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Market Risk Management: The Applicability and Accuracy of Value-at-Risk Models in Financial Institutions

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Abstract

The paper focuses on evaluating the most performant Value-at-Risk models from the perspective of financial institutions. A number of 18 VaR methodologies were used for this purpose, comprising of parametric and non-parametric methods, some of which include time-varying volatilities estimated by means of GARCH and asymmetric GARCH models.

All methods were applied on real P/L data extracted from four commercial banks. Furthermore, we use two backtesting frameworks to validate the results: the Kupiec test (1995) and the newly developed Risk Map (2011). The use of the latter method allowed us to account not only for the number of tail losses but also for the magnitude of these exceptions, demonstrating once again its reliability and more importantly its simplicity in application.

Therefore, the study aims to add to the relatively scarce current literature that investigates the use of VaR models in financial institutions, using real bank data.

The results showed that that models working under the assumption of normality are not the most performant when calculating VaR, models under the assumption of the t-distribution providing more performant VaR estimates.

Parametric models that use GARCH forecasted volatilities have outperformed the other methods, especially the ones using a t-distribution for both the data series and the innovations. These models both passed the backtests and produced some of the lowest average VaRs, which is important for financial institutions in terms of minimizing their capital requirements. Although in some situations the asymmetric GARCH models showed a better performance than the simple GARCH model, the increased efficiency is not significantly superior.

As for non-parametric models, the basic historical simulation passed the backtest for all institutions, even though it is a simple measure and yields a relatively flat structure of the VaR forecasts over time. In addition, the newly developed HS-VIX model also worked well for all the data series, in providing accurate VaR estimates throughout the backtesting window.

Keywords: Value-at-Risk, Historical Simulation, Normal Distribution, t-distribution, EWMA, GARCH, Asymmetric GARCH, Implied volatility, Backtesting, HS-VIX, Risk Map.

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1. Introduction

Market risk is one of the several financial risks that banks are exposed to, along with operational risk, liquidity risk, model risk or credit risk. A definition of market risk would be „the risk of losses on financial instruments arising from changes in market prices”[1]. Commercial banks hold portfolios of trading positions on different assets and are therefore exposed to price changes. Because they are financial intermediaries, the loss of value on their trading portfolios affect the income and capital of deposit-takers.[2] Market risk has four components: equity risk, interest rate risk, foreign exchange or currency risk and commodity risk.[3]

Value-at-Risk is the most commonly used measure for quantifying market risk. Its popularity can also be motivated by its introduction in the regulatory framework by Basel I and II as a standardized measure for market risk. VaR is, further on, used to determine the capital requirements for banks' trading portfolios. However, the Basel Committee does not impose a specific VaR estimation model to be used by banks, allowing them to use their own internal models, calibrated to their specific operations and portfolios.

This motivates the paper's focus on different models of estimating VaR for financial institutions trading portfolios and their accuracy. VaR has been a rather controversial measure since its introduction, receiving much criticism over the years. Its simplistic interpretation and implementation have been seen as a dual facet, both as an advantage and as a drawback. Many empirical studies have questioned the use of VaR as a tool for understanding and managing market risk, thus shedding doubt on the entire Basel regulatory framework. Yet, even though alternative risk measures such as Expected Shortfall have been proven to provide superior results, VaR still holds its position as the most widely used measure for market risk.

The literature on VaR estimation models fitted to real bank data is scarce so far, the two most notable papers in this field being the ones written by Berkowitz and O'Brien (2002) and Perignon and Smith (2008). Both studies focus on the use of VaR models by banks as a market risk management tool and the performance of these models when applied on real trading portfolio returns. Thus, our paper builds on these two previous studies, through its objective to analyze and determine the VaR estimation models that are the most efficient in determining the market risk of bank's trading portfolios. Nevertheless, the present study adds to the current research literature due to the use of several VaR estimation methods on real

[1] IMF (2004)
[2] Idem
[3] Dowd(2005)

profit/loss data, extracted from four European and American banks, including a series of models that have not been used in the previous papers. These methodologies comprise of parametric, non-parametric and semi-parametric approaches, including models that account for time-varying volatility of returns. What is more, in addition to the traditional Kupiec (1995) backtesting framework, we also employ the Risk Map (2011) methodology. This is a recently developed tool for validating VaR models, which takes into account the magnitude of the VaR exceptions as well as their frequency.

Furthermore, the recent financial crisis has put pressure and brought to light several negative issues concerning Value-at-Risk as a standardized risk measure. One of the downsides of VaR models is that they tend to not perform well during financial crises: many financial institutions have experienced considerable trading losses during 2007-2009 period that were not captured in their market risk exposures. Thus, we try to assess the actual performance of the different VaR methods during this time span.

1.1.Purpose

The purpose of this thesis is to evaluate the accuracy of several VaR estimation methods using real data from four commercial banks.

1.2.Delimitations

The time limit set for this study is rather restrictive, therefore a few specifications are necessary.

Firstly, financial institutions have a policy of selectively disclosing their profit/loss data, VaRs and its composition on a daily basis to the wide public. Therefore the access to the high frequency data needed for the study is restrained, as well as time consuming to collect. We managed to compile daily profit/loss data series from four commercial banks, using a graph data digitizing software applied on P/L plots disclosed by these banks in their annual reports or their risk management reports. However, the data was available only starting with 2007 for three of the banks, which limits our time series to a span of 4 years of daily observations. Another consequence of data disclosure policies is that banks do not make public the actual composition of their portfolios. This is a problem for two reasons. The first reason is that by not having access to the composition of the portfolio, we cannot analyze the impact each individual market risk factor has on the total market risk of the banks, which would have been interesting to investigate on real data. The second reason derives from the fact that the portfolio components do not have constant weights over time, since the banks rebalance their trading portfolios on a regular basis according to different criteria. Hence, the P/L data series basically results from a number of different portfolios over time, and not from one single portfolio. Banks may choose to limit the total risk of their portfolio when it becomes too large by investing in correlated assets and thus hedge the risk. The difference in VaR measures in 2 consecutive days may be related more to this adjustment in portfolio composition than the actual volatility of returns.

Secondly, a larger number of banks could have been included in the study which could have offered the analysis a cross-sectional feature. Nevertheless, the time limit combined with the relative time-consuming procedure of extracting the data from each graph has constrained us to the use of only four banks.

Thirdly, while there is a considerable number of VaR methodologies that are available, we are limited to the 18 models we are using due to the time constraint imposed in developing this study.

1.3.Structure of the paper

The paper is structured as follows. In the second part we present the theoretical background that lies behind the VaR measure. Section 3 presents the methodology used to conduct the study, and describes the models used. The data is examined in section 4 and the incurring results are described and interpreted in section 5. Finally, we conclude the study in section 6 and suggest further research venues in section 7.

2. Theoretical background

Value-at-Risk measures "the worst expected loss over a given horizon, under *normal market conditions* at a given confidence level".[4]

A mathematical definition of VaR is the following:

$$\Pr(R \leq -VaR) = 1 - \alpha$$

where R denotes the future P/L and α is the confidence level for which VaR is computed.

VaR computations rely on 2 parameters: the confidence level and the holding period.

The choice of confidence level is given by the purpose of the risk measure for which is needed. Institutions generally prefer to set high confidence levels, generally in the range of 95% to 99% when using VaR for determining capital requirements, setting risk limits, reporting or comparison purposes. However, lower confidence levels are favored for backtesting purposes.[5]

The holding period depends on the assumption that the portfolio composition is constant over this time interval. The parameter depends on the purpose for which VaR is calculated as well. Institutions can select their holding period depending on their investment or reporting horizon or the liquidity of the market in which it operates. The most commonly used holding periods are one day or one month. Financial institutions are regulated by the Bank of International Settlements (BIS) capital adequacy rules which state that banks should operate with a holding period of 10 working days.[6]

Thus, the optimal choice of VaR parameters depends on the particular characteristics of each institution and their estimation purposes.

The use of VaR as a market risk measure has a number of advantages. A first attraction of VaR is that it provides a "common, consistent measure of risk across different positions and risk factors"[7]. That is to be said that it can be applied to any asset class and it also allows for comparisons between the risks of different portfolios or assets. Furthermore, it integrates the correlation between risk factors in its computation. This leads to another benefit of VaR, in that it allows for risk aggregation. In addition, VaR is a probabilistic measure, hence providing valuable information about the probabilities associated with the computed

[4] Jorion (2001)

[5] Dowd(2005),pg 29-30

[6] Idem, pg 30-31

[7] Ibid., pg 12

losses. VaR is also a holistic measure. It takes into account a multitude of driving risk factors and it focuses on a portfolio approach rather than individual positions.[8]

Moreover its simplistic and intuitive interpretation makes it an easy concept to grasp and apply. VaR is basically a number that refers to a maximum amount of money that can be lost with a certain probability.

Although since its introduction VaR has established itself as a dominant measure of market risk, it has also received much criticism, with many authors warning about the dangers of relying on it.

One such criticism refers to the fact that it is not a coherent risk measure. The main consequence of this fact is that it does not always encourage diversification.

Another drawback is that it does not say anything about the size of the losses should a tail event occur. Thus, losses can exceed the VaR estimates by far, which would prove highly inconvenient for institutions that base their market risk management on VaR.

Moreover, VaR was accused of being destabilizing during crises. David Einhorn compared it to "an airbag that works all the time, except when you have a car accident"[9]. VaR is after all defined as the worst expected loss give normal market conditions[10], and this is where its weakness lies.

2.1.Backtesting

As more and more financial institutions have developed their own internal models to assess the exposure to market risk, backtesting has become increasingly necessary in verifying if these models perform adequately and give accurate estimates of risk. This is particularly important since banks' capital requirements (under Basel Capital Accords) are computed based on their results and an underestimation could lead to difficulties in covering losses.

Several backtesting techniques were developed, each having its pros and cons. These approaches lean on different ideas, being based on the frequency of tail losses or on the size of the tail losses, while others turn to forecast evaluations in backtesting. Moreover the saddlepoint technique[11] was proposed as an alternative to the more "classic" methods.

Since most of our subsequent analysis will be based on the Kupiec test and the Risk Map technique, the following subsections will reveal some of the intuitions behind these methods, and also their advantages and faults.

[8] Dowd(2005)
[9] Einhorn,D; Brown, A.(2008)
[10] Jorion (2001)
[11] Wong(2008)

2.1.1.Kupiec approach to backtesting

The Kupiec (1995) test represents one of the basic frequency-of-tail-losses models and consists of comparing the actual, observed tail losses with those predicted by the model given a confidence level. Basically, if the number of tail losses yielded by the VaR-model is grossly larger than the expected number of exceptions then the model underestimates the risk assumed by the bank and this will lead to a rejection. On the other hand, a smaller number of deviations from what is expected means that the bank overestimates the risk that it takes on. This bears its own costs as it will give the appearance that the bank is riskier than it actually is, and the institution will have to keep a higher amount of its funds as capital requirements, thus forfeiting the high returns it could earn on these funds by placing them as e.g. credits.

The null hypothesis of the Kupiec test, that the model is consistent, states that the tail losses follow a binomial distribution. The probability of having *x* or more deviations is given by the following formula:

$$\Pr(x|n,p) = \binom{n}{x} p^x (1-p)^{n-x}$$

where *n* is the number of observations, while *p*, the predicted frequency of the tail losses, is equal to 1 minus the expected level of confidence assumed for the model.[12]
The likelihood ratio test statistic used when employing the Kupiec test is formulated below:[13]

$$LR = -2ln[p^x(1-p)^{n-x}] + 2ln\left[\left(\frac{x}{n}\right)^x \left(1-\frac{x}{n}\right)^{n-x}\right]$$

The model is not rejected when the estimated probability associated with the test statistic is greater than the significance level for which VaR is calculated (usually *p* equals 1% - corresponding to a 99% confidence level, or *p* equals 5% - for 95% confidence zone).
The main advantages of this test are the easiness in application and the fact that it does not require a significant amount of information. [14]

There are however some shortcomings that arise when using this test. Most notable are the necessity of large sample size (otherwise the test loses its reliability and becomes more prone to type I and type II errors in hypothesis tests[15])[16], and the fact that it ignores violations

[12] Dowd (2005)
[13] Jorion (2001)
[14] Dowd(2005)
[15] See Brooks (2008) p. 64 for definitions of type I and type II errors
[16] Kupiec(1995)

clustering[17]. Another drawback is represented by the lost of valuable information regarding the size of the loss and also about their occurrence pattern over time[18].

Two solutions that take into account the first issue were proposed[19]: firstly the use of a smaller confidence level (for instance 95% instead of 99%) and secondly applying the test to a longer historical sample period. The violations clustering drawback was solved by Christoffersen (1998) who developed a conditional backtesting approach which accounts for exceptions clustering.

2.1.2.The Risk Map Approach

One of the main criticisms brought to the previous backtesting techniques (e.g. the Kupiec test) is the fact that they only account for the number of VaR exceptions, thus completely neglecting the amplitude of these violations[20]. Therefore a new method called the *Risk Map* was developed which takes into consideration both the number of exceptions and their magnitude[21].

The basic idea behind this approach is to recalculate VaR at a much higher confidence level (99.8% in this case), as it is very likely that a violation that occurs at a regular degree of confidence (e.g. 99%) to appear at the higher level as well.[22]

In this scope, it introduces a new variable called the VaR "super exception" which is defined by means of the following expression, similarly to a regular model violation:

$$R_t < -VaR_{t,99.8\%}$$

It also defines an indicator variable associated with the $VaR_{t,99.8\%}$, as seen below:

$$I_t(99.8\%) = \begin{cases} 1, & if\ R_t < -VaR_{t,99.8\%} \\ 0, & otherwise \end{cases}$$

Both the regular $VaR_{99\%}$ and the $VaR_{99.8\%}$ models are then backtested using a standard method (e.g. the Kupiec test), verifying if they appease its requirements. Moreover, the method strives for displaying the backtesting results in a graphical manner, presenting the areas where both tested hypotheses cannot be rejected, or where one of the hypotheses can be

[17] Perignon et al (2008)
[18] Dowd(2005)
[19] Perignon et al. (2008)
[20] Berkowitz (2002)
[21] Colletaz et al (2011)
[22] The 99.8% level is arbitrary, any other high level for the VaR calculations could be chosen based on the data series used.

rejected and the other cannot as well as the zone corresponding to a rejection for both test statistics.

The log-likelihood test statistic for the unconditional coverage hypothesis is written as[23]:

$$LR_{UC}(\alpha') = -2\ln\left[(1-\alpha')^{T-N'}\alpha'^{N'}\right] + 2\ln\left[\left(1-\frac{N'}{T}\right)^{T-N'}\left(\frac{N'}{T}\right)^{N'}\right]$$

where *α'* represents the probability that a super exception can occur (it equals *1 – confidence level*), *N'* signifies the number of super exceptions whereas *T* equals the total number of VaR forecasts.

Using multiple testing approach does not allow for controlling the nominal size of the test (which could result in rejecting a valid model) and, thus, the suggestion is to apply a joint test of both VaR exceptions and super exceptions under the form of a multivariate unconditional coverage test[24] as proposed by Perignon and Smith (2008).

The likelihood test statistic is given by the following formula[25]:

$$LR_{MUC}(\alpha,\alpha') = 2\left\{N_0 ln\left(\frac{N_0}{T}\right) + N_1 ln\left(\frac{N_1}{T}\right) + N_2 ln\left(\frac{N_2}{T}\right)\right.$$
$$\left. - \left[N_0\ln(1-\alpha) + N_1\ln(\alpha'-\alpha) + N_2\ln(\alpha')\right]\right\}$$

where N_i represents the total number of ones counted for each of the below indicator variables[26] (each of them showing how many times a bank's profit/loss has fallen in a particular VaR interval):

$$J_{1,t} = \begin{cases} 1, & if\ \ -VaR_{t,99.8\%} < R_t < -VaR_{t,99\%} \\ 0, & otherwise \end{cases}$$

$$J_{2,t} = \begin{cases} 1, & if\ \ R_t < -VaR_{t,99.8\%} \\ 0, & otherwise \end{cases}$$

and

$$J_{0,t} = 1 - J_{1,t} - J_{2,t}$$

[23] Colletaz et al.(2011)
[24] Idem
[25] Idem
[26] Ibid.

There are a number of advantages that this technique has over other methods employed in backtesting.[27] The most important one is the fact that it accounts for the magnitude of VaR exceptions and at the same time manages to keep its simplicity, being in this respect comparable to the standard approaches to model validation. Furthermore, the Risk Map can be applied to a variety of VaR models, such as credit-risk VaR, operational risk VaR, or single asset, portfolio, business line and bank VaR models, but it can also be used to backtest systemic risk measures. In addition, its hypothesis testing structure makes the Risk Map a formal backtesting procedure.

2.2.Basel Capital Agreements

The goal of the Basel capital agreements is to promote the stability of the banking system and to increase its flexibility related to the occurrences of unexpected shocks.

The first capital regulations, known as Basel I, were initially developed in 1988, but they only came into effect in 1992, with the main scope of limiting credit risk and ensuring sufficient capital to take in losses without allowing it to spread systemically. It imposed a capital requirement of 8% on the bank's assets, but it did not take into consideration their maturities or their volatility.

These initial rules were revised in 1996 through an amendment that recognized the importance of market risk in a bank's trading portfolios. The banks were allowed to decide whether to use the Supervising Committee's standardized approach or an alternative approach, when it came to measuring risk models that were the base for calculating the capital requirements.

Basel II was released in 2004 in an attempt to deal with some of the issues in Basel I, most notably the problem of regulatory arbitrage[28], that gave banks the possibility to shift assets off balance sheet, which allowed them to take in increasing amounts of risks. Basel II brought four significant changes in calculations of risk weighted assets[29]. Firstly, Basel II is much more detailed than the first agreement with respect to the break-down of assets categories. Secondly, a significant role was allocated to the ratings provided by major rating agencies. Thirdly, it gave global banks the possibility to use their internally-developed risk models, under the provision of disclosing these risk estimates to the monetary authorities. Last

[27] Colletaz et al (2011)
[28] Jackson(1998)
[29] Elliott(2010)

but not least, Basel II stated a different method to be used for calculating the risk of assets that were held in a bank's trading accounts.

Despite all these improvements, several problems persisted[30]. First of all, it did not deal with the problem of portfolio invariance, ignoring the benefits of diversification. Second, it only uses a single global risk factor, not recognizing thus the country specific risk. Furthermore, Basel II continues to ignore the contagion and counterparty risks and it does not take care of the pro-cyclicality in the banking regulations (it underestimates risks in good times and overestimates them in bad ones).

Also Basel II was not in full effect prior to the start of the financial crisis, since for instance in the United States (where the crisis originated) the implementation of these rules was postponed until 2010 and only to a limited number of banks, while in Europe most banks deferred these regulations until 2008.[31]

Both Basel I and II failed to fulfill their goal of avoiding systemic events in the banking system and the recent financial crisis emphasized once more the need to revise the current capital coverage rules. Thus Basel III was developed, stating the following modifications to the capital regulations[32]. First and foremost is the increase in the capital ratios (banks being, under these new rules, required to raise the capital coverage of their assets) and the use of the leverage ratio as a safety net (as the crisis red-flagged the problems in using risk weighted calculations). Other provisions of the new capital accord are the tougher risk weightings for trading assets, the elimination of softer forms of capital, the capital requirements for counterparty credit risk, new liquidity requirements or counter-cyclical capital requirements.

2.3.Previous research

For a more detailed description of the VaR models we use in this thesis we recommend to review a number of articles, including Kuester et al.(2005) which presents several of the most used VaR methods for financial data, or more detailed textbooks such as Dowd (2005) or Jorion (2001). For further information on the backtesting procedures, we also recommend a series of articles such as Kupiec (1995) and Colletaz et al. (2011).

Previous research about the use of VaR models and their accuracy in financial institutions is rather limited compared to research on VaR in general. Furthermore, most

[30] Blundell-Wignall and Atkinson(2010)
[31] McAleer(2009)
[32] Elliott(2010)

studies focus on either individual assets or simulated portfolios, mainly due to the non-disclosure policies adopted by banks. Nonetheless we mention three studies that focused on comparing VaR models from a financial institution perspective and that set a precedent for this thesis.

Berkowitz & O'Brien (2002). The authors used real daily P/L data and the associated VaR measures from six US banks with large trading portfolios. They employed several backtesting frameworks to assess the statistical accuracy of the banks' internal VaR models and then compared the results to those generated by a simple ARMA(1,1)-GARCH(1,1) model. They concluded that banks do tend to overestimate their VaRs. Nonetheless, in spite of the fact that VaR internal models utilized by commercial banks are complex and incorporate very detailed information, they did not manage to outperform the forecasts generated by the ARMA-GARCH model employed by the authors.

Perignon & Smith (2008). The empirical study carried out by Perignon and Smith also employs daily bank trading revenues, from 5 commercial banks. Several VaR estimation methods were applied to the data and the results were backtested using a framework developed by the authors in the article. The conclusion is that non-parametric VaR models, particularly GARCH-based methods or filtered Historical Simulation seem to be the most accurate when it comes to the banks' trading portfolios. This is consistent with the practice at commercial banks, which also use non-parametric methods, such as the HS, as a basis for their VaR estimations.

Billinger & Eriksson (2009). In their master thesis written for the Finance Program at Lund University, the authors performed a comparative study using a total of 10 VaR models, using a theoretical portfolio that was built to mimic the trading portfolio of Bank of America and the S&P500 index. Their intent was to determine the most precise model for bank's trading portfolios. The results were backtested using the framework developed by Cristoffersen. Their findings showed that the models which have the highest performance are the ones with leptokurtic features and time-varying volatility, the GARCH(1,1)-t model being the most accurate. Models that assume normality perform poorly, regardless of the use of volatility dynamics.

Our study builds on and compares with these previous papers, through its function of comparatively analyzing different VaR frameworks from the perspective of financial institutions. Nonetheless, our paper adds to the current literature due to the use of a multitude of VaR estimation methods on real financial data, including methods that have not been applied in these articles. Also, the use of real data permits us to assess the models in an empirically relevant context and adds a practical dimension to the study.

Furthermore, we use a backtesting framework that has been recently developed, the Risk Map, which takes into consideration the magnitude of the violations and not only their frequency.

3. Methodology

3.1. Characteristics of financial data

When determining the most accurate VaR models to use we must take into account the characteristics exhibited by the financial returns. One of the most important such feature is that financial returns are not normally distributed. This is emphasized by three facts: (i) volatility clustering, reflected by high autocorrelation in absolute and squared returns; (ii) high kurtosis, manifested by a return distribution that is more peaked around the center and has fatter tails than the normal distribution; and (iii) skewness in the returns.[33] Therefore, VaR estimation models that are based on the normality assumption may not be able to capture the actual risk that the institutions are exposed to. Consequently, models that take these deviations from the normal distribution into consideration should prove to have a better fit in estimating the potential losses. Non-parametric methods have an advantage in this case, in the sense that they do not assume a certain distribution for the returns, but use the data from the past to model the forecasts. Furthermore, models that are based on time-varying volatility tend to perform better that models that assume constant volatility.[34]

Financial data series also tend to exhibit a leverage effect which means that a negative shock to financial time series is likely to cause volatility to rise by more than a positive shock of the same magnitude. VaR models that use volatility forecasts that account for this asymmetry are thus expected to perform better than models that ignore the "leverage effect".

3.1.1.Estimating volatility

Volatility is measured by the standard deviation or variance of returns and it is often used as a simple measure of the total risk of financial assets. Many VaR models require the estimation or forecast of a volatility parameter.

Historical volatility is the simplest estimate and it simply involves calculating the variance of returns in the usual way over some historical period which then becomes the volatility forecast for all future periods.[35] The problem with this model is that it assumes constant volatility over time. Financial data is known to exhibit volatility clusters across time, which means that a constant volatility assumption will not manage to correctly capture the dynamics of the financial returns over time. Hence, a number of alternative models that

[33] Kuester et al. (2005)
[34] Berkowitz & O'Brien (2002), Cristoffersen et al.(2008).
[35] Brooks(2008)

estimate time-varying volatility have been introduced. These include the Exponentially Weighted Moving Average (EWMA) model, GARCH models and implied volatility models.

EWMA. The exponentially weighted moving average (EWMA) is an extension of the historical average volatility measure, which allows more recent observations to have a stronger impact on the forecast of volatility than older observations. Therefore, the latest observation will have the largest weight, while weights associated with previous observations decline exponentially over time.[36] This volatility model is used by J.P. Morgan's Risk Metrics Department in computing their VaR.

GARCH. The GARCH model was developed independently by Bollerslev (1986) and Taylor (1986). The model allows the conditional variance to be dependent upon previous own lags. A benefit of these models is that they account both for volatility clusters and leptokurtosis.

GARCH(1,1) is the most popular model from the GARCH family used in estimating time-varying volatility, which is most often motivated by the fact that it fits the data well most of the times.[37]

Nevertheless, the simple GARCH(1,1) model is based on a symmetric distribution, thus failing to capture the "leverage effect" that financial data often exhibits. Consequently, non-linear extensions of GARCH have been proposed, such as Exponential GARCH (EGARCH) model by Nelson (1991) or the GJR-GARCH model by Glosten, Jagannathan and Runkle (1993). These methods will be discussed in more detail in section 3.3.

Implied volatility. The volatility forecast over the lifetime of an option implied by the option's valuation model can be determined using the price at which the option is traded. For example by inverting the Black-Scholes option pricing formula and using observable market variables as inputs (spot price, strike price, time to maturity, risk-free interest rate) the market's volatility forecast over the lifetime of the option can be obtained.[38]

The fact that implied volatility is a forward-looking volatility forecast over the lifetime of the option provides two advantages of this method over the historical volatility: (i) it incorporates information that other methods will ignore, unless present in the historical data; and (ii) provide estimates of volatility in which experienced market players are confident enough to bet money on.[39]

[36] Brooks(2008)
[37] Idem
[38] idem
[39] Dowd(2005)

In the recent years volatility indices have been created with the express purpose of capturing implied volatility. A good example would be the VIX index which is computed as a weighted average of different prices of options written on the S&P500 index. VIX is calculated and published by the Chicago Board Options Exchange (CBOE). The VIX represents by its definition the market's best estimate of the volatility of the S&P500 index over the remaining life of the option.

3.2.Non-parametric methods

The non-parametric methods are the most attractive techniques when it comes to estimating value-at-risk, as they do not require strong assumptions about which distribution the returns follow. Another strength is the a-theoretical approach, as they use data from the past in forecasting risks in the future, and do not assume a theoretical distribution.[40]

3.2.1.Basic Historical Simulation

Since most of the models currently used by the commercial banks are centered on this method[41], it is thus important to understand its theoretical background and how well it performs compared to other models.

The historical simulation is considered to be the simplest way to estimate VaR, using sample quantile estimate based on historical returns[42]:

$$VaR_t = q(\alpha)$$

When using a 99% confidence interval, the value-at-risk measure is represented by the loss which is equaled or exceeded in 1% of the time. For instance, if a number of 250 observations are used then the $VaR_{99\%}$ will be equal to the third worst loss.

Two motivations are considered responsible for the wide use of this method today. Firstly, the size and complexity of the trading positions at commercial banks make parametric VaR methods hard to implement in practice[43] and, secondly, both banks and regulators do not want huge changes in risk market charges, opting for a smoother evolution through time[44].

[40] Dowd(2005)
[41] Perignon & Smith(2006)
[42] Kuester et al.(2005)
[43] Perignon & Smith (2006)
[44] Jorion(2001)

Among the advantages of this method the more important are the simplicity in implementation, as it relies on actual returns and also the lack of assumptions made about the shape of the distribution of the risk factors.[45]

There are also disadvantages when employing this method. One such drawback is that historical simulation does not capture volatility dynamics and as a result can cause clustering in VaR violations[46]. As a solution, Berkowitz as well as Perignon propose using a model that does not assume constant volatility, such as a GARCH model.

The slow reaction to shocks can be considered a shortcoming[47], but the historical simulation's slow adjustment speed can also be seen as an advantage as it avoids frequent adjustments to the coverage capital[48].

Other deficiencies are the assignment of equal probability weights to each return[49] and also the fact that historical simulation is rather a conservative model, as it overpredicts VaR[50].

3.2.2.Age Weighted Historical Simulation

The age weighted historical simulation is a hybrid method which blends benefits from various methods of VaR estimation (specifically RiskMetrics and Historical Simulation), allowing for weighting the probabilities associated with each observation with reference to time. Hence newer observations are given more weight opposed to older ones. Moreover, this method takes into consideration the properties of the observed data series, such as e.g. skewness.[51]

The idea behind this method is to replace the constant weights that were used in the basic historical simulation with the age-weighted probabilities. This means that instead of having a weight equal to $1/n$ for all values, each observation will now have a weight equal to $w(i)$, which is decaying exponentially with the rate λ (λ close to 1 indicates a slow rate of decay, while λ close to 0 indicates a high rate of decay)[52]. Thus, $w(1)$ corresponds to the newest observation, $w(2)$ to the second newest observation (can be written as $\lambda w(1)$), $w(3)$ to the third (can also be $\lambda^2 w(1)$) and so on. Then $w(1)$ can solved for, keeping in mind that the

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[45] Pritsker (2001)
[46] Christoferssen et al (2008)
[47] Perignon et al(2008)
[48] Christoferssen et al (2008)
[49] Pritsker (2001)
[50] Danielsson(1997)
[51] Boudoukh et al. (1998)
[52] Dowd (2005)

sum of the weights must be equal to 1. This can be expressed mathematically in the following manner:

$$w(1)$$
$$w(2) \to \lambda w(1)$$
$$w(3) \to \lambda^2 w(1)$$
$$\dots$$
$$w(N) \to \lambda^{N-1} w(1)$$

All these weights must sum to 1: $\sum_{i=1}^{N} w(i) = 1$, which can also be written as:

$$w(1) \sum_{i=1}^{N} \lambda^{i-1} = 1$$

Solving for *w(1)* in the above equation yields the following result:

$$w(1) = \frac{1-\lambda}{1-\lambda^N}$$

A three-step implementation process is further proposed for this approach[53]. The first step is to associate a weight to each observation (presented above). Secondly, the returns are sorted in ascending order, while keeping the association with the corresponding weight, and, thirdly, obtaining the *x%* VaR measure, by starting from the lowest return and then keep accumulating weights until the selected *x%* level is reached.

There are four advantages of age weighted historical simulation[54]. The first one is that this method is a generalization of basic historical simulation. Second, if λ is chosen correctly, VaR estimates have a more rapid response to large losses. Third, it reduces the distortion caused by rarely occurring events and diminishes the ghost effects. Last but not least, the age weighting can be modified to increase the efficiency of the risk estimates and to eliminate the remaining ghost effects[55].

As any other estimation method, age weighted historical simulation has its own shortcomings. Even if the approach allows for stochastic volatility, it does so indirectly and inefficiently[56]. It also has the tendency to understate the tail risk, as it does not associate large profits with risk, and the inability to react to changes in conditional volatility.[57]

[53] Boudoukh et al. (1998)
[54] Dowd(2005)
[55] Ghost effects refer to the effect of an extreme event that lasts for an increased period of time
[56] Hull(1998)
[57] Pritsker (2001)

3.2.3.Volatility Weighted Historical Simulation

This is another method to compute VaR, which takes into consideration the volatility of returns. The main idea behind this approach is that the returns are rescaled in order to account for the recent changes in volatility.

$$y_t^* = \frac{y_t}{\sigma_t}\sigma_T$$

During intervals with high volatility, the returns are scaled upwards while during low-volatility periods the returns are scaled downwards. Thus the method produces risk estimates that display an accurate sensitivity to current volatility estimates.[58]

The volatility that is incorporated in the HS model can be estimated through different methods. In this study we will estimate the model employing volatility forecasts obtained from GARCH, EGARCH, GJR-GARCH methodologies as well as using implied volatility.

HS-VIX. The model, introduced by Nossman and Vilhelmsson (2011) is built on a filtered volatility weighted historical simulation method that uses the implied volatility given by the VIX index.

The portfolio returns are rescaled using the option implied volatility:

$$y_t^* = \frac{y_t}{VIX_{t-1}}VIX_{T-1}$$

The major advantages of the method lie in the fact that it is completely non-parametrical, forward-looking and is based on what is supposed to be the best variance forecast that exists.

However, there are some drawbacks that the authors note. Firstly, the volatility forecast includes a variance risk premium. Nonetheless, the method is proved to be immune to the variance risk premium under the assumption that the premium is proportional to the VIX level. Secondly, the variance forecast, as defined by the index, has a horizon of 22 working days. This would not be a problem if the variance of returns was constant, because the one-day volatility could simply be rescaled by dividing the implied volatility by 22. Nonetheless, when it comes to time-varying volatility as is the case with financial returns this becomes problematic. The solutions proposed by the authors are either to build an implied variance forecast scaled to the desired horizon, should one have access to option price data, or to use

[58] Dowd(2005)

some parametric assumption about the volatility process and adjust the forecast accordingly. Lastly, the model should, theoretically, be valid only for the S&P500. However, this could be solved if the volatility of the asset the model is applied to can be written as a constant multiplied by the level of the VIX. Another solution would be to use a different volatility index, that may suit better the asset or portfolio the method is applied on.

The authors tested their model using returns of the S&P500 index and compared the accuracy of the HS-VIX with a basic Historical Simulation and a HS-GARCH model. Their conclusion was that their model had a satisfying performance compared to the other methods tested. Hitherto, we are also applying the HS-VIX model on our portfolio to investigate its accuracy when it comes to forecasting volatilities for other assets than the S&P500.

3.3.Parametric methods

There are several alternative approaches to estimating risk by means of parametric methods, such as the normal, lognormal, student-t, extreme value theory and other various distributions. Dowd (2005) considers these methods to be more powerful than the non-parametric ones as they allow for the additional information in the distribution function to be used. Dowd (2005) also brings forward the fact that these approaches are easy to use, but warns about their error vulnerability when assuming a wrong density function.

3.3.1.Normal distribution

The normal distribution is one of the most widespread methods and what makes it attractive[59] is the fact it requires data for only two variables, more specifically the mean and the variance or standard deviation. Thus, the formula used to calculate the value-at-risk can be written in the following manner, as an expression of the two aforementioned parameters:[60]

$$VaR = -\mu - t_{\alpha}\sigma$$

where μ represents the mean, σ is the standard deviation and t_{α} is the corresponding critical value to the assumed confidence level α.

Even though many models continue to work under the assumption of a normal distribution, it was proven that financial data usually exhibits skewness greater than 0 and a kurtosis in excess of three and as a consequence this could cause an under or overestimation

[59] Dowd (2005)
[60] Idem

of risk.[61] Also, a normal distribution allows a return to have any value, thus raising the possibility of incurring losses so large that we stand to lose more than what we have invested[62].

3.3.2.Student-t distribution

As opposed to the normal distribution which is characterized by the first two moments, the student-t distribution can accommodate, besides the mean and the standard deviation, a third moment, more specifically excess kurtosis. The formula employed to calculate value-at-risk under the assumption of a student-t distribution is:

$$VaR(\alpha) = -\mu + t(\alpha, \nu)\sqrt{\frac{\nu - 2}{\nu}}\sigma$$

From this formula it is obvious that the VaR based on student-t distribution is characterized by three parameters, namely the mean (μ), the standard deviation (σ) and the degrees of freedom (ν). In this case t represents the corresponding statistic given a confidence level (α) and the degree of freedom (ν).

An advantage of the student-t distribution could be that is a generalization of the normal distribution when the number of degrees of freedom is very large, but when ν is finite it has the ability to account for higher than normal kurtosis[63]. Another advantage is that this method will provide better predictive densities[64]. As a deficiency, the possibility of producing fallacious high risk estimates can be pointed out[65]. Also, another problem with the t-distribution is its instability, which means that VaR forecasts over long periods of time are not reliable.

3.3.3.Exponentially Weighted Moving Average (RiskMetrics)

The exponentially weighted moving average method aims at capturing the movements in volatility in the short run[66] and considers the more recent observations to have a greater importance than older observations, and as a consequence the former will carry more weight. This weight decays exponentially as we go further back in the past.

[61] Angelidis et al. (2004)
[62] Dowd (2005)
[63] Dowd(2005)
[64] Danielsson (1997)
[65] Dowd(2005)
[66] Hendricks (1996)

The formula for the volatility under the exponentially weighted moving average approach is given by the expression[67]:

$$\sigma_t{}^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)\eta_{t-1}^2$$

where σ_t is the volatility at the moment t, λ is the decay factor and η_{t-1} represents the percentage change at $t-1$.

The exponentially weighted moving average is simply a combination of two components.[68] The first one is the previous day's weighted average, with λ as its corresponding weight, while the second one is represented by the previous day's percentage change, being given a weight of (1-λ).

Besides the use of decaying weights, another advantage of this method is the fact that it relies on only one parameter (λ), thus making it easy to use (*RiskMetrics Technical Document* estimates a value for λ of 0.94). The reliance on the recent past is seen both as an advantage, since it allows for updating the volatility forecast based on the most recent return, and also as a fault, since it reduces the overall sample size hence resulting in an increased possibility of measurement error.

Dowd (2005) makes the inference that the current volatility estimate is the best forecast of future volatility, ignoring thus the recent evolutions in the data used. A solution to this problem is the use of a GARCH model.

3.3.4.GARCH(1, 1)

Bollerslev (1986) developed a generalized form of the autoregressive conditional heteroskedasticity model introduced by Engle (1982), which allows for a more flexible structure in modeling volatility.

The basic GARCH(p, q) is modeled dependent on previous volatilities and on past returns and is described by the formula[69]:

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \cdots + \alpha_q u_{t-q}^2 + \beta_1\sigma_{t-1}^2 + \cdots + \beta_p\sigma_{t-p}^2$$

In practice however the most used version is the GARCH(1, 1) process, which has the advantage of simplicity and of fitting the data very well, making it less likely to violate the

[67] Hull (2008)
[68] Hendricks (1996)
[69] Brooks (2008)

non-negativity constraints. It also allows accounting for volatility clustering and fat tails in the distribution (leptokurtosis).

The following model specification for calculating VaR, relying on the GARCH(1, 1) process is given[70]:

$$R_t = \mu + u_t$$

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

$$VaR_{99\%t} = -(\hat{R}_t - 2.33\hat{\sigma}_t)$$

Although the standard GARCH model reduces the likelihood of breaching the non-negativity constraints, it does not eliminate this possibility altogether. Also the standard version does not take into account the possibility that a negative shock in the economy can cause the volatility to rise by more than a positive shock of the same amplitude will (asymmetries known as leverage effects).

These drawbacks are dealt with by employing extensions of the GARCH model that account for possible asymmetries, such as the Glosten, Jagannathan and Runkle (1993) model (GJR model) or the exponential GARCH proposed by Nelson (1991).

The EGARCH model. The model was introduced by Nelson in 1991. Its superiority in modeling volatility over a simple GARCH model is given by the fact that it allows asymmetries under its formulation.[71]

Nelson's EGARCH model is defined in the following manner:[72]

$$\ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$$

Several empirical studies have shown that EGARCH models have a better forecasting performance than symmetric GARCH models in what concerns financial data. Alberg et al. (2008) conducted a study on two Tel Aviv Indexes and concluded that EGARCH skewed student-t model gave the most precise characterization of the dynamic behavior of the index returns. This was due to the fact that the model managed to capture the serial correlation, asymmetric volatility clustering and leptokurtic innovation. Su (2010) reached a similar

[70] Perignon et al(2008)
[71] Brooks(2008)
[72] Brooks(2008)

conclusion after carrying out a comparative study on the Chinese stock market. EGARCH models turned out to fit the sample data better than symmetric GARCH in modeling volatility of the Chinese stock returns.

GJR-GARCH model. The model introduced by Glosten, Jagannathan and Runkle (1993) is a simple extension of GARCH with an additional term added to account for possible asymmetries.

The equation for a GJR-GARCH(1,1) is:

$$\sigma_t^2 = K + \delta\sigma_{t-1}^2 + \alpha\, \varepsilon_{t-1}^2 + \phi\varepsilon_{t-1}^2\, I_{t-1}$$

$$\varepsilon_t = \sigma_t Z_t \quad where\ Z_t \quad is\ iid$$

$$I_{t-1} = \begin{cases} 0 & if\ \ \varepsilon_{t-1} \geq 0 \\ 1 & otherwise \end{cases}$$

Su et al. (2009) carried out a study that employed several symmetric and asymmetric GARCH models in computing Value at Risk forecasts on the QQQQ index returns. Their results showed that GJR-GARCH models tended to outperform both the symmetric GARCH models and the asymmetric NA-GARCH models used. Furthermore, ARMA(1,1)-GJR-GARCHM(1,1) turned out to be the best market risk management for financial portfolios in terms of the smallest violation number and the least capital charge.

4. Data

Sample period. The data consists of daily P/L series taken from 4 commercial banks: Bank of America, Deutsche Bank, Danske Bank and Swedbank and has a length of 982 observations, during the January 2007-December 2010 interval.

The data series have been extracted from P/L plots disclosed by the aforementioned banks in their annual and risk management reports. To obtain the data points from the graphs we used a software application called GetData Graph Digitizer.

The use of real data from commercial banks confers a practical dimension to the study. Furthermore, the 4 year time interval and the high frequency of the data allow for a more precise analysis of the models during different market conditions, including turbulent periods such as the global financial crisis in 2008.

To estimate the VaR models we use a rolling window and a recursive window, depending on the methodology we are testing. The in-sample period consists of one year which equals 250 daily observations, while the out-of sample period consists of the remaining 732 observations. In the case of the recursive widow, the in-sample period length will expand as we go forward through time with the VaR forecasts. The in-sample period length is consistent with the Basel Committee recommendations, while the out of sample period is extensive enough to allow a proper application of the backtesting frameworks. Using a shorter in-sample interval could be motivated by the desire to capture short-term movements in the underlying risk of the trading portfolio, while a longer in-sample period increases the accuracy of the historical percentiles estimation.[73]

Hence, a recursive window is applied to the HS models, since it helps to avoid the loss of observations while the rolling window is applied to the remainder of the models.

Software. The data was extracted from graphs disclosed in the annual reports of the four banks, using GetData Graph Digitizer.

The data series for the VIX, VDAX and VSTOXX volatility indices were extracted from Google Finance, Frankfurt Stock Exchange and EuroStoxx respectively, over the same January 2007- December 2010 period.

Microsoft Office Excel 2007 was employed to compute the VaR models, either manually or by using VBA functions. The age-weighted HS and the volatility weighted HS were computed using VBA functions in Microsoft Office Excel.

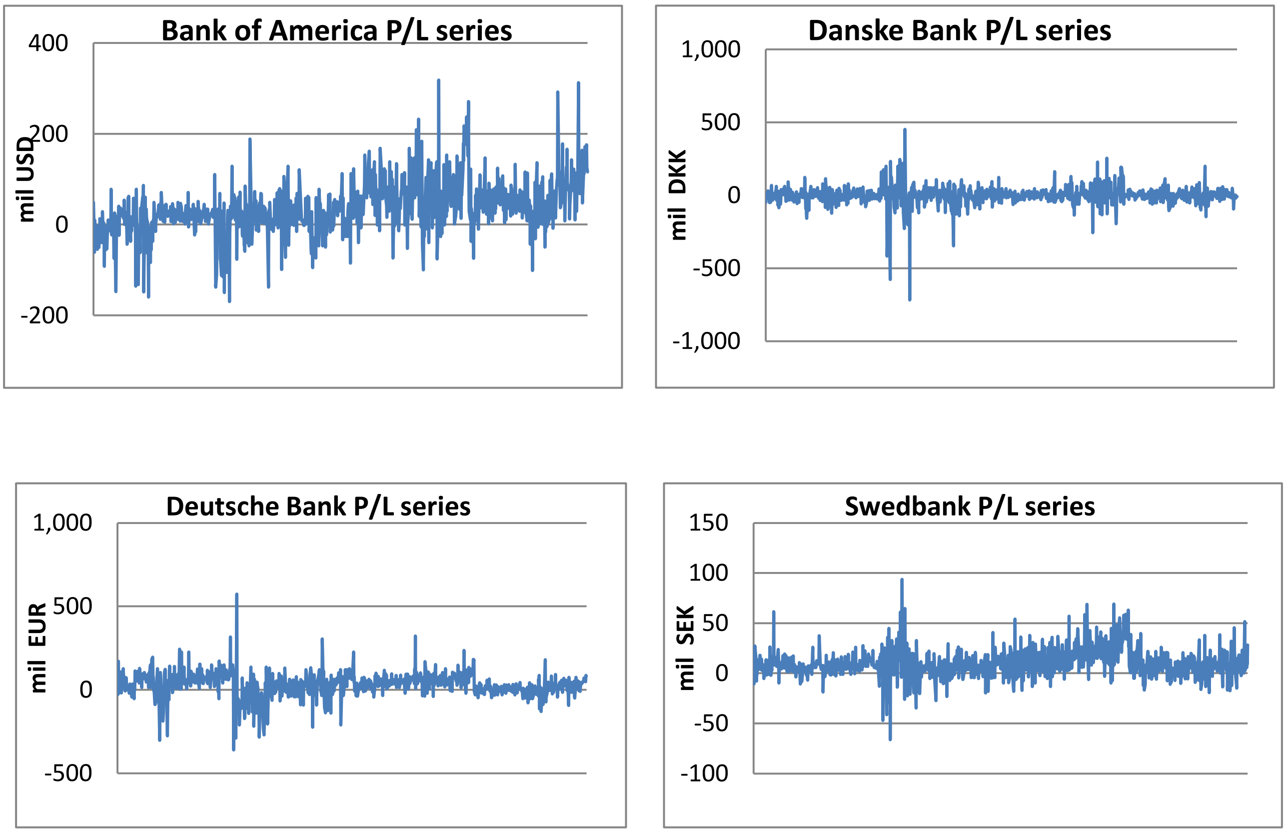
[73] Hendricks (1996)

Time varying volatilities modeled through GARCH processes were estimated through a function built in Matlab 2010, also using the Econometrics Toolpack.

Eviews 7 was used in the data analysis: Jarque Berra test for normality and unit root tests for the P/L data series.

Descriptive statistics. From the P/L graphs presented below a volatility clustering phenomenon can be observed in the banks' portfolio returns series. Large movements in volatility are followed by equally large movements, while small variations in volatility are followed by equally small variations. The analysis went further with checking for autocorrelation in P/L observations by assessing the autocorrelation partial autocorrelation functions[74]. From the correlograms it can be inferred that the data series for all the banks included in the study exhibit autocorrelation.

Figure 1.Daily P/L plots



In order to properly confirm the presence of time-varying volatility deducted from the P/L graphs and the correlograms, two formal heteroskedasticity tests were applied to the data series: the Ljung-Box-Pierce Q-test and the Engle ARCH test.

[74] The correlograms are presented in Appendix 1

The Ljung-Box-Pierce Q-test tests for a departure from randomness based on the ACF of the data. The Q-test statistic is asymptotically Chi-Square distributed, under the null hypothesis of no serial correlation.[75] The null hypothesis of Engle's ARCH test is that a time series is a random sequence of Gaussian disturbances (no ARCH effects exist) and the test statistic is also Chi-Square distributed.[76]

The tests were conducted by extracting the sample mean from the actual P/L data. Both tests yielded the same results for the P/L data series of the four banks.[77] The null hypothesis was rejected in all cases, which implies the existence of significant serial autocorrelation in the data series, and thus supports the presence of time varying volatility in the P/L data.

The conclusion that can be drawn is that VaR models that assume constant volatility of returns over time should prove less accurate than models that take into consideration this change in volatility.

Table 1.Summary statistic for the P/L distributions

	BANK OF AMERICA (mil USD)	DANSKE BANK (mil DKK)	DEUTSCHE BANK (mil EUR)	SWEDBANK (mil SEK)
Mean	35.607	-1.054	29.807	10.249
Median	29.700	-2.885	33.470	8.356
Maximum	318.499	452.310	573.684	93.603
Minimum	-169.619	-717.877	-359.330	-66.402
Std. Dev.	56.976	66.464	72.534	14.222
Skewness	0.423	-1.778	-0.354	0.716
Kurtosis	5.598	27.340	9.495	6.957
Jarque-Bera	305.4184	24758.7	1746.677	724.7141
Probability	0.00000	0.00000	0.00000	0.00000

The normality assumption of the data series is rejected by the Jarque-Berra test in all four cases, as shown in Table 1. All the P/L series exhibit a leptokurtic distribution, with Danske Bank displaying the highest excess kurtosis coefficient. In addition, all banks have a slightly asymmetric portfolio return distribution. Bank of America and Swedbank P/L distributions are skewed to the right (positive asymmetry) while Danske Bank and Deustche Bank exhibit negatively skewed distributions.

[75] Box et al (1994)
[76] Engle(1982)
[77] Appendix 2

Hence, VaR estimation methods that are based on the assumption of a normal distribution of returns are expected to have a lower forecasting power than the models that are based on distributions which are closer to the actual P/L distribution in our case. It will be interesting to analyze this difference, as well as the accuracy of parametric models versus non-parametric models from this perspective.

It can also be observed from the P/L graphs that the portfolio returns seem to be stationary for all the banks over time. This is also confirmed by the Augmented Dickey Fuller test conducted on the series.[78] The null hypothesis of the series having a unit root is rejected in all cases, thus the P/L series are stationary.

[78] Results of the ADF test are presented in Appendix 3

5. Empirical Results

In this section the results are first summarized structured by the VaR model used, and then they are categorized by bank. We believe this is the best approach since we are aiming at finding the most accurate model, but different banks seem to deem different models as more appropriate. The investment portfolios characteristics are specific to each bank, depending on the policies, preferences and particularities of the institution, and it is only natural that these trademarks should have an impact on the VaR methodologies used. Consequently, it is important to contrast and compare the models not only among themselves, but also in the context of each P/L series. The graphs for all VaR estimations for all the banks and models plotted against the original P/L series can be analyzed in Appendix 7, over the backtesting window. All the VaR forecasts were computed for a 99% confidence level and a 1 day holding period.

5.1.Model analysis

A series of both parametric and non-parametric models were employed to estimate VaR for the four P/L data series, a part of these models including time-varying volatility forecasts. All models were backtested using both the Kupiec framework and the Risk Map model for a confidence level of 99%. The backtesting window varies in size, the models based on time-varying volatility forecasted through GARCH models having a shorter window than the rest of the models. This is due to the fact that a window of 482 observations was used to estimate volatilities through the GARCH models. We preferred to use a longer estimation window rather than 250-day one because it offers more stability to the estimated volatilities and it increases the probability of the models converging to a solution. Nonetheless, this reduces the backtesting window to 500 observations for parametric methods and 250 for volatility weighted HS. The latter is due to the fact that we also use a recursive window starting with 250 observations to determine the VaR, while the total number of observations left after estimating GARCH volatilities is equal to 500. The 250 observations backtesting window, while not ideal, is still consistent with Basel requirements and it should still be able to produce accurate results.

We also want to note that in constructing the Risk Map's graphical rendition it is very important to account for the number of VaR forecasts that are yielded by a model. Thus, when the model is used to realize VaR predictions over, e.g. one year, the Risk Map's boundaries, regarding the number of super exceptions and the total number of violations, need to be computed based on forecasts' sample size. Correspondingly, these limits will be expanded as the number of VaR predictions increases. In our study, for the one year map, the maximum value that can be taken by the super exceptions (N') is 3, while the number of total exceedances (N) needs to be between 1 and 7. The two years map has the limit of super tail losses set at 4 and N is required to be between 1 and 11 to not reject the model. Finally, or the three years worth of VaR forecasts, the map frontiers are N' strictly lower than 6 and N in the interval [2, 15]. It is also necessary to mention that these values hold at the 99% confidence interval for the two test statistics and a decrease in the degree of confidence will result in tighter intervals for the map's boundaries (see Colletaz et al. (2011) for a map example at the 95% confidence level).

Time-varying volatilities were computed using a series of GARCH and asymmetric GARCH models, under the assumptions of normally or t-distributed innovations. The conditional mean for the GARCH models tested is an ARMA(1,1). This was chosen due to the fact that it had both all coefficients statistically significant and the lowest information criteria out of the other estimated AR/MA/ARMA models. Furthermore, the ARMA(1,1) model proved to be best for all the banks taken into consideration. From the analysis of the correlograms for the four data series, it could be observed that Bank of America and Danske Bank seem to require an ARMA(p,q) model while the Deutsche Bank and Swedbank data series appeared to require a pure AR(p) model. Nevertheless, after estimating several mean models, the ARMA(1,1) proved to be the best fit for all four institutions.

In what concerns the conditional variance, the GARCH models specifications were fit to each data series. After estimating a series of GARCH specifications on a window of the most recent 482 observations in our data series, we chose those models that had most or all coefficients statistically significant.[79] Moreover, between two models with a different number of coefficients but with all coefficients statistically significant we chose the model that had the largest number of coefficients. This is due to the fact that, in this case, a larger number of significant coefficients can better break down the evolution of the variance. However, some of the asymmetric GARCH models exhibit statistically insignificant leverage coefficients for some of the banks. This means that the leverage effect does not have a statistically significant influence on the dependent variable, in this case the conditional variance. Nevertheless, we resumed to apply these models, but keeping in mind their drawbacks.

[79] A selection of the results is presented in Appendix 4.

After choosing the appropriate GARCH models for each data series, an Engle ARCH test was conducted on the standardized innovations in order to determine whether there is any autocorrelation left.[80] While in the pre-estimation analysis the ARCH test indicated rejection of the null hypothesis of no ARCH effects, the results show the opposite for the post-estimation analysis. The ARCH test indicates non-rejection of the null hypothesis, with highly significant p-values, confirming the explanatory power of the models employed.

The reason for using a 482 window of the most recent observations in the data series for estimating the GARCH coefficients is because we wanted to choose the models that best fit the variance at the current moment. However, the selected models were also tested on a series of random windows consisting of 482 observations from inside the data series, yielding similar results. The same approach was used for the conditional mean testing.

The results of the backtesting procedures are summarized in Appendix 6, in terms of acceptance or rejection of the models. More detailed results will be presented and commented on in the following subchapters that analyze the model fit for each bank.

5.1.1.Non-parametric models

The non-parametric models were computed using a recursive window that starts with 250 observations. The Age weighted HS was computed using a rather conservative decay factor λ equal to 0.999.

In order to compute VaR using the volatility weighted HS, we used volatility forecasts derived from a series of GARCH and asymmetric GARCH (EGARCH, GJR-GARCH) models, both under the assumption of a normal and a Student-t distribution for innovations.

The models seem to yield different results, according to the P/L series they were applied to.

Basic HS. VaR estimates using the basic HS method follow a rather flat structure over time, with few variations in value. The basic HS model tends to react slowly to volatility fluctuations, which can lead to a large number of violations due to a sudden increase in volatility. From the graphs of Swedbank and Danske Bank it can be observed that most violations happen at the beginning of the backtesting window, this being a period characterized by high volatility, consistent with the financial crisis of 2008. However, the model is deemed suitable by both the Kupiec test and the Risk Map backtesting framework and for all the data series. The unconditional coverage test for both normal exceptions and "super-exceptions" do not reject the model, as does the multivariate unconditional coverage

[80] The results are presented in Appendix 5.

test. Thus, the simplest VaR estimation method seems to provide a good performance in the case of banks' portfolios. The downside however could be judged from the perspective of capital requirements. The flat structure of the basic HS results implies a higher, almost constant over time capital charge. This is also confirmed by the average VaR yielded by this model, which is among the largest out of all the methods tested, for all the banks. Methods which are able to follow the P/L series more closely in terms of volatility fluctuations will yield a lower average VaR and thus will require lower capital charges. Hence, while the basic HS is not rejected by the backtesting frameworks, there may be models that are equally good in what concerns the coverage tests but, at the same time, fit the data series better, thus outperforming the basic HS.

The good performance of the basic historical simulation, also noticed by other authors[81], could help explain its popularity as it is the most used VaR estimation method in banks around the world[82]. Moreover, the conclusion of historical simulation's slow adjustment speed drawn from our results[83] is also confirmed by previous research[84]. Historical simulation's non-rejection remark presented above is also backed by the backtesting techniques used by Berkowitz et al. in their paper.

While the Basic HS was accepted as being fit by all institutions, and by both procedures, the rest of the models do not perform as well.

Age weighted HS. The age-weighted HS looks like it underperforms the basic HS in terms of acceptance. Deutsche Bank's Kupiec test rejects the model, due to a higher number of VaR violations than expected. Nonetheless, for the other banks, this model outperforms the basic HS in that it yields a lower average VaR, and therefore imposes lower capital requirements for the financial institutions. The VaR estimates still follow a rather flat structure in time, and are not able to quickly adapt to changes in volatility, as is the case with the basic HS. By attributing higher weights to more recent data it was expected that the age weighted HS would be able to react in a more timely manner to volatility fluctuations than the basic HS. Still, graphically, the age weighted HS does not seem to be much more flexible than the basic HS in adjusting to changes in market conditions.

HS-VIX. In the case of the volatility weighted HS using volatility indexes such as VIX, VDAX and VSTOXX the results are interesting to analyze. The HS-VIX model seems to work for all the banks, in what concerns the Kupiec test. While the Risk Map measures are

[81] Kuester et al. (2005)
[82] Perignon & Smith(2006)
[83] See graphs in Appendix 7
[84] Nossman and Vilhelmsson (2011), Perignon and Smith (2010), Jorion (2001),

not available for Bank of America and Deutsche Bank due to the lack of violations in the test for super-exceptions, the framework does not reject the VaR method for Danske Bank and SwedBank. However the method yields higher VaR estimates than both the basic and age weighted HS. In the context of capital requirements, this method underperforms the previous ones. Still, this model seems to adapt faster to the volatility fluctuations unlike the previous models, which can prove very useful during turbulent times.

Two other volatility indexes were used to cross-compare the performance of volatility weighted HS, namely VDAX and VSTOXX. VDAX was used for the Deutsche Bank data series, since it is built to characterize the German market, while the VSTOXX was used for Danske Bank and Swedbank. While Danske Bank and Deutsche Bank do no reject their respective models, even outperforming the HS-VIX, by a lower average VaR, in the case Swedbank the Kupiec test rejects the model and the multivariate statistic for the RiskMap accepts it, yet with a very small probability.

The performance of the volatility weighted HS using volatility indexes may be influenced by several factors. Firstly, the volatility indexes do not perfectly mimic the actual volatility of the banks' portfolios, since they were built for their respective market indexes. Thus the volatility estimates from the index may not be consistent with the real volatility evolution of the portfolio. Secondly, there is also a possibility that the P/L data is not perfectly aligned with the calendaristic dates, due to the data extraction procedure used. The graphs the data was extracted from were not dated for each observation, but only yearly. Therefore, we had to assume the number of observations for each year, which may have resulted in more or less observations than in the real portfolio. Hence, the data series is probably loosely aligned to the volatility index returns, which can make a difference in the overall assessment of the method.

Volatility weighted HS. The volatility weighted HS, using volatility forecasts modeled through GARCH models have dramatically different results according to the data series they are applied to. While for Bank of America and Danske Bank all the six models pass the backtest, Deutsche Bank rejects all the models and Swedbank only accepts the HS estimated with a GARCH(1,1) volatility.

The HS-EGARCH(1,1)-t is the most performant model in terms of minimizing capital requirements for Bank of America, while the HS-GARCH(1,1)-t is the best one in the case of Danske Bank. These models estimated the minimum average VaR of all the volatility weighted HS models using GARCH volatility forecasts. Furthermore, it can be observed that the VaR estimated through these methods manage to better fit the P/L series tend to be more sensitive to the volatility changes in the data series, especially compared to the basic and age weighted HS. Therefore, they also yield a much lower average VaR than the above-mentioned models, in this case more than twice as small. This is important for banks, since it lowers their capital requirements.

5.1.2.Parametric methods

The VaR estimates were computed using time-varying volatilities forecasted through an EWMA model and several GARCH models, both under the assumption of a normal and *t*-distribution. Hence, the models are supposed to be able to capture and adapt promptly to the fluctuations in the volatility of the data series.

The EWMA volatility forecasts were computed using the decay factor recommended by RiskMetrics for daily observations, which is equal to 0.94.

Again, the results are very different across banks. There are institutions for which all the models pass the test as opposed to banks for which only a very small number of models are not rejected.

Normal distribution. Under the normal distribution assumption, all the models were deemed suitable by both backtesting frameworks both for Bank of America and Danske Bank, while for Swedbank only the parametric EWMA model passed the test. All models were rejected in the case of Deutsche Bank, however. Neither data series follows a normal distribution, yet models were rejected for some of the series while passing the tests for the others.

T-distribution. The number of degrees of freedom for the *t*-distribution calculation was determined as being the closest integer to (4k-6)/(k-3) where k is the kurtosis coefficient for the return distribution. [85]

Since the analysis of the P/L data series showed that the observations do not follow a normal distribution, but exhibit excess kurtosis, we expect these models to perform better than the parametric models under the normal distribution. Furthermore, the VaR estimates are computed using volatilities derived from GARCH models that use the *t*-distribution for the innovations when estimating the conditional volatility.

Indeed these models show a much better performance than the models under the normal distribution. While for Bank of America and Danske Bank all models pass the backtests, the true difference can be noticed in the case of Deutsche Bank and Swedbank. The latter accepts all the models under the Kupiec framework (the RiskMap results are not

[85] Dowd(2005)

available due to the lack of violations in the test for super-exceptions), while the former accepts two of the three models, namely the GARCH and GJR models, and rejects the EGARCH method. The rejection of the parametric EGARCH model may be due to the fact that the leverage coefficient for the conditional volatility estimate is highly insignificant, therefore the volatility forecasts may not be as performant as for the other models.

In what concerns the parametric VaR computed with EWMA volatilities, using the *t* distribution in determining the estimates does not make a difference in the performance of the test. The model is still rejected for Deutsche Bank and passes the tests for all the others.

To sum up this model analysis, it can be said that the models perform differently across data series. Nonetheless, there are a few methods that yield satisfactory results for all the banks' P/L series included in this study, such as the basic HS or the parametric models using a t-distribution and time-varying volatility derived from symmetric GARCH-t and asymmetric GJR-GARCH-t frameworks. The parametric models tend to estimate more accurate VaRs, since they are able to fit the data series volatility fluctuations better. Moreover, the VaR estimates from these models are lower than is the case of basic HS, which is important for lowering capital requirements.

This concurs with the results of Berkowitz and O'Brien, showing that a GARCH-based model was better in forecasting volatility changes and that its VaR estimates are lower than those of other models. Perignon and Smith also proved that the parametric VaR GARCH-based models fit the data best.

In the following subchapters the results will be analyzed in the context of each data series (for each bank), since it is necessary to determine how the characteristics of financial data influence these models.

5.2. Bank of America

In the case of Bank of America all models are validated by the two backtesting techniques employed. Both the Kupiec test statistic and the Risk Map multivariate unconditional coverage test show significance at the 99% confidence level, since all p-values are greater than 0.01.

These findings can be easily displayed in a graphical representation, by constructing the Risk Maps for the three different numbers of volatility forecasts. In this way the backtesting results can be effortlessly interpreted and this validation technique can be conveniently applied to various VaR estimation models, with diverse numbers of volatility forecasts. The Risk Map can also be used to realize a cross-sectional analysis of the efficiency of a given VaR model, across a large number of banks. The application of this graphic technique allows for quickly identifying the areas where the models are rejected at the 99% and 99.8% confidence levels, the surfaces where either the super exceptions test or the VaR violations test cannot be rejected and more importantly the zone that corresponds to a non-rejection by the tests. Thus when looking at the mentioned Risk Maps[86] it can be very easily observed that all of the models used to calculate VaR for Bank of America lie in the green zone of non-rejection for both tests.

It can also be observed that the parametric GARCH-t methods are proven to have a better fit for the data, when rescaling the number of violations and super exceptions to account for the sample size. This comes in opposition to the non-parametric approaches in the sense that even though the number of tail losses that exceeds VaR is similar for both, the GARCH-t methods yield a lower average VaR. In addition, the basic historical simulation performs better than more complex methods like the Exponentially Weighted Moving Average (EWMA) model or even the normally distributed parametric GARCH models.

Table 2: Bank of America

		Model	N	N'	N 1y	N' 1y	Average VaR	Kupiec		Risk Map Disjointed				Risk Map Multivariate	
								LR_{UC} (99%)	p-value	LR_{UC} (99%)	p-value	LR_{UC} (99.8%)	p-value	LR_{MUC}	p-value
Normal	Volatility Weighted HS	GARCH(1,1)	2	1	2	1	-80.94	0.097	0.756	0.097	0.756	0.399	0.527	0.989	0.610
		EGARCH(1,1)	3	1	3	1	-55.94	0.108	0.743	0.108	0.743	0.399	0.527	0.400	0.819
		GJR(1,1)	2	1	2	1	-135.54	0.097	0.756	0.097	0.756	0.399	0.527	0.989	0.610
t		GARCH(1,1)	2	1	2	1	-77.16	0.097	0.756	0.097	0.756	0.399	0.527	0.989	0.610
		EGARCH(1,1)	3	1	3	1	-135.53	0.108	0.743	0.108	0.743	0.399	0.527	0.400	0.819
		GJR(1,1)	2	1	2	1	-177.86	0.097	0.756	0.097	0.756	0.399	0.527	0.989	0.610
		Historical Simulation	4	1	1	0	-122.70	1.821	0.177	1.821	0.177	0.166	0.684	1.880	0.391
		AWHS	2	2	1	1	-126.72	5.489	0.019	5.489	0.019	0.176	0.675	N/A	N/A
		HS-VIX	2	0	1	0	-76.51	5.489	0.019	5.489	0.019	N/A	N/A	N/A	N/A
		EWMA - Normal	7	2	2	1	-63.38	0.014	0.905	0.014	0.905	0.176	0.675	0.308	0.857
		EWMA - t	5	1	2	0	-52.63	0.836	0.361	0.836	0.361	0.166	0.684	0.863	0.649
Normal		GARCH(1,1)	6	2	3	1	-57.11	0.198	0.656	0.198	0.656	0.783	0.376	0.783	0.676
		EGARCH(1,1)	8	2	4	1	-64.50	1.563	0.211	1.563	0.211	0.783	0.376	1.681	0.432
		GJR(1,1)	7	2	3	1	-62.37	0.735	0.391	0.735	0.391	0.783	0.376	1.028	0.598
t		GARCH(1,1)-t	3	1	1	0	-57.38	0.927	0.336	0.927	0.336	0.000	0.997	1.219	0.544
		EGARCH(1,1)-t	3	1	1	0	-58.29	0.927	0.336	0.927	0.336	0.000	0.997	1.219	0.544
		GJR(1,1)-t	3	1	1	0	-57.65	0.927	0.336	0.927	0.336	0.000	0.997	1.219	0.544

*where **N** represents the number tail losses exceeding VaR and **N'** shows how many of these violations are also super exceptions. **N 1y** represents the number of violations for a VaR forecast sample size of one year, while **N' 1y** gives the number of super exceptions over the same time frame (in theory, the expected **N 1y** should be 2 and **N' 1y** should be 0). **N/A** means that the test statistic is not defined as either N or N' is equal to 0. The p-values presented in bold-italic signify a model rejection.*

[86] Appendix 8

Among the non-parametric approaches, the model that leads to the least number of violations at the 99% and the 99.8% confidence levels is the HS-VIX which has only two tail losses exceeding VaR, none of which is a super exception. Even though this model does not yield any super exceedances it has the highest average VaR of all the methods used. The large average VaR holds for most of the non-parametric models, but, surprisingly, the volatility weighted historical simulation models are proven to give the lowest average VaRs while at the same time the number of violations remains low.

Another interesting observation is that when it comes to the parametric GARCH models, those that are under the assumption of a *t* distribution work better than the models under the normal distribution. This was to be expected as it was shown in section 4 that Bank of America's P/L series exhibits a leptokurtic distribution, and not a normal one. Also, when discussing GARCH based models, it can be inferred that the simple GARCH(1, 1) gives similar results to the asymmetric ones, regardless of the distribution.

Berkowitz and O'Brien reached a similar conclusion, demonstrating that more complex methods used by banks did not outperformed the results of a simple ARMA (1, 1) – GARCH(1, 1) model[87].

In addition, it can be noticed that, as it was also pointed out by Colletaz et. Al (2011), when an exception occurs at a standard degree of confidence it is very likely that the respective loss to exceed a VaR calculated at a very high confidence level. For example, when applying the Age Weighted Historical Simulation it results in two losses that are greater than the VaR estimated by the model and both of these violations also account as a super exception.

The detailed results, with the tail losses that exceed VaR, the super exceptions and the test statistics for the Kupiec, Risk Map disjointed and Risk Map multivariate unconditional coverage backtesting procedures, with their associated p-values, are presented in the table above.[88]

5.3.Danske Bank

None of the models used to estimate VaR on the Danske Bank data series can be rejected either by the Kupiec test or the Risk Map multivariate unconditional coverage test. Nonetheless in this case the Exponentially Weighted Moving Average (Risk Metrics) model is rejected when computing VaR at the 99.8% confidence level, meaning that the number of

[87] Berkowitz and O'Brien (2002)

[88] See Appendix 9 for a ranking of the models used to estimate VaR for Bank of America

super exceptions do not satisfy the required restriction. This can be very quickly detected just by pinpointing this model's coordinates on the three year sample size map, as seen in Appendix 9. All the other models are placed in green zone of non-rejection.

There can also be seen an important shift in what concerns the efficiency of the one year normally distributed GARCH based VaR forecasts[89] as these models display an increased efficiency, while the parametric GARCH models with a *t* distribution continue to provide the best fit. In the case of Bank of America the non-parametric methods ranked above the normal GARCH models. Moreover, the volatility weighted historical simulation continues to produce the lowest average VaR, although its effectiveness decreased. HS-VIX, just as before, gives the worst average VaR, albeit for Danske Bank its results are poorer, lodging it in the lower echelons. Even when using the VSTOXX index instead of the VIX index the model's efficiency does not improve, but on the contrary, it worsens, proving thus that VIX volatility forecasts fit better the portfolio of the bank than the European-based index.

Table 3: Danske Bank

		Model	N	N'	N 1y	N' 1y	Average VaR	Kupiec		Risk Map Disjointed				Risk Map Multivariate	
								LR_{UC} (99%)	p-value	LR_{UC} (99%)	p-value	LR_{UC} (99.8%)	p-value	LR_{MUC}	p-value
Normal	Volatility Weighted HS	GARCH(1,1)	7	2	7	2	-215.01	5.608	0.018	5.608	0.018	2.591	0.107	5.902	0.052
		EGARCH(2,1)	5	1	5	1	-113.49	2.018	0.155	2.018	0.155	0.399	0.527	2.018	0.364
		GJR(1,1)	7	1	7	1	-221.82	5.608	0.018	5.608	0.018	0.399	0.527	5.763	0.056
t		GARCH(1,1)	7	1	7	1	-185.75	5.608	0.018	5.608	0.018	0.399	0.527	5.763	0.056
		EGARCH(1,1)	5	1	5	1	-108.07	2.018	0.155	2.018	0.155	0.399	0.527	2.018	0.364
		GJR(1,1)	7	1	7	1	-142.64	5.608	0.018	5.608	0.018	0.399	0.527	5.763	0.056
		Historical Simulation	8	2	3	1	-76.40	0.062	0.803	0.062	0.803	0.176	0.675	0.180	0.914
		AWHS	11	3	4	1	-71.84	1.619	0.203	1.619	0.203	1.236	0.266	1.955	0.376
		HS-VIX	11	4	4	1	-72.30	1.619	0.203	1.619	0.203	2.978	0.084	3.198	0.202
		HS-VSTOXX	13	4	4	1	-72.79	3.617	0.057	3.617	0.057	2.978	0.084	4.461	0.107
		EWMA - Normal	11	6	4	2	-72.77	1.619	0.203	1.619	0.203	7.883	***0.005***	8.005	0.018
		EWMA - t	8	4	3	1	-77.99	0.062	0.803	0.062	0.803	2.978	0.084	3.698	0.157
Normal		GARCH(1,1)	6	2	3	1	-107.97	0.198	0.656	0.198	0.656	0.783	0.376	0.783	0.676
		EGARCH(2,1)	5	3	2	1	-160.72	0.000	0.993	0.000	0.993	2.616	0.106	3.819	0.148
		GJR(1,1)	5	2	2	1	-194.82	0.000	0.993	0.000	0.993	0.783	0.376	1.047	0.593
t		GARCH(1,1)-t	2	1	1	0	-126.06	2.329	0.127	2.329	0.127	0.000	0.997	3.221	0.200
		EGARCH(1,1)-t	2	1	1	0	-125.68	2.329	0.127	2.329	0.127	0.000	0.997	3.221	0.200
		GJR(1,1)-t	2	1	1	0	-127.68	2.329	0.127	2.329	0.127	0.000	0.997	3.221	0.200

*where **N** represents the number tail losses exceeding VaR and **N'** shows how many of these violations are also super exceptions. **N 1y** represents the number of violations for a VaR forecast sample size of one year, while **N' 1y** gives the number of super exceptions over the same time frame (in theory, the expected **N 1y** should be 2 and **N' 1y** should be 0). **N/A** means that the test statistic is not defined as either N or N' is equal to 0. The p-values presented in bold-italic signify a model rejection.*

The historical simulation continues to function better than the EWMA models and in this case it is more appropriate than age weighted historical simulation or HS-VIX models.

[89] Appendix 9

Furthermore, the results obtained for Danske Bank come to confirm the inference made when analyzing Bank of America findings, i.e. the models under the normality assumption perform worse than those that suppose a *t* distribution for the returns. Also, all the versions of the GARCH models for a *t* distribution yielded the same number of violations and super exceptions.

The table above summarizes the figures obtained for Danske Bank, presenting the average VaR for each model as well as the number of violations and super exceptions, and the test statistics with their p-values.

5.4.Swedbank

After backtesting the results obtained for Swedbank there are several observations to be brought forward, clearly evident from the table below as well as from the Risk Maps presented in Appendix 9.

Table 4: Swedbank

								Kupiec		Risk Map Disjointed				Risk Map Multivariate	
		Model	N	N'	N 1y	N' 1y	Average VaR	LR_{UC} (99%)	p-value	LR_{UC} (99%)	p-value	LR_{UC} (99.8%)	p-value	LR_{MUC}	p-value
Normal	Volatility Weighted HS	GARCH(1,1)	6	3	6	3	-13.92	3.642	0.056	3.642	0.056	5.837	0.016	6.319	0.042
		EGARCH(1,3)	10	3	10	3	-12.91	13.142	***0.000***	13.142	***0.000***	5.837	0.016	13.705	***0.001***
		GJR(1,1)	9	4	9	4	-13.15	10.390	***0.001***	10.390	***0.001***	9.770	***0.002***	13.132	***0.001***
t		GARCH(1,1)	9	2	9	2	-12.84	10.390	***0.001***	10.390	***0.001***	2.591	0.107	10.417	***0.005***
		EGARCH(1,1)	11	3	11	3	-11.86	16.102	***0.000***	16.102	***0.000***	5.837	0.016	16.438	***0.000***
		GJR(1,1)	8	4	8	4	-13.06	7.870	***0.005***	7.870	***0.005***	9.770	***0.002***	11.440	***0.003***
		Historical Simulation	6	2	2	1	-25.14	0.256	0.613	0.256	0.613	0.176	0.675	0.841	0.657
		AWHS	5	3	2	1	-23.93	0.836	0.361	0.836	0.361	1.236	0.266	4.655	0.098
		HS-VIX	12	5	4	2	-28.11	2.533	0.111	2.533	0.111	5.228	0.022	5.451	0.066
		HS-VSTOXX	16	5	5	2	-30.21	7.768	***0.005***	7.768	***0.005***	5.228	0.022	8.896	0.012
		EWMA - Normal	12	3	4	1	-30.32	2.533	0.111	2.533	0.111	1.236	0.266	2.711	0.258
		EWMA - t	10	1	3	0	-33.81	0.889	0.346	0.889	0.346	0.166	0.684	1.678	0.432
Normal		GARCH(1,1)	13	1	6	0	-18.16	9.039	***0.003***	9.039	***0.003***	0.000	0.997	10.562	***0.005***
		EGARCH(1,3)	16	7	8	3	-19.27	15.557	***0.000***	15.557	***0.000***	15.364	***0.000***	20.176	***0.000***
		GJR(1,1)	14	1	7	0	-18.71	11.068	***0.001***	11.068	***0.001***	0.000	0.997	12.883	***0.002***
t		GARCH(1,1)-t	9	0	4	0	-21.06	2.645	0.104	2.645	0.104	N/A	N/A	N/A	N/A
		EGARCH(1,1)-t	9	0	4	0	-23.66	9.039	0.104	2.645	0.104	N/A	N/A	N/A	N/A
		GJR(1,1)-t	9	0	4	0	-21.32	2.645	0.104	2.645	0.104	N/A	N/A	N/A	N/A

where ***N*** *represents the number tail losses exceeding VaR and* ***N'*** *shows how many of these violations are also super exceptions.* ***N 1y*** *represents the number of violations for a VaR forecast sample size of one year, while* ***N' 1y*** *gives the number of super exceptions over the same time frame (in theory, the expected* ***N 1y*** *should be 2 and* ***N' 1y*** *should be 0).* ***N/A*** *means that the test statistic is not defined as either N or N' is equal to 0. The p-values presented in bold-italic signify a model rejection.*

Firstly, out of the 18 different models applied to compute this bank's VaR, 9 of these are rejected by the two backtesting techniques used to validate the models' figures. It is apparent that the age weighted historical simulation and the basic historical simulation return the best VaR forecasts as they have the lowest number of violations. Despite continuing to give the lowest average VaR, the Volatility Weighted HS persists in performing poorly, as all but the normally distributed GARCH(1,1) based model were rejected by both the Kupiec and the multivariate unconditional coverage tests.

Secondly, just as was the case for the banks above, the models that assume a normal distribution for the profits/losses tend to show an inferior performance than the models that presume a *t* distribution for the returns. The parametric GARCH-*t* models remain ranked among the top performing models, whereas the normally distributed parametric GARCHs are rejected by the Kupiec test as well as the Risk Map.

Thirdly, using the VIX index yields better results than when the model is run by utilizing the VSTOXX index, confirming the finding presented for Danske Bank. More so, the latter is rejected by the validation techniques as it yields too many losses larger than VaR.
To be remarked as well is the fact that the parametric GARCH-*t* models do not have any super exceptions over the entire forecast sample.

Last but not least, an improvement in the performance of the EWMA models can be noticed, even though they continue to return large average VaRs. The poor performance of Risk Metrics was also noticed by Perignon and Smith whose tests rejected the model in almost every situation[90].

5.5.Deutsche Bank

The backtesting procedures applied to the same models as above but now for Deutsche Bank's profit/loss series show that most of them perform inadequately since from the 18 models employed to forecast VaR, 13 of them were rejected, including all the volatility weighted HS models, the EWMA as well as all the parametric normal GARCH models. Surprisingly, the parametric EGARCH-*t* model was also invalidated as it led to too many violations.

Another observation that can be made regarding the rejected models is that out of the 13 invalid models 11 of these were situated in the red zone of the Risk Maps, which means that these models breached not only the restrictions imposed on *N* but also those for the number of super exceptions (*N'*).

Otherwise, the age weighted HS was also rejected, since, even though the number of violations is equal to 1 it does not lie in the non-rejection interval of [2, 15].

[90] Perignon and Smith (2008)

The basic HS seems to fit Deutsche Bank’s data best, leading to the lowest number of violations, of which only one is a super exception. It is closely followed by the HS-VIX and the parametric GARCH-*t* model. For this bank as well, we have tried to investigate whether using another volatility index (specifically for this case the VDAX index was employed) instead of the VIX index will lead to better results in VaR forecasting. Just as for the two other banks (Danske Bank and Swedbank) this replacement did not improve the model’s forecasting power.

Table 5: Deutsche Bank

		Model	N	N'	N 1y	N' 1y	Average VaR	Kupiec		Risk Map Disjointed				Risk Map Multivariate	
								LR_{UC} (99%)	p-value	LR_{UC} (99%)	p-value	LR_{UC} (99.8%)	p-value	LR_{MUC}	p-value
Normal	Volatility Weighted HS	GARCH(2,1)	14	8	14	8	-48.25	26.068	***0.000***	26.068	***0.000***	29.773	***0.000***	35.375	***0.000***
		EGARCH(1,2)	17	7	17	7	-57.54	37.407	***0.000***	37.407	***0.000***	24.277	***0.000***	41.367	***0.000***
		GJR(1,1)	17	9	17	9	-53.62	37.407	***0.000***	37.407	***0.000***	35.529	***0.000***	46.439	***0.000***
t		GARCH(1,1)	18	8	18	8	-50.21	41.449	***0.000***	41.449	***0.000***	29.773	***0.000***	46.933	***0.000***
		EGARCH(1,1)	16	10	16	10	-54.16	33.491	***0.000***	33.491	***0.000***	41.515	***0.000***	47.187	***0.000***
		GJR(1,1)	19	9	19	9	-52.15	45.612	***0.000***	45.612	***0.000***	35.529	***0.000***	52.758	***0.000***
		Historical Simulation	5	1	2	0	-243.12	0.836	0.361	0.836	0.361	0.166	0.684	0.836	0.658
		AWHS	1	0	0	0	-231.83	8.714	***0.003***	8.714	***0.003***	N/A	N/A	N/A	N/A
		HS-VIX	8	0	3	0	-283.90	0.062	0.803	0.062	0.803	N/A	N/A	N/A	N/A
		HS-VDAX	11	0	4	0	-246.31	1.619	0.203	1.619	0.203	N/A	N/A	N/A	N/A
		EWMA - Normal	34	16	11	5	-126.05	52.066	***0.000***	52.066	***0.000***	47.744	***0.000***	64.584	***0.000***
		EWMA - t	22	6	7	2	-143.60	19.358	***0.000***	19.358	***0.000***	7.883	***0.005***	20.763	***0.000***
Normal		GARCH(2,1)	14	6	7	3	-73.28	11.068	***0.001***	11.068	***0.001***	11.592	***0.001***	14.830	***0.001***
		EGARCH(1,2)	24	14	12	7	-71.90	38.189	***0.000***	38.189	***0.000***	48.341	***0.000***	55.115	***0.000***
		GJR(1,1)	23	11	11	5	-70.12	35.010	***0.000***	35.010	***0.000***	33.037	***0.000***	43.932	***0.000***
t		GARCH(1,1)-t	8	2	4	1	-85.99	1.563	0.211	1.563	0.211	0.783	0.376	1.681	0.432
		EGARCH(1,1)-t	14	3	7	1	-82.76	11.068	***0.001***	11.068	***0.001***	2.616	0.106	11.085	***0.004***
		GJR(1,1)-t	10	3	5	1	-82.36	3.954	0.047	3.954	0.047	2.616	0.106	4.518	0.104

*where **N** represents the number tail losses exceeding VaR and **N’** shows how many of these violations are also super exceptions. **N 1y** represents the number of violations for a VaR forecast sample size of one year, while **N’ 1y** gives the number of super exceptions over the same time frame (in theory, the expected **N 1y** should be 2 and **N’ 1y** should be 0). **N/A** means that the test statistic is not defined as either N or N’ is equal to 0. The p-values presented in bold-italic signify a model rejection.*

The volatility weighted HS continues to underestimate the VaR over the one year forecasting period, as these models yield just as above the lowest average VaR while at the same time they have the largest number of violations and super exceptions.

Consistent with the figures obtained for the three other banks, in this case as well the non-parametric methods return the highest average VaR. Furthermore, the remark regarding the superiority of *t* distribution based model over the ones that work under the assumption of normality, is backed by the findings in this case.

The use of asymmetric GARCH models does not seem to significantly improve the estimation ability of a model as the GARCH model’s volatility forecasts were shown to be better in this case than those of GJR and EGARCH. Consequently, all the VaR estimation methods based on the asymmetric GARCHs resulted in a larger number of exceptions.

6. Conclusions

This paper focused on the performance analysis of several VaR estimation methodologies applied to real data collected from four financial institutions. The use of real profit/loss data confers this study practical amplitude, allowing us to make relevant assessments of a large number of VaR estimation models.

Our findings showed that models working under the assumption of normality are not the most performant when calculating value-at-risk since they will lead to larger number of exceptions. From the capital requirements point of view this means that a bank which uses a poor performing model will find itself in the situation of being penalized by the terms of the Basel Accord when the model returns a large number of violations. An increase in the scaling factor will automatically generate higher capital requirements for the banks, which is not in their best interest. Taking into consideration the results of this study, it can be inferred that the models that assume *t* distributed returns and even the non-parametric models give better VaR forecasts than models which use a *normal* distribution. This is consistent with previous studies[91] which also found that VaR estimation models using the assumption of a *t* distribution are much more performant than the ones under the assumption of a normal distribution of returns.

Furthermore, the parametric GARCH VaR models seem to have the best fit for the data. For two of the four banks they ranked the highest, giving the lowest number of exceptions, while for the other two they followed closely behind the non-parametric methods. The basic historical simulation, the HS-VIX, along with two parametric methods (GARCH-*t* and GJR-*t* based VaR models) were the only models that were not rejected by either backtesting technique, for all the four data series. Also, the EGARCH-*t* VaR model was only rejected by the Risk Map in the case of Deutsche Bank as it yielded too many super exceptions.

The basic historical simulation, despite its simplicity and slow adjustment speed to market conditions, had a good overall performance, passing the backtest for all the banks. This may be one of the reasons that could explain its popularity and widespread use in financial institutions.

However, the volatility weighted historical simulation had the worst overall results. Its severe underestimation of the risk lead to a high number of tail losses exceeding the worst expected loss and thus to the rejection of the model by the backtesting frameworks employed.

[91] Kuester et al. (2005), Berkowitz et al. (2009)

Another conclusion that can be drawn from the results presented is the fact that even though in some situations the asymmetric GARCH models showed a better performance than the simple GARCH model, the increased efficiency is not significantly superior.

The ranking of the models used by performance in terms of both number of exceedances and minimizing of the average yielded VaR differs from bank to bank. Nonetheless the parametric GARCH models under a t-distribution assumption both for returns and innovations generally have very high positions in the ranking.

It is important to mention as well that the backtesting techniques employed in this study allowed the models to yield a larger number of exceptions than one would expect to see when calculating VaR at a certain confidence level. Thus, for a 99% degree of confidence and a sample of VaR forecasts over two years (500 observations), it is expected that the number of violations to be equal to 5, while the number of super exceptions should be 1. But, in our case the validation procedures rejected the models when the super exceptions were in an amount larger than 4 and N lied outside the [1, 11] interval.

Finally, the use of the newly developed backtesting technique called the Risk Map allowed us to account not only for the number of tail losses but also for the magnitude of these exceptions, demonstrating once again its reliability and more importantly its simplicity in application. It was shown that the method can be applied to various VaR estimation models as well as to different number of VaR forecasts.

7. Future research

Kuester et al. in their 2005 paper analyze the performance of a large number of VaR forecasting methods, by applying these models on daily return data for the Nasdaq composite index. It will be interesting to investigate how all these models work on real profit/loss data from commercial banks and to compare the results with those in their study. Moreover, a series of other methods can be introduced, such as methods using volatilities estimated through GARCH models with Markov-switching parameters. The backtesting techniques could also be supplemented by adding several other model validating procedures, such as the Christoffersen backtesting framework.

It will also be intriguing to jointly test the significance of each particular model across the results for all the four banks. This would offer a better understanding of how the model works from an overall perspective.

Furthermore, the analysis would be more detailed and accurate if the actual composition of the portfolios were known. This would allow for an examination of the way each risk factor contributes to the market risk of the portfolio. In addition, the implementation of stress tests on the trading portfolios would be possible, in order to get a better insight in the portfolio’s weaknesses as well as back up the VaR forecasts.

As a final point, a larger number of financial institutions could be used to test the model, conferring the study a more complex cross-sectional dimension.

References

Alberg, D.; Shalit, H; Yosef, R.(2008), "Estimating Stock Market Volatility Using Asymmetric GARCH models", *Journal of Applied Financial Economics*, 18, 15, 1201-1208.

Alexander, C. and Sheedy, E. (2008), "Developing a stress testing framework based on market risk models", *Journal of Banking & Finance*, 32, 10, 2220–2236.

Alexander, C. (1999), "Correlation and cointegration in energy markets", *RISK Publications*, 291-304.

Angelidis, T., Benos, A., and Degiannakis, S. (2004), "The use of GARCH models in VaR estimation", *Journal of Statistical Methodology*, 1, 1-2, 105-128.

Bank of America Annual Reports and Risk Management Reports 2007-2010.

Berkowitz, J. (2001), "Testing Density Forecasts, with Applications to Risk Management" *Journal of Business and Economic Statistics*, 19, 4, 465-474.

Berkowitz, J. and O'Brien, J. (2002), "How Accurate Are Value-at-Risk Models at Commercial Banks?", *The Journal of Finance*, 57, 3, 1093-1111.

Blundell-Wignall, A. and Atkinson, P. (2010), "Thinking beyond Basel III: necessary solutions for capital and liquidity", *OECD Journal: Financial Market Trends*, Volume 2010, Issue 1.

Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics*, 31, 3, 307–327.

Box, G. E. P.; Jenkins,M. ;Greinsel, G . C. (1994), „Time Series Analysis: Forecasting and Control.", 3rd ed.

Bodoukh, J.; Richardson, M.; and Whitelaw, R. F. (1998) „The Best of Both Worlds: A Hybrid Approach to Calculating Value at Risk".

Brooks, C. (2008), "Introductory econometrics for finance", 2nd edition, *Cambridge University Press*.

Cabedo, J.D. and Moya, I. (2003), "Estimating oil price 'Value at Risk' using the historical simulation approach", *Journal of Energy Economics*, 25, 3, 239–253.

Campbell, R., Huisman, R., and Koedijk, K. (2001), "Optimal portfolio selection in a Value-at-Risk framework", *Journal of Banking & Finance*, 25, 9, 1789–1804.

Christoffersen, P. (1998), “Evaluating Interval Forecasts”, *International Economic Review*, 39, 4, 841–862.

Christoffersen, P., Berkowitz, J., and Pelletier, D. (2008) "Evaluating Value-at-Risk Models with Desk-Level Data", CREATES Research Papers 2009-35, School of Economics and Management, University of Aarhus.

Christoffersen, P. and Pelletier, D. (2004), “Backtesting Value-at-Risk: A Duration-Based Approach”, *Journal of Financial Econometrics*, 2, 84-108.

Colletaz, G., Hurlin, C., and Perignon, C. (2011), “The Risk Map: A New Tool for Backtesting Value-at-Risk Models”, *Working Papers Series (SSRN)*.

Coronado, M. (2009), “Comparing Different Methods for Estimating Value-at-Risk (VaR) for Actual Non-Linear Portfolios: Empirical Evidence”, *European Journal of Finance*.

Damodaran, A. (2007), “Strategic risk taking. A framework for risk management”, 1st edition, *Wharton School Publishing*

Danielsson, J. and de Vries, C. G. (1997), “Value-at-Risk and Extreme Returns”, *Annales d’Economie et de Statistique*, 60, 239–270.

Danske Bank Annual Reports and Risk Management Reports 2007-2010

Deutsche Bank Annual Reports and Risk Management Reports 2007-2010

Dowd, K. (2005), “Measuring Market Risk”, 2nd edition, *John Wiley & Sons, Ltd*.

Einhorn,D; Brown, A. (2008), “Private Profits and Socialized Risk”, *Global Risk Association of Risk Professionals*.

Elliott, D.J. (2010), “Basel III, the banks, and the economy”, *The Brookings Institution*.

Engle, R.F.(1982), "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation", *Econometrica* Vol. 50, pg 987–1007.

Glosten, L.R., Jagannathan, R., and Runkle, D.E. (1993), “On the relation between the expected value and the volatility of the nominal excess return on stocks”, *Journal of Finance*, 48, 5, 1779-1801.

Gustafsson, M. and Lundberg, C. (2009), “An empirical evaluation of Value-at-Risk”, University of Gothenburg, School of Business, Economics and Law, Master Thesis.

Härdle, W. K., Hautsch, N., and Overbeck, L. (2009), “Applied Quantitative Finance”, *Springer Berlin Heidelberg*.

Hendricks, D. (1996), “Evaluation of Value-at-Risk Models Using Historical Data”, *Journal of Economic Policy Review*, Volume 2, Issue 1.

Huckins, N. and Rai, A. (1999), “Market Risk for Foreign Currency Options: Basle's Simplified Model”, *Journal of Financial Management*, Volume 28, Issue 1.

Hull, J.C. (2008), “Options, Futures and Other Derivatives”, 7th edition, *Prentice Hall*.

Hull, J.C. and White, A. (1998), “Incorporating volatility updating into the historical simulation method for value at risk”, *Journal of Risk*, Volume 1, Issue 1, Fall 1998.

IMF (2004), “Compilation Guide on Financial Soundness Indicators”, *IMF, Washington DC*, Appendix VII, Glossary.

Jackson, P., Maude, D., and Perraudin, W. (1998), “Bank capital and value-at-risk”, *Bank of England*, Working Paper No. 79.

Jorion, P. (2001), “Value at Risk: The New Benchmark for Managing Financial Risk”, 2nd edition, *McGraw-Hill*.

Kerkhof, J. and Melenberg, B. (2004), “Backtesting for risk-based regulatory capital”, *Journal of Banking and Finance*, 28, 8, 1845–1865.

Kuester, K., Mittnik, S., and Paolella, M.S., (2005), “Value-at-Risk Prediction: A Comparison of Alternative Strategies”, *Journal of Financial Econometrics*, 4, 1, 53-89.

Kupiec, P. (1995), “Techniques for Verifying the Accuracy of Risk Measurement Models” *Journal of Derivatives*, 3, 2, 73-84.

McAleer, M., Jiménez-Martín, J.A., and Pérez-Amaral, T. (2009), “Has the Basel II Accord Encouraged Risk Management During the 2008-09 Financial Crisis?”, *Working Papers Series (SSRN)*.

Nelson, D.B. (1991), “Conditional heteroskedasticity in asset returns: A new approach”, *Econometrica: Journal of the Econometric Society*, 59, 2, 347-370.

Nossman, M. and Vilhelmsson, A. (2011), “Non-parametric Future Looking Value-at-Risk”, *Working Paper Series (SSRN)*.

Pérignon, C. and Smith, D.R. (2006), “ The Level and Quality of Value-at-Risk Disclosure by Commercial Banks”, *Working Papers Series (SSRN)*.

Pérignon, C. and Smith, D.R. (2007), "Diversification and Value-at-Risk", *Journal of Banking & Finance,* 34, 1, 55–66.

Pérignon, C. and Smith, D.R. (2008), "A New Approach to Comparing VaR Estimation Methods", *Working Papers Series (SSRN).*

Pérignon, C., Deng, Z.Y., and Wang, Z.J. (2006), "Do banks overstate their Value-at-Risk?", *Journal of Banking & Finance*, 32, 5, 783-794.

Pritsker, M. (2001), "The Hidden Dangers of Historical Simulation", *Journal of Banking & Finance,* 30, 2, 561–582.

Su, C. (2010), "Application of EGARCH Model to Estimate Financial Volatility of Daily Returns: The empirical case of China", Gothenburg University, Master Thesis.

Su, Y.; Lin, C.; Chen, P. (2009) – Asymmetric GARCH Value at Risk for QQQQ, *Working Paper Series (SSRN).*

St. Pierre, E.F. (1998) "Estimating EGARCH-M models: Science or art?", *The Quarterly Review of Economics and Finance*, 38, 2, 167-180.

Swedbank Annual Reports and Risk Management Reports 2007-2010.

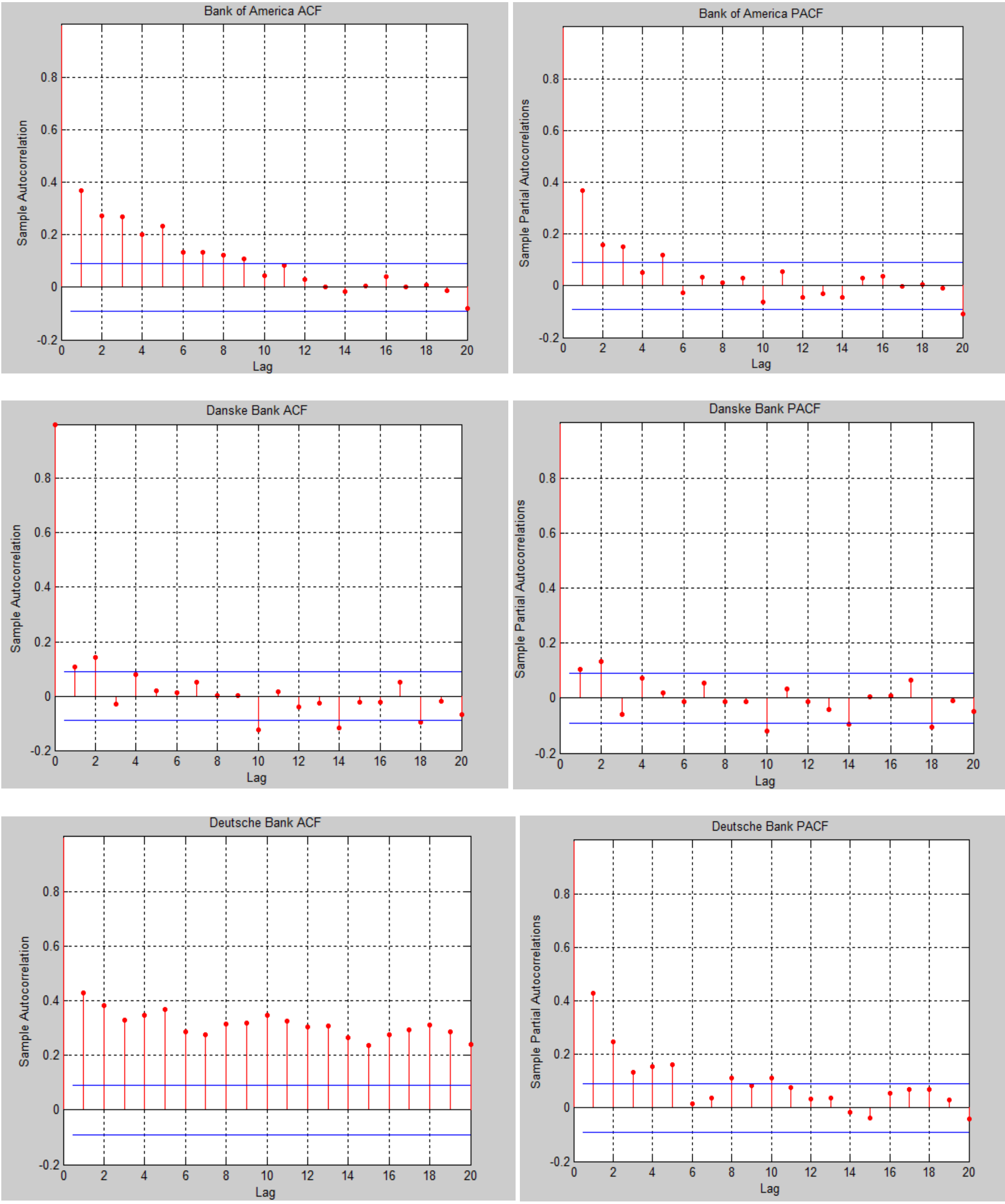
Trenca, I. (2009), "The Use in Banks of Value-at-Risk Method in Market Risk Management", *Scientific Annals of the "Alexandru Ioan Cuza" University of Iasi: Economic Sciences Series*, LVI, 186-196.

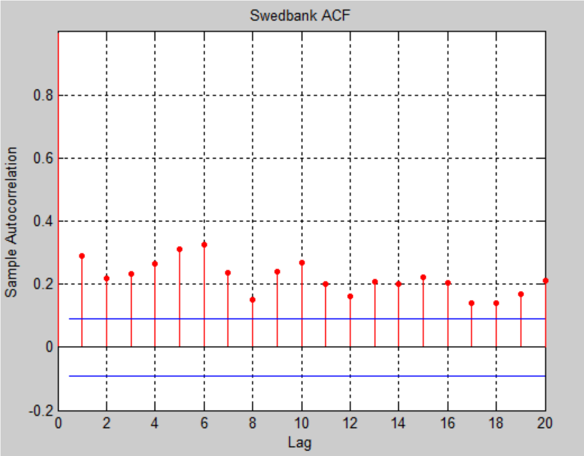
Varotto, S. (2011), "Liquidity Risk, Credit Risk, Market Risk and Bank Capital", *Working Papers Series (SSRN).*

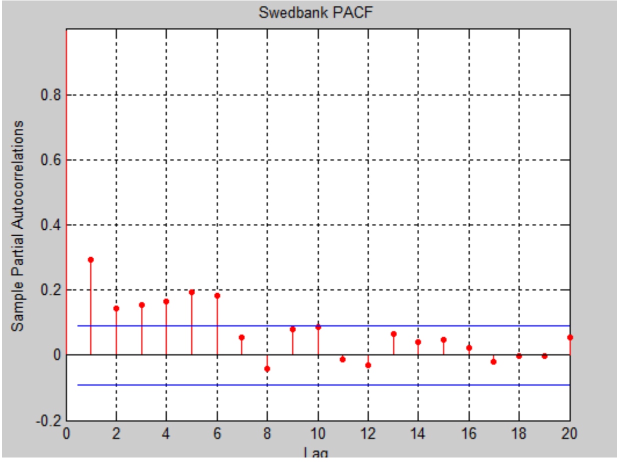
Wong, W.K. (2010), "Backtesting value-at-risk based on tail losses", *Journal of Empirical Finance,* 17, 3, 526–538.

Appendix

Appendix 1. Autocorrelation and Partial Autocorrelation Functions for the P/L series







Appendix 2. Heteroskedasticity tests results – Model pre-fit

Ljung-Box Test

Bank of America				
Lags	H	p-value	Statistic	Critical Value
10	1	0	1397.4	23.2093
15	1	0	1707.4	30.5779
20	1	0	1987.7	37.5662

Danske Bank				
Lags	H	p-value	Statistic	Critical Value
10	1	0	80.6298	23.2093
15	1	0	91.5256	30.5779
20	1	0	98.115	37.5662

Deutsche Bank				
Lags	H	p-value	Statistic	Critical Value
10	1	0	826.4	23.2093
15	1	0	1016.7	30.5779
20	1	0	1168.3	37.5662

Swedbank				
Lags	H	p-value	Statistic	Critical Value
10	1	0	476.6159	23.2093
15	1	0	607.4351	30.5779
20	1	0	678.0991	37.5662

Engle ARCH Test

Bank of America				
Lags	H	p-value	Statistic	Critical Value
10	1	0	85.2168	23.2093
15	1	0	85.4262	30.5779
20	1	0	89.8562	37.5662

Danske Bank				
Lags	H	p-value	Statistic	Critical Value
10	1	0	100.9642	23.2093
15	1	0	102.1193	30.5779
20	1	0	104.7544	37.5662

Deutsche Bank				
Lags	H	p-value	Statistic	Critical Value
10	1	0	175.4165	23.2093
15	1	0	178.6246	30.5779
20	1	0	184.9489	37.5662

Swedbank				
Lags	H	p-value	Statistic	Critical Value
10	1	0	117.2434	23.2093
15	1	0	127.4896	30.5779
20	1	0	128.5644	37.5662

Appendix 3. ADF test results

Null Hypothesis: BA has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 4 (Automatic - based on SIC, maxlag=21)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-8.849919	0.0000
Test critical values:	1% level	-3.967469	
	5% level	-3.414420	
	10% level	-3.129341	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: DANSKE has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=21)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-26.38752	0.0000
Test critical values:	1% level	-2.567321	
	5% level	-1.941146	
	10% level	-1.616482	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: SWED has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 6 (Automatic - based on SIC, maxlag=21)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-6.224616	0.0000
Test critical values:	1% level	-3.967489	
	5% level	-3.414429	
	10% level	-3.129346	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: DEUTSCHE has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 8 (Automatic - based on SIC, maxlag=21)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-4.813389	0.0004
Test critical values:	1% level	-3.967508	
	5% level	-3.414439	
	10% level	-3.129352	

*MacKinnon (1996) one-sided p-values.

Appendix 4. GARCH coefficient estimation results

Bank of America

GARCH(1,1)

	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	8.35531	2.94142	2.84057	0.00469
AR(1)	0.86821	0.04409	19.6932	0
MA(1)	-0.61928	0.07651	-8.09409	0
ARCH(1)	0.03228	0.01206	2.67647	0.00769
GARCH(1)	0.94511	0.02567	36.81709	0

GARCH(1,1)-t

	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	7.09363	2.41137	2.94175	0.00342
AR(1)	0.87947	0.03848	22.85237	0
MA(1)	-0.65956	0.06392	-10.318	0
ARCH(1)	0.04393	0.02302	1.90791	0.057
GARCH(1)	0.93477	0.03564	26.226	0

EGARCH(1,1)

	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	9.01852	3.00827	2.99791	0.00286
AR(1)	0.85779	0.0469	18.28953	0
MA(1)	-0.62572	0.07709	-8.11731	0
ARCH(1)	0.06976	0.0285	2.44787	0.01473
GARCH(1)	0.96772	0.01679	57.62844	0
Leverage(1)	0.04468	0.02265	1.97212	0.04917

EGARCH(1,1)-t

	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	7.24716	2.41932	2.99554	0.00288
AR(1)	0.87794	0.03939	22.2899	0
MA(1)	-0.66693	0.06416	-10.3945	0
ARCH(1)	0.092	0.05135	1.79155	0.07383
GARCH(1)	0.96924	0.0233	41.58975	0
Leverage(1)	0.04682	0.03223	1.45288	0.14691

GJR-GARCH(1,1)

	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	1.15032	0.53117	2.16561	0.03083
AR(1)	0.92607	0.03018	30.68939	0
MA(1)	-0.77725	0.0486	-15.9912	0
ARCH(1)	0.11123	0.03207	3.46887	0.00057
GARCH(1)	0.8784	0.01565	56.1195	0
Leverage(1)	-0.026	0.04447	-0.58462	0.55908

GJR-GARCH(1,1)-t

	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	7.18069	2.42422	2.96207	0.00321
AR(1)	0.87838	0.0388	22.64162	0
MA(1)	-0.65992	0.06407	-10.3005	0
ARCH(1)	0.04831	0.02907	1.66199	0.09717
GARCH(1)	0.93429	0.03656	25.55591	0
Leverage(1)	-0.01201	0.03609	-0.33292	0.73934

Danske Bank

GARCH(1,1)	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.5397	0.49664	1.08671	0.27771
AR(1)	0.78898	0.1275	6.18828	0
MA(1)	-0.70422	0.15709	-4.48282	0.00001
ARCH(1)	0.25299	0.03361	7.52613	0
GARCH(1)	0.74701	0.02559	29.19285	0

GARCH(1,1)-t	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.08686	0.5056	0.1718	0.86367
AR(1)	0.7382	0.16901	4.36789	0.00002
MA(1)	-0.64628	0.19249	-3.35745	0.00085
ARCH(1)	0.26097	0.07488	3.4852	0.00054
GARCH(1)	0.70819	0.05847	12.11244	0

EGARCH(2,1)	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.70329	0.67274	1.04541	0.29636
AR(1)	0.74055	0.20072	3.68949	0.00025
MA(1)	-0.66921	0.22137	-3.02304	0.00264
ARCH(1)	0.52552	0.0995	5.28142	0
ARCH(2)	-0.18687	0.10901	-1.71418	0.08714
GARCH(1)	0.96653	0.01094	88.35915	0
Leverage(1)	-0.10016	0.06145	-1.62988	**0.10378**
Leverage(2)	0.19198	0.05567	3.44873	0.00061

EGARCH(1,1)-t	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.4411	0.64517	0.68365	0.49452
AR(1)	0.676	0.19678	3.43499	0.00064
MA(1)	-0.5733	0.21827	-2.62642	0.0089
ARCH(1)	0.3841	0.08767	4.38158	0.00001
GARCH(1)	0.929	0.02699	34.41661	0
Leverage(1)	0.1211	0.0647	1.87186	0.06183

GJR-GARCH(1,1)	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.55575	0.59322	0.93684	0.34931
AR(1)	0.78805	0.14019	5.6211	0
MA(1)	-0.7047	0.16613	-4.24179	0.00003
ARCH(1)	0.26189	0.05648	4.63667	0
GARCH(1)	0.74935	0.02647	28.30502	0
Leverage(1)	-0.02247	0.08833	-0.25437	0.79932

GJR-GARCH(1,1)-t	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.25125	0.53755	0.46739	0.64043
AR(1)	0.73339	0.16281	4.50454	0.00001
MA(1)	-0.63529	0.18575	-3.42008	0.00068
ARCH(1)	0.38666	0.12584	3.07257	0.00224
GARCH(1)	0.70326	0.05987	11.74666	0
Leverage(1)	-0.25291	0.14432	-1.75239	0.08035

Deutsche Bank

GARCH(2,1)	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.42536	0.39775	1.06941	0.28542
AR(1)	0.98731	0.01024	96.44769	0
MA(1)	-0.84955	0.03287	-25.84832	0
ARCH(1)	0.13765	0.0577	2.38568	0.01743
ARCH(2)	0.10823	0.0431	2.51077	0.01237
GARCH(1)	0.67179	0.03848	17.45766	0

GARCH(1,1)-t	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.61004	0.35955	1.69668	0.09041
AR(1)	0.98536	0.00939	104.8963	0
MA(1)	-0.84723	0.02815	-30.09733	0
ARCH(1)	0.16136	0.06151	2.62346	0.00898
GARCH(1)	0.73018	0.08946	8.16168	0

EGARCH(1,2)	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.88771	0.53557	1.6575	0.09807
AR(1)	0.97903	0.0127	77.1125	0
MA(1)	-0.79113	0.03491	-22.66418	0
ARCH(1)	0.27368	0.04882	5.60604	0
GARCH(1)	1.3949	0.09382	14.86741	0
GARCH(2)	-0.61892	0.0879	-7.04126	0
Leverage(1)	0.06171	0.02729	2.2614	0.02418

EGARCH(1,1)-t	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.54869	0.35988	1.52462	0.12801
AR(1)	0.98634	0.00927	106.43369	0
MA(1)	-0.84538	0.02798	-30.21291	0
ARCH(1)	0.36392	0.11731	3.10224	0.00203
GARCH(1)	0.80708	0.09837	8.20488	0
Leverage(1)	-0.0134	0.06556	-0.20438	**0.83814**

GJR-GARCH(1,1)	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.51952	0.41769	1.24379	0.21418
AR(1)	0.98787	0.01061	93.10942	0
MA(1)	-0.84975	0.0329	-25.82909	0
ARCH(1)	0.23429	0.03535	6.62814	0
GARCH(1)	0.71442	0.05057	14.12641	0
Leverage(1)	-0.0846	0.04992	-1.69447	0.09083

GJR-GARCH(1,1)-t	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.5736	0.35271	1.62626	0.10455
AR(1)	0.98606	0.00925	106.57933	0
MA(1)	-0.84896	0.0279	-30.42849	0
ARCH(1)	0.14285	0.06953	2.05469	0.04045
GARCH(1)	0.72935	0.08847	8.24388	0
Leverage(1)	0.03858	0.09414	0.40987	**0.68208**

Swedbank

GARCH(1,1)

	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.28148	0.1881	1.49648	0.13518
AR(1)	0.97777	0.01385	70.60026	0
MA(1)	-0.88009	0.03235	-27.20535	0
ARCH(1)	0.04551	0.01722	2.64361	0.00847
GARCH(1)	0.92718	0.03426	27.06038	0

GARCH(1,1)-t

	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.28414	0.18147	1.5658	0.11805
AR(1)	0.97265	0.01461	66.58366	0
MA(1)	-0.87747	0.03268	-26.8523	0
ARCH(1)	0.03814	0.01995	1.91198	0.05647
GARCH(1)	0.94604	0.03151	30.02661	0

EGARCH(1,3)

	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.37324	0.21673	1.72214	0.08569
AR(1)	0.97013	0.01652	58.72724	0
MA(1)	-0.87694	0.03571	-24.55576	0
ARCH(1)	0.08287	0.03136	2.643	0.00849
GARCH(1)	1.08593	0.03794	28.62146	0
GARCH(2)	-1.04651	0.04731	-22.11808	0
GARCH(3)	0.90572	0.04164	21.75004	0
Leverage(1)	0.07172	0.02879	2.4914	0.01306

EGARCH(1,1)-t

	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.2842	0.17428	1.63072	0.10361
AR(1)	0.97464	0.01389	70.17437	0
MA(1)	-0.88962	0.02937	-30.29307	0
ARCH(1)	0.03038	0.03565	0.85232	**0.39446**
GARCH(1)	0.97682	0.01542	63.33731	0
Leverage(1)	0.05097	0.02491	2.04613	0.04129

GJR-GARCH(1,1)

	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.30614	0.19594	1.56241	0.11885
AR(1)	0.97601	0.01466	66.56148	0
MA(1)	-0.88164	0.03349	-26.32551	0
ARCH(1)	0.04375	0.02052	2.13174	0.03354
GARCH(1)	0.94344	0.02875	32.81056	0
Leverage(1)	-0.02457	0.02358	-1.04213	0.29788

GJR-GARCH(1,1)-t

	Coeff	StError	T-stat	p-value
	-------	---------	--------	-------
C	0.28016	0.17608	1.59104	0.11226
AR(1)	0.97401	0.01416	68.79794	0
MA(1)	-0.88365	0.03132	-28.21722	0
ARCH(1)	0.04069	0.02154	1.88902	0.05949
GARCH(1)	0.95749	0.02485	38.53663	0
Leverage(1)	-0.04069	0.0296	-1.37465	0.16988

Appendix 5. Post-estimation ARCH test results – Selection

Bank of America - GARCH(1,1)

Lags	H	p-value	Statistic	Critical Value
10	0	0.8997	4.8706	23.2093
15	0	0.9802	5.9744	30.5779
20	0	0.9883	8.4688	37.5662

Danske Bank GARCH(1,1)

Lags	H	p-value	Statistic	Critical Value
10	0	0.9317	4.3203	23.2093
15	0	0.2257	18.7421	30.5779
20	0	0.2131	24.7005	37.5662

Deutsche Bank GARCH(2,1)

Lags	H	p-value	Statistic	Critical Value
10	0	0.9429	4.0963	23.2093
15	0	0.8706	9.1308	30.5779
20	0	0.8875	12.7585	37.5662

Swedbank GARCH(1,1)

Lags	H	p-value	Statistic	Critical Value
10	0	0.9118	4.675	23.2093
15	0	0.955	7.0926	30.5779

Appendix 6. Backtesting results (Acceptance/Rejection)

		BA			Danske			Deutsche			Swedbank		
	Model	Kupiec	RM 99.8%	RM Multivariate UC	Kupiec	RM 99.8%	RM Multivariate UC	Kupiec	RM 99.8%	RM Multivariate UC	Kupiec	RM 99.8%	RM Multivariate UC
	Historical Simulation	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
	AWHS	Accept	Accept	N/A	Accept	Accept	Accept	Reject	N/A	N/A	Accept	Accept	Accept
	HS-VIX	Accept	N/A	N/A	Accept	Accept	Accept	Accept	N/A	N/A	Accept	Accept	Accept
	HS-VIX(VSTOXX and VDAX)				Accept	Accept	Accept	Accept	N/A	N/A	Reject	Accept	Accept
Normal	GARCH	Accept	Accept	Accept	Accept	Accept	Accept	Reject	Reject	Reject	Reject	Accept	Reject
Normal	EGARCH	Accept	Accept	Accept	Accept	Accept	Accept	Reject	Reject	Reject	Reject	Reject	Reject
Normal	GJR	Accept	Accept	Accept	Accept	Accept	Accept	Reject	Reject	Reject	Reject	Accept	Reject
t	GARCH-t	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	N/A	N/A
t	EGARCH-t	Accept	Accept	Accept	Accept	Accept	Accept	Reject	Accept	Reject	Accept	N/A	N/A
t	GJR-t	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	N/A	N/A
Volatility Weighted HS	GARCH	Accept	Accept	Accept	Accept	Accept	Accept	Reject	Reject	Reject	Accept	Accept	Accept
Volatility Weighted HS	EGARCH	Accept	Accept	Accept	Accept	Accept	Accept	Reject	Reject	Reject	Reject	Accept	Reject
Volatility Weighted HS	GJR	Accept	Accept	Accept	Accept	Accept	Accept	Reject	Reject	Reject	Reject	Reject	Reject
Volatility Weighted HS	GARCH-t	Accept	Accept	Accept	Accept	Accept	Accept	Reject	Reject	Reject	Reject	Accept	Reject
Volatility Weighted HS	EGARCH-t	Accept	Accept	Accept	Accept	Accept	Accept	Reject	Reject	Reject	Reject	Accept	Reject
Volatility Weighted HS	GJR-t	Accept	Accept	Accept	Accept	Accept	Accept	Reject	Reject	Reject	Reject	Reject	Reject
	EWMA - Normal distribution	Accept	Accept	Accept	Accept	Reject	Accept	Reject	Reject	Reject	Accept	Accept	Accept
	EWMA - t distribution	Accept	Accept	Accept	Accept	Accept	Accept	Reject	Reject	Reject	Accept	Accept	Accept

Appendix 7. Graphical representation of VaR estimates against the P/L data series

Bank of America – Non-parametric models

Figure 1: Bank of America - P/L series, Basic Historical Simulation, Age Weighted Historical Simulation

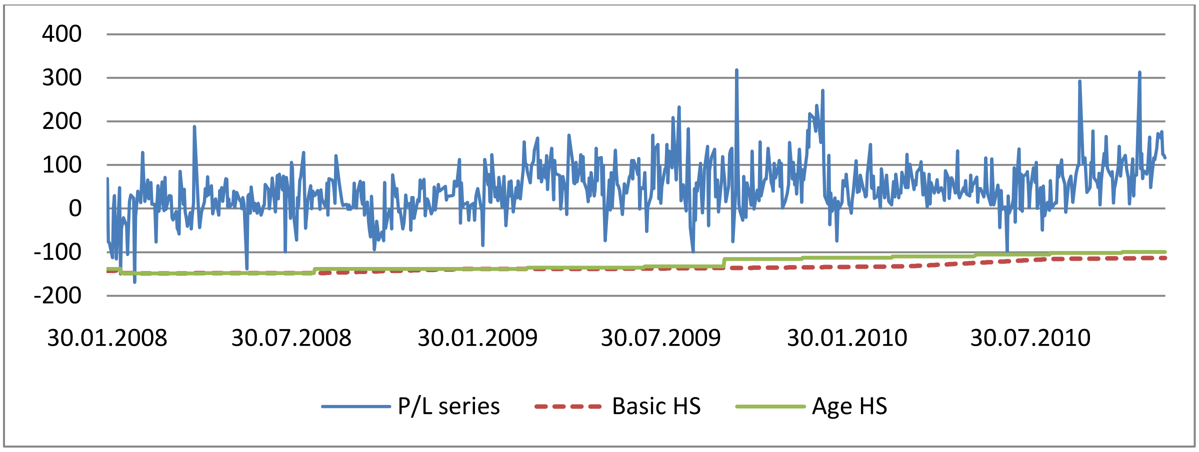


Figure 2: Bank of America - P/L series, HS-VIX

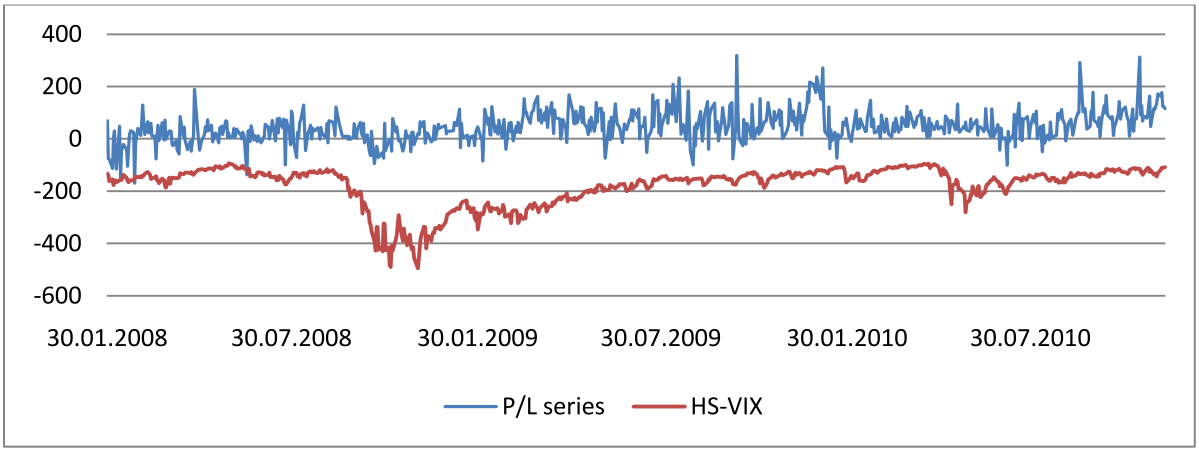


Figure 3: Bank of America - P/L series, HS-GARCH(1,1), HS-EGARCH(1,1)

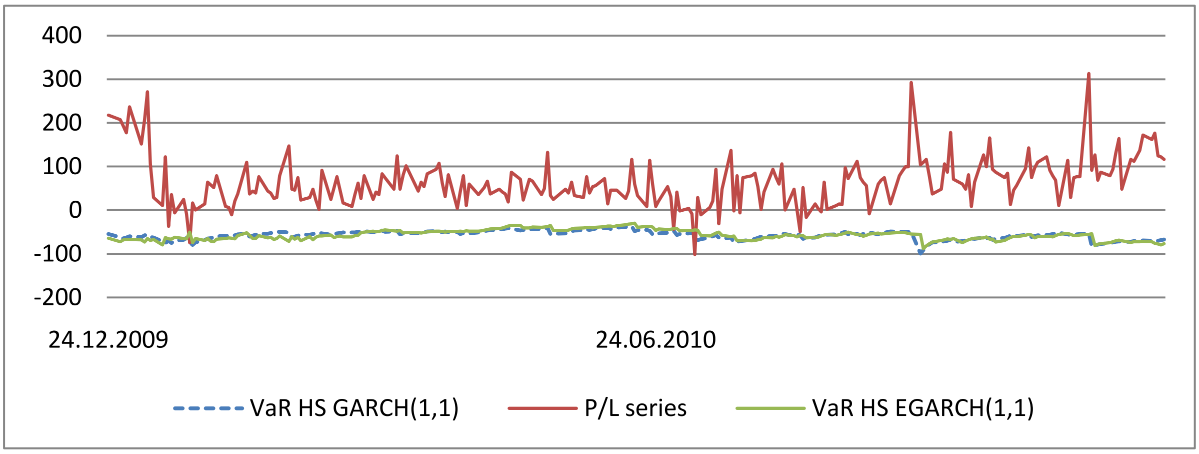


Figure 4: Bank of America - P/L series, HS-GARCH(1,1)-t, HS-EGARCH(1,1)-t

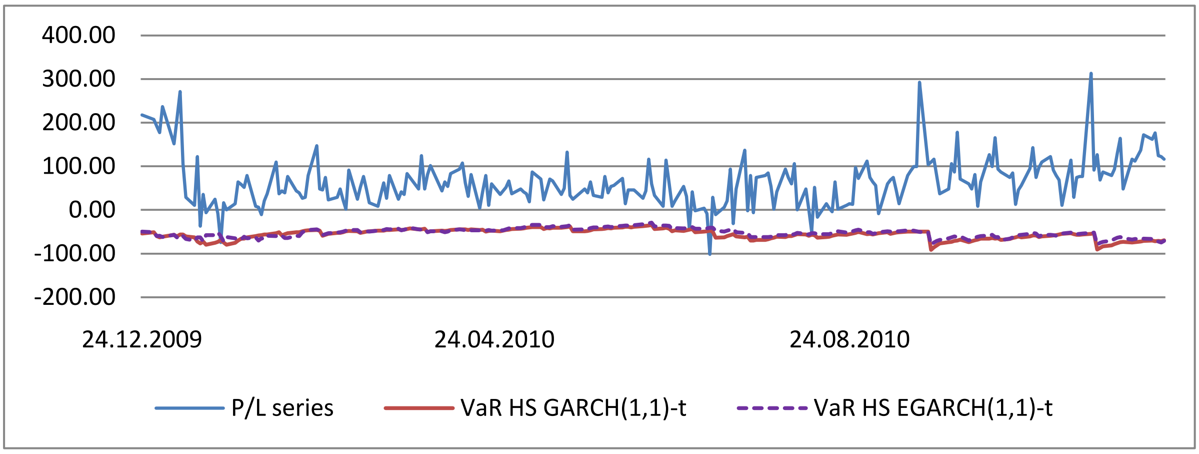
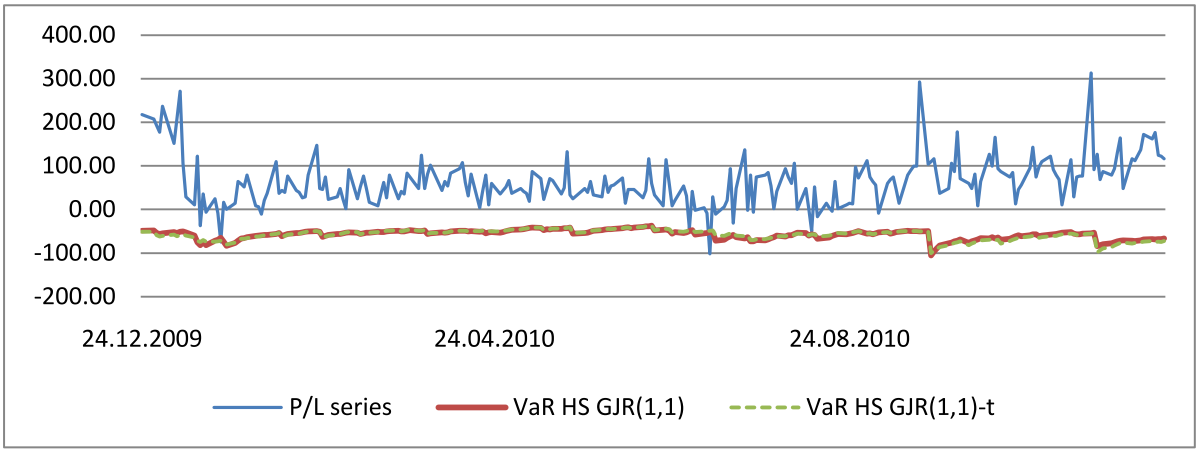


Figure 5: Bank of America - P/L series, HS-GJR(1,1), HS-GJR(1,1)-t



Bank of America – Parametric models

Figure 6: Bank of America - P/L series, EWMA-normal, EWMA-t

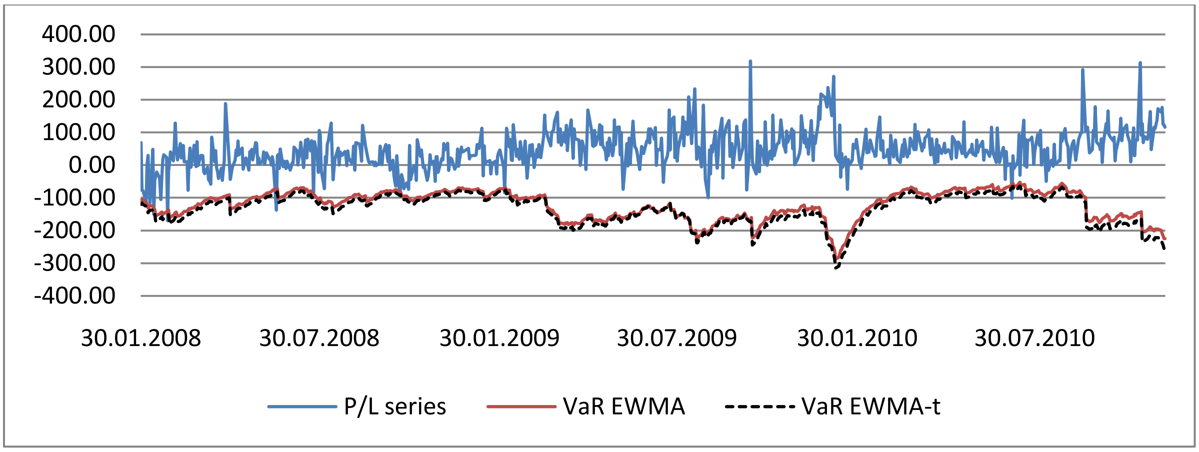


Figure 7: Bank of America - P/L series, GJR(1,1), GJR(1,1)-t

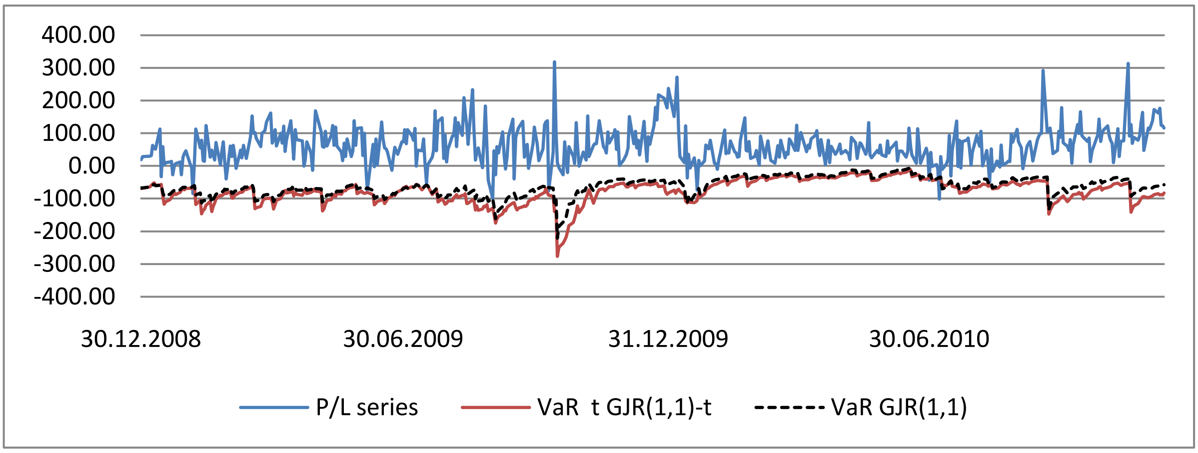


Figure 8: Bank of America - P/L series, EGARCH(1,1), EGARCH(1,1)-t

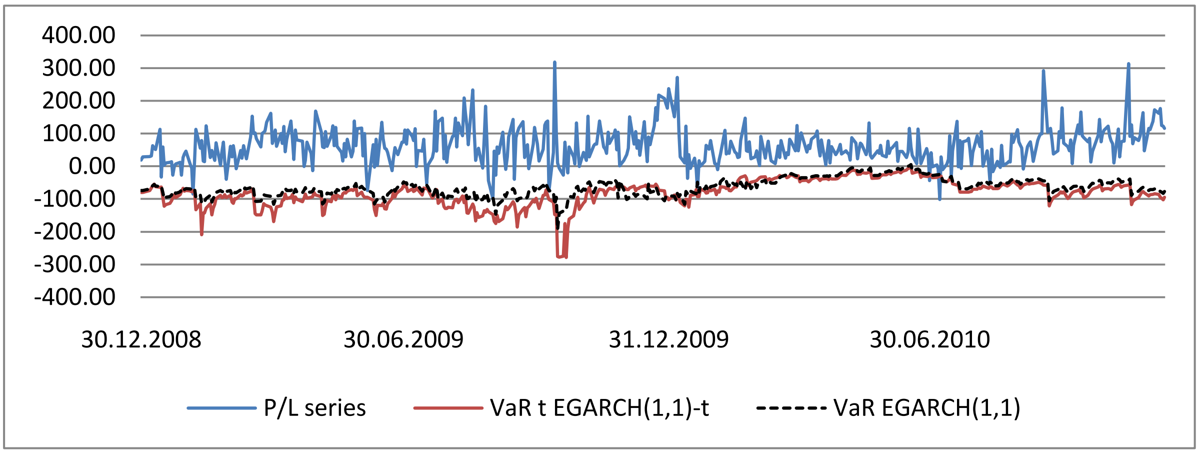
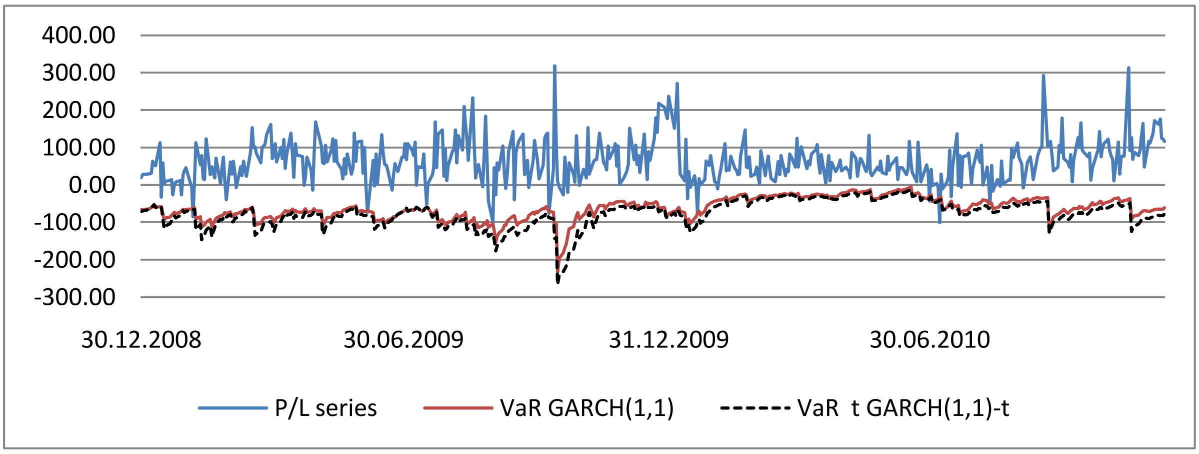


Figure 9: Bank of America - P/L series, GARCH(1,1), GARCH(1,1)-t



Danske Bank – Non-parametric models

Figure 10: Danske Bank - P/L series, Basic Historical Simulation, Age Weighted Historical Simulation

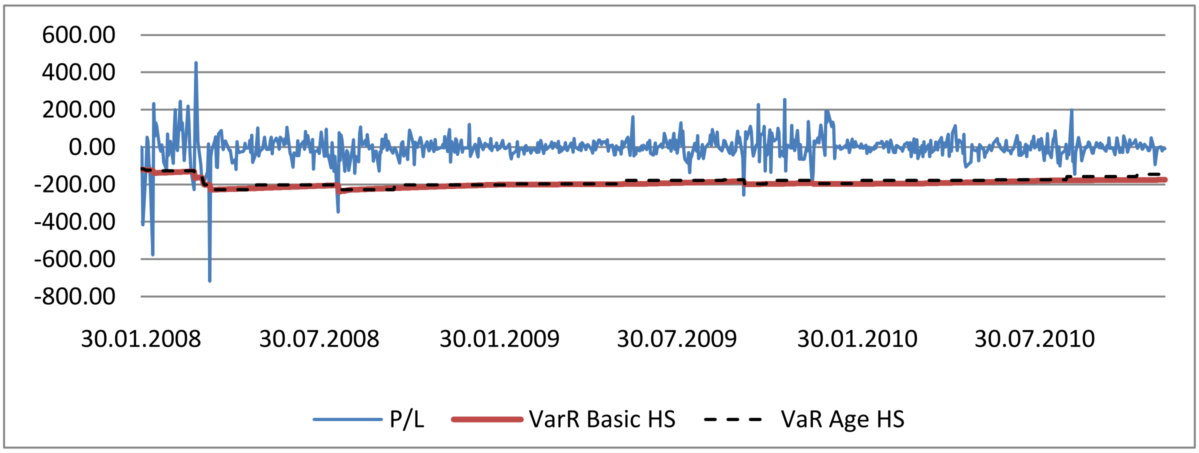


Figure 11: Danske Bank - P/L series, HS-VIX, HS-VSTOXX

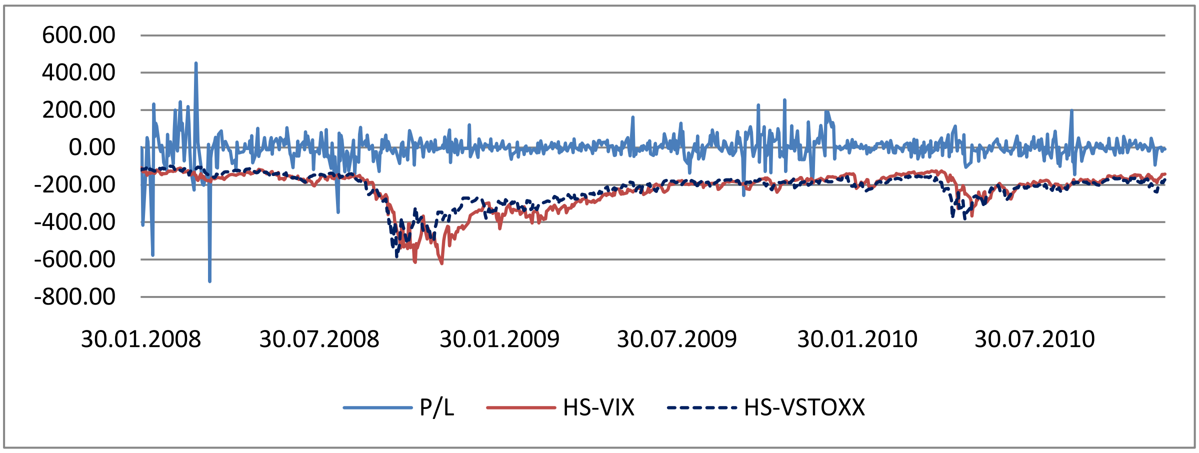


Figure 12: Danske Bank - P/L series, HS-GARCH(1,1), HS-EGARCH(2,1)

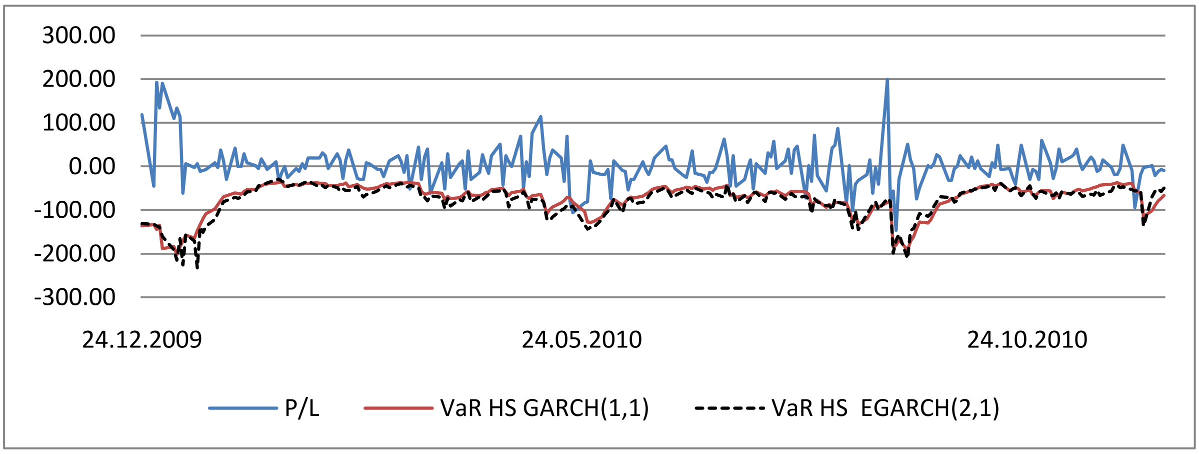


Figure 13: Danske Bank - P/L series, HS-GARCH(1,1)-t, HS-EGARCH(1,1)-t

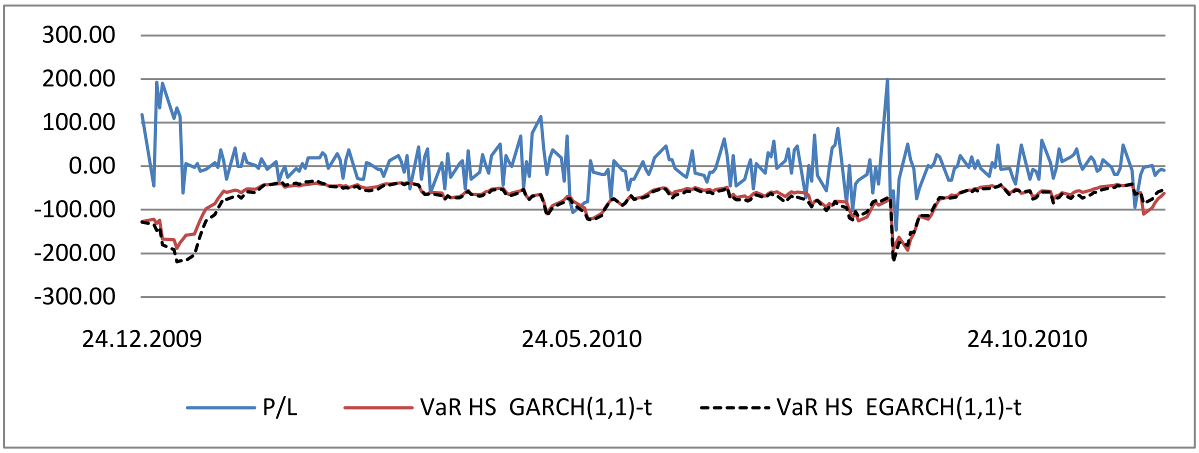
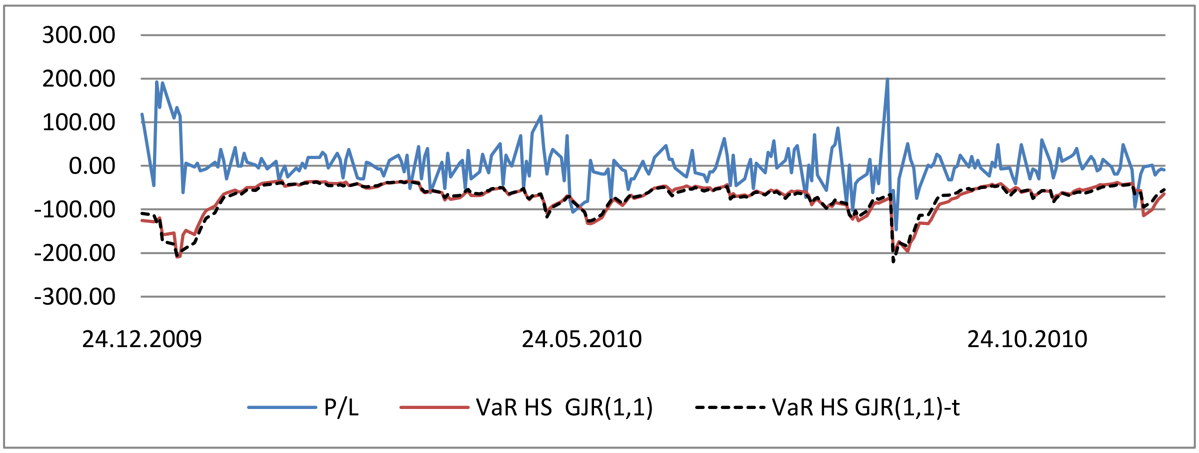


Figure 14: Danske Bank - P/L series, HS-GJR(1,1), HS-GJR(1,1)-t



Danske Bank – Parametric models

Figure 15: Danske Bank - P/L series, EWMA-normal, EWMA-t

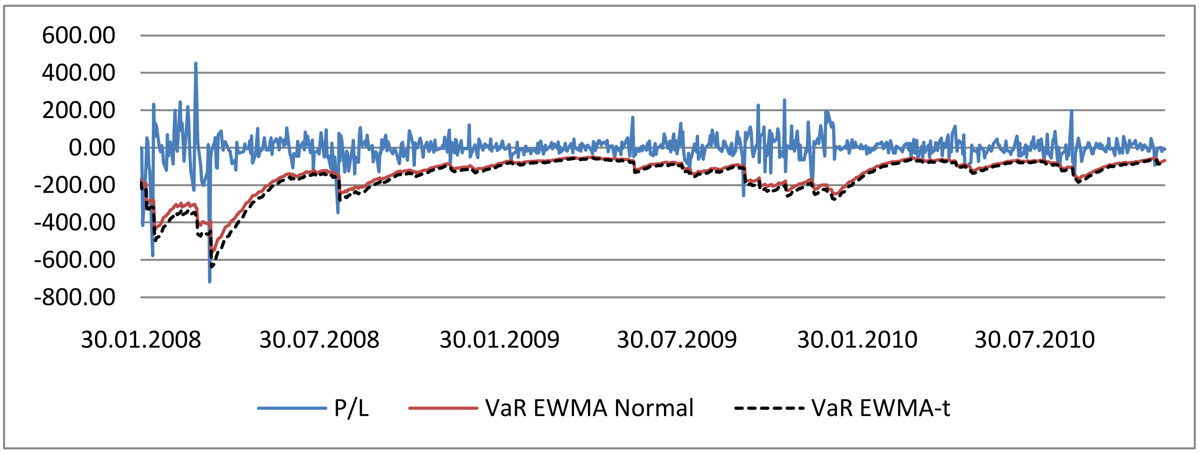


Figure 16: Danske Bank - P/L series, GJR(1,1), GJR(1,1)-t

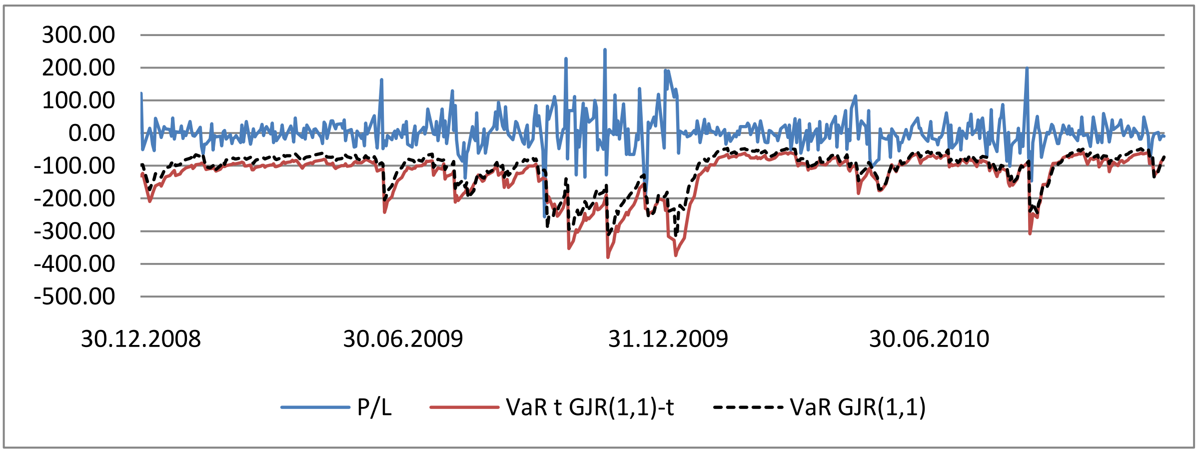


Figure 17: Danske Bank - P/L series, EGARCH(2,1), EGARCH(1,1)-t

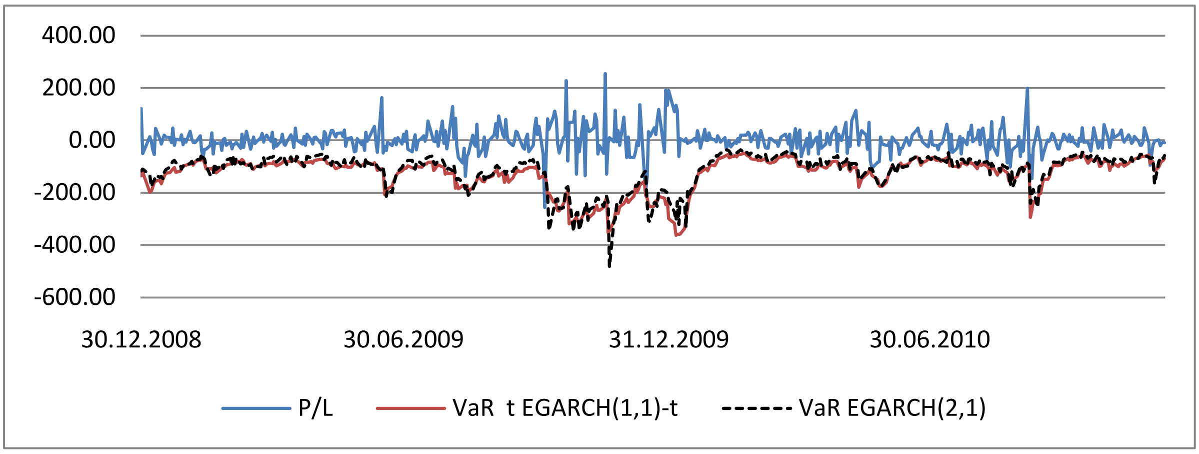
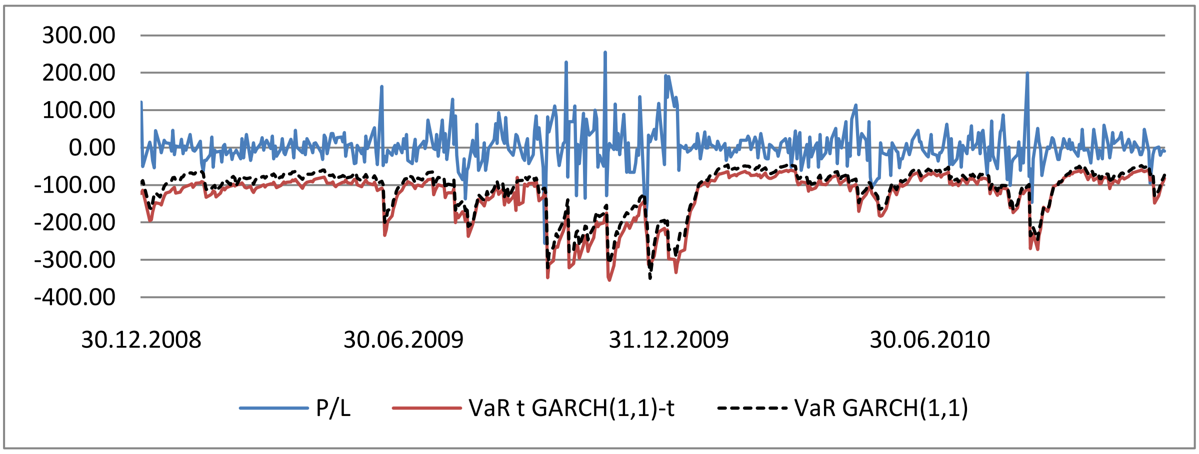


Figure 18: Danske Bank - P/L series, GARCH(1,1), GARCH(1,1)-t



Deutsche Bank – Non-parametric models

Figure 19: Deutsche Bank - P/L series. Basic Historical Simulation, Age Weighted Historical Simulation

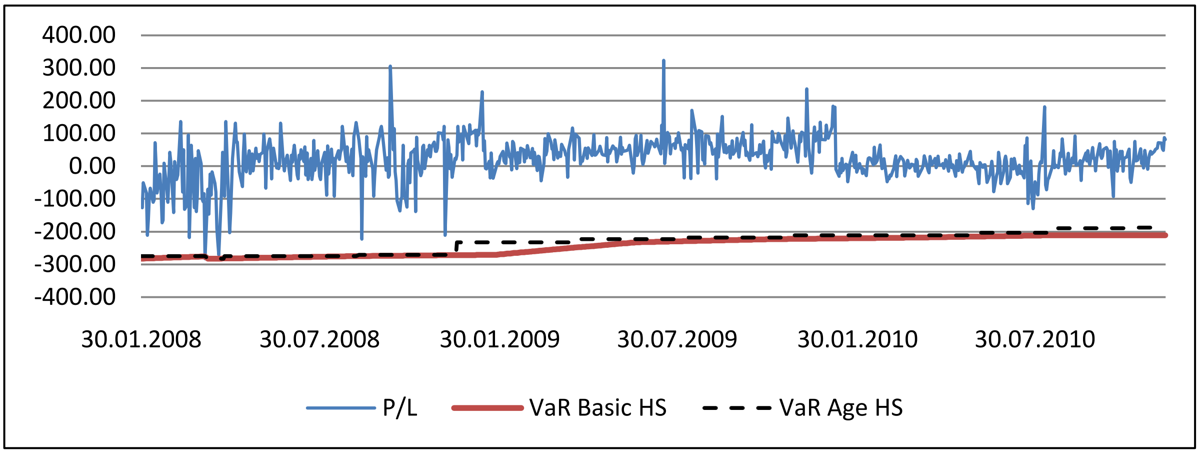


Figure 20: Deutsche Bank - P/L series, HS-VIX, HS-VDAX

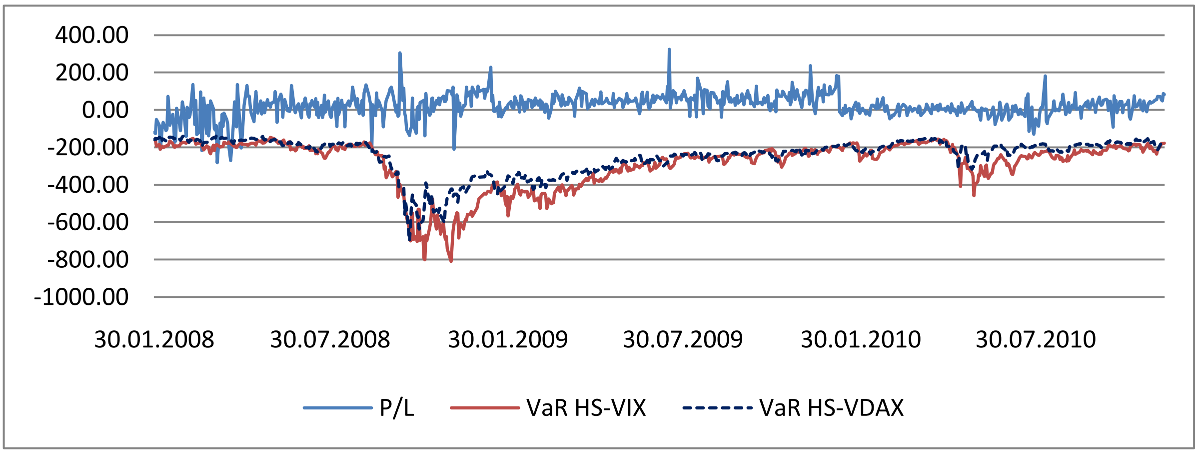


Figure 21: Deutsche Bank - P/L series, HS-GARCH(2,1), HS-EGARCH(1,2)

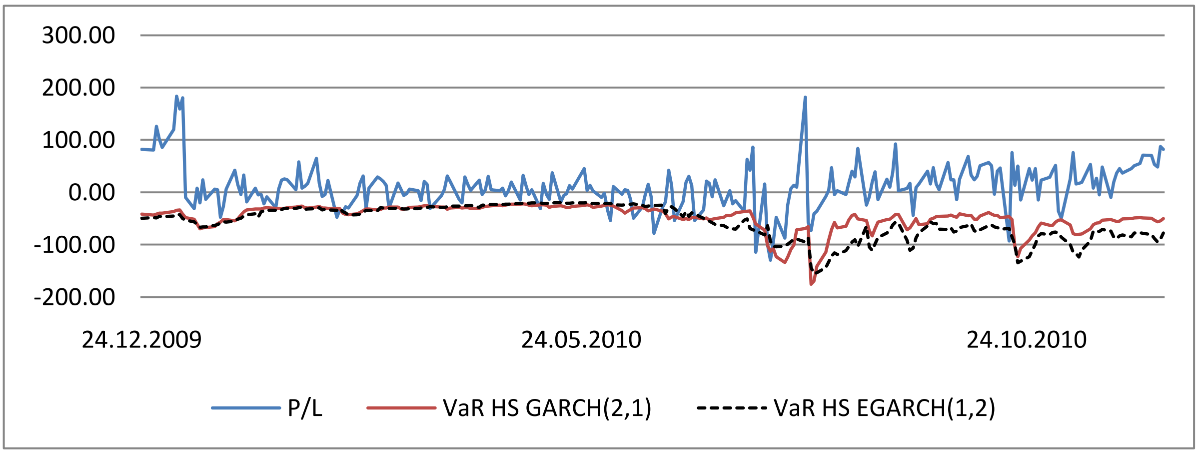


Figure 22: Deutsche Bank - P/L series, HS-GARCH(1,1)-t, HS-EGARCH(1,1)-t

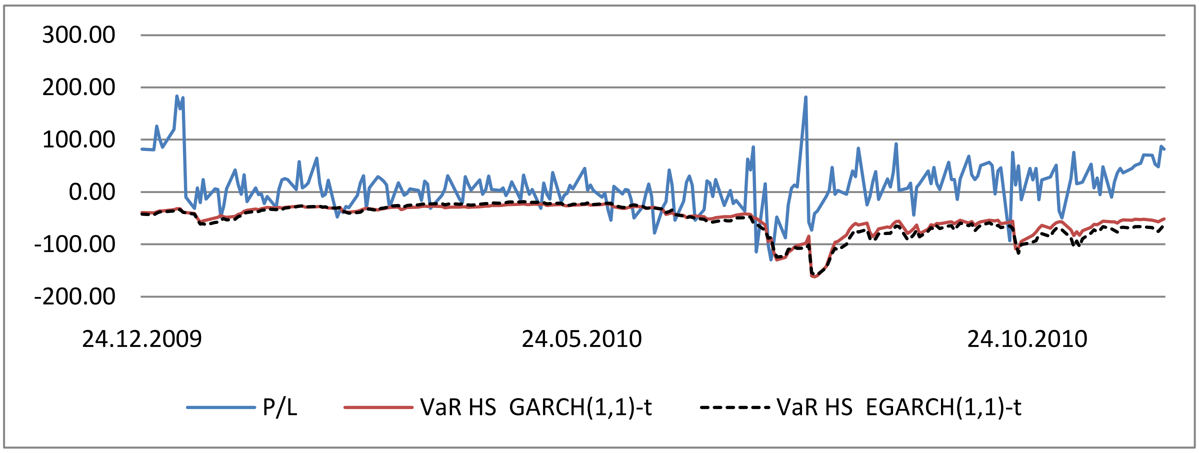
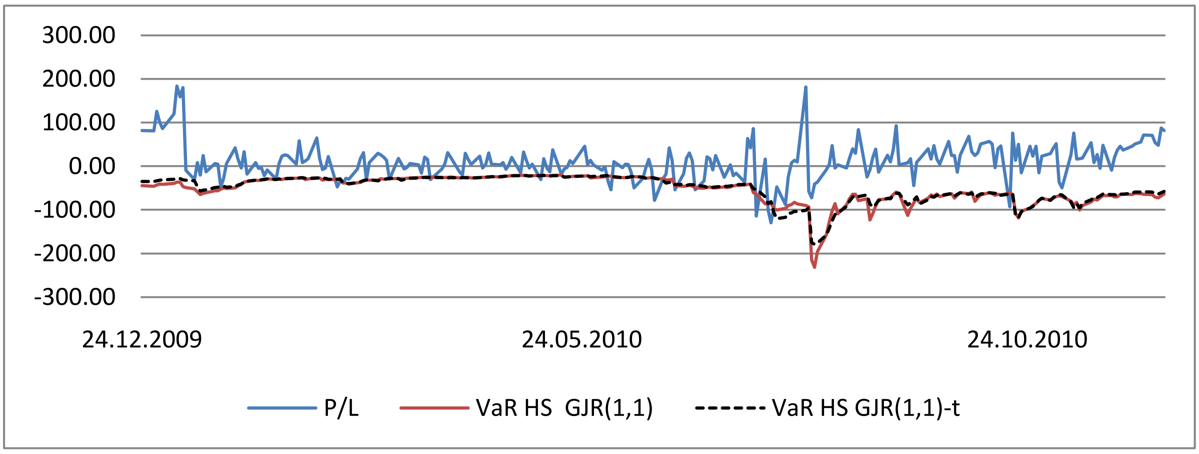


Figure 23: Deutsche Bank - P/L series, HS-GJR(1,1), HS-GJR(1,1)-t



Deutsche Bank – Parametric models

Figure 24: Deutsche Bank - P/L series, EWMA-normal, EWMA-t

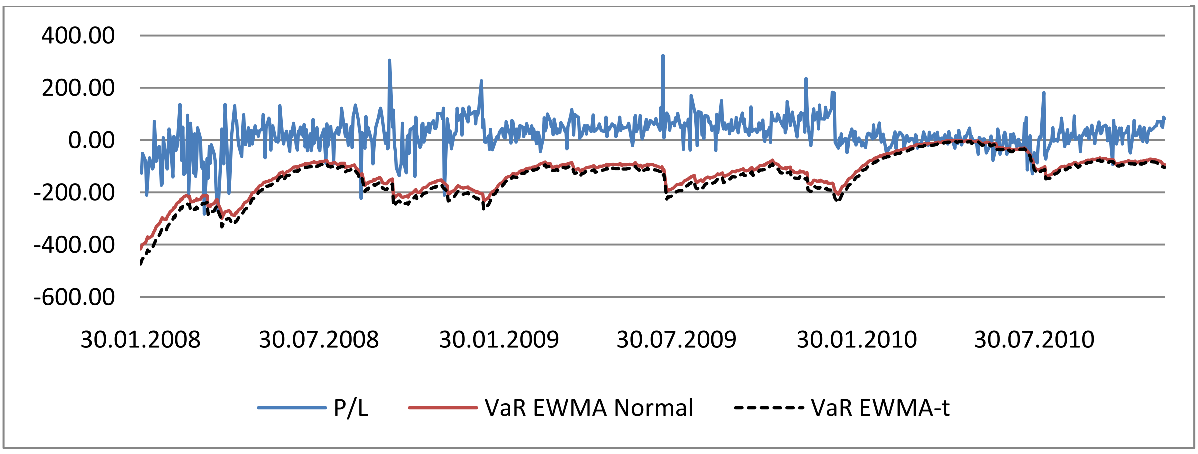


Figure 25: Deutsche Bank - P/L series, GJR(1,1), GJR(1,1)-t

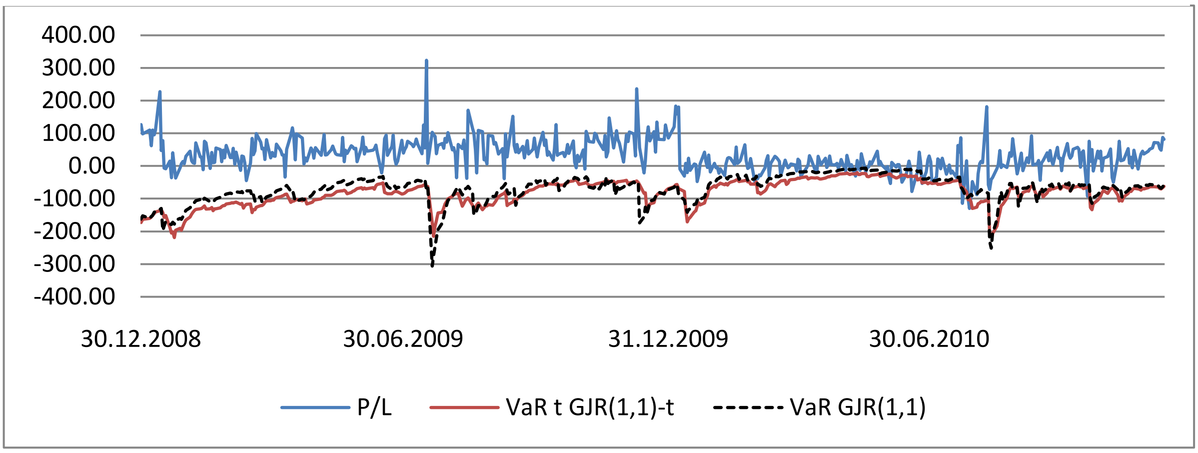


Figure 26: Deutsche Bank - P/L series, EGARCH(1,2), EGARCH(1,1)-t

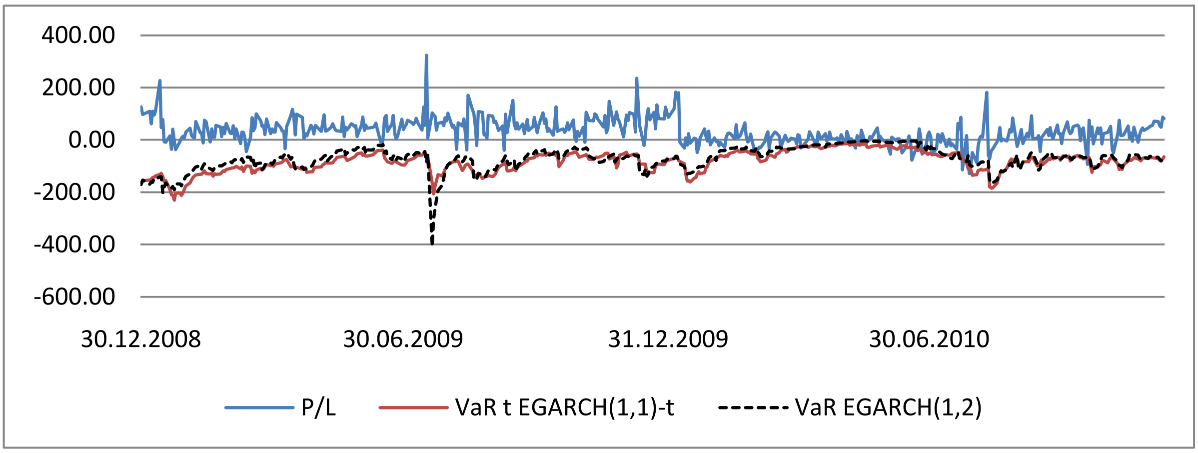
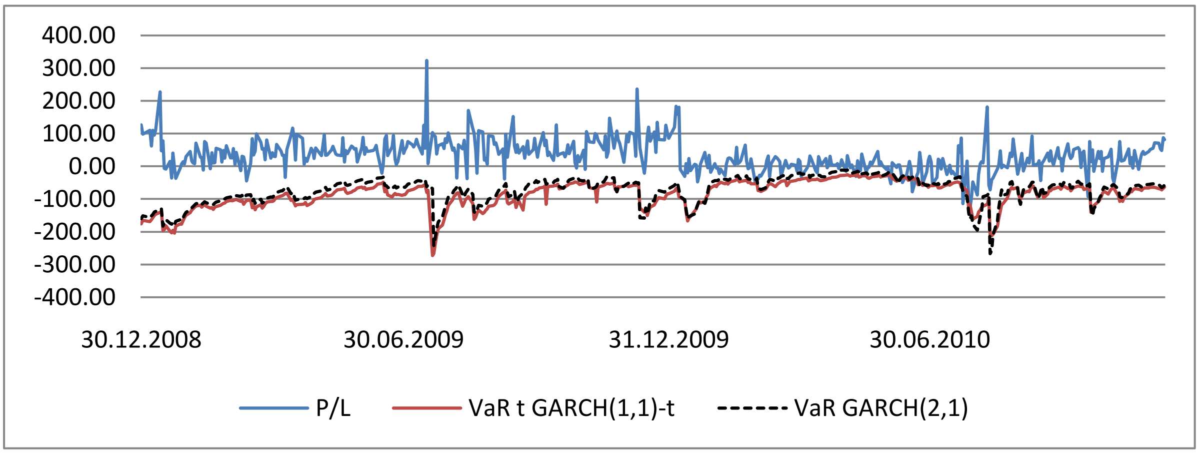


Figure 27: Deutsche Bank - P/L series, GARCH(2,1), GARCH(1,1)-t



Swedbank – Non-parametric models

Figure 28: Swedbank - P/L series, Basic Historical Simulation, Age Weighted Historical Simulation

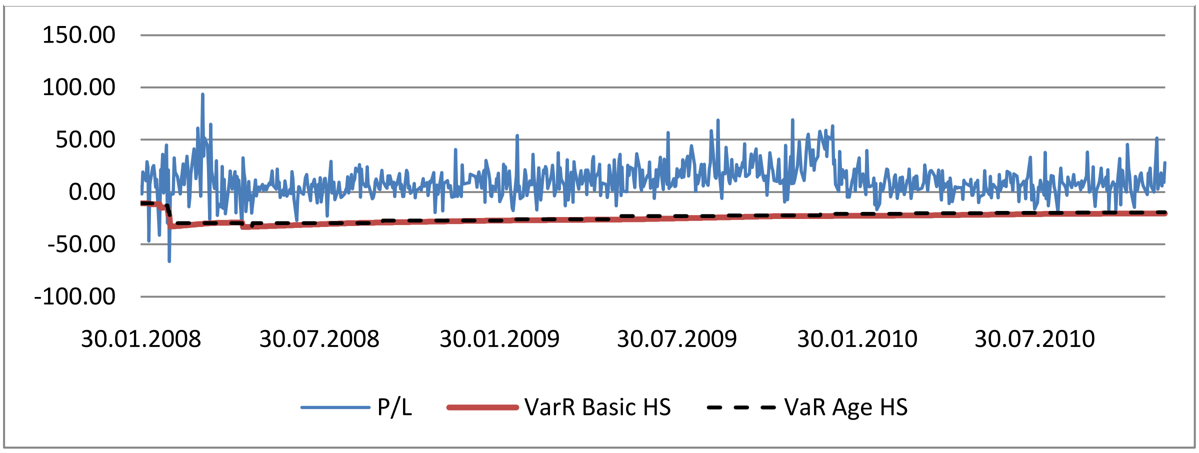


Figure 29: Swedbank - P/L series, HS-VIX, HS-VSTOXX

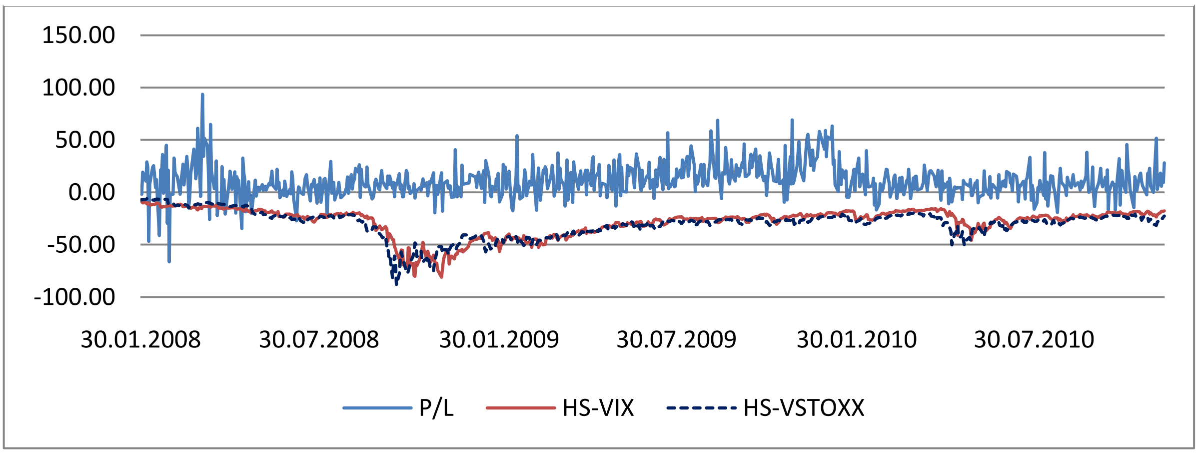


Figure 30: Swedbank - P/L series, HS-GARCH(1,1), HS-EGARCH(1,3)

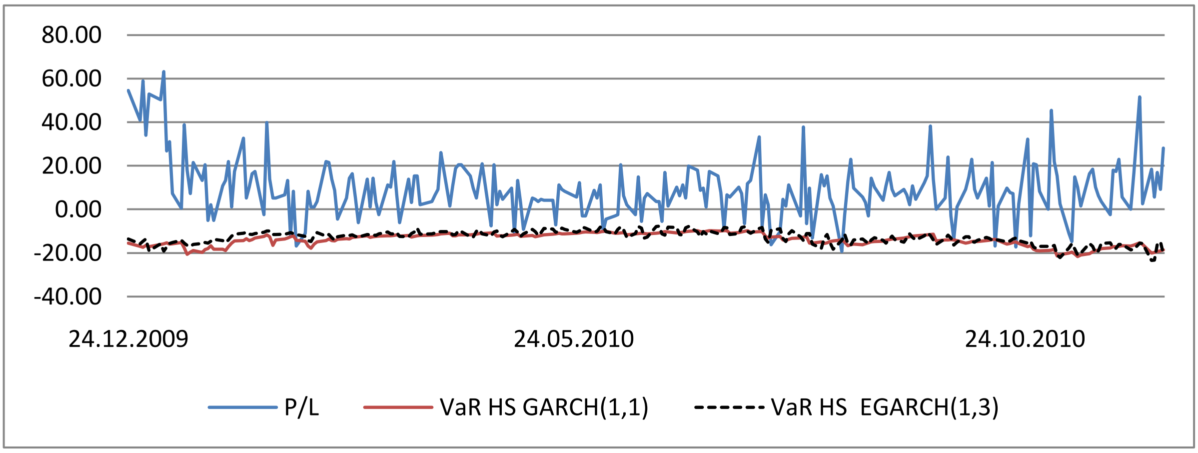


Figure 31: Swedbank - P/L series, HS-GARCH(1,1)-t, HS-EGARCH(1,1)-t

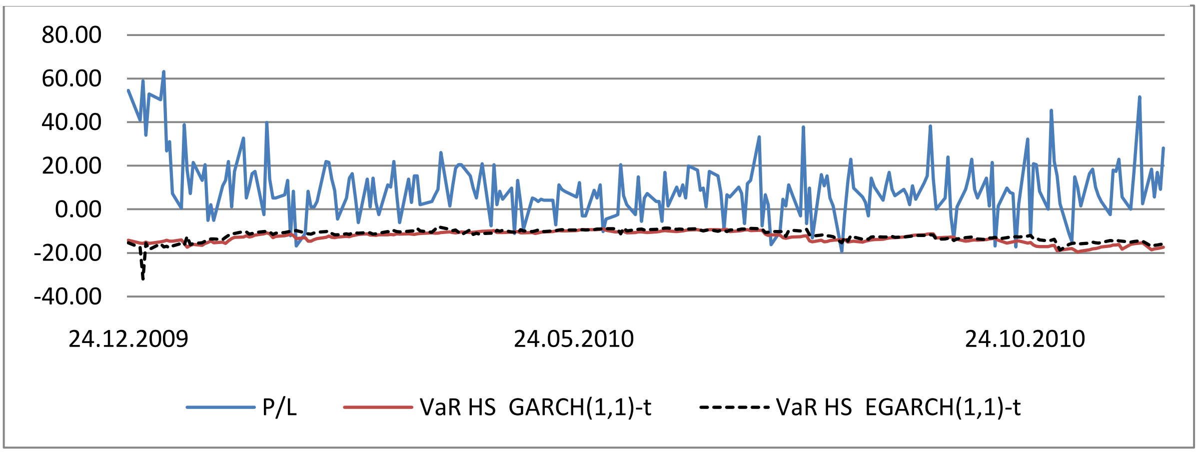
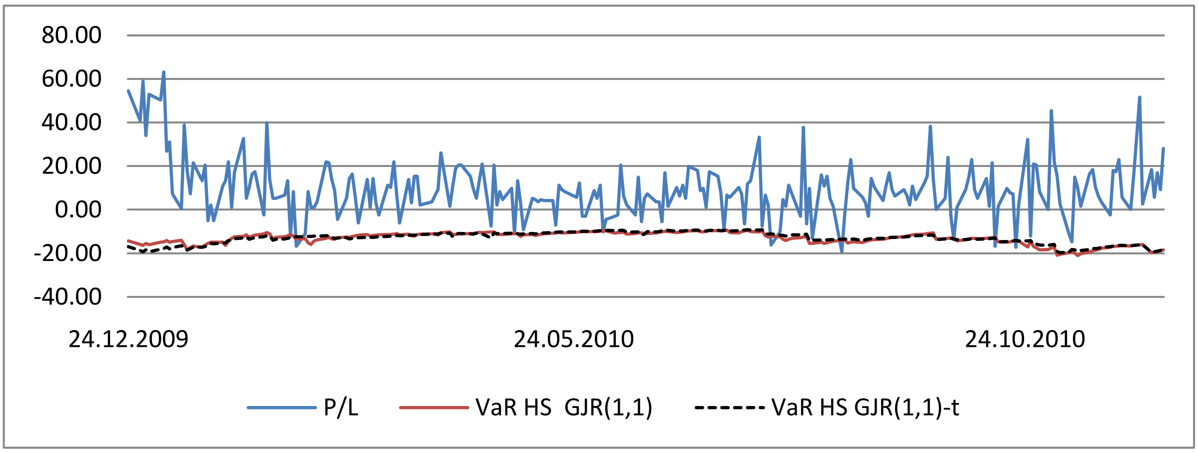


Figure 32: Swedbank - P/L series, HS-GJR(1,1), HS-GJR(1,1)-t



Swedbank – Parametric models

Figure 33: Swedbank - P/L series, EWMA-normal, EWMA-t

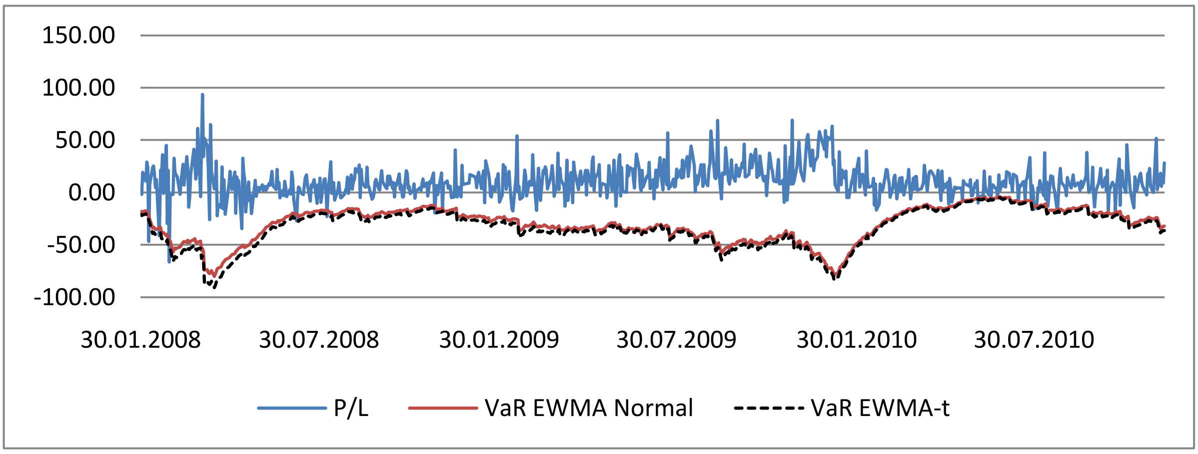


Figure 34: Swedbank - P/L series, GJR(1,1), GJR(1,1)-t

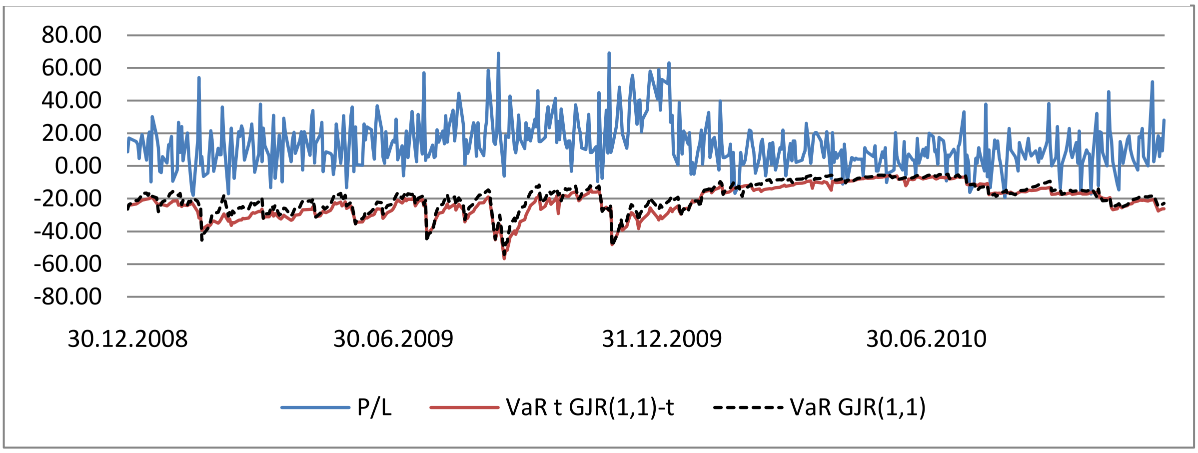


Figure 35: Swedbank - P/L series, EGARCH(1,3), EGARCH(1,1)-t

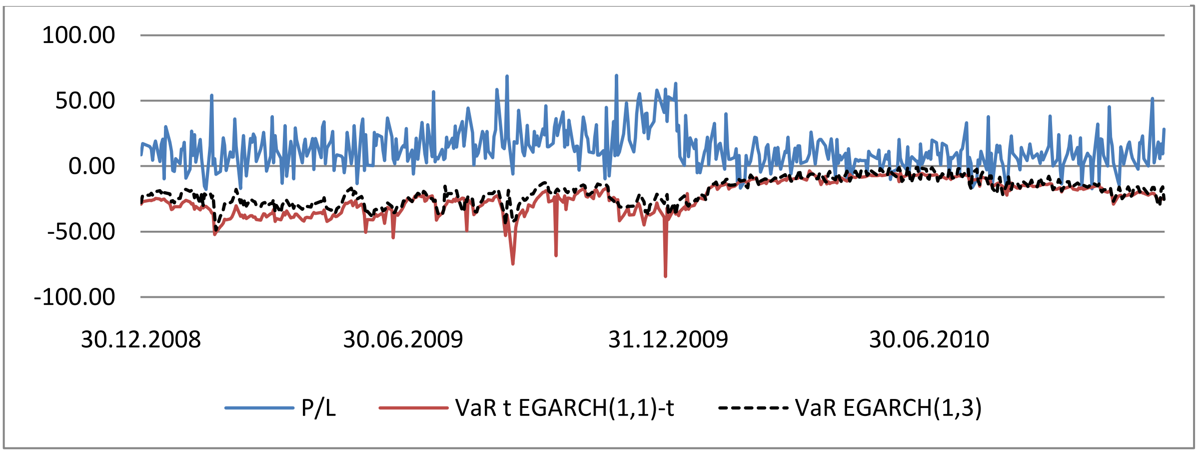
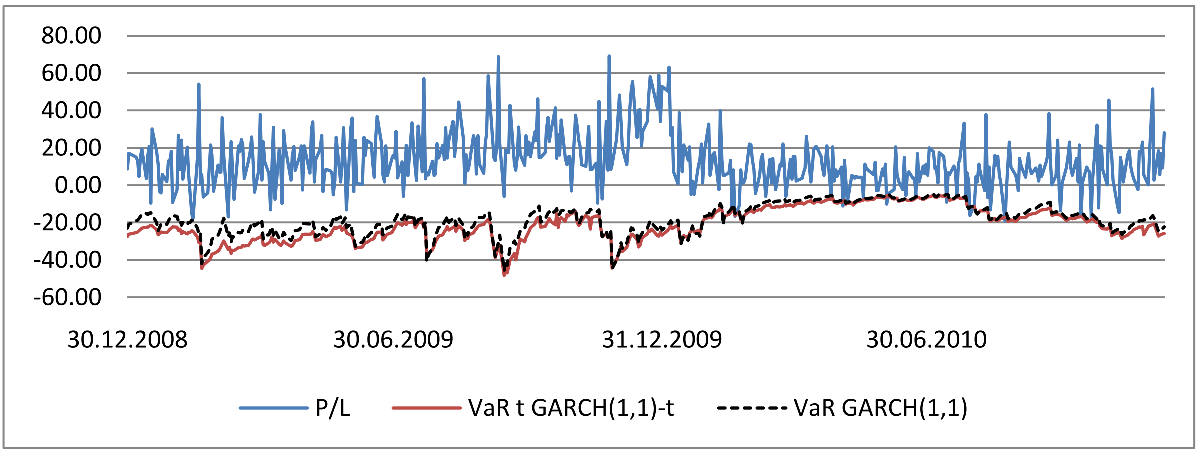


Figure 36: Swedbank - P/L series, GARCH(1,1), GARCH(1,1)-t



Appendix 8. Risk Maps

Figure 3: Risk Map for three years VaR forecasts

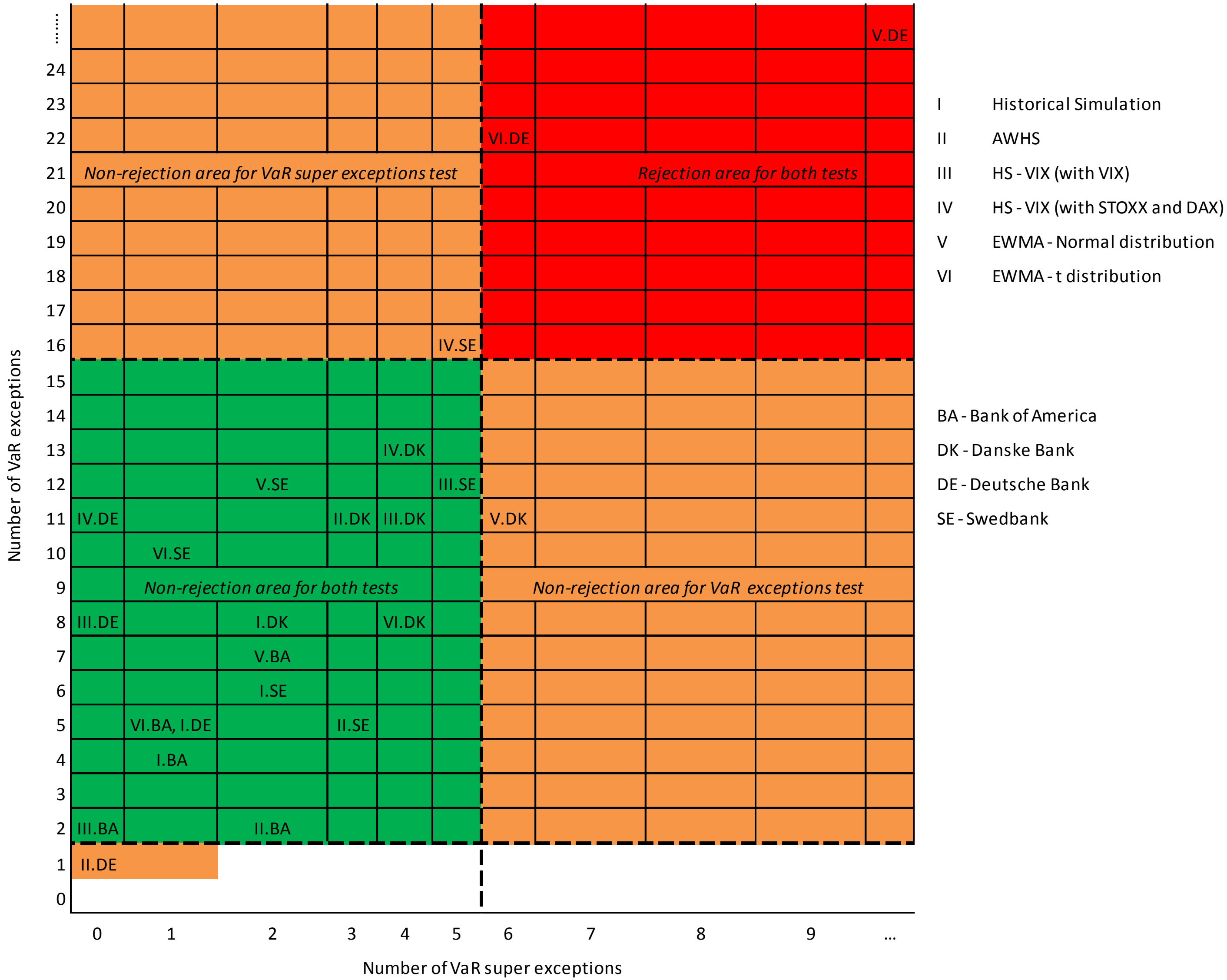
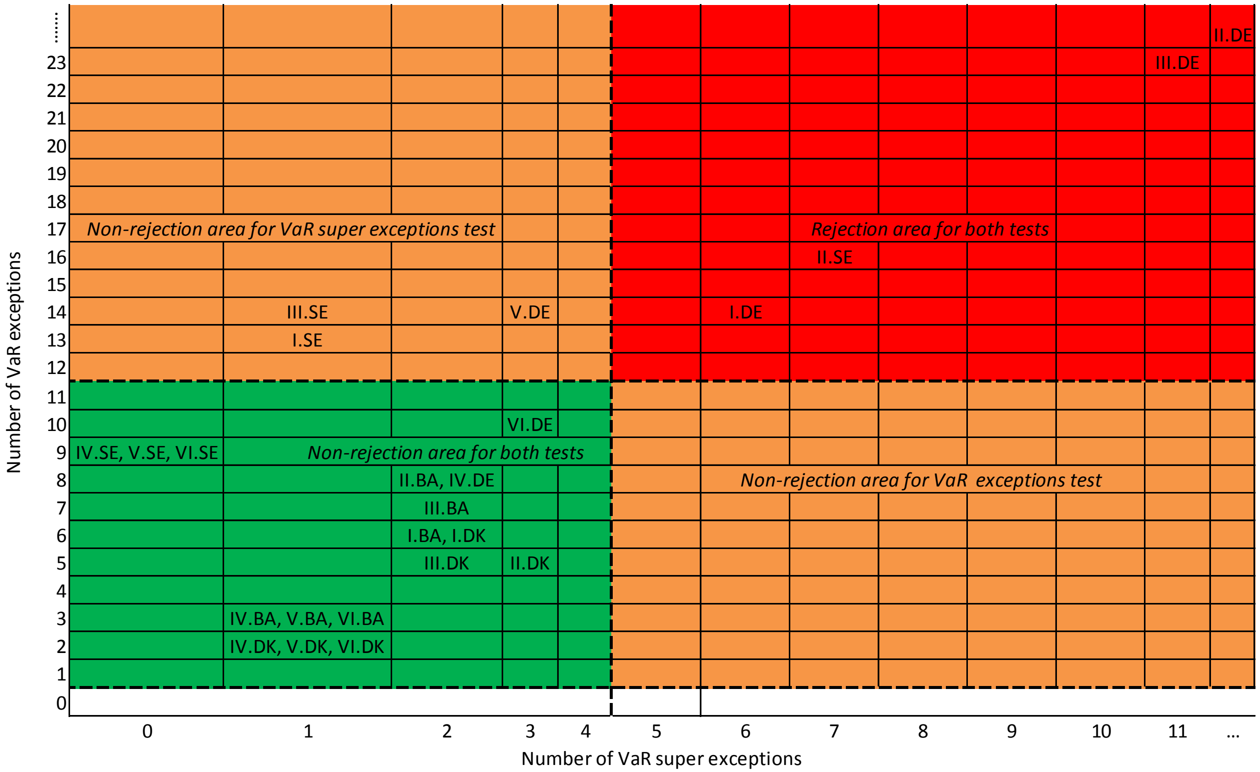


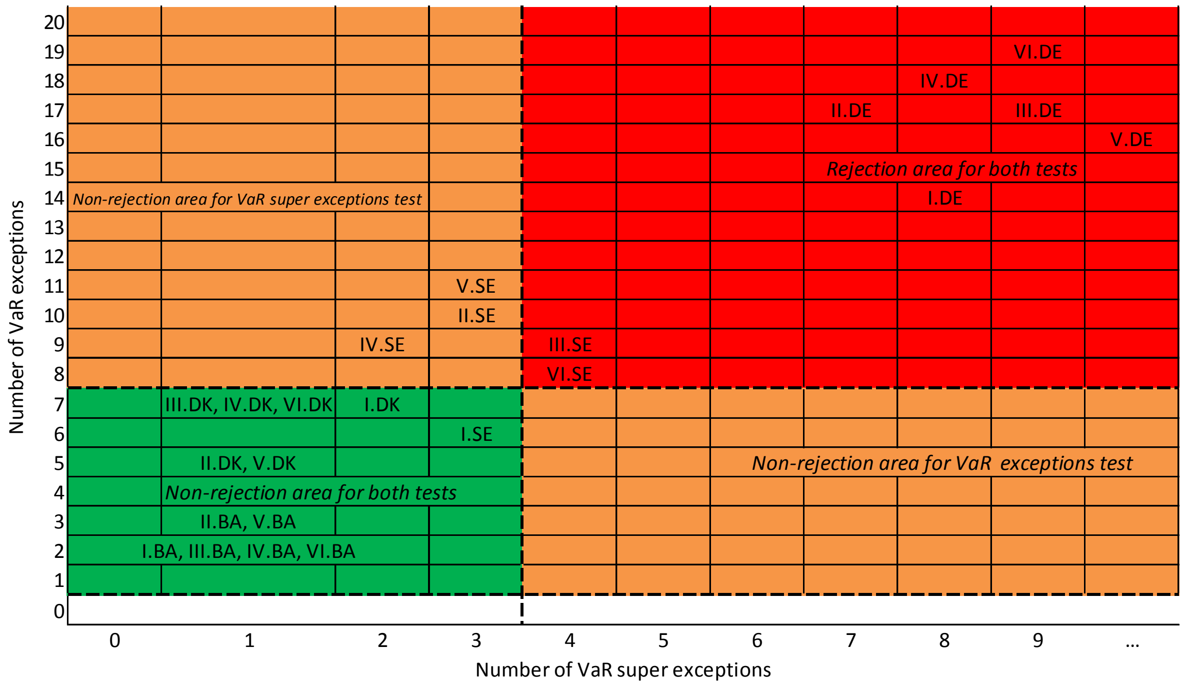
Figure 4: Risk Map for two year VaR forecasts



I	GARCH(p, q) - Normal distribution
II	EGARCH(p, q) - Normal distribution
III	GJR(p, q) - Normal distribution
IV	GARCH(p, q) - t distribution
V	EGARCH(p, q) - t distribution
VI	GJR(p, q) - t distribution

BA - Bank of America
DK - Danske Bank
DE - Deutsche Bank
SE - Swedbank

Figure 5: Risk Map for one year VaR forecasts



I VWHS - GARCH(p, q) - Normal distribution
II VWHS - EGARCH(p, q) - Normal distribution
III VWHS - GJR(p, q) - Normal distribution
IV VWHS - GARCH(p, q) - t distribution
V VWHS - EGARCH(p, q) - t distribution
VI VWHS - GJR(p, q) - t distribution

BA - Bank of America
DK - Danske Bank
DE - Deutsche Bank
SE - Swedbank

Appendix 9. Model rankings

Table 1: Bank of America - 1Y - model ranking (by N, N' and then average VaR)

Model	N	N'	Average VaR
GARCH(1,1) - t	1	0	-76.51
GJR(1,1) - t	1	0	-77.16
EGARCH(1,1) - t	1	0	-80.94
Historical Simulation	1	0	-135.54
HS-VIX	1	0	-177.86
AWHS	1	1	-126.72
EWMA - t	2	0	-135.53
VWHS GARCH(1,1) - t	2	1	-55.94
VWHS GARCH(1,1) - Normal	2	1	-57.38
VWHS GJR(1,1) - Normal	2	1	-57.65
VWHS GJR(1,1) - t	2	1	-58.29
EWMA - Normal	2	1	-122.70
VWHS EGARCH(1,1) - t	3	1	-52.63
VWHS EGARCH(1,1) - Normal	3	1	-57.11
GJR(1,1) - Normal	3	1	-62.37
GARCH(1,1) - Normal	3	1	-63.38
EGARCH(1,1) - Normal	4	1	-64.50

Table 2: Bank of America - 1Y - model ranking (by average VaR)

Model	N	N'	Average VaR
VWHS EGARCH(1,1) - t	3	1	-52.63
VWHS GARCH(1,1) - t	2	1	-55.94
VWHS EGARCH(1,1) - Normal	3	1	-57.11
VWHS GARCH(1,1) - Normal	2	1	-57.38
VWHS GJR(1,1) - Normal	2	1	-57.65
VWHS GJR(1,1) - t	2	1	-58.29
GJR(1,1) - Normal	3	1	-62.37
GARCH(1,1) - Normal	3	1	-63.38
EGARCH(1,1) - Normal	4	1	-64.50
GARCH(1,1) - t	1	0	-76.51
GJR(1,1) - t	1	0	-77.16
EGARCH(1,1) - t	1	0	-80.94
EWMA - Normal	2	1	-122.70
AWHS	1	1	-126.72
EWMA - t	2	0	-135.53
Historical Simulation	1	0	-135.54
HS-VIX	1	0	-177.86

Table 3: Danske Bank - 1Y - model ranking (by N, N' and then average VaR)

Model	N	N'	Average VaR
EGARCH(1,1) - t	1	0	-125.68
GARCH(1,1) - t	1	0	-126.06
GJR(1,1) - t	1	0	-127.68
GJR(1,1) - Normal	2	1	-108.07
EGARCH(2,1) - Normal	2	1	-113.49
Historical Simulation	3	1	-194.82
EWMA - t	3	1	-160.72
GARCH(1,1) - Normal	3	1	-107.97
AWHS	4	1	-185.75
HS-VIX	4	1	-221.82
EWMA - Normal	4	2	-142.64
HS-VSTOXX	4	1	-215.01
VWHS EGARCH(1,1) - t	5	1	-76.40
VWHS EGARCH(2,1) - Normal	5	1	-77.99
VWHS GARCH(1,1) - t	7	1	-71.84
VWHS GJR(1,1) - Normal	7	1	-72.30
VWHS GJR(1,1) - t	7	1	-72.77
VWHS GARCH(1,1) - Normal	7	2	-72.79

Table 4: Danske Bank - 1Y - model ranking (by average VaR)

Model	N	N'	Average VaR
VWHS GARCH(1,1) - t	7	1	-71.84
VWHS GJR(1,1) - Normal	7	1	-72.30
VWHS GJR(1,1) - t	7	1	-72.77
VWHS GARCH(1,1) - Normal	7	2	-72.79
VWHS EGARCH(1,1) - t	5	1	-76.40
VWHS EGARCH(2,1) - Normal	5	1	-77.99
GARCH(1,1) - Normal	3	1	-107.97
GJR(1,1) - Normal	2	1	-108.07
EGARCH(2,1) - Normal	2	1	-113.49
EGARCH(1,1) - t	1	0	-125.68
GARCH(1,1) - t	1	0	-126.06
GJR(1,1) - t	1	0	-127.68
EWMA - Normal	4	2	-142.64
EWMA - t	3	1	-160.72
AWHS	4	1	-185.75
Historical Simulation	3	1	-194.82
HS-VSTOXX	4	1	-215.01
HS-VIX	4	1	-221.82

Table 5: Swedbank - 1Y - model ranking (by N, N' and then average VaR)

Model	N	N'	Average VaR
AWHS	2	1	-23.93
Historical Simulation	2	1	-25.14
EWMA - t	3	0	-33.81
GARCH(1,1) - t	4	0	-21.06
GJR(1,1) - t	4	0	-21.32
EGARCH(1,1) -t	4	0	-23.66
EWMA - Normal	4	1	-30.32
HS-VIX	4	2	-28.11
HS-VSTOXX	5	2	-30.21
GARCH(1,1) - Normal	6	0	-18.16
VWHS GARCH(1,1) - Normal	6	3	-13.92
GJR(1,1) - Normal	7	0	-18.71
EGARCH(1,3) - Normal	8	3	-19.27
VWHS GJR(1,1) - t	8	4	-13.06
VWHS GARCH(1,1) - t	9	2	-12.84
VWHS GJR(1,1) - Normal	9	4	-13.15
VWHS EGARCH(1,3) - Normal	10	3	-12.91
VWHS EGARCH(1,1) - t	11	3	-11.86

Table 6: Swedbank - 1Y - model ranking (by average VaR)

Model	N	N'	Average VaR
VWHS EGARCH(1,1) - t	11	3	-11.86
VWHS GARCH(1,1) - t	9	2	-12.84
VWHS EGARCH(1,3) - Normal	10	3	-12.91
VWHS GJR(1,1) - t	8	4	-13.06
VWHS GJR(1,1) - Normal	9	4	-13.15
VWHS GARCH(1,1) - Normal	6	3	-13.92
GARCH(1,1) - Normal	6	0	-18.16
GJR(1,1) - Normal	7	0	-18.71
EGARCH(1,3) - Normal	8	3	-19.27
GARCH(1,1) - t	4	0	-21.06
GJR(1,1) - t	4	0	-21.32
EGARCH(1,1) -t	4	0	-23.66
AWHS	2	1	-23.93
Historical Simulation	2	1	-25.14
HS-VIX	4	2	-28.11
HS-VSTOXX	5	2	-30.21
EWMA - Normal	4	1	-30.32
EWMA - t	3	0	-33.81

Table 7: Deutsche Bank - 1Y - model ranking (by N, N' and then average VaR)

Model	N	N'	Average VaR
AWHS	0	0	-231.83
Historical Simulation	2	0	-243.12
HS-VIX	3	0	-283.90
HS-VDAX	4	0	-246.31
GARCH(1,1) - t	4	1	-85.99
GJR(1,1) - t	5	1	-82.36
EGARCH(1,1) - t	7	1	-82.76
EWMA - t	7	2	-143.60
GARCH(2,1) - Normal	7	3	-73.28
GJR(1,1) - Normal	11	5	-70.12
EWMA - Normal	11	5	-126.05
EGARCH(1,2) - Normal	12	7	-71.90
VWHS GARCH(2,1) - Normal	14	8	-48.25
VWHS EGARCH(1,1) - t	16	10	-54.16
VWHS EGARCH(1,2) - Normal	17	7	-57.54
VWHS GJR(1,1) - Normal	17	9	-53.62
VWHS GARCH(1,1) - t	18	8	-50.21
VWHS GJR(1,1) -t	19	9	-52.15

Table 8: Deutsche Bank - 1Y - model ranking (by average VaR)

Model	N	N'	Average VaR
VWHS GARCH(2,1) - Normal	14	8	-48.25
VWHS GARCH(1,1) - t	18	8	-50.21
VWHS GJR(1,1) -t	19	9	-52.15
VWHS GJR(1,1) - Normal	17	9	-53.62
VWHS EGARCH(1,1) - t	16	10	-54.16
VWHS EGARCH(1,2) - Normal	17	7	-57.54
GJR(1,1) - Normal	11	5	-70.12
EGARCH(1,2) - Normal	12	7	-71.90
GARCH(2,1) - Normal	7	3	-73.28
GJR(1,1) - t	5	1	-82.36
EGARCH(1,1) - t	7	1	-82.76
GARCH(1,1) - t	4	1	-85.99
EWMA - Normal	11	5	-126.05
EWMA - t	7	2	-143.60
AWHS	0	0	-231.83
Historical Simulation	2	0	-243.12
HS- VDAX	4	0	-246.31
HS-VIX	3	0	-283.90

Appendix 10. VBA programming

Age Weighted HS

```
function ageHS(confidence, pl, n, lambda)
Dim i As Integer, j As Integer
Dim v(1 To 1000, 1 To 1000) As Double, w1 As Variant
Dim t As Double, w As Double, k As Double
w1 = ((1 - lambda) / (1 - (lambda ^ n)))
v(1, 1) = w1
v(1, 2) = pl(1)
'build the array with the weights on column 1 and returns on column 2

 For i = 2 To n
        v(i, 1) = w1 * lambda ^ (i - 1)
        v(i, 2) = pl(i)
    Next i

'sort the returns
For i = 1 To n - 1
    For j = i To n
        If v(i, 2) > v(j, 2) Then
            t = v(i, 2) 'interchange the returns
            v(i, 2) = v(j, 2)
            v(j, 2) = t
            w = v(i, 1) 'interchange the weights
            v(i, 1) = v(j, 1)
            v(j, 1) = w
         End If
    Next j
Next i

'determine the associated VaR value
k = 0
i = 1
Do
k = k + v(i, 1)
i = i + 1
Loop Until (k > (1 - confidence))

ageHS = v(i, 2)
End Function
```

HS-VIX

```
Function vvixhs(vix, pl, n As Integer, confidence As Double)
Dim k As Integer
Dim resc(1 To 1000) As Double
For k = 1 To n - 1
resc(k) = (pl(k) * vix(2)) / vix(k + 1)
Next k
vvixhs = Application.Percentile(resc, (1 - confidence))
End Function
```

EWMA

```
Function ewma(pl, sigma1, lambda)
Dim var(1 To 250) As Variant
Dim i As Integer
var(250) = sigma1 ^ 2
For i = 1 To 249
var(249 - i + 1) = lambda * var(249 - i + 2) + (1 - lambda) * (pl(249 - i +
2) ^ 2)
Next i
ewma = Sqr(var(1))
End Function
```

Volatility weighted HS

```
Function volhs(vol, pl, n As Integer, confidence As Double)
Dim k As Integer
Dim resc(1 To 500) As Double
For k = 1 To n
resc(k) = (pl(k) * vol(1)) / vol(k)
Next k
volhs = Application.Percentile(resc, (1 - confidence))
End Function
```

Appendix 11. Matlab programming

Volatility estimates through GARCH models

```
function [c] = garchforecast(series, rw, spec )
for i=1:(numel(series)-rw)
    s=series(i:(i+rw));
    Coeff = garchfit(spec, s);
   [sigmaforecast]=garchpred(Coeff,s,1);
    c(i)=sigmaforecast;
end
end
```