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Analysis of a coordinated multi-echelon inventory control system

-A case study on its performance compared to the current inventory control system at

syncron

MANAGE YOUR GLOBAL SUPPLY CHAIN – EASILY

A master thesis project

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Preface

This master thesis was performed during a time period of 20 weeks from September 2009 until January 2010. The project corresponds to 30 ECT and it is the final part of the Master of Science program Industrial Engineering and Management at Lund Institute of Technology, Lund University. The study was executed at Syncron in cooperation with the division of Production Management at Lund Institute of Technology.

The master thesis was performed with the knowledge that we have learned during our four years of studding at Lund University and the goal has always been that both Syncron and the University will benefit from the results.

There are several people who have made this master thesis possible. First we would like to thank our supervisors at Syncron, Cecilia Wiman and Daniel Martinsson. We also would like to give great thanks to Sara Östborg, who has answered our questions well about the SCP. At Lund Institute of Technology we would like to thank our supervisor Peter Berling for all the time he has invested in this project. We also would like to thank Johan Marklund who has helped us with the analytical model during his paternity leave. Finally we would like to thank everybody at Syncron in Malmö, where the major part of the work has been done.

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Abstract

Even though the coordinated inventory control is becoming more well-known, it is relative unused by companies, whose advantages of using it should be obvious. This master thesis illustrates the result of using coordinated inventory control compared to a currently used non-coordinated inventory control.

There exist precise coordinated methods for control of a multi-echelon inventory system, but they are too computationally complex to use in practice. Approximations are needed to allow a coordinated inventory control in practice and not just in theory. The basic idea with the coordinated model evaluated in this master thesis is to introduce an induced backorder cost at the central warehouse allowing decompose of the multi-echelon inventory system to several single-echelon inventory systems. The inventory system which the model decomposes is a distribution system with one central warehouse and N different retailers. The coordinated model has earlier been used and tested in real case scenarios and other master thesis, but since then it has been developed to improve its performance.

The study began by selecting a number of articles that represent the actual material flow within the studied inventory system. Then the reorder points of all the selected articles were calculated with the coordinated model. The calculated reorder points were then compared, by simulations in the simulation software Extend, with the current reorder points obtained from Syncron. The reorder points from Syncron are calculated without any coordinated inventory control.

The results of the project have shown that by using a coordinated inventory control of the inventory system, the total inventory in the system is reduced significantly, with about 35%, while the service level most of the times is improved or at least maintained. Most of the inventory within the inventory system has shifted from the central warehouse out to the retailers.

Sammanfattning

Vid lagerstyrning av ett flernivålagersystem används idag för det mesta inte en koordinerad lagerstyrning. Trots att den koordinerade lagerstyrningen börjar bli mer utbredd så är den relativ oanvänd ute hos företagen, vars nytta av att använda den skulle vara påtaglig. Detta examensarbete visar på resultatet av att använda en koordinerad lagerstyrning jämfört med en icke koordinerad lagerstyrning.

Det finns exakta koordinerade metoder för styrning av flernivålagersystem men de är för beräkningstunga för att kunna användas i praktiken. Genom att ta fram och använda sig av approximationer kan den teoretiska koordinerad lagerstyrning användas praktiskt. Grundtanken med den koordinerade modellen som analyseras i detta examensarbete är att införa en fiktiv bristkostnad som möjliggör nedbrytning av ett flernivålagersystem till flera enkla lagersystem. Lagersystemet som bryts ned består av ett centrallager och N stycken olika återförsäljare. Den koordinerade modellen har använts och testats tidigare på flera verkliga scenarier och i andra examensarbete, men sedan dess har modellen utvecklats för att förbättra dess resultat.

Studien startade med att välja ut ett antal artiklar som representerar det verkliga flödet av material i det studerade lagersystemet. Därefter beräknades beställningspunkter fram för alla valda artiklar med hjälp av modellen för koordinerad lagerstyrning. De beräknade beställningspunkterna jämfördes därefter med de nuvarande beställningspunkterna från Syncron, som är framtagna utan koordinerad lagerstyrning, i simuleringsmjukvaran Extend

Resultaten från studien visar att användandet av en koordinerad styrning av ett lagersystem kan reduceras det totala lagret i systemet kraftigt, med i genomsnitt 35 %, samtidigt som servicenivån i systemet i de flesta fall förbättras eller i alla fall bibehålls. Den stora skillnaden som sker inom lagersystemet är att lagernivån sänks hos centrallagret och istället förskjuts ut till de olika återförsäljarna.

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1 Introduction

In this chapter the company Syncron is described. The problem background and problem definition, delimitations, objective, purpose, target group and report outline are discussed.

1.1 Syncron

Syncron is a global company with offices in the most of the world such as Japan, United Kingdom, Australia, India, Italy and Sweden. They deliver software and services for global supply chain planning, fulfillment and supply. The company has been in the supply chain business for over 15 years and Syncron has delivered significant results for industry leaders such as Volvo, Tetra Pak and Astra Zeneca (Manage your global supply chain easily - Syncron-, 2009).

Syncron has developed core values that influence the whole company. Whatever they do the core values have been chosen to help them do it the right way. The core values are (Manage your global supply chain easily - Syncron-, 2009):

- *Customer success,* Syncron is developing the best solution for the customer, in order to create a maximum value.
- *Always ahead,* the customer will gain access to thought leadership that will help them to stay ahead of competition.
- *Global perspective*, no matter where the customer is located, Syncron is there for them.
- *Make a difference*, Syncron takes pride in their work and strive to always exceed expectations.
- *Fairness and respect*, Syncron treat each other with respect and maintain fairness in all relationships.

Syncron has five solutions they sell to their customers. The solutions easily enable global processes across the extended supply chain whilst leveraging the customers' existing investment in ERP systems (Manage your global supply chain easily - Syncron-, 2009). The solutions can be overviewed in Figure 1 below:

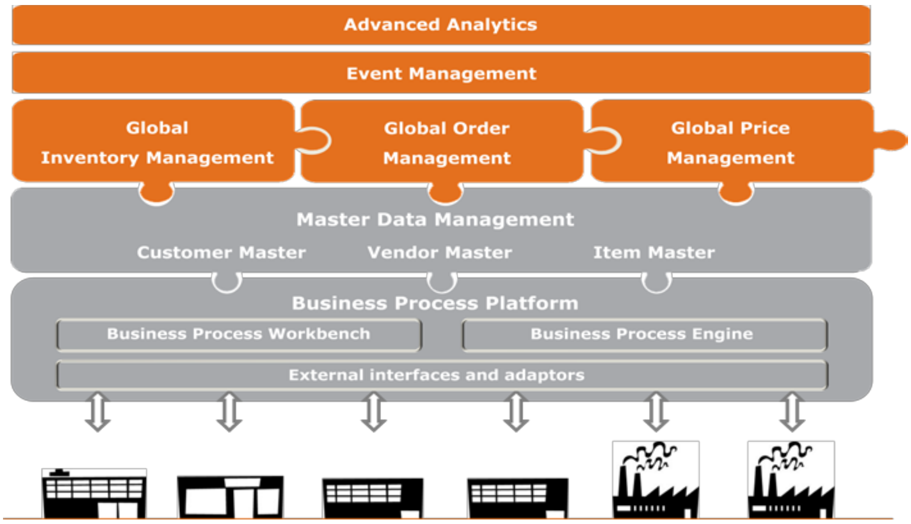


Figure 1 – Syncrons solutions (Manage your global supply chain easily - Syncron-, 2009)

Business Process Platform

Syncrons Business Process Platform (BPP) is based on Service Oriented Architecture (SOA) which means that the platform is able to adapt after its customers processes. All Syncrons solutions are built on the BPP. Hence, everyone that implements someone of Syncrons solutions needs the BPP. The platform consists of three major parts (Manage your global supply chain easily - Syncron-, 2009):

- Business process workbench, which is used to design the services and process in what is called Workflows and Micro flows.
- Business process engine, which is responsible for executing the Workflows and Micro flows.
- External interfaces and adaptors.

Master data management

In many companies their data is stored in different databases that sometimes are spread all over the world. This makes it hard to reach the data and sometimes the data could disappear, which can affect and reduce the operational efficiency. Syncron Master Data Management (MDM) brings together all the dispersed master data into one master store, available from everywhere (Manage your global supply chain easily - Syncron-, 2009).

Global inventory management

The Global Inventory Management (GIM) optimizing the customers global inventories and ensures that the right goods are always at the right place at the right time and in the right quantity. Through an interface that is simple to use, large number of products in a complex global chain is made easy to manage (Manage your global supply chain easily - Syncron-, 2009).

Global order management

The Global Order Management (GOM) manages the customers global order fulfillment with a single process. It integrates the internal and external business systems so Syncrons customers can provide their customers with real time information, which enable for an example track and trace information (Manage your global supply chain easily - Syncron-, 2009).

Global price management:

With increasing globalization it is important to be able to adjust prices to stay competitive. The Global Price Management (GPM) supports the different steps in a pricing process from data gathering to price setting and execution. It helps the customer to quickly analyze and synchronize new prices across the organization, with significantly reduced administrative costs (Manage your global supply chain easily - Syncron-, 2009).

1.2 Problem background

Today, there is no effective and simple method to optimize the reorder points in a multi-echelon inventory system. Syncron is currently controlling the ordering process (i.e. the reorder points) in a decentralized manner without any direct coordination between the different echelons. The division of Production Management at Lund University has developed a procedure for calculating the reorder points in a similar manner but with a great potential for improved coordination. This is done with the introduction of an induced backorder cost at the central warehouse, allowing the multi-echelon inventory system to be broken down into several single-echelon inventory systems. The coordinated model has earlier been used and tested in real case scenarios and other master thesis, the problem then was that the service level was not achieved. But since then the model has been developed to improve its performance, this has yet to be verified and tested though.

1.3 Problem definition

This master thesis evaluates the model originally developed at the division of Production Management at Lund University. The following issue should be answered:

- What is the potential of this new coordinated model compared with the current uncoordinated method used by Syncron?

1.4 Objective

The objective is to evaluate a specific model, developed at the division of Production Management at Lund University, for the control of a multi-echelon inventory system. Furthermore, the reduction of inventory level will be analyzed, with the simulation software Extend, when a coordinated method is used instead of an uncoordinated method. The project also evaluates how well the model fulfills the service levels defined by Syncron.

1.5 Purpose

The purpose is to carry out the evaluation in a proper and independent point of view, and to create a report where the potential and the underlying theory of the model are described. The purpose is also that the outcome will be of value for both the University and Syncron.

1.6 Delimitations

The study includes a multi-echelon inventory system with a central warehouse linked to the maximum of 11 retailers. All the demand data in the multi-echelon inventory system is taken from one of Syncron customers and all the data is limited over the time period of one year. All the demand data comes from a multi-echelon inventory system that handles spare parts. The study includes 135 articles that have been restricted down from about 39,000 articles. There is no direct demand from the central warehouse, all the demand goes through the retailers.

1.7 Target Group

The target groups of this master thesis are primarily Syncron, the division of Production Management at Lund University and other NGIL partners. Also students, especially those studying inventory management are the target group.

1.8 Report outline

The report is divided into following chapters:

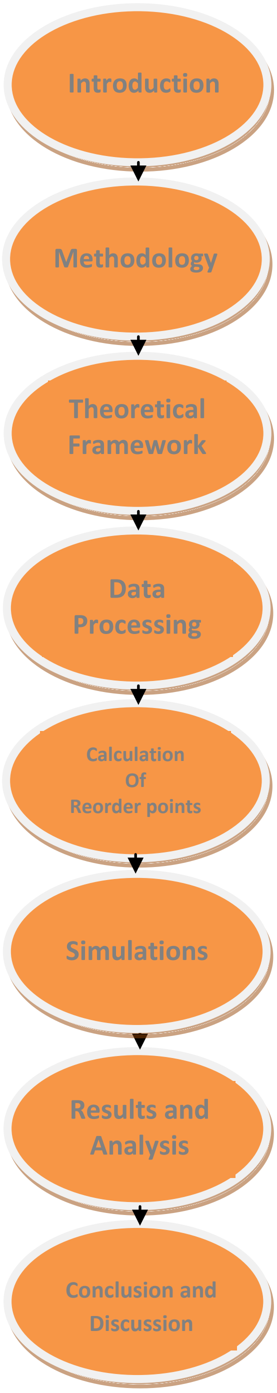


Figure 2 – Report outline

Chapter 1 – Introduction

The first chapter provides a description of Syncron, it also provides a description of the problem and the objective with this master thesis.

Chapter 2 – Methodology

The second chapter presents the methodology, which this master thesis is based on.

Chapter 3 – Theoretical framework

In this chapter the theoretical framework needed to understand this master thesis is presented to the reader.

Chapter 4 – Data processing

In this chapter the data obtained from Syncron and how the chosen articles were selected is presented.

Chapter 5 – Calculation of reorder points

This chapter describes how reorder points is calculated with the analytical model, programmed in Excel and Visual Basics.

Chapter 6 – Simulations

This chapter contains a brief description of the simulation model structure and the assumptions made, as well as input and output parameters.

Chapter 7 – Results and analysis

In this chapter the results from the simulations are presented and analyzed.

Chapter 8 – Conclusion and discussion

This chapter contains the conclusion of the results and a discussion around some different aspects and assumptions that might affect the results of the simulations.

2 Methodology

In this chapter the methodology which this master thesis is based on is presented. First the general methodology in the form of scientific approach, data gathering, method of analysis and credibility is presented, followed by approach depending on knowledge and our practical approach.

2.1 Scientific approach

During a paper, there are two main directions of approach that should be chosen between, *inductive* and *deductive*. The inductive approach starts with the empirical and based on that, models and theories are created. The deductive approach begins with the theory and an empirical study is carried out so that the theory is tested. An approach going back and forth between the two above-mentioned theories is called *abduction* (Björklund & Paulsson, 2003, p. 62), (Wallén, 1996, p. 47).

2.1.1 Scientific approach used in this project

In this master thesis a deductive approach was used. First, the theory of multi-echelon inventory systems and specific models were understood and described, and then data was collected and simulated to evaluate the model.

2.2 Data gathering

The purpose of the study determine when choosing a method to collect data. There are two different methods to choose from, *quantitative* and *qualitative* data collection method. All information in quantitative studies can be measured or evaluated numerically while qualitative studies aims at providing a comprehensive picture of the situation. Mathematical models are usually suitable for quantitative studies and interviews are often useful for qualitative studies (Björklund & Paulsson, 2003, p. 63).

Depending on what kind of information that is collected, data can be divided into two different groups, *primary data* and *secondary data*. Primary data is collected to be used in the current study and secondary data is data collected for a purpose other than the current study (Björklund & Paulsson, 2003, pp. 67-68).

2.2.1 Literature review

Examples of literary studies are magazines, books and newspapers. Any form of written material is literature and the literature is mostly secondary data. When the literature is used in a study, it is important to remain critical because it is easy to manipulate the texts and it is not always that the text is comprehensive. Some positive things about literature are that much information can be addressed quickly, it is usually easy to access and it can be accessed with small economic resources (Björklund & Paulsson, 2003, pp. 67,69).

2.2.2 Data gathered by others

Sometimes it's very difficult to get hold of data and sometimes it is impossible because the authority to collect it is denied. Then it is appropriate to use data that others have collected. By using data gathered by others the investigation time can be reduced. There are generally four different types of data collected by others (Höst, Regnell, & Runeson, 2006, p. 98):

- *Processed material*: Data collected by others and processed for an example in academic publications and theses.
- *Available statistics*: Data collected and processed without any conclusions. For an example, data from Statistics Sweden.
- *Index data*: Data collected for any purpose and is available in unprocessed form. For an example data in a customer database.
- *Archive Data*: Data that is not systemized. For an example, protocol.

2.2.3 Data gathering used in this project

In the beginning of this master thesis data was collected directly from Syncron. All this data can be measured or evaluated numerically, which makes it quantitative. This data was not created for this master thesis, which makes it secondary data. During the process of the work a lot of data came as output from the simulations. This data is also quantitative; it can be measured or evaluated numerically. This is new data and it was the key to answer the problem definition, which makes it primary data.

Literature review

Literature studies are the basis for any theory used in the thesis. Sources were carefully analyzed to be sure that no false information was used and that the authors understood the problem.

Data gathered by others

Since this master thesis builds on real customer demand data received from a third company, all data used came from Syncron and was stored in their SCP software. This means that all data is index data which is not processed in any way.

Criticism to gathered data

The data gained from Syncron and their SCP software is as previously said secondary information and its accuracy cannot be verified by the authors. But since it is index data it is assumed to be correct and that no modifications have been done with it. One problem encountered during the thesis work was that the obtained data from Syncron could distinguish because of different settings in the SCP software. An example of the settings was if trends were allowed. However after tests with different setting in the SCP and checks and comparisons with own calculations, all data is seen as correct. Syncron helped out a lot with the tests with different settings in the SCP, though it is also in Syncrons interest that the study is carried out on the correct data so that they can benefit from the outcome.

All literature used in the project is considered to have high credibility. Articles are taken from respected international journals where the research work are refereed and must be of a high standard to be published. The books and internet sources used is believed to be accurate because the authors possess the knowledge to determine this, which they can do because of their knowledge from their education.

2.3 Methods of analysis

Data can be analyzed in several different ways to answer the purpose of the project. Several methods of data analysis are available and what/which to use depends primarily on the nature of the data collected; if it is quantitative or qualitative (Höst, Regnell, & Runeson, 2006, p. 110). Since the data used in this master thesis is only quantitative, only those three methods are described in more detail.

The analysis of quantitative data includes the following three different methods; *use of analytical models*, *statistical processing* and *modeling/simulation*. Analytical models are used to structure and evaluate the collected data. The models can be either strictly adhere to the theory or be specifically customized for the analyzed situation. By statistical processing of the data collected, new information can be obtained from the current data such as mean and standard deviation of the data. A correlation analysis can also be carried out between different variables which can indicate strength in the relationship between variables. This processing can be done manually, but mostly some form of computer software specifically adapted for this purpose is used. With the help of simulation tools different scenarios are tested and the results compared (Björklund & Paulsson, 2003, pp. 71-73).

2.3.1 Method of analysis used in this project

To answer the purpose of this master thesis a couple of analyzes have been made off the collected data. First a huge amount of simulations where done to be able to have some output data to which statistical data processing was used. Microsoft Excel was an important tool used to retrieve, for example mean and standard deviation on the data which was the foundation for the upcoming data sorting process. From the statistical processing of the output, key figures were obtained and used in the comparison between the different methods evaluated in this project.

2.4 Credibility

When simulation models are used to evaluate different methods peer performance, it is required that the model and its results are "correct". A high level of credibility of the research project is obtained when three different aspects are met: *validity*, *reliability* and *objectivity* (Björklund & Paulsson, 2003, p. 59).

2.4.1 Validity

Validity is defined as "to what extent something really measures what it intends to measure" (Björklund & Paulsson, 2003, p. 59). A model may have validity for an experiment, but not for another, i.e. a model is developed specifically for one purpose, and the validity is determined from this. For the validation of a simulation model there are four general angles, *performing self-validation*, *validation is performed by the model user*, *a third party performing the validation*, *validation is performed using a scoring model* (Sargent, 2004).

The most common way to perform validation is that the developers do it themselves. This is a subjective decision based on the results of a number of tests carried out during the model development process. However, credibility will be suffering in this approach because the developer's objectivity is questionable. To increase the objectivity and for the most part, the number of persons performing the validation of the model, the users can carry out the validation. Even in this case, however, objectivity can be questioned. By allowing an independent third party to perform validation, commonly known as "independent verification and validation (IV&V), an objective validation is obtained. This adds credibility and is often used when large costs are associated with the development of the model. When IV & V is used, it is most straightforward to only evaluate the validation that has already been done. Finally, a scoring model can be used. This method is rarely used in practice (Sargent, 2004).

Which focus the validation has depends on the model's character. When the model's underlying theories and assumptions has to be assured a *conceptual validation* is carried out. It also decides if the model is consistent with its purpose. *Computerized model validation* is used when it should be ensured that the programming and implementation of the model is correct. *Operational validation* aims to establish that the model's output is sufficiently consistent with why the model was created (Sargent, 2004).

2.4.2 Reliability

That different measurement, of the same kind and on the same objects, produces the same result is called reliability. This means that the measurements do not contain random errors and that the instrument is reliable. Comparing the differences between the maximum and minimum value can assess reliability of a series of measurements and a reliability coefficient can be obtained by calculating the correlation between two different measurement series (Wallén, 1996, p. 67).

2.4.3 Objectivity

Objectivity is the extent to which values influence the study. The objectivity may be increased if the reader all the time gets all reasoning clearly explained to them and thus take its own position on the outcome. By reproduce sources properly and avoid distortion of the underlying facts as the example to use value-charged words, objectivity is further increased (Björklund & Paulsson, 2003, pp. 61-62).

2.4.4 Credibility in this project

The authors have made all their decisions and assumptions in this project with a continuous target of maintaining the credibility.

Validity

In this master thesis, its developers and its users make the validation of the models. The focus will be on computer-based and operational validation. Since this project is based on quantitative data, i.e. measurable numbers, there is no scope for measurement error. All models have therefore been validated by at least one of the first two of Sargents angles for the validation of a simulation model. In those cases where assumptions were made that could affect or even reduce the validity, it has been carefully commented upon in the report.

The analytical model is created and tested by researchers in the area and it is therefore considered to be very valid. In cases where inconsistencies have emerged that the authors could not explain, they were shown for the creators who subsequently have been able to find the problem area. The simulation model used in this project is an expansion of an existing and well-tested model. In order to assure that the results of the new simulation model is correct; the results from a number of test simulations were compared with the existing model. When the same input resulted in the same output in both models, the expanded simulation model is considered to have a high validity.

Reliability

By conducting several experiments in a steady state procedure in the simulations, the reliability is achieved in this master thesis. Since simulations are based on historical data from a limited period of time, it is a great possibility that the input data used may change if a similar study is carried out in the future. A change in demand, lead time or fillrate over time is highly likely which in this case might affect the results a bit. That the results will differ markedly, however, is not likely as the result of this report is based on a variety of demand patterns, lead times, fillrates, etc. Therefore it is not considered that a change in the parameters affect the reliability.

Objectivity

The authors tried much as possible to let their own values stand aside. All tests were carried out with a neutral approach and all choices were reasoned well. It is therefore considered that report have a high level of objectivity.

2.5 Approach depending on knowledge

What level of ambition a research project has depends largely on the level of knowledge held within the area. The literature distinguishes between four different studies, exploratory study, descriptive study, explanatory study and normative study, which in turn lead to the study carried out in various ways (Wallén, 1996, p. 46).

When the study aims to obtain basic knowledge about the problem and its nature, an *exploratory study* is carried out. As an example, typical cases and variables are specified, and concepts that are relevant to the problem are determined. A project that aims to determine the characteristics of the research project uses a so-called *descriptive study*. Here, data is collected and systematized in order to determine values of the variables and their interaction. An *explanatory study* is used when the scope of the project is to "explain". Cause-effect and systemic effects are some of the explanations that may be relevant to illuminate. Finally described are *normative studies,* where the results of the study will provide a norm- or action proposal. In these studies often value issues, ethical issues and political issues comes in. Their disagreement is here presented as well as various proposals for action and its impact on the various parties involved (Wallén, 1996, pp. 46-47).

2.5.1 Approach depending on knowledge used in this project

In this master thesis, the approach based on the level of knowledge has been as an explanatory study. The various methods for calculating the reorder point is evaluated against each other and why difference in outcome occurs is described.

2.6 Practical approach

The approach of this master thesis can be defined with an operations research. An operations research can be divided into six stages (Hillier & Lieberman, 2005, p. 8):

1. Defining the problem and collect relevant data
2. Create a mathematical model representing the problem
3. Develop a methodology to develop a solution to the problem of model
4. Test the model and improve it
5. Prepare to implement the model
6. Implementation

In the first step the problem area is studied and based on that study a problem is defined. Once that's done, it is important to involve all partners and make them understand that the problem exists and that it needed to be resolved. After that, data is collected to create an overall picture of the problem. Another reason for collecting data is to ensure that raw data is available to put into the model created in step two (Hillier & Lieberman, 2005, pp. 8-11).

In the second step a mathematical model based on the significantly of the problem is created. Here it is important to start with a simple version and then improve it gradually to finally have a model that represents this problem well (Hillier & Lieberman, 2005, pp. 12-14).

In the third step a method (often computer-based) is created to develop a solution that represents the problem in the model. It's easy to believe that this step is what takes most time, but this is often not the case. Already developed programs, for an example Excel and Visual Basics, which easily can model the problem, is often used here (Hillier & Lieberman, 2005, pp. 15-17).

In the fourth step, the model is tested and improved. Almost always when big mathematical models have been built bugs occurs that need to be resolved. The more accurate the model is tested and the more bugs that are eliminated, the greater the validity of the model will be. The model can be tested in different ways, for an example it can be tested, like in this case, with a simulation program. (Hillier & Lieberman, 2005, pp. 17-19).

In the fifth step a well-documented system is created to prepare for implementation of the model. The system will contain the model, solution method of the model and worked procedures for implementation. Often, this is a computerized system that needs a number of computer programs to work (Hillier & Lieberman, 2005, pp. 19-20).

In the sixth step, the system is implemented. Here it is important that the team who worked on the model is participating because they know the model best. During implementation, it is important to constantly provide feedback how implementation is progressing. If major differences arise from the tests, a decision must be taken about changing the model (Hillier & Lieberman, 2005, p. 21).

2.6.1 Practical approach used in this project

This master thesis follows the work procedure of an operations research project described above. The first step, the problem definition, was at first established among all those involved in the project so that everybody understood which data was relevant to have in order to solve the problem. Then data was received, sorted and fitted to the prebuilt analytical model. Most of the work in this master thesis has been devoted to the 4th step where the analytical model has been validated and tested in conjunction to an existing inventory control model (Syncrons current model) with the simulation program Extend. The result of step five, which is this report, evaluates the two inventory control models against each other and offer advice for future implementations. The sixth step falls outside the scope of this master thesis and is therefore delimitated.

Some of the steps where already done by others or existing software could be used and therefore a limited amount of time in this project has been put on these steps. E.g. step two and three where the analytical model was developed and programmed at the division of Production Management at Lund University.

3 Theoretical framework

In this chapter the theoretical framework for the master thesis is presented. The theoretical framework is important for the understanding of the project. First some general definitions are presented, followed by theory of statistical distributions and different inventory systems. The main thing described in the theoretical framework is decompose of a multi-echelon inventory system in to several single echelon inventory systems by introducing an induced shortage cost.

3.1 General definitions

Holding costs: By having stock, capital is tied-up. These costs are primarily capital costs and the cost of warehouse buildings, insurance, and rejects are included here (Axsäter, 1991, p. 39).

Setup costs: When an order shall be produced, setup costs arise. This depends on setup costs and running cost for different machines (Axsäter, 1991, p. 39).

Ordering costs: For new orders, administrative costs of purchase and shipping and handling costs arise. Ordering costs and shortage costs are balanced against holding costs to determine Q (order quantity) (Axsäter, 1991, p. 39).

Shortage costs: These costs are costs that arise when one cannot deliver directly when one unit is demanded. It is very difficult to assess the costs as they are difficult to measure. These costs are balanced against holding costs to determine R (reorder point) (Axsäter, 1991, p. 39).

Lead time: The time from order to delivery and it includes possible delays e.g. due to stockouts at previous echelons (Axsäter, 1991, p. 13).

Service level: When shortages costs are difficult to measure a different concept has developed; service requirements. Three common definitions of service requirement are defined below ($SERV_1$, $SERV_2$ and $SERV_3$). In this master thesis $SERV_2$

and $SERV_3$ is used. (Axsäter, 1991, p. 40). More definitions exist.

$SERV_1$: Probability of no stock out during an order cycle (Axsäter, 1991, p. 68).

$SERV_2$: Fraction of demand that can be satisfied immediately from stock on hand, also known as fillrate (Axsäter, 1991, p. 68).

$SERV_3$: Fraction of time with positive stock on hand, also known as ready rate (Axsäter, 2006, p. 94).

Backorder: Units that have been demanded but not yet delivered (Axsäter, 2006, p. 46).

Inventory position: Stock on hand + outstanding orders - backorders (Axsäter, 2006, p. 46).

3.2 Statistical distributions

Customer demand is normally uncertain but it still has to be described in some manner. A common way of doing so is to use statistical distribution functions. This section describes the distribution functions used in this master thesis to describe the customer demand when reorder points are calculated, see chapter 5 for a description of how the calculation is done.

3.2.1 Normal distribution

A normal distribution, also called Gauss distribution, is a continuous distribution with a density function that can be described as a bell-shaped curve that is symmetric around μ, see Figure 3 (Vännman, 2002, pp. 115-116), (Ross, 1985, p. 34).

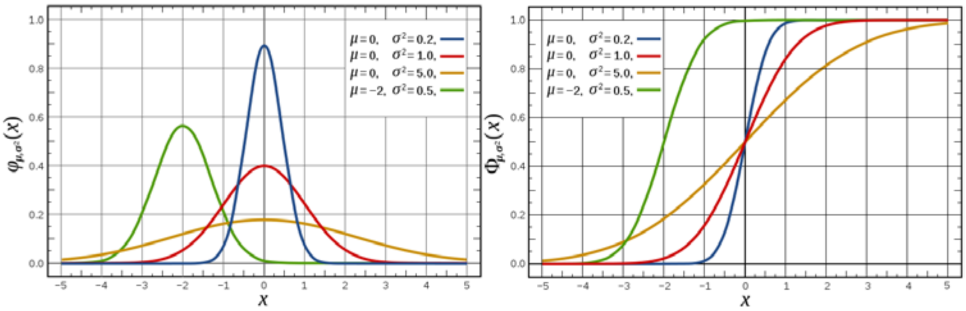


Figure 3 – Density and cumulative distribution function for a normal distribution (Wikipedia, den fria encyklopedin, 2009)

The normal distribution is perhaps the best known of all continuous distributions and it is suitable to use as a stochastic model if phenomenon that can be understood as the sum of many random variables are studied (Vännman, 2002, pp. 115, 165). The central limit theorem says that under very general conditions, a sum of many independent random variables will have a distribution that is approximately normal. In many situations, the demand comes from many independent customers, and their demand can then be represented by a normal distribution (Axsäter, 2006, p. 85). One problem that might arise when a continuous distribution is used as an approximation of the reality is the undershoot problem described in section 3.4.4.

The normal distribution is suitable to use as an approximation when the demand is high, but not when the demand is low. A good rule to follow is that the ration between the mean and the standard deviation shall be above two (Berling & Marklund, 2009). The reason for not using the normal distribution when demand is low is the high probability of negative demand (Axsäter, 1991, pp. 66-67). In such situation the demand is better approximated using other distribution functions e.g. Poisson distribution (described in section 3.2.3) or Compound Poisson distribution (described in section 3.2.4). The normal distributions density function and cumulative distribution function describes as follows (Vännman, 2002, p. 155):

Density function:

$$\varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty \qquad (3.1)$$

Cumulative distribution function:

$$\phi(t) = f(t)dt \tag{3.2}$$

The parameters μ and σ is the expected value and the standard deviation of demand (Vännman, 2002, p. 155).

3.2.2 Exponential distribution

An exponential distribution is a continuous distribution with a density function that can be described as a ski slope; see Figure 4 below (Vännman, 2002, p. 113).

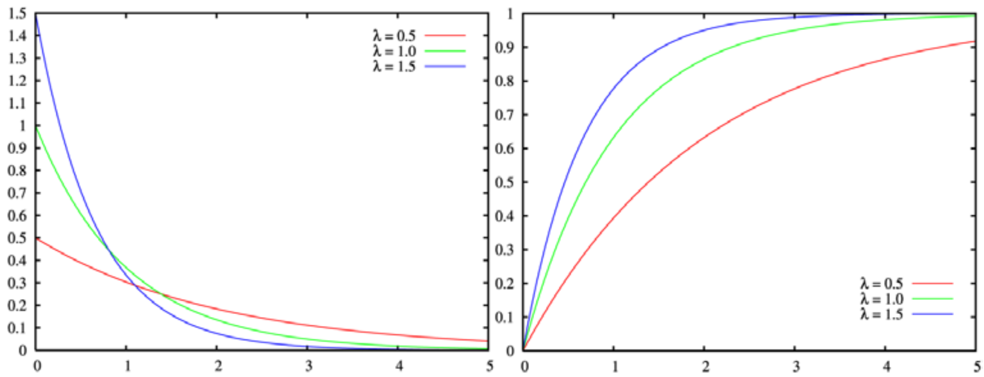


Figure 4 - Density and cumulative distribution function for an exponential distribution (Wikipedia, den fria encyklopedin, 2009)

Time between events that occur randomly and are independent of each other is often exponential distributed (Vännman, 2002, p. 113). An example is time between customer arrivals. They occur randomly and often independently of each other. The exponential distributions density function and cumulative distribution function describes as follows (Ross, 1985, p. 33), (Vännman, 2002, p. 113):

Density function:

$$\varphi(x) = \begin{cases} \lambda e^{-\lambda x}, & x \leq 0 \\ 0, & x > 0 \end{cases} \tag{3.3}$$

Cumulative distribution function:

$$\phi(x) = \begin{cases} 0, & x \leq 0 \\ 1 - e^{-\lambda x} & x > 0 \end{cases} \tag{3.4}$$

The parameter λ is the number of arrivals over a period of time, and 1/ λ is the arrival intensity.

3.2.3 Poisson distribution

A Poisson distribution is a discrete distribution. The distribution can be described as a series of events that occur randomly and independently of each other during a time interval (Vännman, 2002, p. 85). When time between customer arrivals are exponential distributed and the customers only demands one unit at the time the demand follows a Poisson process (Law & Kelton, 2000, pp. 325-326). The Poisson distribution is easy to handle and it is appropriate to use when the demand is relative low. An example of low frequency demand is spare parts (Axsäter, 1991, p. 146), like in the studied inventory system in this master thesis.

The number of independent Poisson distributed customer arrivals, where each customer demands one unit, over a period of time t, can be described as follows (Axsäter, 2006, p. 78):

$$P(k) = \frac{\lambda t^k}{k!} e^{-\lambda y}, k = 0,1,2 \dots \quad (3.5)$$

The parameter λ indicates the average number of events during time period t (Vännman, 2002, p. 85).

It's very convenient and computationally efficient to use a Poisson distribution. But it is important that the variance (σ^2) divided by the mean (μ) is approximately equal to one. A good rule that can be used to determine whether a Poisson distribution fits the demand process is $0.9 \le (\sigma^2)/\mu \le 1.1$ (Axsäter, 2006, p. 85).

3.2.4 Compound Poisson distribution

During a Poisson distributed demand, each customer only request one unit. While for a compound Poisson distributed demand, each customer can request one or several units at the time. This is the big difference between the Poisson distribution and the compound Poisson distribution. The distribution of demand size in the compound Poisson distribution is stochastic and is called the compounding distribution (Axsäter, 2006, pp. 78-79).

f_j^k: Probability that k customers give the total demand j

D(t): Stochastic demand in the time interval t.

$$P(D(t) = j) = \sum_{k=0}^{\infty} \frac{\lambda t^k}{k!} e^{-\lambda t} f_j^k \quad (3.6)$$

The variance (σ^2) divided by the mean (μ) must be > 1 to use a compound Poisson distribution and the compound Poisson distribution is appropriate to use when the relation above is (σ^2/μ) > 1.1 (Axsäter, 2006, p. 78).

If the studied event is customer arrivals, then the time between customer arrivals is defined as 1/λ where λ is calculated through the following expression (Axsäter, 2006, p. 79):

μ: average demand per unit of time

f_j: probability of demand size j (j = 1,2, …).

$$\mu = \lambda \sum_{j=1}^{\infty} jf_j \leftrightarrow \lambda = \frac{\mu}{\sum_{j=1}^{\infty} jf_j} \qquad (3.7)$$

When a Compound Poisson distribution is fitted to the demand, the compounding distribution can be very complex and computational complex. If that is the case, it is easy to fit a predefined distribution to the compounding distribution. When a logarithmic compounding distribution is fitted to the compound Poisson distribution the distribution is called a negative binominal distribution (Axsäter, 2006, p. 78). This is the compounding distribution used to approximate customer demand in the SCP software and for computationally complex articles in the analytical model.

3.2.5 Gamma distribution

A gamma distribution is a continuous distribution. The distribution has two input parameters, a shape parameter (α) and a scale parameter (β). If the shape parameter (α) = 1 the gamma distribution is the same as an exponential distribution. The exponential distributions density function and cumulative distribution function describes as follows (Law & Kelton, 2000, pp. 301-303):

Density function:

$$\varphi(x) = \frac{\beta^{-\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}}}{\Gamma(\alpha)}, if\ x > 0, otherwise\ 0 \qquad (3.8)$$

Cumulative distribution function:

$$\phi(x) = 1 - e^{-\frac{x}{\beta}} \sum_{j=0}^{\alpha-1} \frac{\left(\frac{x}{\beta}\right)^j}{j!}, if\ x > 0, otherwise\ 0 \qquad (3.9)$$

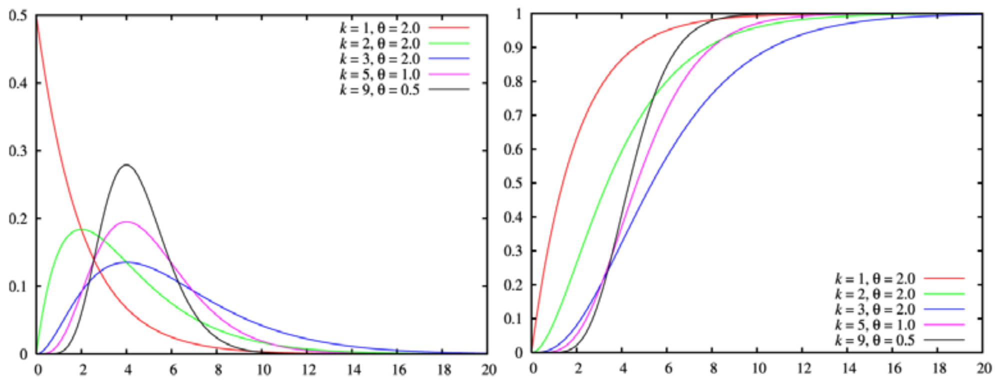


Figure 5 - Density and cumulative distribution function for a gamma distribution (Wikipedia, den fria encyklopedin, 2009)

If the variance (σ^2) divided by the mean (μ) is < 1, but not too far from one, there is a risk for negative demand when using a normal distribution (see section 3.2.1). The normal distribution is often a good alternative even though the variance (σ^2) divided by the mean (μ) is close to one, but an alternative in these cases is to use a gamma distribution where the demand always is nonnegative (Axsäter, 2006, p. 86).

3.3 Single-echelon inventory systems

An inventory system considered as a single-echelon inventory system is characterized by two properties (Axsäter, 1991, p. 38):

- Different types of articles should be controlled independently.
- Articles are kept in stock only in a single-echelon inventory system, not in multi-echelon inventory system.

A single-echelon inventory system is described below, see Figure 6:

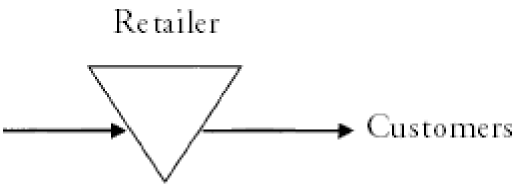


Figure 6 – A single-echelon inventory system

Traders are examples where single-echelon inventory systems are used. They often sell products from a single output stock and can then manage their warehouse through a single-echelon inventory system. Various cost parameters that are optimized and considered in a single-echelon inventory system are (Axsäter, 1991, pp. 38-39):

- Holding costs
- Ordering costs and Setup costs
- Shortage costs or Service level

A common reordering point system for single-echelon inventory system is called an (R, Q)-policy. When the stock position is equal to or less than the reorder point (R), the order quantity (Q) is ordered. The review and the demand can be booth periodic and continuous, se Figure 7 for a (R, Q)-policy with continues review and a continuous demand. (Axsäter, 1991, p. 42).

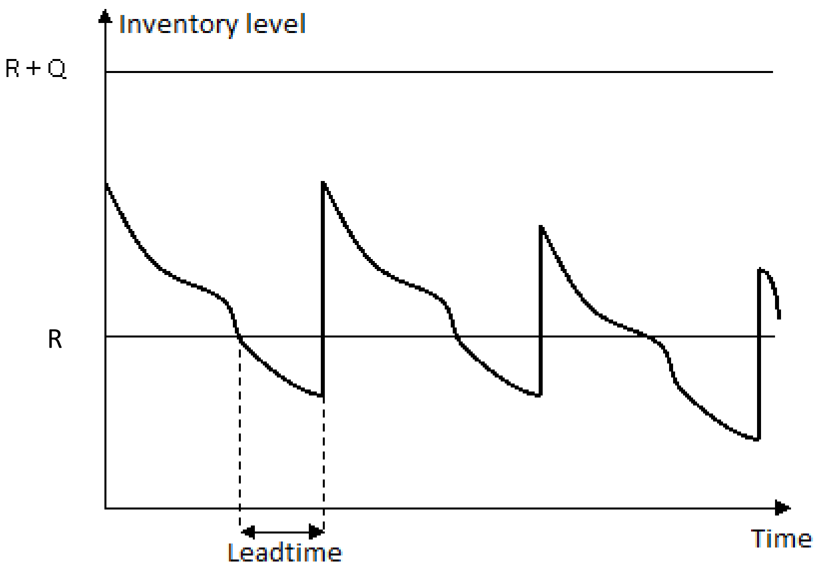


Figure 7 - (R, Q)-policy with continues review and continuous demand.

3.3.1 Optimization of a reorder point in a single-echelon inventory system

There are many ways to optimize reorder points in a single-echelon inventory system. For example it could depend upon if the demand is relative low or high, if cost optimization or fillrate optimization is used (Axsäter, 2006, pp. 77, 94). Two methods used within this master thesis are optimization against holding and backorder cost and meeting a fillrate constraint, which are described below.

To optimize a single-echelon inventory system with a normal or a compound Poisson distributed demand, a (R,Q)-policy with a given batch quantity Q, constant lead times, continuous inspection and a backorder system the following notations is needed (Axsäter, 2006, p. 91).

Q:	order quantity
R:	reorder point
h:	holding cost per unit and time unit
p:	shortage cost per unit and time unit
μ’:	mean of lead-time demand
σ’:	standard deviation of lead-time demand
ϕ():	distribution function of the normal distribution
φ(x):	density function of the normal distribution
f_k:	probability for demand size k for the compounding distribution
k:	positive demand size
j:	positive inventory level

Normal distribution

First the fillrate optimizing procedure is described. For a normal distributed demand the distribution function of the inventory level is (Axsäter, 2006, p. 91):

$$F(x) = P(IL \leq x) = \frac{1}{Q}\int_{R}^{R+Q}\left[1 - \phi(\frac{u - x - \mu^{'}}{\sigma^{'}})\right]du \qquad (3.10)$$

The lost cost function, which is a function that measures the degree of wrongness, i.e. the difference between estimated and the true value, is defined as (Axsäter, 2006, p. 91):

$$G(x) = \varphi(x) - x(1 - \Phi(x)) \qquad (3.11)$$

and

$$G^{'}(x) = \Phi(x) - 1 \tag{3.12}$$

Using (3.12), (3.10) can be reformulated as (Axsäter, 2006, p. 92):

$$\begin{aligned} \mathrm{F(x)} &= \frac{1}{Q}\int_{R}^{R+Q}\left[-G^{'}\left(\frac{u-x-\mu^{'}}{\sigma^{'}}\right)\right]du \\ &= \frac{\sigma'}{Q}\left[\mathrm{G}\left(\frac{\mathrm{R-x-\mu^{'}}}{\sigma^{'}}\right)-\mathrm{G}\left(\frac{\mathrm{R+Q-x-\mu^{'}}}{\sigma^{'}}\right)\right] \end{aligned} \tag{3.13}$$

For a continuous distributed demand like above, $SERV_2 = SERV_3$. $SERV_3$ = Prob(IL>0) = 1-Prob(IL<0) so $SERV_2$ can then be expressed like (Axsäter, 2006, p. 98):

$$SERV_2 = SERV_3 = 1 - F(0) = 1 - \frac{\sigma'}{Q}\left[\mathrm{G}\left(\frac{\mathrm{R-\mu^{'}}}{\sigma^{'}}\right)-\mathrm{G}\left(\frac{\mathrm{R+Q-\mu^{'}}}{\sigma^{'}}\right)\right] \tag{3.14}$$

For a given $SERV_2$ a reorder point (R) can be calculated.

Secondly the cost optimization procedure is described. For a normal distributed demand the expected cost is (Axsäter, 2006, p. 104):

$$C = h\left(\frac{Q}{2} + R - \mu^{'}\right) + (h+p)\cdot\frac{\sigma^{'2}}{Q}\left[H\left(\frac{R-\mu^{'}}{\sigma^{'}}\right) - H\left(\frac{R+Q-\mu^{'}}{\sigma^{'}}\right)\right] \tag{3.15}$$

where

$$H(x) = \frac{(x^2+1)\big(1-\phi(x)\big) - x\varphi(x)}{2} \tag{3.16}$$

By differentiate the cost function (3.15) with respect to R the following expression to determine the service level is achieved (Axsäter, 2006, p. 105):

$$\frac{dC}{dR} = -p + (h+p)SERV_3 = -p + (h+p)SERV_2 \tag{3.17}$$

The cost function C is a convex function of R and thus the optimal R is obtained when dC / dR = 0 which correspond to (Axsäter, 2006, p. 105):

$$SERV_2 = \frac{p}{h+p} \tag{3.18}$$

Knowing this (3.14) can be used to determine R.

Compound Poisson distribution

First the fillrate optimizing procedure is described. For a compound Poisson distributed demand the probability function of the inventory level is (Axsäter, 2006, p. 90):

$$P(IL=j)=\frac{1}{Q}\sum_{k=\max(R+1,j)}^{R+Q} P(D(L)=k-j), j \leq R+Q \tag{3.19}$$

If the probabilities have been obtained for one reorder point (R), the probability can be obtained for any given R by a simple conversion (Axsäter, 2006, pp. 90-91):

$$\begin{aligned} P(IL=j|R=r) &= \frac{1}{Q}\sum_{k=\max(R+1,j)}^{R+Q} P(D(L)=k-j) \\ &= \frac{1}{Q}\sum_{k=\max(1,j-r)}^{Q} P\big(D(L)=k-(j-r)\big) \\ &= P(IL=j-r|R=0) \end{aligned} \tag{3.20}$$

When consider a customer demand, $SERV_2$ is the ratio between the expected satisfied quantity and the expected total demand quantity (Axsäter, 2006, pp. 97-98):

$$SERV_2=\frac{\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\min(j,k)*f_k*P(IL=j)}{\sum_{k=1}^{\infty}f_k} \tag{3.21}$$

If R ≤ -Q the stock will never be positive, then $SERV_2$ will be zero. For any given $SERV_2$ a reorder point (R) can be calculated by starting with R = -Q and increase R by one until $SERV_2$ is obtained (Axsäter, 2006, p. 98).

Secondly the cost optimization procedure is described. For a compound Poisson distributed demand the expected cost is (Axsäter, 2006, p. 102):

$$C=-p\left(R+\frac{Q+1}{2}-\mu'\right)+(h+p)\sum_{j=1}^{R+Q} jP(IL=j) \tag{3.22}$$

To be able to find the optimal reorder point R, the cost difference between the reorder point R + 1 and R is used. According to (Axsäter, 2006, p. 102) the following expression is obtained:

$$
\begin{aligned}
&C(R+1) - C(R) \\
&= -p + (h+p) \sum_{j=1}^{R+1+Q} P(IL = j | R+1) \\
&= -p + (h+p) SERV_3(R+1)
\end{aligned} \tag{3.23}
$$

To find the optimal R, the procedure starts with R = - Q and increases R by one unit at the time until the cost are increasing. It is possible to start the optimization at R = - Q because $SERV_3 = 0$ for R ≤ - Q and therefore values of R < - Q are not interesting. Interesting to note is that a similar relationship between p and (h+p) exists, see (3.18).

3.4 Multi-echelon inventory systems

It is seldom that a single-echelon inventory system exists in practice. Instead several inventory levels are linked together, which is a multi-echelon inventory system. These systems are more difficult to manage and control. Since one must take into account the link between the different inventory levels. (Axsäter, 1991, p. 107).

There exist many different multi-echelon inventory systems and an example is a two-level distribution system that can be seen in Figure 8 below, this is the system considered in this master thesis. The system consists of two levels where the central warehouse represents one level and a number of parallel retailers represent the second level. One thing that distinguishes a distribution system is that each layer has exactly one predecessor. The best allocation of stock level between the different layers in the multi-echelon distribution system depends on the system structure, demand variations, lead times and different cost functions. (Axsäter, 1991, pp. 108-109).

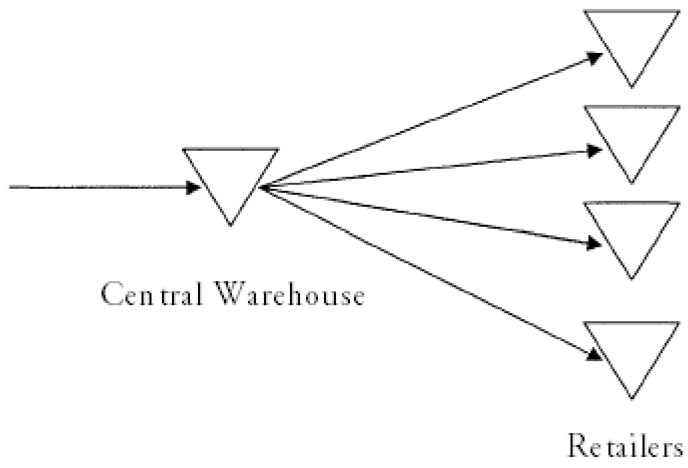


Figure 8 – A multi-echelon distribution system

3.4.1 Determine a reorder point in multi-echelon inventory systems

Some theoretical results on how the exact cost can be determined for a one-warehouse-multi-retailer system with a (R,Q)-policy exist. An example can be seen in (Axsäter, 2000). The drawback for this exact method is that it becomes computationally complex and it is almost impossible to use for larger systems with high demand and/or many retailers.

The analytical model used in this project to calculate the reorder points in a multi-echelon inventory system is based on an approximation and is therefore not an exact technique to determine the reorder points. The results will not be as good as for the exact model. However, the analytical model will work for larger systems with high demand and/or many retailers. The analytical model will be described in the following sections. The model, which is designed and developed at the division of Production Management at Lund University, is based on the research presented in three different research articles. All articles use an induced backorder cost at the central warehouse the differences are how this cost is determined and certain model assumptions. In the first article the backorder cost is determined through an iterative procedure in a multi-echelon inventory system with identical retailers and were the stochastic lead-times at the retailers are replaced with the correct averages obtained with Little's formula (Andersson, Axsäter, & Marklund, 1998). The second article uses the same procedure but in a model with non-identical retailers, the average lead-time is then more complicated to compute and approximations are used (Andersson & Marklund, 2000). The third and last article examines how a simple closed form expression for estimating an induced backorder cost of the central warehouse which makes the model conceptually and computationally simpler to use (Berling & Marklund, 2006). In section 3.5, there is a more thorough description of how this analytic model works.

3.5 Model for heuristic coordination of a decentralized inventory system

Before the analytical model is described in detail, the inventory system and the assumptions made will be explained. All the assumptions made are from the three articles described in section 3.4.1. During the following description of the analytical model, the notations below will be used:

N:	number of retailers
Q:	largest common divisor of all order quantities in the system
q_i:	order quantity at retailer i, expressed in units of Q
Q_i:	order quantity at retailer i, expressed in number of units $(Q_i = q_i Q)$
Q_0:	warehouse order quantity, expressed in units of Q
h_0:	holding cost per unit and time unit at the warehouse
h_i:	holding cost per unit and time unit at retailer i
p_i:	shortage cost per unit and time unit at retailer i
L_0:	constant lead-time for an order to arrive at the warehouse
l_i:	constant transportation time between the warehouse and retailer i
L_i:	lead-time for an order to arrive at retailer i
$\bar{L}_i$:	expected lead-time for an order to arrive at retailer i
$D_i(t)$:	customer demand at retailer i during time period t, stochastic variable
μ_i:	expected demand per time unit at retailer i
μ_0:	expected demand per time unit at the warehouse = $\sum_{i=1}^{N} \mu_i$
σ_i:	standard deviation of the demand per time unit at retailer i
$D_0(t)$:	retailer demand at the warehouse during the time period t, stochastic variable
R_i:	reorder point for retailer i
R_0:	warehouse reorder point in units of Q
$B_0^i(R_0)$:	expected number of backordered units at the warehouse designated for retailer i when the reorder point is R_0
$B_0(R_0)$:	expected number of backordered units at the warehouse given R_0
C_i:	expected cost per time unit at retailer i
C_0:	expected warehouse cost per time unit

TC: expected total system cost per time unit

The model deals with an inventory system with one central warehouse and N retailers; similar to the inventory system described in Figure 8. Customer demand in the system takes place at the retailers who replenish their stocks from the central warehouse. Transportation times from the central warehouse to the retailers are considered to be constant, but delays may occur due to stockouts at the central warehouse. The perceived stochastic lead-times at the retailers are replaced by an estimate of their mean. The central warehouse replenishes its stock from an outside supplier where the lead time is constant, i.e. the supplier always has the required units in stock. Stockouts at all echelons is handled in accordance to a first-come-first-served policy and all facilities apply a (R,Q)-policy with continuous review. In addition partial deliveries are assumed.

The different costs that the model takes into account are the holding costs for all echelons and shortage costs at the retailers, which is proportional to the time until delivery. All orders quantities is considered to be predetermined and fixed, which results in that the only decision variables to be considered is the reorder points. This limitation can be considered as a weakness of the model, but in reality the order quantity often is limited by containers or pallet size. There are indications that the savings that can be obtained by varying the optimal order quantity is marginal given that the reorder points are properly set (Zheng, 1992). Furthermore, the initial inventory position, the reorder point and the batch size at the central warehouse are integer multiples of Q. The inventory position at the central warehouse is always non-negative, i.e. $R_0 \geq -1$ so that the maximum delay is no more than L_0. This is an original assumption, but the process in the model in this master thesis cannot guarantee that $R_0 \geq -1$, so this assumption is not used. This also means that an order placed at time t is independent of demand and retailer orders occurring after time t.

The objective with this model is to optimize the reorder points for the whole inventory system so that the total cost is minimized. This total cost for the system can be divided into two parts, the cost at the warehouse and at the different retailers:

$$TC = C_0 + \sum_{i=1}^{N} C_i \qquad (3.24)$$

3.5.1 Optimizing of the reorder points

The approach to solve this coordinated problem is to use an induced backorder cost, β, at the central warehouse (CW), which make it possible to decompose the multi-echelon inventory problem into N+1 single echelon problems that are relatively easy to solve. The analytical method to optimize the reorder points for a multi-echelon inventory system can be summarized in the following five steps:

1. Determine demand distribution (D_0) at CW
2. Determine the "induced" backorder cost (β) at CW
3. Determine R_0 by cost minimization given D_0 and β treating the CW as a single-echelon inventory model
4. Given R_0 determine the expected lead-time to each retailer (L_i)
5. Determine R_i by cost minimization given L_i treating each retailer as a single-echelon inventory model (with constant lead-time)

Determine demand distribution at CW

The demand distribution at the central warehouse is obtained by linking the demand arising from the various retailers together. This demand is in turn based on each retailer's perceived demand, D_i, and predetermined order quantity, Q_i, independent of R_i. (Andersson, Axsäter, & Marklund, 1998, p. 381). There are a few different alternatives to achieve this. First, the exact demand distribution at the retailers can be used which is preferable but it can be rather time consuming when there are large differences in Q_i and a small common denominator (Q). Secondly, an approximation with correct mean and variance can be used where three different distributions are working. The normal distribution is fast and works well when the ratio between the mean and the standard deviation is above two. A distribution that works for every ratio between the mean and the standard deviation is the Gamma distribution. It is usually very fast but for extremely low ratios between the mean and the standard deviation it can be slow. This depends on numerical problems in Excel. The negative Binomial distribution works for the extremely low ratios but is instead a little bit slower than the gamma distribution (Berling & Marklund, 2009).

Determine the "Induced" Backorder Cost at CW

The first-proposed method of calculating the induced backorder cost is an iterative process that can be very computational demanding which is shown in (Andersson, Axsäter, & Marklund, 1998, p. 381), especially when it comes to non-identical retailers (Andersson & Marklund, 2000). This is not a practical approach when a coordinated inventory control of larger systems wants to obtained. By instead creating a simpler method for estimating a near optimal backorder cost, β*, it is possible to get around this problem. Such a method has been suggested in (Berling & Marklund, 2006, pp. 297,301) and extensive numerical studies also show that this estimate of β* performs well for both identical and non-identical retailers, and for normally distributed and compound Poisson distributed customer demand. The expression of β_i* is as follows:

$$\beta_i^* = g(Q_i, p_i) \cdot \sigma_i^{k(Q_i, p_i)} \tag{3.25}$$

where

$$g(Q_i, p_i) = min\left\{g_a(p_i) \cdot Q_i^{g_b(p_i)}, G(p_i)\right\} \tag{3.26}$$

and

$$k(Q_i, p_i) = max\left\{1, min\left(k_a(p_i) \cdot Q_i^{k(p_i)}, K(p_i)\right)\right\} \tag{3.27}$$

The values $g_a(p_i)$, $g_b(p_i)$, $G(p_i)$, $k_a(p_i)$, $k(p_i)$ and $K(p_i)$ are estimated analogously as functions of p_i which is explained in more detailed in (Berling & Marklund, 2006, p. 301). The values, i.e. *k* and *p,* do not need to be calculated as above but can also be obtained from tables.

To estimate the induced backorder cost at the central warehouse, a simple weighting approach is used (Berling & Marklund, 2006, p. 303):

$$\beta^* = \sum_{i=1}^{N} \frac{\mu_i}{\mu_0} \cdot \beta_i^* \tag{3.28}$$

Determine R_0 by treating CW as a single-echelon inventory system

Using the estimated β* for the central warehouse, the optimal R_0 that minimize (3.24) is obtained by solving this single-echelon inventory problem. In (Berling & Marklund, 2006, p. 297) it is described for a complete delivery approach further down it is described for a partial delivery approach:

$$\min_{R_0} \tilde{C}_0\,(R_0) = \min_{R_0}\{C_0(R_0) + \beta^* B_0(R_0)\} \tag{3.29}$$

where

$$B_0(R_0) = \frac{1}{Q_0} \sum_{y_0=R_0+1}^{R_0+Q_0} E_{D_0(L_0)}[(D_0(L_0) - (y_0 Q))^+] \tag{3.30}$$

and

$$C_0(R_0) = h_0(E[I_0]) \tag{3.31}$$

in which

$$E[I_0] = \frac{1}{Q_0} \sum_{y_0=R_0+1}^{R_0+Q_0} E_{D_0(L_0)}[(y_0 Q - D_0(L_0))^+] \tag{3.32}$$

This means that the holding cost at the central warehouse consist of the holding cost for inventory on hand, I_0. The term y_0 describes the inventory position at the central warehouse (inventory on hand + outstanding orders – backorders) (Axsäter, Lagerstyrning, 1991, p. 40).

Determine the expected lead time to each retailer

With an optimal reorder point at the central warehouse, R_0^*, calculated as above, the expected lead-time, $\bar{L}_i$, to each retailer can be determined. This is done either by using Little's formula or the "METRIC" approach described in (Sherbrooke, 1968) where the stochastic lead-time is replaced with its mean value or an estimate at the mean value. Little's formula uses a partial delivers policy and is described below (Axsäter, 1991, p. 76):

$$\bar{L}_i(R_0) = \frac{Average\ number\ of\ backorders}{\lambda_0} \tag{3.33}$$

The expected lead-time is calculated usin the following approximation (Andersson & Marklund, 2000, p. 497):

$$\bar{L}_i(R_0^*) = \frac{E[B_0]Q}{\mu_0}\left(1 + \frac{\mu_i(Q_i - Q) - (\eta_i/N)}{\eta_i}\right) + \left(\frac{E[B_0^r]Q}{\mu_0}\right) \cdot \left(\frac{\mu_i(Q_i - Q)}{\eta_i}\right) + l_i \tag{3.34}$$

where

$$\eta_i = \sum_{i=1}^{N} \mu_i(Q_i - Q) \tag{3.35}$$

and

$$E[B_0^r] = E\left[\left\lceil\frac{B_0}{\bar{Q}}\right\rceil \bar{Q}\right] - E[B_0] \tag{3.36}$$

in which

$$\bar{Q} = \sum_{i=1}^{n} \frac{\mu_i}{\lambda_0} \tag{3.37}$$

where

$$\lambda_0 = \sum_{i=1}^{n} \frac{\mu_i}{Q_i} \tag{3.38}$$

Determine R_i by treating each retailer as a single-echelon inventory system.
With a given lead-time, $\bar{L}_i(R_0^*)$, for each retailer their reorder points now can be determined as a single-echelon inventory problem. Either each reorder point is calculated to match the target fillrate or with a cost optimization approach, booth described in section 3.3.1.

3.6 "Under-shoot"-adjustment

In practice, there is a common problem in inventory control, namely that the service level achieved is below the target service level used when calculating the reorder points. The reasons for this can be summarized in two hypotheses (Berling, Reorder point adjustment, 2009):

1. The lead time is not constant and deterministic, as often assumed, but instead it is stochastic and varies over time.
2. The demand is not continuous (or unit by unit), as often assumed, but instead it varies with the customer order sizes.

According to some preliminary studies, it is in this case the latter of these two assumptions that is the main reason for the arising service level problems. This conclusion could be drawn as the ready rate was close to the target service level while the fillrate was way below. If the demand is continuous these two measurements would coincide. If demand is not continuous and customer demands more than one unit each time, orders will be placed when the inventory level is below rather than at the reorder point, which ultimately leads to a reduced service level (Berling, Reorder point adjustment, 2009), see Figure 9.

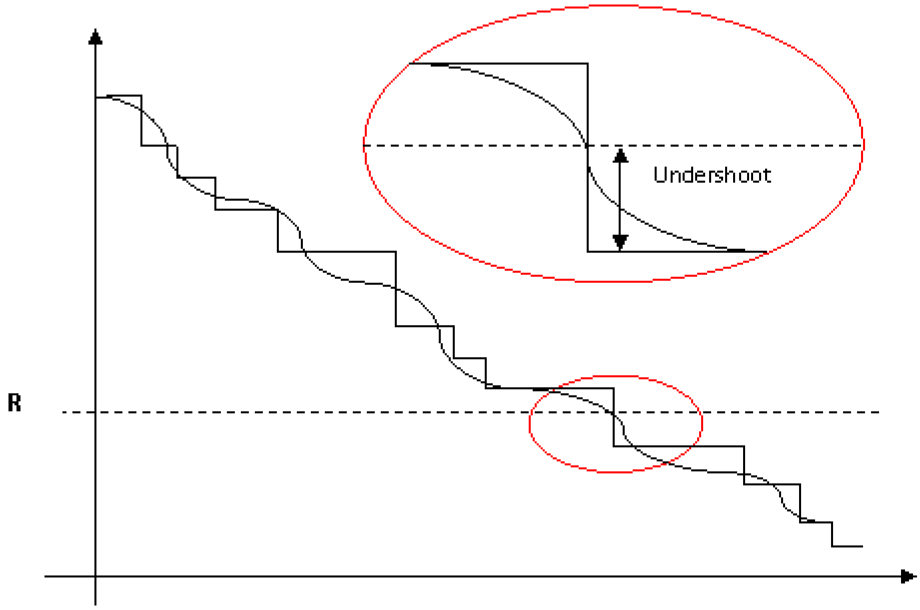


Figure 9 – The undershoot problem with non-continuous demand.

Five different ways to compensate for undershoot to be used for a normal approximation has been developed at the division of Production Management at Lund University. Three of the undershoot compensation methods make use of the observation that when a (R, Q)-policy is used, the inventory position is uniformly distributed between R+1 and R+Q. With this knowledge, the undershoot distribution function is easily calculated when the distribution function for customer order sizes are known. The other two methods instead uses the order sizes to compensate for undershoot. Two of the five different methods will be described in detail below, and to do that some necessary notes are defined (Berling, Reorder point adjustment, 2009):

μ_D: mean demand during lead-time
σ_D: standard deviation of the demand during lead-time
μ_U: average size of undershoot
σ_U: average standard deviation of undershoot
$r(\mu, \sigma)$: reorder point calculated assuming μ and σ and continuous review

U1

When the undershoot distribution function is known, it can be used to calculate the actual reorder point, i.e. the inventory position when the order is actually placed. From the actual reorder point a "real" service level is obtained. When the distribution function for the actual reorder points is known, R is can be adjusted so that the weighted service level (over all actual reorder points) is consistent with the target service level (Berling, Reorder point adjustment, 2009).

U5

By using the actual undershoot standard deviation instead of the standard deviation of the customer order size, the reorder point could be obtained from the following function (Berling, Reorder point adjustment, 2009):

$$R_{U5} = r\left(\mu_D + \mu_U, \sqrt{\sigma_D^2 + \sigma_U^2}\right) \qquad (3.39)$$

Looking at formula (3.13) the following assumptions can be made:

$$\mu^{'} = \mu_D + \mu_U \qquad (3.40)$$

$$\sigma^{'} = \sqrt{\sigma_D^2 + \sigma_U^2} \qquad (3.41)$$

which leads to

$$R_{U5} = r(\mu^{'}, \sigma^{'}) \qquad (3.42)$$

4 Data processing

In this chapter all data obtained from Syncron is presented. The sorting process of how the chosen articles were selected depending on different demand distributions is described and how current reorder points from Syncron is achieved are mentioned.

4.1 Received data

Data provided by Syncron contained:

- Weekly sales for one year (from week 36 2008 to week 36 2009).
- Standard deviation of the average customer demand over a month for each article and retailer.
- Mean of the average customer demand for each article and retailer.
- Transportations times from the central warehouse to each retailer for all articles.
- Lead-times from suppliers to the central warehouse.
- Order quantities for the central warehouse and for each article and retailer.
- A multiple of the order quantities that needs to be ordered for each article and retailer.
- A minimum order quantity that needs to be ordered for each article and retailer.
- Target fillrate for each article and retailer.

4.2 Sorting

The provided data set contained about 39,000 articles. This was too many to simulate in the given time span and a sorting was needed to reduce the number of articles to about 100. The sorting was done in five steps. Before the actual sorting began a pivot table of the data was created. The columns consisted of the article numbers and the rows of the monthly demand for each retailer for the actual article number, see Appendix 1. This facilitated the handling of the large volumes of data.

Step 1

The total demand for each article over all retailers was summarized. All articles with a total demand of < 10 were dismissed, since these articles were considered as articles to be handled manually.

In the first step around 25,000 articles were sorted out.

Step 2

Articles that contained any retailer with an overall negative demand over a period of a month were sorted out. Negative demand arises when the customers return units. The analytical model cannot handle a negative demand like the SCP is able to do, which is why these articles were sorted out.

In this step around 2,000 articles disappeared.

Step 3

Some articles had some missing data. For some of the articles lead times from the central warehouse to a retailer were missing and some articles were missing order quantities for any retailer. These articles were dismissed.

In the third step around 2,000 articles disappeared.

Now about 10,000 articles reminded that were considered representative to analyze. To get a good selection of about 100 articles that represented the 10,000 articles, the 10,000 articles were sorted further in two steps (step 4 and 5). The two steps were to select different articles depending on their demand patterns. The three demand patterns that were recognized in the two steps below were high-demand articles (can be modeled with a normally distributed demand), slow-demand articles (can be modeled with a Poisson distributed demand) and lumpy demand articles (can be modeled with a compound Poisson distributed demand).

Mean and standard deviation for each article were calculated on a retailer level. These values were then compared with the ones obtained from Syncron. This is to be sure that Syncron calculated their reorder points on the same data used in the analytical model and the simulations. Otherwise the results from the simulations in Extend would not be comparable. Mean and standard deviation were calculated by the following formulas:

X: demand for each retailer within each article

$$\bar{X} = Mean = \frac{X_1 + X_2 + \ldots + X_N}{N} \qquad (4.1)$$

$$\sigma = Stddev = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2} \quad (4.2)$$

The mean and the standard deviation were calculated on monthly basis. The weekly sales were combined into months, after that the mean and standard deviation were calculated with formula 4.1 and 4.2 above. Month was used to minimize the risk of negative demand.

Step 4

In step four 35 high-demand articles were sorted out. This was done by taking a ratio between the mean and standard deviation for each article and retailer. The 35 articles with a relation as above (mean / standard deviation) of at least 2.2 in at least one of the retailers were selected. Some of the 35 articles had more than one retailer with at least the ratio of 2.2 and some had just one retailer. None of the articles only contained high demand retailers.

Step 5

In step five 57 slow demand articles and 43 lumpy demand articles were sorted out. The 57 slow demand articles were sorted out by calculating the ratio between the variance (standard deviation^2) and the mean. All articles with at least two retailers with the ratio described above (variance / mean) which were equal to one were sorted out. Subsequently, articles were sorted, from largest to smallest, depending on how many retailers for every article that had the ratio equal to one. Based on that, 57 articles were selected and the distribution is as follows:

- *19 articles with 75-83% of the retailers variance/mean = 1.*
- *19 articles with 57-75% of the retailers variance/mean = 1.*
- *19 articles with 50-57% of the retailers variance/mean = 1.*

Just as in step four, there were no articles that only contained slow demand retailers.

The 43 lumpy demand articles were sorted out by the same method as the slow demand articles were. The different was that the ratio instead should be greater than one. Subsequently, articles were sorted from largest to smallest, depending on the mean of the variance of all the retailers for every article. Based on that, 43 articles were selected and the distribution between the articles was as follows:

- *15 articles with a mean variance at the retailers between 35 and 250.*
- *20 articles with a mean variance at the retailers between 250 and 300.*
- *8 articles with a mean variance at the retailers between 300 and 10,000.*

Just as in step four and as in the sorting of slow demand articles, there were no articles that only contained lumpy demand articles. Like mentioned, there is no article which only have retailers to be considered as high-demand, slow demand and lumpy demand. Often, each article has the mixture of all three different classifications.

When the 135 articles were selected they were checked so they represented the 10,000 articles well. The high demand articles were a bit overrepresented among the 135 articles but the slow demand and the lumpy demand articles comprised the largest proportion just as among the 10,000 articles. The reason of the overrepresentation of the high demand articles was to make sure that the selected part represented the high demand products among the 10,000 articles. A smaller part could give misleading results. There was also a good spread of other variables among the 135 articles such as, lead time, order quantity, target fillrate and the number of retailers that were represented in each article.

4.3 Current reorder points

Syncron calculated their reorder points for the selected articles according to their standard operating procedures. Syncrons reorder points were later compared in the simulation program Extend (see chapter 6) with the calculated reorder points from the analytical model (see chapter 5).

Like mentioned in section 1.1, Syncron has five different solutions they sell to their customers. It is under the solution Global Inventory Management (GIM) that Syncron calculates their reorder points using a tool called Supply Chain Planner (SCP). The SCP has many extra features who have been disabled to make the result from simulation comparable. Some examples of the features are trend adjustments and classifications. When these features are used, they only affect the calculations of the mean and the standard deviation of the articles. This means that a future implementation of the analytical model is no more influenced by these settings than the current algorithm for calculating reorder points in the SCP. The results can therefore be seen as applicable even though these features are disabled.

5 Calculation of reorder points

This chapter describes how reorder points were calculated with the analytical model, programmed in Excel and Visual Basic. Areas that are described in more detail are required input data, which settings that are made and how articles are grouped.

5.1 Input data

In order to calculate a new reorder point by using the analytical model described in section 3.4.3, the approach has been programmed in Microsoft Excel using Visual Basic for the more difficult calculations. This Excel program is developed by the division of Production Management at Lund University and it is hereinafter referred to as the Excel-model. A picture of its interface can be seen in Appendix 2. The following input parameters are required in the Excel-model:

- Order quantities for the central warehouse and each retailer.
- Transportation times between the central warehouse and the retailers, the lead-time between the supplier and the central warehouse.
- Target fillrate for each retailer.
- Holding cost at each installation.
- Shortage cost at each retailer, which is achieved from formula (3.18).
- Average customer demand per time unit at each retailer.
- Standard deviation of customer demand at each retailer.
- Probability for every possible customer order size at each retailer.

Most of these parameters could be used directly from the data received from Syncron, and a few of them needed to be adjusted to fit in the requirements of the Excel-model. Mean and standard deviation of customer demand was calculated on a monthly basis as described in section 4.2. In the Excel-model, however, all data must be based on the same time unit and when transportation times were in days the mean and standard deviation of customer demand where adjusted for days. This was done by divide the mean customer demand with the average number of days per month (365/12), and the standard deviation of customer demand with the square root of the average number of days per month. A compound distribution over the size of what each customer demanded was also created, i.e. the probability that a customer demands a certain number of units were calculated. This distribution is also put into the Excel-model but on another sheet than the one in Appendix 2.

5.2 Settings

In addition to selecting the number of retailers and set all the input parameters in the Excel-model, there are also eight other settings that must be taken into account before an estimate can be made. As seen in Appendix 2, they are:

- Leadtime_choice
- CW_demand
- Ret_demand
- Choice
- Cost_FR_opt
- Local_search
- Undershoot comp.
- Order size dist

Leadtime_choice

Leadtime_choice determines how the lead-time between the central warehouse and the retailers will be calculated. The option that works best here is the classical Little's formula (option code 0) and therefore, this choice has been used in all calculation. Little's formula is an exact method when estimating the lead-times for a system with identical retailers but because it is valid in the most general situations it can also be used in a partial deliveries solution. The other two choices, Partial AM and A2 AM, are two methods to estimate the lead-time based on the equation (3.33).

CW_demand

CW_demand determine which statistical distribution that describes the demand at the central warehouse. There are four possible options to choose between and they are described in more detail in section 3.4.3. In this study the negative Binominal distribution (option code 4) has been used for all the articles because sometimes extreme ratios between the standard deviation and the mean of the demand occur. To make the results as comparable as possible, only one approximate demand distribution among the articles was used.

Ret_demand

Ret_demand determine which statistical distribution to describe demand at the retailers and here there are 5 different choices. In the first option, Normal (option code 0), customer demand is adapted to a normal distribution with the mean and standard deviation of each retailer. This option is simple to calculate and has proven to be robust. However, in cases where the variance is much larger than the mean, which means that the probability of negative values is increasing, it can work less well. For these cases, it may work better if NegBin (option code 1) is used, which adjusts customer demand to a negative binomial distribution with the mean and standard deviation of each retailer. In the next two options, Compound Poisson-Geometric (option code 2) and Poisson (option code 3), customer demand adapts to, as in the previous cases, the distribution the name suggests. The most accurate customer demand distribution in relation to reality is given by the option Compound Poisson-Empirical (option code 4). Depending on how the relationship between variance and mean is for the retailer's demand, this option use one of three different distributions to describe the customer demand. If variance is less than the mean, the Excel-model will choose a normal distribution (i.e. the same as option code 0) as customer demand at that retailer. In cases where the variance is equal to the mean, a Poisson distribution (option code 3), will represent the demand. In other cases, when the variance is greater than the mean, customer demand at the retailers will be represented by a compound Poisson distribution which uses the actual customer demand distribution for each retailer that is entered in the Excel-model. The settings for the customer demand at the retailers used in this thesis are primarily Normal (option code 0) and Compound Poisson-Empirical (option code 4), but also NegBin (option code 1) has been used in a number of occasions.

Choice

The Choice setting determines how the induced backorder cost (β) at the central warehouse is set, see section 3.4.3. Either this is calculated with an equation (option code 3) or by using a table (option code 4). In this thesis, all calculations are done with choice number 3.

Cost_FR_opt

Cost_FR_opt setting determines whether to optimize the reorder points for cost (option code 0) or after fillrate (option code 1). Since it was important to achieve the target fill rate than to have low costs in this study, all reorder points are optimized against fillrate. As equation (3.18) is an approximation, the result can be unreliable if there is a low demand and a high target service level. Because of this, there is a high probability that the cost optimization selects R one unit to low which results in a service level far below the target.

Undershoot comp

Undershoot comp. setting determines the type of undershoot compensation that will be used when demand is approximated with a normal distribution. The two at issue in this study is option 1 and option 5, see section 3.4.4 for details of these two. The reason that only these two were used was that these were considered to be the most reliable of the five different methods developed according to results in a pre-study.

Local_search and Order size dist

These are settings that have not yet been programmed. The Local_search will give a possibility to test R_0+1 to see if the total cost is reduced and the Order size dist has been replaced by the Ret_demand choice.

5.3 Calculations depending on undershoot and demand

For each article, three different calculations of the reorder points where made. The first two were made to base a decision to determine which of the two undershoot compensation method it was worthwhile to pursue with. In these calculations, the customer demand at the retailers where set to be normal distributed. The only thing separating these two calculations is the undershoot compensation method used. The third and final calculation was made with the selected undershoot compensation and with the Compound Poisson-Empirical approach for the customer demand at the retailers. The undershoot compensation is needed for this calculation because some of the retailers within the article can be approximated with a normal distributed demand. As this setting was more computationally complex than the normal distribution case, a maximum time was placed on how long a calculation could take. If no answer was received after 20 minutes, the calculation was terminated, and instead the NegBin setting was used. For a couple of articles this approach also took too long time, and therefore there are a few articles that only were calculated twice. Worthwhile to mention is that all articles may be calculated for all the different settings but it can sometimes take up to 12 hours before an answer is received. This seemed unreasonable for a calculation which is to be made each month to tens of thousands of articles. That was also why a time limit was introduced. If a batch quantity, see Appendix 2, larger than one can be used, the calculation time will be heavily reduced.

With these three calculations as a basis, the articles were divided in to four different groups depending on the settings used. This is described in section 7.1.2.

6 Simulations

This chapter contains a brief description of the simulation model structure and the assumptions made during the simulations. How the simulation model is verified, runtime of the simulations and input and output parameters are described as well.

6.1 The simulation model

To simulate a real-world performance over time for the articles in this study, an existing simulation model, which is created at the division of Production Management at Lund University, has been expanded and improved to handle up to eleven retailers. An overview of the simulation model can be seen in Appendix 3. The model represents an inventory system with a central warehouse and up to eleven retailers where all installations replenish their stock as an (R,Q)-policy.

Customer demand in the simulation model is created by two blocks, which can be seen in Appendix 8. By using an exponential distribution with the correct time between customer arrivals, the generator block creates the customer arrivals in the model. Connected to each one of the customer is a random amount of units, based on the compound probability distribution of customer demand sizes. This set makes each customer arrival, and the number of items demanded by the customer, independent of each other, which make the model consistent with reality.

The retailers are model by two blocks, retailer trigger and retailer inventory, which can be seen in Appendix 8 and Appendix 9. What these two blocks do is to convert the customer to its demand size, i.e. a customer who demands more than one unit is transformed and become as many units as demanded. Then when the inventory level reaches the reorder point, it triggers an order to the central warehouse.

In the central warehouse, see appendix 10, the demand from the retailers is handled and fulfilled as long as there are units in stock. The units are sent to each retailer with a delay corresponding to the transportation time. If an order cannot be fulfilled completely, as many units as there are in stock will be sent to the retailer and the remaining units will be backordered. When the inventory level reaches down to or below the reorder point an order of new units is sent from the supplier, who is represented by a delay block and delivers new units when the length of the lead time is expired.

6.1.1 Assumptions made in the simulation

In order to clarify the assumptions made in the simulation model the following list is presented. Some of the assumptions are new but most of them are mention earlier in this report.

- Customer demand is assumed to follow a compound Poisson distribution.
- Demands that cannot be satisfied directly from stock are recorded as backorders.
- Each installation use a continuous review policy, i.e. as soon as the inventory level reaches the reorder point an order is triggered. This might not represent reality in full, where periodic review is much more common. Periodic review means that the inventory is controlled perhaps one to two times per day, which results in a delay of the lead-time with a half day up to a day. Since the SCP software uses continuous review, the results will be most comparable if this also is applied in the simulation model.
- Transportation times from the central warehouse to the retailers are constant which means that the only thing that can affect the transportation time is stockouts at the central warehouse.
- The supplier always has stock on hand and can therefore always satisfy an order from the central warehouse, which therefore will experience the lead time from the supplier as constant.
- The central warehouse follows a partial deliveries policy an always fulfills as much of an order as it has stock on hand.
- Order at the central warehouse is handled according to a first come first served policy and an order that cannot be met directly is recorded as a backorder.
- All order quantities at all installations are constant.

6.1.2 Verification of the simulation model

To verify that the new simulation model gives accurate results, a few test series where done and the results then were compared with the original model. Since the original model is developed by a team at the division of Production Management at Lund University and used in several similar studies, it is considered to be valid and give accurate results. The comparison was made with the same input data fed into the two models, which then were simulated over a long period of time. As the means of the two models were within each other's standard deviation, it is certain to say that they deliver the same results and the new modified model can therefore be assumed to function correctly. The small variations in the results between the two models arose probably because of different random numbers during the simulations. The same random seed number was used in both models but since the number of blocks in the two models is different, some random numbers in some of the blocks may be different between the two models.

The new simulation model has also been tested so it generates the right demand compared with the real customer demand. By measuring the cumulative size of all demand that the model generates, and then divide it by the total simulation time, the mean demand per time unit is obtained and can be compared with the true mean demand per time unit. Since the deviations between these two values were very low for all retailers in a number of different simulations, it can be assumed that the model also generate an accurate customer demand.

6.1.3 Runtime of the simulations

All articles have been simulated with a customized simulation time, which depends on the mean customer demand and the size of the order quantity. Each simulation was conducted over a given period of time, which consisted of 30 blocks and can be seen in more detail in Figure 10. With this arrangement, 30 smaller simulations could be made for each simulation and the results obtained are therefore averages over these 30 blocks. To be sure that almost all possible outcomes are realized over a block length and that the blocks are independent of each other, the block length was set to 20 expected order cycles. An order cycles is defined as the order quantity divided by the mean customer demand at each retailer. For each article the longest expected order cycle, i.e. the highest ratio between the order quantity and the mean customer demand, among the retailers were used as base for the block length. This value was then rounded upward to the nearest hundred. A lower limit of 500 time units was set for the block length, which means that each block is a simulation over at least barely a year and a half. However, in most cases each block length is a lot longer.

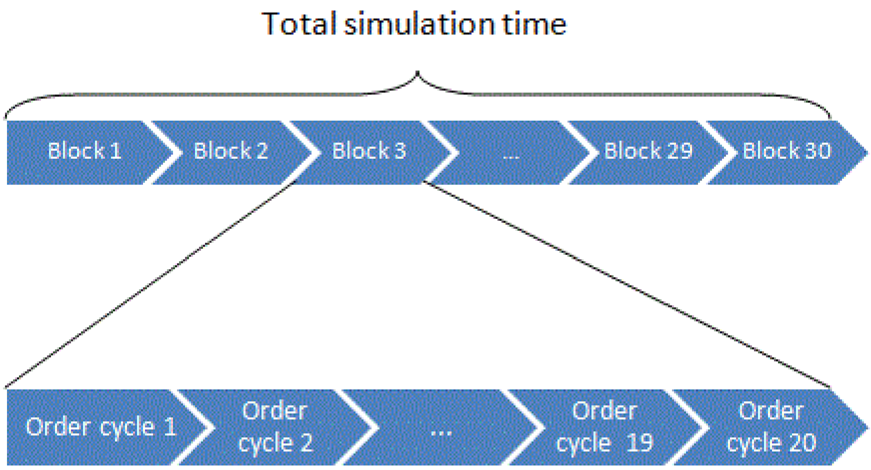


Figure 10 – Overview on the simulation time setup. The total simulation time consist of 30 blocks. One block consist of 20 order cycles and one order cycle is at least 500 time units long, often longer.

6.2 Input parameters

Several input parameters are required for the simulation model to run properly, see Appendix 13 for an overview of how data is fed into the simulation model. They are described briefly below:

- Order quantity for each installation (Q).
- Start position for the inventory level, here set to reorder point plus order quantity (R+Q).
- Transportation time between the central warehouse and retailers as well as transportation time between the subcontractor and the central warehouse.
- Holding cost and shortage cost for each installation.

Due to these parameters differ for each articles, the parameters must be changed for each new simulation. To facilitate this, the Extend file is linked to an Excel sheet into which data is copied. In addition, the mean time between customer arrivals (1/λ), see section 3.2.3, and the probability function for the customer demand size is fed into the Extend blocks that generate customer demand for each retailer, see Appendix 8.

6.3 Output parameters

The results needed from the simulations to answer the purpose of this study are expected fillrate and mean inventory level for all retailers and the central warehouse. Fillrate is calculated in the simulation model according to this formula:

$$1 - \frac{requested\ amount\ of\ units\ that\ not\ can\ be\ satisfied\ directly\ from\ stock}{total\ amount\ of\ requested\ units} \quad (6.1)$$

Expected mean inventory level is calculated as the mean inventory on hand per time unit, which is represented by the number of units in the cost calculation block, see Appendix 12, divided by the block length in time.

As can be seen in Appendix 13, some other results also will come as output from the simulations but these are excluded and not used in this study. Among those is the mean and standard deviation of the expected lead-time from the central warehouse to each retailer, the expected costs for each installation and the expected total cost of the system. Lead-time may be interesting to study further to understand more of what lies behind certain behaviors, and for those times when it is important that the units arrive on time. But since this is not an objective of this master thesis, the lead-time values are not analyzed in detail. The expected costs of the various installations are very interesting to look at when there are clear differences in the holding cost of various retailers. This, for example may be due to lower warehouse costs or increase in product value due to transportation closer to the end customer. Since none of these costs or similar differential costs was available for the articles studied, the holding cost was set to one (1) for all articles and installations. This implies that the expected cost is equal to the expected inventory level which makes the expected cost results unnecessary to look at in this study.

7 Results and analysis

In this chapter the results from the simulations will be presented. First, an overview of the key figures used in the analysis is made. Secondly, the results are described and analyzed. This starts with the choice of undershoot method and is followed by a better analyze of the results were the articles are divided into several different subgroups depending on demand type and demand size.

7.1 Key figures and grouping

To be able to get a good and simple overview of the results key figures were compiled and the articles were grouped depending on demand, model settings etc.

7.1.1 Key figures

As mentioned above in section 6.3 fillrate and inventory level are two parameters, which are analyzed to develop strengths/weaknesses in the analytical model and the SCP. These two parameters were used to generate key figures that can be seen in Table 1 below.

Key figures / Model
Mean of difference from target fillrate (%)
Stdev. of mean of difference from target fillrate (%)
Mean of weighted difference from target fillrate (demand, %)
Stdev. of mean of weighted difference from target fillrate (demand, %)
Mean of weighted difference from target fillrate (picks, %)
Stdev. of mean of weighted difference from target fillrate (picks, %)
Max positive difference from target fillrate at retailers (%)
Mean of max positive difference from target fillrate at retailers (%)
Stdev. of mean of max positive difference from target fillrate at retailers (%)
Max negative difference from target fillrate at retailers (%)
Mean of max negative difference from target fillrate at retailers (%)
Stdev. of mean of max negative difference from target fillrate at retailers (%)
Reduction in inventory at CW by using the new model relative to SCP (%)
Reduction in inventory at Retailers by using the new model relative to SCP (%)
Total reduction in inventory by using the new model relative to SCP (%)

Table 1 – Key figures

The target fillrate is obtained from Syncron and it is the fillrate that needs to be maintained to keep the customers satisfied. Within each simulated article the difference from target fillrate (simulated fillrate - target fillrate) was calculated for each retailer. The mean of these values for each article were then calculated and will henceforth be called retailer mean. The weighted mean depending on demand and number of picks were also calculated and will henceforth be called the weighted demand of retailer mean and the weighted picks of retailer mean. The weighted means were calculated for each article by multiplying each retailer's difference from target fillrate with the correct demand/picks weight and summarize the multiplications for all retailers within the article. A positive value of the difference means that the calculation method meets or is above the target fillrate while a negative value means that the target fillrate is not achieved. To every calculated mean, a standard deviation has been calculated to see how reliable the result is. If the standard deviation is small the result is solid and vice versa. A description of the key figures in Table 1 follows:

Mean of difference from target fillrate (%): This is the mean in % of the retailer mean for all current articles.

Mean of weighted difference from target fillrate (demand, %): This is the mean in % of the weighted demand of retailer mean for all current articles.

Mean of weighted difference from target fillrate (picks, %): This is the mean in % of the weighted picks of retailer mean for all current articles.

Max positive difference from target fillrate at retailers (%): This is the maximum positive value in % of the difference from target fillrate for all retailers within the current articles.

Mean of max positive difference from target fillrate at retailers (%): This is the mean in % of the max positive difference from target fillrate for all current articles.

Max negative difference from target fillrate at retailers (%): This is the maximum negative value in % of the difference from target fillrate for all retailers within the current articles.

Mean of max negative difference from target fillrate at retailers (%): This is the mean in % of the max negative difference from target fillrate for all current articles.

Reduction in inventory at CW by using the new model relative to SCP (%): This is the mean in % of all current articles calculated with equation 7.1, at the CW.

$$\frac{Expected\ inventory\ with\ the\ SCP - Expected\ inventory\ with\ the\ analytical\ model}{Expected\ inventory\ with\ the\ SCP} \quad (7.1)$$

Reduction in inventory at Retailers by using the new model relative to SCP (%): This is the mean in % of all current articles calculated with equation 7.1, for all the retailers.

Total reduction in inventory by using the new model relative to SCP (%): This is the mean in % of all current articles calculated with equation 7.1, for both the CW and all the articles.

7.1.2 Grouping

The 135 articles have been grouped depending on the settings in the analytical model (see section 5.3) to be able to analyze the result from various angles. Each article is represented in two of the four groups bellow.

- Group 1. Contains all the 135 articles in the study. These were calculated with normal distributed approach as customer demand at the retailers and both undershoot compensations methods were used.
- Group 2. Contains 109 articles out of the 135 in Group 1 that in addition to been calculated with normal distributed approach as customer demand at the retailers, also were calculated with the Compound Poisson-Empirical approach as customer demand at the retailers.
- Group 3. Contains 15 articles out of the 135 in Group 1 that were not possible to calculate with the Compound Poisson-Empirical approach and were instead calculated with the Negative Binominal approach as customer demand at the retailers.
- Group 4. Contains 11 articles out of the 135 in Group 1 in which the reorder point could not be calculated for anymore settings than those in Group 1.

The articles in the second group were further grouped two times depending on classification from the SCP and their amount of demand. The first group that was created from Group 2 was with the classification from the SCP, which depends on demand patterns. The demand patterns represented in the SCP among the selected articles are fast articles, slow articles and lumpy articles. Just like in 4.2, there are not many articles that only have retailers with one demand pattern.

Often, each article has the mixture of all three different classifications. The number of each demand patterns and how they are specified follows:

- 43 fast articles are represented. In a fast article more than 50% of the retailers within the article must be classified as fast.
- 45 slow articles are represented. In a slow article 50% or more of the retailers within the article must be classified as slow.
- 21 lumpy articles are represented. In a lumpy article 50% or more of the retailers within the article must be classified as lumpy.

The amounts of fast, slow and lumpy articles are not the same as in section 4.2 (where high demand corresponds to fast, slow to slow and lumpy to lumpy). This is because the SCP does not have the same rules for how the classification is made. However, all the high demand articles from section 4.2 are among fast articles from the SCP. One thing that is different is that some of the slow- and lumpy demand articles from section 4.2 are among the fast articles from the SCP.

The second group that was created from Group 2 was depending on the total amount of demand of each article. They were grouped in the following four different groups:

- 30 articles with a total demand of each article < 20.
- 21 articles with a total demand of each article between 20 and 100.
- 30 articles with a total demand of each article between 100 and 1000.
- 28 articles with a total demand of each article between 1000 and 5000.

7.2 Choose of undershoot method

The first thing that was done was to simulate all the articles in Group 1. This was done to determine a choice of the undershoot method, so that the number of future simulations could be reduced. The most important key figures are presented below in Table 2, a full accounting of all the key figures can be seen in Appendix 14.

Key figures / Model	U1	U5
Mean of difference from target fillrate (%)	0,7%	0,6%
Mean of weighted difference from target fillrate (demand, %)	-1,4%	-1,6%
Mean of weighted difference from target fillrate (picks, %)	-1,7%	-1,9%
Max positive difference from target fillrate at retailers (%)	27,2%	27,2%
Mean of max positive difference from target fillrate at retailers (%)	7,4%	7,4%
Max negative difference from target fillrate at retailers (%)	-34,8%	-34,8%
Mean of max negative difference from target fillrate at retailers (%)	-5,6%	-5,9%
Total reduction in inventory by using the new model relative to SCP (%)	34,8%	35,4%

Table 2 – Key figures, choose of undershoot method 1 or 5

According to these values, both U1 and U5 are above the target fillrate when looking at the mean of difference from target fillrate. When the values are above the target fillrate, more inventory than necessary is kept in stock, which results in too much tied-up capital. The two weighted mean is however both below the target fillrate. U1 has an advantage here since it is not as much under the target fill as U5. Because of the negative value, the service level to the customers is not achieved, which consequences are difficult to measure and can cost much money. Regarding the two extreme values of the maximum difference from target fillrate, both positive and negative, U1 and U5 are almost identical. U5 is a bit lower regarding the mean of maximum negative difference from target fill rate, which is reasonable when U5 also is lower when it comes to the weighted means. In relation to the SCP, U5 reduces the total inventory a little more than U1, which also is reasonable because the U5 is slightly lower in all means of difference from target fillrate. This means that U5 does not tie up as much capital in stock.

The choice of the undershoot method became U1. This is because U5 does not meet the target fillrate at the weighted mean values as good as U1, the major criterion is to meet the target fillrate. U5 reduces the stock a little more than U1, but the consequence of unsatisfied customers can cost very much money, booth in lost income and badwill. Implementation time and computation time of the two different methods were not taken into account when the choice of the undershoot method was made.

7.3 Different demand approaches compared to the SCP

This section analyzes the potential of the analytical model with a normal distributed demand and the selected undershoot method, henceforth referred to as Approach 1, the results obtained when the compound Poisson approach was used as customer demand, henceforth referred to as Approach 2, and the current performance of the SCP software. See section 5.2 for a detailed description of what these settings mean.

7.3.1 Normal distribution settings as customer demand

Table 3 below presents the key figures of the articles in Group 1, a complete table can be seen in appendix 14.

Key figures / Model	Approach 1	SCP
Mean of difference from target fillrate (%)	0,7%	-3,0%
Mean of weighted difference from target fillrate (demand, %)	-1,4%	-4,1%
Mean of weighted difference from target fillrate (picks, %)	-1,7%	-2,4%
Max positive difference from target fillrate at retailers (%)	27,2%	10,0%
Mean of max positive difference from target fillrate at retailers (%)	7,4%	4,7%
Max negative difference from target fillrate at retailers (%)	-34,8%	-66,3%
Mean of max negative difference from target fillrate at retailers (%)	-5,6%	-16,3%
Total reduction in inventory by using the new model relative to SCP (%)	34,8%	

Table 3 – Key figures, normal distributed settings as customer demand

Approach 1 performs a result closer to the target fillrate than the SCP does calculated as a straight mean or a weighted mean, see Figure 11. When it comes to the two weighted cases, Approach 1 is performing best when the deviation is weight towards demand. SCP, however, performs best, compared with itself, when the deviation is weighted towards picks. The reason for this may be due to what is considered important, either to the inventories with few big orders having a high level of service or to the inventories with many small orders having a high level of service. What is chosen depends, among others, on the company's market situation. Have the company important customers who buy large quantities seldom or many smaller customers who buy smaller quantities often and therefore considered as valuable to the company. If the two models are compared, it makes quite clear that the analytical model considers that the demand is the most important and therefore the inventories with few big orders have as high a service as possible. SCP, however, believes that the number of picks is the most important and therefore the SCP tries to have as much service as possible on the most visited inventories. A likely reason that the SCP controls after number of picks is that this inventory systems handle spare parts, which usually means that customers are buying smaller quantities more times. However, by a small reprogramming in the analytical model, it can control after number of picks instead of demand.

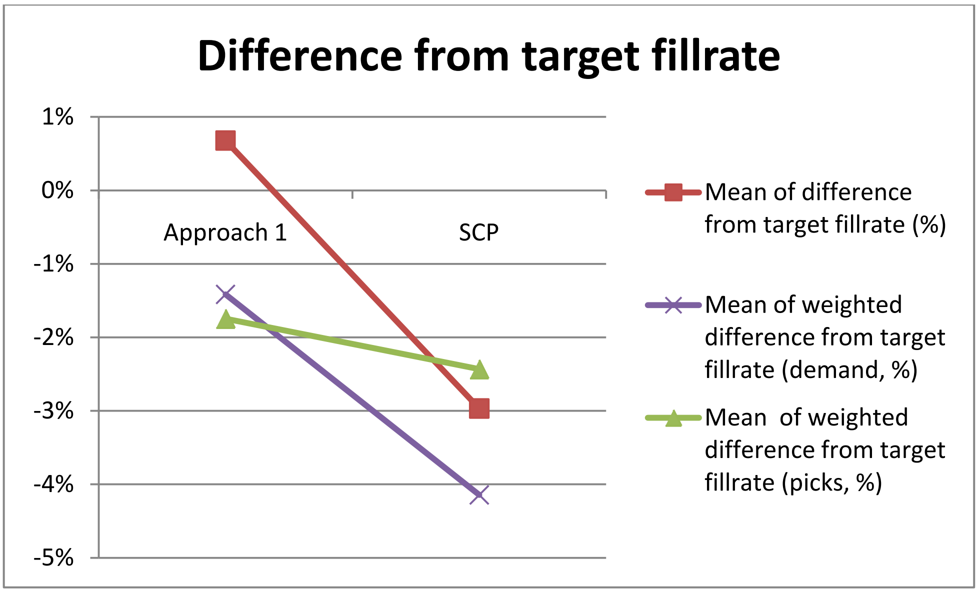


Figure 11 – Difference from target fillrate, normal distributed settings as customer demand

Regarding the spread from the target fill rate (see Figure 12 below), the results are mixed. Approach 1 has both a higher mean and a higher max difference than the SCP regarding positive inventory, which is advantageous for the SCP. But Approach 1 has a smaller difference than the SCP in the case of mean and max difference for negative inventory, which is advantageous for Approach 1.

The reason for the high max negative difference from target fillrate for the analytical model largely depends on the reorder points at the central warehouse. The analytical model only considers reorder points at the central warehouse to be above -1, which results in the maximum lead-time between the central warehouse and a retailer is equal to the transportation time from the supplier to the central warehouse plus the transportation time from the central warehouse out to the retailer. If the reorder point is less than -1, more backorders than calculated will arise at the central warehouse which will lead to longer lead-times out to the retailers which then cannot maintain the target fillrate. In this study, this has been seen for articles with retailers that experience a much lower fillrate then the target, where the reorder point at the central warehouse is often much below -1. According to the creators of the model, this is a known problem but it can be resolved quite easy by for example implement a rule that only allows reorder points equal or above -1. The result will be an increase of the total inventory.

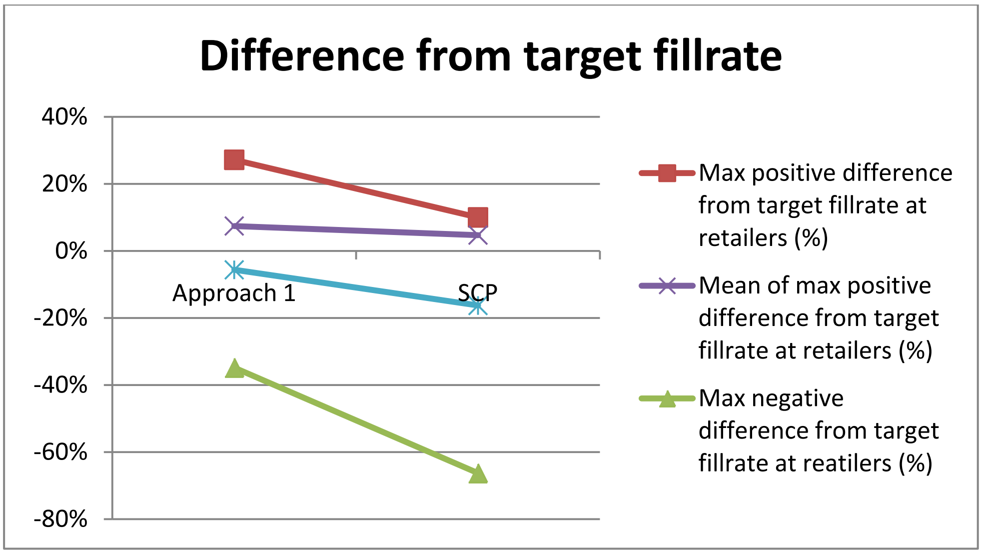


Figure 12 – Spread from target fillrate, normal distributed settings as customer demand

A remarkable value is that Approach 1 lowers the total inventory in the system with approximately 35% compared with the SCP. This releases a lot of capital previously tied up in excess inventories. If the both models were improved so they would hit fillrate better, the SCP would need to add more total inventory in the system compared to Approach 1. This is because Approach 1 has approximate the same positive and negative mean of max difference from target fillrate, while the SCP has much more negative than positive. The total reduction in inventory by using Approach 1 compared to the SCP would then be even greater.

7.3.2 Compound Poisson settings as customer demand

Table 4 below shows the overall results for the articles in Group 2, which mean the most important key figures. A complete table of this can be seen in Appendix 14.

Key figures / Model	Approach 1	SCP	Approach 2
Mean of difference from target fillrate (%)	1,9%	-2,6%	0,8%
Mean of weighted difference from target fillrate (demand, %)	-0,4%	-4,3%	-1,1%
Mean of weighted difference from target fillrate (picks, %)	-0,8%	-2,3%	-1,2%
Max positive difference from target fillrate at retailers (%)	27,2%	10,0%	20,6%
Mean of max positive difference from target fillrate at retailers (%)	8,7%	5,6%	5,6%
Max negative difference from target fillrate at retailers (%)	-30,5%	-66,3%	-27,7%
Mean of max negative difference from target fillrate at retailers (%)	-4,4%	-16,2%	-3,9%
Total reduction in inventory by using the new model relative to SCP (%)	33,9%	-	35,7%

Table 4 – Key figures, compound Poisson settings as customer demand

From these results it can be seen that both Approach 1 and Approach 2, on average, performs above the target fillrate while the SCP does not really reach the target fillrate. If the mean difference from target fillrate is weighted against demand or number of picks at each retailer, none of the models meet the target but Approach 1 is closest. What can be seen here is that the analytical model still performs better on both dimensions compared to the SCP, but again it performs best when the difference is weighted by demand.

It is also important to look at the spread of the fillrate of the different models. In Table 4, it can be seen that the SCP is the one that has the greatest mean of the maximum negative difference while Approach 2 is the method that has the greatest concentration of the results around the target fillrate. The difference between Approach 1 and 2 is highest in the mean of the maximum positive difference, which can be interpreted as Approach 1 in more cases has a higher fillrate than target. This is not always optimal because it results in more stock-keeping units and thus, increased tied-up capital. It is also possible that the maximum positive difference of the fillrate is reflected in the average difference from target fillrate for the system, which then can receive a higher average value than it should have, based on the other simulated articles. Overestimated fillrate in Approach 1 is in large extent probably due to the demand which is continuous and giving rise to reorder points that are not integers. To be sure that the fillrate is achieved, these are rounded up to nearest whole number, which in some cases may be the cause of the overestimation. Because of this it cannot be determined which of the two approaches that is preferable only by looking at the average fillrate. An important aspect to consider is therefore the robustness of the method. This robustness reflects the difference between the upper and lower line in Figure 13 below.

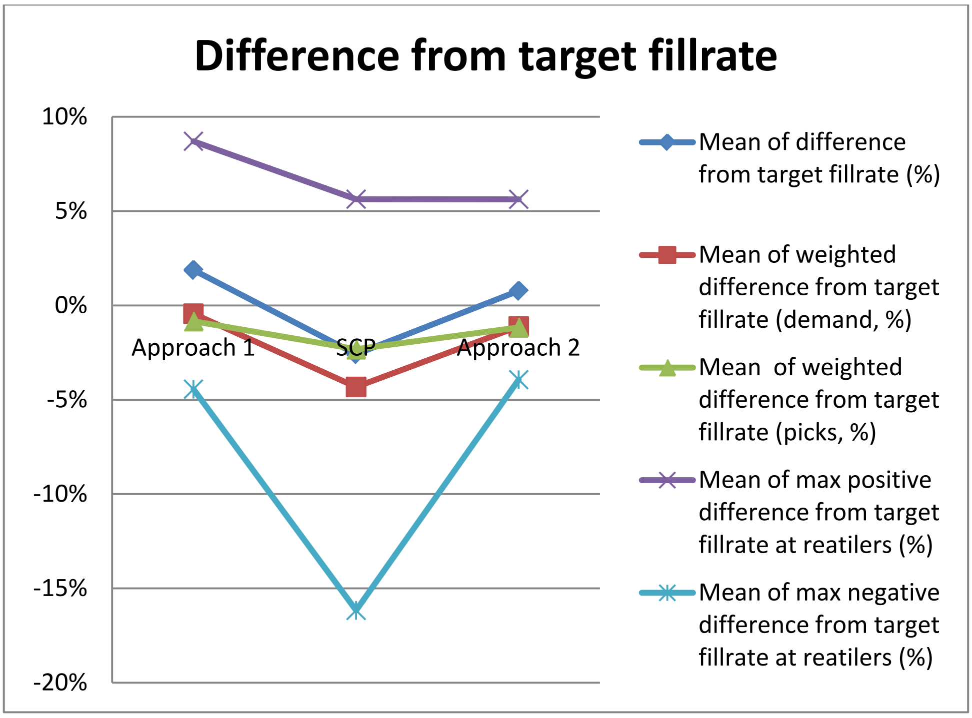


Figure 13 – Difference from target fillrate, compound Poisson settings as customer demand

Both approaches in the analytical model lower the mean inventory in the system with approximately 35% compared to the SCP. The difference of just two percentages that can be seen in Table 4 can be directly linked to the differences that also exist in the service level. Approach 1 lowers the mean inventory in the system a bit less than Approach 2, but this result is instead displayed in a higher average service level to the customers.

In order to do a better analysis of where the different approaches are working well and which demand types they are best suited for, an analysis is made with respect to the different groupings made in section 7.1.2 above.

Fast articles

The results from the simulations of the articles, classified as fast by the SCP, can be seen below in Table 5, and in Appendix 14 where the complete results are found. Notably, the SCP performs better than the analytical model for the two weighted differences in fillrate, while it is vice versa when the results are compared without any weighting. However, none of the models meet the predetermined service level. The robustness of the analytical model's both approaches may be considered better than the SCP. This is because the average of the maximum negative values for the SCP ports more than twice as much lower than what they do for the analytical model. At the same time, the analytical model's two approaches reduce the inventory level by an average of just over 40%.

Key figures / Model	Approach 1	SCP	Approach 2
Mean of difference from target fillrate (%)	-1,6%	-4,2%	-1,7%
Mean of weighted difference from target fillrate (demand, %)	-4,3%	-2,8%	-3,8%
Mean of weighted difference from target fillrate (picks, %)	-4,3%	-2,5%	-3,8%
Max positive difference from target fillrate at retailers (%)	22,6%	10,0%	20,6%
Mean of max positive difference from target fillrate at retailers (%)	6,2%	2,9%	3,7%
Max negative difference from target fillrate at retailers (%)	-30,5%	-55,5%	-27,7%
Mean of max negative difference from target fillrate at retailers (%)	-8,3%	-16,9%	-7,0%
Total reduction in inventory by using the new model relative to SCP (%)	43,1%	-	41,1%

Table 5 - Key figures, compound Poisson settings as customer demand, fast articles

When Approach 1 and Approach 2 are compared with one another, the similarity is distinct. As before, Approach 2 is more concentrated around the target fillrate when the mean of the maximum differences is compared. Approach 1 has a higher mean of the maximum positive difference of the fillrate. In Figure 14 below are the analyzed values plotted. Unlike when the results for the entire Group 2 were demonstrated, Approach 2 is now performing better on the weighted differences. As discussed before, the higher mean of the maximum positive difference for Approach 1 also may result in a higher mean difference. This, together with the smaller difference between the mean of the maximum positive and negative values makes Approach 2 to feel more robust than Approach 1. Approach 1, however, reduces the mean inventory by two percentage points more compared to Approach 2.

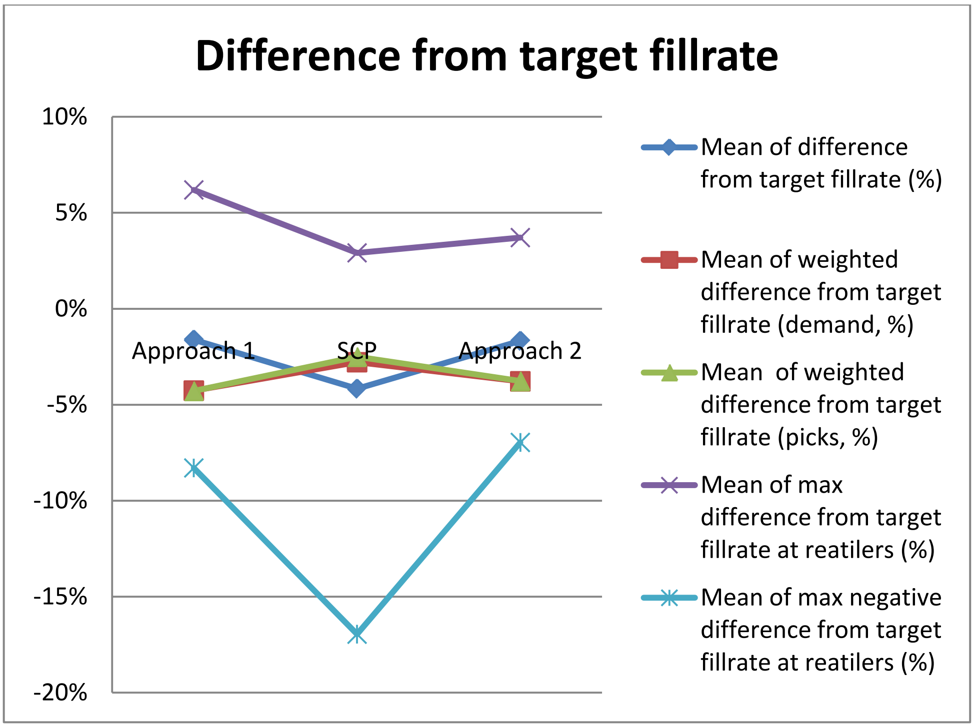


Figure 14 - Difference from target fillrate, compound Poisson settings as customer demand, fast articles

Slow Articles

Table 6 below shows the results of the simulations for articles that were classified as slow by the SCP. Here as well, the more detailed results are seen in Appendix 14.

Key figures / Model	Approach 1	SCP	Approach 2
Mean of difference from target fillrate (%)	4,1%	5,0%	3,0%
Mean of weighted difference from target fillrate (demand, %)	1,7%	1,4%	1,0%
Mean of weighted difference from target fillrate (picks, %)	1,7%	2,3%	1,1%
Max positive difference from target fillrate at retailers (%)	19,6%	10,0%	10,0%
Mean of max positive difference from target fillrate at retailers (%)	8,8%	9,8%	7,5%
Max negative difference from target fillrate at retailers (%)	-11,7%	-43,2%	-10,3%
Mean of max negative difference from target fillrate at retailers (%)	-1,3%	-3,5%	-1,5%
Total reduction in inventory by using the new model relative to SCP (%)	45,1%	-	47,3%

Table 6 - Key figures, compound Poisson settings as customer demand, slow articles

Slow articles are handled well with both approaches in the analytical model and the SCP. The three different mean values of difference from the target fillrate all meet the target fillrate. Maximum deviations of the two approaches are not so large, where Approach 1 only has slightly higher mean of the maximum positive difference than Approach 2. This probably depends on the previously discussed continuous demand distribution which can be very decisive for the slow articles when determine the reorder point. As an example the reorder point can be either one or two units depending on how the rounding is performed. For an article with only a few demanded units per year, one unit in difference in the reorder point will have a great influence on the fillrate and also on the inventory level. The biggest difference in the experienced fillrate is that the SCP has a wider spread of the fillrate than the different approaches in the analytical model, which can be seen very clearly in Figure 15 below. This affects the robustness of the model and makes the fillrate values unsure than the ones in the analytical model.

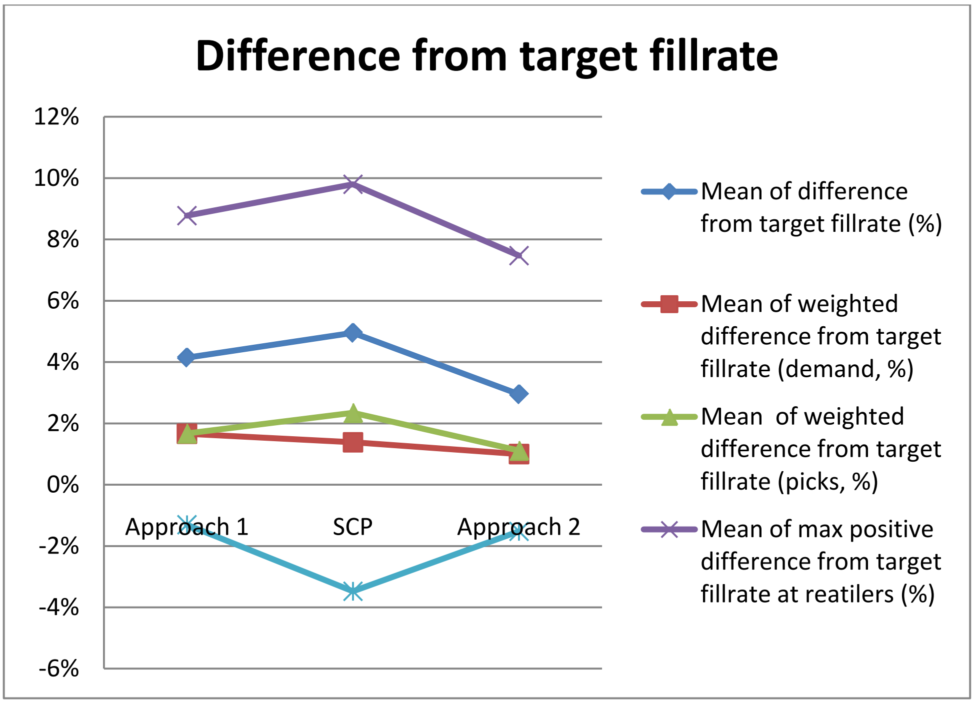


Figure 15 - Difference from target fillrate, compound Poisson settings as customer demand, slow articles

The main difference between the analytical model and the SCP for the slow parts are still the inventory reduction achieved with the analytical model. The largest reduction is obtained with Approach 2, which reduces the total mean inventory in the system by over 47%, two percentage points more than Approach 1.

Lumpy articles

For lumpy articles, it is a big difference between the performance of the analytical model and the SCP. The results of the simulations for the SCP for the various mean differences from the target fillrate are between 10-20 percent below target fillrate. The analytical model output, however, meets the target fillrate quite well for the mean difference as seen in Table 7 below and in Appendix 14.

Key figures / Model	Approach 1	SCP	Approach 2
Mean of difference from target fillrate (%)	3,9%	-15,0%	1,0%
Mean of weighted difference from target fillrate (demand, %)	2,6%	-18,9%	-0,4%
Mean of weighted difference from target fillrate (picks, %)	0,6%	-11,5%	-0,9%
Max positive difference from target fillrate at reatilers (%)	27,2%	10,0%	11,7%
Mean of max positive difference from target fillrate at reatilers (%)	13,4%	2,3%	5,5%
Max negative difference from target fillrate at reatilers (%)	-21,4%	-66,3%	-19,8%
Mean of max negative difference from target fillrate at reatilers (%)	-3,5%	-40,6%	-3,1%
Total reduction in inventory by using the new model relative to SCP (%)	-6,4%	-	1,9%

Table 7 - Key figures, compound Poisson settings as customer demand, lumpy articles

When the difference from target fillrate is weighted against demand and number of picks, Approach 1 performs slightly better than Approach 2, whose mean of the differences ports just below the target fillrate. The mean of the positive maximum difference for Approach 1 is, however, almost twice that of Approach 2, while their mean of the maximum negative difference in much is equivalent. As discussed earlier, the size of the mean of the maximum positive difference is important and must be taken into account when the various means of the difference from the target fillrate is analyzed. This is to minimize misinterpretations based on values that can be misleading for the whole group. Because of this there is a chance that Approach 2 performs better than Approach 1 where a potential high positive difference raises the mean differences from the target fillrate over the values achieved with Approach 2. Figure 16 below, shows clearly how the values for Approach 2 is concentrated near the target fillrate while the SCP differ much in the mean of the maximum negative difference and for Approach 1 it is vice versa for the mean of the maximum positive difference.

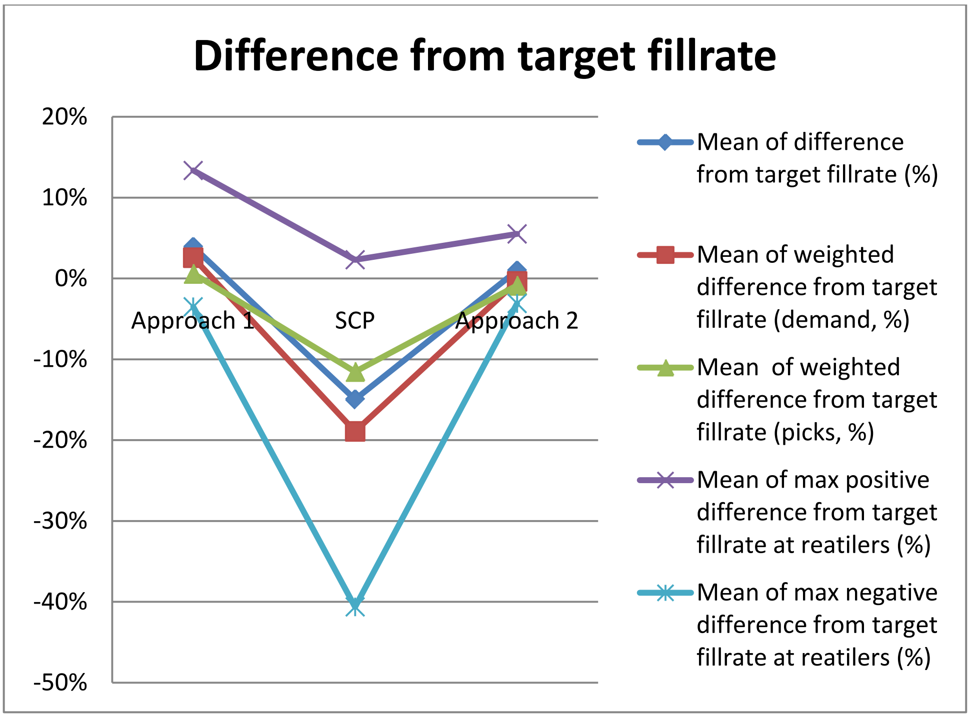


Figure 16 - Difference from target fillrate, compound Poisson settings as customer demand, lumpy articles

The total reduction of inventory in the system for these articles is not as high as for the previous two groups. Approach 1 increases the total inventory in the system with just over six percent while Approach 2 reduces inventory by nearly two percent. This suggests that the lumpy articles are difficult to control and the SCP probably has too low safety-stocks for these articles, given that the method in general not meets the target fillrate for these articles. However, these values are likely to be slightly too low due to the high mean of the maximum negative difference achieved by the SCP.

Different demand sizes

To get an idea of how the analytical model performs at different total demand sizes, the results for the four groups depending on demand size is presented here. In Appendix 14, the key figures of each group are shown in four different tables.

Items with a total demand of less than 20 units in a full year can be broadly comparable to the slow articles which are described earlier. A comparison between the mean differences from target fillrate is very similar, which is illustrated very clearly when Figure 17 below is compared with Figure 15 above. As can be seen, both the analytical model and the SCP, on average, meets or is slightly above the target fill rate for both the mean and the two weighted means of the difference from the target fillrate. As for the slow articles, Approach 2 has the smallest spread around the target fillrate while it reducing the total average stock by almost 47 percent.

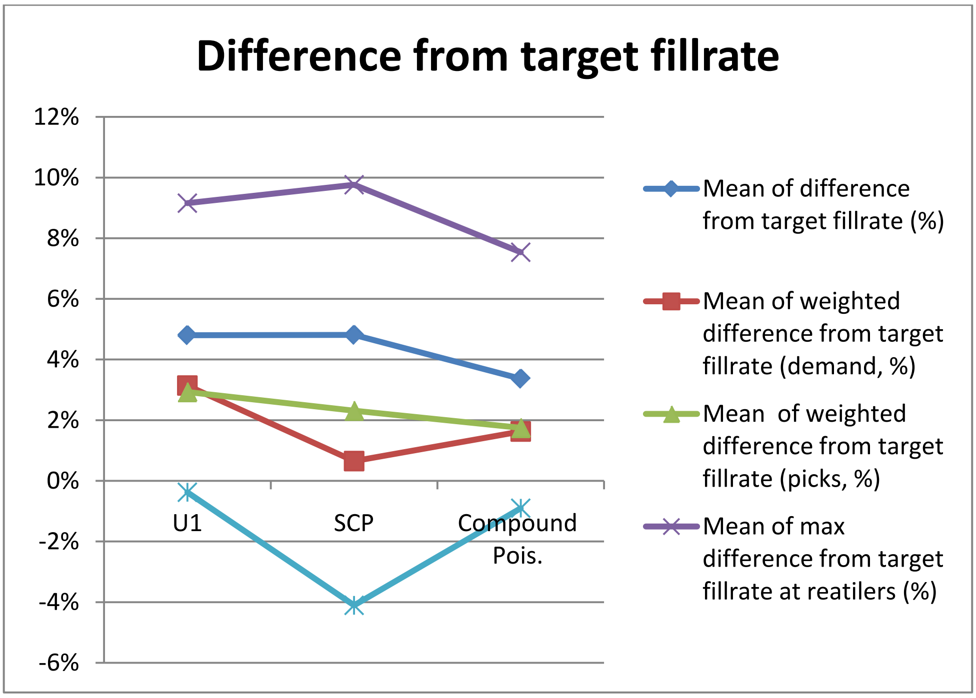


Figure 17 - Difference from target fillrate, compound Poisson settings as customer demand, demand < 20

Articles with a total demand of 20 to 100 units per year are handled much like the articles with a total demand of under 20 units per year, which was described above. Visualization of how the methods meet the fillrate, in Figure 18 below, is very similar for both the slow articles and the articles with a demand of fewer than 20 units per year. The inventory reduction of almost 45% with Approach 2 for these articles is also in line with the two other groups. One conclusion to be drawn from these results is that most of the 45 articles classified as slow by the SCP are in these two groups, with a total of 51 articles and a total demand of up to 100 units per year. The method that manages these articles the best is the analytical model with Approach 2, where the difference to Approach 1 is primarily the lower mean of the maximum positive difference.

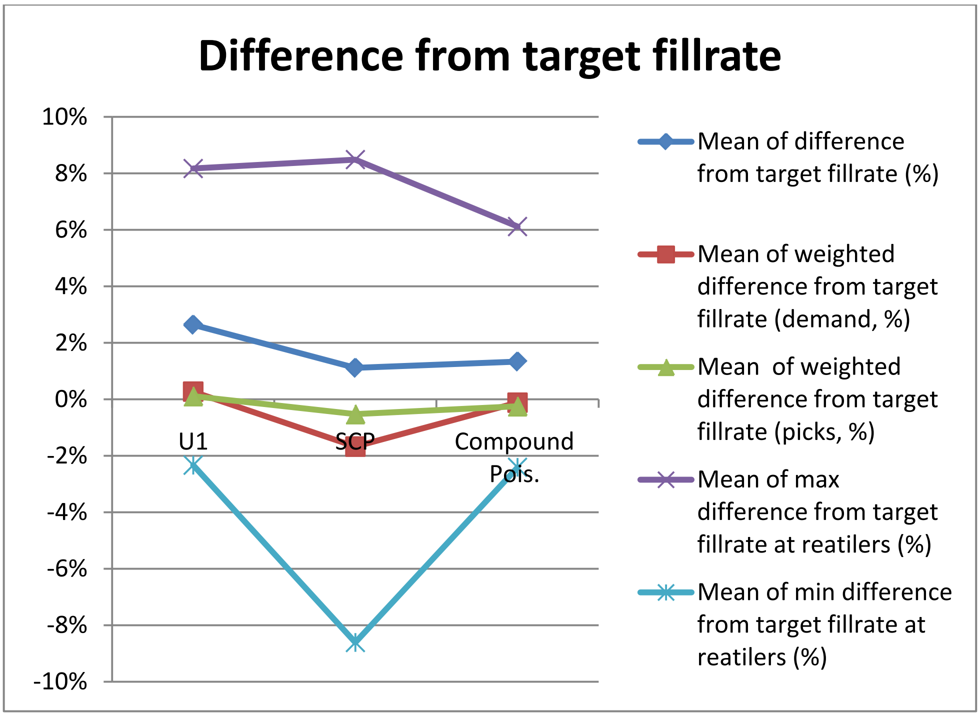


Figure 18 - Difference from target fillrate, compound Poisson settings as customer demand, 20 < demand < 100

For the group of articles with a total demand of between 100 and 1000 units per year, the result of the simulations are shown in Figure 19 below. Even for this group, a parallel can draw to a demand-type, namely the lumpy articles. The similarities between the graphs in Figure 19 below and Figure 16 for lumpy articles are striking. However, the maximum deviation in this group is a bit larger than for lumpy articles and mean differences from the target fillrate is slightly lower. That 16 of the 21 articles classified as Lumpy by the SCP is in this group is yet another sign that these are difficult articles to control and they affect the outcome even if they only are in a slight majority (16 out of a total of 30 articles in the group, the majority of the remaining articles were classified as fast). The difference between Approach 1 and 2 of this group is like in most of the other groups that Approach 1 has a higher mean of the maximum positive difference from the target fillrate than Approach 2 has. In other, the approaches are fairly similar but Approach 2 reduces the inventory a bit more, about 16%, compared with about 12% for Approach 1. Because of the lower mean of maximum positive difference for Approach 2, it must be seen as the best option for this group.

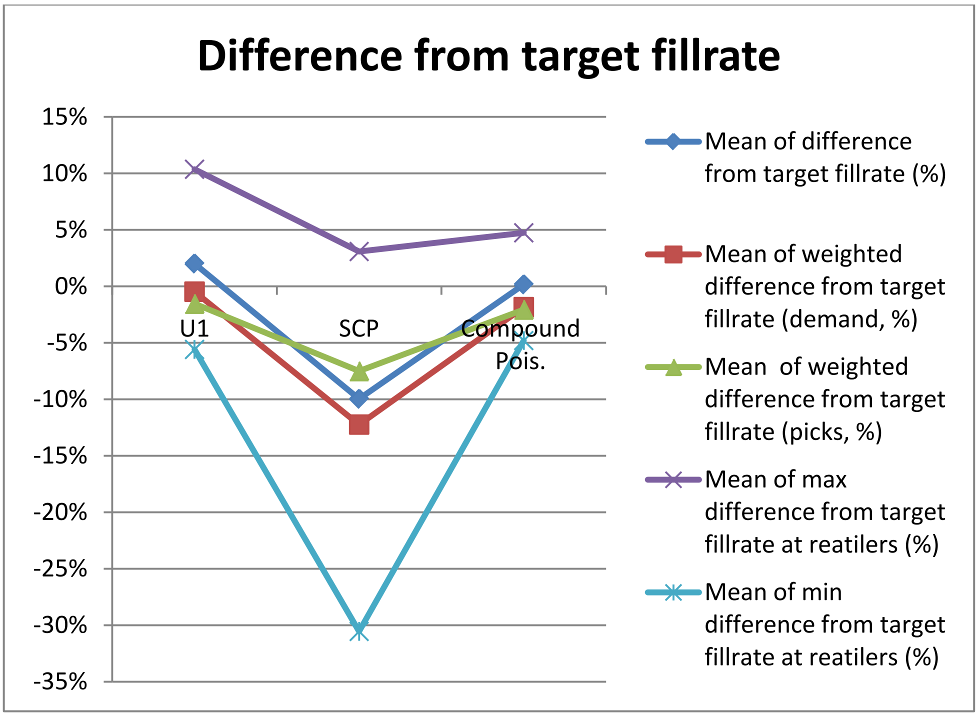


Figure 19 - Difference from target fillrate, compound Poisson settings as customer demand, 100 < demand < 1000

The group of articles with a high demand per year, from 1000 to 5000 units, can be substantially similar to the group of fast articles. The majority of all fast articles are included here as the results also suggest when Figure 20 below is compared with Figure 14, which describes the differences in fillrate for the fast articles. None of the methods meet the target fillrate as shown in Figure 20. The similarities between Approach 1 and 2 are high for these articles as well and the difference is once again in the mean of the maximum positive difference. Inventory reduction in Approach 1 is here more than 42% while it is almost 39% for Approach 2.

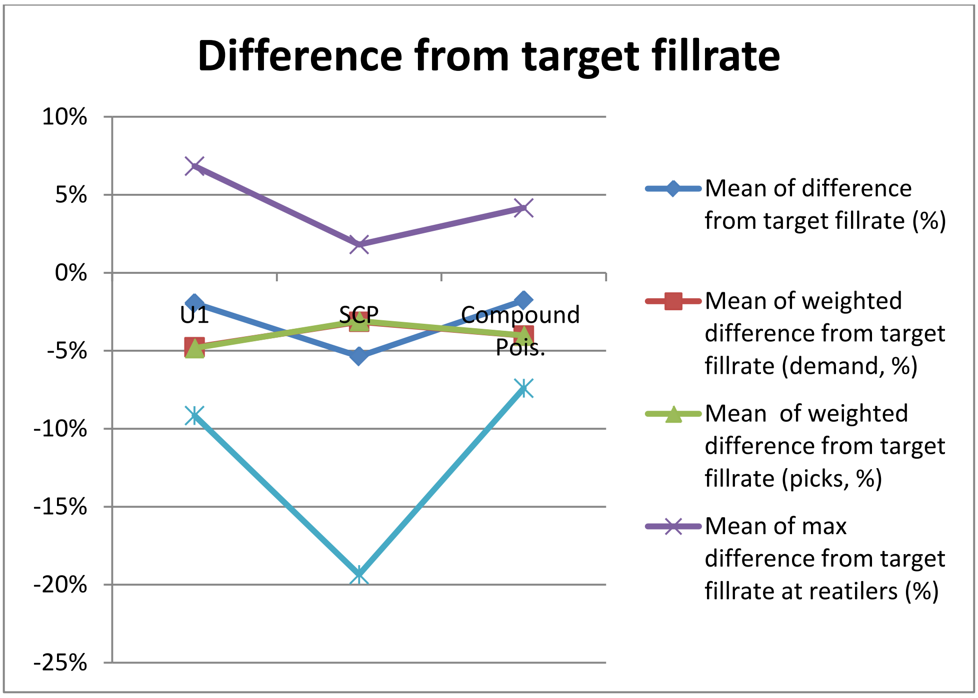


Figure 20 - Difference from target fillrate, compound Poisson settings as customer demand, 1000 < demand < 5000

According to the results from these groups depending in the demand size, it can be seen that the greater the demand is, the more difficult the methods have to meet the target fillrate. The weighted values for the difference from the target fillrate are for the analytical model consistently under the not weighted values. This indicates that the articles/retailers with high customer demand are more difficult to control, because they in this case have a higher impact on the result. One possible reason for this may be the lead-time estimate for the central warehouse. The analytical model use an average value for this lead-time, which in reality may vary from either the transportation time from central warehouse or to the lead time from the supplier plus transportation time from the central warehouse. If the lead time will be longer than expected for an article with high demand, it is a high probability that demand during this extended lead-time becomes larger than stock-on-hand, resulting in a reduced fillrate.

7.3.3 Negative Binominal setting and problem articles

As described in section 5.2, some articles are calculated with a negative Binomial distribution as customer demand, which will be called Approach 3 from now on. Table 8 below shows the results for these articles. A more detailed table can be seen in Appendix 14.

Key figures / Model	Approach 1	SCP	Approach 3
Mean of difference from target fillrate (%)	-3,9%	-4,1%	-3,2%
Mean of weighted difference from target fillrate (demand, %)	-5,0%	-3,5%	-4,3%
Mean of weighted difference from target fillrate (picks, %)	-5,0%	-3,1%	-4,3%
Max positive difference from target fillrate at retailers (%)	18,9%	6,0%	14,9%
Mean of max positive difference from target fillrate at retailers (%)	2,7%	0,8%	2,8%
Max negative difference from target fillrate at retailers (%)	-25,3%	-38,3%	-26,5%
Mean of max negative difference from target fillrate at retailers (%)	-9,6%	-14,3%	-9,8%
Total reduction in inventory by using the new model relative to SCP (%)	39,7%	-	34,7%

Table 8 - Key figures, Negative Binominal settings as customer demand

Here it can be seen that Approach 3 is barely a percentage point closer to meet target fillrate than Approach 1, for the mean differences from target fillrate. Otherwise, perform of the two approaches are very similar when it comes to spread of the fillrate. The SCP results of these articles are in line with the analytical model for the mean difference from the target fillrate while its spread is offset downward, which is clearly visible in Figure 21 below. Inventory reduction for Approach 1 is nearly 40%, which is five percent more than for Approach 3.

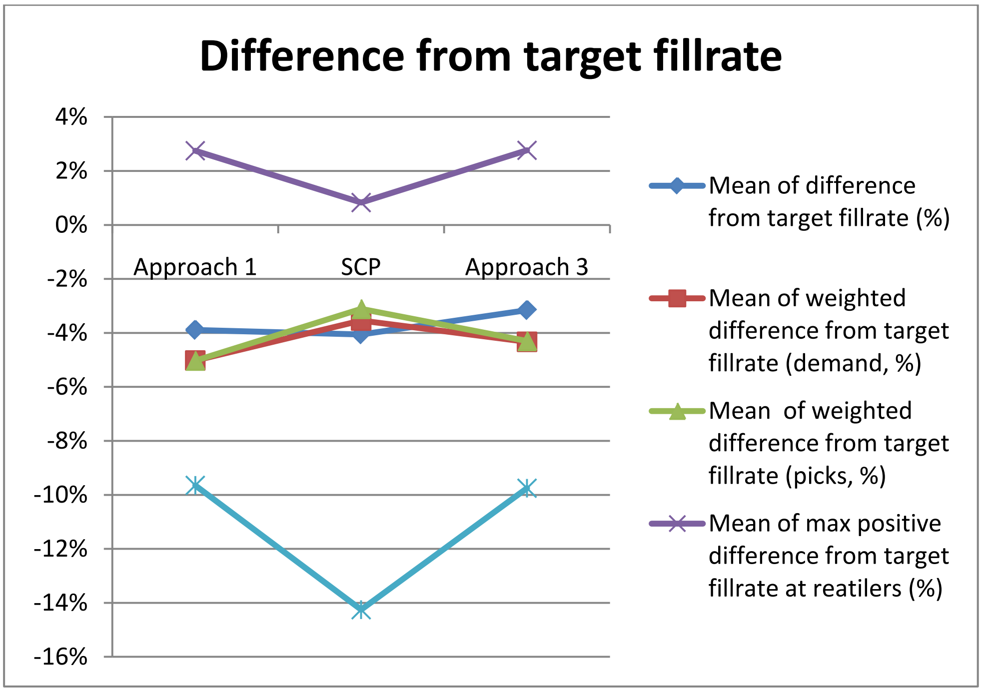


Figure 21 - Difference from target fillrate, Negative Binominal settings as customer demand

The problem articles that could not be calculated using other than a normal distribution approximation for customer demand is the result of the simulations shown in Table 9 below (more detailed in Appendix 14). Here, none of the models meet the target fillrate. Approach 1 port, on average, slightly closer to the target fillrate in general over all the articles, while the SCP performs better when the results are weighted against the demand and the number of picks. Approach 1 and SCP has about the same maximum differences and the inventory reduction with Approach 1 is barely 37%. The values of these problems articles of Approach 1 is approximately 5 percent under the differences from the target fillrate obtained when all of the articles in this study was simulated with Approach 1. Inventory reduction, however, is close to the performance of all articles (35%).

Key figures / Model	Approach 1	SCP
Mean of difference from target fillrate (%)	-5,0%	-5,3%
Mean of weighted difference from target fillrate (demand, %)	-6,2%	-3,2%
Mean of weighted difference from target fillrate (picks, %)	-6,3%	-2,6%
Max positive difference from target fillrate at retailers (%)	16,4%	6,1%
Mean of max positive difference from target fillrate at retailers (%)	1,4%	1,0%
Max negative difference from target fillrate at retailers (%)	-34,8%	-35,5%
Mean of max negative difference from target fillrate at retailers (%)	-9,6%	-7,2%
Total reduction in inventory by using the new model relative to SCP (%)	36,6%	-

Table 9 - Key figures, Problem articles

All articles in these two groups were classified as fast by the SCP. The reason that these articles could not be calculated with Approach 2, and that those differences from the target fillrate are below the average for Group 1, is to a large extent the distribution of the demand sizes. That the size of the number of requested products which vary from one unit up to a few hundred and sometimes up to 1000 units may be seen as unusual.

7.4 Inventory allocation

When using Approach 1 and 2 compared to the SCP, the total inventory in the system has decreased in almost every group of articles, which has been described earlier. But it is not only the total inventory that has been decreased with the two different approaches compared to the SCP. The inventory has also been reallocated from the central warehouse to the different retailers. Inventory reduction for Group 1 can be seen in Figure 22 below:

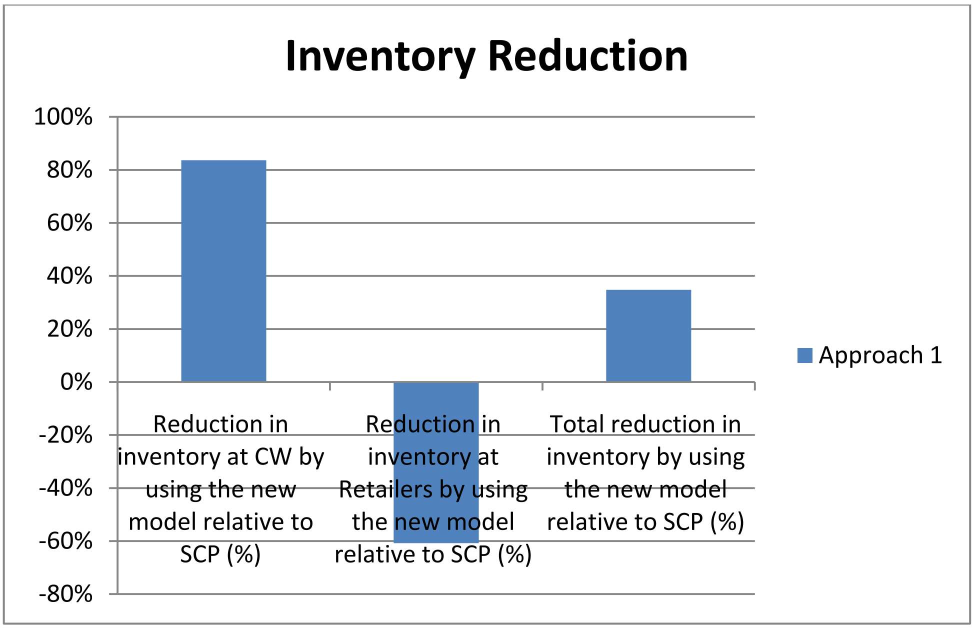


Figure 22 – Inventory reduction for Group 1

It is easy to believe that the difference between the reduction at the CW and the increase at the retailers is the total reduction, but that is not the case, it is just a percentage increase or decrease. The inventory level at the central warehouse has been reduced on average by about 84%, while the inventory level has increased at the retailers by about 61%. The total inventory has been reduced by approximately 35%, which means that a reallocation of the total inventory from the central warehouse to the retailers allows a reduction of the total inventory, while the service level to end customer is maintained. It is not only the allocation of inventory that the two approaches changed compared with the SCP, but also reorder points which in turn leads to changed service levels at the central warehouse and the retailers. In order to show this, one example article has been sorted out which represents the general results good and has a small difference from target fillrate, both with the SCP and the two approaches. The article will henceforth be known as the example article and its key figures can be seen in Table 10 and the visualization of its inventory reduction can be seen in Figure 23 below.

Key figures / Model	Approach 1	SCP	Approach 2
Weighted difference from target fillrate (demand, %)	0,1%	1,3%	0,8%
Weighted difference from target fillrate (picks, %)	0,3%	1,6%	0,9%
Reduction in inventory at CW by using the new model relative to SCP (%)	89,5%	-	89,5%
Reduction in inventory at Retailers by using the new model relative to SCP (%)	-12,1%	-	-12,1%
Total reduction in inventory by using the new model relative to SCP (%)	32,7%	-	32,7%

Table 10 – Key figures, example article

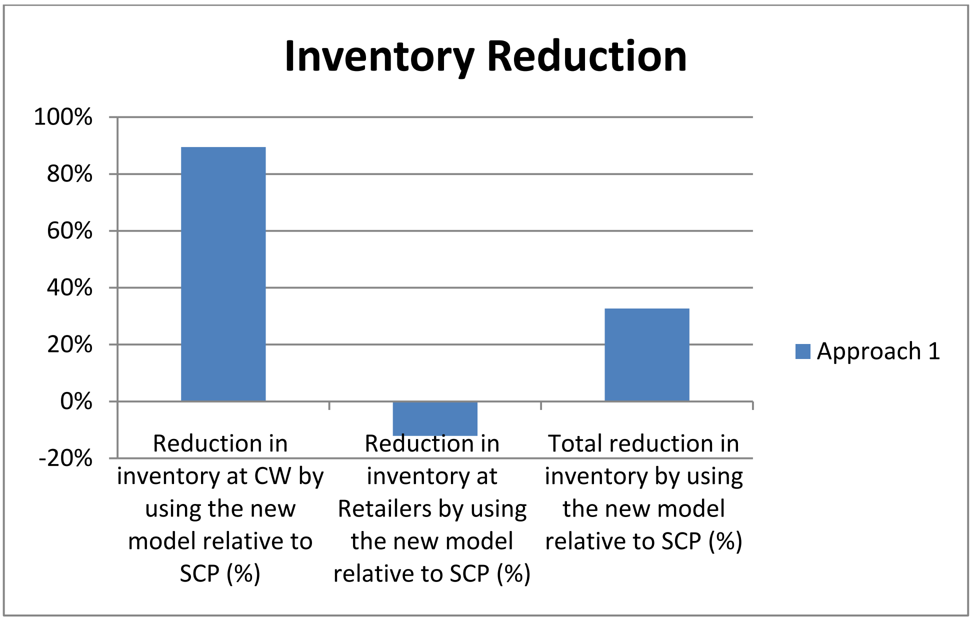


Figure 23 – Inventory reduction, example article

In the example article the reduction of the total inventory and reduction of the inventory at the central warehouse is about the same as the overall average for Group 1. But the increase of inventory at the retailers is not equal to the overall average in Group 1. This is because the group lumpy articles raise the average reduction at the retailers, which is shown when Figure 26 is compared to Figure 22. How the reorder points are affected at the example article can be seen in Table 11 below:

Allocation / model	Approch 1	SCP	Approch 2
Warehouse	-2	10	-2
Retailer 1	5	2	6
Retailer 2	1	1	1
Retailer 3	3	2	3
Retailer 4	1	1	0
Retailer 5	1	1	1

Table 11 – reorder points, example article

The reorder points are reduced at the central warehouse and increased at the retailers when the two approaches are compared with the SCP. This is explaining the inventory allocation seen above. Service level is also affected, which can be seen in Table 12 below:

Allocation / model	Approch 1	SCP	Approch 2	Target fillrate
Warehouse	33,4%	98,9%	33,4%	-
Retailer 1	96,3%	96,8%	98,1%	98,5%
Retailer 2	96,0%	99,1%	96,0%	90,0%
Retailer 3	98,2%	99,4%	98,2%	99,0%
Retailer 4	98,8%	100,0%	89,8%	90,0%
Retailer 5	95,9%	99,3%	95,9%	90,0%

Table 12 –Service level, example article

The two approaches, compared with the SCP, reduce the service level noticeably at the central warehouse, while the most important service level, the service level at the retailers which is experienced by the customers is maintained.

Within the three different classifications of articles from the SCP in Group 2, interesting properties can be seen on the inventory reduction, which I shown in the figures below.

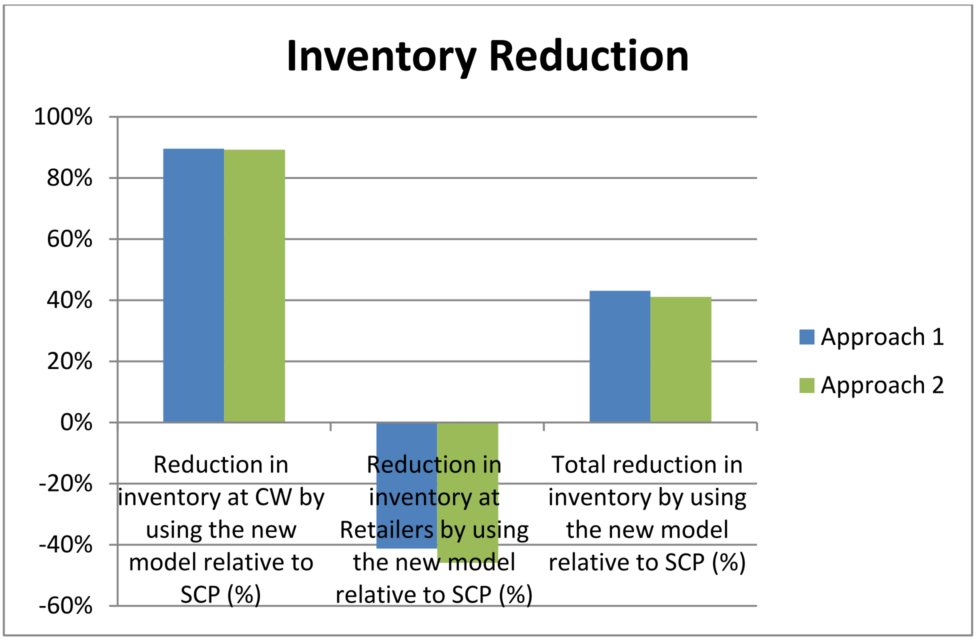


Figure 24 – Inventory reduction, fast articles within Group 2

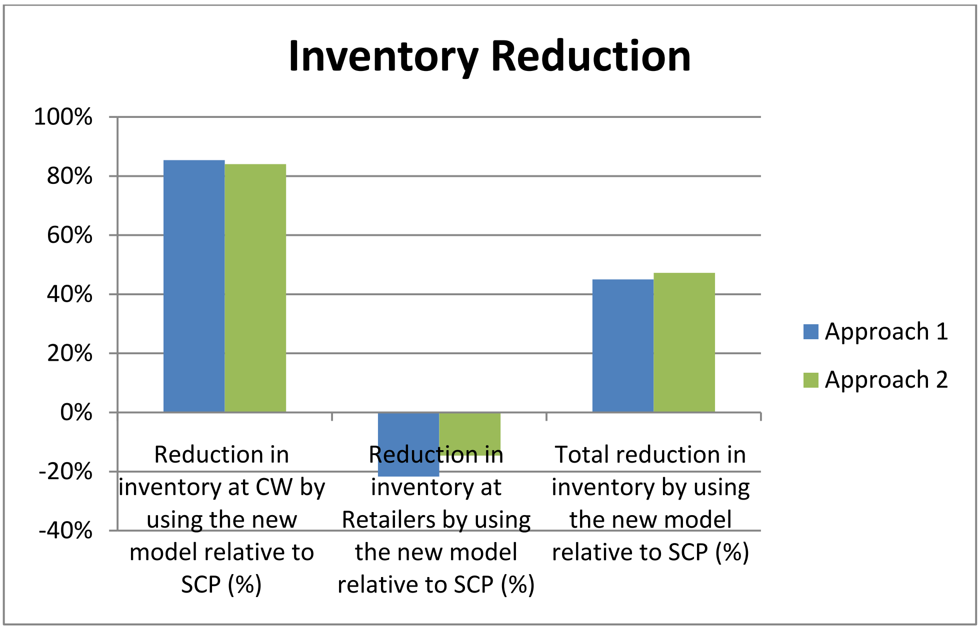


Figure 25 - Inventory reduction, slow articles within Group 2

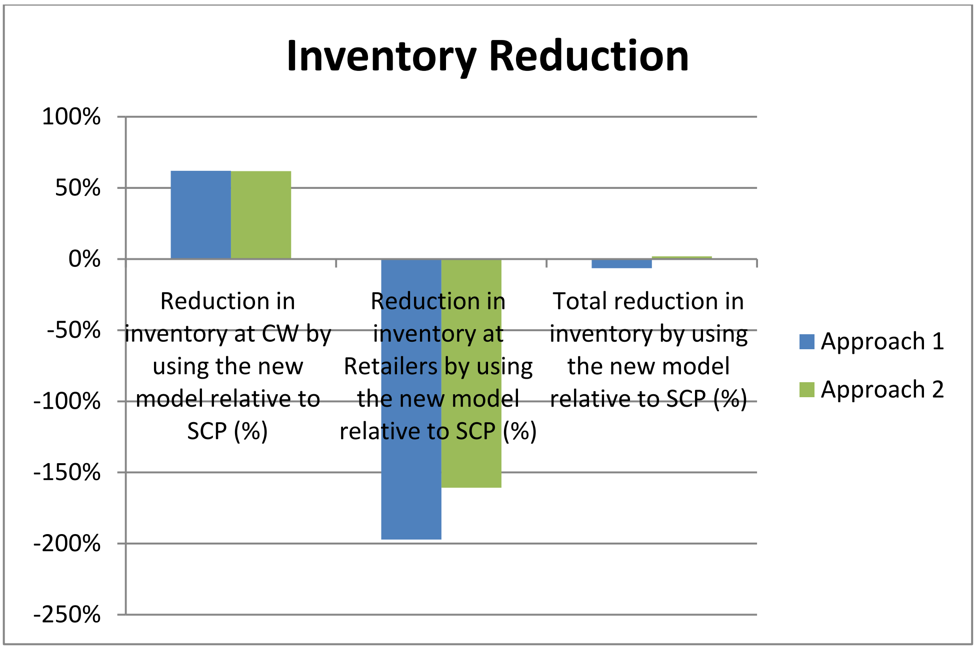


Figure 26 - Inventory reduction, lumpy articles within Group 2

All three key figures for inventory reduction for the fast articles are similar to the overall average in Group 1, see Figure 22. Slow articles are about the same as in Group 1 in two of the three key figures. What differs is the less increase in average inventory at the retailers, which results in a higher reduction of the total inventory. Lumpy articles differ to all three key figures relative to Group 1. Lumpy articles reduce the total inventory at the central warehouse less than Group 1, but the big difference is that they increase the inventory at the retailers by approximately three times compared with Group 1. The total inventory reduction is also different where it is approximately 0% of lumpy articles. Linked to earlier results and that the total inventory reduction is 0% indicates that the SCP probably sets the safety stock too low at retailers with lumpy demand.

8 Conclusions and discussion

This chapter answers the purpose of this master thesis by summarizing the results of the simulations which leads to a conclusion. A discussion around some different aspects that might affect the results more or less is made. Among these are the models differences from reality, how costs are set and thoughts around a future implementation in the SCP software. Suggestions to future studies on the coordinated inventory control model are given in the end.

The purpose of this master thesis has been to evaluate the performance of a model for coordinated inventory control of a multi-echelon inventory system developed by the division of Production Management at Lund University in a real-life setting. This has been achieved by comparing the results in the simulation software Extend, for the coordinated inventory control method with the current uncoordinated inventory control method in the SCP software which is developed by Syncron.

From the simulation results, a conclusion can be drawn that the articles classified as slow and lumpy by the SCP should be controlled by the analytical model's Approach 2 (compound-Poisson distributed setting as customer demand). If this is done, the target fillrate is achieved while the inventory is reduced by around 47% for the slow articles and approximately 2% for the lumpy articles. 2% for the lumpy articles does not seem much, but for these articles Approach 2 meets target fillrate while the SCP is 11 - 19% below target fillrate.

The results also shows that 26 out of the 135 articles that could not be calculated quick enough with Approach 2 were all classified as fast by the SCP. Articles classified as fast by the SCP, should according to these results be controlled with Approach 1 (normal distributed setting as customer demand), which gives an inventory reduction on almost 40% compared to the SCP. The reason for this is mainly that Approach 1 is less computationally complex in order to generate reorder points than Approach 2 and Approach 1 can therefore be used for all articles classified as fast. The distinction between the approaches used for the different demand types minimize the risk of a reorder point calculation taking too long or crash when all the fast articles are calculated with Approach 1, even though Approach 2 sometimes gives a slightly better results for the fast articles that can be calculated with Approach 2.

The analytical model not only reduces the total inventory in the system, it also reallocates inventory from the central warehouse to the retailers. This affects service levels at the central warehouse, which is reduced significantly compared to the SCP, which may seem wrong. However, it must be observed that it is only the service level at retailers that is important for the customer. A shortage in the central warehouse is therefore not disastrous as long as the service level is maintained at the retailer level.

The major advantage of the coordinated method compared to the uncoordinated method used by Syncron is its great savings potential by reducing the inventory at the same time as the service level most of the time is improved or at least maintained. The investment needed to implement the coordinated method in the SCP is relative small for Syncron. However the investment for the potential customers is unknown, which can be very big.

8.1 Discussion

Though a lot of time has been spent on making a good selection of articles that represent all the 39,000 articles from the start well, the result in this master thesis for these articles would have been more accurate if all the articles from the start were simulated. This is however unreasonable since it had taken far too long time and some articles are not even possible to simulate. But this is something that should be kept in mind when the results are reviewed.

8.1.1 Holding costs and shortage costs

No value of h (holding cost) and p (shortage cost) has been obtained from Syncron. These values are needed in both the analytical model and the simulation model. To be able to do all the calculations and simulations, the parameter h has been set to one and after that p is obtained by the equation (3.18). By setting h to one the cost of keeping inventory would be the same in all inventories located all over the world and the value of all articles would be equal. For an example an engine and a nail has equal value when h is set to one, they are both seen as an article among others. This is not the case in reality. Holding costs will vary depending on where the article is held in stock and a nail is not worth as much and never will be as an engine. By estimating an exact value of h and p, more accurate results will be obtained. This is an area for further investigation. But is there not any big difference in the relative savings between the products, then this is not a problem.

8.1.2 The reality differ

All the results and analysis have come from and been developed on values from the simulation program Extend. The results from simulation models are always a bit different from the reality, it is impossible to reflect the reality exact and it is important to keep this in mind when the results are reviewed. This section will discuss two parameters that not reflect the reality very well.

Demand

The demand has been estimated from sales data that Syncron received from one of its customers. This may be a bit misleading, as customers who had intended to buy an article, while the article was out of stock, have not been able to buy the article and has not been registered as a customer. This probably makes the demand in the reality a bit higher than the estimated demand from sales data.

Lead time

All the lead times used in this project are assumed to remain constant and the supplier of the central warehouse assumes to have a fillrate of 100%. This is not the case in reality. Lead times can vary for an example because of delayed shipments. That the supplier of the central warehouse always has products available is also unlikely, just as with the retailers it is impossible to maintain a service level of 100%. Better estimates can be made on the lead times. A new analysis with the selected articles with more exact lead times will generate more accurate results.

When backorders are used at the retailers, the customers that have not been served would have to wait very much longer on its orders with the new inventory structure compared to if the service level had been higher in the central warehouse. I.e. the customer may have to wait $l_i + L_0$ and sometimes even longer if R at the central warehouse is < -1 instead of waiting at most l_i when the service level is high at the central warehouse. Cost of this is difficult to measure.

8.1.3 Implementation

When recommendations are given and conclusions drawn the implementation- and calculation times are not taken into account. This is because the authors have no relevant knowledge within this area. These aspects should probably not be a problem as there is not much that needs to be changed in the SCP in order to implement the analytical model. All calculations are made on the same data that already exists in the SCP today. It is almost just the calculations of reorder points that need to be reprogrammed.

The implementation of the coordinated approach does not fit all type of companies. It requires that the companies have control over the entire supply chain from the central warehouses to the retailers. If that control does not exist, it is very difficult to influence the various components of the supply chain to the change that is required. Other companies that the coordinated model is not suitable for are companies with many retailers (above 50) linked to a central warehouse. An implementation of such a company is difficult and costs a lot of money because of the many inventories, particularly in the beginning, when all the retailers must be connected to achieve the synergy created by the coordinated inventory control.

Different costs that may arise within the companies during an implementation have not been taken into account. These costs can for example consist of reconstruction of both the central warehouses and the retailers. The central warehouse will get over capacity, while the retailers will get a under capacity due to the new inventory structure.

8.2 Future research

The results of this study have not filled all the gaps and some issues still remains. Above all among these are the analytical model and its estimate of reorder points. Several cases have emerged where the simulated fillrate ended pretty far from the target fillrate. Of particular interest are results where the simulation does not reach the target fillrate. A future research area is therefore considered to be why this occurs. The pattern that has been observed in this study is that the problem increases the greater the total demand becomes and when R is below -1 at the central warehouse the difference from target fillrate is the highest.

9 Source reference

Andersson, J., & Marklund, J. (2000). Decentralized inventory control in a two-level distribution system. *European journal of operations research , 127*, 483-506.

Andersson, J., Axsäter, S., & Marklund, J. (1998). Decentralized multiechelon inventory control. *Production and operations managment , 7* (4), 370-386.

Axsäter, S. (2000). Exact analysis of continious review (R,Q)-policies in two-echelon inventory systems with compound Poisson demand. *Operations Research , 48* (5), 686-696.

Axsäter, S. (2006). *Inventory control.* Lund University, Lund: Springer Science+Business Media, LLC.

Axsäter, S. (1991). *Lagerstyrning.* Lund: Studentlitteratur.

Berling, P. (2009). Reorder point adjustment. Lund University, Lund.

Berling, P., & Marklund, J. (2006). Heuristic coordination of decentralized inventory systems using induced backorder costs. *Production and operations managment , 15* (2), 294-310.

Berling, P., & Marklund, J. (2009). Multi.echelon inventory control at Syncron., (p. 6). Lund University, Lund.

Björklund, M., & Paulsson, U. (2003). *Seminarieboken - att skriva, presentera och opponera.* Lund: Studentlitteratur.

Hillier, F., & Lieberman, G. (2005). *Introduction to operations research.* New York, USA: McGraw-Hill Companies Inc.

Höst, M., Regnell, B., & Runeson, P. (2006). *Att genomföra examensarbete.* Lund: Studentlitteratur.

Law, A. M., & Kelton, D. W. (2000). *Simulation Modeling and Analysis.* Georgia, USA: The McGraw.Hill Companies, Inc.

Manage your global supply chain easily - Syncron-. (2009, 12 16). Retrieved from http://www.syncron.se/

Ross, S. M. (1985). *Introduction to Probability Models.* Orlando, USA: Academic Press inc.

Sargent, R. G. (2004). Validitation and verification of simulation models. *Simulation Conference, 2004. Proceedings of the 2004 Winter* .

Sherbrooke, C. (1968). METRIC: A multi-echelon technique for recoverable item control. *Operations Research* , 122-141.

Wallén, G. (1996). *Vetenskapsteori och forskningsmetodik.* Lund: Studentlitteratur.

Wikipedia, den fria encyklopedin. (2009, 12 17). Retrieved from http://sv.wikipedia.org/wiki/Portal:Huvudsida

Vännman, K. (2002). *Matematisk statestik.* Lund: Studentlitteratur.

Zheng, Y. (1992). On properties of stochastic inventory sytems. *Managment science , 38*, 87-103.

Appendix 1 - Pivot table

Radetiketter	2															2 Totalt	3		
	2008				2008 Total	2009								2009 Totalt			2008		
	9	10	11	12		1	2	3	4	5	6	7	8				9	10	11
1030152-0000	1	1	0	0	2	0	0	0	0	0	0	0	0	0	2		0	0	0
1110083-0002	1009	1337	825	1246	4417	945	1288	1177	736	803	823	1009	1389	8170	12587		0	0	0
1110390-0000	66	44	416	0	526	400	0	20	420	20	50	20	420	1350	1876		0	0	0
1110391-0000	1110	162[illegible]	933	735	5400	904	1234	2015	1330	1813	1053	1774	1680	11803	17203		3390	3670	3926
1110936-0000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0
1139732-0000	0	0	0	[illegible]	0	0	1	0	0	0	0	0	0	1	1		0	0	0
1144683-0000	0	0	0	2	2	0	0	1	0	1	0	0	0	2	4		0	0	0
118582-0000	4	2	0	0	6	0	0	4	0	0	2	0	6	12	18		16	15	8
12242-0001	487	1636	1236	1713	[illegible]	516	1208	2059	251	859	512	1464	252	7121	12193		820	839	903
1255462-0000	0	6	0	0	6	0	0	[illegible]	[illegible]	0	0	1	0	6	12		0	0	0
1260548-0000	1	3	1	2	7	1	1	1	5	0	1	2	4	15	22		0	0	0
1273544-0000	2	0	0	0	2	0	1	0	0	0	0	0	0	1	3		0	0	0
1287426-0000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0
1294818-0001	0	[illegible]	[illegible]	0	0	0	0	0	0	1	0	0	0	1	1		0	0	0
1348529-0000	1	0	1	1	3	0	0	0	1	0	0	0	0	1	4		0	0	0
1348955-0000	42	32	47	23	144	34	42	41	13	11	13	12	22	188	332		0	0	0
1363297-0000	0	0	3	0	3	0	1	0	0	0	1	0	0	2	5		0	0	0
1490177-0000	0	1	0	2	3	0	0	0	0	1	0	1	0	2	5		0	0	0
1497953-0000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0
1499369-0000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0
1520564-0003	990	740	200	560	2490	769	900	1210	700	640	1166	0	1160	6545	9035		4285	1520	2420
1524018-0000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	1
1528411-0000	2	1	0	0	3	0	0	1	0	0	0	0	1	2	5		0	1	2
2033-0008	12	4	35	53	104	25	110	5	72	90	45	57	20	424	528		7	53	182
2033-0009	0	0	28	0	28	14	45	13	0	0	0	18	4	94	122		224	39	54
232979-0000	121	102	72	141	436	152	82	120	28	92	44	55	20	593	1029		83	143	96
23376-0000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0

Retailer 2

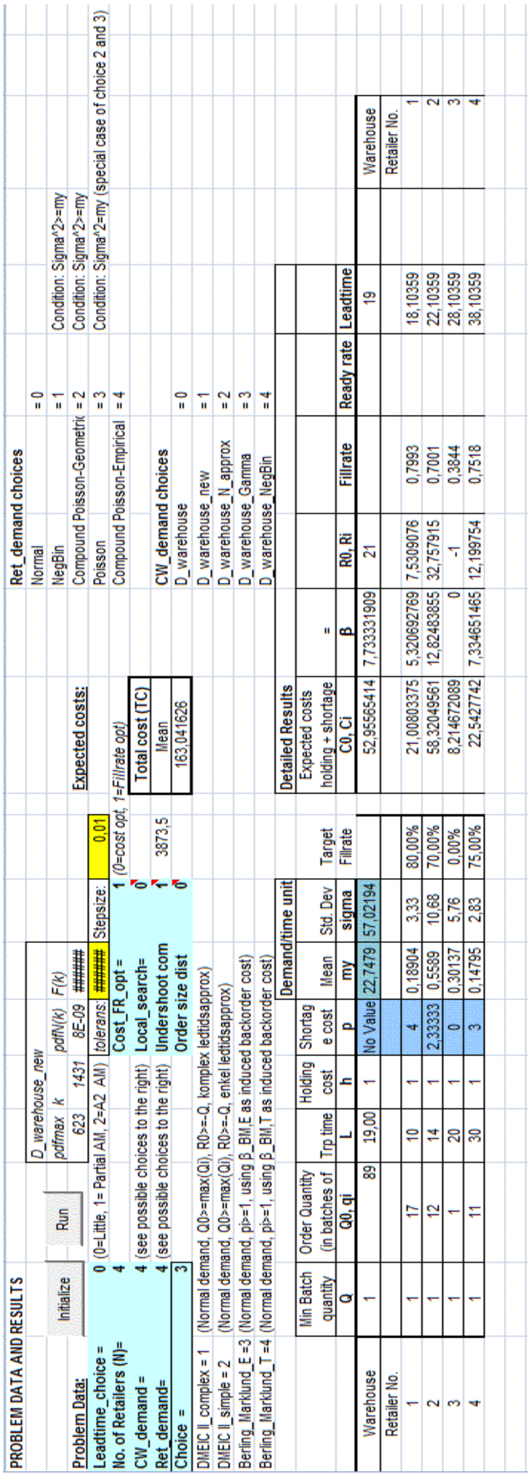
Year

Month

Order quantity

Article number

Appendix 2 - Interface of the Excel-model



PROBLEM DATA AND RESULTS

D_warehouse_new			
pdfmax	k	pdfN(k)	F(k)
623	1431	8E-09	######

Problem Data:

Leadtime_choice =	0	(0=Little, 1= Partial AM, 2=A2 AM)	tolerans:	######	Stepsize:	0,01	
No. of Retailers (N)=	4		Cost_FR_opt =			1	(0=cost opt, 1=Fillrate opt)
CW_demand =	4	(see possible choices to the right)	Local_search=			0	
Ret_demand=	4	(see possible choices to the right)	Undershoot com			1	3873,5
Choice =	3		Order size dist			0	

DMEIC II_complex = 1 (Normal demand, Q0>=max(Qi), R0>=-Q, komplex ledtidsapprox)
DMEIC II_simple = 2 (Normal demand, Q0>=max(Qi), R0>=-Q, enkel ledtidsapprox)
Berling_Marklund_E =3 (Normal demand, pi>=1, using β_BM,E as induced backorder cost)
Berling_Marklund_T =4 (Normal demand, pi>=1, using β_BM,T as induced backorder cost)

Expected costs:

Total cost (TC)
Mean
163,041626

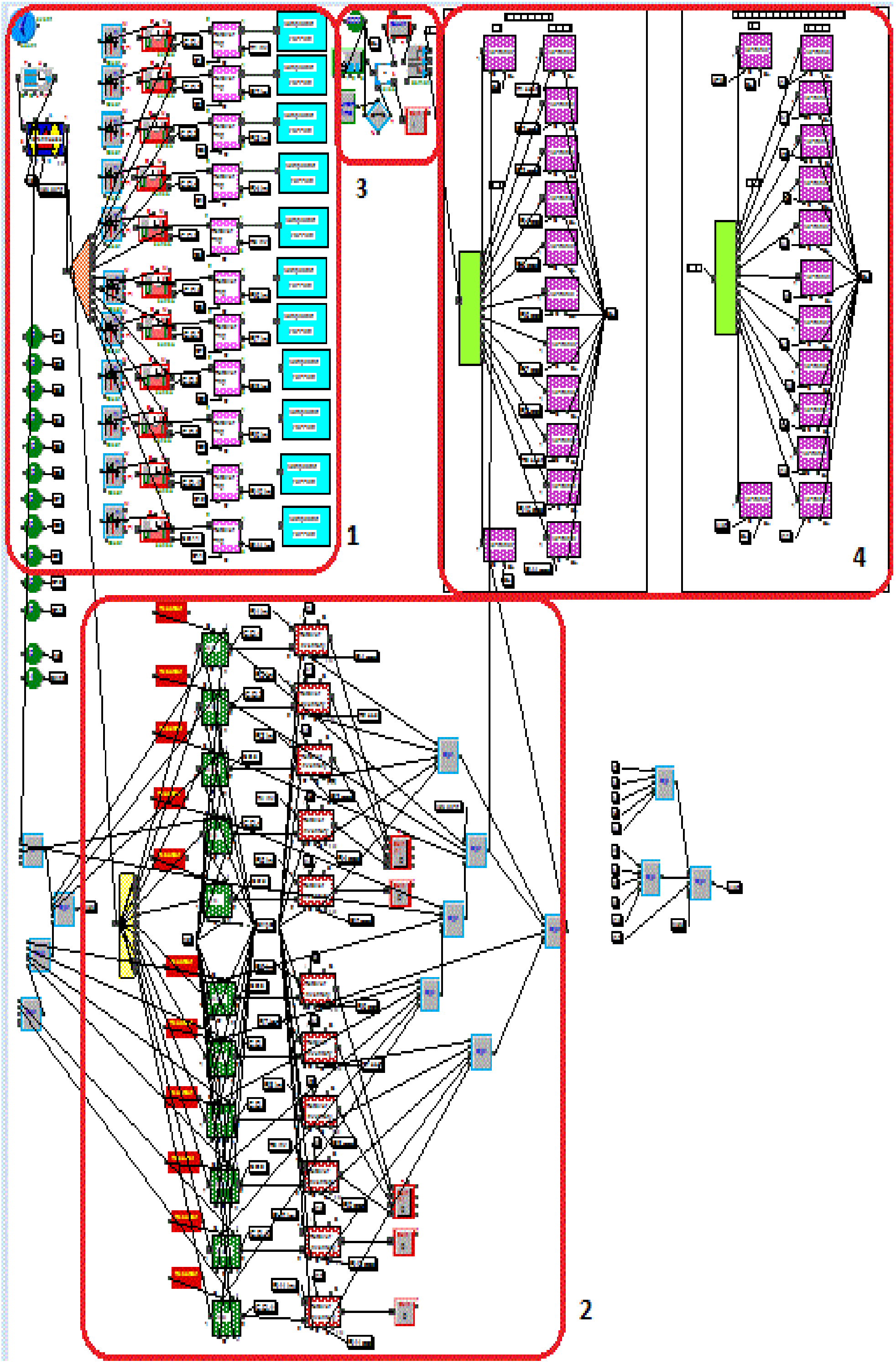
Ret_demand choices		
Normal	= 0	
NegBin	= 1	Condition: Sigma^2>=my
Compound Poisson-Geometric	= 2	Condition: Sigma^2>=my
Poisson	= 3	Condition: Sigma^2=my (special case of choice 2 and 3)
Compound Poisson-Empirical	= 4	

CW_demand choices	
D_warehouse	= 0
D_warehouse_new	= 1
D_warehouse_N_approx	= 2
D_warehouse_Gamma	= 3
D_warehouse_NegBin	= 4

	Min Batch quantity	Order Quantity (in batches of	Trp time	Holding cost	Shortage cost	Demand/time unit Mean	Std. Dev	Target Fillrate
	Q	Q0, qi	L	h	p	my	sigma	
Warehouse	1	89	19,00	1	No Value	22,7479	57,02194	
Retailer No.								
1	1	17	10	1	4	0,18904	3,33	80,00%
2	1	12	14	1	2,33333	0,5589	10,68	70,00%
3	1	1	20	1	0	0,30137	5,76	0,00%
4	1	11	30	1	3	0,14795	2,83	75,00%

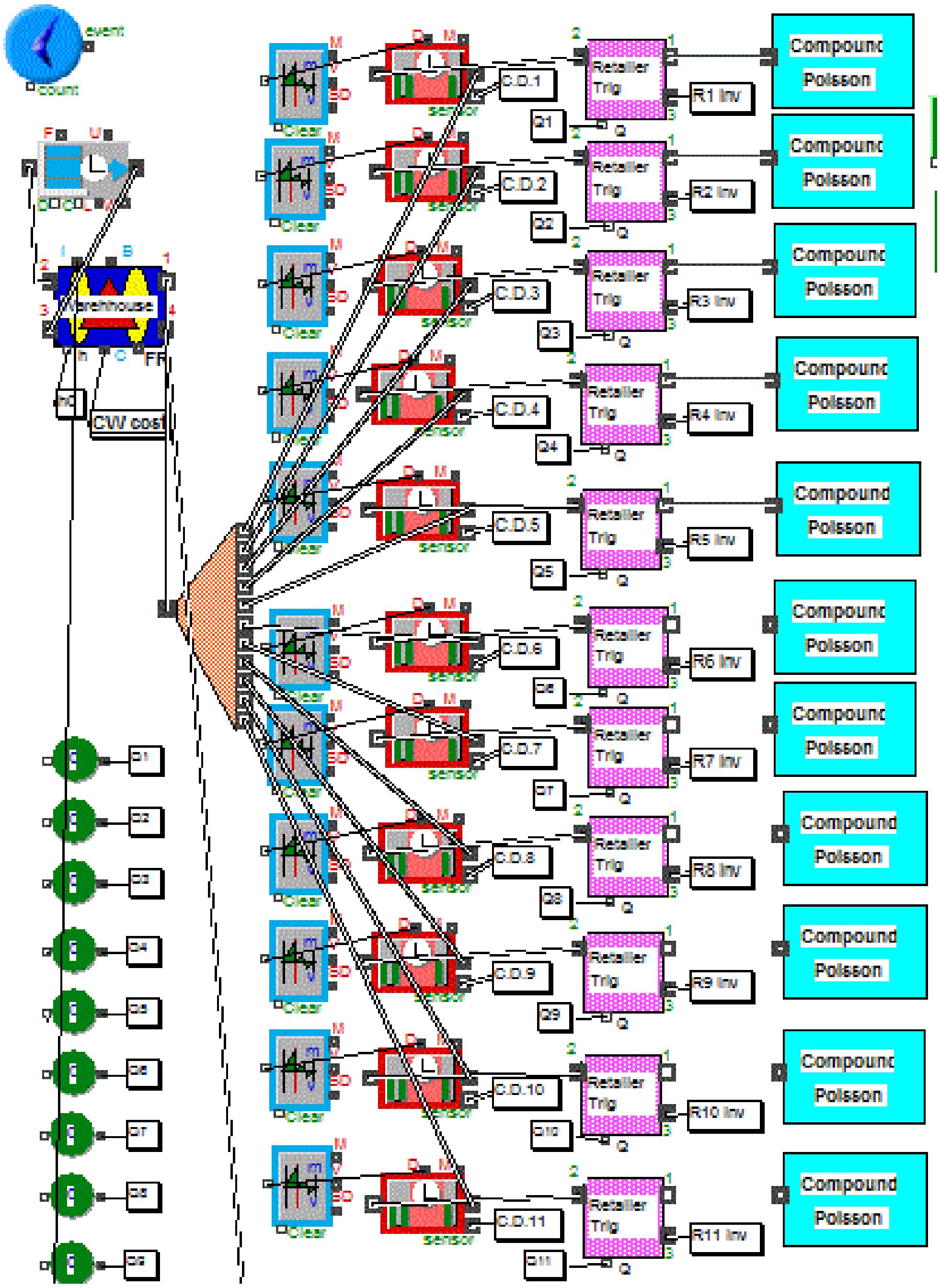
Detailed Results: Expected costs holding + shortage	=					
C0, Ci	β	R0, Ri	Fillrate	Ready rate	Leadtime	
52,95565414	7,733331909	21			19	Warehouse
						Retailer No.
21,00803375	5,320692769	7,5309076	0,7993		18,10359	1
58,32049561	12,82483855	32,757915	0,7001		22,10359	2
8,214672089	0	-1	0,3844		28,10359	3
22,5427742	7,334651465	12,199754	0,7518		38,10359	4

Appendix 3 - Extend model: overview

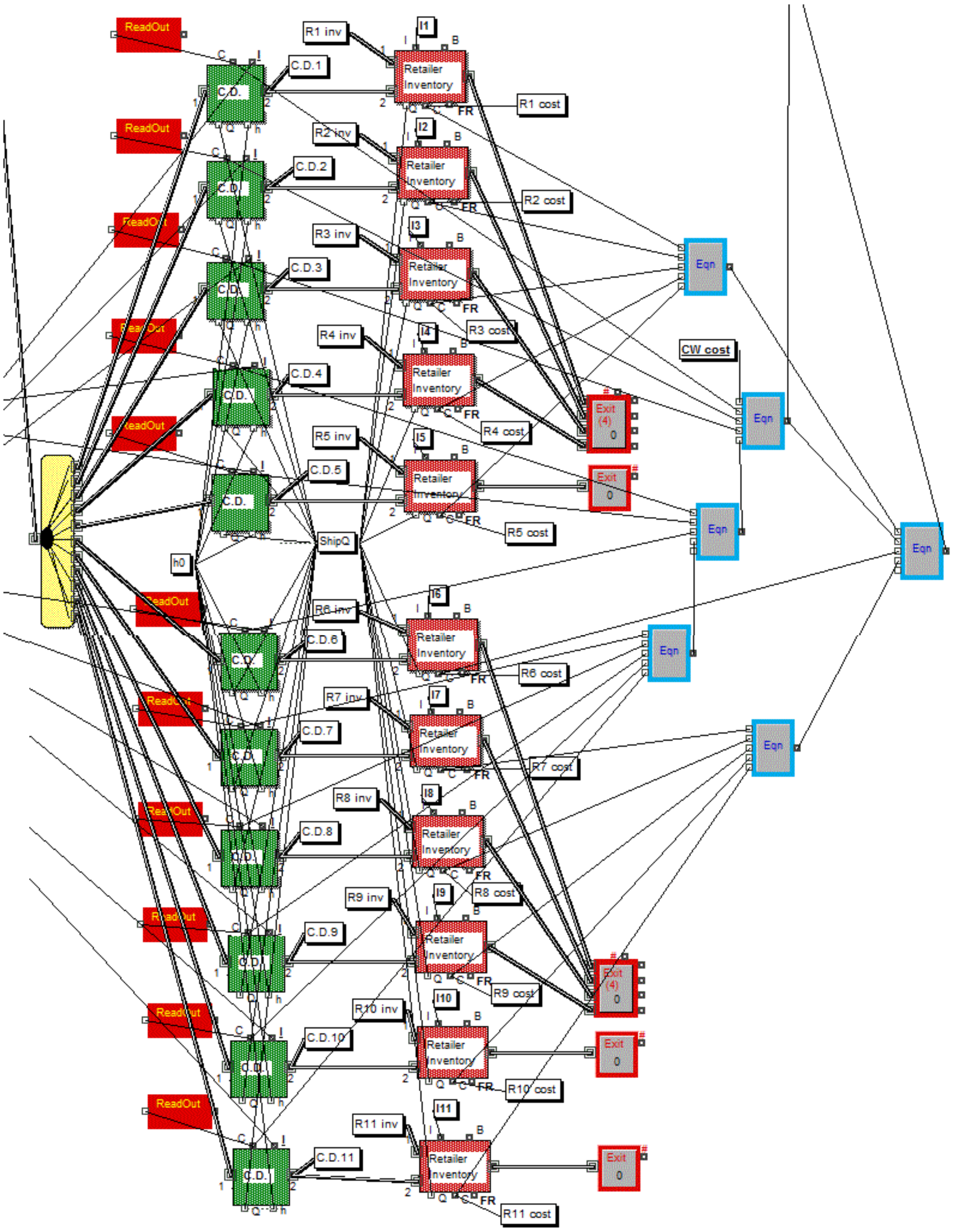


1. This part contains the customer demand generator block, the retailer order trigger block and the block for the central warehouse.
2. This part contains a block that splits the deliveries out to each retailer and the block which represents the retailer inventory.
3. This part divides the total simulation time into smaller sub-batches which are used to measure the results.
4. This part contains the cost calculation and the calculations for the expected inventory.

Appendix 4 - Extend model: part 1

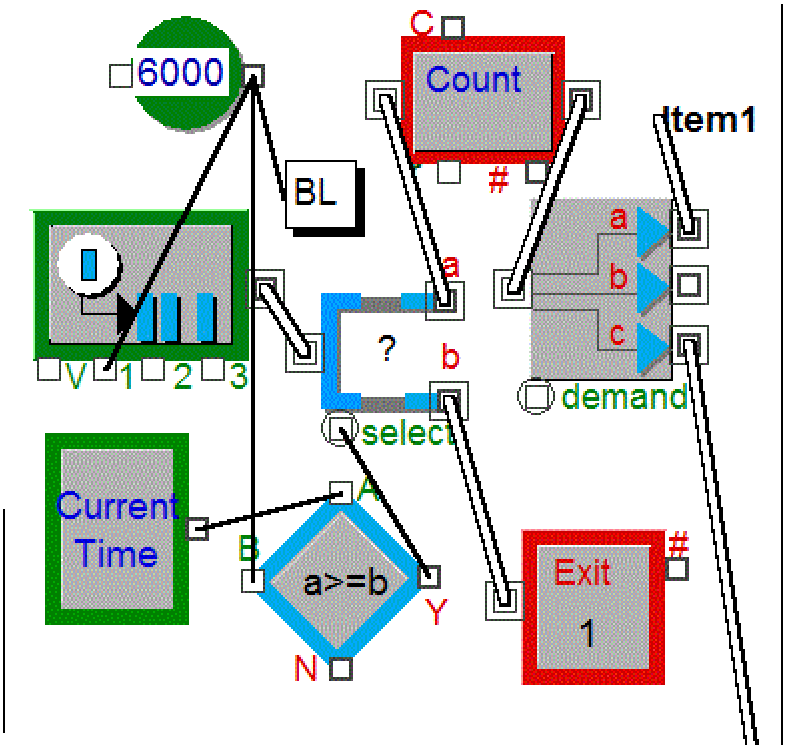


Appendix 5 - Extend model: part 2

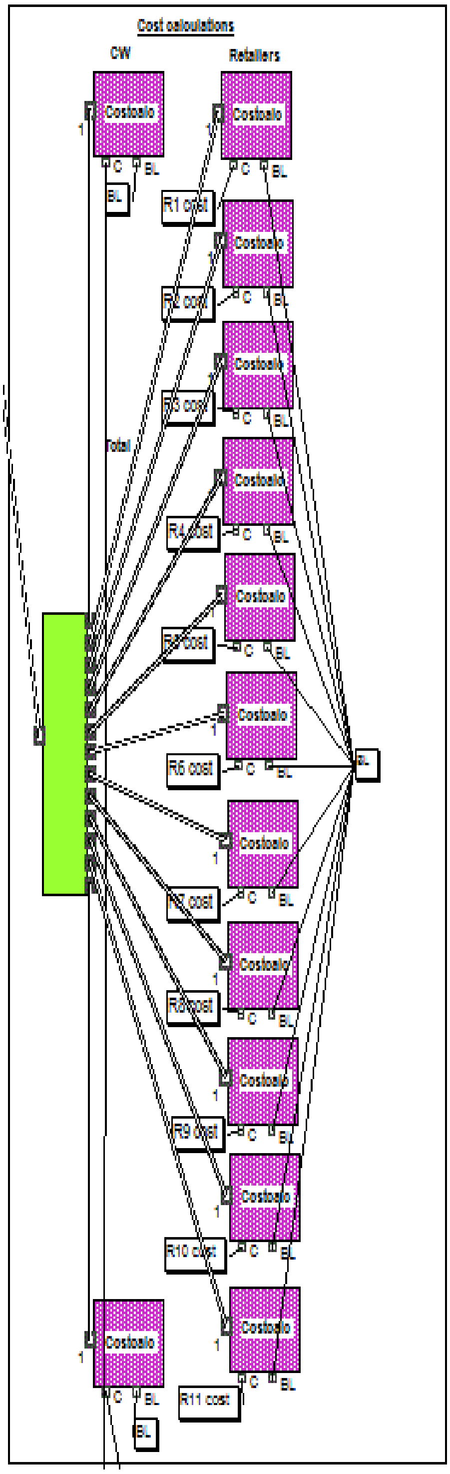


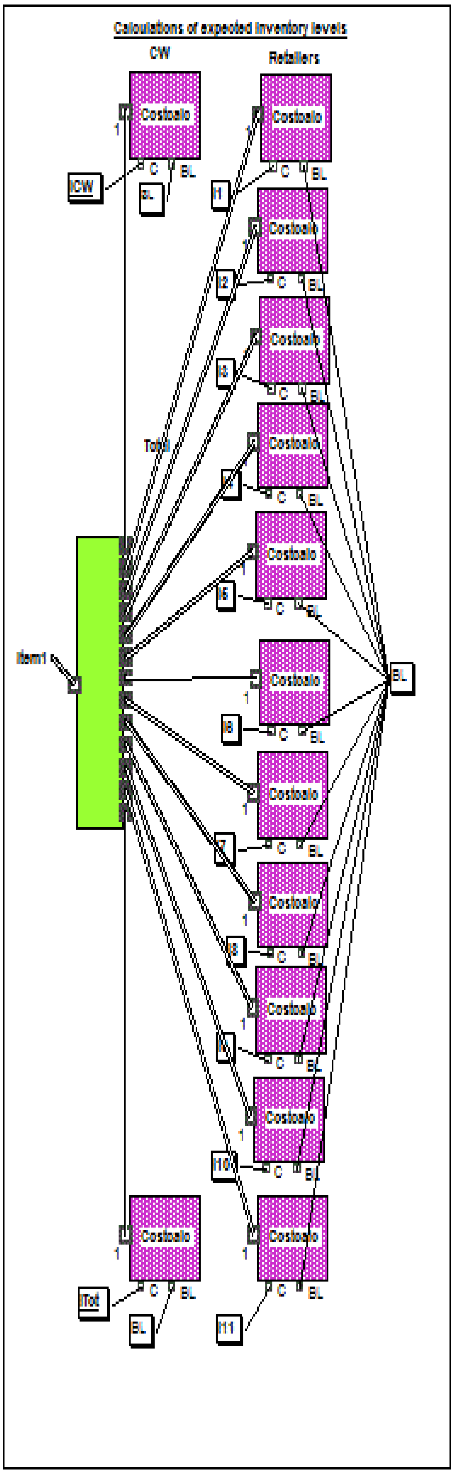
Notation: C.D blocks (accumulates the items to a certain retailer for complete deliveries) are not used when the simulations are carried out with partial deliveries.

Appendix 6 - Extend model: part 3

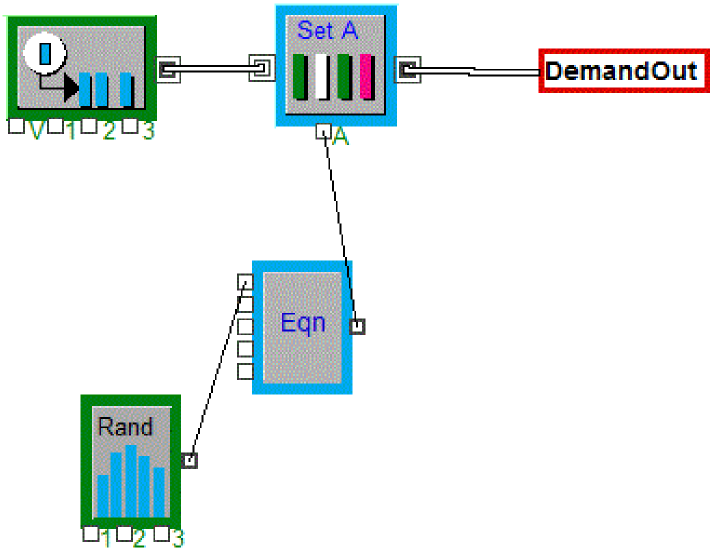


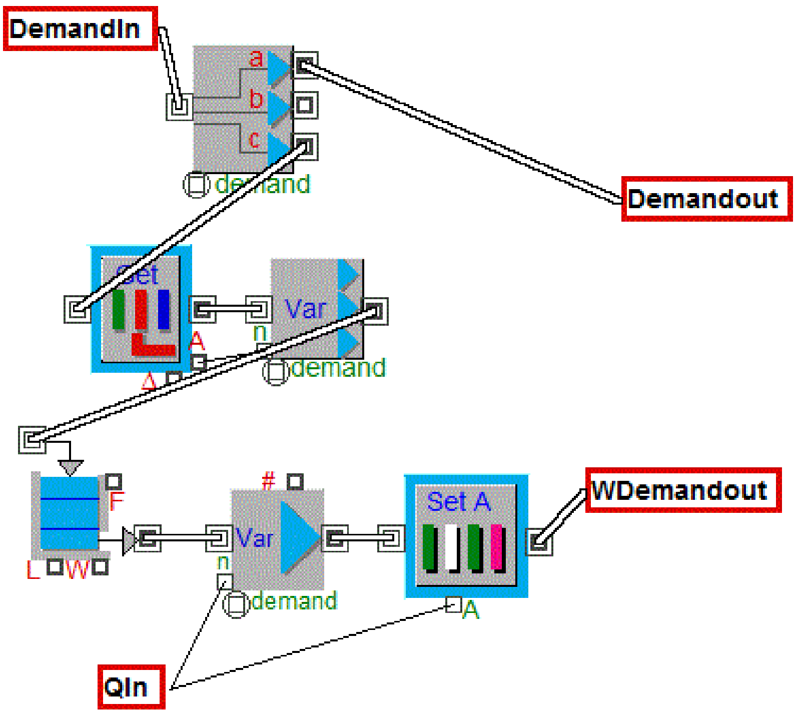
Appendix 7 - Extend model: part 4



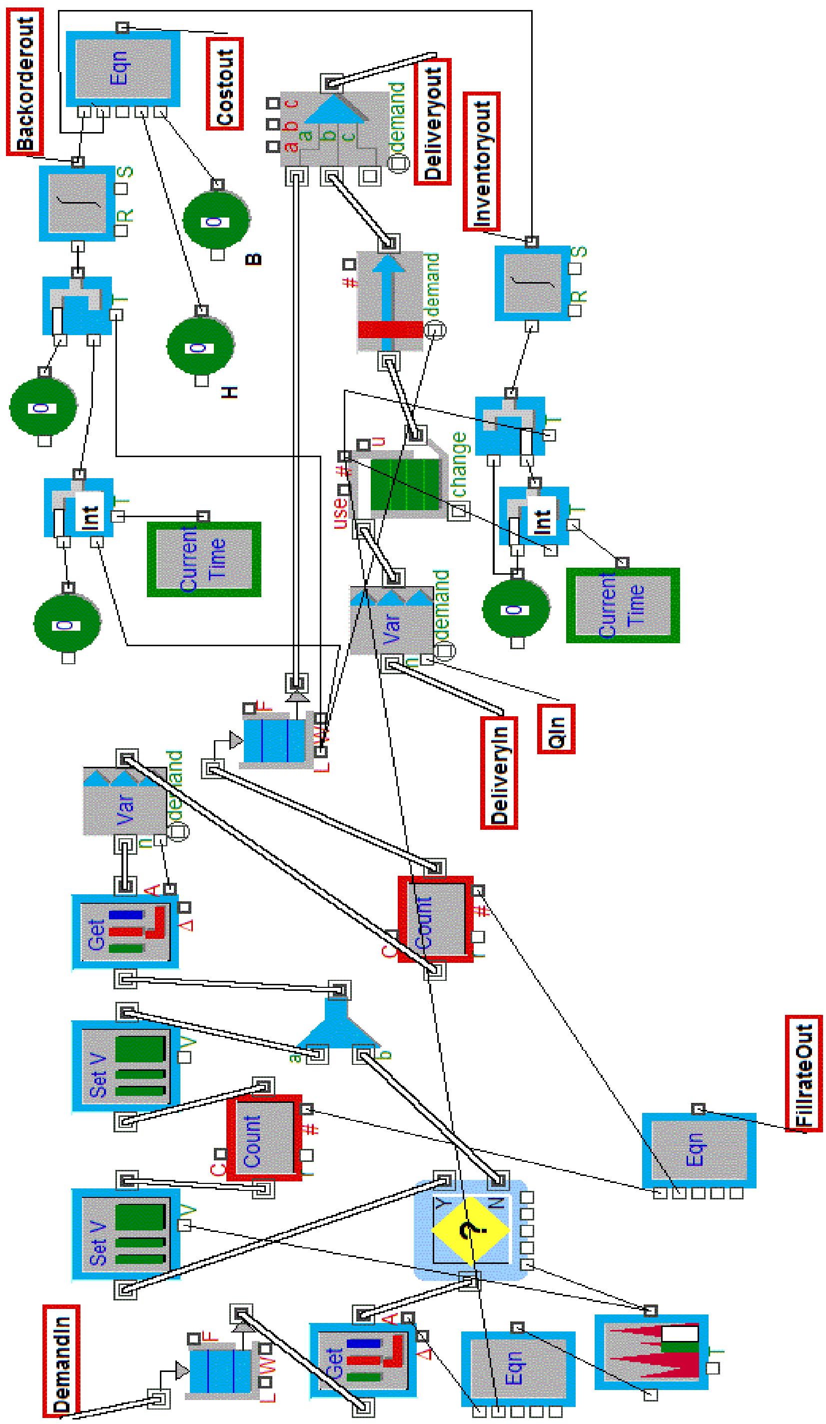


Appendix 8 - Extend model: customer demand generator block and retailer trigger block

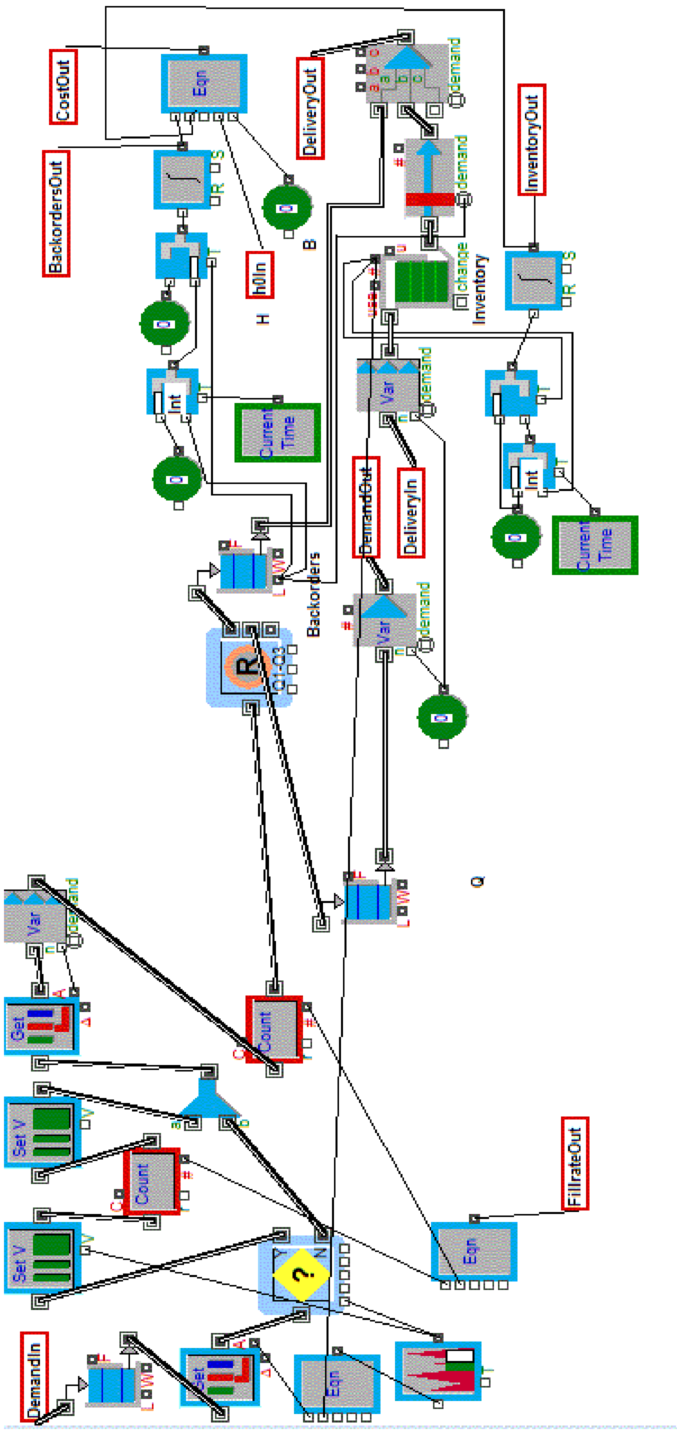




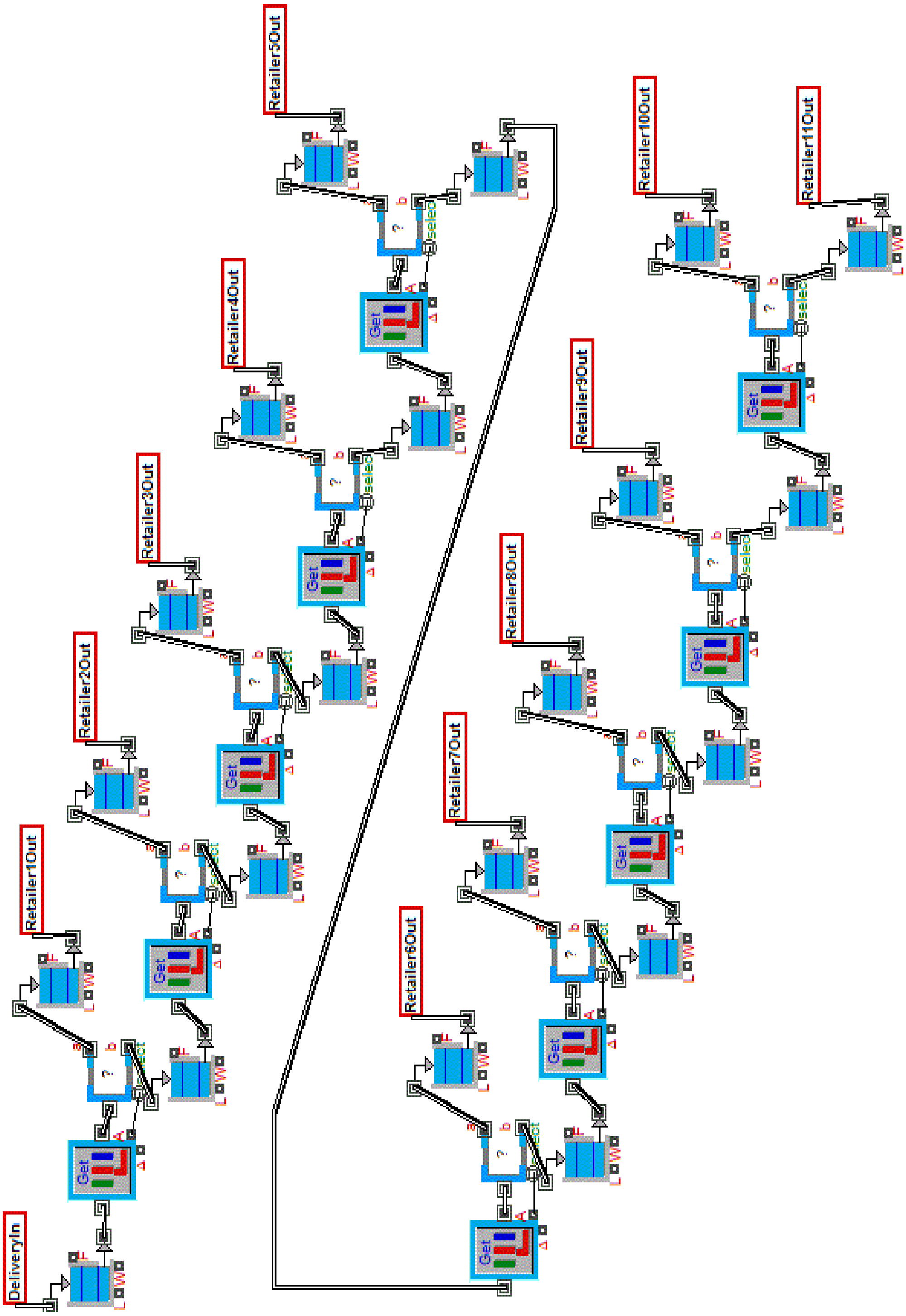
Appendix 9 - Extend model: retailer inventory block



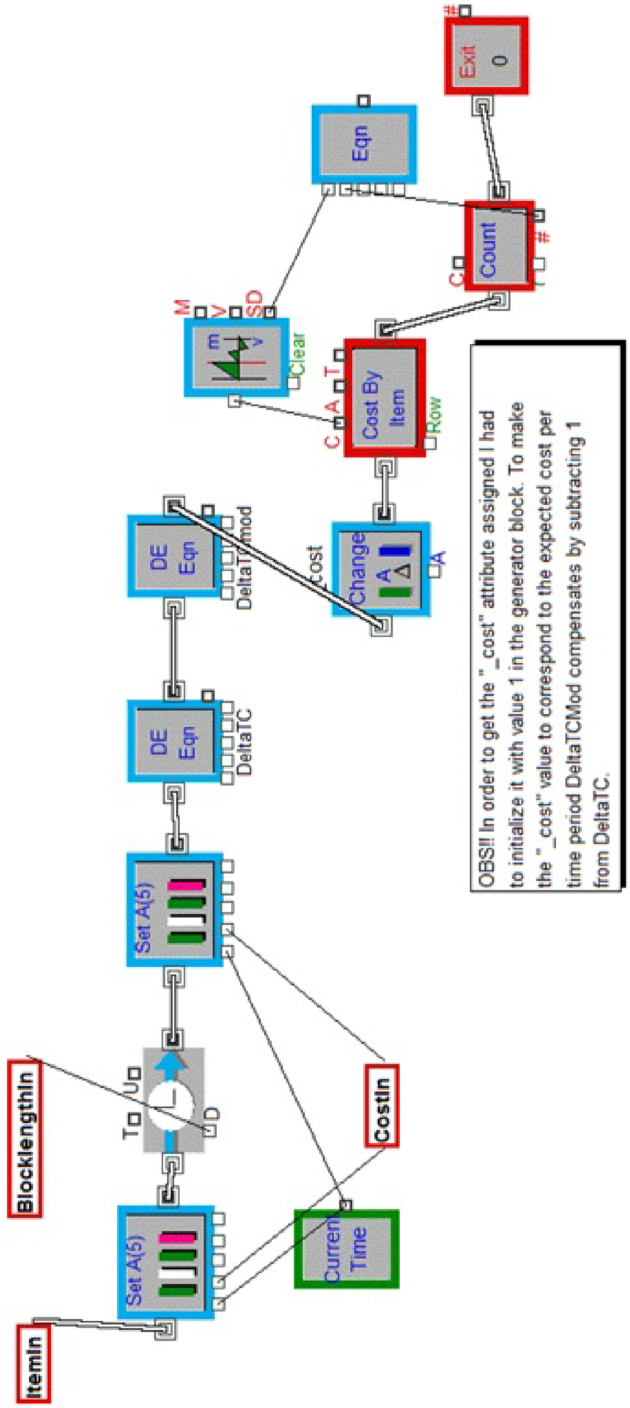
Appendix 10 - Extend model: central warehouse block



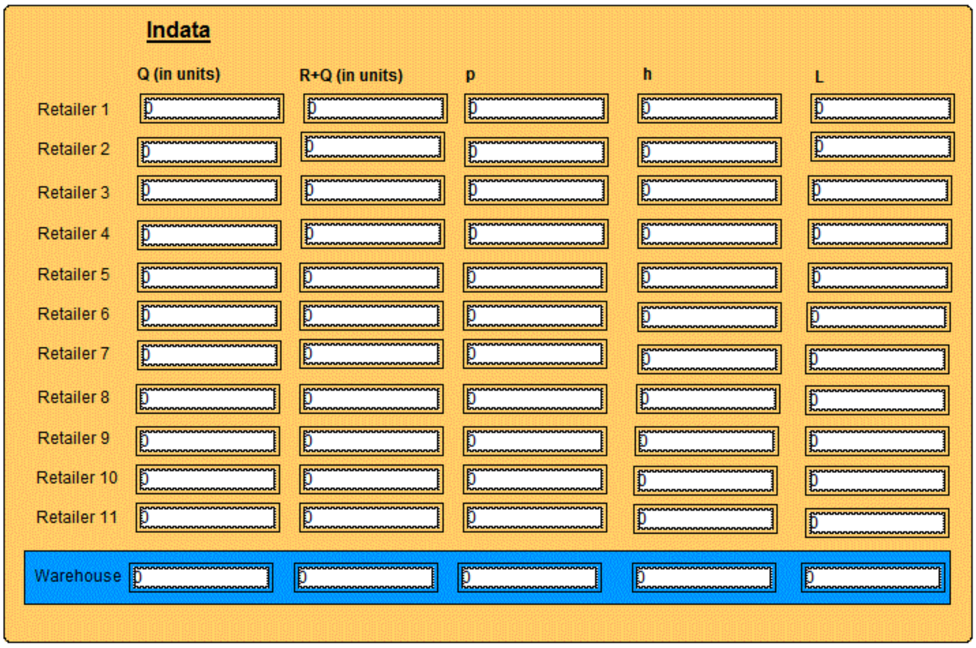
Appendix 11 - Extend model: splitter block

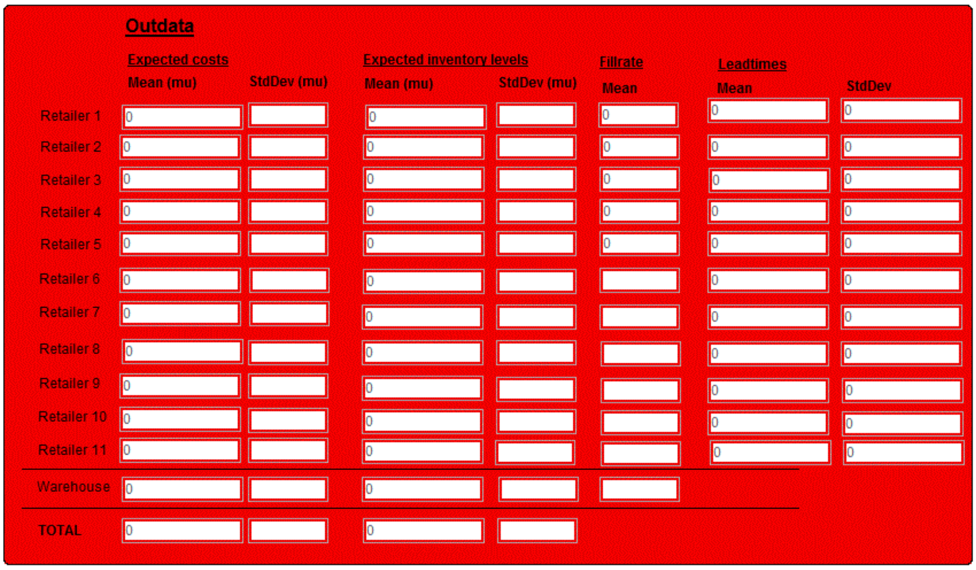


Appendix 12 - Extend model: cost calculation block



Appendix 13 - Extend model: indata and outdata





Appendix 14 – Key figures

Key figures for choose of undershoot method 1 or 5.

Key figures / Model	U1	U5
Mean of difference from target fillrate (%)	0,7%	0,6%
Stdev. of mean of difference from target fillrate (%)	6,0%	6,1%
Mean of weighted difference from target fillrate (demand, %)	-1,4%	-1,6%
Stdev. of mean of weighted difference from target fillrate (demand, %)	2,1%	2,0%
Mean of weighted difference from target fillrate (picks, %)	-1,7%	-1,9%
Stdev. of mean of weighted difference from target fillrate (picks, %)	1,8%	1,8%
Max positive difference from target fillrate at retailers (%)	27,2%	27,2%
Mean of max positive difference from target fillrate at retailers (%)	7,4%	7,4%
Stdev. of mean of max difference from target fillrate at retailers (%)	6,9%	6,7%
Max negative difference from target fillrate at retailers (%)	-34,8%	-34,8%
Mean of max negative difference from target fillrate at retailers (%)	-5,6%	-5,9%
Stdev. of mean of min difference from target fillrate at retailers (%)	7,8%	7,8%
Reduction in inventory at CW by using the new model relative to SCP (%)	83,7%	83,7%
Reduction in inventory at Retailers by using the new model relative to SCP (%)	-60,8%	-58,7%
Total reduction in inventory by using the new model relative to SCP (%)	34,8%	35,4%

Key figures for normal distributed setting as customer demand.

Key figures / Model	Approach 1	SCP
Mean of difference from target fillrate (%)	0,7%	-3,0%
Stdev. of mean of difference from target fillrate (%)	6,0%	11,0%
Mean of weighted difference from target fillrate (demand, %)	-1,4%	-4,1%
Stdev. of mean of weighted difference from target fillrate (demand, %)	2,1%	4,0%
Mean of weighted difference from target fillrate (picks, %)	-1,7%	-2,4%
Stdev. of mean of weighted difference from target fillrate (picks, %)	1,8%	2,4%
Max positive difference from target fillrate at retailers (%)	27,2%	10,0%
Mean of max positive difference from target fillrate at retailers (%)	7,4%	4,7%
Stdev. of mean of max difference from target fillrate at retailers (%)	6,9%	5,5%
Max negative difference from target fillrate at retailers (%)	-34,8%	-66,3%
Mean of max negative difference from target fillrate at retailers (%)	-5,6%	-16,3%
Stdev. of mean of min difference from target fillrate at retailers (%)	7,8%	18,0%
Reduction in inventory at CW by using the new model relative to SCP (%)	83,7%	
Reduction in inventory at Retailers by using the new model relative to SCP (%)	-60,8%	
Total reduction in inventory by using the new model relative to SCP (%)	34,8%	

Key figures for normal distributed setting and compound Poisson setting as customer demand, all articles.

Key figures / Model	Approach 1	SCP	Approach 2
Mean of difference from target fillrate (%)	1,9%	-2,6%	0,8%
Stdev. of mean of difference from target fillrate (%)	6,2%	11,7%	4,5%
Mean of weighted difference from target fillrate (demand, %)	-0,4%	-4,3%	-1,1%
Stdev. of mean of weighted difference from target fillrate (demand, %)	2,1%	4,4%	1,4%
Mean of weighted difference from target fillrate (picks, %)	-0,8%	-2,3%	-1,2%
Stdev. of mean of weighted difference from target fillrate (picks, %)	1,7%	2,7%	1,4%
Max positive difference from target fillrate at retailers (%)	27,2%	10,0%	20,6%
Mean of max positive difference from target fillrate at retailers (%)	8,7%	5,6%	5,6%
Stdev. of mean of max positive difference from target fillrate at retailers (%)	6,3%	5,6%	4,4%
Max negative difference from target fillrate at retailers (%)	-30,5%	-66,3%	-27,7%
Mean of max negative difference from target fillrate at retailers (%)	-4,4%	-16,2%	-3,9%
Stdev. of mean of max negative difference from target fillrate at retailers (%)	6,9%	19,3%	6,0%
Reduction in inventory at CW by using the new model relative to SCP (%)	82,3%		81,6%
Reduction in inventory at Retailers by using the new model relative to SCP (%)	-64,7%		-56,2%
Total reduction in inventory by using the new model relative to SCP (%)	33,9%		35,7%

Key figures for normal distributed setting and compound Poisson setting as customer demand, fast articles.

Key figures / Model	Approach 1	SCP	Approach 2
Mean of difference from target fillrate (%)	-1,6%	-4,2%	-1,7%
Stdev. of mean of difference from target fillrate (%)	6,4%	8,9%	5,0%
Mean of weighted difference from target fillrate (demand, %)	-4,3%	-2,8%	-3,8%
Stdev. of mean of weighted difference from target fillrate (demand, %)	2,0%	1,5%	1,8%
Mean of weighted difference from target fillrate (picks, %)	-4,3%	-2,5%	-3,8%
Stdev. of mean of weighted difference from target fillrate (picks, %)	2,0%	1,5%	1,7%
Max difference from target fillrate at retailers (%)	22,6%	10,0%	20,6%
Mean of max difference from target fillrate at retailers (%)	6,2%	2,9%	3,7%
Stdev. of mean of max difference from target fillrate at retailers (%)	6,4%	4,5%	5,3%
Max negative difference from target fillrate at retailers (%)	-30,5%	-55,5%	-27,7%
Mean of max negative difference from target fillrate at retailers (%)	-8,3%	-16,9%	-7,0%
Stdev. of mean of max negative difference from target fillrate at retailers (%)	8,0%	14,0%	7,3%
Reduction in inventory at CW by using the new model relative to SCP (%)	89,6%		89,3%
Reduction in inventory at Retailers by using the new model relative to SCP (%)	-41,3%		-45,9%
Total reduction in inventory by using the new model relative to SCP (%)	43,1%		41,1%

Key figures for normal distributed setting and compound Poisson setting as customer demand, slow articles.

Key figures / Model	Approach 1	SCP	Approach 2
Mean of difference from target fillrate (%)	4,1%	5,0%	3,0%
Stdev. of mean of difference from target fillrate (%)	4,7%	7,4%	4,2%
Mean of weighted difference from target fillrate (demand, %)	1,7%	1,4%	1,0%
Stdev. of mean of weighted difference from target fillrate (demand, %)	1,5%	2,6%	1,2%
Mean of weighted difference from target fillrate (picks, %)	1,7%	2,3%	1,1%
Stdev. of mean of weighted difference from target fillrate (picks, %)	1,3%	2,0%	1,1%
Max positive difference from target fillrate at retailers (%)	19,6%	10,0%	10,0%
Mean of max positive difference from target fillrate at retailers (%)	8,8%	9,8%	7,5%
Stdev. of mean of max difference from target fillrate at retailers (%)	2,8%	0,8%	2,5%
Max negative difference from target fillrate at retailers (%)	-11,7%	-43,2%	-10,3%
Mean of max negative difference from target fillrate at retailers (%)	-1,3%	-3,5%	-1,5%
Stdev. of mean of max negative difference from target fillrate at retailers (%)	3,9%	10,2%	3,2%
Reduction in inventory at CW by using the new model relative to SCP (%)	85,4%		84,1%
Reduction in inventory at Retailers by using the new model relative to SCP (%)	-21,7%		-14,6%
Total reduction in inventory by using the new model relative to SCP (%)	45,1%		47,3%

Key figures for normal distributed setting and compound Poisson setting as customer demand, lumpy articles.

Key figures / Model	Approach 1	SCP	Approach 2
Mean of difference from target fillrate (%)	3,9%	-15,0%	1,0%
Stdev. of mean of difference from target fillrate (%)	8,1%	20,5%	4,2%
Mean of weighted difference from target fillrate (demand, %)	2,6%	-18,9%	-0,4%
Stdev. of mean of weighted difference from target fillrate (demand, %)	3,1%	8,8%	1,2%
Mean of weighted difference from target fillrate (picks, %)	0,6%	-11,5%	-0,9%
Stdev. of mean of weighted difference from target fillrate (picks, %)	1,9%	4,7%	1,3%
Max positive difference from target fillrate at retailers (%)	27,2%	10,0%	11,7%
Mean of max positive difference from target fillrate at retailers (%)	13,4%	2,3%	5,5%
Stdev. of mean of max positive difference from target fillrate at retailers (%)	8,4%	7,5%	3,8%
Max negative difference from target fillrate at retailers (%)	-21,4%	-66,3%	-19,8%
Mean of max negative difference from target fillrate at retailers (%)	-3,5%	-40,6%	-3,1%
Stdev. of mean of max negative difference from target fillrate at retailers (%)	6,0%	18,7%	5,2%
Reduction in inventory at CW by using the new model relative to SCP (%)	62,1%		61,8%
Reduction in inventory at Retailers by using the new model relative to SCP (%)	-197,2%		-160,8%
Total reduction in inventory by using the new model relative to SCP (%)	-6,4%		1,9%

Key figures for normal distributed setting and compound Poisson setting as customer demand, total demand < 20 units per year.

Key figures / Model	Approach 1	SCP	Approach 2
Mean of difference from target fillrate (%)	4,8%	4,8%	3,4%
Stdev. of mean of difference from target fillrate (%)	4,8%	8,9%	4,3%
Mean of weighted difference from target fillrate (demand, %)	3,1%	0,6%	1,6%
Stdev. of mean of weighted difference from target fillrate (demand, %)	2,2%	4,4%	1,5%
Mean of weighted difference from target fillrate (picks, %)	2,9%	2,3%	1,7%
Stdev. of mean of weighted difference from target fillrate (picks, %)	1,7%	3,2%	1,4%
Max positive difference from target fillrate at retailers (%)	19,6%	10,0%	11,7%
Mean of max positive difference from target fillrate at retailers (%)	9,2%	9,8%	7,5%
Stdev. of mean of max positive difference from target fillrate at retailers (%)	3,2%	1,0%	2,6%
Max negative difference from target fillrate at retailers (%)	-10,2%	-43,2%	-10,2%
Mean of max negative difference from target fillrate at retailers (%)	-0,4%	-4,1%	-0,9%
Stdev. of mean of max negative difference from target fillrate at retailers (%)	3,8%	12,1%	3,1%
Reduction in inventory at CW by using the new model relative to SCP (%)	83,4%		83,7%
Reduction in inventory at Retailers by using the new model relative to SCP (%)	-19,4%		-7,8%
Total reduction in inventory by using the new model relative to SCP (%)	41,9%		46,7%

Key figures for normal distributed setting and compound Poisson setting as customer demand, total demand 20 – 100 units per year.

Key figures / Model	Approach 1	SCP	Approach 2
Mean of difference from target fillrate (%)	2,6%	1,1%	1,3%
Stdev. of mean of difference from target fillrate (%)	5,0%	9,9%	3,8%
Mean of weighted difference from target fillrate (demand, %)	0,3%	-1,7%	-0,1%
Stdev. of mean of weighted difference from target fillrate (demand, %)	1,1%	2,2%	0,8%
Mean of weighted difference from target fillrate (picks, %)	0,1%	-0,5%	-0,2%
Stdev. of mean of weighted difference from target fillrate (picks, %)	0,8%	2,3%	0,7%
Max positive difference from target fillrate at retailers (%)	17,2%	10,0%	11,1%
Mean of max positive difference from target fillrate at retailers (%)	8,2%	8,5%	6,1%
Stdev. of mean of max positive difference from target fillrate at retailers (%)	3,9%	4,3%	3,9%
Max negative difference from target fillrate at retailers (%)	-11,7%	-46,4%	-8,5%
Mean of max negative difference from target fillrate at retailers (%)	-2,3%	-8,6%	-2,4%
Stdev. of mean of max negative difference from target fillrate at retailers (%)	4,0%	12,9%	2,6%
Reduction in inventory at CW by using the new model relative to SCP (%)	83,8%		80,0%
Reduction in inventory at Retailers by using the new model relative to SCP (%)	-38,2%		-28,7%
Total reduction in inventory by using the new model relative to SCP (%)	43,5%		44,5%

Key figures for normal distributed setting and compound Poisson setting as customer demand, total demand 100 – 1000 units per year.

Key figures / Model	Approach 1	SCP	Approach 2
Mean of difference from target fillrate (%)	2,0%	-10,0%	0,2%
Stdev. of mean of difference from target fillrate (%)	7,3%	16,7%	4,1%
Mean of weighted difference from target fillrate (demand, %)	-0,5%	-12,2%	-1,8%
Stdev. of mean of weighted difference from target fillrate (demand, %)	2,7%	6,7%	1,3%
Mean of weighted difference from target fillrate (picks, %)	-1,5%	-7,5%	-2,1%
Stdev. of mean of weighted difference from target fillrate (picks, %)	1,9%	2,9%	1,4%
Max positive difference from target fillrate at retailers (%)	27,2%	10,0%	9,9%
Mean of max positive difference from target fillrate at retailers (%)	10,4%	3,1%	4,7%
Stdev. of mean of max positive difference from target fillrate at retailers (%)	8,6%	6,3%	3,7%
Max negative difference from target fillrate at retailers (%)	-30,5%	-66,3%	-24,3%
Mean of max negative difference from target fillrate at retailers (%)	-5,6%	-30,5%	-4,8%
Stdev. of mean of max negative difference from target fillrate at retailers (%)	7,1%	23,4%	6,3%
Reduction in inventory at CW by using the new model relative to SCP (%)	73,1%		72,9%
Reduction in inventory at Retailers by using the new model relative to SCP (%)	-148,1%		-128,9%
Total reduction in inventory by using the new model relative to SCP (%)	11,5%		15,7%

Key figures for normal distributed setting and compound Poisson setting as customer demand, total demand 1000 – 5000 units per year.

Key figures / Model	Approach 1	SCP	Approach 2
Mean of difference from target fillrate (%)	-2,0%	-5,4%	-1,8%
Stdev. of mean of difference from target fillrate (%)	7,0%	8,8%	5,5%
Mean of weighted difference from target fillrate (demand, %)	-4,8%	-3,1%	-4,0%
Stdev. of mean of weighted difference from target fillrate (demand, %)	2,0%	1,5%	1,8%
Mean of weighted difference from target fillrate (picks, %)	-4,8%	-3,1%	-4,0%
Stdev. of mean of weighted difference from target fillrate (picks, %)	2,1%	1,7%	1,9%
Max positive difference from target fillrate at retailers (%)	22,6%	9,8%	20,6%
Mean of max positive difference from target fillrate at retailers (%)	6,8%	1,8%	4,2%
Stdev. of mean of max positive difference from target fillrate at retailers (%)	7,1%	4,6%	6,0%
Max negative difference from target fillrate at retailers (%)	-29,4%	-46,4%	-27,7%
Mean of max negative difference from target fillrate at retailers (%)	-9,2%	-19,3%	-7,4%
Stdev. of mean of max negative difference from target fillrate at retailers (%)	8,0%	13,4%	7,7%
Reduction in inventory at CW by using the new model relative to SCP (%)	89,9%		89,7%
Reduction in inventory at Retailers by using the new model relative to SCP (%)	-43,6%		-50,7%
Total reduction in inventory by using the new model relative to SCP (%)	42,1%		38,9%

Key figures for negative binomial distributed setting as customer demand.

Key figures / Model	Approach 1	SCP	Approach 3
Mean of difference from target fillrate (%)	-3,9%	-4,1%	-3,2%
Stdev. of mean of difference from target fillrate (%)	5,0%	5,7%	5,3%
Mean of weighted difference from target fillrate (demand, %)	-5,0%	-3,5%	-4,3%
Stdev. of mean of weighted difference from target fillrate (demand, %)	1,2%	1,0%	1,2%
Mean of weighted difference from target fillrate (picks, %)	-5,0%	-3,1%	-4,3%
Stdev. of mean of weighted difference from target fillrate (picks, %)	1,0%	0,8%	1,0%
Max positive difference from target fillrate at retailers (%)	18,9%	6,0%	14,9%
Mean of max positive difference from target fillrate at retailers (%)	2,7%	0,8%	2,8%
Stdev. of mean of max positive difference from target fillrate at retailers (%)	6,6%	2,3%	4,0%
Max negative difference from target fillrate at retailers (%)	-25,3%	-38,3%	-26,5%
Mean of max negative difference from target fillrate at retailers (%)	-9,6%	-14,3%	-9,8%
Stdev. of mean of max negative difference from target fillrate at retailers (%)	7,1%	9,3%	7,3%
Reduction in inventory at CW by using the new model relative to SCP (%)	91,9%		91,8%
Reduction in inventory at Retailers by using the new model relative to SCP (%)	-40,5%		-53,9%
Total reduction in inventory by using the new model relative to SCP (%)	39,7%		34,7%

Key figures for problem articles, only calculated with normal distributed setting as customer demand.

Key figures / Model	Approach 1	SCP
Mean of difference from target fillrate (%)	-5,0%	-5,3%
Stdev. of mean of difference from target fillrate (%)	5,8%	8,2%
Mean of weighted difference from target fillrate (demand, %)	-6,2%	-3,2%
Stdev. of mean of weighted difference from target fillrate (demand, %)	2,5%	1,2%
Mean of weighted difference from target fillrate (picks, %)	-6,3%	-2,6%
Stdev. of mean of weighted difference from target fillrate (picks, %)	2,7%	0,5%
Max positive difference from target fillrate at retailers (%)	16,4%	6,1%
Mean of max positive difference from target fillrate at retailers (%)	1,4%	1,0%
Stdev. of mean of max positive difference from target fillrate at retailers (%)	7,8%	2,2%
Max negative difference from target fillrate at retailers (%)	-34,8%	-35,5%
Mean of max negative difference from target fillrate at retailers (%)	-9,6%	-7,2%
Stdev. of mean of max negative difference from target fillrate at retailers (%)	11,7%	12,0%
Reduction in inventory at CW by using the new model relative to SCP (%)	86,1%	
Reduction in inventory at Retailers by using the new model relative to SCP (%)	-49,9%	
Total reduction in inventory by using the new model relative to SCP (%)	36,6%	