010-09-14 21:59
	annika_t	2	2010-09-14 21:59
2	Eldrene	3	2010-09-14 21:59
	annika_t	2	2010-09-14 21:58
	Eldrene	1	2010-09-14 21:17

The outcome in example 2 is not efficient since only one of the two bidders with the highest revealed willingness to pay was awarded a train ticket. Peters & Severinov (2006) show that an efficient outcome could have been reached if all bidders had acted according to a *cross-bidding strategy*, in which case there exists an efficient Bayesian Nash equilibrium.

Definition 4 *A cross-bidding strategy involves placing a bid on the auction with the current lowest price until becoming the highest bidder or until reaching a price equal to the valuation of the good plus the minimum increment where the bidder exits the bidding process.*

If we return to example 2 it is clear that only four of the nine bids can be interpreted as cross-bids. The first and only bid by beartraveller was a cross-bid since it was placed in an auction with the lowest current standing price[12]. The same goes for the first bid placed by Eldrene. At that stage, the standing price in both auctions was 1 SEK, so the first bid placed by annika_t was also a cross-bid. That increased the standing price of the second auction to

[11] As will be explained in section 3.1, beartraveller makes use of the proxy-bidding agent which is why the bid history shows a higher bid.

[12] This bid does not conform to the cross-bidding strategy if one employs a strict interpretation since it is then prescribed (as in definition 4) that a cross-bid shall be placed in the auction with the lowest current standing price *and* be raised only with the minimum increment. We will however disregard this requirement since it is only really of importance when considering price-uniformity (see e.g. Peters & Severinov 2006, Andersson et al. 2009).

2 SEK, which means that the second bid placed by Eldrene was not a cross-bid – in order for it to be classified as a cross-bid it should instead have been placed in the first auction. annika_t's next bid was a cross-bid since it was placed in the first auction where the current standing price was 1 SEK at the time. The rest of the bids placed by annika_t were however not cross-bids.

The 'correct' behavior for Eldrene would have been to place bid number two in the first auction making the current standing price 2 SEK for both auctions. annika_t would then place her next bid in either auction, raising the price in that auction to 3 SEK. Eldrene would then place a bid in the other auction raising the price of that auction to 3 SEK as well. The bidders would then continue to take turns to place bids accordingly until one of them reaches their valuation and exits the bidding process.

An important note to be made here is that six of the bids were submitted in the last minute of the auctions, which both closed at 22.00 on September 14, 2010. Although not all bids were cross-bids in this auction, this still points out that one or more bidders may have wanted to place more bids but simply did not have time to do so. Clearly this is a weakness in simultaneous auctions with fixed ending time.

The theoretical model in Peters & Severinov (2006) has been tested empirically by Anwar et al. (2006) and Andersson et al. (2009). Both find support for the model but there are important differences in their methods and data sets that are useful to discuss. Anwar et al. use data from auctions on CPU's from different sellers and since the auctions are not perfectly simultaneous[13] they divide them into three groups with ending times within one day, one hour and one minute. While this makes it possible to asses the impact of the degree of simultaneity (or overlap which would be a more correct term) it does not fully reflect the idea of the auctions ending at the same time. Keeping this in mind, the results show that 32 percent of participants cross-bid in the minute sample, with lower figures for the hourly (31 percent) and daily sample (19 percent). It should be noted that the these figures apply to auctions where multi-unit bidders are included, the corresponding figures for the sample with strictly one-unit bidders are 20 percent for the minute and hourly samples and 14 percent for the daily sample. Andersson et al. (2009) use data on SJ train ticket auctions on Tradera.com (i.e. the same type of data used in this essay) and claim this to be superior to the data set in Anwar et al. (2006) with respect to the demands given in Peters & Severinov (2006) – the good is completely homogenous, the auctions end at exactly the same time and the reserve price (1 SEK) is likely to equal the sellers true marginal cost of having one more traveller on board. They focus on efficiency and price uniformity and find that 75 percent of auctions containing only cross-bids are efficient while the figure for the full sample is 33,6 percent. They also report that the mean fraction of cross-bids in all groups of competing auctions is 0.699.

[13]According to definition 2 and as noted in footnote 8 the auctions in their data are overlapping, not simultaneous.

2.3.3 Overlapping Auctions

Given the above, it seems that simultaneous auctions are fairly common, at least for specific auction types such as consumer electronics and train tickets, and that when faced with simultaneous auctions an efficient Bayesian Nash equilibrium can be reached if all participants adopt a cross-bidding strategy – a practice that to some extent is confirmed empirically. We have also noted that the simultaneous ending times constitutes a potential threat to achieving an efficient outcome.

Let us now ask what happens if auctions are not simultaneous but overlap so that for some time there exist concurrent auctions of identical goods but they will start and end in sequence as specified in definition 2[14]. Overlapping auctions are at the frontier of current research on competing online auctions and are therefore somewhat understudied, but there have been a few attempts to elucidate the workings of this design feature.

Stryszowska (2006) construct models for both simultaneous and overlapping auctions and in contrast to Peters & Severinov (2006) she introduces fixed ending times in the model for simultaneous auction which results in a theoretical prediction that for groups of simultaneous auctions there exist both efficient and inefficient equilibria. The inefficient equilibria can arise if bids are placed in the last minute before the auction closes when there is a positive probability that not all bids will be registered. For overlapping auctions, however, Stryszowska show that there exist only efficient equilibria since there is then enough time for bidders to reallocate between auctions.

Huang et al. (2007) propose a theoretical model for overlapping auctions[15] based on the ascending-bid design. Their prediction is that there exists an efficient Bayesian Nash equilibrium in which bidders always bid on the auction that ends first, and then start to bid on the auction first to end of those that remain. With respect to last minute bidding they argue that it is consistent with their model if there is uncertainty regarding future auctions selling the same object. In this respect they differ from the possible explanations given in Stryszowska (2006) and Bajari & Hortaçsu (2004). We will return to this issue in the analysis (section 6).

To the best knowledge of the author of this essay there have been no (or at least very few) empirical tests of the predictions of the theoretical models of overlapping auctions presented in Stryszowska (2006) and Huang et al. (2007), except for the brief empirical test included in the latter. However, an experimental study has been carried out by Hoppe (2008), which draws on the theoretical findings in Stryszowska (2006) and Huang et al. (2007). While one should be careful when interpreting experimental studies[16], the results are consistent with the theoretical predictions, i.e. efficiency is higher in overlapping auctions than in simultaneous auctions. Hoppe also finds that revenue is higher in overlapping than in simultaneous auctions

[14] It its true that Anwar et al. (2006) used data on auctions that can be interpreted as overlapping, it was however not their stated goal to investigate such data and their theoretical foundation assumed simultaneous auctions (with non-fixed ending time).

[15] In their paper Huang et al. refer to overlapping auctions as parallel auctions.

[16] The subjects participating in the experiments may e.g. not be experienced bidders or there may not exist proper economic incentives to mimic real-world behavior (Bajari & Hortaçsu 2004).

which is not predicted by Stryszowska or Huang et al. This leads us to the next section where we will discuss price in the context of simultaneous and overlapping auctions.

Table 2: Summary of research on competing auctions

Article	**Type**	**Design**	**Prediction/Results**
Peters & Severinov (2006)	T	S	Simultaneous auctions are efficient if non-fixed ending time and bidders adopt cross-bidding strategy.
Stryszowska (2006)	T	S/O	Simultaneous auctions are efficient if non-fixed ending time, may be inefficient if fixed ending time. Overlapping auctions are always efficient.
Huang et al. (2007)	T/E	O	Overlapping auctions are efficient if bids are placed on the auction first to end. No cross-bidding exists.
Anwar et al. (2006)	E	S	Find support for theoretical predictions in Peters & Severinov.
Hoppe (2008)	Exp	O	Find support for theoretical predictions in Stryszowska.
Bapna et al. (2009)	T/E	O	Hypothesize and find support for the hypothesis that overlap and institutional bidding (cf. cross-bidding) affects price negatively while participative bidding affects price positively.
Andersson et al. (2009)	E	S	Find support for theoretical predictions in Peters & Severinov.

Abbreviations: T - Theoretical, E - Empirical, Exp - Experimental, S - Simultaneous, O - Overlapping

2.4 On Price in Competing Auctions

We are only sparsely equipped with predictions for price that are based on theoretical models. We know from the press release regarding the change in auction design for train tickets that SJ plan to submit twice as many tickets in the post-change period which according to standard economic theory should mean a negative price effect due to increased supply (SJ 2010). At the same time findings in Bapna et al. (2009) suggests that a higher share of *institutional bidders* who behave similar to the cross-bidding strategy will put a downward pressure on prices while the share of *participatory bidders* who are more active in the bidding process put an upward pressure on prices. The same authors also hypothesize that overlap affects price negatively by providing information that creates an option value in the possibility of choosing other auctions than the focal auction, i.e. the current auction of interest. This is also something that Huang et al. (2007) discuss in addition to their prediction that cross-bids should be significantly lower in overlapping auctions than in simultaneous auctions, meaning in effect that we expect the bidding activity to be higher in the former design. Again standard economic theory suggests this would lead to an increase in demand which should elevate prices.

2.5 Hypotheses

To sum up the theoretical discussion we can conclude that existing research on competing auctions suggests that for simultaneous auctions it is possible to reach an efficient equilibrium if bidders adopt a cross-bidding strategy. This does however require a non-fixed ending time that allows for all bids to be registered safely, as specified in the model suggested by Peters & Severinov (2006). When a fixed ending-time is introduced, as in Stryszowska (2006), an efficient equilibrium is not guaranteed even if bidders use a cross-bidding strategy and this is simply because there is a positive probability that some bids are submitted to late. This is an important implication since it is often the case that a high share of the bids in an auction is submitted close to ending time (see e.g. Roth & Ockenfels 2002). A crucial factor here is clearly that since all auctions end at the same time, there is a risk of there not being enough time to fully follow the cross-bidding strategy. Overlapping auctions, which by definition are separated in ending time, does not suffer from this problem and should therefore allow for losing bidders to submit their bid in another auction. According to Stryszowska (2006) this leads to an efficient outcome, which means that under the assumption that late bidding is common in simultaneous auctions, using a design with overlapping auctions should be more efficient.

Hypothesis 1 *Switching from a design with simultaneous auctions to one with overlapping auctions will lead to a more efficient outcome.*

Huang et al. (2007) arrives at the same conclusion as Stryszowska (2006), however since they use a slightly different theoretical model their prediction is that the cross-bidding strategy will never be used when bidders are faced with overlapping auctions. They argue that the optimal strategy should instead be to bid on the auction that ends first and if one does not win one should continue to bid on the auction first to end of those remaining. Although it is not likely that cross-bidding stops all together and all at once, this does imply that the cross-bidding activity should be lower in overlapping auctions as compared to simultaneous auctions.

Hypothesis 2 *Cross-bidding will be non-existent or significantly lower in overlapping auctions than in simultaneous auctions, i.e. the fraction of institutional bids will be lower and the fraction of participatory bids will be higher.*

Further, if the model specified by Huang et al. (2007) is correct, we expect bids to always be submitted to the auction that ends first. As with cross-bidding, this behavior cannot be expected to be universal, but it is still meaningful to investigate since it gives a hint of which model that best explains bidder behavior.

Hypothesis 3 *In overlapping auctions, bids will be placed on the auction that ends first within the group of competing auctions.*

When it comes to prices we know that Bapna et al. (2009) hypothesize that information on auctions that precedes and follows a given auction, i.e. overlapping, lowers the price of the focal auction and that they find support for this in their data. The authors also hypothesize and find support for the idea that higher participation in a given auction leads to an increase in the price of the good, which is in line with standard economic theory where we expect an increase in demand to affect price positively. Although they use data from an auction site that is different from Tradera, their results are interesting when considering any price difference in the data prior to and following the change in design for auctions of train tickets on Tradera. What we have then are two effects in a state of opposition where the aggregate effect depends on which of the two are the strongest. A cautious expectation is that prices will be higher in the overlapping setting, which is the result reached using the experimental design in Hoppe (2008).

Hypothesis 4 *Prices will on average be higher in overlapping auctions as compared to simultaneous auctions.*

3 Institutional Features

This section contains a brief explanation of how the bidding process works on the Tradera auction site and what rules apply for train tickets that are sold through auctions.

3.1 The Tradera Bidding Process

In order to place a bid on Tradera one needs to be a registered user and as such one also need to specify a unique user name and a user ID will be assigned. Once this is in place, it is possible to place bids on auctions. A bid must be at least as high as the current standing price plus a minimum increment which varies with the current standing price[17]. When placing a bid on a Tradera auction it is also possible to make use of their proxy-bidding agent, a feature that makes it possible to approximate online auctions to the second-price sealed bid auction design. The idea of proxy bidding is that the buyer submits only one bid and the automated proxy-bidding agent then automatically raises the bid with the minimum increment if the buyer is overbid. This process continues until the buyer becomes the highest bidder or until some other buyer places a bid higher than the maximum bid in which case the bidding agent will exit the auction on behalf of the buyer and send a notification by e-mail. The proxy-bidding feature is automatically activated if the bid is larger than the current standing bid plus the minimum increment, making it practically impossible to pay more than is necessary to win the auction.

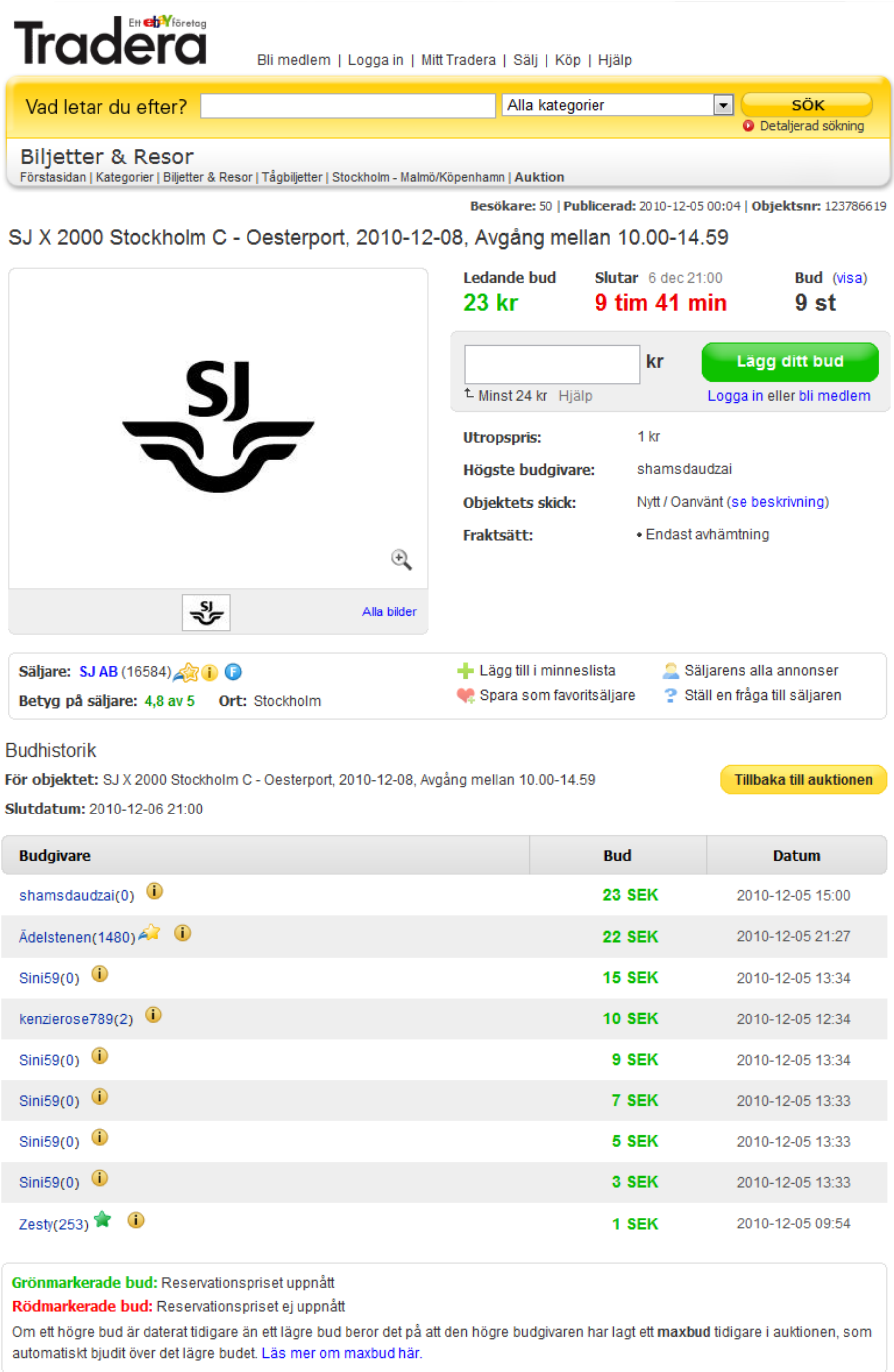
Another feature is the 'buy now'-offer, which gives prospective buyers the opportunity to buy the good to a fixed price and which, if used, immediately ends the auction and awards the good to the buyer who makes use of the offer. Since this feature is not part of the auction mechanism we will not deal with it here. In case SJ have used it when selling train tickets those deals have simply been excluded in the data collecting process.

As for the way auction details are presented on Tradera, images of the auction environment on the website are provided in figure 2. The top part depicts the bidding page of an auction and the lower part shows the bid history. The latter shows all bids that have been placed in that specific auction up until the moment it is being viewed. If a bidder has used the proxy-bidding agent to place a bid, the actual bid is not revealed except if someone places a higher bid (Tradera 2011*a*).

It is through the bidding history of auctions that belong to the same group that it is possible to extract information on the bidders' revealed willingness to pay. As explained in section 2.1, we refer to willingness to pay as revealed since there is no way to know what each bidder was actually willing to pay for a particular ticket. We simply have to use the highest bid provided as a proxy for their willingness to pay.

[17] 1 SEK for intervals $1-99$ SEK, 5 SEK for $100-249$ SEK, 10 SEK for $250-999$ SEK, 25 SEK for $1000-2499$, 50 SEK for $2500-4999$ SEK and 100 SEK for 5000 SEK and up.

Figure 2: Example of a train ticket auction



3.2 Buying Train Tickets on Tradera

Train tickets auctions do not differ greatly from auctions selling other goods and follow the general bidding procedure on Tradera as described above. It is however worth to note that it is only the process of selling tickets through auction that has been outsourced to Tradera. When a bidder wins an auction that person receives an e-mail where he or she is declared the winner of that particular auction and attached is a hyper link which leads to the SJ website where the winner pays for the ticket and selects how it should be delivered.

Before the Change in Design As explained earlier, the pre-change environment featured auctions where for each auction a ticket to a specified departure was offered. Auctions offering tickets to the same departure is here defined as being members of the same group of competing auctions (GCA).

After the Change in Design With the change in design that was announced on November 15, 2010 SJ switched from using simultaneous auctions to using overlapping auctions. The degree of overlap was specified to be two minutes, but the actual data show that the degree of overlap has been everything from non-existent, i.e. equivalent to simultaneous auctions, to more than ten minutes.

Apart from the change in ending times for auctions with identical goods, SJ also began grouping departures together into time blocks so that for a given group of auctions it is now no longer possible to know exactly for what departure the ticket is valid – that information is not revealed until the auction closes. Three different time blocks have been defined; between 5.00 and 9.59, between 10.00 and 14.59 and finally between 15.00 and 20.59. Clearly this makes it harder to evaluate the change in design ceteris paribus, simply because it is no longer the only variable that has been changed. Other changes that was announced by SJ in conjunction with the change in design are the number of tickets being offered for sale through auction, which according to the press release will be doubled, and ending times, which are moved from six hours prior to departure to approximately two days prior to departure with all auctions now ending between 21.00 and 22.00 two days before departure (SJ 2011*a*).

Perhaps the most important consequence of the change in design is that the possibility to easily sort auctions belonging to the same group was lost. This was in all likelihood not changed on purpose and it has now been taken care of. However, for the data set analyzed, or rather the post-change part of the data-set, the sorting mechanism was not in working order which together with the increased number of auctions in each group probably made it a challenge to find and keep track of all auctions belonging to the same group.

Apart from the main change in design from simultaneous auctions to overlapping auctions, the other changes are also likely to affect efficiency and price and this will be more exhaustively discussed in the analysis in section 6.

4 Methodology

This section will cover a description of the data set as well as the methodology used for collecting and analyzing the data. The latter is divided into two parts; the preparing of data for analysis through data manipulation and statistical analysis.

4.1 Data and Descriptive Statistics

The data collected before the change in auction design consist of 17 264 auctions which are divided into 3 892 groups of competing auctions (GCA). These groups can further be divided into block groups of competing auctions (BGCA) that correspond to the time blocks used for auctions after the change in design[18]. There are 2 192 such groups. The data collected after the change in auction design consist of 6 671 auctions which can be divided into 930 groups[19].

In the complete data set there are 15 046 unique bidders and a total of 200 732 bids. Not all auctions were bid on however, and the 1 818 auctions that received zero bids have been removed. In some cases entire groups of auctions have not received any bids which is why the revised number of groups used in the analysis is 4 701. This summary information and more can be found in table 3, which is grouped according to route.

The pre-change data set contains auctions of train tickets on 31 different routes while there in the post-change data set exist 37 routes. This is most likely due to the extension of the usage of auctions to sell train tickets (SJ 2010). In order to provide better conditions for comparison between the two periods, auctions of tickets that are valid for routes that do not exist in the pre-change data set are excluded from the post-change data set[20].

[18] See section 3.2 for details on the time blocks.

[19] Note that the groups in the post-change part of the data set are similar to the block groups in the pre-change part, we will however for simplicity refer to them as groups.

[20] Route 3 and 4 (Stockholm C - Malmö Syd and the inverse route) do not exist in the post-change data set and these routes have therefore been included in routes 13 and 17 (Stockholm C - Malmö C and vice versa).

Table 3: Descriptive statistics

ID	Route	Auctions S	Auctions O	GCA (BGCA) S	GCA (BGCA) O	Mean price (SEK) S	Mean price (SEK) O
1	Stockholm C - Göteborg C	2 206	863	508 (130)	73	132.13	176.02
						(3.22)	(6.01)
2	Göteborg C - Stockholm C	2 220	751	529 (138)	66	120.42	185.45
						(3.18)	(5.44)
5	Stockholm C - Falun C	895	268	191 (117)	47	84.46	73.72
						(2.36)	(3.63)
6	Falun C - Stockholm C	776	306	186 (98)	43	105.43	68.65
						(2.44)	(3.26)
7	Stockholm C - Halmstad C	16	14	7 (7)	4	18.75	4.86
						(6.70)	(2.19)
8	Karlstad C - Stockholm C	1 200	242	259 (141)	40	95.79	101.59
						(3.03)	(6.39)
9	Göteborg C - Oesterport	489	214	102 (66)	45	73.00	72.50
						(2.97)	(3.79)
10	Koebenhavn H - Göteborg C	305	115	62 (38)	21	52.28	63.27
						(2.84)	(4.41)
11	Göteborg C - Koebenhavn H	358	157	73 (45)	29	81.53	67.26
						(2.75)	(4.31)
12	Oesterport - Göteborg C	605	269	120 (76)	46	68.95	61.03
						(2.32)	(3.25)
13	Stockholm C - Malmö C	850	302	213 (159)	51	151.86	195.99
						(4.99)	(8.12)
14	Stockholm C - Oesterport	476	205	128 (89)	33	177.98	231.48
						(6.76)	(10.71)
15	Koebenhavn H - Stockholm C	136	67	33 (33)	17	122.02	190.57
						(10.62)	(15.17)
16	Oesterport - Stockholm C	277	150	86 (66)	31	161.88	244.85
						(8.92)	(11.43)
17	Malmö C - Stockholm C	1 020	380	256 (138)	53	154.99	173.22
						(4.59)	(7.26)
18	Stockholm C - Östersund C	349	133	78 (78)	30	148.13	108.41
						(6.66)	(8.93)
19	Stockholm C - Sundsvall C	961	470	231 (117)	54	110.42	96.77
						(3.76)	(4.59)
20	Sundsvall C - Stockholm C	938	457	212 (116)	59	122.96	99.36
						(4.02)	(4.60)
21	Stockholm C - Karlstad C	975	208	185 (158)	48	115.10	97.08
						(3.58)	(6.83)

Table 3: Descriptive statistics (cont.)

		Auctions		GCA (BGCA)		Mean price (SEK)	
ID	**Route**	**S**	**O**	**S**	**O**	**S**	**O**
22	Östersund C - Stockholm C	310	142	78 (78)	32	175.65	118.49
						(7.90)	(8.39)
23	Luleå C - Kiruna C	166	17	57 (57)	7	87.15	64.29
						(6.61)	(14.97)
24	Narvik - Luleå C	197	32	57 (57)	16	69.72	58.22
						(5.52)	(13.26)
25	Luleå C - Narvik	179	37	54 (54)	17	53.42	49.46
						(4.73)	(14.44)
26	Kiruna C - Luleå C	173	55	58 (58)	23	98.78	51.16
						(5.77)	(7.95)
27	Stockholm C - Koebenhavn H	13	33	5 (4)	9	192.15	233.18
						(30.51)	(21.21)
28	Malmö C - Göteborg C	19	22	4 (4)	6	48.58	62.05
						(15.07)	(9.55)
29	Stockholm C - Odense	17	4	5 (5)	1	90.18	169.75
						(32.76)	(32.78)
30	Odense - Stockholm C	48	23	15 (15)	5	41.90	144.91
						(7.10)	(23.9)
31	Halmstad C - Stockholm C	3	4	2 (2)	1	113.67	1.00
						(56.17)	(0.00)
Total/Overall		16 177	5 940	3 794 (2 144)	907	116.79	133.22
						(0.99)	(1.74)

Abbreviations: S - Simultaneous, O - Overlapping, GCA - Group of competing auctions, BCGA - Block group of competing auctions (these are given in parenthesis after the values for GCA). Standard errors are given in parenthesis beneath the mean values for price.

4.1.1 Data Collection

A practical feature of online auction sites such as eBay and Tradera is that all bits of information on completed auctions are saved for some period of time, in the case of Tradera this period is two months. During this period it is possible to retrieve the information on a particular auction, if one has the auction ID, or perform a search for completed auctions which will then provide a list of auctions from which the auction ID's can be extracted. Taking advantage of this service, the data needed for this essay have been acquired in the following way. First, a search for completed auctions was performed at the Tradera website with the added specification that only objects submitted by seller 'SJ AB' should be considered. This produced a list of all completed auctions of train tickets submitted by SJ from the time of the query and approximately two months back. By using the unique auction ID, the source code of each individual auction web page was then downloaded and saved in text files using a custom made web crawler[21] type of application. From these files, containing all bits of information on an auction that are publicly available, the relevant information was extracted and saved to a database[22]. Due to the amount of data and the need to create a custom tool for collection, the data collection process has been a major part of the essay.

4.2 Data Manipulation

Some of the variables used for analysis in this essay cannot readily be produced by statistical software, and need therefore be constructed in a more custom fashion[23]. For each variable a tailored web-based program have been used which will briefly be explained here.

Efficiency As defined in section 2.1, efficiency is a binary measure taking on the value 1 if a group of auctions is efficient and 0 otherwise. Practically, this is determined by taking all bids submitted to each auction in a given group, eliminating all but the highest bid for each bidder and sorting these in a descending order. Then, the x highest bids, where x is the number of auctions in the group, should match the bids placed by the actual winners of the auctions. If all of the highest bidders – i.e. those with the highest revealed willingness to pay – did indeed win, the outcome is efficient. To get the measure of relative efficiency, one simply calculates the fraction of the highest bidders to win (cf. Andersson et al. 2009). Formally,

$$\text{relative efficiency} = \frac{\text{no. of high-bidders to win}}{\text{no. of auctions in group}}$$

[21] A web crawler is a program that automatically searches ('crawls') the Internet and is generally used to find and index web pages to facilitate searching. The one used here has been created for targeted search of specific auctions at Tradera and is written in PHP.

[22] The method used was inspired by similar solutions used by Anwar et al. (2006) and Andersson et al. (2009), however the author of this essay is responsible for the design and construction of the solution used here.

[23] It is pointed out if a variable is measured at the group level or block group level – if no indication of the measuring level is given, it is measured at the auction level.

The measures of efficiency is used in the Mann-Whitney rank-sum tests (both absolute and relative measures) as well as the logit regressions (only the absolute measure).

Overlap (GCA) As can be derived from definition 2, overlap is the time an active auction shares with one or more other active actions, either at the beginning of the auction (shared with the end of previous auction(s)) or at the end of the auction (shared with the beginning of subsequent auction(s)). For simplicity this is operationalized as the difference in ending time (in minutes) between the first and last auction in a group of competing auctions (cf. Bapna et al. 2009). Formally,

$$\text{overlap} = \text{ending time of auction N} - \text{ending time of auction 1}$$

The measure of overlap of auctions is used in both the logit regression (efficiency) and the OLS regression (price). When used in the price regression, all auctions in a GCA are assigned the group-level value.

Cross-bids (GCA) The share of cross-bids is measured in a similar way as efficiency. We want to find out if a particular bid was placed in the auction with the lowest current standing price so we need to take all bids submitted to each auction in a given group and sort them in ascending order so that we start with the first bid that was placed, which will of course always be a cross-bid. We then basically replay the course of events by taking each bid and submit it to the auction that it was originally submitted to. The crucial middle step is that we for each bid check if it was submitted to the auction that the cross-bidding strategy specifies, i.e. the one with the current lowest standing bid (cf. e.g. Peters & Severinov 2006, Andersson et al. 2009). When we know the share of cross-bids for each auction, a mean is calculated at the group level. This measure is used in the logit regression.

Cross-bids This measure is the same as the above measure of cross-bids, with the only difference that it concerns the share of cross-bids placed in a given auction rather than the group mean. It is used in the price regression.

Bids placed on auction first to end (GCA) [Bids-AFTE] This measure is in a way very similar to the cross-bid measures. All bids placed in the auctions in a given group are selected and sorted in ascending order. We then check if, by the time the bid was submitted, it was submitted to the auction first to end (cf. Huang et al. 2007). Like the cross-bid measures, it is a fraction measure computed as the share of all bids in an auction that is placed on the auction first to end. The measure is used in the logit regression.

4.3 Statistical Analysis

Although some of the analysis, or rather data manipulation, is of ad hoc fashion as we have seen above, basically all of the results rely on conventional statistical analysis. The measures of efficiency and the basic measures of price are simple mean values, which requires no further explanation. We will however briefly describe how comparisons of these measures have been made and also how regression analysis have been used in order to derive factor-specific effects for both efficiency and price.

4.3.1 Equality Tests

The most important comparisons we want to make are to see if there is a statistically significant difference between simultaneous and overlapping auctions for the measures of efficiency and price. For this task we could use a simple t-test, however since the variable *efficient* is binary we will instead use the non-parametric Wilcoxon-Mann-Whitney rank-sum test (Mann-Whitney test henceforth). The non-parametric property lets us use data regardless of its distribution, meaning that we do not need to assume it to be normally distributed which would be unlikely for a binary measure such as absolute efficiency (see e.g. Tanizaki 2004). Regardless of variables we will test the null hypothesis that two mean values are equal.

4.3.2 Regression Analysis

In order to better understand what affects efficiency and price, we use regression analysis to estimate the significance of the variables that we have determined theoretically to be important. Since efficiency is a binary measure we should not use the ordinary least squares (OLS) method, although it is possible to do so by using the linear probability model (LPM). There are however several problems with this approach, of which two are particularly important since there is no easy way to deal with them. First, if using OLS the estimated values for efficiency need not be in the range between 0 and 1 but can take on negative values as well as values larger than 1. Second, LPM assumes that the probability of a GCA being efficient increases linearly with the independent variables, which might not be the case. The solution is to instead use the logit model which, with individual level data, use the maximum likelihood estimator (Gujarati 2006).

When it comes to price we should be able to use OLS to find out what effect the explanatory variables have. We should however test the assumptions required for the OLS method to deliver consistent and unbiased estimates. This includes making sure that the explanatory variables are exogenous and not exactly linear and that the error term is normally distributed, has a zero mean, is homoskedastic and does not exhibit autocorrelation. Finally, we need to check that we use the correct functional form. The tests are conducted using standard statistical methods, of which most can easily be performed using statistical software such as STATA. When running a regression STATA will automatically exclude variables should it detect perfect collinearity. For homoskedasticity and normality in the error term a graphic examination is used in combination with calculating the mean and running the regression with robust standard errors. To test for endogeneity, we save the residuals from the regression and check to see if they are correlated with any of the explanatory variables. Finally, by looking at the correlation between price and the explanatory variables as well as scatter graphs, we should be able to determine if the right functional form has been used (ibid).

4.3.3 Variables

In addition to the variables described in section 4.2, the following variables will be used for the statistical analysis[24].

Bidders This variable is used in the standard multiple linear regression model estimated with OLS which is used for the price regression, where it concerns the number of bidders in an auction.

Bidders (GCA) This variable is used in the logit model used for the efficiency regression, where it concerns the number of bidders in a group of competing auctions.

Bids Used in the price regression, where it concerns the number of bids in an auction.

Auctions (GCA) Represents the number of auctions in a group of competing auctions and is used in the logit regressions.

Auctions (BGCA) Represents the number of auctions in a block group of competing auctions and is used in the price regression.

Intercity Dummy variable that takes the value 1 if the train type is intercity and 0 otherwise, i.e. if train type is X2000[25]. Used in the price regression.

Period1 Dummy variable that takes the value 1 for pre-change data and 0 otherwise, i.e. for post-change data. Used in the price regression and the logit regressions.

rt* Dummy variables for routes that takes the value 1 if an auction concerns a ticket for a train that operates on that route and 0 otherwise. The asterisk functions as a wild card representing a number between 1 and 31, which is the number of routes in the data set (excluding routes 3 and 4 as explained in footnote 20). Used in the price regression.

[24] All variables have been put together by simple database operations or, as in the case of the dummy variables, generated directly in STATA.

[25] X2000 is the designation used by SJ for their express trains which usually operates on longer distances making few stops, while Intercity is slower and makes more frequent stops. See SJ (2011*b*) for more details on this.

5 Results

This section presents the results of the empirical investigation of efficiency and price in simultaneous and overlapping auctions.

5.1 Efficiency

As described in section 4.2 a GCA is considered efficient if all auctions are won by the highest bidders. It is a binary measure, meaning that it can only take the values *efficient* or *not efficient* and it is therefore likely to be sensitive to factors such as the number of auctions in a group and the number of bidders involved. This is quite intuitive – more auctions and more bidders ought to make it more difficult to achieve an efficient outcome[26]. This should be kept in mind when considering the following results which are based on mean values, a measure that does not account for these factors. See table 4 for a summary of the results and test statistics (Mann-Whitney test is used unless stated otherwise).

In the simultaneous setting 36.4 percent of the groups of competing auctions are efficient (excluding 98 groups of competing auctions that received zero bids). This means that for roughly one third of the GCA, the winning bids were placed by the bidders with highest revealed willingness to pay. If we assign the existing groups to super groups according to which time block they belong to (BGCA), the figure drops to 25.9 percent. One can think of this as a possible remedy to the problem of comparison that arise from the fact that for the simultaneous auctions each group contains auctions for only one departure time whereas the overlapping auctions are grouped together according to time blocks which results in a higher average number of auctions per group[27]. We will return to this issue when discussing determinants of efficiency in section 5.1.2 below.

Having a result for simultaneous auctions we may now compare this to the results for overlapping auctions for which 20.4 percent of the GCA are efficient (excluding 6 groups of competing auctions that received zero bids). At a glance this suggests that simultaneous auctions are more efficient, both in the original groups and when grouped into block groups. Statistical tests confirm the difference at one percent significance level in both cases.

As discussed above, it is not surprising that the figure for overlapping auctions is lower since it is likely that the number of auctions per group is higher, which we also confirmed above (see footnote 27). Further, the mean number of bids placed in each auction is higher for overlapping auctions with 11.1 (0.11) against 8.3 (0.06) as is the mean number of bidders with 4.4 (0.03) against 3.5 (0.02), standard errors in parenthesis. The differences are statistically significant at the one percent level[28].

[26] The effect of the number of auctions might be ambiguous since a high number of auctions combined with a low number of bidders suggests that is should be easier to reach an efficient outcome.

[27] The mean number of auctions in each group is 4.4 (0.02) for simultaneous auctions and 7.3 (0.17) for overlapping auctions, standard errors in parenthesis. A Mann-Whitney test also reveals that the difference is significant at the one percent level (z=15.567).

[28] Mann-Whitney statistic z=25.051 for bids and z=26.495 for bidders

5.1.1 Relative Efficiency

Following Andersson et al. (2009) in acknowledging efficiency as a dichotomous and somewhat rigid measure, we now introduce the alternative measure of relative efficiency. Using this slightly softer measure the level of efficiency is taken into account by calculating the fraction of the bidders with the highest (revealed) willingness to pay that wins an auction, which results in a more nuanced picture of efficiency revealing that most GCA are quite close to satisfying the more strict condition of absolute efficiency. Putting it in figures; for simultaneous auctions an average of 79.7 percent of the winning bids are placed by high-bidders, for overlapping auctions the figure is 75.8 percent. The difference is statistically significant at the one percent level. This result holds when grouping the simultaneous auctions into time blocks corresponding to those for overlapping auctions in which case the mean fraction of the highest bidders in a GCA to win drops to 77.4 percent, yet stays significantly higher than the figure for overlapping auctions. Evidently, introducing the measure of relative efficiency – and thereby considering the number of auctions in a group – does not change the result that groups of auctions in the simultaneous setting are more efficient than groups in the overlapping setting.

Table 4: Summary of efficiency

	Absolute eff. (%)		**Relative eff. (%)**	
	GCA	**BGCA**	**GCA**	**BGCA**
Simultaneous	36.4	25.9	79.7	77.4
Overlapping	20.4		75.8	
Mann-Whitney statistic	-9.574^{**}	-3.519^{**}	-6.572^{**}	-4.595^{**}

Significance levels : † : 10% $*$: 5% $**$: 1%

5.1.2 Determinants of Efficiency

Regardless of the use of measure, the results above show a higher level of efficiency for simultaneous auctions as compared to overlapping auctions. This is not in line with what can be expected from the theoretical framework and so we need to take a closer look at the determinants of efficiency, i.e. what affects whether a given GCA is efficient or not. From the theoretical discussion in section 2.3 we know that the number of auctions and bidders per group, the degree of overlap and the share of crossbids as well as bids on the auction first to end are good candidates concerning what affects efficiency.

As a pre-test of the theoretical predictions, we first investigate the correlations between the determinants of efficiency and efficiency itself – the results are shown in table 5 and 6. It is clear that the correlation between efficiency and the number of auctions (GCA) and bidders (GCA) as well as the degree of overlap is negative whereas the correlation between efficiency and the share of crossbids as well as the share of bids placed on the auction first to end is positive.

Table 5: Correlations for pre-change data

	efficient	**auctions (GCA)**	**bidders (GCA)**	**overlap**	**cross-bids**	**bids-AFTE**
efficient	1					
auctions (GCA)	-0.1003	1				
bidders (GCA)	-0.4318	0.1472	1			
overlap (GCA)	.	.	.	.		
cross-bids (GCA)	0.3573	-0.1584	-0.5217	.	1	
bids-AFTE	.	.	.	.	.	.

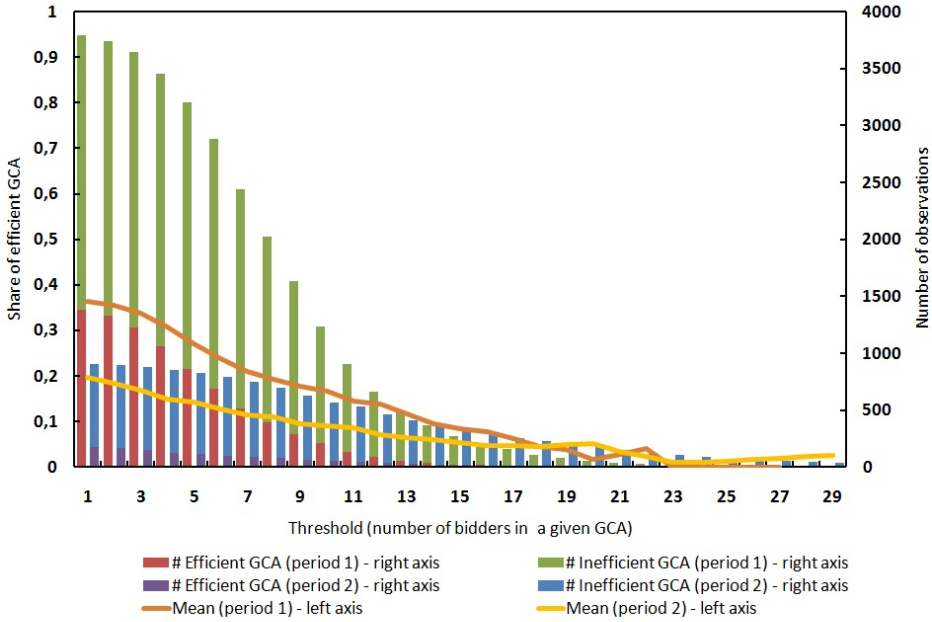
Table 6: Correlations for post-change data

	efficient	**auctions (GCA)**	**bidders (GCA)**	**overlap**	**cross-bids**	**bids-AFTE**
efficient	1					
auctions (GCA)	-0.2471	1				
bidders (GCA)	-0.3686	0.6095	1			
overlap (GCA)	-0.3263	0.5235	0.4376	1		
cross-bids (GCA)	0.4254	-0.4532	-0.5406	-0.4838	1	
bids-AFTE	0.1775	-0.4022	-0.4322	-0.2715	0.3423	1

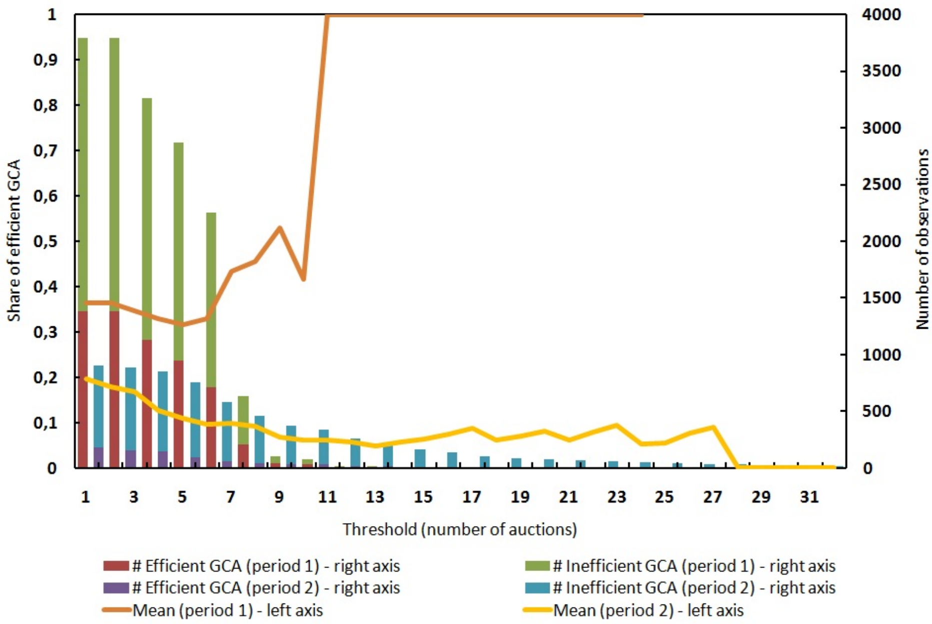
A problem with the correlation measure is that it only detects linear relationships, it is therefore also a good idea to graphically examine the relationship between the variables. Figure 3 depicts each independent variable and its relationship to the dependent variable. The graphs have been created by iterated calculation of mean values of efficiency while gradually increasing the value of the independent variable. As can be seen in the respective figures, the graphs follow the measures of correlation in showing that bidders (GCA), cross-bids and bids on the first auction to end are the main determinants of efficiency. The number of auctions (GCA) and the degree of overlap have smaller, yet still distinguishable effects.

Figure 3: Effects on efficiency when varying different determinates in isolation

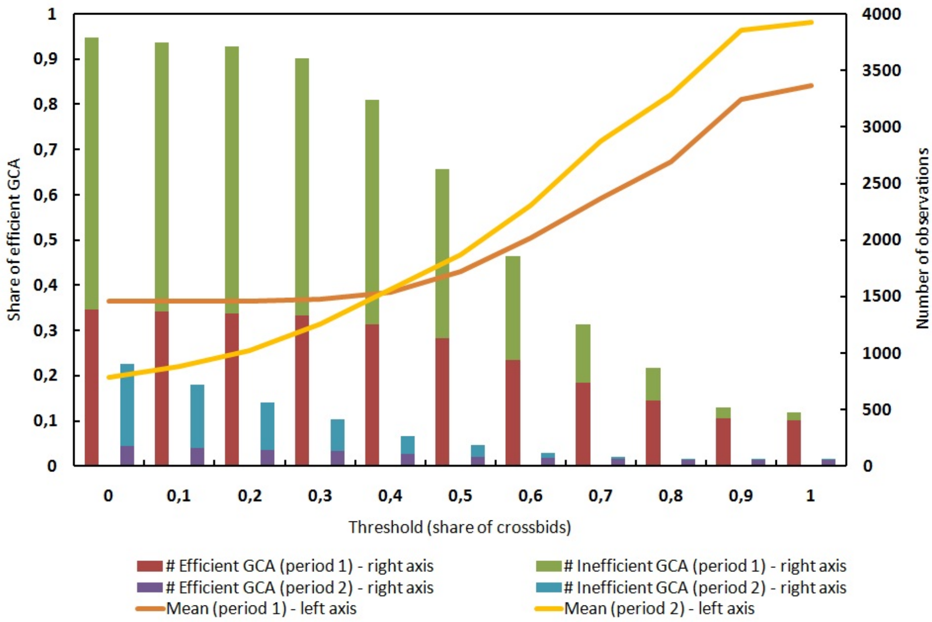
(a) Number of bidders in a GCA



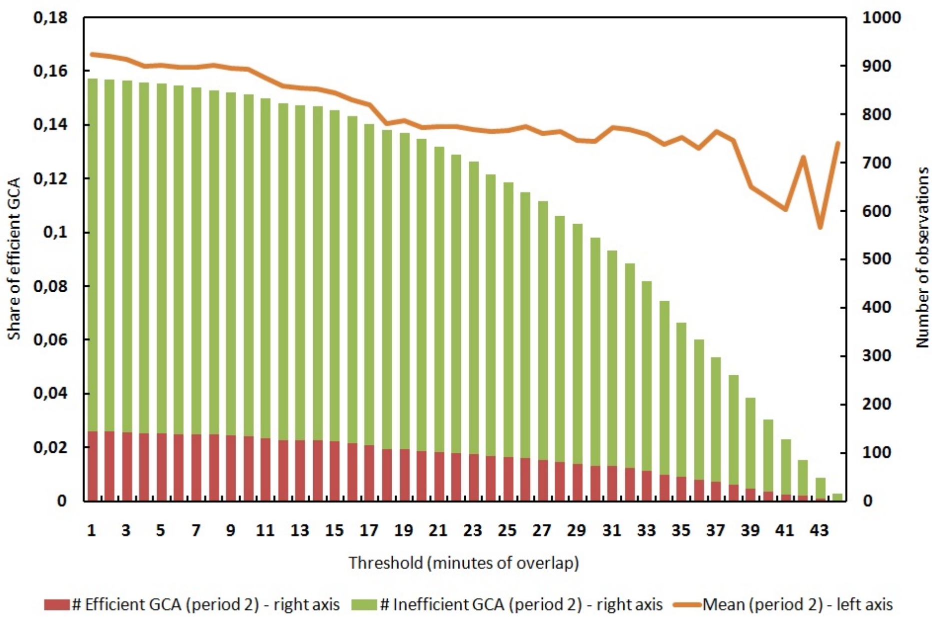
(b) Number of auctions in a GCA



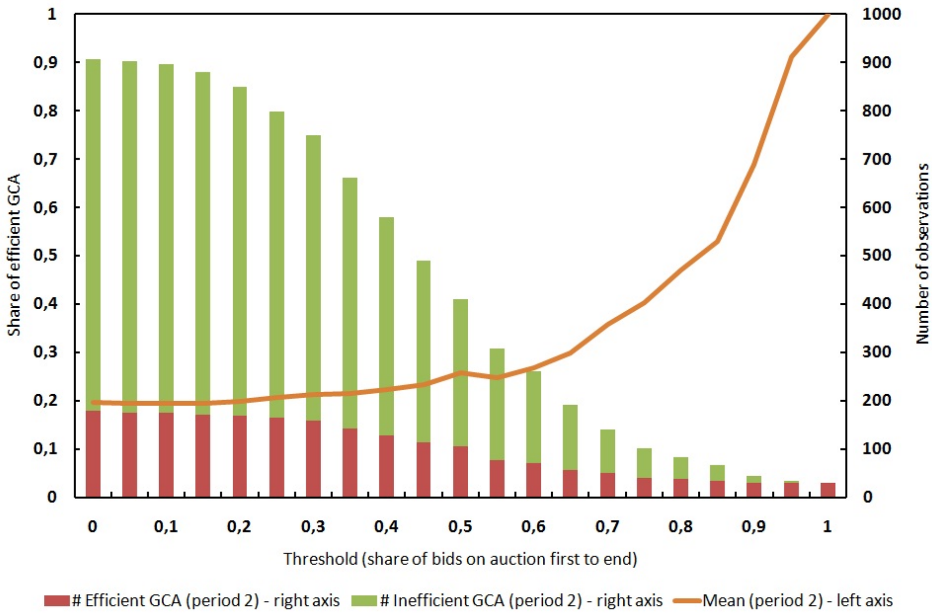
(c) Share of cross-bids in a GCA



(d) Degree of overlap in a GCA



(e) Share of bids placed on the auction first to end in a GCA



In order to find out what effect the explanatory variables have on efficiency we run a regression with *efficient* being the dependent variable and using *auctions (GCA)*, *bidders (GCA)*, *overlap (GCA)*, *cross-bids (GCA)* and share of *bids on auction first to end* as independent variables. Because the variables *overlap* and share of *bids on auction first to end* have little effect on the auctions before the change in design and since, in the same way, *cross-bids (GCA)* have a presumed less significant effect on the auctions after the change in design, we run two separate regressions – the first with only pre-change data and the second with only post-change data. We then also add the dummy variable *period1* and run a third regression using the full data set. Since *efficient* is a binary variable, we use logistic regression.

The results are presented in table 7 and as can be seen the results are quite mixed. From theory we expect *overlap (GCA)*, *cross-bids (GCA)* and the share of *bids on auction first to end* to increase the probability of an efficient outcome while we expect the number of bidders in a GCA to have a decreasing effect. We also discussed that the role of the number of auctions in a group is dubious since a high number of auctions should be positive for efficiency if there are few bidders, but if there are many bidders a high number of auctions may negatively affect the probability of reaching an efficient outcome. The only coefficients that are unambiguous are those for the share of cross-bids which has a clear positive effect on efficiency and the number of bidders which has a clear negative effect.

Table 7: Estimation results : Determinants of efficiency

Variable	Coefficient1 (Std. Err.)	Coefficient2 (Std. Err.)	Coefficient3 (Std. Err.)
period1	–	–	-0.903*
			(0.395)
overlap (GCA)	–	-0.023*	-0.012
		(0.010)	(0.009)
auctions (GCA)	-0.060*	0.019	-0.024
	(0.026)	(0.035)	(0.022)
bidders (GCA)	-0.281**	-0.163**	-0.253**
	(0.015)	(0.026)	(0.013)
cross-bids (GCA)	1.560**	1.860**	1.669**
	(0.204)	(0.432)	(0.184)
bids-AFTE	–	-0.368	-0.847†
		(0.474)	(0.477)
Intercept	0.853**	0.289	1.326**
	(0.239)	(0.527)	(0.436)

Significance levels : † : 10% ∗ : 5% ∗∗ : 1%

5.2 Price

A simple Mann-Whitney test reveals a statistically significant difference in prices, with auctions in the overlapping design having the higher mean price (133.22 SEK) compared to the simultaneous design (116.79 SEK). This is in line with what we expect from the theoretical prediction. In order to find out what factors contribute to this difference we regress *price* on the dummy variable *period1* and control for train type (*intercity*), degree of overlap (*overlap (GCA)*), number of bids (*bids*), number of bidders (*bidders*), mean share of cross-bids in a given auction (*cross-bids*), number of auctions in a given blockgroup (*auctions (BGCA)*), and finally, route (*rt**). The results are presented in table 8.

As can be seen, *period1* is positive, suggesting that the price is higher in the pre-change auctions. This result is however not statistically significant and the only conclusion here is that, given the data we have and controlling for the factors included in the regression, we cannot tell if there is a difference in prices for auctions before and after the change in design. We can on the other hand see that all of the other factors have the expected effect. The coefficient for the dummy-variable *intercity* has a value of -31.5 suggesting that – ceteris paribus – a ticket for an Intercity train is on average 31.5 SEK cheaper. Since X2000 trains are considered superior in comfort and travel time (SJ 2011*b*), this result is expected. A more important result is that of the degree of overlap. As seen in table 8 the coefficient reads -0.888, suggesting that for each minute of overlap the price is on average reduced by 0.89 SEK. To give an idea of the magnitude of this effect, the mean overlap for the post-change

Table 8: Estimation results : Price

Variable	Coefficient	(Std. Err.)
period1	0.610	(4.363)
intercity	-31.507**	(1.555)
overlap (GCA)	-0.888**	(0.124)
auctions (BGCA)	-4.141**	(0.102)
bids	4.351**	(0.105)
bidders	24.604**	(0.387)
cross-bids	-6.729**	(2.112)
rt1	92.162**	(17.660)
rt2	85.582**	(17.660)
rt3	0.000	(0.000)
rt4	0.000	(0.000)
rt5	27.347	(17.718)
rt6	34.056†	(17.736)
rt7	-33.117	(22.793)
rt8	36.101*	(17.642)
rt9	0.129	(17.811)
rt10	-2.544	(17.982)
rt11	-14.914	(17.902)
rt12	-2.764	(17.763)
rt13	37.738*	(17.655)
rt14	45.781**	(17.759)
rt15	-5.410	(18.364)
rt16	22.720	(17.915)
rt17	36.198*	(17.647)
rt18	39.357*	(17.873)
rt19	32.678†	(17.623)
rt20	26.855	(17.629)
rt21	27.250	(17.653)
rt22	42.900*	(17.897)
rt23	23.992	(18.524)
rt24	16.943	(18.329)
rt25	16.937	(18.373)
rt26	19.998	(18.331)
rt27	48.427*	(21.112)
rt28	-21.241	(21.554)
rt29	0.000	(0.000)
rt30	-47.755*	(19.902)
rt31	-1.995	(34.958)
Intercept	20.504	(17.964)

Significance levels : † : 10% ∗ : 5% ∗∗ : 1%

auctions is 29.5 minutes which gives a price reduction of 26.25 SEK. This is without doubt a non-negligible amount and the effect is also in line with theory (see section 2.4).

For the next three variables – *auctions (BGCA)*, *bids* and *bidders* – we are provided with predictions from standard microeconomic theory. As discussed in section 2.4, the number of auctions can be thought of as supply while the number of bidders and bids are synonymous to demand. The regression result conforms to these predictions; the coefficient for *auctions (BGCA)* is −4.141 and for *bids* and *bidders* the coefficients are 4.351 and 24.604 respectively. This means that an increase in the number of auctions by one leads to an average decrease in the price of a ticket by just over 4 SEK while an increase in bids or bidder by one unit (i.e. 1 bid or 1 bidder) leads to an average increase in price by approximately 4.35 SEK and 24.60 SEK respectively. Note the magnitude of the effect of an extra bidder which clearly shows the importance of this variable.

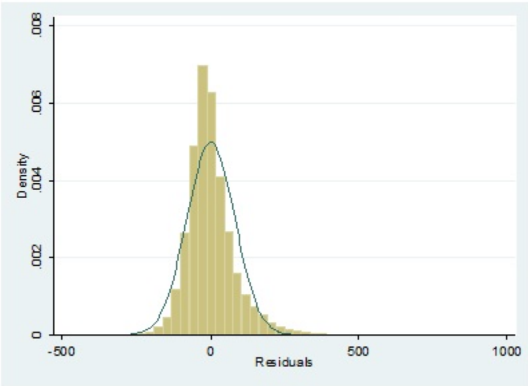
Finally, to control for price effects due to the popularity of different routes, the 29 dummy variables for routes have been included, each taking the value 1 if the ticket being auctioned is valid for travel on this route and 0 otherwise. In order to avoid multicollinearity, the variable *rt29* has been excluded which means that the price differences for other routes are relative to route 29. From the results it is clear that some routes have more effect on the price than do other routes, e.g. routes 1 (Stockholm - Göteborg), 2 (Göteborg - Stockholm), and 14 (Stockholm - Oesterport) have a positive price effect of roughly 92 SEK, 86 SEK, and 46 SEK respectively, all being significant at the one percent level.

Summarizing, we see that the regression results for price conform to the theoretical predictions; *intercity*, *overlap (GCA)*, *cross-bids*, and *auctions (BGCA)* exert a negative pressure on price while *bidders* and *bids* exert a positive pressure. Further, we cannot confirm that there is a statistically significant difference between the two periods, however it is likely that the positive effects of the increase in *bidders* and *bids* outweigh the negative effect of overlap between auctions, which would mean that the higher (average) number of bidders and bids in the post-change period is an important factor for the higher mean price of the post-change period. In effect, the increased competition induced by more active bidders as opposed by the more careful institutional bidders, leads to higher prices since this effect is greater than the overlap effect. As for the route-dependent effects, it is quite reasonable and intuitive that tickets for routes connecting the main cities – which can be expected to be the most busy routes – have a higher price.

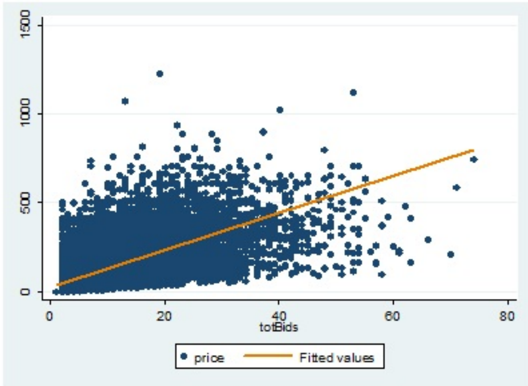
When interpreting these results we also need to consider the limitations of the OLS method as discussed in section 4.3.2. In order to check if the data set meets the demands of the model, we run a few tests. We first consider a histogram and summary statistics for the regression residuals. As can be seen in figure 4, the residuals looks fairly normal with a mean close to zero. In the same figure are also scatter plots of price and the explanatory variables. Finally, looking at the correlation coefficients for the regression residuals and the explanatory variables, these show zero correlation which indicates that endogeneity should not be a problem.

Figure 4: Graphical illustration of price regression variables

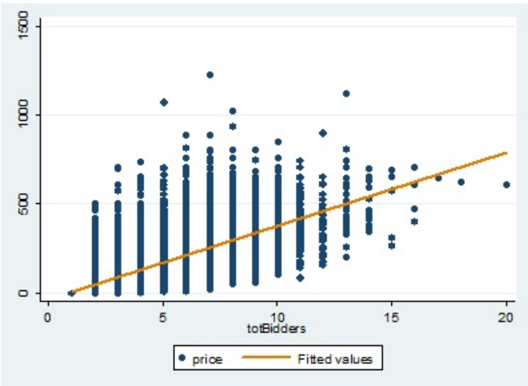
(a) Histogram of residuals



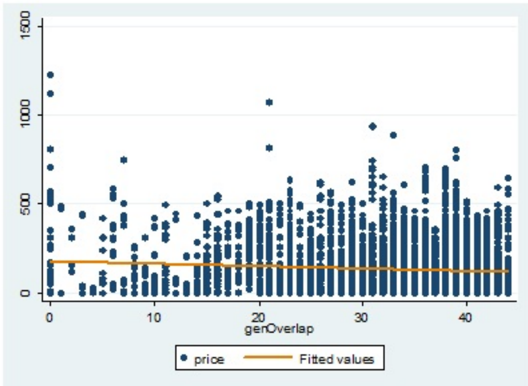
(b) Price/Bids



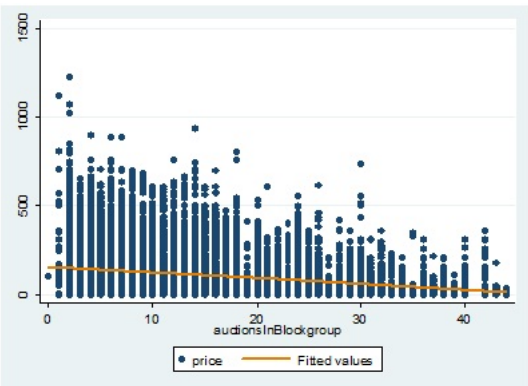
(c) Price/Bidders



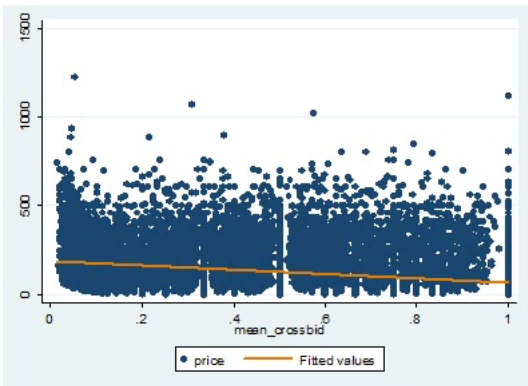
(d) Price/Overlap



(e) Price/Auctions



(f) Price/Crossbids



6 Analysis

This section will evaluate and analyze the empirical results on the basis of the theoretical framework. We will discuss how statistical results should be interpreted and if there are enough grounds for accepting or rejecting the hypotheses.

The theoretical prediction for efficiency is that overlapping auctions should be more efficient than simultaneous auctions. Although different scholars propose slightly different models and are not in agreement on the exact grounds for this prediction, it is still an unanimous prediction per se. The results for efficiency in the data set used does however not confirm the theoretical prediction – both the absolute and the relative measures indicate that simultaneous auctions are more efficient than overlapping auctions, using a significance level of one percent. Clearly the attempt to use relative efficiency to overcome the possible bias of a significantly higher number of auctions per group did not change the outcome, even though the difference in efficiency is considerably smaller for this measure. It gets even smaller when the simultaneous auctions are grouped together into block groups to more resemble the new, post-change design. The difference still remains however, and albeit a slightly less strong test statistic it remains significant at the one percent level.

In an attempt to demonstrate what factors determinate efficiency we used regression analysis to examine the effect of individual explanatory variables. With support from theory and by using graphs to examine relationships between efficiency and the candidate determinates, a number of variables were deemed fit to use in the regression analysis. The result was not as distinct as could be expected from theory – of the variables used as possible determinants only two were consistent and significant through the three regressions that were fitted. The significance of the number of auctions, the degree of overlap and bids submitted to the auction first to end do not seem to have a great effect on efficiency, at least not with the variables being operationalized as they have been here. The share of crossbids and the number of bidders were the only significant variables and considering that the number of bidders are higher in overlapping auctions, this is in line with the lower value for efficiency. Also the share of cross-bids is lower for overlapping auctions, which is what theory predicts. However theory also predicts a positive effect from overlap between auctions as well as from bids being placed on the auction to end first. Regarding overlap there is a possibility that this measure is not correctly operationalized, since it is done in a different way than the research it was adapted from. The reason for using a different operationalization is simply because the data set used here have two parts – before and after the change in design. In this way it is different from the data set used in Bapna et al. (2009), from which the measure is adapted. They calculate overlap between each individual auction, which we have not done because this might have affected the ability to make predictions using the whole data set.

The possible problem with the variable for overlap is related to another possible confounding factor, namely search costs. Haruvy & Popkowski Leszczyc (2010) discuss search costs as a possible explanation to why bidders choose not to bid on the auction with the lowest standing price. Considering that the change in auction design brought about a considerable impairment of the sorting possibilities, this could have affected bidders opportunity to compare auctions. On the other hand, since the auction ending first is showed first by default it should not have affected prescribed behavior to bid on the auction first to end. Nevertheless, since auctions within different time blocks are shown together it might be difficult as well as time consuming to find out what auction one should bid on (cf. Haruvy, Popkowski Leszczyc, Carare, Cox, Greenleaf, Jank, Jap, Park & Rothkopf 2008).

Finally, it should be noted that throughout the essay we have assumed that bidders have private values, i.e. that the valuation of one bidder is independent of other bidders' valuations. If we assume that bidders do not always want to buy train tickets for their own consumption, but plan to re-sell them after the auction has closed, then we would also need to assume that they care about how much others value a ticket since that will be an indicator of its market value (Krishna 2010). It is difficult to say something about this without having investigated further with regards to theory and empirical results. A possible method to examine if there are evidence of interdependent values could be to simply check how many, if any, train tickets that in an obvious way are being re-sold in another auction[29].

To summarize the analysis on efficiency it is clear that we need to reject hypothesis 1 – with the data set used in this essay we cannot say that overlapping auctions are more efficient than simultaneous auctions. We cannot even find evidence that suggest that the level of efficiency is the same for the two types of design. We have discussed the possible effect of search costs due to a faulty sorting feature and interdependent values due to an increased interest in re-selling the tickets bought, both of which affect the results by violating the assumptions that the theoretical models are built on. On the other hand, perhaps the most viable explanation is that there has not gone long enough time since the change in design. Due to time limitations only one month worth of auction data could be collected from the post-change period which might well be too short a time for bidders to adjust to the new design.

It should be said that when we consider the intermediate effects of the change in design, such as the decrease in cross-bidding as specified in hypothesis 2 the results are more positive. We shall return to this in a moment when discussing price, first we shall also note that hypothesis 3 cannot be rejected on the basis of the graphical examination of the effect on efficiency when bids are always placed on the auction first to end. It did however not have the same positive effect when included in the logit regression. We should therefore not reject the hypothesis of the importance of placing bids on the auction first to end, but at the same time we should exercise cautiousness in drawing any general conclusions until it has been

[29]Normally when buying train tickets for SJ operated trains, one must specify the name of the holder of the ticket which reduces the ability to re-sell the ticket. However, for tickets sold through auction at Tradera it is possible to circumvent this limitation and since SJ also changed the time between the closing of an auction and the departure of the train to approximately two days instead of approximately six hours, there is clearly more time available for those who plan to re-sell their ticket in another auction.

further investigated.

When considering price, the results are more in line with what theory predicts. We expect the overlap of auctions to have a negative effect on price, mostly because of the option value of participating in a future auction rather than the current. We also expect the number of auctions in a block group[30] to exert a negative pressure on price and the number of bids and bidders to exert a positive pressure. The empirical results give support to these theoretical findings with significant effects for all three variables, where the number of bidders stands out as the variable with the largest effect.

From theory we also predict that the share of cross-bids should have a negative effect on price since that strategy explicitly states that one should only slowly increase price in order to achieve both efficiency and price-uniformity at the lowest level needed for the bidders with the highest revealed willingness to pay to win. Using a measure of the share of cross-bids placed in a given auction, we found that the result for this variable corresponds to the theorical prediction. The same goes for the dummy variable intercity, which we according to the descriptions given by SJ – that the Intercity train type is somewhat slower and less comfortable than the express train type X2000 – argued should have a negative effect on price, which was confirmed by the data. The final variables used in the regression on price are the 29 dummy variables on route, which not surprisingly indicated a significant disparity in price over different routes.

Having used OLS to estimate the price regression we also performed a number of tests to secure consistent and unbiased estimates. The results show no indication of there being any problem with the usage of OLS, not with regards to normality of the error term nor with endogeneity in the explanatory variables.

Summarizing the investigation of differences in price between simultaneous and overlapping auctions, we have found that it is likely that there is a difference in price as demonstrated both by simple mean values and by regression analysis where we included the explanatory variables motivated by theory. Having found that the share of cross-bids is significantly lower for overlapping auctions, we cannot reject hypothesis 2 which stated this to be true. Combined with the increasing number of bidders to bid on a given auction we have argued that this should lead to an increased upward pressure on prices through an increase in bidding activity, i.e. higher demand. We have thereby good grounds to confirm hypothesis 4, stating that the average price is higher in overlapping auctions as compared to simultaneous auctions.

[30] Note that block groups are used here instead of the regular groups, this is in order to make the conditions of the two periods as similar as possible

7 Conclusion

In the introduction to this thesis, two research questions were specified. First, are overlapping auctions more efficient than simultaneous auctions, and second, what happens with prices when switching from one design to another, in this case from simultaneous to overlapping? In this section we present conclusions of our findings and answers to those questions.

The Theoretical Framework It is suggested in theory that overlapping auctions are more efficient than simultaneous auctions, mostly due to the opportunity to bid in another auction should one lose the current one. In order to investigate whether this theoretical prediction is correct, we downloaded data on train tickets sold by SJ through auctions on Tradera. A simple rank-sum comparison of means show that simultaneous auctions are significantly more efficient than overlapping auctions. In order to account for the increase in the number of auctions that came with the change in design, we use relative efficiency – which measures the share of auctions in a group of auctions with identical goods that are won by the bidders with the highest revealed willingness to pay. This measure also points to simultaneous auctions as being significantly more efficient. To examine what variables are behind this result we use a logistic regression model which indicates that of the variables included, it is basically only the share of crossbids and the number of bidders that have a significant effect on efficiency, the first increases the possibility of an efficient outcome whereas the latter decreases it. More research is needed to see if the results here are representative and more research is also needed to explain what affects efficiency in competing online auctions.

The First Research Question The answer to the first question must therefore be that with the data set used here, we do not find evidence to support the theoretical prediction that overlapping auctions are more efficient that simultaneous auctions. As discussed in the analysis in the previous section, overlapping auctions are closing in on simultaneous auctions for every more sophisticated measure used, but it remains less efficient and this is statistically significant at the one percent level in all of the measures used.

The Second Research Question Regarding price we find evidence in the data set used that overlapping auctions on average have a higher price than do simultaneous auctions. This has been tested by using mean values and also by estimating a regression model using the OLS method. The results from the regression confirm the theoretical predictions – price is affected positively by the number of bids and bidders while the degree of overlap and the number of auctions in a block group affects price negatively. This is consistent with theory since an increase in demand increases price while an increase in supply decreases price. Also the share of crossbids is found to be a significant factor that affects price and as theory predicts an increase in the share of crossbids decreases price since it is believed that bidders adopting the cross-bidding strategy submit fewer bids in a slower pace. In order to avoid inconsistent and biased results the assumptions of the OLS are tested and the results are found to be satisfactory.

The answer to the second question is thus that all results and tests indicate that prices are higher in overlapping auctions than in simultaneous auctions. Although we get strong results and almost all variables are significant, it is important to remember that the theoretical support for this question is not that strong, and more research is needed in order to get reliable results.

7.1 Further Research

A good starting point for future research would be to replicate the tests in this essay using a larger set of data, i.e. data that covers a period when it can be assumed that bidders have grown accustomed to the new design. Also, it would be interesting to investigate the importance of the sorting mechanism that was failing during the time this essay was written. Perhaps most important would be to focus on further theoretical research on competing online auctions. As may have been noticeable on the occasionally somewhat thin theoretical advances in the area of overlapping auctions, this would be an excellent area on which to focus future research. Combined with empirical investigations this is instrumental for a solid understanding of the workings of competing online auctions.

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