

Abstract

IKEA's business idea is to offer a wide range of well designed, functional home furnishing products at prices so low that as many people as possible will be able to afford them. To be able to do this the costs must be kept as low as possible. Achieving desired customer service levels is although the key objective while cost reductions are a secondary focus.

To be able to retain desired customer service levels companies use safety stock to compensate for variations in customer demand and lead time.

This Master Thesis objective is to develop a method to evaluate statistical distribution based on their ability to achieve pre-defined levels of service. Furthermore simulations of the upstream supply chain will be performed to verify and quantify the impact of the results generated through the developed method.

The results indicate that the current setup of statistical distributions is not considered optimal and a new setup is recommended, which consists of normal distribution on DCG level and gamma distribution on store level. This new setup generates more accurate inventory control in terms of obtained service levels throughout the supply chain.

This study is based on a small selection of articles on one single market. The recommendation is hence to continue the study with a wider scope in order to investigate whether these conclusions are applicable in a global context.

Acknowledgement

This master thesis has been written at the department of Industrial Management and Logistics, Lund Institute of Technology. We would like to address our advisor Professor Stig-Arne Mattsson with a great deal of thankfulness for his straight-forward guidance and dedication throughout the work process. Second to him, simulation expert Christian Howard deserves recognition for his support in the creation of the simulation models.

We would also like to thank our advisors at IKEA of Sweden. Paul Björnsson who have accompanied along the way and mentored us during this project and Birgitta Elmquist, who have helped us with finding all information and data needed.

Furthermore we would like to thank the kind librarians of Malmö City Library who cooped with us during our occupation the hot summer months of 2009.

Lund, October 2009

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1 Introduction

This chapter will give a background to the purpose and choice of scope for this Master Thesis. Problem discussion, objectives, delimitations, target audience and tools will also be introduced.

1.1 Presentation of IKEA

IKEA's business idea is to offer a wide range of well designed, functional home furnishing products at prices so low that as many people as possible will be able to afford them.¹

The company is divided into three corporate groups: the IKEA Group, the Inter IKEA Centre Group and the Ikano Group.² To ensure long term success by conserving corporate culture and values both the IKEA Group and the IKEA Centre Group are owned and controlled by foundations.³ IKEA of Sweden, IoS, which is the focus for this report, is a part of the IKEA Group. IoS is responsible for product R&D and global supply chain management within IKEA.⁴

1.2 Background

As part of the overall aim to meet customer success IKEA is gradually turning into a more process based organization and three main processes were defined in 2002. The supplying process' mission for customer success is to make the IKEA range available for customers by buying, producing and distributing it to the lowest cost possible. Achieving desired customer service levels is however the key objective while cost reductions through

¹ Kamprad, I. (2007) p. 8

² Peterson, L-G. (09-06-09)

³ www.ikea-group.ikea.com (09-06-09)

⁴ Björnsson, P. (09-06-04)

inventory control efficiency are a secondary focus, and hence desirable as long as customer service levels are retained. A key element for success in this mission is precision in planning and execution throughout the supply chain.⁵

One of the core processes in the Supplying process is Plan and Secure Supply, PSS, which objective is to execute decisions concerning inventory planning.⁶ An important aspect in this context is to statistically predetermine average demand during lead time to ensure efficient inventory control.

Today IoS manage this based on assumption of normal distribution on store level and lognormal distribution on DCG level. Two pre-studies, conducted by students at Lund University and Växjö University respectively, indicate that the current setup might not be optimal.^{7,8}

1.3 Problem Discussion

To be able to retain defined customer service levels companies use safety stock to compensate for variations in customer demand and lead time deviations. One risk with this is the accumulation of capital generated by excess safety stock. It is hence important to apply efficient and accurate tools and methods for determining appropriately sized safety stocks.

⁵ Björnsson, P. (09-06-04)

⁶ Ericson, B. (09-06-04)

⁷ Stjernfeldt, E. and Johansson, L. (2009)

⁸ Dahlberg-Hildén, A. et al. (2009)

The safety stock is directly related to the set re-order point. To determine re-order points, statistical distributions are often applied. When an inventory control system is implemented an important question is therefore the determination of which statistical distribution that provides most accurate safety stock.

Traditionally the statistical distributions are evaluated based on their general conformity with the empirical distribution of historical demand observations. An alternative concept could be to start backwards by determining the re-order point, based on the empirical observations, which will generate the pre-defined service. The statistical distributions could then be evaluated based on the deviation between the re-order points they suggest and the empirical ditto.

This concept would put the core problem of service retainment in focus since the evaluation would be based on performance deviation where it really matters. As in the world of sports, it does not matter who is in the lead during the first 89 minutes of a soccer game. The winner is the team with most goals after the 90th.

1.4 Objectives

This thesis aims to:

- Review and develop a robust method to find and evaluate the most appropriate statistical distribution (on STORE and DCG level) based on their ability to achieve pre-defined service based on a concept developed by Stig-Arne Mattsson. Present the outcome of the applied method. (Report Part I)

- Build a two-step simulation model to verify the method in Part I in comparison to current control. Conduct evaluation of measure frequency, indicated economical impact and lead time deviation sensitivity. (Report Part II)
- Suggest recommendations based on above results

1.5 Delimitations

The project is geographically limited to a pre-defined market. IKEA's representation on this market consists of 17 stores supported by one DCG.

As of today a significant share of IKEA's total goods flow is distributed through DCGs whereas the rest is delivered directly from supplier to store.⁹ This study will only focus on DCG goods flows, as illustrated in Figure 1.1.

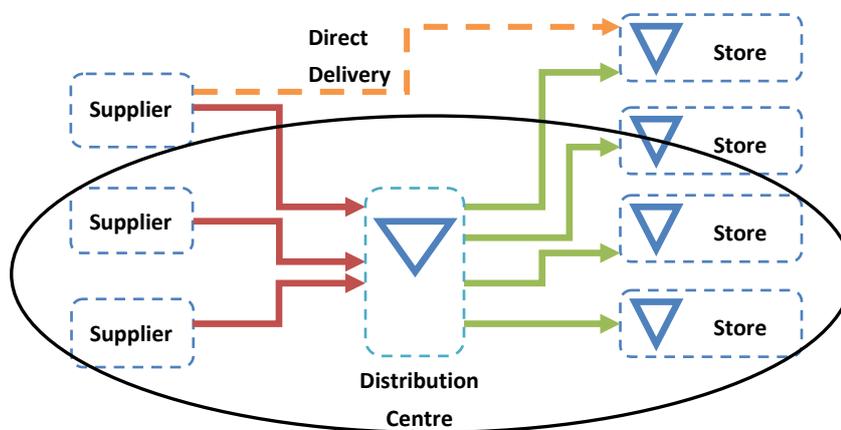


Figure 1.1 IKEA Supply chain overview

⁹ Goods Flow Advance (09-10-19)

Articles that are produced by, and sourced from, multiple suppliers have been excluded. Multiple sourcing implies two different lead times which increases model complexity. Since only seven percent of the selected articles are sourced from dual suppliers it is not considered time-efficient to include this dimension in the analysis.

IKEA's products are divided into several different ranges. This study will only include articles within the self-serve stock, which is replenished according to mathematical models.

Throughout this thesis sales data is approximated as demand data, why it further on is referred to as demand data.

The study evaluates the normal, gamma and lognormal distributions. The reason for this selection is that the normal and lognormal are the statistical distributions implemented at IKEA today. The gamma distribution is indicated as an interesting alternative in two pre-studies.^{10,11}

The articles are selected prior to the study by representatives from IoS to represent IKEA's range of products. The number of articles included in the study was initially 270 but have been reduced to 105 due to insufficient data. This procedure is explained in Chapter 4.

The upstream supply chain is divided into two levels which are handled individually, as illustrated in Figure 1.2 below. This is done in order to generate results and conclusions which can be implemented in IKEA's current inventory control systems.

¹⁰ Stjernfeldt, E. and Johansson, L. (2009)

¹¹ Dahlberg-Hildén, A. et al. (2009)

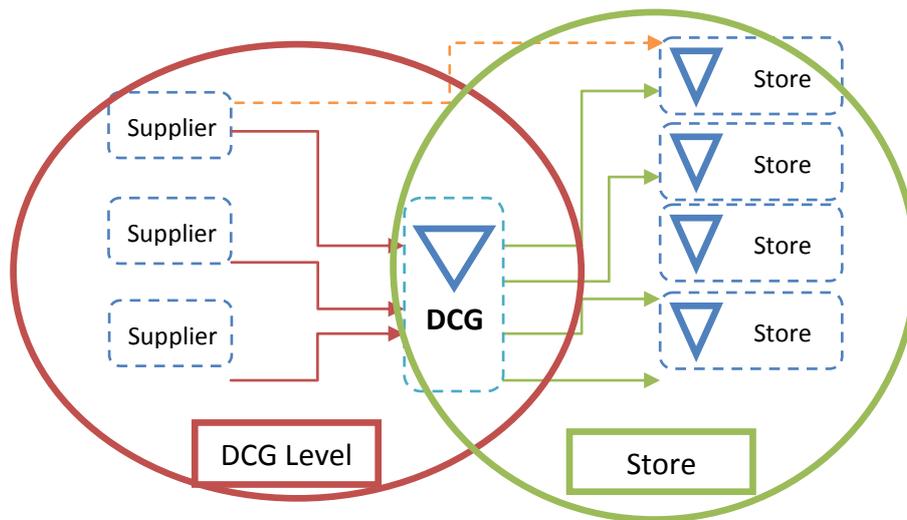


Figure 1.2 Supply Chain Levels

Because of hardware capacity limitations the number of articles analyzed in the dynamical simulation model is reduced through a sample selection. This implies a 98 % reduction of input parameter rows and makes it possible to perform necessary simulations and analysis within the project time frame.

1.6 Target audience

This thesis is directed to people connected to the Supplying process at IoS, engineering students and staff at the Department of Industrial Management and Logistics at Lund University, Faculty of Engineering.

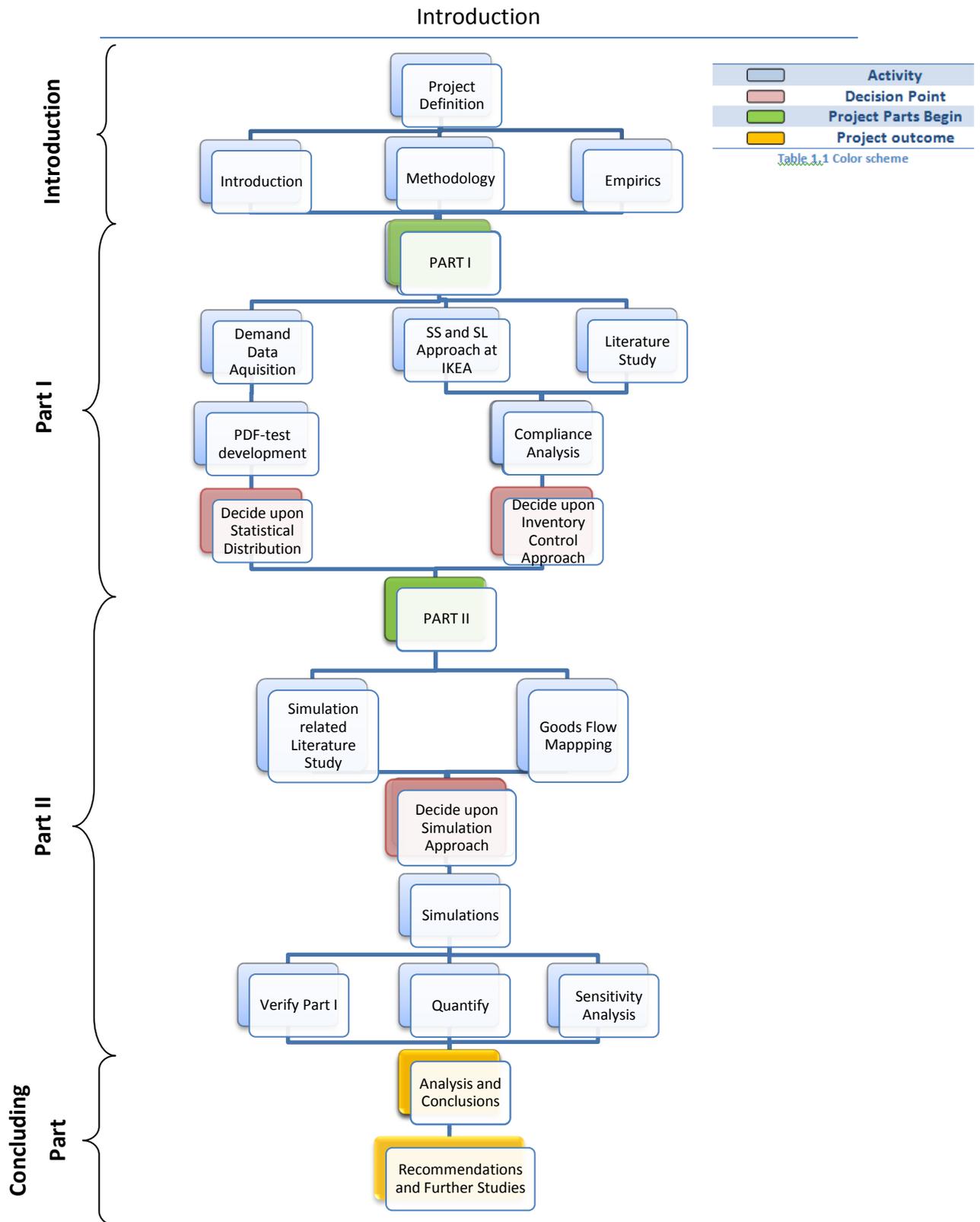
1.7 Tools

The demand data is analyzed using Microsoft Excel and simulation models are built and executed in simulation software Extend.

1.8 Notes to reader

The report is divided into four sections as illustrated in the model of procedure in Figure 1.3 below.

- The Introductory Part provides background and introductory information to the thesis. It also introduces the methodology and theory applied and the empirics that describe the situation at IoS today.
- Part I contains the static simulation, which is the method developed to determine the most appropriate statistical distribution based on ability to determine accurate re-order points. These tests are further on referred to as Probability Distribution Fit-tests or in short PDF-tests. Results and analysis summarize this part.
- Part II contains the dynamic simulation modelling and evaluation of the method in Part I. The results in Part I are quantified regarding the service levels and average inventory levels. Evaluations of measure frequency and lead time deviation sensitivity are also performed. Results and analysis summarize this part.
- The Concluding Part contains conclusions and recommendations of the project results and a discussion of overall reflections is presented.



2 Methodology

This chapter will introduce the methodology used for assessing the purpose of the thesis. The approach for collecting data will be introduced and the credibility discussed.

2.1 Deductive or inductive

Inductive and deductive are the two main approaches to render a picture of reality in a study.

A deductive study is based on a theoretical literature study in which a view and expectations of the future are rendered. Later, empirical observations are collected to verify or reject these biases. An inductive study is conducted in the opposite way, and is hence based on empirical observations which are the foundation for later creation and formulation of theory.¹²

Deduction is, considering the nature of the study, the most suitable description of this aspect of methodology in the report.

2.2 Quantitative or qualitative approach

Quantitative and qualitative are the two main techniques when assessing data and observations for a study. The objective of quantitative research is to develop and employ scientific models and theories on the topic examined. Measurement and observations is a central process of quantitative research since it provides the fundamental connection between empirical observations and theoretical models. Quantitative studies are most often based on relatively large random samples.¹³

¹² Merriam, S. B. (1994) p. 33

¹³ Jacobsen, D. I. (2002) p. 42

Qualitative research aims to gain deeper understanding of the studied topic and the underlying reasons for different events. The questions why and how are more central than what, where and when. Qualitative studies are typically based on smaller, but more focused, samples of observations.¹⁴

This study is primarily of quantitative nature and analyses a large amount of data observations in a statistical framework.

2.3 Credibility

Credibility is an expression describing to what extent the results of a study are reliable. In this thesis this measure is described in terms of validity, reliability and objectivity.¹⁵

2.3.1 Validity

Validity measures whether the results describes what it is intended to. High level of validity is reached by eliminating systematic errors and taking all essential parameters into consideration.¹⁶

This study can be regarded as valid from most perspectives. Models with solid theoretical foundations have consistently been chosen. Data has been acquired on first hand basis directly from IoS on request.

The original 270 articles provided from IoS are gathered from three different Business Areas, BA, in order to ensure sample variation. The articles have different characteristics. There are no indications of biased selection or tendencies for systematic loss of certain types of articles in the

¹⁴ Jacobsen, D. I. (2002) p. 42

¹⁵ Björklund, M. and Paulsson, U. (2003) p. 59

¹⁶ Ibid p. 59

reduction. Hence, the final sample of 105 articles is still considered to be representative for the BA:s and the validity of the method is maintained.

One aspect that decreases the level of validity is the necessary approximation that sales data represent actual demand in the models. There has been no feasible possibility to measure actual demand of self-serve articles. The reader should therefore be aware that this approximation has been done when the conclusions are evaluated.

Furthermore, the validity is decreased by the absence of historical sales data on a daily basis. This means that the lead time had to be approximated as seven days on store level in the static analysis in Part I, which generally implies a multiplication of 3.5 of the actual lead time. This presumption characterizes the evaluation of the demand distributions.

In Part II a simulation model is created. A discussion of the validity in this model is presented in Section 8.6.

2.3.2 Reliability

Reliability refers to the level of accuracy in a measurement. This gives an indication of to which extent the results would be equal if the study was repeated.¹⁷

Due to the vast amount of articles in IKEAs range a sample selection have been selected by IKEA representatives to serve as basis for this study. This selection represent around 1 % of the total range which implies that the study should be seen as an indication of the possible result of further studies.

¹⁷ Björklund, M. and Paulsson, U. (2003) p. 59

A sales volume classification of the articles is made based on number of sold pieces in order to divide the articles into three categories; low, medium and high. Classification limits are chosen to make the groups approximately equally sized. The purpose of this classification is to identify potential differences in demand patterns between different articles. The selected limits are only applicable for this specific selection of articles and are not to be seen as general guidelines.

2.3.3 Objectivity

The objectivity of a report is a measure of to what extent personal values and opinions of the authors or the subjects of interviews are reflected in the results.¹⁸

Compared to other common methods of evaluating statistical distributions the Probability Distribution Fit-tests, described and conducted in Part I, are more objective because of their foundation in the absolute deviation in generated re-order points. Alternative methods often rely on graphical or otherwise subjective ways of evaluation.

The models and theory applied are partly developed by the tutor assigned to this study, Stig-Arne Mattsson, Adjunct Professor in Supply Chain Management at the Department of Logistics and transport at Chalmers University of Technology, which might inflict a level of subjectivity in choice of methodology.

The selection of articles from the IKEA range has been made by IKEA representatives implying a risk for conditionality. There have however been no indications of such tendencies.

¹⁸ Björklund, M. and Paulsson, U. (2003) p. 60

2.4 Sources of information

Information presented in this thesis is collected from interviews, literature and electronic sources or based upon analysis of data acquired on first hand basis from IoS. All sources and references are presented in the bibliography.

2.4.1 Primary data

Primary data is data collected for the specific purpose of the actual study.¹⁹

Interviews

A series of interviews have been conducted with Stig-Arne Mattsson. These have primarily been concerning PDF-test methodology but have also covered various issues in the fields of inventory control and demand prognosis.

Further interviews have been conducted with Christian Howard, Extend simulation expert at the Department of Industrial Management and Logistics at Lund University, Faculty of Engineering. The interview gave valuable information about the approach towards building the simulation model and continuous contact was maintained during the whole creation process.

2.4.2 Secondary data

Secondary data is previously collected data presented in reports or literature. It is most often very time consuming to collect, interpret and analyze primary data, why the use of secondary data in most cases is to be preferred when applicable.²⁰

¹⁹ Björklund, M. and Paulsson, U. (2003) p. 68

²⁰ Ibid p. 67

Data

Sales data for the period March – 08 to March – 09 for the selected articles were acquired. The corresponding lead times, order quantities and service levels were also received. The data was collected by Birgitta Elmqvist, Project Leader at Supply Chain Planning, IoS.

Literature and reports

Scientific literature has been used to gain a deeper understanding within the fields of inventory control and demand forecasting. Literature related to the simulation software Extend has also been applied.

Electronic sources

The major difference between printed publications and electronic sources is that the latter are more elusive. It is easy to publish material on the Internet, and most of the material is not reviewed to the same extent. This implies that it is important to view material found online even more critical than in general.

2.4.3 Criticism of sources

When collecting primary as well as secondary data a critical attitude towards the sources is necessary. The collected data should be pre-viewed regarding the aspect of credibility and its relevance for the purpose. If the sources are not pre-viewed the collected data might be inapplicable, incorrect or misleading, which will affect the final result.²¹

Literature

Literature used in this thesis are published work from well known and acknowledge authors, why they are considered trustworthy. Some of the theories were published a long time ago, but they are still considered up to date and are frequently used in the industry.

²¹ Björklund, M. and Paulsson, U. (2003) p. 68

Electronic sources

The number of internet sources in this thesis has been limited in order to maintain good credibility. The primal sources have been IKEA's official website together with Wolfram's Mathworld. Both these are considered to provide accurate and reliable information. Furthermore, a limited number of statistical graphs have been collected online. These have been compared to corresponding graphs in literature.

2.5 Simulation software Extend

2.5.1 Model classification

A simulation model can be generally described through three classifications.²²

Static vs. Dynamic

In a static model time is insignificant whereas a dynamical is totally time-based.

Deterministic vs. Stochastic

A deterministic model presents output directly based on input. It is hence possible to derive input based on output through reverse-engineering. A stochastic model generates output through random events.

Discrete vs. Continuous

A discrete model jumps from one event to the next without intermediate change. A continuous model is modified throughout execution.

Thus, the simulation model developed to validate this study is dynamic, stochastic and discrete. This type of model is often created through a method called discrete-event simulation.

²² Laguna, M. and Marklund, J. (2005) p. 224

2.5.2 Model development

In order to create an accurate dynamic simulation model, much effort must be put at research of theory and empirics respectively to provide a sufficient knowledge basis to make necessary assumptions. A gradual overview of the development of the simulation is presented in this section. The approach used is influenced by Bank's 12-step guide to simulation which has been modified to suit the needs of this study.²³ The following steps have been used in this thesis;

1. Conceptual modelling

Before building a simulation model it is highly significant to define the purpose and expected functionality of the simulations. A conceptual model is a graphical draft defining input parameters, functional layout and outputs of the model.

2. Model programming

The conceptual model is translated and built in the simulation software Extend.

3. Input data

All necessary input data are observed and calculated in the first part of the study. During this step the data are organized in a spreadsheet linked to Extend in order to enable automate simulation runs.

4. Output data

The outputs from the simulation are defined.

²³ Banks, J. (1998) pp. 15-18

5. Validation

The simulation model is technically and conceptually audited to guarantee functionality.

6. Experiment design

The tests that are to be performed are defined.

7. Simulation

Simulations are performed.

8. Results

Results are compiled and presented

9. Analysis

Analysis of the results

3 Theoretical framework

In this chapter relevant theory concerning inventory control, statistical distributions and the simulation software Extend will be presented.

3.1 Table of parameter abbreviations

Table 3.1 introduces and explains terms and abbreviations that are needed to present the theories in this chapter.

Table 3.1 Table of parameters and abbreviations used in this chapter

Abbreviation	Definition
SS	Safety stock
σ_{LT}	Variation in lead time
σ_D	Variation in customer demand during lead time
μ_{LT}	Mean demand during lead time
R	Re-order point
Q	Order quantity
k	Safety factor
$\Phi()$	Probability density function of a statistical distribution < 1
IP	Inventory position
IL	Inventory level
SL	Service level
CV	Coefficient of variation
r	Coefficient of correlation

3.2 Safety factor

The safety factor is used to illustrate the relation between the safety stock and the standard deviation of demand during lead time. This is the variable used for distribution comparison in the PDF-test. The k-value is calculated as in Equation 3.1 below.²⁴

$$k = \frac{R - \mu_{LT}}{\sigma_D} = \frac{SS}{\sigma_D}$$

Equation 3.1

²⁴ Lumsden, K. (2006) p. 312

For the normal distribution the operation in Equation 3.1 corresponds to standardization of the parameters, which implies that the k-value for this distribution can be calculated according to Equation 3.2.²⁵

$$SL = P(X < R) = \Phi\left(\frac{R - \mu}{\sigma}\right) = \Phi(k) \rightarrow$$
$$\rightarrow k = \Phi^{-1}(SL)$$

Equation 3.2

The normal distribution generates k-values which are independent of time period. In order to calculate k-values during lead time for the lognormal and gamma distributions the parameters are scaled using Equation 3.3 and Equation 3.4.²⁶

$$\mu_{LT} = \mu_{week} \times \frac{LT(days)}{7}$$

Equation 3.3

$$\sigma_{LT} = \sigma_{week} \times \sqrt{\frac{LT(days)}{7}}$$

Equation 3.4

3.3 Safety stock definitions

Safety stock is defined as extra units of stock carried as protection against unforeseen events causing insufficiencies in the cycle stock. The two primary causes for this are variations in lead time and in the customer demand during lead time. By carrying an adequately sized safety stock a company compensates for these variations without affecting customer service and the need to place expensive express orders is diminished.²⁷

²⁵ Blom, G. et al. (2005) p. 147

²⁶ Ibid p. 228

²⁷ Lumsden, K. (2006) p. 312

3.3.1 Safety stock calculations

Determination of the safety stock size is either based on the shortage cost or the service level. Companies more often apply service based inventory control due to the complexity in determining the shortage cost.²⁸ The safety stock can be calculated as seen in Equation 3.5, which is a conversion of Equation 3.1, given a safety factor representing the degree of service. Lead time is assumed to be constant.²⁹

$$SS = R - \mu_{LT} = k * \sigma_D$$

Equation 3.5

3.4 (R, Q) – Policy

The (R,Q)-policy is a re-order point system that is commonly used in the industry. When the inventory position reaches the re-order point a constant quantity is ordered, as illustrated in Figure 3.1. In case of single piece demand during continuous review orders will always be placed exactly on the re-order point. If, on the other hand, review is periodic or more than one piece is bought at a time the inventory level will often drop below the re-order point before the order is triggered. The average inventory level is calculated as in Equation 3.6 below.³⁰

²⁸ Axsäter, S. (1991) p. 68

²⁹ Lumsden, K. (2006) p. 312

³⁰ Axsäter, S. (2006) p. 48

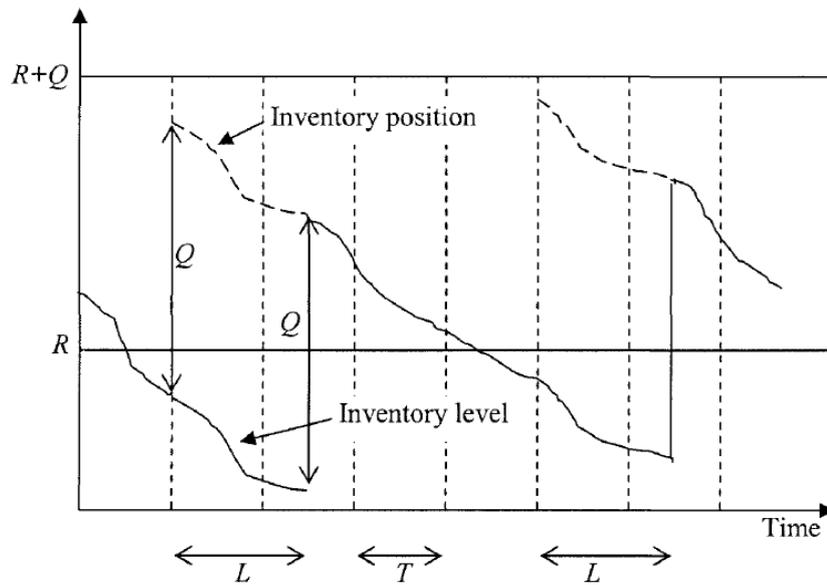


Figure 3.1 (R,Q)-policy with periodic review and continuous demand.³¹

$$\text{Average IL} = \frac{R + Q - \mu_{LT} + R - \mu_{LT}}{2} = SS + \frac{Q}{2}$$

Equation 3.6

3.5 Service level definitions

There are three different frequently used definitions of service level; $SERV_1$, $SERV_2$ and $SERV_3$.

$SERV_1$

$SERV_1$ = Probability of no stock-out per inventory cycle³², which is defined in Equation 3.7.³³

$$SERV_1 = \Phi\left(\frac{SS}{\sigma_D}\right) = \Phi(k)$$

Equation 3.7

³¹ Axsäter, S. (2006) p. 48

³² Axsäter, S. (1991) p. 68

³³ Axsäter, S. (2006) p. 94

SERV₁ is a popular service level measure, much because of the simple calculations. One important advantage is that it is aligned with the customer perspective of service since it measure probability of stock-out rather than shortage quantity.³⁴

The measure is not affected by order quantity, meaning that two articles *cetiris paribus* but with different minimum order quantities will be assigned equal safety stocks in order to maintain a certain customer service level. The service experienced by customers would however differ since the order quantities affects average inventory levels, as seen in Equation 3.6. Articles with smaller order quantities and therefore shorter stock cycles will hence suffer from stock-outs more frequently.³⁵

SERV₂

SERV₂ = "Fill rate"- fraction of demand that can be satisfied immediately from stock on hand.³⁶

SERV₂ is calculated through Equation 3.8 below.³⁷

$$SERV_2 = 1 - \frac{\sigma_D}{Q} * \left(G\left(\frac{SS}{\sigma_D}\right) - G\left(\frac{SS+Q}{\sigma_D}\right) \right)$$

Equation 3.8

³⁴ Axsäter, S. (1991) p. 68

³⁵ *Ibid* p. 68

³⁶ Axsäter, S. (2006) p. 94

³⁷ Axsäter, S. (1991) p. 72

The loss function $G(x)$ used in Equation 3.8 is defined as in Equation 3.9.³⁸

$$G(x) = \int_x^{\infty} (v - x)\varphi(v) dv = \varphi(x) - x(1 - \Phi(x))$$

Equation 3.9

$$G'(x) = \Phi(x) - 1$$

Equation 3.10

Equation 3.10 shows that $G'(x)$ is negative and increasing. Consequently, $G(x)$ is convex and decreasing which can be seen in Figure 3.2.³⁹

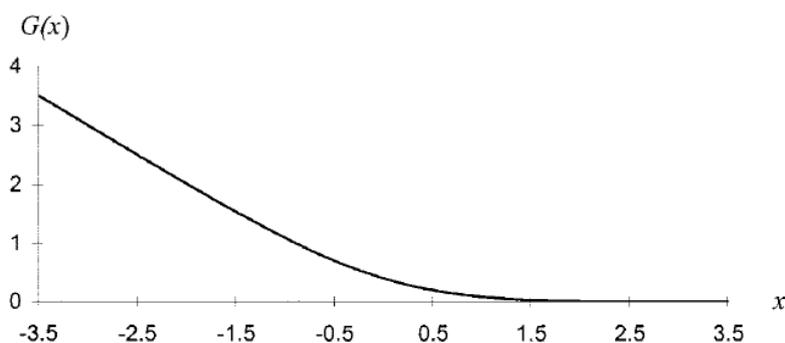


Figure 3.2 Loss function $G(x)$.⁴⁰

When function $G\left(\frac{SS+Q}{\sigma_D}\right)$ is relatively small the term is negligible and $SERV_2$ can be defined as in Equation 3.11.⁴¹

$$SERV_2 = 1 - \frac{\sigma_D}{Q} * G\left(\frac{SS}{\sigma_D}\right) = 1 - \frac{\sigma_D}{Q} * G(k)$$

Equation 3.11

³⁸ Axsäter, S. (2006) p. 91

³⁹ Ibid p. 92

⁴⁰ Ibid p. 92

⁴¹ Axsäter, S. (1991) p. 72

A common condition for this approximation is $\frac{SS+Q}{\sigma_D} > 1.94$ which gives $G\left(\frac{SS+Q}{\sigma_D}\right) < 0.01$, see Figure 3.2 above.

Unlike $SERV_1$ the $SERV_2$ measure account for order quantity which allows estimation of shortage quantities. The calculations are more complex than for $SERV_1$, making this definition less frequently used in the industry. Despite this, $SERV_2$ is although widely considered more appropriate among inventory control theorists.⁴²

The size of the order quantity has a large impact on the service level. The smaller the quota between the standard deviation and the order size is, the higher the service level gets as illustrated in Figure 3.3.⁴³

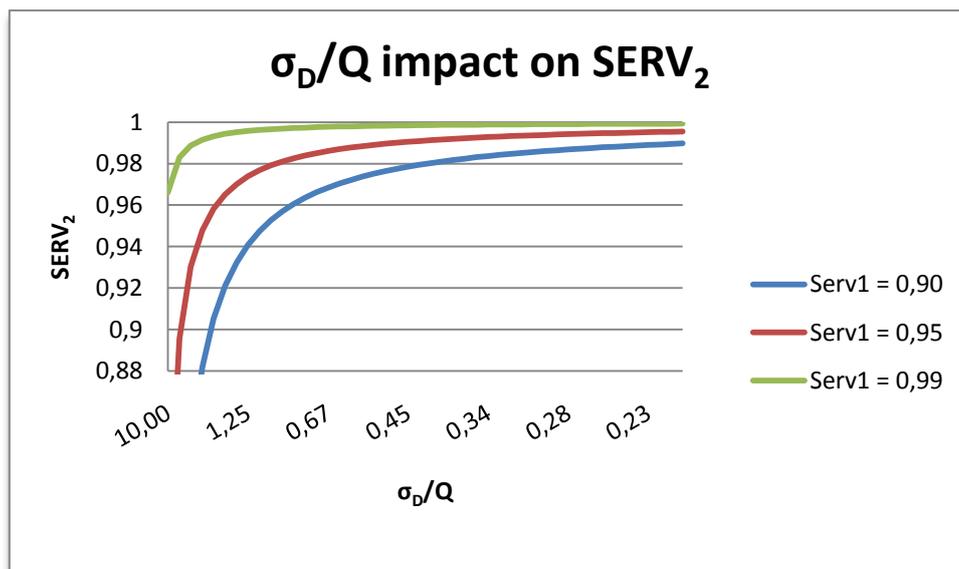


Figure 3.3 The quotation $\frac{\sigma_D}{Q}$ impact on $SERV_2$

⁴² Mattsson, S-A. (2007) p. 3

⁴³ Axsäter, S (1991) p. 72

SERV₃

SERV₃ = "ready rate"- fraction of time with positive stock on hand.⁴⁴

This definition can only be used as a measurement and not to control the inventory. SERV₃ is defined as the mean of numerous accumulated observations noted as zero (0) if the stock is empty and one (1) if it is not. If there is something in stock at all inspections the service is considered to be 100 %, if there is no stock-out at four out of five times the service level is 80% etc. Generally accuracy increases with observation frequency.⁴⁵

3.5.1 Relations between service level measures

Equation 3.12, derived from Equation 3.11, serves as an approximation of the relation between SERV₁ and SERV₂.⁴⁶

$$SERV_2 = 1 - \frac{\sigma_D}{Q} * G(\Phi^{-1}(SERV_1))$$

Equation 3.12

SERV₃ is identical to SERV₂ in the case of continuous demand or Poisson demand and continuous review and can therefore serve as a measurement of the actual service under these premises. In cases of periodical review or discrete demand the measures are still familiar and can be considered approximately equal within reasonable limits. If it is possible for one customer to buy several pieces simultaneously the relation between the measures weakens further. Measure correlation does also decrease with increased periods in between reviews. This reduction is explained by stock-outs occurring during the inter-inspection periods and which will hence not be noted in the SERV₃ measure. Because of this, it is important to denote the inspection interval when a SERV₃ measure is presented as achieved

⁴⁴ Axsäter, S. (2006) p. 94

⁴⁵ Björnsson, P. (09-06-18)

⁴⁶ Axsäter, S. (1991) p. 74

level of $SERV_2$. The practical interpretation of continuous review varies but measures on order line basis are the most common.⁴⁷

3.5.2 Factors influencing the safety stock

A given level of safety stock accumulates a corresponding amount of capital. This creates a situation where the degree of inventory service level offered has to weigh against the costs the stock entail and to which extent these can be embedded in product prices.⁴⁸

The accumulation of capital through safety stock increases exponentially with increased level of service, as can be seen in

Figure 3.4 - Figure 3.6. The quota between the mean value and standard deviation affects the safety factors for the gamma and lognormal distribution, as illustrated in Figure 3.5 - Figure 3.6. This reasoning is based on the relation between k-values and safety stock defined in Equation 3.5.⁴⁹

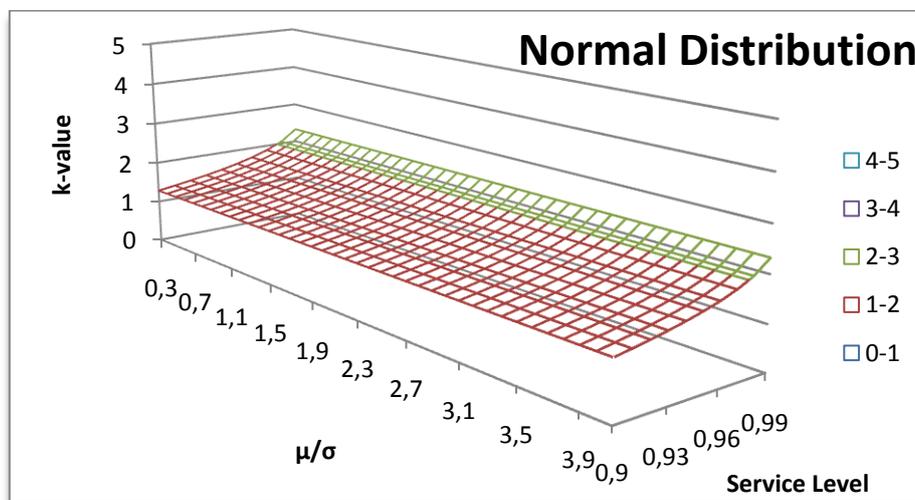


Figure 3.4 K-value as a function of the service and the quota μ/σ

⁴⁷ Axsäter, S. (2006) p. 95

⁴⁸ Axsäter, S. (1991) pp. 68-69

⁴⁹ Mattsson, S-A. (09-09-03)

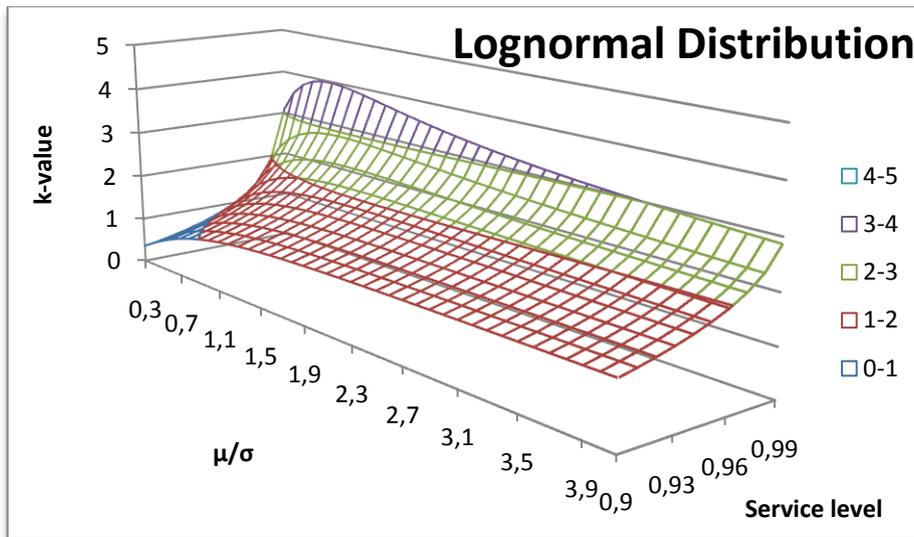


Figure 3.5 K-value as a function of the service and the quota μ/σ

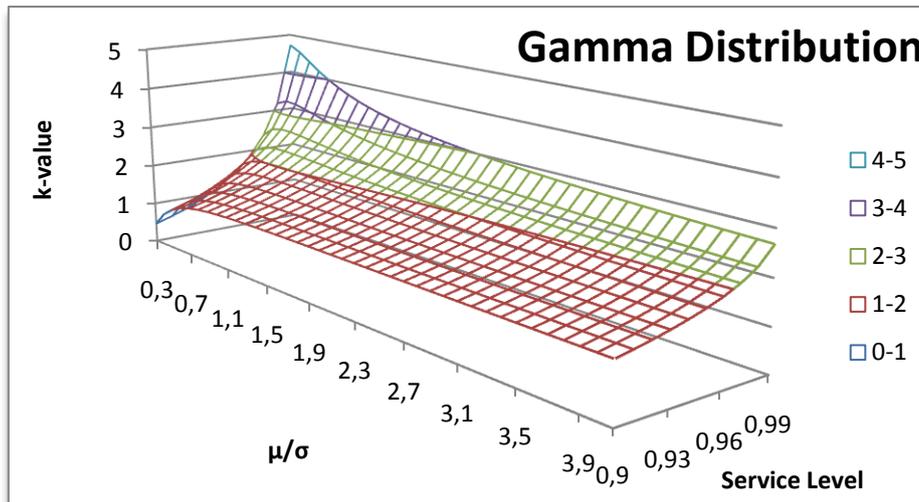


Figure 3.6 K-value as a function of the service and the quota μ/σ

3.6 Statistical Distributions

Statistical distributions are either discrete or continuous, while customer demand can be considered as discrete.

Opinions about the importance of using the correct distribution when estimating the demand differ. Some recognized researchers such as Wilkingson together with Silver and Peterson advocate that it is almost always valid to assume a normal distribution. Lau, Zotteri and Mattsson assert that this assumption will generate vast economical inefficiencies because of deviations in between achieved and desired service levels.⁵⁰

3.6.1 Different methods to evaluate statistical distribution accuracy

Statistical distribution evaluation can be performed through a number of different approaches with the common goal to find the distribution that most accurate describes the empirical data. A common prerequisite is that the data needs to be uncorrelated.⁵¹

Graphical methods use graphs to plot the empirical distribution against the statistical distributions that are being tested. The distribution which is most similar in shape to the empirical distribution is considered to be the best fit. Goodness-of-fit tests are a family of statistical hypothesis tests used to assess whether a number of observations are an independent sample of a certain distribution. The tests either confirm that a statistical distribution is similar to the empirical distribution or not.⁵² These tests are subjective and dependent on personal evaluations. A new method, the Probability Distribution Fit – test, PDF-test, has been developed by Stig-Arne Mattsson to objectively compare distributions. This method has been further developed during this project and is thoroughly explained in Section 5.3.⁵³

⁵⁰ Mattsson, S-A. (2003b) pp. 1-2

⁵¹ Kelton, W.D. and Law A. M. (2000) p. 329

⁵² Kelton, W.D. and Law A. M. (2000) p. 356

⁵³ Mattsson, S-A. (09-06-10)

3.6.2 Empirical distribution

The empirical distribution is a series of true observations in opposite to the theoretical distributions, which represents approximations of the reality. The empirical distribution is limited to the data observed in a study while the theoretical distributions can generate new data based on parameters describing the empirical data. This fact makes the empirical distribution more demanding when it comes to the amount of collected data that is needed.⁵⁴

The Boot Strapping method

The Boot strapping method is used to extend the number of observations by generating more based on randomly combining existing observations. This method is also applicable to estimate demand during lead time, when the lead time exceeds the time period on which the data is collected and hence can be expressed as a multiple of such periods.⁵⁵

The boot strapping procedure in practice:⁵⁶

- Input data, consisting of the original observations, is collected and placed in a vector
- Randomly a number of data points from the vector, corresponding to the length of the lead time, are accumulated.
- The procedure is repeated until a desired amount of data points have been generated.

The random selection of data points is conducted according to the principle of sampling with replacement, which implies that all observations are

⁵⁴ Kelton, W.D. and Law A. M. (2000) p. 301

⁵⁵ Mattsson, S-A. (09-07-21)

⁵⁶ Bookbinder, J. and Lordahl, T. (1989) pp. 302-312

equally likely to be selected in each draw. A larger amount of draws implies a more representative distribution of generated “lead time demands”.⁵⁷

3.6.3 Normal Distribution

The normal distribution is a continuous probability distribution that describes data clustered around a mean value with a certain standard deviation. Parameters are estimated through Equation 3.13 and Equation 3.14.⁵⁸

$$\mu = \frac{\sum_{i=1}^n x_i}{n}$$

Equation 3.13

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^n |x_i - \mu|^2}$$

Equation 3.14

The probability density function adopts a symmetrical bell-shape, peaking at the mean.⁵⁹ It is of importance to notice that the normal distribution is defined for both positive and negative values, implying a risk for negative demand during lead time in inventory control contexts, as illustrated in Figure 3.7.⁶⁰

⁵⁷ Smart, C. and Willemain, T. (2000) pp. 64-68

⁵⁸ Blom, G et al. (2005) p. 228

⁵⁹ Sheldon, R. (1989) p. 35

⁶⁰ Axsäter, S. (2006) p.85

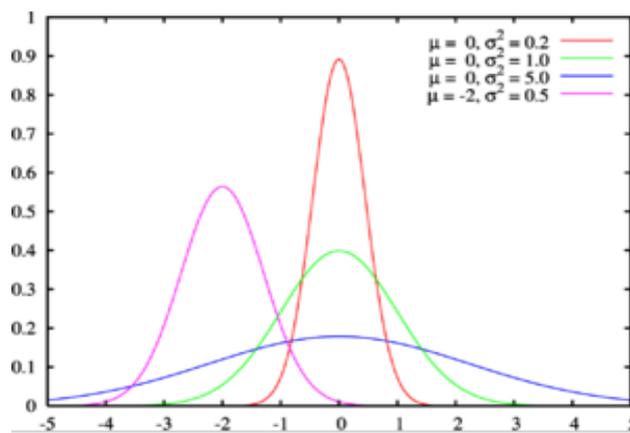


Figure 3.7 The probability density function of the normal distribution⁶¹

The normal distribution is fairly general and can according to the central limit theorem be applied in most cases provided that the parameters are estimated from a sufficient number of independent observations.⁶² As can be seen in Figure 3.4 in Section 3.5.2 the normal distribution does not account for the relation between the mean value and the standard deviation when calculating the safety stock. Furthermore, the sum of many independent normally distributed observations is also normally distributed.⁶³

3.6.4 Lognormal distribution

The lognormal distribution is closely related to the normal distribution and most properties can therefore be derived from the normal distribution.⁶⁴ A random variable Y is said to be lognormal distributed, with parameters μ_{LN} and σ_{LN} as defined in Equation 3.15 and Equation 3.16 below, if $X = \ln(Y)$ is normal distributed with parameters μ_x and σ_x , as defined in Equation 3.13 and Equation 3.14 above.⁶⁵

⁶¹ www.mathworld.wolfram.com (09-09-01)

⁶² Kelton, W.D. and Law A. M. (2000) p. 254

⁶³ Ibid p. 254

⁶⁴ www.mathworld.wolfram.com (09-09-01)

⁶⁵ Ibid (09-09-01)

$$\mu_{LN} = LN(\mu_x) - 0.5 * LN\left(1 + \frac{\sigma_x^2}{\mu_x^2}\right)$$

Equation 3.15

$$\sigma_{LN} = \sqrt{LN\left(1 + \frac{\sigma_x^2}{\mu_x^2}\right)}$$

Equation 3.16

An important difference from the normal distribution is that the lognormal distribution is bounded by zero to the left; hence there is no risk for negative values, as seen in Figure 3.8.⁶⁶

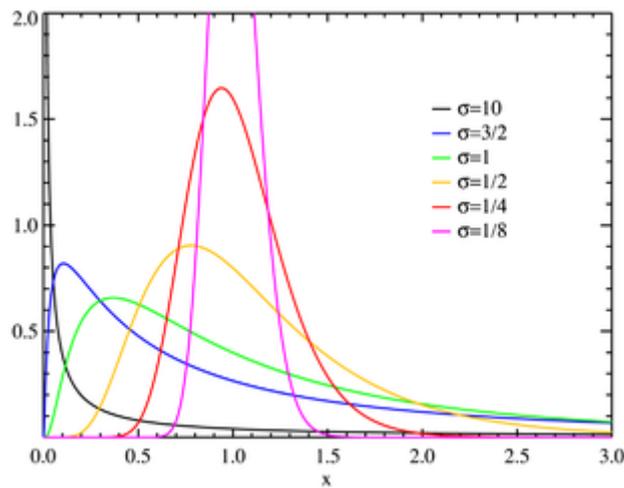


Figure 3.8 The probability density function of the lognormal distribution⁶⁷

When using the lognormal distribution for safety stock calculations it accounts for the relation between the mean value and the standard deviation, which can be seen in Figure 3.5 in Section 3.5.2. The sum of several independent lognormal distributed observations is not lognormal distributed.⁶⁸

⁶⁶ Axsäter, S. (2006) p.88

⁶⁷ www.mathworld.wolfram.com (09-09-01)

⁶⁸ Ibid (09-09-01)

3.6.5 Gamma distribution

The gamma distribution is a continuous distribution bounded by zero to the left and unbounded on the right; hence there is no risk for negative demand, as illustrated in Figure 3.9 below. The probability for very high demand is larger than for the normal distribution.⁶⁹

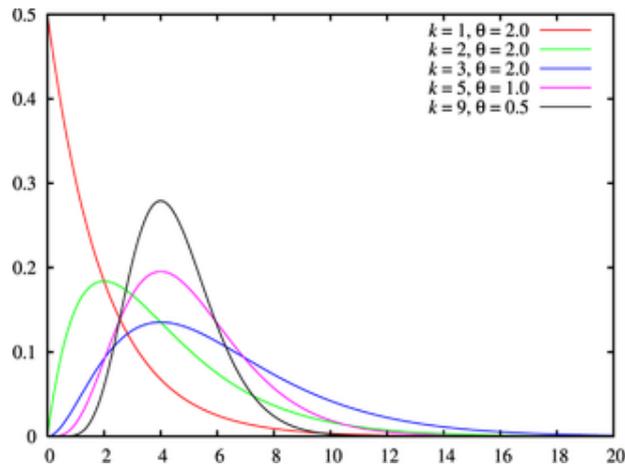


Figure 3.9 The probability density function of the gamma distribution⁷⁰

The gamma distribution is defined by a scale parameter, α , and a shape parameter, β . The estimation of the mean and standard deviation is presented in Equation 3.17 and Equation 3.18 below. It is very flexible and can be used for a wide variety of purposes. It is often used to render inter-arrival times and life times but has also proven to be accurate in inventory control models.⁷¹

$$\mu_x = \frac{\beta}{\alpha}$$

Equation 3.17

⁶⁹ Mattsson, S-A. (2007) p. 10

⁷⁰ www.mathworld.wolfram.com (09-09-01)

⁷¹ Mattsson, S-A. (2007) p. 10

$$\sigma_x = \frac{\sqrt{\beta}}{\alpha}$$

Equation 3.18

When using the gamma distribution for safety stock calculations it accounts for the relation between the mean value and the standard deviation, which can be seen in Figure 3.5 in Section 3.5.2. Furthermore the sum of many independent gamma distributed variables with the same shape parameter is also gamma distributed.⁷²

3.6.6 Poisson distribution

The Poisson distribution is a discrete distribution that is frequently used when studying phenomenon that occur randomly in a given time or area.⁷³ The Poisson distribution is not defined for negative values, hence there is no risk for negative demand as seen in Figure 3.10 below.

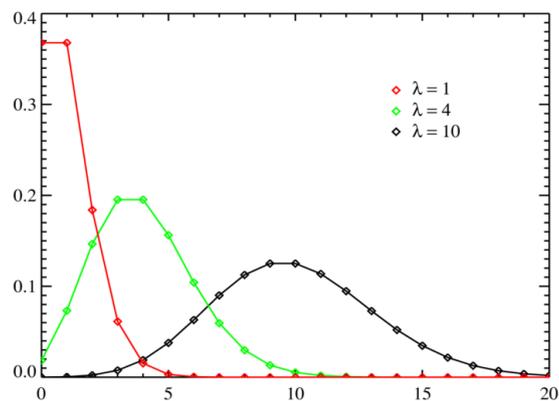


Figure 3.10 The probability density function of the Poisson distribution⁷⁴

⁷² www.mathworld.wolfram.com (09-09-01)

⁷³ Blom, G. (2005) p. 180

⁷⁴ www.wikipedia.org (a) (09-11-03)

The mean value is the sum of the discrete random variable X multiplied with its probability mass function $p(x)$, as seen in Equation 3.19 and is also referred to as lambda.⁷⁵ The standard deviation is the square root of the mean value, as seen in Equation 3.20 below.⁷⁶

$$\mu = \sum_{i=0}^{\infty} x_i * p(x_i) = \lambda$$

Equation 3.19

$$\sigma = \sqrt{\mu}$$

Equation 3.20

The sum of many independent poisson distributed variables is also poisson distributed.⁷⁷

3.7 Statistical concepts

3.7.1 Coefficient of variation

The coefficient of variation, CV, illustrates the relative spread of the demand and hence the risk for negative values when applying the normal distribution. The CV is defined as in Equation 3.21 below.⁷⁸

$$CV = \frac{\sigma_D}{\mu_{LT}}$$

Equation 3.21

If the mean value of the demand during lead time is much greater than its standard deviation, the CV is small and the probability for negative demand

⁷⁵ www.wikipedia.org (b) (09-11-03)

⁷⁶ Blom, G. (2005) p. 182

⁷⁷ Ibid p. 183

⁷⁸ Kelton, W.D. and Law, A.M. (2000) p. 333

is relatively low.⁷⁹ However, shorter stock cycles or greater variation of demand during these entails increased risk for generation of negative demand when the normal distribution is applied.⁸⁰

3.7.2 Exponential smoothing

Exponential smoothing is a technique in statistics that can be applied on time series of data. The result is a smoother series of data, which often is desirable in forecasting. Historical observations, s_{t-1} , are assigned an exponentially decreasing weight, $(1-\alpha)$. High levels of α implies more weight on the latest observation while lower levels implies more weight on historical observations, as can be seen in Equation 3.22 and Equation 3.23.⁸¹

$$s_0 = x_0$$

Equation 3.22

$$s_t = \alpha x_t + (1 - \alpha)s_{t-1}$$

Equation 3.23

3.7.3 Coefficient of Correlation

The coefficient of correlation, r , is a measure which indicates the strength and direction of the relation in between two random variables. The coefficient is calculated according to Equation 3.24, which is based on the co-variance c_{xy} calculated in Equation 3.25.⁸²

⁷⁹ Mattsson, S-A. (2003b) p. 9

⁸⁰ Mattsson, S-A. (2006) p. 250

⁸¹ Axsäter, S. (1991) p. 19

⁸² Blom. G (2005) p. 234

An increasing total dependency generates $r = 1$, a decreasing $r = -1$. Other relations generate values in between. The coefficient $r = 0$ indicates totally independent variables.⁸³

$$r = \frac{c_{xy}}{s_x \times s_y}$$

Equation 3.24

s_x and s_y are the standard deviation of the sample selections.

$$c_{xy} = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$$

Equation 3.25

3.7.4 Confidence Interval

A confidence interval is an interval defined to cover the value of a stochastic variable with a certain probability. Assume a random, independent sample $X = (X_1, X_2, \dots, X_n)$ of the normal distributed, stochastic, variable θ with mean μ' and standard deviation σ' . Interval I_θ is defined to cover μ' with the probability $1-\alpha$ based on the estimated parameters. This is expressed in Equation 3.26 below.⁸⁴

$$I_\theta = \mu' \pm \phi^{-1} \left(1 - \frac{\alpha}{2} \right) \times \frac{\sigma'}{\sqrt{n}}$$

Equation 3.26

⁸³ Blom. G (2005) p. 234

⁸⁴ Ibid pp. 287-289

3.8 Extend Simulation

Extend is a simulation software which is used to analyze and design business processes. It provides a graphical modelling environment where pre-programmed blocks represent different activities. The different blocks are interconnected and form a model of the reality. User defined parameters are assigned to the blocks, which make it easy to generate and compare different scenarios. All system variables are updated after each event and a timer is run to trigger the next step.⁸⁵

3.9 Simulating demand

In order to perform simulation based tests of inventory control it is of great importance to generate realistic demand patterns in the model. A common assumption is that non-decreasing stochastic processes with stationary and mutually independent increments serves as a good approximation of cumulative demand in stochastic inventory models. This type of process can be modelled by an appropriate sequence of compound Poisson processes.⁸⁶

3.9.1 Compound Poisson processes

Compound Poisson processes are often applied to generate customer demand. Customers are assumed to arrive according to a Poisson process and buy a stochastic quantity of an article. There are different approaches to estimate distribution parameters but one decision rule is that compound Poisson demand with a logarithmic compounding distribution is applicable when $\frac{(\sigma')^2}{\mu'} > 1.1$ for the empirical data. Under this assumption, parameters for arrival and quantities are based on observations according to the parameters α and λ in Equation 3.27 and Equation 3.28.⁸⁷

⁸⁵ Laguna, M. and Marklund, J. (2005) p. 226

⁸⁶ Axsäter, S (2006) p. 77

⁸⁷ Ibid p. 81

$$\alpha = 1 - \frac{\mu_{LT}}{\sigma_{D(LT)}}$$

Equation 3.27

$$\lambda = -\mu_{LT} \frac{(1 - \alpha) \ln(1 - \alpha)}{\alpha}$$

Equation 3.28

The α -parameter is used to generate the quantity demanded by one customer according to a logarithmic process. The λ -parameter is the parameter for Poisson distributed customer arrivals. Inter-arrival times are hence exponentially distributed with mean $\frac{1}{\lambda}$.⁸⁸

⁸⁸ Axsäter, S (2006) p. 81

4 Empirics

In this chapter the service level approach and measurement frequency at IoS today is presented as well as the inventory control models that are being used. The data gathered for this thesis is discussed and the sorting of the data is explained.

4.1 Service levels at IKEA

IKEA has a traditional focus on minimized total costs to be able to provide home furniture to the many people. Meanwhile, inventory service level measures are prioritized KPIs since insufficient availability generates lost sales and unsatisfied customers. This is particularly important for IKEA since the customers' perspective of service is closely related to article availability rather than inter-personal perspectives. A natural consequence of this is an aspiration to minimize capital accumulation generated by excess safety stock while maintaining inventory service levels.⁸⁹

Today IoS manages inventory control through $SERV_1$ but measures using the $SERV_3$ definition. Measurements are conducted for each article once a week and the results from all stores are accumulated to determine the service level.⁹⁰

4.2 Safety stock policies at IKEA

IoS are controlling inventory replenishment based on an (R,Q)-policy under the assumptions of normal distributed demand on store level and lognormal distributed demand on DCG level. This is done through applying the so-called statrules 4 and 5 in the inventory control systems. The desired service level generates a re-order point which gives a safety stock

⁸⁹ Björnsson, P. (09-06- 18)

⁹⁰ Ibid (09-06- 18)

according to Equation 4.1, equivalent to Equation 3.5 presented in Section 3.3.1.⁹¹

$$SS = k \times \sqrt{\text{Var}(D)}$$

Equation 4.1

4.3 Lead time deviation measure at IKEA

The lead time measure used at IKEA only notes orders arriving later than expected, i.e. orders arriving earlier than expected is considered punctual.⁹² This is exemplified in Table 4.1 below.

Table 4.1 Lead time deviation measure at IKEA

Planned LT	30	30	30
Actual LT	30	32	28
LT deviation	0	2	0

4.4 Sales data

The following data has been gathered by IoS prior to this study;

- Sales data for 270 articles during the period March 2008 - March 2009
 - Store level
Weekly sales in 17 stores during the period
 - DCG level
Outbound delivery records on DCG level

⁹¹ Björnsson, P. (09-05-19)

⁹² Ibid (09-09-22)

For each of the articles the following attributes were requested;

- Lead time data in days
 - Supplier → DCG
 - DCG → Store

- Minimum order quantity
 - Store level
 - DCG level

- Number of min order quantities ordered each time, which are predetermined by IoS
 - Store level
 - DCG level

- Service Level classification
 - The same on store and DCG level

- Price level classification
 - The same on store and DCG level

The sales data observations are assumed to be uncorrelated based on tests on similar data conducted in the two pre-studies.^{93,94}

⁹³ Stjernfeldt, E. and Johansson, L. (2009)

⁹⁴ Dahlberg-Hildén, A. et al. (2009)

4.5 Data sorting

4.5.1 Primary data handling and selection

In order to generate a valid statistical analysis the following criteria are applied on the input data;

1. Sufficient sales data (Observations > 50% of the weeks)
 - 215 articles qualified
 - 55 were discarded
 - Incomplete data records (scattered gaps)
 - Ramp-up products (clustered gaps in the beginning of the period)
 - Ramp-down products (clustered gaps in the end of the period)

Complete data regarding lead time, minimum order quantity, total order quantities and service levels are required for each article. An extract from the compilation of these parameters is found in Appendix I. Articles for which any of these parameters are unavailable have been discarded from the study.

2. Complete article parameters
 - 113 articles qualified
 - 102 were discarded

Out of these articles a relatively small number are sourced from multiple suppliers. Because of the increased complexity in analysis and simulation in relation to the number of articles concerned these articles have been discarded.

3. No multiple sourcing
 - 105 articles qualified
 - 8 were discarded

In approximately 1 % of the observations the number of returns exceed sales, resulting in negative sales figures for that week. Because of the approximation that sales equals demand, these observations have been set to zero since the phenomenon negative demand is not defined. This approach is feasible since the number of returns can be considered to be randomly distributed over the period but are for most weeks disguised by a greater amount of sales. The presence of returns is therefore accounted for in the demand distributions. Weeks with negative sales can hence be seen as weeks with low or zero sales and a stochastic amount of returns.

4.6 Article classification

To provide deeper analysis the articles have been classified regarding three different aspects.

The desired $SERV_1$ levels are determined by IKEA and are set to 90, 95 and 99%, depending on article.⁹⁵ Furthermore IKEA has made a three-level price classification of the articles, based on the article store prices.⁹⁶

The average sales volume in pieces per article and week is calculated individually on store level and on DCG level. Three intervals have been defined on each level so that the number of pieces is approximately equally distributed among them. The intervals are presented in Table 4.2 and Table 4.3 below.

⁹⁵ Björnsson, P. (09-06-18)

⁹⁶ Elmqvist, B. (09-06-20)

Table 4.2 Classification of sales volumes in pieces on store level

Sales volumes	Pieces per week = X
Low	$X < 4$
Medium	$4 \leq X < 15$
High	$X \geq 15$

Table 4.3 Classification of sales volumes in pieces on DCG level

Sales volumes	Pieces per week = X
Low	$X \leq 65$
Medium	$65 < X < 220$
High	$X \geq 220$

The number of articles in each classification is presented in Table 4.4 on store level and in Table 4.5 on DCG level. The distribution of articles among the levels is considered even for all three classifications.

Table 4.4 Distributions of articles within classification, Store level

	Service Level	Price Level	Sales Volume
90% / Low	23	32	40
95% / Medium	36	43	33
99% / High	46	30	31

Table 4.5 Distribution of articles within classification, DCG level

	Service Level	Price Level	Sales Volume
90% / Low	23	32	34
95% / Medium	36	43	36
99% / High	46	30	35

Part I

5 Method to evaluate statistical distributions

In this chapter the different steps in the PDF-test are described and the results and analysis are presented.

5.1 Introduction

The working process for applying the PDF-test on empirical data is illustrated in Figure 5.1 below.

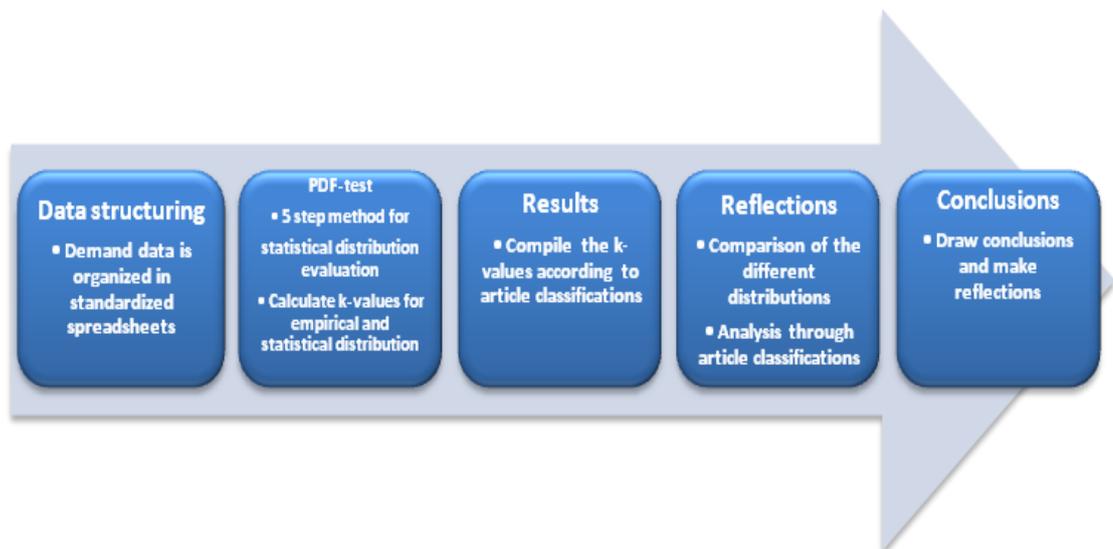


Figure 5.1 Work process, Part I

5.2 Data structuring

In order to simplify processing and analysis of the articles a consistent data structure is required. Data for the articles, which qualified in the data selection process, is sorted by store. This generates a 17 x 52 (stores x weeks) cell matrix for each article used as input for the PDF-test. An example sheet is presented in Appendix II.

The article matrices are placed in individual worksheets sorted by desired $SERV_1$ level.

Much of the data handling and structuring have been carried out through tailored VBA-macros in order to minimize time consumption and the risk for human errors as well as to enable multiple analysis runs.

5.3 Probability Distribution Fit - Tests

The PDF-test is a static simulation method developed by Stig-Arne Mattsson. The general concept of this method is to evaluate statistical distributions based on their ability to determine accurate re-order points. This is measured by deviation from the re-order point determined by the empirical distribution through comparison of k-values. It is the re-order point that separates the statistical distributions and induces the differences in between the k-values. The method, which is divided into five steps, is schematically explained in Section 0 below.

The CV-criterion, defined in Section 3.6.3, is applied on a 2.5 %-level to limit the risk for negative demand forecasts. According to Equation 5.1 below this implies that for articles with a $CV \geq 0.5$ the normal distribution is automatically rejected as candidate for the most accurate distribution since it is considered too likely to generate negative demand. For these articles, the gamma and lognormal distributions are hence by design ranked higher than the normal distribution.

$$CV \geq \frac{1}{\Phi(0.975)} = \frac{1}{1.96} \rightarrow CV \geq 0.5$$

Equation 5.1

5.3.1 Schematic description of the PDF-test

1. Data sorting

Sort the empirical observations of demand during lead time in ascending order in a vector, as illustrated in Table 5.1.

Table 5.1 Ascending demand during lead time data

Demand _{LT}	...	24	24	25	26	26	27	...
----------------------	-----	----	----	----	----	----	----	-----

2. Index value determination

Calculate the index value corresponding to the desired $SERV_1$ for the article. The index value corresponds to the proportion of lead time observations that the safety stock shall be set to satisfy, in accordance with the definition of $SERV_1$. To obtain a service level of 90% based on 52 observations, the corresponding index value is $0.9 \times 52 = 46.8 \approx 47$, as illustrated in Table 5.2.

Table 5.2 Index value

Index value	44	45	46	47	48	49	50	51
Demand _{LT}	23	24	24	25	26	26	27	28

3. Re-order point readout

Collect the demand observation at the corresponding position of the index value, as illustrated in

Table 5.3. Since the vector represent observations of demand during lead time sorted in ascending form, the value in this position represents a quantity that satisfies demand in 90% of the observed periods. Hence, this value is an appropriate re-order point for the article at the specific service level.

Table 5.3 Value in the vector at the position of the index value

Index value	44	45	46	47	48	49	50	51
Demand _{LT}	23	24	24	25	26	26	27	28

Steps 1 – 3 are also illustrated in Figure 5.2 below.

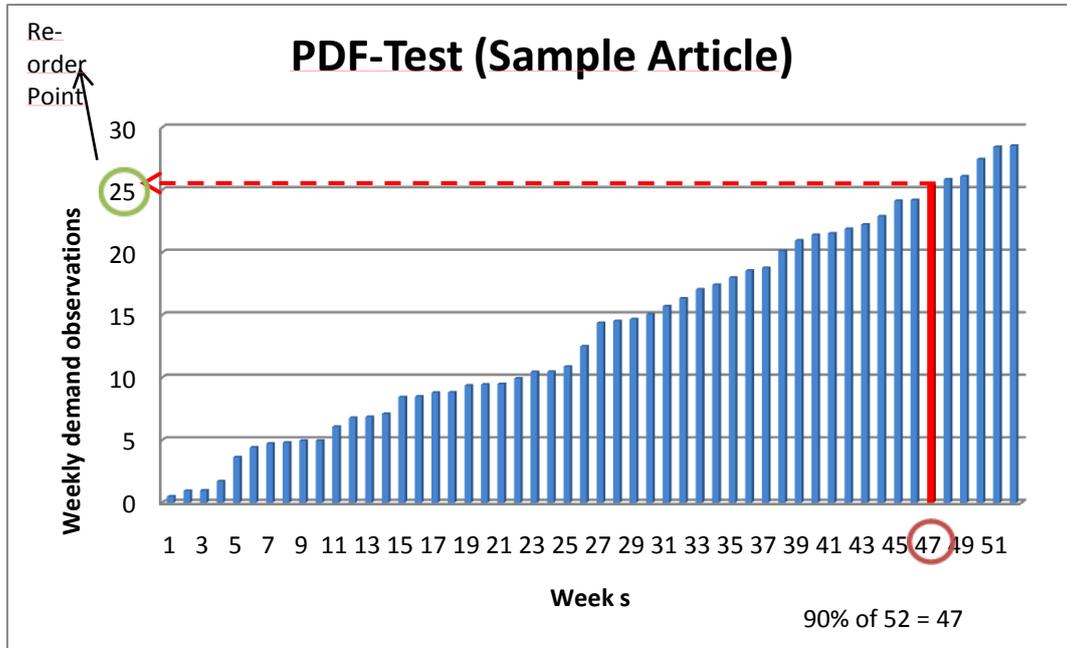


Figure 5.2 Step 1-3 of the PDF-method

4. K-value calculation

When the re-order point is determined, the k-value for the empirical distribution can be calculated according to Equation 3.1. K-values for the statistical distributions are also calculated according to Equation 3.1, with mean value and standard deviation estimated from the demand data and re-order points generated by the statistical distributions respectively.

5. K-value comparison

The k-values generated by the statistical distributions are compared to the empirical k-value. The results are compiled and the distribution that generates the k-value with least absolute deviation from the empirical k-value is considered as the most suitable.

Minor modifications of the method are required to make it applicable on store and DCG-level. These are presented in Sections 5.3.2 and 0 correspondingly.

5.3.2 Applying PDF-tests on Store level

On this level the 105 articles are distributed through 17 stores. Comparison of the empirical distribution and the three statistical distributions evaluated in this study generates $17 * 105 * 4 = 7140$ k-values. In order to be able to handle and analyze such volumes of data and spreadsheets task specific VBA Macros have been developed to automate sorting and execution of calculations, which are presented in Appendix III. An example of the corresponding Data and PDF-calculation sheets are presented in Appendix IV and Appendix V. The pre-programmed Excel commands that were used to calculate the k-values are presented in Appendix VI.

All the articles on store level have lead times shorter than one week. The only available observations of actual sales are on weekly basis. In order to apply the model the lead times have been set to one week throughout the calculations. This approach emanates from the fact that it is not possible to statistically correct scale down empirically observed data.⁹⁷

⁹⁷ Mattsson, S-A. (09-08-10)

5.3.3 Applying PDF-tests on DCG level

The amount of data is considerably smaller on this level since customer demand for 17 stores is accumulated and distributed through one single DC. Modified versions of the analytical VBA Macro tools are used to consolidate data and execute calculations.

The lead times from the suppliers to the DCG consequently exceeds one week which generates a reversed scaling problem compared to on store level. Several weeks demand has to be compounded to enable evaluations of the statistical distributions based on demand during lead time. To achieve this in a statistically correct way the Boot strapping method presented in Section 3.6.2 is applied. The method requires lead times to be noted as integers, why these have been consistently rounded up to whole weeks. This ensures that the obtained k-value generates a service level slightly above the desired. Based on the 52 available observations 1000 data points were generated, providing a broader data basis and better prerequisites to find the most appropriate distribution.

6 Results of the PDF-tests

In this chapter the results from the PDF-tests are compiled. The results on store and DCG level are presented in different sections to avoid misconceptions. On both levels, the results are presented in general as well as with respect to the different classifications previously presented.

6.1 Store Level

6.1.1 General result

The total result on store level shows that the gamma distribution is the most accurate distribution for 49% of the tested articles as visualized in Figure 6.1 below. The CV-criterion is not fulfilled for a large number of the articles, as can be seen in Table 6.1. This means that these articles only are tested for the gamma and lognormal distributions.

Table 6.1 Normal rejection in the total result on store level

Total store	
Normal Rejected	68%

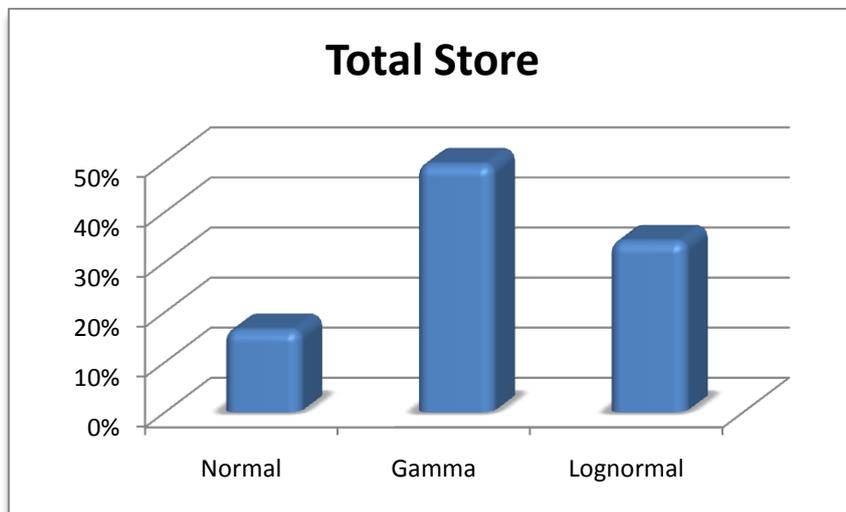


Figure 6.1 Compilation of the total result on store level

6.1.2 Service level aspect

The gamma distribution proves to be most accurate for two of the three service levels. In the third case it is second to the lognormal distribution by 6 percentage points. The results are illustrated in Figure 6.2. The number of normal CV-based normal rejections is remarkably high for all service levels as can be seen in Table 6.2.

Table 6.2 Normal rejections sorted by service level

Service level	90%	95%	99%
Rejected Normal	85%	60%	63%

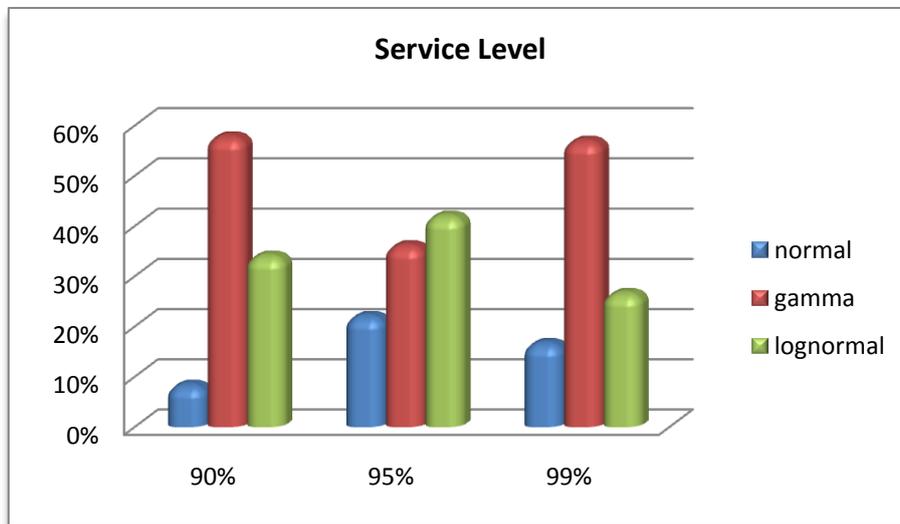


Figure 6.2 Compilation of the result from a service level aspect

6.1.3 Price level

The gamma distribution consistently proves to be most accurate throughout the price levels. The lognormal distribution is the second best alternative for all levels and the normal distribution is least accurate. The results are presented in Figure 6.3 below. The number of rejected normal distributions tends to increase for more expensive articles as shown in Table 6.3.

Table 6.3 Normal rejections sorted by Price level

Price level	L	M	H
Rejected Normal	50%	71%	82%

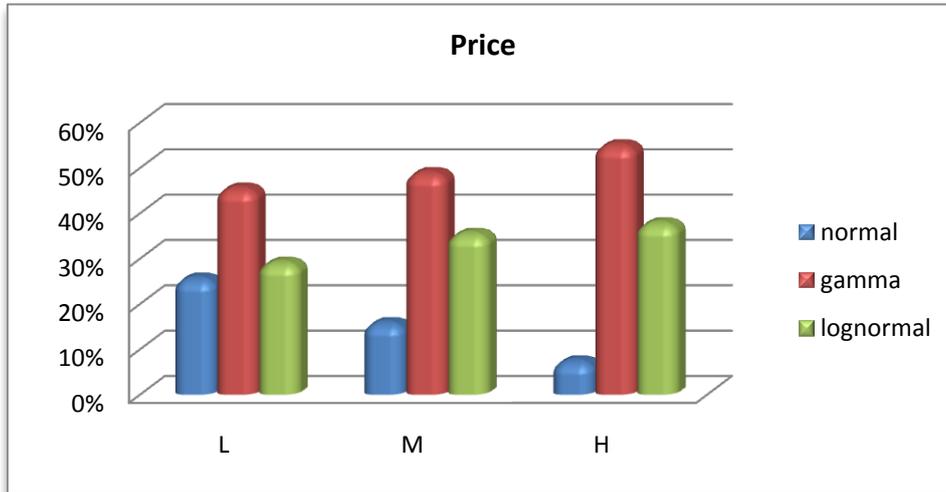


Figure 6.3 Compilation of the results from a Price aspect

6.1.4 Sales volume aspect

The gamma distribution is the best performing distribution among low and medium volume articles. For high volume articles the distributions perform notably equally, although the normal distribution is slightly more accurate and over performs the gamma by 5 percentage points. The results are illustrated in Figure 6.4 below. The share of rejected normal distributions is close to 100% for low volume articles but decreases rapidly as sales volumes increase, as presented in Table 6.4.

Table 6.4 Normal rejections sorted by Sales volume

Sales volume	L	M	H
Rejected Normal	98%	69%	27%

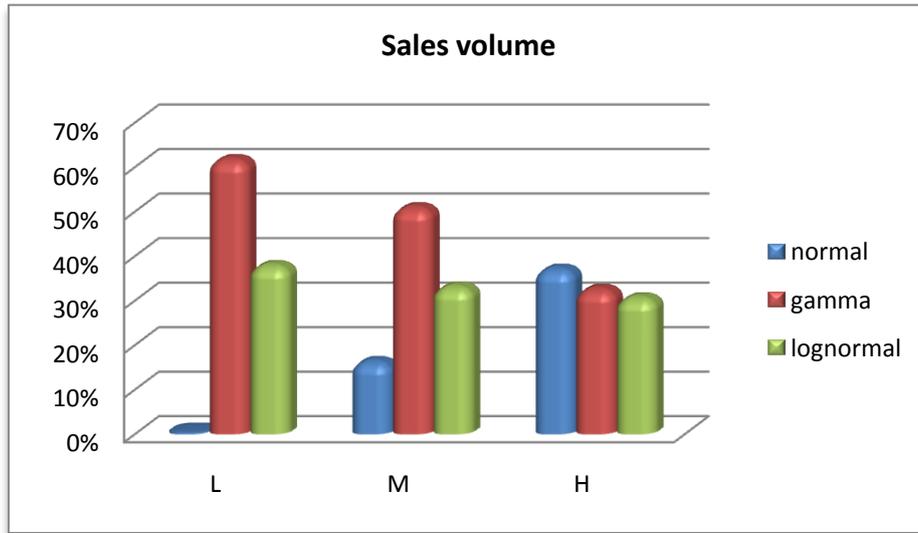


Figure 6.4 Compilation of the results from a Sales volume aspect (pieces)

6.2 DCG level

6.2.1 General result

The overall result on DCG level shows that the normal distribution is the most appropriate distribution to represent demand during lead time on this level. The normal distribution is an obvious winner with a margin of 13 percentage points to the second best, which is the gamma distribution and 25 percentage points to the third best, which is the lognormal distribution. The results are visualized in Figure 6.5. The number of rejected normal distributions is substantially lower than on store level which can be seen in Table 6.5.

Table 6.5 Normal rejections in the total result on DCG level

Total DCG	
Normal Rejected	12%

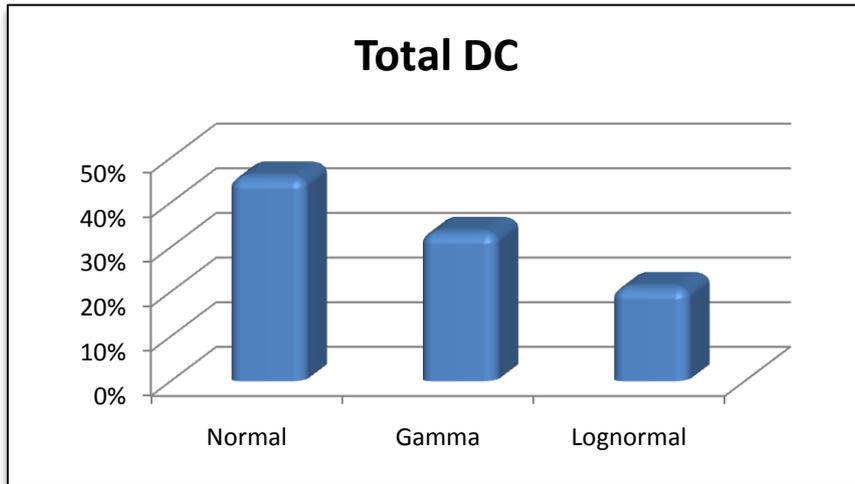


Figure 6.5 *Compilation of the total result on DCG level*

6.2.2 Service level aspect

The normal and gamma distribution perform equally for service level 90 % while lognormal is slightly below the other two. For higher service levels gamma maintains the same share while normal increases its share and lognormal decreases. The results are illustrated in Figure 6.6 below. The number of rejected normal distributions decreases with increased service level, which can be seen in Table 6.6.

Table 6.6 *Normal rejections sorted by Service level aspect*

Service level aspect	90%	95%	99%
Rejected Normal	22%	14%	7%

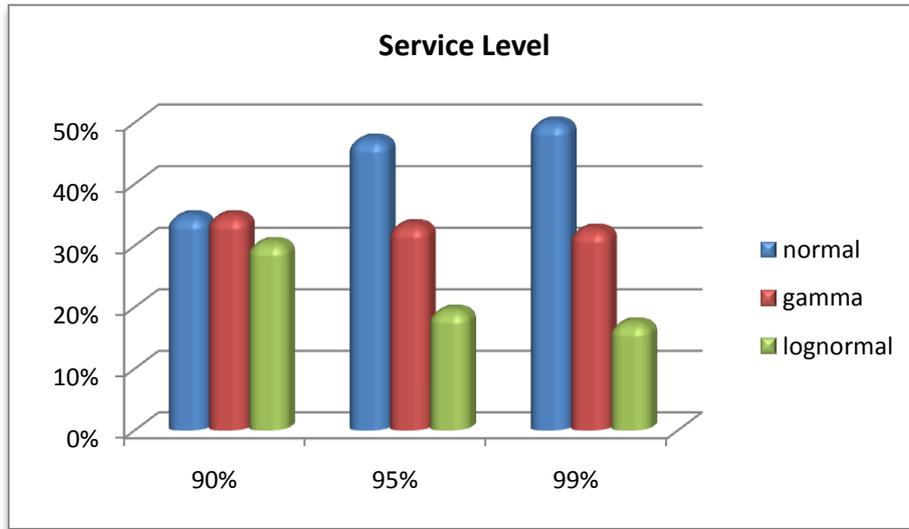


Figure 6.6 *Compilation of the results from a service level aspect*

6.2.3 Price aspect

The most suitable distribution from the price perspective is the normal distribution for low and medium level and gamma for the high level. The results are illustrated in Figure 6.7 below. The amount of rejected normal distributions is relatively low, as can be seen in Table 6.7.

Table 6.7 *Normal rejections sorted by Price aspect*

Price level	L	M	H
Rejected Normal	24%	5%	11%



Figure 6.7 *Compilation of the results from a price perspective*

6.2.4 Sales volume aspect

Gamma generates the most accurate control for low volume articles, but is second to the normal distribution for medium and high volume articles. The results are visualized in Figure 6.8 below. The amount of rejected normal distributions is considered to be low, especially for the high and medium volume articles, as can be seen in Table 6.8.

Table 6.8 *Normal rejections sorted by Sales volume aspect*

Sales volume aspect	L	M	H
Rejected Normal	21%	11%	6%

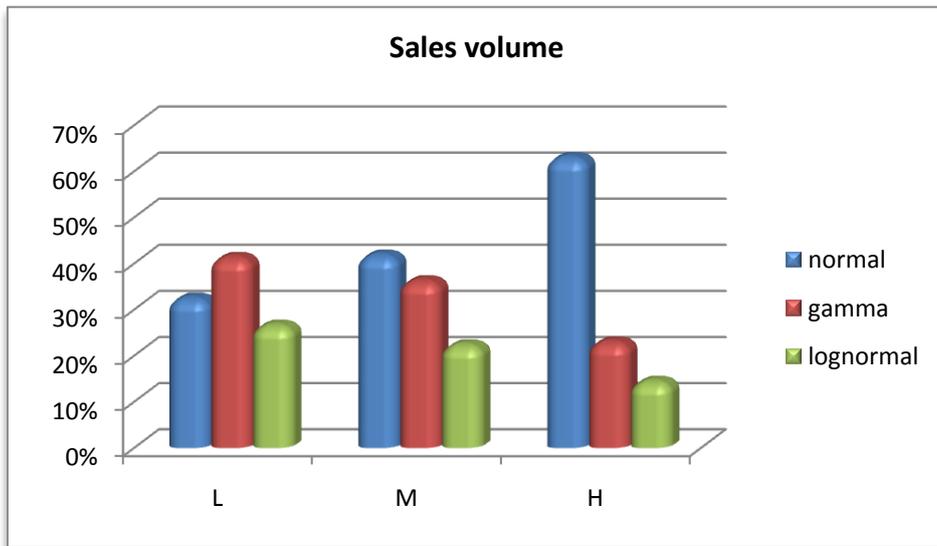


Figure 6.8 *Compilation of the results from a Sales volume aspect (pieces)*

7 Part I - Analysis

In this chapter the PDF-test results are analyzed and interpreted with respect to the theoretical framework. The analysis will be presented separately for the store and DCG level.

7.1 Store Level

The gamma distribution is generally considered to be the most suitable distribution on store level. There are however two exceptions when the gamma is not the best option.

The first occasion is when the service level is set to 95 % and the lognormal distribution is the best fit. The Gamma distribution is for this characteristic better for service level 90 % and 99 %, which indicates that the good performance by the lognormal distribution is more of coincidental nature than an explainable phenomenon. Worth mentioning is that the gamma distribution also is considered a good fit at a service level of 95 % and is only six percentage points behind the lognormal distribution as seen in Figure 6.2.

The second exception is for high volume articles and is more interesting. For these articles the normal distribution is the best fit in consensus with the Central Limit Theorem. Furthermore the rate of CV rejections decreases with increased volumes, which is clearly stated in Table 6.4. The two other distributions also perform well and all three contestants are within a range of 6 percentage points from each other, which is also seen in Figure 6.4.

At all occasions, despite exception two, the amount of rejected normal distributions are very high, which partially explains the poor results of the normal distribution. This indicates that the articles have a low demand and/or high standard deviation which imply a non-tolerable risk for

negative demand forecasts if the normal distribution is applied. This phenomenon can especially be noted for low frequency articles for which almost 100% of the normal distributions are rejected. It is also apparent for the articles with high price, for which the demand can be assumed to be lower than for the cheaper articles.

The adaptive ability of the gamma distribution is noteworthy, as can be seen in Figure 3.6 in Section 3.5.2. It outperforms the other two for low volume articles and is only marginally second to the normal distribution for the high volume articles. There is however an obvious trend indicating that the difference would increase if the sales volume would increase further.

7.2 DCG Level

Overall on DCG level the normal distribution is considered to generate the most accurate re-order points. This is in accordance with the obvious trend of increased accuracy of the normal distribution for higher volumes which is noted on store level. The volumes on DCG level are higher since they represent accumulated demand in 17 stores and the fact that lead times are significantly higher amplifies them furthermore. These circumstances are greatly beneficial for the normal distribution according to the Central Limit Theorem.

There are three cases when the normal distribution is not the best distribution and that is when the service level is 90%, the article price is high and the sales volume is low. In these cases the gamma distribution is equal to or better than the normal distribution. In the two first cases the gamma is about ten percentage points better than the normal and in the last case they are equal. These three aspects are assumed to be correlated to an unknown extent since articles with 90% service level and high prices tend to have small volumes, as illustrated in Table 7.1. Hence, the exceptions can be regarded to be caused by gammas better accuracy for

lower volumes. The lognormal distribution is considered to be the least appropriate candidate due to its poor performance for all aspects and levels.

Table 7.1 Compilation of articles regarding the volume aspect for service level 90%

Service level 90%	Volume aspect	Product Area 1-5
	Low	14
	Medium	10
	High	3

The indication of volume as a significant aspect of distribution selection that was noted on store level is enhanced by the results on DCG level.

Part II

8 Method for developing the simulation model

The model development presented in the methodology chapter is elaborated and the different steps in the simulation process are explained.

8.1 Introduction

In order to quantify the impacts of the recommendations generated through the PDF-tests in Part I two dynamical simulation models, representing the store and DCG level, are constructed. This part of the report presents the construction process of these models together with the tests that are conducted and the results generated.

The first purpose of this dynamical simulation is to verify the method and generated results in Part I.

The second purpose is to enable a comparison of the setup suggested in Part I and the current. This is performed through measuring the achieved levels of service and the average differences in generated stock for the two setups respectively.

Furthermore two specific subtests are conducted. The accuracy of IoS's weekly $SERV_3$ measures is evaluated through comparison with continuous measures in the models in a service level measure frequency analysis. Secondly, a sensitivity analysis of lead time variation is conducted on DCG level. The work process is summarized in Figure 8.1 below.

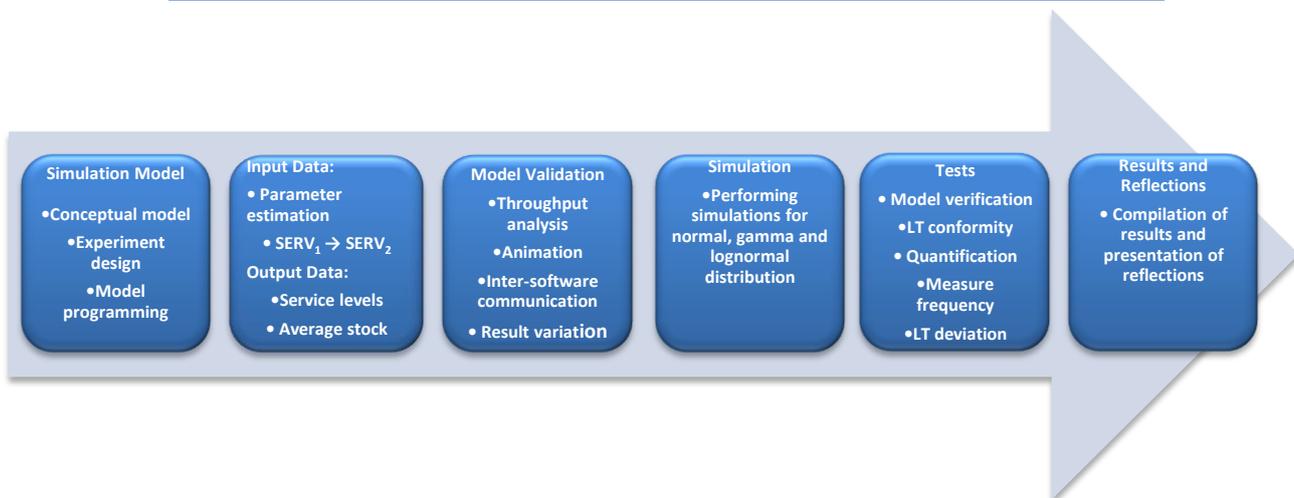


Figure 8.1 Working process, Part II

8.2 Conceptual modelling

Since the upstream supply chain has been divided into two levels, as discussed in Section 1.5, separate simulation models are built to represent the two systems.

Both models consists of the following three fundamental functions;

- Generating demand
- Withdrawal from stock
- Refilling inventory

The generated demand is served by an inventory. If the inventory is empty the demanded piece is registered as a lost sale. In accordance to the (R, Q)-policy an order of quantity Q is placed when the re-order point is reached and delivered after a delay representing the system lead time. In this model the assumption that the DCG never is out of stock is made. Order quantities are predetermined by IoS while the re-order points are calculated in Part I. On store level, the simulation is run for each store individually, as illustrated in Figure 8.2.

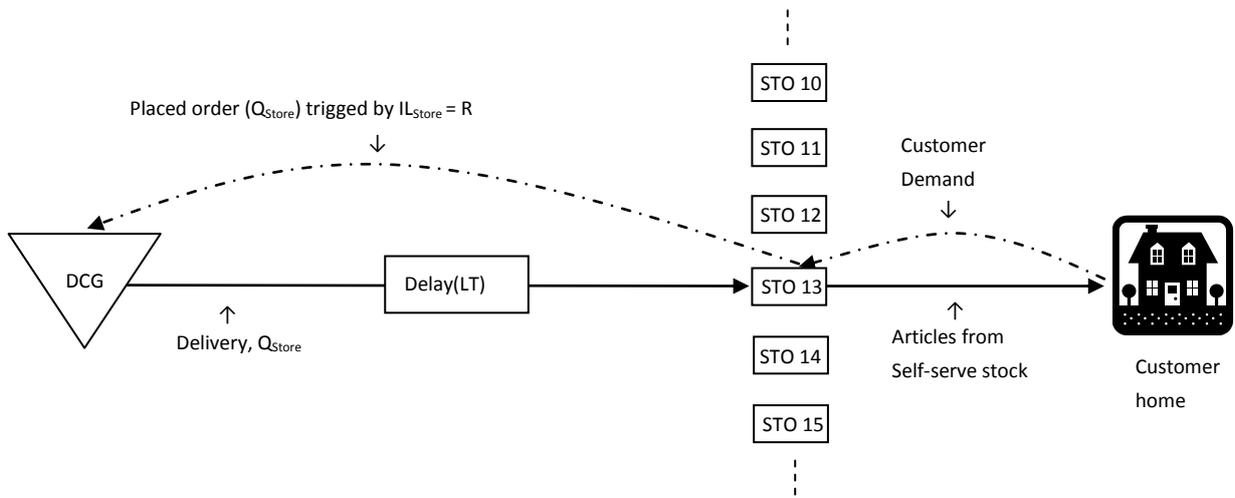


Figure 8.2 Conceptual model, Store level

On DCG level the outbound deliveries to stores are handled in batches of Q_{Store} , often corresponding to a pallet or box. The number of such batches represents the accumulated demand from the 17 stores, which are compiled and generated as one. Upon generation of this demand, expressed as a multiple of Q_{Store} , a corresponding volume is withdrawn from the inventory and sent to the stores. If the inventory is empty the demanded batch is registered as a lost sale. According to the same principles as on store level, the DCG inventory is replenished according to an (R, Q)-policy with a predetermined Q_{DCG} and re-order point generated by the distributions. In this model the assumption that the supplier never is out of stock is made. The model is illustrated in Figure 8.3.

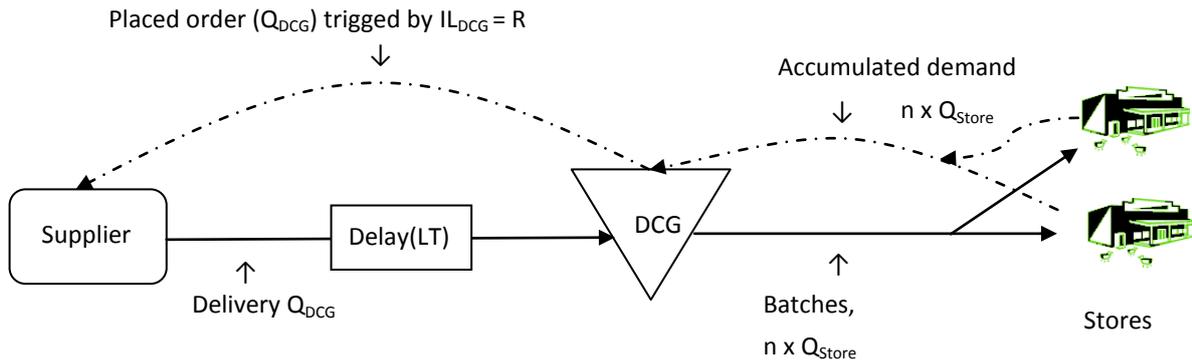


Figure 8.3 Conceptual model, DCG level

8.3 Model programming

In the model programming part the conceptual model is transformed into a dynamic simulation model. The two systems are built separately as in the conceptual model. For both levels the three distributions are tested simultaneously, which entails that they are exposed to exactly the same customer demand and hence are evaluated on equal terms.

The concept of logarithmic compound Poisson is applied to generate demand which is expressed in terms of pieces on store level and in batches on DCG level. This approach is applicable, as discussed in Section 3.9.1, since the condition $\frac{(\sigma)^2}{\mu} > 1.1$ is true for all articles. The model is constructed to treat a customer who demands multiple pieces, or store that demands multiple batches, as a corresponding amount of individual customers in order to measure the achieved service levels. This is not a perfect model of reality, but enhances the connection between service level control and measure in accordance with Section 3.5.1.

A simulation run is initiated by filling the inventory up to a level of $R + Q$. Each demanded piece is then withdrawn from the inventory individually and registered. Continuous review enables that an order is placed exactly when the inventory level R is reached. The order is then delayed according to the lead time.

The simulation model used for the simulations and the explanation for the individual blocks are presented in

Appendix V.

8.4 Input data

All necessary parameters for the distributions are calculated in Part I. In some of the experiments parameters need modification in order to fit the purpose of the test, as illustrated in Table 8.1 below.

Table 8.1 Modified parameters

Part I (LT = 1 week)	Adjusted according to	Part II (LT = Actual)
μ	Equation 3.3	μ_{LT}
Σ	Equation 3.4	σ_{LT}
K	Section 3.2	k_{LT}
R	Equation 3.1	R_{LT}
SERV ₁	Equation 3.12	SERV ₂
Q _{DCG} = Pieces		Q _{DCG} = Batches

To create a realistic simulation model it is important to generate demand patterns similar to the real situation. In this model, demand is generated on a daily basis through a compound Poisson process based on parameters estimated from the demand data.

On DCG level withdrawals from inventory are only made in batches with volumes equal to multiples of the minimum order quantity, Q_{Store}. Since the received data from IoS is expressed in pieces, this is handled by dividing the number of outbound pieces with Q store, prior to the estimation of compound Poisson process. This implies that demand on DCG level is generated in terms of batches rather than individual pieces

To ensure measurability of the inventory control, the SERV₁ levels assigned to the articles by IoS need to be translated into corresponding SERV₃ levels, which is a service level measure that can be measured in the simulation model. The transformation steps are illustrated in Figure 8.4 below. For all

the articles $\frac{SS+Q}{\sigma_{D(LT)}} > 1.94$, why the approximation of $SERV_2$ in Equation 3.11 is valid and the transformation of $SERV_1$ to $SERV_2$ in Equation 3.12 can be legitimately performed. The $SERV_2$ measure is approximately equivalent to $SERV_3$, as discussed in Section 3.5.1, why control and measurement compatibility is successfully achieved through this procedure.

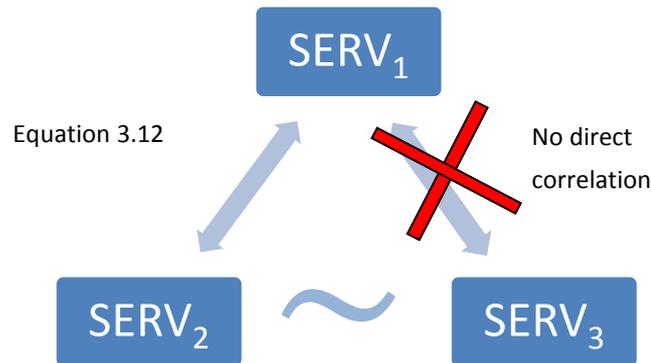


Figure 8.4 Service level measure relations

In the PDF-tests, the evaluation is based on k-values with a large number of decimals. In order to generate control parameters for the dynamical simulation the re-order points are consequently rounded up according to industry best practice⁹⁸. This is intuitive due to the discrete nature of re-order points, but decreases the accuracy of the evaluation since this round up often implies that two or more distributions suggests equal re-order points.

In order to conduct automated simulation runs the input data is required to be presented in a consequent structure and a logical sequence. A spreadsheet structure that is coordinated with receive- and send functions in Extend is created, as seen in Appendix VI. In this structure every article is assigned with attributes for order quantity, lead time, re-order point and

⁹⁸ Stig-Arne Mattsson (09-09-10)

the compound Poisson process parameters determining demand generation.

8.5 Output data

Three different service level measurements and the average inventory levels are observed in the dynamic simulation model;

- $SERV_3$ –weekly: as done today by IoS
- $SERV_3$ – daily: closer to $SERV_2$, and hence more correct
- $SERV_3$ under continuous review – $SERV_2$
- Average inventory level

The outputs from the dynamic simulation models are the same for both simulation levels. $SERV_3$ under continuous review and demand is equivalent to $SERV_2$. This is also true when the demand is described through a poisson process. When inspection frequency decreases, the two measures can be considered approximately equal. The accuracy of this approximation decreases with increased inter-inspection intervals. These assumptions are more closely discussed in Section 3.5.1. $SERV_3$ under daily review is hence closer to the real service level than weekly, but since IoS uses weekly review this measurement is also performed.

$$SERV_2 \sim SERV_3 = 1 - \frac{\text{Observations with } IL = 0}{\text{Total number of observations}}$$

Equation 8.1

The data that is used to calculate order line service is registered every time a customer is demanding an article. If the inventory level is positive for the specific article it is registered as a served customer demand and if it is not it is registered as not served. The service level is then calculated as shown in Equation 8.2.

$$SERV_2 = SERV_3 \text{ under continuous review} = 1 - \frac{\text{Observations with } IL = 0}{\text{Total number of observations}}$$

Equation 8.2

Since different statistical distributions generate different re-order points the inventory level is affected and that is why the average articles in stock are registered. The distribution which generates service levels closest to the desired is considered the best.

8.6 Validation

The simulation model is developed in collaboration with simulation programming expert Christian Howard at the department of Industrial Management and Logistics, Production Management at LTH. Both the authors have thoroughly examined the model design, during as well as after the construction process, in order to find and eliminate errors. All identified discrepancies have been corrected.

In order to validate some aspects of the simulation model design, the animation feature in Extend is used. This feature animates the simulation run and enables stepwise observation of the process. Tests have been conducted during sample runs under which the following aspects were closely observed;

- **Import of input parameters from Microsoft Excel**

The import is monitored and reviewed. Continuous comparison between the applied parameters in Extend and the corresponding parameters in Excel was conducted.

No errors detected.

- **Generation of customer demand**

Generated demand is compared to the empirical average demand. Since the demand generation is based on parameters derived from the empirical demand there should be consistency in order of magnitude. The deviations from the generated and empirical demand are shown in Table 8.2 below and the detailed results are presented in Appendix IX.

Table 8.2 Deviation between theoretical demand and empirical demand

	Store	DCG
Mean error	0.01%	0.1%
Standard deviation	3%	1.2%

Mean error and standard deviation is considered to be within acceptable boundaries.

- **Comparison of theoretical and empirical average inventory level**

According to Equation 3.6 the theoretical average inventory level is calculated for each article. The theoretical average inventory levels of the simulated articles are accumulated and compared to the accumulated average inventory level measured in the dynamic simulation of the same articles. In

Table 8.3 the deviation between the accumulated inventory levels are presented. Full results of this test are illustrated in Appendix X.

Table 8.3 Deviation between theoretical and simulated average *IL*

	Normal	Gamma	Lognormal
Theoretical	2054	2339	2319
Simulation	2126	2323	2314
Deviation	-3%	1%	0%

Deviations are considered to be within acceptable boundaries.

- **Generation of initial inventory level as R + Q**

The initial inventory fill is monitored through the animation feature in Extend.

No errors detected.

- **Service level measurement**

The service level measurements are monitored through the animation feature in Extend.

The SERV3-measures in the model are feasible and observed with the correct frequencies. The level of service consistently decreases with increased measure frequency (week – day – OS) as expected.

- **Inventory depletion and replenishment**

The imitation of inbound deliveries and extractions from stock are monitored through the animation feature in Extend.

No errors detected.

- **Export of results to Excel**

The export is monitored and reviewed. Continuous comparison between results in Extend and corresponding cells in Excel.

No errors detected.

- **Variation in between simulation runs**

30 runs were performed for each article. The variation between the results was monitored.

The results vary in between runs, but the deviations are consistently within tolerated ranges.

There are three important aspects of validity in the conceptual simulation model that are worth special notice;

One factor that decreases model validity is that the re-order points are calculated from the sales observations and tested against a generated sample demand.

The generation of demand through compound Poisson processes emanates that the observations of sales volume are uncorrelated. This has been proved for a significant sample of the articles in one of the pre-studies.⁹⁹ The three distributions are simultaneously evaluated in the model which means that they are tested against identical generations of demand. This provides fair and more exact evaluation of the distributions and therefore increases validity.

The validity of the model is hereby regarded as acceptable.

⁹⁹ Dahlberg et al., (2009) pp.31-33

8.7 Experiment design

To fulfill the study objectives, a number of simulations with different setups are needed. These setups are described below. Independent of these differences, the main model is common and the basic experiment design is kept consistent throughout the simulations.

All simulations are run for two years with the global time unit days. The first year is to give the model time to tune in and stabilize. All observations are made during the second year. In order to evaluate the stability of the results every simulation is run 30 times.¹⁰⁰

Because of hardware capacity limitations the total number of simulations is reduced through selection of a representative sample consisting of nine articles and three stores, presented in Table 8.4 and Table 8.5 below. The articles and are selected with regard to volume classification rather than desired level of $SERV_1$, since this aspect proved to be the most significant characteristic in the PDF-tests. In some experiments, simulations are conducted for a smaller amount of articles denoted as Group 1. Group 1 is presented under the title *Notes* in Table 8.4.

Table 8.4 Sample articles based on volume classification

Sample articles			
Article	$SERV_1$	Volume	Notes
1	90	Low	Group 1
2	95	Low	
3	99	Medium	
4	95	Low	Group 1
5	95	High	
6	99	Medium	
7	99	High	Group 1
8	99	High	
9	99	Medium	

¹⁰⁰ Laguna, M. and Marklund, J. (2005) p. 358

Table 8.5 Volume classification of the represented stores

Store	Volume classification
A	High
B	Medium
C	Low

The following experiments are conducted;

8.7.1 Test one – Verification of the PDF-test

Store level and DCG level

The purpose with this test is to verify the developed method in Part I by performing an independent test, in this case a dynamic simulation. The statistical distributions are evaluated on basis of their ability to achieve pre-defined service. The overall preferred distributions on both levels are compared. The output will be an indication whether or not the method in Part I is reliable or not. The lead time setup is identical to the one used in the PDF-tests, i.e. one week on store level and actual on DCG level. This test is conducted on the limited selection of articles in Group 1 on store level and for all nine articles on DCG level.

8.7.2 Test two – Conformity between LT = 1 week and LT = 2 days

Store level

The method in Part I is based on a lead time that is equal to the time unit that the demand data is collected for, in our case one week. The simulation model opens up for a study with the actual lead time, which for all the stores in our study is two days. In this test the conformity between the obtained service levels for the two cases is evaluated on an individual article basis. The purpose is to get an indication whether the results from the PDF-tests still are valid when the actual lead time of two days is used. This test is conducted on the limited selection of articles in Group 1.

8.7.3 Test three – Quantification

Store level and DCG level

This test compares the amount of stock that the current setup generates to the amount of stock the recommended setup generates. The purpose is to get an indication of the economic impact in a change of setup.

When estimating the impact on store level, the three sample stores have been considered as a representative sample of the stores, why the accumulated stock is scaled by the factor $\frac{17}{3}$. This is a rough estimation of reality, but provides an indication of the impact the recommended setup would have. This test is conducted on all sample articles with actual lead times on both levels.

8.7.4 Test four – Service level measure frequency

Store level and DCG level

In this test the reliability in the current service level measurement, which is performed once a week, is evaluated. The test will perform service level measurements once a week, once a day and on an order line basis. To evaluate the level of accuracy the correlation and the quota between the continuous and the two discrete measures are calculated. This is done in order to give an indication of the size of the measure error when discrete measuring is applied. This test is conducted on all sample articles with actual lead time on both levels.

8.7.5 Test five – Lead time deviation

DCG level

In this test the simulation system is exposed to variations in the lead time. Unlike IKEAs way of measuring lead time deviation, both shorter and longer lead times are accounted for. In this test the lead time is set to vary within $\pm 10\%$ and measurement of achieved service levels are performed. The achieved levels of service are compared to corresponding measures for

static lead times in previous tests. The purpose is to establish in which magnitude the service levels are affected by lead time deviation. This test is conducted on the limited selection of articles in Group 1 with actual lead time.

9 Results

The results from the simulations are compiled and presented in tables and graphs. On both levels the results are presented with respect to the different simulations previously presented.

9.1 Test one – Verification of the PDF-test

Store level

The distribution selection generated through the PDF-tests in Part I is compiled for the selected articles in the selected stores. As seen in Table 9.1 below the overall preferred distribution for this selection of articles is the gamma, which is chosen in 67% of the cases. As also can be seen in Table 9.1 the dynamic simulation also indicates that the gamma distribution is the overall most appropriate distribution on this level and is chosen in 78% of the cases. This indicates that the two methods generate conform results. More detailed test results are presented in Appendix XI.

Table 9.1 Model verification results

Model Verification			
Article	Store	PDF-Test	Dynamic Simulation
1	A	Lognormal	Gamma/Normal
	B	Gamma	Gamma/Normal
	C	Gamma	Gamma/Normal
4	A	Gamma	Gamma
	B	Gamma	Gamma/Lognormal
	C	Gamma	Gamma
7	A	Lognormal	Gamma
	B	Normal	Lognormal
	C	Gamma	Lognormal
Overall Preferred distribution		Gamma, 67%	Gamma, 78%

DCG level

In Table 9.2 the preferred distributions from the PDF-tests and the dynamic simulation is presented. The PDF-test showed that the lognormal distribution was to be preferred for this specific, limited sample of articles, and was chosen in 44% of the cases. In the dynamic simulation the lognormal distribution was considered the best or equal to the best in 67% of the cases. This indicates that both the methods generate conform results. More detailed test results are presented in Appendix XI.

Table 9.2 Model verification results

Model Verification		
	PDF-Test	Dynamic Simulation
1	Lognormal	Lognormal
2	Lognormal	Gamma
3	Lognormal	Lognormal
4	Gamma	Normal
5	Lognormal	Lognormal/Gamma
6	Normal	Normal
7	Normal	All
8	Gamma	Lognormal/Gamma
9	Normal	Lognormal
Overall Preferred distribution	Lognormal, 44%	Lognormal, 67%

9.2 Test two – Conformity between LT = 1 week and LT = 2 days

Store level

The test results from the dynamic simulations with a lead time of one week and two days are compiled into bar charts. It is only for article 1 in store C, 4 in store A and C that there is a slight deviation for some of the distributions.

For the normal distribution article 1-C deviates with 0.75 percentage points and 0.57 percentage points for article 4-A, shown in Figure 9.1 below.

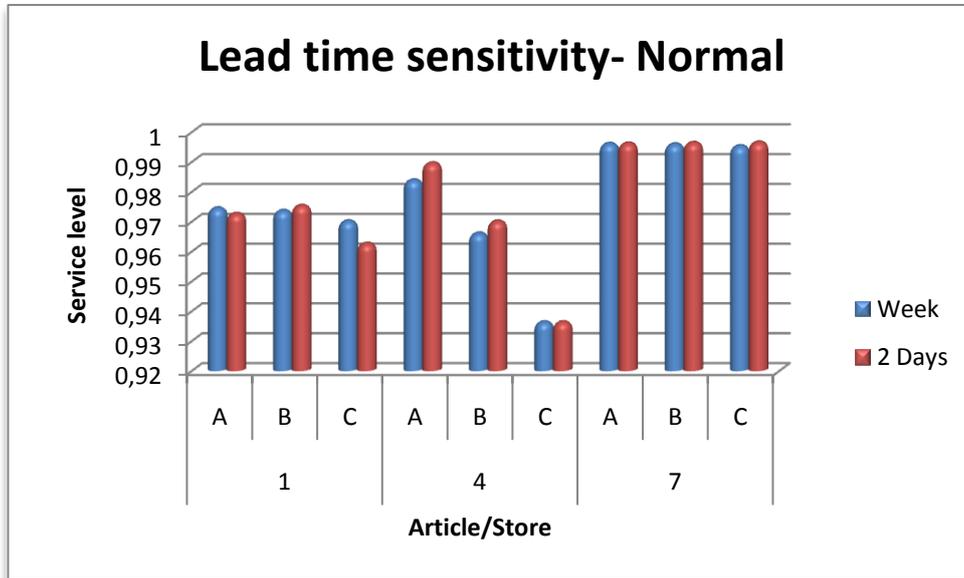


Figure 9.1 Conformity between the service level based on LT = one week and LT = 2 days

For the lognormal distribution the service levels are very similar as shown in Figure 9.2. It is only article 1-C that notably deviates and the deviation is 1.18%.

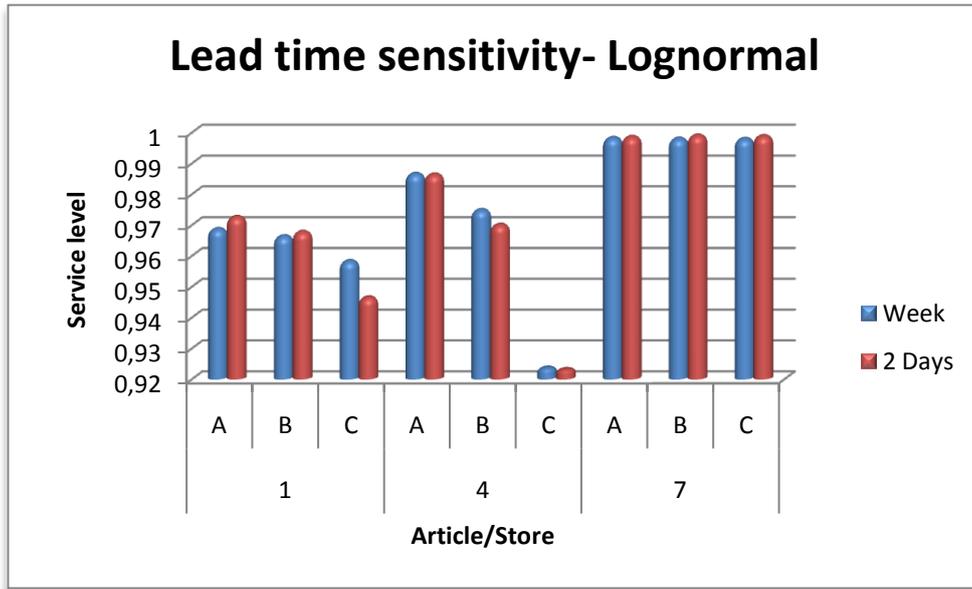


Figure 9.2 Conformity between the service level based on LT = one week and LT = 2 days

The gamma distribution also delivers corresponding results and the consistent trend that article 1-C deviate is notable also in this test, as illustrated in Figure 9.3. The deviation is slightly larger for the gamma distribution than for the other distributions with a deviation in the service level of 2.36 percentage points. For article 4-C the deviation is in the magnitude of 1 percentage point.

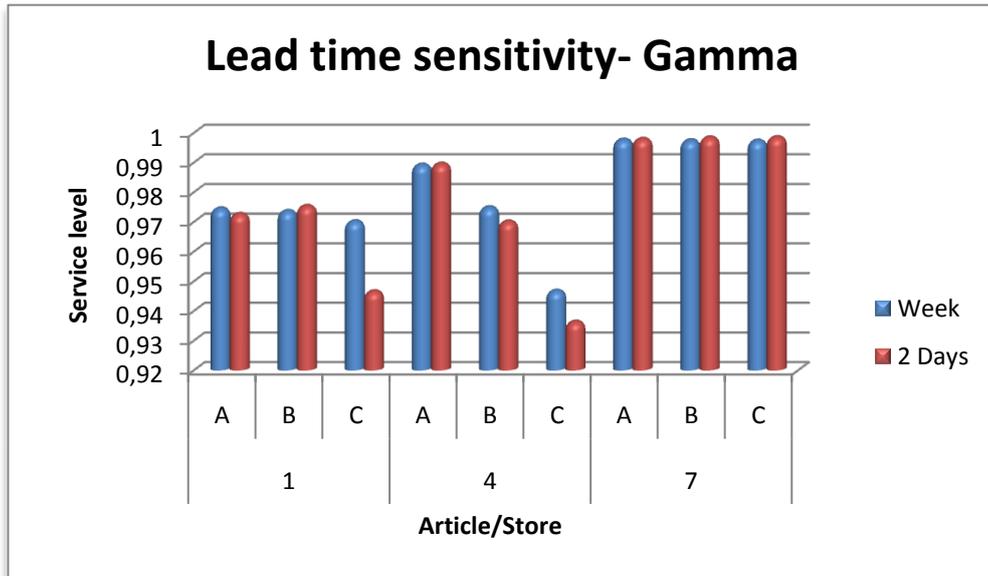


Figure 9.3 Conformity between the service level based on LT = one week and LT = 2 days

9.3 Test three – Quantification

Store level

As we can see in the Figure 9.4 below the recommended distribution from Part I generate a larger stock than the current distribution. The average stock keeping increases with about 9 %. This calculation includes predicted stock in all 17 stores, based on the indicative simulation results for the three sample stores.

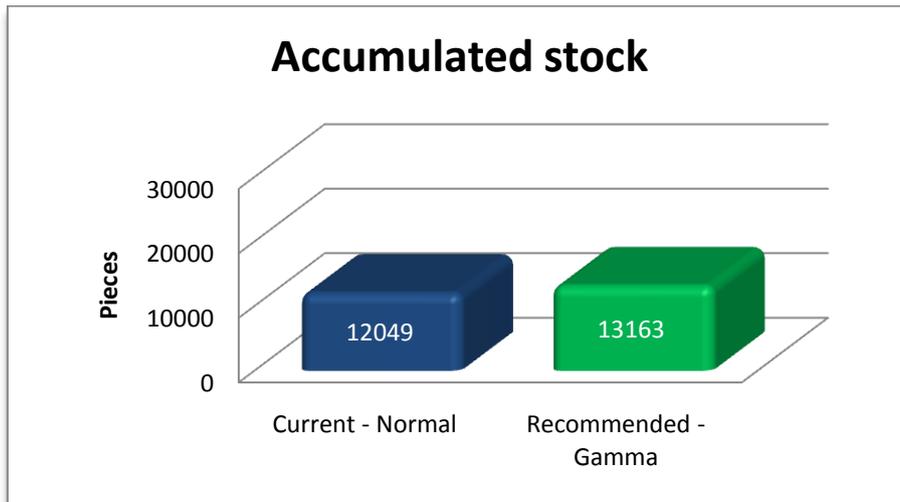


Figure 9.4 Comparison of generated stock for the new and old setup on store level

DCG level

On DCG level the introduction of the recommended distribution in Part I would generate a decrease of the average stock keeping with 8 % in comparison to the current distribution, as illustrated in Figure 9.5 below.

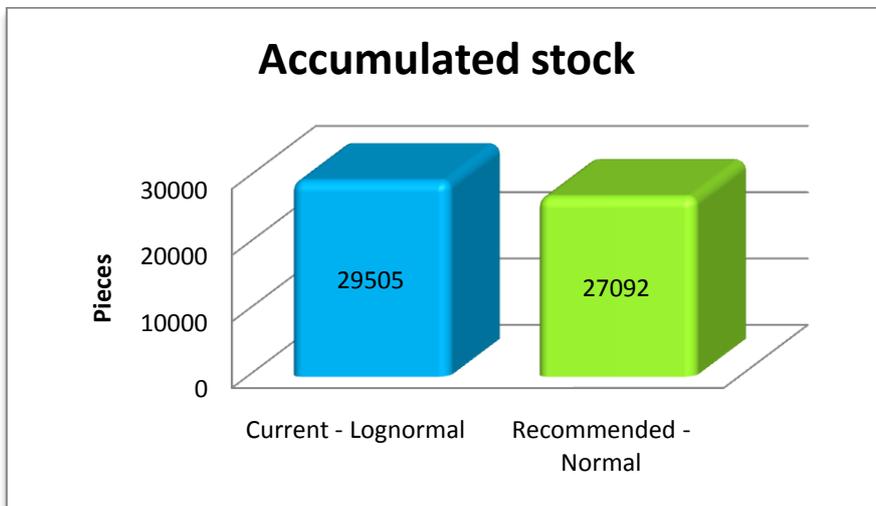


Figure 9.5 Comparison of generated stock for the current and recommended setup on store level

Total

To get a perspective of the total impact a change of setup could generate on the stock keeping the two levels are compiled in Figure 9.6. The recommended setup implies indications of a marginal decrease of the average total stock keeping.

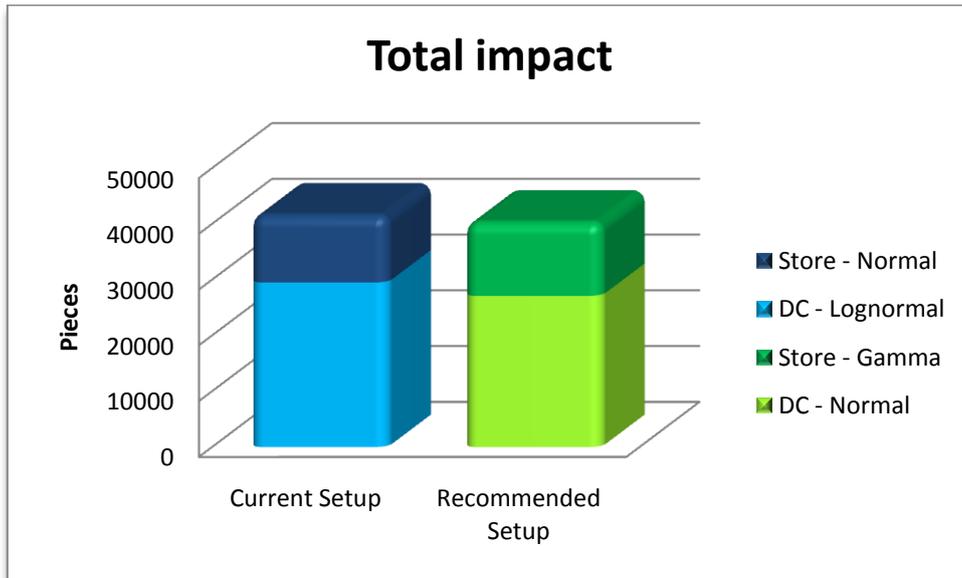


Figure 9.6 Total comparison of generated stock for the current and recommended setup

The total impact of a setup change on both store and DCG level is illustrated for each of the simulated articles individually in Figure 9.7. A change generates reduction for all articles, deviating between 0.1 – 13.3%.

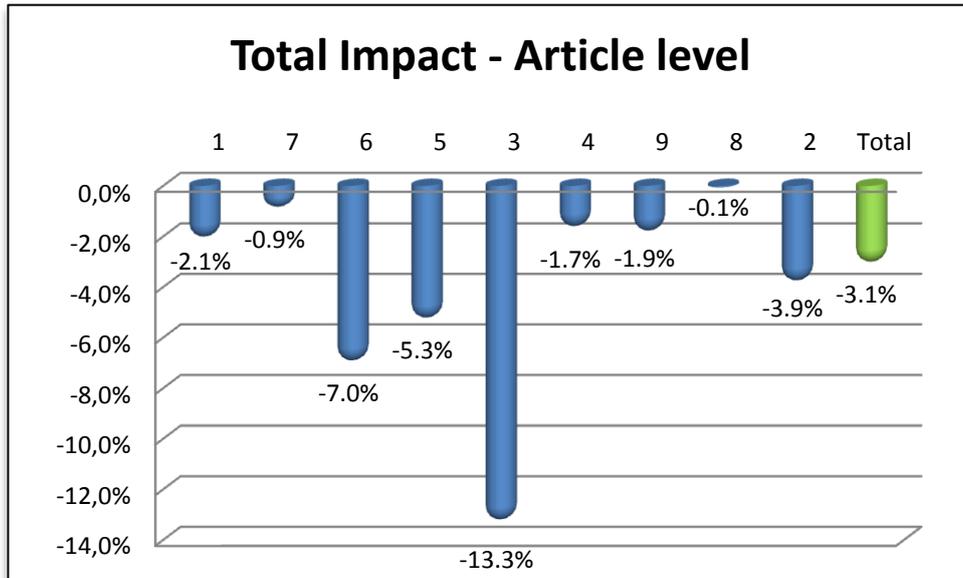


Figure 9.7 Article specific impact of setup change on both Store and DCG level

A confidence interval fitted to have its upper boundary at status quo = 0 reaches 97.5%, as seen in Table 9.3 below. This indicates that the hypothesis of inventory reduction is true on a 98.75% level, since half of the expected values outside the interval are located below the lower boundary and hence also represent a reduction.

Table 9.3 Impact and confidence interval

Impact	Data
Total impact	-3.1%
97.5 %-confidence	±3.1%

9.4 Test four – Service level Measure frequency

Store level

The correlation between different service level measure frequencies is illustrated in Figure 9.8. The correlation between measuring on an order line basis and on a daily basis is much higher than the correlation between the order line basis and weekly basis. The coefficients of correlation are within the interval 0.1 – 0.5, which indicates that the measures are weakly correlated.

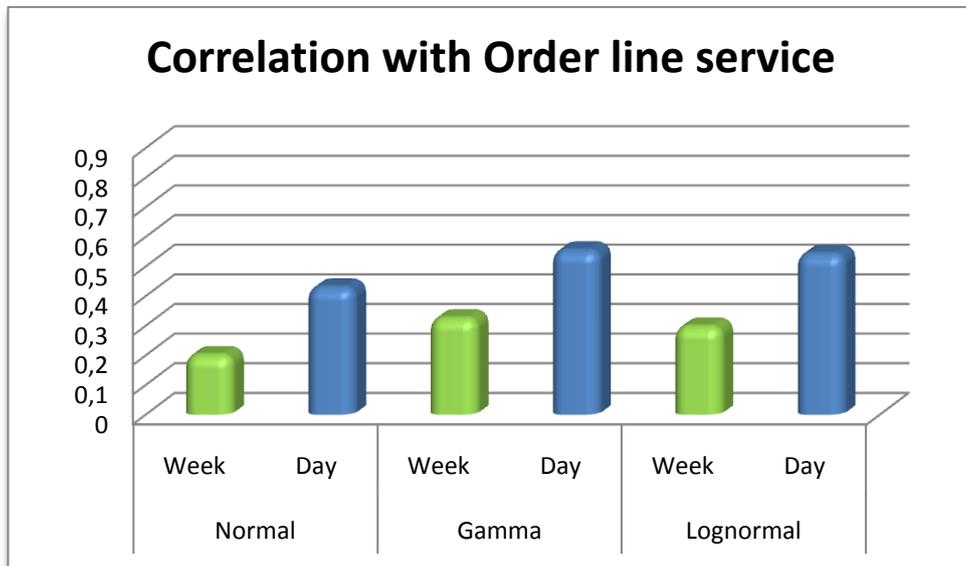


Figure 9.8 Correlation between different frequencies in service level measurement, Store level

The quotas between continuous and discrete measures are presented in Table 9.4. The quota is, independent of frequency and distribution, around 98.8%. This implies that the average measure error is slightly above 1 % when applying discrete measures if the $SERV_3$ under continuous review is considered equal to $SERV_2$.

Table 9.4 Quotas between continuous and daily/weekly measures, Store level

Normal		Gamma		Lognormal	
Week	Day	Week	Day	Week	Day
98.8%	98.7%	98.9%	98.8%	98.7%	98.7%

DCG level

On DCG level the correlation between the frequencies is higher and varies from 0.65-0.8. Daily measures are still higher performing than weekly, but the difference has decreased significantly, as illustrated in Figure 9.9 below.

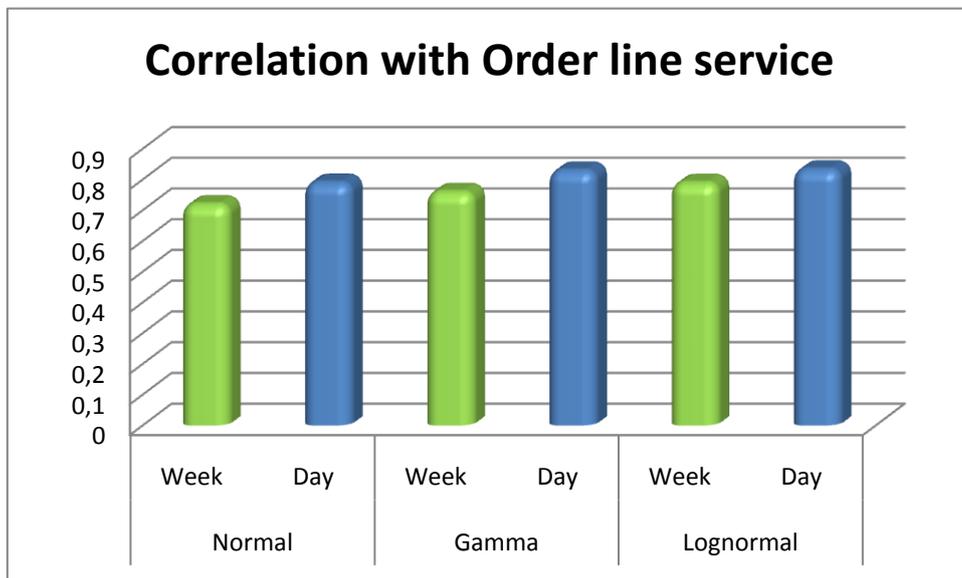


Figure 9.9 Correlation between different frequencies in service level measurement, DCG level

The quotas between continuous and discrete measures are presented in Table 9.5. The quota is, independent of frequency and distribution, around 99.6% which implies that the average measure error is about 0.4 % when applying discrete measures.

Table 9.5 Quotas between continuous and daily/weekly measures, DCG level

Normal		Gamma		Lognormal	
Week	Day	Week	Day	Week	Day
99.5%	99.5%	99.6%	99.6%	99.6%	99.7%

9.5 Test five – Lead time deviation

DCG level

Lead time variation implies service level reductions for all three articles, as can be seen in Figure 9.10 - Figure 9.12 below. The variation is in the range of 0 – 1.25 percentage points for all articles and distributions.

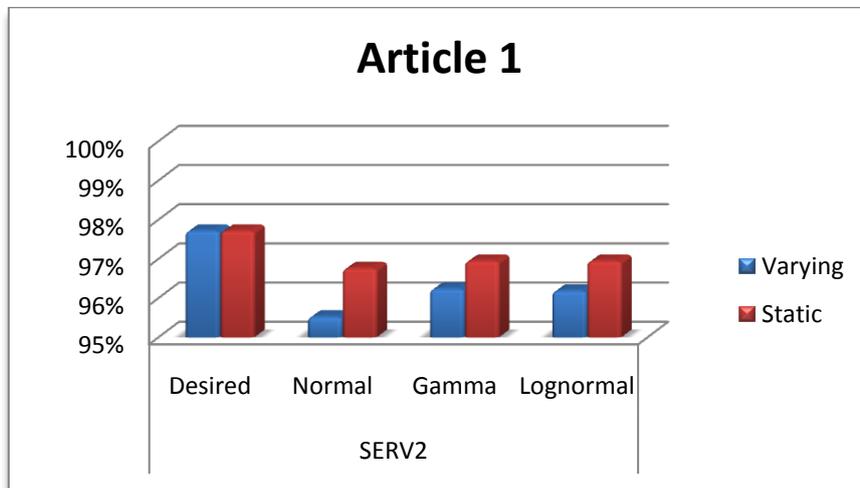


Figure 9.10 Comparison between static and dynamic lead time for article 1

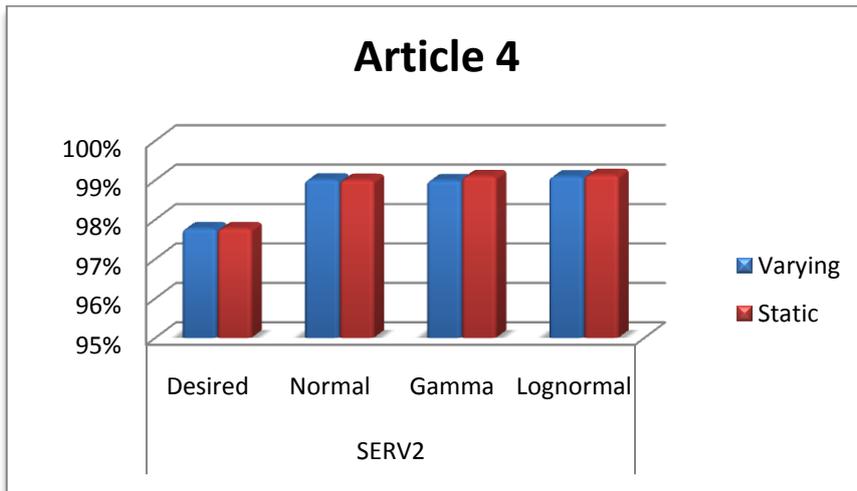


Figure 9.11 Comparison between static and dynamic lead time for article 4

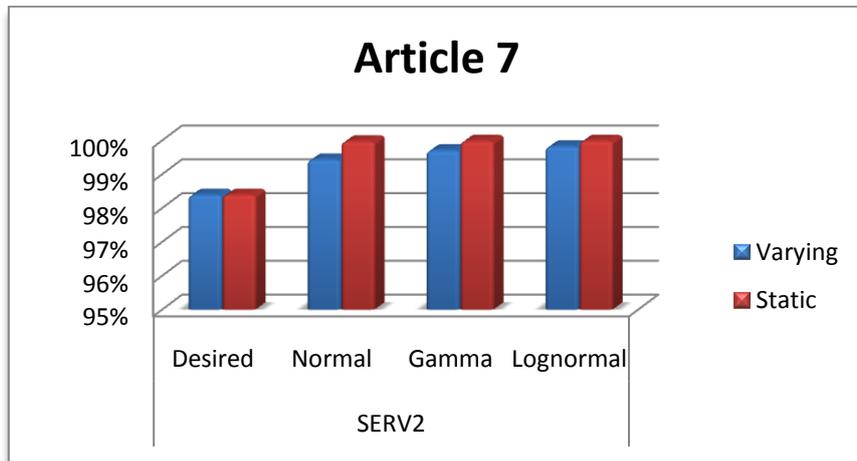


Figure 9.12 Comparison between static and dynamic lead time for article 7

10 Part II - Analysis

This chapter provides analysis of the results of Part II and discusses the identified trends.

10.1 Overall reflections

In the dynamic simulation the re-order points are rounded up, which generates higher safety stocks and hence higher service level than the PDF-test suggests. Despite this the desired service levels are not achieved in most cases when simulating the system. One reason for this is that the service measured applied in control, $SERV_1$, is not equivalent to the measure applied in measurement, $SERV_3$. When expressing the pre-defined levels of $SERV_1$ as $SERV_2$ an indication of the actual service levels that IKEA strives to obtain is revealed. These are, for all articles selected for simulation, extremely high which implies that the desired service levels are extremely expensive in practice.

A more feasible approach for IKEA to control service would be to translate desired $SERV_2$ levels into corresponding $SERV_1$ levels. This would provide a more realistic approach to inventory control within the company and enable continued measurement through the current $SERV_3$ methodology.

10.2 Test one – Verification of the PDF-test

Store and DCG level

The test results from the PDF-tests and the dynamic simulation generate conform results on both store and DCG level. For low-volume articles, several distributions often generate equal service even if they perform differently in the PDF-tests, as can be seen in Table 9.1 and Table 9.2. This is derived from the consequent round up of the re-order point which for these articles often leads to equal re-order points. The conformity in between the results from the PDF-tests and the dynamic simulation

indicates that the PDF-tests generate realistic preferred distributions. Thereby the result for all the 105 articles used in the PDF-test and the recommendation from Part I can be considered trustworthy.

10.3 Test two – Conformity between LT = 1 week and LT = 2 days

Store level

For all distributions the overall service level conformity between the lead times of one week and two days is very good. There are however small consistent differences between the received service levels for the two lead times. The reason for these differences lies in the fact that the re-order points are consistently rounded up to be applicable in the dynamic simulation model. For a lead time of one week the re-order points are rounded up with one margin and for two days with another. The reason for the service deviation lies in the size of these round up margins. The lead time that generates the largest margin also generates a higher service, which is illustrated in Table 10.1 below for an example article, store and distribution.

Table 10.1 Re-order points round up margin for LT = one week and LT = two days

Article 1-C		LT = one week	LT = 2 days
Gamma	Exact	3.26	0.96
	Rounded up to	4	1
	Round up margin	0.74	0.04
	Service level (%)	97.02	94.66

10.4 Test three – Quantification

Store level

Since IoS prioritize customer service levels rather than decreased costs, a decrease of capital accumulation through inventory control is only of interest as long as service levels are maintained. The recommended setup implies more accurate control, based on the PDF-test results, despite

decreased total stock-keeping. The simulations indicate that this would generate savings in the magnitude of 3.1%. This reduction of average stock level is directly caused by reduction of the total amount of safety stock, a reduction which hence is of considerably greater magnitude.

It is important to bear in mind that these results only are indicative because of the limited amount of included articles, and could deviate if the study was extended to include more articles and stores. There are substantial variations in the stock reduction between different articles, why the average could shift in both directions. The generation of a 97.5%-confidence interval indicate that the conclusion of a decrease of total stock is valid on 98.75%-basis, implying that the probability of reducing average stock is very high.

As seen in this thesis the approximated $SERV_2$ levels are very high, almost 100% in most cases, which create extremely high inventory levels and therefore high capital accumulation, as seen in Figure 3.6. This is something that should be further looked into since such levels of desired service are very hard to motivate.

10.5 Test four – Measure frequency

Store and DCG level

There are two feasible explanations of the relation between decreased measure frequency and accuracy. Naturally, a decreased number of observations imply that stock-outs have occurred during the inter-observation periods without being noticed. Because of this possible misallocation and the fact that each of the relatively few observations has a greater impact on the service level the accuracy is decreased. Since the approximation that $SERV_2 = SERV_3$ attenuates when inspection frequency declines the relation between simulation model control and measure is weakened for daily and weekly measures.

On store level the $SERV_3$ under continuous review is weakly correlated to the discrete service measures with levels reaching from 0.1 – 0.5. This indicates that the measures are related, but only to a certain extent. Variations in one measure are hence not necessarily mirrored in the other two. The great differences in between the levels of correlation for daily and weekly measurements imply that a significant increase of measure reliability is achievable through applying daily measure.

On DCG level the correlation factors are significantly higher and more equal for the two frequencies. The high correlation factors imply that the gain of conducting more frequent measurements is small.

One feasible explanation for the differences between store and DCG levels is that the accumulation of all 17 stores demands on DCG level decrease variance in demand patterns which generates more stable control.

From the quotas between order line service and the two discrete measures it is possible to estimate the average measure error when applying daily or weekly observations. These quotas are consistently in the magnitude of 98.8% on store level and 99.7% on DCG level independent of distribution and frequency. This implies that the weekly observations of $SERV_3$ on store level conducted today are about one percentage point higher than the actual service level on order line basis. On DCG level the deviation can be considered insignificant and hence negligible.

10.6 Test five – Lead time deviation

DCG level

The results for this test was as expected better for the static lead time than for the varying lead time. Lead time variation implies volatility in the demand during lead time and increases the risk that the pre-calculated safety stock based on constant lead time is insufficient. The impact of

variation in customer demand during lead time can be seen in Equation 3.8.

No significant difference in between the three distributions can be noticed, even if the normal distribution tends to differ marginally more than its two competitors. Overall the deviations are considered to be within reasonable limits and there is no cause for immediate action in this aspect.

11 Conclusions

In this chapter the outcome of the objectives are compiled and presented in a concise and structured order corresponding to the objectives of study.

11.1 Part I

A robust method has been developed through review and modifications of a concept created by Stig-Arne Mattsson.

The conclusion that can be drawn from the results in Part I is that volume is strongly indicated as the critical characteristic for which distribution that is consider most suitable. The results indicate that the gamma distribution is most accurate for low and medium volume articles and that the normal distribution is to be preferred for higher volumes.

There are two possible conceptual setups for implementing this. Either all articles on each level can be controlled by one distribution according to Table 11.1

Table 11.1 Current setup versus recommended setup

	DCG level	Store level
Current setup	Lognormal	Normal
Recommended setup	Normal	Gamma

The second alternative is to sequence the articles according to a volume criterion on each level respectively and hence divide the articles in two groups on each level and control the low volume articles with gamma and high volume articles with normal, as illustrated in Table 11.2.

Table 11.2 Optional setup due to the volume aspect

	Volume aspect	DCG level	Store level
Current setup	Low	Lognormal	Normal
	Medium	Lognorma	Normal
	High	Lognorma	Normal
Recommended	Low	Gamma	Gamma
optional setup	High	Normal	Normal

11.2 Part II

A two-step simulation model has been successfully created and validated and has been used to verify the method and results of Part I.

The dynamical model is compared to the static PDF-tests and result conformity in between the two methods is ensured. The transition into actual lead times is also verified.

The setup recommended in Part I is indicated to generate a stock reduction of 3.1%. The hypothesis of stock reduction is true on 98.75-level based on the sample selection of articles included in the dynamical simulation study. Regarding the service level measure frequency the weekly measure applied today is indicated to generate substantial loss in accuracy compared to more frequent measures on store level. To resolve this, the measure frequency could be increased or approximate correction for the error, which consistently is about 1%, could be applied. On DCG level the measure accuracy is considered sufficient.

When the distributions are exposed to lead time deviation on DCG level, the normal distribution is affected the most. The results are not to be considered as alarming and no immediate action is needed.

The $SERV_2$ levels calculated indicates that the experienced service IoS strives to retain is extremely high. This implies capital accumulation on levels which are hard to motivate.

11.3 Recommendations

- The current statistical distribution setup for determining demand during lead time is not considered optimal.
 - Embrace the indications of the possibility for a more optimal setup, hence broaden the study further, in regard to the amount of articles and markets included, to ensure general validity.
 - Gather sales data on a daily basis for higher validity in future PDF-tests.
- Indications of decreased inventory levels are presented.
 - Broaden the amount of articles in the study and include all stores for a more general estimate of reductions in the inventory level.
- The simulations showed that the weekly measure of service levels on store level is not sufficient.
 - Increase measure frequency to be performed on a daily basis or approximate manual correction for the error, which consistently is about 1%.
- Current service level control and measure is incompatible.
 - Introduce $SERV_2$ control which is a more suitable service measurement for inventory control than $SERV_1$ and also

- compatible with the present measure procedure, i.e. $SERV_3$.
- If the $SERV_1$ measurement is maintained modification of measure procedure is needed.

12 Discussion

A general discussion of the methods applied and approximations used and to what extent they affect the credibility of the results.

The choice of applying two, complementary, methods to achieve the objectives enhances credibility and enables deeper analysis of the problem. The methods generate conform results and cross-verifies each other.

There are however known aspects that decrease the accuracy of the results. Several approximations influence Part II, such as rounded re-order points and transformed service levels. Out of these approximations the rounded re-order points are considered to have the greatest impact on results, especially for low-volume articles for which this can increase the re-order point with up to 50%. The accumulation of capital generated by these extra pieces in stock is however negligible. The impact of the transformed service levels is hard to estimate.

Furthermore the simplified handling of customer demand in the simulation model by interpreting one customer demanding multiple pieces as an equivalent amount of customers demanding one piece implies a divergence from reality.

A more significant aspect affecting both Part I and II is that the demand analysis is based upon figures of weekly sales, which implies two levels of insecurity. Primarily, sales do not equal demand. It is unknown to which extent customers have been unable to buy desired articles because of stock-outs, lack of information in the store or simply because they were misplaced. Secondly, much information of the demand structure is lost because of the accumulation of demand during the long periods. The PDF-tests in Part I are coined by the approximation that lead time is one week

on store level which decreases validity. Daily sales figures would have provided a more detailed picture and enabled use of actual lead time through bootstrapping.

This method optimizes re-order points in an (R,Q)-system according to the most apt distribution based on a series of parameters. Most of these parameters are uncontrollable as imposed in the method but there is one exception. The order quantities can be adjusted by IoS. To obtain total optimality in the system, re-order points and order quantities should be optimized simultaneously.

One weakness bound to the method is that it does not account for trends or seasonal variations. Either goods with major fluctuations could be forecasted through other methods, or be bounded with a manually forecasted extra stock accounting for such variations. An intuitive thought is that self-serve stock, such as the studied articles, are of more consistent nature and hence is less affected than other ranges.

In further studies it is also possible to gradually increase model complexity by e.g. including direct delivery, multiple sourcing and multiple DCGs. Furthermore it could be of interest to include more distributions in the evaluation.

Due to the limited framework of the project only a selection sample of all IKEA articles has been processed in this Master thesis. Further broadening of the study based on the findings in this thesis would therefore be of great interest.

Despite these aspects the indication of increased performance with the recommended setup is considered as worthy of attention since it implies significantly more accurate control. The statistically invigorated hypothesis of decreased inventory levels further enhances this point of view.

13 Bibliography

In this chapter the complete information of the sources is presented.

13.1 Published sources

Axsäter, S. (2006) *Inventory Control*. New York: Springer.

Axsäter, S. (1991) *Lagerstyrning*. Lund: Studentlitteratur.

Banks, J. (1998) *Handbook of simulation: Principles, methodology, advances, applications and practice*. New York: Wiley.

Björklund, M. and Paulsson, U. (2003) *Seminarieboken- att skriva, presentera och opponera*. Lund: Studentlitteratur.

Blom, G. (2005) *Sannolikhets teori och statistikteori med tillämpningar*. Lund: Studentlitteratur.

Bryman, A. and Bell, E. (2005) *Företagsekonomiska forskningsmetoder*. Malmö: Liber ekonomi.

Eriksson L. and Wiedersheim-Paul, F. (2001) *Att utreda, forska och rapportera*. Malmö: Liber Ekonomi.

Jacobsen, D. I. (2002) *Vad, hur och varför: Om metodval i företagsekonomi och andra samhällsvetenskapliga ämnen*. Lund: Studentlitteratur.

Kamprad, I. (2007) *En möbelhandlares testamente: Lilla IKEA Ordboken*. Inter IKEA systems.

Kelton, W. D. and Law, A. M. (2000) *Simulation modeling and analysis*. New York: McGraw-Hill.

Laguna, M. and Marklund, J. (2005) *Business process modeling, simulation and design*. USA: Pearson prentice hall.

Lumsden, K. (2006) *Logistikens Grunder*. Lund: Studentlitteratur.

Merriam S. B. (1994) *Fallstudien som forskningsmetod*. Lund: Studentlitteratur

Sheldon, R. (1989) *Introduction to probability models*. USA: Academic Press.

13.2 Articles

Bookbinder, J. and Lordahl, A. (1989) *Estimation of inventory re-order levels using the bootstrap statistical procedure*. Ontario: IIE Transactions.

Dahlberg-Hildén, A. et al. (2009) *A project on how to find a distribution to estimate the demand during lead time*. Unpublished Local research at the Department of Industrial Management and Logistics, Engineering logistics. Lunds University and IKEA

Mattsson, S-A. (2007) *Användning av cykelservice för säkerhetslagerberäkning*. Local research at the Department of Technology Management and Economics. Chalmers University.

Mattsson, S-A. (2006) *Lagerstyrning vid korta ledtider*, Local Research at the department of Industrial Management and Logistics. Lund University

Mattsson, S-A. (2003a) *Användning av verklig efterfrågefördelning*, Local research at the Department of Industrial Management and Logistics, Engineering logistics. Lunds University.

Mattsson, S-A. (2003b) *Efterfrågefördelning för bestämning av säkerhetslager*. Local Research at the department of Industrial Management and Logistics. Lund University

Smart, C. and Willemain, T. (2000) *Get Real - A new way to forecast intermittent demand*, Apics: The performance Advantage.

Stjernfeldt, E. and Johansson, L. (2009) *The probability distribution of demand – A case study at IKEA of Sweden*. Local research at the School of Economics and Management, Växjö University

13.3 Electronic sources

<http://www.ikea-group.ikea.com/?ID=25> , About the IKEA group, (09-06-09)

<http://mathworld.wolfram.com/topics/ContinuousDistributions.html>,
Continuous distributions (09-09-01)

<http://www.informs-sim.org/wsc00papers/042.PDF>, The Extend simulation environment (09-09-15)

http://en.wikipedia.org/wiki/Poisson_distribution (a) (09-11-03)

http://en.wikipedia.org/wiki/Expected_value (b) (09-11-03)

Goods Flow Advance, IKEA intranet (2009-10-15)

13.4 Oral Sources

Björnsson Paul, Process Leader

Christian Howard, PhD Student Departement of Industrial Management and Logistics, Lund Institute of Technology, Lund University

Elmqvist Birgitta, Project Leader

Ericsson Bosse, Business analysis manager

Mattsson Stig-Arne, Adjunct Professor in Supply Chain Management at the Department of Logistics and transport at Chalmers University of Technology

Pettersson Lars – Göran, Supplier union representative/Head of conserving corporate culture

Appendix I

Complete data regarding lead times, minimum order quantity, total order quantities and service levels for an example article.

Article 4	Min order quantity	Number of Q ordered each time	Lead time	Mean demand during a week	Standard deviation during a week	Service level classification	Price level classification
Storenumber	Q	Q0,qi	LT (days)	My (pieces)	Sigma	S	P
DC	1	68	22	38,56	21,75	95%	M
STO A	1	18	2	5,10	4,11	95%	M
STO B	1	4	2	2,02	2,36	95%	M
STO C	1	4	2	1,38	2,39	95%	M
STO D	1	4	3	1,96	2,73	95%	M
STO E	1	9	2	3,33	3,52	95%	M
STO F	1	10	2	3,13	3,00	95%	M
STO G	1	4	2	1,52	2,15	95%	M
STO H	1	5	2	1,81	1,87	95%	M
STO I	1	4	2	0,92	1,30	95%	M
STO J	1	5	2	2,42	2,74	95%	M
STO K	1	4	2	2,13	2,18	95%	M
STO L	1	5	2	2,35	2,58	95%	M
STO M	1	5	2	2,35	2,70	95%	M
STO N	1	5	2	2,83	2,39	95%	M
STO O	1	4	2	1,40	2,01	95%	M
STO P	1	4	2	2,52	3,12	95%	M
STO Q	1	4	2	1,10	1,49	95%	M

Appendix II

The demand data for the articles that qualified in the data selection process is sorted by store.

Article 4										
Week	STO A	STO B	STO C	STO D	STO E	STO F	STO G	STO H	STO I	...
1	13	5	10	2	4	2	3	3	4	6
2	13	4	5	9	0	6	0	2	2	8
3	8	3	7	4	6	2	8	2	2	5
4	5	9	2	3	2	2	3	3	4	4
5	12	7	1	7	5	7	1	4	3	10
6	5	9	1	0	0	1	2	0	0	4
7	6	2	2	4	8	2	2	5	0	6
8	1	0	3	1	4	0	2	3	0	1
9	9	5	7	1	2	2	2	2	1	3
10	4	4	3	2	2	4	7	0	5	4
11	2	2	6	2	6	0	1	0	2	5
12	4	5	3	5	1	2	4	5	2	3
13	2	3	2	1	16	3	0	1	1	1
14	1	0	2	1	6	4	2	1	2	2
15	6	5	3	7	2	1	6	1	1	3
16	8	4	0	1	6	4	3	2	1	6
17	3	2	8	6	4	4	1	0	1	2
18	5	2	2	3	9	7	2	1	2	1
19	8	3	2	1	2	3	3	4	2	4
20	1	2	1	2	2	6	2	2	2	1
21	5	1	2	2	3	5	3	3	1	1
22	4	3	0	2	4	4	1	5	4	5
23	5	2	0	3	2	6	9	2	2	3
24	10	1	0	9	6	10	2	8	2	7
25	0	4	0	11	6	7	2	1	1	2
26	21	2	0	1	10	2	4	6	1	0
27	10	2	0	2	6	5	2	2	0	5
28	3	2	0	6	14	3	2	1	0	4
29	0	2	0	2	1	10	0	4	0	1
30	2	2	0	2	2	0	0	2	0	5
31	1	6	0	0	4	2	0	1	0	10
32	1	2	0	0	2	4	0	5	0	2
33	4	0	0	0	1	1	0	1	0	2
34	5	0	0	0	6	6	0	2	0	0
35	5	0	0	0	1	6	0	3	0	0
36	4	0	0	0	4	4	0	2	0	0
37	11	0	0	0	2	11	0	1	0	0
38	9	0	0	0	4	3	0	2	0	0
39	3	0	0	0	5	9	0	1	0	0
40	3	0	0	0	3	3	0	1	0	0
41	6	0	0	0	0	0	0	0	0	0
42	4	0	0	0	0	0	0	0	0	0
43	7	0	0	0	0	0	0	0	0	0
44	3	0	0	0	0	0	0	0	0	0
45	2	0	0	0	0	0	0	0	0	0
46	6	0	0	0	0	0	0	0	0	0
47	8	0	0	0	0	0	0	0	0	0
48	3	0	0	0	0	0	0	0	0	0
49	1	0	0	0	0	0	0	0	0	0
50	3	0	0	0	0	0	0	0	0	0
51	0	0	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0	0	0	0

Appendix III

Microsoft Visual Basic Code

Sub Automatisering PDF-test ()

Application.ScreenUpdating = False

Dim variable As String

Dim NormalWeek As Integer

Dim NormalLT As Integer

Dim GammaWeek As Integer

Dim NormalReject As Integer

Worksheets(1).Activate

NbrOfSheets = Worksheets.Count

For n = 1 To NbrOfSheets - 1

variable = n

Collects the service level for the article being analyzed

Worksheets(1).Activate

Cells(2, 2).Select

Selection.copy

Sheets("Beräkningar").Activate

Cells(8, 6).Select

ActiveSheet.Paste

For k = 0 To 16

Worksheets(n).Activate

Copies store name from data sheet into calculation sheet

Range("A" & 3 + k).Select

Selection.copy

Sheets("Beräkningar").Select

Range("D" & 28 + 7 * k).Select

ActiveSheet.Paste

Collects data from data sheet into calculation sheet

Sheets(n).Select

Range(Cells(3 + k, 2), Cells(3 + k, 53)).Select

Selection.copy

```

Sheets("Beräkningar").Select
Range("A3").Select
Selection.PasteSpecial
Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
:=False, Transpose:=True

```

Data sorting

```

Range("A3:A54").Select
ActiveWorkbook.Worksheets("Beräkningar").Sort.SortFields.Clear
ActiveWorkbook.Worksheets("Beräkningar").Sort.SortFields.Add
Key:=Range("A3") _
, SortOn:=xlSortOnValues, Order:=xlAscending,
DataOption:=xlSortNormal
With ActiveWorkbook.Worksheets("Beräkningar").Sort
.SetRange Range("A3:A54")
.Header = xlNo
.MatchCase = False
.Orientation = xlTopToBottom
.SortMethod = xlPinYin
.Apply
End With

```

Copy and paste the calculated k-values together with corresponding store name

```

Range("E12:G16").Select
Selection.copy
Range("E" & 27 + 7 * k).Select
Selection.PasteSpecial
Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
:=False, Transpose:=False

```

If the CV for the normal distribution > 0.5 it is marked by the color red

```

If (Cells(3, 10) > 0.5) Then
Range("E" & 29 + 7 * k).Select
With Selection.Font
.Color = -16776961
.TintAndShade = 0
End With
End If

```

The best distribution is decided, given that the normal was considered ok

```
If (Cells(3, 10) < 0.5) Then
  Range("F21:F23").Select
  Selection.copy
  Range("L20").Select
  Selection.PasteSpecial
  Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
    :=False, Transpose:=False
  Range("M23").Select
  Selection.copy
  Range("F24").Select
  Selection.PasteSpecial
  Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
    :=False, Transpose:=False
```

Registration of best performed distribution

```
If (Cells(24, 6) = "Normal") Then
  NormalWeek = NormalWeek + 1
End If
If (Cells(24, 6) = "Gamma") Then
  GammaWeek = GammaWeek + 1
End If
If (Cells(24, 6) = "Lognormal") Then
  LogWeek = LogWeek + 1
End If

Else
```

The best distribution is decided, given that the normal was not considered ok

```
Range("F22:F23").Select
Selection.copy
Range("I21").Select
Selection.PasteSpecial
Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
  :=False, Transpose:=False
Range("J23").Select
Selection.copy
Range("F24").Select
```

```
Selection.PasteSpecial  
Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks_  
:=False, Transpose:=False
```

Registration of best performed distribution

```
If (Cells(24, 6) = "Gamma") Then  
    GammaWeek = GammaWeek + 1  
End If  
If (Cells(24, 6) = "Lognormal") Then  
    LogWeek = LogWeek + 1  
End If  
  
NormalReject = NormalReject + 1  
  
End If
```

Copy and paste the deviation for the different distributions k-values from the empirical k-value

```
Range("F19:F24").Select  
Selection.Copy  
Range("H" & 27 + 7 * k).Select  
Selection.PasteSpecial  
Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks_  
:=False, Transpose:=False  
  
Next k
```

Presents the compilation of the best performed distributions and the number of rejected normal

```
Cells(29, 12) = NormalWeek  
Cells(30, 12) = GammaWeek  
Cells(31, 12) = LogWeek  
Cells(32, 12) = NormalReject
```

Copy the compiled results from the calculation sheet and paste in the data sheet

```
Range("D26:M146").Select  
Selection.Copy  
Sheets(n).Select  
Range("A22").Select  
ActiveSheet.Paste
```

Color the red normal into black for the next article to come

```
Sheets("Beräkningar").Select  
Range("E28:E143").Select  
With Selection.Font  
    .ColorIndex = xlAutomatic  
    .TintAndShade = 0  
End With
```

The variables are emptied

```
NormalWeek = 0  
GammaWeek = 0  
LogWeek = 0  
NormalReject = 0
```

Next n

```
Application.ScreenUpdating = True
```

End Sub

Appendix VI

Excel functions

Excel is used to calculate the re-order points and k-values. The different commands that are used for the different distributions are presented in the sections below.

Empirical distribution

In Excel the following commands are used to find the index value and the corresponding re-order point:

PositionInVector = CEILING ("service level"*52;1), which calculates the index-value.

Re-order point = OFFSET ("StartOfVector";"PositionInVector";0), which collects the re-order point.

Normal distribution

To obtain the k-value in excel the inverse of the normal cumulative distribution is calculated, with the desired service as input according to Equation 3.2;

K-value = NORMINV (service level)

Re-order point = $k * \sigma_x + \mu_x$

Gamma distribution

The re-order point is calculated using the Excel command below with the input parameters: service level, alpha and beta. The re-order point is further used to calculate the k- value which is further used to calculate the k-value according to Equation 3.1.

Re-order point = GAMMAINV (service level; α ; β)

K-value= (Re-order point - μ_x)/ σ_x

Lognormal distribution

To obtain the re-order point the Excel command below is used with the input parameters: service level, mean_{LN} and standard deviation_{LN} according to Equation 3.15 and Equation 3.16. Further the re-order point is used to calculate the k-value according to Equation 3.1.

Re-order point = LOGINV (service level; μ_{LN} ; σ_{LN})

K-value = (Re-order point - μ_x)/ σ_x

The loss function G(x)

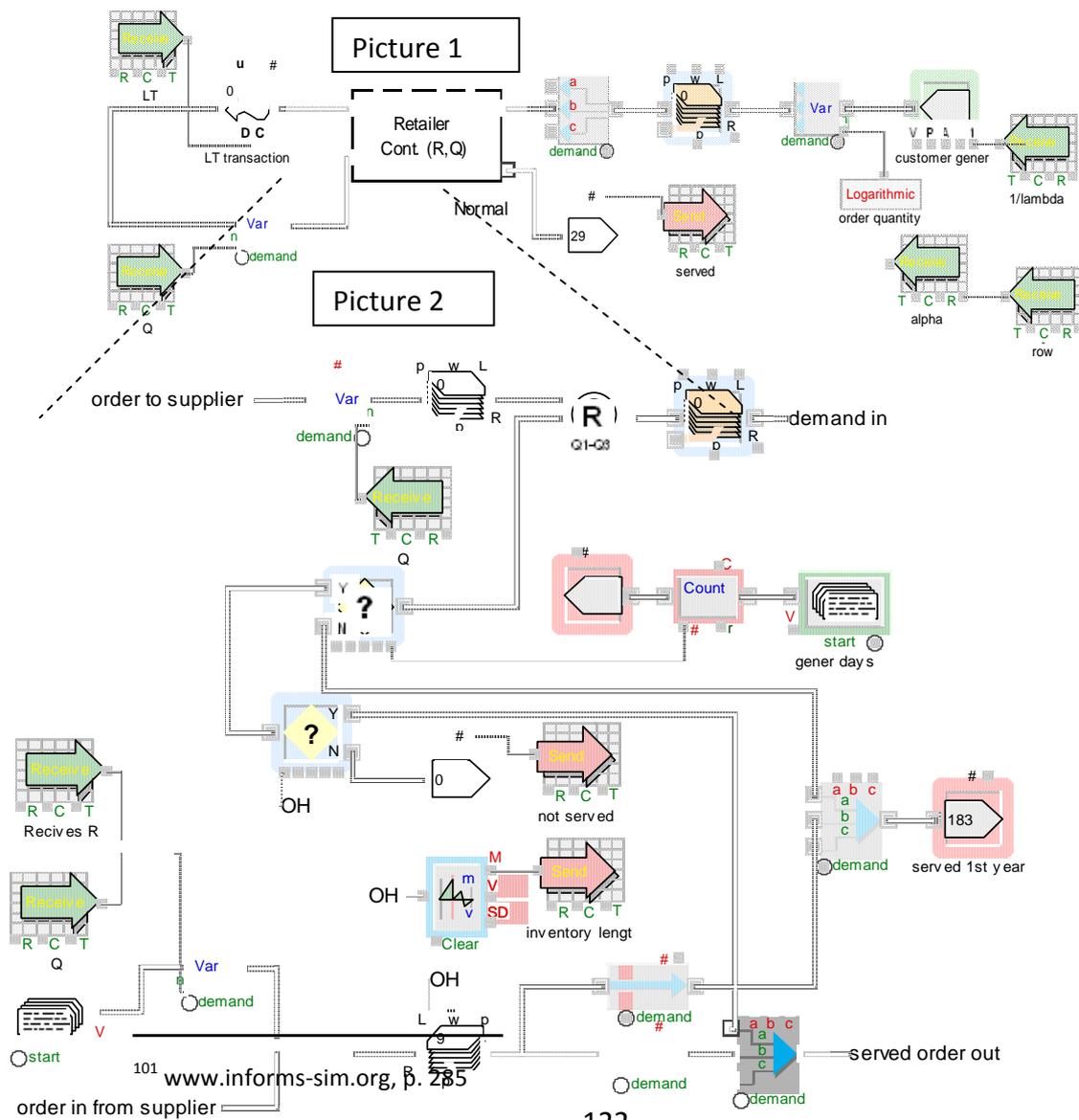
To calculate the loss function for a given service level the following formula applies

= NORMDIST (X;0;1;FALSE)-X*(1-NORMDIST(X;0;1;TRUE))

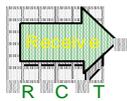
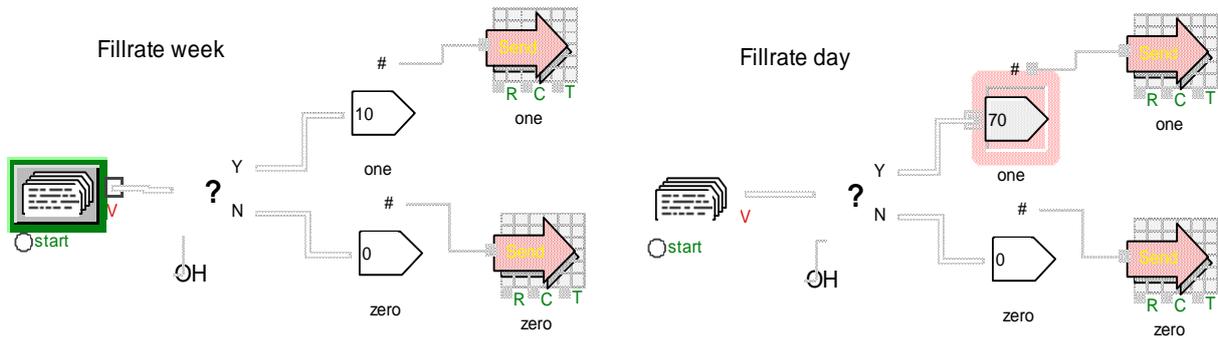
In which $X = \phi^{-1}(SL)$

Appendix VII

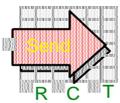
The simulation model has been built hierarchically¹⁰¹, which means that some blocks has been consolidated and replaced by a larger block (in our model block “Retailer Cont. (R, Q)” to make the simulation model easier to follow, picture 1. The content of the hierarchical block is overviewed in the picture 2 below. In picture 3 the model that measures the fill rate per day and week is shown. These models belong to picture 2.



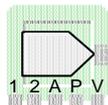
Picture 3



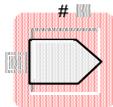
This block receives values from a spreadsheet in Excel



This block sends values to a spreadsheet Workbook in Excel



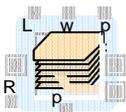
Represents the arrival of items from outside the scope of a model at a constant or a random rate



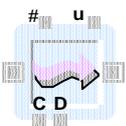
Passes items out of the simulation.



Selects one of two paths based on one or more conditions calculated in an equation.



Takes in items and holds them until they are requested by other blocks in the model.



The Transaction block performs transactions on multiple items at the same time.



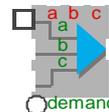
Generates random numbers based on the logarithmic distribution.



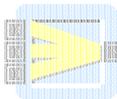
Performs mathematical calculations of the inputs and outputs

a
b
c

Produces several items from a single input item.



Allows items from several sources to be joined as a single stream



Receives items from up to three sources and merges them into a single stream.

#

Passes an item only when certain conditions exist at the demand input



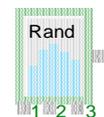
Generator block with scheduled arrival times rather than random.



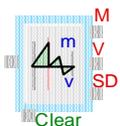
Turns a single item into a stream of identical items



Allows a variable number of items to be joined into a single item



Generates random integers or real numbers based on the selected distribution



Calculates the mean, variance, and standard deviation of the values input during the simulation.

Appendix VIII

Simulation input parameters

Common data	Article	STO	SERV1	SERV2	LT (weeks)	LT (days)	Q						
	1	A	0,9	0,98	0,29	2	7						
	1	B	0,9	0,98	0,29	2	7						
	1	C	0,9	0,99	0,29	2	7						
	4	A	0,95	1,00	0,29	2	18						
	4	B	0,95	0,99	0,29	2	4						
4	C	0,95	0,99	0,29	2	4							

Compound poisson generation	Article	mean(day)	std(day)	alpha (day)	r	lambda (day)	1/lambda						
	1	0,44	1,16	0,67	0,22	0,24	4,16						
	1	0,35	0,91	0,58	0,25	0,22	4,62						
	1	0,17	0,67	0,62	0,10	0,10	10,14						
	4	0,73	1,55	0,70	0,32	0,38	2,65						
	4	0,29	0,89	0,64	0,16	0,17	6,02						
4	0,20	0,90	0,76	0,06	0,09	11,13							

Calculation of R-values	Article	mean	std	normal	gamma	lognormal	normal	gamma	lognormal	normal	gamma	lognormal
		week	week	k-värden (week)			R (week)- exact			R (week)- rounded up		
	1	3,10	3,08	1,28	1,30	1,06	7,04	7,11	6,35	8	8	7
	1	2,42	2,41	1,28	1,30	1,06	5,51	5,57	4,97	6	6	5
	1	1,17	1,77	1,28	1,18	0,82	3,44	3,26	2,62	4	4	3
	4	5,10	4,11	1,64	1,96	1,85	11,85	13,16	12,70	12	14	13
4	2,02	2,36	1,64	2,01	1,70	5,91	6,77	6,05	6	7	7	
4	1,38	2,39	1,64	1,98	1,43	5,31	6,10	4,79	6	7	5	

Calculation of R-values	Article	mean	std	normal	gamma	lognormal	normal	gamma	lognormal	normal	gamma	lognormal
		LT	LT	k-värden(LT)			R(LT) - exact			R(LT) - rounded up		
	1	0,88	1,64	1,28	1,06	0,68	2,99	2,62	2,01	3	3	3
	1	0,69	1,29	1,28	1,05	0,68	2,34	2,05	1,57	3	3	2
	1	0,34	0,95	1,28	0,66	0,44	1,55	0,96	0,75	2	1	1
	4	1,46	2,20	1,64	2,00	1,53	5,07	5,85	4,83	6	6	5
4	0,58	1,26	1,64	1,87	1,22	2,66	2,94	2,12	3	3	3	
4	0,40	1,28	1,64	1,49	0,89	2,49	2,30	1,52	3	3	2	

Appendix IX

Validation of the simulation model through comparison of theoretical throughput and throughput in the simulation model.

Store - Throughput					DC - Throughput			
Article	Store	Theoretical	Simulation	Deviation	Article	Simulation	Theoretical	Deviation
7	A	12480	11757	6%	1	279	279	0,1%
7	B	7713	7696	0%	4	5745	5771	-0,4%
7	C	3399	3292	3%	7	1187	1187	0,0%
8	A	5268	5159	2%	2	847	827	2,4%
8	B	1869	1864	0%	6	3365	3385	-0,6%
8	C	1586	1567	1%	5	15563	15623	-0,4%
4	A	266	265	0%	8	222	217	2,0%
4	B	105	106	-1%	9	10942	11074	-1,2%
4	C	72	79	-8%	3	2504	2519	-0,6%
2	A	30	30	0%				
2	B	55	56	-1%				
2	C	43	43	0%				
9	A	1092	1108	-1%				
9	B	492	491	0%				
9	C	270	273	-1%				
1	A	161	161	1%				
1	B	126	128	-2%				
1	C	61	63	-3%				
5	A	10685	9945	7%				
5	B	10350	11238	-8%				
5	C	4570	4626	-1%				
6	A	420	410	3%				
6	B	311	304	2%				
6	C	236	237	0%				
3	A	696	700	-1%				
3	B	488	493	-1%				
3	C	501	495	1%				

Appendix X

Validation of the simulation model through comparison of theoretical average inventory level and average inventory levels generated in the simulation model.

Theoretical average inventory level					Simulation average inventory level				
Article	Store	Normal	Gamma	Lognormal	Article	Store	Normal	Gamma	Lognormal
1	A	6	5	5	1	A	7	7	7
1	B	5	5	4	1	B	6	6	6
1	C	5	4	4	1	C	5	5	5
2	A	2	2	1	2	A	3	2	2
2	B	2	2	2	2	B	3	3	3
2	C	2	2	2	2	C	3	3	2
3	A	17	30	28	3	A	23	29	28
3	B	9	13	14	3	B	11	14	14
3	C	12	19	18	3	C	15	19	19
4	A	13	13	12	4	A	13	13	13
4	B	4	4	4	4	B	5	5	5
4	C	4	4	3	4	C	6	6	6
5	A	367	388	375	5	A	376	389	381
5	B	201	210	209	5	B	214	219	219
5	C	306	313	310	5	C	310	315	312
6	A	16	17	17	6	A	17	18	17
6	B	8	9	8	6	B	9	10	9
6	C	8	9	8	6	C	10	10	10
7	A	136	161	174	7	A	149	165	175
7	B	109	128	136	7	B	114	126	133
7	C	85	97	102	7	C	86	96	100
8	A	283	385	363	8	A	292	360	343
8	B	192	213	214	8	B	188	208	209
8	C	189	209	209	8	C	184	202	202
9	A	39	49	50	9	A	39	47	48
9	B	26	33	33	9	B	26	32	31
9	C	10	14	14	9	C	12	14	14
Sum		2054	2339	2319	Sum		2126	2323	2314

Appendix XI

Store level

Parameters			Orderline service, LT = 1 week			Deviation			PDF-test
Article	Store	SERV2	Normal	Gamma	Lognormal	Normal	Gamma	Lognormal	Winner
1	A	97,92%	97,45%	97,45%	96,88%	-0,47%	-0,47%	-1,05%	Lognormal
1	B	98,37%	97,37%	97,37%	96,64%	-1,00%	-1,00%	-1,73%	Gamma
1	C	98,81%	97,02%	97,02%	95,84%	-1,79%	-1,79%	-2,97%	Gamma
7	A	99,79%	99,61%	99,77%	99,83%	-0,18%	-0,02%	0,04%	Gamma
7	B	99,86%	99,60%	99,75%	99,81%	-0,26%	-0,11%	-0,05%	Gamma
7	C	99,92%	99,54%	99,74%	99,80%	-0,39%	-0,19%	-0,12%	Gamma
4	A	99,52%	98,39%	98,93%	98,67%	-1,13%	-0,59%	-0,86%	Lognormal
4	B	98,76%	96,61%	97,49%	97,49%	-2,15%	-1,27%	-1,27%	Normal
4	C	98,75%	93,63%	94,68%	92,37%	-5,12%	-4,07%	-6,38%	Gamma

DC level

Parameters			Orderline service, LT = actual			Deviation			PDF-test
Article	Store	SERV2	Normal	Gamma	Lognormal	Normal	Gamma	Lognormal	Winner
1	DC	97,71%	96,76%	96,95%	96,95%	-1,00%	-0,80%	-0,80%	Lognormal
4	DC	97,77%	99,00%	99,09%	99,13%	1,20%	1,30%	1,40%	Lognormal
7	DC	98,39%	99,97%	99,98%	99,98%	1,60%	1,60%	1,60%	Lognormal
2	DC	98,68%	97,82%	98,28%	98,07%	-0,90%	-0,40%	-0,60%	Gamma
6	DC	99,42%	99,70%	99,86%	99,91%	0,30%	0,40%	0,50%	Lognormal
5	DC	99,58%	98,35%	98,88%	98,84%	-1,20%	-0,70%	-0,70%	Normal
8	DC	99,84%	99,74%	99,81%	99,87%	-0,10%	0,00%	0,00%	Normal
9	DC	99,94%	98,99%	99,29%	99,42%	-0,90%	-0,70%	-0,50%	Gamma
3	DC	100,00%	98,26%	99,07%	99,35%	-1,70%	-0,90%	-0,60%	Normal