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Lund University
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Division of Production Management
Department of Industrial Management and Logistics

Inventory Control

-at Maintenance Unit Tetra Pak in Lund

Authors: Johan Holstein
Andreas Olofsson

Supervisors: Johan Marklund, LTH
Martin Martell, Maintenance Unit Tetra Pak

Acknowledgment

This master thesis is the concluding part of our education as Mechanical Engineers at Lund Institute of Technology. The idea of conducting our Master Thesis at Maintenance Unit, Tetra Pak AB, started to grow the summer of 2005, when we were working in the production at Maintenance Unit. The question about an appropriate assignment was posed to Martin Martell and he told us that Maintenance Unit needed to create a model for their inventory control. The work with the thesis did not start until the beginning of September 2006.

We would like to take this opportunity and thank everybody who has contributed to this thesis. Especially our supervisors, Johan Marklund at the department of Industrial Management and Logistics at Lund Institute of Technology and Martin Martell at Maintenance Unit Tetra Pak AB. They have assisted us throughout the work and always taken the time to discuss different matters and given us important input and guidance. Without your devotion the master thesis would not have been what it is today. We are also grateful for all the support and information that all the members of the staff at Maintenance Unit have given us throughout our work, thanks guys!

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Johan Holstein & Andreas Olofsson

Abstract

Maintenance Unit is one out of many divisions at Tetra Pak in Lund. They are producing spare parts for the dairies to enable a quick and easy way of serving the packaging machines. When assembling the spare parts, approximately 30 % of the components used are reconditioned components, lowering the price for the spare part. Today they have 1000 different spare parts in their product portfolio and due to this they have a warehouse with approximately 6000 different components.

The objective of this master thesis is to improve the inventory control system at Maintenance Unit. The overall objective can be broken down into two parts. The first is to create a model for controlling and optimising the safety stock of new components. The second is to create a model for joint inventory control of new and reconditioned components.

The thesis can be seen as a quantitative study since the majority of the information is gathered from Maintenance Units' business system. It contains some elements from a qualitative study such as interviews. These have been conducted to verify the information from the business system and to gather new information.

The majority of the theories used are well known within the subject of inventory control such as ABC inventory analysis. These theories are complemented by other less known theories. There are not many papers covering the subject remanufacturing and reverse logistics. Therefore the formulas used in the part concerning reconditioned articles had to be deduced.

The steps required to reach the conclusion were the following:

- categorisation of the components into homogenous groups with consideration to different factors.
- determine the distribution for the demand.
- establish different fill rates and the holding costs associated with these.

The results are that the components can be categorised in several dimensions into homogenous groups with a certain fill rate. These groups can be visualised by cubes where the axles are the dimensions in which the components are categorised by. It is up to the decision maker to determine the appropriate fill rates for the different groups.

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1 Introduction

This chapter starts with a presentation of Maintenance Unit and the background to this master thesis. The section that follows will give the reader more in-depth knowledge about the problem at hand, the purpose and limitations of the thesis.

1.1 Background[1]

Tetra Pak is one of the world's leading companies in processing, packaging and distributing provisions. Within the company there are several divisions and one of them is Maintenance Unit. Maintenance Unit is a workshop who provides the dairies with spare parts to the packaging machines. The dairies need the spare parts when they perform maintenance services on the machines. Maintenance Unit consist of two workshops, one in Lund, Sweden, and one in Hocheim, Germany. In Lund there are 9 workshop technicians who assemble the spare parts and three persons are working with administrative tasks.

Before the workshop was founded in 1997, the service engineer dismantled the spare parts in the dairy and the machine was idle during this time. This way of performing service on the machines was expensive and time consuming for the dairies and therefore the demand for ready-made spare parts increased. The workshop in Lund initially produced 600 spare parts/year. Today it is producing 3000 spare parts/year. The main market for Maintenance Unit is Europe but there are customers all over the world.

The benefits with a standardised workshop are that:

- Tetra Pak can offer the same warranty and the same price all over Europe for the spare parts.
- the spare part is tested and ready to be installed directly into the machine, minimising the idle time.
- it is minimising the risk for insufficient components.
- it reduces the space required for inventory and workshops at the market company.

When the Maintenance Unit workshop assembles spare parts both new and reconditioned components are used. The new components are delivered from the central warehouse. They are either delivered to meet the demand of a specific order or they are delivered to the regional warehouse at the department where they are stored and assembled at a later stage.

The group of components that can not be reconditioned and therefore are always taken as new when the spare parts are assembled, will from now on be referred to as non-reco components. The group of components, both new and reconditioned, that can be reconditioned will be referred to as reco components. The components that have been reconditioned in this group are referred to as reconditioned components.

[1] Martell, M. 2006-09-06

The main task for the regional warehouse is to cover the shortages of the central warehouse, see Figure 1.1.

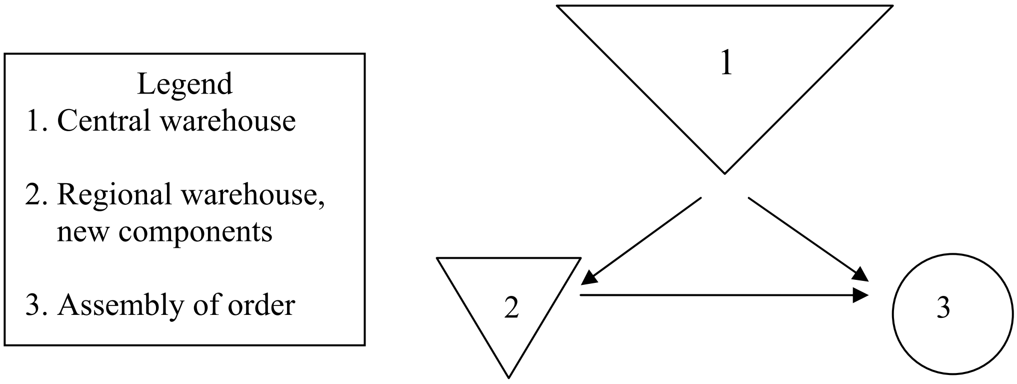

Figure 1.1. The material flow for non-reco components. The regional warehouse covers the shortages that may arise at the central warehouse.

The non-reco components represent approximately 70 % of the total amount of components in the spare part. When the technician at Maintenance Unit assembles the order, he always uses the reconditioned components, which are recovered from old spare parts first. When the order is completed and the spare part is assembled it is sent to the dairy, through a market company, who performs the service on the machine. The replaced spare part is returned to Maintenance Unit, enabling them to disassemble it and recondition some of its components. The reconditioned components are stored and used the next time the technician is assembling a similar unit, see Figure 1.2.

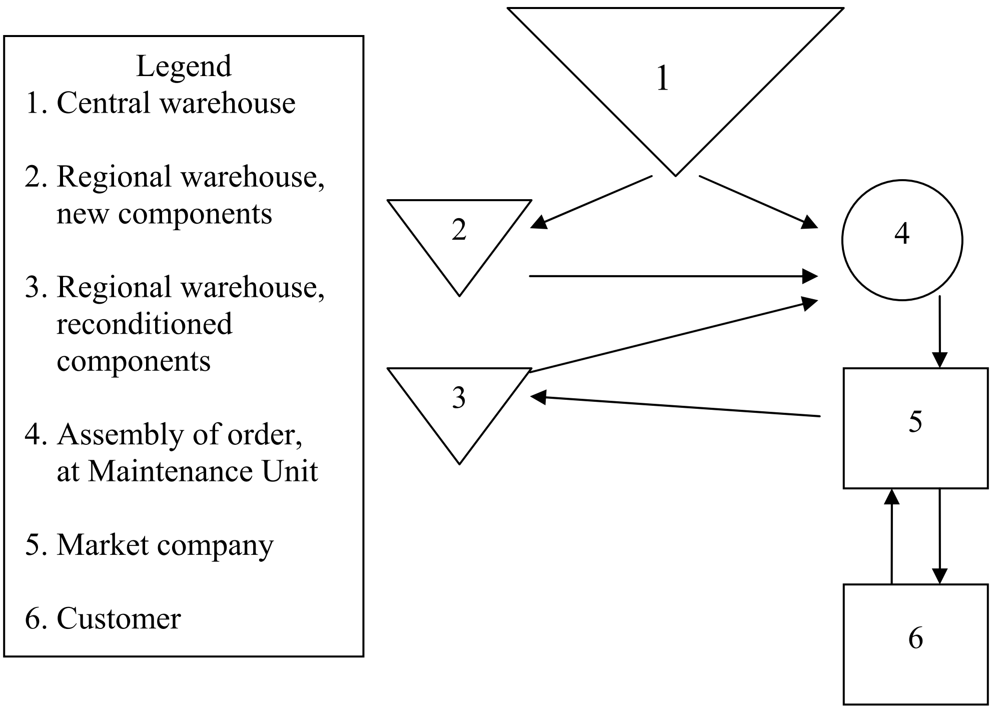

Figure 1.2. The material flow for the components. The central warehouse delivers components directly to a specific order (4) or to the regional warehouse of new components (2). When the spare part is assembled (4), components are used from the regional warehouse for new (2) or reconditioned components (3). The spare part is assembled (4) and sent to the diary (6) through a market company (5). The replaced spare part is returned to Maintenance Unit through the market company.

Maintenance Units' product portfolio consists of approximately 1000 different spare parts and in the future they are going to grow in capacity and size. There are several reasons why the portfolio will increase further:

- new machine types are developed.
- new spare parts are created on old machines.

These issues create new spare parts to be included in the product portfolio.

The quantities of new components in the warehouse are increased due to the fact that new spare parts are introduced in the product portfolio. Today there are approximately 6000 different components in the regional warehouse at Maintenance Unit. The components tie up approximately seven million SEK and the assembled spare parts tie up approximately six million SEK.[2] Therefore it is very important that the inventory levels are optimized. The benefits are that:

- if the tied up capital is reduced, the division can invest in other projects with the money that are saved.
- the margins are increased for the different spare parts.

Maintenance Unit replaced their old business system (Jeeves) with SAP R3 in may 2006. In connection to this they started to receive the non-reco components from the central warehouse just in time to meet the demand from a specific order. These components are only kept as safety stock at Maintenance Unit from now on. The reco components are held as inventory at Maintenance Unit and are used every time there is a need for one.

1.2 Problem description

When Maintenance Unit changed their business system there was a need to determine a proper reorder point and ordering quantity. The old settings resulted in that huge quantities of new components were ordered in the new system. A quick and dirty method was applied, which resulted in that these orders disappeared, but the method did not take essential information into account. For the non-reco components the inventory levels were unnecessary high. Maintenance Unit needs an inventory control model that takes the ordered quantity and the reorder point into consideration. The model needs to answer:

- in what quantities should the different components be purchased and what reorder points should be used?
- how will this affect the cost for the inventory and the service level that they have to maintain?

It is very important to balance the trade off between having too much stock at hand with an appropriate service level. Large inventory levels will increase the holding costs but render the possibility to supply the diaries with spare parts. If the inventory levels are decreased, it will decrease the holding costs but increase the risk of running short of components. Maintenance Unit have to supply many new spare parts each year, meaning that the number of different components that they must keep in stock is increasing.

[2] Wågnert, C. 2006-11-13

1.3 Objective

The objective of this master thesis is to improve the inventory control system at Maintenance Unit, Tetra Pak Technical Service AB in Lund. The objective can be broken down into two parts. The first is to create a model for controlling and optimizing the safety stock of new components. The second part is to create a model for joint inventory control of new and reconditioned components.

1.4 Delimitations

The major limitations for this master thesis are the following:

- the thesis is primary based on material available in the business system Jeeves. If the data needed was not attainable in Jeeves, interviews were conducted. The figures that have been used in the different models are based on historical data. Only components that were used during the year 2005 are included in the data material. This is to prevent old components that are not in use anymore to be included. It is important to monitor changes in the input data (demand, cost etc.) and update the models accordingly. Changes in the demand structure depend on the number of new spare parts that are introduced and produced and the size of the market share in existing and future markets.
- no components with an order quantity of 100 or more in the old business system are included.
- the time available is often limited. If each component would be investigated in depth the result would be better compared to the suggested approach. However there is often a time restriction that makes this impossible.

1.5 Target group

The primary target group for this work is the management team at Maintenance Unit, Tetra Pak in Lund, teachers, researchers and students at Lund Institute of Technology.

1.6 Chapter overview and guidance for the reader

This chapter aims at giving the reader an overview of the different chapters. Each chapter can be read separately although the authors suggest the reader to read the entire paper to increase the understanding of the different matters discussed.

Chapter 1: Introduction
This chapter provides essential information about the background of the master thesis. It describes the problem and specifies the purpose of the thesis. The chapter also includes information about the delimitations and the target group.

Chapter 2: Methodology
The chapter discusses methodological choices that are available and the methods that were chosen throughout the thesis.

Chapter 3: Theoretical frame of references
This chapter presents the theories that are of importance to understand the forthcoming analysis. It aims to give the reader the information necessary to comprehend the problems that arise in the data collection analysis.

Chapter 4: Data collection analysis of non-reco components and analysis of the findings
This chapter presents how the authors developed an inventory control model for the safety stock for components which cannot be reconditioned and are only used as new. The information that was required when developing the model will be presented. The chapter also presents an analysis of the findings from the data collection with consideration taken to the theoretical framework. The authors own conclusions are drawn with respect to the theoretical base.

Chapter 5: Data collection analysis of reco components and analysis of the findings
This chapter illustrates how the authors developed an inventory control model for components that can be reconditioned. In this chapter the authors own conclusions are drawn with consideration to the theoretical framework. It will also analyse the results of the findings from the data collection for reconditioned components.

Chapter 6: Development of the model
This chapter develops the model used in chapter 5 to fit to a (R,Q) system.

Chapter 7: Conclusions
The chapter presents the results of the findings making sure that the thesis´ objectives are met.

Chapter 8: Recommendations for implementations and follow up
The final part presents some recommendations when implementing the inventory control models.

2 Methodology

This chapter starts with a description about different scientific approaches for conducting a study. The sections that follow contain information about the relation between theory and empirics and different approaches. The difference between qualitative and quantitative studies will be discussed and different information sources will be presented. Finally, statistical credibility is discussed and critique of the sources is provided. At the end of each section a motivation of the authors choices will be presented.

2.1 Different scientific approaches

A project depends on the level of knowledge that is available in the area[3]. It is common to separate different studies into the following categories depending on the knowledge available in the specific area[4].

Explorative study. If little knowledge is available the explorative study may be used. There is not much knowledge in the specific area available and therefore the main purpose with explorative studies is to gain as much knowledge as possible. It is important to illuminate the problem from different angles. It is also vital to be creative since many of these studies form a base for further investigations.[5]

Descriptive study. If there is already some knowledge about the problem at hand the investigation tends to be descriptive. When conducting a descriptive study, only a few aspects will be investigated, but these aspects will be considered in depth. There can be either a description about each aspect on its own or the connection between them.[6] A descriptive study tries to map the facts.[7]

Explanatory study. When going one step further with a study, attempts will be made trying to elucidate the cause and effect between different parameters. How different factors are connected, how they affect each other and give rise to the investigated condition will be identified and explained.[8]

Diagnostic study. Investigations of this kind try to make a prediction about the future development for the problem at hand.[9]

Normative study. When the purpose for the project is to result in a recommendation, the project is of a normative nature. It is important for the person conducting the research to clarify his or hers standpoint, recommendation of action and the consequences for the involved parties.[10]

The authors’ standpoint is that if an explorative study is used one tries to achieve a holistic view. This will result in that the study will not be able to penetrate deeper into the problems.

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[3] Wallén, G. (1996) *”Vetenskapsteori och forskningsmetodik”*
[4] Patel, R. & Davidson, B. (2003) *”Forskningsmetodikens grunder”*
[5] Ibid.
[6] Ibid.
[7] Lekvall, P. & Wahlbin, C. (2001) *”Information för marknadsföringsbeslut”*
[8] Ibid.
[9] Ibid.
[10] Wallén, G. (1996) *”Vetenskapsteori och forskningsmetodik”*

The further down the list one goes the more penetrated the problems will become, but the holistic view may be lost to some degree. The last couple of scientific approaches in the list will be very detailed.

More knowledge is required the further down the list one goes. To be able to do a descriptive study one needs to obtain the knowledge that is gained from the explorative study. To be able to do an explanatory study, knowledge from the explorative and the descriptive levels needs to be obtained and so forth. Often this makes it hard to distinguish the different types of investigations and an investigation might have several purposes at the same time.[11]

The purpose of this study is to give Maintenance Unit a recommendation how to improve their inventory control system. This master thesis can therefore be characterised as a normative study although the study contains elements from all the different categories. The decisions and delimitations that will be made are stated in a clear way enabling the reader to comprehend under what conditions the assumptions will be valid.

2.2 The relation between theory and empirics[12]

This chapter is based on the book "Information för marknadsföringsbeslut" by Lekvall and Wahlbin and it clarifies that a part of a researchers work is to relate theory and empirics to one another. There are three different ways of doing this namely deduction, induction and hypothetic-deduction.

A person who is using a *deductive* approach in his or her field starts with already existing theories and principles. From these a conclusion is drawn for the specific case. From the existing theories hypotheses are derived and tested empirically in the current case. This means that an existing theory decides what information is to be gathered, how it is to be interpreted and how one is to relate the results to the existing theory. This way of working gives the research high objectivity since it has its origin in existing theories. There is however an increased risk that new findings will not be discovered. The existing theories might lead the person who is conducting the research in the wrong direction. People who work in science tend to favour the deductive approach.

An *inductive* approach implies that a person can perform an investigation without the research having the support from an existing theory. From the gathered material a theory can then be outlined. There is a risk that the generalisation of the theory is not valid, since it is based on empirical material that is specific for the given situation. Even though the starting point is not from an existing theory, the work that is performed will not be unprejudiced. All people have ideas and conceptions that will affect the theories that are produced. Among people who work with social sciences the inductive approach is often preferred.

The third way of connecting theory with empirics is the *hypothetic-deductive* approach. This is a combination of the other two ways presented in the previous parts of this chapter. The hypothetic-deductive approach implies that from a specific case an attempt is made to create a hypothetical pattern, which can explain the case. This first part is an inductive approach. The next step is to try to verify the theories or hypothesis that were created in the fist step by

[11] Lekvall, P. & Wahlbin, C. (2001) *"Information för marknadsföringsbeslut"*
[12] Patel, R. & Davidson, B. (2003) *"Forskningsmetodikens grunder"*

applying them to new cases. This is the deductive part of the approach. This way the original hypothesis or theory can evolve over time and become more general. The hypothetic-deductive approach has the benefit that it does not steer the development as strictly as the other ones do. Even in this case the persons working with the project have experiences that will affect them and thus no research will start unbiased. Figure 2.1 illustrates the different ways of relating theories and empirics that have been presented in this chapter.

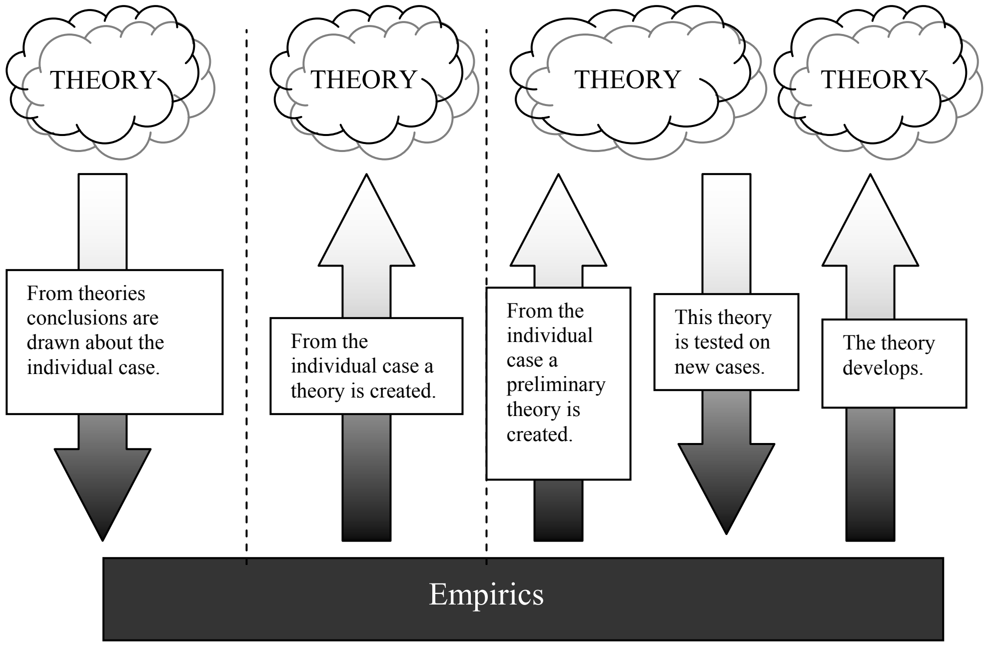

Figure 2.1. Three different ways of illustrating the relation between theories and empirics.[13]

One of the purposes for this thesis is to determine the safety stock for non-reco components. This has been done many times before and there is a vast amount of literature that describes how it can be achieved in different situations. The approach when determining the safety stock therefore has a clear element of a deductive relation between theories and empirics. When developing the model for joint inventory control of reco components a deductive approach was used. The theories covering the topic remanufacturing are more scarce but none-the-less the starting point was in the available theories.

2.3 Different approaches

The different approaches that the authors find relevant are described in this chapter. These are the time series approach, the case study approach, a combined survey and case study approach and the operations research modeling approach. They decide how a study is technically going to proceed. They differ in the way they attain the material needed to make any conclusions. The different approaches have their advantages and disadvantages depending on the results that are to be concluded. One should decide the amount of items to be included, depending if a broader or more in depth result is desirable. A decision should also be made whether quantitative or qualitative data should be used.[14] Chapter 2.4 will describe quantitative and qualitative studies more in detail.

[13] Patel, R. & Davidson, B. (2003) *"Forskningsmetodikens grunder"*
[14] Lekvall, P. & Wahlbin, C. (2001) *"Information för marknadsföringsbeslut"*

2.3.1 Case study

A case study can be described as an in depth study focusing on a single object. Characteristic for the case study is that the focus of attention is aimed at detailed descriptions of the case. It is hard to draw any profound conclusions about the underlying population. This is one of its weaknesses since the few objects investigated imply that the possibility to generalise is limited.[15] The great advantage is that it enables the person who is conducting the research to concentrate on a specific event and attain knowledge about the factors that influence this event[16]. This enables the person conducting the study to understand why certain results may appear, more than just finding out what the results are. Several different methods may be used when carrying out a case study which is an advantage.[17] The case study is very useful when doing an explorative study[18].

The authors' objective is to create a model that can be implemented at Maintenance Unit. Maintenance Unit is a special workshop within Tetra Pak due to the fact that currently no other workshops within the company use reconditioned components like they do. Hence the opportunity to generalise and adapt the model to other departments is limited. Our objective is to create an inventory model for Maintenance Unit, not to create a model to draw conclusions about the population of workshops within and outside Tetra Pak.

2.3.2 A combined survey and case study approach

A combined survey and case study approach may be used when several objects are studied with the purpose of comparing them among themselves and from this, draw conclusions for larger populations. The approach is normally divided into two separate approaches, the survey approach and the experimental approach. The major difference between the two is that in a survey approach the reality is observed the way it is, without trying to affect it. In the experimental approach, one tries to control and steer the studied reality to be able to illuminate the factors that are of specific interest.[19]

In a *survey approach* one measures the conditions that were decided to be measured in advance[20]. The purpose is to attain information that can be compiled into patterns to enable statistical comparisons[21]. In many cases it is of interest to make a statement about the entire population, not just the items that were investigated. The possibilities to draw conclusions from the items that have been investigated to the entire population depend on several factors. These are the size of the selection, the selection procedure and the decline that were received in the survey.[22] It is important that the selected items are representative for the entire population[23]. The survey approach is common for investigations where the goal is to reach a descriptive level.[24]

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[15] Lekvall, P. & Wahlbin, C. (2001) *"Information för marknadsföringsbeslut"*
[16] Bell, J. (2000) *"Introduktion till forskningsmetodik"*
[17] Denscombe, M. (2000) *"Forskningshandboken"*
[18] Lekvall, P. & Wahlbin, C. (2001) *"Information för marknadsföringsbeslut"*
[19] Ibid.
[20] Ibid.
[21] Bell, J. (2000) *"Introduktion till forskningsmetodik"*
[22] Lekvall, P. & Wahlbin, C. (2001) *"Information för marknadsföringsbeslut"*
[23] Bell, J. (2000) *"Introduktion till forskningsmetodik"*
[24] Lekvall, P. & Wahlbin, C. (2001) *"Information för marknadsföringsbeslut"*

The purpose of conducting an *experiment* is to isolate different factors and in detail study their effects, either to test existing theories or examine new issues[25]. In an experimental approach the variables can be manipulated, making sure that the value is known for each item investigated. It also implies that disturbing factors can either be eliminated or neutralised by determining that the different classifications are homogenous.[26]

2.3.3 Time series approach

In a time series approach, centre of attention is different patterns which evolve over time. It focuses on one variable at the time and time is the only explanation variable. The time series approach can be used in studies with different purposes. All the approaches, the case study, the combined survey and case study approach and the time series approach may be used in a mixed form.[27]

2.3.4 Operations research modeling approach

This approach consists of a number of steps that should be performed. These are the following[28]:

- define the problem and gather data of importance.
- formulate a mathematical model to represent the problem.
- develop a computer based procedure for deriving solutions to the problem
- test and refine the model.
- prepare for the ongoing application of the model.
- implement the model

This approach is convenient to use when a more mathematical approach is preferred.

A great deal of information has been gathered to create an understanding for the matters that vary with this specific case. The authors´ approach is similar to a case study since one object (Maintenance Units inventory) is considered in depth. No attempts will be made to generalise the models although they may be quite general. The approach the authors chose has some similarities with the survey approach as well. Information was gathered and compiled into patterns to enable statistical comparison when different distributions were determined. In some aspect the approach the authors used may be seen as experimental. For example some of the parameters were manipulated to see if the data could be applied to a certain distribution. The information was gathered from the year 2005, but the time was decided not to be an explanation variable. Therefore the time series approach does not have many similarities to the chosen approach. It is quite evident that the approach chosen is primarily based on the operations research modeling approach since the objective for the thesis is to create a model for controlling the inventory. The first four steps were performed in the thesis. The last steps were not included since the implementation part is not a part of the objective.

[25] Denscombe, M. (2000) *”Forskningshandboken”*
[26] Lekvall, P. & Wahlbin, C. (2001) *”Information för marknadsföringsbeslut”*
[27] Ibid.
[28] Hillier, F. & Lieberman, G. (1995) *”Introduction to operations research”*

2.4 Qualitative and quantitative studies

The two main distinctions between qualitative and quantitative studies are pertaining to how the gathered data is collected, presented (number or words) and how the first part of the analysis is carried out (statistical compilation or verbal argument).[29] Quantitative methods transform the data into numbers and from this material statistical analysis are performed. In a qualitative study, it is the scientists' interpretation of the information that is the most important thing.[30]

An advantage with a *qualitative study* is that it takes the entire situation into consideration. This holistic view provides an increased understanding. In these types of investigations it is essential to be flexible. For instance during an interview it is important to be able to rephrase a question if it was not formulated correctly. Qualitative studies aim at understanding the separate issues and their special situation. Which information that becomes central in a study depends to a great extent on the information source. From this flexibility and closeness to the information source, relevant interpretations ought to be made. However there will be a problem to compare the information from the different sources.[31]

A Quantitative study is designed to be able to make statistical generalisations. This enables the person who is carrying out the investigation to make a statement concerning the entire population by gathering material from the sources that were selected. This can be done by standardising the way to perform a survey. Quantitative information is gathered in a way that is characterised by the selection of objects and that there is no space for individual adaptation.[32]

In many cases a combination of qualitative and quantitative studies may generate multiple advantages. One of these might be if both procedures give the same result the validity of the gathered information is increased. This implies that it does not matter if the qualitative or the quantitative study is used. One should start from the problem description and try to find the method that is suitable for the purpose of the investigation.[33]

In our work quantitative data has been gathered from the business system and several calculations have been carried out to establish different statements. Many different statistical analyses have been performed to determine the demand distribution. This quantitative approach has been supported by material that was gathered in a more qualitative way. The qualitative data gathered from different people have been used when the authors could not collect the desired data from the business system. Qualitative information was also used to determine if the gathered data from the business system (quantitative data) was reasonable. Apart from this, qualitative material was gathered to create a sense of the complexity of the problem at hand in the beginning of the thesis.

When data is collected in a study it may be hard to determine if the data is quantitative or qualitative. This is due to the fact that the data can be gathered in a qualitative way and be presented as quantitative. For example the data can be collected by questioners and be presented in statistical tables.

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[29] Lekvall, P. & Wahlbin, C. (2001) *"Information för marknadsföringsbeslut"*.
[30] Magne Holme, I. & Krohn Solvang, B. (1997) *"Forskningsmetodik"*
[31] Ibid.
[32] Ibid.
[33] Ibid.

The objective of the thesis is to create a model for a specific inventory control system and not to study a general phenomenon. The model will solve problems at Maintenance Unit regarding what order quantity and reorder point to use. The majority of the data used in this master thesis is of a quantitative nature and a mathematical model is created and therefore the approach can be described as a quantitative modelling approach.

2.5 Information sources

There are two kinds of data; primary and secondary data. The primary data is information that is collected for the specific task at hand, e.g. interviews and questionnaires. Secondary data is information that is already gathered and written for another purpose e.g. websites, literature and CD records.[34]

2.5.1 Primary data

Primary data is the data that the authors gathers for a specific purpose. It can be interviews, questionnaires or observations.[35]

Interviews are a way of gathering information through questions or dialogue. There are different ways of conducting an interview. Two examples are:

- the author meets the interviewee face to face. This is a common way to perform an interview.
- the interviewer can also carry out the interview over the phone.

It is important that the questions asked are open since this reduces the risk of steering the respondent in a certain direction.[36]

One of the advantages with *questionnaires* is that the persons who are answering the questions are answering the same questions in the same order. Because of this, questionnaires are often used to establish generalizations or comparisons of the results.[37]

Observation is another way of gathering information and it has to be done in a structured way. The information which is gathered has to be registered in an organised manner. Observations are commonly used when the topic relates to explorative investigations. There are different ways to carry out an observation. It can be done through an observation scheme, where the studied objects are determined in advance, or it can be done with an investigative purpose. The investigative purpose uses observations to gain as much knowledge as possible.[38]

The matters that have been discussed in this chapter are based on a social economic point of view. The authors have used the mathematical approach described in Chapter 2.3.4, where the primary data was collected by interviews. Observations are not used in the thesis due to the

[34] Patel, R. & Davidson, B. (2003) *”Forskningsmetodikens grunder”*
[35] Ibid.
[36] Ibid.
[37] Ibid.
[38] Ibid

time restraint. Questionnaires are not either used since the information could be gathered in an easier way by interviews. They are however good illustrators of primary data and are therefore presented in this chapter. The data from the business system is primary data if it is collected for this specific purpose. An example might be if sensors are placed where the different events occur and simultaneously the information is stored. In the authors´ case the data from the business system was stored earlier. Therefore it is considered to be secondary data which is defined in the following chapter.

2.5.2 Secondary data

Secondary data refers to information which is already gathered and the information is produced for another purpose. To get basic information about a specific topic the existing literature is always important. Besides books there are websites, daily press and protocols.[39]

Internet is a growing source of information and it is of great importance when a person is writing an essay. When gathering information from the internet it is essential to bear in mind that the data could have been published without any examination. The authors who use information from the internet must therefore examine the data even more carefully than if it is collected from a book.[40]

"*The Press*" constitutes an important information source because it is up to date. The value of the information depends on:

- the journalist's expertise of knowledge.
- the newspaper's specialization.
- the inside information which the correspondent can reveal.

There are of course many newspapers who are publishing poor objective reports that leaks objectivity. Here it is the author's common sense that decides the value of the information.[41]

The main secondary source that the authors have used is data from the business system. When an event occur the data for that event is saved in the business system. Different parameters were used to attain the information that was stored in the database that the business system uses.

Both primary and secondary information have been used when this thesis was written. The secondary information used is found in the literature, the database and the internet. The internet has been used very sparsely when the safety stock for new components was determined. When used, the sources have been investigated carefully and attempts have been made to verify the information. Concerning the theories of how to determine the safety stock for non-reco components the majority of the information was collected from books. The area is well known and there is a vast amount of literature covering the area. This is why it was relatively easy to find relevant information in books. Most of the books were found in the library in Lund. Course literature used in classes at LTH has also been used. Our supervisor at

[39] Bell, J. (2000) *"Introduktion till forskningsmetodik"*
[40] Denscombe, M. (2000) *"Forskningshandboken"*
[41] Ibid.

LTH gave us recommendations with some well know authors and books that he thought could be useful.

When gathering information to optimise the inventory levels for reco components the internet was used in a wider range. This is because the area is relatively new and it was hard to find relevant facts in books. Articles found on the internet were carefully examined. The articles gave the authors basic knowledge about the area. The main primary data source that has been used is information gathered from interviews. In most of the interviews the interviewers met the interviewee face to face. Information collected from interviews and the business system was used throughout the thesis. As mentioned earlier the main secondary source of information used is material collected from the business system. Many questions have been asked to verify the data from the business system and the other way around.

2.6 Statistical credibility

The gathered information has to be reviewed to achieve credibility. There are three dimensions to consider, reliability, validity and objectivity. The relationship between reliability and validity can be illustrated with the game of dart, see Figure 2.2 below. High reliability is the result if the darts are gathered to the same location and high validity is illustrated when the darts are located in the middle of the target.[42]

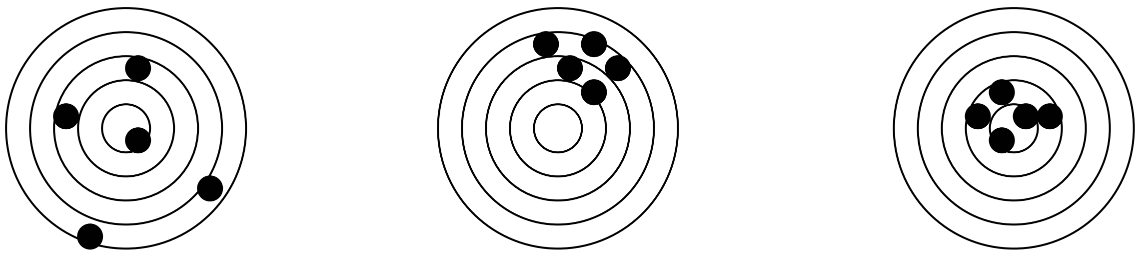

Figure 2.2. The picture to the left illustrates low reliability and low validity. The picture in the middle shows high reliability but low validity. The picture to the right illustrates high validity and high reliability.[43]

2.6.1 Reliability

Reliability is a measurement which shows if the same results are attained at different times, if the same circumstances apply[44]. There are a number of factors which can influence the answer e.g. if the interviewee has talked to a colleague and the colleague has influenced the interviewee[45].

The result can depend on the interviewer. If the questions are unclear they might be interpreted in different ways. As a result, it is very important that the interviewer is well trained to achieve as high reliability as possible. There are many ways to minimise the influence from the interviewer. The interview can for example be performed with two persons

[42] Björklund, M & Paulsson, U. (2003) *"Seminariehandboken"*
[43] Ibid.
[44] Bell, J. (2000) *"Introduktion till forskningsmetodik"*
[45] Ibid.

conducting the interview. This method leads to a conversation between the interviewers and the interviewee and the result is that the reliability is increased.[46]

2.6.2 Validity

Validity explains to what extent the measurement really measure the things one intends to measure[47]. The conception is more complicated than reliability and it is hard to achieve good validity if the reliability is poor[48]. It is important to critically examine the questions to achieve high validity, making sure that other persons who examine the questions come to the same conclusions[49].

From a mathematical point of view the validity may be to determine that the right data is attained from the database. Information can be collected in many different ways in the business system and it is easy to make a mistake that results in incorrect information. This is a major issue that the authors have taken into consideration when this thesis was written. The information that has been gathered is verified by asking the appropriate staff that works with these issues if it is realistic, thus increasing the validity. Several searches were performed to determine that the right result was achieved to increase the validity. Another matter that affects the validity is the assumptions that are made in the model. If the assumptions are poorly chosen, the measurement will not measure the intended aim and the validity will be low.

2.6.3 Objectivity

Objectivity is to what extent personal values are influencing the study. It is important that the authors are open minded and making sure that they do not steer the person who reads the essay into a given direction. This will enable the reader to make his own opinion about the results of the study. If the reader has an open mind throughout the study it will result in high objectivity for the study.[50]

All persons have perceptions about different matters. Therefore it is important when performing interviews that the questions are open, making sure that the interviewers own opinion does not affect the interviewee. It is also important that the answers are unbiased.

It is crucial for the authors to achieve a high credibility and therefore aspects discussed under the statistical credibility chapter have been adopted by the authors. To ensure that a high reliability and validity is met the authors asked questions to members of the staff that posses the right knowledge, e.g. to determine if the figures that had been collected from the business system were reasonable. If the figures were not in the right size they were discussed with the authors´ supervisor at Tetra Pak and in some cases new searches in the database were performed. The questions that were asked both in interviews and when reassuring that the information was correct were discussed between the authors making sure that the questions were asked in an appropriate way. This lowers the risk for misinterpretations. Both authors

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[46] Patel, R. & Davidson, B. (2003) *”Forskningsmetodikens grunder”*
[47] Björklund, M & Paulsson, U. (2003) *”Seminariehandboken”*
[48] Bell, J. (2000) *”Introduktion till forskningsmetodik”*
[49] Ibid.
[50] Björklund, M. & Paulsson, U. (2003) *”Seminariehandboken”*

have been present at all interviews to minimise the risk of misinterpretations. Except for the interviews that took place over the phone.

When conducting searches in Query, the database that Jeeves is built upon, the results were always compared to the results in the business system. This was the authors´ way of determining that the right data was attained, since the data would differ if some errors had been made. If there was a difference in the results this would have resulted in low reliability. This comparison increased the reliability and validity. The comparison also led to that the authors developed the methods for searching the database and the result was improved, i.e. a smaller difference between the data from the business system and the database equals a higher reliability.

2.7 Critique of sources[51]

There are many potential errors during a study and therefore it is important to view the information in a critical way, otherwise the entire study can be in vain. The authors of the book *"Att utreda, forska och rapportera"* discuss four issues to consider: demand of contemporary, criticism of tendency, criticism of dependency and genuineness.

Demand for contemporaneousness. It is important to investigate if the authors have written the information at the same time as the events have occurred. A log book is a good source concerning the demand of contemporaneousness but a memoir is poorer due to the difference in time i.e. the time difference between when the event occurred and when it was written.

Criticism of tendency. To get an answer about the tendency in a piece of information, it is common to use the question "Which interests does the person who provides the information in this question have?".

Criticism of dependency. It is vital to determine if the source is what it is stated to be. This area is a growing problem due to the internet. The importance of the internet is increasing concerning the gathering of information and with the internet it is harder to be sure that the source is reliable.

Genuineness. In this area it is essential to establish the authenticity. One method is to try to sort out the information - the bad should be thrown away and the good should be stored.

No attempts have been made by the authors to investigate the demand of contemporaneousness. It was considered hard, since the time difference required to determine when the events occurred and when they were written was not available. The literature used was written by well know authors in their fields, meaning that the criticism of tendency and dependency is considered to be low. The authors increased the genuineness of the material gathered from the business system by comparing the material with other sources. All the internet articles are written by well known authors and the articles are linked to their homepages. The link between the articles and the homepage is a proof of its genuineness. The issues discussed above are something the authors have been aware of while gathering the material.

[51] Eriksson, L-T. & Wiedersheim-Paul, F. (2001) *"Att utreda, forska och rapportera"*

3 Theoretical frame of references

The theories presented in this chapter provide the necessary background to fully comprehend the data collection and analysis chapters that follows. A wide area of different distributions as well as inventory theories will be presented. This will enable the reader to understand the complexity of the problem that will be discussed in the following parts of the thesis.

3.1 Safety Stock

One of the objectives of the thesis is to determine the safety stock for non-reco components. This is one issue of the overall objective and therefore it is of importance to know the purpose of a safety stock.

The purpose of the safety stock is to cover the arbitrary variations of the demand during the lead time.[52] Normally the safety stock is not to be used and therefore it essentially constitutes a constant expected cost for the component. Two factors that influence the size of the safety stock are how uncertain the lead-times are and how irregular the sales for the component are. If these two factors are large the safety stock has to be increased.[53]

3.2 Different inventory systems

It is important to determine what kind of inventory system one has since it will affect which formulas to use for different calculations.

When optimising inventory it is a common opinion that the optimum solution can be determined, but this is not always the case. If the system is of a complex nature more approximations has to be used, since the connection between different warehouses has to be considered. The different echelon systems can be divided into two different models, a single echelon system and a multi echelon system.[54]

Multi echelon systems will not be considered further because the models treated in this master thesis are solely single echelon models.

3.2.1 Single echelon system[55]

Single echelon systems can often be solved exactly or with very small approximations. The single echelon system is characterised by qualities such as:

- various components can be controlled independently of each other.
- components are stored in a single location (as opposed to a multi echelon system).

[52] Axsäter, S. (1991) *"Lagerstyrning"*
[53] Pewe, U. (1993) *"Lönsam Logistik"*
[54] Axsäter, S. (1991) *"Lagerstyrning"*
[55] Ibid.

Often the first condition is fulfilled for a system, but not the second one, thus deeming it to be a multi echelon system. In a factory there is usually a central warehouse and different local warehouses or storage locations with strong interdependencies. These systems are then classified as multi echelon systems. Sometimes the connection is weak between the different warehouses and it is advantageous to use methods for a single echelon system even though it is defined to be a multi echelon system.

3.3 Different ordering systems[56]

It is of great importance to know when an order is to be placed and what kind of different ordering policies that are available. This is the reason why the authors have included this chapter in the thesis.

An inventory control system needs to provide guidance as to when and in what quantities to order. Typically the decision is based on the stock situation, the demand, and different cost parameters. One may use a service requirement instead of shortage cost if this cost is hard to establish. A decision about refilling the stock should not only be based on the level of the stock on hand, but should also consider other factors such as outstanding orders and backorders. This is why the inventory position is defined as:

Inventory position (IP) = stock on hand + outstanding orders – backorders *Eq. 3.1*

It is the inventory position that describes when an order should be placed. If the customers can order products in advance and reserve items, these should be subtracted in the inventory position. Holding and shortage costs are based on the inventory level. The inventory level can be defined as:

Inventory level (IL) = stock on hand – backorders *Eq. 3.2*

There are two different ways of monitoring the inventory position, continuous review and periodic review. When using continuous review the inventory position is monitored continuously and as soon as it reaches a certain value, an order is placed. The items that are ordered will be delivered after a certain time, the lead-time (L). The lead-time is the time that elapses from the ordering decision until the ordered items are delivered. The alternative way of monitoring the inventory position is to do it at certain time intervals. This is called periodic review. Both techniques have their advantages and disadvantages. Depending on which approach that is used, the inventory position has to protect against variations in demand under different time periods.

There are different ordering policies concerning inventory control of single echelon systems. Two of the most commonly used are the (R,Q) policy and the (s,S) policy. When the inventory position reaches or goes below the reorder point R, in a (R,Q) policy, an order of the size Q is placed. In case of continuous review, the order will be placed exactly when reorder point is struck[57]. The (s,S) policy is quite similar to the (R,Q) policy. When the inventory position reaches the reorder point s, an order up to the maximum level S is placed, meaning that the ordering quantity is not fixed as in the (R,Q) system. If continuous review

[56] Axsäter, S. (2006) *"Inventory control"*
[57] Ibid.

and continuous demand apply, the two policies are equivalent[58]. If a continuous review is used and the demand is always one unit at a time, the policy is an order-up-to (S-1,S) policy i.e. a (s,S) policy with s = S-1, see Figure 3.1.[59] In periodic review, a variation of the (s,S) policy is the order-up-to-S policy, which means that an order is always placed unless the demand for the period is zero, raising the inventory position to S even if the reorder point is not reached.

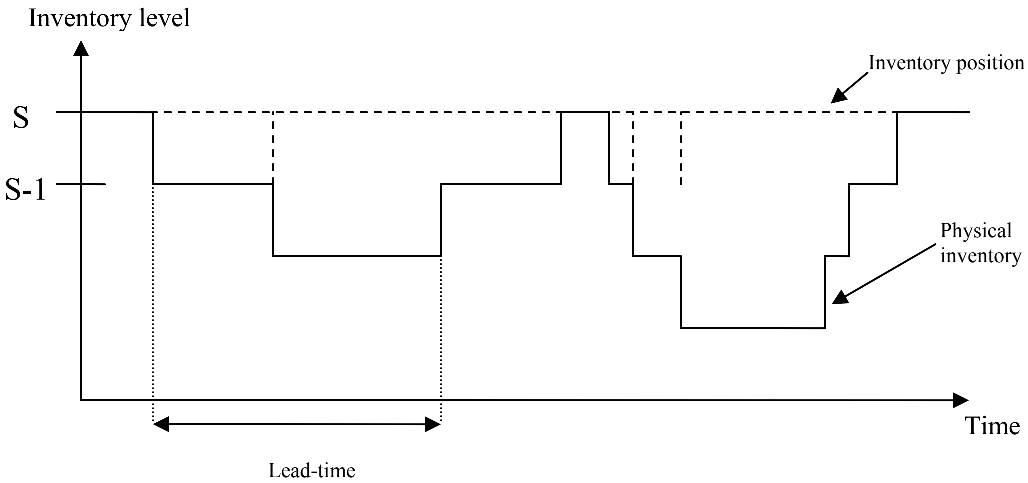

Figure 3.1. An illustration of the ordering policy (S-1,S).

Let t represent an arbitrary point in time, IP(t) = the inventory position at that time, L = the lead-time, and D(t,t+L) = the demand during the lead-time. At time t + L everything ordered at t have been delivered. Nothing ordered after time t has yet been delivered due to the lead-time. There is a relation between the inventory position, the inventory level and the demand during the lead-time. This inventory balance equation can be stated as follows:[60]

$$IL(t+L) = IP(t) - D(t,t+L) \qquad \textit{Eq. 3.3}$$

The demand during the time period, L, is stochastic. If the distribution of the inventory position at time t and the distribution of the demand under the lead-time (t,t+L) is known, it is relatively easy to determine the distribution of the inventory level at time t + L. If a (S-1,S) policy and continuous review is used the inventory position is constant.[61]

3.4 Inventory related costs

When optimising the inventory levels one has to determine the costs associated with carrying inventory and the shortage costs. Therefore it is of importance to understand what should be included in these costs. The shortage costs are often hard to determine per se, and an alternative is to use a service level constraint. This is very common in practise, and it makes it justifiable to present a quite extensive chapter about different service levels. The last cost that is presented here is the ordering cost, which should be balanced against the holding cost to determine the size of the quantities to purchase.

[58] Axsäter, S. (2006) *"Inventory control"*
[59] Axsäter, S. (1991) *"Lagerstyrning"*
[60] Axsäter, S. (2006) *"Inventory control"*
[61] Ibid.

There are however more costs that can be of importance to consider such as the cost of operating the inventory control system and purchasing costs if they are affected by quantity discounts etc. Costs that ought to be included are those that vary with the inventory level.[62] The chapters to come will discuss the inventory related costs such as holding cost, shortage cost, ordering cost and different service levels.

3.4.1 Holding cost

Holding cost can be defined as the cost associated with having one unit in inventory for one period of time. It can be divided into four categories[63]:

- capital cost
- inventory service cost
- storage space cost
- inventory risk cost

When carrying inventory there is an opportunity cost for the capital that is tied up in inventory[64]. There is always an alternative use for the money and thus there always is an opportunity cost. Money that is not tied up in an investment can be used to give a certain yield.[65] This is one reason why the capital costs should be closely related to the return on an alternative investment. It does not have to be equal to the expected return of the alternative investment since the financial risk associated with this investment has to be considered.[66] The opportunity cost ought to be based on the return from the best alternative investment that possesses the same risk as the inventory, which in the normal case is a low risk investment.[67] The major contributor to the holding cost is normally considered to be the capital cost.[68] An alternative to determine the capital cost by using the opportunity cost is to use the firm's average return. The holding cost is normally calculated as a percentage, κ, times the replacement cost per unit, C.[69]

$$h = \kappa \cdot \mathrm{C} \qquad \textit{Eq. 3.4}$$

The formula depends on the statement earlier that the holding cost is primarily based on the capital cost. In the literature the range for the values for κ is quite large and few recommendations are given for what percentage to use. This makes it possible to justify many different inventory levels from an economic point of view.[70] It is also important to state that κ in many cases can differ for different items[71]. The capital cost can be determined by a similar expression to Eq. 3.4. It can be formulated as follows[72]:

[62] Axsäter, S. (2006) *"Inventory control"*
[63] Berling, P. (2005) *"On Determination of Inventory Cost Parameters"*
[64] Axsäter, S. (2006) *"Inventory control"*
[65] Persson, I. & Nilsson, S-Å. (2001) *"Investeringsbedömning"*
[66] Axsäter, S. (2006) *"Inventory control"*
[67] Berling, P. (2005) *"On Determination of Inventory Cost Parameters"*
[68] Axsäter, S. (2006) *"Inventory control"*
[69] Berling, P. (2005) *"On Determination of Inventory Cost Parameters"*
[70] Ibid.
[71] Axsäter, S. (2006) *"Inventory control"*
[72] Berling, P. (2005) *"On Determination of Inventory Cost Parameters"*

$$Capital\ cost = \kappa_C \cdot C \qquad Eq.\ 3.5$$

where κ_C is affected by several parameters such as the interest rate, inflation, cost changes and financial risk. There are several methods to determine κ_C. In practise the cost of lending capital is the one most frequently used.[73]

The other components that affect the holding cost are the inventory service cost, storage space cost and the inventory risk cost that are sometimes called out-of pocket holding cost as a common name. Tax and insurance costs represents the inventory service cost if they exist. The insurance cost is based on the average inventory, thus it should be included in the cost. Rent, heating and lightning of a facility are examples of storage space costs. The storage space cost should also include handling and transportation. These should only be included if they vary with the inventory on hand. Costs for obsolescence and damage of the components are categorised under inventory risk costs. The company cannot control most of these factors. The inventory risk cost is the cost that arises for example when the inventory changes its physical attributes and due to this the value is decreased. The financial risk is not a part of the inventory risk cost since it should be considered when determining the capital cost.[74]

3.5 Cost associated with failure to perform the expected service[75]

If a company is unable to meet customers´ demand, there will in the normal case arise costs that are called shortage costs. There is a trade off between these costs and the extra holding cost that arise due to the additional inventory that is kept to decrease the risk of shortages during the lead time, the safety stock. In production it is quite difficult to determine the shortage cost since it is hard to anticipate the effect of a shortage of a component. A shortage cost is normally divided into one of the following three categories:

- B_1 = shortage cost per item and period of time.
- B_2 = shortage cost per item.
- B_3 = shortage cost per period of time.

An alternative to use shortage costs to determine the safety stock is to use a service level constraint, making sure that a certain percentage of the customers´ demand are meet.

3.5.1 Service levels

The decision concerning the safety stock or the reorder point can be based either on meeting a certain service level constraint or a cost minimization. It is hard to determine the shortage cost per se and therefore it is often easier from a practical point of view to work with a service level constraint. The service level should be based on the shortage costs and the cost for giving an appropriate service. Three different service levels will be defined.[76]

[73] Berling, P. (2005) *"On Determination of Inventory Cost Parameters"*
[74] Ibid.
[75] Ibid.
[76] Axsäter, S. (2006) *"Inventory control"*

"Cycle service level" (S_1). This service level describes the probability of no stock out per order cycle i.e. the probability that an order arrives before there is a stock out.[77] This definition of service level is easy to use but it has one problem, it does not take the order quantity into consideration. A result of this is that if the order quantity is large and covers the demand for a longer period of time, the experienced service level might be high even if the cycle service level is low, due to the fact that the number of orders will be quite few. The other way around, the experienced service level might be low even if the definition of the service level is high when the order quantity is low.[78]

"Fill rate" (S_2). The definition of this service level is as follows: fraction of demand that can be satisfied instantly from stock on hand.[79] The fill rate is somewhat more complex to work with compared to the cycle service level, but is normally a better way of measuring the experienced service level.[80]

"Ready rate" (S_3). This service level is defined as: fraction of time with positive stock on hand. The ready rate is a bit more complex to work with compared to the cycle service level, but it will provides a better measurement of the experienced service level. If the demand is continuous or described by a Poisson distribution, the fill rate and the ready rate will be identical.[81]

At present time none of the service levels above have been used to optimise the inventory levels at Maintenance Unit. The authors' models are based on the fill rate since it measures the service level for the components in an appropriate way.

When the service level is agreed upon, the problem is to determine the reorder point. One also has to ensure that the service level is sufficiently large to cover the demand during the lead time.[82]

The cycle service level is often used with continuous review and continuous demand. If the demand during the lead time is normal distributed with an average of μ` and standard deviation of σ` the following model can be used:[83]

$$P(D(L) \le R) = cycle\_service\_level = \Phi((R-\mu`)/\sigma`) = \Phi(\mathrm{SS}/\sigma`) \qquad \textit{Eq. 3.6}$$

To ensure that the cycle service level is reached, the ratio SS/σ`, which is called the safety factor (k), has to be of appropriate size. When the safety factor is agreed upon the safety stock can be determined as:[84]

$$SS = k \cdot \sigma` \qquad \textit{Eq. 3.7}$$

If the demand has a Poisson distribution or if it is continuous, the fill rate and ready rate are equal.[85] This statement will be used throughout the thesis.

[77] Axsäter, S. (2006) *"Inventory control"*
[78] Axsäter, S. (1991) *"Lagerstyrning"*
[79] Axsäter, S. (2006) *"Inventory control"*
[80] Axsäter, S. (1991) *"Lagerstyrning"*
[81] Axsäter, S. (2006) *"Inventory control"*
[82] Ibid.
[83] Ibid.
[84] Ibid.
[85] Ibid.

Ready rate is easily determined in the case of a compound Poisson or Poisson demand. Since the definition of ready rate is the fraction of time with positive stock on hand, the following connection between the inventory level and the service level exist:[86]

$$ready\_rate = P(IL > 0) \qquad \text{Eq. 3.8}$$

In the case of a compound Poisson demand, it is more complicated to determine the fill rate since the demand size can vary.[87] The fill rate can be identified as:[88]

$$fill\_rate = \frac{E\left[Quantity_{Satisfied}\right]}{E\left[Quantity_{Demanded}\right]} \qquad \text{Eq. 3.9}$$

where

$$E\left[Quantity_{Demanded}\right] = \sum_{k=1}^{\infty} k \cdot f_k = E[K] \qquad \text{Eq. 3.10}$$

f_k represents the probability of the demand size k (k = 1, 2, …).[89] The satisfied demand depends on two things, the inventory level and the ordered quantity. If the inventory level is j and the demanded quantity is k > 0, then the satisfied quantity is j if k is greater than j and the quantity is k if j is greater or equal to k. If the inventory level and the demanded quantity are independent the satisfied demand can be expressed as:[90]

$$E\left[Quantity_{Satisfied}\right] = \sum_{k=1}^{\infty}\sum_{j=1}^{\infty} \min(j,k) \cdot f_k \cdot P(IL = j) \qquad \text{Eq. 3.11}$$

Equation 3.9 can be rewritten into the following:

$$fill\_rate = \frac{\sum_{k=1}^{\infty}\sum_{j=1}^{\infty} \min(j,k) \cdot f_k \cdot P(IL = j)}{\sum_{k=1}^{\infty} k \cdot f_k} \qquad \text{Eq. 3.12}$$

These three definitions of service levels are commonly used,[91] but there are many other ways of defining a service level. The important thing to bear in mind is that it is defined in a clear way, enabling a common interpretation for the people involved.[92]

In the most cases it is not appropriate to apply the same service level to all items that are kept in stock. It is also difficult to give each component an individual service level. To deal with this problem it is customary to divide the different items into classes and give each class a

[86] Axsäter, S. (2006) *"Inventory control"*
[87] Ibid.
[88] Marklund, J. 2006-09-12
[89] Axsäter, S. (2006) *"Inventory control"*
[90] Marklund, J. 2006-09-12
[91] Berling, P. (2005) *"On Determination of Cost Parameters"*
[92] Axsäter, S. (2006) *"Inventory control"*

service level that is appropriate. It is also important that the service level is based upon the shortage costs and the costs that are associated with providing an appropriate service level.[93]

3.6 Ordering cost[94]

When an order is placed, fixed costs that are associated with the order and independent of the size of the purchase might arise. These costs must be agreed upon to determine the quantity to order. Costs that should be included in this cost are administration work related to preparing, realizing, monitoring and receiving the order. The cost can be based on either the actual or planned utilization of the staff. From a theoretical point of view, the ordering cost should only include the marginal cost of placing the specific order that is investigated. Therefore the ordering cost will change with time depending on the utilisation of the staff. In practise this is not viable since it is not realistic to recompute the ordering cost every time it will be used.

3.7 The Economic Order Quantity

The following section will present how the order quantity is determined and the theory will be used for the reco components.

The most commonly used formula in inventory control is the Economic Order Quantity (EOQ), also know as the Wilson formula. The main advantage with this formula is that the mathematical calculations are easy to perform. The disadvantage is that it is has a restricted practical usability since it is based on a several conditions[95].
The assumptions that the Wilson formula is based upon are[96]:

- the demand is constant and continuous.
- the ordering quantity does not need to be an integer.
- the entire ordering quantity is delivered at the same time.
- no shortages are allowed.
- ordering and inventory costs are constant.

If applying the simplifications described above, the figure over the inventory level becomes easier to construct, see Figure 3.2 below[97].
The authors used the following notations:

- h = inventory cost per item and period of time.
- A = ordering cost
- d = demand per period of time
- Q = ordering quantity
- C = cost per period of time

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[93] Axsäter, S. (2006) *"Inventory control"*
[94] Berling, P. (2005) *"On Determination of Inventory Cost Parameters"*
[95] Widén, G. (1973) *"Lagerteori"*
[96] Axsäter, S. (2006) *"Inventory control"*
[97] Ibid.

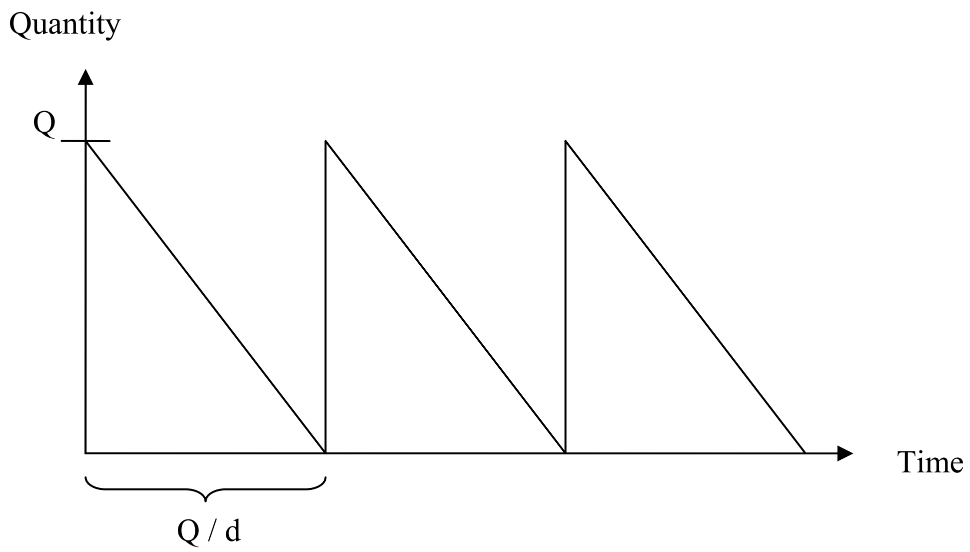

Figure 3.2. The figure illustrates the inventory level over time. The lead-time equals zero and therefore the inventory level and the inventory position are identical. There is no safety stock in this figure.[98]

When the figure is constructed, it is possible to express the total cost per time unit, which can be described as Eq. 3.13[99]:

$$C = \frac{Q}{2}h + \frac{d}{Q}A \qquad \textit{Eq. 3.13}$$

The first part of the formula above is referring to the cost of carrying inventory and the second term is referring to the ordering cost. The cost of carrying inventory is determined by multiplying the average inventory with the inventory cost per item per period of time. The ordering cost is determined by multiplying the ordering cost with the amount of orders per period of time. When the total cost is established it is possible to minimise the cost with consideration taken to the quantity, see Eq. 3.14 below.[100]

$$\frac{dC}{CQ} = \frac{h}{2} - \frac{d}{Q^2} \cdot A = 0 \qquad \textit{Eq. 3.14}$$

Eq. 3.14 above leads to the Wilson formula Eq. 3.15 below[101].

$$Q' = \sqrt{\frac{2 \cdot A \cdot d}{h}} \qquad \textit{Eq. 3.15}$$

If Eq. 3.13 is combined with Eq. 3.14 a new equation that minimises the cost, C, is created, see Eq. 3.16[102].

[98] Axsäter, S. (2006) *"Inventory control"*
[99] Axsäter, S. (1991) *"Lagerstyrning"*
[100] Ibid.
[101] Ibid.
[102] Ibid.

$$C' = \sqrt{\frac{A \cdot d \cdot h}{2}} + \sqrt{\frac{A \cdot d \cdot h}{2}} = \sqrt{2 \cdot A \cdot d \cdot h} \qquad \text{Eq. 3.16}$$

In many cases the items can not be divided into pieces. Therefore the condition that the order quantity, Q, does not have to be an integer does not need to be strictly valid. In practise it is common to round off Q to an integer. It will not have a large input on the results, particularly if Q is large.[103]

3.8 ABC inventory analysis

Time is often a scarce resource and in many cases it is appropriate to divide components into homogenous classifications. This reduces the time required for implementation and follow up of the results. This approach will be used in the thesis.

Each component can be controlled in a specific way. Since there are often thousands of components, this approach can be hard to apply. It is easier to divide the components into a number of classes and control the components in a given class the same way.[104] The easiest way of controlling the movement of material is to apply a common approach for all the different components. However, by dividing the components into different classes enables a person to control each class in the best possible way.[105]

When resources are scarce this differentiation allows the effort to be oriented towards the classification with the highest priority. By sorting the components e.g. after their profitability, the most important components can be prioritized.[106] If the control system is fully computerised not much additional effort has to be spent to include all the components, once the system has been implemented. However, if the control system contains manual elements it is important that the class with the highest priority is in focus. It is always important to monitor the class with the highest priority closely, even if the control system is computerised[107]

One way of doing the categorisation is to use the so called ABC classification. When using this method it is common practise to use the SEK volume, i.e., the volume that is spend during a year times the value of the product, as a base for the categorisation. The products are divided into three different classes – a relatively small A class, which contains the most essential products and the SEK volume for the A class represents approximately 60-80 % of the accumulated SEK volume. The B class includes a larger number of components compared to the A class and represents approximately 15-30 % of the accumulated SEK volume. The last class, the C class, is the largest of the classes (concerning the number of components) and it represents 5-15 % of the accumulated SEK volume, see Figure 3.3 below.[108] The A class containing components with a high SEK volume are often the most important ones and this class should be given the highest priority[109]. The A-components should be ordered with a high frequency and the safety stock should be kept at a minimum since they often are expensive to

[103] Axsäter, S. (1991) *”Lagerstyrning”*
[104] Axsäter, S. (2006) *“Inventory control”*
[105] Aronsson, B. & Ekdahl, B. & Oskarsson, B. (2004) *”Modern logistik”*
[106] Ibid.
[107] Axsäter, S. (2006) *“Inventory control”*
[108] Aronsson, B. & Ekdahl, B. & Oskarsson, B. (2004) *”Modern logistik”*
[109] Axsäter, S. (2006) *“Inventory control”*

keep in stock. Concerning the C-products, they should be ordered more rarely and the safety stock can be large as it will not be as costly since they have a low SEK volume. A great deal of time can be saved to avoid a control system that goes into detail for these products.[110]

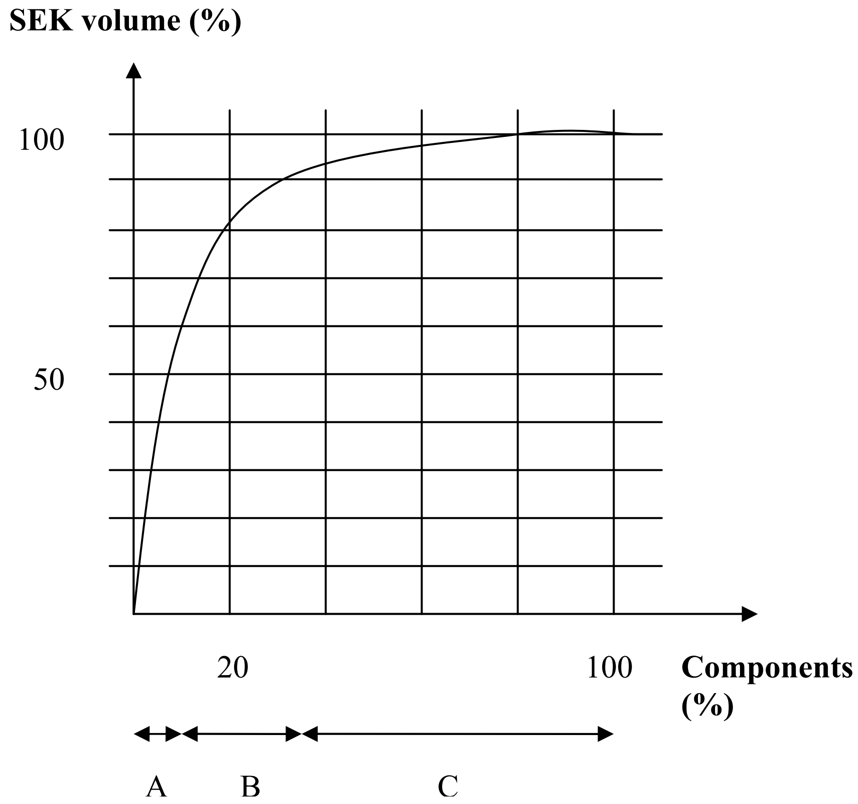

Figure 3.3. An example of an ABC inventory analysis.

Often the result of the ABC classification is that a small amount of the components represents a large portion of the total amount of e.g. the SEK volume. This is called the Pareto-principle which states that approximately 20 percent of the components represents 80 percent of the SEK volume.[111]

All components in the spare parts that Maintenance Unit assembles are equally important. If one component is missing the spare part can not be assembled. Therefore the components in the C group will be given the highest fill rate since they are the cheapest. If the lead-time is used as classification it makes sense to have a higher fill rate for the A group since it takes longer time to receive the ordered components.

The ABC inventory analysis has been given its name from the three categories used. One can decrease or increase the number of classes and the categorisation can also be performed by using several criterions. If several criterions are used the fact that a number of different factors may affect the categorisation is then taken into consideration.[112]

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[110] Aronsson, B. & Ekdahl, B. & Oskarsson, B. (2004) *”Modern logistik”*
[111] Olsson, J & Skärvad, P-H. (2001) *”Företags ekonomi”*
[112] Ibid.

3.9 Lead-time

The demand during the lead-time is an important issue when determining the reorder point. Therefore it is important to know how the lead-time is defined.

One of the most important parameters for controlling inventories is the lead-time, defined as the time between when an order is created by the receiver until the material is available to them. If the lead-time is large the inventory levels have to be large as well to secure the production against demand uncertainties.[113] If the lead-time is increased, the demand uncertainty during this time is increased as well. The size of the safety stock depends on this uncertainty.[114]

As an example, the demand for components with long lead-times used in Maintenance Unit's spare parts are hard to predict. To be able to guard against this uncertainty, the inventory levels are raised.

The lead-time can be divided into two parts, which are called the administrative and the physical part. Both parts are vital to minimise, ensuring that the total lead-time becomes as small as possible. If the lead-time is minimised, the inventory costs can also be minimised. There are many connections between the relationship of the lead-time and the cost associated with inventory. They are the following:[115]

- with a long lead-time the future becomes more uncertain and the inventory levels have to be increased.
- the uncertainty for the demand increases with longer lead-times, meaning that the inventory levels have to be increased.
- for a long lead-time the uncertainty for the lead-time is increased.

3.10 Demand

To make sure that the appropriate demand distributions are used in the authors models it is important to see what the demand structure looks like.

It is important to determine the pattern of demand to achieve satisfactory results. Some examples of different patterns are: constant, season, trend, business cycle and random.[116] A constant demand is preferable compared to varying demands, since it is easier to achieve higher utilization of capacity. It may be possible to influence the demand pattern. Thus it is of importance to influence the factors that are changeable to get the best result.[117]

If the demand is stochastic, it can be divided into two categories, discrete and continuous stochastic demand.[118] The distribution is characterised by its cumulative distribution function F(x) which is defined as[119]:

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113 Pewe, U. (1993) *"Lönsam Logistik"*
114 Axsäter, S. (1991) *"Lagerstyrning"*
115 Persson, G. & Virum, H. (1998) *"Logistik"*
116 Ibid.
117 Ibid.
118 http://sv.wikipedia.org/wiki/Sannolikhetsf%C3%B6rdelning 2006-09-18

$F(x) = P(X \leq x), \qquad -\infty < x < \infty$ *Eq. 3.17*

The demand considered in this thesis can not be negative and therefore the first boundary is changed to zero, i.e. $0 < x < \infty$.

3.10.1 Different distributions

This chapter will present some of the most common distributions that the reader should be familiar with to know which options that are available when determining the distribution for the demand.

A discrete distribution has a cumulative distribution function consisting of a sequence of finite steps. This means that it belongs to a discrete random variable X. This variable can only have values which are finite or values that consists of a countable quantity.[120]

A continuous distribution has a continuous cumulative distribution function. This means that it belongs to a continuous random variable X where $P(X = x) = 0$ for all x in space.[121]

The normal distribution is used in two different circumstances. Either when the data has an exact or approximate normal distribution. The latter is appropriate to use when there is a high demand for an item and the demand is independent uniformed stochastic variables. This phenomenon is described by the central limit theorem. The main benefits for the normal distribution is because it is easy to handle and it has many good mathematical qualities. The normal distribution depends on two variables, the mean value, m, and the standard deviation, σ, see Figure 3.4 below for an example. The equation looks like:[122]

$X \in N(m, \sigma)$, with $E(X) = m$, $V(X) = \sigma^2$ and $D(X) = \sigma$ *Eq. 3.18*

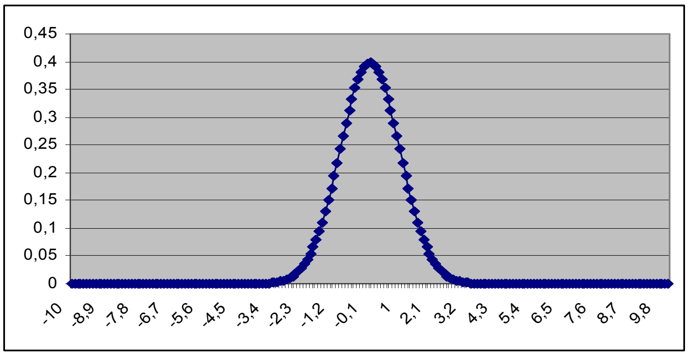

Figure 3.4. A normal distribution with E(X)=0 and D(X)=1.

The normal distribution will not be used in this thesis. The reason for this is if the demand is low, the standard deviation might be larger than the mean value and the outcome will be poor.

It became evident that the Poisson distribution is an appropriate distribution that describes the demand in the situations that are considered in this master thesis. It is a special case of the

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[119] Blom, G (1989) *"Sannolikhetsteori och statistikteori med tillämpningar"*
[120] http://sv.wikipedia.org/wiki/Sannolikhetsf%C3%B6rdelning 2006-09-18
[121] Ibid.
[122] Blom, G. (1989) *"Sannolikhetsteori och statistikteori med tillämpningar"*

Compound Poisson distribution. To understand the formula that describes the probability for a certain inventory level for a Poisson distribution, the general case will be described first i.e. the Compound Poisson distribution.

Compound Poisson distribution is a discrete distribution. The distribution is used when the demand is low and random. This means that the demand occurs independently of each other and at any time. The arrival intensity of the customers and their demand are stochastic variables. The latter means that the customer can order more than one item at a time. The equation to calculate the probability for a certain inventory level is stated below:[123]

$$P(IL = j) = \frac{1}{Q} \sum_{k=\max\{R+1, j\}}^{R+Q} P(D(L) = k - j), \quad j \leq R + Q \qquad \textit{Eq. 3.19}$$

IL = inventory level
Q = ordering quantity
k = inventory position
R = reorder point

The Poisson distribution is determined by a given arrival intensity, λ,[124]:

X ϵ Po (λt), with E(X) = λt, V(X) = λt and D(X) = $(\lambda t)^{0.5}$ *Eq. 3.20*

The difference between the Poisson and the Compound Poisson distribution is that with a Poisson distribution the customer is restricted to order one item at a time. If an order-up-to-S policy is used, Eq. 3.19 above becomes easier to handle because of changes in the variables:[125]
Q = 1
R = S-1
k = S, the inventory position is always S.

Eq. 3.19 can then be rewritten into the following[126]:

$$P(IL = j) = \frac{1}{Q} \sum_{k=\max\{R+1, j\}}^{R+Q} P(D(L) = k - j) = \frac{1}{1} \sum_{S}^{S} P(D(L) = S - j) = \qquad \textit{Eq. 3.21}$$
$$= P(D(L) = S - j), \qquad j \leq S$$

3.11 Histogram[127]

Histograms are used in this thesis to graphically investigate what kind of distribution the data has. The histograms enables the authors to determine that the distribution chosen is reasonable before different goodness of fit tests are performed.

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[123] Axsäter, S. (1991) *"Inventory Control"*
[124] Blom, G. (1989) *"Sannolikhetsteori och statistikteori med tillämpningar"*
[125] Axsäter, S. (1991) *"Inventory Control"*
[126] Ibid.
[127] Laguna, M. & Marklund, J. (2005) *"Business Process Modeling, Simulation, and Design"*

The histogram is created by dividing the data range into certain equal intervals. The next step is to determine the frequency of the data in each interval. The relative frequencies between the different intervals are an estimate of the probability that a value will fall into this interval. The shape of the histogram will approach the shape of the distribution for the data. Since the number of intervals and their width will affect the shape of the histogram it is crucial that they are selected wisely to enable to find a probability distribution function (pdf) that adequately describes the data attained.

3.12 Goodness of fit tests

To be able to perform any kind of calculations on the data gathered, the distributions of the different data material have to be determined. To statistically test the hypothesis that a certain random variable adequately describes the available data material one can use so called goodness of fit tests. Two different tests were conducted in the data collection analysis. Both tests are well known but none the less they are presented here to show the formulas and guidelines that will be used later on in the thesis.

3.12.1 The Chi-Square Test

To determine a distribution for a sample of data, it is common to use a test called the Chi-Square test. This test compares the distribution with a theoretical distribution and decides if it is to be rejected or not. If it is not possible to reject the distribution when the Chi-Square test is used, it can possibly be assumed to be the tested distribution.[128] The Chi-Square test is used on data which can be classified into different classes. It is important that there is a sufficiently large quantity of data for the classification. A guideline is that there should be at least five numbers of data in each class. When the categorization is completed, the next step is to compare the observed quantity with the expected quantity for the determined distribution in each interval for a given degree of freedom, see Eq. 3.22 below.[129]

$$\chi^2 = \sum_{i=1}^{r} \frac{(x_i - np_i)^2}{np_i} \qquad \textit{Eq. 3.22}$$

In Eq. 3.22, n is the total amount of observations, x_i is the number of observations in interval number i, r is the number of intervals and p_i is the probability value for each interval.
The χ^2 is calculated from the formula above, and if:

$\chi^2 > \chi^2_\alpha(f)$, f = r-k-1 :the distribution is to be rejected
$\chi^2 < \chi^2_\alpha(f)$, f = r-k-1 :the distribution can be assumed

k is the number of parameters in the theoretical distribution. The value $X^2_\alpha(f)$ can be found in a statistical table.[130]

[128] Laguna, M & Marklund, J. (2005 "*Buisness Process Modeling, Simulation, and Design*")
[129] Blom, G. (1989) *"Sannolikhetsteori och statistikteori med tillämpningar"*
[130] http://www.ets.kth.se/personal/lina/docs/2C4030NoterKursdel2Dag2.pdf#search=%22funktion%20kolmogorov%20smirnov%20test%22 2006-10-20

3.12.2 The Kolmogorov-Smirnov Test[131]

An alternative to testing the distribution using the Chi-Square test is to use the Kolmogorov-Smirnov test. The advantage with this test compared to the Chi-Square test is that it may be used for smaller amounts of data. In this method each data value is assessed individually instead of in classes as in the Chi-Square test. The steps for constructing the test are the following:

- sort the data in ascending order i.e. $x_1 < x_1 < \ldots < x_{n.}$
- calculate D^+ and D^- using the theoretical cumulative distribution, $\hat{F}(x)$:

$$D^+ = \max_{1 \le i \le n}\left[\frac{i}{n} - \hat{F}(x_i)\right] \qquad \textit{Eq. 3.23}$$

$$D^- = \max_{1 \le i \le n}\left[\hat{F}(x_i) - \frac{i-1}{n}\right] \qquad \textit{Eq. 3.24}$$

- calculate $D = \max(D^+, D^-)$.
- find the Kolmogorov-Smirnov critical value by using the level of significance and the sample size n.
- if the critical Kolomogorov-Smirnov value is greater or equal to D, then the hypothesis is not rejected.

3.13 Birth and death processes

It is important that the probabilities for different inventory positions can be calculated. The theory in this section will be used to calculate the probabilities for different inventory positions for reco components.

Queuing theory is based on the birth and death process. Birth is when a customer arrives to the system and death is when the customer departures from the system. A birth-and-death process may be used to describe a queuing situation. It is possible from the birth-and-death process to calculate the stationary probability for a given number of customers in the system, it can be 0, 1, 2… number of customers. When the stationary probabilities have been obtained it is easy to calculate the expected number of customers in the system or in the queue.[132]

The birth-and-death process can be illustrated graphically in a so called state diagram, see Figure 3.5.[133]

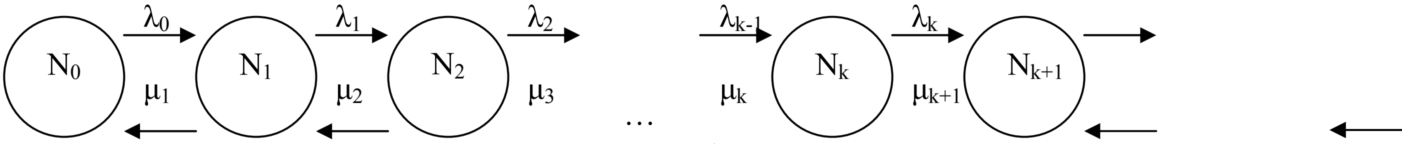


Figure 3.5. A state diagram.

[131] Laguna, M & Marklund, J (2005) *"Business Process Modeling, Simulation, and Design"*
[132] Olsson, G & Rosen, C (2005) *"Industrial Automation"*
[133] Ibid.

By looking at the arrows in the figure one can tell how the events can occur. For example if the system is in state N_1 the transition to N_2 is determined by an exponential distribution with a mean arrival rate λ_1.[134]

To determine the stationary probabilities $P_0, P_1 \ldots$ the Rate In = Rate Out principle is applied together with the fact that the sum of the stationary probabilities must equal 1, $P_0 + P_1 \ldots = 1$. The balance equations can be expressed as:[135]

State 0: $\lambda_0 * P_0 = \mu_1 * P_1$
State 1: $\lambda_0 * P_0 + \mu_2 * P_2 = \mu_1 * P_1 + \lambda_1 * P_1$
State 2: $\lambda_1 * P_1 + \mu_3 * P_3 = \mu_2 * P_2 + \lambda_2 * P_2$
…

3.13.1 The M/M/1 model[136]

The M/M/1 model is a special case of the birth-death-process which is described in the text above. The model is simple to apply and it is used in several different areas. The following conditions have to be satisfied for this process to be valid:

- calling population: the population that can arrive to the system can be considered to be infinitely large.
- arrival process: the arrival process is descried as a Poisson process with λ as a mean rate. The arrival process is independent of the state in which the system is in.
- queue configuration: the queue is a single queue and the length has the possibility to be infinite.
- queue discipline: the queue is considered to be a FIFO queue.
- service mechanism: the process consists of a single server. The service time is exponentially distributed with μ as a mean rate. The server is independent of the number of customers in the system.

The system reaches a steady state if the mean arrival rate, λ, is strictly less than the mean demand rate, μ, otherwise the system will grow to an infinite size. If the system has steady-state characteristics the stationary probability, P_0, in the system can be calculated as follows.

$P_0 = 1 - \rho$ *Eq. 3.25*

where $\rho = \lambda / \mu$.

Any arbitrary selected stationary probability, P_k, in the system can be calculated as follows.

$P_k = P_0 * \rho^k = (1 - \rho) * \rho^k$ *Eq. 3.26*

[134] Olsson, G & Rosen, C (2005) *"Industrial Automation"*
[135] Laguna, M & Marklund, J (2005) *"Business Process Modeling, Simulation and Design"*
[136] Ibid.

4 Data collection of non-reco components and analysis of the findings

This chapter presents the steps required to solve the problem at hand in the order they were performed. First the assumptions concerning non-reco components are stated, and the lead-time for the components is determined. A classification is conducted and a plot is constructed to visualise trends for the demand of components. The next step is to establish the distribution for the demand for certain components. To determine an appropriate distribution, histograms and two goodness of fit tests are used. The following section describes how the fill rates and the associated costs are calculated for a component to enable a comparison between different fill rates. The final section presents a sensitivity analysis.

4.1 Model overview

This model consists only of non-reco components, and the movements of the components are illustrated in Figure 4.1 below.

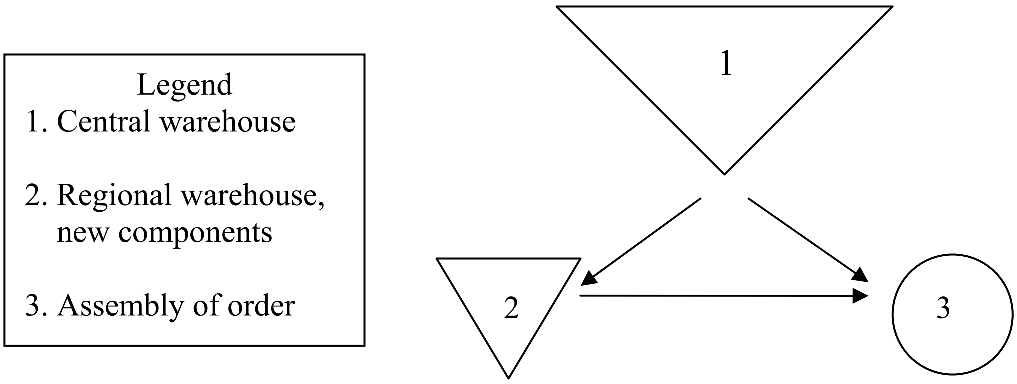


Figure 4.1. The material flow for non-reco components. The regional warehouse covers the shortages at the central warehouse.

4.2 Assumptions

In this thesis several model assumptions will be used. The assumptions for the model concerning the non-reco components will be stated here. They will be discussed more in depth in other parts of the thesis. The assumptions are:

- the model is based on a single echelon system.
- the demand for components is stationary over the year.
- the distribution of the demand for components can be described as a Poisson distribution.

A few more assumptions will be presented later in the thesis concerning non-reco components. These assumptions need a greater understanding of the model before they are discussed.

4.3 Lead-time for the components

To calculate the lead-time for different components, the dates when an order was created and received was used. The data was attained from Jeeves database. In Excel a pivot table was created from the data collected. In this table the time between an order was created and received was calculated. To get the lead-time expressed in number of working days the weekends were subtracted.

To calculate the mean value, the different lead-times for the orders that contained the component during the year 2005 were added and divided by the total amount of orders containing the component during that year. The standard deviation was also calculated. For an example see Table 4.1 below.

Art nr:	Order created	Order received	Number of working days
x	2005-03-08 13:12	2005-03-09 00:00	1
x	2005-03-15 08:01	2005-03-17 00:00	2
x	2005-03-23 11:09	2005-03-31 00:00	6*
x	2005-05-17 12:50	2005-05-18 00:00	1
x	2005-05-19 10:53	2005-05-20 00:00	1
x	2005-06-28 15:24	2005-06-29 00:00	1
x	2005-08-12 13:35	2005-08-16 00:00	2*
x	2005-09-15 15:53	2005-09-20 00:00	3*
x	2005-11-03 14:22	2005-11-04 00:00	1
x	2005-12-09 09:10	2005-12-12 00:00	1*
x	2005-12-09 10:43	2005-12-12 00:00	1*
x	2005-12-14 13:20	2005-12-15 00:00	1
		Average lead-time:	1,75
		Std dev:lead-time	1,484771179

*Table 4.1. Determination of the lead-time. * indicates a situation where the weekends are subtracted from the lead-time.*

When calculating the lead-time, it is important to examine the result because there are many factors that can influence the result in a critical way. Examples of this might be that the schedule for the workers may be tight, resulting in that they do not prioritise goods receiving. This means that the goods have actually arrived to the system and the lead-time for the components increases because delayed reporting even if they are physically available to the system. This can be misleading when the lead-time is verified. It may occur when there are many orders for spare parts to be assembled and there is no components missing. Then assembling the components will be prioritised and the reporting will be delayed. This implies that there is no direct need for the components received at this time. There can also be some faults in the data material that are misleading, e.g. there can be an order that takes abnormally long time. It is important to observe if there are any anomalies, and if there are, investigate why these exists. There were some cases where an order that was created later, was received days before orders created earlier. This delivery pattern is very strange and the only explanation that the authors can think of is that the first order was forgotten and stored somewhere and not found until days later.

4.4 ABC inventory analysis of new components

To enable an efficient and effective way of controlling the inventory, an ABC classification of the components was performed. Only non-reco components were considered in this first categorisation. The objective for the classification is to create homogenous groups. Each group consists of components with similar characteristics and can therefore be controlled the same way. The categorisation ought to depend on matters that affect the service level that is going to be accomplished. These are the demand, the lead-time and the SEK volume (demand times the price) for the components.

4.4.1 Parameters used to attain the necessary data from the business system

Different parameters in the business system (Jeeves) were needed to attain the necessary data and enable a classification of the components. Some of the parameters used to attain the required information were quite obvious. More parameters were added when tests were performed and the data was not satisfactory. The parameters were found in Jeeves and they were used in Query, which is the database that Jeeves is built upon. The parameters assured that only non-reco components were included. The different parameters were used to:

- separate new and reconditioned components.
- sort out components that were not to be included, such as the inventory of new spare parts. This was done by securing that components included had the right storage location.
- avoid components that are bought in large quantities. These are outside the scope of the task for the master thesis according to the supervisor at Tetra Pak. The parameter was used to sort out components that were ordered in quantities of 100 or more.
- return the number of components that had been used during a certain time interval.
- establish that all investigated events took place during the year 2005.
- determine the type of transactions that had been used, since there may be many reasons why a component is removed from inventory.
- establish the price for the component.
- determine when an order was placed and when it was received to be able to calculate the lead-time.
- determine the amount of a specific component that is included in the different spare parts.

With these parameters data was obtained from the database. The data was used to create three tables, one with the frequency, one with the SEK volume and one with the lead-time for the components.

The parameters used to obtain the required information had a few delimitations. The first one was that only components that have a storage location will be included in the material, since the selection was conducted with the names of the storage locations. There are however components that do not have a certain place in the warehouse. These components are quite few and they will not affect the classification to a greater extent. It is impossible to include these, since there is no way of separating the components if the storage locations would not have been used. Another limitation is that only components with an order quantity of less than

100 will be included. The components that have a higher order quantity should be kept at a service level very close to 100 % since the holding cost are low compared to the ordering cost.

The search was performed in a way that only components used during the year 2005 are registered. For example, if only new and no reconditioned components have been used during the year the component will be labelled as if it is only used as new. These components are regarded to be few. One also has to bear in mind that components can be added/subtracted to the classes and change the boundaries over the years. An example might be when new spare parts are included in Maintenance Units portfolio, or if the number of components that are being reconditioned increase or decrease.

When changing business system many things can go wrong, and the data would be unreliable. Because of this no data from the last couple of months with the old business system and no data from the new business system were used.

4.4.2 Creating an artificial component

The spare parts that have the highest turnover are the different Cutting jaws and Pressures jaws. These spare parts represent approximately 50-60 % of the total number of spare parts sold each year[137]. These spare parts are sold and shipped in a pair, one right and one left, except for breakdowns. Some of the components in these spare parts are the same. The result of these two things is that in many cases there is a demand for several, say q, components at the same time. This amount q for the specific component is always the same. Therefore it is possible to create an artificial component where the demand is expressed in units of q instead of units of one.

This will only affect the demand and not affect the SEK volume since the price for the artificial component will increase.

It is natural to use an artificial component when there is a demand for the same amount q of components all the time. This reduces the complexity when performing calculations to determine the demand distribution. Consideration to the fact that many spare parts contain a specific amount of a component is taken when the frequency was calculated. It was harder to determine how often a spare part is sold together with another spare part containing that component. It was proven to be quite time consuming, since it requires manual calculations for each component. After consultations with the authors´ supervisor at Tetra Pak it was considered reasonable to divide the total demand for a component with two, to take this into consideration. This reduction of the demand is an assumption that will affect the model.

4.4.3 Stock outs and discrepancies at the central warehouse

The material gathered from the database represents the entire amount of components that were used during 2005. From May 2006 the central warehouse is delivering the non-reco components just in time for specific orders. There is a need to establish the amount of components that they do not deliver due to stock outs and discrepancies to determine the

[137] Martell, M. 2006-10-10

safety stock of non-reco components. The material concerning this question was gathered from people who work in the logistic and quality area at the central warehouse.

According to the logistic department at Tetra Pak, the central warehouse managed to deliver 11023 of the ordered 15019 components to Maintenance Unit during a certain time interval.[138] The amount of discrepancies for the components from the central warehouse is 0,37 %.[139] This means that on average 0,37 % of the picked order rows from the central warehouse contains faults i.e. wrong components or a wrong amount of components. Therefore on average they fail to deliver $1-(11023/15019)\cdot(1-0,0037)=26,88\%$ of the components. The safety stock at Maintenance Unit has to buffer against these uncertainties. The demand per lead-time for the non-reco components in the safety stock should therefore be 26,88 % of the demand per lead-time when the components were not sent to specific orders. All the data material affected by the frequency is reduced with 73.12 % except the material used to determine the distribution. When calculations are performed to determine the distribution and when searching for trends the total demand is used because otherwise there would not be enough material to conduct the tests. The greater the amount of data used the greater the possibility of finding a distribution that fits the data material. This reduction of the demand is an assumption that will affect the inventory model.

4.4.4 What value to use when calculations are performed

Two different values could be used for the different classes when calculations are performed, either the mean value or the median. There might be some odd numbers that appears due to e.g. a correction of the inventory in the business system, if it did not match the physical inventory level. This was the main reason for choosing the median as the typical value for the different classes. The median values for the demand, SEK volume and lead-time in each group was chosen and used to calculate the fill rates for different classes. The values were chosen with consideration to the boundaries for the different classes.

To determine the cost for a component in a class when different fill rates are calculated, one needs to know how the demand has been decreased when creating the artificial component in Chapter 4.4.2. To do this, the SEK volume and the factor that the demand had been decreased with for the components were arranged in a decreasing order. From this the median value for the factor in different groups is determined. The price is determined the same way from the SEK volume.

4.4.5 ABC diagrams

To construct the diagram for the demand frequency a table was created. In this table the components were organised with decreasing frequencies. The next step was to calculate the percentage of how much of the total frequency a specific component represents. These percentages where then presented in an accumulated form that represents the y-axis in the frequency diagram. The frequency diagram is created by plotting the y-axis against the x-axis which is the percentage of the components that the component represents. The result can be

[138] Siversson, J. 2006-11-06
[139] Nordgren, L. 2006-11-07

seen in Figure 4.2 below. The SEK volume diagram was created the same way and is visualised in Figure 4.3.

The diagram for the lead-time was constructed in a different way compared to the frequency and the SEK volume. The y-axis consisted of a table with the lead-times arranged in a decreasing order. The y-axis was plotted against the x-axis which was constructed the same way as it was done before. The result can be seen in Figure 4.4. From the table containing the lead-times one day was subtracted from each lead-time due to the fact that the components arrive one day before they are to be assembled. This is done because the technicians will be able to assemble the spare part on time even if the components are delayed one day from the central warehouse.

The lead-time was presented differently compared to the frequency and the SEK volume. The frequency and lead-time diagrams were constructed the way they were because it is important to know which components represent 80 % of the accumulated value. It is not important to know which components represent 80 % of the accumulated lead-time.

The greater the number of groups the diagrams are divided into the more complex the situation becomes. The diagrams where divided into three different groups to be able to make them homogenous and still be manageable. There is not an exact figure what the boundaries for the different groups should be. The important thing to bear in mind is that the groups that are created are homogenous. The boundaries were chosen after a discussion with the supervisor at Tetra Pak.

Frequency diagram
The frequency diagram is visualised in the figure below.

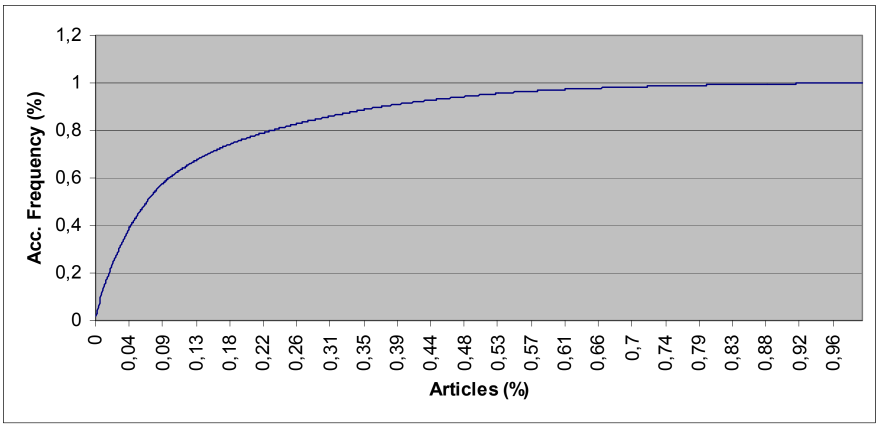


Figure 4.2. ABC diagram for the frequency with consideration to the fact that many components of the same kind are sent simultaneously with different spare parts.

The frequency boundaries were set to 8 % and 25 % of the components. In numbers they were the following:

- A-Component boundary: frequency is > 20 components per year. Representing high frequency components, approximately 8 % of the components.
- B-Component boundaries: frequency is 5 < and ≤ 20 components per year. Representing intermediate frequency components, approximately 17 % of the components.

- C-Component boundary: frequency is ≤ 5 components per year. Representing low frequency components, approximately 75 % of the components.

It is important to bear in mind that when determining in which class a component will end up, one has to decrease the demand frequency for the component with consideration to the factor discussed in Chapter 4.4.2.

The values for the different groups were the following:

- A-component: The median value for the demand frequency is 38 for the artificial component.
- B-component: The median value for the demand frequency is 8 for the artificial component.
- C-component: The median value for the demand frequency is 1 for the artificial component.

SEK volume diagram
The SEK volume diagram is visualised in the figure below.

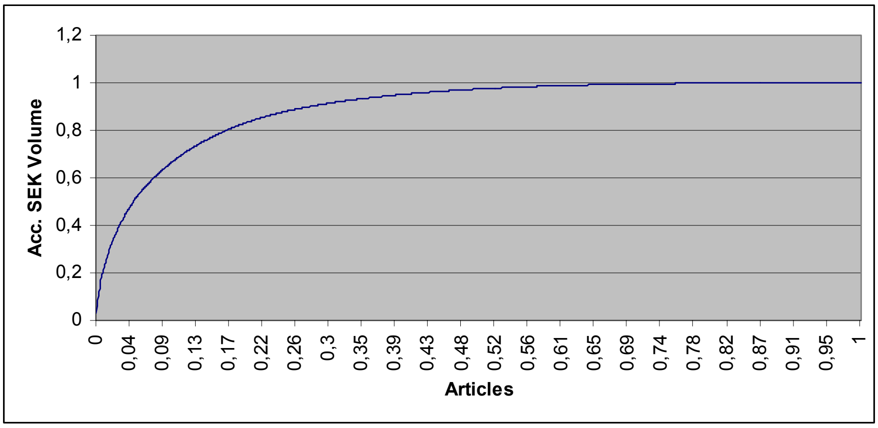


Figure 4.3. ABC diagram with SEK volume as a classification.

The SEK volume boundaries were set to 15 % and 35 % of the components. In numbers they were the following:

- A-Component boundary: > 3080 in SEK volume. Representing high SEK volume components, approximately 15 % of the components.
- B-Component boundaries: 700 < and ≤ 3080 in SEK volume. Representing intermediate SEK volume components, approximately 20 % of the components.
- C-Component boundary: ≤ 700 in SEK volume. Representing low SEK volume components, approximately 65 % of the components.

The values for the different groups were the following:

- A-component: The median value for the SEK volume is 5720.
- B-component: The median value for the SEK volume is 1315.
- C-component: The median value for the SEK volume is 120.

Lead-time diagram
The lead-time diagram is visualised in the figure below.

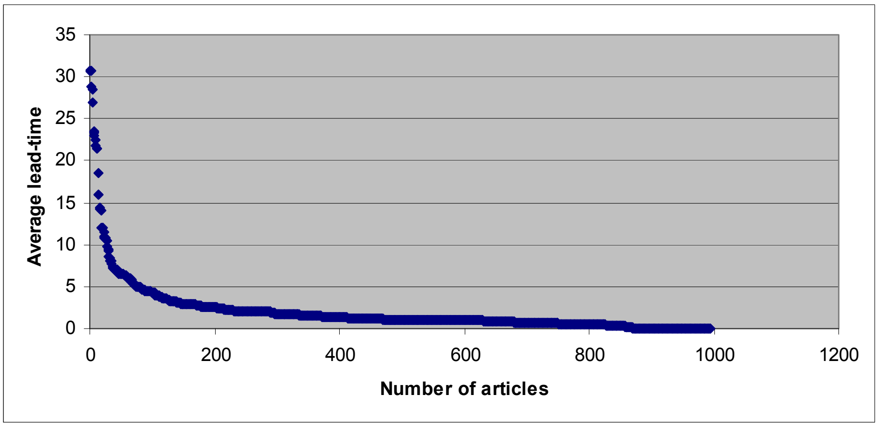


Figure 4.4. The lead-time for the different components plotted in a decreasing order.

The boundaries for the components' lead-times were set to 2 % and 7 % of the components. In numbers they were the following:

- A-Component boundary: lead-time is > 15 days for a component. Representing components with large lead-times, approximately 2 % of the components.
- B-Component boundaries: lead-time is between 5 < and ≤ 15 days for a component. Representing components with intermediate lead-time, approximately 5 % of the components.
- C-Component boundary: lead-time is ≤ 5 days for a component. Representing components with low lead-times, approximately 93 % of the components.

The boundaries for the components' lead-times were chosen differently compared to the other factors. If wider ranges would have been selected the groups would have become less homogenous.

The values for the different groups were the following:

- A-component: The median value for the lead-time is 23.
- B-component: The median value for the lead-time is 7.
- C-component: The median value for the lead-time is 1.

4.4.6 A cube representing the different classes

The boundaries for the SEK volume and the frequency were used to create a matrix. In this matrix each cell represents components with a certain SEK volume and frequency. The boundaries in the matrix are the boundaries in the SEK volume and frequency diagram. By adding a third dimension a cube containing 27 cells was created from the matrix where the third dimension represents the different lead-times, see Figure 4.5.

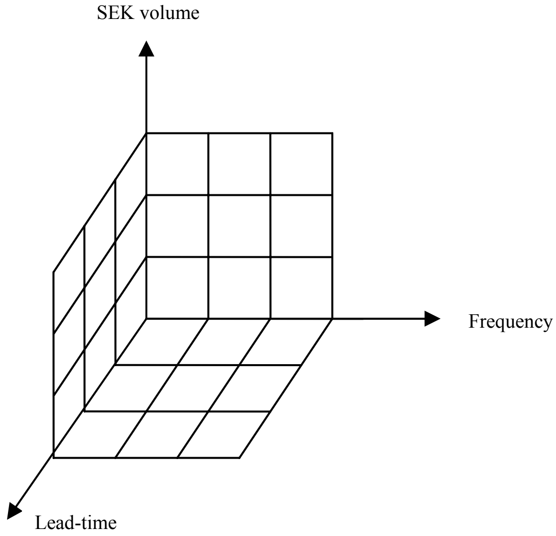


Figure 4.5. A cube in which the factors for the categorisation are represented by the axis and the cells are representing the different classes that a component may end up in.

When examples are made concerning a component belonging to a certain class they will be named with a combination of three letters A, B and C. The A represents components with high levels, B intermediate levels and so forth. The non-reco components will be named with three letters for the three different dimensions. The first letter for example in category ABB represents the frequency, the second the SEK volume and the third the lead-time.

4.5 Pattern of demand for the components

Maintenance Unit changed their business system from Jeeves to SAP R3 in may 2006. They changed the production strategy when they started to use the new business system. Instead of using "make-to-stock" they started to use a "make-to-order" strategy. The authors used Jeeves to attain the data material under the "make-to-order" period. This meant that the dates when the components had been picked for assembly were not the dates when the true demand appeared. This is because the ready made spare parts were kept in inventory with the old production strategy. To determine which spare parts contained a certain component, the bill of material were used. The bill of material is where all components are listed for the given spare part. This approach was applied because it was important to see when the true demand occurred. There was no data available on how the components were sent. Instead the time when the spare parts containing the components were shipped were used. This made it possible to see how the components were shipped by looking at the spare parts´ bill of material. If the data from when the components were collected from the warehouse would have been used, which was the authors' first instinct to do, the dates of the true demand would have been incorrect. The spare parts were in some cases produced in batches with the old production strategy. With the new strategy they will be assembled as the orders enters the system.

This way of looking through the bill of materials to get the dates when the demand occurred was quite time consuming. It will only be used to determine what kind of distribution the demand has or to spot trends in sales. The demand that will be used in the classification is obtained from when the components are collected from the warehouse. The distribution does not matter in this case. This is due to the fact that the correct dates when they are consumed is not important. On average the amount of components that are shipped per year should be approximately the same as the amount removed from the warehouse during a year.

To produce the date and quantity of the true demand the authors needed two lists from the business system. The first list included which spare parts contained the investigated component. The second list created contained all the spare parts that were sold during the year 2005. A comparison between these two was performed to generate a new list. This list contained the dates when a spare part, containing the component, was sold and in what quantities. This was done with nine different components from separate classes. The quantity and the dates were plotted against each other to visualise if there was a trend in the demand pattern, see Appendix 1 and 2.

The demand is obtained in two ways in this master thesis. The first way used the bill of materials and the number of spare parts sold during a year. The second method is to focus on when components are removed from the warehouse. When comparing these, there was sometimes a difference in the quantity. This difference can be explained by the spare parts that were sold during 2005 did not have to be produced within this year. It is also possible that the component was broken during the assembly of the spare part and it had to be replaced by a new.

The demand obtained from the bill of materials for one of the studied components is plotted in Figure 4.6. The example shows that the demand for this component is constant over time since the dots that represent the demand are spread even over the year. This was the case for all of the nine components that were investigated. The diagram is a quick and easy way to visualise if the demand of a component has increased since the dots will be closer together. In the normal case the component is demanded two at a time since many spare parts are sold in pairs. In some cases a spare part is sold by itself. This might be the case for example if a breakdown occurs at the diary. Another example is that the dairy disregards when to perform service on the machines and changes only one spare part when there is a need for it. When the demand is greater or lower than an integer times two, they must either be disregarded or raised/lowered to a higher/lower level. Since the normal case is dominating this action is appropriate. The authors chose to always raise the demand a higher level to increase the margins against stock outs. It is an advantage to have a rule to deal with these situations the same way every time they appear. These cases will always exist so it is vital to deal with them.

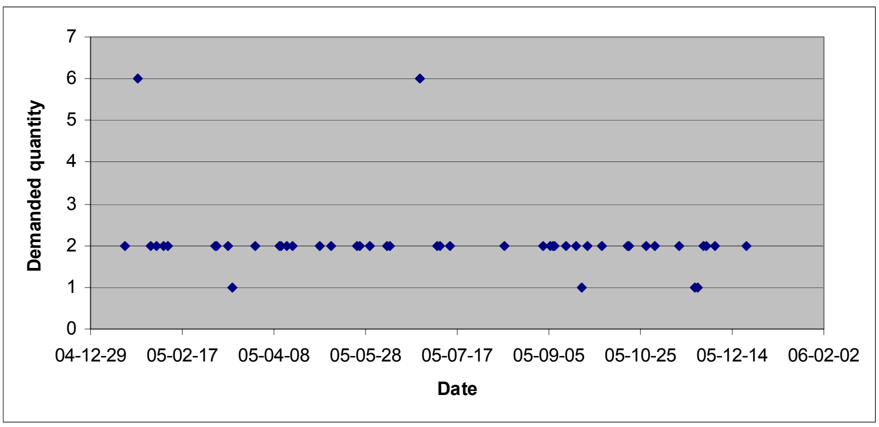


Figure 4.6. The diagram visualises the demand for a component over a year.

A good way to visualise trends in another way compared to Figure 4.6 is to plot the days between two orders, see Figure 4.7. If the orders arrive at a constant time interval this diagram will be a straight line. This is not expected to happen since the orders arrive randomly. The diagram can be used with some sort of control limit that gives a signal if a certain amount of orders after each other are beyond the limits. If this is the case there is a good reason to investigate it further and establish if the demand has changed for a reason. The diagram shows that there are some differences in time between two orders but no trends can be spotted.

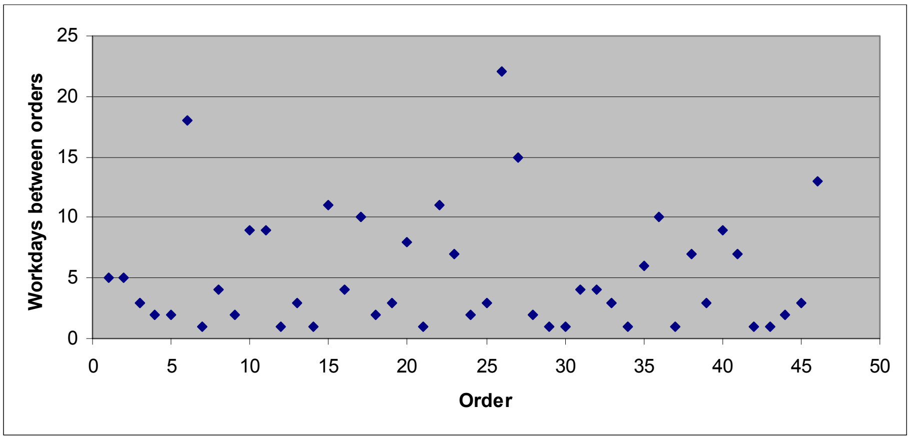


Figure 4.7. The diagram visualises the number of days between two orders.

4.6 Determination of the demand distribution

The authors´ hypothesis is that the demand has a Poisson distribution. This will be tested by creating histograms and performing Chi-Square tests and Kolmogorov-Smirnov tests. In the case that each order represents a demand of q components or an integer times q the demand for the components is divided by q. Then every time there is a demand for q components this represents one artificial component.

To attain the dates when different spare parts were sent a code in Matlab was written. The code computed the number of times an order containing a specific component was placed during a lead-time. The values that had been generated from the code were used to draw the histogram for the events that had taken place.

4.6.1 Visualising the histograms

The first step to determine the distribution for the demand was to compare two histograms. One with the observed demand during the lead-time and one with the expected demand during the lead-time. As mentioned earlier the Poisson distribution was used to calculate the expected values.

The procedure described in Chapter 3.11 was performed. The results are visualised in Table 4.2 below. To determine if there are differences for the different classes, components from different classes were tested.

Founded on observed demand:

Component: AAA			Component: ACA		
Days / year	365		Days / year	365	
Number of weekends	104		Number of weekends	104	
Number of working days	261	Factor, f	Number of working days	261	Factor, f
Number of intervals	261	2	Number of intervals	261	2
Value	Observed value	Value after the factor	Value	Observed value	Value after the factor
0	196	196	0	147	147
1	3	51	1	1	83
2	48	11	2	82	21
3	1	3	3	0	10
4	10	0	4	21	0
5	1	0	5	0	0
6	2	0	6	10	0
7	0	0	7	0	0
8	0	0	8	0	0
9	0	0	9	0	0
10	0	0	10	0	0
Tot. demand	159	82	Tot. demand	309	155

Founded on expected demand:

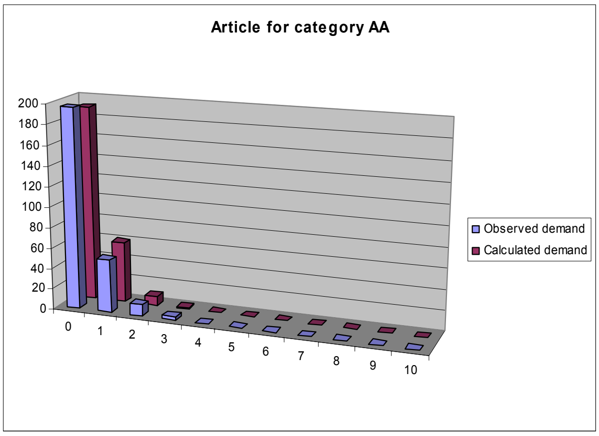
Calculated	Average intensity Probability	0,318008 Value	Sort	Calculated	Average intensity Probability	0,59387 Value	Sort
0	0,727597	189,9029	191	0	0,552186	144,1206	144
1	0,231381	60,39057	60	1	0,327927	85,58888	86
2	0,036791	9,602332	9	2	0,097373	25,41432	25
3	0,0039	1,017872	1	3	0,019276	5,030932	5
4	0,00031	0,080923	0	4	0,002862	0,74693	1
5	1,97E-05	0,005147	0	5	0,00034	0,088716	0
6	1,05E-06	0,000273	0	6	3,36E-05	0,008781	0
7	4,75E-08	1,24E-05	0	7	2,85E-06	0,000745	0
8	1,89E-09	4,93E-07	0	8	2,12E-07	5,53E-05	0
9	6,67E-11	1,74E-08	0	9	1,4E-08	3,65E-06	0
10	2,12E-12	5,54E-10	0	10	8,3E-10	2,17E-07	
		Tot. number of intervals	261			Tot. number of intervals	261
		Tot. demand	81			Tot. demand	155

The comparison

Value\Component	Observed demand	Calculated demand	Value\Component	Observed demand	Calculated demand
0	196	191	0	147	144
1	51	60	1	83	86
2	11	9	2	21	25
3	3	1	3	10	5
4	0	0	4	0	1
5	0	0	5	0	0
6	0	0	6	0	0
7	0	0	7	0	0
8	0	0	8	0	0
Tot. demand	82	81	Tot. demand	155	155

Table 4.2. This table visualises the procedure to produce a comparison between an observed demand and a calculated demand.

When the calculation is made it is possible to draw the histogram for the calculated demand and compare it to the histogram of the real demand. This is done in Figure 4.8 below.



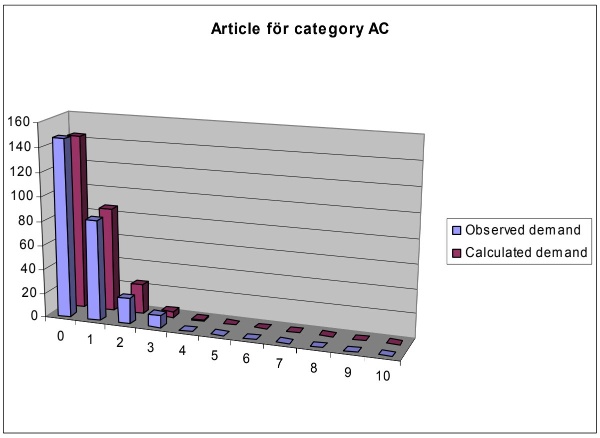


Figure 4.8. The figures show the comparison between the observed and the calculated theoretical demand.

This way of guessing the distribution was used because it is a quick way to determine if there is a possibility that the sampled data can be fitted to a certain distribution or not. It is easy to produce and visualise the histograms and compare them to each other. The disadvantage of the method is that it is not a reliable way of determining a certain distribution. It is hard to know if the demand belongs to a certain distribution or not. Visualising the histograms can therefore be a first step to determine the distribution and to achieve a satisfactory result. This method needs to be combined with statistical methods to determine if the distribution has a good fit with the data. The histograms of the different classes are available in Appendix 3.

4.6.2 Chi-Square test

The Chi-Square test is a method that was used in this thesis because of its accuracy to reject/assume a distribution for a given data material. The test requires few data values in a wide range to achieve satisfactory results.

To be able to perform a Chi-Square test it is necessary to create two histograms. The histograms created in the previous chapter will be reused here. When the histograms have been created the procedure described in Chapter 3.12.1 was performed. The results from the Chi-Square tests are presented in Table 4.3.

Component AAA:

Observed value	Expected value	(Diff^2)/expected value	Sort
196	191	0,130890052	0,1308901
52	60	1,066666667	1,0666667
11	9	0,444444444	0,4444444
3	1	4	4
0	0	#DIV/0!	0
0	0	#DIV/0!	0
0	0	#DIV/0!	0
0	0	#DIV/0!	0
0	0	#DIV/0!	0
0	0	#DIV/0!	0
0	0	#DIV/0!	0
			χ^2:
			5,6420012

Component ACA:

Observed value	Expected value	(Diff^2)/expected value	Sort
147	144	0,0625	0,0625
83	86	0,104651163	0,1046512
21	25	0,64	0,64
10	5	5	5
0	1	1	1
0	0	#DIV/0!	0
0	0	#DIV/0!	0
0	0	#DIV/0!	0
0	0	#DIV/0!	0
0	0	#DIV/0!	0
0	0	#DIV/0!	0
			χ^2:
			6,8071512

Table 4.3. The table shows the calculations of Chi-square tests for two components.

This result is compared to a Chi-Square distribution table, see Chapter 3.12.1.

It was vital to assure the right factor was used to decrease the demand for the components to achieve a satisfactory result. If the factor was incorrect the Chi-Square test was with high probability going to reject the distribution even if it was the right one. The difference between the plots with different factors can be seen in Figure 4.9 below.

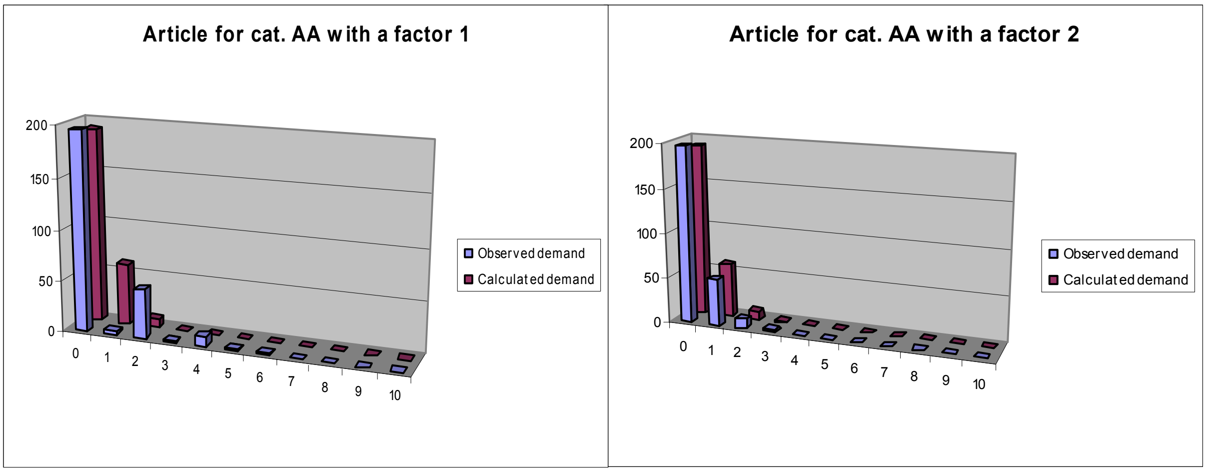


Figure 4.9. The difference between the figures is that the factor for the purple histogram is 1 respectively 2.

The result when applying the test on the data material was mixed. In some classes, those who had a wide range and many values, the results were satisfactory and the hypothesis of Poisson demand could not be rejected. With the histograms that only contained few values in the intervals the result was poorer. The findings are illustrated in Appendix 4. To investigate if the demand distribution is a Poission distribution a Kolmogorov-Smirnov test was used to assure that the right distribution was applied.

4.6.3 Kolmogorov-Smirnov test

The authors investigated the difference in arrival time between two orders and determined if this time can be described as an Exponential distribution. If the time between two orders can be described as an Exponential distribution the data has a Poisson distribution.

The first step was to calculate the time between the orders. After that the cumulative distribution function values for the theoretical distribution were calculated as follows:

$$\hat{F}(x_i) = 1 - e^{-x_i / average\_x} \qquad \textit{Eq. 4.1}$$

where x is the time between two orders and the average_x is the average time between two orders. After this step the other stages were preformed according to Chapter 3.12.2. An example is illustrated in Appendix 5.

The result when applying the Kolmogorov-Smirnov test was mixed on the different classifications, see Table 4.4.

Classification	dof	D+	D-	Value	Table value
AA	78	0,1923	0,0852	**0,1923**	**0,1846**
AB	39	0,1282	0,0994	**0,1282**	**0,261**
AC	32	0,0898	0,1772	**0,1772**	**0,285**
BA	63	0,0989	0,1689	**0,1689**	**0,2054**
BB	14	0,0989	0,1994	**0,1994**	**0,418**
BC	7	0,2383	0,35	**0,35**	**0,577**
CA	11	0,1184	0,1948	**0,1948**	**0,468**
CB	3	0,175	0,3241	**0,3241**	**0,828**
CC	17	0,2754	0,2754	**0,2754**	**0,381**

Table 4.4. The result for the different categories when applying the Kolmogorov-Smirnov Test.

Table 4.4 shows that there is one value that exceeded its table value and that is class AA. The other classes can be assumed to have the applied distribution. The disadvantage with this test is that it rejects the distribution more often than desired. This together with that the value was very close to the table value motivates to use a Poisson distribution for class AA.

The conclusion was drawn on the basis of the combination of the two tests. Using the Kolmogorov-Smirnov test and the Chi-Square test the hypothesis of Poisson demand could not be rejected for the components in the different classes.

4.7 Deciding the fill rate for the different classes

The fill rate is vital when it comes to deciding the inventory level for the components. It is used in the final step when calculating how high the inventory levels shall be. In this final step it is compared to the inventory cost.

The first step towards calculating the fill rate is to decide the average intensity, a_i.

a_i = demand per year / (number workdays per year / lead-time)

The probability for a certain demand was calculated by using an average demand intensity for a Poisson distribution, see the formula below.

$P(n) = Po(n, a_i)$, where n is the probability for a demand and a_i is the average intensity.

The different demand levels were then used to calculate the different probabilities. For a given value of the inventory position the probabilities can be used to calculate the fill rate. The fill rate is the total probabilities of the different inventory levels which are greater than zero see the derivation and Eq. 4.2 below.

$$S_2 = P(IL > 0) = P(S - D(t, t+L) > 0) = P(S > D(t, t+L)) = P(D(t, t+L) < S) = \\ = \sum_{i=0}^{S-1} P(D(t, t+L) = i) \qquad \text{Eq. 4.2}$$

where IL = inventory level
S = order up to level
D(t,t+l) = demand per lead-time

When the fill rate is calculated, it is compared to the inventory cost. The inventory cost is calculated by multiplying the probability for a given inventory level with the inventory level and summarised. The summation consists of all the terms that have positive inventory levels and is the average inventory level. This sum is then multiplied with the price, the factor that describes the number of components that are a part of the artificial component and the investment rate. The result of this multiplication is the expected inventory cost, see Eq. 4.3 below.

$$IC = E[IL] \cdot p \cdot f \cdot ir \qquad \textit{Eq. 4.3}$$

where IC = inventory cost, IL = inventory level, p = price, f = factor and ir = investment rate.

The investment rate, ir, at Tetra Pak is 7.5 %[140] and it is multiplied with the price. When the inventory cost is calculated it is possible to compare it with the fill rate. The comparison is used as a basis when making a decision on which level the inventory position shall be. An example of how much the fill rate changes relative the inventory cost, when changing the order up to level (inventory position) is illustrated in Table 4.5 below.

Example	Average intensity 0,401521287						
	Order up to level, S:	1	2	3	4	5	6
Demand, D(t,t+L)	Probability, P(D(t,t+L))						
0	0,669301072	1	2	3	4	5	6
1	0,268738628	0	1	2	3	4	5
2	0,05395214	-1	0	1	2	3	4
3	0,007220978	-2	-1	0	1	2	3
4	0,000724844	-3	-2	-1	0	1	2
5	5,82081E-05	-4	-3	-2	-1	0	1
6	3,8953E-06	-5	-4	-3	-2	-1	0
7	2,23435E-07	-6	-5	-4	-3	-2	-1
8	1,12142E-08	-7	-6	-5	-4	-3	-2
9	5,00306E-10	-8	-7	-6	-5	-4	-3
	Fill rate:	0,669301	0,93804	0,99199	0,99921	0,99994	0,999996
Price, p							
55	Tied up capital	147,2	353,6	572	791,6	1011,6	1231,6
Factor, f = 4	**Inventory cost, IC**	**11,0**	**26,5**	**42,9**	**59,4**	**75,9**	**92,4**

Table 4.5: The table illustrates an example of how the fill rate changes relative to the inventory cost.

Table 4.6 below shows the fill rate and the total inventory cost for different order up to levels, given a component for class AAA. This comparison is vital when it comes to making a decision on which level the inventory position shall be. If the total inventory cost is low, it is better to have high value for the fill rate rather than a low fill rate which can lead to a shortage

[140] Wågnert, C. 2006-11-13

cost. If the price on the component is high i.e. high total inventory cost and the component has a tiny demand/lead-time it may be better to take the risk of a shortage cost than to have many of this specific component in inventory. It is up to the decision-maker to make that choice. Table 4.6 will guide the person to the appropriate levels.

Example in cat. AAA							
Order up to level, k:	1	2	3	4	5	6	7
Fill rate:	0,6693	0,9380	0,9919	0,9992	0,99994	0,999996	1
Tot. inventory cost:	11,0	26,5	42,9	59,4	75,9	92,4	108,9

Table 4.6: The table illustrates an example of how much the total inventory cost and the fill rate will increase for a component when the inventory position increases.

It is important to notice that the fill rate increases a great deal per step when the inventory position is low. When the fill rate is getting closer to one the marginal utility becomes smaller per step. The inventory cost rises with a high rate in the beginning as well but not to the same extent. So when the fill rate is approaching the value one it becomes more expensive relative the increase in fill rate. A decision has to be made regarding what the inventory position should be making sure the inventory cost is optimised relative the shortage cost.

4.8 Sensitivity analysis of the demand

So far the main focus has been on the fill rates and the costs associated with these for different components. This is because of Maintenance Unit have to be able to deliver the spare part with a probability of 95 %.[141] To attain the fill rate for the spare part, the fill rates for the components in the spare part are multiplied. The assumption for this to be valid is that the fill rates for the components are independent. Due to the multiplication the fill rates for the components have to be substantially higher than 95 %.

It is important to know how the fill rate for the spare part is affected if e.g. the demand for the components in a spare part are unexpectedly high. This sensitivity analysis of the demand will visualise how the fill rate for the spare part is affected if the demand for the components are higher than expected.

Two fictitious spare parts were used in the sensitivity analysis. One spare part consisted of 5 components in each of the classes AAB, BBB and CCB. The second spare part consisted of 20 components in each of the classes AAB, BBB and CCB. Appropriate order up to levels were chosen for the different classes to give a suitable fill rate for the spare part with the expected demand. When the fill rate and the cost for the spare part had been calculated the median demand for the components in class A (demand) was altered from 38 to 80. The components with medium respectively low demand were altered from 8 to 19 and 1 to 4. When the demand had been altered, a new fill rate and cost for the spare parts were calculated.

The different outcomes when the higher values and the median values for the components were used could be compared. It is vital to consider that not all components in a spare part have the higher value that is used when calculations were performed. This example becomes a

[141] Martell, M. 2006-12-01

worst case scenario. It is also vital to bear in mind that not all components have the median value that is used in the authors´ model. The true values for the components are a mixture of different values, high and low. Therefore it is important to perform a sensitivity analysis and determine what the outcome can be.

	Different fill rate, same cost
5 artnbr. From AAB, BBB, CCB st. values	0,996341733
5 artnbr. From AAB, BBB, CCB high demand	0,883681112
20 artnbr. From AAB, BBB, CCB st. values	0,985447033
20 artnbr. From AAB, BBB, CCB high demand	0,609792797
	Same fill rate, different cost
5 artnbr. From AAB, BBB, CCB st. values	2859
5 artnbr. From AAB, BBB, CCB high demand	4272
20 artnbr. From AAB, BBB, CCB st. values	11436
20 artnbr. From AAB, BBB, CCB high demand	17088

Table 4.7: The table displays the changes in the fill rate and cost if the demand changes for the components.

The approach in the sensitivity analysis is that the median value is used to determine the initial cost and the initial fill rate for the spare part. Either the fill rate or the cost for a spare part is kept at a constant level when the demand is altered for the components. The first part in Table 4.7 illustrates the changes in the fill rates and the second the changes in the cost. The result is that the fill rate for a spare part is more sensitive for changes in the demand if it consists of many components. Therefore it is vital to keep the fill rates at a higher level when the spare part consists of many components.

5 Data collection of reco components and analysis of the findings

The chapter presents the assumptions made concerning reco components. A classification is created and the distribution for the reconditioned components is determined. The chapter ends with an example that illustrates how the inventory level can be optimised for a component using a (S-1,S) system.

5.1 A description of the model for reco components

The model for reco components is described in Figure 5.1 below. Maintenance Unit handles both new and returned spare parts. An important part of the workshop is where they disassemble the returned spare part and store the reconditioned components. Since the model consists of returned spare parts, it is important to take the reusability frequency for the components and how often the spare parts are returned into consideration. These issues are vital for the model, to be able to describe the reality as good as possible. The dashed lines contain the warehouse for the reco-components.

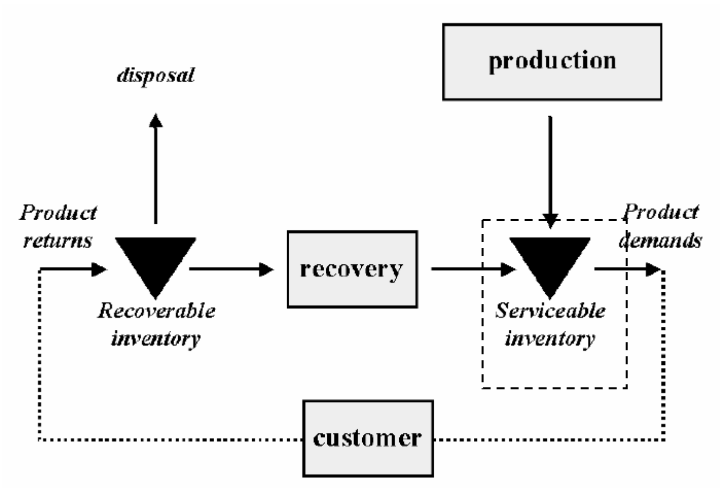


Figure 5.1. The figure describes the spare parts movements for the workshop.[142]

5.2 Assumptions

The model assumptions made for the reco components is stated below. They will be discussed more in depth in chapters to come. In a (S-1,S) system the authors´ assume that:

- the model is based on a single echelon system.
- the demand for components is constant over the year.
- the distribution of the demand for components can be described as a Poisson distribution.
- an artificial component can be used to simplify the calculations. This can be done because the components are always demanded in the same quantity q.
- the return rates for incoming spare parts are constant.

[142] Teunter, R & van der Laan, E. (2003) *”Valuation of inventories in systems with product recovery”*

- a Poisson distribution can be used for the incoming spare parts (even though some markets consolidate the spare parts before they are returned).
- The lead-time for reconditioning and ordering new components are equal.
- 70 % of the sent spare parts are returned to Maintenance Unit.
- the reconditioned components do not wear any inventory holding costs according to Tetra Pak.
- the lead-time to purchase new components is equal to the time that the returned spare parts spend on the shelves at Maintenance Unit until they are disassembled and stored.
- new reco components are only used when there are no reconditioned components available when the spare parts are assembled.
- the movements of the components are considered to be a Markov chain with single server i.e. a M/M/1 queue, to attain different stationary positions.

When generalising the model to a (R,Q) system an additional assumption will be made.

- When calculating the inventory cost some parts of the flow of the components are considered to be only new components, even though they are a mixture of new and reconditioned components.

5.3 ABC inventory analysis of reco components

To enable an efficient and effective way of controlling the inventory, an ABC classification of the reco components was performed. Apart from the factors that were used when the non-reco components were classified; frequency, SEK volume and lead-time, an additional factor, the reusable frequency of the reconditioned components, was used. The reusability frequency affects the inventory level in a direct way. If a component can be reused several times, the demand for new components decrease. The discussion presented in Chapter 4.4.4 about which values to use when performing calculations is valid for this classification as well. The parameters that were used to derive the data and their limitations were the same that were presented in Chapter 4.4.1.

5.3.1 ABC diagrams

Tables that contained similar information as in the classification for non-reco components were created. They were used to produce the ABC diagrams for the frequency, the SEK volume, the lead-time and reusability frequency for reco components.

The frequency and lead-time tables
The frequency table, consists of the total amount of new and reconditioned components that have been used during the year 2005. There is still a need to reduce the demand for components by converting a number of old components into new artificial components, according to Chapter 4.4.2. The table for the lead-time was constructed same way as it was in the classification for non-reco components.

The SEK volume table
All reconditioned components have a cost but it is hard to determine. There is no method to determine the cost at the present time. One approach could be to decide the amount of time that is spent on one reconditioned component i.e. disassembling, washing, examine and

reporting the component to the system. This figure should be multiplied with the labour cost for a member of the staff. This would result in that each reconditioned component will have the same cost, which gives an awry picture of the true cost. There is another disadvantage with costs for reconditioned components. If a certain service level is achieved additional components that are stored will increase the storage cost but not the service level in a noticeable way. This may lead to a situation where components that can be reconditioned are thrown away, because it would be too expensive to store them. This is not the case in reality, since they strive for reusing as much as possible. These are the motivations why only new components used during the year 2005 are included in the SEK volume table and no consideration is taken to the amount of reconditioned components. The table itself was constructed the same way as in the classification for non-reco components.

The reusability frequency table

There were two alternative approaches to classify the reusability. The first alternative is to base the classification on the amount of reconditioned components stored during a year. The second alternative is to use the number of times a component can be reused together with the demand for the component. The technicians had constructed a list for the reusability frequency for different components. They gave each component a number from 1-6, Number 1 represents components that are only reused once. Number 2 represents components that could be reused twice in average and so forth. Number six was given to components that could be reused 6 times or more.

There is a clear connection between the components that are sent and returned. If there is a high demand for a component there will be a high return rate as well. This is because if the number of spare parts sent are increased, the number of returned spare parts will increase as well. The reusability frequency describes the number of times a component can be reused. The reusability frequency only has six different options available and therefore it is divided into two groups and not three like the other parameters are classified into. If it is combined with the demand during a year and the percentage of the spare parts that are returned, they together illustrate the amount of reconditioned components that are stored during that year. If using the amount of reconditioned components that are stored, it would have been classified into three different groups. The demand has to be used since it is essential to know the information it provides. If the reusability frequency is used instead of the amount of reconditioned components that are stored the number of groups and the complexity is reduced. This was the reason for creating a table with the reusability frequency which was used to create a diagram.

Number six in the reusability frequency represents components that can be reused six times or more. This causes some problems when determining the median value for group number six. There is no way of gathering information about how many times these components can be reused, except from asking the technicians to do a more thorough investigation. This is not an option due to the time restraint that they have. Therefore the authors assume that all components that have the reusability frequency 6 can be reused 6 times. By making this assumption there is a risk of rising the inventory levels slightly more than necessary but the risk of having too low inventory levels is reduced.

Frequency diagram
With help from the ABC diagrams the boundaries were set to create homogenous categories. The frequency diagram is visualised in the figure below.

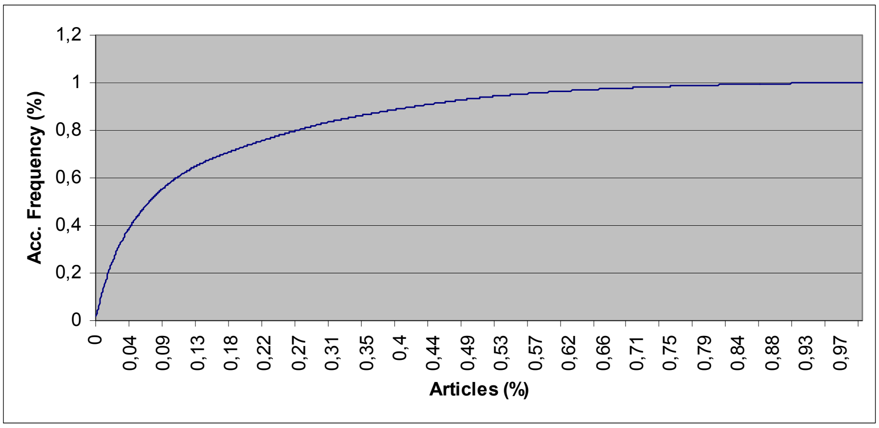


Figure 5.2. ABC frequency diagram for reco components with consideration taken to the factor that decreases the frequency.

The frequency boundaries were set to 15 % and 35 % of the components. In numbers the boundaries were the following:

- A-Component boundary: frequency is > 22 components per year. Representing components with high frequency, approximately 15 % of the components.
- B-Component boundaries: frequency is between 9 < and 22 ≤ components per year. Representing components with intermediate frequency, approximately 20 % of the components.
- C-Component boundary: frequency is ≤ 9 components per year. Representing components with low frequency, approximately 65 % of the components.

When using the frequency diagram and its table it is important to bear in mind that the component's frequency is reduced. It should be reduced with the factor that depends on the amount of specific components that are a piece of the spare parts and the number of spare parts that are sold together.

The values for the groups were the following:

- A-component: The median value for the demand frequency is 52 for the artificial component.
- B-component: The median value for the demand frequency is 15 for the artificial component.
- C-component: The median value for the demand frequency is 3 for the artificial component.

SEK volume diagram
The SEK volume diagram is visualised in the figure below.

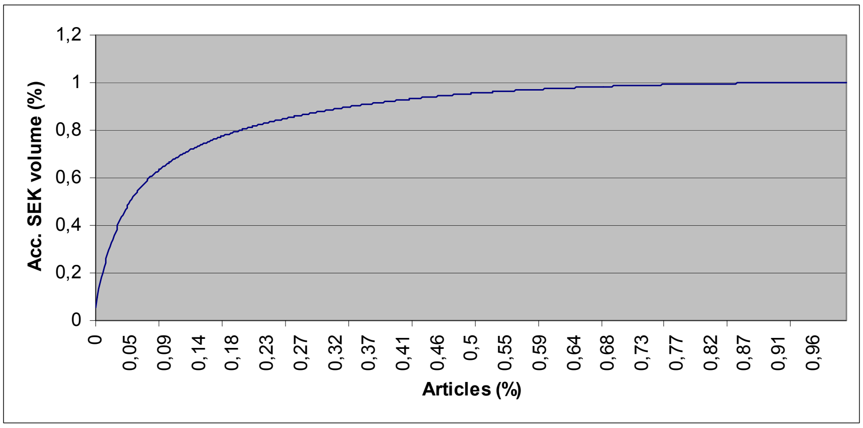


Figure 5.3. The diagram shows the ABC curve for the SEK volume with reco components, where the frequency is represented by the amount of new components that have been used.

The SEK volume boundaries were set to 15 % and 35 % of the components. In numbers the boundaries were the following:

- A-Component boundary: > 8500 in SEK volume. Representing components with high SEK volume, approximately 15 % of the components.
- B-Component boundaries: between 2850 < and ≤ 8500 in SEK volume. Representing components with intermediate SEK volume, approximately 20 % of the components.
- C-Component boundary: ≤ 2850 in SEK volume. Representing components with low SEK volume, approximately 65 % of the components.

The values for the groups were the following:

- A-component: The median value for the SEK volume is 18260.
- B-component: The median value for the SEK volume is 4600.
- C-component: The median value for the SEK volume is 740.

The boundaries were chosen after a discussion with the supervisor at Tetra Pak.

Lead-time diagram
The lead-time diagram is visualised in the figure below.

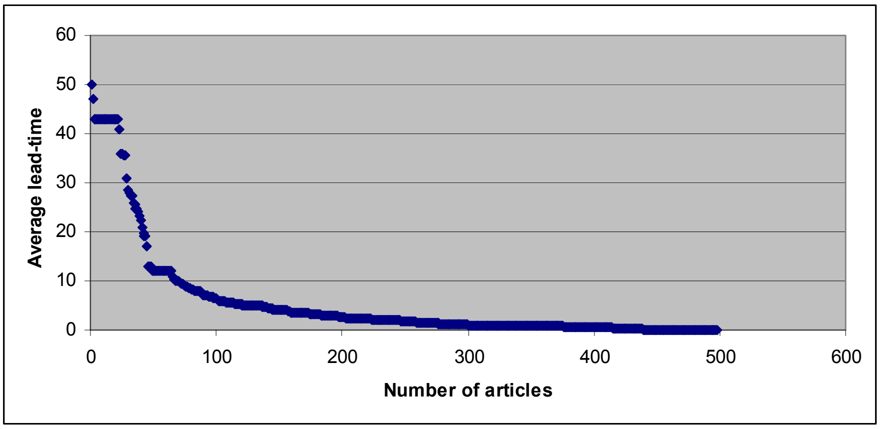


Figure 5.4. This diagram illustrates the lead-time and are organised with a decreasing lead-time.

The lead-time boundaries were set to 9 % and 25 % of the components. In numbers the boundaries were the following:

- A-Component boundary: lead-time is > 15 days for a component. Representing components with high lead-time, approximately 9 % of the components.
- B-Component boundaries: lead-time is between 5 < and ≤ 15 days for a component. Representing components with intermediate lead-time, approximately 16 % of the components.
- C-Component boundary: lead-time is ≤ 5 days for a component. Representing components with low lead-time, approximately 75 % of the components.

The values for the groups were the following:

- A-component: The median value for the lead-time is 41.
- B-component: The median value for the lead-time is 8.
- C-component: The median value for the lead-time is 1.

There are a few components that have longer lead-time than 15 days, but quite many of them have substantially longer lead-time. It is considered wise to control these components manually. Maintenance Unit are continuously working with the components that have a long lead-time. In the future they will hopefully have decreased these lead-times considerably. When an order is received, the technicians at Maintenance Unit have approximately 15-20 days to build the spare part. This means that the median lead-time for the components in the A group is not acceptable.

Reusability frequency diagram
The reusability frequency diagram is visualised in the figure below.

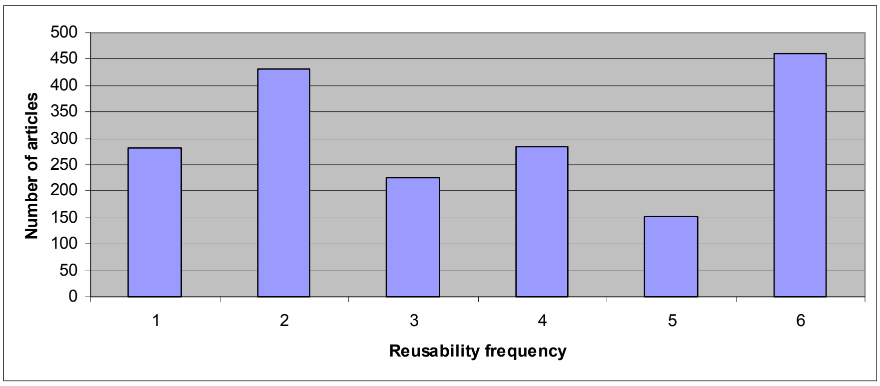


Figure 5.5. The diagram illustrates the number of components that can be reused 1 to 6 times. Column number 6 contains components that can be reused 6 times or more.

The reusability frequency boundary was set to 50 % of the components. In numbers the boundaries were the following:

- A-Component boundaries: between 3 < and ≤ 6 times in reusability frequency. Representing components with high reusability frequency, approximately 50 % of the components.
- B-Component boundary: ≤ 3 times in reusability frequency. Representing components with low reusability frequency, approximately 50 % of the components.

The values for the groups were the following:

- A-component: The value for the reusability frequency is 5.
- B-component: The value for the reusability frequency is 2.

The average and not the median values were used when calculations were performed. In this case there are no extreme values since the components can only be reused 1-6 times. With this in mind it is better to use the mean value, since the motive for using the median value was that there could be extreme values which would affect the mean value in a non desirable way.

5.3.2 A cube representing the different classes

In accordance to Chapter 4.4.6 the frequency and the SEK volume was used to create a matrix. This matrix was developed into a cube when adding the third dimension, the lead-time. The last step was to create an additional cube, where the first cube represents components with a low reusability frequency. The second cube represents components that have a high reusability frequency. The result is two cubes with 18 cells in each cube. Each cell represents components with similar frequency, SEK volume, lead-time and reusability frequency, see Figure 5.6.

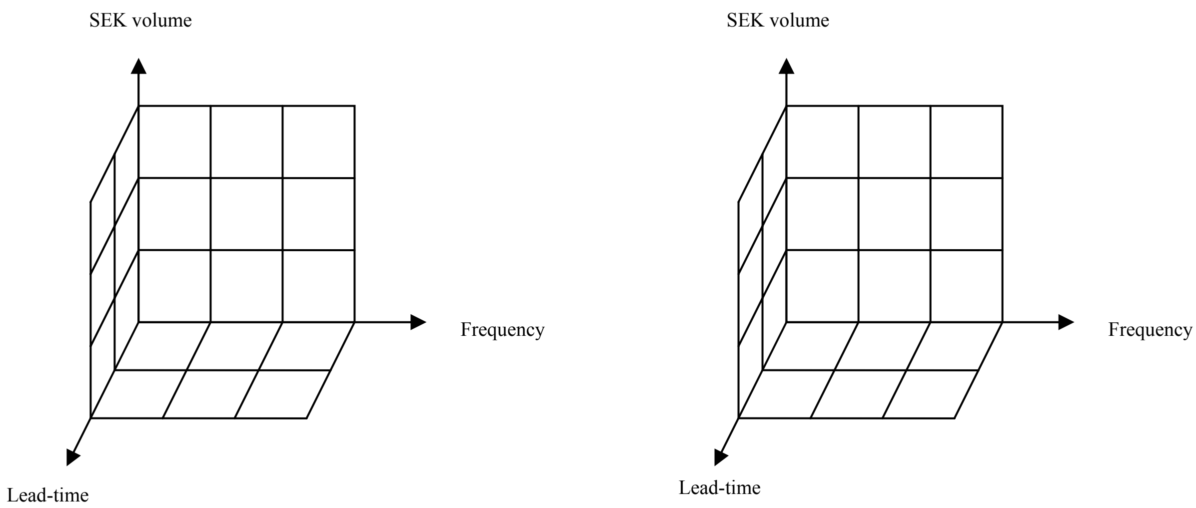


Figure 5.6. Two cubes where each cell represents a class of components with different values for the factors.

5.4 A given distribution for the demand

The distribution for the frequency concerning non-reco components was determined to be a Poisson distribution. The distribution was established by investigating how the spare parts were shipped. The reco components are a part of the same spare parts as the non-reco components and hence the distribution should be identical. The quantity of a given component in a spare part has no significance in this matter since it is only a factor that does not influence the distribution. The distribution for the frequency in this part is determined to be a Poisson distribution due to the facts presented in Chapter 4.6.

5.5 Determine the distribution of incoming spare parts

An important part in inventory control is to determine the appropriate distribution for the demand and the frequency of incoming components. In this chapter the distribution for incoming components is to be established. The first step is to determine the pattern of the incoming spare parts.

5.5.1 The behaviour for incoming spare parts

The pattern for the received spare parts is a bit different compared to the pattern for the spare parts that are sent, especially from markets like Spain and Russia[143]. The market companies in these countries consolidate the spare parts before they are returned. These markets represent approximately 30% of the total market share for the workshop in Lund. Therefore they have a large impact on the structure of the incoming spare parts. It is important to know how the market companies return the spare parts to the workshop, due to the fact that an even flow is desirable. To determine the fact that these markets are consolidating the spare parts before they are returned the authors have compared the average number of spare parts/shipment that were sent and returned by the workshop from these markets, see Table 5.1.

	Spain	Russia
Number of spare parts sent	965	320
Number of shipment / year	137	67
Number of spare parts / shipment	**7,0**	**4,8**
Number of returned spare parts	719	191
Number of shipment / year	53	11
Number of spare parts / shipment	**13,6**	**17,4**

Table 5.1: A comparison between the average number of spare parts/shipment that were sent and returned to the workshop from two markets.

Table 5.1 illustrates that the received quantity per shipment was twice the amount that were sent from Spain. The received quantities per shipment were three times larger for Russia compared to what they sent. The conclusion is that these markets consolidate the spare parts. In the future the workshop strives after a behaviour where the marketing companies return their spare parts directly without consolidating them into larger quantities. In the future it may be possible to steer the activities in a certain direction and reward these activities with bonuses[144]. The authors find it more important that the model will work in the future compared to these circumstances which are temporary. This leads to the conclusion that the compiling behaviour when the spare parts are returned is ignored.

[143] Hansson, S. 2006-10-10
[144] Persson, U. 2006-10-27

5.5.2 Establishing the Poisson distribution for incoming components

Spain and Russia are large markets for the workshop in Lund and they consolidate the spare parts before they are returned. Therefore it is a bit difficult to predict a Poisson distribution for the spare parts. To establish that the right distribution is determined the authors included data material from the workshop in Germany. They do not have the problem with large markets that consolidates the spare parts in larger quantities.

The components that were chosen to establish the distribution had different characteristics such as different reusability frequencies and demand. The process for determining the distribution was the same as when the distribution for the demand was determined for non-reco components, see Chapter 4.6.

The data material was sampled and different histograms were constructed. It was hard to determine the distribution only by eyeballing the histograms, but it seemed like it could be a Poisson distribution. After this step a Chi-Square test was conducted, and this test rejected one out of six components, see Appendix 9. The next step was to make a Kolmogorov-Smirnov test, and this test rejected five out of six components. Even if the Chi-Square test and the Kolomogorov-Smirnov test showed mixed results the hypothesis that demand is Poisson distributed is maintained. When data material from Germany was included the results were more satisfactory compared to when data material only from Lund was used. The consolidating behaviour will fade away in the future and the distribution will be a Poisson distribution as in Germany. The mixed results are caused by the fact that the markets consolidate their spare parts before they are returned.

5.6 The inventory position can vary due to incoming components

The inventory position in a (S-1,S) system without any reconditioned components are at a constant level as stated in Chapter 3.3. However the inventory position can vary if reconditioned components arrive to the system. The inventory position can never be lower than the S level, but when reconditioned components arrive to the system the inventory position increases to a higher level. To which level depends on the amount of components that are returned and can be reused and what the pattern of demand looks like at the given time, see Figure 5.7 that follows.

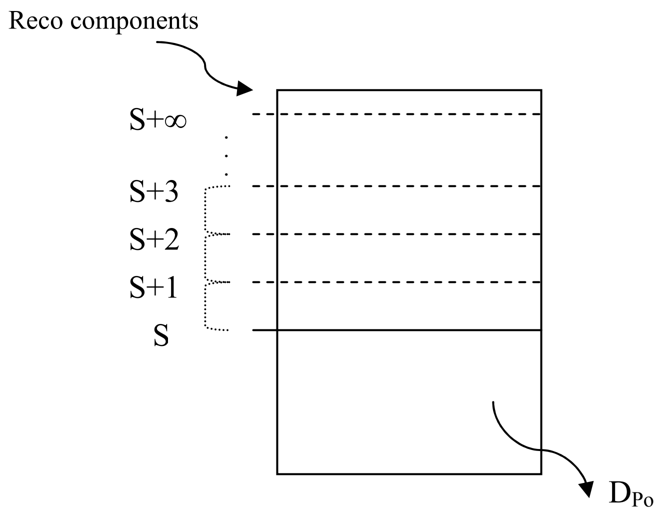


Figure 5.7. The figure shows how the inventory position can vary depending on the amount of reconditioned components that are returned and the demand for the component.

The lead-time for the replenishment of new components and the lead-time for reconditioned components are in most cases quite similar and is assumed equal in the considered model by the authors. The lead-time for the reconditioned components is the time between when a spare part is returned and the time it is stored. Setting the lead-times to be identical simplifies the model but the assumptions may be relaxed without any severe consequences. In the general case the lead-times might be higher for the reconditioned components compared to the new ones. However the technicians are able to prioritise which spare parts to disassemble. This prioritising will decrease the lead-time for the components to approximately a day. This will make the assumption valid because the lead-time for new components is in average a few days.

The different inventory position can be considered as a simple queuing system, M/M/1. When the system is considered to be a M/M/1 system it is easy to calculate the different stationary probabilities. This is due to that the calculations are performed in a standardised way using the well known formulas in the theory that is available about the subject, see Chapter 3.13.1.

$P_0 = 1 - \rho$, where $\rho = \lambda / \mu$ *Eq. 3.25*

And for a certain stationary state, n,

$P_n = P_0 * \rho^n = (1 - \rho) * \rho^n$ *Eq. 3.26*

The components have different characteristics depending on which class they belong to and will effect the inventory position and its fill rate.

Due to the complexity when calculating the inventory position, the authors have decided that only some components, with specific characteristics, are going to be displayed when calculating the fill rate. The extreme characteristics for the components are illustrated in Figure 5.8 below.

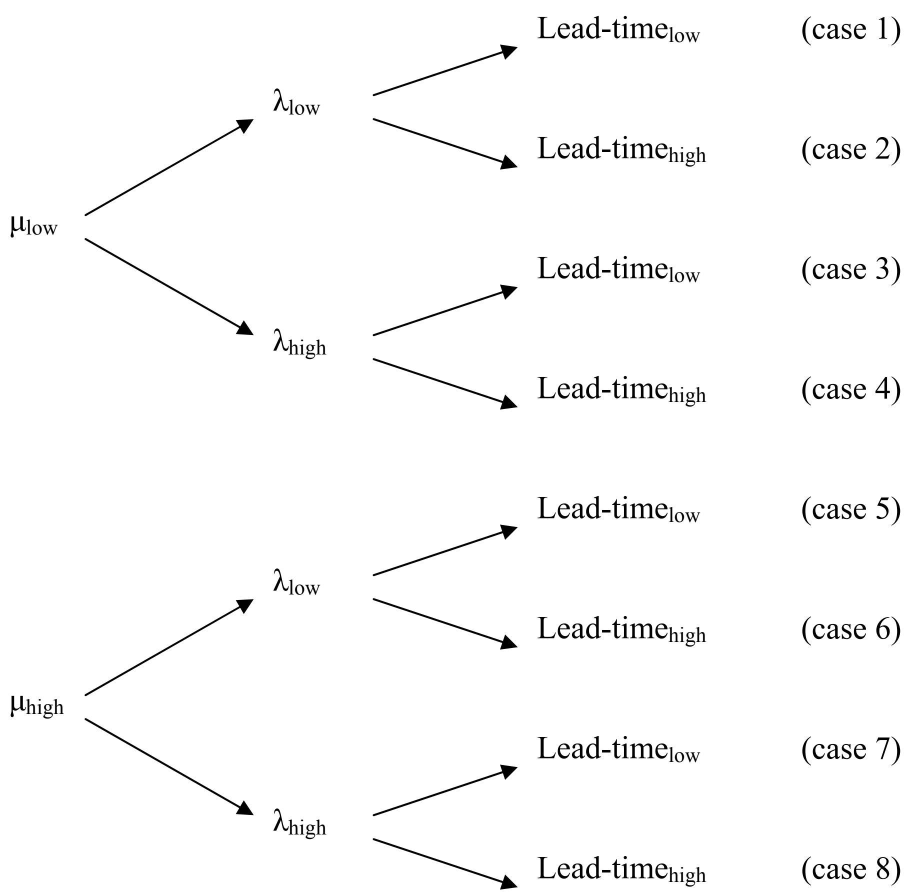


Figure 5.8. An illustration of the different events with extreme values that can exist.

In reality the third and the fourth case is not likely to occur since $\rho \geq 1$ is not allowed. The authors have chosen case number six as an example. It is harder to optimise a case that has a more frequent demand than the opposite and it is interesting to see how the lead-time and the reconditioned frequency affect the fill rate.

Not all spare parts are returned. Extractions from the business system show that approximately 70 % of the sent spare parts are returned. This return rate is included when calculations concerning the reusability frequency are performed. [145]

5.6.1 An example for optimising the inventory level for a reco component

The first step is to calculate the different stationary probabilities. The values for this example where chosen to be the following:

- lead-time$_{high}$ = 8 days
- λ_{low} = (reconditioned components/year)*return rate / ((number of working days/year) / lead-time) = 2,86*0,7/(261/8) = 0,0613
- μ_{high} = (demand/year) / ((number of working days) / lead-time) = 52/(261/8) = 1,59391

[145] Martell, M. 2006-12-01

By using Eq. 3.25 and Eq. 3.26 above the stationary probabilities, p_n, are calculated. The results are displayed in Table 5.2 below.

Reconditioned intensity, λ	Demand intensity, μ
0,0613	1,5939

Stationary probability

p_0	p_1	p_2	p_3	p_4	p_5	p_6	p_7
0,9615	0,0370	0,0014	5E-05	2E-06	8,1E-08	3,1E-09	1,2E-10

Table 5.2. The different stationary probabilities for a component.

When this is carried out the next step is to calculate the fill rate for a fixed lower inventory position, see Eq. 4.2. This step is similar to the one made for non-reco components in that chapter. The only difference is that in this part the matrix is increased due to the fact that the inventory position can alter.

Reconditioned intensity, λ	Demand intensity, μ
0,0613	1,59387

Demand, D(t,t+L)	Order up to level, S: 1 Probability, P(D(t,t+L))	IP(1)	IP(2)	IP(3)	IP(4)	IP(5)	IP(6)	IP(7)	IP(8)	IP(9)
0	0,2031	0,2031	0,2031	0,2031	0,2031	0,2031	0,2031	0,2031	0,2031	0,2031
1	0,3238		0,3238	0,3238	0,3238	0,3238	0,3238	0,3238	0,3238	0,3238
2	0,2580			0,2580	0,2580	0,2580	0,2580	0,2580	0,2580	0,2580
3	0,1371				0,1371	0,1371	0,1371	0,1371	0,1371	0,1371
4	0,0546					0,0546	0,0546	0,0546	0,0546	0,0546
5	0,0174						0,0174	0,0174	0,0174	0,0174
6	0,0046							0,0046	0,0046	0,0046
7	0,0011								0,0011	0,0011
8	0,0002									0,0002
9	0,0000									
	Fill rate:	0,2031	0,5269	0,7849	0,9220	0,9767	0,9941	0,9987	0,9997	1,0000

Table 5.3: This table shows the different fill rates at a given inventory positions with a specific order up to level S.

The inventory levels are calculated by taking the inventory position and subtracting the demand per lead-time. The probability is founded on a Poisson distribution with fix average demand intensity. The fill rates presented at the bottom of Table 5.3 are calculated by adding the probabilities in the columns for the different inventory positions. The final step is to calculate the total fill rate. This is done by multiplying the probability for a specific inventory position with its fill rate i.e. a combination of Table 5.2 and 5.3, and sum the different states together, see Eq. 5.1 below.

$$Fillrate = P(IL > 0) = p_0 \cdot Fillrate(S(1)) + p_1 \cdot Fillrate(S(2)) + ... + p_n \cdot Fillrate(S(n+1)) =$$

$$= 0{,}9615 \cdot 0{,}2031 + 0{,}0370 \cdot 0{,}5269 + 0{,}0014 \cdot 0{,}7849 + ... = 0{,}21598 \qquad \textit{Eq. 5.1}$$

In this case the total fill rate is not particular high due to the low initial inventory position. If the order up to level S is set higher the fill rate will increase, see the table below.

Order up to level, S=	Total fill rate
1	0,2160
2	0,5370
3	0,7903
4	0,9242
5	0,9773
6	0,9942
7	0,9987

Table 5.4. This table shows the fill rates of different order up to levels.

When the total fill rate is calculated, the next step is to determine the total cost for the inventory. This needs to be done to balance the total fill rate with the inventory cost, see Chapter 1.2.

The total inventory cost can be calculated by using the formula below:

$$IC = \sum_{n=0}^{\infty} \left(p_n \cdot E\left[IL\left(sp_n\right)\right] \cdot p \cdot f \cdot ir \right) \qquad \textit{Eq. 5.2}$$

Where:
IC = inventory cost
p_n = stationary probability
$E\left[IL\left(p_n\right)\right]$ = average inventory level for a given stationary probability, where IL≤S (order up to level S)
p = price
f = factor
ir = investment rate

When the summation in Eq. 5.2 above is calculated, it is important that the inventory level never exceeds the initialising inventory position i.e. IL≤S (in the calculation). This can occur in practise, if the amount of reconditioned components that have arrived to the inventory system is greater than the demand for the given time period. If the inventory level is greater than the initialising inventory position the inventory level will be the same as the initialising inventory position when calculations are being made. This is due to the reconditioned components are not included in the cost, and should therefore be excluded from the cost calculations.

Table 5.5 shows the values used in this example.

Reconditioned intensity, λ	0,0613
Demand intensity, μ	1,5939
Price:	55
Factor:	4
Order up to level, S:	2

Table 5.5. This table shows which parameters that are used in this example to calculate the inventory cost.

Table 5.6 illustrates how the inventory costs are calculated for different inventory positions with a given order up to level S.

		IP = 2			IP = 3		
D(t,t+L)	Probability, P(D(t,t+L))	IL(t,t+L)	P(IL>0)	IC(IL(t,t+L))	IL(t,t+L)	P(IL>0)	IC(IL(t,t+L))
0	0,2031	2	0,2031	6,0930	3	0,2031	6,0930
1	0,3238	1	0,3238	4,8570	2	0,3238	9,7132
2	0,2580	0	0	0	1	0,2580	3,8704
3	0,1371	-1	0	0	0	0	
4	0,0546	-2	0	0	-1	0	
5	0,0174	-3	0	0	-2	0	
			Fill rate:	IC:		Fill rate:	IC:
			0,5269	10,9500		0,7849	19,6766

		IP = 4			IP = 5		
D(t,t+L)	Probability, P(D(t,t+L))	IL(t,t+L)	P(IL>0)	IC(IL(t,t+L))	IL(t,t+L)	P(IL>0)	IC(IL(t,t+L))
0	0,2031	4	0,2031	6,0930	5	0,2031	6,0930
1	0,3238	3	0,3238	9,7132	4	0,3238	9,7132
2	0,2580	2	0,2580	7,7409	3	0,2580	7,7409
3	0,1371	1	0,1371	2,0564	2	0,1371	4,1127
4	0,0546	0	0		1	0,0546	0,8194
5	0,0174		Fill rate:	IC:		Fill rate:	IC:
			0,9220	25,6034		0,9767	28,4792

Table 5.6. The table illustrates how the fill rate and its associated cost are calculated for different inventory positions.

Table 5.7 shows how the total inventory cost is calculated by multiplying the stationary positions with the inventory costs. This is the final part in Eq. 5.2 and the result is the total inventory cost for the given order up to S level.

Inventory position	Stationary prob.	Inventory cost	Stat. prob.*Inventory cost
2	0,9615	10,9500	10,528425
3	0,037	19,6766	0,728034348
4	0,0014	25,6034	0,035844794
5	0,0001	28,4792	0,002847921
		Tot. IC:	11,29515206

Table 5.7. The table shows the last step for calculating the total inventory cost.

The example in the table above is not a complete calculation; it only illustrates the procedure for determining the total inventory cost.

6 Development of the model

So far a (S-1,S) system has been used. This system might not be the best alternative when there is a large demand for the components. The model will therefore be extended into a (R,Q) system where an order does not have to be placed every time there is a demand. The parameters in the Wilson formula will be determined. A model to determine the inventory positions, fill rates and the associated inventory costs for a (R,Q) system will be established. The last section in this chapter presents a sensitivity analysis.

6.1 Determination of the order quantity using the Wilson formula

To establish the probabilities for different inventory positions and to calculate the fill rate the order quantity, Q, is needed. The order quantity is determined from the Wilson formula:

$$Q^* = \sqrt{\frac{2Ad}{h}} \qquad \text{Eq. 6.1}$$

Where A is the ordering cost, d is the demand per lead-time and h is the holding cost per item and time period.

6.1.1 Ordering cost

The ordering cost should include costs that are independent of the size for the order. In this case costs such as the time to prepare the order and the administration work concerning the receiving of the components should be included. The time that this requires is approximately 10 minutes all together.[146] The price for an hours labour for a member of the staff can be calculated as follows: $(25000 \cdot 12{,}16 + 257000)/(220 \cdot 8) \approx 320 SEK/h$.[147] The first two figures are the price for the company, in which the salary and the holiday pay are included for the member of the staff. The large figure in the calculation includes overhead costs per employee. This cost includes the costs for the facilities, IT, human resources department, economy department and the salary for the president of the company. The last two figures are the total number of working hours for a person per year. The ordering cost is then determined to be approximately 53 SEK per order, derived from the 10 minutes that an additional order requires.

The ordering cost is an approximation. Even though the factors that are included in this part are estimates, e.g. the salary varies depending on who conducts the release or the receiving of the order, they provide a reasonable estimate of the ordering cost. The other figures have been used in similar cases when calculations have been performed at Tetra Pak. One can discuss if it is correct not to include the cost for the transportation that is needed when an order is placed. The reason that it is not included is that on average 10-20 orders are delivered to Maintenance Unit per day.[148] In most cases it does not require an additional transportation if an extra order is placed.

[146] Wågnert, C. 2006-11-13
[147] Ibid.
[148] Hansson, S. 2006-10-10

6.1.2 Demand for new components

The demand for new components is used in the Wilson formula to determine the order quantity. This demand is used since only new components are ordered and have a cost. To calculate the demand for new components Eq. 6.2 below was used.

$$D_{new} = D_{tot} \cdot (1 - P_{ret}) + (D_{tot} \cdot P_{ret}) / (RF + 1) \qquad \textit{Eq. 6.2}$$

Where:
D_{new} = the demand for new reco-components
D_{tot} = the total demand for reco-components
P_{ret} = the factor that describes the probability of how many spare parts that are returned
RF = the reusability factor

The first part of Eq. 6.2 describes the number of components that are used as new due to the fact that not all the spare parts are returned to Maintenance Unit. The second part describes the reconditioned components which are disposed of and have to be replaced by new components when spare parts are assembled.

6.1.3 Holding cost

The holding cost, h, is defined as the inventory cost per item and time period. The calculations are made by multiplying the price per item with the investment rate. The price depends on which class the component belongs to and the investment rate which is 7.5%. To determine the price for components in a class the median value for the SEK volume was used. By dividing this value with the demand for new components and multiplying this with 7.5 %, the holding cost for an item in a specific group was determined. The formula for calculating the holding cost is described by Eq. 6.3 below.

$$HC = \left(\frac{SEKvolume}{D_{new}} \right) \cdot ir \qquad \textit{Eq. 6.3}$$

6.1.4 Example for determining the order quantity

In this example the ordering quantity for components that have a high demand, high SEK volume and high reusability factor is determined. The numbers that have been used are the median values that were established in Chapter 5.3.1. The order cost is 53 SEK / order, the demand is calculated by using Eq. 6.2 and the result is $D_{new} = 52 \cdot 0{,}3 + \frac{52 \cdot 0{,}7}{6} = 21{,}7$ components / year. The holding cost is calculated by using Eq. 6.3 and it is determined to HC $= (18260 / 21{,}7) \cdot 0{,}075 = 63$ SEK / component and year. If Eq. 6.1 is used the order quantity is determined to be $\sqrt{(2 \cdot 53 \cdot 21{,}7) / 63} = 6{,}04 \approx 6$ components per order. If implementing this system an order quantity has to be determined for each class.

6.2 Determination of the inventory position in a (R,Q)-system

The first step to determine the inventory position is to draw the Markov chain for the specific process. In this case the Markov chain is visualised in the following rate graph.

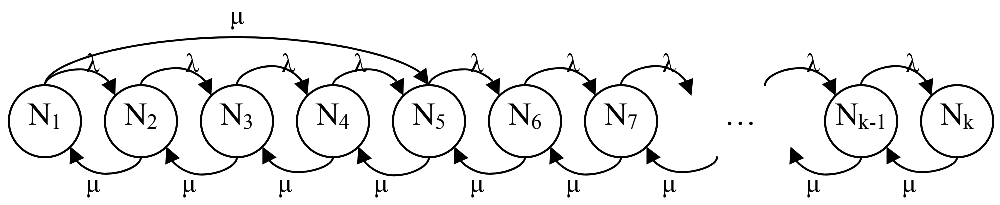


Figure 6.1: A Markov chain for a R,Q-system with new and reconditioned components.

The large arrow, that starts from position one and ends in position five, symbolises the order quantity, Q, in the system. This arrow will change in size depending on the size of Q. The small arrows at the top of each circle, λ, symbolises the reconditioned components arrival intensity. The arrows at the bottom of each circle, μ, symbolises the demand intensity for the component. The first state, N_1, is the lowest inventory position possible i.e. R+1.

To calculate this process the formulas that are available in the Chapter 3.13 have to be modified. Two formulas were developed. Eq. 6.4 calculates the stationary probability in the range of the large arrow i.e. the states one to five. Eq. 6.5 calculates the remaining stationary probabilities i.e. the states larger than five.

P_n is defined as the stationary probability, λ is the arrival intensity for reconditioned components and μ is the demand intensity for reco components.

The formula derives from Figure 6.1, see the calculations below.

The derivation of Eq. 6.4:

$$P_1\lambda + P_1\mu = P_2\mu \Rightarrow P_2 = \frac{\lambda}{\mu}P_1 + \frac{\mu}{\mu}P_1 = P_1(A+B), A = \frac{\lambda}{\mu}, B = \frac{\mu}{\mu}$$

$$P_1\mu + P_2\lambda = P_3\mu \Rightarrow P_3 = P_2A + P_1B = P_1(A+B)A + P_1B = P_1(A^2 + BA + B)$$

$$P_1\mu + P_3\lambda = P_4\mu \Rightarrow P_4 = P_3A + P_1B = P_1A(A^2 + BA + B) + P_1B = P_1(A^3 + BA^2 + BA + B)$$

$$\dots$$

The result of this derivation is illustrated below in Eq. 6.4.

$$P_n = \left(\left(\frac{\lambda_1}{\mu}\right)^{n-1}\right)P_1 + \left(\frac{\lambda_2}{\mu}\cdot\sum_{k=0}^{n-2}\left(\frac{\lambda_1}{\mu}\right)^k\right)P_1 \qquad \textit{Eq. 6.4}$$

If this equation is used together with the fact that the sum of all the stationary probabilities has to be equal to one, the different stationary probabilities can be calculated.

In the following formula R is the reorder point and the other parameters are the same as in the previous formula. In the derivation the authors have used Figure 6.1. The result of the derivation is Eq. 6.5.

$$P_5\lambda = P_6\mu \Rightarrow P_6 = AP_5 = A(A^4 + BA^3 + BA^2 + BA + B)P_1 = \left(A^5 + BA^4 + BA^3 + BA^2 + BA\right)P_1$$
$$P_6\lambda = P_7\mu \Rightarrow P_7 = AP_6 = A(A^5 + BA^4 + BA^3 + BA^2 + BA)P_1 = \left(A^6 + BA^5 + BA^4 + BA^3 + BA^2\right)P_1$$
$$P_7\lambda = P_8\mu \Rightarrow P_8 = AP_7 = A(A^6 + BA^5 + BA^4 + BA^3 + BA^2)P_1 = \left(A^7 + BA^6 + BA^5 + BA^4 + BA^3\right)P_1$$
$$\cdots$$

The result of this derivation is illustrated in Eq. 6.5:

$$P_n = \left(\left(\frac{\lambda_1}{\mu}\right)^{n-1}\right)P_1 + \left(\frac{\lambda_2}{\mu}\cdot\sum_{k=1}^{n-2}\left(\frac{\lambda_1}{\mu}\right)^{k}\right)P_1 - \left(\frac{\lambda_2}{\mu}\cdot\sum_{k=R+1}^{n}\left(\frac{\lambda_1}{\mu}\right)^{(k-R-1)}\right)P_1 \qquad \textit{Eq. 6.5}$$

If this equation is used together with the fact that the sum of all the stationary probabilities has to be equal to one, the different stationary probabilities can be calculated.

The final step is to use the formulas above and calculate the different stationary probabilities.

6.2.1 Calculating the inventory position for a given class, an example

This part consists of an example to illustrate the method for determining the inventory position. The following parameters were chosen:

RF = reusability factor
D_{tot} = total demand for reco components
P_{ret} = percentage of how many spare parts that are returned
L = lead-time
WD = working days per year

The numbers that are used in the following example represents a component belonging to a certain class with a high demand for reco components, high reusability factor and medium lead-time.

λ = number of reconditioned components per lead-time =

$$\frac{\left(D_{tot}\cdot RF\cdot P_{ret}\right)}{\left(\frac{WD}{L}\right)} = \frac{\left(\frac{5}{6}\cdot 52\cdot 0{,}7\right)}{\left(\frac{261}{8}\right)} = 0{,}9298$$

μ = demand per year/(working days per year/lead-time) = $\frac{D_{tot}}{\left(\frac{WD}{L}\right)} = \frac{52}{\left(\frac{261}{8}\right)} = 1{,}5939$

The same component used in Chapter 6.1.4 is used here and therefore the order quantity was determined to be:

$$Q^* = \sqrt{\frac{2 \cdot A \cdot d}{h}} = \sqrt{\frac{2 \cdot 53 \cdot 21{,}7}{63}} = 6{,}04 \approx 6$$

The result of the different stationary inventory positions are illustrated in Table 6.1, they are calculated by using Eq. 6.4 and Eq. 6.5.

Inventory position, IP	Probability for the given IP	%
0+R	0,0695	6,95
1+R	0,1100	11,00
2+R	0,1336	13,36
3+R	0,1474	14,74
4+R	0,1554	15,54
5+R	0,1601	16,01
6+R	0,0934	9,34
7+R	0,0545	5,45
8+R	0,0318	3,18
9+R	0,0185	1,85
10+R	0,0108	1,08
11+R	0,0063	0,63
12+R	0,0037	0,37
13+R	0,0021	0,21
14+R	0,0013	0,13
15+R	0,0007	0,07
16+R	0,0004	0,04
17+R	0,0002	0,02
18+R	0,0001	0,01

Table 6.1: The different stationary inventory positions, where R is the reorder point.

The table illustrates that if there is a high reusability frequency the inventory position varies a great deal. This leads to that a lower Q is necessary compared to a system with a low reusability.

6.3 Determining the fill rate for different classes

To determine the fill rate one has to calculate the probabilities for different inventory levels and add the probabilities for inventory levels greater than 0. Eq. 3.21 was modified to fit the given situation where components can be reused, see Eq. 6.6 below. P_n is defined as the stationary inventory position n, R is the reorder point, Q is the order quantity, IL is the inventory level, D(L) is the demand during the lead-time and m is any given number in the Markov chain in Figure 6.1.

$$P(IL = j) = \sum_{k=\max\{R+1,j\}}^{R+Q+m} P(D(t,t+L) = k - j) \cdot P_k \qquad j \le R + Q + m \qquad \textit{Eq. 6.6}$$

First the order quantity has to be determined. In this case the component used in Chapter 6.1.4 will be used again. Therefore the order quantity is 6. The next step is to try different reorder points and see how the fill rate changes.
Calculations needed were performed in Excel and are illustrated in Table 6.2. In this example the reorder point was determined to be one. The component belongs to a group with high frequency, medium lead-time and high reusability. The probabilities for the different demands required to have e.g. one in inventory level was determined. These probabilities were multiplied by the appropriate stationary probabilities. The result of this is the column called accumulated probability (Acc. Prob.). If the probabilities in the column are added the total probability for a certain inventory level (one in this case) is calculated.

IL=0				IL=1				IL=2			
D	Prob. D	Prob. Sta.	Acc. Prob.	D	Prob. D	Prob. Sta.	Acc. Prob.	D	Prob. D	Prob. Sta.	Acc. Prob.
2	0,258	0,069	0,018	1	0,324	0,069	0,022	0	0,203	0,069	0,014
3	0,137	0,110	0,015	2	0,258	0,110	0,028	1	0,324	0,110	0,036
4	0,055	0,134	0,007	3	0,137	0,134	0,018	2	0,258	0,134	0,034
5	0,017	0,147	0,003	4	0,055	0,147	0,008	3	0,137	0,147	0,020
6	0,005	0,155	0,001	5	0,017	0,155	0,003	4	0,055	0,155	0,008
7	0,001	0,160	0,000	6	0,005	0,160	0,001	5	0,017	0,160	0,003
8	0,000	0,093	0,000	7	0,001	0,093	0,000	6	0,005	0,093	0,000
9	0,000	0,054	0,000	8	0,000	0,054	0,000	7	0,001	0,054	0,000
10	0,000	0,032	0,000	9	0,000	0,032	0,000	8	0,000	0,032	0,000
11	0,000	0,019	0,000	10	0,000	0,019	0,000	9	0,000	0,019	0,000
12	0,000	0,011	0,000	11	0,000	0,011	0,000	10	0,000	0,011	0,000
13	0,000	0,006	0,000	12	0,000	0,006	0,000	11	0,000	0,006	0,000
14	0,000	0,004	0,000	13	0,000	0,004	0,000	12	0,000	0,004	0,000
15	0,000	0,002	0,000	14	0,000	0,002	0,000	13	0,000	0,002	0,000
16	0,000	0,001	0,000	15	0,000	0,001	0,000	14	0,000	0,001	0,000
17	0,000	0,001	0,000	16	0,000	0,001	0,000	15	0,000	0,001	0,000
		Total:	0,043773			Total:	0,080792			Total:	0,116175

Table 6.2. The table illustrates the probabilities for three different inventory levels.

These calculations are performed for several inventory levels and together they represent the fill rate for a certain reorder point and order quantity.

Probability, IL=1	0,080792	Probability,IL=9	0,028047
Probability, IL=2	0,116175	Probability, IL=10	0,016361
Probability, IL=3	0,135606	Probability, IL=11	0,009544
Probability, IL=4	0,143146	Probability, IL=12	0,005567
Probability, IL=5	0,138023	Probability, IL=13	0,003248
Probability, IL=6	0,117112	Probability, IL=14	0,001894
Probability, IL=7	0,082425	Probability, IL=15	0,001105
Probability, IL=8	0,048081	Probability, IL=16	0,000643
		Fill rate:	0,928472

Table 6.3. The table illustrates the fill rate for a component that belongs to a group with high frequency, medium lead-time and high reusability.

6.4 Inventory cost for a (R,Q) system consisting of reco components

It is a complex task to calculate the inventory cost for a (R,Q) system that consists of new and reconditioned components. This is due to the fact that the number of reconditioned components can vary. Because of this it is hard to determine the number of new components in a certain inventory position. Therefore approximations will be used.

When the fill rate was determined the different probabilities for inventory levels were calculated, see Chapter 6.3. These probabilities will be used in this section. The authors have divided the problem into three different parts 1, 2 and 3, see the figure below.

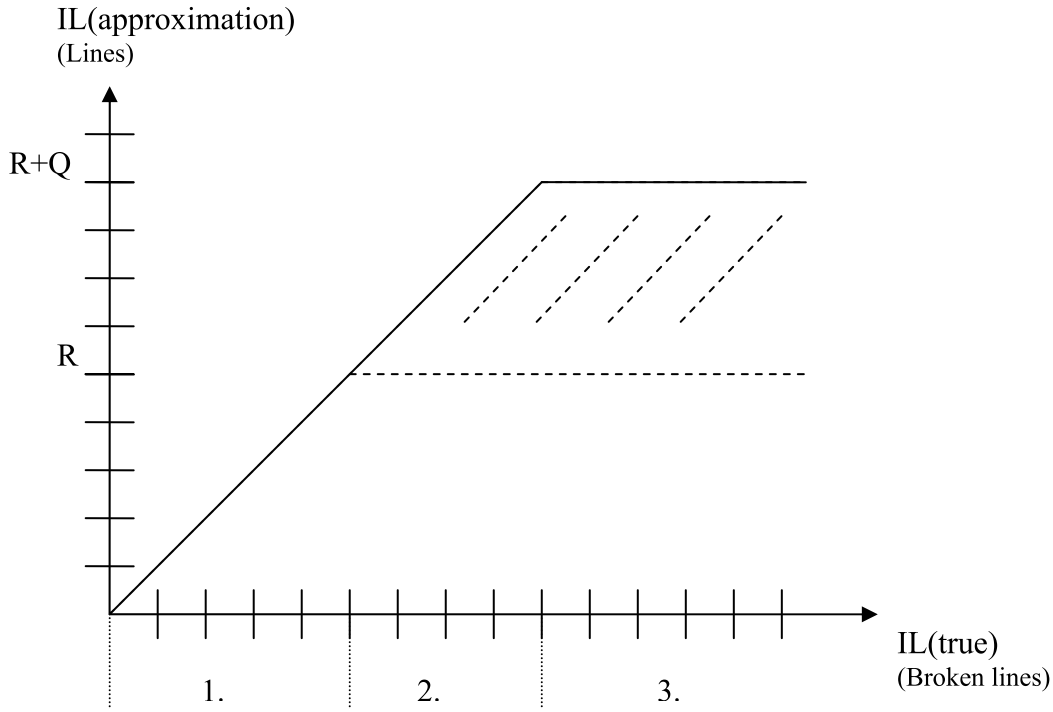


Figure 6.2: An illustration of the difference between the true inventory level and the approximated inventory level for new components. The true inventory level can vary when the IL>R+1. This is illustrated in the figure by the marked area.

Part 1, IL ≤ R:
This part can be calculated in an exact way without any approximations. This is due to the fact that with these inventory levels all the components are new. This is based on that the lead-time for ordering new components equals the lead-time for reconditioning the components. The part includes every inventory level less then R+1. The formula describing the cost for this part is Eq. 6.7, where IC_1 = inventory cost part 1, p = price, f = factor and ir = investment rate.

$$IC_1 = p \cdot f \cdot ir \cdot \sum_{i=1}^{R} i \cdot P(IL = i) \qquad \textit{Eq. 6.7}$$

Part 2, R+1 ≤ IL ≤ R+Q:
This part can not be calculated in an exact way and approximations are necessary, due to the fact that reconditioned components are stored. It is hard to calculate the exact pattern of

incoming components and because of this the authors have decided that all the incoming components are treated as new. This approximation results in a higher inventory cost compared to the true cost. The formula describing part 2 is visualised below:

$$IC_2 = p \cdot f \cdot ir \cdot \sum_{i=R+1}^{R+Q} i \cdot P(IL = i) \qquad \textit{Eq. 6.8}$$

Part 3, IL > R+Q:
The inventory levels greater than R+Q are included in this section. The inventory levels in this part can never consist of only new components since the maximum number of new components in the system is R+Q. The authors are using the maximum number of new components when the inventory cost is calculated. This approximation is used since this event has the greatest probability compared to the events where the number of new components are less then R+Q. This approximation will result in a higher inventory cost compared to the true value. The calculations are made by the following formula:

$$IC_3 = p \cdot f \cdot ir \cdot (R+Q) \cdot \sum_{i=R+Q+1}^{\infty} P(IL = i) \qquad \textit{Eq. 6.9}$$

Total inventory cost:
The total inventory cost is calculated by using, $IC = IC_1+IC_2+IC_3$, which represents the upper bound on the true expected cost.

This cost should be compared with the fill rate for the specific reorder point and order quantity that were used. New calculations should be produced with new reorder points and order quantities to result in new costs and fill rates. A comparison between all of these should be done to generate the optimum solution.

6.5 Sensitivity analysis of the demand

As stated earlier in the sensitivity analysis of the demand for non-reco components, the spare parts have to be delivered with a 95 % probability. The fill rates for the components have to be substantially higher than this, since they are multiplied with each other to attain this value. The components that belong to a class with a high SEK volume are determined to have a lower fill rate compared to classes with lower SEK volume. The reason for choosing different fill rates for the classes in this example is that it is less expensive to store components that belongs to classes with low SEK volume. This sensitivity analysis will visualise how the fill rate for the spare part is affected if the demand for the components is unexpectedly high.

The same two fictitious spare parts used in Chapter 4.8 will be reused here i.e. two spare parts consisting of 5 respectively 20 components in each of the classes AAB, BBB and CCB. The reorder points where chosen to give an appropriate fill rate for the spare parts with the expected demand. When the fill rate for the spare part had been calculated the median demand was altered from 52 to 80 for the components with high demand. The components with medium respectively low frequency were altered from 15 to 21 respectively 3 to 8. When the demand had been changed the new fill rates were calculated. The table below illustrates how the fill rate for the spare part decreases if the cost is kept at a constant level and the demand for the components is unexpectedly high.

	Different fill rate, same cost
5 artnbr. From AAB, BBB, CCB st. values	0,995666
5 artnbr. From AAB, BBB, CCB high demand	0,968644
20 artnbr. From AAB, BBB, CCB st. values	0,982775
20 artnbr. From AAB, BBB, CCB high demand	0,880354

Table 6.4: The table displays the changes in the fill rate if the demand is unexpectedly high for the components.

The outcome for the different spare parts can be compared. The conclusion was the same as in the previous sensitivity analysis i.e. it is more important to use a higher fill rate for a class if there are many components in that class in a spare part. This sensitivity analysis shows an extreme case, i.e. when all the components have an unexpectedly high demand. In the normal case the spare part will consist of components that are a mixture of higher and lower values, compared to the median value, in the different classes. It is important to bear this in mind when the fill rates for the different classes are determined.

7 Conclusion

The conclusion summarises the findings from this master thesis.

7.1 A model for controlling the safety stock of non-reco components

The purpose for the first part of the thesis was to create a model for control and optimisation of the safety stock for new components, where no reconditioning is possible (so called non-reco components). This was achieved by using an ABC classification that categorised the data on frequency, SEK volume and the lead-time. A (S-1,S) system was chosen due to the fact that the system was going to cover the shortages of the central warehouse. Therefore batch ordering is not an option. To determine the fill rate for different components, the lead-time and the distribution of the demand were needed. To establish the distribution several tests were used. The final step of the model is to determine different fill rates and the costs for a class of components and compare these. This step is vital since otherwise it will not be possible to achieve the right cycle service level for the spare part.

If the central warehouse always can deliver components in time Maintenance Unit do not need a safety stock. Therefore it is very important that the central warehouse have a high fill rate reducing the inventory levels at Maintenance Unit.

In the previous business system the inventory levels were set to a level where at least one spare part could be produced. If the model of this thesis is implemented it will result in more appropriate inventory levels. The result will be a higher fill rate for the components which results in a higher cycle service level for the spare parts.

7.2 A model for joint inventory control of reco components

The second part of the thesis focuses on creating a model to control and optimise the inventory that consists of new and reconditioned components (so called reco components). The steps required to attain the necessary results were similar to those in the previous part such as an ABC classification. In addition to the parameters (frequency, SEK volume and lead-time) that were used earlier an additional parameter was chosen, the reusable frequency of the reconditioned components. Additional activities were the determination of the distribution for the demand and incoming spare parts. In this part a (S-1,S) system was used. The major difference between this part and the model for non-reco components is that the inventory position can vary.

The model was in a later stage developed into a (R,Q) system. This enhanced model is therefore based on the findings from the (S-1, S) system of components that can be reconditioned. The fill rate and cost are harder to calculate compared to the model for non-reco components.

The prior business system that Maintenance Unit used did not take the reconditioned components into consideration which this model does. The result will be a better way to control the inventory and balance the service level and inventory costs.

8 Recommendations for implementation and follow up

This chapter presents different matters that are of importance to acknowledge when implementing the models that are the findings from this master thesis.

The models are based on data collected from the year 2005. Thus it is important to update the data when the models are used. One should also bear in mind that the models are based on theories and it is essential to advance slowly when implementing the models.

It is not necessary to optimise the entire inventory with these models. They can also be used to determine the right inventory levels for specific components or to be used as complement to another model.

8.1 Implementation of the models

If the inventory is going to be optimised using these models it is essential to consider the following information when implementing the models.

When implementing new reorder points and order quantities there are several important aspects that need to be taken into careful consideration. One of these is that when the different parameters are changed in the system it should be done step by step, to make sure that everything is working properly. It may be wise to include just a few components when the models are implemented and then in a gradual manner increase the number of components. This is a slow process, and it takes time to implement it, so patience is the keyword.

Another thing of importance is that the person who is performing the implementation should be well aware of what the different parameters and boundaries represent. It is important due to the fact that otherwise it is hard to be able to react if a component is not behaving as expected. The components that have extreme values should be investigated further and be handled manually. An example of these might be the ones with extreme lead-times and high frequencies, since they will differentiate themselves from the classes they belong to. To some extent some of the classes may disappear. For example if all components with low SEK volume and low frequency are cheap they can be bundled into one group and be given the fill rate 100 %.

The models have been based on historical data, since this was the only data available. In the future when updating the models it can be a possibility that the new reorder points and order quantities are based on forecasts to be proactive instead of reactive. The follow up of the different factors that affect the reorder point and the order quantity have to be performed on continuous basis to notice differences. It is especially important to control the components which are:

- fading away, making sure that these components can be removed from the system.
- experiencing large changes in the demand, price or lead-time structure. This is important because the component may be altered to another class.

When the implementation is completed and the routines are set for the follow up, the model can be an important tool for the inventory control at Maintenance Unit.

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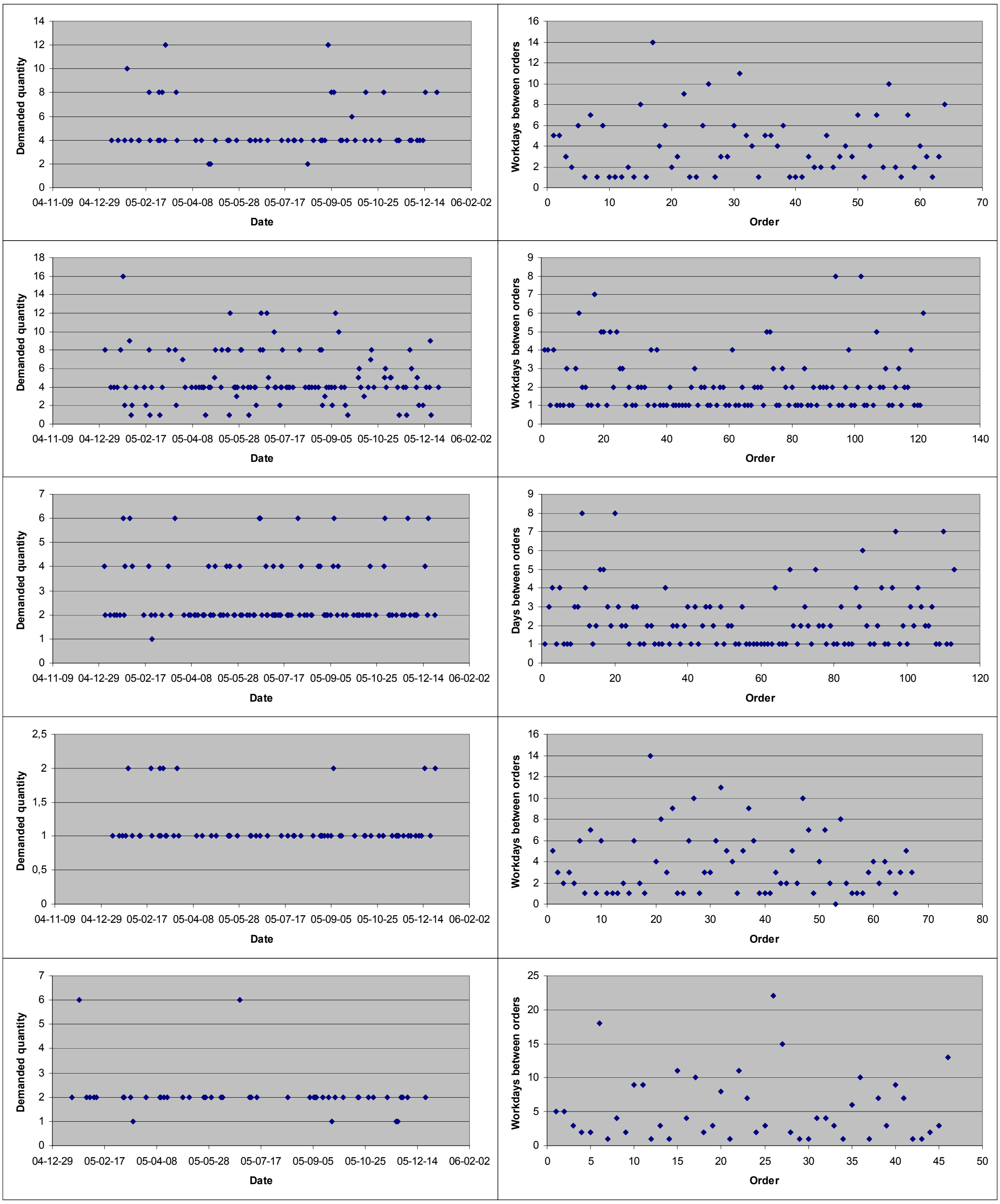
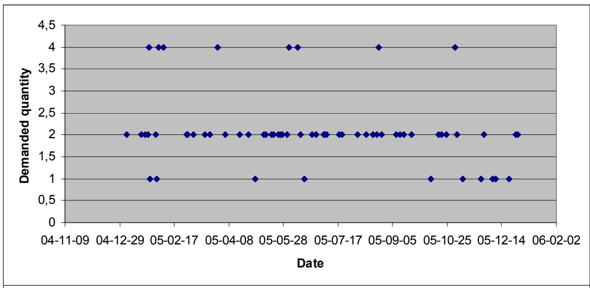
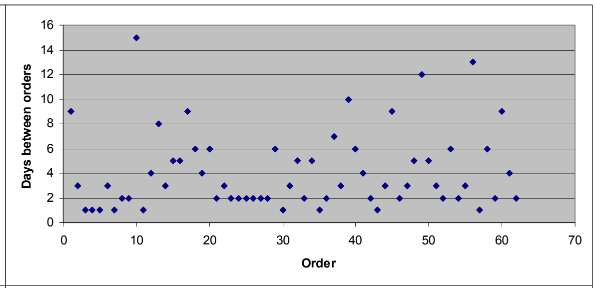
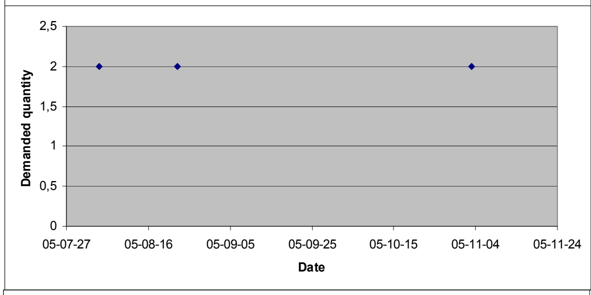
Appendix

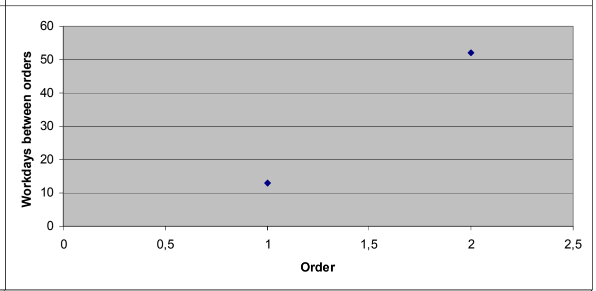
Appendix 1 & 2: Demand patterns for different components.

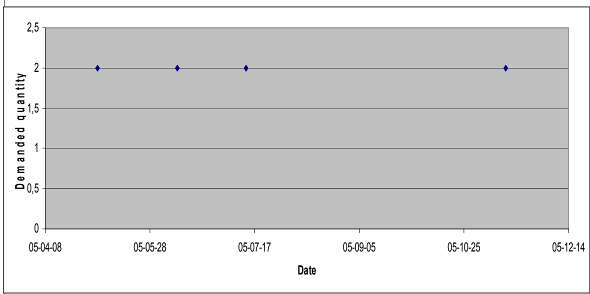



Demanded quantity
0 0,5 1 1,5 2 2,5 3 3,5 4 4,5
04-11-09 04-12-29 05-02-17 05-04-08 05-05-28 05-07-17 05-09-05 05-10-25 05-12-14 06-02-02
Date

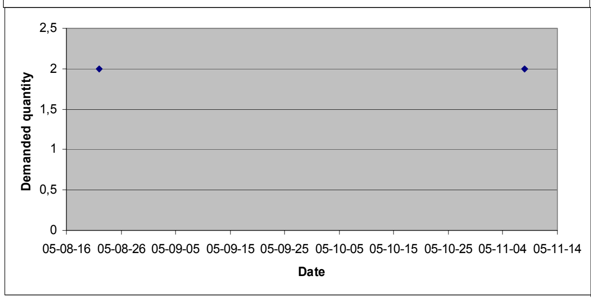

Days between orders
0 2 4 6 8 10 12 14 16
0 10 20 30 40 50 60 70
Order

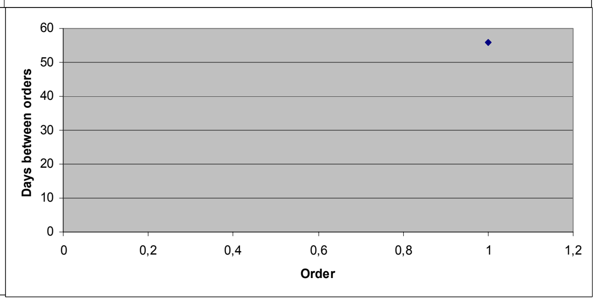

Demanded quantity
0 0,5 1 1,5 2 2,5
05-07-27 05-08-16 05-09-05 05-09-25 05-10-15 05-11-04 05-11-24
Date


Workdays between orders
0 10 20 30 40 50 60
0 0,5 1 1,5 2 2,5
Order


Demanded quantity
0 0,5 1 1,5 2 2,5
05-04-08 05-05-28 05-07-17 05-09-05 05-10-25 05-12-14
Date

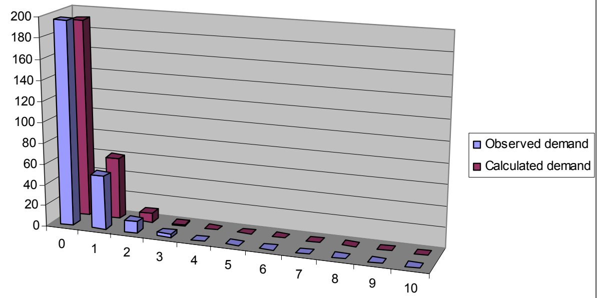

Workdays between orders
0 10 20 30 40 50 60 70 80 90 100
0 1 2 3 4
Order


Demanded quantity
0 0,5 1 1,5 2 2,5
05-08-16 05-08-26 05-09-05 05-09-15 05-09-25 05-10-05 05-10-15 05-10-25 05-11-04 05-11-14
Date

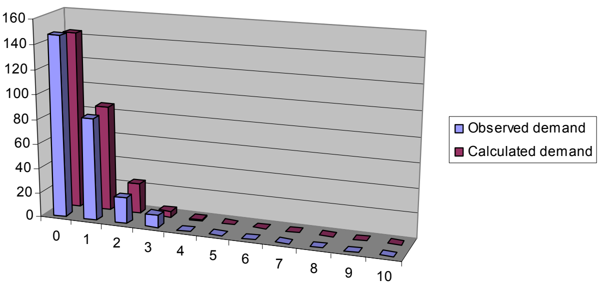

Days between orders
0 10 20 30 40 50 60
0 0,2 0,4 0,6 0,8 1 1,2
Order

Appendix 3: Comparison between the observed and the expected demand for different components.

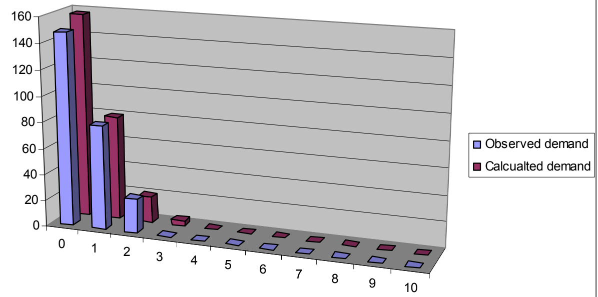
Article for category AA



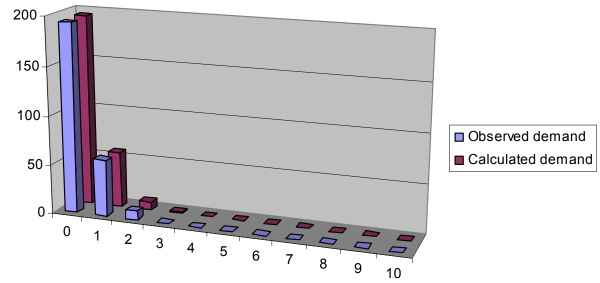
Article för category AC



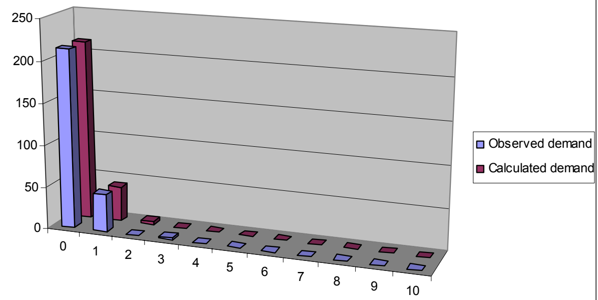
Article for category AB



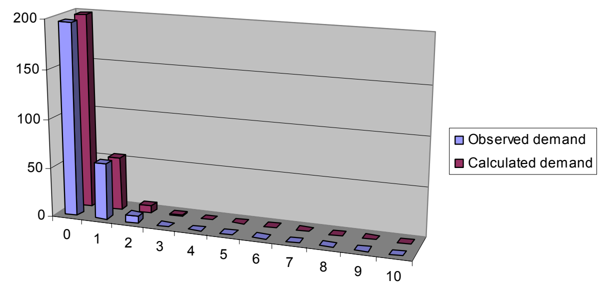
Article for category BA



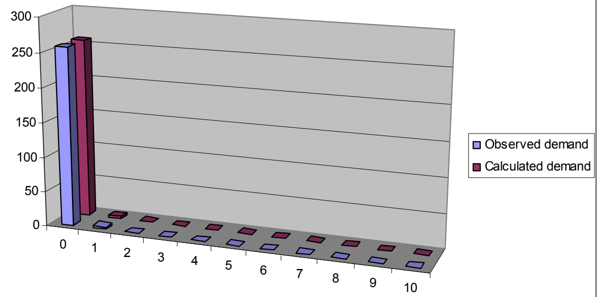
Article for category BB



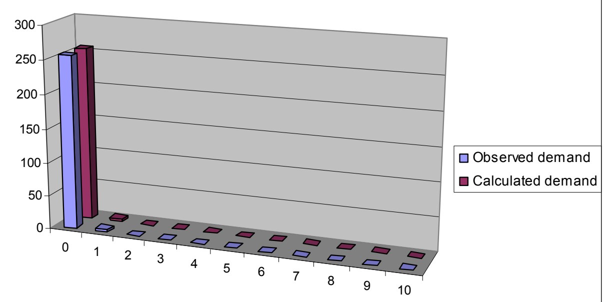
Article for category BC

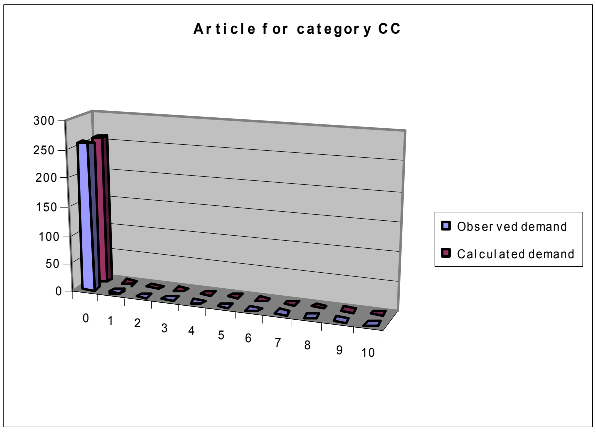


Article for category CA



Article for category CB




Article for category CC
300
250
200
150
100
50
0
0
1
2
3
4
5
6
7
8
9
10
Observed demand
Calculated demand

Appendix 4: The results of Chi-Square tests for the different components.

Component aaa:

Observed value	Calculated value	(Diff^2)/calc.value	Sort
196	191	0,130890052	0,1308901
52	60	1,066666667	1,0666667
11	9	0,444444444	0,4444444
3	1	4	4
			x^2:
			5,6420012

Component aba:

Observed value	Calculated value	(Diff^2)/calc.value	Sort
148	157	0,515923567	0,5159236
80	80	0	0
26	20	1,8	1,8
0	4	4	4
			x^2:
			6,3159236

Component aca:

Observed value	Calculated value	(Diff^2)/calc.value	Sort
147	144	0,0625	0,0625
83	86	0,104651163	0,1046512
21	25	0,64	0,64
10	5	5	5
0	1	1	1
			x^2:
			6,8071512

Component baa:

Observed value	Calculated value	(Diff^2)/calc.value	Sort
194	195	0,005128205	0,0051282
58	57	0,01754386	0,0175439
9	8	0,125	0,125
0	1	1	1
			x^2:
			1,1476721

Component bba:

Observed value	Calculated value	(Diff^2)/calc.value	Sort
214	215	0,004651163	0,0046512
45	42	0,214285714	0,2142857
0	4	4	4
2	0	#DIV/0!	0
			x^2:

4,2189369

Component bca:

Observed value	Calculated value	(Diff^2)/calc.value	Sort
197	199	0,020100503	0,0201005
57	54	0,166666667	0,1666667
7	7	0	0
0	1	1	1
			x^2:
			1,1867672

Component caa:

Observed value	Calculated value	(Diff^2)/calc.value	Sort
258	258	0	0
3	3	0	0
			x^2:
			0

Component cba:

Observed value	Calculated value	(Diff^2)/calc.value	Sort
257	257	0	0
4	4	0	0
			x^2:
			0

Component cca:

Observed value	Calculated value	(Diff^2)/calc.value	Sort
259	259	0	0
2	2	0	0
			x^2:
			0

Appendix 5: The result of a Kolmogorov-Smirnov test for a component.

Art nbr	**Total orders**	**Aver. tbo**				
9005-60	32	7,46875				
Order delivered	**Time between orders**	**Sort**	**i/n**	**fxi**	**D+**	**D-**
			0			
2005-01-14	11	0	0,03125	0	0,03125	0
2005-01-31	5	1	0,0625	0,12531479	-0,06281	0,094065
2005-02-07	7	1	0,09375	0,12531479	-0,03156	0,062815
2005-02-16	7	2	0,125	0,23492578	-0,10993	0,141176
2005-02-25	2	2	0,15625	0,23492578	-0,07868	0,109926
2005-03-01	6	2	0,1875	0,23492578	-0,04743	0,078676
2005-03-09	15	3	0,21875	0,33080089	-0,11205	0,143301
2005-03-30	15	3	0,25	0,33080089	-0,0808	0,112051
2005-04-20	8	3	0,28125	0,33080089	-0,04955	0,080801
2005-05-02	7	4	0,3125	0,41466143	-0,10216	0,133411
2005-05-11	16	4	0,34375	0,41466143	-0,07091	0,102161
2005-06-02	6	5	0,375	0,48801301	-0,11301	0,144263
2005-06-10	0	6	0,40625	0,55217255	-0,14592	0,177173
2005-06-10	9	6	0,4375	0,55217255	-0,11467	0,145923
2005-06-23	18	6	0,46875	0,55217255	-0,08342	0,114673
2005-07-19	9	7	0,5	0,60829195	-0,10829	0,139542
2005-08-01	12	7	0,53125	0,60829195	-0,07704	0,108292
2005-08-17	1	7	0,5625	0,60829195	-0,04579	0,077042
2005-08-18	2	8	0,59375	0,65737876	-0,06363	0,094879
2005-08-22	2	8	0,625	0,65737876	-0,03238	0,063629
2005-08-24	4	8	0,65625	0,65737876	-0,00113	0,032379
2005-08-30	8	8	0,6875	0,65737876	0,030121	0,001129
2005-09-09	8	9	0,71875	0,70031427	0,018436	0,012814
2005-09-21	1	9	0,75	0,70031427	0,049686	-0,01844
2005-09-22	6	11	0,78125	0,77071817	0,010532	0,020718
2005-09-30	8	12	0,8125	0,79945058	0,013049	0,018201
2005-10-12	17	15	0,84375	0,8657925	-0,02204	0,053293
2005-11-05	3	15	0,875	0,8657925	0,009207	0,022043
2005-11-10	4	16	0,90625	0,88261069	0,023639	0,007611
2005-11-16	3	16	0,9375	0,88261069	0,054889	-0,02364
2005-11-21	3	17	0,96875	0,8973213	0,071429	-0,04018
2005-11-24	16	18	1	0,91018846	0,089812	-0,05856
2005-12-16						
					0,089812	0,177173
			Table value:	0,28	**max:**	0,177173

Appendix 6: A compilation of Kolmogorov-Smirnov tests for the demand for different components.

	Total orders; dof	Max. value	Critical value
Component 1	78	0,19	0,184561157
Component 2	39	0,13	0,261008891
Component 3	32	0,18	0,27
Component 4	63	0,17	0,205360697
Component 5	14	0,2	0,418
Component 6	7	0,35	0,577
Component 7	11	0,19	0,468
Component 8	3	0,32	0,828
Component 9	17	0,28	0,381

Appendix 7: Comparing the inventory cost with the fill rate for different components.

Class aaa

Inventory position, k:	1	2	3	4	5	6
Fill rate:	0,669301	0,93804	0,991992	0,999213	0,999938	0,999996
Tot. Inventory cost	14,72462	35,3615	57,18532	79,168	101,1666	123,1665

Class aba

Inventory position, k:	1	2	3	4	5	6
Fill rate:	0,680955	0,942618	0,992891	0,999331	0,999949	0,999997
Tot. Inventory cost	3,513727	8,377636	13,50095	18,6575	23,81724	28,97722

Class aca

Inventory position, k:	1	2	3	4	5	6
Fill rate:	0,763847	0,969618	0,997334	0,999823	0,999991	1
Tot. Inventory cost	0,31776	0,721122	1,136013	1,551939	1,967935	2,383935

Class baa

Inventory position, k:	1	2	3	4	5	6
Fill rate:	0,863236	0,99019	0,999525	0,999983	0,999999	1
Tot. Inventory cost	50,06767	107,4987	165,4711	223,4701	281,4701	339,4701

Class bba

Inventory position, k:	1	2	3	4	5	6
Fill rate:	0,869287	0,991058	0,999587	0,999986	1	1
Tot. Inventory cost	10,25759	21,95208	33,74721	45,54704	57,34703	69,14703

Class bca

Inventory position, k:	1	2	3	4	5	6
Fill rate:	0,895772	0,994369	0,999795	0,999994	1	1
Tot. Inventory cost	0,945936	1,995989	3,051773	4,107767	5,163767	6,219767

Class caa

Inventory position, k:	1	2	3	4	5	6
Fill rate:	0,978605	0,999769	0,999998	1	1	1
Tot. Inventory cost	234,8652	474,8099	714,8095	954,8095	1194,809	1434,809

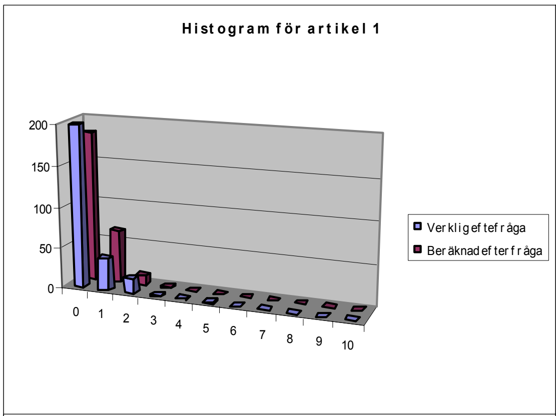
Class cba

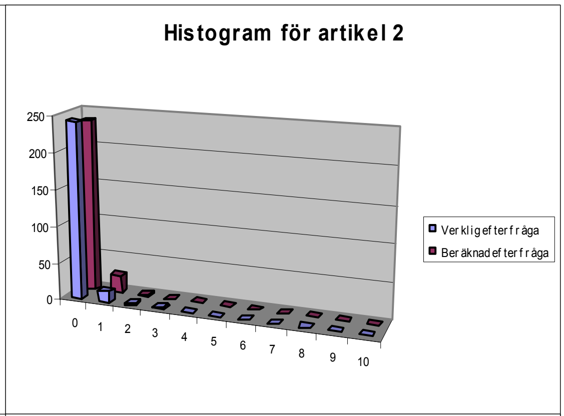
Inventory position, k:	1	2	3	4	5	6
Fill rate:	0,985685	0,999897	1	1	1	1
Tot. Inventory cost	49,38282	99,47766	149,5776	199,6776	249,7776	299,8776

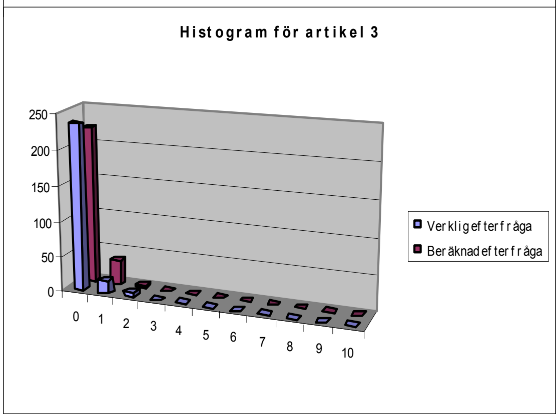
Class cca

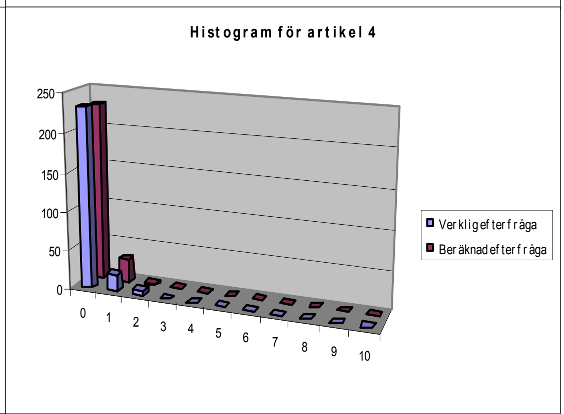
Inventory position, k:	1	2	3	4	5	6
Fill rate:	0,97469	0,999677	0,999997	1	1	1
Tot. Inventory cost	7,017767	14,21544	21,41542	28,61542	35,81542	43,01542

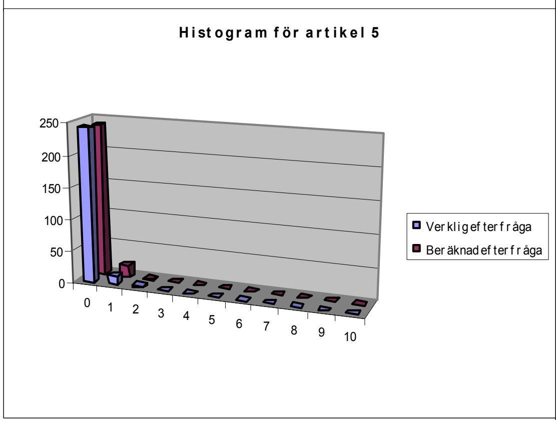
Appendix 8: Comparing histograms by visualising incoming components.

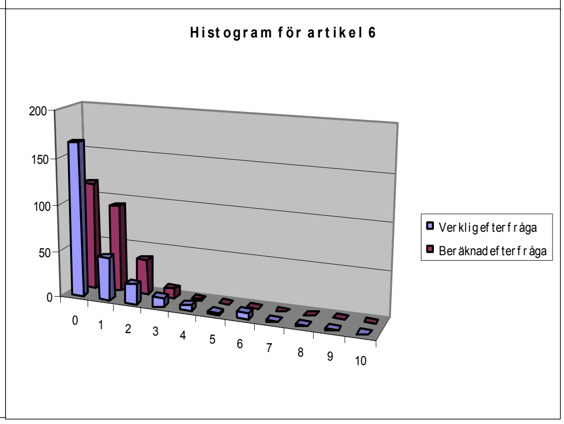












Appendix 9: Chi-Square tests for incoming components.

Component 1:			
Observed value	Calculated value	(Diff^2) / calc.value	Sort
199	184	1,2228	1,2228
40	65	9,6154	9,6154
18	12	3	3
1	1	0	0
2	0	#DIV/0!	0
1	0	#DIV/0!	0
0	0	#DIV/0!	0
			x^2: 13,8382

Component 2:			
Observed value	Calculated value	(Diff^2) / calc.value	Sort
241	235	0,1532	0,1532
15	24	3,375	3,375
3	1	4	4
2	0	#DIV/0!	0
0	0	#DIV/0!	0
0	0	#DIV/0!	0
0	0	#DIV/0!	0
			x^2: 7,5282

Component 3:			
Observed value	Calculated value	(Diff^2) / calc.value	Sort
235	223	0,6457	0,6457
18	35	8,2571	8,2571
6	3	3	3
0	0	#DIV/0!	0
			x^2: 11,9029

Component 4:			
Observed value	Calculated value	(Diff^2) / calc.value	Sort
233	229	0,0699	0,0699
22	30	2,1333	2,1333
6	2	8	8
0	0	#DIV/0!	0
			x^2: 10,2032

Component 5:			
Observed value	Calculated value	(Diff^2) / calc.value	Sort
243	241	0,0166	0,0166
15	19	0,8421	0,8421
3	1	4	4
0	0	#DIV/0!	0
0	0	#DIV/0!	0
0	0	#DIV/0!	0
0	0	#DIV/0!	0
			x^2: 4,8587

Component 6:			
Observed value	Calculated value	(Diff^2) / calc.value	Sort
167	116	22,4224	22,4224
46	94	24,5106	24,5106
21	38	7,6053	7,6053
10	10	0	0
6	2	8	8
2	1	1	1
6	0	#DIV/0!	0
			x^2: 63,5383

Appendix 10: An example of a Kolmogorov-Smirnov test for an incoming component.

Art nbr	Total orders	Average time between orders
490798	36	6,27778

Order delivery date	Time between orders	Sort	i/n	Fxi	D+	D-
			0			
2005-01-05	6	0	0,027778	0	0,027778	0
2005-01-13	3	0	0,055556	0	0,055556	-0,02778
2005-01-18	3	0	0,083333	0	0,083333	-0,05556
2005-01-21	1	0	0,111111	0	0,111111	-0,08333
2005-01-24	8	0	0,138889	0	0,138889	-0,11111
2005-02-03	0	0	0,166667	0	0,166667	-0,13889
2005-02-03	0	0	0,194444	0	0,194444	-0,16667
2005-02-03	5	0	0,222222	0	0,222222	-0,19444
2005-02-10	0	0	0,25	0	0,25	-0,22222
2005-02-10	20	0	0,277778	0	0,277778	-0,25
2005-03-10	4	0	0,305556	0	0,305556	-0,27778
2005-03-16	30	1	0,333333	0,147253	0,186081	-0,1583
2005-04-27	9	1	0,361111	0,147253	0,213858	-0,18608
2005-05-10	79	1	0,388889	0,147253	0,241636	-0,21386
2005-08-29	12	2	0,416667	0,272822	0,143845	-0,11607
2005-09-14	4	2	0,444444	0,272822	0,171622	-0,14384
2005-09-20	0	2	0,472222	0,272822	0,1994	-0,17162
2005-09-20	0	3	0,5	0,379901	0,120099	-0,09232
2005-09-20	0	3	0,527778	0,379901	0,147877	-0,1201
2005-09-20	0	3	0,555556	0,379901	0,175655	-0,14788
2005-09-20	0	4	0,583333	0,471212	0,112121	-0,08434
2005-09-20	8	4	0,611111	0,471212	0,139899	-0,11212
2005-09-30	4	4	0,638889	0,471212	0,167677	-0,1399
2005-10-06	0	4	0,666667	0,471212	0,195454	-0,16768
2005-10-06	4	4	0,694444	0,471212	0,223232	-0,19545
2005-10-12	1	5	0,722222	0,549078	0,173145	-0,14537
2005-10-13	5	5	0,75	0,549078	0,200922	-0,17314
2005-10-20	3	6	0,777778	0,615477	0,162301	-0,13452
2005-10-25	0	6	0,805556	0,615477	0,190078	-0,1623
2005-10-25	1	8	0,833333	0,720384	0,11295	-0,08517
2005-10-26	2	8	0,861111	0,720384	0,140728	-0,11295
2005-10-28	4	9	0,888889	0,761558	0,127331	-0,09955
2005-11-03	2	12	0,916667	0,852142	0,064524	-0,03675
2005-11-07	2	20	0,944444	0,958657	-0,01421	0,04199
2005-11-09	6	30	0,972222	0,991594	-0,01937	0,047149
2005-11-17	0	79	1	0,999997	3,43E-06	0,027774
2005-11-17						
					0,305556	0,047149
					max:	0,305556

Appendix 11: A compilation of Kolmogorov-Smirnov test for incoming components.

	Total orders; dof	Max. value	Critical values
Component 1	151	0,596	0,1326
Component 2	47	0,5957	0,2378
Component 3	36	0,3056	0,2717
Component 4	63	0,5714	0,2054
Component 5	20	0,1274	0,356
Component 6	214	0,5654	0,1114