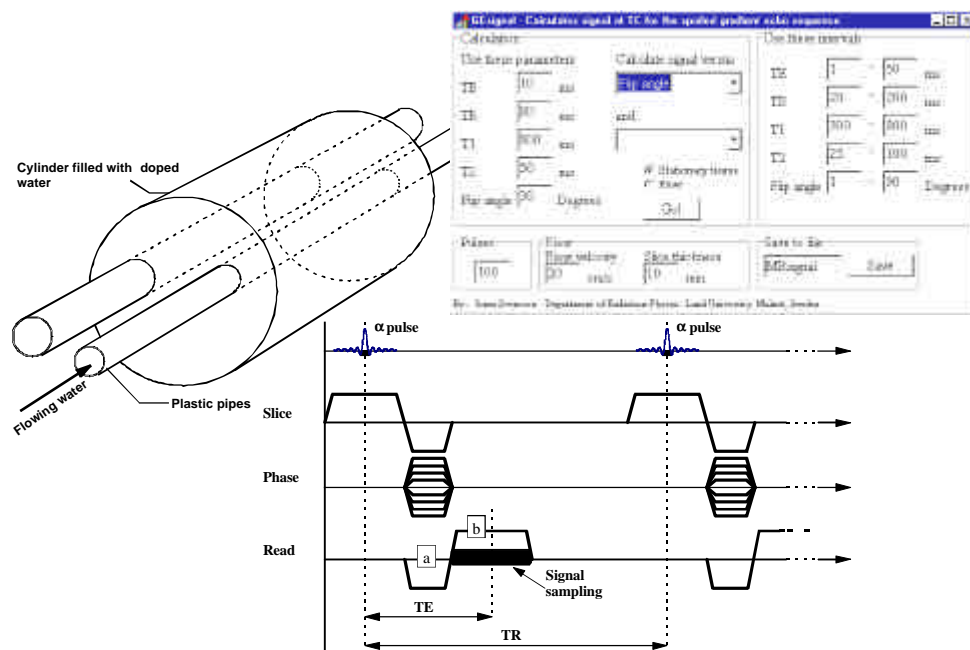


# -MR Flow Imaging-

## A Study of Flow Effects on the Gradient Echo Sequence



Thesis for the degree of Master of Science  
Jonas Svensson

Department of Radiation Physics, Lund University, Lund  
Department of Radiation Physics, UMAS, Malmö

## **Abstract**

Effects of flow are a fundamental part of magnetic resonance imaging (MRI). Since almost every part of the body contains some kind of flow, the effects are seen in generally all MR images. The presence of flow in an image slice may be a vast source of artefacts but the effects of flow can also be used for diagnostic purposes, for example in magnetic resonance angiography (MRA). In MRA, flow effects are utilised in order to obtain vascular information from a patient. MRA has become an effective diagnostic tool capable of obtaining high resolution morphological and physiological vascular information throughout the body.

The aim of this thesis has been to study the origin of flow effects in MRI, and to investigate the behaviour of MR signal when flow is present. Of special interest was effects in the spoiled gradient echo pulse sequence, since it is the most common sequence used in MRA.

The signal equations for the spoiled gradient echo sequence were used to study signal behaviour when flow is present and sequence and tissue parameters are changed. Calculations were performed assuming both stationary and flowing tissue. A program for MS Windows 95™ called GEsignal was developed especially for this work. It was written in a visual object oriented program language (Borland Delphi). GEsignal calculates the signal of the spoiled gradient echo sequence. It can be used for both stationary tissue and for flow perpendicular to an image slice.

Phase effects on the magnetisation vector due to flow were also studied. A spreadsheet (MS Excel) was set up to illustrate these effects. The spreadsheet can be used to vary up to four gradient lobes in time and amplitude strength. It is updated in real time to show phase effects due to flow in the direction of the gradients.

Simulations and flow measurements were performed to verify the calculations. The Simulations were carried out using "The MRI Simulator", a PC based program that simulates a real MR scanner (Petersson et al, 1993). The flow measurements were carried out at a Siemens Magnetom Vision 1.5T MR scanner.

Calculations, simulations and measurements agreed fairly well qualitatively. Quantitatively, calculations and simulations agreed well. The signal difference was a few percent except for the combination of high flip angles and high flow velocity, where the difference may be as high as 10 to 15%.

Measured data differed rather much from calculated results. However, if the calculated signal was modified to take into account a gaussian flip angle variation over the image slice, the results agreed very well with measured signal. The signal difference between calculations and measurements was reduced from tens of percent to a few percent by introducing a gaussian flip angle distribution.

Both GEsignal and the spreadsheet are useful tools to study flow effects qualitatively. If a quantitative study is to be performed, the method used in GEsignal appears too restricted. However, the result may be modified to agree well with measured data. The spreadsheet finds its best application as a tool to understand the concept of gradient moment nulling.

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# 1. Introduction

This thesis is the result of a graduate project for a degree of master of science in radiation physics. The project has been carried out at the Department of Radiation Physics, Lund University, Malmö University Hospital, during 1996-1997.

Flow effects are a fundamental part of magnetic resonance imaging (MRI), and therefore physiologic flow plays an important role when performing imaging. Since almost every part of the body contain some kind of flow, the effects are seen in almost all MR images. Presence of flow in an image slice may be a severe source of artefacts but the effects of flow can also be used for diagnostic purposes, for example in magnetic resonance angiography (MRA).

In MRA, flow effects are utilised in order to obtain vascular information from a patient. Since MRA was first suggested, it has become an effective diagnostic tool capable of obtaining high resolution morphological and physiological vascular information throughout the body.

There are two main techniques of MRA: The Time Of Flight (TOF) technique and the Phase Contrast (PC) technique. Both of them are implemented using gradient echo pulse sequences.

TOF (section 3.6.1) make use of the fact that spins flowing through an image slice experience fewer RF pulses than the stationary tissue. The more RF pulses a spin isocromat has experienced the lower signal it will yield. Thus the flowing spins leave a higher signal than the stationary tissue. The phenomenon is referred to as inflow or washin and is discussed in more detail in section 3.4.1. Calculations concerning inflow effects are presented in section 4.1.

PC techniques (Section 3.6.2) make use of the phase information of the signal. For stationary spins the transverse magnetisation is totally rephased at the echo time (TE) of a gradient echo sequence. Spins in motion, on the other hand, will have a transverse magnetisation somewhat out of phase at TE. For example, spins flowing with constant velocity in the direction of a bipolar magnetic field gradient will have accumulated a phase proportional to their flow velocity at TE. This difference in transverse magnetisation phase between stationary and flowing tissue can be used to create an image. Such an image will ideally contain signal only in areas with flow and have a zero signal elsewhere.

In this thesis the origin of the flow effects in the spoiled gradient echo sequence was investigated. Calculations were performed to illustrate TOF effects, using the signal equations for the spoiled gradient echo sequence. These equations are simplified solutions to Bloch's equation. To apply the signal equations to flowing spins as well as to stationary tissue, a program for MS Windows 95™, called GESignal, was developed during the project. The influence of flow on transverse magnetisation phase was also investigated. Calculations were carried out showing both effects on phase caused by flow and how these may be corrected for. Calculations on inflow and phase effects were combined to obtain Phase contrast signal.

Simulations were carried out using "The MRI Simulator" a computer program for MS Windows 95™. It implements the k-space formalism and a FFT algorithm to generate NMR signal and images. The obtained data were compared with results from calculations and measurements.

Flow effects in imaging were also studied experimentally. Measurements were carried out using a Siemens Magnetom Vision 1.5T MR scanner and a flow phantom.

Both inflow effects and phase contrast techniques were investigated and the results were compared with calculations and simulations.

## **2. Aim**

The aim of this thesis has been to study the origin of flow effects in MRI, and to investigate the behaviour of MR signal when flow is present. Of special interest was effects in the spoiled gradient echo sequence, since it is the most common sequence used in MRA.

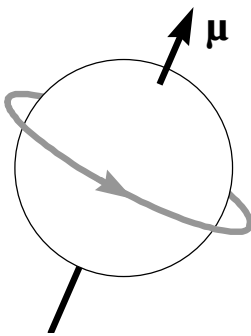
It was also of interest to investigate whether or not the simple signal equations for the spoiled gradient echo sequence are adequate to describe signal behaviour when flow is present. The intention was to compare the results from calculations and measurements.

The liability of the simulation program “The MRI Simulator” developed at the department by J. S. Petersson was also intended to be tested. Simulations were to be performed and the results should be compared with both calculations and real measurements.

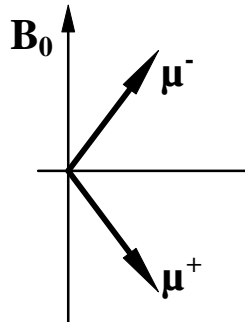
### 3. Theory

#### 3.1 Basics of MRI

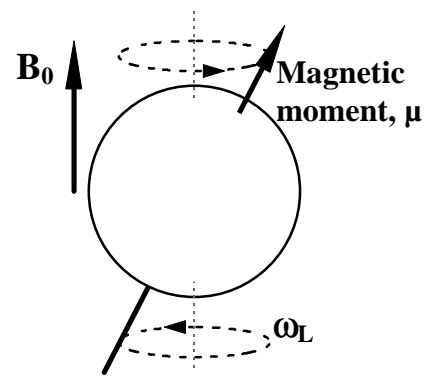
MRI is made possible through the fact that all nuclei with either an uneven mass number or an uneven atomic number possess an angular momentum and have a characteristic nonzero spin quantum number. In classical physics, the spin can be thought of as the nucleus being a spinning sphere (Figure 1). Since it is charged, it will induce a magnetic field with an axis coincident with the axis of rotation. The magnetic moment  $\mu$  is a vector with a direction and a magnitude.



**Figure 1:** Classical description of spin. A spinning nucleus will bear angular momentum. Since the nucleus is charged, it will possess a magnetic moment,  $\mu$ .



**Figure 2:** The two possible discrete orientations for a  $^1\text{H}$ -nucleus in a static magnetic field  $B_0$ .



**Figure 3** frequency of a spinning nucleus about the axis of a static magnetic field,  $B_0$ .

Normally the magnetic moments of a group of spins will be randomly oriented. However, if a static magnetic field ( $B_0$ ) is applied, the spins tend to assume discrete orientations. In MR imaging the nucleus of the hydrogen atom ( $^1\text{H}$ ) is the explicit nucleus of interest. The  $^1\text{H}$  nucleus is simply a proton and the number of discrete orientations it can undertake is two. One with (parallel,  $\mu^-$ ) and one against (antiparallel,  $\mu^+$ ) the applied magnetic field (Figure 2). The parallel orientation corresponds to a lower energy state than the antiparallel state.

The alignment of the magnetic moment with the applied magnetic field is not totally parallel. The nuclei will rotate around the axis of the applied field with a precise frequency called the Larmor frequency,  $\omega_L$  (Figure 3).  $\omega_L$  is determined from the relation

$$\omega_L = \gamma B_0$$

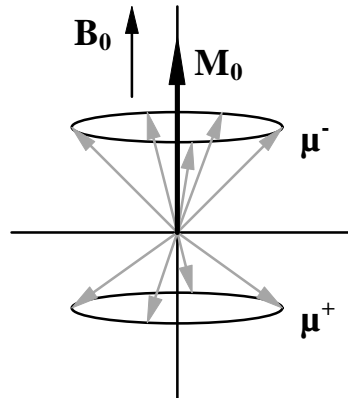
Where  $\gamma$  is the gyromagnetic ratio, a constant of proportionality dependent on the specific nucleus.

For a large group of nuclei in a magnetic field, the distribution of spins in the antiparallel and the parallel states will almost cancel out. This is due to the fact that the energy difference between the two energy states is small compared with the thermal energy at room temperature. However, there is a slight predominance to the parallel orientation. The small imbalance creates a net magnetisation vector,  $M_0$  (Eq. 3.1-1), completely parallel to the applied magnetic field (Figure 4).



$$M_0 = \sum \mu^- - \sum \mu^+$$

Eq. 3.1-1

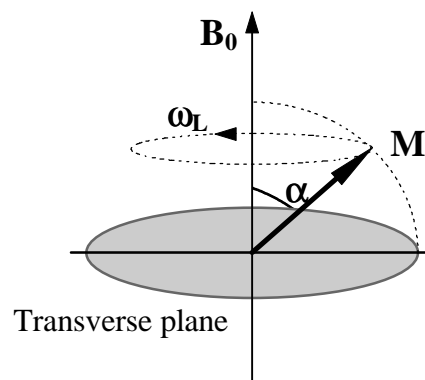


**Figure 4:** The magnetic moments of individual nuclei yield a total net magnetisation,  $M_0$ , in the direction of the magnetic field.

Even though the magnetic moment,  $\mu$ , of a single nucleus can be divided in one component parallel to  $B_0$  and one perpendicular to  $B_0$ , there will be no component of  $M$  perpendicular to the magnetic field. This is the result of the different nuclei having a randomised precession distribution.

A nucleus in the  $\mu^-$  state can be excited to the  $\mu^+$  state if irradiated with electromagnetic radiation having a frequency corresponding to the Larmor frequency. The required frequency is typically in the order of tens of MHz and in MRI the term radio frequency (RF) pulse is commonly used.

In a group of nuclei being irradiated, the total net magnetisation vector,  $M$ , will rotate an angle  $\alpha$  around the axis of the direction of  $B_0$  (Figure 5), and precess at the Larmor frequency.



**Figure 5:** The net magnetisation vector,  $M$ , under influence of an RF pulse.  $M$  rotates an angle  $\alpha$  around the axis of the  $B_0$  field and precesses at the Larmor frequency.

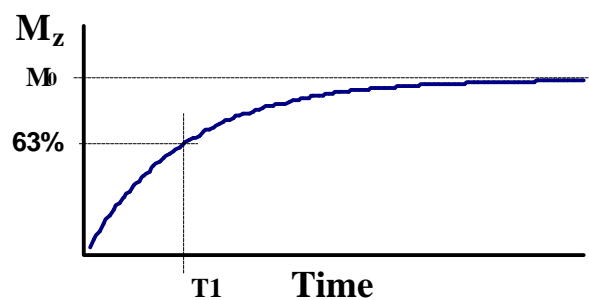
At this point  $M$  can be thought of as being the vector sum of two vector components. One along the  $B_0$  axis called  $M_z$  and one in the transverse plane,  $M_{xy}$ ,

that is

$$\bar{M} = \bar{M}_{xy} + \bar{M}_z$$

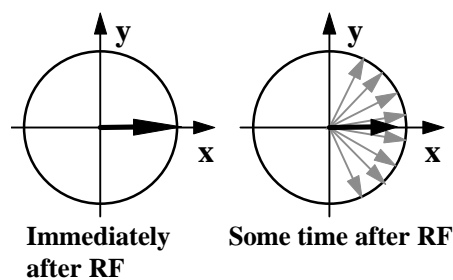
The rotation angle  $\alpha$  of  $M$  is a function of the RF pulse duration and amplitude. In MRI  $\alpha$  is called flip angle.

When the pulse is turned off, the excited spins will deexcite to the lower energy state. The redundant energy will be emitted to the nuclei surroundings. Deexcitation is determined by probability and the process is called T1-relaxation. The result of a randomised deexcitation is a growth of  $M_z$  (Figure 6). T1 is a time constant corresponding to the time it takes for  $M_z$  to grow to 63% of its original size  $M_0$ . Typical values of T1 in tissue are in the range of 300 - 3000 ms.

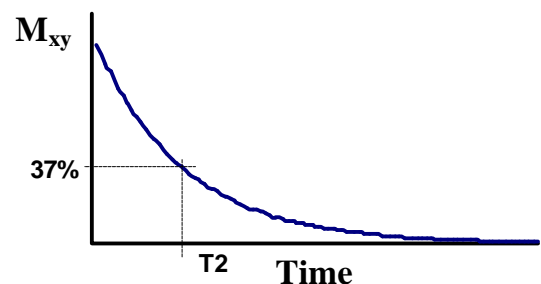


**Figure 6: Relaxation of  $M_z$  vector after RF-pulse excitation. Relaxation is due to deexcitation of nuclei to the lower energy state.**

In any tissue, the interactions between individual spins create random local variations in the static magnetic field. These variations lead to fluctuations in the precession frequency of the individual nuclei. The variation in precession frequency dephases the spins (Figure 7). This leads to a decrease of  $M_{xy}$  with time (Figure 8) called T2-relaxation or spin-spin relaxation. T2 is the time for  $M_{xy}$  to fall to 37% of its maximum (initial) value.



**Figure 7: Dephasing of spin magnetisation after a RF pulse. The spreadout yields a decrease of  $M_{xy}$  with time.**



**Figure 8: T2 relaxation of  $M_{xy}$  vector after RF pulse. Relaxation is due to spread out of spin magnetisation in the transverse plane**

It is impossible to create a perfectly homogenous magnetic field in an MR scanner. The external inhomogeneities increase the rate of dephasing. When referring to T2 relaxation and meaning the total effect of true T2 relaxation, effects of external field inhomogeneities etc, the denotation  $T2^*$  is often used. Typical T2-values in

tissue are 50 - 200 ms, while  $T2^*$  often is much shorter.  $T2^*$  is related to both the tissue and the imaging device used, while true T2 just depends on the tissue. If the external field inhomogeneities and the true T2 are known,  $T2^*$  can be calculated, (taking into account the inhomogeneities) using Eq. 3.1-2 (IPSM, 1985).

$$\frac{1}{T2^*} = \frac{1}{T2} + g \frac{|\Delta B_0|}{2} \quad \text{Eq. 3.1-2}$$

## 3.2 Pulse sequences

An MR pulse sequence is a set of RF and gradient magnetic field pulses, used in conjunction with signal sampling to produce an MR image. There is a wide range of pulse sequence types available for imaging, for example spin echo sequences, gradient echo sequences and inversion recovery sequences.

The production of an image with spatial resolution in two dimensions, requires that

- signal is sampled at a number of discrete points, typically 128, 256 or 512.
- The pulse sequence is repeated, typically 128, 256 or 512 times.

The time to complete one cycle of a sequence is called Time of Repetition (TR). The time from the first RF pulse in a cycle to the centre of the corresponding signal sampling interval is called the Time of the Echo (TE).

### 3.2.1 Spin echo sequences

The most common sequence in clinical MRI is the spin echo sequence. It was first implemented for use in MR spectroscopy as early as in 1950. The sequence starts with an RF pulse flipping the magnetisation vector an angle  $90^\circ$ . Immediately after the pulse, the transverse magnetisation,  $M_{xy}$ , starts to decrease due to dephasing of spins (Figure 7). After a time corresponding to  $TE/2$  another RF pulse is applied, this time a  $180^\circ$  pulse. The  $180^\circ$  pulse will flip the magnetisation of individual nuclei in a mirror-like way around the x axis. The dephasing process before the  $180^\circ$  pulse is thus turned into a rephasing process.  $M_{xy}$  will grow to a maximum value and thereafter decay. The recovery of  $M_{xy}$  is called a spin echo.

By applying a  $180^\circ$  pulse, the dephasing effects introduced by external field inhomogeneities are reversed, leaving only effects of true T2 relaxation.

### 3.2.2 Gradient echo sequences

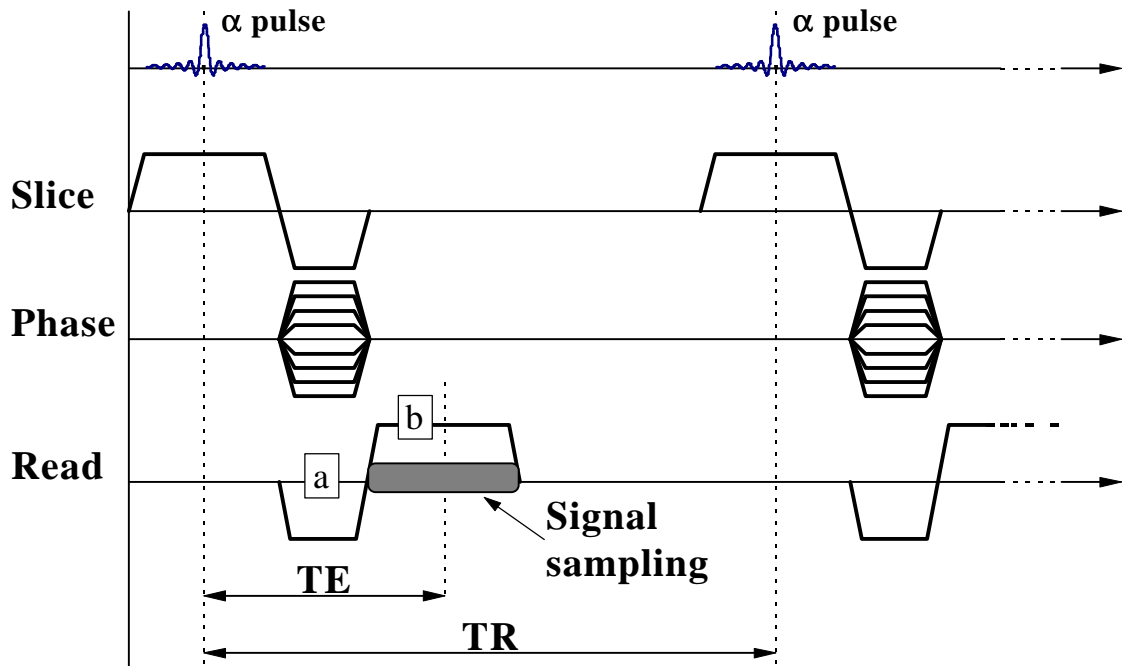
A spin echo sequence has a fairly long TR due to the use of a  $180^\circ$  RF pulse and the time needed for longitudinal magnetisation to grow back. A long TR leads to long acquisition times and thus there is a wish for sequences with a shorter TR. The use of flip angles less than  $90^\circ$  will leave some remaining longitudinal magnetisation after the pulse. The time needed for T1 relaxation is thereby reduced and TR can be shortened. This technique is implemented in gradient echo sequences.

The basic idea of gradient echo sequences is to obtain the echo only with the help of gradient rephasers, omitting the  $180$  pulse used in spin echo sequences. The sequence starts with an RF pulse flipping the magnetisation vector an angle  $\alpha$ , where  $0 < \alpha < 90^\circ$  (Figure 9). When flip angles of less than  $90^\circ$  are used, the  $180^\circ$  rephasing

pulse is omitted, since such a pulse would not only affect the  $M_{xy}$ -magnetisation but also the remaining  $M_z$ -magnetisation.

An initial negative lobe of the frequency encoding gradient (“Read” in Figure 9) dephases transverse magnetisation (a). The negative lobe turns to a positive lobe, equal in size (b), that rephases the transverse magnetisation and forms a gradient echo at time TE, the echo time. A gradient pair like this is called a bipolar gradient.

The slice selecting gradient is not allowed to leave any phase information, thus the positive slice selecting gradient is also compensated by a corresponding negative lobe.



**Figure 9: A Gradient echo sequence. The uppermost line indicates when the RF pulse is applied. Line two, “slice”, shows the slice selecting gradients. The phase encoding gradient is seen in line three, “Phase”; its amplitude is changed as the sequence is repeated. In line four, “read”, the gradient de/re-phasing is seen, these gradient lobes form a gradient echo at TE. Signal is collected during the rephasing lobe of the echo. At the bottom, the concepts of TE and TR is presented.**

In spin echo techniques most of the time in a TR interval is used to allow for sufficient recovery of the longitudinal magnetisation  $M_z$ . In gradient echo techniques, flip angles less than  $90^\circ$  are used. Such a flip angle leaves some portion of  $M_z$  remaining after the pulse. This obviates the need for a long TR to provide sufficient T1 relaxation and the TR therefore can be shortened dramatically.

If a short TR and a large flip angle is used, there will be magnetisation left in the transverse plane when the following RF pulse is emitted. This results in artefacts in the final image. To avoid these artefacts, a technique called spoiling is often used. Spoiling is a way to destroy (spoil) the magnetisation remaining in the transverse plane after signal sampling and before the next RF pulse (Stark et al, 1992). There are two main ways of spoiling transverse magnetisation: RF spoiling and gradient spoiling.

- If RF spoiling is used, the RF pulse excites the spins with a different phase increment each time. That is,  $M_{xy}$  is placed in different directions for each new

RF pulse.

- Gradient spoiling is accomplished through a large dephasing gradient at the end of the sequence, before the next RF pulse. The gradient will spread out the phase of individual spins.

The two techniques can be used either one at the time or simultaneously.

The absence of the 180° pulse makes gradient echo sequences sensitive to static magnetic field inhomogeneities, this is a great disadvantage. As a result,  $M_{xy}$  will decrease with a time constant  $T2^*$  rather than with true T2.  $T2^*$  is much shorter than T2 and forces the transverse magnetisation to decay faster than it would do with T2-relaxation. This more rapid relaxation yields signal loss but by using short echo times, the problem may be partly solved.

Signal evolution during the gradient echo sequence can be calculated using Bloch's equation. With proper approximations (no magnetic field inhomogeneities, no influence of gradients and assuming the RF pulses being delta functions) Bloch's equation can be solved analytically for the gradient echo sequence. The transverse as well as the longitudinal magnetisation,  $M_{xy}(t)$  and  $M_z(t)$ , can be calculated for any time  $t$  ( $0 < t < TR$ , where  $t = 0$  is the time when the RF pulse is delivered).

Immediately after an RF pulse, Bloch's equation yields:

$$M_{xy,after} = M_{xy,before} \cos(\alpha) + M_{z,before} \sin(\alpha) \quad \text{Eq. 3.2-1}$$

$$M_{z,after} = -M_{xy,before} \sin(\alpha) + M_{z,before} \cos(\alpha) \quad \text{Eq. 3.2-2}$$

$M_{xy,before}$  is the transverse magnetisation just before an RF pulse and  $M_{z,before}$  is the corresponding longitudinal magnetisation. In the same way  $M_{xy,after}$  and  $M_{z,after}$  is the magnetisation right after an RF pulse. The equations describe the rotation of the magnetisation vector caused by the RF pulse (Figure 5).

A time  $t$  after the RF pulse, the magnetisation is

$$M_y(t) = M_{xy,after} e^{-(t/T2)} \quad \text{Eq. 3.2-3}$$

$$M_z(t) = 1 + (M_{z,after} - 1) e^{-(t/T1)} \quad \text{Eq. 3.2-4}$$

These equations describe the relaxation of the magnetisation vector (Figure 6 and Figure 8).

In a spoiled gradient echo sequence, the transverse magnetisation is removed, after each echo and before the new RF pulse. This corresponds to setting  $M_{xy,before}$  to zero (if perfect spoiling is assumed) in Eq. 3.2-1 and Eq. 3.2-2 above. They simplify to

$$M_{xy,after} = M_{z,before} \sin(\alpha) \quad \text{Eq. 3.2-5}$$

$$M_{z,after} = M_{z,before} \cos(\alpha) \quad \text{Eq. 3.2-6}$$

As the pulse sequence is repeated, multiple RF pulses will be delivered. TR is most often short compared to T1, which leads to incomplete T1 relaxation between

pulses. The result will be a decrease in signal as the number of pulses increases. This decrease will continue until a state of equilibrium is reached. The state of equilibrium is called steady state and the signal at steady state can be calculated using Eq. 3.2-3 to Eq. 3.2-6. The result is (Stark et al, 1992)

$$S \propto \frac{\left(1 - e^{-\frac{TR}{T_1}}\right)}{1 - \cos(\alpha) e^{-\frac{TR}{T_1}}} \sin(\alpha) e^{-\frac{TE}{T_2^*}} \quad \text{Eq. 3.2-7}$$

At steady state, the gradient echo sequence will have an optimal flip angle, the Ernst angle  $\alpha_E$ , that yields the highest signal (Potchen E. James et al, 1993):

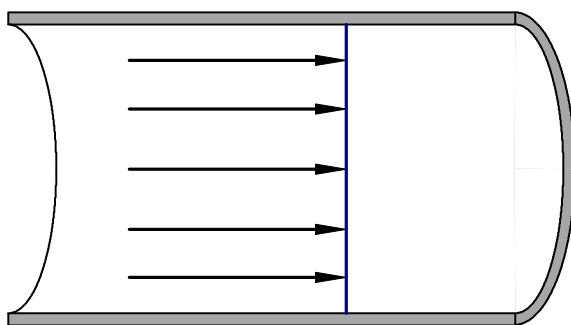
$$\cos(\alpha_E) = e^{-\frac{TR}{T_1}} \quad \text{Eq. 3.2-8}$$

### 3.3 Flow models

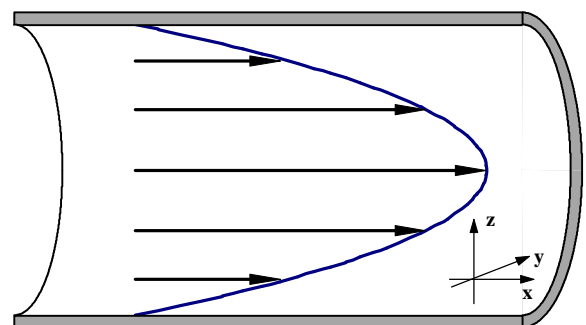
In the human body there are several levels of physiological flow present, for example, macroscopic flow (flowing blood in larger blood vessels), diffusion and perfusion (capillary flow) (Stark et al. 1992). However, this work is concerned only with the effects of macroscopic blood flow.

A blood vessel with blood flowing within it, may be approximated by a straight pipe containing a flowing viscous liquid. The liquid will then have some kind of velocity distribution over a cross section of the pipe.

The most simple velocity distribution to use is the so called plug flow model. It assumes that the velocity distribution inside the pipe is uniform, that is, the flow velocity is the same everywhere over a cross section of the pipe (Figure 10). This model is in most cases not very realistic but it is practical and easy to implement when performing calculations of flow effects in MRI.



**Figure 10: Plug flow model.** A liquid flowing in a straight pipe is assumed to have constant velocity over a cross section of the pipe. This model is in most cases not very realistic, but easy to implement when performing calculations of flow effects in MRI



**Figure 11: Laminar flow model.** If laminar flow is established in a pipe, the velocity distribution over a cross section of a pipe is parabolic. This model is realistic for a viscous liquid flowing in a straight pipe at not too high velocities

A more realistic model of a blood vessel is the laminar flow model. If flow in a

circular pipe can be assumed laminar, the radial flow velocity distribution will have the form of a parabola (Figure 11). For flow with constant velocity,  $v$ , in the direction of the pipe the laminar flow model can be written (with co-ordinate axes as in Figure 11 and the origin at the centre of the pipe)

$$v_x = v_x(y,z) = v_{\max} \left( 1 - \frac{y^2 + z^2}{R^2} \right) \quad y^2 + z^2 < R^2 \quad \text{Eq. 3.3-1}$$

$v_{\max} = v(0,0)$  is the maximum velocity (or twice the mean velocity since the flow velocity distribution is parabolic) and  $R$  is the radius of the pipe.

If a viscous liquid flowing in a straight circular pipe will have a laminar flow distribution or not, depends on parameters such as flow velocity, liquid viscosity and pipe diameter. For low flow velocities, the flow will indeed be laminar. However, as the velocity increases, the laminar flow gradually breaks down due to an, in space and time, randomised flow that adds to the original flow. The flow is said to become turbulent.

To describe the grade of turbulence, the Reynolds number ( $Re$ ) is introduced

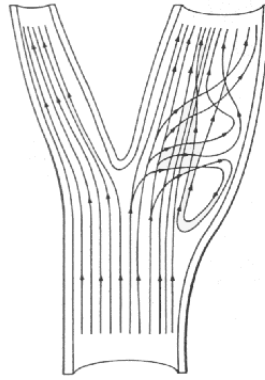
$$Re = \frac{\rho v d}{\mu} \quad \text{Eq. 3.3-2}$$

$\rho$  is the density of the liquid,  $v$  is the mean flow velocity,  $d$  is the diameter of the pipe and  $\mu$  is the liquid's viscosity.  $Re$  is dimensionless and for steady flow in a smooth-walled pipe, the flow will be laminar for  $Re$  up to approximately 2100 whereas for higher values of  $Re$ , flow will be more or less turbulent (Stark et al. 1992). At really high flow velocities the flow will be completely turbulent. The flow velocity distribution for completely turbulent flow looks almost like in the plug flow model (Figure 10).

If laminar flow is established, the flow velocity distribution is not a function of the fluid's density and viscosity (Eq. 3.3-1). Therefore, if the maximum (or the mean) velocity is known, and laminar flow can be assumed, the flow velocity at any point inside the pipe is also known.

Density and viscosity of the liquid only matters before laminar flow is established (for example after a pipe bending or a vascular stenosis).

The simple laminar model is not sufficient to describe the blood flow in a vessel. The blood flow velocity will vary during the cardiac cycle of diastole and systole. Venous flow is, however, less pulsatile than arterial flow. Blood vessels are not straight pipes, they bend and bifurcates. In connection to a vessel bending or a bifurcation, blood will have a complex flow behaviour, with separation of laminar streamlines as a result (Figure 12). Sometimes this complex flow will even force blood to go backwards.



**Figure 12: Complex behaviour of blood flow at carotid bifurcation. Separation of flow stream lines is seen, as well as blood flowing backwards.**

### 3.4 Flow effects in MRI

In this section two main flow effects in MR images are discussed: inflow effects and effects on the phase of the rotating magnetisation vector.

Inflow effects (also called washin or time of flight effects) arise from the fact that flowing spins experience fewer RF pulses than stationary spins. As a result of that, signal from flow will be higher than signal from stationary tissue. Phase effects appear when spins move along a magnetic field gradient. Their magnetisation vector will then accumulate a net phase angle.

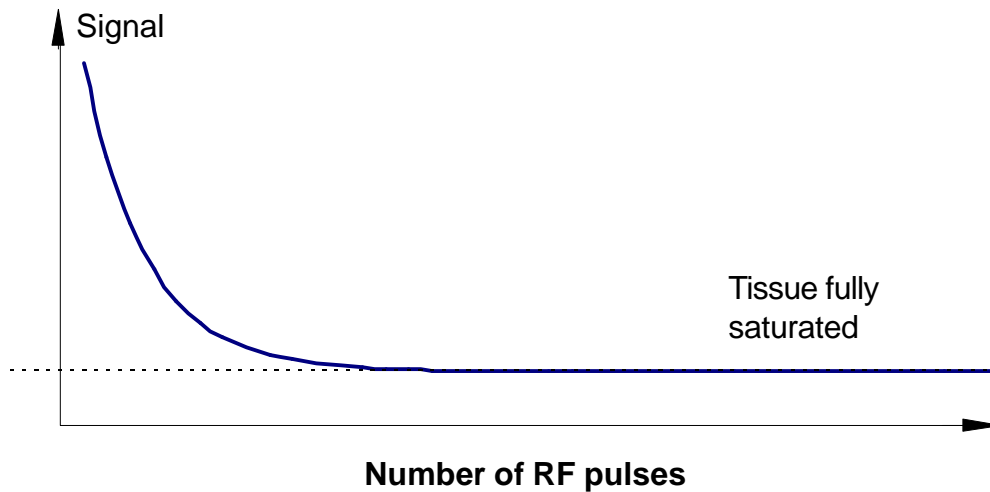
Both these effects may be used for MR angiography. Inflow effects are used in TOF angiography (3.6.1) and the phase effects are used in phase contrast angiography (3.6.2).

The effects of flow are not always welcome. Especially the phase effects, they cause both signal loss and artefacts in the final image.

#### 3.4.1 Inflow effects

During the sequence, the stationary tissue in the image slice is excited with RF pulses a number of times separated a time TR. In gradient echo sequences the TR used is too short to allow the longitudinal magnetisation to fully relax between the RF pulse repetitions, and the result is loss of signal intensity in the echoes following the first one. The signal will decrease as the number of delivered RF pulses increases until a state of equilibrium is reached (Figure 13). The number of RF pulses needed to reach equilibrium is a strong function of both flip angle and T1.





**Figure 13: A short TR does not allow full T1 relaxation between pulse repetitions. The result is a decrease in signal intensity with increasing number of delivered RF pulses. The decrease will go on until a state of equilibrium is reached, the tissue is then referred to as fully saturated.**

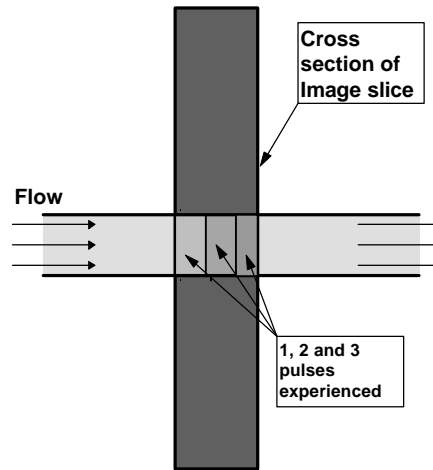
If flow is present, for example blood flowing perpendicular to the image slice, there will continuously be new, unexcited spins entering the slice. These spins have not experienced any pulses before, i.e. they carry full magnetisation when they experience their first RF pulse and give a stronger signal than the surrounding saturated stationary tissue. This phenomenon is referred to as “inflow” or “washin”, and many flow imaging sequences rely on this kind of suppression of stationary tissue and the inflow of “fresh” spins into the image slice.

Since the signal decreases when the number of experienced pulses increases, it is of interest to be able to calculate the number of experienced pulses for flowing spins. A first approximation of this, is to assume that all flowing spins in an image slice have received the maximum possible number of RF pulses. If there is flow present with a constant velocity,  $v$ , through an image slice with thickness  $d$ , and a gradient echo sequence with repetition time  $TR$  is used, the maximal number of pulses a voxel (a small volume element), flowing perpendicular to the imaging plane, can experience is

$$\# RF = \frac{d}{vTR} \tag{Eq. 3.4-1}$$

The number of pulses must be an integer, since the RF pulse is approximated to have an infinitesimally short duration. The right side of the equation is thus rounded to nearest larger integer.

Obviously, all spins flowing perpendicularly to an image slice will not have experienced the same number of RF pulses. As long as the flow velocity is not too high, spins on the entering side of the slice will have felt less pulses than spins leaving the slice (Figure 14).



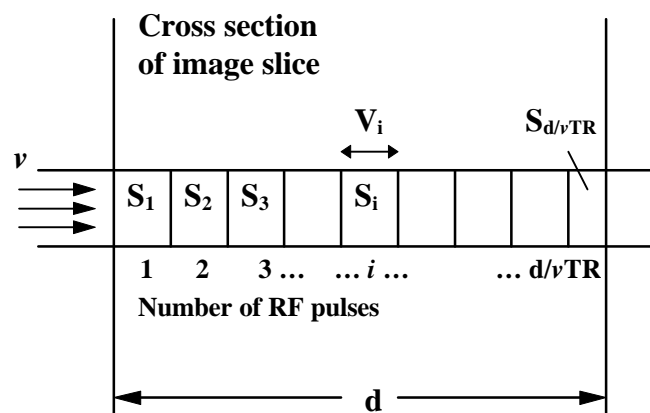
**Figure 14: Flow through an image slice. Spins at the beginning of the slice passage have felt just one pulse and yield higher signal at TE than spins leaving the slice. Stationary spins yields even lower signal. The total signal from areas with flow will be an average of signal from voxels all over the flowing volume. Inflow signal will be dependent on flow velocity, slice thickness and TR.**

With increasing flow velocity, the signal will increase, since the average number of pulses experienced will decrease. This holds up to the point where

$$vTR = d \tag{Eq. 3.4-2}$$

that is when the maximum number of experienced pulses is one. For even higher flow velocities, no further increase of inflow effects will be seen, and signal will be constant (for plug flow, that is). Since the RF pulse is approximated by an infinitesimally short pulse, the same amount of spins will experience a pulse even for high flow velocities.

Calculations of the total signal (or the size of the magnetisation vector  $M$  at TE) from flow volumes may be performed. These are most easily performed using the plug flow model (Section 3.3). The number of experienced pulses in volumes with flow will be in the range from 1 pulse to the number of pulses given by Eq. 3.4-1 (Figure 15).



**Figure 15: Calculation of signal from flow perpendicular to the image slice.  $S_i$  means signal from volume that have experienced  $i$  pulses.  $V_i$  is the relative volumes yielding  $S_i$ . The total signal is a summation of  $S_i$  weighted with their relative volumes.**

Signal  $S_1$  to  $S_{d/vTR}$  can be calculated using Eq. 3.2-3 to Eq. 3.2-6. The respective signals,  $S_i$ , have to be weighted together to obtain the total signal. Weighting factors for each  $S_i$  are given by the relative volume,  $V_i$ , containing spins that have received  $i$  pulses, The total signal is thus

$$S_{tot} = \sum_{i=1}^{d/vTR} S_i V_i \quad \text{Eq. 3.4-3}$$

$V_i$  is given by

$$V_i = vTR, \quad 1 \leq i < \frac{d}{vTR} \quad \text{Eq. 3.4-4}$$

$$V_i = \left( \text{frac}\left(\frac{d}{vTR}\right) \right) vTR, \quad i = \frac{d}{vTR} \quad \text{Eq. 3.4-5}$$

where *frac* is a function that returns the fractional part of the argument inside the parenthesis. For example, if the argument  $z$  equals 2.37 then  $\text{frac}(z)$  is 0.37.

Inflow effects may be suppressed by the use of presaturation pulses. These pulses excite spins outside the actual image slice, on the inflow side. If the flip angle of these pulses is large enough, the result will be a saturation of the spins flowing into the image slice and a decrease in signal contrast between flowing and stationary tissue.

If the sequence used contains more than one slice selecting RF pulse per repetition an effect called washout or outflow may be observed. Washout is signal loss in volumes containing fast flow. It is due to spins leaving/entering the slice between the first and the second RF pulse within the same TR. Since this work is concerned only with effects in gradient echo sequences the washout effect is of no interest here.

### 3.4.2 Flow induced phase effects

Magnetic field gradients used during image acquisition dephase the magnetisation vector for stationary spins. To correct for this, gradients are usually applied in equal and opposite pairs in order to restore the initial phase at TE ("a" and "b" in Figure 9). For reasons explained below, this method is not applicable to moving spins. Flow will, if it is not taken care of, cause phase effects that may lead to signal loss and artefacts in the final image. The artefacts appear as ghost images of the moving spins spread out in the phase encoding direction (the direction in the image corresponding to direction of the phase encoding gradient).

The phase angle of a general rotating system at a time  $t$  is defined as

$$\Phi(t) = \int_{t_0}^t \omega(t) dt \quad \text{Eq. 3.4-6}$$

where  $\omega$  is the angular frequency of which the system rotates. For a spin magnetisation vector,  $M$ ,  $\omega$  will be the Larmor frequency corresponding to the present

magnetic field strength. The Larmor frequency can be written

$$\omega(t) = \gamma \overline{B(t, x(t))} = \gamma \overline{G(t)x(t)} \quad \text{Eq. 3.4-7}$$

where the latter equality is true in a co-ordinate system rotating at the Larmor frequency of  $B_0$ .  $B$  is the time and position dependent magnetic field strength,  $\gamma$  is the gyromagnetic constant and  $x(t)$  is the position of spins at time  $t$ .  $G(t)$  is the gradient amplitude. Combining Eq. 3.4-6 and Eq. 3.4-7 yields the phase shift equation:

$$\Phi(t) = \gamma \int_{t_0}^t \overline{G(t) x(t)} dt \quad \text{Eq. 3.4-8}$$

for the transverse magnetisation of spins at position  $x$ . The phase shift equation describes how the transverse magnetisation vector for a group of spins accumulates a phase angle due to the influence of a magnetic field gradient applied for a certain time. It holds for stationary spins as well as for moving spins, since the movement is described by  $x(t)$ .

Stationary spins feeling a positive gradient  $G$  for a certain time  $t-t_0$  will acquire a certain phase shift  $\Phi(t)$  according to Eq. 3.4-8. Since  $x(t)$  is constant for stationary spins, a second gradient lobe equal in size and shape to the first one but with opposite sign will give the same negative phase shift. The two lobes together form a bipolar gradient. The total phase shift after a bipolar gradient is thus zero for stationary spins.

Spins moving in the direction of a bipolar magnetic field gradient will also acquire a phase shift but since they move they will not feel the same gradient strength during the whole sequence. Thus, for moving spins the phase shift will not totally cancel out after a bipolar gradient. Consider spins moving with constant velocity  $v$  in the direction of a bipolar gradient. The induced phase shift after the two lobes is then directly proportional to the velocity. Eq. 3.4-8 yields

$$\Phi(v) = \gamma v T A_g \quad \text{Eq. 3.4-9}$$

where  $T$  is the time interval (centre to centre) between the individual gradient lobes and  $A_g$  is the area (in terms of gradient strength and time) of one of the lobes.

Thus, stationary spins will form an echo at TE since they are totally rephased and have a zero phase angle at that time. The flowing spins will not be in phase after the bipolar gradient, and the remaining phase shift (Eq. 3.4-9) can give rise to ghosting artefacts or, due to a velocity distribution within separate voxels, lead to signal loss.

### 3.5 Gradient Moment Nulling (GMN)

Neither the artefacts nor the signal loss due to flow induced phase effects are welcome. MR images are obtained for diagnostic purposes and if the volume of interest contains flow, some kind of flow compensation is of interest. The most common way to deal with the problem is to replace the original bipolar gradients with three or more modified gradient lobes. These lobes should be distributed in such a way that the phase disturbance induced by flow cancel out at the same time as the stationary spins form an echo, at TE that is. The method is called Gradient Moment Nulling (GMN) and does not affect the inflow phenomena explained in section 3.4.1.

The purpose of GMN is to eliminate the flow induced phase shift. To explain how it is done, it is convenient to use the properties of the Taylor series. Every continuous differentiable function may be expanded into its Taylor series, the time dependent position of a group of spins is no exception:

$$x(t) = \sum_{n=0}^m \frac{1}{n!} x_0^{(n)} (t - t_0)^n \quad \text{Eq. 3.5-1}$$

$x_0^{(n)}$  is the nth derivative of  $x(t_0)$ . This means that  $x_0^{(0)}$  is the initial position of the spins,  $x_0^{(1)}$  is the spins initial velocity and so on so that every term corresponds to one type of motion. When this is put into the phase shift equation (Eq. 3.4-8) it yields

$$\Phi(t) = g \int_{t_0}^t \bar{G}(t) \sum_{n=0}^{n=m} \frac{1}{n!} x_0^{(n)} (t - t_0)^n dt \quad \text{Eq. 3.5-2}$$

Switching the order of integration and summation is allowed and if the integral is split up in terms after such a reorder, it looks like in (Eq. 3.5-3)

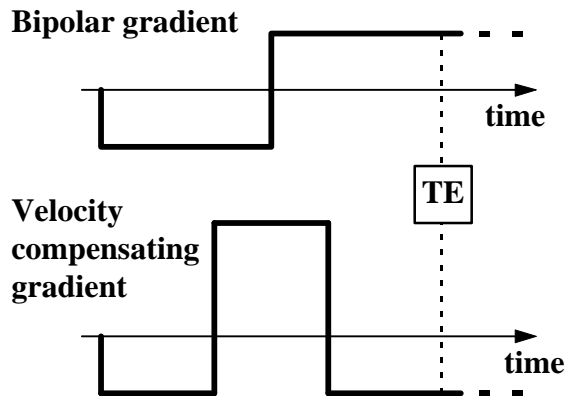
$$\begin{aligned} \Phi(t) &= g \sum_{n=0}^{n=m} \frac{1}{n!} x_0^{(n)} \int_{t_0}^t \bar{G}(t) (t - t_0)^n dt = \\ &= g x_0 \int_{t_0}^t \bar{G}(t) dt + g v_0 \int_{t_0}^t \bar{G}(t) (t - t_0) dt + \frac{g a_0}{2} \int_{t_0}^t \bar{G}(t) (t - t_0)^2 dt - \end{aligned} \quad \text{Eq. 3.5-3}$$

In GMN, the appearance of the time varying gradient  $G(t)$  is changed, in order to set  $\Phi(t)$  to zero at time  $t$ . The phase angle at time  $t$  will be zero if the integrals are zero.

Assume a bipolar gradient and spins flowing with a constant velocity in its direction. The first integral will be zero since the gradient lobes compensate each other. The second integral will not be zero, it will result in an accumulated phase angle according to Eq. 3.4-9. Integrals of higher order will be zero since the spins have no initial acceleration, jerk etc. If, instead, the gradient lobes are applied in the same way as the flow compensating gradient in Figure 16, the second integral will be zero, that is

$$g \frac{1}{1!} v_0 \int_{t_0}^{TE} \bar{G}(t) (t - t_0)^1 dt = 0 \quad \Rightarrow \quad \Phi(x_0^{(1)}, TE) = 0 \quad \text{Eq. 3.5-4}$$

The first integral will also be zero since the total area of the lobes is zero. Higher order integrals are still zero since no acceleration/jerk etc. was assumed.



**Figure 16: Bipolar gradient and a gradient shape compensating for phase effects from flow with constant velocity. The relative strength of the lobes in the velocity compensating gradient is -1 : 2 : -1.**

According to the same principle, one can adjust the gradients so the phase angle gets independent of constant acceleration and constant jerk too at TE. However, for these cases the gradient wave forms will be more complicated.

Note that GMN will only affect flow induced phase effects, inflow effects will still remain in an image containing flow.

### 3.6 MR Angiography

Images of the cardiovascular system are called angiograms and the technique used to produce them is called angiography. Conventional angiography is based on x-ray techniques and a contrast agent distributed in the cardiovascular system. Even though conventional angiographic techniques have been used for a long time they still contain some major drawbacks, for example

- They are invasive.
- Although they provide a dynamic presentation of flow through a region of the body, a conventional angiogram contains no quantitative information of flow velocity or organ perfusion.

MRI offers non-invasive methods for both imaging of vascular anatomy and quantification of blood flow velocity (Potchen et al, 1993; Stark David D. et al, 1992).

The techniques used in MR angiography are based on two different types of flow effects and the use of gradient echo sequences. In the TOF (Time Of Flight) techniques the stationary tissue is suppressed by multiple RF pulses while the flowing spins experience fewer pulses and therefore leave a higher signal. The effect is called inflow or washin and is described in more detail in section 3.4.1. PC (Phase Contrast) techniques make use of phase effects caused by flow (section 3.4.2) to separate signal from stationary spins and flowing spins. Signal in the PC image is more related to flow velocity than to tissue parameters.

#### 3.6.1 TOF techniques

In TOF Angiography, the signal from stationary tissue is suppressed, or saturated, by applying RF pulses very rapidly. The saturation is a result of the TR being shorter than T1; the shorter the TR is, the greater is the saturation effect. As the number of applied RF pulses is increased, the degree of saturation will increase until a

state of equilibrium is reached.

Having suppressed the signal of background tissues, now consider the signal of blood flowing in to the image volume. RF pulses are applied only to the image slice (assuming single slice imaging and that there are no saturation pulses applied outside the image slice). Blood outside this slice is fully relaxed and possesses its full magnetisation. As the blood enters the image slice it experiences its first RF pulse and gives a high signal compared with the suppressed stationary tissue. If the blood stays long enough in the slice to experience more RF pulses, it will decrease in signal strength, but it will still have a higher signal than the surrounding tissue (if it does not stay long enough to reach its equilibrium). The effect of inflowing blood having a brighter signal intensity than the stationary tissue is referred to as inflow effect, and is described in more detail in section 3.4.1.

The amount of contrast enhancement from inflow effects is a function of many parameters. Mostly it is a question of suppressing the stationary tissue rather than enhancing the inflowing blood.

- The dependence on the flip angle is very complex. For gradient echo sequences and stationary spins at equilibrium, the flip angle, yielding the highest signal, is given by the Ernst angle (Eq. 3.2-8). For flowing spins this optimal flip angle will increase with increasing velocity.
- An increase in flow velocity yields an increase in inflow signal, since the spins experience fewer pulses the higher velocity they have.
- The effects of TR and flip angle are interrelated. A large flip angle will saturate tissue at a relatively long TR, while a small flip angle will not saturate tissue even at very short TR.
- A short T1 of the stationary tissue permits a more complete relaxation between RF pulses and therefore decreases the effect of saturation. This effect is of course also interrelated with TR.
- Lowering T1 of the inflowing blood (by administering a contrast agent) will yield an increase in blood signal.

### 3.6.2 Phase contrast techniques

In section 3.4.2 it is stated that the induced phase angle at TE for the magnetisation vector of spins flowing with constant velocity  $v$  in the direction of a bipolar gradient field is

$$\Phi(v) = \gamma v T A_g \tag{Eq. 3.4-9}$$

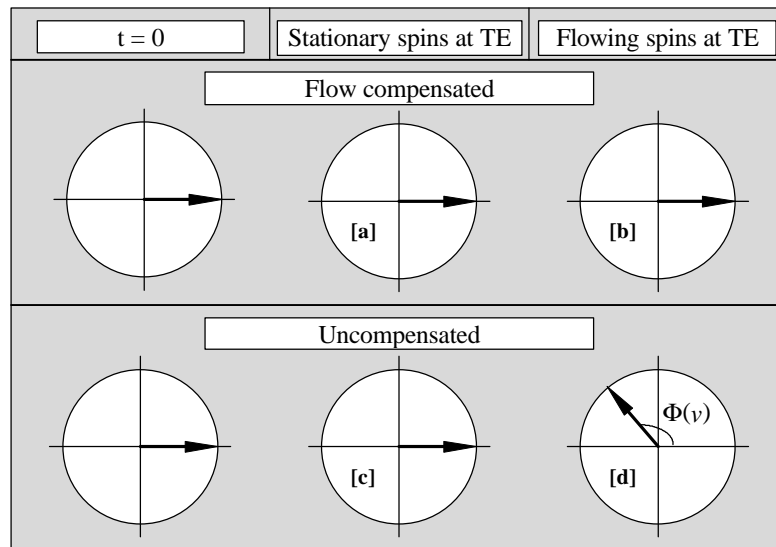
For stationary spins the phase angle is zero at TE.

If, instead, the gradient lobes are organised to give flow compensation (for example with help of GMN, section 3.5), both flowing and stationary spins will have zero phase at TE.

The information contained in the signal matrices from the two sequences, may be used to create an angiogram. The technique is called Phase Contrast (PC) angiography and is explained in this section

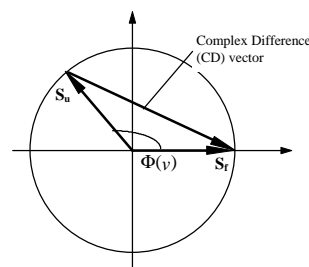
The signal from each voxel of the object after Fourier transformation is a complex number. This number can be expressed as a vector with a magnitude and a direction. The magnitude is seen as the signal intensity in the final image and the direction of the vector is described by a phase angle. For stationary tissue the phase angle is ideally zero and flowing spins accumulate a phase angle according to Eq. 3.4-

9 (if bipolar gradients are used). Figure 17 shows the ideal signal vector for the two sequences in case of both flow and stationary spins. At  $t = 0$  the phase angle can be assumed zero for both sequences. At  $t = TE$  the flow compensated sequence will give zero phase again for both stationary [a] and flowing [b] spins. In the uncompensated sequence, the stationary spins [c] still have zero phase at TE but flowing spins [d] will have accumulated a phase  $\Phi(v)$  according to Eq. 3.4-9.



**Figure 17: Signal vectors for flowing spins, assuming the vector to have a zero phase angle at  $t = 0$ . At TE the stationary spins yield a totally rephased signal vector both in the flow compensated and the uncompensated sequence. Flowing spins are totally rephased in the flow compensated sequence whereas in the uncompensated sequence a phase angle  $\Phi(v)$  is accumulated.**

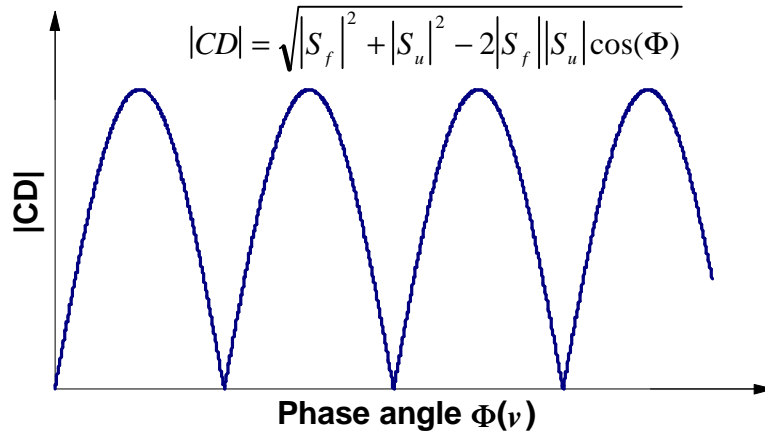
The signal vector matrix from the flow compensated acquisition can be subtracted pixel by pixel from the matrix corresponding to the uncompensated acquisition. Pixels containing signal from stationary spins will leave a zero vector ( $[c] - [a]$ ), but in the case of flowing spins the result will be a new vector ( $[d] - [b]$ ), called Complex Difference (CD) vector (Figure 18).



**Figure 18: Complex Difference (CD) vector. When subtracting signal matrix elements from a flow compensated sequence ( $S_f$ ) and an uncompensated sequence ( $S_u$ ), flowing spins leave a new vector called CD vector**

The magnitude of this CD vector is dependent on the flow velocity. Low flow velocity yields a small CD vector. As the velocity increase, the length of the vector will too until  $\Phi(v) = 180^\circ$ . For even higher flow velocities the magnitude of the vector will decrease. If the velocity is increased further, a periodical behaviour will be seen (Figure 19).





**Figure 19: Periodical behaviour of the magnitude of the CD vector (  $|CD|$  ) as the flow velocity (and thereby the accumulated phase angle,  $\Phi$ , Eq. 3.4-9) is increased.  $|CD|$  may be calculated using the equation in the figure (the cosine theorem).  $S_f$  is the signal from the flow compensated sequence and  $S_u$  is the signal vector from the uncompensated sequence. Plug flow is assumed.**

A Phase Contrast Complex Difference (PC CD) angiogram is an image showing the magnitude of the CD vector as pixel intensity. This kind of angiogram will ideally have no signal intensity in pixels with stationary spins. Signal intensity in pixels with flowing spins will vary with flow velocity according to Figure 19.

The maximum signal intensity is obtained when the flow induced phase angle  $\Phi$  equals  $(2n+1)\pi$ , where  $n = 0, 1, 2, \dots$

Another form of PC angiogram is the so called PC PD angiogram. PD means Phase Difference and displays  $\Phi$  as signal intensity in the image instead of the magnitude of the CD vector. In a PC PD angiogram, the signal intensity will be directly proportional to flow velocity (Eq. 3.4-9) as long as the requirement -  $180^\circ < \Phi < 180^\circ$  is true.

The flow velocity yielding a phase shift of  $180^\circ$  is often called the  $v_{enc}$  of a PC sequence. The  $v_{enc}$  can be varied by changing the area of the gradient lobes (according to Eq. 3.4-9) and thereby making the sequence sensitive to flow of another velocity.

The most simple form of a PC sequence is sensitive to flow in one direction. Such a sequence has to be based on at least two other sequences as described above. More advanced types of sequences will be sensitive to flow in all directions.

## 4. Calculations

Calculations of the spoiled gradient echo signal were performed. Sequence parameters as well as tissue parameters were varied and signal was calculated for both stationary and flowing spins. The obtained results are used not only to increase the understanding of signal behaviour, but also to compare the theoretical signal equations with measurements (Section 7).

The calculations performed in this work are mostly concerned with the signal behaviour when flow is present. Especially the inflow effects and the phase effects of flow are investigated. Calculations of signal in phase contrast sequences are also performed.

The resulting diagrams are plotted in a special section (called Diagrams) at the end of this work.

### 4.1 Inflow effects

#### 4.1.1 Method

The signal equations for the spoiled gradient echo sequence (Eq. 3.2-3 to Eq. 3.2-6) were used to calculate the signal. Calculations were performed for flowing as well as stationary tissue.

A program for MS Windows 95™ called GEsignal (Appendix A) was developed especially for this work. It was written in Borland Delphi, a visual object oriented programming language. GEsignal was designed to illustrate the signal effects when varying the tissue parameters and parameters of the spoiled gradient echo sequence. When performing flow calculations, GEsignal implements the concept in section 3.4.1 (Figure 15 and Eq. 3.4-3 to Eq. 3.4-5) and the plug flow model and assumed (section 3.3).

The user can choose to calculate the signal for stationary tissue or for flow perpendicular to the image plane. All basic parameters (tissue and sequence dependent) are set by the user and signal is calculated as a function of at least one, in some cases two, of these parameters. In the case of flow calculations, flow velocity and slice thickness are also set by the user. The result is saved as a text-file and is preferably imported to a spreadsheet for further evaluation. Due to the graphical user interface the program is very easy to use and self instructing.

All calculations in this section are carried out using GEsignal.

#### 4.1.2 Suppression of stationary tissue

In image acquisition, multiple RF pulses are delivered with short intervals. The longitudinal magnetisation is not given time to fully relax between the pulse repetitions, and the signal therefore decreases as the number of experienced pulses increases.

A small flip angle yields a small transverse magnetisation after an RF pulse and the time needed to recover before next pulse is short. A large flip angle, on the other hand, yields a large transverse magnetisation but the time needed to recover is long.

The result is high signal for large flip angles after one or a few experienced RF pulses, but as the number of pulses increases the signal will fall rapidly due to the long time needed to recover. Small flip angles, will show a fairly low initial signal,

but the recovering time is short and therefore the signal decrease will not be as rapid as for large flip angles (Diagram 1-2).

In tissue with a short T1 the longitudinal magnetisation needs less time to recover between pulses than in matter with a long T1. This results in a higher steady state signal for tissue with short T1 than for tissue with long T1 (diagram 3-4).

After a large flip angle pulse, almost no longitudinal magnetisation remains and the recovery is more dependent on T1 than if a small flip angle pulse is used. Due to this effect, steady state signal from acquisitions performed with a small flip angle will contain minor T1 effects whereas acquisitions using a large flip angle will have a more pronounced dependence on T1. This is seen if diagrams 1 and 2 are compared. Signal from a 20° flip angle will not decrease as much as signal from an 80° flip angle when T1 is increased from 100 ms to 1000 ms.

#### 4.1.3 Inflow signal and flip angle

In stationary tissue the steady state signal from a spoiled gradient echo sequence will yield highest the signal at a flip angle called the Ernst angle (Eq. 3.2-8). This optimal flip angle is dependent on both TR of the sequence and the T1 of the tissue being imaged (Diagram 5).

If flow perpendicular to the image slice is present, the steady state criteria will no longer hold in areas with flow since the flowing spins do not experience as many pulses as the stationary spins. The result is a higher signal from flowing tissue than from stationary tissue, the so called inflow effect (Section 3.4.1).

The optimal flip angle in areas with flow does not only depend on TR and T1 but also on slice thickness and flow velocity. Low flow velocity allows the flowing spins to experience many RF pulses. The signal behaviour will be close to that of stationary spins, yielding an optimal flip angle close to the Ernst angle. A high flow velocity results in few experienced pulses for the spins. This favours a larger flip angle according to the result in section 4.1.2. Increase of flow velocity will also result in a total signal increase due to the inflow effect. Calculations in diagram 6 show both flip angle dependence and inflow effect for a variety of flow velocities.

#### 4.1.4 Inflow signal and T1

Tissue with a short T1 needs less time to recover between pulses than tissue with a longer T1. This results in two effects of interest.

- The optimal flip angle increases. Even if a large flip angle is used there will be enough longitudinal magnetisation to obtain a high signal when the next pulse is delivered.
- A higher steady state signal. The faster the longitudinal magnetisation is recovered, the more magnetisation will end up in the transverse plane when the next pulse is delivered.

These effects on signal are seen for flowing spins (Diagram 7) as well as for stationary tissue.

From a clinical point of view, the influence of T1 on signal is interesting. In order to enhance signal from specific tissues, MR contrast agents are distributed in the patient. These agents induce a signal increase by lowering the tissue's T1 relaxation time. In TOF MR-angiography (section 3.6.1), contrast agents may be used to decrease blood T1, and a combined effect of both inflow and reduced T1 enhances

vascular contrast. The effects of T1 are, however, less pronounced than the inflow effect (Diagram 7).

#### 4.1.5 Inflow signal and TR, flow velocity and slice thickness

Signal from flowing spins, is a strong function of the number of pulses experienced by the spins. For flow perpendicular to the imaging plane, the maximum number of pulses a spin can experience is given by Eq. 3.4-1.

$$\# RF = \frac{d}{vTR} \quad \text{Eq. 3.4-1}$$

where  $d$  is the slice thickness and  $v$  is the flow velocity. The maximum number of pulses that flowing spins can experience will thus vary in similar ways if either one of the three parameters is varied.

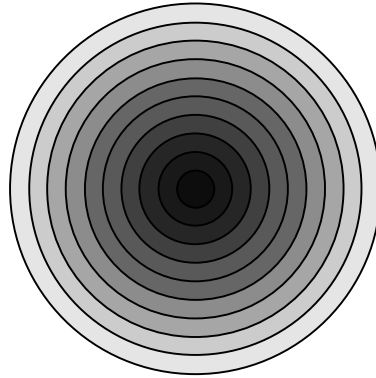
Due to the above, the total inflow signal will vary in the same manner if TR is varied as if  $v$  is (diagram 8-9). If  $d$  is varied however, there will be an effect in signal not only due to inflow, but also from the fact that the number of spins giving signal is changed (diagram 10). If the slice thickness is doubled and the flow velocity is doubled at the same time, no inflow signal enhancement will appear, but the total signal is doubled since the total number of spins giving signal is twice as high (diagram 11).

#### 4.1.6 Laminar flow

All calculations performed in section 4.1.2 through 4.1.5 are carried out assuming plug flow. However, the plug flow approximation does not describe the flow in an adequate way, instead the laminar flow model is more suitable (see section 3.3 for flow models). Calculations were performed to see how the total signal from flow perpendicular to the image slice is changed when laminar flow instead of plug flow is assumed.

the flow velocity distribution over a cross section of a straight circular pipe is parabolic in a perfect laminar model (Eq. 3.3-1 and Figure 11). In the plug flow model the flow velocity is assumed constant everywhere (Figure 10).

To calculate the total signal from a straight circular pipe with laminar flow, a cross section of the pipe was divided into ten sections. Each section was assumed to have an individual constant flow velocity calculated using Eq. 3.3-1 (Figure 20).



**Figure 20: Cross section of a straight circular pipe, containing flow. The pipe is divided into ten parts, assuming individual constant flow velocity in each part. Dark grey means high flow velocity and light grey means low flow velocity. The model is used to calculate an approximate total signal from a pipe with laminar flow.**

Signal was calculated for each part of the pipe using GEsignal and the total signal was calculated as the weighted sum of the individual signals. Weighting factors were the relative areas of each part. The total laminar signal was compared with signal calculated for plug flow with a velocity corresponding to the mean velocity of the laminar flow (diagram 12).

Signal was also plotted versus pipe radius for both plug flow and laminar flow (diagram 13).

#### 4.1.7 Flip angle variation over slice profile

GEsignal makes the assumption that all spins in an image slice are subject to the same flip angle when irradiated with an RF pulse. This may not be valid, since the excitation pulse is not an ideal sinc-shaped RF pulse. The excitation pulse in an MR scanner is a truncated sinc pulse. If heavy truncation is used, which is often the case, the pulse shape will be close to the one of a Gauss curve. The Fourier transform of a Gauss curve is a Gauss curve and the flip angle is proportional to the RF pulse amplitude. Thus a heavily truncated sinc excitation pulse makes the flip angle distribution over the image slice similar to the shape of a Gauss curve.

Calculations were performed to illustrate these effects on total signal from an image slice. Both stationary and flowing (plug flow) spins were assumed. A slice was divided into twenty "subslices", each one having a specific flip angle. A Gaussian flip angle distribution over a slice profile was assumed. The desired flip angle was set in the centre of the slice. Signal was calculated for each subslice, using GEsignal, and the total signal was taken as the average of signal from all subslices. The resulting total signal for various flow velocities was plotted versus flip angle (diagram 14).

## 4.2 Phase effects and Gradient Moment Nulling (GMN)

### 4.2.1 Method

The integral of the phase shift equation (Eq. 3.4-8) was solved numerically for a variety of gradient schemes and flow velocities.

A spreadsheet (MS Excel 7.0 for MS Windows 95™) was configured to implement Eq. 3.5-3 and visualise the phase effects when parameters such as gradient

strength, gradient duration and flow velocity are changed. The first three terms (integrals) on the right side of Eq. 3.5-3 are calculated. The sheet therefore takes into account phase effects from stationary spins and spins moving with both velocity and acceleration components. The gradient schemes in the spreadsheet assume the frequency encoding gradient. A more detailed explanation of how the sheet works is given in Appendix B. Tables below are exact copies of the spreadsheet.

The aim was to show phase effects on the spin magnetisation vector  $M$ , for spins flowing in the direction of an applied magnetic field gradient. Especially the phase evolution for spins moving in the direction of a bipolar gradient (Eq. 3.4-9), and gradient schemes compensating for constant velocity and constant acceleration (GMN, section 3.5) was studied.

#### 4.2.2 Bipolar gradient

If flow is present in the direction of a standard bipolar gradient, the induced total phase will not be zero at TE. If the flow has a constant velocity, the induced phase angle will be proportional to flow velocity (Eq. 3.4-9). Stationary tissue will be totally rephased at TE.

A standard bipolar gradient like in Table 1, will result in the gradient scheme and phase evolution presented in Diagram 15-16. The induced phase angle is calculated for stationary tissue and two different flow velocities. The phase at TE is proportional to flow velocity.

	Gr-lobe 1		Gr-lobe 2		Gr-lobe 3		Gr-lobe 4	
	start	duration	start	duration	start	duration	start	duration
Time step	55	15	70	30				
ms	6.545	1.785	8.33	3.57	0	0	0	0
Gradient strength	mT/m	6	mT/m	-6	mT/m		mT/m	
	Range [ms]							
	Time step	ms						
	100	12.1						

**Table 1: The bipolar gradient, no flow compensation**

#### 4.2.3 GMN for constant velocity

By using the concept of GMN (section 3.5) even the flowing spins can be rephased at TE. Assuming only stationary tissue and flow with constant velocity present in the image slice, a total compensation can be performed with the help of three gradient lobes. The lobes should have the relative area 1 : -2 : 1 (Table 2) to provide a total compensation at TE. The use of such gradient lobes yields a result seen in diagram 17-18. A total flow compensation is achieved at TE = 10 ms.

	Gr-lobe 1		Gr-lobe 2		Gr-lobe 3		Gr-lobe 4	
	start	duration	start	duration	start	duration	start	duration
Time step	40	15	55	15	70	30		
ms	4.76	1.785	6.545	1.785	8.33	3.57	0	0
Gradient strength	mT/m	3	mT/m	-6	mT/m	3	mT/m	

Range [ms]	
Time step	ms

**Table 2: Gradients compensating for constant velocity**

#### 4.2.4 GMN for constant acceleration

If the flow present contains not only a constant velocity component but also a constant acceleration component, the situation gets somewhat more complicated. To compensate for all phase effects four gradient lobes have to be used. Then both the effects from flow with constant velocity and flow with constant acceleration will be compensated for. If each gradient lobe is turned on for the same time with the relative strength 1 : -3 : 3 : -1 (Table 3) no remaining phase at TE will be seen. The result of this kind of compensation is seen in diagram 19-20. Total flow compensation is achieved at TE = 10 ms.

	Gr-lobe 1		Gr-lobe 2		Gr-lobe 3		Gr-lobe 4	
	start	duration	start	duration	start	duration	start	duration
Time step	25	15	40	15	55	15	70	30
ms	2.975	1.785	4.76	1.785	6.545	1.785	8.33	3.57
Gradient strength	mT/m	-3	mT/m	9	mT/m	-9	mT/m	3
	Range [ms]							
	Time step	ms						
	100	11.9						

**Table 3: Gradients compensating for constant acceleration**

### 4.3 Phase contrast techniques

#### 4.3.1 Method

Calculations of phase contrast signal were performed using GEsigal and a spreadsheet (MS Excel for MS Windows 95™).

To obtain the magnitude of the signal vectors  $S_f$  and  $S_u$  (see section 3.6.2) signal was calculated with GEsigal and both vectors were assumed to have the same magnitude. The induced phase angle  $\Phi$  was calculated using Eq. 3.4-9. The magnitude of the CD vector was calculated using the equation in Figure 19.

#### 4.3.2 Phase Contrast Complex Difference (PC CD) signal

PC CD signal will vary periodically as the flow velocity is increased, due to the relationship between flow velocity and the flow induced phase angle (Figure 19). Maximum signal is achieved at flow velocities inducing a phase angle  $\Phi = (2n+1)\pi$ ,  $n= 0, 1, 2, \dots$ . Effects of inflow will be seen in phase contrast signal in the same way as in signal from standard gradient echo sequences, since the length of the signal vectors are part of the CD magnitude calculation.

The magnitude of the CD vector was calculated as a function of the flow velocity for different  $v_{enc}$ :s (Diagram 21-22). Plug flow was assumed.

For flow through a straight pipe or for blood in a vessel it is often more

appropriate to assume laminar flow instead of plug flow. *Inflow* signal from a pipe with laminar flow differ from signal from a pipe with plug flow (Diagram 12-13). Therefore it is of interest to study how Phase Contrast CD signal will change when assuming laminar flow instead of plug flow. Calculations were performed using the same method as when calculating for laminar flow in the inflow case (Section 4.1.6), the result was plotted versus flow velocity for some  $v_{enc}$ :s (Diagram 23-24).

If phase contrast signal is plotted versus  $v_{enc}$  instead of versus flow velocity the curves will have a maximum at the  $v_{enc}$  corresponding to the present flow velocity. At higher  $v_{enc}$ :s the PC CD signal will decrease slowly but for lower  $v_{enc}$ :s the behaviour of the signal will be very complex (Diagram 25-26).

The calculation for laminar flow is plotted as a function of  $v_{enc}$  as well (Diagram 27-28). It is only the result for  $v_{enc}$ :s equal to or higher than the corresponding velocity that is of interest since the signal behaviour for lower  $v_{enc}$ :s are way to complex to be seen when so few points are plotted.



## 5. Simulations

Computer simulations of MRI signal behaviour in the presence of flow were performed, using a PC computer based program called The MRI Simulator. The aim was to obtain data that could be compared with results from calculations and measurements.

### 5.1 The MRI Simulator

“The MRI Simulator” is a computer program developed for MS Windows 95™ by J. Stefan Petersson (J. S. Petersson et al 1993). The purpose of the program is, as the name indicates, to simulate an MR imaging device. It implements the k-space formalism and an FFT algorithm to generate the ideal NMR signal. Signal is calculated and the result is “collected” during a virtual signal sampling event, where effects on signal amplitude due to relaxation and excitation are taken care of. Effects on signal phase due to movement or flow as well as artefacts due to stimulated echoes and transversal magnetisation are also included.

There are several built-in sequences that may be used and it is possible to create one’s own or to modify the built-in pulse sequences with the help of a graphical object oriented editor. In the same way, the incorporated drawing device may be used to create one’s own or to modify the present object slice matrices. Different tissue types (in terms of T1, T2, proton density, flow velocity etc.) may be defined within it.

When performing a simulation, the program implements the selected pulse sequence on the selected object matrix, collects signal and performs the necessary mathematical operations to create an image.

The image may be viewed in several different ways. It is possible to view the standard magnitude image, the phase image, the real or the complex part of the image. A simple ROI drawer is incorporated to evaluate mean signal in a rectangular region of an image. It is also possible to step through the image row by row or column by column and view the signal intensity profile.

It is possible to incorporate flow and/or motion into an object matrix. The flow is assumed perpendicular to the image slice (in the slice selection direction). Both plug flow and laminar flow can be implemented.

### 5.2 Method

In order to illustrate inflow effects, a sequence similar to the spoiled gradient echo sequence used in the measurements was constructed. In Siemens scanners the sequence is called `fl2d_12_rb130.wkc`.

Inflow simulations were performed using the same parameters as in the measurements (TE = 12 ms, TR = 24 ms, and slice thickness 10 mm). The tissue matrix was constructed with a large rectangular object in the centre. Parameters for the object was T1 = 700 ms, T2 = 180 ms and p.d. = 100 %. Inside this rectangle a smaller rectangle was drawn with the same parameters but with a variable plug flow assumed, all other pixels were stationary tissue. The flow object had to be rectangular to make evaluation with the built-in ROI drawer possible. Thus, plug flow was used since laminar flow in a rectangular object does not introduce the same flow velocity distribution as in a circular object.

Flow effects on transverse magnetisation phase was studied using two different

sequences. They were both gradient echo sequences with a perfect transverse magnetisation spoiling assumed at the end of each TR. One sequence (Figure 21, page 35) was a standard gradient echo sequence with ordinary bipolar gradients in both frequency encoding direction and slice selection direction. The other sequence (Figure 22, page 35) was compensated for flow with constant velocity using GMN (section 3.5) in the slice selection direction.

Simulations were performed for a number of flow velocities using both the uncompensated and the flow compensated sequence. Parameters used were TE = 14 ms, TR = 26 ms, flip angle = 40°. The same tissue matrix as in the inflow simulations was used.

### 5.3 Inflow effects

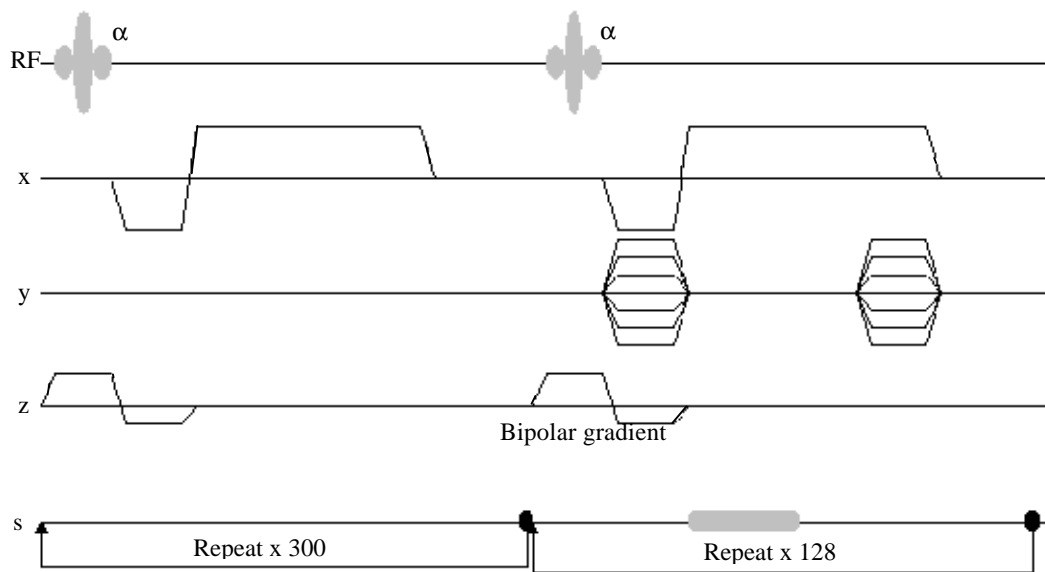
Evaluation of the simulated inflow images were done using the built in ROI drawer. Signal mean was taken over a ROI of the flow area for different flow velocities and flip angle. The result was plotted versus flip angle for the different flow velocities (Diagram 29).

### 5.4 Phase effects

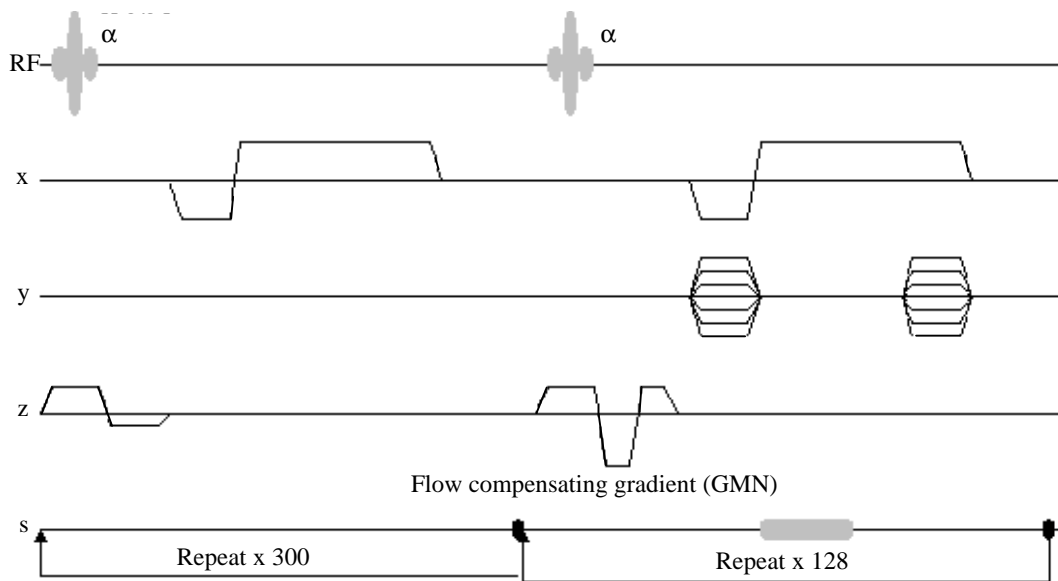
ROI:s were drawn over the flow areas in the phase image of the respective phase effects simulations. The induced transverse magnetisation phase (in relative units) at TE is seen in Table 4.

Flow velocity [cm/s]	Relative transverse magnetisation phase	
	Uncompensated seq.	Flow compensated seq.
0	-0.1	0.0
5	-1.7	-0.3
10	-3.1	-0.2
20	-6.0	-0.1
40	-11.8	0.0

**Table 4: Flow induced phase angles for a variety of flow velocities simulated using both an uncompensated sequence and a flow compensated sequence. For the uncompensated sequence the induced phase is proportional to flow velocity. When using the flow compensated sequence the phase is close to zero.**



**Figure 21: Standard gradient echo sequence.** The first part of the sequence is a so called warmup, to achieve steady state in stationary tissue. Notice the standard bipolar gradient in the slice selection direction, if flow is present in this direction, it will not be compensated for.



**Figure 22: Flow compensated gradient echo sequence.** The first part of the sequence is a so called warmup, to achieve steady state in stationary tissue. Instead of the bipolar gradient used in Figure 21, the gradients are arranged to compensate for flow with constant velocity in the slice selection direction (section 3.5).

## 6. Measurements

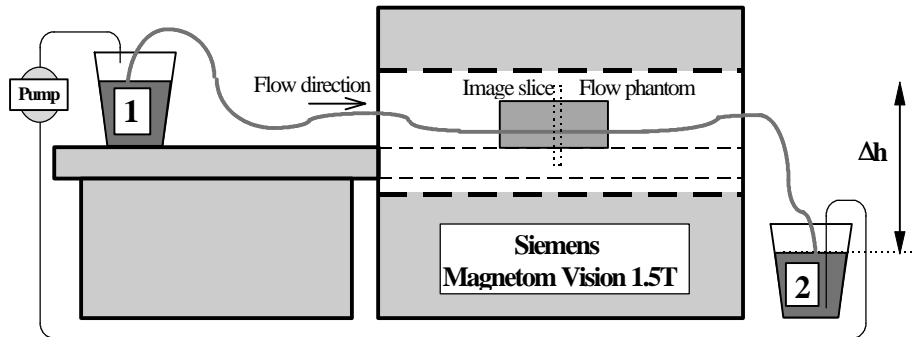
To verify calculations and simulations, measurements were carried out on one of the two clinical MR scanners in the hospital, a Siemens Magnetom Vision 1.5T MR scanner. Both inflow measurements and phase contrast measurements were carried out.

### 6.1 Method

The equipment used is principally described in Figure 23. Flow is established from fluid container no. 1 to container no. 2 by the use of gravity and a height difference  $\Delta h$  between the two containers. The flow velocity is controlled by varying  $\Delta h$ . The fluid is regular tap water doped with  $Mn^{2+}$ -ions to an approximate concentration of 0.145 mM, yielding a T1 of 700 ms and a T2 of 180 ms at 1.5T. Assuming magnetic field inhomogeneities of 0.25 ppm over the imaging volume yields  $T2^* \approx 20$  ms (Eq. 3.1-2).

The same water is used over and over again with the help of a pneumatic pump to transport the water from bucket no. 2 back to bucket no.1. Note that the pump is not affecting the flow through the flow phantom in the MR scanner in any way.

The flow phantom is a plastic cylinder filled with doped water and a number of pipes with 1.0 cm inner diameter. Between the buckets and the flow phantom there are soft plastic tubes connected to one of the flow phantom pipes. The phantom was placed in the scanner inside a head coil.



**Figure 23: Measurement equipment.** A Siemens Magnetom Vision 1.5T MR scanner, a flow phantom placed in a head coil,  $Mn^{2+}$  doped water, two buckets and a pneumatic pump. Flow is established using gravity and a height difference  $\Delta h$  between the two buckets.

The flow velocity was measured using flow quantification sequences available on the scanner. The actual measurements were performed using the sequences listed in Table 5.

Sequence	Usage	Sequence type
fl2d_12_rb130.wkc	Inflow measurements	Spoiled gradient echo sequence
fl2d_pc_5_5_5_14fb108.wkc	Phase Contrast measurements	PC Complex Difference, $v_{enc}$ 5, TE=14
fl2d_pc_10_10_10_11fb108.wkc	Phase Contrast measurements	PC Complex Difference, $v_{enc}$ 10, TE=11
fl2d_pc_20_20_20_10fb108.wkc	Phase Contrast measurements	PC Complex Difference, $v_{enc}$ 20, TE=10
fl2d_pc_30_30_30_9fb108.wkc	Phase Contrast measurements	PC Complex Difference, $v_{enc}$ 30, TE=9
fl2d_pc_45_45_45_9fb108.wkc	Phase Contrast measurements	PC Complex Difference, $v_{enc}$ 45, TE=9
fl2d_pc_60_60_60_8fb108.wkc	Phase Contrast measurements	PC Complex Difference, $v_{enc}$ 60, TE=8

fl2d_pc_80_80_80_8fb108.wkc	Phase Contrast measurements	PC Complex Difference, $v_{enc}$ 80, TE=8
fl2d_pc_100_100_100_8fb108.wkc	Phase Contrast measurements	PC Complex Difference, $v_{enc}$ 100, TE=8

**Table 5: Sequences used during flow measurements**

## 6.2 Inflow effects

Measurements were carried out to illustrate inflow effects. The fl2d\_12\_rb130.wkc sequence was used with the parameter listed in Table 6. The image slices were cut in the transverse plane. Flow was therefore present perpendicular to image plane.

Parameter	Value
TE [ms]	12
TR [ms]	24
Flip angle [Degrees]	10, 20, 30, 40, 50, 60, 70, 80, 90
Slice thickness [mm]	10
Matrix	512 x 512
FOV [mm]	200
Flow velocity [cm/s]	5, 11, 15, 22, 28, 35, 54

**Table 6: Inflow measurement parameters.**

In the images, circular ROI:s were drawn exactly covering the pipe radius. Signal mean from these ROI:s are plotted versus flip angle for the different flow velocities in Diagram 30.

## 6.3 Phase Contrast

Effects of  $v_{enc}$  choice and flow velocity in phase contrast techniques was studied. The phase contrast sequences listed in Table 5 was used for measurements. Sequence parameters used are listed in Table 7

Parameter	Value
TE [ms]	Depending on sequence
TR [ms]	104
Flip angle [Degrees]	30
Slice thickness [mm]	30
Matrix	256 x 256
FOV [mm]	200
Flow velocity [cm/s]	5, 11, 15, 22, 28, 35, 40, 45, 50

**Table 7: Phase contrast measurement parameters.**

The slice thickness and TR was chosen to make inflow effects possible (see Figure 13 and Eq. 3.4-1). In the same way as for the inflow measurements, ROI:s were drawn and signal mean was plotted versus flow velocity (Diagram 31-32) and  $v_{enc}$  (Diagram 33-34).

## 7. Comparisons

In this section, the results from calculations, simulations and real measurements are put together and compared. The results presented here are the same as in section 4-6.

### 7.1 Inflow

The inflow signal was calculated, simulated and measured. The simulations were performed using a sequence similar to the one used for the measurements. However, the sequence used for simulations had no flow compensation, which is the case for the real sequence. Furthermore, the calculations used in this comparison assumed laminar flow, but the simulations were performed using plug flow. Signal plotted versus flip angle for different flow velocities were compared.

The calculated results as well as the simulated results were normalised to the measured results for the curve corresponding to a flow velocity of 22 cm/s. All curves were normalised using the same normalisation constant and plotted in three diagrams containing stationary tissue (Diagram 35), spins with a flow velocity of 22 cm/s (Diagram 36) and spins with a flow velocity of 54 cm/s (Diagram 37).

In section 4.1.7, signal is calculated taking into account that the flip angle varies over the image slice. Some of these calculations were modified to hold for laminar flow. Inflow signal calculations has thus been performed for four different cases

- Plug flow assuming constant flip angle over image slice.
- Laminar flow assuming constant flip angle over image slice.
- Plug flow assuming a flip angle distribution over image slice
- Laminar flow assuming a flip angle distribution over image slice.

Signal for stationary tissue has been calculated for two cases

- Assuming constant flip angle over image slice
- Assuming a flip angle distribution over image slice

The different types of calculations for stationary tissue are compared to the measured result in diagram 38. The measured result together with the four different types of flow calculations used were plotted for two flow velocities (5 cm/s in diagram 39 and 22 cm/s in diagram 40). The measured result in all three cases is normalised to the calculated results, all diagrams use the same normalisation constant (the constant is made to fit the 22 cm/s curves ).

### 7.2 Phase contrast

PC CD signal was both calculated and measured. However, no simulations were performed on phase contrast, since it is not yet possible to implement PC sequences in the MRI simulator. Signal calculated for laminar flow was compared with the measured results. The curves plotted as a function of flow velocity as well as the curves plotted versus  $v_{enc}$  were compared.

In the case of signal plotted versus flow velocity, the calculated signal was normalised to the measured for the  $v_{enc} = 30$  curves. All curves were normalised using the same normalisation constant, and then plotted in four different diagrams (Diagram 38-41).

Signal plotted versus  $v_{enc}$  was normalised for the 22 cm/s curves. Also in this case, the curves were normalised using the same constant and then plotted in three

different diagrams (diagram 42-44).

## 8. Discussion

This thesis deals with signal behaviour when flow is present in MRI. A program called GEsignal was developed to study the signal from the spoiled gradient echo sequence. Calculations were performed using a variety of more or less complex assumptions. The inflow effect in gradient echo sequences was studied and the influence of many different parameters was shown. The calculations were compared with simulations using the MRI simulator and measurements from an MR scanner.

GEsignal was used to calculate inflow signal. It proved to be a useful tool to obtain qualitative results. If quantitative results are required however, the method implemented in GEsignal may be somewhat restricted.

The suppression of signal as a function of the number of experienced pulses was shown to be dependent on both the choice of flip angle and T1 of the tissue.

Signal maximum was obtained for a specific flip angle. This flip angle as well as signal intensity increases with increasing flow velocity. These effects are due to the inflow phenomena. If the flow velocity is high enough to let no spin experience more than one pulse during their stay in the image slice, signal will remain constant for increasing flow velocity.

If T1 is reduced, increase in signal and optimal flip angle similar to inflow is seen. Compared to the inflow effects, the T1 effects are, however, less prominent. A T1 reduction from 1000 ms to 100 ms yields signal increase in the order of ten percent whereas an increase in flow velocity from 5 cm/s to 50 cm/s yields signal increase in the order of hundreds of percents.

Inflow effects are affected in the same manner if TR, flow velocity ( $v$ ) and slice thickness ( $d$ ) are varied. For example, if both  $v$  and  $d$  are halved, no change in signal will appear. If  $d$  is varied though, not only the inflow effects are changed but also the number of spins giving signal, and thereby the total signal, will change.

A spreadsheet was configured to solve the phase shift equation in order to study flow effects on transverse magnetisation phase. The sheet calculates the phase shift on transverse magnetisation for flowing and stationary spins. A phase shift proportional to flow velocity was shown to appear at TE for spins flowing with constant velocity in the direction of a bipolar gradient. Gradient Moment Nulling (GMN) was implemented both for spins flowing at a constant velocity and spins flowing at a constant acceleration. A total compensation was achieved for spins with constant velocity using three gradient lobes with the relative strength 1 : -2 : 1. For spins with constant acceleration a total compensation was achieved using four gradient lobes with the relative strength 1 : -3 : 3 : -1. However, it is seldom necessary to compensate for acceleration components, since the phase effects of acceleration are very small.

Phase contrast signal could be calculated using GEsignal and a spreadsheet. When assuming plug flow, the theoretically predicted result was obtained. That is, maximum signal at flow velocities inducing a phase shift of  $(2n+1)\pi$  radians and zero signal at flow velocities inducing zero phase ( $2n\pi$  radians). If the laminar flow model was assumed, the obtained result was quite different. Since the calculations concerning laminar flow are so complex compared with the plug flow case, signal was only plotted for a few points. This makes the results difficult to interpret, especially for lower  $v_{enc}$ 's. A periodical behaviour of the signal was still seen though. However,



signal never reaches zero. Furthermore, the maximum signal is no longer obtained at flow velocities corresponding to the  $v_{enc}$ , instead it is seen for lower flow velocities.

The reason why signal never reaches zero for laminar flow is the distribution of flow velocities over the flowing area. There will always be some areas yielding signal even if the mean flow velocity yields zero signal. The reason of maximum signal for lower flow velocities than the  $v_{enc}$  is also due to the spreadout in flow velocities. For a mean velocity of half the one of the  $v_{enc}$ , the same magnitude distribution of CD vectors will be present as in the case of a mean flow velocity corresponding to the  $v_{enc}$  (phase angles in the interval  $0 < \Phi < \pi$  yield the same magnitude of the CD vectors as phase angles in the interval  $0 < \Phi < 2\pi$ ). For mean flow velocities between these two, CD vectors from phase angles near  $\pi$  will be more represented than other phase angles. Thus the maximum signal is obtained for a mean flow velocity somewhere between half the one of the  $v_{enc}$  and the one corresponding to the  $v_{enc}$ , instead of at a mean flow velocity corresponding to the  $v_{enc}$ .

Laminar inflow calculations, performed with GEsignal, were compared with simulated plug flow and measured signal. The results from the simulations as well as from the measurements were found to agree qualitatively well with the calculations. Both optimal flip angle increase and signal increase are seen for increasing flow velocity in all three cases.

The calculated and the simulated results also agree quantitatively well. The difference between the resulting curves is a few percents or less. Only when using high flip angles together with high flow velocities, the difference is in the order of 10%. However, the difference seen, is probably due to the different flow models used in the two cases. The difference between calculations and measurements is more pronounced. The most probable explanation is that GEsignal assumes a constant flip angle over the image slice. It would be more adequate to assume a gaussian flip angle distribution over the image slice.

Some calculations were performed taking into account the flip angle variation over the image slice. The result was compared with results from more simple calculations and with measurements. The flip angle was assumed to have a gaussian variation over the slice and with the desired flip angle in the centre. The results from these calculations agree very well with the corresponding measurements. The difference between measured and calculated curves is decreased from tens of percent to a few percents when taking into account a gaussian flip angle variation. The flip angle variations over image slice therefore seem to be much more important than the choice of flow model.

An effect that has not been taken care of in the calculations is the possibility of intravoxel dephasing. When a distribution of flow velocities is present over the image volume, flow velocity will not be constant inside each separate voxel. This will cause an intravoxel phase distribution resulting in signal loss. How strong the effect will be depends on strength and time duration of the gradient pulses in the flow direction, the flow velocity and the type of flow present.

Calculations of the phase contrast signal for laminar flow were compared with results from measurements. In the calculations, laminar flow was assumed but no variation of the flip angle over the image slice was implemented. Since both calculations and measurements were plotted for very few points, it is difficult to interpret the results. However, there is a periodical behaviour, for both cases, when signal is plotted versus flow velocity. For low  $v_{enc}$ 's it is difficult to see where

maximum signal is obtained due to the few points plotted. For higher  $v_{enc}$ :s maximum signal is obtained for a flow velocity lower than the specific  $v_{enc}$ . This effect is seen in both the calculated and the measured curves and is explained above. Furthermore the calculated curves seem to yield maximum signal for even lower flow velocities than the measured curves.

The difference in the measured and calculated curves can at least to some extent be explained by the use of a constant flip angle over the image slice in the calculations.

Both GEsignal and the phase effects spreadsheet have a great educational value. They are easy to use and very self-instructing. They are both useful tools when studying the concepts of inflow signal behaviour and flow induced phase effects.

## 9. Conclusions

The program developed during this project, GEsignal, has proven to be a useful tool when studying the inflow effects in MRI. GEsignal yields qualitatively satisfying results and is very easy to use. However, for use in quantitative studies and comparisons with measurements, the method implemented in GEsignal seems to be somewhat too restricted. The main reason seems to be that GEsignal assumes a constant flip angle over the entire image slice. In measurements, the flip angle will vary over the image slice. The simplified solutions of the signal equations and the flow model used (plug flow model), seem, however, to be adequate for most inflow studies. If the calculated signal from GEsignal is modified to take into account a Gaussian flip angle variation over the slice, the results agree almost fully with measured signal.

GEsignal has a great educational value. It is easy to use and can be of great help when trying to understand the inflow signal behaviour.

The spreadsheet configured to solve the phase shift equation is very easy to use. It is useful to illustrate how the phase shift is accumulated for spins moving in the direction of a gradient shaped as rectangular lobes. It is also a simple tool for understanding the concept of GMN.

Calculations of phase contrast signal have to be performed using a laminar flow model if the results are to provide any quantitative values. The use of the plug flow model yields a too idealised result for PC calculations. The periodical behaviour of PC signal is easily demonstrated with basic flow phantom experiments.

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# Appendix

## Appendix A GEsignal

A Windows 95 based program called GEsignal was developed to illustrate the effects of varying the spoiled gradient echo sequence and tissue parameters. The program can be used for calculating signal for stationary tissue or for flow perpendicular to the image plane. The result is stored in a text file and imported to a spreadsheet for evaluation.

GEsignal was developed using Borland Delphi, a visual object oriented programming language. The code used in Delphi is similar to the programming language PASCAL.

This appendix explains how to use GEsignal and its limitations and errors are also explained. The full Delphi code is plotted as well as a “program flow chart”.

### 9.1 Usage

At startup GEsignal will look like in Figure X. The program window is divided into five smaller areas described below.

The screenshot shows the GEsignal program window with the following parameters and settings:

- Calculation:**
  - Use these parameters: TE (10 ms), TR (80 ms), T1 (800 ms), T2 (50 ms), Flip angle (30 Degrees).
  - Calculate signal versus: Flip angle (selected).
  - and: (empty dropdown).
  - Stationary tissue (selected), Flow (unselected).
  - Go! button.
- Use these intervals:**
  - TE: 1 - 50 ms
  - TR: 20 - 200 ms
  - T1: 300 - 800 ms
  - T2: 25 - 100 ms
  - Flip angle: 1 - 90 Degrees
- Pulses:** 100
- Flow:** Flow velocity (20 cm/s), Slice thickness (10 mm)
- Save to file:** MRsignal, Save button

By: Jonas Svensson Department of Radiation Physics Lund University Malmö, Sweden

Figure X: The program GEsignal

#### 9.1.1 “Calculation”

In this area all the basic parameters is set. Signal is calculated as a function of one parameter (flip angle, TE, TR, T1 or T2), which is also set in this area (“Calculate signal versus”). In some cases signal can be calculated as a function of two parameters. Furthermore, the calculation can be performed for either stationary or

flowing spins, depending on how the buttons “stationary tissue” and “flow” are marked.

When the button labeled “Go!” is pressed, the calculation is carried out and the result is stored in a vector. After the calculation is finished, the “Go!” button changes label and instead reads “Done”.

### 9.1.2 “Use these intervals”

When the calculation is carried out the signal is calculated as a function of one of the parameters. GEsignal has to know in what interval the signal should be calculated. The purpose of this area is to tell GEsignal what interval to use. Even though intervals for all parameters are visible, calculation is only carried out as a function of the parameter in the “Calculation” area.

### 9.1.3 “Pulses”

This area is only used if the calculation is performed for stationary spins. The signal after a certain number of experienced pulses is then calculated. That number is set in this area.

### 9.1.4 “Flow”

If the calculation is performed for flow instead of stationary spins, the number of experienced pulses (and therefore the signal) will be a function of TR, flow velocity and slice thickness. GEsignal will therefore use this area instead of the “Pulses” area to calculate the appropriate total signal.

### 9.1.5 “Save to file”

To be able to view/evaluate the result of the calculation, the result vector has to be written to a text file. In this area the file name can be specified. As the “Save”-button is pressed, the result is saved with the specified file name to the same directory as the program file for GEsignal. The extension “.txt” is automatically added.

## 9.2 Flow calculations

Spins flowing through an image slice will have experienced a varying number of pulses depending on the time they spent within the slice. GEsignal uses the flow velocity, the slice thickness and TR to calculate the total signal for flow perpendicular to an image slice. The concept explained in section 3.4.1 is implemented (Figure 15, Eq. 3.4-1 to 3.4-5).

## 9.3 Limitations

Some limitations of the program

- The program uses the simple signal equations that are solutions to Bloch’s equation under special circumstances. These equations are true in an ideal world with no magnetic field inhomogeneities and no influence of gradients on the signal.
- The plug flow model is assumed instead of the laminar flow model, when performing flow calculations.



- In an MR scanner the image slice will not be cut out perfectly, this is not taken care of in GEsignal. GEsignal assumes a constant flip angle distribution over the image slice while it would be more appropriate to assume a Gaussian flip angle distribution.
- The RF pulse is assumed to have the shape of a delta function.
- The maximum number of experienced pulses GEsignal can handle is 110.
- The maximum TE, TR, T1 and T2 to be used is 1500 ms
- All parameters have to be integer values.

## 9.4 Errors

There are some bugs in GEsignal that have not yet been corrected for.

- Signal can not be calculated as a function of TR when using the flow option. The result will be a file containing incorrect information.
- The only possibility to calculate signal as a function of two parameters is when calculations are performed for stationary tissue, and the second parameter is "Number of pulses". Any other case will leave an empty file for evaluation.
- GEsignal can only handle integer values of TE, TR, T1, T2, Flip angle, Number of pulses, Flow velocity and Slice thickness
- When saving the result, the filename can not be the same as a file already open in another program, for example in a spreadsheet program.

## 9.5 The Code

The total Borland DELPHI code for GESignal is plotted on the following pages.

```
unit GEcode;

interface

uses
  Windows, Messages, SysUtils, Classes, Graphics, Controls, Forms, Dialogs,
  StdCtrls;

type
  TMRsignal = class(TForm)
    intervallbox: TGroupBox;
    TEfrom: TEdit;
    TRfrom: TEdit;
    Tlfrom: TEdit;
    T2from: TEdit;
    FlipFrom: TEdit;
    TEto: TEdit;
    TRto: TEdit;
    Tlto: TEdit;
    T2to: TEdit;
    FlipTo: TEdit;
    Label1: TLabel;
    Label2: TLabel;
    Label3: TLabel;
    Label4: TLabel;
    Label5: TLabel;
    Label6: TLabel;
    Label7: TLabel;
    Label8: TLabel;
    Label9: TLabel;
    Label10: TLabel;
    Label12: TLabel;
    Label11: TLabel;
    Label13: TLabel;
  end;
end;
```

```
Label14: TLabel;  
Label15: TLabel;  
GroupBox2: TGroupBox;  
EditFilnamn: TEdit;  
FilKnapp: TButton;  
GroupBox3: TGroupBox;  
countknapp: TButton;  
Xval: TComboBox;  
Label31: TLabel;  
GroupBox1: TGroupBox;  
Pulser: TEdit;  
Label17: TLabel;  
Label18: TLabel;  
editTE: TEdit;  
Label30: TLabel;  
Label33: TLabel;  
EditTR: TEdit;  
Label34: TLabel;  
Label35: TLabel;  
EditT1: TEdit;  
Label36: TLabel;  
Label37: TLabel;  
EditT2: TEdit;  
Label38: TLabel;  
Label39: TLabel;  
EditFlip: TEdit;  
Label40: TLabel;  
Label16: TLabel;  
Yval: TComboBox;  
Label20: TLabel;  
Flowbox: TGroupBox;  
EditFlow: TEdit;  
EditSkiva: TEdit;  
Flodeshastighet: TLabel;  
Skivtjocklek: TLabel;  
Label21: TLabel;  
Label22: TLabel;  
RadioStationar: TRadioButton;  
RadioFlow: TRadioButton;  
Label23: TLabel;
```

```

Label24: TLabel;
Label25: TLabel;
Label26: TLabel;
procedure countknappClick(Sender: TObject);
procedure FilKnappClick(Sender: TObject);

private
  { Private declarations }
public
  { Public declarations }
end;

var
  MRsignal: TMRsignal;
  Signalen : array[1..6,1..110,1..1500] of real;
  FlowSignalen : array[1..1500] of real;
  MRfil : textfile;
implementation

{$R *.DFM}

Procedure BeraknaTE;
Var  TE, TR, T1, T2, pulsantal : integer;
     fliprad : real;
Begin
TR := Strtoint(MRsignal.EditTR.text);
T1 := Strtoint(MRsignal.editT1.text);
T2 := Strtoint(MRsignal.editT2.text);
Fliprad:= strtoint(MRsignal.EditFlip.text)*pi/180;
For TE := Strtoint(MRsignal.TEfrom.text) to Strtoint(MRsignal.TEto.text) do
  begin
    pulsantal := 1;
    Signalen[1,pulsantal,TE] := Sin(fliprad);
    Signalen[2,pulsantal,TE] := cos(fliprad);
    Signalen[3,pulsantal,TE] := Signalen[1,pulsantal,TE]*exp(-TE/T2);
    Signalen[4,pulsantal,TE] := 1+(Signalen[2,pulsantal,TE]-1)*exp(-TE/T1);
    Signalen[5,pulsantal,TE] := 0;
    Signalen[6,pulsantal,TE] := 1+(Signalen[2,pulsantal,TE]-1)*exp(-TR/T1);
    For pulsantal := 2 to strtoint(MRsignal.pulser.text) do
      begin

```

```

        Signalen[1,pulsantal,TE] := Signalen[6,pulsantal-1,TE]*Sin(fliprad);
        Signalen[2,pulsantal,TE] := Signalen[6,pulsantal-1,TE]*cos(fliprad);
        Signalen[3,pulsantal,TE] := Signalen[1,pulsantal,TE]*exp(-TE/T2);
        Signalen[4,pulsantal,TE] := 1+(Signalen[2,pulsantal,TE]-1)*exp(-TE/T1);
        Signalen[5,pulsantal,TE] := 0;
        Signalen[6,pulsantal,TE] := 1+(Signalen[2,pulsantal,TE]-1)*exp(-TR/T1);
    end;
end;
MRsignal.countknapp.caption := 'Done!';
end;

Procedure BeraknaTR;
Var TE, TR, T1, T2, pulsantal : integer;
    fliprad : real;
Begin
TE := Strtoint(MRsignal.editTE.text);
T1 := Strtoint(MRsignal.editT1.text);
T2 := Strtoint(MRsignal.editT2.text);
Fliprad:= strtoint(MRsignal.EditFlip.text)*pi/180;
For TR := Strtoint(MRsignal.TRfrom.text) to Strtoint(MRsignal.TRto.text) do
begin
    pulsantal := 1;
    Signalen[1,pulsantal,TR] := Sin(fliprad);
    Signalen[2,pulsantal,TR] := cos(fliprad);
    Signalen[3,pulsantal,TR] := Signalen[1,pulsantal,TR]*exp(-TE/T2);
    Signalen[4,pulsantal,TR] := 1+(Signalen[2,pulsantal,TR]-1)*exp(-TE/T1);
    Signalen[5,pulsantal,TR] := 0;
    Signalen[6,pulsantal,TR] := 1+(Signalen[2,pulsantal,TR]-1)*exp(-TR/T1);
    For pulsantal := 2 to strtoint(MRsignal.pulser.text) do
begin
        Signalen[1,pulsantal,TR] := Signalen[6,pulsantal-1,TR]*Sin(fliprad);
        Signalen[2,pulsantal,TR] := Signalen[6,pulsantal-1,TR]*cos(fliprad);
        Signalen[3,pulsantal,TR] := Signalen[1,pulsantal,TR]*exp(-TE/T2);
        Signalen[4,pulsantal,TR] := 1+(Signalen[2,pulsantal,TR]-1)*exp(-TE/T1);
        Signalen[5,pulsantal,TR] := 0;
        Signalen[6,pulsantal,TR] := 1+(Signalen[2,pulsantal,TR]-1)*exp(-TR/T1);
    end;
end;
MRsignal.countknapp.caption := 'Done!';
end;

```

```

Procedure BeraknaT1;
Var TE, TR, T1, T2, pulsantal : integer;
    fliprad : real;
Begin
TE := Strtoint(MRsignal.editTE.text);
TR := Strtoint(MRsignal.editTR.text);
T2 := Strtoint(MRsignal.editT2.text);
Fliprad:= strtoint(MRsignal.EditFlip.text)*pi/180;
For T1 := Strtoint(MRsignal.T1from.text) to Strtoint(MRsignal.T1to.text) do
begin
    pulsantal := 1;
    Signalen[1,pulsantal,T1] := Sin(fliprad);
    Signalen[2,pulsantal,T1] := cos(fliprad);
    Signalen[3,pulsantal,T1] := Signalen[1,pulsantal,T1]*exp(-TE/T2);
    Signalen[4,pulsantal,T1] := 1+(Signalen[2,pulsantal,T1]-1)*exp(-TE/T1);
    Signalen[5,pulsantal,T1] := 0;
    Signalen[6,pulsantal,T1] := 1+(Signalen[2,pulsantal,T1]-1)*exp(-TR/T1);
    For pulsantal := 2 to strtoint(MRsignal.pulser.text) do
        begin
            Signalen[1,pulsantal,T1] := Signalen[6,pulsantal-1,T1]*Sin(fliprad);
            Signalen[2,pulsantal,T1] := Signalen[6,pulsantal-1,T1]*cos(fliprad);
            Signalen[3,pulsantal,T1] := Signalen[1,pulsantal,T1]*exp(-TE/T2);
            Signalen[4,pulsantal,T1] := 1+(Signalen[2,pulsantal,T1]-1)*exp(-TE/T1);
            Signalen[5,pulsantal,T1] := 0;
            Signalen[6,pulsantal,T1] := 1+(Signalen[2,pulsantal,T1]-1)*exp(-TR/T1);
        end;
    end;
end;
MRsignal.countknapp.caption := 'Done!';
end;

Procedure BeraknaT2;
Var TE, TR, T1, T2, pulsantal : integer;
    fliprad : real;
Begin
TE := Strtoint(MRsignal.editTE.text);
T1 := Strtoint(MRsignal.editT1.text);
TR := Strtoint(MRsignal.editTR.text);
Fliprad:= strtoint(MRsignal.EditFlip.text)*pi/180;
For T2 := Strtoint(MRsignal.T2from.text) to Strtoint(MRsignal.T2to.text) do

```

```

begin
  pulsantal := 1;
  Signalen[1,pulsantal,T2] := Sin(fliprad);
  Signalen[2,pulsantal,T2] := cos(fliprad);
  Signalen[3,pulsantal,T2] := Signalen[1,pulsantal,T2]*exp(-TE/T2);
  Signalen[4,pulsantal,T2] := 1+(Signalen[2,pulsantal,T2]-1)*exp(-TE/T1);
  Signalen[5,pulsantal,T2] := 0;
  Signalen[6,pulsantal,T2] := 1+(Signalen[2,pulsantal,T2]-1)*exp(-TR/T1);
  For pulsantal := 2 to strtoint(MRsignal.pulser.text) do
    begin
      Signalen[1,pulsantal,T2] := Signalen[6,pulsantal-1,T2]*Sin(fliprad);
      Signalen[2,pulsantal,T2] := Signalen[6,pulsantal-1,T2]*cos(fliprad);
      Signalen[3,pulsantal,T2] := Signalen[1,pulsantal,T2]*exp(-TE/T2);
      Signalen[4,pulsantal,T2] := 1+(Signalen[2,pulsantal,T2]-1)*exp(-TE/T1);
      Signalen[5,pulsantal,T2] := 0;
      Signalen[6,pulsantal,T2] := 1+(Signalen[2,pulsantal,T2]-1)*exp(-TR/T1);
    end;
  end;
  MRsignal.countknapp.caption := 'Done!';
end;

Procedure BeraknaFlip;
Var TE, TR, T1, T2, Flipvinkel, pulsantal : integer;
    fliprad : real;
Begin
TE := Strtoint(MRsignal.editTE.text);
TR := Strtoint(MRsignal.editTR.text);
T1 := Strtoint(MRsignal.editT1.text);
T2 := Strtoint(MRsignal.editT2.text);
For FlipVinkel := Strtoint(MRsignal.Flipfrom.text) to Strtoint(MRsignal.Flipto.text) do
  begin
    Fliprad:= FlipVinkel*pi/180;
    pulsantal := 1;
    Signalen[1,pulsantal,FlipVinkel] := Sin(fliprad);
    Signalen[2,pulsantal,FlipVinkel] := cos(fliprad);
    Signalen[3,pulsantal,FlipVinkel] := Signalen[1,pulsantal,FlipVinkel]*exp(-TE/T2);
    Signalen[4,pulsantal,FlipVinkel] := 1+(Signalen[2,pulsantal,FlipVinkel]-1)*exp(-TE/T1);
    Signalen[5,pulsantal,FlipVinkel] := 0;
    Signalen[6,pulsantal,FlipVinkel] := 1+(Signalen[2,pulsantal,FlipVinkel]-1)*exp(-TR/T1);
    For pulsantal := 2 to strtoint(MRsignal.pulser.text) do

```

```

begin
  Signalen[1,pulsantal,FlipVinkel] := Signalen[6,pulsantal-1,FlipVinkel]*Sin(fliprad);
  Signalen[2,pulsantal,FlipVinkel] := Signalen[6,pulsantal-1,FlipVinkel]*cos(fliprad);
  Signalen[3,pulsantal,FlipVinkel] := Signalen[1,pulsantal,FlipVinkel]*exp(-TE/T2);
  Signalen[4,pulsantal,FlipVinkel] := 1+(Signalen[2,pulsantal,FlipVinkel]-1)*exp(-TE/T1);
  Signalen[5,pulsantal,FlipVinkel] := 0;
  Signalen[6,pulsantal,FlipVinkel] := 1+(Signalen[2,pulsantal,FlipVinkel]-1)*exp(-TR/T1);
end;
end;
MRsignal.countknapp.caption := 'Done!';
end;

Function TotaltAntalPulser:real;
Begin
  TotaltAntalPulser :=
(StrToInt(MRsignal.editSkiva.text)/10)/(StrToInt(MRsignal.editFlow.text)*(strtoint(MRsignal.editTR.text)/1000));
End;

Function Deltjocklek:real;
Begin
  If StrToInt(MRsignal.editFlow.text)*(strtoint(MRsignal.editTR.text)/1000) <
(strtoint(MRsignal.editSkiva.text)/10) then
    Deltjocklek := StrToInt(MRsignal.editFlow.text)*(strtoint(MRsignal.editTR.text)/1000)
  else
    Deltjocklek := strtoint(MRsignal.editSkiva.text)/10;
End;

Function SnuttTjocklek:real;
Begin
  If StrToInt(MRsignal.editFlow.text)*(strtoint(MRsignal.editTR.text)/1000) <
(strtoint(MRsignal.editSkiva.text)/10) then
    SnuttTjocklek := Frac(TotaltAntalPulser)*Deltjocklek
  else
    SnuttTjocklek := strtoint(MRsignal.editSkiva.text)/10;
End;

Procedure BeraknaTEFlow;
Var TE, pulsantal: integer;
    fliprad : real;
Begin

```



```

For TE := 1 to 1500 do
  Flowsignalen[TE] := 0;
For TE := StrToInt(MRsignal.TEfrom.text) to StrToInt(MRsignal.TEto.text) do
  Begin
    For Pulsantal := 1 to trunc(TotaltAntalPulser) do
      Flowsignalen[TE] := Flowsignalen[TE] + signalen[3,pulsantal,TE]*DelTjocklek;
      Flowsignalen[TE] := Flowsignalen[TE] + signalen[3,trunc(TotaltAntalPulser)+1,TE]*SnuttTjocklek;
    End;
  end;

Procedure BeraknaTRFlow;
Var TR, Pulsantal: integer;
  fliprad : real;
Begin
  For TR := 1 to 1500 do
    Flowsignalen[TR] := 0;
  For TR := StrToInt(MRsignal.TRfrom.text) to StrToInt(MRsignal.TRto.text) do
    Begin
      For Pulsantal := 1 to trunc(TotaltAntalPulser) do
        Flowsignalen[TR] := Flowsignalen[TR] + signalen[3,pulsantal,TR]*DelTjocklek;
        Flowsignalen[TR] := Flowsignalen[TR] + signalen[3,trunc(TotaltAntalPulser)+1,TR]*SnuttTjocklek;
      End;
    end;

Procedure BeraknaT1Flow;
Var T1, Pulsantal: integer;
  fliprad : real;
Begin
  For T1 := 1 to 1500 do
    Flowsignalen[T1] := 0;
  For T1 := StrToInt(MRsignal.T1from.text) to StrToInt(MRsignal.T1to.text) do
    Begin
      For Pulsantal := 1 to trunc(TotaltAntalPulser) do
        Flowsignalen[T1] := Flowsignalen[T1] + signalen[3,pulsantal,T1]*DelTjocklek;
        Flowsignalen[T1] := Flowsignalen[T1] + signalen[3,trunc(TotaltAntalPulser)+1,T1]*SnuttTjocklek;
      End;
    end;

Procedure BeraknaT2Flow;
Var T2, Pulsantal: integer;

```

```

        fliprad : real;
Begin
  For T2 := 1 to 1500 do
    Flowsignalen[T2] := 0;
  For T2 := StrToInt(MRsignal.T2from.text) to StrToInt(MRsignal.T2to.text) do
    Begin
      For Pulsantal := 1 to trunc(TotaltAntalPulser) do
        Flowsignalen[T2] := Flowsignalen[T2] + signalen[3,pulsantal,T2]*DelTjocklek;
        Flowsignalen[T2] := Flowsignalen[T2] + signalen[3,trunc(TotaltAntalPulser)+1,T2]*SnuttTjocklek;
      End;
    end;

Procedure BeraknaFlipFlow;
Var Flip, Pulsantal: integer;
    fliprad : real;
Begin
  For Flip := 1 to 1500 do
    Flowsignalen[Flip] := 0;
  For Flip := StrToInt(MRsignal.Flipfrom.text) to StrToInt(MRsignal.Flipto.text) do
    Begin
      For Pulsantal := 1 to trunc(TotaltAntalPulser) do
        Flowsignalen[Flip] := Flowsignalen[Flip] + signalen[3,pulsantal,Flip]*DelTjocklek;
        Flowsignalen[Flip] := Flowsignalen[Flip] + signalen[3,trunc(TotaltAntalPulser)+1,Flip]*SnuttTjocklek;
      End;
    end;

procedure TMRsignal.countknappClick(Sender: TObject);
begin
  If RadioStationar.checked then
    Begin
      If (Xval.text = 'Flip angle') and (Yval.text = '') then
        BeraknaFlip
      Else If (Xval.text = 'TE') and (Yval.text = '') then
        BeraknaTE
      Else If (Xval.text = 'TR') and (Yval.text = '') then
        BeraknaTR
      Else If (Xval.text = 'T1') and (Yval.text = '') then
        BeraknaT1
      Else If (Xval.text = 'T2') and (Yval.text = '') then

```

```

        BeraknaT2
    Else If (Xval.text = 'Flip angle') and (Yval.text = 'Antal pulser') then
        BeraknaFlip;
    End
Else If RadioFlow.checked then
    Begin
        If (Xval.text = 'Flip angle') and (Yval.text = '') then
            Begin;
                BeraknaFlip;
                BeraknaFlipFlow;
            End
        Else If (Xval.text = 'TE') and (Yval.text = '') then
            Begin;
                BeraknaTE;
                BeraknaTEFlow;
            End
        Else If (Xval.text = 'TR') and (Yval.text = '') then
            Begin;
                BeraknaTR;
                BeraknaTRFlow;
            End
        Else If (Xval.text = 'T1') and (Yval.text = '') then
            Begin;
                BeraknaT1;
                BeraknaT1Flow;
            End
        Else If (Xval.text = 'T2') and (Yval.text = '') then
            Begin;
                BeraknaT2;
                BeraknaT2Flow;
            End
        End;
    end;

procedure TMRsignal.FilKnappClick(Sender: TObject);
var i, Puls: integer;
begin
If RadioStationar.checked then
    Begin
        assignfile(MRfil,EditFilNamn.text + '.txt');
    End;
end;

```

```

rewrite(MRfil);
Writeln(MRfil);
If Yval.text = '' then
  Writeln(MRfil,' Signal as a function of ',Xval.text)
else
  Writeln(MRfil,' Signal as a function of ',Xval.text,' och ',Yval.text);
If (Xval.text = 'TE') and (Yval.text = '') then
  Begin
    writeln(MRfil,'          TR          ',EditTR.text);
    writeln(MRfil,'          T1          ',EditT1.text);
    writeln(MRfil,'          T2          ',EditT2.text);
    writeln(MRfil,'          Flip angle ',EditFlip.Text);
    writeln(MRfil,'          Pulses      ',Pulser.Text);
    writeln(MRfil,'          ',Xval.text,'          Signal');
    For i := StrToInt(TEfrom.text) to StrToInt(TEto.text) do
      writeln(MRfil,'          ',i,'
',signalen[3,strToInt(Pulser.text),i]:5:6,' ')
    end
  else If (Xval.text = 'TR') and (Yval.text = '') then
    Begin
      writeln(MRfil,'          TE          ',EditTE.text);
      writeln(MRfil,'          T1          ',EditT1.text);
      writeln(MRfil,'          T2          ',EditT2.text);
      writeln(MRfil,'          Flip angle ',EditFlip.Text);
      writeln(MRfil,'          Pulses      ',Pulser.Text);
      writeln(MRfil,'          ',Xval.text,'          Signal');
      For i := StrToInt(TRfrom.text) to StrToInt(TRto.text) do
        writeln(MRfil,'          ',i,'
',signalen[3,strToInt(Pulser.text),i]:5:6,' ')
      end
    else If (Xval.text = 'T1') and (Yval.text = '') then
      Begin
        writeln(MRfil,'          TE          ',EditTE.text);
        writeln(MRfil,'          TR          ',EditTR.text);
        writeln(MRfil,'          T2          ',EditT2.text);
        writeln(MRfil,'          Flip angle ',EditFlip.Text);
        writeln(MRfil,'          Pulses      ',Pulser.Text);
        writeln(MRfil);
      End
    end
  end

```

```

        writeln(MRfil,'',Xval.text,'Signal');
        For i := StrToInt(Tlfrom.text) to Strtoint(Tlto.text) do
            writeln(MRfil,'',i,'
',signalen[3,strtoint(Pulser.text),i]:5:6,' ')
        end
    else If (Xval.text = 'T2') and (Yval.text = '') then
        Begin
            writeln(MRfil,'TE',EditTE.text);
            writeln(MRfil,'TR',EditTR.text);
            writeln(MRfil,'T1',EditT1.text);
            writeln(MRfil,'Flip angle',EditFlip.Text);
            writeln(MRfil,'Pulses',Pulser.Text);
            writeln(MRfil,'',Xval.text,'Signal');
            For i := StrToInt(T2from.text) to Strtoint(T2to.text) do
                writeln(MRfil,'',i,'
',signalen[3,strtoint(Pulser.text),i]:5:6,' ')
            end
        else If (Xval.text = 'Flip angle') and (Yval.text = '') then
            Begin
                writeln(MRfil,'TE',EditTE.text);
                writeln(MRfil,'TR',EditTR.text);
                writeln(MRfil,'T1',EditT1.text);
                writeln(MRfil,'T2',EditT2.text);
                writeln(MRfil,'Pulses',Pulser.Text);
                writeln(MRfil,'',Xval.text,'Signal');
                For i := StrToInt(Flipfrom.text) to Strtoint(Flipto.text) do
                    writeln(MRfil,'',i,'
',signalen[3,strtoint(Pulser.text),i]:5:6,' ')
                end
            else If (Xval.text = 'Flip angle') and (Yval.text = 'Number of pulses') then
                Begin
                    writeln(MRfil,'TE',EditTE.text);
                    writeln(MRfil,'TR',EditTR.text);
                    writeln(MRfil,'T1',EditT1.text);
                    writeln(MRfil,'T2',EditT2.text);
                    writeln(mrfile);
                    write(MRfil,'',Yval.text,'',Xval.text,' ');
                    For i := (StrToInt(Flipfrom.text) + 1) to Strtoint(Flipto.text) do

```

```

        write(MRfil,'          ',i,'          ');
Writeln(mRfil);
For Puls := 1 to StrToInt(Pulser.text) do
  Begin
    write(MRfil,'          ',Puls,'          ');
    For i := StrToInt(Flipfrom.text) to Strtoint(Flipto.text) do
      write(MRfil,'          ',signalen[3,Puls,i]:5:6,'          ');
      Writeln(MRfil);
    End;
  end;
closefile(MRfil);
MRsignal.countknapp.caption := 'Go!';
End
Else If RadioFlow.checked then
  Begin
    assignfile(MRfil,EditFilNamn.text + '.txt');
    rewrite(MRfil);
    Writeln(MRfil);
    If (Xval.text = 'TE') and (Yval.text = '') then
      begin
        Writeln(MRfil,'Signal as a function of TE');
        Writeln(MRfil,'Flow ',editflow.text,' cm/s');
        Writeln(MRfil,'Slice thickness ', editskiva.text,' mm');
        If TotaltAntalPulser > 1 then
          Writeln(MRfil,'Spins feel at most          ',Trunc(TotaltAntalPulser),' Pulser')
        Else
          Writeln(MRfil,'Spin does not feel more than 1 pulse');
        Writeln(MRfil);
        Writeln(MRfil,'TE          Signal');
        For i := StrToInt(TEfrom.Text) to Strtoint(TEto.text) do
          Begin
            Writeln(MRfil,'          ',i,'          ',FlowSignalen[i]:5:6);
          End;
        End;
      end;
    If (Xval.text = 'TR') and (Yval.text = '') then
      begin
        Writeln(MRfil,'Signal as a function of TR');
        Writeln(MRfil,'Flow ',editflow.text,' cm/s');
        Writeln(MRfil,'Slice thickness ', editskiva.text,' mm');
        If TotaltAntalPulser > 1 then

```

```

        Writeln(MRfil,'Spins feel at most      ',Trunc(TotaltAntalPulser),' Pulser')
Else
    Writeln(MRfil,'Spin does not feel more than 1 pulse');
Writeln(MRfil);
Writeln(MRfil,'TR                               Signal');
For i := StrToInt(TRfrom.Text) to Strtoint(TRto.text) do
    Begin
        Writeln(MRfil,'',i,'                               ',FlowSignalen[i]:5:6);
    End;
End;
If (Xval.text = 'T1') and (Yval.text = '') then
begin
    Writeln(MRfil,'Signal as a function of T1');
    Writeln(MRfil,'Flow ',editflow.text,' cm/s');
    Writeln(MRfil,'Slice thickness ', editskiva.text,' mm');
    If TotaltAntalPulser > 1 then
        Writeln(MRfil,'Spins feel at most      ',Trunc(TotaltAntalPulser),' Pulser')
    Else
        Writeln(MRfil,'Spin does not feel more than 1 pulse');
    Writeln(MRfil);
    Writeln(MRfil,'T1                               Signal');
    For i := StrToInt(T1from.Text) to Strtoint(T1to.text) do
        Begin
            Writeln(MRfil,'',i,'                               ',FlowSignalen[i]:5:6);
        End;
    End;
If (Xval.text = 'T2') and (Yval.text = '') then
begin
    Writeln(MRfil,'Signal as a function of T2');
    Writeln(MRfil,'Flow ',editflow.text,' cm/s');
    Writeln(MRfil,'Slice thickness ', editskiva.text,' mm');
    If TotaltAntalPulser > 1 then
        Writeln(MRfil,'Spins feel at most      ',Trunc(TotaltAntalPulser),' Pulser')
    Else
        Writeln(MRfil,'Spin does not feel more than 1 pulse');
    Writeln(MRfil);
    Writeln(MRfil,'T2                               Signal');
    For i := StrToInt(T2from.Text) to Strtoint(T2to.text) do
        Begin
            Writeln(MRfil,'',i,'                               ',FlowSignalen[i]:5:6);
        End;
    End;

```

```

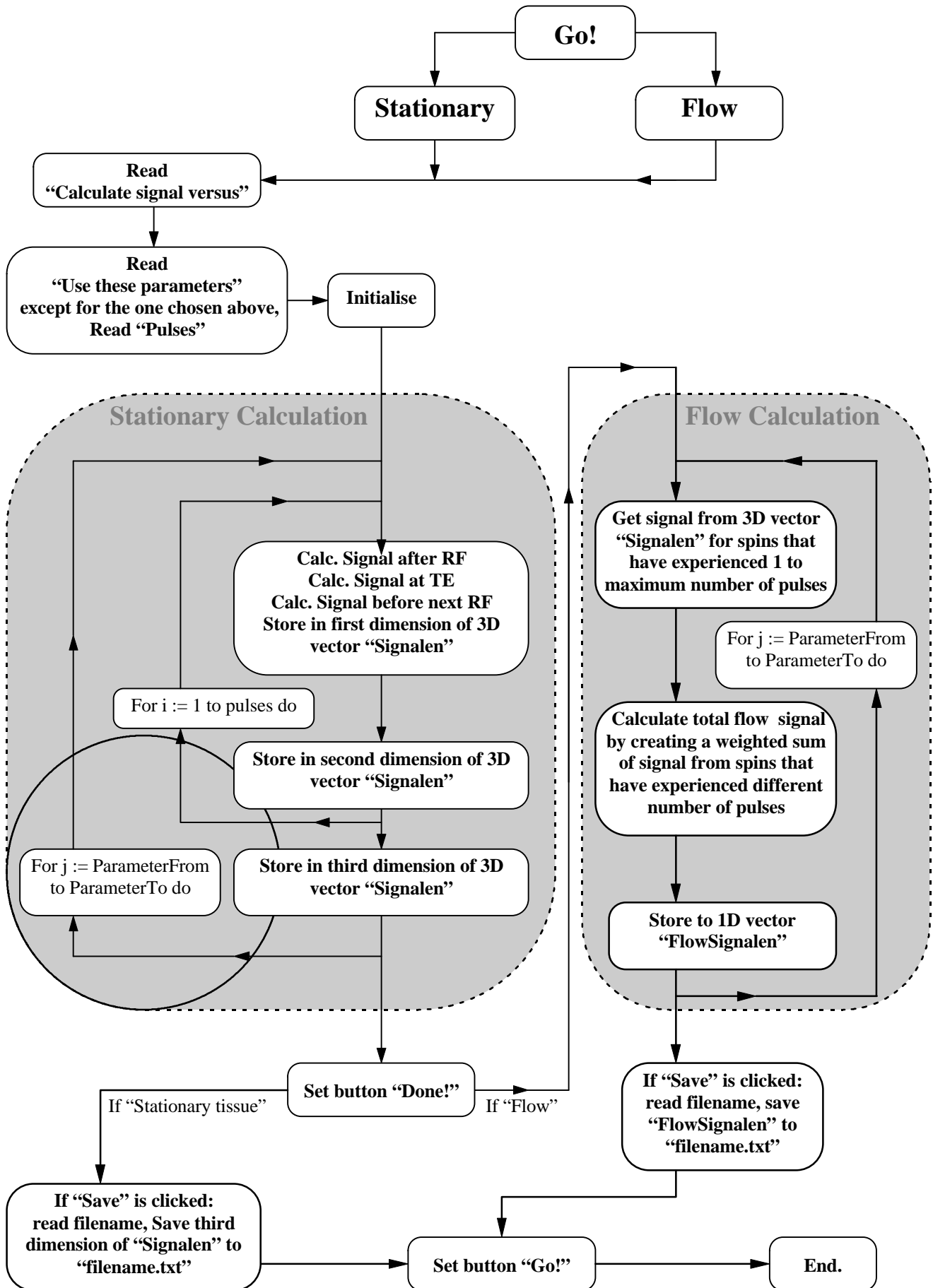
        End;
    End;
If (Xval.text = 'Flip angle') and (Yval.text = '') then
begin
    Writeln(MRfil,'Signal as a function of Flip angle');
    Writeln(MRfil,'Flow ',editflow.text,' cm/s');
    Writeln(MRfil,'Slice thickness ', editskiva.text,' mm');
    If TotaltAntalPulser > 1 then
        Writeln(MRfil,'Spins feel at most          ',Trunc(TotaltAntalPulser),' Pulser')
    Else
        Writeln(MRfil,'Spin does not feel more than 1 pulse');
    Writeln(MRfil);
    Writeln(MRfil,'Flip angle                               Signal');
    For i := StrToInt(Flipfrom.Text) to Strtoint(Flipto.text) do
        Begin
            Writeln(MRfil,' ',i,'                               ',FlowSignalen[i]:5:6);
        End;
    End;
    closefile(MRfil);
    MRsignal.countknapp.caption := 'Go!';
End;
end;

end.

```



## A.6 GEsignal Flow chart



## Appendix B Phase effects Excel sheet

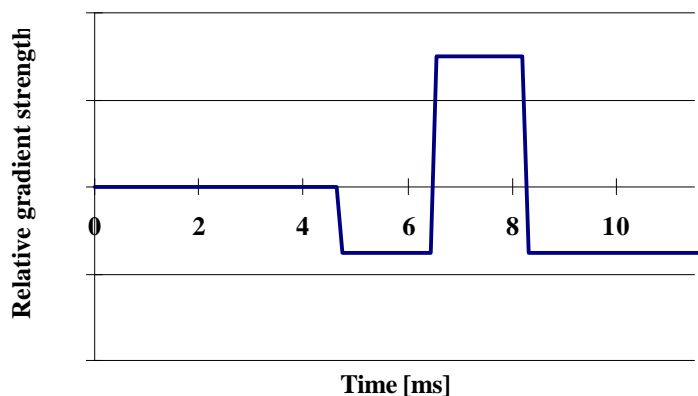
A spreadsheet (MS Excel) was set up using Eq. 3.5.3 to visualize the phase effect when varying different parameters. The first three terms (integrals) of the summation are calculated, letting the sheet show phase effects on the magnetization vector from stationary spins, spins with constant velocity and spins with constant acceleration. The sheet has two parts, one part to set the parameters used in the calculation (Table H) and one part that performs the calculation (Table I).

Col. L	Col. M	Col. N	Col. O	Col. P	Col. Q	Col. R	Col. S	Col. T	Col. U	Col. V	Col. W
	Gr-lobe 1		Gr-lobe 2		Gr-lobe 3		Gr-lobe 4		(3) Flow type		
	Start	Duration	Start	Duration	Start	Duration	Start	Duration	Stationary	x	
(1) Time step	20	20	40	20	60	40			Const. Vel.	x	
ms	10	10	20	10	30	20	0	0	Const. acc		
Grad. strength	mT/m	-3	mT/m	6	mT/m	-3	mT/m		$\gamma$	63.87	MHz/1.5T
	Range [ms]								x(0)	0.01	m
	Time step	ms							v(0)	0.5	m/s
	100	(2) 50							a(0)	1	m/s <sup>2</sup>

**Table H: Parameter change area of the “phase effects spreadsheet”. White cells are possible to vary, grey cells are either part of the calculation or just informative text.**

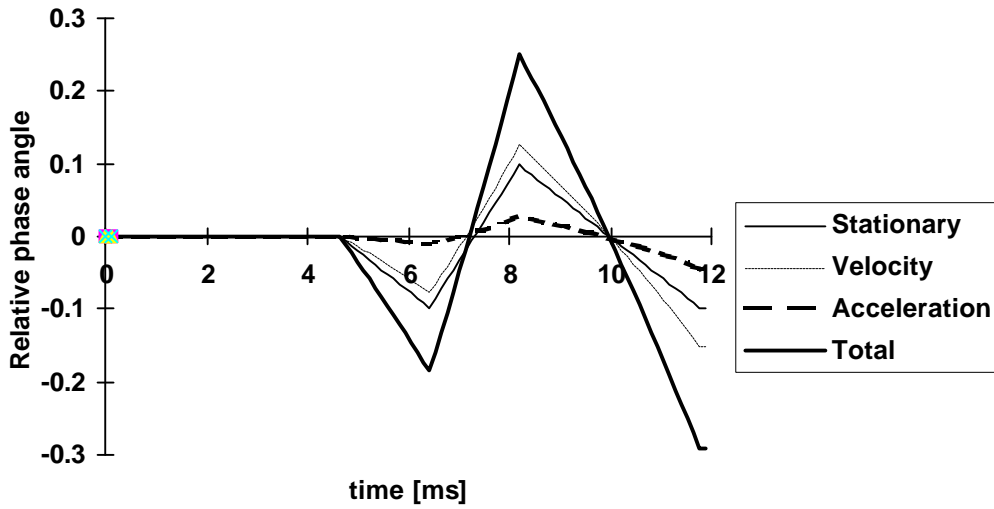
It is possible to vary up to four gradient lobes in both duration and amplitude by changing the numbers in the white cells to the right of “Time step” (1, Col. L, Table H). The grey cells **should not be changed**, they are either part of the calculation or just informative text. Total time range (the time from 0 ms to the end of the calculation) is set in the white cell (2) below “Range” and “ms”. To the right below “Flow type” (3), the user can set which types of motion that should be present by marking the respective cells with an “x”. Initial position and motion components can also be set to the right.

Recalculation is performed as soon as a parameter is changed and the result is illustrated in two diagrams below the parameter change area in the spreadsheet. One of the diagrams shows gradient strength variation during acquisition (Figure Y) and the other show how the phase angle varies due to the different types of motion and gradients applied (Figure Z).



**Figure Y: Gradient time schedule**

**Ackumulated phase for different flow types**



**Figure Z: Phase evolution**

In the gradient time schedule, the gradients appear to have a finite rise and fall time. This is just in the diagram, calculations are performed assuming perfectly rectangular gradients. In an MR scanner though, the rise and fall times are finite.

The calculation part of the sheet is found in columns A-J, these cells should not be changed. A short description of how calculation is performed is seen in Table I below.

Col. A	Col. B	Col. C	Col. D	Col. E	Col. F	Col. G	Col. H	Col. I	Col. J
Time step, used to step the time forward (downwards), total number of steps = 100	Time, recalculates time step to actual time, using column A and cell N8	Gradient strength, uses Cells M3-T3 and column B to calculate the Gradient strength at the time	Calculates the differentials for the first, second and third term (position, velocity and acceleration) in Eq. 2.4.3			Calculates the integrals for the first, second and third term (position, velocity and acceleration) in Eq. 2.4.3. This is the induced phase shift from each motion component			Total phase angle. Adds Column G-I to get the total phase angle for each time step

**Table I: Description of calculation**



# Diagrams

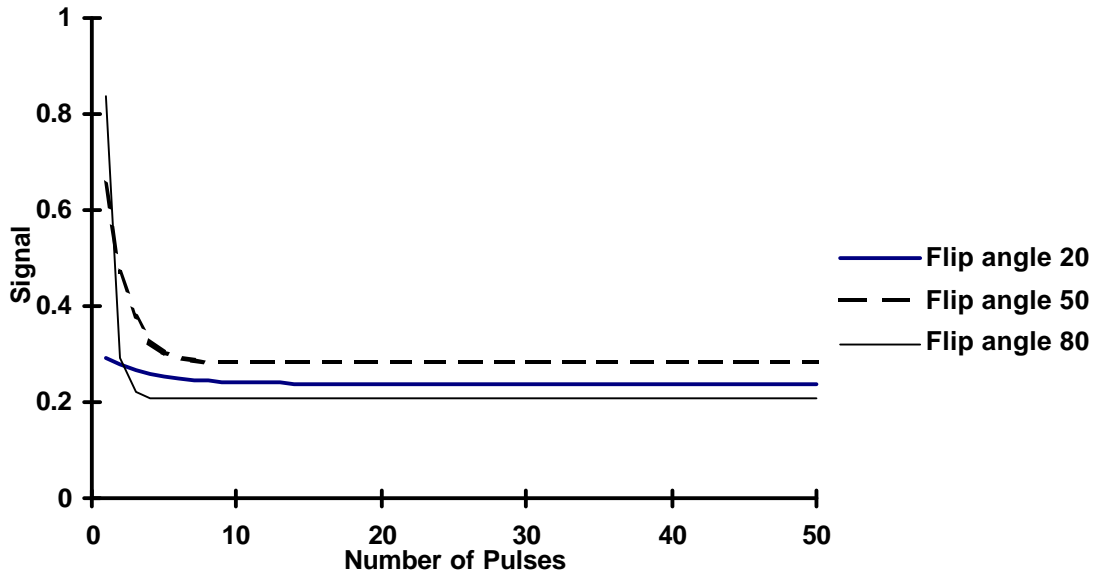


Diagram 1: Signal versus Number of experienced pulses.  $T_1 = 100$  ms,  $T_2^* = 20$  ms,  $TE = 12$  ms,  $TR = 24$  ms.

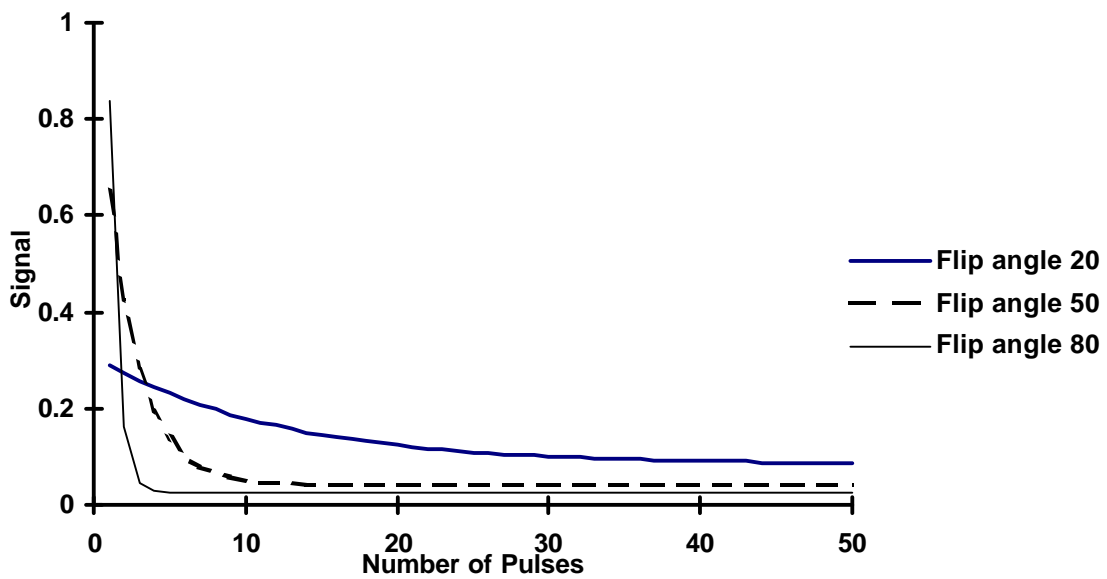


Diagram 2: Signal versus Number of experienced pulses.  $T_1 = 1000$  ms,  $T_2^* = 20$  ms,  $TE = 12$  ms,  $TR = 24$  ms.

High signal for large flip angles after one or a few experienced RF pulses is seen, but as the number of pulses increase the signal will fall rapidly due to the long time needed to recover. Small flip angles instead, show a fairly low initial signal, but the recovering time is short and therefore the signal decrease will not be as rapid as for large flip angles. Steady state signal is higher in tissue with a short  $T_1$  (Diagram 1), than in tissue with long  $T_1$  (Diagram 2). The flip angle yielding highest signal is dependent on the  $T_1$  used.

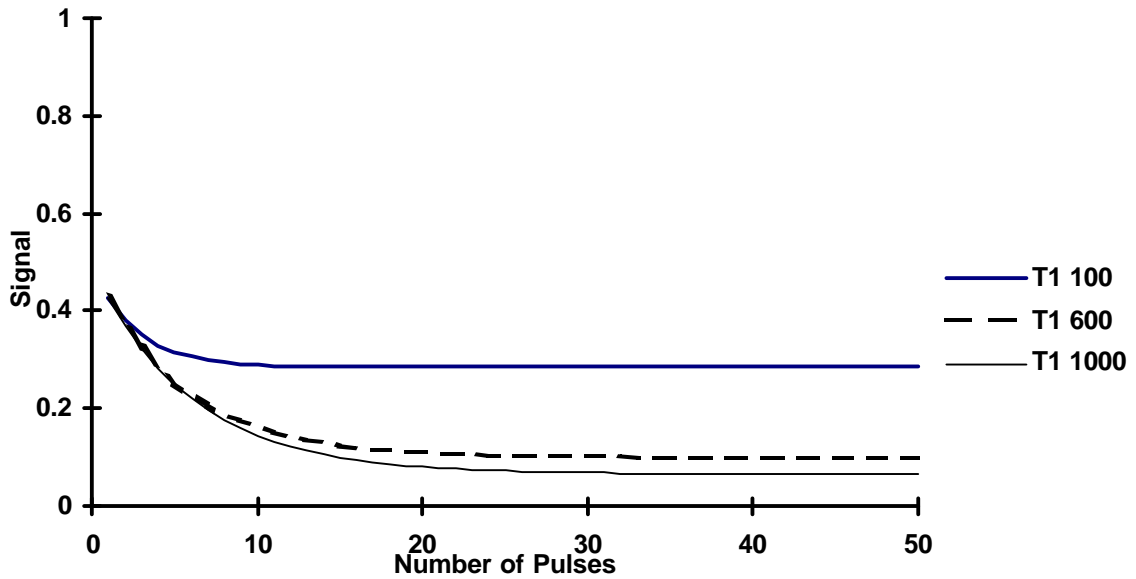


Diagram 3: : Signal versus Number of experienced pulses. Flip angle = 30°, T2\* = 20 ms, TE = 12 ms, TR = 24 ms.

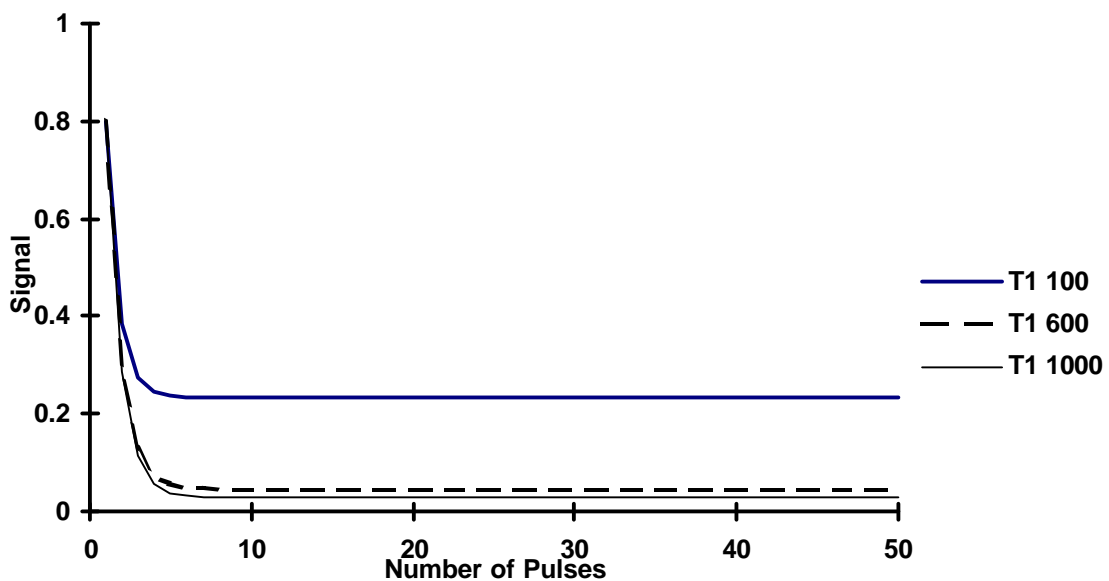


Diagram 4: Signal versus Number of experienced pulses. Flip angle = 70°, T2\* = 20 ms, TE = 12 ms, TR = 24 ms.

In tissue with a short T1, the longitudinal magnetisation needs less time to recover between pulses than in matter with a long T1. This result in a higher steady state signal for tissue with short T1 than for tissue with long T1. The flip angle affects how rapidly signal will fall when multiple pulses are delivered. For a small flip angle (Diagram 3), signal decrease less rapidly than for a large flip angle (Diagram 4) as the number of experienced pulses increases.

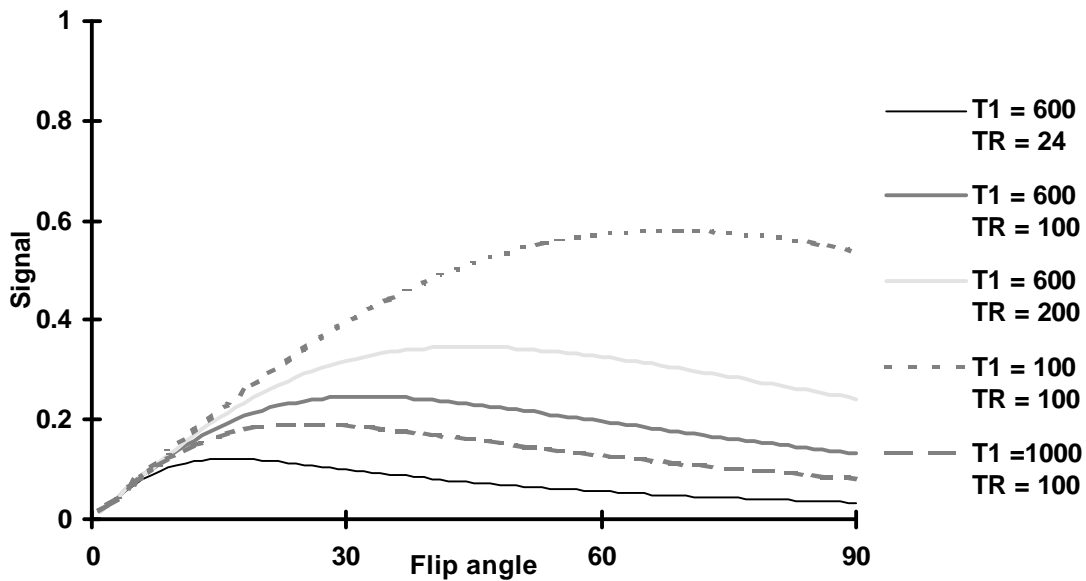


Diagram 5: Signal versus flip angle plotted for various T1 and TR.  $T_2^* = 20$  ms, TE = 12 ms. The steady state signal from a spoiled gradient echo sequence used on stationary tissue will yield highest signal at a flip angle called the Ernst angle. This optimal flip angle is dependent on both TR of the sequence and T1 of the tissue being imaged.

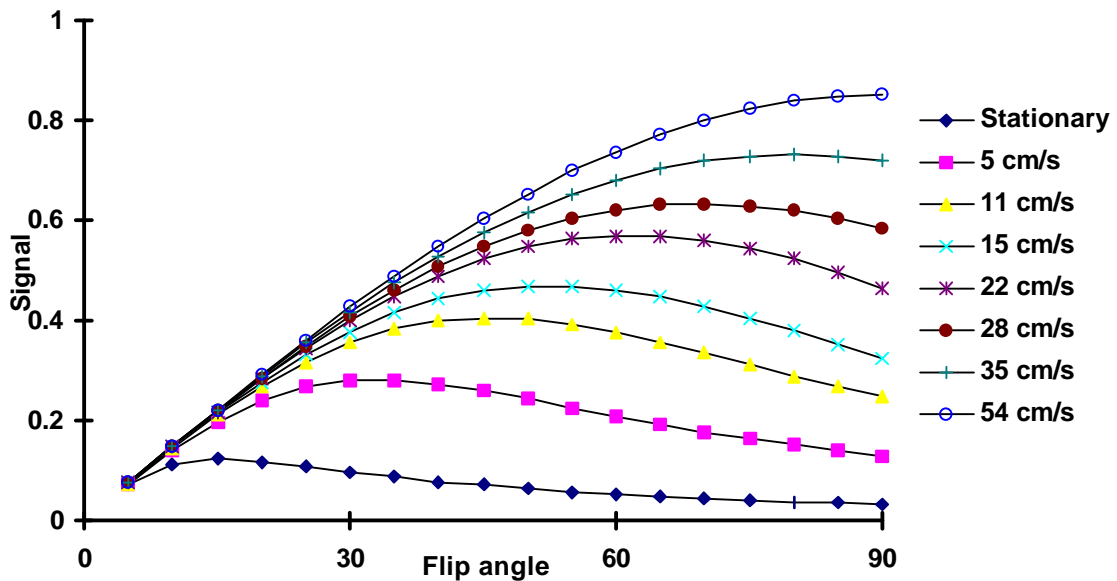
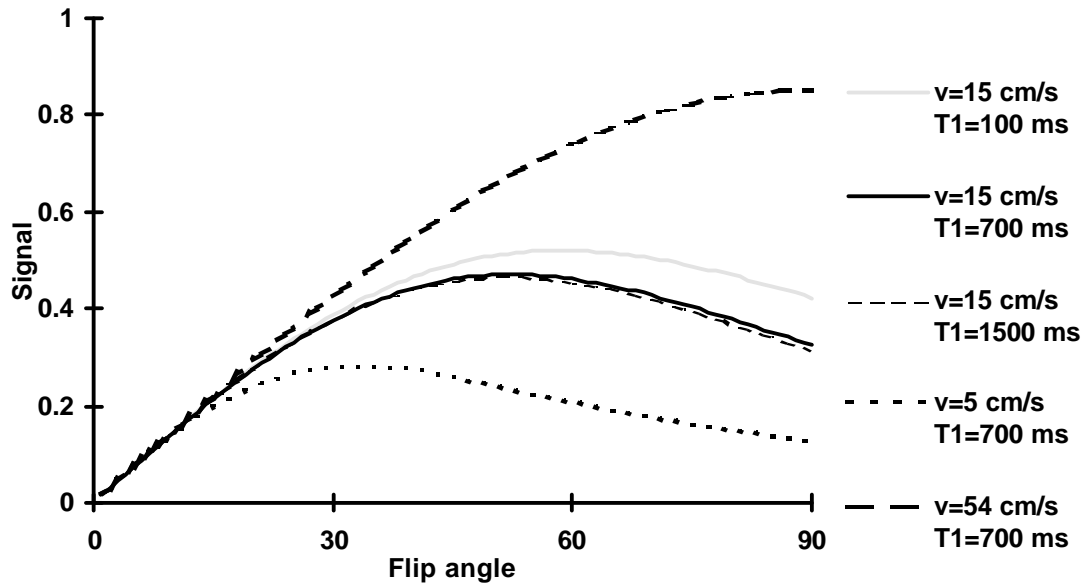


Diagram 6: Signal versus flip angle, plotted for various flow velocities (plug flow assumed). T1 = 700 ms,  $T_2^* = 20$  ms, TE = 12 ms, TR = 24 ms, slice thickness = 10 mm. Flow perpendicular to an image slice results in higher signal in areas with flow than in stationary areas, the so called inflow effect. Inflow signal will increase with increasing velocity until no spins feel more than one pulse ( $v \geq d / TR \approx 40$  cm/s). For higher flow velocities the signal remains constant. The optimal flip angle does not only depend on TR and T1 (Diagram 5) but also on slice thickness and flow velocity. Low flow velocity allows the flowing spins to experience many RF pulses and the signal behaviour will be close to that for stationary spins, yielding an optimal flip angle closer to the Ernst angle. A high flow velocity result in few experienced pulses for the spins. This favours a larger flip angle.





**Diagram 7: Signal versus flip angle for various T1 and flow velocities. TE = 12 ms, T2\* = 20 ms. If T1 is lowered in tissue, the time needed for the longitudinal magnetisation to grow back between pulses is reduced. This results in two effects of interest. 1) The optimal flip angle increases since more of the longitudinal magnetisation is recovered between RF pulses if T1 is lowered. 2) A higher steady state signal. The faster the longitudinal magnetisation is recovered, the more magnetisation will end up in the transverse plane when the next RF pulse is delivered. The diagram show that effects of T1 are less prominent than inflow effects.**

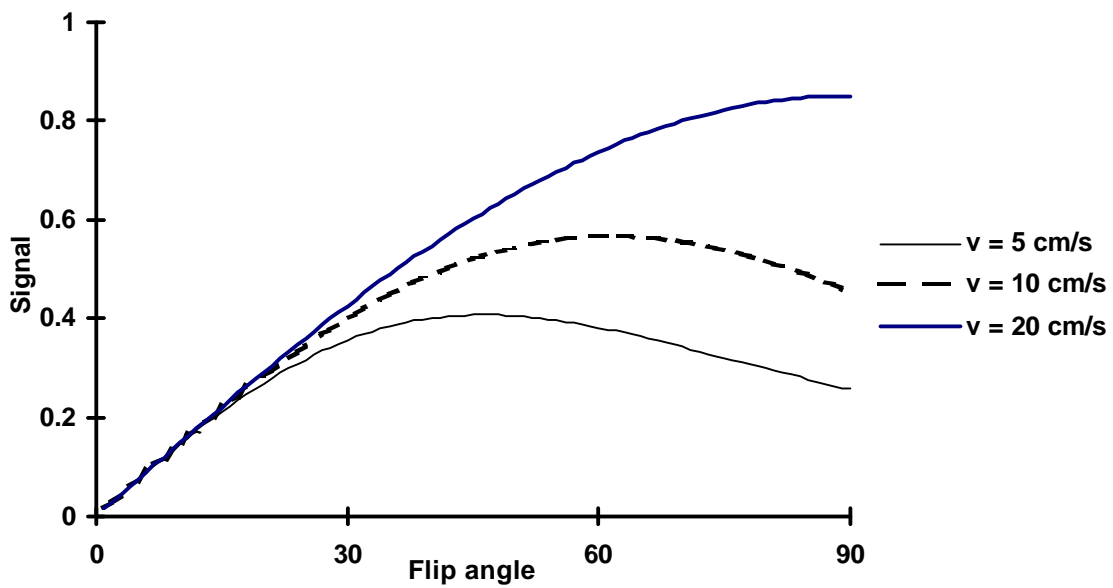


Diagram 8: Signal versus Flip angle for various flow velocities. TE = 12 ms, TR = 50 ms, T1 = 700 ms, T2\* = 20 ms, slice thickness 10 mm.

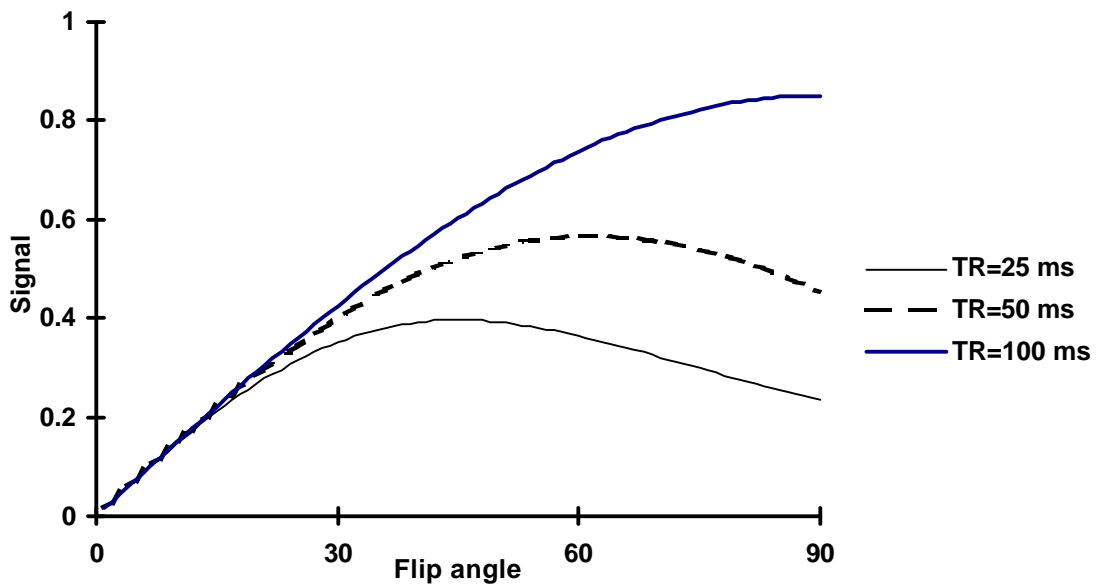


Diagram 9: Signal versus Flip angle for various TR. TE = 12 ms, T1 = 700 ms, T2\* = 20 ms, Flow velocity v = 10 cm/s, slice thickness 10 mm.

The two dashed lines in diagram 8-9 are calculated with the same parameters. The thick lines are the result of calculations performed with flow velocity v and TR respectively halved. Thin lines correspond to the double flow velocity and TR respectively. As seen the total inflow signal vary in the same manner if TR and v are varied. An increase of flow velocity or TR decrease the average number of experienced pulses for the flowing spins and thereby the total signal is increased.

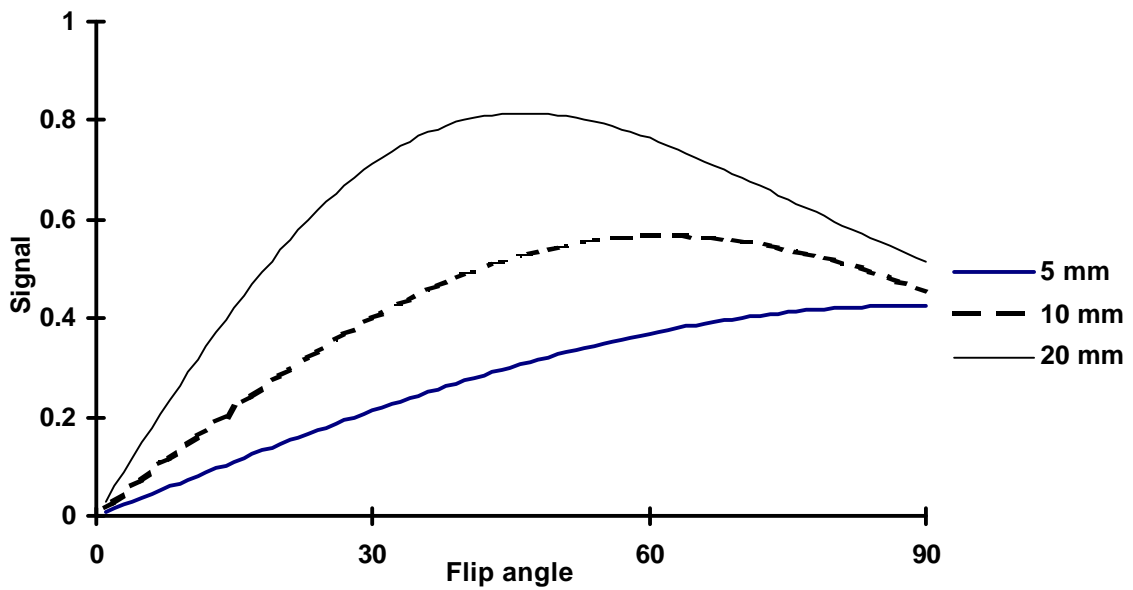


Diagram 10: Signal versus flip angle for different slice thickness. TE = 12 ms, TR = 50 ms T1 = 700 ms, T2\* = 20 ms, Flow velocity v = 10 cm/s.

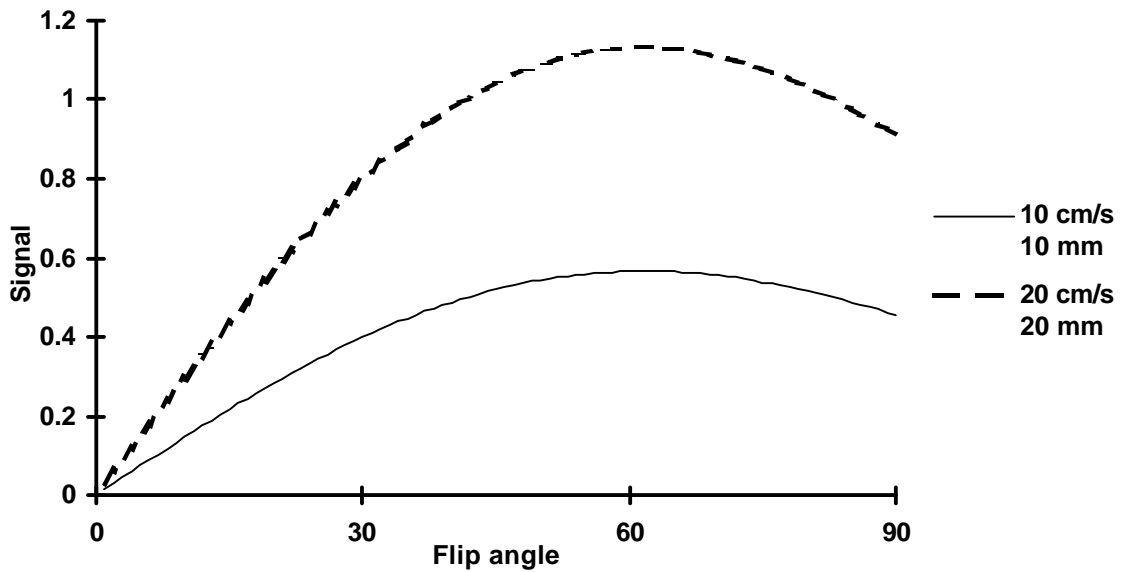


Diagram 11: Signal versus flip angle for different slice thickness and flow velocities. TE = 12 ms, TR = 50 ms T1 = 700 ms, T2\* = 20 ms.

If  $d$  is varied, there will be an effect in signal not only due to inflow, but also from the fact that the number of spins giving signal is changed. In Diagram 10, the dashed line is calculated using the same parameters as in Diagram 8 and Diagram 9, the thick line should yield a signal enhancement due to inflow but is instead lower than the dashed line. The opposite holds for the thin line. If the slice thickness is doubled and the flow velocity is doubled at the same time, no inflow signal enhancement will appear, but the total signal is doubled since the total number of spins are doubled (Diagram 11).

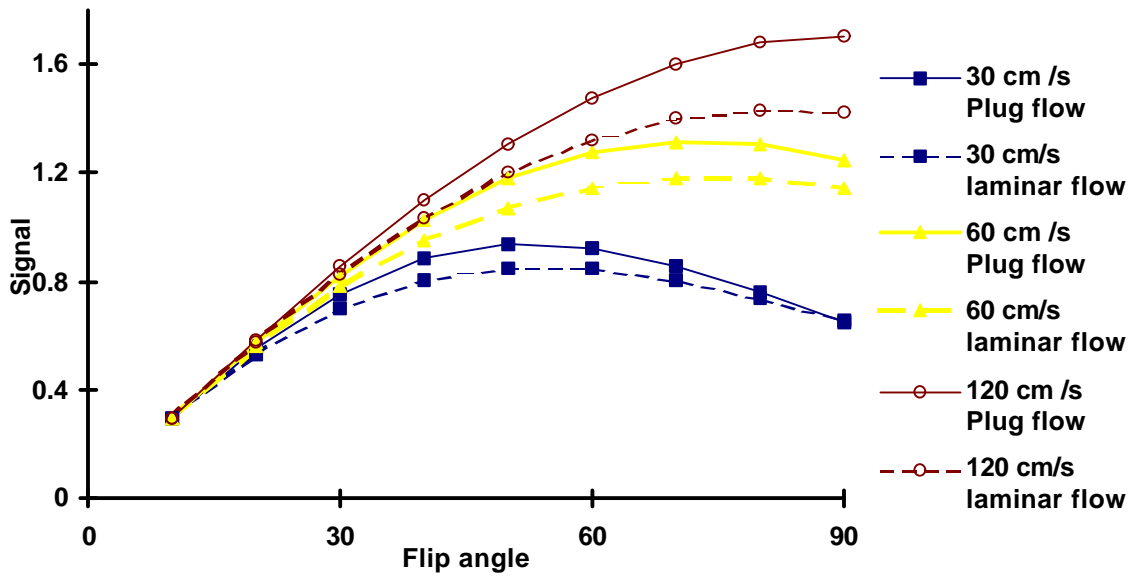


Diagram 12: Signal versus flip angle for different flow velocities calculated for both plug flow and laminar flow in a straight circular pipe. TE = 12 ms, TR = 50 ms T1 = 700 ms, T2\* = 20 ms, slice thickness = 20 mm. Parameters are chosen in order to see how the difference between plug flow and laminar flow varies with flow velocity.

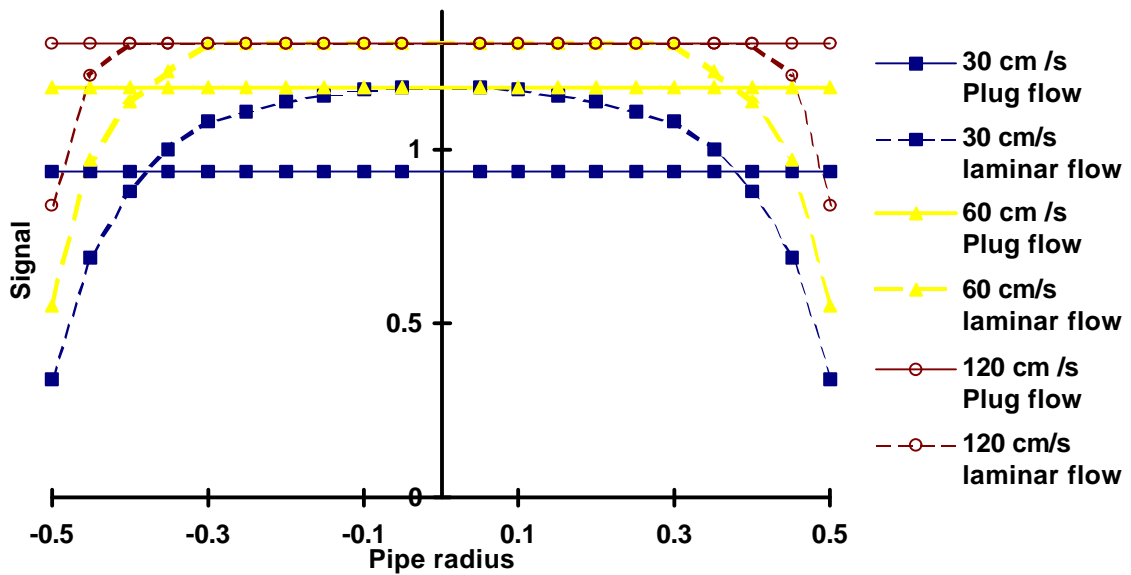
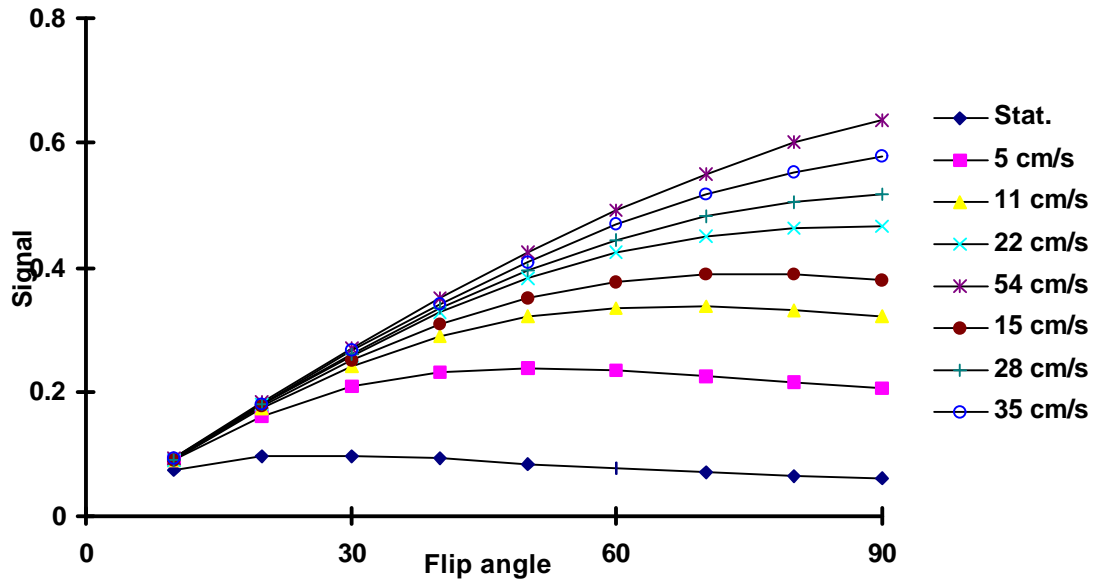


Diagram 13: Signal versus pipe radius for different flow velocity calculated for both plug flow and laminar flow in a straight circular pipe. TE = 12 ms, TR = 50 ms T1 = 700 ms, T2\* = 20 ms, slice thickness = 20 mm, flip angle = 50°.

The higher the flow velocity the more does the plug flow model overestimate the signal compared to laminar flow (Diagram 12). For low velocity laminar flow, signal is high in the middle of the pipe and low in the outer regions. Signal from the outer regions are therefore “compensated” by signal from the centre of the pipe yielding a total signal close to the signal from the mean velocity (used to calculate plug flow signal). In the case of high velocity laminar flow, a great deal of the flowing spins does only experience one RF pulse therefore yielding a constant signal regardless of flow velocity. The plug flow model uses the mean velocity to calculate signal and therefore overestimates the signal, especially in the outer regions.



**Diagram 14: Signal versus flip angle, plotted for various flow velocities (plug flow assumed). The calculation is performed using a Gaussian flip angle distribution over the profile of an image slice.  $T_1 = 700$  ms,  $T_2^* = 20$  ms, TE = 12 ms, TR = 24 ms, slice thickness = 10 mm.**

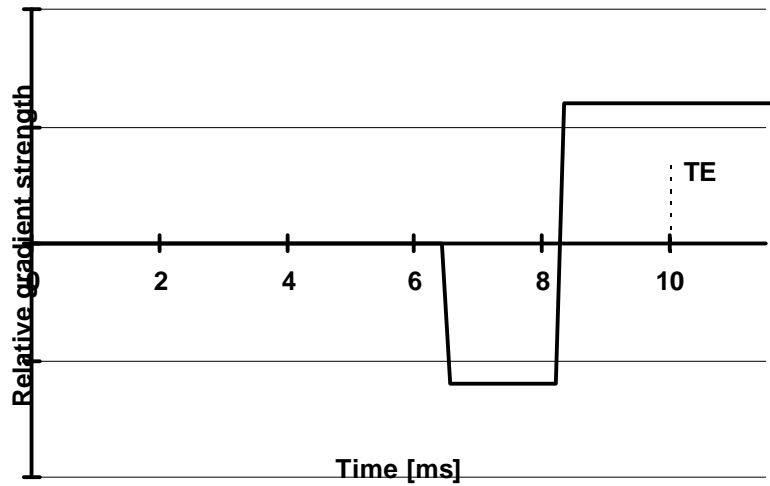


Diagram 15: Gradient strength scheme for a bipolar gradient.

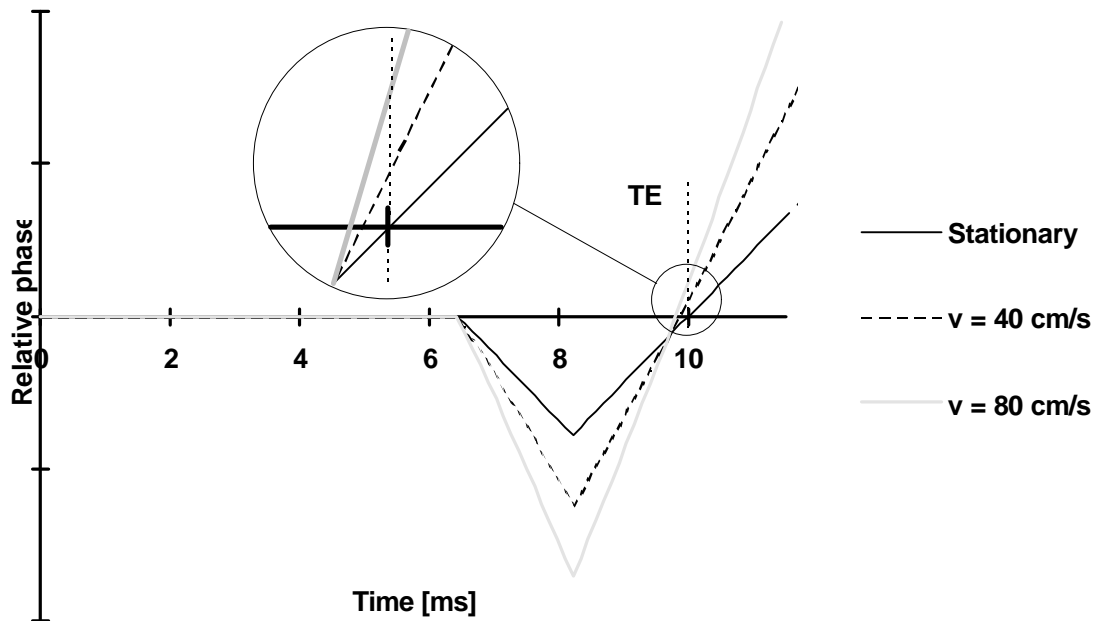


Diagram 16: Phase evolution for stationary spins and for spins moving with constant velocity in the direction of a bipolar gradient. Stationary spins are rephased at  $TE = 10$  ms and will form an echo. Flowing spins are out of phase. The accumulated phase at TE is proportional to flow velocity.

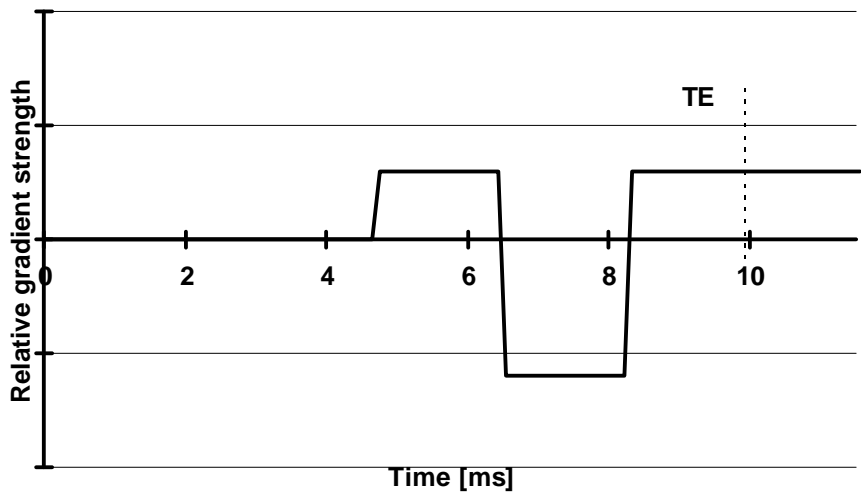


Diagram 17: Gradient strength scheme compensating for flow with constant velocity.

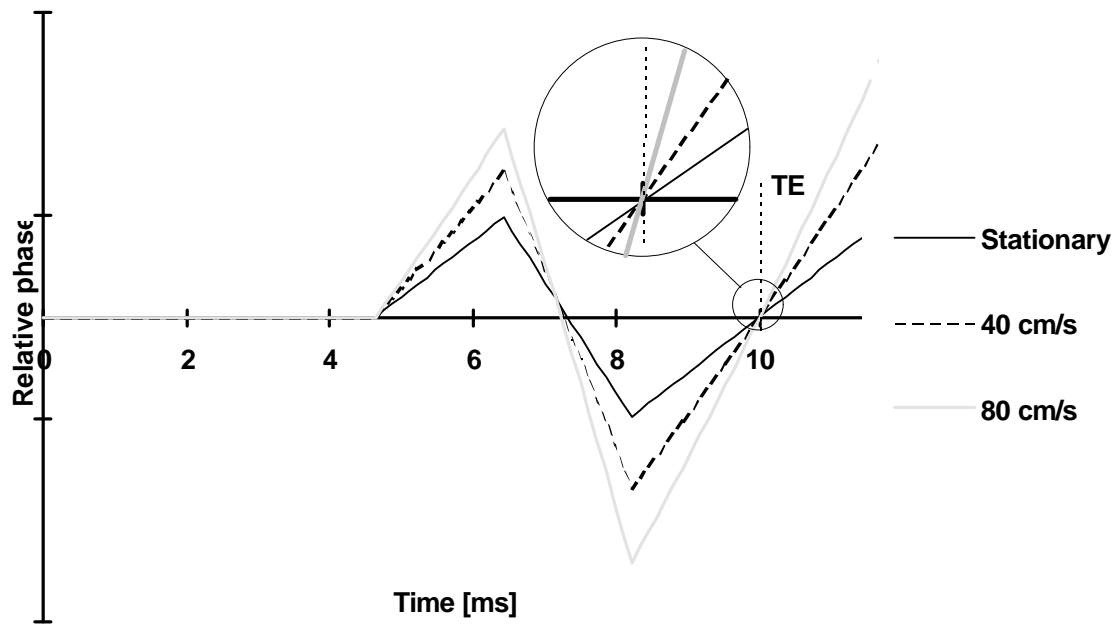


Diagram 18: Phase evolution for stationary spins and for spins moving with constant velocity in the direction of gradient lobes compensating for flow. Flowing spins are assumed to have constant velocity. Both stationary spins and flowing spins (regardless of flow velocity) are rephased at TE = 10 ms and will form an echo.

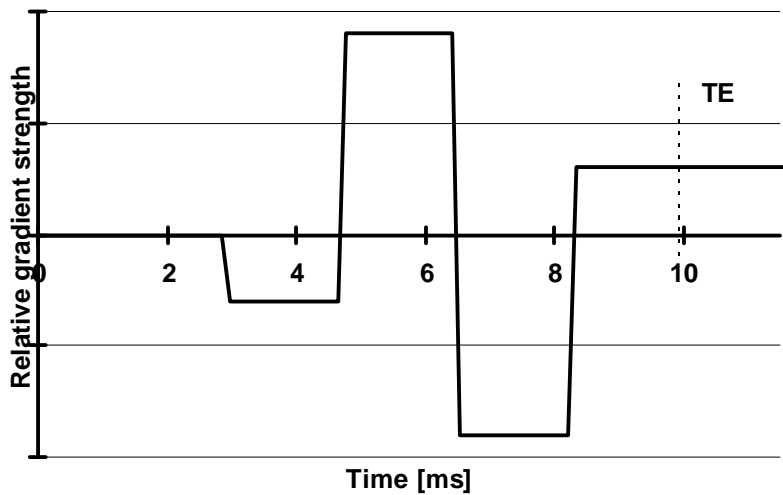


Diagram 19: Gradient strength scheme compensating for flow with constant velocity as well as for flow with constant acceleration.

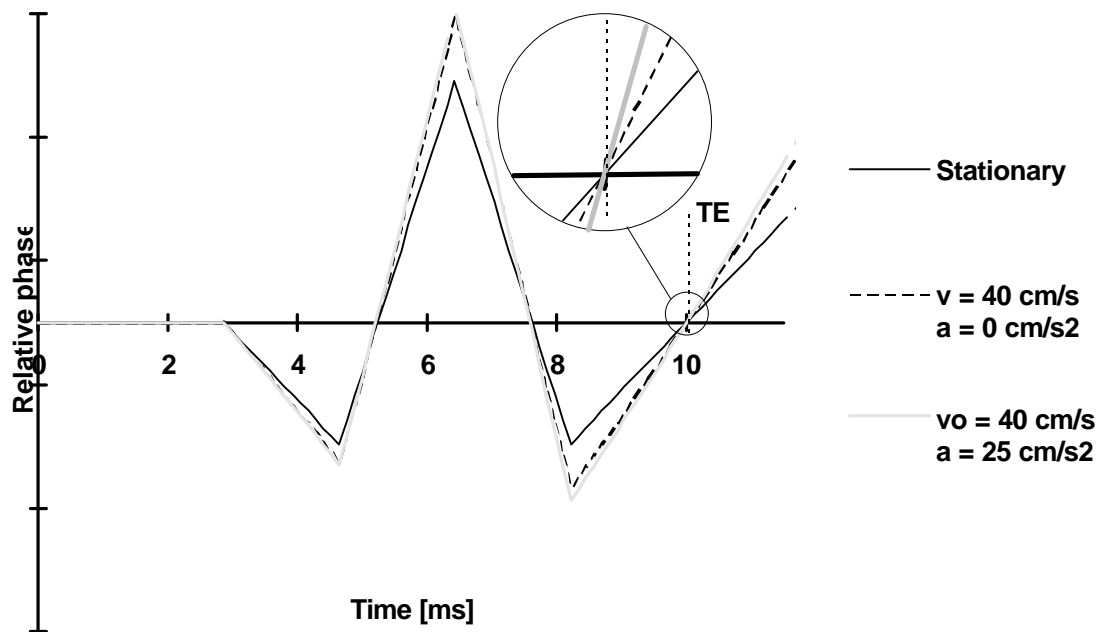


Diagram 20: Phase evolution for stationary spins and for flowing spins. Both spins flowing with constant velocity and spins flowing with constant acceleration are represented.  $v_0$  means the spin velocity at  $t = 0$ . Flow is assumed to be in the direction of the gradient lobes of Diagram 19. Stationary spins as well as flowing spins (regardless of flow velocity and acceleration) are rephased at  $TE = 10$  ms and will form an echo. A spin acceleration of  $25 \text{ cm/s}^2$  is extraordinary high, but even with such a strong acceleration the phase effect of acceleration is minor. The use of acceleration compensation is thus not very common.



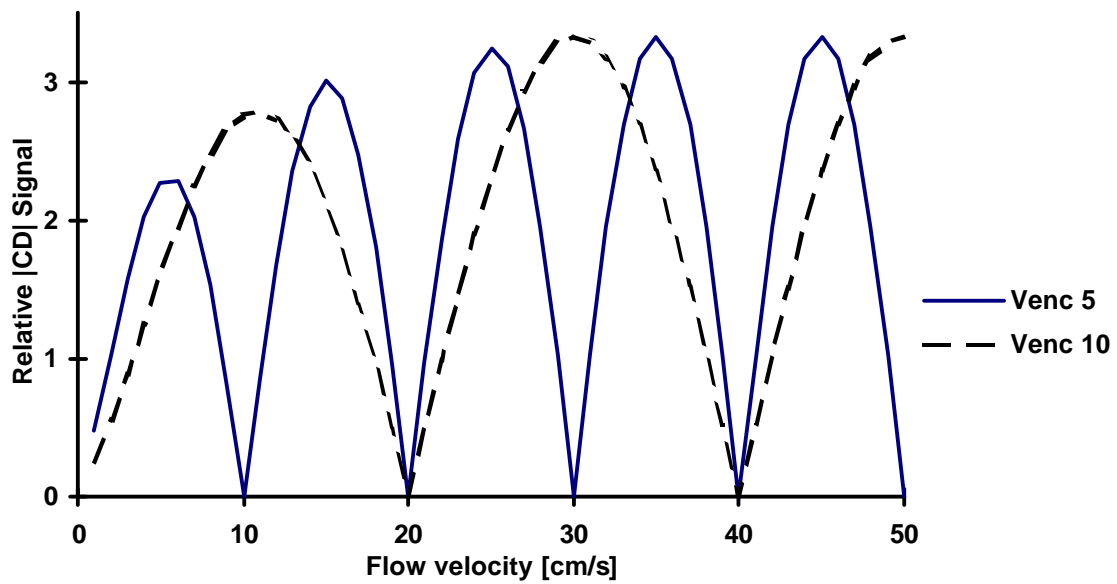


Diagram 21: Phase Contrast Complex Difference signal plotted versus flow velocity. Plug flow is assumed. TE = 11 ms, TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 40°, slice thickness d = 30 mm.

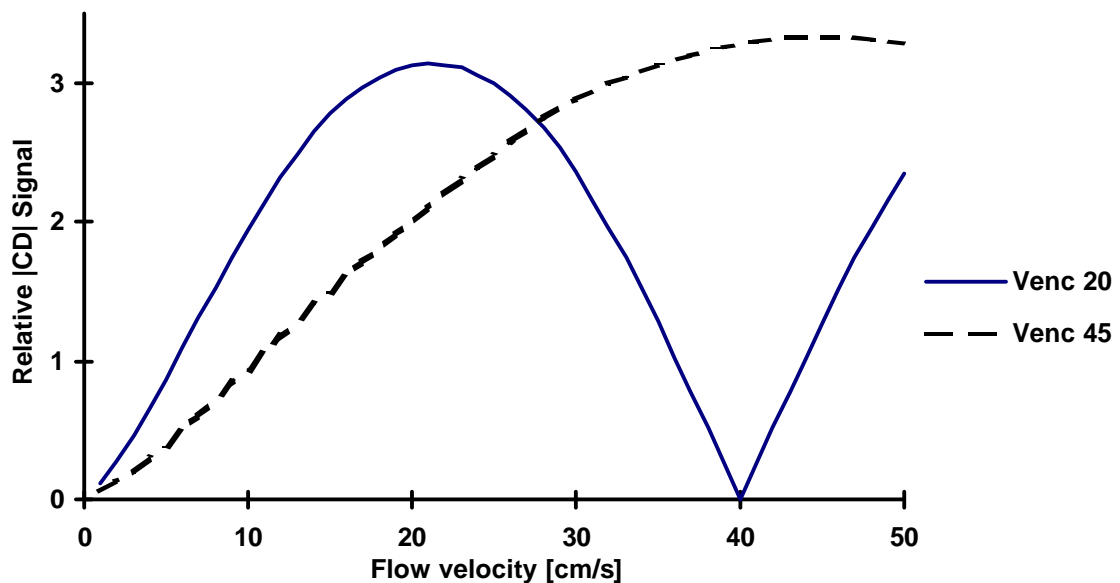


Diagram 22: Phase Contrast Complex Difference signal plotted versus flow velocity. Plug flow is assumed. TE = 11 ms, TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 40°, slice thickness d = 30 mm.

For ideal plug flow the CD signal has maxima at  $2(n+1)$  times the venc velocity and minima at  $2n$  times venc velocity,  $n = 0, 1, 2, \dots$ . Inflow effects are also seen, signal is generally increasing until maximum inflow effect are reached (that is when no spin feel more than one pulse,  $v \geq d / TR \approx 30$  cm/s).

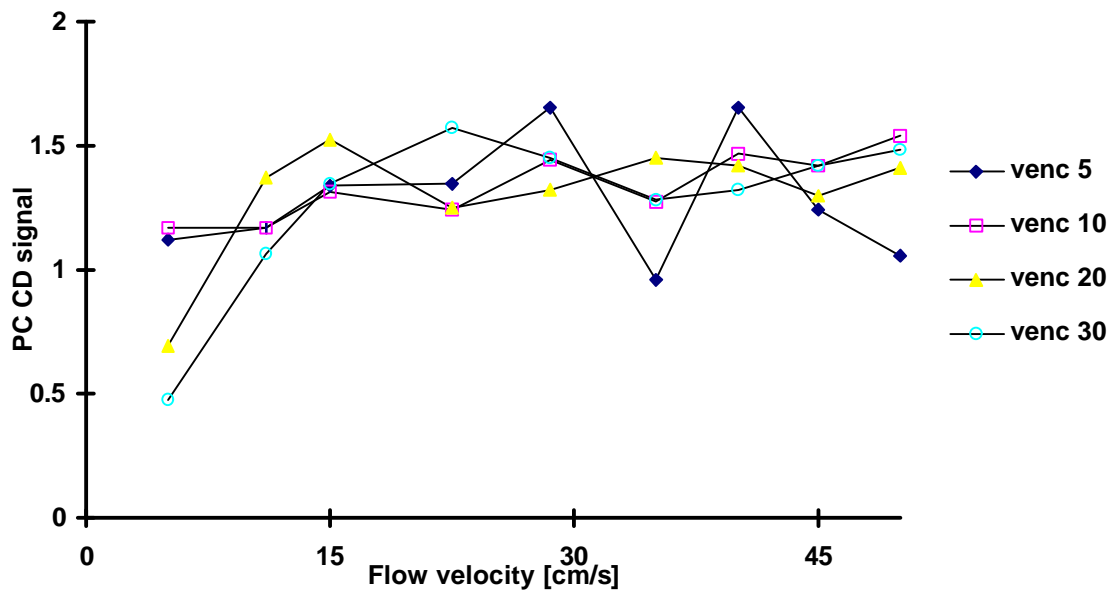


Diagram 23: Phase Contrast Complex Difference signal plotted versus flow velocity. Laminar flow is assumed. TE = 11 ms, TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm.

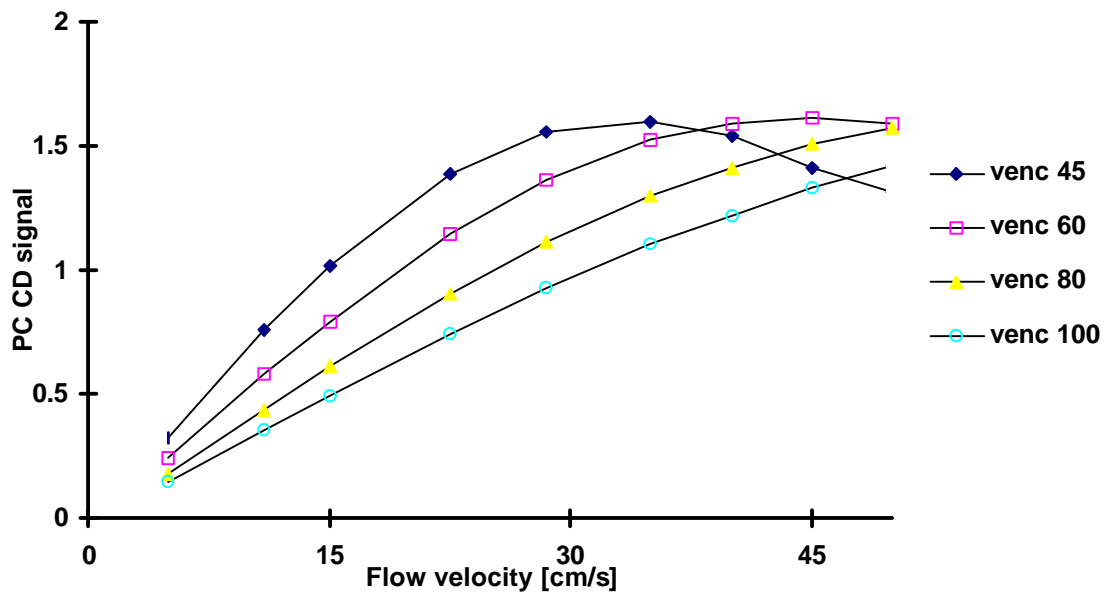
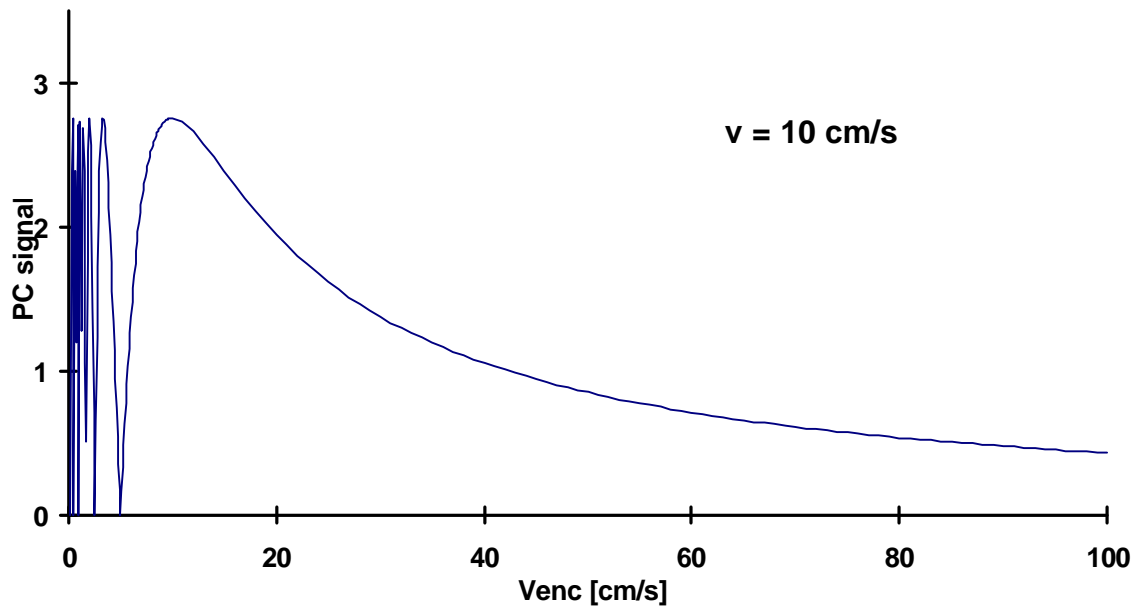


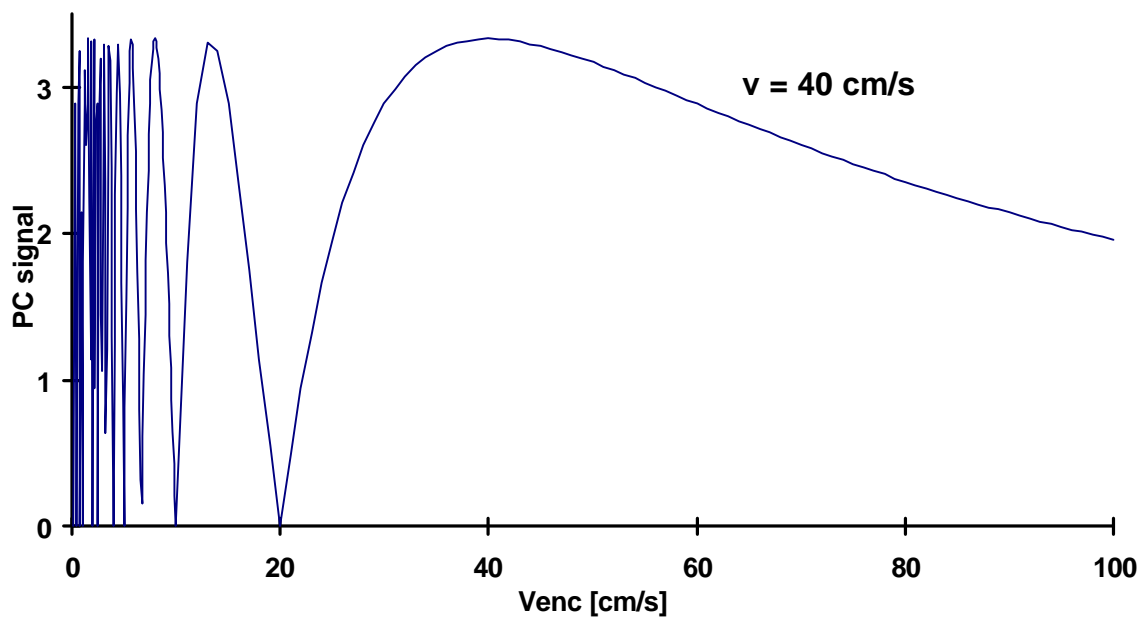
Diagram 24: Phase Contrast Complex Difference signal plotted versus flow velocity. Laminar flow is assumed. TE = 11 ms, TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm.

Since the calculation to obtain PC signal for laminar flow is so complex, the curves are not plotted for as many points as in the case of plug flow (Diagram 21-Diagram 22). This should be kept in mind when studying Diagram 23-24. Since there is a velocity distribution in a pipe with laminar flow, the total signal loss at velocities yielding an accumulated phase of  $n2\pi$ ,  $n=0, 1, 2, \dots$  (Diagram 21-22) is not seen. After the first maximum, the signal never seem to go under half its

**maximum. The maximum signal is no longer obtained at the velocity corresponding to the  $v_{enc}$ . For these calculations, the first signal maximum is obtained at lower velocities than the  $v_{enc}$ .**



**Diagram 25: Phase Contrast Complex Difference signal plotted versus  $v_{enc}$  for a flow velocity of 10 cm/s. Plug flow is assumed. TE = 11 ms, TR = 104 ms, T1 = 700 ms,  $T2^* = 20$  ms, Flip angle = 30°, slice thickness = 30 mm.**



**Diagram 26: Phase Contrast Complex Difference signal plotted versus  $v_{enc}$  for a flow velocity of 40 cm/s. Plug flow is assumed. TE = 11 ms, TR = 104 ms, T1 = 700 ms,  $T2^* = 20$  ms, Flip angle = 30°, slice thickness = 30 mm.**

When plug flow is assumed, PC CD signal has a maximum at the  $v_{enc}$  corresponding to the present flow velocity. Signal decreases with increasing  $v_{enc}$  if the  $v_{enc}$  is higher than the flow velocity. For lower  $v_{enc}$ :s the signal behaviour is very complex.

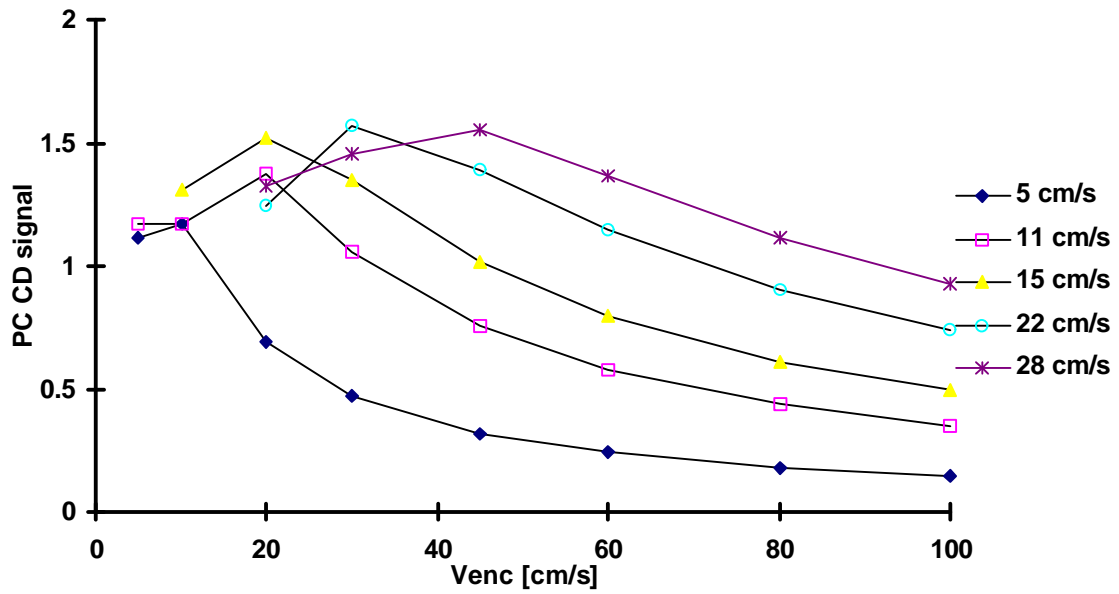


Diagram 27: Phase Contrast Complex Difference signal plotted versus  $v_{enc}$  for low velocities. Laminar flow is assumed. TE = 11 ms, TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm.

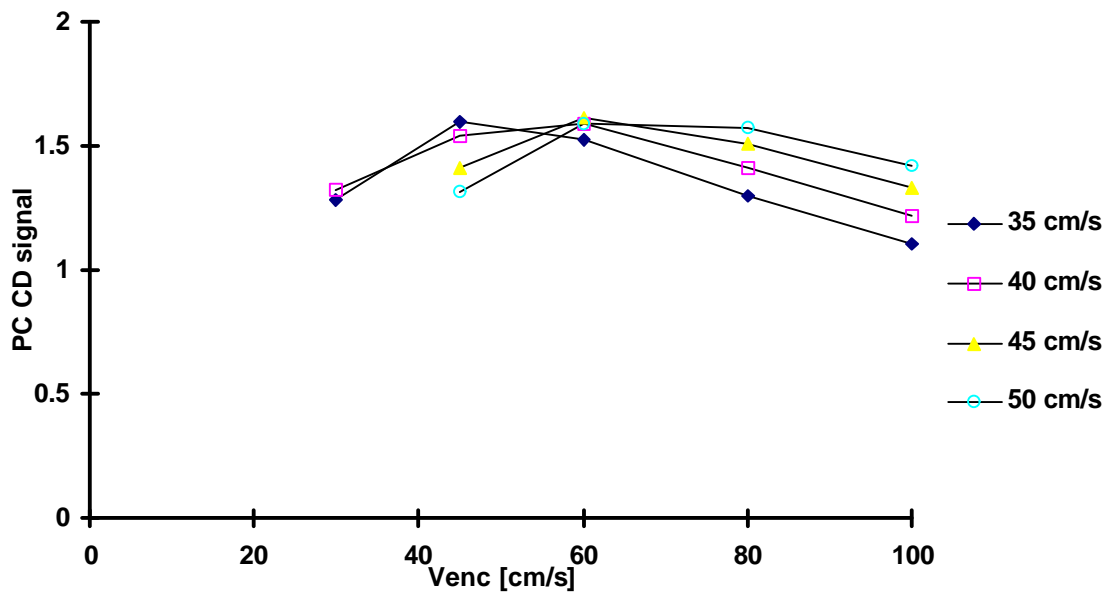


Diagram 28: Phase Contrast Complex Difference signal plotted versus  $v_{enc}$  for high flow velocities. Laminar flow is assumed. TE = 11 ms, TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm.

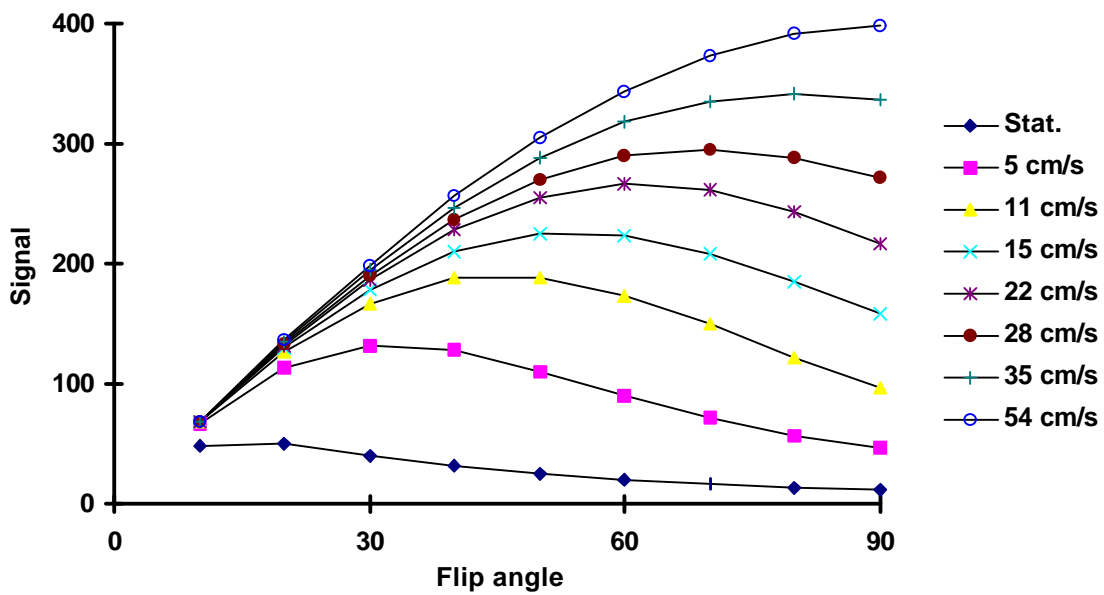


Diagram 29: Simulated signal plotted versus flip angle for various flow velocities. Plug flow perpendicular to the image slice was assumed.  $T_1 = 700$  ms,  $T_2^* = 20$  ms,  $TE = 12$  ms,  $TR = 24$  ms, slice thickness = 10 mm.

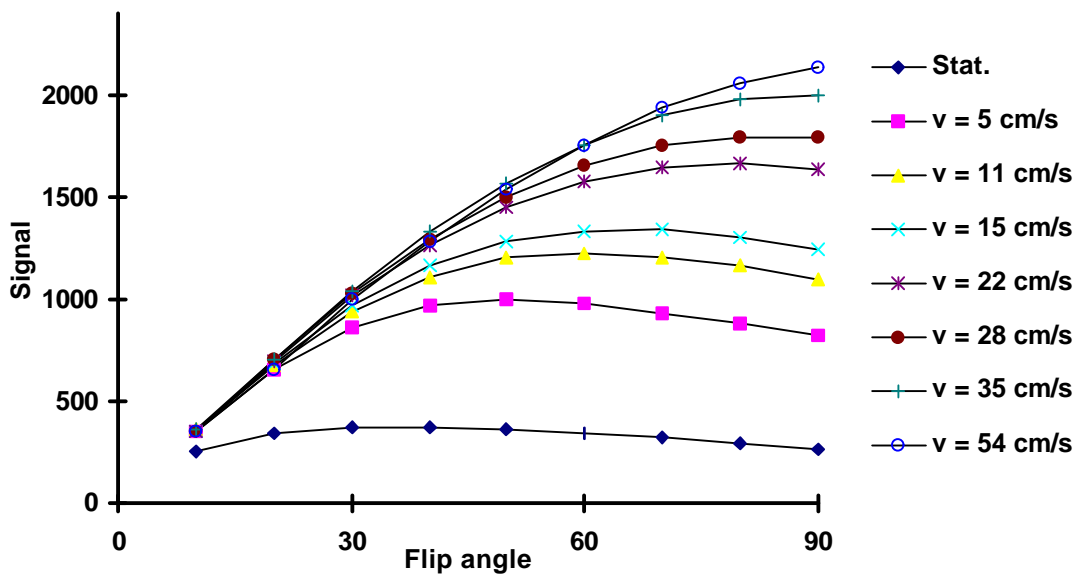


Diagram 30: Measured signal versus flip angle, plotted for various flow velocities.  $T_1 = 700$  ms,  $T_2^* = 20$  ms,  $TE = 12$  ms,  $TR = 24$  ms, slice thickness = 10 mm. Flow perpendicular to an image slice result in higher signal in areas with flow than in stationary areas, the so called inflow effect. Inflow signal increase with increasing velocity until no spins feel more than one pulse ( $v \geq d / TR \approx 40$  cm/s). For higher flow velocities signal remains constant.

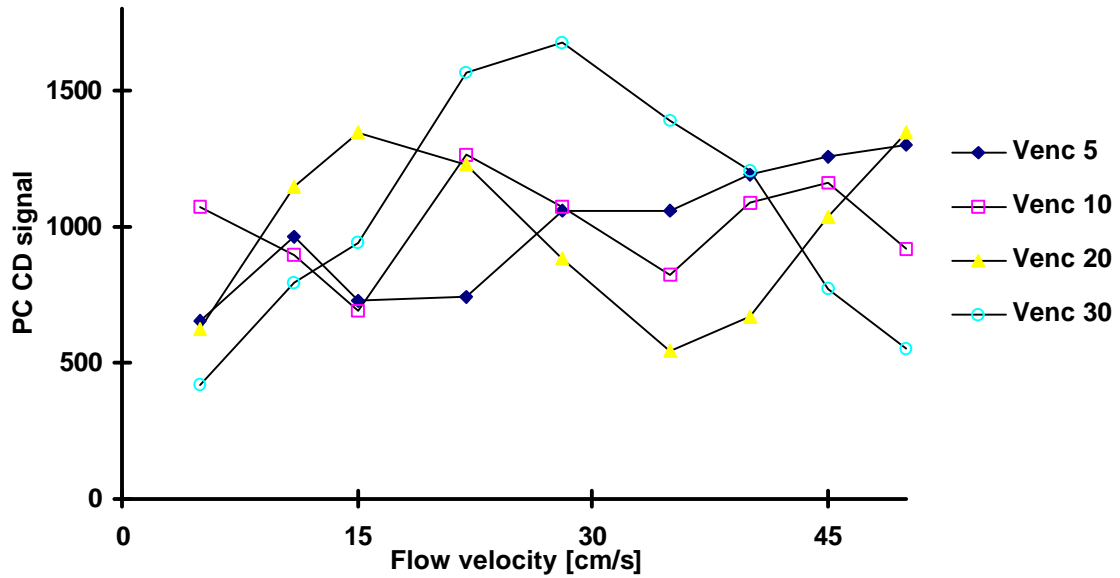


Diagram 31: Measured Phase Contrast Complex Difference signal plotted versus flow velocity for different  $v_{enc}$ :s. TE = 7-14 ms (depending on the sequence used), TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm.

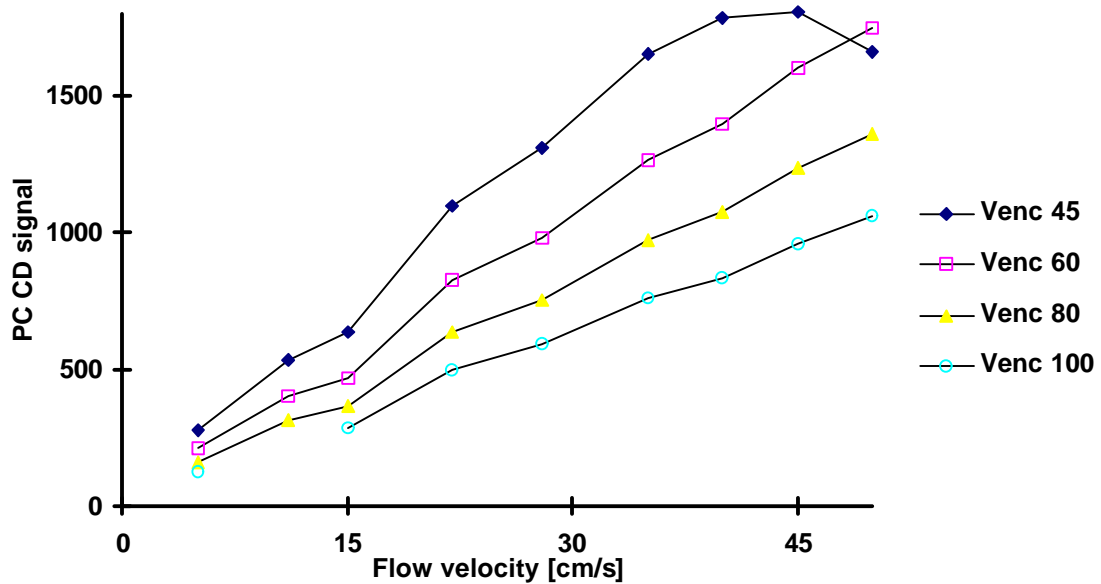


Diagram 32: Measured Phase Contrast Complex Difference signal plotted versus flow velocity for different  $v_{enc}$ :s TE = 7-14 ms (depending on the sequence used), TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm.

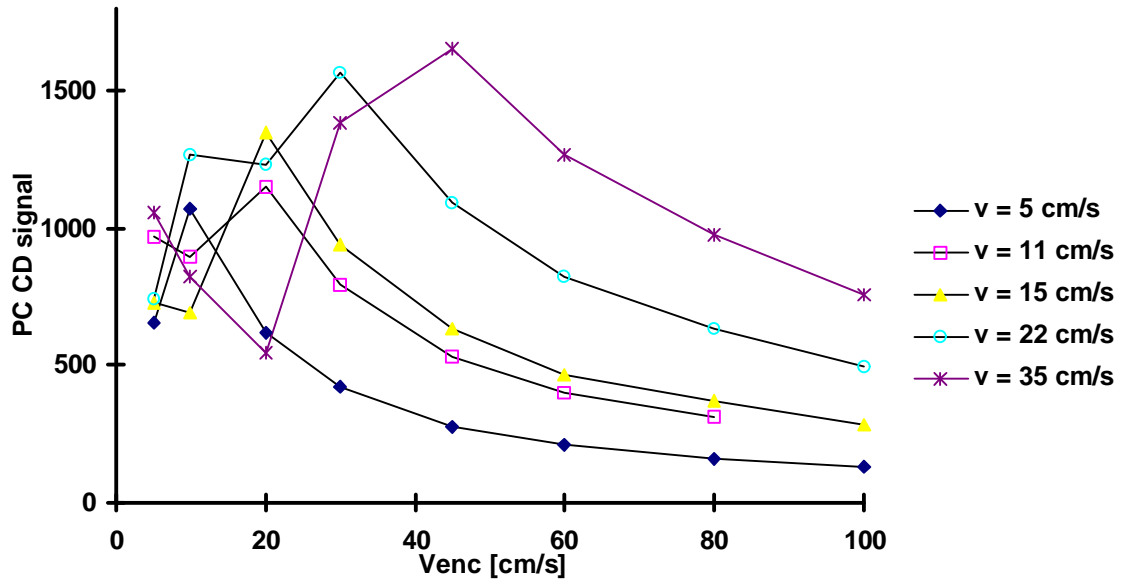


Diagram 33: Measured Phase Contrast Complex Difference signal plotted versus  $v_{enc}$ . TE = 7-14 ms (depending on the sequence used), TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm

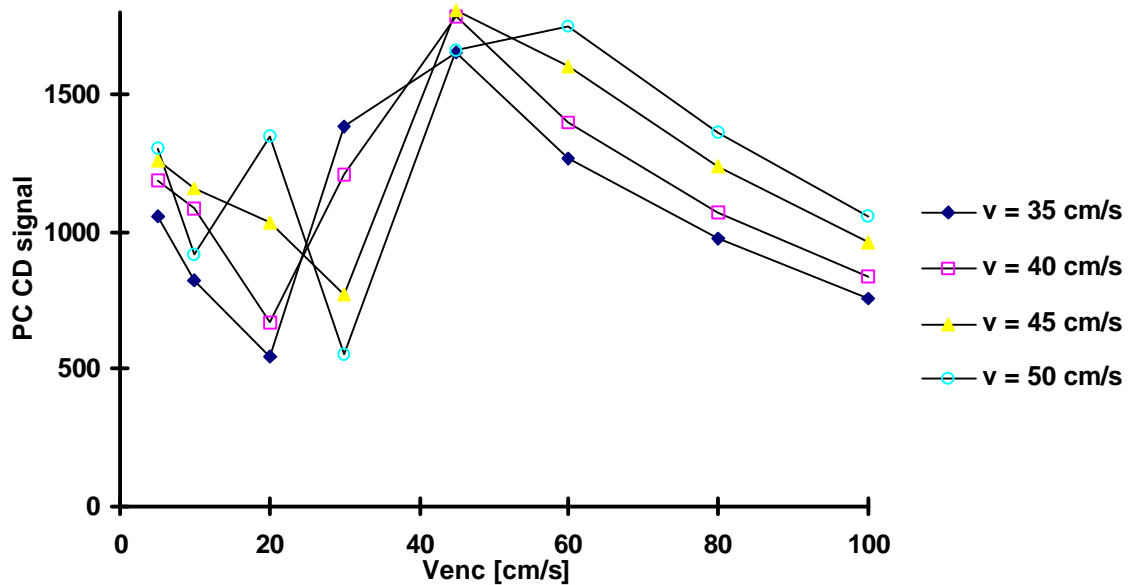


Diagram 34: Measured Phase Contrast Complex Difference signal, plotted versus  $v_{enc}$ . TE = 7-14 ms (depending on the sequence used), TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm.



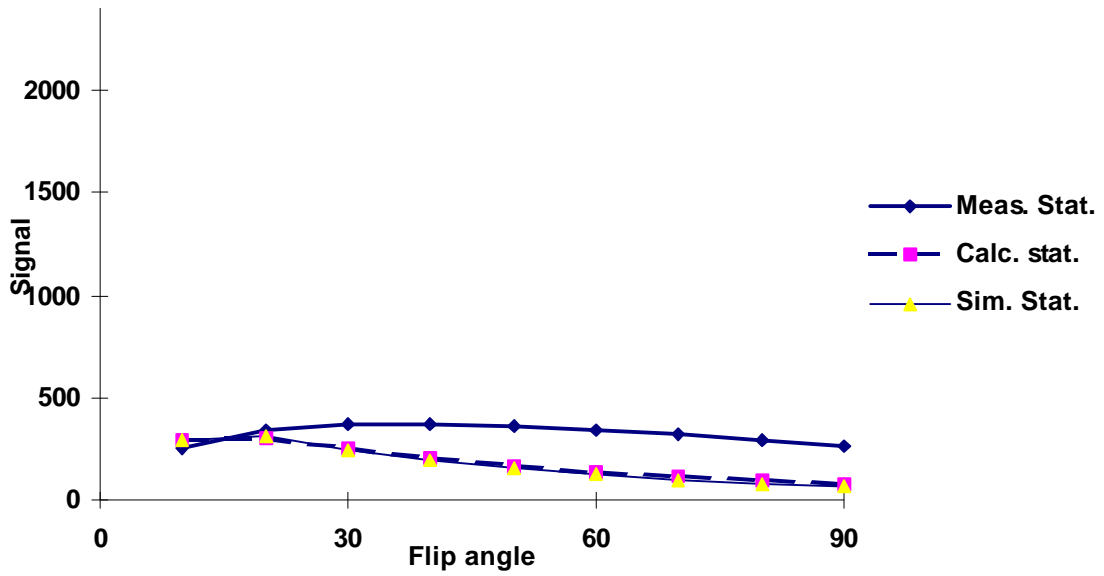


Diagram 35: Measured, calculated and simulated inflow signal compared for stationary tissue. T1 = 700 ms, T2\* = 20 ms, TE = 12 ms, TR = 24 ms, slice thickness = 10 mm.

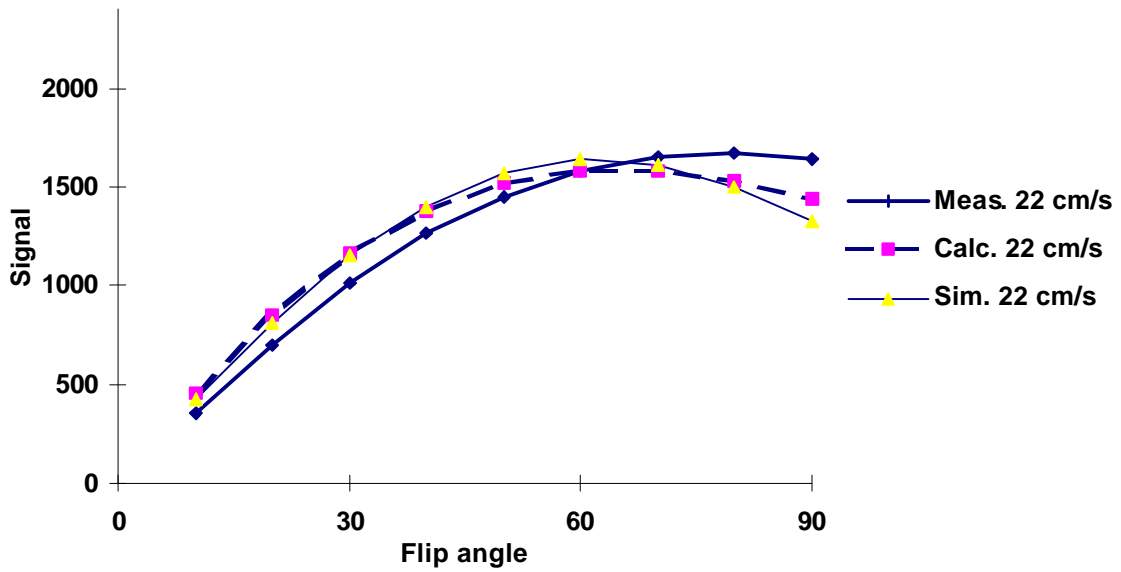
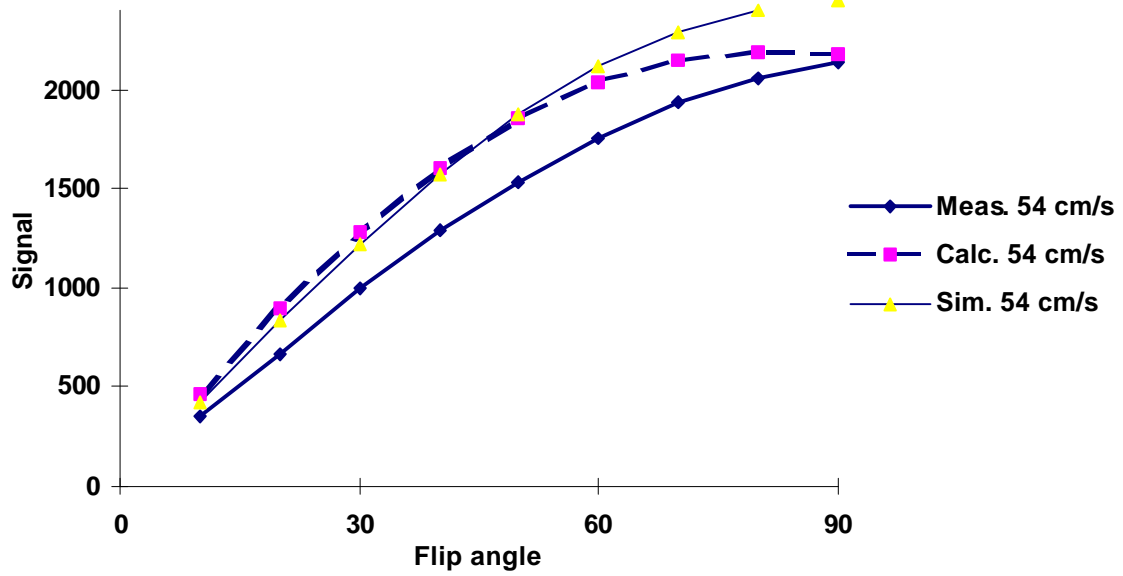
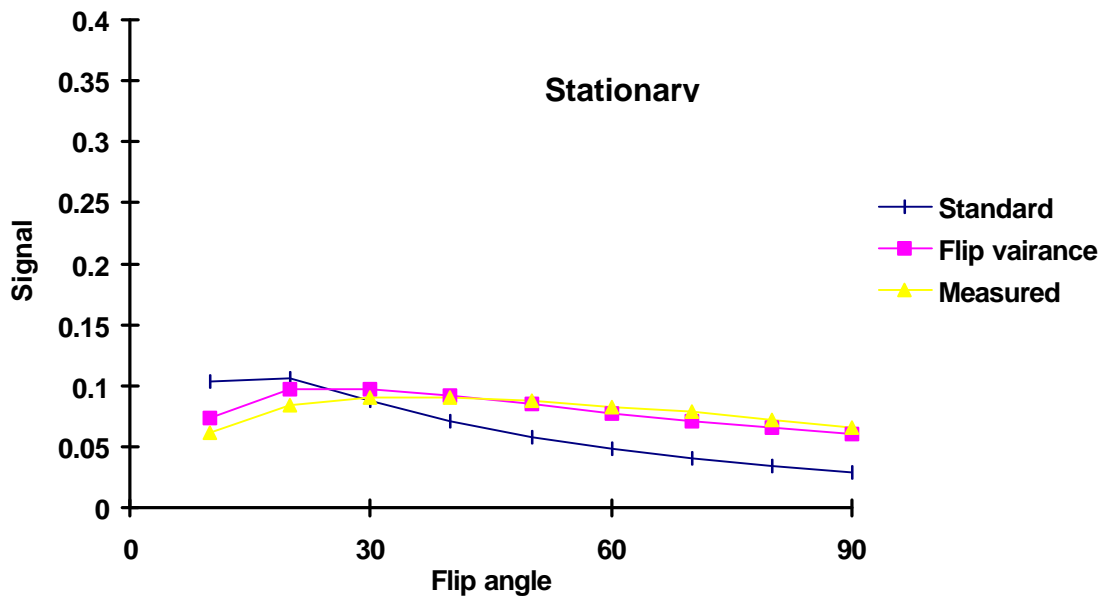


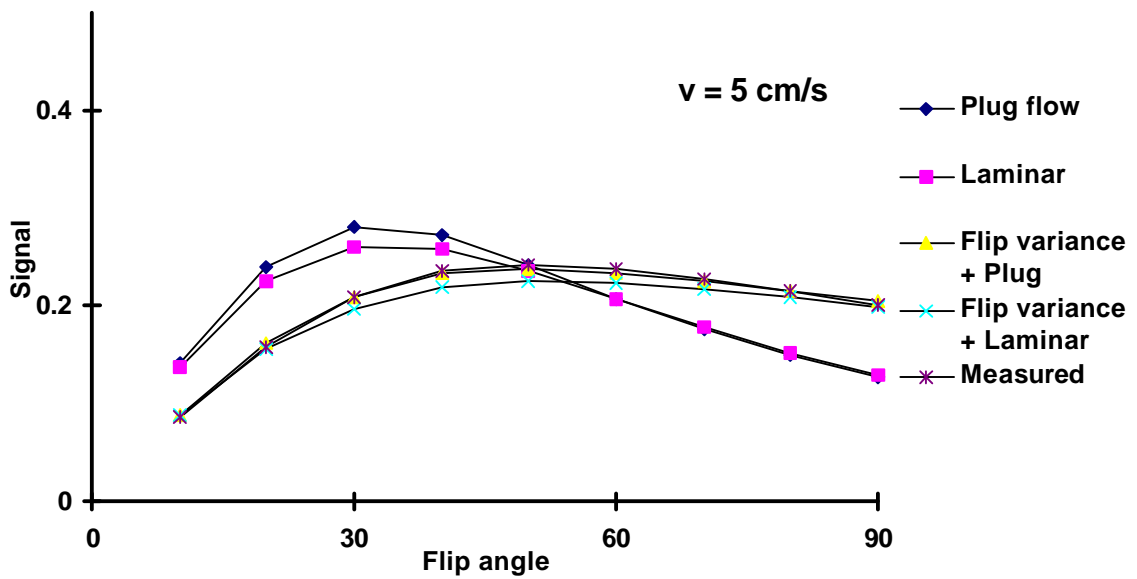
Diagram 36: Measured, calculated and simulated inflow signal compared for a flow velocity of 22 cm/s. Calculated data are assumed laminar and plug flow is assumed for simulated data. T1 = 700 ms, T2\* = 20 ms, TE = 12 ms, TR = 24 ms, slice thickness = 10 mm.



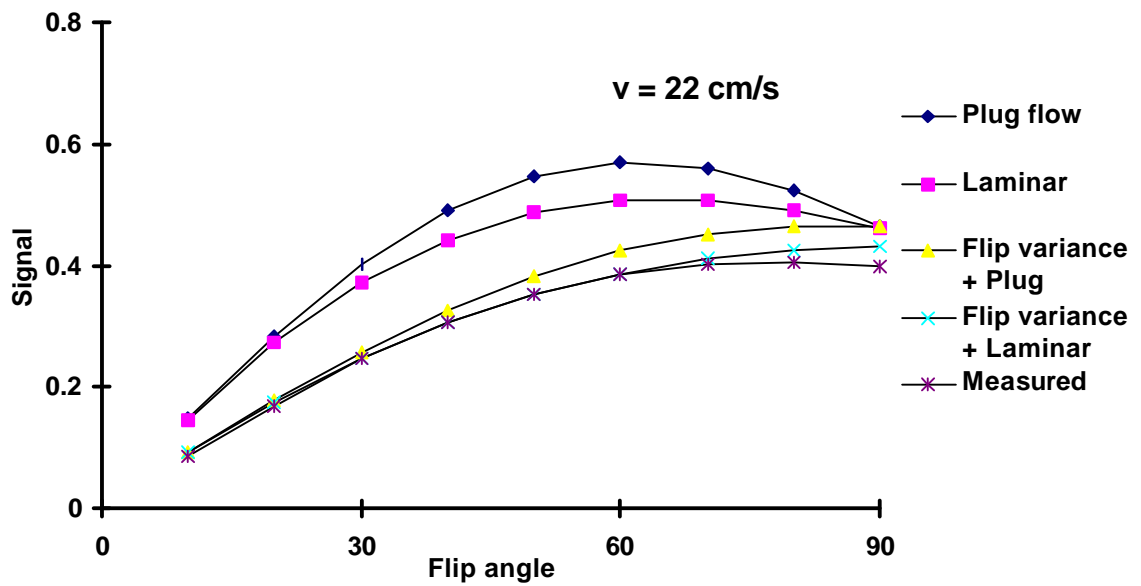
**Diagram 37: Measured, calculated and simulated inflow signal compared for a flow velocity of 54 cm/s. Calculated data are assumed laminar and plug flow is assumed for simulated data.  $T1 = 700$  ms,  $T2^* = 20$  ms,  $TE = 12$  ms,  $TR = 24$  ms, slice thickness = 10 mm.**



**Diagram 38:** Different types of calculations compared with results from a real measurement for stationary tissue. See next page for comments.



**Diagram 39:** Different types of calculations compared with results from a real measurement. Flow velocity of 5 cm/s. See next page for comments.



**Diagram 40:** Different types of calculations compared with results from a real measurement. Flow velocity 22 cm/s.

**Diagram 38-40:**  $T_1 = 700$  ms,  $T_2^* = 20$  ms,  $TE = 12$  ms,  $TR = 24$  ms, slice thickness = 10 mm. In Diagram 38 "Standard" means signal from GEsigal without any modification, "Flip variance" means signal calculated using the assumption that the flip angle has a gaussian distribution over the image slice. As seen the curves taking into account a flip angle variance over the slice are very similar to the results of the real measurement. It seems to be much more important to take into account the flip angle variance than assuming laminar flow instead of plug flow.

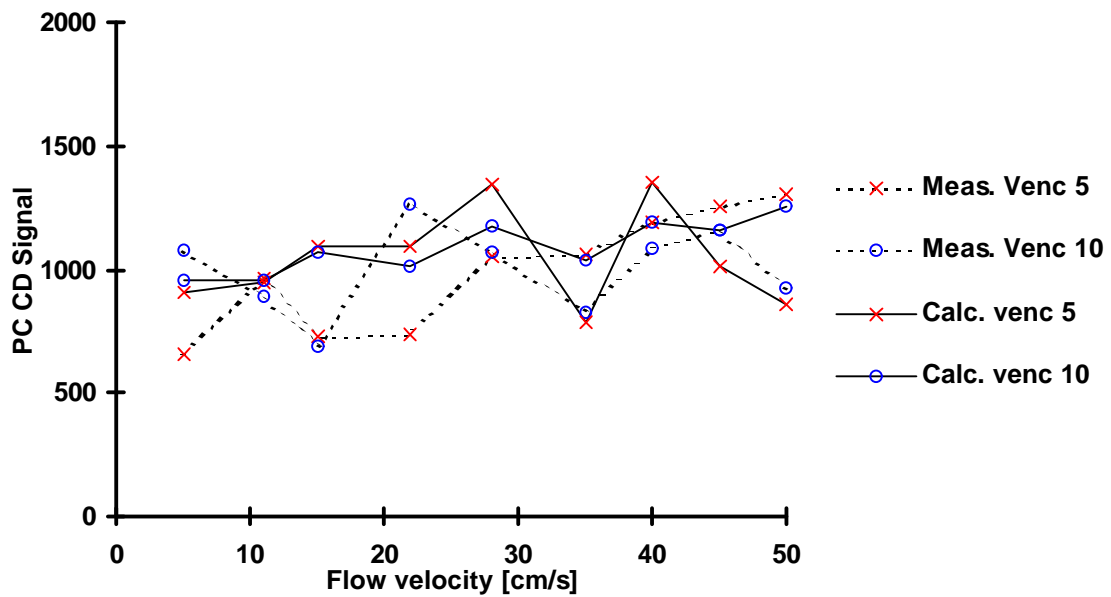


Diagram 41: Measured and calculated Phase Contrast Complex Difference signal, plotted versus flow velocity for low  $v_{enc}$ :s. Calculations are performed for laminar flow. TE = 11 or 14 ms (depending on the sequence used), TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm.

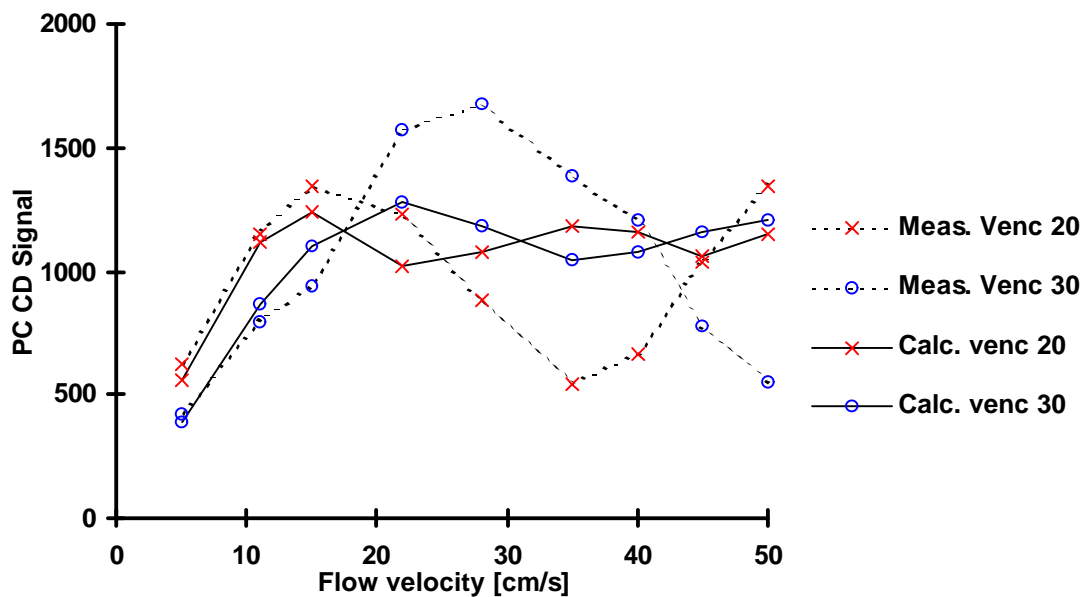


Diagram 42: Measured and calculated Phase Contrast Complex Difference signal, plotted versus flow velocity for intermediate  $v_{enc}$ :s. Calculations are performed for laminar flow. TE = 9-10 ms (depending on the sequence used), TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm.

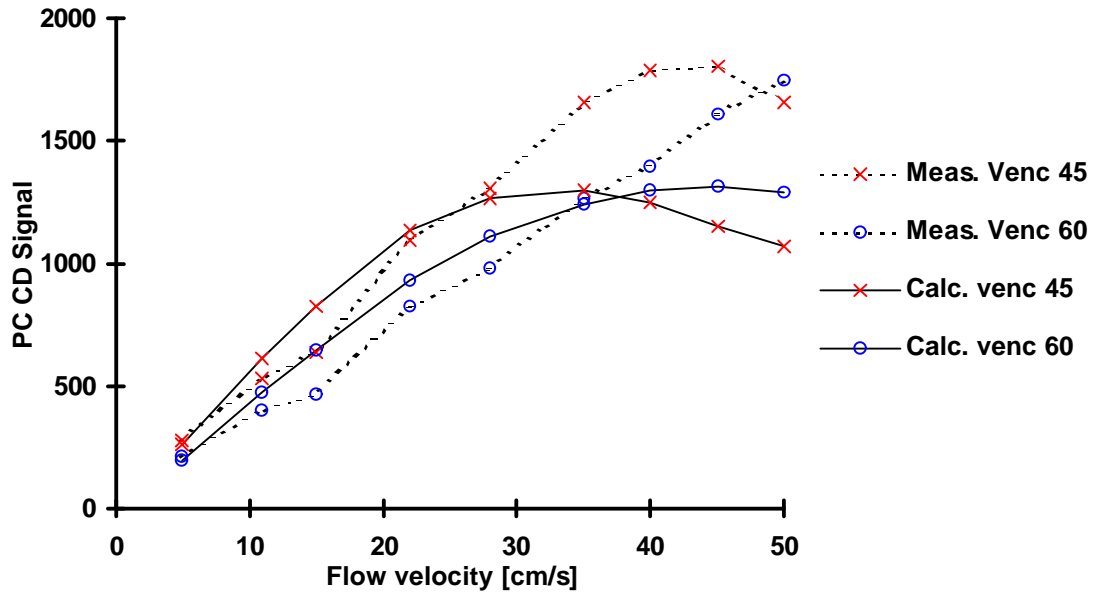


Diagram 43: Measured and calculated Phase Contrast Complex Difference signal, plotted versus flow velocity for intermediate  $v_{enc}$ :s. Calculations are performed for laminar flow. TE = 8-9 ms (depending on the sequence used), TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm.

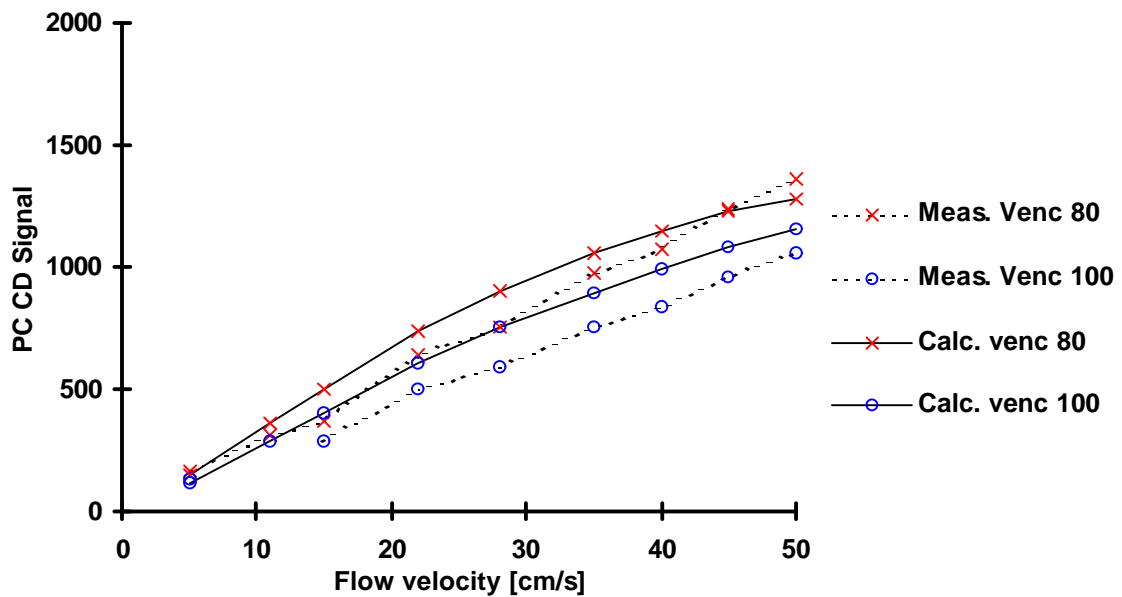


Diagram 44: Measured and calculated Phase Contrast Complex Difference signal, plotted versus flow velocity for high  $v_{enc}$ :s. Calculations are performed for laminar flow. TE = 8 ms, TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm.

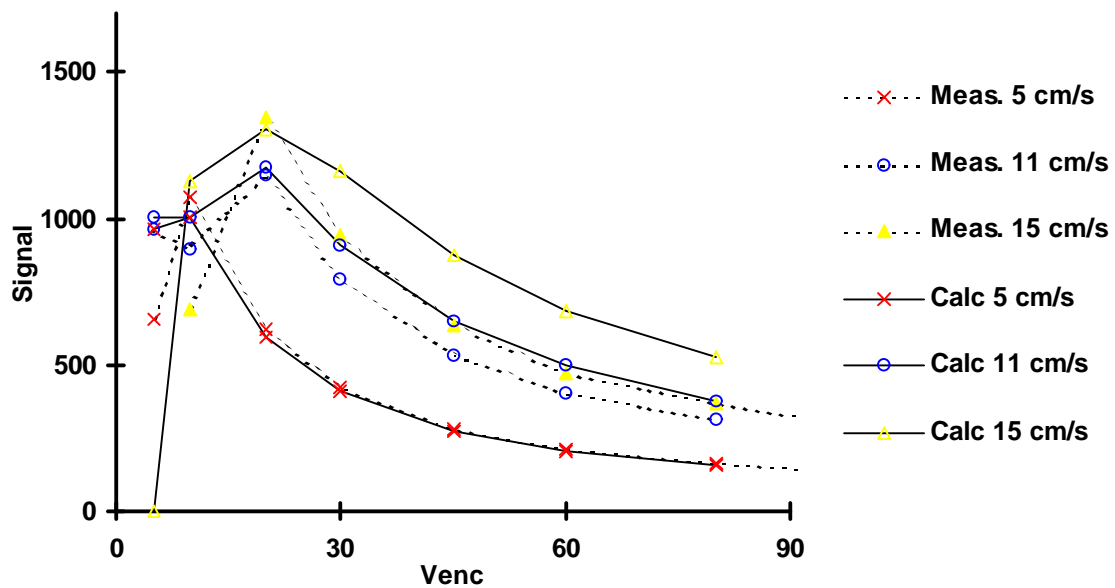


Diagram 45: Measured and calculated Phase Contrast Complex Difference signal, plotted versus  $v_{enc}$ 's for low flow velocities. Calculations are performed for laminar flow. TE = 7-14 ms (depending on the sequence used), TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm.

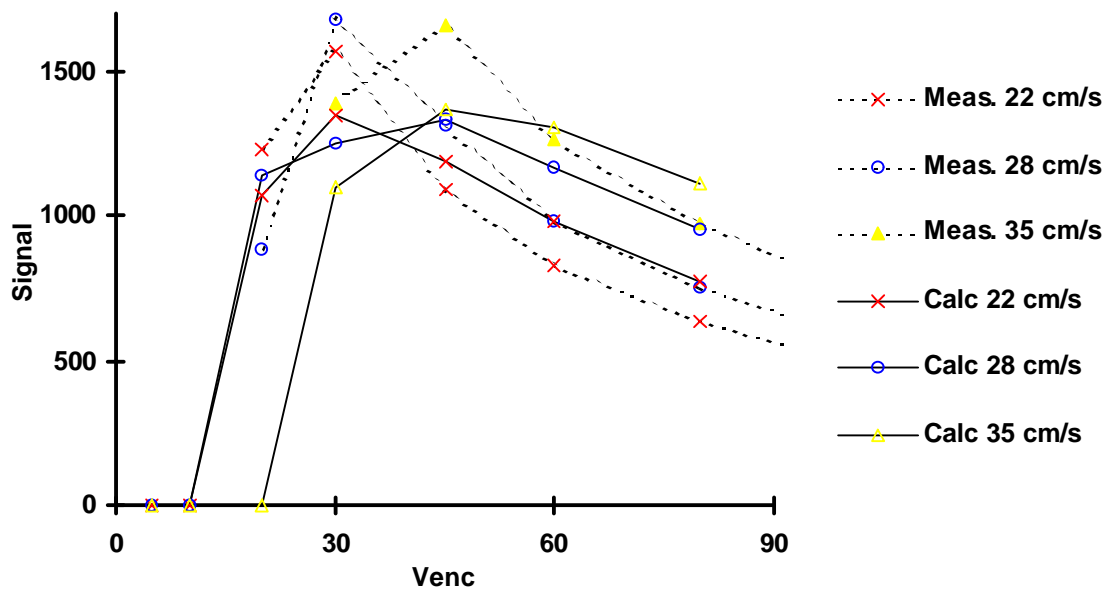
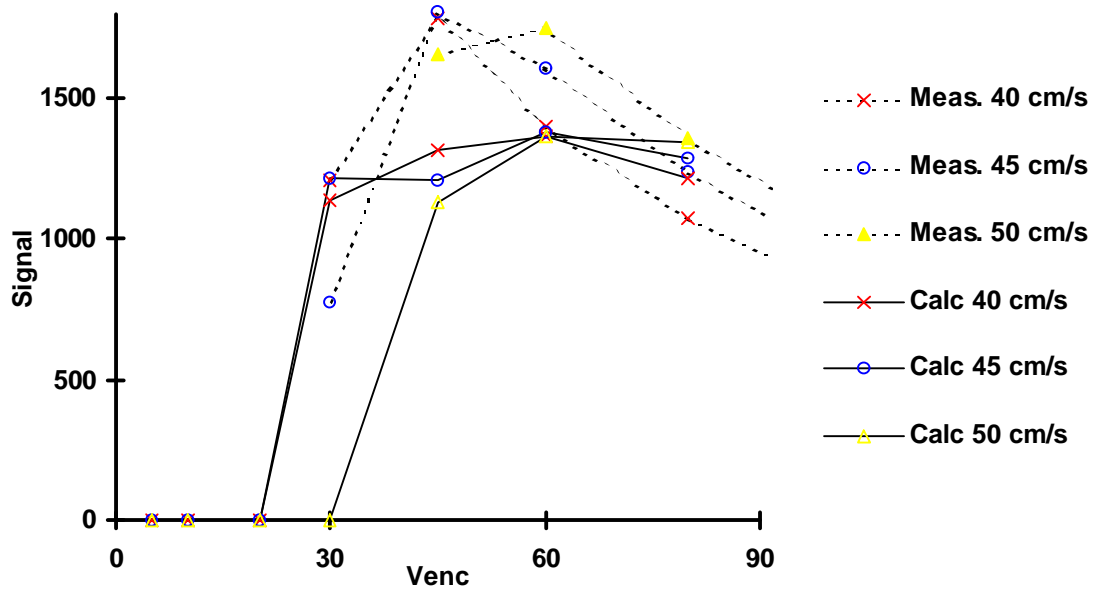


Diagram 46: Measured and calculated Phase Contrast Complex Difference signal, plotted versus  $v_{enc}$ 's for intermediate flow velocities. Calculations are performed for laminar flow. TE = 7-14 ms (depending on the sequence used), TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm.



**Diagram 47: Measured and calculated Phase Contrast Complex Difference signal, plotted versus  $v_{enc}$ 's for high flow velocities. Calculations are performed for laminar flow. TE = 7-14 ms (depending on the sequence used), TR = 104 ms, T1 = 700 ms, T2\* = 20 ms, Flip angle = 30°, slice thickness = 30 mm.**



