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European Optimal Currency Portfolio

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Abstract:

This paper will analyze the construction of an optimal currency portfolio using mean variance portfolio analysis, in order to determine what would have been the most stable monetary union for the European Union prior to the complete transition to the Euro currency on January 1, 2002. The analysis calculates the minimum variance portfolio of the potential European Union members by using the sovereign bond yield to maturity as a proxy for the return and variance of the asset. The mean variance model used was subject to variable upper and lower bound constraints of the portfolio weights, that dependant on the size of the country's GDP to total GDP of the portfolio.

The data obtained from the calculation of the efficient minimum variance portfolios indicates that the Eurozone did not form an optimal currency portfolio that provided the most stability. The analysis also determined that Greece, Portugal, and Ireland were the countries that were most frequently left out of the optimal minimum variance portfolio, implying that they could contribute to instability within the optimal currency portfolio. Even though the data period is from 1993 until 2001 this analysis accurately represents potential countries that would cause instability within the currency union, currently seen in the 2010/2011 sovereign debt crisis.

Keywords: Eurozone, Euro, Portfolio Optimization, Optimal Currency Portfolio, MVP Portfolio

Acknowledgements

tack så mycket

Table of Contents

1 Introduction

This paper will analyze the construction of an optimal currency portfolio using mean variance portfolio analysis; in order to determine what would have been the most stable monetary union for the European Union prior to the complete transition to the Euro currency on January 1, 2002.

1.1 Background

As the world becomes an increasingly integrated through globalization countries desire more stability when engaging in international trade. By forming trade unions such as North American Free Trade Agreement (NAFTA) and the European Union (EU) countries can benefit from lower barriers to trade, but they can still be exposed to unsuspected movements in the foreign exchange market. One resolution this problem is to enter a monetary union with trading members that fit the criteria for an optimal currency area. It has been theorized that certain trade blocs could already be or become Optimal Currency Areas. The groundbreaking theory behind optimal currency areas was developed by Robert Mundell (1961), and is considered one of the most influential works for international economic integration. The study of Optimal Currency Areas always examines how individual countries fare in potential economic development, and the economic criteria countries must obtain to go from having a floating currency to a monetary union. This theoretical framework is what was used in the construction of what is known today as the Euro; the official currency of the Eurozone currently 17 of the 27 European Union (EU) member states. A key question to ask is if the Eurozone member states formed an optimal currency area that provides stability; if not, what would be the optimal currency area of the European Union.

It should not be a question of whether an area meets the criteria for an optimal currency area, is really a question whether the countries contributing to a monetary union form an optimal currency portfolio. By focusing on the individual country's financial and macroeconomic data mean variance portfolio analysis will be applied to find the appropriate combination of European Union member states that make up the minimum variance portfolio.[1] The modern portfolio theory, which is used to find the optimal portfolio by maximize the return given a certain level of risk or to minimize risk given an expected return, was pioneered by

[1] It is important to understand the historical background of the Euro in order to understand why certain constraints are applied to the mean variance. This will be discussed further in section 2.1 under background history.

Markowitz (1952). This theory mathematically revolutionized the concept of diversification in finance. He was able to show that investing in a set of assets would have lower risk then investing in an individual asset. The mean variance analysis uses an assets expected return and its standard deviation in order calculate the optimal portfolio given a certain level of risk or expected return. The minimum variance portfolio, which has the lowest risk is used in this analysis because it is assumed that it is the most stable portfolio. The minimum variance portfolio will solve the optimization problem by finding the portfolio with the lowest variance. The mean variance portfolio framework has been used to apply to all kinds of asset classes, including construction of equity, bond, currency, commodity and option portfolios, as well as a portfolios consisting of a combination of the different assets. In other words, this paper examines the optimal currency portfolios in order to determine a stable currency union for EU member states.

1.2 Problem Discussion

Though an economic and monetary union provides stability for trade and movement of goods and services, it also takes away from a countries ability to control their own economic policies. Today the main goal of the European Central Bank is keep economic stability and a target inflation of about two percent across the Eurozone member states.[2] By taking away monetary policy from individual states within the union you are limiting ways in which countries can achieve macroeconomic stability. For example if the average inflation across the Eurozone members is 2.2% the ECB would consider raising interest rates to maintain the target inflation of the Euro, even though an individual country could have inflation of 0.5% and raising rates could cause economic destruction within the country.

When looking at an optimal currency portfolio for the EU, it is important that you are including the members that will contribute to economic stability. What this paper aims to do is to examine the current composition of the Eurozone against other possible compositions using mean variance portfolio analysis. Given the 2011 sovereign debt crisis plaguing the Eurozone it is unknown whether the monetary union is comprised of the optimal member states. Examining the efficiency of the composition of the Eurozone is important for future enlargement, and could assist in examining other potential optimal currency portfolios.

2 This is according to the European Central Bank (ECB) website.

1.3 Purpose

This paper will analyze the construction of an optimal currency portfolio using mean variance portfolio analysis; in order to determine what would have been the most stable monetary union for the European Union prior to the complete transition to the Euro in 2002.

1.4 Method

The methodology used in this paper uses the efficient set mathematics developed by Best and Grauer (1990) for application of constraints on mean variance portfolio optimization. While Markowitz (1956, 1959) and Sharpe (1972) applied general linear constraints to the model their only binding constraint applied were that the sum of the weights be equal to one. Best and Grauer (1990) modified Markowitz model with no riskless asset that allows for constraints is modified and then applied to our specific requirements in order to properly formulate the minimum variance portfolio.

When using the mean variance framework for optimal currency portfolios there are a few constraints that must be applied. A country has two options when it comes to the Eurozone, they can either be in or out. Since you cannot short sell a country the first constraint that must be applied is a short selling constraint on all assets. When constructing the mean variance portfolio to the optimal currency portfolio it would not make sense to have a country such as Germany to have for example a 5% weight and a country like Finland to have 35% weighting. To better illustrate this Germany currently has a population and Gross Domestic Product (GDP) that is approximately 1550% and 1600% larger than Finland respectively, so by allowing Finland to have a larger weight then Germany would be unreasonable. So during the construction of the optimal portfolio upper and lower bound constraints will be applied to ensure that smaller economic countries will not have a larger impact then the larger economies. The European Economic Community (EEC) solved this issue by creating the European Currency Unit (ECU), which was the accounting unit, comprised of a basket of the currencies based on the respective country's share on the EEC's Gross National Product (GNP).[3]

Due to the complex nature of applying the constraints in calculating the minimum variance portfolio Microsoft Excel with the assistance of Visual Basic for Applications will be created

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[3] Ungerer (1997)

to allow for variable upper and lower bounds. There will be three separate calculations of the optimal portfolio over two different time periods.

The sovereign bonds from the individual countries are used as a proxy for the expected returns. Though government bonds are considered a risk free investment because a country theoretically could raise taxes or print more money in order to pay off the debts. There are however not completely risk free, government bonds carry three kinds of risk. Credit risk is the risk associated with a country defaulting on their debt. Though this is a rare occurrence it has happened before, most recently during the Russian Financial Crisis in 1998 who defaulted on their domestic debt or the Argentine debt restructuring in 2002 where the government defaulted on part of their external debt due to the inability to pay it back. These bonds are still exposed to Inflation risk is if inflation for the country is higher than expected, causing the investor to have less purchasing power. Most countries issue an inflation-indexed bond,[4] which increases the interest rate it pays to the investor as the inflation in the country increases. Government bonds can also be subject to currency risk for foreign investors. The expected return on the bond could be lower or higher depending if your home currency has appreciated or depreciated during the investment. This is agreeable with Aguiar, Mark and Gita Gopinath (2006) and Hilscher, Jens and Yves Nosbusch (2010) countries that have a higher and lower risk of default have higher and lower yields on sovereign debt respectively. The standard deviation used in the optimization problem will be the standard deviation of the sovereign bonds plus the standard deviation of the inflation. Due to the need for constraints as illustrated above there will be no short selling and the upper and lower bounds will be dependant on the assets included percent share of the optimal Eurozone's nominal GDP plus and minus the standard deviation of the expected GDP growth, plus the currency fluctuation band for that time period.[5]

The optimal portfolio will also use a similar method to how the European Economic Community constructed the ECU. The expected return will be the nominal growth rate of GDP and the standard deviation will be the standard deviation of the nominal GDP. Just like

[4] Inflation-indexed bonds are not used because there has been instances where the yield on these bonds have been less than zero due to investors expectation of high inflation.
[5] Chapter under Background History will provide a better understanding of the currency bands applied.

the optimization method above there will be a short selling constraint, and an upper and lower bond constraint.[6]

1.5 Shortfalls

One aspect of the theory behind optimal currency portfolios is that countries should have similar business cycles. If countries do not have similar business cycles then it provides the central bank with difficult decisions when trying control inflation in upturns and promotes growth during downturns. When looking closely at the sub-additive property of the diversification effect; two assets combined together will never increase the total risk.[7] Thus it can be assumed that the mean variance portfolio optimization applied here does not take into account business cycles. However, since the ECB said that all of the countries included in this study were in a "stable state" during the period of the analysis it is assumed that all the countries studied have the same cycles.

1.6 Disposition

In section two a brief historical background of the how the Euro came to be in existence. It is important to understand how exchange rates between the European countries have undergone many alterations and exchange rate regimes leading up to the Euro. The exchange rate regimes, specifically the currency bands between the European countries must also be applied when investigating the optimal portfolio of the Eurozone. This will be followed by a continued review of optimal currency portfolios and mean variance portfolio analysis. In section three, information will be provided explaining why certain data was used and its source. Section four will provide the reader with a step-by-step understanding of the methodology used throughout this paper in order to properly examine the mean variance analysis. The significant results obtained from these tests will be posted and explained under in section five. From the results conclusions will be made in section six. References are presented in section seven and section eight will have the appendix which will provide the charts, tables, and any other useful information not included in the main body of the essay.

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[6] This implies that the asset weight (w) can either be w=0, or lower bound $<w<$ upper bound, where the bounds are dependant on percent share of optimal Eurozone GDP.

7 In examining portfolio analysis it can be seen that the subadditive property shows that the variance of two assets will never combine to be larger than the individual asset variances added: $E(\sigma_a^2) + E(\sigma_b^2) \leq E(\sigma_{a+b}^2)$ for further explanation see Steele (1997)

2 Literary Review

This section will go over the historical background of the Eurozone, and the theoretical framework of the mean variance portfolio theory.

2.1 Historical Background

The foreign exchange market used to be controlled by the United States Dollar who through most of history used some form of the gold standard. This meant that for every dollar that was in the market the government held a certain percentage of the dollar equivalent in gold. If the US government printed more money then the value of the dollar would decline and the price of gold would increase. This was later replaced by a version of the gold standard called the Bretton Woods system. Under this system the US dollar was the reserve currency and other nations would peg themselves to the US dollar and allow their currency to fluctuate within a tight band. In order to keep the confidence in the US dollar it was voluntarily pegged to gold. Following the collapse of the Bretton Woods system in the early 1970´s with what is know as the "Nixon Shock", the United States Dollar became fiat money.[8]

In 1968, prior to the "Nixon Shock" the members of the European Economic Community (EEC) were presented with the Barre Report which called for further organization of the members economic policies and move toward "monetary co-operation".[9] A three step plan to be able to reach a European monetary union called the "Werner Plan" was presented to the European Community by Pierre Werner, the Prime Minister and Finance Minister of Luxembourg in October of 1970.[10] In 1971 the Group of Ten comprised of 7 EEC member states signed the Smithsonian Agreement ending the fixed exchange rate system, and allowed for a currency band to be applied. The countries agreed to first devalue the US dollar and then to allow their currency to float within a band of ±2.25% against the US dollar, also referred to as the "tunnel".[11] Even though this would provide stability between the EEC currencies and the US dollar it allowed for large fluctuations between the EEC member states. For example if French Franc started at the low end of the band and appreciated 4.5% and Germany Mark started at the high end of their band and depreciated 4.5% this would mean that the French France would appreciate by 9% versus the German Mark. Because of this the EEC took their

[8]Fiat money is a monetary unit that has no intrinsic value, but has its value is based on the confidence and demand of the country's goods and resources.
[9] Ungerer (1997)
[10] Ungerer (1997)
[11] Ungerer (1997)

first steps toward monetary integration by applying the “snake in the tunnel” which essentially pegged all the EEC currencies to one another. The system allowed the EEC currencies to fluctuate within a band of ±2.25% between one another.[12]

The European Economic Community (EEC) member states had a difficult time maintaining the currency band with the US dollar, so by 1973 it was abandoned letting the US dollar to float freely. The failure to maintain the “tunnel” with the US dollar made it extremely difficult to maintain the “snake” currency band between the EEC countries, causing them to float freely amongst one another. This was the end of the Werner Plan; however was not the end of attempted monetary union within the European Community.

In 1979, the European Community (EC) created the European Monetary System, which implemented two main concepts in the shift toward monetary integration. First it created the European Currency Unit (ECU), an accounting unit whose value was determined by a weighted average basket of currencies from the contributing EC member states. The weighting of national currencies in this basket was a fixed percentage equal to their country's share of the EEC's Gross National Product (GNP). The composition of the ECU was adjusted three times during in 1979, 1984 and 1989 and finally locked into place in 1993 when the European Monetary Institute (EMI) made the participating member countries to make 20% of their gold and dollar reserves available. The ECU was pegged at a value of 1:1 to the Euro based on the weighting scheme established in 1989 even though the countries participation with the Eurozone changed leading up to introduction of the Euro. The United Kingdom withdrew from the ECU in 1992 due to speculative attack; Greece joined the Euro in 2001; also Austria and Finland were founding members of the Euro but were not included in the ECU currency basket since they did not join the ERM until 1995. Most importantly the European Exchange Rate Mechanism (ERM) was created to achieve exchange rate and monetary stability in Europe. The ERM allowed the exchange rates of the countries participating to fluctuate within a band of ±2.25% against the ECU, with the exception of Italy which allowed a ±6.00% of the Lira versus the ECU.[13]

Had the ECU been reweighted after the expansion of the EU in 1994, and the United Kingdom leaving the exchange rate regime due to speculative attack; it would include

[12] Spahn (2001)
[13] Spahn (2001)

member states that would eventually be part of the original Eurozone such as Finland and Austria. In Figure 1 and Figure 2 the differences in the weighting is extremely different, while France has the lowest difference of about a 7% increase, Italy has about a 50% increase and Greece has about a 450% increase from the ECU basket in 1989 and the Eurozone basket in 1995.

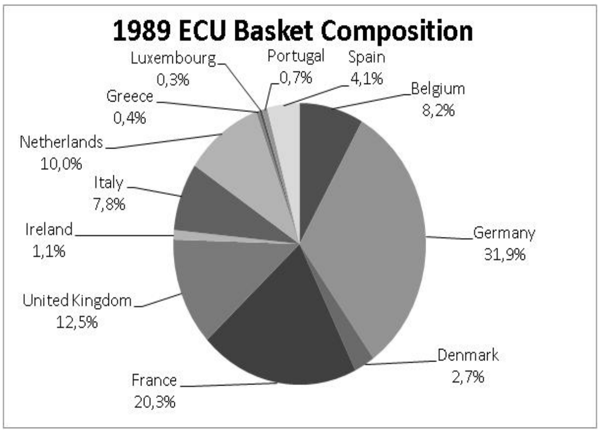

Figure 1. ECU weighting in 1989

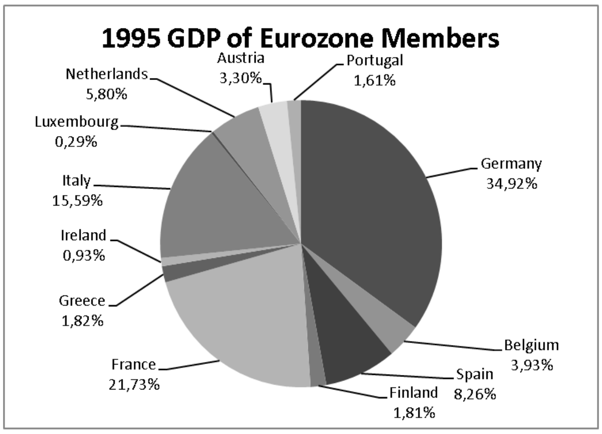

Figure 2. Percent of total GDP for 1995 Eurozone members

Appendix 2.1 contains pie charts that show the historical weighting of each country in the ECU basket, and the ECU basket and EU member states weighted GDP for a given year.

2.2 Optimization Problem

For this paper the indirect utility function is used to solve the mean variance optimization problem, where the utility is a function of the expected return and variance also referred to as the risk. It is assumed that the investor is risk averse; therefore it is used to maximize the investors' utility based on the market conditions.

$$\mathbb{U}(\mu, \sigma^2) = t\mu - \frac{1}{2}\sigma^2 = t \cdot \mathbf{w}'\boldsymbol{\mu} - \frac{1}{2}\mathbf{w}'\mathbf{V}\mathbf{w}$$

Where; $\mathbb{U}$ is the indirect utility function of the expected return and variance; μ is the expected return of the portfolio; and σ^2 is the variance of the portfolio; t is the risk tolerance; $\mathbf{w}'$ is an $(1xN)$ vector of weights; $\boldsymbol{\mu}$ is an $(Nx1)$ vector of expected returns; and V is the variance covariance matrix.

In Sharp's critical line method the upper bound and lower bound inequality constraints are used to find the optimal weights of the portfolio.[14] In order to do this the indirect utility function has to be maximized:

$$\max_{\{\mathbf{w}\}} \ \mathbb{U}(\mu, \sigma^2) \quad = \quad t \cdot \mathbf{w}'\boldsymbol{\mu} - \frac{1}{2}\mathbf{w}'\mathbf{V}\mathbf{w}$$

$$\mathbf{w}'\mathbf{I} = 1, \ lb \leq w_i \leq ub \ \forall i$$

Where t is the risk tolerance; $\mathbf{w}'$ is an $(1xN)$ vector of weights; $\boldsymbol{\mu}$ is an $(Nx1)$ vector of expected returns; and V is the variance covariance matrix. This model is subject to constraints when calculating the weights for a particular asset. Like the traditional portfolio optimization the weights of the portfolio must sum to one.

For this model the investors risk tolerance t is the inverse of relative risk aversion, thus it implies that as t increases so does the investors risk tolerance. An investor that is most risk averse will have zero risk tolerance where; $t = 0$, thus being equal to the mvp portfolio since is it has the lowest risk.

2.3 Mean Variance Framework

The traditional mean variance model presented by Markowitz (1952) simplifies the optimization process by classifying assets by their expected return and variance. The model can be used to maximize the expected return given a certain level of risk, or to minimize risk given an expected return.

When using the mean variance framework to solve and optimization problem it is important to be aware of the assumptions the model makes. There are as follows:

- There are $N \geq 2$ risk bearing assets[15].
- No redundant assets included.
- All assets *i* have finite returns and variances σ_i^2.
- The expected returns are not identical.

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[14] http://www.stanford.edu/~wfsharpe/mia/mia.htm

[15] In this paper the assets are defined as a sovereign entity or a measurable economic area such as a country.

- No transaction costs.
- No short selling allowed
- There is no riskless asset present.

The expected return of the portfolio is the sum of the weighted assets expected returns computed as:

$$E(r_P) = \sum_{i=1}^{n} w_i E\left(r_i\right)$$

The return on the portfolio is simply defined as the weighted average of the returns of the assets in the portfolio. The variance of the portfolio is the summation of the expected return of the portfolio minus the mean of the portfolio squared divided by the number of observations. The standard deviation of the portfolio is the square root of the variance.

$$\sigma_p^2 = \frac{1}{n}\sum_{i=1}^{n} E(r_p - \mu)^2$$

$$\sqrt{\sigma_p^2} = \sigma_p$$

The variance of a portfolio can also be written as the weighted sum of a combination of assets covariance's of the individual asset returns as seen here:

$$\mathrm{Var}\left(r_p\right) = \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \mathrm{Cov}\left(r_i, r_j\right)$$

Where the covariance of two assets is the correlation between the two assets times their respective standard deviations.

$$\mathrm{Cov}\left(r_i, r_j\right) = \rho_{ij}\sigma_i\sigma_j = \sigma_{ij}$$

Where ρ_{ij} is the correlation coefficient between asset i,j and σ_i , and σ_j represent standard deviations of r_i and r_j respectively.

The variance covariance matrix of the asset returns is defined as:

$$V \equiv \begin{bmatrix} \sigma_{11} & \sigma_{12} & . & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & . & . \\ . & . & . & . \\ \sigma_{N1} & . & . & \sigma_{NN} \end{bmatrix}, V^{-1} \equiv \begin{bmatrix} s_{11} & s_{12} & . & s_{1N} \\ s_{21} & s_{22} & . & . \\ . & . & . & . \\ s_{N1} & . & . & s_{NN} \end{bmatrix}$$

$$\sigma_{ii} \equiv \sigma_i^2 \Leftrightarrow \sqrt{\sigma_i^2} \equiv \sigma_i$$

Therefore the variance of the portfolio with N assets consisting of weights w is written in the quadratic form as:

$$\sigma_p^2 = w_p' V w_p = w_1^2 \sigma_1^2 + 2w_1 w_2 \sigma_{12} + 2w_1 w_3 \sigma_{13} + ..$$

$$w' \equiv [w_1, w_2, \dots, w_N]'$$

From this framework set up by Markowitz (1952) the optimal weights are they calculated based on a certain level of risk or given a certain expected return. As previously stated this paper will attempt to find the optimal weights for the minimum variance portfolio.

3 Data

Though the European Union currently consists of 27 member states, and dozens of other countries that have the potential to become members; this analysis of the Eurozone must start from the beginning. Although the majority of the Eurozone members had their currencies pegged to one another in 1999 the Euro banknotes were not in circulation until January 1, 2002; where only 12 of the 15 EU member states at the time adopted the Euro[16]. Therefore this is where the analysis must begin in order to understand if the countries in the Eurozone formed an efficient optimal currency portfolio.

Reuters Datastream was used to gather quarterly data from 1970-2011, although the analysis in this paper uses the time period 1993-2001 to investigate the optimal portfolio. The reason this time period was used is because that is when the Maastricht Treaty became affective establishing the Copenhagen criteria; which set economic guidelines for the currency union. [17]

The 10 year government bond's yield to maturity was used to represent the return on the sovereign entity for that particular time period. The 10 year bond is typically considered the most stable government note, and is assumed to provide an accurate representation of a

[16] The Euro 12 founding members in 2001 were: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain. The European Union also included: Denmark, Sweden, and United Kingdom.

[17] Copenhagen Criteria

country's risk premium. The data was gathered on a quarterly basis since macroeconomic data for these countries like GDP is only published on a quarterly basis. Below in Figure 3 the yield to maturities are presented for the EU 15 countries from 1993 to 2001.

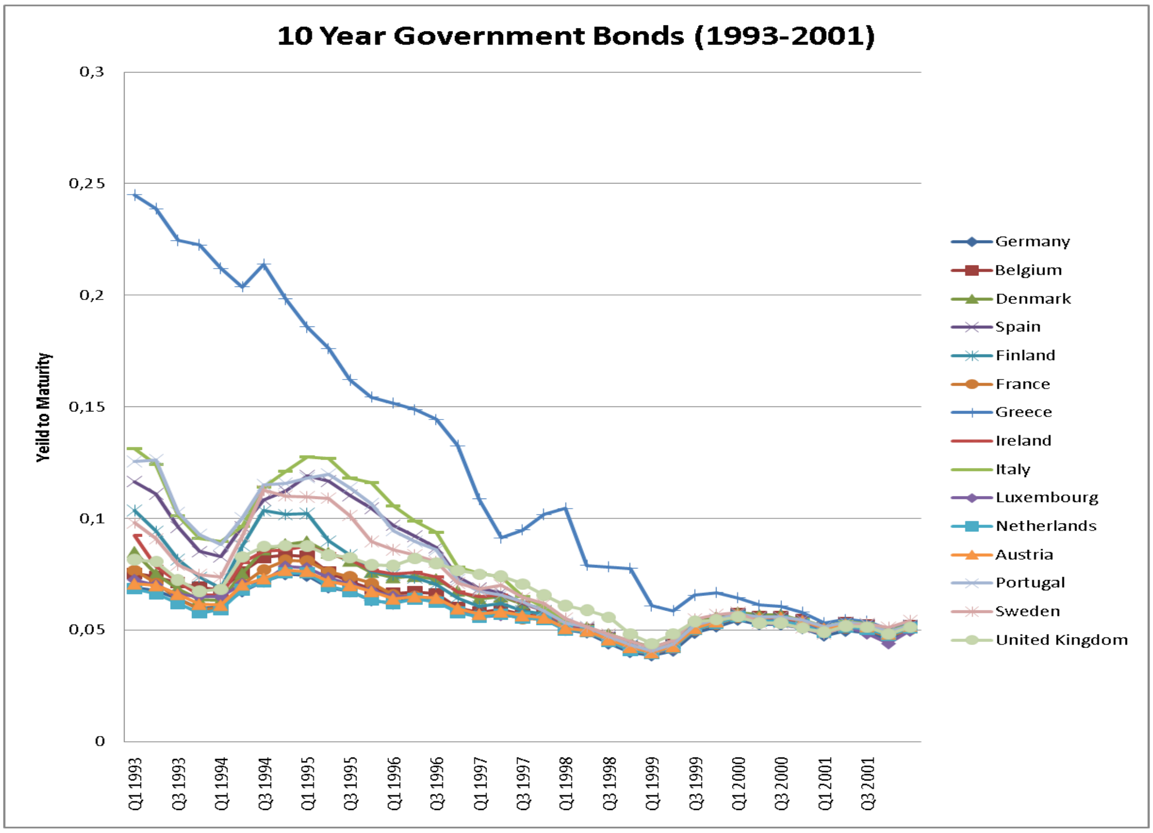

Figure 3. YTM of 10yr sovereign bond.

It appears that during this period that Greece has had the largest yield followed by Italy and Portugal. This would imply based upon the assumptions of this paper that these countries would be considered the most risky since their risk premium is higher than the other countries presented here. Appendix 2.1 contains the yield to maturity on the 10 year government bonds for the EU 15 countries from 1993 to 2011.

Though GDP was gathered on a quarterly basis, the total yearly GDP was used when solving the optimization problem in order to prevent any biases in GDP reporting during a calendar year. Given that all the countries had different currencies during the analysis period all of the GDP information had to be analyzed in a base currency; therefore the United States Dollar was used to represent the value of GDP for each respective country as seen in Figure 4.

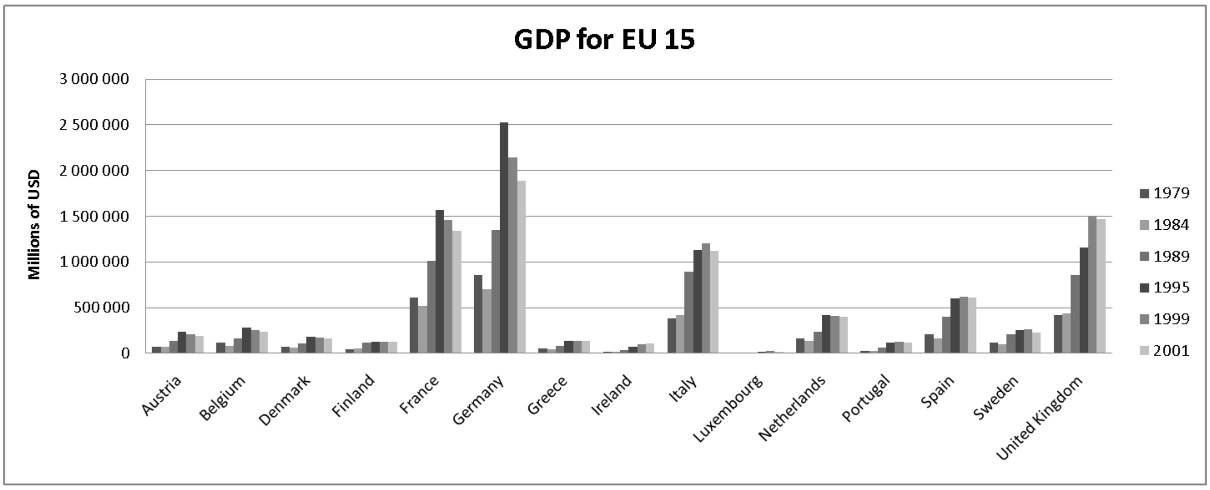

Figure 4. GDP of the Original EU Members

This paper uses GDP as an approximate value of an economy. Figure 4 shows that Germany has the largest GDP followed by France, United Kingdom, and Italy. Although it appears that the GDP totals are affected by the exchange rate with the USD the GDP prices will still reflect that particular countries total share on the portfolio.[18]

Central limit theorem says that under normal circumstances the sum a large number of variables will be normally distributed. Although the time period used in this paper has only 36 data points they are normally distributed at the 10% confidence interval level.

4 Methodology

This section describes the method used to find the minimum variance portfolios weights, which are constrained with variable upper and lower bounds, and a short selling restriction. In order to do this we must maximize the indirect utility function:

$$\max_{\{\mathbf{w}\}} \mathbb{U}(\mu, \sigma^2) = t \cdot \mathbf{w}'\boldsymbol{\mu} - \frac{1}{2}\mathbf{w}'\mathbf{V}\mathbf{w}$$

$$\mathbf{w'I} = 1, \; w_i = 0; \text{or} \;\; lb \leq w_i \leq ub \; \forall i$$

Where t is the risk tolerance; $\mathbf{w}'$ is an $(1xN)$ vector of weights; $\boldsymbol{\mu}$ is an $(Nx1)$ vector of expected returns; and V is the variance covariance matrix. This model is subject to constraints when calculating the weights for a particular asset. Like the traditional portfolio optimization the weights of the portfolio must sum to one. However the weights for the individual assets

[18] For further reference see Triangular arbitrage.

can either be zero, or greater than or equal to the lower bound while also being less than or equal to the upper bound. In this paper the bounds are a function of the country's GDP size within the optimal portfolio.

First the expected returns and standard deviations of the assets were calculated using the yield to maturity on the 10 year government bonds of each country. Therefore the expected returns on the assets are calculated by:

$$\mu_i = \frac{1}{n}\sum_{i=1}^{n} r_i$$

Where μ_i is the expected return on asset i; and M is the number of observations.

Additionally the variances of the assets are calculated by:

$$\sigma_i^2 = \frac{1}{n}\sum_{i=1}^{n} E(r_i - \mu)^2$$

Where; the variance is the sum of the standard deviations from the mean squared.

4.1 Variance Covariance matrix

The variance covariance matrix is an (NxN) matrix of the covariance's between assets i, j , in this paper it was calculated by pre and post multiplying the diagonal matrix of standard deviations with the correlation matrix. The variance covariance matrix of the asset returns is defined as:

$$V \equiv \begin{bmatrix} \sigma_{11} & \sigma_{12} & . & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & . & . \\ . & . & . & . \\ \sigma_{N1} & . & . & \sigma_{NN} \end{bmatrix}, V^{-1} \equiv \begin{bmatrix} s_{11} & s_{12} & . & s_{1N} \\ s_{21} & s_{22} & . & . \\ . & . & . & . \\ s_{N1} & . & . & s_{NN} \end{bmatrix}$$

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Therefore the variance of the portfolio with N assets consisting of weights w is written in the quadratic form as:

$$\sigma_p^2 = w_p' V w_p = w_1^2\sigma_1^2 + 2w_1w_2\sigma_{12} + 2w_1w_3\sigma_{13} + ..$$

$$w' \equiv [w_1, w_2, \ldots, w_N]'$$

It was determined that the covariance matrix used in the optimization problem was calculated correctly since the diagonal of the matrix is equal to the variance of the respective assets.

4.2 Efficient set constants

The efficient set constants were constructed; which are positive scalars; were calculated in order to simplify the optimization equation. .

$$A \equiv \mu' V^{-1} I$$
$$B \equiv \mu' V^{-1} \mu$$
$$C \equiv I'V^{-1} I$$
$$D \equiv BC - A^2$$

4.3 Computing the unconstrained MVP portfolio

The optimal portfolio which is represented by w_p or $w(t)$ when solving for a particular risk tolerance.

$$\boldsymbol{w}(t) = V^{-1}\,(t\boldsymbol{\mu} - \gamma(t)\mathbf{I})$$

By substituting in the scalars calculated the optimization problem it simplifies the calculation process.

$$\mathrm{w}(t) = \mathbf{V}^{-1}\left(t\mu - \left(-\frac{1}{C} + t\frac{A}{C}\right)\mathbf{I}\right)$$

$$\mathrm{w}(t) = \frac{1}{C}\mathbf{V}^{-1}\mathbf{I} + t\left(\mathbf{V}^{-1}\left(\mu - \frac{A}{C}\mathbf{I}\right)\right)$$

$$\mathbf{w}(t) = \mathbf{h}_0 + \mathrm{t}\mathbf{h}_1$$
$$\mathbf{h}_0 \equiv \frac{1}{C}\mathbf{V}^{-1}\mathbf{I}$$
$$\mathbf{h}_1 \equiv \mathbf{V}^{-1}\left(\boldsymbol{\mu} - \frac{A}{C}\mathbf{I}\right)$$
$$\mathbf{h}_0'\mathbf{I} = 1 \qquad \mathbf{h}_1'\mathbf{I} = 0$$

From this equation the mvp portfolio was calculated with the only restriction that the sum of the weights are equal to one. Since the risk tolerance of the mvp portfolio is equal to zero, the weights of the mvp portfolio are equal to the $\mathbf{h}_0$ vector as shown below.

$$\mathbf{w}(t) = \mathbf{w}_{\text{mvp}} + \text{t}\mathbf{h}_1$$

$$\mathbf{w}_{\text{mvp}} = \mathbf{h}_0$$

$$\mathbf{w}_{\text{mvp}} = \frac{1}{\text{C}}\mathbf{V}^{-1}\mathbf{I}$$

After obtaining the weights of the mvp portfolio the expected return and variance was calculated.

$$\mu_{mvp} = \boldsymbol{\mu}'\boldsymbol{w}$$

Where μ_{mvp} is the return on the mvp portfolio; $\boldsymbol{\mu}'$ is a $(1xN)$ vector of returns; $\boldsymbol{w}$ is a $(Nx1)$ vector of portfolio weights.

$$\sigma^2_{mvp} = \boldsymbol{w}'\boldsymbol{V}\boldsymbol{w}$$

Where σ^2_{mvp} is the variance of the mvp portfolio; $\boldsymbol{w}'$ is an $(1xN)$ of the weights; $\boldsymbol{V}$ is the (NxN) variance covariance matrix; and $\boldsymbol{w}$ is a $(Nx1)$ vector of portfolio weights. The square root of the variance is the standard deviation of the mvp portfolio written as:

$$\sqrt{\sigma^2_{mvp}} = \sigma_{mvp}$$

4.4 Constrained MVP portfolios

However, the unconstrained mvp portfolio allows short selling of the assets and the weights of the portfolio are not representative of the size of the country's economy.

Therefore the following restrictions are applied;

$$\mathbf{w}'\mathbf{I} = 1,\ w_i = 0; \text{or}\ \ lb \leq w_i \leq ub\ \forall i$$

$$lb_{it} \cong \frac{(GDP_{it} - (GDP_{it} * E(\sigma_{GDP})))}{\sum GDP_{it}}$$

$$ub_{it} \cong \frac{(GDP_{it} + (GDP_{it} * E(\sigma_{GDP})))}{\sum GDP_{it}}$$

Where GDP_{it} is the nominal GDP for a country i included in the optimal portfolio at time t; $E(\sigma_{GDP})$ is the expected variance of the EU 15 members GDP which was found to be approximately 10%. This will be further explained in the results section of this paper with an example.

4.5 Solver

An algorithm from Excels Solver is used to explore the set of variables in order to determine the optimal portfolio structure. In order to do this the algorithm will have to maximize utility function with a risk tolerance of zero; while not violating the constraints on the model. The model used is unique and complex since it allows for variable bound constraints that are dependant on the country's weighted value on the portfolio. The weighted value is the percentage of GDP of the individual country from the total GDP of the optimal portfolio at time t. What this tool does is maximizes the target cell, in this case the Utility function, by change the portfolio weights that are subject to the constraints above. By solving the function with a risk tolerance of zero; the mvp portfolio is found for each time period for the specific combination of countries. Section 5 will present the results for all the minimum variance portfolios that were calculated by maximizing the Utility function calculation.

4.6 Benchmark Portfolios

Due to the absence of a risk free rate benchmark portfolios needed to be calculated in order to analyze the results obtained from the mvp portfolio calculations. The benchmark portfolios were calculated by:

$$BM_{pt} = \sum_{i=1}^{N} \left(\left(\frac{GDP_{it}}{GDP_{pt}} \right) * E(r_i) \right)$$

Where; BM_{pt} is the benchmark for portfolio p at time t; GDP_{it} is the GDP of country i at time t; GDP_{pt} is the total GDP of portfolio p at time t; and $\mathrm{E(r_i)}$ is the expected return of the yield to maturity of country i at time t. In order to speed up the collection of weights from the benchmark portfolios a computer program written in VBA was used, and can be seen in Appendix 4.1.

4.7 Information Ratio

Proper analysis of data is almost as important as obtaining it. The traditional mean variance framework often uses the Sharpe ratio which takes the expected return of the portfolio less the risk free rate, then is divided by the standard deviation of the portfolio.[19] This determines how much return the portfolio is generating given the level of risk taken. Given that the model used in this paper does not have a risk free rate the information ratio also identified simply as IR is used.

The information ratio is the expected return of a portfolio minus the expected return on the benchmark portfolio, divided the tracking error.

$$IR = \frac{E(R - R_b)}{\sigma}$$

Where; R is the expected return of the portfolio; R_b is the return on the benchmark portfolio; and σ is the tracking error also known as the standard deviation of the active return. The IR measures the excess return over the benchmark relative to the standard deviation of those excess returns i.e. it shows how well the portfolio performs given the risk taken.[20]

5 Results

This section will present the results obtained from running the constrained quadratic programming model in Excel and Visual Basic for Applications which calculated the minimum variance portfolios. The results include the weights obtained from the min variance portfolios by maximizing the utility function calculation with GDP data from 1995 and 2001 and three combinations of the 15 EU member states from the time period.

5.1 Expected Return, Variance, and Standard Deviation

Below in Table 1 the 15 EU countries the expected return, variance and standard deviation are presented. They were calculated from the yield to maturity on their 10 year government bonds between the first quarter in 1993 to the first quarter in 2002, for a total of 36 periods.

[19] Sharpe, "Macro-Investment Analysis"
[20] Information Ratio

	Expected Return	Variance(93-01)	Std.Dev.(93-01)
Germany	5,67054%	0,00956%	0,97782%
Belgium	6,07946%	0,01392%	1,17981%
Denmark	6,34784%	0,01828%	1,35199%
Spain	7,36324%	0,06740%	2,59607%
Finland	6,59649%	0,03571%	1,88969%
France	5,86541%	0,01316%	1,14737%
Greece	12,19514%	0,42009%	6,48145%
Ireland	6,35568%	0,02146%	1,46499%
Italy	7,76973%	0,08830%	2,97145%
Luxembourg	5,85432%	0,01100%	1,04887%
Netherlands	5,70189%	0,00860%	0,92712%
Austria	5,85892%	0,00953%	0,97643%
Portugal	7,54054%	0,07804%	2,79362%
Sweden	7,02297%	0,04610%	2,14717%
UK	6,62973%	0,02033%	1,42571%

Table 1. Expected Return and Variance

From this table it is evident that Greece has the highest return on their bond, about 4.4% spread over the second highest return represented by Italy. Germany has the lowest expected return is about 5.67%, which was expected due to the size of the German economy and their strict fiscal policy. Accordingly Germany has one of the lowest variances and standard deviations, along with Austria and the Netherlands.

5.2 Unconstrained MVP portfolio

The unconstrained mvp portfolio calculation allowed for the countries to have a negative weight. When the efficient portfolio was calculated with the utility function it results in a negative weight, which means that you must sell the asset in order to generate the portfolios expected return and variance. In table 2 the unrestricted portfolio weights for the EU 15 countries along with the expected return, variance, and standard deviation for the mvp portfolio are presented.

	Asset w	Asset μ	
Germany	-1,666262067	0,0567054	
Belgium	0,130650912	0,0607946	μ(mvp)
Denmark	0,906373018	0,0634784	5,2967088%
Spain	-0,693418893	0,0736324	
Finland	-0,85877457	0,0659649	σ²(mvp)
France	1,625240172	0,0586541	0,0011545%
Greece	0,120097447	0,1219514	
Ireland	-0,601010427	0,0635568	σ(mvp)
Italy	-0,073402727	0,0776973	0,3397790%
Luxembourg	-1,034344282	0,0585432	
Netherlands	3,318773146	0,0570189	
Austria	-0,427758646	0,0585892	
Portugal	0,643293497	0,0754054	
Sweden	-0,685093562	0,0702297	
UK	0,295636982	0,0662973	

Table 2. Unconstrained MVP portfolio.

The expected return of the unconstrained mvp portfolio is approximately 5.3% with a very low standard deviation of approximately 0.34%. However, according to the portfolio weights over 331% of The Netherlands would have to be bought and over 166% of Germany would need to be sold to form this minimum variance portfolio. In forming an optimal currency portfolio this would be impossible. Therefore, constraints must be applied to the model in order to get an accurate representation of an optimal currency portfolio that has minimum variance, i.e. indicating stability.

5.3 Constrained MVP Portfolios

Using the constrained portfolio weights the utility function was calculated. As previously stated our risk tolerance for our efficient portfolios is zero which will be equal to the mvp portfolio. Therefore our maximum utility calculation only consisted of the portfolio weights and the variance covariance matrix.

$$\max \ \mathbb{U} = -\frac{1}{2}\mathbf{w}'\mathbf{V}\mathbf{w}$$

By maximizing the Utility like this it would still provide the same answer as the unconstrained mvp portfolio, as a result linear constraints are applied to the model. As a reminder the constraints are as follows:

$$\mathbf{w}'\mathbf{I} = 1,\ w_i = 0; \text{or} \ \ lb \leq w_i \leq ub \ \forall i$$

$$lb_{it} \cong \frac{(GDP_{it} - (GDP_{it} * E(\sigma_{GDP})))}{\sum GDP_{it}}$$

$$ub_{it} \cong \frac{(GDP_{it} + (GDP_{it} * E(\sigma_{GDP})))}{\sum GDP_{it}}$$

These complex constraints provide a better representation of an individual country within the mvp portfolio by using their respective GDP be the determinant of their portfolio weight. In other words a country included in the analysis will only be able to have a weighting of zero or a weight that is proportional to their GDP with respect to the other countries involved in the analysis. This also means that if 15 countries are included in the analysis but only 12 are determined to be contributors in the mvp portfolio construction, then the bands in which the portfolio weights constrained will change based on the new total GDP. The weights of the countries are calculated based on their GDP divided by the total GDP. The bands are calculated by adding and subtracting approximately 10% of the respective country's GDP divided by the total GDP for the portfolio. An example of the variable bounds can be seen in Table 3 and Table 4.

	GDP*	Weights	LowBand	UpBand
Germany	189097,09	23,2324%	20,9092%	25,5557%
Belgium	23215,48	2,8523%	2,5670%	3,1375%
Denmark	16047,62	1,9716%	1,7744%	2,1688%
Spain	60910,78	7,4835%	6,7351%	8,2318%
Finland	12456,20	1,5304%	1,3773%	1,6834%
France	133976,29	16,4603%	14,8143%	18,1063%
Greece	13103,17	1,6099%	1,4489%	1,7708%
Ireland	10463,59	1,2856%	1,1570%	1,4141%
Italy	111735,85	13,7279%	12,3551%	15,1006%
Luxembourg	2019,89	0,2482%	0,2233%	0,2730%
Netherlands	40065,41	4,9224%	4,4302%	5,4147%
Austria	19015,53	2,3362%	2,1026%	2,5699%
Portugal	12003,28	1,4747%	1,3273%	1,6222%
Sweden	22735,95	2,7933%	2,5140%	3,0727%
UK	147089,10	18,0714%	16,2642%	19,8785%
Total	813935,23	1		

Table 3. Weights and Bounds EU 15

	GDP*	Weights	LowBand	UpBand
Germany	189097,09	30,1080%	27,0972%	33,1188%
Belgium	23215,48	3,6964%	3,3267%	4,0660%
Denmark	0,00	0,0000%	0,0000%	0,0000%
Spain	60910,78	9,6982%	8,7284%	10,6680%
Finland	12456,20	1,9833%	1,7849%	2,1816%
France	133976,29	21,3317%	19,1985%	23,4648%
Greece	13103,17	2,0863%	1,8777%	2,2949%
Ireland	10463,59	1,6660%	1,4994%	1,8326%
Italy	111735,85	17,7906%	16,0115%	19,5696%
Luxembourg	2019,89	0,3216%	0,2894%	0,3538%
Netherlands	40065,41	6,3792%	5,7413%	7,0171%
Austria	19015,53	3,0276%	2,7249%	3,3304%
Portugal	12003,28	1,9112%	1,7200%	2,1023%
Sweden	0,00	0,0000%	0,0000%	0,0000%
UK	0,00	0,0000%	0,0000%	0,0000%
Total	628062,56	1		

Table 4. Weights and Bounds Euro

In Table 3 all of the countries are included therefore they all have a weight in the portfolio based on their GDP to total GDP of the EU 15. Table 4 on the other hand only includes the countries that were in the Eurozone in 2001. Since Denmark, Sweden, and the United Kingdom are not included the portfolio weights for the countries included will change based on the new total GDP. Germany that has the largest weight in Table 3 of approximately 23.2% and in Table 4 has a weight of approximately 30.1%. What is most interesting that the bands increase from Table 3 to Table 4 although they are still proportionally to the share of the country's GDP to total GDP for the portfolio. For instance Germany's weight can fluctuate within a band of approximately 4.6% in Table 3 with 15 countries, while in Table 4 it can fluctuate within a band of approximately 6%.

The benchmark portfolios are simply the percentage of a country's GDP with respect to the total GDP of the beginning portfolio. The country weights in Table 3 and Table 4 are examples of benchmark portfolio weights for the entire EU 15 member states, and the original Eurozone members respectively.

A total of twelve portfolios are calculated from the 15 EU countries using GDP information from year end 1995 and 2001, six mvp portfolios and six corresponding benchmark portfolios. The reason that the analysis takes place in 1995 is because that is the year the final members of the original Euro members joined the European Union and subsequent EMU. Furthermore, 2001 is used as an analysis year because that is the final year prior to the circulation of the Euro banknotes. Although the majority of the Original Euro members had their currencies pegged to one another in 1999, this is not used as the analysis year because Greece was denied entry until 2001 for not meeting the convergence criteria at the time.

The first two mvp portfolio calculated used the ECU original member states consisting of Germany, Belgium, Denmark, France, United Kingdom, Ireland, Italy, Luxembourg, and The Netherlands based on their 1995 and 2001 GDP values. Since we also have an implied weight for these countries i.e. their weight in the ECU basket this will also be used in the analysis.

After solver was used to maximize the utility function the results for the weights of the mvp portfolio were gathered. Based off the 1995 GDP data Denmark, Ireland, and Italy were not included in the optimal portfolio; this means that if the optimal currency portfolio was based off this data they would not be included. The 2001 GDP data only includes Germany, France, The Netherlands, and Luxembourg in the mvp portfolio, therefore only these four make up the optimal currency portfolio.

Microsoft Excel 12.0 Answer Report
Worksheet: Original ECU based on 1995 GDP
Report Created: 2011-05-19 20:19:35

Target Cell (Max)

Cell	Name	Original Value	Final Value
$S$47	Utility Function	-0,061467094	-0,058838614

Adjustable Cells

Cell	Name	BM Weights	MVP Weights	ECU Weights
$Q$44	Germany constrained	0,34329464	0,450815896	0,3298
$Q$45	Belgium constrained	0,038665265	0,050775381	0,0964
$Q$46	Denmark constrained	0,024763829	0	0,0306
$Q$47	U constrained	0	0	0
$Q$48	Finland constrained	0	0	0
$Q$49	France constrained	0,213638938	0,258249862	0,1983
$Q$50	Greece constrained	0	0	0
$Q$51	Ireland constrained	0,009125448	0	0,0115
$Q$52	Italy constrained	0,153228615	0	0,0949
$Q$53	Luxembourg constrained	0,002813585	0,003694811	0
$Q$54	Netherlands constrained	0,057012182	0,074868626	0,1051
$Q$55	Austria constrained	0	0	0
$Q$56	Portugal constrained	0	0	0
$Q$57	Sweden constrained	0	0	0
$Q$58	United Kingdom constrained	0,157457498	0,161595424	0,1334

Expected Return
0,056705
0,060795
0,063478
0,073632
0,065965
0,058654
0,121951
0,063557
0,077697
0,058543
0,057019
0,058589
0,075405
0,07023
0,066297

	benchmark
return(μ)	0,06226
variance	0,000382
stdev	0,019544

	mvp portfolio
μ(mvp)	0,058997
Variance	0,000235
stdev	0,015341

	ECU portfolio
μ(ECU)	0,061077
Variance	0,000322
stdev	0,017933

In Appendix for chapter 5 there is a comprehensive results section for the results tables in this section, which outlines the constraints applied and whether the result obtained was restricted in finding the mvp portfolio. By taking a quick look at Appendix 5.1 it is apparent that the only binding constraint applied to the portfolio weights was for the weights that were equal to zero. The countries included in the mvp portfolio were all in between the upper and lower bounds, hence they were not binding.

Microsoft Excel 12.0 Answer Report
Worksheet: Original ECU based on 2001 GDP
Report Created: 2011-05-19 19:58:55

Target Cell (Max)

Cell	Name	Original Value	Final Value
$S$47	Utiltiy Function	-0,053946874	-0,05199727

Adjustable Cells

Cell	Name	BM Weights	MVP Weights	ECU Weights	Expected Return
$Q$44	Germany constrained	0,280680116	0,569633997	0,3298	0,056705
$Q$45	Belgium constrained	0,034459144	0	0,0964	0,060795
$Q$46	Denmark constrained	0,023819757	0	0,0306	0,063478
$Q$47	Spain constrained	0	0	0	0,073632
$Q$48	Finland constrained	0	0	0	0,065965
$Q$49	France constrained	0,198863342	0,30358869	0,1983	0,058654
$Q$50	Greece constrained	0	0	0	0,121951
$Q$51	Ireland constrained	0,015531289	0	0,0115	0,063557
$Q$52	Italy constrained	0,165851471	0	0,0949	0,077697
$Q$53	Luxembourg constrained	0,002998162	0,006084702	0	0,058543
$Q$54	Netherlands constrained	0,059469794	0,120692612	0,1051	0,057019
$Q$55	Austria constrained	0	0	0	0,058589
$Q$56	Portugal constrained	0	0	0	0,075405
$Q$57	Sweden constrained	0	0	0	0,07023
$Q$58	United Kingdom constrained	0,218326925	0	0,1334	0,066297

	benchmark
return(μ)	0,063101
variance	0,000409
stdev	0,020231

	mvp portfolio
μ(mvp)	0,058997
Variance	0,000235
stdev	0,015341

	ECU portfolio
μ(ECU)	0,061077
Variance	0,000322
stdev	0,017933

Although the mvp for the original ECU based on the 2001 GDP begins with nine countries, only four are included in the optimization solution. Appendix 5.2 shows that Belgium, Denmark, Ireland, Italy, and the United Kingdom all have binding constraints for their weights to equal zero. Additionally, Germany, The Netherlands, and Luxembourg are constrained to the upper bounds, while France still lies within the upper and lower bounds.

In the 1995 original ECU chart it appears that Italy is visibly underweighted in the ECU basket while Belgium and The Netherlands are noticeably overweighed. Similar results are seen in the 2001 ECU chart in addition to a visible underweighting of the United Kingdom in the ECU basket. Thought the ECU basket used here was based on the GNP of the county in 1989 it is interesting to see that it has a lower variance and standard deviation then the benchmarks from 1995 and 2001.

The most important analysis in this paper is the evaluation of the original Eurozone members in order to determine if they in fact were an optimal currency portfolio based on future financial stability. The benchmark portfolio for the 1995 and 2001 GDP data only includes the original 12 Eurozone member states in the analysis.

Microsoft Excel 12.0 Answer Report
Worksheet: Euro members based off 1995 GDP
Report Created: 2011-05-19 18:50:26

Target Cell (Max)

Cell	Name	Original Value	Final Value
$S$47	Utility Function	-0,125196975	-0,093022401

Adjustable Cells

Cell	Name	BM Weights	MVP Weights
$Q$44	Germany constrained	0,349244677	0,397835216
$Q$45	Belgium constrained	0,039335418	0,044808169
$Q$46	Denmark constrained	0	0
$Q$47	Spain constrained	0,082611639	0,026391364
$Q$48	Finland constrained	0,018093538	0,020610899
$Q$49	France constrained	0,217341762	0,247580601
$Q$50	Greece constrained	0,018234533	0
$Q$51	Ireland constrained	0,009283611	0,010575244
$Q$52	Italy constrained	0,155884398	0,145286714
$Q$53	Luxembourg constrained	0,002862351	0,00326059
$Q$54	Netherlands constrained	0,058000326	0,066069933
$Q$55	Austria constrained	0,032991194	0,037581271
$Q$56	Portugal constrained	0,016116552	0
$Q$57	Sweden constrained	0	0
$Q$58	United Kingdom constrained	0	0

Expected Return
0,056705405
0,060794595
0,063478378
0,073632432
0,065964865
0,058654054
0,121951351
0,063556757
0,077697297
0,058543243
0,057018919
0,058589189
0,075405405
0,07022973
0,066297297

	benchmark
return(μ)	0,063768
variance	0,00025
stdev	0,015824

	mvp portfolio
μ(mvp)	0,061228
Variance	0,000186
stdev	0,01364

When examining Appendix 5.3 one would say that it exemplifies the complexity of this optimization problem. Both Greece and Portugal have their weights binding to equal zero, whereas Spain is binding to the lower bound constraint. Furthermore it shows that Germany, Belgium, Finland, Ireland, France, Luxembourg, The Netherlands, and Austria are all binding to the upper bound constraint, and Italy is not binding and lies in between its upper and lower bounds.

Microsoft Excel 12.0 Answer Report
Worksheet: Euro members based off 2001 GDP
Report Created: 2011-05-21 22:34:13

Target Cell (Max)

Cell	Name	Original Value	Final Value
$S$47	Utility Function	-0,00013965	-0,000101274

Adjustable Cells

Cell	Name	BM Weights	MVP Weights
$Q$44	Germany constrained	0,301080025	0,344978344
$Q$45	Belgium constrained	0,036963644	0,042353048
$Q$46	Denmark constrained	0	0
$Q$47	Spain constrained	0,096982031	0,111122286
$Q$48	Finland constrained	0,019832733	0,022724401
$Q$49	France constrained	0,213316785	0,244418975
$Q$50	Greece constrained	0,020862837	0
$Q$51	Ireland constrained	0,016660107	0,019089198
$Q$52	Italy constrained	0,177905602	0,103844742
$Q$53	Luxembourg constrained	0,003216069	0,003684981
$Q$54	Netherlands constrained	0,063792075	0,07309314
$Q$55	Austria constrained	0,030276487	0,034690885
$Q$56	Portugal constrained	0,019111606	0
$Q$57	Sweden constrained	0	0
$Q$58	UK constrained	0	0

Expected Return
0,056705405
0,060794595
0,063478378
0,073632432
0,065964865
0,058654054
0,121951351
0,063556757
0,077697297
0,058543243
0,057018919
0,058589189
0,075405405
0,07022973
0,066297297

	benchmark
return(μ)	0,064747761
variance	0,0002793
stdev	0,01671228

	mvp portfolio
μ(mvp)	0,06185199
Variance	0,000202548
stdev	0,014231943

From the above tables the mvp portfolios that were calculated form both the 1995 and the 2001 GDP data shows that Greece and Portugal are not included in the optimal currency portfolio. The results in appendix 5.6 indicate that Greece and Portugal binding to the constraint for the portfolio weight to equal zero.

Austria, Finland, and Sweden joined the European Union in 1995 bringing the membership of the economic and political union to a total of 15 member states. All three of the new member states were required to join the monetary union in accordance with the Accession Treaty in signed in 1994[21]. Although the United Kingdom and Denmark had an opt out for the Euro they are included in the analysis of the EU 15 countries since they also had the option to join the Euro.

Microsoft Excel 12.0 Answer Report
Worksheet: EU 15 based on 1995 GDP
Report Created: 2011-05-19 18:17:49

Target Cell (Max)

Cell	Name	Original Value	Final Value
$S$47	Utility Function	-0,271325093	-0,092152195

Adjustable Cells

Cell	Name	BM Weights	MVP Weights	Expected Return
$Q$44	Germany constrained	0,238886972	0,322799343	0,056705405
$Q$45	Belgium constrained	0,028587181	0,036356881	0,060794595
$Q$46	Denmark constrained	0,019420541	0,023285384	0,063478378
$Q$47	Spain constrained	0,071060113	0,076356161	0,073632432
$Q$48	Finland constrained	0,02059142	0,016723468	0,065964865
$Q$49	France constrained	0,177896035	0,200884316	0,058654054
$Q$50	Greece constrained	0,013443274	0	0,121951351
$Q$51	Ireland constrained	0,006715675	0	0,063556757
$Q$52	Italy constrained	0,157884944	0,044080596	0,077697297
$Q$53	Luxembourg constrained	0,001756205	0	0,058543243
$Q$54	Netherlands constrained	0,042225826	0,053608454	0,057018919
$Q$55	Austria constrained	0,023277197	0,030493051	0,058589189
$Q$56	Portugal constrained	0,010538078	0,014896183	0,075405405
$Q$57	Sweden constrained	0,036067159	0,032459156	0,07022973
$Q$58	United Kingdom constrained	0,15164938	0,148057007	0,066297297

benchmark	
return(μ)	0,065131
variance	0,000267
stdev	0,016349

mvp portfolio	
μ(mvp)	0,061988
Variance	0,000184
stdev	0,013576

Using the GDP data from 1995 the utility function was maximized to find the mvp portfolio for the entire European Union. The results from this mvp portfolio provide the lowest variance and standard deviation out of all the portfolios analyzed. This mvp portfolio does not include Greece, Ireland, and Luxembourg since the optimal portfolio binds them to the constraint that their respective weights be equal to zero as seen in appendix 5.5. Also seen in appendix 5.5 is that every other country except Italy is binding to the upper bounds, while Italy is binding to the lower bounds.

[21] Eichengreen, B. and F. Ghironi, 2001

Microsoft Excel 12.0 Answer Report
Worksheet: EU 15 based off 2001 GDP
Report Created: 2011-05-19 17:20:09

Target Cell (Max)

Cell	Name	Original Value	Final Value
$S$47	Utility Function	-0,131003311	-0,097186488

Adjustable Cells

Cell	Name	BM Weights	MVP Weights	Expected Return
$Q$44	Germany constrained	0,232324495	0,263690684	0,056705405
$Q$45	Belgium constrained	0,028522516	0,032373349	0,060794595
$Q$46	Denmark constrained	0,019716085	0,022377959	0,063478378
$Q$47	Spain constrained	0,074834926	0,084938409	0,073632432
$Q$48	Finland constrained	0,015303671	0,017369823	0,065964865
$Q$49	France constrained	0,16460313	0,18682624	0,058654054
$Q$50	Greece constrained	0,016098537	0	0,121951351
$Q$51	Ireland constrained	0,012855556	0,014591188	0,063556757
$Q$52	Italy constrained	0,13727855	0,055812562	0,077697297
$Q$53	Luxembourg constrained	0,002481638	0,002816685	0,058543243
$Q$54	Netherlands constrained	0,049224327	0,055870116	0,057018919
$Q$55	Austria constrained	0,023362459	0,02651663	0,058589189
$Q$56	Portugal constrained	0,014747223	0	0,075405405
$Q$57	Sweden constrained	0,027933365	0,031704656	0,07022973
$Q$58	United Kingdom constrained	0,180713523	0,205111702	0,066297297

	benchmark
return(μ)	0,065156
variance	0,000262
stdev	0,016187

	mvp portfolio
μ(mvp)	0,062692
Variance	0,000194
stdev	0,013942

The mvp portfolio analysis for the EU 15 countries based on the 2001 GDP provides the most stable composition of an optimal currency portfolio just prior to the circulation of the Euro banknotes and coins. According to the convergence report from 2001 all of the countries within the European Union met the criteria to join the monetary union, however according to the results from table above it indicates that Greece and Portugal would not be included within the optimal currency portfolio. [22]

5.4 Analysis Table

The results from above are combined and presented in the table that follows. In this table it presents a clearer picture of the twelve portfolios analyzed by splitting them into two sections, one with the mvp portfolios calculated, and the other consisting of the benchmark portfolios. In the mvp portfolio section, the dark grey boxes with the number 0 inside represent the assets that were eliminated from the mvp portfolio calculation. Greece was the worst performing country since they were not selected for any of the four efficient portfolios calculations. The second worst performing country was Portugal since it was only selected in one of the four

[22] Eichengreen, B. and F. Ghironi, 2001

efficient portfolios calculations. The third worst performing country in the efficient portfolio calculations was Ireland, since they were only selected in three out of the six portfolios.[23]

MVP	1995 EU	2001 EU	1995 EURO	2001 EURO	1995 O. ECU	2001 O.ECU
Germany	0,3227993	0,26369068	0,3978352	0,3449783	0,4508159	0,569634
Belgium	0,0363569	0,03237335	0,0448082	0,042353	0,05077538	0
Denmark	0,0232854	0,02237796	0	0	0	0
Spain	0,0763562	0,08493841	0,0263914	0,048185	0	0
Finland	0,0167235	0,01736982	0,0206109	0,0227244	0	0
France	0,2008843	0,18682624	0,2475806	0,244419	0,25824986	0,30358869
Greece	0	0	0	0	0	0
Ireland	0	0,01459119	0,0105752	0,0190892	0	0
Italy	0,0440806	0,05581256	0,1452867	0,1667821	0	0
Luxembourg	0	0,00281668	0,0032606	0,003685	0,00369481	0,0060847
Netherlands	0,0536085	0,05587012	0,0660699	0,0730931	0,07486863	0,12069261
Austria	0,0304931	0,02651663	0,0375813	0,0346909	0	0
Portugal	0,0148962	0	0	0	0	0
Sweden	0,0324592	0,03170466	0	0	0	0
UK	0,148057	0,2051117	0	0	0,16159542	0
RETURN	0,0619879	0,06269242	0,0612285	0,0621078	0,05899654	0,05734601
Std. Dev. σ	0,0135759	0,01394177	0,0136398	0,0144564	0,01534127	0,01442183

BM	1995 EU	2001 EU	1995 EURO	2001 EURO	1995 O. ECU	2001 O.ECU
Germany	0,238887	0,2323245	0,3492447	0,30108	0,34329464	0,28068012
Belgium	0,0285872	0,02852252	0,0393354	0,0369636	0,03866527	0,03445914
Denmark	0,0194205	0,01971608	0	0	0,02476383	0,02381976
Spain	0,0710601	0,07483493	0,0826116	0,096982	0	0
Finland	0,0205914	0,01530367	0,0180935	0,0198327	0	0
France	0,177896	0,16460313	0,2173418	0,2133168	0,21363894	0,19886334
Greece	0,0134433	0,01609854	0,0182345	0,0208628	0	0
Ireland	0,0067157	0,01285556	0,0092836	0,0166601	0,00912545	0,01553129
Italy	0,1578849	0,13727855	0,1558844	0,1779056	0,15322862	0,16585147
Luxembourg	0,0017562	0,00248164	0,0028624	0,0032161	0,00281359	0,00299816
Netherlands	0,0422258	0,04922433	0,0580003	0,0637921	0,05701218	0,05946979
Austria	0,0232772	0,02336246	0,0329912	0,0302765	0	0
Portugal	0,0105381	0,01474722	0,0161166	0,0191116	0	0
Sweden	0,0360672	0,02793336	0	0	0	0
UK	0,1516494	0,18071352	0	0	0,1574575	0,21832693
RETURN	0,0651312	0,06515589	0,0637683	0,0647478	0,06225999	0,06310143
Std. Dev. σ	0,0163489	0,01618662	0,0158238	0,0167123	0,01954426	0,02023131

IR	1,1335253	1,09738724	1,1629237	1,1702413	0,77645789	0,99069316

Also included in this table is the information ratio (IR), which calculated the excess return over the benchmark portfolio relative to the standard deviation of the returns. According to Grinold and Kahn (1995) portfolio managers that have an IR above 0.75 is considered great, and above 1.0 is considered exceptional. The IR for the EU 15 analysis was 1.1335 and 1.0974 for 1995 and 2001 respectively, since this is above 1.0 it is determined that the mvp portfolio performs better then the benchmark portfolio. The best results are found in the Euro analysis where the IR is 1.163 and 1.17 for 1995 and 2001 respectively. While the analysis on

[23] Although Ireland and Denmark both were selected in only 50% of their efficient portfolios calculations, Ireland was not selected in three out of the four common efficient portfolio calculations.

the original ECU member states are underperforming against the benchmark with an IR of 0.7765 and 0.9907 for 1995 and 2001 respectively.

In order to illustrate that the mvp portfolios do in fact perform better then their benchmark portfolios they are charted in Figure 5.

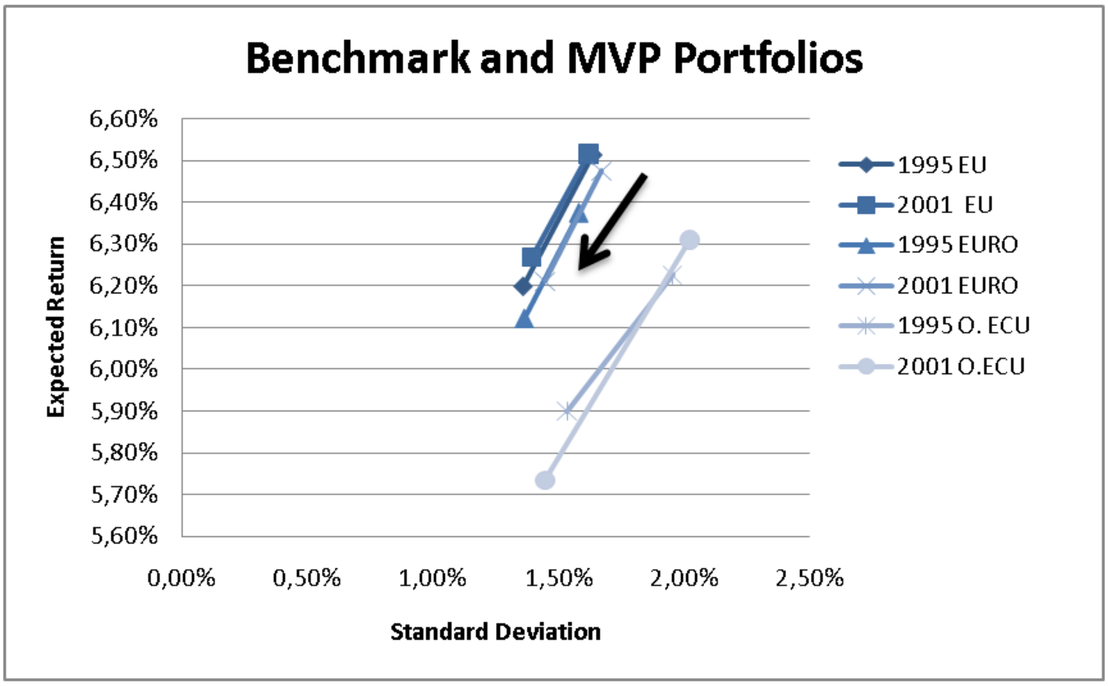


Figure 5. Scatter Plot of Benchmark and MVP Portolios

The lines on the chart link the benchmark portfolio to its mvp portfolio, with the black arrow indicating the movement from the benchmark to the efficient mvp portfolio. By looking closely at this chart it can be seen that given the specific year i.e. 1995 or 2001 the most efficient portfolios are created with all of the EU 15 member states.

6 Conclusion

The framework of mean variance portfolio optimization problem that was developed Markowitz (1952) has been developed over the years to allow for the calculation of many different asset classes. Furthermore, the development of algorithmic programming has allowed for the advancement in the calculation of complex optimization problems. Applying the theory of optimal currency areas to the mean variance portfolio theory has permitted the assessment of the legendary monetary union of the European Union; the Eurozone. Where main policies and goals set by the European Central Bank are intended ensure maximum stability within the Eurozone.

In this paper, it is assumed that maximum stability is equivalent of an optimal portfolio with a risk tolerance of zero, which by definition would be the minimum variance portfolio of the monetary union. By using a basket of EU currencies based on their respective economic weight, and their sovereign bond yields and deviations; the minimum variance portfolios were successfully constructed, despite the complexity of the optimization problem that contained variable restrictions.

The data obtained from the calculation of the efficient minimum variance portfolios indicates that the Eurozone did not form an optimal currency portfolio that provided the most stability. Furthermore, it was also determined that the original European Economic Community members that were originally used in the basket of currencies called the European Currency Unit (ECU), and the 15 members of the European Union for the period just prior to the implementation of the Euro, also did not form optimal currency portfolios that would provide the most stability had they been used to create a currency union.

The analysis also determined that Greece, Portugal, and Ireland were the countries that were most frequently left out of the optimal minimum variance portfolio, implying that they could contribute to instability within the optimal currency portfolio if it were included in the creation of the Eurozone. Even though the data period is from 1993 until 2001 this analysis accurately indicates the potential countries that would cause instability within the currency union, as seen during the current 2010/2011 sovereign debt crisis. These three countries had to receive a bailout from the European Union in order to prevent default on their government debt. Because of this there was an increase in their sovereign risk premiums, which increased due to concern over their increasing government deficits and debt levels.

However, other European Union member states that were included in the minimum variance portfolios are also experiencing economic hardships due to the debt crisis. For example, Spain and Italy whose economies are also suffering due to sovereign debt problems were found to be included in most of the optimal portfolio calculations. In those calculations however they were often binding to the lower bound constraint. This implies that for those portfolios the optimal weight without the lower bound would have been between zero and the lower bound, meaning that the variance by including the country is less then by not including them.

Another possible reason as to why Italy was included in the optimal portfolios is because they had a relatively normal expected return, but a large variance due to the fact that prior to the Euro the Italian Lira was allowed to fluctuate more then the other countries. This also contributed to a lower correlation coefficient between Italy and the other countries, thus making Italy more desirable to the minimum variance portfolio due to the diversification effect.

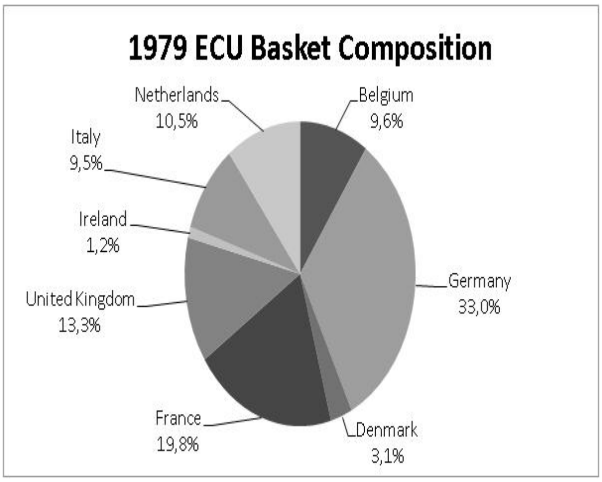
Although the Eurozone may be considered and optimal currency area based on the criteria created by Mundell, it does not account for the strict ECB policies that are in place to provide stability. However, this paper concludes that the Eurozone was not an optimal currency portfolio since the combination of countries used to create the monetary union would not be the portfolio that provided the most stability. The analysis presented in this paper clearly shows that the most stable optimal currency portfolio will not include countries that have excess economic risk.

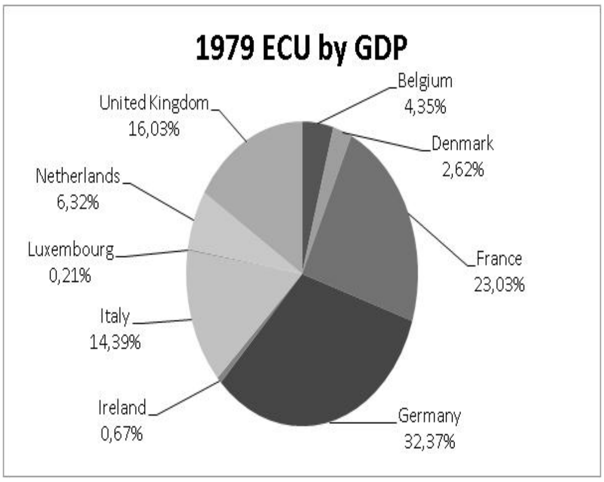
Future research to be done on this topic should include the enlargement of the Eurozone and whether the expansion of the Eurozone causes instability in monetary union. Additionally, this analysis could be used to examine other possible optimal currency portolios, such as Southeast Asia.

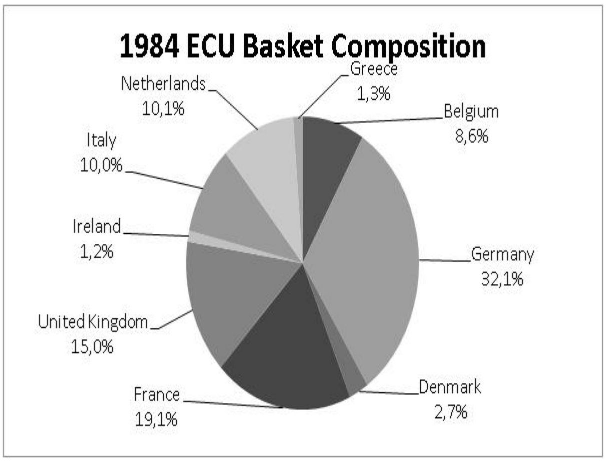
7 APPENDIX

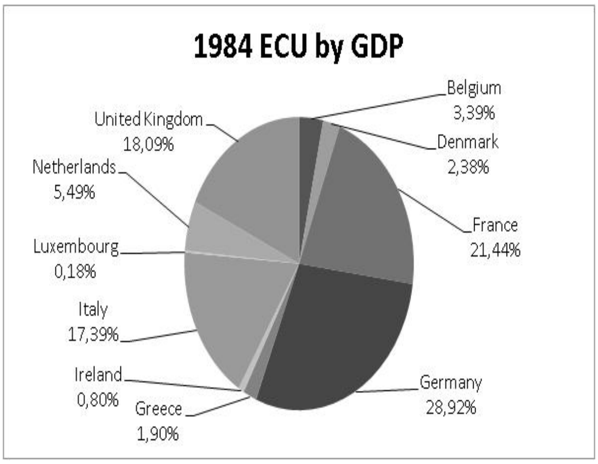
Appendix 2.1

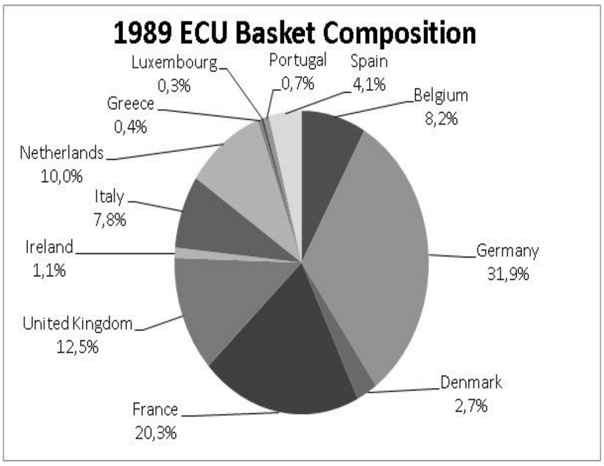
Below are charts based on different weighting schemes of the Eurozone and the EU based off GDP for a given year, as well as the historical weighting of the ECU.

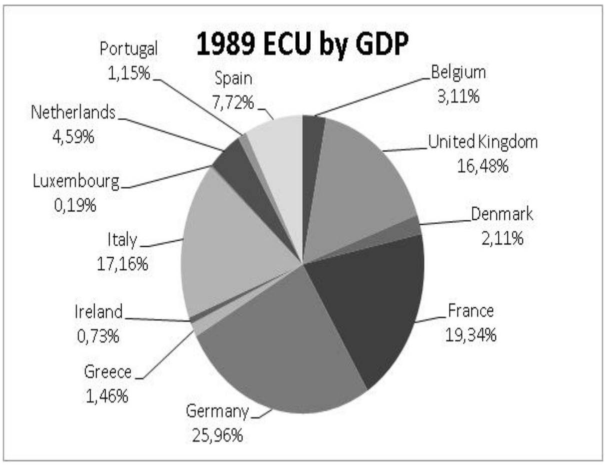




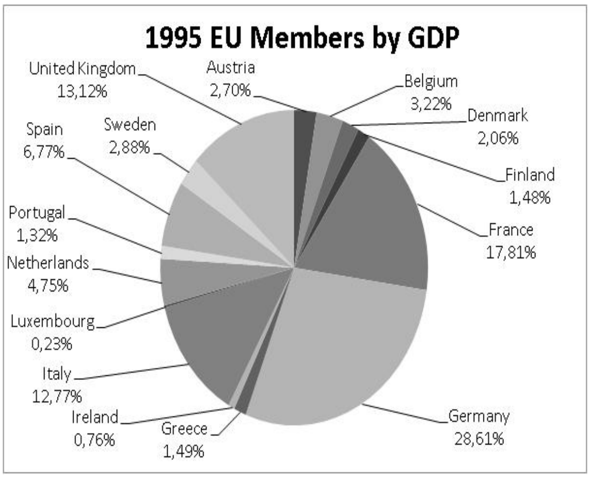


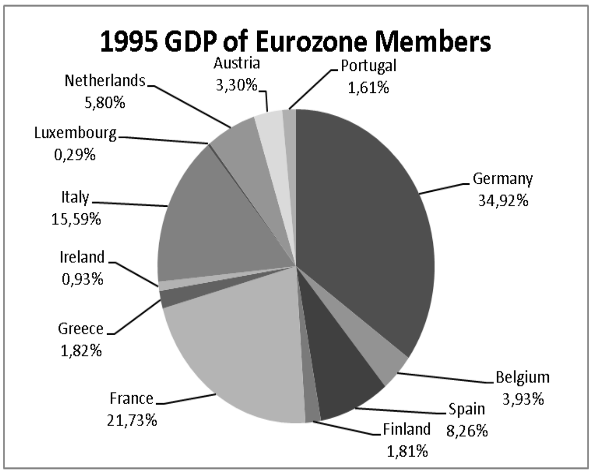


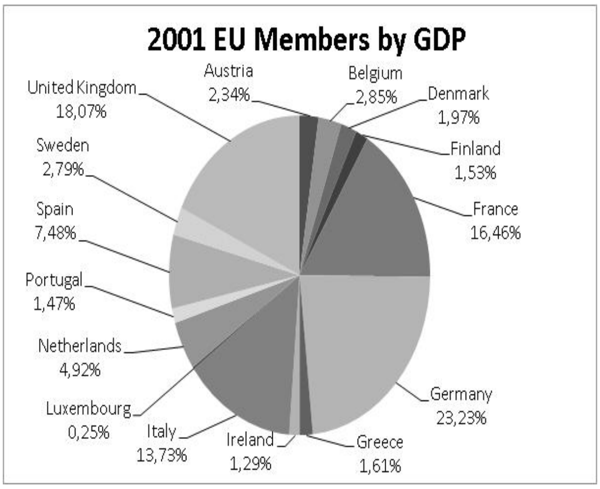


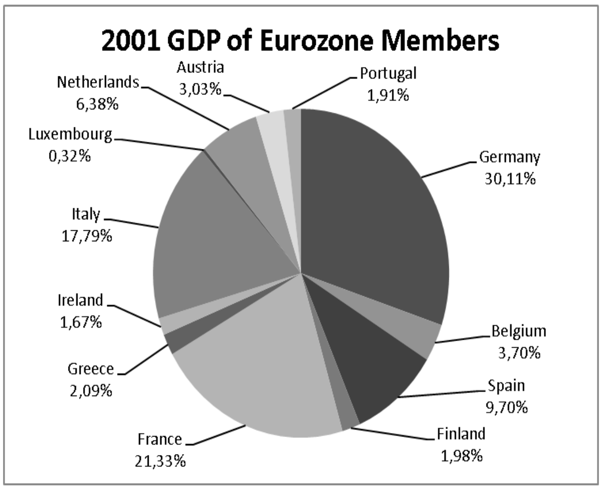


Appendix 2.1 (continued)

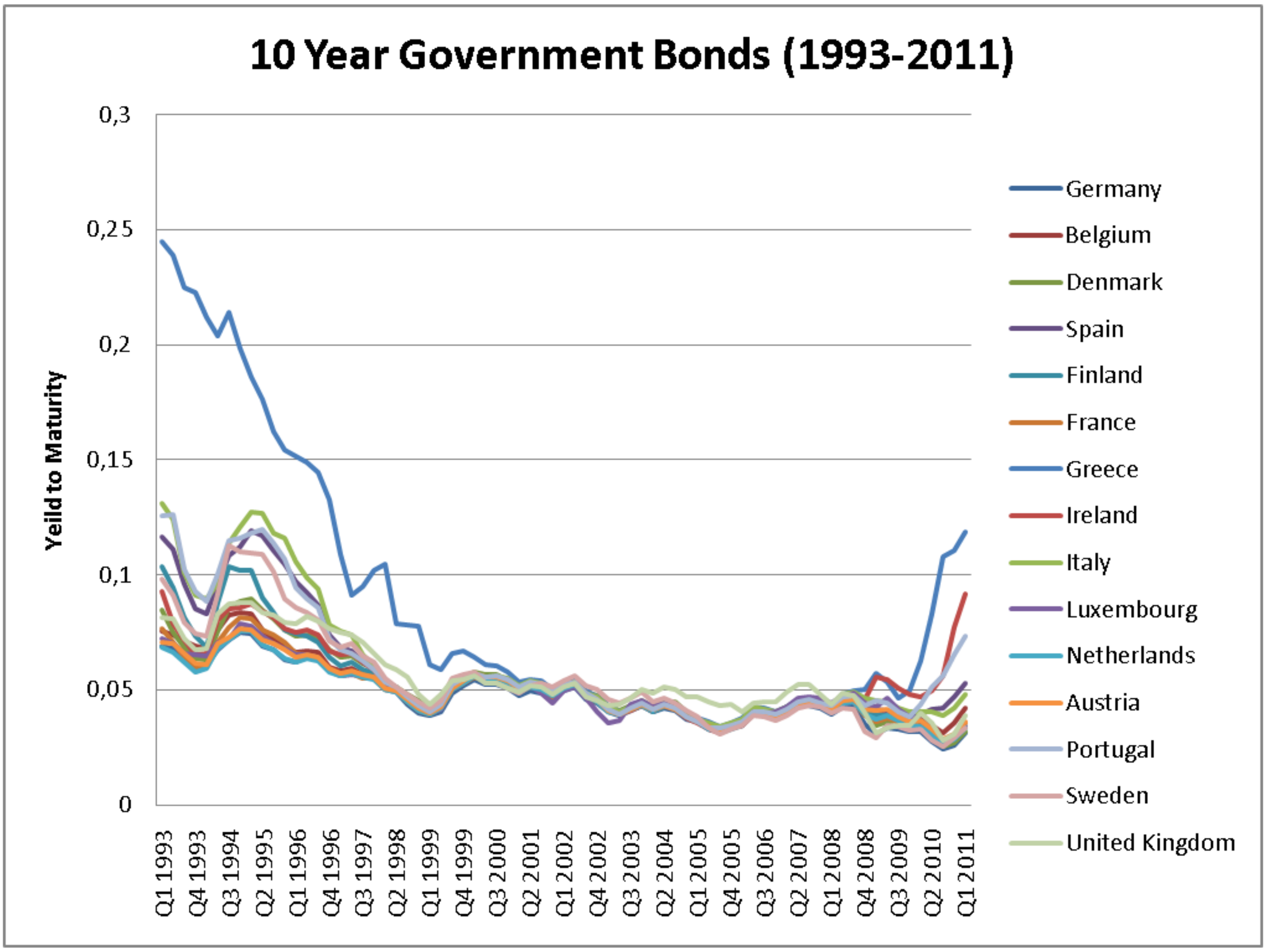








Appendix 3.1



This table shows the yield to maturity quarterly from 1993 to 2011 on the European Union (15) members analyzed in this paper.

Appendix 4.1
This is the VBA code used to gather the Benchmark Portfolio weights for a particular GDP year, and countries included in the benchmark.

```
Attribute VB_Name = "BenchMarkModel"
Sub BenchmarkFinder()
Attribute BenchmarkFinder.VB_Description = "This should find the
benchmark portfolio based on gdp percent of the year compared with the
make up of original potential euro states"
Attribute BenchmarkFinder.VB_ProcData.VB_Invoke_Func = "b\n14"
'
' BenchmarkFinder Macro
' This should find the benchmark portfolio based on gdp percent of the
year compared with the make up of original potential euro states
'
' Keyboard Shortcut: Ctrl+b
'
    Range("D58:D60").Select
    Selection.Copy
    Range("Q42").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
    Range("G58:G60").Select
    Application.CutCopyMode = False
    Selection.Copy
    Range("Q38:Q40").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
    Range("B39").Select
    Application.CutCopyMode = False
    ActiveCell.FormulaR1C1 = "0"
    Range("B42").Select
    ActiveCell.FormulaR1C1 = "0"
    Range("B52").Select
    ActiveCell.FormulaR1C1 = "0"
    Range("G58:G60").Select
    Selection.Copy
    Range("O38").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
    Range("D58:D60").Select
    Application.CutCopyMode = False
    Selection.Copy
    Range("O42").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
    Range("B50").Select
    Application.CutCopyMode = False
    ActiveCell.FormulaR1C1 = "0"
    Range("B51").Select
    ActiveCell.FormulaR1C1 = "0"
    Range("D58:D60").Select
```

```
    Selection.Copy
    Range("N42").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
    Range("G58:G60").Select
    Application.CutCopyMode = False
    Selection.Copy
    Range("N38").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
    Range("B45").Select
    Application.CutCopyMode = False
    ActiveCell.FormulaR1C1 = "0"
    Range("D58:D60").Select
    Selection.Copy
    Range("M42").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
    Range("G58:G60").Select
    Application.CutCopyMode = False
    Selection.Copy
    Range("M38").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
    Range("B39").Select
    Application.CutCopyMode = False
    ActiveCell.FormulaR1C1 = "=R[-18]C[4]"
    Range("B39").Select
    Selection.AutoFill Destination:=Range("B39:B53"),
Type:=xlFillDefault
    Range("B39:B53").Select
    Range("B41").Select
    ActiveCell.FormulaR1C1 = "0"
    Range("B53").Select
    ActiveCell.FormulaR1C1 = "0"
    Range("B52").Select
    ActiveCell.FormulaR1C1 = "0"
    Range("B53").Select
    ActiveWindow.SmallScroll Down:=9
    Range("G58:G60").Select
    Selection.Copy
    Range("R38").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
    Range("D58:D60").Select
    Application.CutCopyMode = False
    Selection.Copy
    Range("R42").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone,
SkipBlanks _
        :=False, Transpose:=False
End Sub
```

Appendix 5.1

Microsoft Excel 12.0 Answer Report
Worksheet: Original ECU based on 1995 GDP
Report Created: 2011-05-19 20:19:35

Target Cell (Max)

Cell	Name	Original Value	Final Value
$S$47	Utility Function	-0,061467094	-0,058838614

Adjustable Cells

Cell	Name	BM Weights	MVP Weights	ECU Weights	Expected Return
$Q$44	Germany constrained	0,34329464	0,450815896	0,3298	0,056705
$Q$45	Belgium constrained	0,038665265	0,050775381	0,0964	0,060795
$Q$46	Denmark constrained	0,024763829	0	0,0306	0,063478
$Q$47	U constrained	0	0	0	0,073632
$Q$48	Finland constrained	0	0	0	0,065965
$Q$49	France constrained	0,213638938	0,258249862	0,1983	0,058654
$Q$50	Greece constrained	0	0	0	0,121951
$Q$51	Ireland constrained	0,009125448	0	0,0115	0,063557
$Q$52	Italy constrained	0,153228615	0	0,0949	0,077697
$Q$53	Luxembourg constrained	0,002813585	0,003694811	0	0,058543
$Q$54	Netherlands constrained	0,057012182	0,074868626	0,1051	0,057019
$Q$55	Austria constrained	0	0	0	0,058589
$Q$56	Portugal constrained	0	0	0	0,075405
$Q$57	Sweden constrained	0	0	0	0,07023
$Q$58	United Kingdom constrained	0,157457498	0,161595424	0,1334	0,066297

	benchmark
return(μ)	0,06226
variance	0,000382
stdev	0,019544

	mvp portfolio
μ(mvp)	0,058997
Variance	0,000235
stdev	0,015341

	ECU portfolio
μ(ECU)	0,061077
Variance	0,000322
stdev	0,017933

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$Q$59	constrained	1	$Q$59=1	Not Binding	0
$Q$44	Germany constrained	0,450815896	$Q$44<=$X$44	Not Binding	0,01373
$Q$45	Belgium constrained	0,050775381	$Q$45<=$X$45	Not Binding	0,00155
$Q$46	Denmark constrained	0	$Q$46<=$X$46	Not Binding	0,03351
$Q$47	U constrained	0	$Q$47<=$X$47	Binding	0
$Q$48	Finland constrained	0	$Q$48<=$X$48	Binding	0
$Q$49	France constrained	0,258249862	$Q$49<=$X$49	Not Binding	0,03085
$Q$50	Greece constrained	0	$Q$50<=$X$50	Binding	0
$Q$51	Ireland constrained	0	$Q$51<=$X$51	Not Binding	0,01235
$Q$52	Italy constrained	0	$Q$52<=$X$52	Binding	0
$Q$53	Luxembourg constrained	0,003694811	$Q$53<=$X$53	Not Binding	0,00011
$Q$54	Netherlands constrained	0,074868626	$Q$54<=$X$54	Not Binding	0,00228
$Q$55	Austria constrained	0	$Q$55<=$X$55	Binding	0
$Q$56	Portugal constrained	0	$Q$56<=$X$56	Binding	0
$Q$57	Sweden constrained	0	$Q$57<=$X$57	Binding	0
$Q$58	United Kingdom constrained	0,161595424	$Q$58<=$X$58	Not Binding	0,05148
$Q$44	Germany constrained	0,450815896	$Q$44>=0	Not Binding	0,45082
$Q$45	Belgium constrained	0,050775381	$Q$45>=0	Not Binding	0,05078
$Q$46	Denmark constrained	0	$Q$46>=0	Binding	0
$Q$47	U constrained	0	$Q$47>=0	Binding	0
$Q$48	Finland constrained	0	$Q$48>=0	Binding	0
$Q$49	France constrained	0,258249862	$Q$49>=0	Not Binding	0,25825
$Q$50	Greece constrained	0	$Q$50>=0	Binding	0
$Q$51	Ireland constrained	0	$Q$51>=0	Binding	0
$Q$52	Italy constrained	0	$Q$52>=0	Binding	0
$Q$53	Luxembourg constrained	0,003694811	$Q$53>=0	Not Binding	0,00369
$Q$54	Netherlands constrained	0,074868626	$Q$54>=0	Not Binding	0,07487
$Q$55	Austria constrained	0	$Q$55>=0	Binding	0
$Q$56	Portugal constrained	0	$Q$56>=0	Binding	0
$Q$57	Sweden constrained	0	$Q$57>=0	Binding	0
$Q$58	United Kingdom constrained	0,161595424	$Q$58>=0	Not Binding	0,1616
$Q$52	Italy constrained	0	$Q$52=0	Not Binding	0

Appendix 5.2

Microsoft Excel 12.0 Answer Report
Worksheet: Original ECU based on 2001 GDP
Report Created: 2011-05-19 19:58:55

Target Cell (Max)

Cell	Name	Original Value	Final Value
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Adjustable Cells

Cell	Name	BM Weights	MVP Weights	ECU Weights	Expected Return
$Q$44	Germany constrained	0,280680116	0,569633997	0,3298	0,056705
$Q$45	Belgium constrained	0,034459144	0	0,0964	0,060795
$Q$46	Denmark constrained	0,023819757	0	0,0306	0,063478
$Q$47	Spain constrained	0	0	0	0,073632
$Q$48	Finland constrained	0	0	0	0,065965
$Q$49	France constrained	0,198863342	0,30358869	0,1983	0,058654
$Q$50	Greece constrained	0	0	0	0,121951
$Q$51	Ireland constrained	0,015531289	0	0,0115	0,063557
$Q$52	Italy constrained	0,165851471	0	0,0949	0,077697
$Q$53	Luxembourg constrained	0,002998162	0,006084702	0	0,058543
$Q$54	Netherlands constrained	0,059469794	0,120692612	0,1051	0,057019
$Q$55	Austria constrained	0	0	0	0,058589
$Q$56	Portugal constrained	0	0	0	0,075405
$Q$57	Sweden constrained	0	0	0	0,07023
$Q$58	United Kingdom constrained	0,218326925	0	0,1334	0,066297

	benchmark
return(µ)	0,063101
variance	0,000409
stdev	0,020231

	mvp portfoli
µ(mvp)	0,057346
Variance	0,000208
stdev	0,014422

	ECU portfoli
µ(ECU)	0,061077
Variance	0,000322
stdev	0,017933

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
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$Q$47	U constrained	0	$Q$47<=$X$47	Binding	0
$Q$48	Finland constrained	0	$Q$48<=$X$48	Binding	0
$Q$49	France constrained	0,30358869	$Q$49<=$X$49	Not Binding	0,1
$Q$50	Greece constrained	0	$Q$50<=$X$50	Binding	0
$Q$51	Ireland constrained	0	$Q$51<=$X$51	Not Binding	0,0315
$Q$52	Italy constrained	0	$Q$52<=$X$52	Not Binding	0,3366
$Q$53	Luxembourg constrained	0,006084702	$Q$53<=$X$53	Binding	0
$Q$54	Netherlands constrained	0,120692612	$Q$54<=$X$54	Binding	0
$Q$55	Austria constrained	0	$Q$55<=$X$55	Binding	0
$Q$56	Portugal constrained	0	$Q$56<=$X$56	Binding	0
$Q$57	Sweden constrained	0	$Q$57<=$X$57	Binding	0
$Q$58	United Kingdom constrained	0	$Q$58<=$X$58	Not Binding	0,4431
$Q$44	Germany constrained	0,569633997	$Q$44>=0	Not Binding	0,5696
$Q$45	Belgium constrained	0	$Q$45>=0	Binding	0
$Q$46	Denmark constrained	0	$Q$46>=0	Binding	0
$Q$47	U constrained	0	$Q$47>=0	Binding	0
$Q$48	Finland constrained	0	$Q$48>=0	Binding	0
$Q$49	France constrained	0,30358869	$Q$49>=0	Not Binding	0,3036
$Q$50	Greece constrained	0	$Q$50>=0	Binding	0
$Q$51	Ireland constrained	0	$Q$51>=0	Binding	0
$Q$52	Italy constrained	0	$Q$52>=0	Binding	0
$Q$53	Luxembourg constrained	0,006084702	$Q$53>=0	Not Binding	0,0061
$Q$54	Netherlands constrained	0,120692612	$Q$54>=0	Not Binding	0,1207
$Q$55	Austria constrained	0	$Q$55>=0	Binding	0
$Q$56	Portugal constrained	0	$Q$56>=0	Binding	0
$Q$57	Sweden constrained	0	$Q$57>=0	Binding	0
$Q$58	United Kingdom constrained	0	$Q$58>=0	Binding	0

Appendix 5.3

Microsoft Excel 12.0 Answer Report
Worksheet: Euro members based off 1995 GDP
Report Created: 2011-05-19 18:50:26

Target Cell (Max)

Cell	Name	Original Value	Final Value
$S$47	Utility Function	-0,125196975	-0,093022401

Adjustable Cells

Cell	Name	BM Weights	MVP Weights
$Q$44	Germany constrained	0,349244677	0,397835216
$Q$45	Belgium constrained	0,039335418	0,044808169
$Q$46	Denmark constrained	0	0
$Q$47	Spain constrained	0,082611639	0,026391364
$Q$48	Finland constrained	0,018093538	0,020610899
$Q$49	France constrained	0,217341762	0,247580601
$Q$50	Greece constrained	0,018234533	0
$Q$51	Ireland constrained	0,009283611	0,010575244
$Q$52	Italy constrained	0,155884398	0,145286714
$Q$53	Luxembourg constrained	0,002862351	0,00326059
$Q$54	Netherlands constrained	0,058000326	0,066069933
$Q$55	Austria constrained	0,032991194	0,037581271
$Q$56	Portugal constrained	0,016116552	0
$Q$57	Sweden constrained	0	0
$Q$58	United Kingdom constrained	0	0

Expected Return
0,056705405
0,060794595
0,063478378
0,073632432
0,065964865
0,058654054
0,121951351
0,063556757
0,077697297
0,058543243
0,057018919
0,058589189
0,075405405
0,07022973
0,066297297

	benchmark
return(μ)	0,063768
variance	0,00025
stdev	0,015824

	mvp portfolio
μ(mvp)	0,061228
Variance	0,000186
stdev	0,01364

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$Q$59	constrained	1	$Q$59=1	Not Bind	0
$Q$44	Germany constrained	0,397835216	$Q$44<=$X$44	Binding	0
$Q$45	Belgium constrained	0,044808169	$Q$45<=$X$45	Binding	0
$Q$46	Denmark constrained	0	$Q$46<=$X$46	Binding	0
$Q$47	U constrained	0,026391364	$Q$47<=$X$47	Not Bind	0,067714063
$Q$48	Finland constrained	0,020610899	$Q$48<=$X$48	Binding	0
$Q$49	France constrained	0,247580601	$Q$49<=$X$49	Binding	0
$Q$50	Greece constrained	0	$Q$50<=$X$50	Not Bind	0,020771511
$Q$51	Ireland constrained	0,010575244	$Q$51<=$X$51	Binding	0
$Q$52	Italy constrained	0,145286714	$Q$52<=$X$52	Not Bind	0,032285937
$Q$53	Luxembourg constrained	0,00326059	$Q$53<=$X$53	Binding	0
$Q$54	Netherlands constrained	0,066069933	$Q$54<=$X$54	Binding	0
$Q$55	Austria constrained	0,037581271	$Q$55<=$X$55	Binding	0
$Q$56	Portugal constrained	0	$Q$56<=$X$56	Not Bind	0,018358854
$Q$57	Sweden constrained	0	$Q$57<=$X$57	Binding	0
$Q$58	United Kingdom constrained	0	$Q$58<=$X$58	Binding	0
$Q$52	Italy constrained	0,145286714	$Q$52>=$W$52	Binding	0
$Q$44	Germany constrained	0,397835216	$Q$44>=0	Not Bind	0,397835216
$Q$45	Belgium constrained	0,044808169	$Q$45>=0	Not Bind	0,044808169
$Q$46	Denmark constrained	0	$Q$46>=0	Binding	0
$Q$47	U constrained	0,026391364	$Q$47>=0	Not Bind	0,026391364
$Q$48	Finland constrained	0,020610899	$Q$48>=0	Not Bind	0,020610899
$Q$49	France constrained	0,247580601	$Q$49>=0	Not Bind	0,247580601
$Q$50	Greece constrained	0	$Q$50>=0	Binding	0
$Q$51	Ireland constrained	0,010575244	$Q$51>=0	Not Bind	0,010575244
$Q$52	Italy constrained	0,145286714	$Q$52>=0	Not Bind	0,145286714
$Q$53	Luxembourg constrained	0,00326059	$Q$53>=0	Not Bind	0,00326059
$Q$54	Netherlands constrained	0,066069933	$Q$54>=0	Not Bind	0,066069933
$Q$55	Austria constrained	0,037581271	$Q$55>=0	Not Bind	0,037581271
$Q$56	Portugal constrained	0	$Q$56>=0	Binding	0
$Q$57	Sweden constrained	0	$Q$57>=0	Binding	0
$Q$58	United Kingdom constrained	0	$Q$58>=0	Binding	0

Appendix 5.4

Microsoft Excel 12.0 Answer Report
Worksheet: Euro members based off 2001 GDP
Report Created: 2011-05-21 22:34:13

Target Cell (Max)

Cell	Name	Original Value	Final Value
$S$47	Utility Functi	-0,00013965	-0,000101274

Adjustable Cells

Cell	Name	BM Weights	MVP Weights
$Q$44	Germany cor	0,301080025	0,344978344
$Q$45	Belgium cons	0,036963644	0,042353048
$Q$46	Denmark cor	0	0
$Q$47	Spain constr	0,096982031	0,111122286
$Q$48	Finland cons	0,019832733	0,022724401
$Q$49	France const	0,213316785	0,244418975
$Q$50	Greece cons	0,020862837	0
$Q$51	Ireland const	0,016660107	0,019089198
$Q$52	Italy constrai	0,177905602	0,103844742
$Q$53	Luxembourg	0,003216069	0,003684981
$Q$54	Netherlands	0,063792075	0,07309314
$Q$55	Austria const	0,030276487	0,034690885
$Q$56	Portugal con	0,019111606	0
$Q$57	Sweden cons	0	0
$Q$58	UK constrain	0	0

Expected Return
0,056705405
0,060794595
0,063478378
0,073632432
0,065964865
0,058654054
0,121951351
0,063556757
0,077697297
0,058543243
0,057018919
0,058589189
0,075405405
0,07022973
0,066297297

	benchmark
return(μ)	0,064747761
variance	0,0002793
stdev	0,01671228

	mvp portfolio
μ(mvp)	0,06185199
Variance	0,000202548
stdev	0,014231943

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$Q$59	Total GDP fo	1	$Q$59=1	Not Binding	0
$Q$44	Germany cor	0,344978344	$Q$44<=$X$44	Binding	0
$Q$45	Belgium cons	0,042353048	$Q$45<=$X$45	Binding	0
$Q$46	Denmark cor	0	$Q$46<=$X$46	Binding	0
$Q$47	U constraine	0,111122286	$Q$47<=$X$47	Binding	0
$Q$48	Finland cons	0,022724401	$Q$48<=$X$48	Binding	0
$Q$49	France const	0,244418975	$Q$49<=$X$49	Binding	0
$Q$50	Greece cons	0	$Q$50<=$X$50	Not Binding	0,023904698
$Q$51	Ireland const	0,019089198	$Q$51<=$X$51	Binding	0
$Q$52	Italy constrai	0,103844742	$Q$52=>$X$52	Binding	0
$Q$53	Luxembourg	0,003684981	$Q$53<=$X$53	Binding	0
$Q$54	Netherlands	0,07309314	$Q$54<=$X$54	Binding	0
$Q$55	Austria const	0,034690885	$Q$55<=$X$55	Binding	0
$Q$56	Portugal con	0	$Q$56<=$X$56	Not Binding	0,021898132
$Q$57	Sweden cons	0	$Q$57<=$X$57	Binding	0
$Q$58	UK constrain	0	$Q$58<=$X$58	Binding	0
$Q$44	Germany cor	0,344978344	$Q$44>=0	Not Binding	0,344978344
$Q$45	Belgium cons	0,042353048	$Q$45>=0	Not Binding	0,042353048
$Q$46	Denmark cor	0	$Q$46>=0	Binding	0
$Q$47	U constraine	0,111122286	$Q$47>=0	Not Binding	0,111122286
$Q$48	Finland cons	0,022724401	$Q$48>=0	Not Binding	0,022724401
$Q$49	France const	0,244418975	$Q$49>=0	Not Binding	0,244418975
$Q$50	Greece cons	0	$Q$50>=0	Binding	0
$Q$51	Ireland const	0,019089198	$Q$51>=0	Not Binding	0,019089198
$Q$52	Italy constrai	0,103844742	$Q$52>=0	Not Binding	0,103844742
$Q$53	Luxembourg	0,003684981	$Q$53>=0	Not Binding	0,003684981
$Q$54	Netherlands	0,07309314	$Q$54>=0	Not Binding	0,07309314
$Q$55	Austria const	0,034690885	$Q$55>=0	Not Binding	0,034690885
$Q$56	Portugal con	0	$Q$56>=0	Binding	0
$Q$57	Sweden cons	0	$Q$57>=0	Binding	0
$Q$58	UK constrain	0	$Q$58>=0	Binding	0

Appendix 5.5

Microsoft Excel 12.0 Answer Report
Worksheet: EU 15 based on 1995 GDP
Report Created: 2011-05-19 18:17:49

Target Cell (Max)

Cell	Name	Original Value	Final Value
$S$47	Utility Function	-0,271325093	-0,092152195

Adjustable Cells

Cell	Name	BM Weights	MVP Weights	Expected Return
$Q$44	Germany constrained	0,238886972	0,322799343	0,056705405
$Q$45	Belgium constrained	0,028587181	0,036356881	0,060794595
$Q$46	Denmark constrained	0,019420541	0,023285384	0,063478378
$Q$47	Spain constrained	0,071060113	0,076356161	0,073632432
$Q$48	Finland constrained	0,02059142	0,016723468	0,065964865
$Q$49	France constrained	0,177896035	0,200884316	0,058654054
$Q$50	Greece constrained	0,013443274	0	0,121951351
$Q$51	Ireland constrained	0,006715675	0	0,063556757
$Q$52	Italy constrained	0,157884944	0,044080596	0,077697297
$Q$53	Luxembourg constrained	0,001756205	0	0,058543243
$Q$54	Netherlands constrained	0,042225826	0,053608454	0,057018919
$Q$55	Austria constrained	0,023277197	0,030493051	0,058589189
$Q$56	Portugal constrained	0,010538078	0,014896183	0,075405405
$Q$57	Sweden constrained	0,036067159	0,032459156	0,07022973
$Q$58	United Kingdom constrained	0,15164938	0,148057007	0,066297297

	benchmark
return(μ)	0,065131
variance	0,000267
stdev	0,016349

	mvp portfolio
μ(mvp)	0,061988
Variance	0,000184
stdev	0,013576

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$Q$59	constrained	1	$Q$59=1	Not Bindir	0
$Q$44	Germany constrained	0,322799343	$Q$44<=$X$44	Binding	0
$Q$45	Belgium constrained	0,036356881	$Q$45<=$X$45	Binding	0
$Q$46	Denmark constrained	0,023285384	$Q$46<=$X$46	Binding	0
$Q$47	U constrained	0,076356161	$Q$47<=$X$47	Binding	0
$Q$48	Finland constrained	0,016723468	$Q$48<=$X$48	Binding	0
$Q$49	France constrained	0,200884316	$Q$49<=$X$49	Binding	0
$Q$50	Greece constrained	0	$Q$50<=$X$50	Not Bindir	0,016853787
$Q$51	Ireland constrained	0	$Q$51<=$X$51	Not Bindir	0,008580642
$Q$52	Italy constrained	0,044080596	$Q$52=>$X$52	Binding	0
$Q$53	Luxembourg constrained	0	$Q$53<=$X$53	Not Bindir	0,002645609
$Q$54	Netherlands constrained	0,053608454	$Q$54<=$X$54	Binding	0
$Q$55	Austria constrained	0,030493051	$Q$55<=$X$55	Binding	0
$Q$56	Portugal constrained	0,014896183	$Q$56<=$X$56	Binding	0
$Q$57	Sweden constrained	0,032459156	$Q$57<=$X$57	Binding	0
$Q$58	United Kingdom constrained	0,148057007	$Q$58<=$X$58	Binding	0
$Q$44	Germany constrained	0,322799343	$Q$44>=0	Not Bindir	0,322799343
$Q$45	Belgium constrained	0,036356881	$Q$45>=0	Not Bindir	0,036356881
$Q$46	Denmark constrained	0,023285384	$Q$46>=0	Not Bindir	0,023285384
$Q$47	U constrained	0,076356161	$Q$47>=0	Not Bindir	0,076356161
$Q$48	Finland constrained	0,016723468	$Q$48>=0	Not Bindir	0,016723468
$Q$49	France constrained	0,200884316	$Q$49>=0	Not Bindir	0,200884316
$Q$50	Greece constrained	0	$Q$50>=0	Binding	0
$Q$51	Ireland constrained	0	$Q$51>=0	Binding	0
$Q$52	Italy constrained	0,044080596	$Q$52>=0	Not Bindir	0,044080596
$Q$53	Luxembourg constrained	0	$Q$53>=0	Binding	0
$Q$54	Netherlands constrained	0,053608454	$Q$54>=0	Not Bindir	0,053608454
$Q$55	Austria constrained	0,030493051	$Q$55>=0	Not Bindir	0,030493051
$Q$56	Portugal constrained	0,014896183	$Q$56>=0	Not Bindir	0,014896183
$Q$57	Sweden constrained	0,032459156	$Q$57>=0	Not Bindir	0,032459156
$Q$58	United Kingdom constrained	0,148057007	$Q$58>=0	Not Bindir	0,148057007

Appendix 5.6

Microsoft Excel 12.0 Answer Report
Worksheet: EU 15 based off 2001 GDP
Report Created: 2011-05-19 17:20:09

Target Cell (Max)

Cell	Name	Original Value	Final Value
$S$47	Utility Function	-0,131003311	-0,097186488

Adjustable Cells

Cell	Name	BM Weights	MVP Weights	Expected Return
$Q$44	Germany constrained	0,232324495	0,263690684	0,056705405
$Q$45	Belgium constrained	0,028522516	0,032373349	0,060794595
$Q$46	Denmark constrained	0,019716085	0,022377959	0,063478378
$Q$47	Spain constrained	0,074834926	0,084938409	0,073632432
$Q$48	Finland constrained	0,015303671	0,017369823	0,065964865
$Q$49	France constrained	0,16460313	0,18682624	0,058654054
$Q$50	Greece constrained	0,016098537	0	0,121951351
$Q$51	Ireland constrained	0,012855556	0,014591188	0,063556757
$Q$52	Italy constrained	0,13727855	0,055812562	0,077697297
$Q$53	Luxembourg constrained	0,002481638	0,002816685	0,058543243
$Q$54	Netherlands constrained	0,049224327	0,055870116	0,057018919
$Q$55	Austria constrained	0,023362459	0,02651663	0,058589189
$Q$56	Portugal constrained	0,014747223	0	0,075405405
$Q$57	Sweden constrained	0,027933365	0,031704656	0,07022973
$Q$58	United Kingdom constrained	0,180713523	0,205111702	0,066297297

benchmark	
return(μ)	0,065156
variance	0,000262
stdev	0,016187

mvp portfolio	
μ(mvp)	0,062692
Variance	0,000194
stdev	0,013942

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$Q$59	constrained	1	$Q$59=1	Not Bindir	0
$Q$44	Germany constrained	0,263690684	$Q$44<=$X$44	Binding	0
$Q$45	Belgium constrained	0,032373349	$Q$45<=$X$45	Binding	0
$Q$46	Denmark constrained	0,022377959	$Q$46<=$X$46	Binding	0
$Q$47	U constrained	0,084938409	$Q$47<=$X$47	Binding	0
$Q$48	Finland constrained	0,017369823	$Q$48<=$X$48	Binding	0
$Q$49	France constrained	0,18682624	$Q$49<=$X$49	Binding	0
$Q$50	Greece constrained	0	$Q$50<=$X$50	Not Bindir	0,018272005
$Q$51	Ireland constrained	0,014591188	$Q$51<=$X$51	Binding	0
$Q$52	Italy constrained	0,055812562	$Q$52=>$X$52	Binding	0
$Q$53	Luxembourg constrained	0,002816685	$Q$53<=$X$53	Binding	0
$Q$54	Netherlands constrained	0,055870116	$Q$54<=$X$54	Binding	0
$Q$55	Austria constrained	0,02651663	$Q$55<=$X$55	Binding	0
$Q$56	Portugal constrained	0	$Q$56<=$X$56	Not Bindir	0,016738249
$Q$57	Sweden constrained	0,031704656	$Q$57<=$X$57	Binding	0
$Q$58	United Kingdom constrained	0,205111702	$Q$58<=$X$58	Binding	0
$Q$44	Germany constrained	0,263690684	$Q$44>=0	Not Bindir	0,263690684
$Q$45	Belgium constrained	0,032373349	$Q$45>=0	Not Bindir	0,032373349
$Q$46	Denmark constrained	0,022377959	$Q$46>=0	Not Bindir	0,022377959
$Q$47	U constrained	0,084938409	$Q$47>=0	Not Bindir	0,084938409
$Q$48	Finland constrained	0,017369823	$Q$48>=0	Not Bindir	0,017369823
$Q$49	France constrained	0,18682624	$Q$49>=0	Not Bindir	0,18682624
$Q$50	Greece constrained	0	$Q$50>=0	Binding	0
$Q$51	Ireland constrained	0,014591188	$Q$51>=0	Not Bindir	0,014591188
$Q$52	Italy constrained	0,055812562	$Q$52>=0	Not Bindir	0,055812562
$Q$53	Luxembourg constrained	0,002816685	$Q$53>=0	Not Bindir	0,002816685
$Q$54	Netherlands constrained	0,055870116	$Q$54>=0	Not Bindir	0,055870116
$Q$55	Austria constrained	0,02651663	$Q$55>=0	Not Bindir	0,02651663
$Q$56	Portugal constrained	0	$Q$56>=0	Binding	0
$Q$57	Sweden constrained	0,031704656	$Q$57>=0	Not Bindir	0,031704656
$Q$58	United Kingdom constrained	0,205111702	$Q$58>=0	Not Bindir	0,205111702

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