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Market risk in volatile times

A comparison of methods for calculating Value at Risk.

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Abstract:

Three standard approaches of finding Value at Risk; age weighted historical simulation, volatility weighted historical simulation and t – distribution were compared with Value –at – Risk calculated using generalized extreme value theory during the period 2000 to 2010 to see which approach that would give the most accurate forecast of actual losses during the two volatile times within the time frame. One important aspect was also to see if the results would differ depending on the holding period and quantile used. The Value- at – Risk estimates were updated once per month for the three standard approaches and once per five month for the extreme value approach during the entire ten year period. Value – at- Risk during this period was calculated for two different confidence levels, 95% and 99% and two different holding periods, one and ten days. As previous research had shown that extreme value theory gave the most accurate forecasts for the period during the Global Financial Crises similar results were expected but, at the contrary, the approach to provide the most accurate forecasts of actual losses differed according to Kupiec test for every holding period and confidence level used. As the Basel II Accord states that the Value –at- Risk should be calculated for a holding period of ten days using a 99% confidence level the results from Kupiec test, which tested the model for each approach during a two year period from 2000-2009, accepted all standard approaches for every two year period but with highest p values for volatility weighted approach during all but one of the two year periods. The only approach which the model was not accepted for all two year periods was generalized extreme value theory. As the purpose was to see if the two volatile periods during the chosen ten year time frame could be best estimated with the same approach regardless of the holding period or the confidence level used the conclusion that can be drawn from these result states that no such approach existed. The result depend on the window chosen for the historical simulation approaches, the accuracy of the estimated parameters to calculate the volatility in the volatility weighted historical simulation, the accuracy of the parameters estimated to calculate Value- at- Risk according to generalized extreme value theory and the assumptions made about the underlying profit/loss distribution. The generalized extreme value theory assumes a distribution based on the highest value in every block but when approaching the time of the Global Financial Crises that distribution seemed to change giving lower Value- at- Risk estimates than, for example, volatility weighted historical simulation which adapted to the new market conditions of higher volatility faster.

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1. Introduction

Value at Risk and its different approaches used in risk managing as a result of volatile market prices are investigated in this paper. The introductory chapter will specify the purpose of the paper, give some information about the financial markets specifically during the last decade and give a summary of the results from previous studies concerning Value at Risk as a risk measurement tool. The chapter will conclude with a disposition of the rest of the paper.

Uncertainties in the financial markets require tools to predict the instability in the market prices and estimate potential losses (Baran and Witzany, 2011). The standard method for estimating unexpected losses today is Value at Risk (VaR), which measures the worst loss potential when holding a financial asset over specific time period with a certain probability. According to the Basel Committee working paper number 19 (2011), its use in managing market risk is due to the simplicity of the measurement as it can be applied to all asset classes and it also gives an overview of the risk and is conceptually easy to understand.

The purpose of this paper is to investigate which of the following parametric- and non parametric approaches of calculating Value-at-Risk gives the closest estimate to the true loss during the period of January 2000 to December of 2010. The parametric approach chosen is when an assumption of t distributed profit/losses is made and the non- parametric approaches are age weighted historical simulation and volatility weighted historical simulation. These standard methods of calculating Value-at-risk are compared to Value-at-Risk estimations made using Extreme Value Theory to see if a consistent result can be gained for a period containing both high and low price volatility. The period of interest is year 2000 to 2010 because during this time frame there were two periods of high volatility, caused by the IT – bubble and Global Financial Crises, that are set on each side of a few years with economical stability in the US stock market, which, in this paper, is represented by the S&P500 index. The chosen years therefore provide two different volatile periods and it would be interesting to see if the same approach would provide the best estimates for both of them.

This paper will contribute to the current research about Value at Risk as a risk measure by comparing which of the studied approaches give the most accurate forecast during two different volatile times, calculated for different holding periods, at different confidence levels. Different holding periods will be used to see if the approach giving the best estimates will differ when taking that into consideration. The holding periods that will be used to calculate VaR are one day- and ten-days. This paper will use the S&P 500 index from the years of 1981 to 2011 to calculate VaR, since appropriate approximations of starting values needed to perform the calculations within extreme value theory can be found based on that market (Dowd, 2008).

Previous studies have been made in this field where much of the focus has been on the advantages of using extreme value theory over traditional Value at Risk methods. Gencay and Selcuk (2004) state that extreme value theory gives more accurate estimates at higher quantiles than traditional methods in emerging markets and Kourouma et al (2010) has equal results when testing the VaR models during the financial crises according to Basel two regulatory ten days holding period. According to Kuester, Mittnik and Paolella, (2006) the growing need for financial institutes to manage financial risk gives risk prediction a more prominent role. Based on their research where different approaches have been used to calculate Value at Risk using 30 years of daily return NASQAQ index data the approaches seemed inadequate, although current regulations accepted most of them. According to this research the model with the most accurate results was a hybrid between a heavy tailed GARCH filter and an extreme value theory-based approach. Extreme value theory has previously been used in hydrology and climatology but during the last decade also in financial research. Later Embrechtes did an overview of the theory as a tool for managing risk (Gencay and Selcuk, 2004). Beder (1995) states that VaR is a necessary tool for managing and controlling market risk, but because of its vulnerability to incorrect assumptions, parameters and data used it is not enough to manage every aspect of market risk. A previous study regarding value at risk during the Asian crises of 1997 to1998 in emerging markets containing a reality check test where both filtered and unfiltered approaches were used concluded that approaches within extreme value theory that was filtered worked best during the crises period while harmful during tranquil periods (Bao, Lee and Salto Lu, 2006).

Going back 20 to 30 years the volatility of financial time series such as short-term and long-term interest rates, stocks, exchange rates and corporate spreads have at times displayed different volatility during different periods. Only the last decade the volatility of these series started off relatively high after the burst of the IT bubble followed by a period of low volatility between 2004 and 2006 in both emerging and industrialized countries despite several economical and sociopolitical shocks like increased oil prices, deterioration of creditworthiness of the US car industry, war and natural disasters (Basel Committee, 2011). The last couple of years the Global Financial Crises, with its roots in the US subprime market, caused bank crises with asset classes' correlation reaching unprecedented heights and the financial sector reported a sudden and drastic increase in the number of days the predicted VaR estimations were exceeded by actual losses. The record losses that followed are partly a result of the inability of anticipating and measuring the market risk which proves the inadequacy of the VaR models (Contreras, 2010). During this time the S&P500 index displayed a decrease in value of 56% from October of 2007 to March 2009 further proving the drastic changes in volatility not taken into account in the standard Value-at-Risk models (Mandan, 2010).

The paper will be divided into nine sections including the introduction followed by a brief background section about the Value at Risk and the Basel regulatory framework. The third section will cover the theory behind the different approaches for calculating VaR, maximum likelihood for estimating parameters and backtesting. The fourth chapter covers the methodology used for this paper and in chapter five the results are presented. Chapter six contains the analysis along with an analysis and the conclusion of this essay will be presented in chapter seven. The reference will be presented in chapter eight and the appendix in chapter nine.

2. Background

This chapter contains some background information about Value at Risk and the implementation of the Basel accord, one and two. To conclude the chapter some research about the appropriate holding period to use when calculating VaR, according to the Bank of International Settlement (BIS), is presented.

2.1 Value at Risk in the Basel Accord

During the 1970s the phrase “risk management” was mainly used when evaluating a firm’s or bank’s exposure to credit risk. This changed in the early 1990 as Value at Risk (VaR), the latest advancement in the area of risk measurement on Wall Street, became a measurement firms could use to measure their exposure to markets around the world (Beder, 1995). Value at Risk as a measurement has today become a standard way of measuring market risk and has worked as a replacement for measurements as volatility or standard deviation of financial time series. The measurement helps firms determine which business decisions that holds the highest return at the lowest risk to control and unify the procedures for financial institutes and banks (Urbani).

The Basel Accord contains specifications for the capital requirements of banks and other financial institutions. According to a 1996 amendment to the Basel Accord first created in 1988 by the Bank of International Settlement (BIS), banks with good risk managing skills were given the opportunity to use internal models to determine the regulatory capital requirement needed. A non specific daily capital requirement can be calculated using equation one where VaR_t is the VaR forecast for day t and k is a hysteria factor that, depending on the backtesting results of the bank, multiplies VaR_t within a range of 3-4. As the back testing results get better the hysteria factor is lowered (Dowd, 2008).

$$max\left\{VaR_t, \frac{k}{60}\sum_{i=1}^{60} VaR_{t-i+1}\right\} \qquad (1)$$

Another requirement according to the 1996 amendment is that VaR should be calculated at a 99% confidence level for a holding period of ten day although the ten day period has nothing to do with a bank’s solvency and increasing the solvency was the main purpose of using capital requirements from the beginning. To calculate VaR for the ten day holding period the rules allow banks to multiply the square root of the holding period, h, with the one day VaR (Dowd, 2008). The Basel amendment, which allowed banks to use internal models to calculate their own risk threshold with Value at Risk models, was based on the criticism of the standardized approach used to calculate VaR as the forecasts often had been overly pessimistic contributing to higher daily capital charges (McAleer, 2009). Early on, users of the measurement showed radically different results depending on which approach to calculate VaR that was used (Beder, 1995).

The Basel II Accord, which consists of three main pillars to take into account the calculation of regulatory minimum capital requirements, regulatory supervision to enforce minimum capital requirement and rules on disclosure of capital structure and risk exposure, is an extension of the Basel I Accord (Saunders and Cornett, 2008). Financial institutes forecast their Value at Risk each day. These forecasts are in turn handed over to a regulatory authority, usually the central bank in each country and are compared to the actual return for that day. When the number of violations, defined as the actual return exceeding the Value at Risk forecast over a 250 trading day period, increases a penalty for disproportionate risk taking can be imposed on the bank, hence the Basel II Accord rewards superior risk forecasting and management. To test the accuracy of the internal models used by the banks a backtesting procedure was introduced and if the models showed a higher number of violations than to be expected, given the confidence level chosen, a higher capital ratio was required (McAleer, 2009).

After the outbreak of the Financial Crises the weakness of the risk managing regulatory system became apparent as the internal models permitted by The Basel II Accord partly provided banks an opportunity to increases their risk taking using their internal VaR forecasts and changes in the regulatory system are to be expected, especially concerning handling excessive risk (McAleer, 2009).

2.2 Holding periods for Value at Risk

When using Value at Risk to calculate the regulatory capital required for banks the horizon of the holding period should be taken into account. The Basel regulatory framework has set a minimum holding period of ten days over which the calculations should be made. In the academic world discussion of whether this is an appropriate horizon or not has taken place and although no single answer has been found a consensus that the time horizon should differ depending on the position of the bank/financial institute exists. According to Christofferssen and Diebold (2000) the relevant time horizon will change depending on which asset classes that make up the portfolio (equity or fixed income), where the portfolio is placed within the firm, for example, with the CFO or the trading desk and that the time horizon should therefore be decided based on the situation (Basel Committee, 2011). When taking this into consideration the general idea of a basic ten day horizon praxis is not optimal. Although the literature suggests a varying time horizon from case to case instead of the fixed ten days it is also important to recognize that others, like Danielsson (2002) , states that a VaR calculated for ten at a confidence level of 99% predict the happening of an event that happens about 25 times per decade while a liquidity crises, as during the Global Financial Crises, is not likely to happen even once per century (Basel Committee, 2011).

3. Theoretical framework

The chapter will explain the theoretical framework for calculating Value-at- Risk using different approaches. A short introduction of the approaches used in this paper will be followed by theoretical framework for each approach chosen in this paper. Special focus will be on how to use GARCH to estimate the parameters in the volatility weighted simulation, on how to calculate Value –at –Risk using Generalized extreme value theory as well as the maximum likelihood method needed to estimate the parameters for the latter approach.

3.1 Value at Risk

Value at Risk is a measurement of market risk, which at a certain confidence level, estimates the loss the holder of a financial asset will suffer from holding that specific asset for a certain period. The mathematical definition of Value – at- Risk can be seen in equation two where Pr represents the probability that the return, R, will be more negative than the Value at Risk estimate.

$$Pr(R \leq -VaR) = 1 - \alpha \qquad (2)$$

VaR can be calculated using different approaches, parametric- , non parametric and semi parametric. When deciding which approach that will give the best estimate the specifics of the financial time series has to be taken into account. Typically financial time series display extra kurtosis, hence an underlying profit/loss distribution with a parameter that takes extra kurtosis into account would be a more realistic assumption than using a normal distribution. The approach that takes the extra kurtosis into account is the student t distribution which is one of the parametric approaches that can be used to calculate VaR.

Another approach when calculating VaR is using an empirical distribution, the historical observations of the return. Basic historical simulation is one of the non parametric approaches where each past observation used is given equal weight, the latest observations is therefore equally important as the distant ones. This approach does not allow recent large losses to have a higher impact when estimating VaR and when the window with past observations is moved for each calculation and observations disappear sudden jumps of the estimated VaR can occur. More realistic versions of the non parametric approach, the age weighted- and volatility weighted can therefore be used.

3.2 The advantages and shortcomings of Value-at-Risk

Value at risk has become the most vividly used measure of market risk due to its simplicity and prevalence in the industry, despite this fact the measurement suffers from disadvantages.

The non-coherency of the measurement, the problem of underestimating the risk as VaR only measures the losses per quantile and the fact that it can be destabilizing in crises are some of the most serious disadvantages (Dowd, 2008). Other problems has to do with the fact there is no unique solution for finding the appropriate time horizon to calculate VaR over. Despite its drawbacks Value at risk as a measurement of market risk has the advantage of being conceptually easy to understand (Basel Committee, 2011). The measurement can be used on all asset classes, it gives an overview of the risk and during periods of normal volatility gives a fairly correct estimate of the loss from holding a financial asset with a certain probability during a certain time horizon (Dowd, 2008). According to Beder (1995) Value at Risk have extreme dependence on the estimation of parameters, which data is used, assumptions and methodology.

The non parametric approaches of historical simulation for finding VaR is based on the assumption that the return of the near future can be determined by the historical observations, in other words, the future is set to repeat itself and the simulation is therefore the distribution's empirical quantile at a specific probability level (Marimoutou et al, 2009). Non parametric approaches are mainly trustful when the market climate is stable because they cannot respond fast enough to shifts in volatility, which can lead to unreliable estimations of VaR (Dowd, 2008).

The parametric approaches base their estimations on theoretical distributions such as Student-t distribution and generalized extreme value (GEV) distribution and model the financial series accordingly. When the assumptions made about the distribution reflects the data properly it's easy to make forecasts of the actual future loss with VaR (Dowd, 2008).

3.3 Age-weighted Historical Simulation

Age-weighted HS can provide a better generalization than basic HS[1], since it has a decay factor, λ, of less than one, which means that the latest observations go towards one while the earliest go towards zero (Boudoukh, Richardson and Whitelow, 1998). Traditional HS does not have this decay factor, so all observations will be given the same weight, hence equal affect when calculating Value-at-Risk. Equation three is used to find the weights where n is the number of observations and if λ is chosen properly, VaR can react well to large losses and deal better with clusters of losses.

[1] The weight(w) for every observation in basic historical simulation is w = 1/n, where n is the number of observations. (1- α) percent of the highest losses is chosen as an estimation of the one- day VaR, where α is the confidence level.

$$W(i)= \frac{\lambda^{i-1}(1-\lambda)}{1-\lambda^{n}} \quad (3)$$

As long as the weight in this model falls from λ^{n} w(1) to zero instead of from 1/n to zero, used in traditional HS formula, it can decrease the ghost effects[2] and the rare events preceding distortions. In addition; diminishing the influence of past extreme events by the passage of time helps the sample size increase and to give better estimations of risk. The new weights are multiplied by each return and Value at risk is found in the same way as for basic historical simulation (Dowd, 2008).

3.4 Volatility-weighted Historical Simulation

Another way to weight the observations is based on the volatility where volatility- weighted HS reflects the recent changes in volatility (Hull and White, 1998).

$$r_{t,i} = \left(\frac{\sigma_{T,i}}{\sigma_{t,i}}\right) r_{t,i} \quad (4)$$

As can be seen in equation four, where $r_{t,i}$ is the historical return on asset i in day t, $\sigma_{T,i}$ shows the latest forecast of asset i and $\sigma_{t,i}$ the forecast for the asset i made in day t-1, if the latest observations display measures of higher volatility than the previous period the product preceding $r_{t,i}$ will be higher than one which will increase the value of the latest P/L and the other way around if the latest observations have low volatility. Value at risk is found in the same way as in basic historical simulation (Dowd, 2008).

3.4.1 GARCH

A GARCH model can be used to forecast volatility changes and their effects on data. To make GARCH forecasts in general a GARCH (1, 1) model will be sufficient to capture the volatility clustering in the data. A GARCH (1,1) model can be seen in equation five where σ_t^2 is the conditional volatility with the mean equation below.

(Conditional variance) $\sigma_t^2 = \alpha_0 + \alpha_1 \mathcal{E}_{t-1}^2 + \beta_0 \sigma_{t-1}^2$, $\varepsilon_t \sim N(0, \sigma_t^2)$, (5)

(mean equation) $r_t = \mu + \varepsilon_t$

$$\sigma^2 = \frac{\alpha_0}{1-(\alpha_1 - \beta_0)} \quad (6)$$

[2] Ghost effects are the jumps that are created when the oldest observations fall out of the sample if they are more extreme.

Where $\alpha_1 + \beta_0 < 1$ restriction presents the stationarity of the GARCH model and the unconditional variance can be seen in equation six. The parameters (α_0, α_1 , β_0) are estimated based on the information available for each period of study (Brooks, 2008).

3.5 Student t distribution

Student t distribution takes into account the extra kurtosis existing in financial time series by using different degrees of freedom, v, where a low degree will display a stronger case of kurtosis. VaR can be calculated according to equation seven, where h stands for the holding period, α for the confidence level, $\mu_{P/L}$ for the mean value of the profit/losses and $\sigma_{P/L}$for the standard deviation.

$$VaR(h, \alpha) = h\mu_{P/L} + \sqrt{h}\,\sqrt{\frac{v-2}{v}}\,\sigma_{P/L}t_{\alpha,v} \qquad (7)$$

The t distribution is a parametric solution where the shape of the P/L curve is assumed (Dowd, 2008).

3.6 Extreme Value Theory

Extreme value theory focuses on finding the rare tail events of extreme losses to estimate future extreme losses. There are two ways of using the extreme value approach; the peaks-over-threshold (POT) approach using the generalized pareto distribution and the generalized extreme-value theory. Although the POT approach seems to use a more direct approach to calculate value at risk with only two parameters that need to be estimated in order to calculate VaR, it requires an approximation of a threshold value. All observations with a higher value than the threshold should be considered extreme observations and be used to create a distribution consisting only of extreme values. Since the approximation of the threshold is problematic the other approach, the Generalized Extreme Value approach, was used (Gencay and Selcuk, 2004).

3.6.1 Generalized Extreme Value(GEV)

If X is assumed to be an identically distributed independent variable with an unknown distribution according to equation eight;

$$F(x) = Prob(X \leq x) \qquad (8)$$

This unknown distribution can be used to find extreme risks with a location parameter, μ, a scale parameter, σ and a tail index; ξ. the first two parameters are related to the mean and standard deviation of the distribution but still distinct from them (Dowd, 2008). A sample of n observations can be chosen from the unknown distribution and divided into blocks of observations. M_n represents the extreme value in every block and as n increases the distribution will converge, according Fisher- Tippett theorem, into the Generalized extreme value (GEV) distribution shown below in equation nine.

$$H_{\xi,\mu,\sigma}, \begin{cases} exp\left[-\left(1+\ \xi\frac{x-\mu}{\sigma}\right)^{-1/\xi}\right] & \xi \neq 0 \\ exp\left[-\ exp\left(\frac{x-\mu}{\sigma}\right)\right] & \xi = 0 \end{cases} \tag{9}$$

The limitation of the GEV distribution is that x satisfies expression ten.

$$1+\xi(x-\mu)/\sigma > 0 \tag{10}$$

The GEV approach is a parametric approach since a predetermined distribution is used. The tail parameter, ξ, determines the shape of the distribution and there are three specific shapes defined. The first is called the Frechét distribution which is valid when the tail index has a value strictly over zero. In the Fréchet distribution the tail is determined by the power function and is, as in the t-distribution, heavy. This distribution has been most useful for financial data series they often display heavy tails in a distribution with a positive tail index.

The second distinct case is where the distribution is called the Gumble distribution and the shape of the curve is determined by a tail index with a value of zero. The implication of a zero value tail index is the distribution’s slimmer tail, containing less extreme values. The shape of the distribution is generally more like the a normal- or log normal distribution.

The third special case is when the tail index takes a value below zero. The distribution is then named Weibull distribution and describes the F(x) distribution as a distribution with lighter than normal tails as most of the observed losses can be found in the middle of the distribution, which is unlikely for financial series.

Both Frétchet- and Gumble distributions under standardized conditions are slightly skewed to the right, with the Frétchet distribution more so than Gumble with a longer right hand side tail. This implies a higher probability for producing large X values compared to Gumble. The reason the extreme losses are represented on the right hand side of the distribution when using extreme value theory compared to the other approaches where the left hand side represents the downside risk, hence the losses, is that the distribution is made up of only the highest losses from an underlying parent distribution. The left hand side of the extreme value distribution will represent the smallest of the extreme losses and the right hand side the highest of the extreme losses while the extreme values are extracted from the "parent" distribution on the left hand side (Dowd, 2008).

Value at Risk, assuming either Frechét or Gumble distribution (depending on the value of the tail index), can be seen in equations eleven and twelve where α is defined as the confidence level, n is the sample size of extreme values, $\xi_n$is the estimated tail index parameter for the specific sample, σ_n is the estimated scale parameter and $\mu_n$is the estimated location parameter (Dowd, 2008).

$$VaR\ (Fréchet) = \mu_n - \frac{\sigma_n}{\xi_n}\left[1 - (-n(\ln(\alpha))^{-\xi_n}\right] \tag{11}$$

$$Var\ (Gumble) = \mu_n - \sigma_n \ln[\ -n \ln(\alpha)] \tag{12}$$

3.7 Maximum likelihood

Maximum likelihood is a method that can be used to find the most probable parameter estimators given the underlying data and performed using a maximum likelihood function. Then maximum likelihood function can be constructed using log-like function and look different depending on the shape of the GEV distribution. If the distribution is a Fréchet distribution the maximum likelihood function contains a tail parameter that is larger than zero and can be seen in equation thirteen below.

$$l(\mu_n, \sigma_n, \xi_n) = -mln(\sigma_n) - \left(1 + \left(\frac{1}{\xi_n}\right)\right) \sum_{i=1}^{m} ln\left[1 + \xi_n\left(\frac{M_n - \mu_n}{\sigma_n}\right)\right] - \sum_{i=1}^{m}\left[1 + \xi_n\left(\frac{M_n - \mu_n}{\sigma_n}\right)\right]^{-1/\xi} \tag{13}$$

Where m is the number of extreme observations and M_n represents the extreme observation. The constrain existing concerning equation thirteen is that for any extreme value, M_n^i , expression fourteen should be true.

$$1 + \xi\left(M_n^i - \mu\right)/\sigma > 0 \tag{14}$$

When the tail parameter is less than zero equation thirteen is simplified to equation fifteen where only two parameters need to be estimated, the scale parameter and location parameter defined as $\sigma$and μ.

$$l(\mu_n, \sigma_n) = -mln(\sigma_n) - \sum_{i=1}^{m} exp\left[1 + \xi_n\left(-\frac{M_n-\mu_n}{\sigma_n}\right)\right] \qquad (15)$$

The advantage of using the maximum likelihood approach is that is statistically well grounded, the parameter estimates can easily be tested with hypothesis and the estimated parameters are asymptotically normal as long as $\xi_n > -1/2$. However, problems do exist and one of the greatest disadvantages is that the approach doesn't have a closed-form solutions for the parameters. Another problem is connected to the fact that the parameters estimated are asymptotic while the sample used to calculate might be small (Dowd, 2008).

3.8 Backtestning

In the area of risk management, risk managers and banking regulators need to be sure that the results of losses based on VaR models are not biased. Backtesting is the solution for this issue and can help managers to diagnose problems with the risk models and improve them. In fact, it is a crucial way to prove accuracy and identify VaR approaches in which improvement is needed (Dowd, 2008).

To implement the backtesting, the standard way, named Kupiec's test, is suggested. Basically Kupiec test wants to see whether the observed violation frequency is close to the nominal violation frequency for the VaR model and specific confidence interval. The null hypothesis is that the model is correct, and the violations have a binomial distribution. The precision of the model can be estimated using equation sixteen where x is the number of violations, p is the probability of violation for a given confidence level and n is the number of observations.

$$\Pr(x|n,p) = \binom{n}{x} p^x (1-p)^{n-x} \qquad (16)$$

Consequently, if the estimated probability is above the desired null significance level, the model is accepted. If the estimated probability is below the significant level, the model is rejected.

Although this method can be applied easily without a great deal of information, it has some drawbacks that diminish the accuracy of the results. This test should be performed for a large sample to give reliable results moreover Kupiec cannot consider the information of the sizes of tail losses only the frequency. The test does not cover volatility clustering (Dowd, 2008).

3.9 Jarque- Bera

Jarque- Bera normality test with the test statistics presented in equation seventeen can be preformed to see if data is normally distributed.

$$W = T\left[\frac{b_1^2}{6} + \frac{(b_2 - 3)^2}{24}\right] \quad (17)$$

This test statistics follows a $\chi 2$ distribution with two degrees of freedom and the null hypothesis of a normal distribution. T equals the sample size and b_1 and b_2 are the coefficients of skewness and excess kurtosis. If theses coefficients are equal to zero, the distribution follows a normal distribution which is symmetric and mesokurtic and the coefficients will be correctly estimated (Brooks 2008).

4. Methodology

The following section starts with a description of the data followed by detailed descriptions of how the calculations for Value –at- Risk were performed using age weighted historical simulation, volatility weighted historical simulation, t- distribution and extreme value theory. To conclude the chapter the backtesting methodology used for this paper will be presented.

4.1 Data

The data chosen to calculate VaR was the S&P 500 Index for which data can be found going back to the third of January 1950 (Yahoo Finance). The chosen index shows two specific volatile periods during the years of 2000 to 2010 which was optimal for this essay since the purpose is to see which approach to calculate VaR would give the best estimates during both volatile periods to see if they are similar or not. The ten year period was divided into 120 smaller periods, each representing a different month. Prior to every month a window containing approximately 500 observations (two years) of the actual returns of the S&P 500 Index was used to estimate the Value at Risk for both of the simulation approaches as well as the t- distribution approach. This window was moved one month at the time, updating the Value at Risk estimates twelve times per year. The 500 observation window was chosen based on the fact that the estimations should be made based on information that is fairly new.

The data used to calculated VaR estimates with extreme value theory was approximately 4000 observations (from May 1983 to 31 of December 1999) to get enough data to divide up into blocks containing approximately 100 observations each. The observations are then sorted from lowest to highest within each of the 40 blocks and the highest loss for each block was used to create a new maximum loss distribution. The new distribution was used to find the three unknown parameters in Fréchet distribution (equation thirteen) by maximum likelihood. The scale-, location and tail parameters, hence the Value at Risk, were not updated as often as the other approaches because of the complexity of the calculations. The Value at Risk estimate was updated 27 times using generalized extreme value approach, once every five months, compared the other approaches where an update was made once per month.

The data was tested with Jarque- Bera normality test to see if the returns for the chosen period were normally distributed and the result can be seen in appendix 9.3.

4.2 VaR with Age Weighted Historical Simulation

To calculate VaR using age weighted historical simulation equation three, containing a factor with a slow rate of decay (0,999), was used to find the appropriate weight for every observation used to estimate Value at Risk. This was done for every two year period preceding every month during the years 2000 to 2010 starting with January of 2000. Each weight was then multiplied with the observations starting with the newest, hence giving the latest observation the highest weight. The weighted observations were then sorted for each two year period from the lowest to the highest value. Interpolation was then used to find the observation representing the Value at Risk Value. Depending on which confidence level (α) is used, (1- α) percent of the highest losses is chosen as an estimation of the one- day VaR. Finally, to find the VaR for different holding periods '*square-root of time rule'* is used where the result of one day VaR is multiplied by the square root of the holding period (h) according to equation eighteen (Dowd, 2008).

$$\text{VaR}(h,\alpha) = \sqrt{h} * \text{1-day VaR}(\alpha) \quad (18)$$

4.3 VaR with Volatility Weighted Historical Simulation

To estimate VaR based on this approach, the volatility for different periods are first calculated. To forecast the volatility for every trading day during the chosen two year windows a GARCH (1,1) model, as described in equation five with the mean equation of AR(1)[3], is used. The GARCH parameters are updated yearly instead of monthly. The calculations are mainly done using Eviews except for the year of 2003 as the ARCH coefficient displayed a negative value. Since volatility cannot be negative the last GARCH calculation was done using excel where non- negative restrictions can be imposed. Maximum Likelihood method was applied to find the unknown parameters which satisfying the non negative restrictions for α_0 , α_1 , and β_0.(Brooks, 2008).

Once the parameters were obtained the variances for the observations within the sample and out of the sample (one day ahead) were calculated followed by the scaling factor, $\left(\frac{\sigma_{T,i}}{\sigma_{t,i}}\right)$, where $\sigma_{T,i}$ represents the out of sample volatility forecast and $\sigma_{t,i}$ represents every variance forecast within the sample. As describes in section 4.2 the new, scaled observations are sorted and the Value at Risk representing the one day risk chosen depending on the confidence level.

4.4 VaR with T- distribution

The t distribution approach was used to take advantage of the fact that financial time series often have extra kurtosis. When using this approach the mean, standard deviation and kurtosis need to be estimated in order to calculate the degrees of freedom needed in equation seven in

[3] AR(1) :$_t y_t = \mu + \phi y_{t-1} + u_t, \sim N(0, \sigma_t^2)$

section 3.5 to calculate the Value at Risk. The degrees of freedom were calculated using equation nineteen (Dowd, 2008).

$$kurtosis = 3(v - 2)/(v - 4) \quad (19)$$

When the approximation of the degrees of freedom is negative, as within the period June 2005 to March 2006, suggesting lighter than normal tails in the distribution, a high value of degrees of freedom was chosen to convert the t distribution into a normal distribution .

4.5 VaR with Generalized Extreme Value approach

When using the Generalized Extreme Value approach a new distribution based on the extreme losses in the original "parent" distribution must be computed. The extreme values were obtained using blocks containing approximately 100 observations of which the highest loss in each block represented an extreme value. 40 blocks were created prior to January 2000 going from May 1983 to December 1999. To find the highest value in each block the observations were divided into blocks of five months, approximately 100 trading days. These were then used to create a distribution and by maximum likelihood find the three unknown parameters (μ, ξ, σ) by using equation thirteen in section 3.7. When the estimated tail parameter had a value higher than zero the generalized extreme value distribution converts into the Fréchet distribution as the sample size, n, increases. For each following five month period one more block, hence one more maximum value, was added. The value at risk calculated using GEV are updated once per five month compared to the other approaches used that were updated once each month. Value at Risk was then calculated using equation eleven in section 3.6.1.

4.6 Back testing methodology

The purpose of using Kupiec test is to see if the model chosen is the correct one for each time period with the null hypothesis accepting the model. The one day VaR models were estimated for each year since the number of exceedances would be low if only the 20 trading days were used. The exceedances are denoted x in equation twenty below and q is one subtracted with the confidence level.

$$LR = -2\ln[(q^x(1-q)^{n-x}] + 2\ln\left[\left(\frac{x}{n}\right)^x \left(1-\frac{x}{n}\right)^{n-x}\right] \quad (20)$$

When testing the models; age weighted historical simulation, volatility weighted historical simulation, t-distribution or generalized extreme value for Value at Risk calculated for ten days, a different time horizon is used to provide enough observations for the test. The ten day value at risk models were tested using a period of two years, giving 48 observations in each test. The chi2 distribution was then used to find the p value.

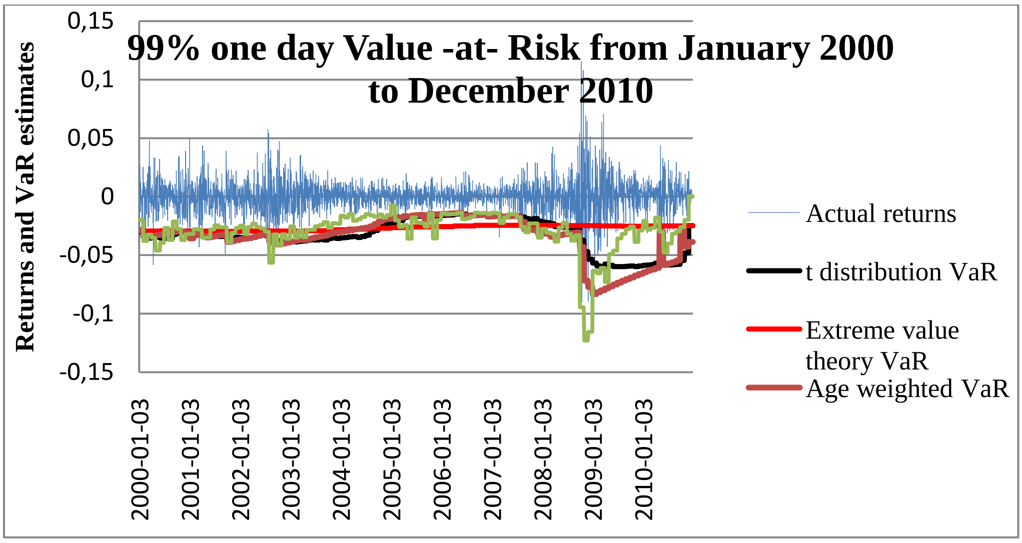
5. Results

In this chapter the results are presented starting with graphs showing the estimated Value at Risk for every holding period with the actual returns during the same period. The results from Kupiec test is then presented where the accepted models for each one year and two year period.

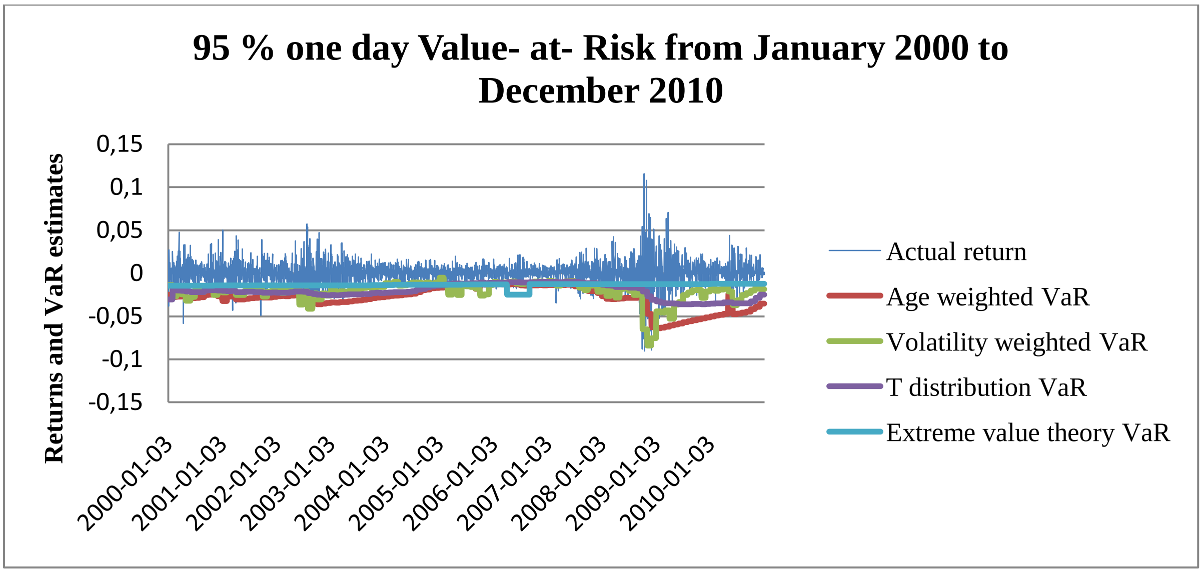
5.1 Graphs

Graph one and two below shows the estimated VaR values for one day during the period of January 2000 to December 2010. As can be seen the actual returns are more volatile during the first and last part of the period while the middle shows a more stable economic growth. When the 95 % confidence level was used, as can be seen in graph one, compared to the 99% confidence level, in graph two, little of which approach provides the best estimating can be determine visually. The historical simulation approaches (age weighted and volatility weighted) seem to follow the actual returns better during both volatile periods compared to the other approaches.

Graph 1.

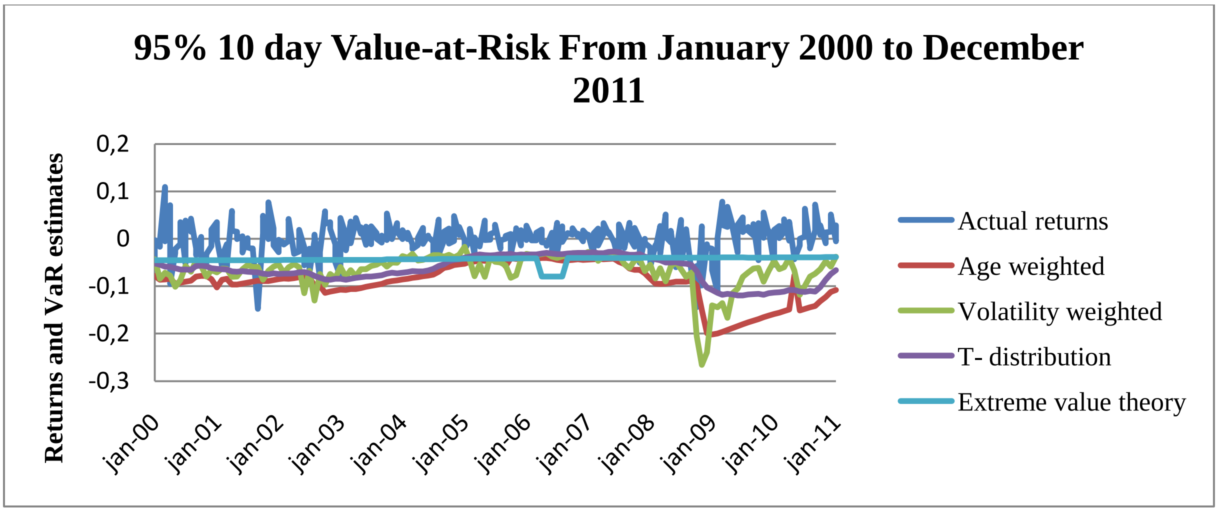


Graph 2.



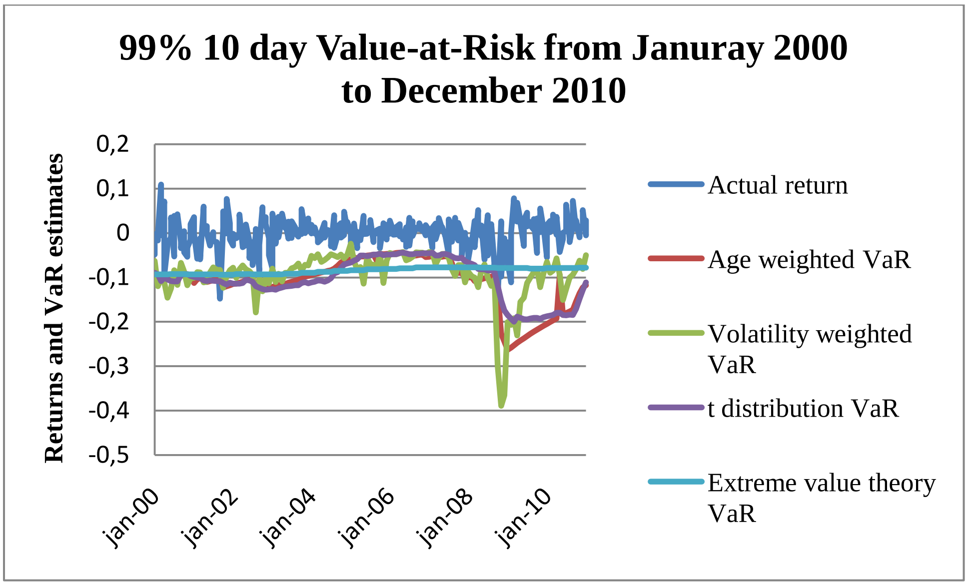
In both graph one and two the VaR estimates based on Generalized extreme value theory seem to follow the actual returns poorly during both volatile periods.

Graph 3



When looking at graph three, which contain the plotted VaR estimates calculated with every chosen approach in addition to the actual aggregated ten days return, the observations are fewer therefore easier to visually detect the difference between the approaches. Similar to what could be detected in graph one and two the simulation approaches seem to follow the actual return better than the other two approaches with fewer exceedances. The volatility weighted approach gives the highest VaR during the financial crises for both confidence levels as can be seen both in graph three and four which displays the ten day VaR but with a confidence level of 99%.

Graph 4.



5.2 Backtesting Results

Table 1. Kupiec test for 95% one day VaR models

P values for the four different approaches when testing 95% one day VaR during the period 2000- 2010				
Kupiec test	**Age weighted**	**Volatility weighted**	**T distribution**	**Generalized Extreme Value Theory**
2000	0,0782	0,9084	0,2261	**0,0000**
2001	0,2986	0,4627	0,4627	**0,0000**
2002	**0,0039**	0,1553	0,2738	**0,0000**
2003	**0,0000**	0,0782	**0,0039**	0,0843
2004	**0,0000**	0,9084	**0,0000**	0,4363
2005	**0,0009**	**0,0000**	**0,0128**	**0,0009**
2006	**0,0133**	0,4446	0,6468	**0,0002**
2007	0,2203	**0,0001**	**0,0000**	**0,0001**
2008	**0,0034**	**0,0034**	**0,0000**	**0,0000**
2009	**0,0000**	0,2738	0,2738	**0,0000**
2010	0,0782	0,6367	**0,0000**	**0,0000**

According to table one the model where age weighted historical simulation with a confidence level of 95% was used to calculate Value at Risk for one day could only be accepted for 2000-

2001, 2007 and 2010. The model was rejected for the remaining years between 2000 and 2010 as the p- value for Kupiec test had a value lower than 0,05, hence the alternative hypothesis, that the model was incorrect, was accepted. The models based on the volatility weighted approach could be accepted for all years except 2005, 2007 and 2008. The model where t distribution was assumed was accepted for 2000-2002, 2006 and 2009. The model based on the generalized extreme value approach could only be accepted for 2003-2004. Whenever the p- value is written as zero it is simply because the number was to low so four decimals were not enough.

Table 2. Kupiec test for 99% one day VaR models

P values for the four different approaches when testing 99% one day VaR during the period 2000- 2010				
Kupiec test	**Age weighted**	**Volatility weighted**	**T distribution**	**Generalized Extreme Value Theory**
2000	0,7680	0,3880	0,7327	0,7680
2001	0,3730	**0,0183**	0,7480	**0,0052**
2002	0,3880	**0,0199**	0,7680	**0,0057**
2003	**0,0244**	0,7327	**0,0244**	0,2732
2004	**0,0244**	0,7327	**0,0244**	**0,0244**
2005	0,7680	**0,0199**	**0,0244**	**0,0244**
2006	0,3843	0,3843	0,1640	**0,0247**
2007	**0,0014**	**0,0003**	**0,0000**	**0,0195**
2008	**0,0004**	**0,0015**	**0,0000**	**0,0000**
2009	**0,0244**	0,7327	**0,0244**	**0,0000**
2010	0,7327	0,1662	**0,0244**	**0,0199**

Table two shows the p-values for the one day VaR models using a confidence level of 99%. According to table two the model based on the age weighted approach could be accepted for two more years compared to the model using a confidence level of 95%, although the years differ somewhat. The model was accepted for the years of 2000-2002, 2005- 2006 and 2010 and rejected for the remaining years during the period of interest. The model using volatility weighted approach was rejected for 2001 and 2002 in addition to the years rejected using a confidence level of 95%. Similar results was found when comparing the models using t distribution for the two different confidence levels as the model used during 2009 was rejected in addition to the same years as calculated with a confidence level of 95% in table one. The models using generalized extreme value theory were rejected for all years except 2000 and 2003.

Table 3.Kupiec test for 99% ten day VaR models

P values for the four different approaches when testing 99% ten day VaR during the period 2000- 2009				
Kupiec test	**Age weighted**	**Volatility weighted**	**T distribution**	**Generalized Extreme Value Theory**
2000 - 2001	0,0952	0,4994	0,0952	**0,0128**
2002 - 2003	0,3260	0,3260	0,3260	0,3260
2004 - 2005	0,3260	0,3260	0,3260	0,3260
2006 - 2007	0,3260	0,3260	0,3260	0,3260
2008 - 2009	0,5102	0,3260	0,5102	0,0992

The p- values from Kupic test performed for the ten day holding period with a confidence level of 99% are presented in table three accepting all the two year models based on 48 observations except the model for 2000-2001 based on the generalized extreme value approach.

Table 4. Kupic test for 95% ten day VaR models

P values for the four different approaches when testing 95% ten day VaR during the period 2000- 2009				
Kupic test	**Age weighted**	**Volatility weighted**	**T distribution**	**Generalized Extreme Value Theory**
2000 - 2001	0,6760	0,8102	0,6760	**0,0000**
2002 - 2003	**0,0265**	0,7016	0,2961	0,1301
2004 - 2005	**0,0265**	**0,0265**	**0,0265**	**0,0265**
2006 - 2007	0,2961	0,7016	0,3314	**0,0265**
2008 - 2009	0,2961	0,7016	**0,0432**	**0,0007**

Table four contains the p-values from the ten day VaR Kupic test based on a 95 % confidence level. More of the models are rejected comparing to the same test with a different confidence level. The model based on the age weighted approach is rejected for the years of 2002-2003 and 2004- 2005. When using the volatility weighted approach the model is only rejected during the two year period 2004 to 2005 as in the case when using the t- distribution approach. When using generalized extreme value the model is rejected for every two year model except the years 2002 and 2003.

The approaches that gave the best estimates during the volatile times of 2001-2002 and 2007-2009 when using a holding period of only one day were the Volatility weighted- and t-distribution approach at the lower confidence level and this seemed accurate when comparing to the ten day holding period for the same confidence level. These two were chosen based on their high p value indicating a higher probability that the models, when these approaches are used, are accurate. However, looking at the ten day period the approaches deemed to be the best must have models that are accepted in both period 2000-2001 and 2002-2003 to be said to work in the first volatile period. During the financial crises no clear result could be gained by studying table one as the model using age weighted approach was only accepted in 2007 and volatility weighted only in 2009. When using the longer holding period at the lower confidence level the age weighted and volatility weighted approaches shows the highest p – values during the Global Financial Crises.

When looking at the results from the 99% confidence level for one day Value at Risk the best approaches for the first volatile period are age weighted and t distribution and for the second volatile period only volatile weighted approach can be accepted 2009 but no model was accepted in 2007-2008. The ten day holding period for this confidence level accepted all the models for every two year period except the model based on generalized extreme value theory 2000-2001.

6. Analysis

In this last chapter the result presented in the previous section will be discussed with special focus on which models based on the different approaches will be accepted during this ten year period and if the results differ for different holding periods and confidence levels. The validity of the results and tests used will also be discussed.

When considering the first volatile period, 2001-2002, which didn't contain actual losses as extreme, the t –distribution seemed to work for both the one and ten days holding period regardless of the confidence level and also used during the ten day holding period at the higher confidence level. At the lower confidence level volatility weighted provided the best p values.

The results cannot provide a clear answer to which approach that works best during volatile times as the best approach to use for the first volatile period were not the same as the second volatile period as the best approach changes depending on the confidence level and the holding period used. Most of the models for each year in the Kupiec test were rejected when using one day Value at Risk during the Global Financial Crises. According to the Basel Accord, as earlier mentioned, the 99% confidence level is preferred when calculated market risk and if only that is taken into account the volatility weighted approach would be preferred although the model cannot be accepted during every year of the Global Financial Crises and age weighted or t- distributed approach for the first volatile period when testing the one day VaR model per year.

6.1 Non-Parametric Approaches

When using the volatility weighted approach the results of one day Value at Risk for both confidence levels are the highest estimated loss of all during the Global Financial Crises. One reason for that can be that the rolling window used for these calculations contains two years with 500 observations, so during a financial crises the observations used in calculating VaR are still high which result in high estimated losses. As it is shown in tables one and two, this method is rejected during years 2005, 2007 and 2008 for both confidence level but for 95% confidence it is also rejected for the years 2001-2002. This suggests that the model cannot be used to accurately predict the actual losses but, as can be seen in graph one and two, it over estimates the loss during the crises which in practice would lead to higher capital requirement but with a lower number of violations.

According to table three and four the volatility weighted approach for both confidence levels can provide good estimates for ten day Value at Risk except during years 2004 and 2005 for 95% confidence level. Basically, a lower confidence level leads to higher numbers of exceedances which could decrease the validity of the model.

According to table three and four the age weighted approach cannot predict the loss well especially during the time of financial crises and the years preceding and following it. When looking at the one day approaches for both confidence levels the results indicate that the age weighted approach works better at higher confidence levels according to table one and two. Generally the age weighted historical simulation doesn't seem to adjust fast enough to a changing market climate between periods of crisis and periods without them so a lag effect is noticed. This is mainly because this approach bases its estimations on the historical observations without considering the changing volatility of the market, however since only a two year window is used the changes are notices sooner when moving from a non volatile to a volatile period.

It is shown that volatility weighted historical simulation can estimate the losses in crises time better than age weighted historical simulation. It is mostly because volatility weighted historical simulation takes into consideration the latest volatility changes directly so the risk estimates are depending on the latest volatility leading to historical simulation of value at risk according to GARCH estimation. This method can answer faster to the market changes. Finally it can get estimated losses that exceed the other estimation historical estimates.

6.2 Parametric Approaches

When the t-distribution approach is used for calculating VaR for the different confidence levels and holding periods a problem occurred between June of 2005 and March 2006 where the approximation of kurtosis couldn't be made since more observations existed in the middle of the distribution than in the tails, making the distribution more light tailed than normal. The t-distribution was therefore converted into a normal distribution by including a high number of degrees of freedom. When looking at the graphs for every confidence level and holding period investigated the VaR estimates during that low volatile period are closer to the actual values which were fairly positive and non volatile.

When Kupiec test was performed to test the models used to calculate VaR during the ten year period the models were tested once per year for the one day VaR calculations. As the result stated the model using t distribution was accepted for the first couple of year regardless of the confidence level used. The t- distribution was based on approximately 500 observations which is hoped to be enough observations for the symmetry assumption about the distribution to hold. As long as the actual returns are distributed with equal degree of volatility over both of the years used as a window to estimate VaR the model is accepted, which it is for the years 2000-2002, 2006 for both of the chosen confidence levels and 2009 in addition for confidence level 95%. The years 2000-2002 and 2006 models are all based on data that display an equal degree of volatility. The model for 2009 is based on data with a higher degree of volatility for the year of 20008 than 2007 and is accepted at the lower confidence level.

When looking at the results from testing the models of every approach with the ten day holding period the model, this time for every two year period, is accepted for every period when assuming a confidence level of 99% except for period 2000-2001 when using the GEV approach. When this result is compared to the models when VaR is calculated with 95% more models are rejected at the latter confidence level as can be seen in table four. This result can seem a bit strange as intuitively more models should be accepted at a lower confidence level but when looking at the ten day VaR for each confidence level the values are much lower at the 95% level resulting in many more exceedances when calculating the LR statistic for the Kupiec test and to many exceedeances leads to a rejection of the model.

The Value at Risk calculated using generalized extreme value approach was expected to have showed the predictions of the highest loss especially during the volatile times. As can be seen in graph one to four the values for Value at Risk calculated with the GEV approach are lower than VaR calculated with the other approaches regardless of the holding period except during a short period around 2006. This is the opposite result compared to what previous research has shown as Kourouma et al (2010) and Gencay and Selcuk (2004) states that extreme value approaches give more accurate estimates, at higher quantiles for example 99,9%, compared to more traditional approaches. In this paper 99% confidence level was considered high enough since it’s the highest of the confidence intervals most used. The result can be affected by the block size and number of block chosen to perform the calculations. In this paper the 4000 observations chosen to create the 40 blocks used to calculate VaR for January to May 2000 were taken from the middle of 1983 up till December 1999. If the highest return from each block during this period wasn’t high enough compared to the following actual loss the Value at Risk are meant to forecast the Fréchet distribution created from the highest observations in each block wouldn’t contain extreme values compared to the future loss. A trade- off between how many observations the maximum likelihood method should need to get accurate values for the three parameters needed to calculate VaR assuming a Fréchet distribution and how many observations the blocks need to contain so the highest loss in each block always represent an extreme loss exist. As only one more extreme value was added every five month after January 2000 the extreme losses from 2002 and 2007-2008 couldn't affect the parameter estimations compared to the remaining "extreme values" from 1983 and forward. When testing the model based on this approach it was rejected more times compared to the other approaches although it was expected to be accepted during the most volatile times suggesting a change in the created Fréchet distribution over time as it approaches the Global Financial Crises. An approach that could have given a more accurate result could have been to use a rolling window of 40 observations instead of increasing it over time letting the latest extreme observation have a weight of 1/40 when coming closer to 2010 instead of 1/66.

According to the results in table one and two and similar to what Gencay and Selcuk (2004) claimed extreme value theory model is accepted more at higher quantiles. When comparing the results presented in table three and four a similar conclusion can be drawn as the model is accepted for all two year periods when using 99% confidence level compared to only once using 95% confidence level.

The validity of Kupiec test is questionable as fewer observations are used as in the case when testing the ten day VaR models for two years containing two ten day periods per month compared to the test for one day VaR over each year containing approximately 250 observations. When comparing the results for VaR calculated over different holding periods the models are more accepted for the longer holding period. This can be explained by looking at graph one and two compared to graph three and four as the fewer observations in three and four doesn't exceed the VaR values as often. The problem with the longer period in these tests are the fact that as VaR was updated for each month only the first twenty trading days with the actual return of each month could be used as the remaining trading days that month weren't enough to be aggregated into a new ten day period within the same month. Aggregation for a ten day period into the following month was not possible in this paper as the aggregated ten day period are compared to a month specific Value at Risk value. This means that for almost every month the actual return for a few trading days are not including which may affect the validity of the results. It should be stressed that Kupiec test cannot show the reliability of the results. As mentioned in section 3.8 it cannot cover volatility clustering, hence a better and more accurate model such as Christoffersen which test the unconditional coverage, like Kupiec test, as well as the independently assumption of losses in its second sub-hypothesis is suggested to solve this problem.

When holding a financial asset over a longer period to get statistically better results the aggregated ten days could have been to use recursive estimation window giving the same amount of observations as used when Kuiepc test was performed for the one day holding period per year.

6.3 The estimated parameters

The estimated parameters from the GARCH model used to find the in- and out of sample volatility were plotted and presented in appendix 9.2. This shows a high beta value which shows the persistency of volatility, but in 2004-2006, during the non volatile period, beta is lowered. A high α_1, which exist during 2004-2006, shows that the volatility of the returns responds fast to the latest market climate. The parameter that is fairly constant is α_0 which is not affecting the forecasted volatility of the returns.

The scale parameter estimated to calculate Value at Risk with GEV is fairly constant during the entire ten year period while the other two, the tail parameter and the location parameter, change as the volatility of the market changes. This is expected as the distressed period would display higher and more losses affecting the tail of the distribution to become heavier and move the mean. Although changes in both the scale parameter and the tail parameter did occur, they were both high during the beginning and the end of the ten year period, and did not cause the Value at Risk to fluctuate that much as the only moved in the rage 0,05 to 0,5. The graph for the parameter estimations during the ten year period can be seen in appendix 9.2.

7. Conclusion

Uncertainties in the financial markets require tools to predict the instability in the market prices and estimate potential losses and the standard method today is Value at Risk (VaR), which measures the worst loss potential when holding a financial asset over specific time period with a certain probability. The simplicity of the measurement is the main part of its appeal as it can be applied to all asset classes although it does suffer from drawback as the problem of estimating the loss if a tail event occurs. The purpose of this paper was to investigate which of four chosen approaches to calculate risk that would give the most accurate result during the two volatile periods during the years 2000-2010. The approaches chosen were Value at Risk according to volatility weighted historical simulation, age weighted historical simulation, assuming t- distributed profit/losses and assuming generalized extreme value distribution.

The result could not clearly indicate that one approach was better than the others regardless of confidence level or holding period used but if chosen according to the Basel II Accord where ten day Value at Risk estimates calculated with a 99% confidence level are required all standard approaches were accepted. This indicates, to the contrary of what was expected, that extreme value approach was not the best approach during these economically distressed times. This result have been affected by the windows used to calculate VaR for every approach and as the historical simulations were done with only approximately 500 observations only the newest data was taken into account. This created larger Value at Risk estimates for the volatility weighted approach as that approach respond quickly to the change in market volatility and can predict losses higher than the previous historical observations contain. The GEV approach was the only approach that was not based on a two year rolling window. Instead blocks with one extreme value each were added over time only updating the VaR estimation once per five months. According to the graphs the distribution of the extreme values does not change much over time although the actual returns in the parent distribution does. A more accurate result could perhaps been obtained using a rolling window of equal number of extreme values to, in the end, calculate VaR.

The accuracy of the results are questionable, especially when testing models for ten day Value at Risk since returns after the twentieth trading day of each month fell out of the sample when aggregating two ten day periods per month to compare to the calculated Value at Risk for that month for each approach.

We encourage future studies within this subject, especially after the publication of the Basel III Accord, as it would be interesting to calculate Value at Risk for the holding periods stated there. Studies in Extreme Value Theory would also be interesting especially replicating this study but using a rolling window of the same number of blocks to estimate the tail-, scale- and location parameters once per month as well to see if the result would differ radically.

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9. Appendix

9.1 Value at Risk estimates for every confidence level and holding period

1-day VaR α=%99				
Date	Age weighted	Volatility weighted	t-distrubution	GEV
Jan , 2011	0.037294809	0.015872562	0.036879691	0.024714734
Dec , 2010	0.038682276	0.025700448	0.04205776	0.024714734
Nov , 2010	0.042831295	0.019738882	0.0480306	0.024714734
Oct , 2010	0.048727373	0.025834384	0.054426203	0.024981306
Sep , 2010	0.054672234	0.029577588	0.057916742	0.024981306
Aug , 2010	0.055863351	0.031682148	0.058000672	0.024981306
Jul , 2010	0.057075644	0.039958776	0.058277511	0.024981306
Jun , 2010	0.058319124	0.047674892	0.058185273	0.024981306
May , 2010	0.029666345	0.029978954	0.056712673	0.024964775
Apr , 2010	0.060816878	0.017993347	0.056764129	0.024964775
Mar , 2010	0.062147064	0.026641528	0.058060343	0.024964775
Feb , 2010	0.063426919	0.028263574	0.058256031	0.024964775
Jan , 2010	0.064644166	0.020678858	0.058652604	0.024964775
Dec , 2009	0.06602227	0.028924538	0.059095849	0.025323991
Nov , 2009	0.067387567	0.038638124	0.059834621	0.025323991
Oct , 2009	0.068918861	0.025456684	0.059309458	0.025323991
Sep , 2009	0.070317706	0.027843634	0.059446023	0.025323991
Aug , 2009	0.07187661	0.031756931	0.059848098	0.025323991
Jul , 2009	0.073442557	0.035697918	0.059872581	0.024980834
Jun, 2009	0.075042615	0.046022434	0.059630907	0.024980834
May , 2009	0.076629562	0.048972736	0.0586049	0.024980834
Apr , 2009	0.07822076	0.07294112	0.057506265	0.024980834
Mar , 2009	0.079961566	0.060733571	0.060821643	0.024980834
Feb , 2009	0.081496135	0.065369136	0.059116631	0.024878003
Jan , 2009	0.083143299	0.063345591	0.056832346	0.024878003
Dec , 2008	0.077289272	0.115505125	0.053579605	0.024878003
Nov , 2008	0.071736415	0.122959905	0.046814711	0.024878003
Oct , 2008	0.042777109	0.094488796	0.03693351	0.024878003
Sep , 2008	0.030447412	0.033069278	0.027781919	0.02469733
Aug , 2008	0.031122419	0.037638718	0.026587408	0.02469733
Jul , 2008	0.031785891	0.031808499	0.026786411	0.02469733
Jun , 2008	0.032165966	0.022380342	0.026234032	0.02469733
May , 2008	0.032864002	0.026507401	0.026175958	0.02469733
Apr , 2008	0.033549212	0.038497684	0.025703851	0.024514035
Mar , 2008	0.034274406	0.031630324	0.023146744	0.024514035
Feb , 2008	0.031620872	0.030909673	0.02228312	0.024514035
Jan , 2008	0.031715079	0.027588986	0.021697801	0.024514035
Dec , 2007	0.031651337	0.035038413	0.020822833	0.024514035
Nov , 2007	0.028543369	0.022960495	0.018856893	0.024503732
Oct , 2007	0.028353814	0.022794265	0.019168172	0.024503732
Sep , 2007	0.028924476	0.030623099	0.018803128	0.024503732
Aug , 2007	0.021536644	0.025768667	0.017463624	0.024503732
Jul , 2007	0.017172058	0.017570042	0.016561994	0.024503732
Jun , 2007	0.016572763	0.014912904	0.015981684	0.02452767
May , 2007	0.016875076	0.014883396	0.016129207	0.02452767
Apr , 2007	0.017224043	0.01772136	0.016958633	0.02452767
Mar , 2007	0.016703934	0.02247457	0.016724563	0.02452767
Feb , 2007	0.0165161	0.01405948	0.014972878	0.02452767
Jan , 2007	0.016849916	0.013413409	0.015224076	0.024485337
Dec , 2006	0.017206246	0.014941992	0.015225724	0.024485337
Nov , 2006	0.015284891	0.013901557	0.01522985	0.024485337
Oct , 2006	0.015617897	0.01407004	0.01546492	0.024485337
Sep , 2006	0.015940863	0.013668292	0.015520114	0.024485337
Aug , 2006	0.016304465	0.017032854	0.015792759	0.025065644
Jul , 2006	0.016067362	0.018449887	0.015643263	0.025065644
Jun , 2006	0.016094389	0.019388542	0.014945674	0.025065644
May , 2006	0.013688209	0.013616956	0.014742702	0.025065644
Apr , 2006	0.013963702	0.014510467	0.014907124	0.025065644
Mar , 2006	0.014288753	0.015038779	0.015742172	0.025475624
Feb , 2006	0.014577551	0.015048427	0.015655177	0.025475624
Jan , 2006	0.014560136	0.014096347	0.015952198	0.025475624
Dec , 2005	0.014876105	0.019754674	0.016144138	0.025475624
Nov , 2005	0.015178045	0.035582883	0.016487378	0.025475624

Oct , 2005	0.0147311	0.014332893	0.015704749	0.02588534
Sep , 2005	0.019709614	0.025580708	0.016289033	0.02588534
Aug , 2005	0.015379972	0.022515683	0.016283545	0.02588534
Jul , 2005	0.015705217	0.022076826	0.016898834	0.02588534
Jun , 2005	0.01604738	0.017698393	0.017386804	0.02588534
May , 2005	0.016750339	0.03600596	0.017129858	0.02649638
Apr , 2005	0.016598689	0.024311776	0.017536685	0.02649638
Mar , 2005	0.01801015	0.025809366	0.020304909	0.02649638
Feb , 2005	0.017428882	0.020357339	0.021180333	0.02649638
Jan , 2005	0.018735375	0.007580018	0.02238488	0.02649638
Dec , 2004	0.019143553	0.015623163	0.0228206	0.027053307
Nov , 2004	0.019541063	0.017976955	0.023537527	0.027053307
Oct , 2004	0.02111986	0.015253182	0.026470588	0.027053307
Sep , 2004	0.023422561	0.017143421	0.028821038	0.027053307
Aug , 2004	0.025399204	0.015972633	0.029836084	0.027053307
Jul , 2004	0.026407186	0.015019567	0.033034199	0.027667618
Jun , 2004	0.026955524	0.017325971	0.03416849	0.027667618
May , 2004	0.02752556	0.019298364	0.034995569	0.027667618
Apr , 2004	0.028122889	0.020579276	0.034164455	0.027667618
Mar , 2004	0.028737999	0.015023451	0.034216685	0.027667618
Feb , 2004	0.02928952	0.017859912	0.034912092	0.028357625
Jan , 2004	0.029895208	0.016330956	0.035275639	0.028357625
Dec , 2003	0.030532518	0.02313348	0.035750989	0.028357625
Nov , 2003	0.031147024	0.022704944	0.035030961	0.028357625
Oct , 2003	0.031872075	0.026672273	0.03569345	0.028357625
Sep , 2003	0.033221469	0.0216543	0.036971563	0.029108055
Aug , 2003	0.033957999	0.024132084	0.036645813	0.029108055
Jul , 2003	0.034697814	0.025221557	0.036996425	0.029108055
Jun , 2003	0.035434545	0.028230571	0.036893071	0.029108055
May , 2003	0.036203524	0.028282991	0.036964744	0.029108055
Apr , 2003	0.036955268	0.034551246	0.03778058	0.029405695
Mar , 2003	0.03689109	0.028925322	0.038144136	0.029405695
Feb , 2003	0.03759908	0.035024423	0.038603859	0.029405695
Jan , 2003	0.038397413	0.025433254	0.038328974	0.029405695
Dec , 2002	0.039194712	0.035875602	0.038912428	0.029405695
Nov , 2002	0.040005246	0.032160854	0.038680276	0.029406157
Oct , 2002	0.040917725	0.041555869	0.037725801	0.029406157
Sep , 2002	0.040590837	0.032058246	0.037119011	0.029406157
Aug , 2002	0.038800809	0.056482031	0.036055973	0.029406157
Jul , 2002	0.032072635	0.029595535	0.033272895	0.029406157
Jun , 2002	0.03275091	0.027262479	0.032645496	0.029289319
May , 2002	0.033479783	0.02587071	0.032319607	0.029289319
Apr , 2002	0.034594566	0.023091633	0.034682451	0.029289319
Mar , 2002	0.035342391	0.025725403	0.035416876	0.029289319
Feb , 2002	0.036037185	0.031934703	0.03546491	0.029289319
Jan , 2002	0.036342316	0.024752225	0.035117716	0.029608874
Dec , 2001	0.037110876	0.026591059	0.034720528	0.029608874
Nov , 2001	0.037898843	0.03071211	0.03575113	0.029608874
Oct , 2001	0.038745508	0.039022153	0.034970373	0.029608874
Sep , 2001	0.03211674	0.026203403	0.033892422	0.029608874
Aug , 2001	0.032849306	0.026233149	0.034008484	0.029374107
Jul , 2001	0.033546789	0.02439564	0.033627581	0.029374107
Jun , 2001	0.034274783	0.028069176	0.033717499	0.029374107
May , 2001	0.035005469	0.034798149	0.034104619	0.029374107
Apr , 2001	0.033576478	0.03511589	0.033886768	0.029374107
Mar , 2001	0.032243855	0.028069679	0.032135433	0.029384067
Feb , 2001	0.032862657	0.02783804	0.032269721	0.029384067
Jan , 2001	0.035706114	0.031945683	0.031833259	0.029384067
Dec , 2000	0.030674966	0.03219156	0.03131381	0.029384067
Nov , 2000	0.030139224	0.037171209	0.030573193	0.029384067
Oct , 2000	0.029489706	0.027496351	0.030924084	0.029213601
Sep , 2000	0.030099529	0.021158275	0.032068201	0.029213601
Aug , 2000	0.030846441	0.036926498	0.03572915	0.029213601
Jul , 2000	0.031498754	0.026610886	0.035661649	0.029213601
Jun , 2000	0.03219976	0.039570371	0.03553459	0.029213601
May , 2000	0.032886209	0.046046149	0.034352215	0.029512245
Apr , 2000	0.03346709	0.035058196	0.03320541	0.029512245
Mar , 2000	0.034230457	0.032522412	0.035439982	0.029512245
Feb , 2000	0.034094669	0.037840105	0.031195974	0.029512245
Jan , 2000	0.028306388	0.019971133	0.030351494	0.029512245

1 Day VaR α=0.95				
Date	Age weighted	Volatility weighted	t-distrubution	GEV
Jan , 2011	0.034126928	0.011876717	0.022512411	0.012245499
Dec , 2010	0.035464685	0.018499064	0.02491617	0.012245499
Nov , 2010	0.038880784	0.014904565	0.028538367	0.012245499
Oct , 2010	0.041554068	0.020168532	0.032523292	0.012283093
Sep , 2010	0.044730946	0.0231605	0.034747367	0.012283093
Aug , 2010	0.045705477	0.025230788	0.034749036	0.012283093
Jul , 2010	0.046697334	0.031281205	0.034975165	0.012283093
Jun , 2010	0.047714706	0.037304867	0.034882927	0.012283093
May , 2010	0.024247283	0.02111562	0.033992435	0.012344405
Apr , 2010	0.047205826	0.012724468	0.033997505	0.012344405
Mar , 2010	0.048238311	0.018875738	0.034821936	0.012344405
Feb , 2010	0.04923173	0.020324243	0.034989306	0.012344405
Jan , 2010	0.050176553	0.014717118	0.035245451	0.012344405
Dec , 2009	0.051246231	0.021344224	0.035531761	0.012473186
Nov , 2009	0.05230597	0.028563409	0.036052677	0.012473186
Oct , 2009	0.053494554	0.019387125	0.035711091	0.012473186
Sep , 2009	0.054580332	0.019759351	0.035793613	0.012473186
Aug , 2009	0.055790348	0.02228556	0.036048507	0.012473186
Jul , 2009	0.05700583	0.025283724	0.036146907	0.012455545
Jun, 2009	0.058247788	0.033330426	0.03601806	0.012455545
May , 2009	0.05947957	0.036281307	0.035420197	0.012455545
Apr , 2009	0.060714652	0.052775725	0.034803543	0.012455545
Mar , 2009	0.062065859	0.042958086	0.035068217	0.012455545
Feb , 2009	0.063256986	0.045652797	0.034021105	0.012517238
Jan , 2009	0.063894796	0.044387172	0.032637888	0.012517238
Dec , 2008	0.062954841	0.075659873	0.030792974	0.012517238
Nov , 2008	0.04726795	0.084160944	0.026877612	0.012517238
Oct , 2008	0.031000135	0.064891876	0.021094915	0.012517238
Sep , 2008	0.027905749	0.022298654	0.017547774	0.01257875
Aug , 2008	0.028524409	0.025306588	0.016787492	0.01257875
Jul , 2008	0.028482985	0.020340826	0.016537152	0.01257875
Jun , 2008	0.028594052	0.015458771	0.016127905	0.01257875
May , 2008	0.029214573	0.018286996	0.016124114	0.01257875
Apr , 2008	0.029823693	0.02837787	0.015858885	0.01264139
Mar , 2008	0.030078452	0.019618735	0.014584493	0.01264139
Feb , 2008	0.02987196	0.026410731	0.01401337	0.01264139
Jan , 2008	0.026607367	0.017337275	0.01327929	0.01264139
Dec , 2007	0.02299917	0.021434779	0.012733427	0.01264139
Nov , 2007	0.020705309	0.013737121	0.011459363	0.012691861
Oct , 2007	0.020631473	0.01455028	0.011675832	0.012691861
Sep , 2007	0.019838844	0.019505308	0.011472703	0.012691861
Aug , 2007	0.016785195	0.016860968	0.010664409	0.012691861
Jul , 2007	0.015335083	0.011084178	0.010055746	0.012691861
Jun , 2007	0.013221919	0.008858005	0.009684118	0.012737935
May , 2007	0.013426524	0.009135397	0.009776794	0.012737935
Apr , 2007	0.013704177	0.011401458	0.010337135	0.012737935
Mar , 2007	0.013342023	0.014652442	0.010214863	0.012737935
Feb , 2007	0.013582787	0.009544931	0.009913067	0.012737935
Jan , 2007	0.013818554	0.008744971	0.010146341	0.012786278
Dec , 2006	0.014110778	0.010180606	0.010169191	0.012786278
Nov , 2006	0.013682992	0.00949986	0.010187988	0.012786278
Oct , 2006	0.013981098	0.009615439	0.010449217	0.012786278
Sep , 2006	0.014270217	0.011075651	0.01051404	0.012786278
Aug , 2006	0.014595712	0.011791754	0.010801414	0.025065644
Jul , 2006	0.013964237	0.012787697	0.010713322	0.025065644
Jun , 2006	0.013134414	0.011838891	0.010378655	0.025065644
May , 2006	0.012679761	0.009596899	0.010210439	0.025065644
Apr , 2006	0.012807552	0.01026455	0.010422652	0.025065644
Mar , 2006	0.01310569	0.010786264	0.010994676	0.013042212
Feb , 2006	0.012646025	0.010587732	0.010926644	0.013042212
Jan , 2006	0.012852005	0.0099383	0.011140186	0.013042212
Dec , 2005	0.013130906	0.013631945	0.011245463	0.013042212
Nov , 2005	0.013397423	0.024362971	0.011502471	0.013042212

Oct , 2005	0.013081285	0.026057226	0.010910735	0.013114473
Sep , 2005	0.018269824	0.01776668	0.011332293	0.013114473
Aug , 2005	0.013657486	0.015842326	0.011310758	0.013114473
Jul , 2005	0.013946305	0.015382654	0.011754782	0.013114473
Jun , 2005	0.014250147	0.012338053	0.012090451	0.013114473
May , 2005	0.014789103	0.025339628	0.011964227	0.013268911
Apr , 2005	0.014270485	0.016976753	0.011981922	0.013268911
Mar , 2005	0.015185497	0.024980612	0.012768289	0.013268911
Feb , 2005	0.015439136	0.014229481	0.013355237	0.013268911
Jan , 2005	0.016537423	0.005346628	0.014129055	0.013268911
Dec , 2004	0.016945988	0.010600086	0.014654195	0.013388213
Nov , 2004	0.017512735	0.012438318	0.015252545	0.013388213
Oct , 2004	0.018756786	0.010679206	0.016479062	0.013388213
Sep , 2004	0.019498061	0.011828149	0.018053587	0.013388213
Aug , 2004	0.022038191	0.010825747	0.0189946	0.013388213
Jul , 2004	0.023938801	0.010792332	0.020740998	0.013532052
Jun , 2004	0.024435884	0.012483422	0.021524554	0.013532052
May , 2004	0.024952636	0.014015674	0.022061779	0.013532052
Apr , 2004	0.02549413	0.014436745	0.021913943	0.013532052
Mar , 2004	0.026051744	0.010035325	0.021909931	0.013532052
Feb , 2004	0.026551712	0.012481035	0.022379259	0.013720759
Jan , 2004	0.027100784	0.011438712	0.022635609	0.013720759
Dec , 2003	0.027678522	0.016024219	0.022970352	0.013720759
Nov , 2003	0.028235588	0.01577119	0.022731544	0.013720759
Oct , 2003	0.028892865	0.018528824	0.023186047	0.013720759
Sep , 2003	0.030070133	0.01534076	0.024066024	0.013961085
Aug , 2003	0.030736797	0.017176134	0.024133757	0.013961085
Jul , 2003	0.031406434	0.01812383	0.024382142	0.013961085
Jun , 2003	0.03207328	0.020296795	0.024339784	0.013961085
May , 2003	0.032769315	0.020341122	0.024417074	0.013961085
Apr , 2003	0.03344975	0.024872271	0.024953664	0.014028169
Mar , 2003	0.033527569	0.020868859	0.025242759	0.014028169
Feb , 2003	0.034171008	0.025358084	0.025599102	0.014028169
Jan , 2003	0.03382003	0.01867099	0.025147397	0.014028169
Dec , 2002	0.034522283	0.026176395	0.025479514	0.014028169
Nov , 2002	0.035236192	0.023502783	0.025426037	0.014076524
Oct , 2002	0.036039894	0.030636378	0.024870258	0.014076524
Sep , 2002	0.031297547	0.023382519	0.024155967	0.014076524
Aug , 2002	0.031403198	0.041211448	0.023082886	0.014076524
Jul , 2002	0.025576528	0.021245886	0.02160894	0.014076524
Jun , 2002	0.025062887	-0.036320999	0.021134859	0.014124227
May , 2002	0.025620662	0.019052878	0.0211795	0.014124227
Apr , 2002	0.026154824	0.016801027	0.022083454	0.014124227
Mar , 2002	0.026720209	0.018989957	0.022503499	0.014124227
Feb , 2002	0.026379096	0.022920316	0.022532952	0.014124227
Jan , 2002	0.026926844	0.017082972	0.022688475	0.014189375
Dec , 2001	0.027496287	0.018645427	0.022398421	0.014189375
Nov , 2001	0.02808011	0.021373454	0.022743412	0.014189375
Oct , 2001	0.028250683	0.027211478	0.022566762	0.014189375
Sep , 2001	0.027918778	0.018761844	0.021835121	0.014189375
Aug , 2001	0.028555591	0.018682194	0.021867089	0.014238552
Jul , 2001	0.029144816	0.017414606	0.021638436	0.014238552
Jun , 2001	0.029777284	0.020172649	0.021640313	0.014238552
May , 2001	0.03041209	0.025203325	0.021910185	0.014238552
Apr , 2001	0.030410242	0.025428869	0.021743411	0.014238552
Mar , 2001	0.026678685	0.019949281	0.020601089	0.014274266
Feb , 2001	0.027190685	0.019795959	0.020641681	0.014274266
Jan , 2001	0.032463109	0.022344753	0.020604241	0.014274266
Dec , 2000	0.026872298	0.020989708	0.020232587	0.014274266
Nov , 2000	0.024649044	0.025146602	0.019653914	0.014274266
Oct , 2000	0.024743327	0.018120954	0.019580996	0.014310238
Sep , 2000	0.025254998	0.014802089	0.020231436	0.014310238
Aug , 2000	0.027913049	0.022028596	0.021871143	0.014310238
Jul , 2000	0.02850333	0.018087626	0.021826601	0.014310238
Jun , 2000	0.029137672	0.027440385	0.021736798	0.014310238
May , 2000	0.029758843	0.0319316	0.021003984	0.014362024
Apr , 2000	0.026601632	0.02447092	0.020267834	0.014362024
Mar , 2000	0.027003288	0.02264773	0.019904807	0.014362024
Feb , 2000	0.027403547	0.026681376	0.018993196	0.014362024
Jan , 2000	0.024806534	0.013699141	0.018424872	0.014362024

10-day VaR α=0.99				
Date	10 day Age weighted	10 day Volatility weighted	10 day t distribution	10 day GEV
Jan-11	0.117936543	0.050193449	0.111408721	0.078154851
Jan-11	0.117936543	0.050193449	0.111408721	0.078154851
Dec-10	0.122324096	0.081271953	0.128402689	0.078154851
Dec-10	0.122324096	0.081271953	0.128402689	0.078154851
Nov-10	0.135444447	0.062419825	0.148064856	0.078154851
Nov-10	0.135444447	0.062419825	0.148064856	0.078154851
Oct-10	0.084951212	0.081695497	0.170931372	0.078997827
Oct-10	0.084951212	0.081695497	0.170931372	0.078997827
Sep-10	0.172888783	0.093532545	0.184250147	0.078997827
Sep-10	0.172888783	0.093532545	0.184250147	0.078997827
Aug-10	0.176655427	0.100187748	0.183687328	0.078997827
Aug-10	0.176655427	0.100187748	0.183687328	0.078997827
Jul-10	0.180489035	0.126360745	0.18559138	0.078997827
Jul-10	0.180489035	0.126360745	0.18559138	0.078997827
Jun-10	0.184421262	0.150761245	0.184668997	0.078997827
Jun-10	0.184421262	0.150761245	0.184668997	0.078997827
May-10	0.09381322	0.094801775	0.179864912	0.07894555
May-10	0.09381322	0.094801775	0.179864912	0.07894555
Apr-10	0.192319854	0.056899958	0.179588827	0.07894555
Apr-10	0.192319854	0.056899958	0.179588827	0.07894555
Mar-10	0.196526272	0.084247909	0.184509517	0.07894555
Mar-10	0.196526272	0.084247909	0.184509517	0.07894555
Feb-10	0.200573529	0.089377268	0.185983727	0.07894555
Feb-10	0.200573529	0.089377268	0.185983727	0.07894555
Jan-10	0.204422803	0.06539229	0.187555897	0.07894555
Jan-10	0.204422803	0.06539229	0.187555897	0.07894555
Dec-09	0.208780749	0.091467419	0.189313416	0.080081493
Dec-09	0.208780749	0.091467419	0.189313416	0.080081493
Nov-09	0.213098198	0.122184477	0.192987821	0.080081493
Nov-09	0.213098198	0.122184477	0.192987821	0.080081493
Oct-09	0.217940575	0.080501103	0.19086523	0.080081493
Oct-09	0.217940575	0.080501103	0.19086523	0.080081493
Sep-09	0.22236411	0.0880493	0.191309726	0.080081493
Sep-09	0.22236411	0.0880493	0.191309726	0.080081493
Aug-09	0.227293799	0.100424233	0.192821808	0.080081493
Aug-09	0.227293799	0.100424233	0.192821808	0.080081493
Jul-09	0.232245758	0.112886729	0.194326536	0.078996334
Jul-09	0.232245758	0.112886729	0.194326536	0.078996334
Jun-09	0.237305584	0.145535715	0.19383291	0.078996334
Jun-09	0.237305584	0.145535715	0.19383291	0.078996334
May-09	0.242323951	0.154865387	0.190870459	0.078996334
May-09	0.242323951	0.154865387	0.190870459	0.078996334
Apr-09	0.247355763	0.230660074	0.188099384	0.078996334
Apr-09	0.247355763	0.230660074	0.188099384	0.078996334
Mar-09	0.252860674	0.192056416	0.199685924	0.078996334
Mar-09	0.252860674	0.192056416	0.199685924	0.078996334
Feb-09	0.257713409	0.206715358	0.19307217	0.078671152
Feb-09	0.257713409	0.206715358	0.19307217	0.078671152
Jan-09	0.262922196	0.200316348	0.1845231	0.078671152
Jan-09	0.262922196	0.200316348	0.1845231	0.078671152
Dec-08	0.244410137	0.365259277	0.174328257	0.078671152
Dec-08	0.244410137	0.365259277	0.174328257	0.078671152
Nov-08	0.226850464	0.38883336	0.151881878	0.078671152
Nov-08	0.226850464	0.38883336	0.151881878	0.078671152
Oct-08	0.135273095	0.298799808	0.118085069	0.078671152
Oct-08	0.135273095	0.298799808	0.118085069	0.078671152
Sep-08	0.09628317	0.10457424	0.087682784	0.078099815
Sep-08	0.09628317	0.10457424	0.087682784	0.078099815
Aug-08	0.098417731	0.119024076	0.083805042	0.078099815
Aug-08	0.098417731	0.119024076	0.083805042	0.078099815
Jul-08	0.100515813	0.100587305	0.084249788	0.078099815
Jul-08	0.100515813	0.100587305	0.084249788	0.078099815
Jun-08	0.101717717	0.070772854	0.081290201	0.078099815
Jun-08	0.101717717	0.070772854	0.081290201	0.078099815
May-08	0.103925098	0.083823762	0.081681928	0.078099815

May-08	0.103925098	0.083823762	0.081681928	0.078099815
Apr-08	0.106091922	0.121740365	0.080667067	0.077520185
Apr-08	0.106091922	0.121740365	0.080667067	0.077520185
Mar-08	0.108385189	0.100023866	0.072392517	0.077520185
Mar-08	0.108385189	0.100023866	0.072392517	0.077520185
Feb-08	0.099993979	0.097744968	0.069190543	0.077520185
Feb-08	0.099993979	0.097744968	0.069190543	0.077520185
Jan-08	0.100291887	0.087244033	0.066161333	0.077520185
Jan-08	0.100291887	0.087244033	0.066161333	0.077520185
Dec-07	0.100090316	0.110801192	0.063307542	0.077520185
Dec-07	0.100090316	0.110801192	0.063307542	0.077520185
Nov-07	0.090262057	0.072607461	0.056043022	0.077487605
Nov-07	0.090262057	0.072607461	0.056043022	0.077487605
Oct-07	0.089662632	0.072081796	0.057457299	0.077487605
Oct-07	0.089662632	0.072081796	0.057457299	0.077487605
Sep-07	0.091467224	0.096838742	0.056707545	0.077487605
Sep-07	0.091467224	0.096838742	0.056707545	0.077487605
Aug-07	0.068104848	0.081487679	0.052829076	0.077487605
Aug-07	0.068104848	0.081487679	0.052829076	0.077487605
Jul-07	0.054302816	0.055561352	0.04906125	0.077487605
Jul-07	0.054302816	0.055561352	0.04906125	0.077487605
Jun-07	0.052407679	0.047158744	0.046996649	0.077563304
Jun-07	0.052407679	0.047158744	0.046996649	0.077563304
May-07	0.053363675	0.04706543	0.047489312	0.077563304
May-07	0.053363675	0.04706543	0.047489312	0.077563304
Apr-07	0.054467206	0.056039861	0.050962944	0.077563304
Apr-07	0.054467206	0.056039861	0.050962944	0.077563304
Mar-07	0.052822476	0.071070832	0.050625093	0.077563304
Mar-07	0.052822476	0.071070832	0.050625093	0.077563304
Feb-07	0.052228493	0.044459981	0.044535493	0.077563304
Feb-07	0.052228493	0.044459981	0.044535493	0.077563304
Jan-07	0.053284112	0.042416925	0.045865322	0.077429434
Jan-07	0.053284112	0.042416925	0.045865322	0.077429434
Dec-06	0.054410926	0.047250726	0.0456143	0.077429434
Dec-06	0.054410926	0.047250726	0.0456143	0.077429434
Nov-06	0.048335069	0.043960582	0.045333973	0.077429434
Nov-06	0.048335069	0.043960582	0.045333973	0.077429434
Oct-06	0.049388126	0.044493375	0.046298498	0.077429434
Oct-06	0.049388126	0.044493375	0.046298498	0.077429434
Sep-06	0.050409436	0.043222934	0.046679325	0.077429434
Sep-06	0.050409436	0.043222934	0.046679325	0.077429434
Aug-06	0.051559245	0.053862614	0.047784683	0.079264526
Aug-06	0.051559245	0.053862614	0.047784683	0.079264526
Jul-06	0.050809461	0.058343665	0.047861833	0.079264526
Jul-06	0.050809461	0.058343665	0.047861833	0.079264526
Jun-06	0.050894925	0.061311952	0.045420942	0.079264526
Jun-06	0.050894925	0.061311952	0.045420942	0.079264526
May-06	0.043285917	0.043060595	0.044182973	0.079264526
May-06	0.043285917	0.043060595	0.044182973	0.079264526
Apr-06	0.044157102	0.045886125	0.045101867	0.079264526
Apr-06	0.044157102	0.045886125	0.045101867	0.079264526
Mar-06	0.045185005	0.047556794	0.048106228	0.080560996
Mar-06	0.045185005	0.047556794	0.048106228	0.080560996
Feb-06	0.046098264	0.047587306	0.047672908	0.080560996
Feb-06	0.046098264	0.047587306	0.047672908	0.080560996
Jan-06	0.046043192	0.044576562	0.048720946	0.080560996
Jan-06	0.046043192	0.044576562	0.048720946	0.080560996
Dec-05	0.047042373	0.062469765	0.048644843	0.080560996
Dec-05	0.047042373	0.062469765	0.048644843	0.080560996
Nov-05	0.047997191	0.112522955	0.05009166	0.080560996
Nov-05	0.047997191	0.112522955	0.05009166	0.080560996
Oct-05	0.046583829	0.045324588	0.046660873	0.081856632
Oct-05	0.046583829	0.045324588	0.046660873	0.081856632
Sep-05	0.062327271	0.080893302	0.048756796	0.081856632
Sep-05	0.062327271	0.080893302	0.048756796	0.081856632
Aug-05	0.048635743	0.071200842	0.048332408	0.081856632
Aug-05	0.048635743	0.071200842	0.048332408	0.081856632
Jul-05	0.049664257	0.069813054	0.050541764	0.081856632
Jul-05	0.049664257	0.069813054	0.050541764	0.081856632
Jun-05	0.050746272	0.055967231	0.051914594	0.081856632
Jun-05	0.050746272	0.055967231	0.051914594	0.081856632
May-05	0.052969223	0.113860844	0.050819631	0.083788912
May-05	0.052969223	0.113860844	0.050819631	0.083788912

Apr-05	0.052489665	0.076880586	0.050765076	0.083788912
Apr-05	0.052489665	0.076880586	0.050765076	0.083788912
Mar-05	0.056953096	0.081616383	0.059098355	0.083788912
Mar-05	0.056953096	0.081616383	0.059098355	0.083788912
Feb-05	0.055114964	0.064375559	0.062342664	0.083788912
Feb-05	0.055114964	0.064375559	0.062342664	0.083788912
Jan-05	0.059246458	0.023970121	0.066161126	0.083788912
Jan-05	0.059246458	0.023970121	0.066161126	0.083788912
Dec-04	0.060537231	0.04940478	0.06879691	0.085550068
Dec-04	0.060537231	0.04940478	0.06879691	0.085550068
Nov-04	0.061794266	0.056848123	0.070792802	0.085550068
Nov-04	0.061794266	0.056848123	0.070792802	0.085550068
Oct-04	0.066786861	0.048234795	0.079076997	0.085550068
Oct-04	0.066786861	0.048234795	0.079076997	0.085550068
Sep-04	0.074068642	0.054212256	0.088165874	0.085550068
Sep-04	0.074068642	0.054212256	0.088165874	0.085550068
Aug-04	0.080319334	0.0505099	0.091286635	0.085550068
Aug-04	0.080319334	0.0505099	0.091286635	0.085550068
Jul-04	0.083506853	0.047496041	0.101951137	0.087492691
Jul-04	0.083506853	0.047496041	0.101951137	0.087492691
Jun-04	0.08524085	0.054789531	0.106777857	0.087492691
Jun-04	0.08524085	0.054789531	0.106777857	0.087492691
May-04	0.087043462	0.061026786	0.109663564	0.087492691
May-04	0.087043462	0.061026786	0.109663564	0.087492691
Apr-04	0.088932382	0.065077385	0.107657672	0.087492691
Apr-04	0.088932382	0.065077385	0.107657672	0.087492691
Mar-04	0.090877533	0.047508323	0.107105738	0.087492691
Mar-04	0.090877533	0.047508323	0.107105738	0.087492691
Feb-04	0.092621594	0.056478002	0.109741666	0.089674683
Feb-04	0.092621594	0.056478002	0.109741666	0.089674683
Jan-04	0.094536948	0.051643016	0.111329663	0.089674683
Jan-04	0.094536948	0.051643016	0.111329663	0.089674683
Dec-03	0.096552299	0.073154486	0.113397526	0.089674683
Dec-03	0.096552299	0.073154486	0.113397526	0.089674683
Nov-03	0.098495539	0.071799338	0.110223675	0.089674683
Nov-03	0.098495539	0.071799338	0.110223675	0.089674683
Oct-03	0.100788349	0.084345133	0.112788658	0.089674683
Oct-03	0.100788349	0.084345133	0.112788658	0.089674683
Sep-03	0.10505551	0.068476908	0.117798133	0.092047753
Sep-03	0.10505551	0.068476908	0.117798133	0.092047753
Aug-03	0.107384621	0.076312352	0.117904198	0.092047753
Aug-03	0.107384621	0.076312352	0.117904198	0.092047753
Jul-03	0.109724121	0.079757565	0.119379552	0.092047753
Jul-03	0.109724121	0.079757565	0.119379552	0.092047753
Jun-03	0.11205387	0.089272904	0.119557712	0.092047753
Jun-03	0.11205387	0.089272904	0.119557712	0.092047753
May-03	0.114485594	0.089438672	0.120386034	0.092047753
May-03	0.114485594	0.089438672	0.120386034	0.092047753
Apr-03	0.116862818	0.109260634	0.122997117	0.092988972
Apr-03	0.116862818	0.109260634	0.122997117	0.092988972
Mar-03	0.116659871	0.091469899	0.125153503	0.092988972
Mar-03	0.116659871	0.091469899	0.125153503	0.092988972
Feb-03	0.11889873	0.110756951	0.127697063	0.092988972
Feb-03	0.11889873	0.110756951	0.127697063	0.092988972
Jan-03	0.121423281	0.080427011	0.125983956	0.092988972
Jan-03	0.121423281	0.080427011	0.125983956	0.092988972
Dec-02	0.123944561	0.113448615	0.12691237	0.092988972
Dec-02	0.123944561	0.113448615	0.12691237	0.092988972
Nov-02	0.126507696	0.101701549	0.128078602	0.092990433
Nov-02	0.126507696	0.101701549	0.128078602	0.092990433
Oct-02	0.129393208	0.131411197	0.126316448	0.092990433
Oct-02	0.129393208	0.131411197	0.126316448	0.092990433
Sep-02	0.128359498	0.101377076	0.123593751	0.092990433
Sep-02	0.128359498	0.101377076	0.123593751	0.092990433
Aug-02	0.122698933	0.178611866	0.119537517	0.092990433
Aug-02	0.122698933	0.178611866	0.119537517	0.092990433

Jul-02	0.101422576	0.093589298	0.10994541	0.092990433
Jul-02	0.101422576	0.093589298	0.10994541	0.092990433
Jun-02	0.103567473	0.086211527	0.106599818	0.092620959
Jun-02	0.103567473	0.086211527	0.106599818	0.092620959
May-02	0.10587237	0.081810368	0.105739994	0.092620959
May-02	0.10587237	0.081810368	0.105739994	0.092620959
Apr-02	0.109397622	0.073022154	0.112752373	0.092620959
Apr-02	0.109397622	0.073022154	0.112752373	0.092620959
Mar-02	0.111762455	0.081350868	0.114255873	0.092620959
Mar-02	0.111762455	0.081350868	0.114255873	0.092620959
Feb-02	0.113959584	0.100986398	0.114391007	0.092620959
Feb-02	0.113959584	0.100986398	0.114391007	0.092620959
Jan-02	0.114924495	0.078273407	0.113776526	0.093631481
Jan-02	0.114924495	0.078273407	0.113776526	0.093631481
Dec-01	0.117354893	0.084088311	0.111850927	0.093631481
Dec-01	0.117354893	0.084088311	0.111850927	0.093631481
Nov-01	0.119846664	0.09712022	0.115845696	0.093631481
Nov-01	0.119846664	0.09712022	0.115845696	0.093631481
Oct-01	0.122524053	0.123398881	0.112792641	0.093631481
Oct-01	0.122524053	0.123398881	0.112792641	0.093631481
Sep-01	0.101562049	0.082862435	0.108627711	0.093631481
Sep-01	0.101562049	0.082862435	0.108627711	0.093631481
Aug-01	0.103878628	0.082956502	0.108182113	0.092889083
Aug-01	0.103878628	0.082956502	0.108182113	0.092889083
Jul-01	0.10608426	0.077145786	0.10728108	0.092889083
Jul-01	0.10608426	0.077145786	0.10728108	0.092889083
Jun-01	0.108386382	0.088762527	0.106498669	0.092889083
Jun-01	0.108386382	0.088762527	0.106498669	0.092889083
May-01	0.110697013	0.11004141	0.108130415	0.092889083
May-01	0.110697013	0.11004141	0.108130415	0.092889083
Apr-01	0.106178148	0.111046193	0.106927487	0.092889083
Apr-01	0.106178148	0.111046193	0.106927487	0.092889083
Mar-01	0.101964021	0.088764118	0.101046386	0.092920577
Mar-01	0.101964021	0.088764118	0.101046386	0.092920577
Feb-01	0.103920847	0.088031613	0.100599659	0.092920577
Feb-01	0.103920847	0.088031613	0.100599659	0.092920577
Jan-01	0.112912646	0.10102112	0.099140964	0.092920577
Jan-01	0.112912646	0.10102112	0.099140964	0.092920577
Dec-00	0.097002759	0.101798653	0.096831441	0.092920577
Dec-00	0.097002759	0.101798653	0.096831441	0.092920577
Nov-00	0.095308594	0.117545684	0.09258643	0.092920577
Nov-00	0.095308594	0.117545684	0.09258643	0.092920577
Oct-00	0.093254638	0.086951095	0.092585922	0.092381516
Oct-00	0.093254638	0.086951095	0.092585922	0.092381516
Sep-00	0.095183068	0.06690834	0.094597773	0.092381516
Sep-00	0.095183068	0.06690834	0.094597773	0.092381516
Aug-00	0.097545012	0.11677184	0.109026872	0.092381516
Aug-00	0.097545012	0.11677184	0.109026872	0.092381516
Jul-00	0.099607807	0.08415101	0.108763179	0.092381516
Jul-00	0.099607807	0.08415101	0.108763179	0.092381516
Jun-00	0.101824582	0.125132501	0.10816002	0.092381516
Jun-00	0.101824582	0.125132501	0.10816002	0.092381516
May-00	0.103995325	0.145610708	0.104390113	0.093325913
May-00	0.103995325	0.145610708	0.104390113	0.093325913
Apr-00	0.10583223	0.11086375	0.100278898	0.093325913
Apr-00	0.10583223	0.11086375	0.100278898	0.093325913
Mar-00	0.108246209	0.102844896	0.107962982	0.093325913
Mar-00	0.108246209	0.102844896	0.107962982	0.093325913
Feb-00	0.107816809	0.119660919	0.09334836	0.093325913
Feb-00	0.107816809	0.119660919	0.09334836	0.093325913
Jan-00	0.089512658	0.063154268	0.089850858	0.093325913
Jan-00	0.089512658	0.063154268	0.089850858	0.093325913

10-day VaR α=0.95				
Date	10 day Age weighted	10 day Volatility weighted	10 day t distribution	10 day GEV
Jan-11	0.107918822	0.037557478	0.065975394	0.038723667
Jan-11	0.107918822	0.037557478	0.065975394	0.038723667
Dec-10	0.112149181	0.058499176	0.074196222	0.038723667
Dec-10	0.112149181	0.058499176	0.074196222	0.038723667
Nov-10	0.122951835	0.047132374	0.086425001	0.038723667
Nov-10	0.122951835	0.047132374	0.086425001	0.038723667
Oct-10	-0.003297914	0.063778498	0.101668284	0.038842551
Oct-10	-0.003297914	0.063778498	0.101668284	0.038842551
Sep-10	0.141451671	0.073239932	0.11098215	0.038842551
Sep-10	0.141451671	0.073239932	0.11098215	0.038842551
Aug-10	0.14453341	0.079786759	0.110159199	0.038842551
Aug-10	0.14453341	0.079786759	0.110159199	0.038842551
Jul-10	0.147669936	0.098919856	0.111902893	0.038842551
Jul-10	0.147669936	0.098919856	0.111902893	0.038842551
Jun-10	0.15088715	0.117968347	0.110980509	0.038842551
Jun-10	0.15088715	0.117968347	0.110980509	0.038842551
May-10	0.076676642	0.066773454	0.108017212	0.039036437
May-10	0.076676642	0.066773454	0.108017212	0.039036437
Apr-10	0.149277929	0.0402383	0.10759444	0.039036437
Apr-10	0.149277929	0.0402383	0.10759444	0.039036437
Mar-10	0.152542935	0.059690323	0.111023221	0.039036437
Mar-10	0.152542935	0.059690323	0.111023221	0.039036437
Feb-10	0.1556844	0.064270899	0.112407882	0.039036437
Feb-10	0.1556844	0.064270899	0.112407882	0.039036437
Jan-10	0.158672192	0.046539612	0.113535981	0.039036437
Jan-10	0.158672192	0.046539612	0.113535981	0.039036437
Dec-09	0.162054812	0.067496361	0.114797227	0.039443678
Dec-09	0.162054812	0.067496361	0.114797227	0.039443678
Nov-09	0.165406	0.09032543	0.117782709	0.039443678
Nov-09	0.165406	0.09032543	0.117782709	0.039443678
Oct-09	0.169164634	0.061307473	0.116240643	0.039443678
Oct-09	0.169164634	0.061307473	0.116240643	0.039443678
Sep-09	0.172598165	0.062484554	0.116514238	0.039443678
Sep-09	0.172598165	0.062484554	0.116514238	0.039443678
Aug-09	0.17642457	0.070473127	0.117560894	0.039443678
Aug-09	0.17642457	0.070473127	0.117560894	0.039443678
Jul-09	0.180268262	0.079954157	0.119299368	0.039387893
Jul-09	0.180268262	0.079954157	0.119299368	0.039387893
Jun-09	0.184195679	0.10540006	0.119162532	0.039387893
Jun-09	0.184195679	0.10540006	0.119162532	0.039387893
May-09	0.188090916	0.114731566	0.117553989	0.039387893
May-09	0.188090916	0.114731566	0.117553989	0.039387893
Apr-09	0.191996589	0.166891497	0.116307073	0.039387893
Apr-09	0.191996589	0.166891497	0.116307073	0.039387893
Mar-09	0.19626948	0.135845395	0.118246441	0.039387893
Mar-09	0.19626948	0.135845395	0.118246441	0.039387893
Feb-09	0.200036154	0.144366821	0.113713148	0.039582981
Feb-09	0.200036154	0.144366821	0.113713148	0.039582981
Jan-09	0.202053086	0.140364561	0.108013506	0.039582981
Jan-09	0.202053086	0.140364561	0.108013506	0.039582981
Dec-08	0.199080688	0.239257525	0.1022706	0.039582981
Dec-08	0.199080688	0.239257525	0.1022706	0.039582981
Nov-08	0.149474382	0.266140273	0.088835235	0.039582981
Nov-08	0.149474382	0.266140273	0.088835235	0.039582981
Oct-08	0.098031033	0.205206131	0.067999035	0.039582981
Oct-08	0.098031033	0.205206131	0.067999035	0.039582981
Sep-08	0.088245728	0.070514536	0.055319578	0.039777501
Sep-08	0.088245728	0.070514536	0.055319578	0.039777501
Aug-08	0.090202102	0.080026458	0.052814985	0.039777501
Aug-08	0.090202102	0.080026458	0.052814985	0.039777501
Jul-08	0.090071106	0.064323341	0.051838784	0.039777501
Jul-08	0.090071106	0.064323341	0.051838784	0.039777501
Jun-08	0.090422333	0.048884926	0.049331822	0.039777501
Jun-08	0.090422333	0.048884926	0.049331822	0.039777501
May-08	0.092384592	0.057828559	0.049895205	0.039777501

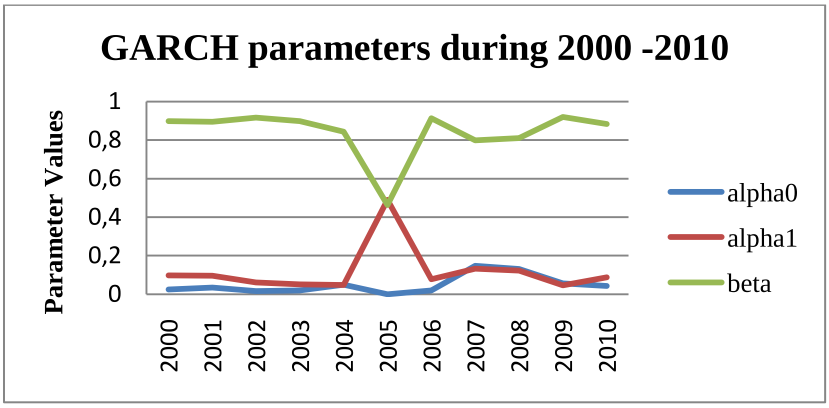
May-08	0.092384592	0.057828559	0.049895205	0.039777501
Apr-08	0.094310798	0.089738704	0.049534553	0.039975585
Apr-08	0.094310798	0.089738704	0.049534553	0.039975585
Mar-08	0.095116416	0.062039888	0.045316299	0.039975585
Mar-08	0.095116416	0.062039888	0.045316299	0.039975585
Feb-08	0.094463432	0.083518064	0.043039295	0.039975585
Feb-08	0.094463432	0.083518064	0.043039295	0.039975585
Jan-08	0.084139881	0.054825277	0.039539662	0.039975585
Jan-08	0.084139881	0.054825277	0.039539662	0.039975585
Dec-07	0.07272976	0.067782723	0.037726595	0.039975585
Dec-07	0.07272976	0.067782723	0.037726595	0.039975585
Nov-07	0.065475937	0.04344059	0.032649978	0.040135189
Nov-07	0.065475937	0.04344059	0.032649978	0.040135189
Oct-07	0.065242447	0.046012027	0.03376444	0.040135189
Oct-07	0.065242447	0.046012027	0.03376444	0.040135189
Sep-07	0.062735933	0.061681199	0.033526704	0.040135189
Sep-07	0.062735933	0.061681199	0.033526704	0.040135189
Aug-07	0.053079448	0.053319063	0.031328072	0.040135189
Aug-07	0.053079448	0.053319063	0.031328072	0.040135189
Jul-07	0.04849379	0.035051249	0.028486687	0.040135189
Jul-07	0.04849379	0.035051249	0.028486687	0.040135189
Jun-07	0.04181138	0.028011472	0.027081995	0.040280887
Jun-07	0.04181138	0.028011472	0.027081995	0.040280887
May-07	0.042458398	0.028888661	0.027401218	0.040280887
May-07	0.042458398	0.028888661	0.027401218	0.040280887
Apr-07	0.043336414	0.036054576	0.030023929	0.040280887
Apr-07	0.043336414	0.036054576	0.030023929	0.040280887
Mar-07	0.042191182	0.04633509	0.030039613	0.040280887
Mar-07	0.042191182	0.04633509	0.030039613	0.040280887
Feb-07	0.042952545	0.030183721	0.028534965	0.040280887
Feb-07	0.042952545	0.030183721	0.028534965	0.040280887
Jan-07	0.043698103	0.027654027	0.029808115	0.040433761
Jan-07	0.043698103	0.027654027	0.029808115	0.040433761
Dec-06	0.044622199	0.032193904	0.02962414	0.040433761
Dec-06	0.044622199	0.032193904	0.02962414	0.040433761
Nov-06	0.043269421	0.030041194	0.029390205	0.040433761
Nov-06	0.043269421	0.030041194	0.029390205	0.040433761
Oct-06	0.044212115	0.030406688	0.03043745	0.040433761
Oct-06	0.044212115	0.030406688	0.03043745	0.040433761
Sep-06	0.045126389	0.035024284	0.03084873	0.040433761
Sep-06	0.045126389	0.035024284	0.03084873	0.040433761
Aug-06	0.046155695	0.037288801	0.032000662	0.079264526
Aug-06	0.046155695	0.037288801	0.032000662	0.079264526
Jul-06	0.044158795	0.040438248	0.032271991	0.079264526
Jul-06	0.044158795	0.040438248	0.032271991	0.079264526
Jun-06	0.041534665	0.03743786	0.030978759	0.079264526
Jun-06	0.041534665	0.03743786	0.030978759	0.079264526
May-06	0.040096924	0.030348058	0.029850701	0.079264526
May-06	0.040096924	0.030348058	0.029850701	0.079264526
Apr-06	0.040501035	0.032459358	0.030920721	0.079264526
Apr-06	0.040501035	0.032459358	0.030920721	0.079264526
Mar-06	0.041443831	0.034109163	0.03309333	0.041243095
Mar-06	0.041443831	0.034109163	0.03309333	0.041243095
Feb-06	0.039990242	0.033481347	0.032719974	0.041243095
Feb-06	0.039990242	0.033481347	0.032719974	0.041243095
Jan-06	0.040641607	0.031427664	0.03350403	0.041243095
Jan-06	0.040641607	0.031427664	0.03350403	0.041243095
Dec-05	0.041523569	0.043107996	0.033153874	0.041243095
Dec-05	0.041523569	0.043107996	0.033153874	0.041243095
Nov-05	0.042366372	0.07704248	0.034327999	0.041243095
Nov-05	0.042366372	0.07704248	0.034327999	0.041243095
Oct-05	0.041366654	0.082400183	0.031500871	0.041471604
Oct-05	0.041366654	0.082400183	0.031500871	0.041471604
Sep-05	0.057774258	0.056183174	0.033082209	0.041471604
Sep-05	0.057774258	0.056183174	0.033082209	0.041471604
Aug-05	0.043188763	0.050097834	0.032607074	0.041471604
Aug-05	0.043188763	0.050097834	0.032607074	0.041471604
Jul-05	0.044102088	0.048644222	0.034274843	0.041471604
Jul-05	0.044102088	0.048644222	0.034274843	0.041471604
Jun-05	0.045062923	0.039016351	0.035166056	0.041471604
Jun-05	0.045062923	0.039016351	0.035166056	0.041471604

May-05	0.04676725	0.080130939	0.034484472	0.041959981
May-05	0.04676725	0.080130939	0.034484472	0.041959981
Apr-05	0.045127234	0.053685206	0.033199372	0.041959981
Apr-05	0.045127234	0.053685206	0.033199372	0.041959981
Mar-05	0.048020758	0.078995631	0.03526547	0.041959981
Mar-05	0.048020758	0.078995631	0.03526547	0.041959981
Feb-05	0.048822835	0.04499757	0.03759754	0.041959981
Feb-05	0.048822835	0.04499757	0.03759754	0.041959981
Jan-05	0.052295924	0.016907522	0.040053915	0.041959981
Jan-05	0.052295924	0.016907522	0.040053915	0.041959981
Dec-04	0.053587918	0.033520415	0.04297247	0.042337246
Dec-04	0.053587918	0.033520415	0.04297247	0.042337246
Nov-04	0.05538013	0.039333415	0.044593386	0.042337246
Nov-04	0.05538013	0.039333415	0.044593386	0.042337246
Oct-04	0.059314165	0.033770616	0.047481019	0.042337246
Oct-04	0.059314165	0.033770616	0.047481019	0.042337246
Sep-04	0.061658281	0.037403892	0.054116206	0.042337246
Sep-04	0.061658281	0.037403892	0.054116206	0.042337246
Aug-04	0.069690879	0.034234019	0.057002851	0.042337246
Aug-04	0.069690879	0.034234019	0.057002851	0.042337246
Jul-04	0.075701136	0.034128349	0.063076623	0.042792104
Jul-04	0.075701136	0.034128349	0.063076623	0.042792104
Jun-04	0.077273049	0.039476047	0.066794219	0.042792104
Jun-04	0.077273049	0.039476047	0.066794219	0.042792104
May-04	0.078907164	0.044321453	0.068763328	0.042792104
May-04	0.078907164	0.044321453	0.068763328	0.042792104
Apr-04	0.080619519	0.045652996	0.068918151	0.042792104
Apr-04	0.080619519	0.045652996	0.068918151	0.042792104
Mar-04	0.082382848	0.031734483	0.068188364	0.042792104
Mar-04	0.082382848	0.031734483	0.068188364	0.042792104
Feb-04	0.083963885	0.039468498	0.070109368	0.043388848
Feb-04	0.083963885	0.039468498	0.070109368	0.043388848
Jan-04	0.085700204	0.036172383	0.071358381	0.043388848
Jan-04	0.085700204	0.036172383	0.071358381	0.043388848
Dec-03	0.087527171	0.050673029	0.072981602	0.043388848
Dec-03	0.087527171	0.050673029	0.072981602	0.043388848
Nov-03	0.089288769	0.049872883	0.071329503	0.043388848
Nov-03	0.089288769	0.049872883	0.071329503	0.043388848
Oct-03	0.091367262	0.058593286	0.073236776	0.043388848
Oct-03	0.091367262	0.058593286	0.073236776	0.043388848
Sep-03	0.09509011	0.048511741	0.076987235	0.044148827
Sep-03	0.09509011	0.048511741	0.076987235	0.044148827
Aug-03	0.097198286	0.054315705	0.078337604	0.044148827
Aug-03	0.097198286	0.054315705	0.078337604	0.044148827
Jul-03	0.099315865	0.057312582	0.079489686	0.044148827
Jul-03	0.099315865	0.057312582	0.079489686	0.044148827
Jun-03	0.101424617	0.064184103	0.079860733	0.044148827
Jun-03	0.101424617	0.064184103	0.079860733	0.044148827
May-03	0.103625672	0.064324276	0.08070682	0.044148827
May-03	0.103625672	0.064324276	0.08070682	0.044148827
Apr-03	0.105777396	0.078653027	0.082434847	0.044360967
Apr-03	0.105777396	0.078653027	0.082434847	0.044360967
Mar-03	0.106023483	0.065993125	0.084355767	0.044360967
Mar-03	0.106023483	0.065993125	0.084355767	0.044360967
Feb-03	0.108058215	0.080189302	0.08657241	0.044360967
Feb-03	0.108058215	0.080189302	0.08657241	0.044360967
Jan-03	0.106948326	0.059042855	0.08430015	0.044360967
Jan-03	0.106948326	0.059042855	0.08430015	0.044360967
Dec-02	0.109169043	0.08277703	0.084433766	0.044360967
Dec-02	0.109169043	0.08277703	0.084433766	0.044360967
Nov-02	0.111426624	0.074322325	0.086165018	0.044513877
Nov-02	0.111426624	0.074322325	0.086165018	0.044513877
Oct-02	0.113968153	0.096880733	0.085663653	0.044513877
Oct-02	0.113968153	0.096880733	0.085663653	0.044513877
Sep-02	0.098971533	0.073942017	0.082601007	0.044513877
Sep-02	0.098971533	0.073942017	0.082601007	0.044513877
Aug-02	0.099305633	0.130322042	0.078513014	0.044513877

Aug-02	0.099305633	0.130322042	0.078513014	0.044513877
Jul-02	0.080880083	0.067185392	0.073060745	0.044513877
Jul-02	0.080880083	0.067185392	0.073060745	0.044513877
Jun-02	0.079255807	-0.114857082	0.070199987	0.044664726
Jun-02	0.079255807	-0.114857082	0.070199987	0.044664726
May-02	0.081019648	0.06025049	0.070511883	0.044664726
May-02	0.081019648	0.06025049	0.070511883	0.044664726
Apr-02	0.082708815	0.053129512	0.072910845	0.044664726
Apr-02	0.082708815	0.053129512	0.072910845	0.044664726
Mar-02	0.084496719	0.060051515	0.07342019	0.044664726
Mar-02	0.084496719	0.060051515	0.07342019	0.044664726
Feb-02	0.083418027	0.072480404	0.073496566	0.044664726
Feb-02	0.083418027	0.072480404	0.073496566	0.044664726
Jan-02	0.085150158	0.0540211	0.074471815	0.044870744
Jan-02	0.085150158	0.0540211	0.074471815	0.044870744
Dec-01	0.086950895	0.058962017	0.072885003	0.044870744
Dec-01	0.086950895	0.058962017	0.072885003	0.044870744
Nov-01	0.088797105	0.067588797	0.074711681	0.044870744
Nov-01	0.088797105	0.067588797	0.074711681	0.044870744
Oct-01	0.089336505	0.086050248	0.073568981	0.044870744
Oct-01	0.089336505	0.086050248	0.073568981	0.044870744
Sep-01	0.088286928	0.059330159	0.070499178	0.044870744
Sep-01	0.088286928	0.059330159	0.070499178	0.044870744
Aug-01	0.090300708	0.059078283	0.069787651	0.045026254
Aug-01	0.090300708	0.059078283	0.069787651	0.045026254
Jul-01	0.092163999	0.055069818	0.069368073	0.045026254
Jul-01	0.092163999	0.055069818	0.069368073	0.045026254
Jun-01	0.094164039	0.063791518	0.068307254	0.045026254
Jun-01	0.094164039	0.063791518	0.068307254	0.045026254
May-01	0.096171472	0.079699913	0.069568229	0.045026254
May-01	0.096171472	0.079699913	0.069568229	0.045026254
Apr-01	0.09616563	0.080413143	0.068526821	0.045026254
Apr-01	0.09616563	0.080413143	0.068526821	0.045026254
Mar-01	0.08436541	0.063085165	0.064571588	0.045139193
Mar-01	0.08436541	0.063085165	0.064571588	0.045139193
Feb-01	0.085984495	0.062600319	0.063828569	0.045139193
Feb-01	0.085984495	0.062600319	0.063828569	0.045139193
Jan-01	0.102657363	0.070660313	0.063631691	0.045139193
Jan-01	0.102657363	0.070660313	0.063631691	0.045139193
Dec-00	0.084977667	0.066375285	0.061789536	0.045139193
Dec-00	0.084977667	0.066375285	0.061789536	0.045139193
Nov-00	0.07794712	0.079520539	0.058056636	0.045139193
Nov-00	0.07794712	0.079520539	0.058056636	0.045139193
Oct-00	0.078245269	0.057303489	0.056715929	0.045252947
Oct-00	0.078245269	0.057303489	0.056715929	0.045252947
Sep-00	0.079863317	0.046808315	0.057166635	0.045252947
Sep-00	0.079863317	0.046808315	0.057166635	0.045252947
Aug-00	0.088268812	0.069660536	0.065204005	0.045252947
Aug-00	0.088268812	0.069660536	0.065204005	0.045252947
Jul-00	0.090135442	0.057198096	0.065012916	0.045252947
Jul-00	0.090135442	0.057198096	0.065012916	0.045252947
Jun-00	0.09214141	0.086774116	0.06452757	0.045252947
Jun-00	0.09214141	0.086774116	0.06452757	0.045252947
May-00	0.094105723	0.100976587	0.062179303	0.045416707
May-00	0.094105723	0.100976587	0.062179303	0.045416707
Apr-00	0.084121748	0.077383844	0.059366692	0.045416707
Apr-00	0.084121748	0.077383844	0.059366692	0.045416707
Mar-00	0.085391896	0.071618409	0.058836445	0.045416707
Mar-00	0.085391896	0.071618409	0.058836445	0.045416707
Feb-00	0.086657626	0.084373918	0.054759788	0.045416707
Feb-00	0.086657626	0.084373918	0.054759788	0.045416707
Jan-00	0.078445149	0.043320489	0.052135567	0.045416707
Jan-00	0.078445149	0.043320489	0.052135567	0.045416707

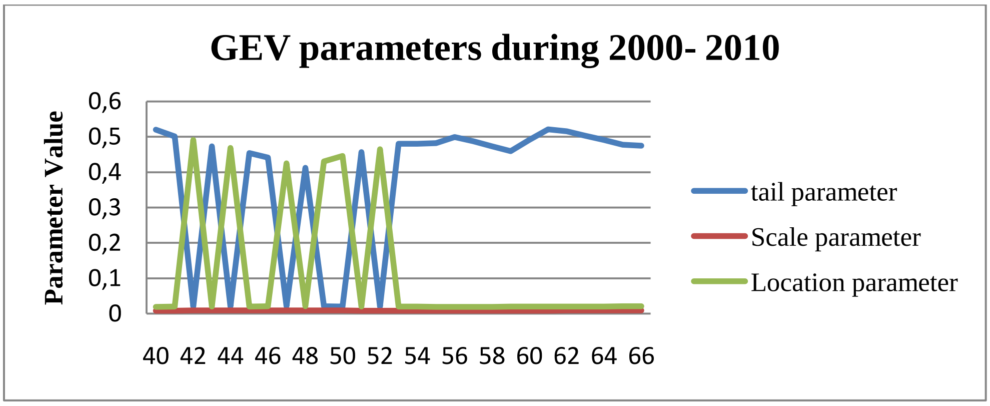
9.2 Calculated coefficients used in GARCH and Generalized Extreme Value Theory

Average GARCH coefficients	
α_0	0.049309051
α_1	0.118810385
β_0	0.839930627



The parameter estimations made using GARCH for every year between 2000-2010

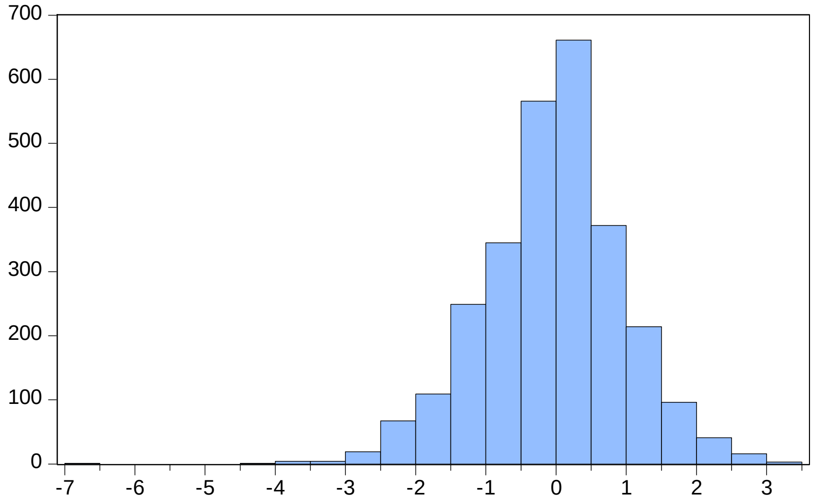
Average GEV Coefficients	
σ	0.008750667
ξ	0.378345103
μ	0.11651145
m	52.96296296



GEV parameters estimated during 2000 to 2010 to calculate Value at Risk where the number of extreme values used in the maximum likelihood function is shown on the x axis.

9.3 Normality test for the S&P 500 returns during 2000 to 2010

This shows that the returns are not normally distributed with a JB value of 182,86 and a p value of 0.



Series: Standardized Residuals
Sample 12/31/1999 12/31/2010
Observations 2768

Mean	-0.042185
Median	0.019172
Maximum	3.349518
Minimum	-6.586959
Std. Dev.	0.999480
Skewness	-0.281758
Kurtosis	4.126065
Jarque-Bera	182.8695
Probability	0.000000