

ABSTRACT

- Title: Trading strategies with market impact constraints
– A case study at SEB
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- Research purpose: Purpose of the master thesis is to develop an algorithmic trading model for a single asset, which is well functioning on the Scandinavian market.
- Deliverables: The main focus has been towards creating an indicating model to quantify market impact, cost of trading and model an efficient trading frontier.
- Methodology: Initially we focused on research papers and literature within our specific field trying to get a deeper knowledge in related theories. Further we had meetings with our project initiators to understand their ambitions with the project. This helped us to formulate the direction of our thesis.
- After broad theoretical studies we started to evaluate different models that are used to quantify market impact and efficient trading. We understood that there is not a general model that is used to quantify these values. At this point we were able to decide on certain models to approach and which simplifications that had to be done. We decided to use the theoretic framework for a market impact model created by US researchers.
- Finally we compiled all research and data into our master thesis. We also had several meetings with experienced individuals from the stock market to discuss the validity of our models that we had developed. In the end we delivered a clear theoretical framework and a model that is simple to use.
- Conclusions: Based on cutting edge research in financial trading theory combined with discussions with traders on the Scandinavian market a pre trade analysing application has been created. The application is today usable to facilitate the decision-making process when minimizing cost at a desired risk level for a large stock order. The application provides the user with cost and risk

estimates. Additional information on how to liquidate the initial portfolio and anticipated market participation is supplied to the user. The information provided by the application is realistic according to experienced professionals.

However manual trading can not be completely replaced. Several orders are of such kind that algorithms are not capable of executing these. Further a good trader takes much information into consideration when trading not possible to capture in algorithms. The conclusion is that traders and algorithms should work side by side to maximize profit. Standardized orders are handled by automatized trading systems allowing traders to focus on more complex orders.

Keywords:

Market Impact, Pre Trade Analysis, Timing Risk, Optimal Trading Trajectories, Trading, Expected Cost, Variance of Cost, Temporary Market Impact, Permanent Market Impact.

OUTLINE:

- Chapter 1: The purpose of this chapter is to give the reader an objective view of SEB's business, organisation, market position and strategy.
- Chapter 2: In this chapter we clarify the background which explains the purpose of the master thesis.
- Chapter 3: The purpose of this chapter is to give the reader an insight in the scientific approach of this master thesis.
- Chapter 4: The ambition of this chapter is to give the reader a deeper knowledge of certain theoretical theories.
- Chapter 5: The development of a useful model is described to the reader in this chapter.
- Chapter 6: The aim of this chapter is to develop and explain the application based on model 1 and model 2.
- Chapter 7: In the following chapter a result summary is presented to give the reader an example of how to utilize the application. Further how well the research purpose is fulfilled is concluded.
- Chapter 8: Our recommendations how to benefit the most from the results found in our research are described and discussed in the following chapter. Further we make recommendation how SEB can take algorithmic trading to the next level.

PREFACE

This master thesis has been a project at SEB Enskilda in Stockholm during the fall of 2006. The main objective of the thesis has been to develop a model that is used for algorithmic trading. It will contribute when trading large volumes of stocks as a pre trade analysis tool and a sales explanatory tool to customers.

Our time at SEB Enskilda has given us great experiences from professional work environment as well as great practice to be part of a project.

Finally, we want to thank all people that has been involved and contributed to this thesis. Our supervisors at SEB Christer Wennerberg and Anders Mårtensson for their real data input, research material and interesting discussions. Further our supervisor at Lund Institute of Technology Ingela Elofsson for her great attitude and remarkable advices. At last, our mathematical support from Lund Institute of Technology, Professor Thomas Guhr and Doctor Rudi Schäfer, who guided us through parts of the hard mathematical framework.

Stockholm January 2007,

Gustaf Tegell

Martin Martinsson

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1. INTRODUCTION

The purpose of this chapter is to give the reader an objective view of SEB's business, organisation, market position and strategy.

1.1 SEB in brief

Skandinaviska Enskilda Banken (SEB) is a North-European retail and investment bank. SEB was founded 1972 through a merger between Stockholm's Enskilda Bank (founded 1856) and Skandinaviska Banken. Their customers are institutions, corporations and private individuals. The head office is located in Stockholm although SEB is represented in 20 countries around the world with more than 20 000 employees. SEB has more than five million customers in northern Europe and in other important financial locations. They have an international business with 50 per cent of the profit generated outside Sweden. SEB has a significant position on the North European retail market with a 16 percent market share in Sweden and 30 percent market share in the Baltic countries. The bank has a great reputation in the financial sector with top rankings in areas as equity research, cash management and private banking. 2005 SEB had an operating profit of 11 223 MSEK and the SEB share is listed on the OM Stockholm Stock Exchange¹.

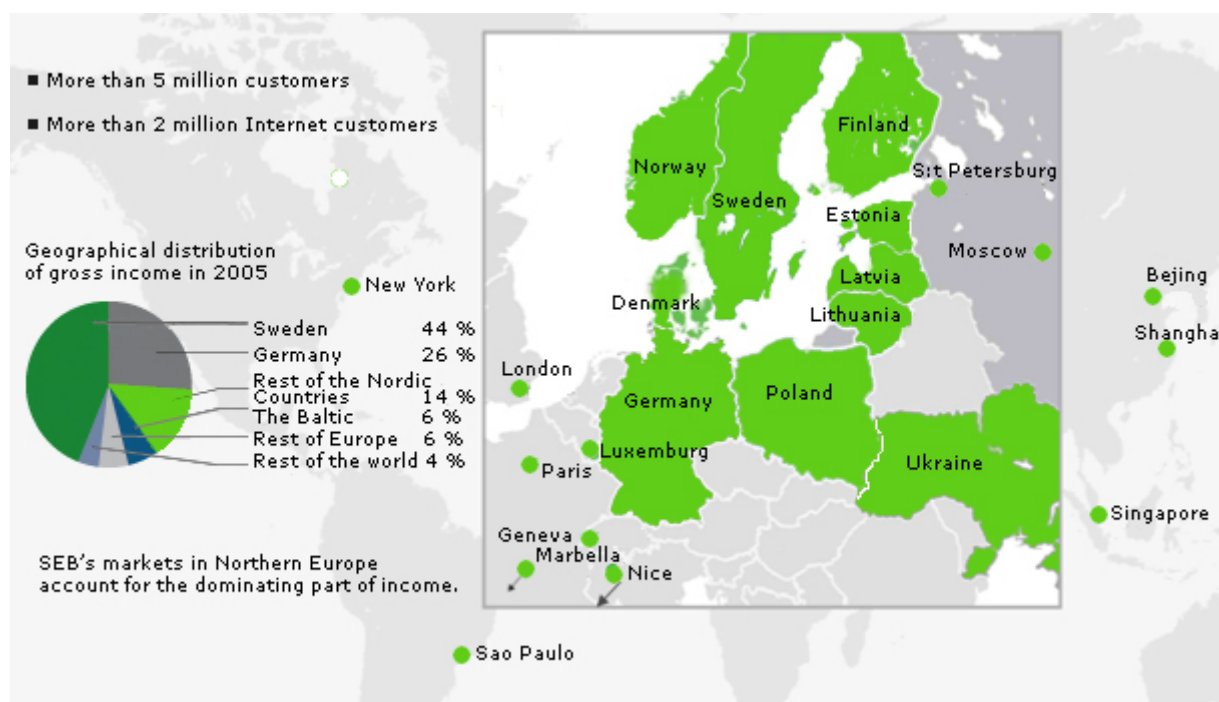


Figure 1.1 SEB market locations and geographical distributions²

¹ <http://www.sebgroup.com/pow/wcp/sebgroup.asp?website=TAB1&lang=en>

² Ibid.

1.2 Vision

SEB has made many strategic moves since the merger in 1972. The competition in the financial industry has become more significant. The vision of becoming the leading North-European bank has demanded several acquisitions to strengthen the bank within life insurance and asset management. As the number of customers has increased in other European countries SEB has expanded mainly in Germany and Eastern Europe. The large expansion resulted in the 3 C-programme (Cost efficiency, increased Customer satisfaction and Cross-servicing within the Group) which purpose is to reach full potential of the acquisitions and mergers³.

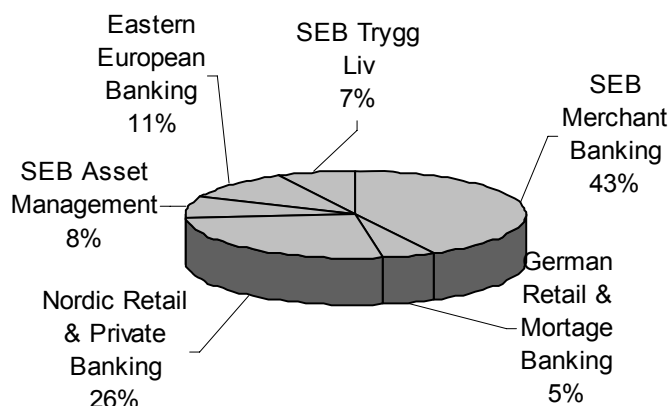


Figure 1.2 Operating profit per division 2005

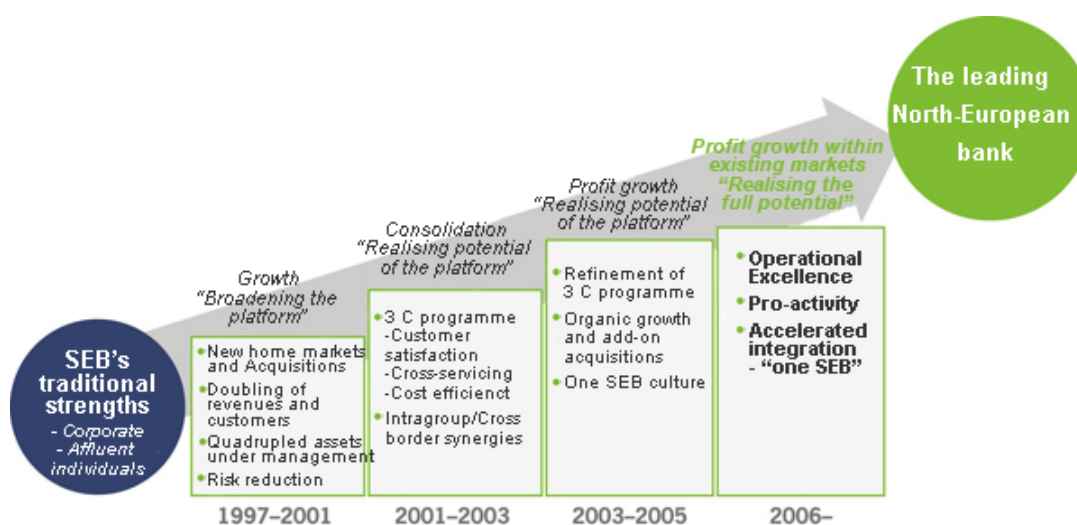


Figure 1.3 Strategic development of SEB⁴

The business concept of SEB is to have satisfied customers and to give investors a competitive return. Further more SEB aims to be considered as a good citizen of society. By providing attractive financial solutions and by local market knowledge SEB aims to achieve these goals.

More precisely the vision of SEB is:

“SEB shall be the leading North-European bank in terms of customer satisfaction and financial performance. Motivated employees and strong cooperation within the Group “One SEB” are prerequisites for reaching the goals⁵.”

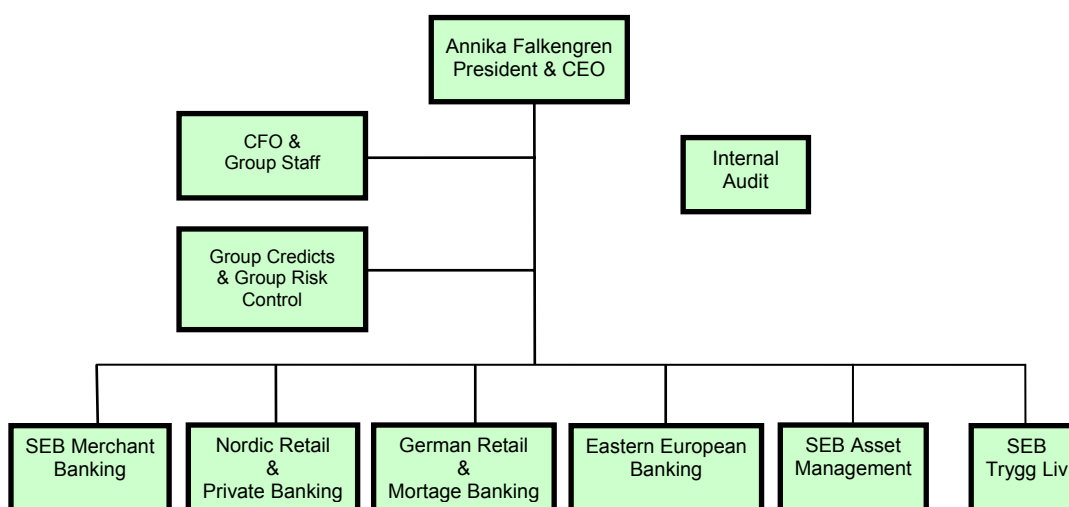
³ <http://www.sebgroup.com/pow/wcp/sebgroup.asp?website=TAB1&lang=en>

⁴ Ibid.

⁵ <http://www.seb.se/pow/wcp/sebgroup.asp?website=TAB1&lang=en>

1.3 Organisation

To meet the needs of a competitive financial market the organisation of SEB has seen a dramatic transformation over the last years. The organisation is divided into six different divisions where SEB Merchant Banking is the largest division with 42% of the total operating profit. This master thesis is within the Merchant Banking division as it is in charge of all stock trades. Today Marcus Wallenberg is Chairman of the Board and Annika Falkengren is CEO for Skandinaviska Enskilda Banken⁶.



Figur 1.4 Organisation of Skandinaviska Enskilda Banken⁷

1.4 SEB Merchant Banking

SEB Merchant Banking is the division that is responsible for large and medium sized corporations, financial institutions and commercial real estate clients. This division consists of two business units; SEB Merchant Banking and Enskilda Securities.

SEB Merchant Banking offers financial services. They have main responsibility for the relationship with SEB’s large and medium size customers. They trade all different kinds of financial instrument e.g. currencies, bonds, derivatives, futures and stocks. Merchant Banking also has a number of advisory, venture capital custody services and other financial services that large and medium size customers demand. SEB Merchant Banking is the largest division of SEB and has reported stable and good results for several years. It has about 2 800 employees in thirteen different countries.

Enskilda Securities is SEB’s investment bank that offers corporate finance services, equity trading and equity research. They have a strong market position on the Nordic market and

⁶ <http://www.sebgroup.com/pow/wcp/sebgroup.asp?website=TAB1&lang=en>

⁷ Ibid.

have also had high rankings in different surveys the last years. Enskilda has about 420 employees in seven different countries.

Both business units trade large volumes of stocks that can affect market equilibrium. Therefore they have a great interest in algorithmic trading and trading strategies for large volume trading which will make their trading more efficient⁸.

⁸ <http://www.sebgroup.com/pow/wcp/sebgroup.asp?website=TAB1&lang=en>

2. BACKGROUND & PROBLEM DISCUSSION

In this chapter we clarify the background which explains the purpose of the master thesis

2.1 Background

Skandinaviska Enskilda Banken (SEB) is today one of the largest players on the Northern European market for retail and investment banking. A key competence is to deliver competitive stock trades. Since the IT revolution the stock market has become an exceptionally efficient and fast moving market. Most stock trades executed by SEB are not large enough to affect the market. There are trades though, that are of such volume that they actually have a market impact. These trades are related to specific strategies which aim to minimize cost and risk for the trade. Traditionally brokers with great experience from the market execute these orders based on their knowledge and experience. If trading is executed during a short time period there will be a large market impact but the broker will not experience a risk that the stock moves in an unfavourable direction. On the opposite trading over a long time period results in a small market impact but a large risk of market movement. Therefore it is of great interest to SEB Merchant Banking to clarify the characteristics of large volume trades and how to execute these efficiently⁹.

2.2 The stock market

The Scandinavian stock market has in the past been dominated by a few large brokers. These players have had a relative comfortable market position since the relationship between customer and broker has been based on a good relationship. Further more, not many actors have had direct access to the stock market, which has enabled the broker to charge high commissions and fees. Usually the communication between customer and broker has been through telecommunication related with a time consuming order process.

Since the expansion of information technology the stock market has seen remarkable changes. Many new brokers have now gained access to the market. These players compete with lower fees and direct access to the market for the end user. The new brokers focus preliminary on small investors with standardised trades.

However, these brokers constantly develop their trading applications with the result of larger market shares. The original brokers have lost their monopoly of stock trading and today focus more and more on large and unique trades.

A market is driven by supply and demand, especially the stock market since it is distinguished by many players and an efficient technology which results in a fast moving market. Today most trades do not affect market equilibrium as the volume of these trades are not large enough. However there are trades that have an impact on the market equilibrium. Depending

⁹ Christer Wennerberg, SEB Merchant Banking, Stockholm 2006-09-15

on the size of the trade as well as the liquidity of the stock, the market can be temporarily or permanently affected¹⁰.

2.3 Algorithmic trading

Algorithmic trading is a tool which uses predefined strategies to facilitate the decision making for a trader. The algorithm uses a model which can be a simple or complex application depending on transaction form. The information technology revolution has developed the usage of algorithmic trading. As mentioned earlier most trades today are done manually, many of these can in the future be executed through automatic trading systems.

SEB has a great interest in building strategies for execution of trades when dealing with large volumes of stocks. The strategies are related to different risks. Therefore SEB would like to quantify the risks related to each trade. Due to the relationship between risk and cost a strategy can be defined that takes both these interests into consideration¹¹.

2.4 Purpose

Purpose of the master thesis is to develop an algorithmic trading model for a single asset, which is well functioning on the Scandinavian market.

2.5 Target Group

The target of this master thesis is primarily two groups, professional people working in the financial industry and students studying financial mathematics:

In the financial industry it is of interest for traders at the Scandinavian Stock Market, professionals working in the business development division and for sales people in the financial industry.

Students studying financial mathematics can read this to understand the basics of market impact, costs of trading and timing risk.

¹⁰ Johan Larsson, SEB Enskilda Equities, Stockholm, 2006-12-05

¹¹ Anders Mårtensson, SEB, Stockholm, 2006-09-05

3. METHODOLOGY

The purpose of this chapter is to give the reader an insight in the scientific approach of this master thesis.

3.1 Introduction

When doing a scientific research it is important to have a structure in the investigations. Several methods how to target the purpose of the research are available. Numerous important circumstances have to be addressed before choosing which method to use. Additionally the purpose or problems investigated in the survey are important factors for which method to use. The choice of method is very dependent on which knowledge is available¹².

3.2 Different scientific approaches

3.2.1 Problem identifying

The first issue when investigating an unknown field is to explore the theoretic framework. When the big picture of the issue is understood a framework of the problem can be stipulated. The first exploration of the subject often results in a deeper research to fully describe or explaining the issues¹³.

Often the purpose of this investigation is to identify a problem from previous premonitions. From this investigation a hypothesis is formulated.

3.2.2 Descriptive, explanatory, diagnostic and normative studies

When the theoretic framework is well known a purpose for the study is to describe rather than understand the problem. The description of the problem varies depending on the utilization of the study. A description of past events as well as present events may be of interest. Further the reasons and relationship between occurrences may be the prime identification target¹⁴.

Besides describing the issues an additional purpose may be to explain these. The depth of the explanation study can vary. An overview of the issues is sometimes sufficient as well as an in depth analyse may be required. The purpose is to identify and analyse the factors that affect the occurrences of interest. Often these studies evaluate previous theories. Frequently this information is used to determine strategies in situations where the factors identified are in use¹⁵.

A diagnostic investigation uses occurrences to establish the driving factors behind the results. The method starts with the results and tries to find solutions to these occurrences. Often a negative outcome of a strategy results in a diagnostic investigation¹⁶.

¹² *Utredningsmetodik för samhällsvetare och ekonomer*. Lundahl, Ulf and Skärvad, Per-Hugo, p. 7-16

¹³ *Utredningsmetodik för samhällsvetare och ekonomer*. Lundahl, Ulf and Skärvad, Per-Hugo, p. 55

¹⁴ *Utredningsmetodik för samhällsvetare och ekonomer*. Lundahl, Ulf and Skärvad, Per-Hugo, p. 48

¹⁵ *Utredningsmetodik för samhällsvetare och ekonomer*. Lundahl, Ulf and Skärvad, Per-Hugo, p. 47

¹⁶ *Utredningsmetodik för samhällsvetare och ekonomer*. Lundahl, Ulf and Skärvad, Per-Hugo, p. 48

A normative investigation makes statements that describe how outcome for certain actions are¹⁷. The investigation is similar to making case recommendations and what the outcome for each action is. The purpose is to be able to make a recommendation on what action to take¹⁸.

3.2.3 Our scientific approach

The approach in our thesis is a combination of describing the phenomenon of market impact and to explain this occurrence. Additionally the problem is of a diagnostic nature. All research started as diagnostic investigations to an occurrence experienced on the stock market, resulting in a framework of theory in the subject.

Traders at SEB have experienced market impact and have intuitions of why this happens. Further the project initiators at SEB have a general idea of the driving factors of the occurrence. The theory is today widely discussed but no straight forwards solutions exist. Our approach is to give a deep description and explanation of the today existing theoretical framework.

3.3 Data gathering

3.3.1 Written sources, interviews, surveys and questionnaires

Written sources are articles, books and research papers which all are examples of secondary data¹⁹.

An interview with experienced people is a central method of the qualitative study to gather primary data.

A survey²⁰ is a way of collecting primary data that could be used to describe a current situation. Surveys can be done market researches, voter surveys, and attitude researches. Data that is collected in a survey is of quantitative character. Parts of a population are examined and used to represent the rest of the population.

If a large number of people are asked a questionnaire is an easier approach to receive a large number of data.

3.3.2 Our approach of data gathering

Data are gathered from different sources; initially we read different research papers and books to get a better knowledge in the subject. SEB's intranet has been used to collect information about SEB in general. Additional interviews have given us a qualitative approach to our survey.

¹⁷ <http://en.wikipedia.org/wiki/Normative>

¹⁸ *Utredningsmetodik för samhällsvetare och ekonomer*. Lundahl, Ulf and Skärvad, Per-Hugo, p. 237

¹⁹ *Utredningsmetodik för samhällsvetare och ekonomer*. Lundahl, Ulf and Skärvad, Per-Hugo, p. 52

²⁰ *Utredningsmetodik för samhällsvetare och ekonomer*. Lundahl, Ulf and Skärvad, Per-Hugo, p. 168-182

3.4 Data types

3.4.1 Primary or secondary and quantitative or qualitative data

Data that is collected by the researchers for a specific purpose is referred as primary data²¹. Data that already exists and collected by other researchers is secondary data.

In research there is a difference between quantitative and qualitative method. In a qualitative study the vital part is verbal whereas a quantitative study is from measurable data and is used to evaluate specific models. Qualitative data is often from a personal interview and quantitative is measures from nature²².

3.4.2 Our data types

To deduct best possible survey we have used both quantitative and qualitative data. Our quantitative survey is done based on a database from Stockholm Stock Exchange. Interviews done at SEB Enskilda are used in the qualitative study where a specific questionnaire is used to evaluate our hypothesis. The data collected from interviews and database is both sources of primary data.

3.5 Reliance of research

3.5.1 Validity, Reliability and Objectivity

The level that the researchers have been able to investigate their research problem is often referred as validity. Further the validity is a measurement of how well the intended measurement is measured. Additionally validity is said to be the correlation of the theoretic definition and the real life definition²³.

Reliability refers to the precision and exactness of a measuring tool. The result of the measurement tool should give the same result regardless of whom and under which circumstances it is executed. The measurement tool should not have a random measurement deviation and should decrease the stochastic deviation²⁴.

It is important that a scientific study is objective and personal values of the authors do not influence and affect the real data and the questions asked. The level of objectivity of the research means the involvement of the authors²⁵.

3.5.2 Reliance of our research

Since the theoretic definition of market impact is how much the price is affected by an order and the real life measurement is directly observed the validity between theory and practice is

²¹ *Utredningsmetodik för samhällsvetare och ekonomer*. Lundahl, Ulf and Skärvad, Per-Hugo, p. 145

²² *Utredningsmetodik för samhällsvetare och ekonomer*. Lundahl, Ulf and Skärvad, Per-Hugo, p. 183

²³ <http://sv.wikipedia.org/wiki/Validitet>

²⁴ *Utredningsmetodik för samhällsvetare och ekonomer*. Lundahl, Ulf and Skärvad, Per-Hugo, p. 152

²⁵ *Utredningsmetodik för samhällsvetare och ekonomer*. Lundahl, Ulf and Skärvad, Per-Hugo, p. 74-75

high. Our investigation results in a quantitative measurement of stock price concession thus answer the theoretical question.

A model can not fully describe real life thus our predicted market impact will deviate from experienced in the market. However the model will give the same results independent of by whom and when it utilized. Unfortunately the model can not be statistically evaluated due to lack of correct data. Our evaluating is based on discussions with experienced stock traders in order to see if the models are reasonably accurate. The purpose of SEB is not to get an exact measurement but an indication for the trade off between market impact and timing risk.

Since we do not have hands on experience from market impact orders no preconceived notions of the end results are made. Further SEB, as the end user of the results, are interested in manage market impact in the most efficient way. They are very keen to get estimates of the real costs not a refined estimate. As a result no pressure to improve the results has been made. However interviews with traders may be affected by their personal experience but should not influence the quantitative results.

3.6 Investigation approaches

3.6.1 Induction, deduction and verification approaches

An inductive approach explores the environment to create a model or theory²⁶.

A deductive approach uses an already existing theory or model to draw certain conclusions for specific events²⁷.

Verification is used to confirm the result of an investigation²⁸.

3.6.2 Our investigative approach

In our master thesis we use a deductive approach. We have collected our empirical data from an already existing database. Models we use have their background in already existing theory from scientific papers and literature. From data and theory we have been able to draw certain conclusions.

3.7 Practical approach

Initially we focused on research papers and literature within our specific field trying to get a deeper knowledge in related theories. Further we had meetings with our project initiators to understand their ambitions with the project. This helped us to formulate the direction of our thesis.

²⁶ *Utredningsmetodik för samhällsvetare och ekonomer*. Lundahl, Ulf and Skärvad, Per-Hugo, p. 41

²⁷ Ibid.

²⁸ Ibid.

After broad theoretical studies we started to evaluate different models that are used to quantify market impact and efficient trading. We understood that there is not a general model that is used to quantify these values. At this point we were able to decide on certain models to approach and which simplifications that had to be done. We decided to use the theoretic framework for a market impact model created by US researchers.

In chapter 4 the theoretical framework of efficient trading and trading strategies is stipulated. Simplifications are necessary to use the framework in real life. Since there is not enough real data available to estimate the functions as well as the constants to the Swedish market we have to rely on the research done on the US market. To use their estimates of the functions and constants we use the same values and create a model to the Swedish market.

Because of the complex mathematical framework we had several discussions with our supervisors from Lund Institute of Technology in order to create possible and well working models. A limitation in the model estimation had to be done due to lack of sufficient data.

We have continuously been in contact with SEB in order to develop the project in the right direction. Additionally we have had several meetings with experienced individuals from the stock market to discuss the validity of our models.

4. THEORETICAL BACKGROUND

The ambition of this chapter is to give the reader a deeper knowledge of certain theoretical theories.

4.1 Pre trade analysis

The last years increase in competitiveness on the financial markets has improved the algorithmic trading activity. A pre trade analysis provides investors with liquidity summaries, cost & risk estimates and trading difficulty which will give them an insight if algorithmic trading is of interest for the specific trade. Not all orders can be handled by algorithms thus requires manual execution, e.g. a stock that is very illiquid is very hard to design an appropriate algorithm for. The idea of a pre trade analyses is to give the trader as much information as possible to develop the best strategy for the trade. Depending on the purpose of the trade there are different strategies to be chosen²⁹.

An essential factor in trading performance is to use a benchmark price. There are different benchmark prices which can be categorized into pre-, intra- and post prices. Pre trade prices are known before the time of trading and are often referred to as implementation shortfall benchmark prices. The most common are the investment decision price, previous night's closing price, opening price and price at time of order entry. Intra day price benchmarks are decided throughout the trade, such as volume weighted average price (VWAP), high and low. The post trade benchmarks are known after the trade has been executed. The most common is the day's closing prices³⁰.

Different investors use the benchmark price that best is coherent with their strategy. An investor using fundamental analyses may want to use the price at decision time, e.g. the investor's belief of the true value of the stock. An investor following the index may be interested in VWAP execution³¹.

When the benchmark price is chosen the next step is to find the optimal strategy for a specific trade. This is often referred to as the trader's dilemma. When deciding the optimal strategy it is necessary to be aware of the costs and risks of the trade. After the trade is carried out it is essential to do a post trade analysis to evaluate the algorithm and traders performance. The cost of a trade is the difference between the benchmark price and the actually received price. A post trade analysis is essential to evaluate the model as well as the trader. If the actual cost largely deviates from the pre trade estimates the model or the trader needs to be questioned³².

4.2 Market impact

Price movement of a stock is affected by several variables, usually divided into macro and micro factors. The macro aspects are e.g. the business cycle, the oil price, political situations and many more. The micro factors are the factors related to the specific stock, such as specific

²⁹ *Understanding the profit and loss distribution of trading algorithms*, Kissel & Malamut, page 2

³⁰ Ibid.

³¹ Ibid.

³² Ibid.

company events. The price movements of the stock caused by a single trade or order are called market impact.

Due to the many factors that may cause stock movements it is hard to distinguish when market impact has occurred. Market impact affects a buyer and seller in different directions, a seller is affected from decreased prices and a buyer from increasing prices³³.

Market impact of an order is often measured as the difference between the stock price at the beginning of the trade and the average received price for the order. A problem with the measurement is that all market activities are accumulated in the price movement. The measurement is actually the total trading cost comprising the stock's natural movements, volatility as well as the market impact of all orders. To be able to use the measurement as market impact an assumption that all other activities are normal occurrences is made. Market impact is therefore fundamentally caused by supply and demand imbalance and information leakage.

4.2.1 Supply-demand imbalance

If the market is in equilibrium any additional order will result an imbalance in the supply and demand equilibrium. Market impact is somewhat driven by basic microeconomics, supply and demand theory. Every time an investor enters an order, either demanded shares or supplied shares, to the market there is change in equilibrium. If there is a difference between the bid and offered price the investor has to attract their counterparties to do the trade, through paying a premium compared to the spread causing the price to move. Figure 4.1 describes supply and demand in a stock market³⁴.

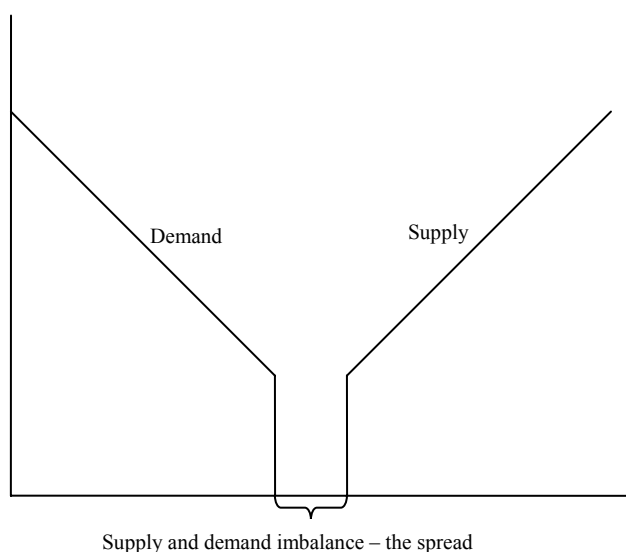


Figure 4.1 Supply and demand in a stock market³⁵

³³ *Optimal Execution of Portfolio Transactions*, Almgren, Robert and Chriss, Neil, page 8

³⁴ Anders Mårtensson, SEB, Stockholm, 2006-09-05

³⁵ Ibid.

4.2.2 Information leakage

Every order that is released to the market expresses information relating to the trade and the specific stock. Investors communicate information to the market concerning its current and future expectations of the stock price through their order size and characteristics of the order. Unusual behaviours often start rumour, which concerns every market participant interested in this specific stock resulting in a chain reaction in supplied and demanded quantity of the stock.

4.2.3 Temporary market impact

Temporary market impact occurs whenever an order is released to the market affecting the equilibrium but is not providing essential information changing the market behaviour or valuation for the specific company. A common description of market impact for a buy order is that an investor triggers a price increase and buys at a higher price followed by a price decline post the trade. Thus temporary market impact is a short lived disturbance in the market followed by a reversion back to the original balance³⁶.

4.2.4 Permanent market impact

Permanent market impact occurs when an order leads the market to believe that future price will be different than original price. An order indicating that the stock is incorrectly priced thus changes the markets belief of the stock is referred to as a permanent market impact order. A large order may start rumours resulting in a large price change. The change often contains both permanent and temporary market impact. Temporary market impact is the part that disappears with the rumours and then there is a price stabilisation and only permanent market impact has remaining effect. At a first stage when a large trade is executed the price equilibrium is changed to attract a counterpart which is shown in figure 4.2. When the trade is finished the share price is stabilised at the new market equilibrium which also is shown in figure 4.2.³⁷

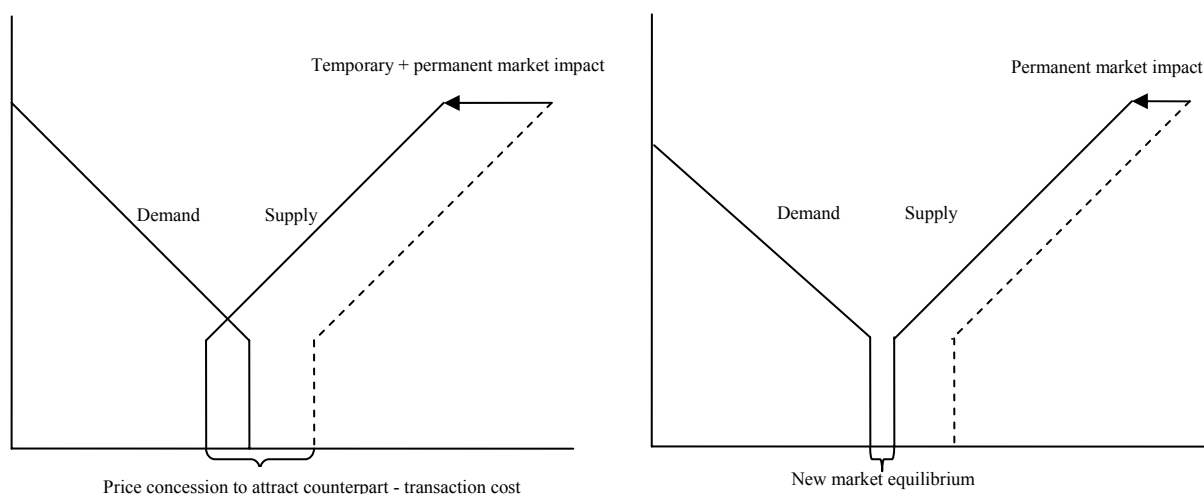


Figure 4.2 Equilibrium change due to market impact³⁸

³⁶ *Optimal Execution of Portfolio Transactions*, Almgren, Robert and Chriss, Neil, page 8

³⁷ *Optimal Trading Strategies*, Kissel & Glantz

³⁸ Anders Mårtensson, SEB, Stockholm, 2006-09-05

4.2.5 Timing risk

Timing risk indicates the uncertainty of the trading cost estimate as there is a risk that the price changes during the trade's execution. The timing risk consists of price volatility, liquidity risk and estimation error surrounding the market impact. The expected price can change depending on news, business information or rumours that change the price from the expected. This is referred to as the price volatility. Volume available in the market is known as liquidity. The risk of not enough available volume is referred to as liquidity risk. For example if it is a large order that shall be executed there has to be a certain volume available. If this volume is less than available volume there will be an imbalance in the market. Definition of timing risk:

“Timing risk is the associated uncertainty in trading cost estimates due to price volatility and liquidity risk. Price volatility affects the price appreciation estimate and liquidity risk affects the market impact estimate³⁹.”

4.2.6 Liquidity

Liquidity is a term that refers to the volume of stocks that are available on the market. A stock that has a high liquidity is easier to trade as there is a larger volume accessible. Liquidity is changed every time there is an order executed, withdrawn or an order is entered to the market. Therefore liquidity is changed throughout the day and is different from day to day.

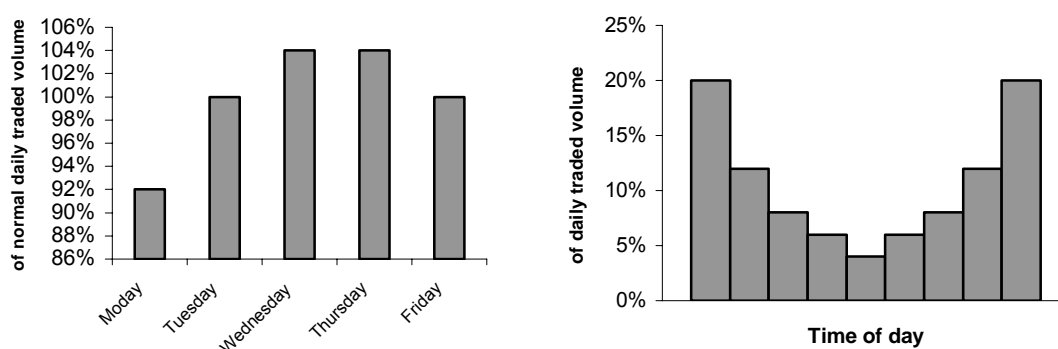


Figure 4.3 Weekly and intra day volume profiles

As shown in figure 4.3 liquidity changes from day to day but there is also an intraday volume profile i.e. the volume that is on the market during a specific time. That means that available volume changes over time during the day. The volume available is the most during post opening and pre closing. That is the first hour and the volume available decreases until an hour after lunch and then volume available starts to increase and has its maximum the last hour pre close.

³⁹ *Optimal Trading Strategies*, Kissel & Glantz, page 118

4.3 Transaction costs

Portfolio managers always aim for the highest possible return. Transaction costs are often underestimated as an important factor for a high return but are often the difference between a first-rate fund and an average fund. Transaction costs in equity markets are defined as costs incurred while implementing an investment decision⁴⁰, see figure 4.2. These costs are related to all necessary actions performed during a trade. This includes the price concession needed to attract a counterpart to the trade. Portfolio managers claim that transaction costs are around 1% but can be as much as 2 to 3% for large orders in difficult market conditions or illiquid stocks⁴¹. Many portfolio managers claim that it is an impossible incident that can not be avoided. However transaction costs can be the difference between a high return and an unsuccessful portfolio.

4.3.1 Transaction costs in global markets

Average transaction costs in global markets are 130bp⁴² which is equivalent to an average loss of 1.3% for every investment. Top researchers in this field are located in the US which has made US portfolio managers more aware of these costs. Thus US portfolio managers have the lowest average transaction costs. This is also due to the very liquid US stock market.

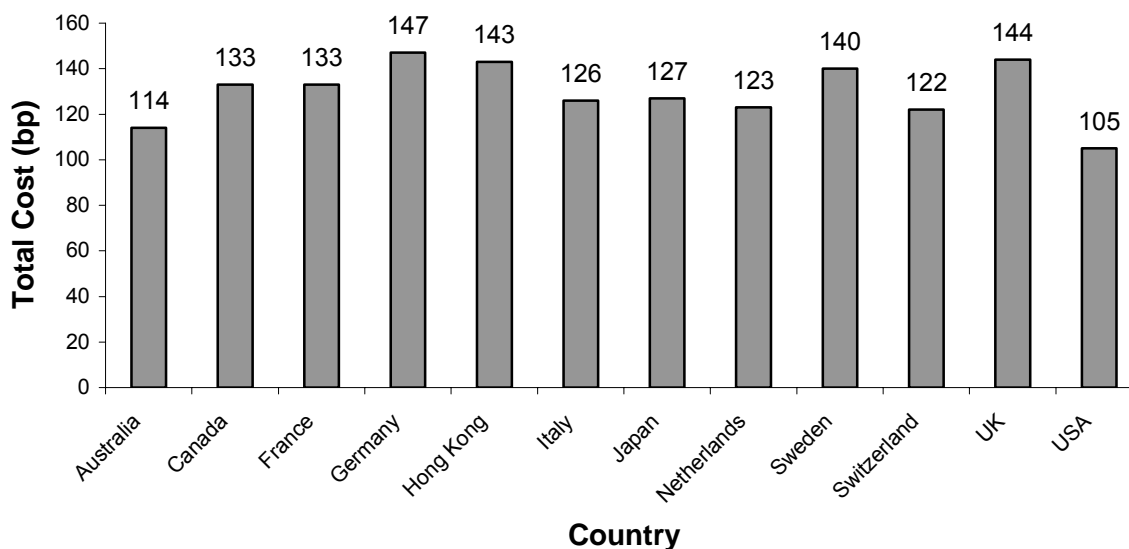


Figure 4.4 Transaction costs in Global Markets⁴³

⁴⁰ *Optimal Trading Strategies*, Kissel & Glantz, page 3

⁴¹ *Optimal Trading Strategies*, Kissel & Glantz, page 3

⁴² *Optimal Trading Strategies*, Kissel & Glantz, page 4

⁴³ *Optimal Trading Strategies*, Kissel & Glantz, page 5

4.3.2 The investment cycle - reasons for transaction costs

The investment cycle can be divided into four stages: asset allocation, portfolio construction, execution services and portfolio attribution.

1. Asset allocation is the stage when the portfolio manager decides which investment classes, i.e. stocks, derivatives, real estate and commodities, to invest in. Already at this stage transaction costs have to be considered to be able to reach expected return.
2. Portfolio construction is closely related to prior investment phase. Throughout this investment phase exact instruments to purchase or sell in each asset class are determined. If stocks are part of the portfolio instruments, the manager has to take transaction costs into consideration as there are differences in liquidity between large, mid and small capitalization stocks and therefore future result will not be as expected.
3. Execution services are the implementation of the investment cycle, also referred to as the trading phase. This is the key phase for low transaction costs. During this phase trading behaviour is decided which will return into different levels of transaction costs.
4. Portfolio attribution is to measure fund performance and determine reasons for missing expected return. This phase can also be referred as post-trade analysis. In the post trade analysis the trading performance can be evaluated. The results are very useful for future trading strategies.

4.4 Optimal trading strategies

Investors participating on the financial market seek an execution for their trades via optimal trading strategies. An optimal trading strategy has the lowest cost for the related level of risk. Often the term “trader’s dilemma” is used to describe the balance between market impact and timing risk. This since a trader knows that market impact is a decreasing function with time and volume while the timing risk increases with time and volume. For instance when trading aggressively the trader will cause a large market impact and have a low timing risk. To passive trading will incur a high timing risk and a low market impact. Therefore there is always a “dilemma” for the trader to determine an optimal strategy somewhere between these two stages and is often declared as:

“Trading too aggressively will lead to higher impact cost but trading too passively will lead to higher risk and may result in even more costly trades⁴⁴.”

This can seem like a trivial procedure but is a complicated mathematical optimisation algorithm. The first person to develop research in this subject was Harry Markowitz who started already in 1952 with mean-variance optimization which concludes the portfolio with the highest expected return for a specified level of risk. The last years focus has been more towards optimisation to minimize price impact under uncertainty. Almgren and Chriss are two of the leading researchers in this field today. Their research approach is a risk-aversion parameter that specifies the investors’ level of risk. An investor with a specified level of risk

⁴⁴ *Optimal Trading Strategies*, Kissel & Glantz, page 18

has a certain trading path that should be followed to optimise the trading cost at a specific level of risk.

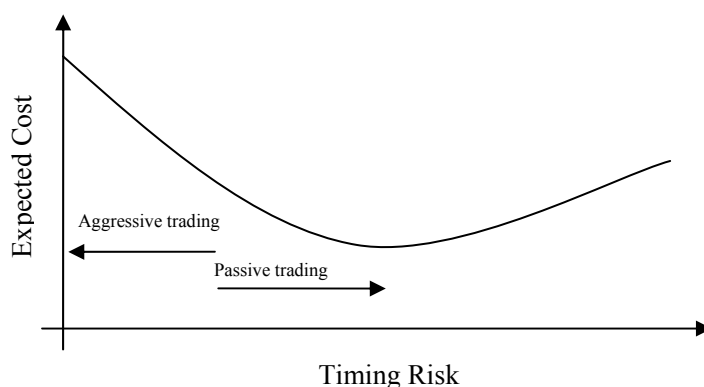


Figure 4.5 Efficient trading frontier

The trading frontier is a function of strategies that contain the lowest cost for a specified quantity of risk. The frontier is solved by optimizing the algorithm for all different risk levels. Figure 4.5 is a trading frontier where the least expected cost occurs at the minimum point. However this point is related to a high risk. All strategies beyond the minimum point are inefficient since there they are dominated by strategies with the same expected cost but a lower risk.

4.5 Existing market impact models

Market impact is a topic extensively discussed. Several researchers discuss the issue and how it affects the performance of a fund, discussed by Kissell and Glantz in *Optimal Trading Strategies* for a good overview. Several working papers discuss what market impact is and what factors influence the size and shape of the impact. Bikker et al (2004) describe in depth the most important variables and the significance of these. Breen et al (2002) tries to forecast price movements based on public data and the information available in order books. Unfortunately this information is not reliable since many actors hide their orders thus not showing their intended actions. There are several corporations focusing on developing these models. However their core business is to manufacture and sell useful analysing tools thus these models are not available to the public.

As a result no straight forward model to quantify market impact and the expected trading costs exist.

The modern foundation in the field was developed by Almgren and Chriss (2000). They create the efficient trading frontier which incorporates both cost and risk of trading. Our models will be based on their foundation and the expansions done by Almgren (2001) and also extended by Almgren, Thum, Hauptmann and Li (2005). The theory described in chapter 4.5.1 - 4.5.3 is a compilation of their work.

4.5.1 Trading Strategies

Initially we hold X shares of an asset, in this thesis the asset is considered to be stocks, which is to be fully liquidated at the end of a time frame. X is positive if we initially hold an asset which is going to be sold and X is negative if we are going to buy X units of the asset. In the thesis we only consider a single asset and not a portfolio of different assets.

Shares holding at time t is denoted by $x(t)$, with $x(0)=X$ and $x(T)=0$. If we ignore serial correlation in the asset price increments the optimal strategy may be determined prior to trading. As Almgren and Chriss (2000) argue there are only marginal improvements in the optimal trading strategy by incorporating correlation thus we ignore this price movement characteristic and are therefore able to create static strategies⁴⁵.

The discrete model is created on the following statements. The time frame of the trade is divided into uniformed trading intervals, τ with $N=T/\tau$ intervals thus $t_k=k\tau$ for $k=0,\dots,N$. The holding of stocks is denoted x_k which corresponds to time t_k , with $x_0=X$ and $x_N=0$. The sales realised between time t_{k-1} and t_k are $n_k=x_{k-1}-x_k$. This corresponds to the sale velocity $v_k=n_k/\tau$ shares per unit time.

$$x_k = X - \sum_{j=1}^k n_j = \sum_{j=k+1}^N n_j, \quad k=0,\dots,N$$

Equation 1

We make no assumptions of the rate of trading within each interval expect that the volume n_k is traded until the next time interval.

4.5.2 Price Dynamics

We denote the market value of the asset by S_k thus the initial value of the portfolio is XS_0 . The price dynamics is characterised by forces regardless of our trading, these are the volatility and the drift of the asset. However the impact of our trading has an effect on the price. The permanent impact contributes to the price dynamics because of the changes in the market equilibrium due to our trade. The price is assumed to follow an arithmetic random walk:

$$S_k = S_{k-1} + \sigma\tau^{1/2}\xi_k - \tau g\left(\frac{n_k}{\tau}\right) = S_0 + \sigma\tau^{1/2}\sum_{j=1}^k \xi_j - \tau\sum_{j=1}^k g(v_j)$$

Equation 2

The random variables ξ_j are independent random variables with zero mean and unit variance σ is the absolute volatility and $g(v)$ is the permanent impact function as a function of the rate of trading during a trading interval. The permanent impact function is assumed to be linear thus it will not affect the optimal trading strategy, which is also empirically determined by Almgren 2005. The drift is neglected in this thesis due to the relative short time horizon of the trades.

⁴⁵ Static strategies are determined in advance of trading

However the temporary impact affects the price of each trade in each interval resulting in a temporary drop in the average price per share. The price per share received at each interval is

$$\tilde{S} = S_{k-1} - h\left(\frac{n_k}{\tau}\right)$$

$$k = 1, \dots, N$$

Equation 3

Where $h(v)$ represent the temporary impact function. The interpretation of the function is that in order to trade $v\tau$ shares in τ we must accept a price concession.

4.5.3 Cost of trading

The revenue of all trades or equivalent the total cash received is often referred to as the capture. This is the sum of all units we sell in each time interval multiplied by the price per share, \tilde{S}_k in each interval:

$$\sum_{k=1}^N n_k \tilde{S}_k = XS_0 + \sigma\tau^{1/2} \sum_{k=1}^{N-1} x_k \xi_k - \tau \sum_{k=1}^N x_k g(v_k) - \tau \sum_{k=1}^N v_k h(v_k)$$

Equation 4

XS_0 represents the initial market value of the portfolio, each additional term represent effects on the revenue due to the different price movement factors. The first term represents the volatility effect, the second the permanent market impact representing the value loss due to price drop caused by selling a piece of the position. The temporary impact represents the value loss acting only on the units of asset we are selling in the k :th period.

The total cost of implementing a trade, also referred to as implementation shortfall, is the difference of the initial market value of the asset and the capture

$$\text{Implementation shortfall} = XS_0 - \sum_{k=1}^N n_k \tilde{S}_k$$

Equation 5

The implementation shortfall is a random variable because of the uncertainty of price movements. In pre-trade analyses, $t = 0$, we use the expected shortfall $E(x)$ and the variance of the shortfall $V(x)$ to optimize the trade. The results are dependent of the number of asset held at each time, x_1, \dots, x_{n-1} .

$$E(x_1, \dots, x_{N-1}) = \sum_{k=1}^N x_k g(v_k) \tau + \sum_{k=1}^N v_k h(v_k) \tau$$

$$V(x_1, \dots, x_{N-1}) = \sum_{k=1}^N \sigma^2 x_k^2 \tau$$

Equation 6

4.6 Summary of the theoretical model

Reducing risks and costs are essential economic targets for a modern business today. Financial actors always seek to decrease their costs through efficient trading especially large orders as they have a market impact which will increase the cost of the trade. Different researchers within this field use different formulas. Therefore a conclusion is that there is not a standard model for the market. In chapter 5 and 6 we develop the theory into practical usable models.

5. CREATING A USEFUL MODEL

The development of a useful model is described to the reader in this chapter.

5.1 Model distinction

As a result of no standard models we decide to create two separate models. The idea is that the models should capture different characteristics thus serve as good complements. Most empiric research indicates that market impact is a non linear function⁴⁶. When optimizing trading strategies with market impact constraints using non linear function result in a not analytical solvable equation system. We chose not to use a numeric optimizing tool but instead create a non linear impact model without optimized strategies and one linear impact model with optimized trading. When using linear impact it is possible to analytical optimize the trading.

The model using non linear impact should result in more accurate pre trade estimates since it is statistically evaluated on the US market. The trading recommended is constant normalized with the intra day volume profile. If 10 % of the daily traded volume is traded between 09.00 and 09.30, 10% of our order will be traded in this time period.

Further we create a model with optimized trading trajectories. In this model we make the simplification of linear impact functions. The model present more complex trading strategies on expense of using empiric rejected impact functions.

5.2 Model 1: Nonlinear Market Impact

This model is based on research done by Almgren et al (2005).

5.2.1 Definitions

The appearances of market impact functions have been widely discussed. Most empiric research indicates that they are power laws. In this model the impact is described by two functions. The permanent impact, I , reflects the information release. The realized impact, J , is the cost actually spent on the trade.

⁴⁶ *Optimal Execution with Nonlinear Impact Functions and Trading-Enhanced Risk* Almgren, Robert

$$I = \frac{S_{post} - S_0}{S_0}$$

$$J = \frac{\tilde{S} - S_0}{S_0}$$

S_0 is the market price at beginning of order

S_{post} is the market price after the trade is executed

\tilde{S} is the average price on the trade

Equation 7

The post trade price should reflect the permanent price change and is measured one time step after the completion of the trade. Earlier measured times will include the temporary time lagging effects.

The impact can be of either sign since the volatility of the stock plays a significant part of the costs. The expected impacts are however considered to be unfavourable thus when executing a buy order the stock price increases and the opposite for a sell order.

The ambition is to describe the impact functions and the cost by a few input variables.

$$X = \sum_{j=1}^n x_j, \text{ order size}$$

$T = \tau_n - \tau_0$, fraction of daily traded volume between t_n and t_0

$T_{post} = \tau_{post} - \tau_0$, fraction of daily traded volume between t_n and t_{post}

Equation 8

In the model a constant rate of execution is assumed, v . However a constant rate defined by dividing the order size by the number of execution periods would not give satisfying results due to the intra day volume profile. Thus all the calculations are done with respect to volume time, τ . The volume time represents the fraction of a daily number of shares that has been traded at time t . At market opening τ equals zero and at closing τ equals one. The volume profile can be measured by several different methods, as discussed in previous chapter. We use a moving average to calculate the specific profiles. The rate of trading is therefore constant in volume time, $v = X/T$.

Additional stock unique variables need to be taken into account.

V , daily traded volume

σ , daily volatility

The volatility, as well as the total daily volume traded, can be estimated through several different methods. However it is important to use a method that captures changes since they have a great influence on the results. We use a moving average for the traded volume and estimate the volatility from the previous thirty days.

In the model the size of the order is measured as the fraction of the order size and the daily traded volume X/V . The traded volume during the time we execute the order is also of

significant importance. Since T is a fraction of daily traded volume between t_0 and t_n we simple multiply T with V . The intuition is that the trader's additional participation of daily traded volume should affect the motion of the stock.

5.2.2 Assumptions in the model

The permanent impact is assumed to be independent of trading time and trading trajectory thus only reflecting the information released to the market by the order. The temporary impact reflects the supply and demand imbalance and the price concession needed to attract counterparties. Therefore the impact is very dependent on trading rate at the specific execution time. The total impact comprises both the temporary and the permanent impact.

The stock is assumed to follow an arithmetic Brownian motion. We assume that trading one stock does not affect others stocks and that the impact functions behave similar over the trading period. Further the motion of the stock is affected by the permanent impact, which is an increasing function of trading rate.

$$dS = S_0 g(\nu)d\tau + S_0 \sigma dB$$

$S_0 g(\nu)d\tau$ is the drift term of the stock thus driven by the permanent impact $g(\nu)$

$S_0 \sigma dB$ is a random function driven by a Brownian motion $B(\tau)$

Equation 9

The expression is integrated in volume time for $0 \leq \tau \leq T$ and $\nu = X/T$ resulting in the permanent impact

$$I = Tg\left(\frac{X}{T}\right) + \sigma\sqrt{T_{post}}\xi, \text{ where } \xi \sim N(0,1)$$

Equation 10

If $g(\nu)$ is a linear function then the accumulated drift at time τ is proportional to the number of shares executed, $X\tau/T$, and the total permanent impact is proportional to total order size X independent of execution time ($\tau = T \rightarrow T * X/T = X$).

Temporary impact affects the price received during the trade. Where the impact is scaled by the starting price of the stock.

$$\tilde{S}(\tau) = S(\tau) + S_0 h\left(\frac{X}{T}\right)$$

Equation 11

Temporary market impact is very influenced by the number of shares that the trader executes in current time period.

Using constant liquidation and time average of the execution price results in an expression for the temporary market impact

$$J - \frac{I}{2} = h\left(\frac{X}{T}\right) + \sigma \left(\sqrt{\frac{T}{12} \left(4 - 3 \frac{T}{T_{post}}\right)} \chi - \frac{T_{post} - T}{2\sqrt{T_{post}}} \xi \right), \text{ where } \chi \sim N(0,1) \text{ independent of } \xi$$

Equation 12

$I/2$ reflects the permanent impact's effect on the price due to traded stocks in previous time periods. The error term arises because of the Brownian motion in the stock price process.

5.2.3 Estimating the impact functions

The form of the impact functions have been widely debated and investigated. A common fact is that they are power law functions. Researches have shown that the temporary impact function most likely is a concave function i.e. exponent less than one. Lillo et al (2003) suggested by empiric research that the function has the exponent of a square root. The permanent function is preferred to be linear since this implies that the impact is independent of trading time. The general appearances of the functions are

$$g(v) = \pm\gamma |v|^\alpha$$

$$h(v) = \pm\eta |v|^b$$

where the sign of g and h are the same as for v ,
negative for a sale order and positive for a buy order

Equation 13

It is possible to estimate these functions and variables for individual stocks and separate buy from sell orders. However to create a common model we make no such distinctions.

The impact functions will vary over time since the properties of the stock changes. To capture the effect of an order size, it is defined as the number of shares relative the flow of shares during the trading time. Impact functions are expressions of the dimensionless amount X/VT , where V is the average number of traded shares per day. The intuition is that trading at a day with low turnover creates a larger impact than on a day with large turnover.

Further the impact functions are scaled with the stock volatility to measure the impacts as a fraction of the normal daily motion of the price instead of a percentage.

To adjust the model to stocks with different liquidity an additional function is added to the permanent impact.

$$L = \left(\frac{\Theta}{V}\right)^\delta$$

where Θ is the total number of shares outstanding

Equation 14

The measurement represents the inverse turnover and has been used in earlier empiric studies. With these modifications the impact functions are

$$I = \gamma\sigma T \left| \frac{X}{VT} \right|^\alpha \left(\frac{\Theta}{V} \right)^\delta + \langle noise \rangle$$

$$J - \frac{I}{2} = \eta\sigma \left| \frac{X}{VT} \right|^\beta + \langle noise \rangle$$

where noise is the error due to the volatility

Equation 15

Almgren et al (2005) estimated the variables in this model on the US stock exchange. They used a sample of 29000 orders done on very liquid stocks. The results are

$$\gamma = 0.314$$

$$\eta = 0.142$$

$$\alpha = 1$$

$$\beta = 3/5$$

$$\delta = 1/4$$

The results are in the 95 percent confidence intervals. One result is that the linear impact function can not be rejected which is of an enormous practical aid. The exponent of the temporary impact function results in a concave function. The intuition is that the bigger the trade is the less additional cost per share is experienced. The positive value of δ indicates that the more traded shares per day the less costs are experienced.

The resulting formulas for the expected cost and the variance of the cost are

$$I = \gamma\sigma \frac{X}{V} \left(\frac{\Theta}{V} \right)^{1/4}$$

$$J = \frac{I}{2} + \eta\sigma \left| \frac{X}{VT} \right|^{3/5}$$

$$V(J) = \frac{1}{3} T \sigma^2$$

Equation 16

The cost received on the order is J and I is the price motion from pre until post trade. The results will vary between different stocks and the most significant variable is the volatility of the stock.

5.3 Model 2: Linear impact with optimized trading trajectories

The aim of this model is to execute a portfolio from a given starting composition to a specified final composition. It seeks to minimize the costs of trading, with a penalty for the uncertainty of the cost, a penalty that estimates an uncertainty cost over time. Depending on the trader's tolerance for risk there are different strategies for trading. A person with a low level of risk will always try to execute an order as fast as possible but to a high expected cost. In contradiction a person with a high level of risk will execute an order during a long time at a low expected cost but with a high uncertainty in final revenue.

In chapter 4 the definition of trading strategies, price dynamics and cost of trading are explained. These are utilized and developed in chapter 5.3.1 - 5.3.3.

5.3.1 Temporary Market Impact

Temporary market impact refers to temporary imbalance in supply and demand caused by our trading which leads to temporary price movements away from equilibrium. For example if a trader decides to sell a certain number of units n_k between times t_{k-1} and t_k . The order is sliced into smaller pieces to locate optimal points of liquidity. If the total number of units is large, the execution price may steadily decrease during a time period. If this effect is short lived and liquidity returns after each time period it is temporary market impact. The temporary impact function $h(v)$ is the temporary drop caused by trading at an average rate v during one timer interval. Given the actual price per share received on sale k is:

$$\tilde{S}_k = S_{k-1} - h\left(\frac{n_k}{\tau}\right)$$

Equation 17

As $h(v)$ only is temporary impact it will not affect the next market price.

5.3.2 Linear Impact Functions

In this model temporary and permanent functions are assumed to be linear which does the computing of optimal trading trajectories significantly easier than with non linear impact functions.

Linear permanent impact $g(v) = \gamma v$, with the constant γ has units of (SEK/share)/share. Each unit that we sell will depress the price per share by γn , regardless of the time we take to sell the units. The permanent impact term of the estimated cost function develops to:

$$\sum_{k=1}^N \tau x_k g\left(\frac{n_k}{\tau}\right) = \gamma \sum_{k=1}^N x_k n_k = \gamma \sum_{k=1}^N x_k (x_{k-1} - x_k) = \frac{1}{2} \gamma \sum_{k=1}^N (x_{k-1}^2 - x_k^2 - (x_k - x_{k-1})^2) = \frac{1}{2} \gamma X^2 - \frac{1}{2} \gamma \sum_{k=1}^N n_k^2$$

Equation 18

5.3.3 The Efficient Frontier of Optimal Execution

As stated earlier a trader will always seek to minimize the expectation of shortfall for a given level of variance of shortfall. A trading strategy is efficient or optimal if there is no strategy that has lower variance for the same or lower level of expected variance. With the unconstrained optimization problem

$$\min_x (E(x) + \lambda V(x))$$

It is possible to sketch an efficient frontier for $\lambda > 0$ and it is strictly convex and has unique solution. The parameter λ is measure of risk aversion. A person with a high value of lambda is a person with a low level of risk and will execute an order faster than a person with a lower lambda. A person with a low level risk has a higher expected cost and a lower variance than a person with a higher level of risk, as it executes faster and therefore has larger impact on the price.

With $E(x)$ and $V(x)$ from chapter 3 it is possible to find a unique global minimum from the combination of $U(x) = E(x) + \lambda V(x)$. By setting its partial derivatives to zero we can find out global minimum.

$$\frac{\delta U}{\delta x_j} = 2\tau \left\{ \lambda \sigma^2 x_j - \tilde{\eta} \frac{x_{j-1} - 2x_j + x_{j+1}}{\tau^2} \right\}$$

Equation 19

For $j = 1, \dots, N-1$. Then $\delta U / \delta x_j = 0$ which equals

$$\frac{1}{\tau^2} (x_{j-1} - 2x_j + x_{j+1}) = \tilde{\kappa}^2 x_j$$

With

$$\tilde{\kappa}^2 = \frac{\lambda \sigma^2}{\tilde{\eta}} = \frac{\lambda \sigma^2}{\eta \left(1 - \frac{\gamma \tau}{2\eta} \right)}$$

From these equations we can get a trading trajectory of the form:

$$x_j = \frac{\sinh(\kappa(T - t_j))}{\sinh(\kappa T)} X \quad j = 0, \dots, N$$

Equation 20

And n_j the sum of sold shares:

$$n_j = \frac{2 \sinh\left(\frac{1}{2}\kappa\tau\right)}{\sinh(\kappa T)} \cosh\left(\kappa\left(T - t_{j-\frac{1}{2}}\right)\right) X \quad j = 1, \dots, N$$

Equation 21

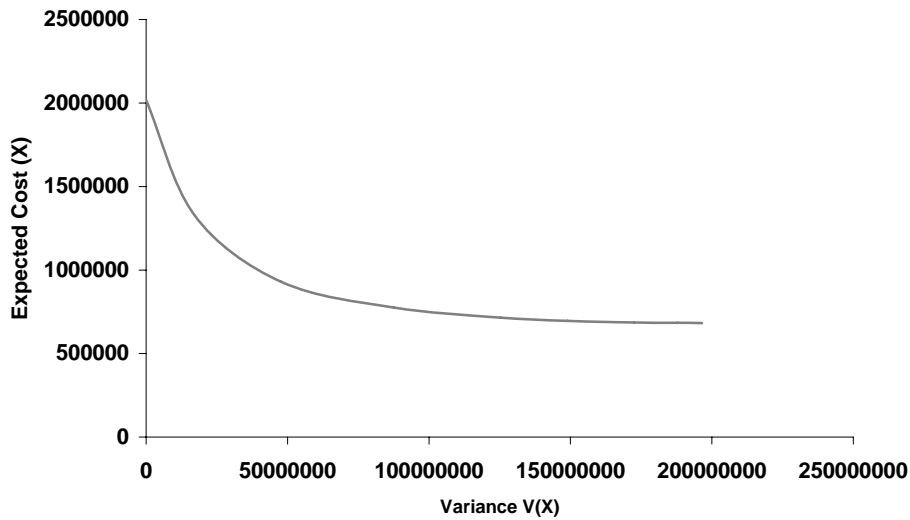
The expectation and variance of the optimal strategy for a given initial portfolio size are the

$$E(X) = \frac{1}{2}\gamma X^2 + \varepsilon X + \tilde{\eta} X^2 \frac{\tanh\left(\frac{1}{2}\kappa\tau\right)(\tau \sinh(2\kappa T) + 2T \sinh(\kappa T))}{2\tau^2 \sinh^2(\kappa T)}$$

$$V(X) = \frac{1}{2}\sigma^2 X^2 \frac{\tau \sinh(\kappa T) \cosh(\kappa(T - \tau)) - T \sinh(\kappa\tau)}{\sinh^2(\kappa T) \sinh(\kappa\tau)}$$

Equation 22

To create an efficient trading frontier the expected cost $E(X)$ is plotted on the y - axis and the variance $V(X)$ on the x - axis. In figure 4.1 an example of a trading frontier is shown. This is for different lambda. All strategies above the front are strategies that have a higher expected cost and variance for the same strategy therefore all efficient strategies are on the efficient front.


Figure 5.1 Efficient Frontier

5.4 Summary of model creation

Models described in this chapter are used in the application presented in chapter 6. The models will create an efficient frontier which is shown as an example in figure 5.1.

6. DEVELOPMENT OF AN USER FRIENDLY APPLICATION

The aim of this chapter is to develop and explain the application based on model 1 and model 2.

6.1 Application interface

An application was created for the user to be able to do an efficient pre-trade analysis in an easy and comprehensive tool. The application allows the user to put in specific stock and order data. The data (Specific Order Data) that has to be entered in to the model is:

- Stock volatility
- Order size
- Initial stock price
- Number of outstanding shares
- Daily turnover

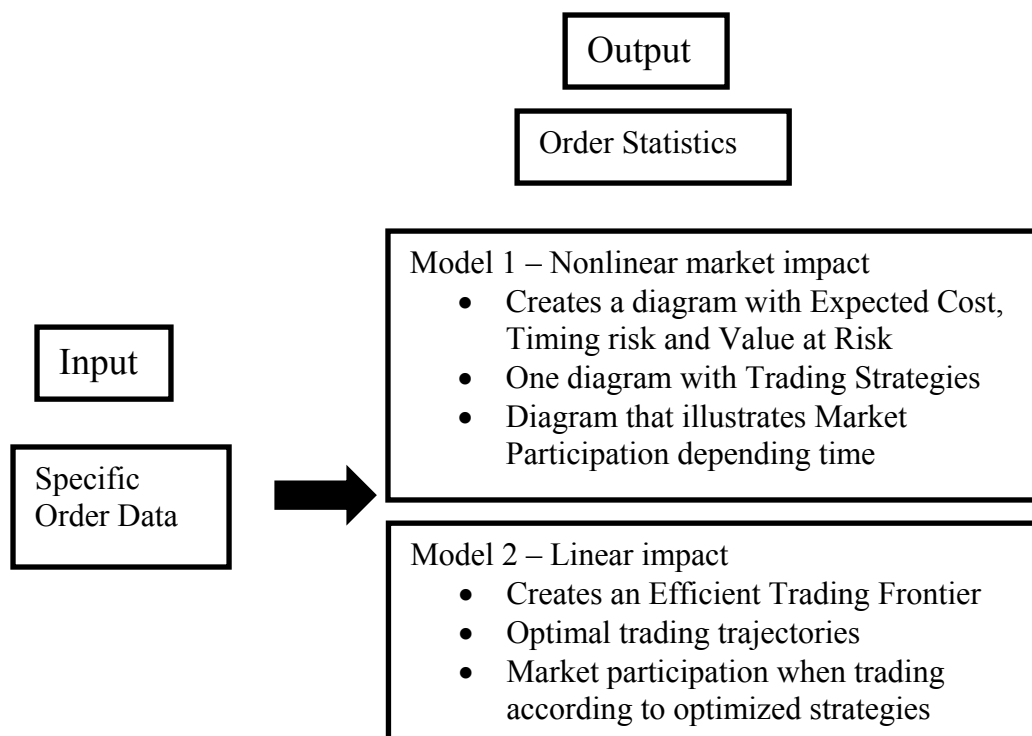


Figure 6.1 Data is put into the application and the two models creates numerous graphs

Parameters are calculated as mentioned in previous chapter. In our application the volume profile is assumed to be identical and constant for all stocks.

Stock & Order specific data			
Yearly volatility	σ		%
Order size	X		ea.
Stock price	S0		SEK
Outstanding shares	Θ		ea.
Daily turnover	V		ea.

This data are put into the application and will create two plots that are generated with the resulting efficient trading frontier. Additional plots representing trading strategies for several risk tolerances are presented in two new plots. Further statistics for the specific order is presented.

Order statistics			
Daily volatility	σ_{daily}		%
Initial portfolio value	$X \cdot S_0$		SEK
Normalized order size	X/V		%

6.2 A graphical interface-Model 1

The first model generates three different graphs, this to clarify the characteristics of a trade. Figure 6.2 demonstrates expected cost, variance of cost and value at risk for a specific trade. They are created from the data that is put into the application. During the pre trade analysis these graphs will clarify which impact different trading behaviours have for a future trade. As expected this graph proves that an aggressive trader will have a large impact on the market i.e. a high expected cost but a lower variance than a passive trader.

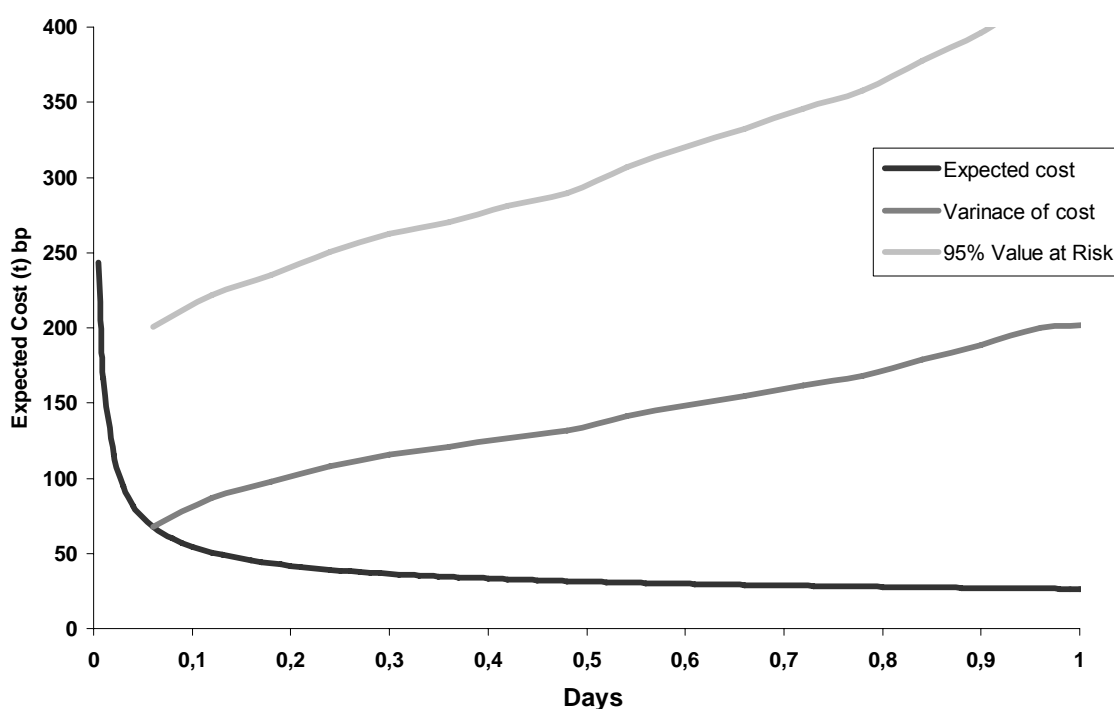


Figure 6.2 Model 1-Constant volume adjusted rate of trading

Figure 6.3 shows volume sizes depending on liquidation horizon. For example if a trader has a certain volume that shall be executed in a set period of time this graph shows what volume that has to be executed within a certain time period.

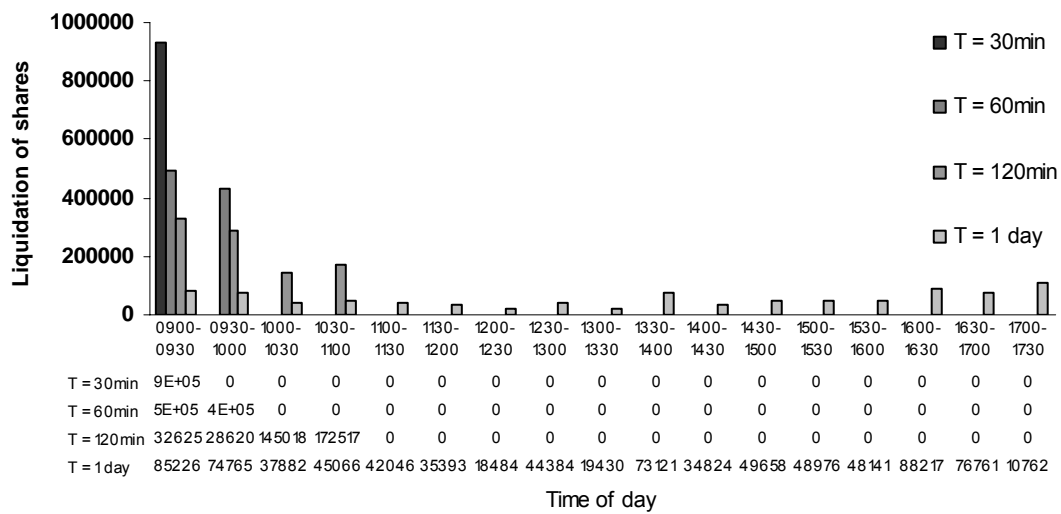


Figure 6.3 Model 1 - Volume adjusted trading depending on liquidation horizon

Figure 6.4 describes which market size a specific trade will have. This information is essential since market participation is an often used restriction by customers.

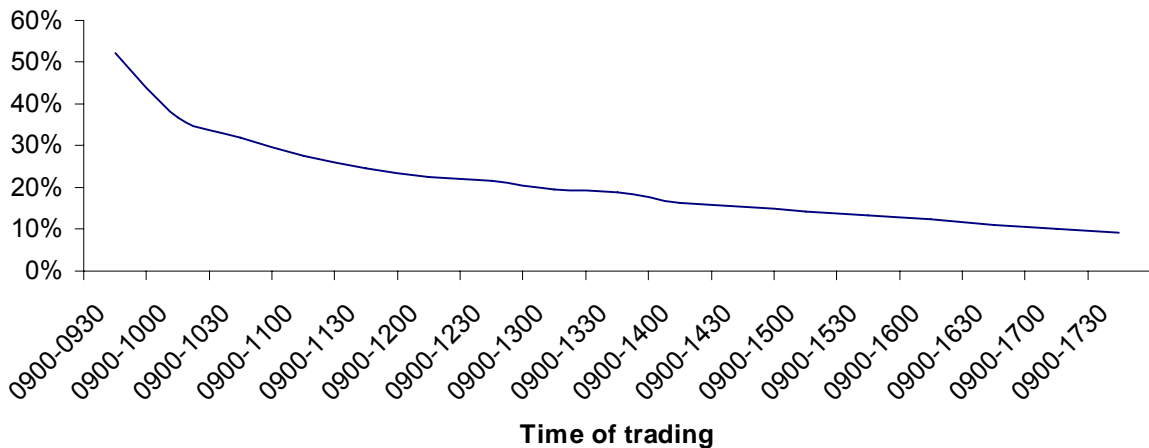


Figure 6.4 Market participation depending on liquidation time

6.3 A graphical interface-Model 2

The second model generates three graphs, to clarify the characteristics of a specific trade. Figure 6.5 depends on the expected cost and the volatility of cost. In this graph the lower line illustrates the efficient trading frontier. An aggressive trading behaviour will result in a high expected cost and to passive trading will cause a high risk. It is created from the data that is put into the application. As explained in earlier chapters every trade on this frontier is efficient to a specific level of risk. A specific level of risk will lead to an optimal trading trajectory which is illustrated in figure 6.6.

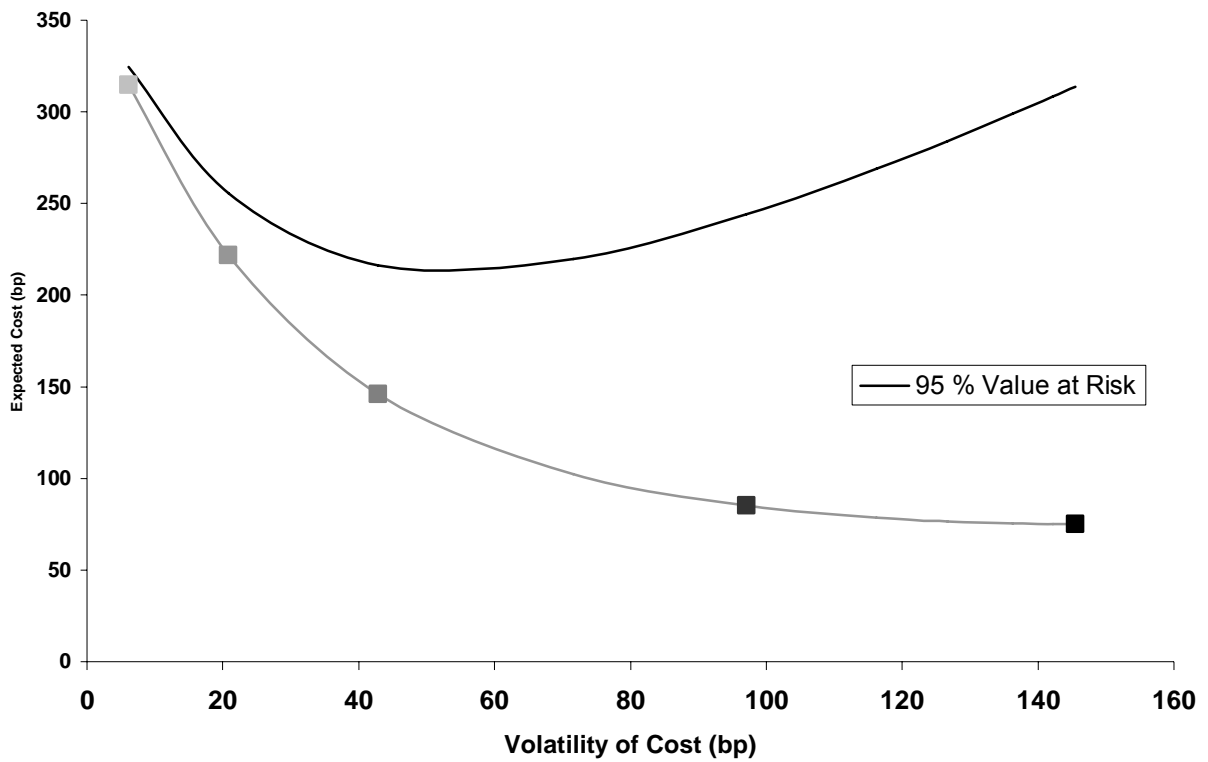


Figure 6.5 Efficient trading frontier with linear impact functions

These optimal trading trajectories that are shown in figure 6.6 are depending on risk level from the efficient frontier in figure 6.5. Values from figure 6.5 relates to different optimal trading trajectories showed in figure 6.6.

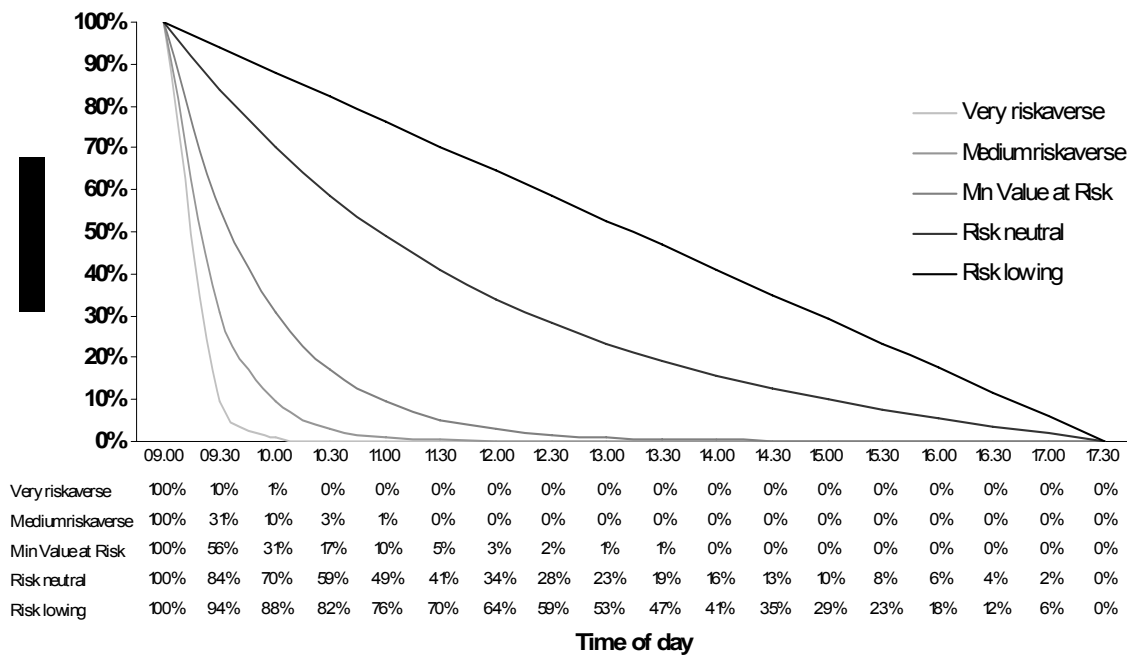


Figure 6.6 Optimal Trading Trajectories

Figure 6.7 illustrates market participation for a specific trade. A person that is very riskaverse will have an aggressive trading behaviour and therefore trade fast. That will lead to a high expected cost as the trade has large market participation during a short time period. The difference to figure 6.4 is that these trades are optimized. It is obvious in this graph that risk lowering person is a person that trades passive with little market impact but has a high risk as the trade has a long duration.

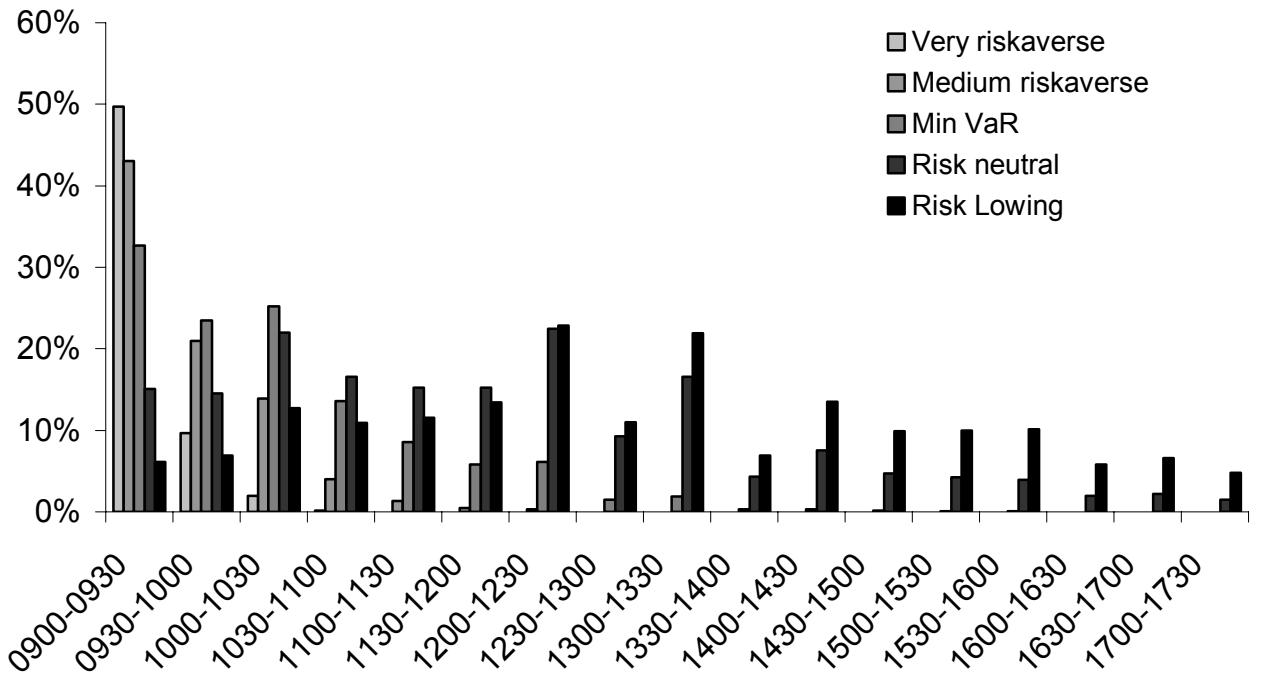


Figure 6.7 Market participation when trading according to optimized strategies

7. RESULTS SUMMARY & CONCLUSIONS

In the following chapter a result summary is presented to give the reader an example of how to utilize the application. Further how well the research purpose is fulfilled is concluded.

7.1 Introduction

The results provided by our models are indications of cost and risk for a specific order. As the models are not evaluated on real trades the estimates are not statistically correct but the behaviour and appearance of the efficient trading frontiers are accurate. As a result the user should not primarily look on the estimated values but focus on the relationship between cost and risk of trading. Thus a user should utilize the application with the purpose to get a feeling on how the trade is best carried out.

The two models are supposed to be used as complements. The main difference, thus the advantage of using the both, is the decision to trade based on a pre defined time frame or a specific risk.

Our first model let the user decide which time horizon to complete the trade within, given estimates of costs and risk measurements. From this time frame a trading strategy is presented. The strategy is based on the intra day volume profile and divides the time horizon into half hour time steps with recommended trading volumes for each.

Our second model is based on the risk aversion of the user. Depending on the risk level of the user, decided from the efficient trading frontier, a recommended strategy is presented. These are optimized based on the risk aversion of the user.

7.2 Model 1-Non linear market impact

In the first model, the non linear market impact model, different values are tested on most sensitive values i.e. yearly volatility, order size and trade duration. In this example it is Nordea, which we have estimated a yearly volatility of 42%, and a daily turnover of 9.3 million shares which is estimated from a ten day moving average. In this specific trade the order size is 10% of daily turnover. These values are for the general trade; they will vary in different tests. This will provide the user with a pre trade analysis; showing market impact for a liquid stock for a specific trade which will simplify the decision process for the decision maker.

In the first test the volatility varies, which results in a higher expected cost with a higher volatility. The value of the volatility is sensitive to changes. Therefore it is of great importance to try to estimate this value as correct as possible. Different methods can be used which is discussed later on.

Order specific data	Yearly volatility	50	40	30	%
	Order size		9,3E+05		ea.
	Normalized order size		10		%
	Trade duration		0,5		Days
Order statistics	Initial portfolio value		9,3E+07		SEK
	Normalized order size		10		%
Model estimates	Expected cost	37	30	22	Bp
	Volatility of cost	169	142	101	Bp
	95% Value at risk	367	306	220	Bp

Table 7.1 Results when trading 10% of daily turnover with a variation in volatility

In the second test there is a variation in the order size. The expected costs increase with a large order this since market impact is larger with a larger order. This since we trade a larger piece of daily turnover. The value at risk measure is higher with a larger order size, this since it is a higher total portfolio value which increase the Value at Risk.

Order specific data	Yearly volatility		40		%
	Order size	9,3E+05	5,4E+05	1,86E+05	ea.
	Normalized order size	10	6	2	%
	Trade duration		0,5		Days
Order statistics	Initial portfolio value		9,3E+07		SEK
	Normalized order size	10	6	2	%
Model estimates	Expected cost	30	19	8	Bp
	Volatility of cost	142	135	135	Bp
	95% Value at risk	306	283	272	Bp

Table 7.2 Results with a variation in order volume size

In the third test the trade duration varieties for the trade. With shorter trade duration market impact increases which results in a higher expected cost. Market impact decreases faster during the start of the trade therefore it is a larger difference between 0.02 days and 0.2 days than between 0.2 days and 0.5 days. This explains that it is important to do a trade aggressive, but not that aggressive that market impact turns to large. The conclusion from the three test trades is that the results are in line with our and other more experienced people's expectations.

Order specific data	Yearly volatility		40		%
	Order size		9,3E+05		ea.
	Normalized order size		10		%
	Trade duration	0,02	0,2	0,5	Days
Order statistics	Initial portfolio value		9,3E+07		SEK
	Normalized order size		10		%
Model estimates	Expected cost	111	39	30	Bp
	Volatility of cost	23	83	135	Bp
	95% Value at risk	111	211	293	Bp

Table 7.3 Results with a variation in trade duration

7.3 Model 2-Linear impact with optimized trading trajectories

In the second model, linear impact with optimized trading trajectories model, different values are tested on most sensitive values in this model it is Kappa and order size. Also for these test the stock is Nordea, for which we have estimated a yearly volatility of 42%, and a daily turnover of 9.3 million shares which is estimated from a ten day moving average. This is 10% of daily turnover.

In this model kappa is the most sensitive parameter as it depends on volatility, lambda, eta, gamma and tau. Therefore this parameter is very responsive to changes and is interesting to test. We keep all variables constant except lambda thus kappa represents different risk tolerances. A high value of kappa represents a low risk tolerance resulting in a higher expected cost.

Order specific data	Yearly volatility		42		%
	Order size		9,3E+05		ea.
	Normalized order size		10		%
	Kappa	10	5	0,5	
Order statistics	Initial portfolio value		9,3E+07		SEK
	Normalized order size		10		%
Model estimates	Expected cost	146	102	75	Bp
	Volatility of cost	43	72	143	Bp
	95% Value at risk	216	220	310	Bp

Table 7.4 Results with a variation in Kappa

In the second test the order size varieties for the trade. As described in earlier chapters the order size has a large impact for a trade. A large order is hard to execute and the market participation will increase therefore our expected costs increases with a large order size.

Order specific data	Yearly volatility		42		%
	Order size	9,3E+05	5,4E+05	1,86E+05	ea.
	Normalized order size	10	6	2	%
	Kappa		10		
Order statistics	Initial portfolio value		9,3E+07		SEK
	Normalized order size	10	6	2	%
Model estimates	Expected cost	146	87	33	Bp
	Volatility of cost	43	43	43	Bp
	95% Value at risk	216	157	103	Bp

Table 7.5 Results with a variation in order volume size

7.4 Conclusions

Purpose of the master thesis was to develop an algorithmic trading model for SEB which is well functioning on the Scandinavian market.

We have based on cutting edge research in financial trading theory combined with discussions with traders on the Scandinavian market created a pre trade analysing application. The application is today usable to facilitate the decision-making process how to best execute a large stock order. The application provides the user with cost and risk estimates. Additional information on how to liquidate the initial portfolio and anticipated market participation is supplied to the user. The information provided by the application is realistic according to experienced professionals.

Implementing the model will enable automatized algorithmic trading. The end results are effort and time saved. Further manual error can be minimized using algorithmic trading.

However all manual trading can not be replaced. Several orders are of such kind that algorithms are not capable of executing these. Further a good trader takes much information into consideration when trading not possible to capture in algorithms. The conclusion is that traders and algorithms should work side by side to maximize profit. Standardized orders are handled by automatized trading systems allowing traders to focus on more complex orders.

8. RECOMMENDATIONS AND DISCUSSION

Our recommendations how to benefit the most from the results found in our research are described and discussed in the following chapter. Further we make recommendation how SEB can take algorithmic trading to the next level.

8.1 How SEB can use the models

There are two main utilizations for SEB the first is to provide traders and risk managers with a support tool when executing an order. Secondly is to use the algorithms discussed in this thesis to develop an automatized trading system. They are not necessarily two different utilization areas. A user should be able to use the generated plots as a decision tool and then decide to trade manually or to chose a recommended strategy and let a computerized system carry out the trading. To fully computerize the trading is difficult since a decision on risk level needs to be taken. This can not be automatized since a universal optimal strategy does not exist.

We make no recommendation that manual trading should be completely replaced by algorithmic trading. Traders take many variables not included in the models into consideration when trading. A specific trading pattern, which is not available through the models, may be requested by the customer. Further many stocks on the Nordic market are illiquid and parameters change constantly thus manual trading can not be replaced. Moreover a good trader can make money when on purpose taking risks.

However our models can give the trader an efficient tool to facilitate the decision making of the trade strategy. Further the models are good explanatory tools for market impact and how it relates to timing risk. This can be of great usage when explaining to customers why a trade was carried out in a specific pattern.

We are confident that the foundation of these models can be used in algorithmic trading. A standard order should be able to be handled by these algorithms.

8.2 Practical recommendations of implementation

The first step, in order to benefit the most from this thesis, is that SEB needs to decide which input data the user should specify and which should be automatically calculated. Users may want to use different methods of calculating these thus SEB needs to decide which are editable and which will be automatically generated. If not to complicate to implement the user should have several choices in the application. Either the user can chose to use a predefined method or put in own calculated data.

The most important variable in the models is the stock volatility. The volatility is the greatest contributor to risk in trading cost and will affect the results significantly. We use a measurement based on a 30 day period. However there are several methods to calculate the volatility of a stock. Several methods try to capture trends and changes in the stock behaviour. The GARCH model captures changes and trends in the volatility thus commonly used in the industry. Another measurement is to see how the market anticipates the volatility and

calculate the implied volatility from options in the specific stock. SEB should decide which measurement to use and if it is editable.

Daily turnover and volume profile should be decided in the same context as the volatility. However the measurements do not affect the results as much as volatility and we recommend a 10 day moving average should be sufficient for turnover and volume profiles may be constant for all shares. Outstanding shares are constant thus not needed to estimate. However this is stock specific and needed to be extracted from a database or be a manually input.

It is necessary that the implemented model take into account which time of the day the algorithms are utilized. If using the application at noon already approximately half of the daily turnover has been traded. Further the volume traded between nine and nine thirty is not the same as between twelve and twelve thirty. Thus the models need to understand when the pre trade analysis is carried out.

When the model is implemented and well working, a further analyse of which orders are capable to be automatized should be carried out. This analyse should investigate if a trader needs to filter all orders in order to decide this or if this can be automatized as well. If decided that some orders are capable to be completely carried out by the algorithms an additional input specifying which of the strategies generated by the two models should be utilized. If a customer desires a time constrained transaction the first model suits the purpose. If the purpose is trade with the most efficient combination of cost and risk the second model is preferable. An additional constraint regarding market participation is necessary to have as an additional input since this is a common request. If using market participation as an input variable the model should exclude all strategies not fulfilling this constraint.

Implementing our recommendations will save time and effort of trading. Since the competition in financial market is increasing automatized trading will allow SEB to decrease the cost related to trading. This will result in higher marginal on each trade when minimizing manual work and. Further lower commissions will be possible helping SEB's increase market shares.

8.3 Model improvements

A great improvement of the model is to estimate the functions and constants to the Scandinavian market. This will improve the pre trade analyses and the trading strategies. A large sample of data is essential to make an accurate estimation. The information of each stock order needed to do the estimation is:

- The stock name (symbol), requested order size and buy or sell order
- The time when the first transaction of an order is sent to the market - the starting time, trading methods (VWAP, limit, market etc).
- Time, sizes and received prices corresponding to each transaction within the order

With this information SEB are able to do the estimation. Perhaps different estimations should be carried out on the different stock lists, dividing stock by large, medium and small capitalization. A statistic survey will determine if it is an advantage to use different model for each market.

In this thesis the trend or momentum of a stock is neglected since trading is carried out in a short time period. Traders usually have an idea of the behaviour of a stock. To capture their belief a drift (momentum) variable can be incorporated into the models. This is a stock specific variable and is integrated into the assumed Brownian motion of the stock thus changes the end results of expected cost and variance. The results will depend on the user's belief of the stock performance thus further the pre trade analysis will be more user adapted.

The optimization model can be improved utilizing intra day volume profiles. The results will be more accurate trading schedule thus simplifying the decision making for the end user. A great improvement is to let market participation be an additional input constraint.

More advanced trading strategies can be incorporated into the models. An optimization of the nonlinear model is possible utilizing mathematical software. Additional strategies can be developed depending on the purpose of utilization. The ambition should be to automatize relatively simple but time consuming trades.

8.4 Academic contribution

Problems in this thesis have their foundation in the financial markets, often markets and especially efficient financial markets are estimated by different academic researchers. This master thesis was initially a project at SEB, although it is closely related to academic theory done by U.S. academic researchers. This thesis aims to give the reader an understanding of which factors influence market impact and their relation to theoretical models that estimates market impact and optimal trading strategies. It also explains complex research in a basic method which helps the reader to understand given theories. It combines quantitative and qualitative research which now can be evaluated empirically. Further we have contributed to develop the theory to a practical usage.

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