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Testing the Black- Litterman Model

Sensitivity of Weight Vector to Variance of Views

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Abstract

The paper investigates sensitivity of the optimal portfolio obtained from the Black Litterman model to the specification of the inputs. Specifically estimation methods of the variances of the views are employed and the results are analysed. For this purpose the MSCI indices of 14 European countries are experienced. The result shows very weak response of the weight vector to the estimation method of the variance matrix of views. Further research suggests the sensitivity analysis concentrated on the prior specification of the equilibrium returns as the beginning point for the Black Litterman model.

Keywords: Black- Litterman model, views, portfolio optimization, variance-covariance matrix

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List of acronyms and abbreviations

CAPM…………………………………………………………Capital Asset Pricing Model

EGARCH…………Exponential Generalized Autoregressive Conditional Heteroskedasticity

GARCH-M………………………………………………………………..GARCH in Mean

HICPs…………………………………………...EU Harmonized Indices of Consumer Prices

MSCI…………………………………........................Morgan Stanley Capital International

1. Introduction

1.1.Background

Optimal asset allocation is the topic that has been widely researched in financial literature. A number of models have been suggested whereas mean-variance and CAPM are the most popular. The simplicity and applicability of the models are important for practitioners. The model will be challenged if the results are not intuitive or not consistent with investor's projections about certain event.

Since the first publication of the Black and Litterman article in 1990, the Black Litternan model has gained wide practical application. However, not much empirical research exists as the outcome of the model depends on the subjective opinion of the portfolio manager. Specifically, Black-Litterman model provides the flexibility of combining the market equilibrium with the views of the investor. In contrast to the mean-variance optimization in which the user inputs a complete set of expected returns and the portfolio optimizer generates the optimal portfolio weights, in the Black-Litterman model the investor uses any number of views, which are statements about the expected returns of any selected portfolios. The model then combines the views with equilibrium, producing both the set of expected asset returns along with the optimal portfolio weights (He, Litterman, 1999)

Although Black Litterman model seems to be more efficient in practice it is difficult to test the model empirically or1 suggest theoretical test analysis to it. Different papers have been developed and many of them suggest newer types of treatment with inputs to the model. However, almost none of them compare two or more different ways of treating the same problem. Coming to the research area of given thesis Meucci (2006,2008,2010), Idzorek (2005), Orlov et al (2007) suggest different methodology about how to treat views and construct better variance matrix of the views.

1.2.Purpose

The purpose of the research paper is to test the sensitivity of the weight vector obtained from the Black Litterman model as response to the different methods of omega (variance matrix of views) estimation. We apply two ways of estimation of the variance matrix for views as described, however not tested, in Walters, J. (2011). Then, we compare the results from the methods applied on the same data (portfolio set).

1.3.Approach

The research applies reverse optimization to construct the initial equilibrium returns as a neutral starting point for BL model. We follow the Idzorek (2005), Bevan, Winkelmann(1998) and the extension by Meucci (2005) to construct equilibrium mean returns and variance matrix. Bayesian approach is used to identify prior distribution. We go further and use two methods described in detail in the methodology section to estimate variance matrix for views to use in BL model. The inputs to the BL model are derived from running EGARCH regression; in particular, we are able to predict the views using such regression.

1.4.Contribution

One of the new features of the paper is the use of EGARCH derived views as proxies for investor views for the Black-Litterman model. Similarly, as Beach, S. and Orlov A. (2007), we employ this model, however, using different factors, such as a dummy variable, SP500 index, inflation and oil price. The benefit of employing this model is that more objective views are obtained, i.e. they are not dependent on the subjective projections of the portfolio manager. In addition, GARCH type models are able to capture characteristics of stock returns.

Finally, no other research has tested different ways of estimation variance matrix of views and their impact on the weight vector.

The outcome of this paper should be of interest both to private portfolio managers and institutions participating in the international equity market, and who want to use an objective methodology for predictions of future returns and volatilities.

1.5.Outline

The outline of the thesis is as follows: Chapter 2 presents a summary of some previous scientific studies as well as a description of the data and the approach used in this paper.

Chapter 3 explains methodology and the background methods that are used in the paper. In chapter 4 we describe the data and the sources of data. Following those chapter, we conclude with empirical results in Chapter 5 and conclusions in Chapter 6.

2. Literature Review

2.1.Background of the Black Litterman Model Development

The theory formulated by Markowitz (1952) combines the two basic objectives of investing: maximizing expected return and at the same time minimizing risk. Markowitz proposed that when the investment is determined, the investor should not only look at the possible payoff of the investment, but also account for the certainty of that payoff. By formulating a mathematical model and making this trade-off explicit it became possible to allocate investments quantitatively. The risk part in the invested portfolio is measured as the standard deviation (or volatility) of returns around their expected value and the result of the portfolio optimization within the theory is an efficient frontier shaped as parabola. The latter indicates the combination of assets with the highest expected return given a certain level of risk.

Although the model has been popular within research and is used by many scholars, it can result in counterintuitive portfolios, poorly reflecting the views of the investor.

Practitioners, asset managers for example, have not widely used this model. One of the disadvantages can be described by Best and Grauer (1991). The authors suggest that if a budget constraint is imposed on the investment decision, the results indicated that mean variance portfolio weights, mean and variance can be very sensitive to the change in the asset means. Investors in the real world also find the weights returned by an optimizer extreme, not intuitive and, hence inappropriate for being implemented in a portfolio (He, Litterman, 1999, Dobrentz 2001).

2.2.Research on Black Litterman- Original Model and Its Extensions

Fischer Black and Robert Litterman improved the original mean-variance model by combining mean variance optimization of Markowitz and CAPM and chose a practitioners' perspective to develop a model for portfolio selection. Therefore, the inputs should be intuitive to the investment manager and the optimized portfolio should reflect the investors´ views. The original model was first proposed in 1990 and a year later the authors elaborated on the tactical asset allocation imbedded with investor's subjective views in a global sense by including bonds, equities and currencies. The authors state that their model does not assume that expected returns are always at the equilibrium as in CAPM. Instead, when expected

returns move away from the mean, imbalances in markets will tend to push them back. Thus, it is suggested that investors may profit by combining his views about returns in different markets with the information contained in the equilibrium (Black, Litterman, 1991).

An important feature of the Black-Litterman framework is that investors should take risk where they have views, and the most risk should be taken where they have the strongest views (Bevan, Winkelman, 1998). The model uses a Bayesian approach to combine the subjective views of an investor regarding the expected returns of one or more assets with the market equilibrium vector of expected returns (the prior distribution) to form a new, mixed estimate of expected returns. The resulting new vector of returns (the posterior distribution), leads to intuitive portfolios with sensible portfolio weights (Izdorek, 2005). Though the model generates more stable results than classical mean-variance optimisation and incorporates return forecasts in a consistent manner, BL characterises risk only by volatility. This might not which may not be applicable for all asset classes (Krishnan, Mains, 2005). The same authors have tried to test the model by including two factors in the Black-Litterman model-- the recession risk (distressed bonds were used as a proxy of this risk) and a beta-neutral index that tracks the excess return of distressed bonds over time. The author´s alternative model optimal portfolio weights vary from the standard BL model to ours. The article concludes that where investor´s confidence is high and asset classes with high beta to recession risk are likely to outperform. Additionally, risk-averse investors and asset classes that have a low or negative beta to recession risk are more likely to hold their value (Krishnan, Mains, 2005).

Some studies have tried to research the asset return distribution and model simplification (i.e. Quian and Gorman (2001), Satchel and Scowcroft, 2000, Meucci (2006), Beach and Orlov (2007)). Qian and Gorman (2001) tried to extend the model by simplifying it: they applied the conditional distribution theory directly to the asset return distribution instead of applying it to the unknown distribution vector of the mean vector. The authors adjust both the mean vector and the covariance matrix. The result is the mean vector returns that are equivalent to those of BL, while the conditional covariance matrix is new. It also reduces the sensitivity of the mean variance optimization to an investor´s volatility estimates.

Satchel and Scowcroft (2000) were also working on simplifying the model and presented its two alternative formulations. One of them takes into account prior beliefs about overall

volatility. In their paper, the authors derived Black-Litterman model using Bayesian method and extended some parameter hypotheses to arrive at a posterior multivariate t-distribution.

Meucci (2006), Beach and Orlov (2007) include non-normally distributed returns and consider fat tails, which is essential for hedge funds and derivatives. Martellini and Zieman (2007) introduced an extension of the BL approach that allows the incorporation of active views of hedge fund strategy performance considering preferences about higher moments of hedge fund return distributions. The authors focused in the on an approximation of the joint distribution of asset returns in terms of third- and fourth-order moments and co-moments (co-skewness and co-kurtosis) and not the distribution itself.

The literature also discussed the problem of estimating an investor's view for the Black-Litterman model. For instance, Izdorek (2005) tried to simplify the BL model for non-quantitative investors. The author allows the investor to specify the confidence in views as the percentage (0-100%) where the confidence measures the change in weight of the posterior from the prior (0%) estimate to the conditional estimate (100%). Becker and Gutler (2009), on the other hand, have explored the estimation of confidence in views with two methods- by using the analysts´ forecasts with the dividend discount model and by Monte Carlo simulation. The article concludes that the implementation of the Black-Litterman model based on the number of analysts' forecasts outperforms all other strategies regarding the Sharpe ratio in constrained and unconstrained cases. Beach, S. and Orlov, A. (2007) have used GARCH-M model to compute the predicted excess returns and to make one-step-ahead forecasts of conditional standard deviations. This information is then used as inputs to produce the covariance matrix of views. Thus, their work introduced more objective inputs to the BL model. The authors also conclude that risk is reduced in the model from using lower values of τ is confirmed by the portfolio risk estimates and the standard deviations (Beach, S., Orlov, A, 2007, p. 14).

Finally, Chiarawongse et al. (2010) have introduced a new approach for qualitative views in the form of linear inequalities that are incorporated into a mean-variance portfolio optimization. The authors computed the expectation of alpha (risk-adjusted measure of the active return on an investment) conditioned on qualitative views that can be combined with a degree of confidence.

Mankert, C. (2006), opposed to the all previous research done on BL model incorporates behavioral finance in her discussion. For example, she brings in the home bias, which increases the riskiness of the foreign asset and influences the portfolio weights through the levels of confidence. An investor who is home biased has less confidence in the views concerning foreign assets than in those concerning domestic assets. This leads to the asset weights being closer to the benchmark weights compared to the weights of the domestic assets (Mankert, 2006, p. 66). Though, a behavioral argument exist as a critique to the Black Litterman model, we rely on the empirical conclusions (which are lacking in the Mankert´s research) to test the four ways to estimate variance matrix of views.

3. Methodology

The foundation for the discussion of the model is embedded in the Bayesian theory and reverse optimization. The Bayesian approach is used to infer the assets' expected returns (Black and Litterman, 1990, 1992), where the expected returns are random variables themselves and are not observable (He and Litterman 2002). Only their probability distribution can be inferred, which begins with a prior belief. In the Black-Litterman model, implied equilibrium returns from reverse optimization are the prior return distribution, the investor's views are the additional information. Combined, they are used to infer posterior distribution of returns.

3.1. Bayesian approach

As previously mentioned, Black-Litterman model uses the Bayesian approach to infer the assets' expected returns (Black and Litterman, 1990, 1992). The Bayes´ theory states that

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \quad (1)$$

P(A|B) is the conditional (or joint) probability of A, given B. It is also known as the posterior probability distribution and it is a probability value that has been revised by using additional information that is later obtained (Triola, n.d.).

P(B|A) is the conditional probability distribution of B given A. also known as the sampling distribution.

P(A) is the probability of A. Also known as the prior distribution and defined as an initial probability value originally obtained before any additional information is obtained (Triola, n.d.)

P(B) is the probability of B, which is also known as the normalizing constant (Walters, 2009).

As stated before, the Bayesian approach is used to combine the probability distribution of the expected returns obtained both from the equilibrium model and the additional views (He, Litterman, 2002).

3.2.Reverse Optimization: Prior Distribution

One of the crucial questions in Black Litterman model is the choice of prior distribution of expected excess returns. As starting point for Black Litterman approach we use "equilibrium" returns implied by reverse optimization method. The idea here is to find neutral starting point, where equilibrium returns clear the market and represent long-run trend (Bevan and Winkelmann, 1998).

Reverse optimization in contrast to classical mean-variance optimization uses portfolio weights as input and produces equilibrium returns. The main reason for reverse optimization being good way to forecast equilibrium returns and use weights as input is that the weights is sometimes easier to predict. Thus, reverse optimization produces more healthy expected returns than those in mean-variance optimization. Secondly, the weights in reverse optimization are easier interpreted by practitioners (Niedermayer, Zimmermann, 2007)

The initial function for Reverse optimization is the same as in traditional models, i.e. traditional utility function. As described in the paper by Walters (2009) the reverse optimization final function comes from the maximization of the following utility function:

$$U = w^T \Pi - (\frac{\lambda}{2})w^T \Sigma w \ (2)$$

where, U is investor's utility, Π is the vector of equilibrium returns, λ (lambda) is the investor's risk-aversion coefficient, w is the variance-covariance matrix of historical returns.

If we maximize the function and take derivatives of the function with respect to *w* we end up with the equation:

$$\Pi = \lambda \Sigma \text{w} \ (3)$$

Risk aversion coefficient represents the level of risk, where the opportunity cost of excess return equals to the variance change. Thus, investors' lambda represents the level of risk taking and shows the level after which investor will prefer lower risk to higher return. The paper uses λ=2.65 as suggested in Orlov (2007).

3.3. Black-Litterman Asset Allocation Model

The aforementioned information set a background for a BL model. Now, we assume that there are N-assets in the market, which may include equities, bonds, currencies and other assets. The returns of these assets have a normal distribution with $\boldsymbol{\mu}$ being the expected return and Σ the covariance matrix (here we follow the description of the model as in He, Litterman, 2002.

r ~ N (μ,Σ)

In equilibrium, all investors hold the market portfolio w$^{\text{eq}}$. The equilibrium, Π, state that if all investors hold the same view, the demand for these assets would be equal to the outstanding supply (Black, 1989). Assuming the average risk tolerance of the world is represented by the risk aversion parameter λ, the equilibrium risk premiums are given by

$\Pi = \lambda \Sigma w_{eq}$ (4) which is identical to the formula (3) presented before. The next step is combining the equilibrium results with the views using Bayesian method. As all the parts were described in previous sections, we go directly to the final formula is used in Black Litterman model:

$$E[R] = \left[(\tau \Sigma)^{-1} + P' \Omega^{-1} P\right]^{-1} \left[(\tau \Sigma)^{-1} \Pi + P' \Omega^{-1} Q\right] \ (5)$$

The formula is the final for the Bayesian combination of the Markowitz approach and views composed by investor and the parameters here are;

τ   is a measure of investor´s confidence in prior estimates

Ω   kxk covariance matrix of views

P   kxn matrix relating views to expected returns

∑   kxk covariance matrix of estimated views

Π   nx1 vector of implied equilibrium returns

Q   kx1 vector of absolute / relative return expectations

3.4.Discussion of Inputs: Investor's Views

One of the main advantages that the Black-Litterman approach provides is the formulation of own views. Views can be relative or absolute (view on the return of only one asset). In addition, views do need to be expressed for every single asset. With traditional mean-variance analysis, however, one can only express absolute views and has to do so for every single asset. The deviation from the market equilibrium will depend on how far our views are from this equilibrium and the degree of confidence we have in our views (Deutsche Bank, 2003). The derivation of the consistent weighting scheme between the subjective view and the equilibrium view is very important in the model. Intuitively, the higher (lower), the degree of confidence, the more (less) the revised expected returns are tilted toward the investor's views. (Zimmerman, et al. 2003, p.272)

 Investor Subjective Views

Investors can express absolute and relative views on assets. However, one can only express linear views on the asset returns. Nonlinear views or views on other important factors like volatility cannot be expressed directly. This limitation was overcome by Meucci (2006) using copula-opinion pooling method. If an investor wants to express a relative view such as: one asset return will outperform another one by 3 percent in the following month in a two asset allocation scenario, then the view matrix Q will be a vector of kx1 where k is number of views (in our case 4x1), and P will be the matrix kxn (4x14) where we give 1 to the assets we have views on.

Specification and Estimation of Views

The views can be described as having the following properties:

- Each view is unique and uncorrelated with another
- The sum of weights in the views is either one (absolute view) or zero (relative view), thus the views are fully invested
- There is no requirement that all assets have a view. It is possible that views conflict; the estimation will merge the views based on the confidence in views and the prior distribution.

To represent k views on n asset several matrices are used.

We will represent the investor's k views on n assets using the following matrices

- P is a kxn matrix of the asset weights within each view. In the previous research the authors computed weights differently. For example, He, Litterman (1999) and Idzorek (2005) used a market capitalization weighed method. On the other hand, Satchell and Scowcroft (2000) used an equal weighted scheme. This thesis uses market capitalization method by following Idzorek.

- Q, a kx1 matrix of the returns for each view.

- Ω is a kxk matrix of the covariance of the views. It is a symmetric matrix with off diagonal elements equal to zero. However, if the investor is confident in the view, the diagonal elements may also be zero. The diagonal feature of the matrix is provided by views´ being independent and uncorrelated. The i-thdiagonal element is equal to ω_i. Ω^{-1}, the inverse of the covariance matrix, is known as the confidence in the investor´s views. Basic Black Litterman model does not provide an explanation why there is such a relationship and it is an investor´s choice to calculate Ω (Walters, 2009).

3.5.Discussion of Inputs: Omega estimation

After the views have been specified, an investor can estimate the Ω matrix in several ways. The following methods are of primary interest in this research.

Proportional to the variance of the prior

Here an assumption is made that the variance of views is proportional to the variance of returns (as in He, Litterman (1999). The variance matrix is calculated as follows

$$\Omega = diag(P(\tau\Sigma)P^T)(6)$$

The alternative method, proportional to the prior variances as in Meucci (2005), could be used to calculate omega. Meucci (2006) ignores diagonalization and suggests a formula below:

$$\Omega = (P\tau P')/c \ (7)$$

In our paper we apply Meucci (2005) way of omega calculation. The result in both methods is similar in the end the only difference that if we include tau initially we ignore it at the latest formula.

Use the variance of residuals in a factor model

The description of this method is presented in Walters (2011) with an extension in Beach, S. and Orlov, A. (2007). The method relies on the factor model to compute the views. Thus, the variance of the residuals from the factor model can be used to estimate the variance of the returns. The general expression for a factor model
of returns is:

$$E(r) = \sum_{i=1}^{n} \beta_i f_i + \epsilon \tag{8}$$

$E(r)$ is the return on the asset

β_i is the factor loading on factor (i)

f_i is the return due to factor (i)

ϵ residual independently and normally distribuited

And the general expression for the variance of the return from a factor model is:

$$V(r) = B V(F) B^T + V(\epsilon) \tag{9}$$

B is the factor loading matrix

F is the vector of returns due to the various factors

Given the above formulas, we can compute the variance of ε directly as part of the regression. Beach and Orlov (2007) use an EGARCH-M model to generate views. The regressors in the EGARCH-M (1,1) mean equation include country-specific Dividend Yield, Premium, Spread, Term and Oil. The variance equation regressors are Inflation, Dividend Yield, Premium, Spread and Term. The GARCH results are used to estimate $\Omega, \Sigma \ and \ Q$ as inputs in BL model.

3.6.Discussion of Inputs: Defining Tau

One of the most discussible parameters in the Black Litterman model is tau, τ, which can be defined as a level of confidence in the views.

Black and Litterman (1990) introduce this parameter as a proportionality constant to scale the variance of the expected return. The resulting τ is close to zero because the uncertainty about the mean is less than the uncertainty of the return itself (Salomons, p. 55). This is the only explanation provided of why it is close to zero. On the other hand, Satchell and Scowcroft (2000) argue for τ around 1. Meucci (2010) proposes a BL model without τ at all. Following from the discussion, we do not decide on a specific value of τ, but instead we test our results both for τ= 0.005 and 0.01 (similar to the research of Beach, Orlov (2007), He, Litterman (1992), Idzorek (2005)).

Appendix 1. provides a list of parameters used in this paper and gives short definitions to each.

3.7.Forecasting with EGARCH (1,1)

In order to forecast the returns and derive proxies for investors´ views, an extension to GARCH (1,1) model is used. Forecasting return volatility with GARCH may be superior to using subjective views of analysts, since GARCH models capture many characteristics of stock returns. The results from the GARCH model are used as inputs in the BL to establish an optimal global portfolio allocation on a rolling basis. Similar methodology is used in Palomba (2006) and Beach, Orlov (2007).

GARCH models are used to estimate the variance of the error terms as a function of past values and additionally of its own past variance (Kroll, 2009). The most widely used GARCH specification asserts that the best predictor of the variance in the next period is a weighted average of the long-run average variance, the variance predicted for this period, and the new information in this period that is captured by the most recent squared residual (Engle, 2002). Using time dependent variance and covariance will enable the model to capture clustering effects in the data. It has been shown that variance in financial markets is high during certain periods and low during other (Brooks, 2008). When the variances change over time it means that the time series has heteroscedasticity, it has a changing volatility. Using GARCH also allows mean reverting i.e. if there is a long term means periods of high volatility mean

reverting will decrease volatility over time and periods with low volatility will increase over time (Brooks, 2008).

Although GARCH(1,1) is able to describe the volatility clustering in and other issues in returns, such as excess kurtosis, the standard GARCH model does not capture other important properties of volatility. Thus, an extension of basic GARCH model, EGARCH, proposed by Nelson (1991) is used instead in the thesis. We arecomputing parameter estimates over a rolling window of a fixed size through the sample. The historical data is split into the estination sample of 140 observations (monthly data) and a prediction sample of 40 observations (monthly data). Then we use the estimation sample and 3-step ahead predictions are made for the prediction sample. In our case we have 14 predictions, since we make an assumption that the excess returns do not change within 3 month period and predict every third month up to February 2011.

In general, any GARCH model can be represented by two equations—one for the conditional mean and the other for the conditional variance. The mean equation of the EGARCH (1,1) model can be represented as:

$$y_t = \mu + \delta\hat{\sigma}_t^2 + \varepsilon_t \quad (10)$$

The variance equation extended to include factors is:

$$log\sigma_t^2 = \omega + \beta log\sigma_{t-1}^2 + \alpha\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + \gamma\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \varphi_1 DV + \varphi_2 SP500 + \varphi_3 INFL +$$
$$\varphi_4 OIL\_PRICE \, (11)$$

DV is the dummy variable for the crisis period, SP500 is the excess return on the SP500 index, INFL is the EU inflation and OIL_PRICE is the price of oil for the correspondent period. The choice of the factors is described in the next section.

4. Data Collection and Description

Previous part of the research provides a detailed description of the research method exploited. Chapter 4 gives a comprehensive outline of the data collection and data processing.

The aim of this paper is to test empirically the effect of different estimation methods of variance covariance matrix Ω. This is achieved by performing a quantitative analysis on index excess returns from 1996 to 2011. Theoretical background was presented and previous studies were consulted. Lund University database is used to search and collect data on

relevant articles for theoretical background. Datastream and MSCI Barra Database are the main sources of data for the research. Finally, from the European Central Bank we are able to retrieve data on risk free rate for the period of interest.

The reference point of every asset allocation process is to identify the universe of the investment. Literature suggests numerical suggestion about how to classify assets into the portfolio, some of them we used while deciding on data to be tested; assets should be homogenous, mutually exclusive, diversifying. The paper uses country specific indices, including the universe of the investment, namely MSCI IsharesIndex for 14 countries retrieved from Morgan Stanley Capital International (MSCI) database. The index provides investment results that correspond generally to the price and yield performance, before fees and expenses, of publicly traded securities in the particular market.

Below in Table 1 we provide a summary statistics for the country-specific returns. In the first column we have the list of all countries. As seen from the table, the mean excess returns are in range between 0.002 and 0.009. The highest return (in column maximum) is observed for Finland, while minimum for Austria. All countries reject the normality based on Jarque-Bera statistic. Negative skewness is observed for all markets. Excess kurtosis is indicated also for all of the return series.

Table1. Summary statistics for the data of excess returns.

Country	Mean	Median	Maximum	Minimum	SD	Skewness	Kurtosis	Jarque-Bera	Probability
Austria	0.003	0.011	0.220	-0.467	0.081	-1.695	10.949	563.232	0.000
Belgium	0.002	0.010	0.162	-0.455	0.071	-2.228	13.387	963.463	0.000
Denmark	0.009	0.020	0.168	-0.297	0.061	-1.196	6.872	156.217	0.000
Finland	0.008	0.011	0.280	-0.382	0.101	-0.498	4.345	21.128	0.000
France	0.005	0.010	0.142	-0.254	0.062	-0.808	4.503	36.713	0.000
Germany	0.006	0.014	0.210	-0.279	0.072	-0.882	5.142	58.092	0.000
Italy	0.003	0.005	0.179	-0.270	0.070	-0.512	4.038	16.047	0.000
Netherlands	0.003	0.009	0.134	-0.290	0.064	-1.175	5.786	100.226	0.000
Norway	0.006	0.014	0.170	-0.406	0.085	-1.503	8.070	262.056	0.000
Portugal	0.003	0.009	0.151	-0.304	0.067	-0.864	5.084	55.279	0.000
Spain	0.007	0.013	0.194	-0.295	0.073	-0.893	5.243	62.009	0.000

Sweden	0.008	0.014	0.205	-0.310	0.080	-0.712	4.727	37.780	0.000
Switzerland	0.005	0.008	0.135	-0.171	0.050	-0.650	3.966	19.773	0.000
UK	0.003	0.007	0.124	-0.212	0.048	-0.733	5.210	53.038	0.000

Note: the data covers 180 months, from 1986 to 2011. The values represent a logarithm of excess returns with a risk free rate taken as Rf of 5%.

Based on the CAPM theory, the stock returns have to be compared to the returns received from the benchmark portfolio, which is MSCI Europe Index in our case. It is a market capitalization weighted index that is designed to measure the equity market performance of the developed countries in Europe, such as consists of the following 16 developed market country indices: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. Ireland and Greece have been excluded from our research.

We use 15 years of monthly data from March 1986 to February 2011 (180 months). The risk free rate is chosen as the rate of the German government long-term bond.

Since we look from a perspective of a German investor and a universe of returns was a European market, thus we consider German long-term bond as a good indicator of a risk free rate.

The EGARCH(1,1) model described in previous section included four factors, such as inflation, oil price, SP500 index, and a dummy variable which gives a value of 1 to the period from July, 2007 to the end of the sample, and 0 otherwise. Recent works done by Brunnermeier (2008) and Cheung et al. (2010) report on two dates of the interest period. The latter authors use July 2007 as the starting point of the global financial crisis. They argue that from that time and on the subprime crisis got more serious as AAA CDS (credit default swaps) got downgraded (Cheung et al., 2010). Though the study done by Brunnermeier (2008) reports another date from February 2007 as the starting point of the subprime mortgage default crisis (indicated by the drop of the ABX index (of CDS) backed by A, BBBB and BBB subprime mortgage). Cheung et al. 2010, however, argue that their alternative benchmark from July 2007 does not alter main results and conclusions. To capture the potential changes within the industries, this work will employ an earlier date from 1 February 2007.

The choice of the factors in the EGARCH model is related to the article by Chen, Roll and Ross (1986) who suggested several macroeconomic variables for the factor model. The original work suggests inflation, treasury-bill rate, long-term government bonds, industrial production, low-grade bonds, equally weighted equities, value-weighted equities, consumption and oil prices as observed economic variables that influence stock returns. Later, Burmeister et al. (2003) have defined similar factors, such as investor confidence, interest rates, inflation, real business activity and a market index. For this paper, we have taken the logarithms of each of the variables of our choice: inflation (EU Harmonized Indices of Consumer Prices (HICPs), oil price returns, SP500 index returns. The consideration for the model choice is based both on the theoretical background as well as various statistics (e.g., adjusted R-squared, Schwarz information criteria). We have also checked the statistics when we omit or change the order of the variables. The factors in the model are significant in most, but not all, regressions. However, as our goal is to forecast the conditional variance, we do not see occasional coefficient insignificance as a problem. Appendix/// presents summary of parameters

5. Empirical results

Returning to the purpose of the paper we try to test the difference in portfolio characteristics based on different omega estimation methods. In other words, we want to see omega sensitivity of portfolio composition in Black Litterman model. As mentioned before, Black-Litterman model possesses important advantages like inclusion of subjective views into the equilibrium model, and producing more reasonable results for investors (Chincarini and Daehwan (2009). However, past research has shown that there are various approaches to simulate the inputs for the model, thus we cannot conclude the general viewpoint without comparison and practical testing. The purpose is to see the effect of the omega (Ω), which is variance matrix of views, on the formulation of portfolios in Black-Litterman model. Mathematically, we want to see how weight vectors of the optimal portfolios produced with the change of methodology used to calculate variances of views.

Below is the summary of the omega estimation methodology employed:
1. We estimate a variance covariance matrix of views based on the historical data. For that purpose, as mentioned in the methodology section, we use formula by Meucci (2005). We use rolling window approach to construct variance covariance matrix for

14 periods beginning from the 140th observation and ending at 180th. We estimate every third period, assuming the results are stable for the next two.

2. We use EGARCH model to obtain forecasted variances for each period mentioned before and use variances from EGARCH to construct omega in the way suggested by Orlov et al. (2007).

Additionally we use different levels of tau which expresses the confidence in prior distribution to see if the result changes with the change of confidence. Thus, we test the result for the tau is equal to 0.005 and 0.01. The Table 3 shows results obtained from the 14 period testing.

Table 3. Weight comparison from the model.

Countries	**Weigths:** *obtained from BL employed EGARCH input*	**Weights:** *obtained from BL employed variances proportional to prior*	**Weights:** *obtained from equilibrium implied returns*	**Weigths:** *obtained from BL employed EGARCH input*	**Weights:** *obtained from BL employed variances proportional to prior*
	tau=0.005	tau=0.005		tau=0.01	tau=0.01
	Average	Average		Average	Average
Austria	0.39	0.38	0.38	0.41	0.28
Belgium	0.44	0.43	0.42	0.45	0.50
Denmark	0.14	0.14	0.13	0.14	-0.19
Finland	0.26	0.25	0.25	0.27	0.27
France	-0.3	-0.29	-0.29	-0.31	-0.12
Norwey	0.7	0.69	0.69	0.7	0.55
Portugal	-0.43	-0.42	-0.42	-0.45	-0.34
Spain	0.63	0.61	0.61	0.65	0.88
Sweden	0.69	0.67	0.67	0.72	0.15
Switzerland	-0.56	-0.54	-0.54	-0.58	-0.31
UK	-2.17	-2.09	-2.08	-2.24	-1.69
Netherlands	0.44	0.42	0.43	0.44	0.35
Germany	0.82	0.79	0.79	0.85	0.67
Italy	-0.04	-0.04	-0.04	-0.04	0.00

Table 3. above summarizes obtained weights for assets. Average weights as seen from the table are not significantly reacting to the change in omega estimation. The first column represents the results where we use variances forecasted from EGARCH model. The variances of views vary between 0.005 and 0.01, thus showing important information about the level of confidence investor is able to put on the variance covariance matrix (Ω) of the views (Idzorek, 2005). In our case the volatility is small, thus influence on the total omega and posterior information is small as well. We can conclude that the variances obtained from the model fit the historical variances well, i.e. intuitively the similar results in both ways of omega estimation are clear. However, one more explanation could be viable in our case. The

assets of the sample countries that are influenced by the active views probably are not significant in the total portfolio. Thus, the small change in their variances doesn't create large influence for the final result.

The other phenomenon is that the results obtained from the Black Litterman model are almost identical to the reverse optimization results (implied returns and weight vector of implied returns). Especially when the confidence level of the views is low the methodology for input does not matter too much, and the result is similar. We can see average example for the weight vectors from BL and reverse optimization models in Chart1.

Chart1. BL and Reverse optimization weight vectors.
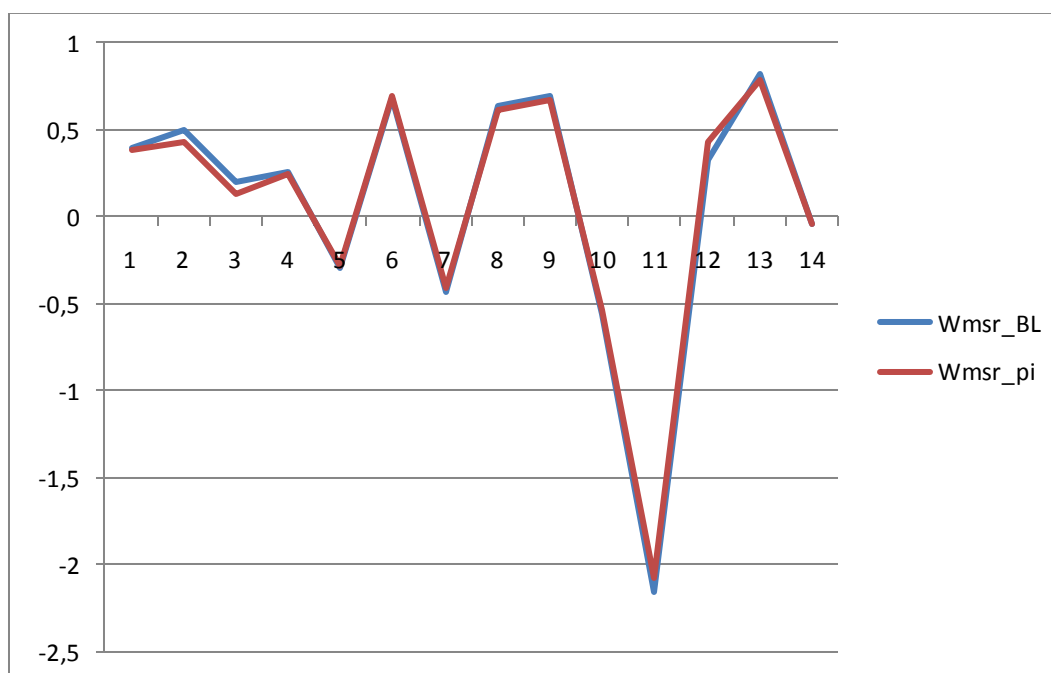

Chart2 below shows the results for the same method, but describing the portfolio average excess returns obtained from the 14 period observations. We conclude that as the omega estimation methodology does not affect weight vectors in average significantly, then the excess returns are similar too. However, when we compare not individual assets but rather the total portfolio returns, we can distinguish outperformance of the Black Litterman model with lower confidence levels. The criteria for outperformance here is the less extreme and less volatile results. Since the higher the confidence level, usually the risk is higher, but investors would prefer lower risks at that point. Reverse optimization implied returns show higher volatility, thus they are more extreme and riskier (higher variances).

Overall the data tested in this paper shows no significant relationship between posterior design of the model and the choice of estimation method of the variances of the applied views by investors.

**Chart2.** Excess Returns Resulted from Black Litterman and Reverse Optimization.
MSCI shares, (1996-2011)
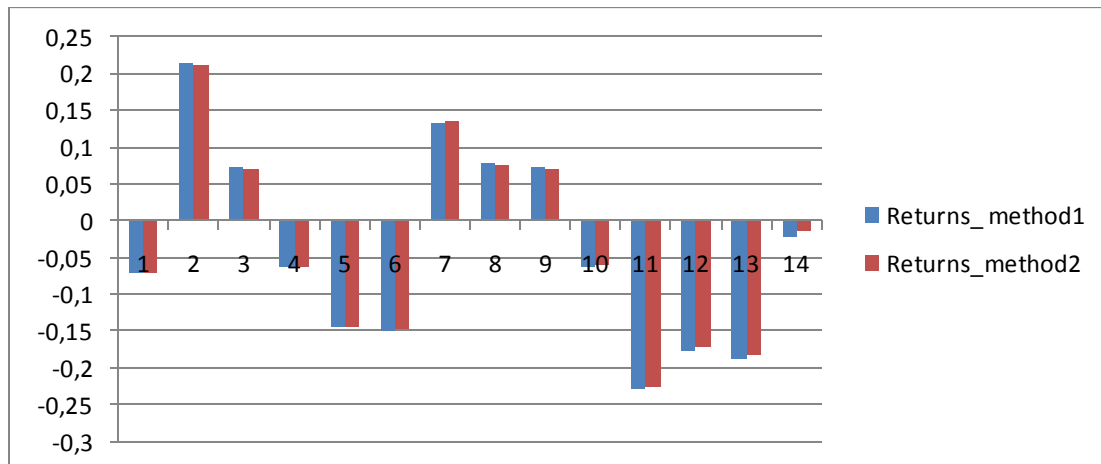
Note: Returns_method1 expresses the portfolio excess returns during 14 periods where we use the variance matrix of views (omega) estimation based on the variances estimated from EGARCH model .Retuns_method2 presents the portfolio excess returns from the 14 period estimation based on the variance matrix of views (omega) calculated proportionally to prior variances.

From the Chart 3 we observe that the portfolio returns obtained from BL model no matter what method is used are smoother in terms of tolerance in variance and generate higher excess returns compared to the neutral equilibrium returns.

Thus, even though the BL model is sometimes difficult to test empirically because of the subjective inputs, it is clear that some of the inputs are more influential than others; play greater role in formulation of final weights.

Chart3. Portfolio returns: comparison to implied returns.
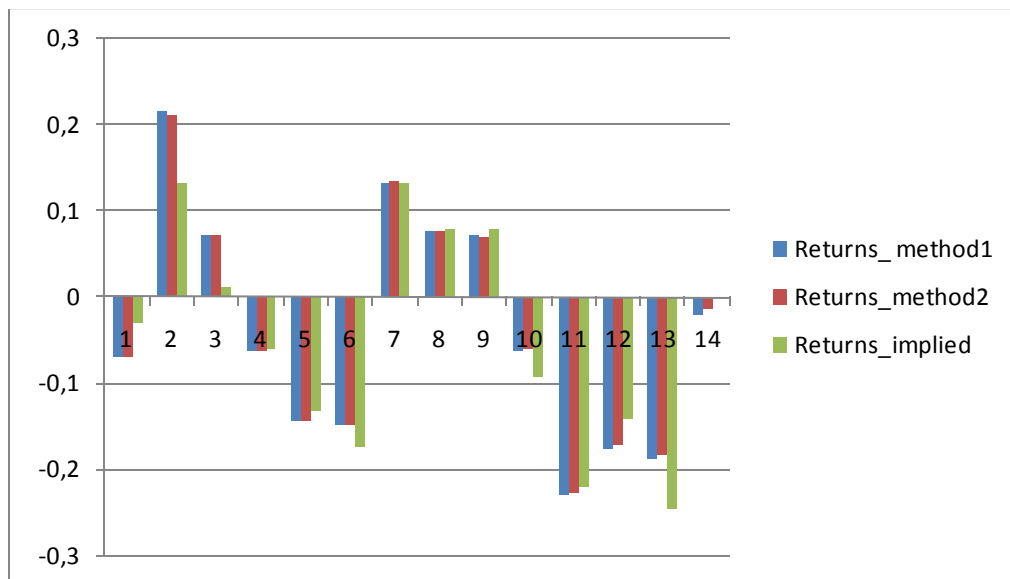
Empirical results from the presented paper show that the difficult ways of input estimation and approach diversification can improve the active portfolio management tactics, however they are not so significantly influential in overall portfolio results. The distribution of prior returns is probably the most important input in the model, however; the assumption could be tested in further research.

6. Conclusion and further discussions

The paper investigated sensitivity of the weight vector obtained from the Black Litterman model as response to the different methods of omega (variance matrix of views) estimation. The result shows that the weights of optimal portfolio are not significantly sensitive to the variances of the views. For the tau being 0.005 we estimated omega in two ways; proportional to the variance covariance matrix of the historical excess returns (both Meucci (2006) and He and Litterman (2002) methods), and omega which employed variances obtained from the EGARCH model estimation. We used rolling window approach and estimated results for the 14 periods (quarterly frequency) ahead. The result shows non-significant reaction to the omega estimation. Secondly, we compare the results to the implied returns. The assets show very similar tails, however; portfolio returns from the BL model outperforms the equilibrium implied return portfolio outcomes. The BL model in all types of estimation method is less extreme and has more intuitive and tolerate weights. The insensitivity to the omega estimation of the weight vectors can be explained in several ways. Firstly, the less investor is confident in his/her views the less the estimation methods for the views make difference. Secondly, if the EGARCH or other employed empirical model fits the data well and captures all information needed for variances, the result will be very similar to the implied returns and similar to the historical variances. Thus, without tau=1 which is very extreme approach the results for different methods will hardly differ.

Further research will be interesting to test the sensitivity of the weight vector to the prior specification of the equilibrium returns. As tested but not presented during this paper, choice of prior model can significantly change the eventual result and design of optimal portfolios.

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Appendix1.

Parameters indicated during the research:

Σ- variance covariance matrix of historical excess returns

V-variance of the parameters using factor model (in our case variances obtained from EGARCH model)

Ω- variance covariance matrix of views

Π- vector of implied returns obtained from reverse optimization

τ- in this paper defined as confidence level in prior distribution of returns

P- is the kxn matrix of views indications (1 or 0 in our case, because we use only absolute views)

Q-kx1 matrix of views (absolute values)

λ- Investor's risk-aversion level