LUND UNIVERSITY

MASTER THESIS

Yaw Gain Optimization

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Preface

This thesis work has been conducted in collaboration with Haldex Traction in Landskrona and the Department of Energy Sciences, Faculty of Engineering, Lund University. The work has been carried out from August 2010 to Februari 2011.

For readers without basic knowledge in vehicle dynamics we refer to the introduction part of this thesis report which includes an introduction to differentials and their affect on handling together with an introduction to tyre dynamics. For readers who already hold this knowledge we refer to *chapter 2* as a starting point. If the reader is interested in certain areas we refer to the bibliography in the end of this thesis report.

Firstly we would like to express our gratitude to Haldex Traction for the opportunity to carry out this thesis work. The time spent at Haldex Traction has been a great experience and given an insight to the exciting world of automotive engineering. This thesis work could not have been done without the help from our supervisors at Haldex, Ola Nicklasson and Niklas Westerlund and Dr. Martin Tunér at Lund University. We would like to thank them for their engagement and all of their help during the entire project. We would also like to thank our examiner Prof. Bengt Johansson and the Department of Energy Sciences, Faculty of Engineering at Lund University.

Contents

Nomenclature

α	Slip angle
$\ddot{\psi}$	Yaw acceleration
δ	Steer angle
κ	Slip ratio
μ	Friction coefficient
σ	Theoretical total slip stiffness
σ_x	Theoretical longitudinal slip stiffness
σ_y	Theoretical lateral slip stiffness
a_y	Lateral acceleration
c_ϕ	Roll stiffness
C_x	Longitudinal slip stiffness
C_y	Cornering stiffness
CGH	Vertical distance to centre of gravity from ground
F_x	Longitudinal force
F_y	Lateral force
F_z	Normal force
I_{zz}	Vertical axis inertia
M_d	Drive shaft torque
M_y	Rolling resistance
R_e	Effective rolling radius
T_w	Track width

Summary

This thesis work has been conducted for a new prototype at Haldex Traction called FXD. It is a transaxle differential coupling for front wheel drive vehicles mechanically similar to Haldex four wheel drive couplings. FXD is mounted between the right driveshaft and the original differential crown wheel i.e. an active limited slip differential for FWD cars. The main part of this thesis has been to create a model which estimates the yaw torque, the torque around the vehicles vertical axis, when giving the FXD a higher or lower torque set point. Depending on the steering wheel angle and if the driver is braking or accelerating a simulation model is able to estimate if the car will behave more or less over steered.

Since the FXD only affect the forces on the front wheels directly these forces will need to be estimated for different torque set points. Consequently different methods needs to be created for estimating the longitudinal and lateral front tyre forces from the cars built in sensors. The computer power required for the different estimation methods is one of the limitation of this thesis work. The simulation model can not be validated without using advanced methods for measuring tyre forces. The models created has instead been validated against a full vehicle simulation model created by Haldex Traction called "Haldex Vehicle Simulation Tool".

The lateral tyre forces were estimated by using the total lateral tyre force and dividing it between the tyres using the calculated coefficient lambda. This coefficient is dependent on the normal load, nominal normal load and difference in slip ratio between the front tyres. For the lateral force estimation the angular velocities of all four wheels, lateral acceleration, yaw rate and steering wheel angle needs to be measured.

The longitudinal forces are calculated using the driver requested engine torque from the CAN-bus, the FXD locking torque and a model of the rolling resistance. The acceleration and inertia of the tyre and drive train have also been included in the calculation. Understanding and estimating the torque transferred through the clutch is one of the more difficult parts of this model. This has been done using a model from H.B.Pajeckas "Tyre and Vehicle Dynamics".

The tyre forces were then used in the the formula for calculating the yaw torque together with the lever arms to the front wheels from the centre of gravity. The lever arms were calculated from a two track vehicle model, using the steering angle and the predefined distances to the centre of gravity.

The next step was to estimate how the yaw torque changes if the set point of the clutch was changed. This was achieved by changing the slip ratio in relation to the change

in torque between the wheels and calculating the longitudinal forces from the change in torque. The new slip ratio was then inserted in the model for the lateral tyre force estimation and the new forces could be calculated. The difference between the new yaw torque and the “current” yaw torque will help to describe how the car will behave.

The model showed rather good results of the yaw torque when compared to the simulation models calculations. The results can be viewed in chapter 6 of this report.

1 INTRODUCTION

1.1 Background

eLSDs (Electronically controlled Limited Slip Differentials) are showing a more popular trend in the passenger vehicle industry. Haldex Traction in Landskrona, mostly famous for their limited slip couplings for AWD (all wheel driven) vehicles, has now started to develop a differential coupling for a FWD (front wheel driven) car.

The control of an eLSD in a FWD car affects the traction capability as well as the handling behaviour. The traction capability was before the start of this thesis project controlled using a slip controller and a yaw controller. This controller measures the driven wheels angular velocity and compares it to the vehicles velocity and then control the differential accordingly.

Another way to control the differential could be to control the angular velocity around the cars vertical axis when driving in a turn. If the car is over steering more than the driver wants the differential might be able to help. This control system would be called “yaw rate control” or abbreviated “yaw control”.

1.2 Project description

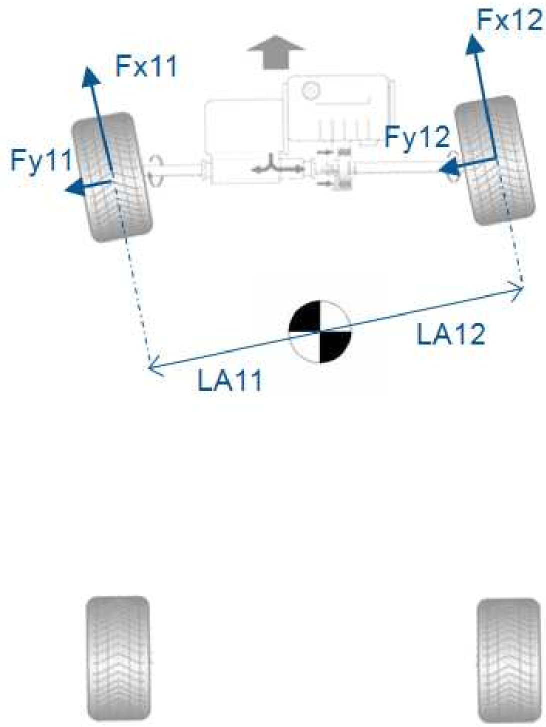

Figure 1.1: Front tyre forces[1]

The idea is to create a yaw torque around the center of gravity that stabilizes the yaw rate. The only way we can control this through the differential coupling is by varying the torque that is transferred through the clutch package. The problem to solve is if more or less torque should be transferred to get closer to the desired yaw rate.

Many problems may arise when developing an algorithm for yaw-control and most of them is due to estimating the cars states in a simple way that do not require to much computing power. The states that need to be calculated are the current lever arms and forces on the front driven wheels that affect the yaw torque.

The lever arms between the centre of gravity and the forces on the front wheels will be rather simple to evaluate geometrically from the position of the centre of gravity, the steering angle and possibly static toe. One problem could be to find an exact centre of gravity position but a rough estimation will probably be good enough.

The longitudinal forces on the front wheels have to be known and can be derived from engine torque and the differential locking torque. When locking the differential, more or less torque is transferred from the high speed wheel to the low speed wheel. Another problem is when the differential has a locking torque close to the maximum locking torque of the current state i.e that the angular velocitys of the right and left wheel are almost equal. It could then be hard to estimate which way torque is transferred through the eLSD and thereby how large each longitudinal force is.

Another related problem is when the clutch package in the differential is completely locked. It will then be hard to know how much torque that really is transferred through the clutch. It could be the maximum torque capacity but it could also be less.

Estimating lateral forces of each wheel often require an advanced model. Using a tyre model like Magic Formula[2] it would be possible to generate the lateral force for each wheel if the normal load, slip angle, slip ratio and ground friction is known. First of all, the Magic Formula[2] uses too much computing power and secondly, factors as ground friction, slip angle and normal load can be very hard to estimate.

The controller will always have to estimate how much traction is available to each driven wheel. If torque is transferred to one wheel because higher longitudinal force on this wheel is needed it must be assured that there is enough traction capability available in the ground contact that the wheel do not spin up. If it spins up the result will be a decrease in longitudinal force on this wheel which is opposite to what was desired. This means that less torque should have been transferred in the first place. This can be the case in an exit of turn with wheel spin on the inner wheel and traction limit on the outer wheel because of high lateral acceleration.

One limitation of the project is that the coefficient of friction is considered to be known. Another limitation is that the calculations only should be valid for situations where positive(drive) torque is applied. To develop an estimation model for this purpose Matlab-Simulink is planned to be used. Haldex has already a full vehicle model that is suitable for this kind of development as a validation tool.

1.3 Aim

The desired result of this project is to estimate the change in yaw torque when locking or opening the differential with a fixed torque, for example 100 Nm in each direction. If this can be estimated well without too much computing power the control structure of a Haldex FXD (differential in FWD) can be changed dramatically and a profit in handling and safety will be reached.

1.4 The differential

The differential distributes the torque from the gearbox to two drive-shafts, either in the front, rear or between the front and rear axles. An "ideal differential" shall preferably be able to fulfil the following criteria according to [3]:

- Allowing speed differentiation, e.g. when travelling in turn.
- Distribute the torque so that all available friction can be used
- Do not transfer more torque than the tyres can transfer to the ground
- Maintaining good vehicle handling
- Its function should be fully automatic
- It should have no energy losses
- The design should not be too complicated
- The cost should be in balance with the higher value for the final product

1.4.1 Open differential

The main idea of a differential is to be able to have a speed difference on the wheels on the same axle. This is desired when travelling through a turn which gives a speed difference between the wheels depending on different turn-radius because of the track-width. If this was not possible, it would create heavy under-steering and create difficulties for the driver during parking manouvers etc. The open differential works well because of its simplicity, low cost and high efficiency.

The power reaches the differential through the crown-wheel that is mounted on the differential housing. The housing is mounted in an outer case with bearings where the drive-shafts are going in to the housing. Both the housing and the drive-shafts are able to rotate individually relative to the outer case. Two differential-pinions are mounted on an axle that goes all the way through the housing and thereby transferring the power from the housing to the side gears mounted on each drive-shaft. The result of this mechanism is that the two side-gears can have difference in speed but regardless transfers almost the same amount of torque. Because of friction between the different gears, gears and housing etc. some torque biasing will be found even in an open differential. Amounts up to 15-25% torque biasing have been observed according to [3].

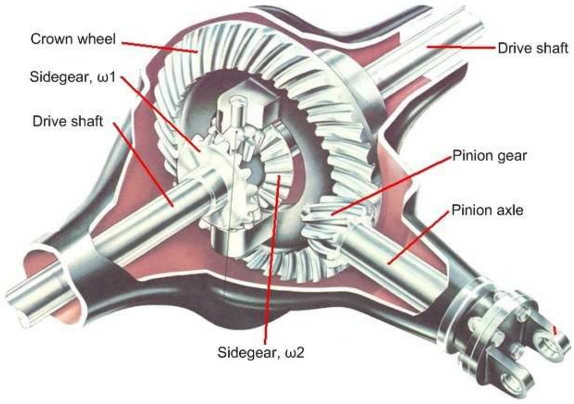

Figure 1.2: Open differential[4]

The speed relationship between the side-gears and the crown-wheel:

$$-1 = \frac{\omega_1 - \omega_r}{\omega_2 - \omega_r} \tag{1.1}$$

$$\omega_r = \frac{\omega_1 + \omega_2}{2} \tag{1.2}$$

A car driving straight ahead on a flat surface with good traction will have the same speed on the two driven wheels on one axle. Regarding to the equation above, if ω_1=ω_2, ω_r=ω_1=ω_2. This means that the side-gears will rotate with the same speed as the crown-wheel and the differential-pinions will only rotate around the side-gears axis.

The drawback of this differential is for example when travelling on a road with good friction on one side and poor friction on the other, so called split-µ. The low- µ wheel will if too much throttle is applied start to spin. This due to the differentials torque splitting effect which divide the torque equal between the two driven wheels. The car will not be able to apply more torque than the torque capacity of the low-µ wheel. A similar situation could appear when reaching high lateral acceleration, see figure 1.3, where one driven wheel loses contact with ground and then starts to spin. A third situation could be when driving off-road. An example is when climbing a hill with only one wheel in contact with ground and the corresponding wheel on the same axle runs out of suspension-travel i.e. loses contact with ground and starts to spin.

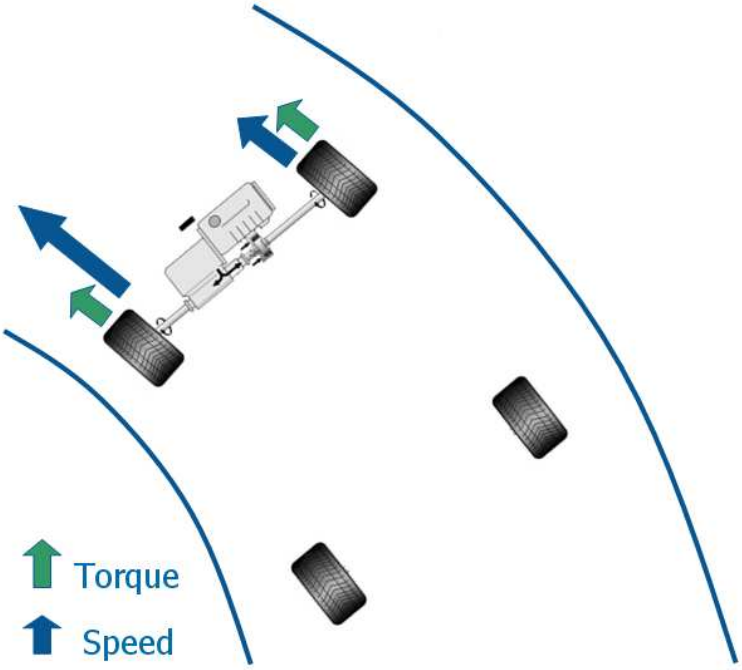

Figure 1.3: Slipping inner wheel[1]

1.4.2 Locked differential

In some situations more torque is preferable on the high friction wheel than on the spinning wheel. This could be done in two ways. Either using a locked differential, which means that the two driven wheels always have the same speed, or using a Limited Slip Differential. The locked differential has a great drawback. It creates under-steering behaviour during cornering but could be favourable for example in off-road situations. The manufacturer Land Rover make it possible for the driver to use a mechanically lockable differential, most often seen in models built for off-road usage.

Another more common vehicle with no speed differentiating is the go-cart which suspension is designed to make the inner rear wheel more or less lift from the surface in turns to make the cart under-steer less.

Tip of the day: This is a reason why you always shall lean outwards in turns if you experiencing under-steering when driving go-cart.

1.4.3 Limited slip differentials, LSD

A more useful way to compensate for the open differentials drawback is to allow some speed differentiating. This could be done by adding a limited slip coupling when transferring torque from the differential housing to one of the drive-shafts, se figure 1.4. Another way is using a complex gear-setup which introduces friction forces when torque is applied from the gearbox, an example of this is the so called Torsen-differential which will be discussed later.

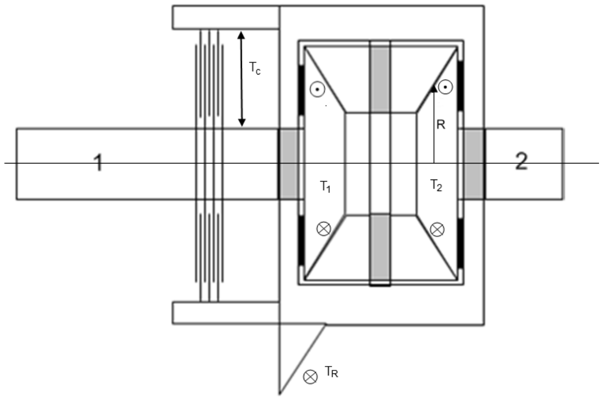

Figure 1.4: Limited slip differential[1]

1.4.3.1 Torque calculations

T_1: torque on side-gear one.

T_2: torque on side-gear two.

T_C: the torque that is transferred through the clutch from the crown wheel to side-gear one.

T_r: torque on the crown-wheel from the gearbox.

F: tangential force on either of the side-gears from the differential-pinions.

R: is the radial length to the differential gear contact.

Consider a case where a car is driven with wheel two on tarmac and wheel one spinning on ice, see figure 1.4. Drive shaft one is connected to the wheel on ice and drive shaft two is connected to the wheel on tarmac with high traction capabillity. Consider the applied torque to the differential to be high enough to saturate the traction grip potential on the low mue side, i.e. drive shaft one will have a higher rotational speed than the differential housing (which will have the average rotational speed of the two drive shafts, 1.2) . This implies existence of a differential speed over the clutch pack of the controllable LSD. In a slipping clutch the direction of the differential speed determines the direction of the clutch package torque transfer. A limited slip clutch device will try to decrease the speed of wheel one with the torque T_c.

The torque applied to driveshaft one is then the torque transferred from the differential-pinions minus the torque transferred through the clutch.

$$T_1 = F * R - T_c => T_1 + T_c = F * R \tag{1.3}$$

The torque on side gear two are:

$$T_2 = F * R \tag{1.4}$$

Equation 1.3 and 1.4 gives us:

$$T_r + T_c = 2 * F * R \tag{1.5}$$

$$T_c = T_2 - T_1 \tag{1.6}$$

We can now calculate the available traction-torque on the two wheels:

$$T_r = -T_c + 2 * F * R = -T_c + 2 * (T_1 + T_c) = 2 * T_1 + T_c \tag{1.7}$$

Torque that is available to propel the car is twice the torque that the low-µ wheel can transfer and the LSD´s torque capacity.

Torque bias is the ratio between the torque on the high u wheel and the low u wheel and is often used to describe the properties of a differential.

Torque bias is calculated in the following way:

$$\frac{T_2}{T_1} = \frac{T_1 + T_c}{T_1} \tag{1.8}$$

Of course does the limited slip clutch have a limit on how much torque that can be transferred through it which means:

$$T_c = min(T_2 - T_1, CapacityofTc) \tag{1.9}$$

Above calculations are taken from [3].

There is no difference in results of the calculations if the clutch package is placed on the same side as side-gear two instead. What will happen if it is the other way around (wheel two spins and wheel one have good traction)? The clutch will instead of slowing side-gear one down, increase the torque and add rotational speed.

Limited slip differentials can be divided into three different categories, torque-sensing, speed-sensing and active differentials. The difference between them is what the amount of transferred torque depends on, torque, speed-difference or a control software using measured signals as inputs.

1.4.4 Torque sensing differentials

The concept of a torque sensing differential is that it activates internal friction surfaces producing a locking torque proportional to the amount of drive torque applied to the crown wheel.

1.4.4.1 Torsen differential

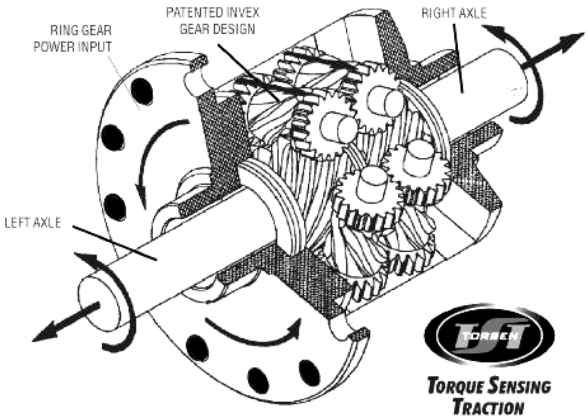

Figure 1.5: Torsen differential[5]

Torsen is a brand of maybe the most common type of passive differentials. It is very popular because of its capability to produce high torque bias which could be up to 14:1 according to [3]. The gear setup on the picture is the most used differential of this type and can have a torque bias between 2.5:1 and 4.5:1 [3].

The torque is transferred from the crown-wheel (ring gear) through the element-gears that have a helical shape to the side-gears. When torque is applied to the crown-wheel the shafts of the element-gears will be forced to rotate around the side-gears axis. Forces will be induced in the contacts between the element-gears and the side-gears. These forces will be normal to the contact points and thereby have components in three directions, which mean that reaction forces will appear between the housing and the side-gear and between the two side-gears. A torque balance between the two element-gears will appear when the two drive shafts have the same speed and will thereby not rotate around its own axis.

The torque transfer between the left and right wheel can occur because of friction forces between the side-gears, between the side-gears and the housing and even some friction in the element gears mounting in the differential housing.

The torque bias depends mostly on the angle of the gears helical-shape and the friction between the two side-gears and their contact with the housing.

1.4.4.2 Ramp actuated differential

Another type of torque sensitive differential is the ramp-differential which often include one or two clutch packages. The difference between this design and the open differential´s is that the shaft that the differential pinions are rotating around is not running all the way through the differential housing. An inner housing in two pieces, which on the outside have direct contact with the clutch plates and on the inside carry the differential pinions and the side-gears, are used. This two piece housing will transfer torque from the outer housing to the differential-pinions shaft. The ends of the shaft for the differential-pinions goes through this two part housing in a v-slot, seen in figure 1.6. This means that when torque is applied the pinions shaft will rise in the v-slots and force the two pieces of the house away from each other and compress the clutch packages. Half of the plates in the clutch package are splined to the outer housing and half of them are splined to the driveshaft. Torque will thereby be transferred directly from the outer housing to the driveshaft.

By changing angles in the v-slots it will be possible to tune the differential to the car. The angle on one side of the slot will affect acceleration and the other one will affect engine braking. There are often also used Belleville washers between the outer clutch plate and the differential-housing to get some preload to the clutch package which give a small amount of constant differential braking.

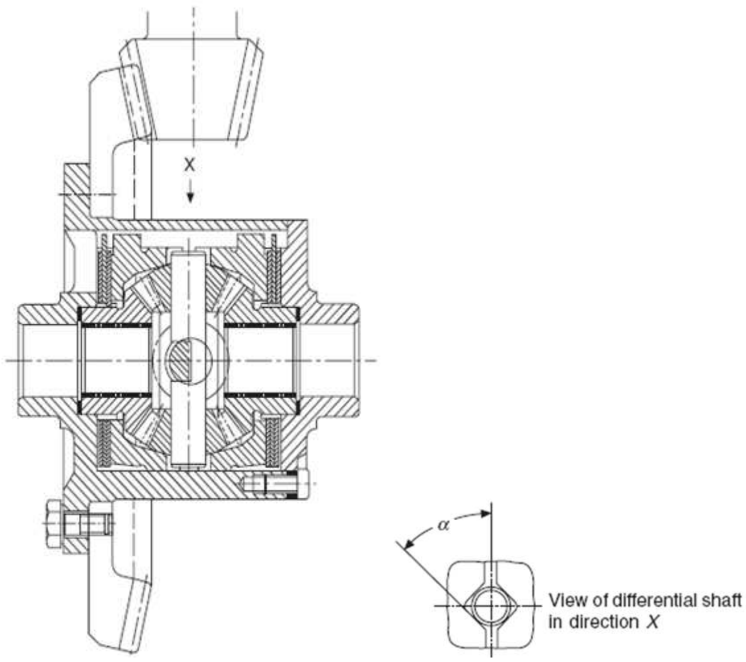

Figure 1.6: Ramp actuated differential[6]

1.4.5 Speed sensing differentials

The speed sensing limited slip differentials are those who transfer torque through the differential depending on speed difference between the left and the right wheel.

1.4.5.1 Viscous differential

Viscous differentials, like the ramp differential, is also a "child" of the open differential. The difference is the limited slip clutch that can transfer torque from the differential housing to one of the drive shafts. The clutch package is compressed by a shear pump that contains washers of a shape that hydraulically pushes them apart when there is a speed difference between them. [7]

Another type of viscous differential looks almost the same with the difference that no pump unit exists. Plates with different patterns on the contact surfaces replace the friction plates in the clutch package. Because of the speed difference on the wheels, hydraulic forces are induced that brakes the discs relative each other. These hydraulic forces grow with speed difference, which means that the more speed difference the more torque is transferred from one wheel to another.

1.4.6 Active limited slip differentials

There exists a number of different active LSD's on the market and different actuation methods. The active hydraulic LSD's uses a hydraulic power unit (HPU) as a pressure source and a hydraulic circuit unit (HCU) to control the differential.

Active electromagnetic LSD's control the differential through an electromagnet, a clutch and a ball-ramp mechanism that works as a mechanical actuator which clamps the clutch to control the torque transfer. A problem with the electromagnetic LSD's is that they have noticeable torque hysteresis according to [8].

Active internal-pump LSD's contains a small integrated pump that is actuated by the relative rotation of one output shaft and the differential case. The flow is generated in proportion to the speed difference of the two. The hydraulic flow pressurizes the piston which clamps the clutch pack. A solenoid valve adjust the oil pressure, this valve is electronically controlled by the differential ECU. An external-pump LSD also uses an ECU to control the the piston that clamps the clutch-package. The difference to an internal pump LSD is that the pump flow is not determined by the speed difference of the housing and shaft.

Active differentials have the ability to in real time control the locking torque of the clutch the same goes for a LSC. LSC stands for Limited Slip Coupling which is a device separated from the differential. Together with the differential they work just like a LSD.

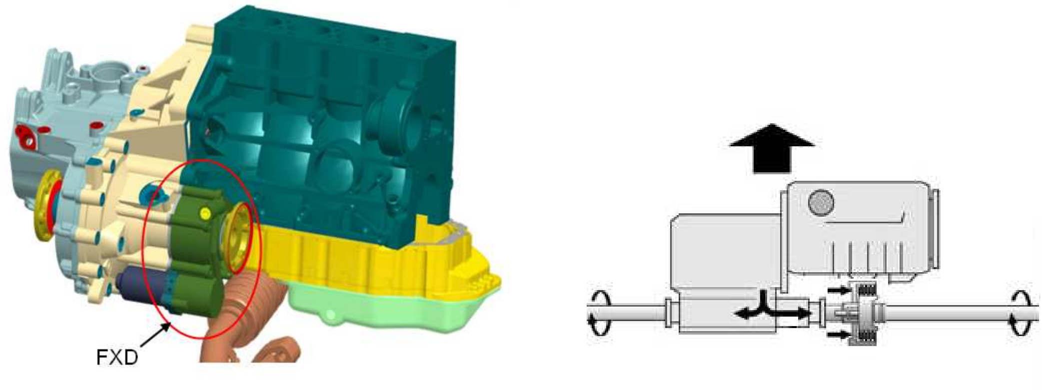

Figure 1.7: Haldex FXD system[1]

1.5 Differentials and their affect on vehicle handling

Different LSDs' affect the traction and the handling of the car in different ways. In this chapter a few driving situations where the LSD have great impact on the behaviour of the car will be presented.

1.5.1 Split mue driving

An ordinary open differential works as a torque balancing device, it transfers an equal amount of torque between the wheels. When driving on a split-µ surface an open differential will create a large difference in wheel velocity if the grip potential on the low-µ is saturated. This since the resistance torque on the low-µ drive shaft balances the torque on the high-µ drive shaft and thus the torque transferred to the ground is limited to twice the torque capacity of the low-µ wheel. The Limited Slip Differential will create a torque bias based on applied torque on crown wheel (torque sensing) or velocity difference (speed sensing) and transfer torque from the low-µ side to the high-µ side. Active differentials will generate torque bias based on software control of the integrated limited slip clutch.

1.5.1.1 Self steer

By transferring torque to the high-µ side this will create a larger longitudinal force which will result in a yaw moment around the cars COG pulling the car into the low-µ side.

1.5.1.2 Torque steer

Unequal traction forces created when using a LSD cause's "torque steer". The distance between the centre of the wheel and the king pin axis (wheel steering axle) will function as a lever arm for the traction forces and thus generate a net difference in steering torque over the steering system. This torque is the necessary counteracting measure of the steering wheel.

The torque steer effect when using an open differential will be negligible due to the torque balancing effect of the open differential. But with unequal traction forces e.g. using a LSD this effect will be apparent. Also a difference in wheel loads will increase the torque steer.

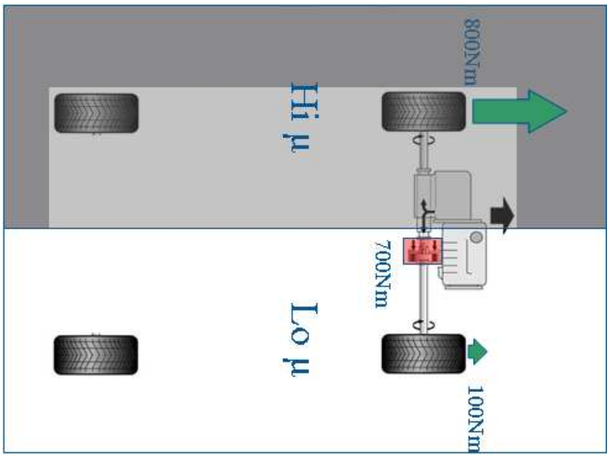

Figure 1.8: Split μ conditions[1]

1.5.2 Steady state cornering

Some LSD's require a larger steering angle at lower lateral accelerations i.e. they increase the natural under steer of the car but they also gives a larger span of lateral acceleration. The increased under steer during lower lateral accelerations compared to the open differential will turn into an over steering moment during larger lateral acceleration. The Viscous Coupling (VC) and the mechanical friction LSD (ML) characteristic when changing from under steering moment to over steering moment is similar to that of an open differential. Since the Viscous Coupling Progressive (VCP) changes from under steer to over steer at lower lateral accelerations the steering angle required at higher lateral accelerations is smaller compared to the other differentials according to [7]. A torque sensing LSD will at lower driving torque work almost as an open differential, it will thereby not require a larger steering angle.

1.5.2.1 Acceleration during steady state cornering

Accelerating out of corners with an open differential often make high powered FWD vehicles spin their inner wheel due to reduced vertical load from lateral load transfer and thus reduced traction force capability. A LSD would minimize or eliminated this problem completely. Due to the weight transfer from the front tyres to the rear during acceleration and reduced traction force capability due to increased longitudinal forces, FWD vehicles tend to under steer more during accelerating in a corner than when running at a steady speed. In this case the LSD will create less under steer in comparison to the open differential often seen in production vehicles. Since the traction capability of the inner tire is saturated a prerequisite for transferring positive driving torque to the outer front wheel is fulfilled. The LSD's self-locking torque characteristic will create higher driving torque at the outer wheel and thus a yaw moment in the turning direction, this will counteract the under steering effect. On top of that if the torque bias created by the LSD is high, reduce the inner wheel excessive slip and the inner wheel will also regain lateral grip – also decreasing the under steer.

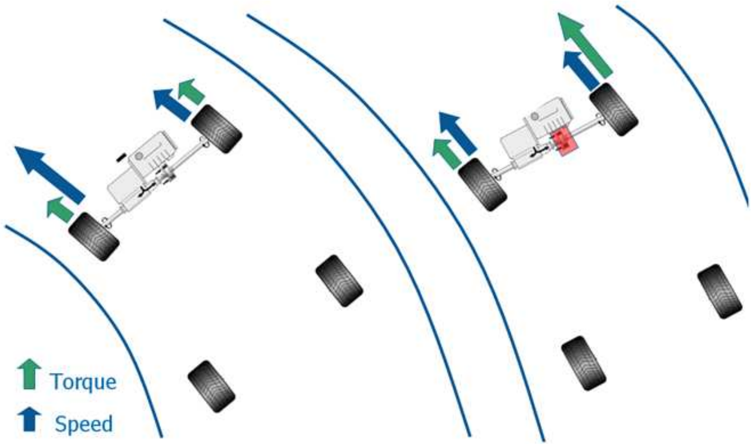

Figure 1.9: FXD system, applied locking torque[1]

When accelerating in a corner a torque sensing LSD will react fast to changes in torque. If the LSD has a high torque bias and the grip potential for the outer wheel is large, the torque transfer can result in an over steering moment. This differential requires a lot of tuning to achieve the right torque bias.

There is a risk that the differential transfers to much torque and thereby spin up both wheels. This will make the car under steer more as the lateral force generation is degraded by the longitudinal slip.

1.5.2.2 Braking or Throttle-off during steady state cornering

The throttle off reaction will create a dynamic weight transfer from the rear axle to the front axle. This will result momentarily in the capability of larger lateral guidance at the front axle, steering the car into a smaller radius. This effect could be counteracted through appling locking torque to the differential.

The effect of braking is the same as throttle off or even worse due to greater longitudinal acceleration. The open differential initially produce a heavy over steering effect which is followed by an over steering reaction. For a torque sensing LSD the inertia in the engine will decide the torque difference, by disengaging the clutch there will be almost no torque applied to the differential.

An active differential can control the locking torque to lock the differential at small deceleration and open the differential at a hard deceleration.

The inner wheel is rotating slower than the outer wheel applying a speed sensing limited slip differential or activating an active LSD will create more braking torque on the outer wheel and less braking torque on the inner wheel. This will create a yaw moment counteracting the normal turn-in reaction i.e. the over-steer will decrease.

1.5.3 Dana's active limited slip differential

1.5.3.1 Short introduction to Dana's active LSD

Sensors detect differences in wheel velocities. By using the bicycle model a theoretical velocity difference can be calculated and be compared to the real velocity difference. If these two do not match the differential will be controlled if possible until the error is near zero. Active LSDs have the ability to function as passive LSDs but also as open differentials. During harsh road conditions the active LSD provides enough torque to handle the situation and during mild handling situations the clutch can disengage and the active LSD works as an open differential to not interfere with the manoeuvre.

Dana has designed an active differential called E-Diff, [8], which uses a gerotor-pump and an ECU to control the clutch package in the differential. It is compact and has a low voltage usage. The following results come from simulations of Dana's Limited Slip Differential compared to an open and passive differential.

1.5.3.2 Hill climb (split-µ conditions)

"Open differential in split-µ hill climb"

With an open differential the wheels will spin up as the torque increases, since the open differential is a torque balancing device it will not be able to utilize the full potential and the car will fail to climb the hill. A considerable amount of torque will leak to the spinning left wheel. The open differential in this test generates a 1400 N longitudinal force to the wheel on the high-µ side [8].

"Passive LSD in Split-µ hill climb"

The passive LSD has the torque capacity of 680Nm, it will limit the torque output to the left spinning wheel. The passive LSD can generate a force of 2200 N to the high-µ wheel, this is an increase of 57% compared to the open differential according to [8].

1.5.3.3 Mild handling performance

"Passive LSD in mild handling"

A mild lane change was performed, as an open loop (no correction). The result showed that the speed was not changed and a small difference in driving wheel spin speed was generated. The LSD generated a noticeable amount of torque, which created a yaw damping effect.

"Active LSD in mild handling"

The active LSD should be able to minimize the unnecessary activation of the differential. A test with a baseline controller showed that the active LSD had more intrusive activation of the differential compared to the passive LSD. To really take advantage of an active differential a simple controller is not enough, a badly controlled active differential will be worse than a passive differential. When using the dynamic bicycle model the difference between the actual delta-rpm and the threshold delta-rpm is small, this will result in a small electric power to activate the differential.

1.5.3.4 Aggressive handling performance

Test properties of [8]: 30 degree steering wheel angle and 400 Nm of torque on the propeller shaft. This generates large amounts of lateral acceleration, the active LSD successfully limits slip, compared to the open differential which allows large amounts of wheel slip.

1.6 Tyres

It is very important to understand, when designing a vehicle, what forces and momentums affect the chassi. To be able to derive these, the designer always ends up with the problem of calculating the forces or momentums generated by the tyres. The tyre that often consists of a rubber-steel compound is very complicated to understand and is something that chassi and drive train engineer never can learn too much about. A short summary of the basics can be read in the following text.

1.6.1 Without rotation and torque

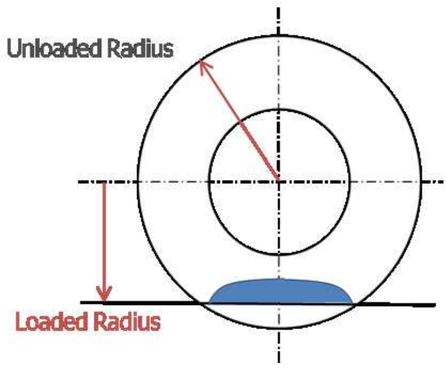

Figure 1.10: Loaded radius[9]

A gas inflated tyre will in an unloaded condition have a nice round shape with an outer radius called "Unloaded Radius". This radius more or less depends on the inflation pressure. When the wheel is mounted on a vehicle which is standing still, the tyre will be exposed to a normal force in the vertical direction. This normal force will be equal to the varying pressure between tyre and road integrated over the contact area. Due to expansion and compression of the tyre, the pressure will vary a bit in the contact patch. But a good approximation of the normal force is that it is equal to the inflation pressure times the contact area. This means that an increase in normal force will demand an increase in contact area, which gives a decrease in height between the centre of the wheel and the road. This height is called "Loaded Radius".

Since the gas in the tyre will compress and expand faster than the tyre, the pressure will vary a lot in dynamic events that makes this theorem a bit more complex. The stiffness of the tyre will make the inflation pressure higher with increase in static normal loads, an effect that also makes its behaviour harder to understand.

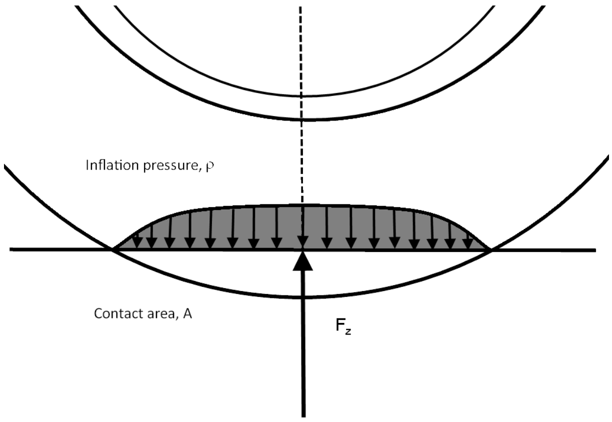

Figure 1.11: Inflation pressure[9]

1.6.2 With rotation, without drive torque

During angular rotation of the wheel, compression and expansion forces will arise along the thread of the tyre. The compression area will occur where the tyre first gets in contact with the road and the expansion area will occur in the end of the contact patch. This because the tyre acts like a spring-damper combination where the air delivers most of the spring characteristics and the rubber gives most of the damping forces.

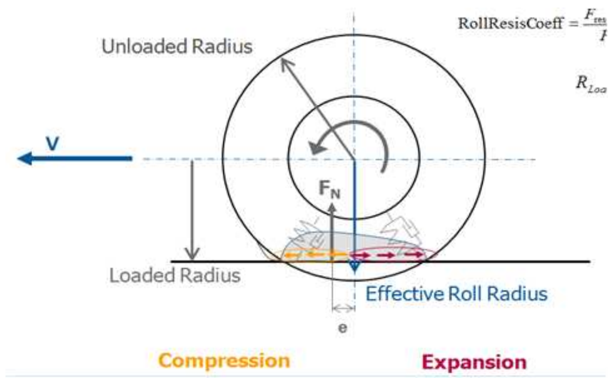

Figure 1.12: Tyre terms

According to [9], higher pressure will occur between the tyre and the road in the compression area than in the expansion area. This means that the resultaning normal force will be placed in front of the centre of the contact patch and in front of the vertical wheel centreline. In other words, a torque on the rim is needed to rotate a wheel with a normal force, this is called rolling resistance and can be calculated with formula 1.10.

$$RollResistanceCoefficient = \frac{F_{resistance}}{F_N} = \frac{F_N * e/R_{loaded}}{F_N} \tag{1.10}$$

Since compression and expansion forces exists in the tyre near the contact patch also shear forces in the contact exists which will result in slip between tire and road. This means that neither the unloaded radius nor the loaded radius can be used to relate the angular velocity to the forward velocity. The radius that is used for this calculation is called the effective rolling radius and is shorter than the unloaded radius and longer than the loaded radius.

1.6.3 With torque

When torque is applied to the rim it will be transferred through the sidewalls of the tyre to the contact patch. Since the sidewalls are not infinitely stiff they will deflect and pulling forces will arise more or less with an angle to the tyres radial line along the sidewalls. Since the tyre circumference will twist a bit around the rim the effective rolling radius and the height between road and wheel centre will decrease a bit.

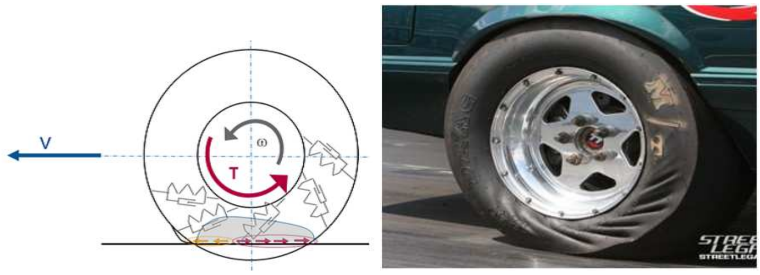

Figure 1.13: Tyre deformation[9]

The applied torque must have a reaction force in the contact between tyre and road. This reaction force will act in the same direction as the earlier discussed expansion forces which will result in an increase of expansion and decrease in compression in the tyre. When the expansion force is larger than the compression force the tyre will be pushed forward. Another effect of higher expansion than compression force is that slip occurs between tyre and road. Higher torque means more slip which can be calculated and expressed either as a ratio or a percentage. Slip ratio is calculated with formula 1.11 that compares the wheels´ forward velocity to its rotational velocity.

$$slip = \frac{\omega * R_e - V_x}{V_x} \tag{1.11}$$

Locked wheel	$\omega = 0$	$slip = -1$
Free rolling	$\omega = \frac{V}{R}$	$slip = 0$
Spinning wheel	$\omega = \frac{2*V}{R}$	$slip = 1$

1.6.4 Longitudinal force

With a constant normal force the longitudinal force for the tyre changes with the slip ratio like in figure 1.14. In the first almost linear part of the curve the force will increase with more throttle, but in the later part where a nonlinear behaviour can be seen the force will decrease with increased throttle. A factor called slip stiffness that depends on a number of factors is used to express longitudinal force in slip ratio, see formula 1.12. The slip stiffness will depend on a number of parameters as normal load, slip angle and of course slip ratio as seen in figure 1.14.

$$F_x = C_\kappa * \kappa \tag{1.12}$$

When applying throttle with a fixed rate the slip will increase in a stable way during the linear part but when the peak of the force vs. slip graph is passed the slip will increase dramatically and the driver will soon experience the situation as wheel spin.

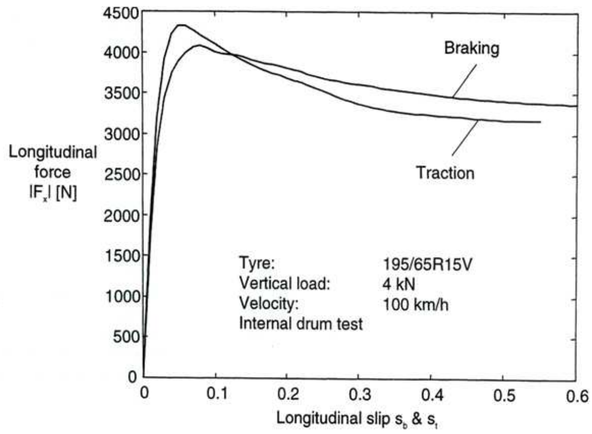

Figure 1.14: Longitudinal slip versus longitudinal force[9]

In figure 1.15 test results of the tyres of a Lamborghini Diablo can be seen. A conclusion is that the maximum longitudinal force is less than proportional to the normal force. With other words, if the normal force is doubled the longitudinal force will almost double. It can also be seen that the longitudinal slip stiffness increases with the vertical load. Since vertical load transfer from inner to outer wheel takes place during vehicle cornering this is an important parameter that influences torque transfer properties of a left-right oriented LSD while cornering.

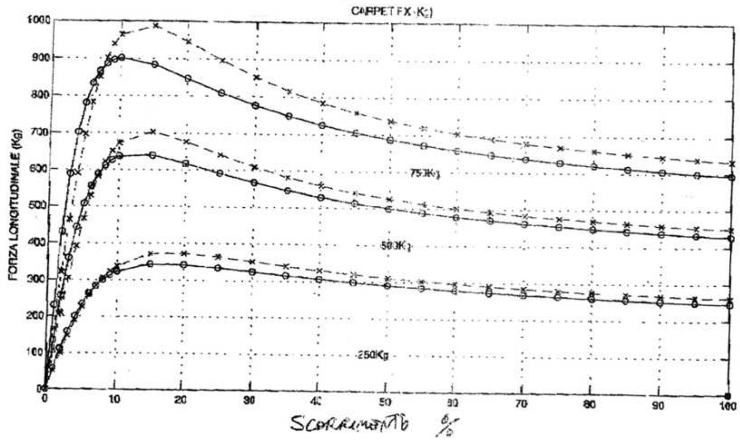

Figure 1.15: Slip ratio versus force, Lamborghini Diablo[9]

The relation between slip ratio, friction and the slip zone is shown in figure 1.16. When torque is applied to a wheel a slip zone will grow from the rear when accelerating and from the front when decelerating. In the rest of the contact patch more or less grip will be available. The slip zone will grow with increasing slip ratio and when the whole contact patch consists of a slip zone the tyre will spin. The reason for that the slip zone will grow in the opposite direction to the torque applied is simply because the normal force between road and tyre is lowest where the expansion force in the tyre is largest.

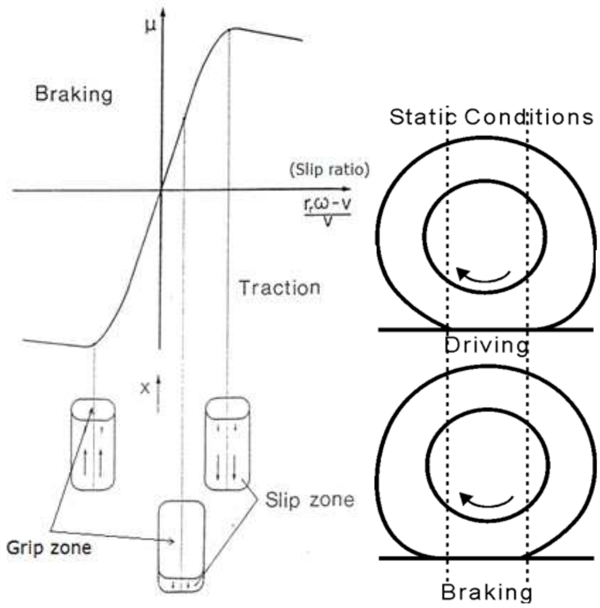

Figure 1.16: Grip zone and slip zone[9]

1.6.5 Lateral force

As mentioned earlier the tyre can be described like a spring-damper combination. The spring characteristics would show that the force is equal to a constant times the displacement. This would mean that with increasing lateral force the deflection of the tyres cross section will increase as seen in figure 1.17.

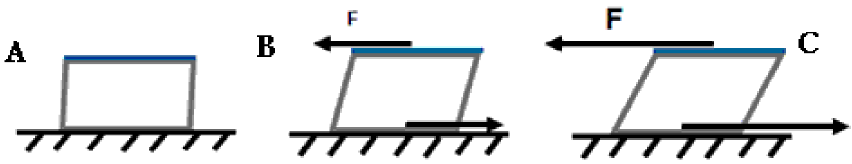

Figure 1.17: Tyre cross section[9]

During rotation the tyre has a constant cross section like the one in figure 1.17A as long as it is not close to the contact patch. A cross section in the beginning of the contact patch will have a small deflection like in figure 1.17B and the longer into the cross section the more deflection will occur. To get this build up in deflection there must be an angular deviation between the direction of the rim centreline orientation and the velocity direction, this is called slip angle and is seen as alpha in figure 1.18. More deflection means higher force which tells that the highest force will be in the rear of the contact patch.

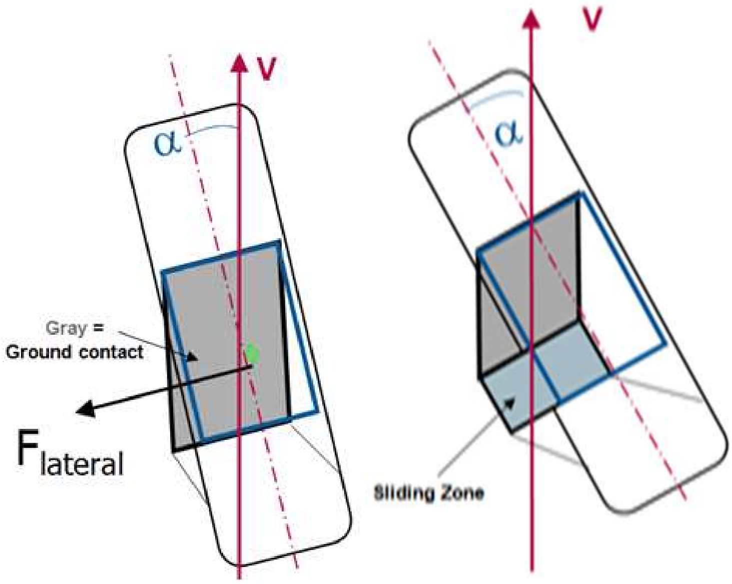

Figure 1.18: Contact patch with sliding zone, the blue line is the contact patch in an unloaded situation, the grey area is the contact with the road[9]

The name "slip angle" can be a bit confusing because with small slip angles, in the linear range where the lateral force grows with increased slip angle, slip (sliding) does not have to be present. It is first when we get close to the nonlinear region that slip(slide) starts to develop. As seen in figure 1.18, slip will rise from the rear of the contact patch when the slip angle is increased and no torque (driving or braking torque) is applied around the y-axis. A maximum lateral force generation excists for a certain slip angle and above that slip angle the lateral force will decrease with increased slip angle.

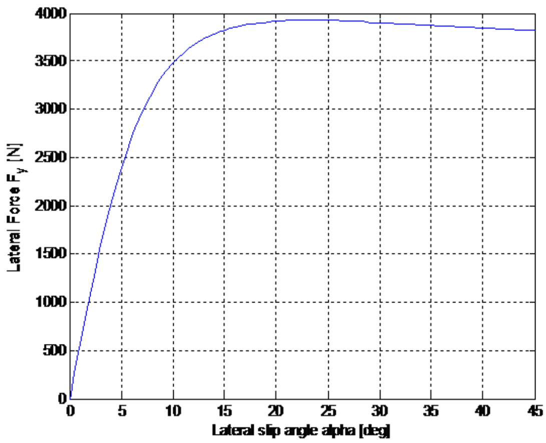

Figure 1.19: Lateral force versus slip angle[9]

As with longitudinal force the lateral force has a relation to slip. The parameter C_y found in formula 1.13 is called cornering stiffness and depends on for example slip angle, normal load and friction coefficient.

$$F_y = C_y * \alpha \tag{1.13}$$

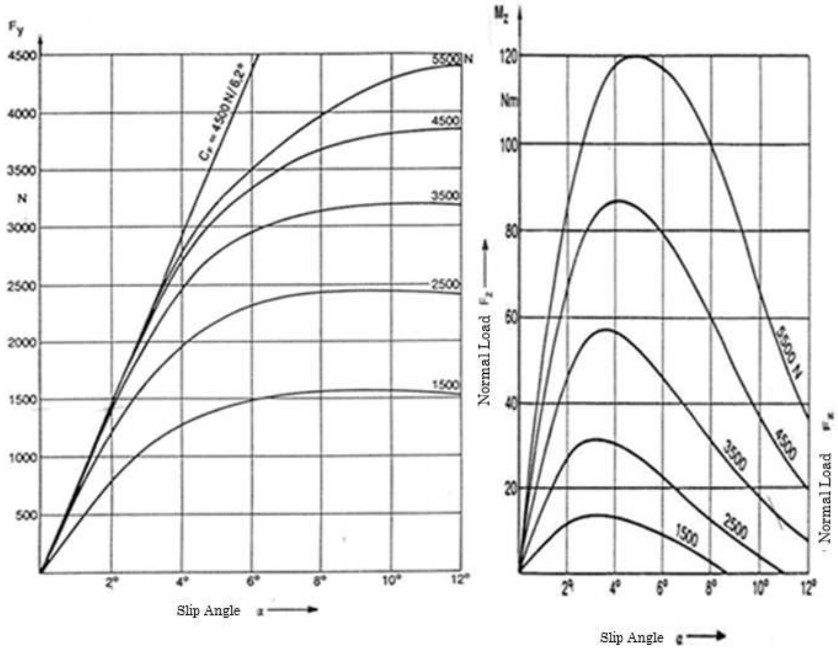

Figure 1.20: Lateral force and aligning torque versus slip angle[9]

Once again, exactly like with longitudinal force the friction is less than proportional to the normal load which can be seen in figure 1.20. Also the aligning torque increases with increased normal load but in this case the torque is more than proportional to the load. With other words, if the load is doubled the torque will be more than doubled. It can also be seen that the cornering stiffness is affected by the vertical load. Increased vertical load increases the cornering slip stiffness. This implies that a vehicle in a cornering situation were the slip angle of the inner and outer wheel is close to equal the outer tire having the highest vertical load will also produce the highest lateral force.

1.6.6 Combined slip

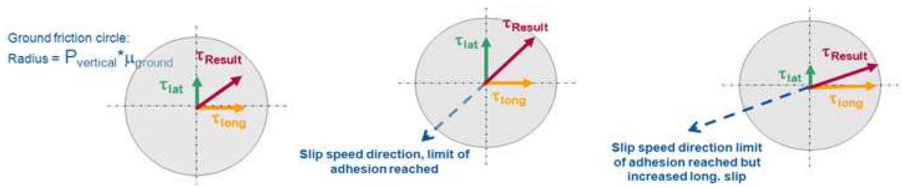

Figure 1.21: The friction circle[9]

To easily describe the relation between longitudinal, lateral and total friction force a simplified model can be used. The total friction force a tyre can develop in any direction is the normal load times the coefficient of friction between tyre and road. We can then visualize the total force like the radius in a circle. If the direction the total force acts in is known then the circle can tell how big the ratio between the longitudinal and the lateral forces are. It is also possible to tell how big lateral or longitudinal force that can be added in a specific moment before the peak of total force is passed. Realize the approximation of this model that the coefficient of friction is the same in all directions.

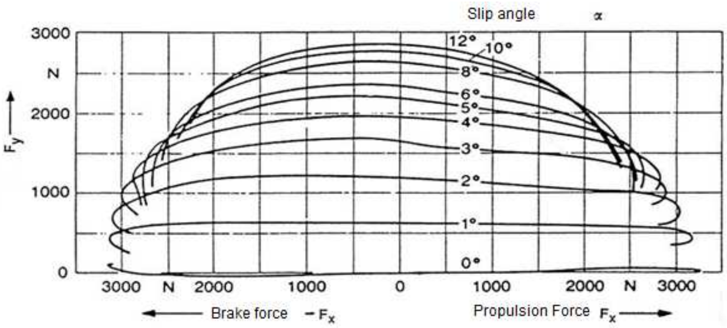

Figure 1.22: Longitudinal force versus lateral force[9]

In figure 1.22, real test results can be seen where the conclusion that a tyre normally can develop more longitudinal force than lateral force can be drawn.

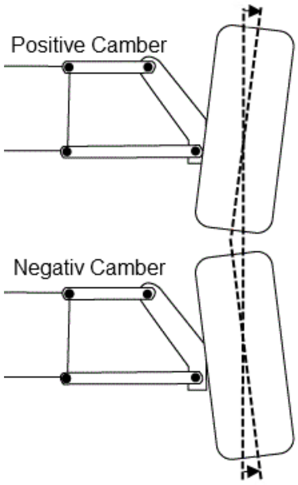

Figure 1.23: Graphic representation of camber

When adding camber to a rolling tyre a new force will arise, called camber thrust. This will affect the vehicle in the same direction as the lateral cornering force on the outer wheels if the tyre has negative camber and in the other direction if positive camber occurs.

Since the effective rolling radius will differ from being small on the inside and larger on the outside like a cone. The outside of the wheel will travel a longer distance than the inside. With other words, when rolling without drive torque there will be propelling forces on the outside of the contact patch and braking forces on the inside of the contact patch.

1.6.7 Tyre models

Tyre models are often used for simulation environments in vehicle models. A number of different models for different purposes have been developed the last century. Many levels of complexity and accuracy exist among these different ways of approach. In figure 1.24, [2], four different ways of approach are presented. The methods in the middle are the least accurate but also the simpler models.

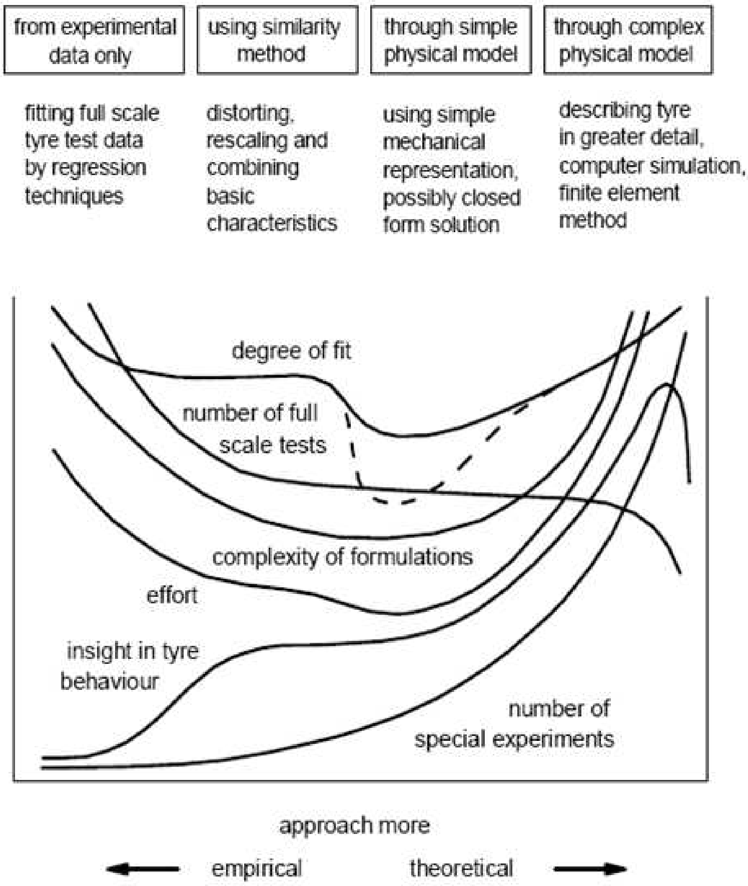

Figure 1.24: Graph representation of different tyre models according to Pacejka[2]

The left models are created using measured data which take their data from tables, formulas and certain interpolation schemes. One of the most famous semi-empirical is the Magic Tyre Formula, [2], which will be discussed later in the text. The right model types are pure mathematical models. These types of models might be quite simple and still deliver sufficient accuracy for a limited field of application. One example of this is the HSRI(Highway Safety Research Institute) model, developed by Dugoff, Fancher and Segel (1970), [2], depicted in figure 1.25. The figure shows a simplification compared to more realistic tyre deformation, this making the model more manageable, still including important characteristics as combined slip and a friction coefficient that decrease with increased sliding speed.

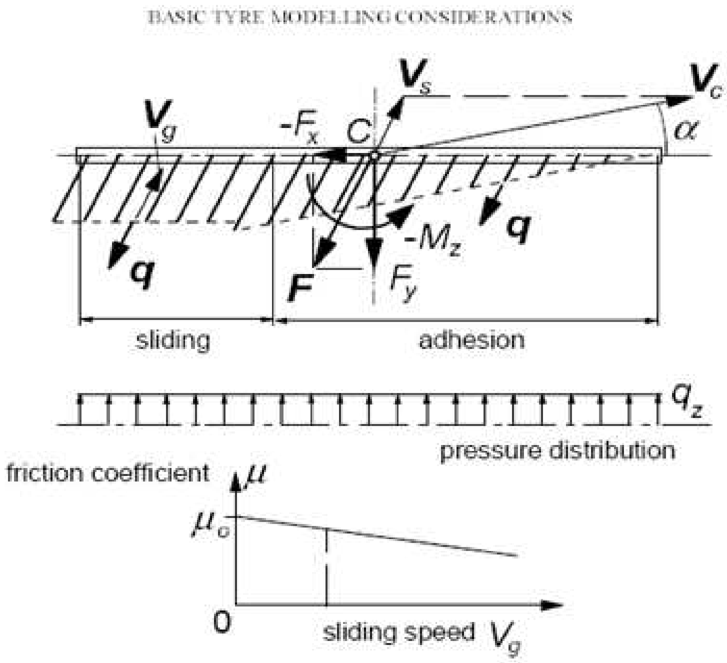

Figure 1.25: HSRI model [2]

With some specific inputs dependent on the complexity of the tyre model, it generates an output vector like the one depicted in the figure below.

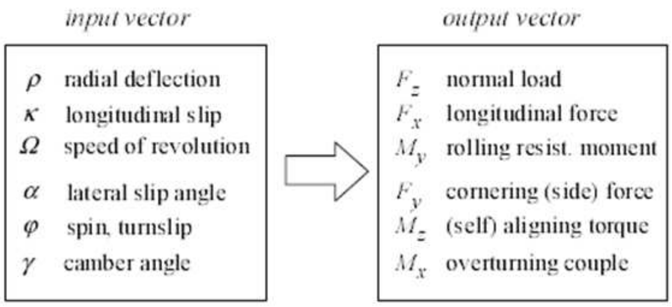

Figure 1.26: Input and output vectors[2]

If the parameters in the diagram are used the tyre is assumed to be uniform and rolling on a flat surface according to [6]. The inputs correspond to the wheels motion relative the ground surface. The output forces and moments can in a simpler case be considered to act on a rigid disc with inertial properties according to those of the tyre in a non-deformed state. It should be considered that dynamic forces acting between the tyre and road such as vibrations would not be taken into account.

The wheel motion, position and attitude are depicted in figure 1.27. Also the torques and forces acting on the wheel is shown in the figure.

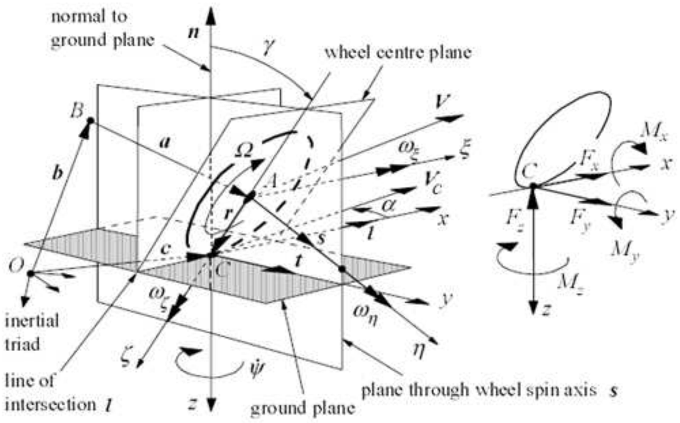

Figure 1.27: Wheel motion, position and attitude [2]

1.6.8 The Brush Model

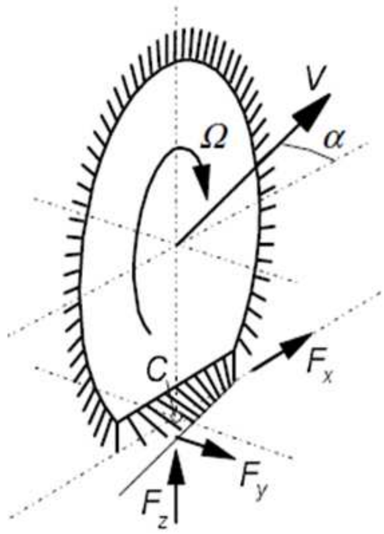

Figure 1.28: Brush representation of a tyres deflection[2]

The brush model is a theoretical tyre simulation model from [6] that calculates longitudinal and lateral tyre forces as a function of slip. The tyre is assumed to be built up by a number of bristles placed in a row like in figure 1.28 where every bristle can deform independently in any direction parallel to the road. By creating a model over how each bristle in the contact patch will deform by amount and angle the resulting force for each bristle can be calculated. The deformation of the bristle will depend on slip angle, slip ratio, normal load, road friction etc.

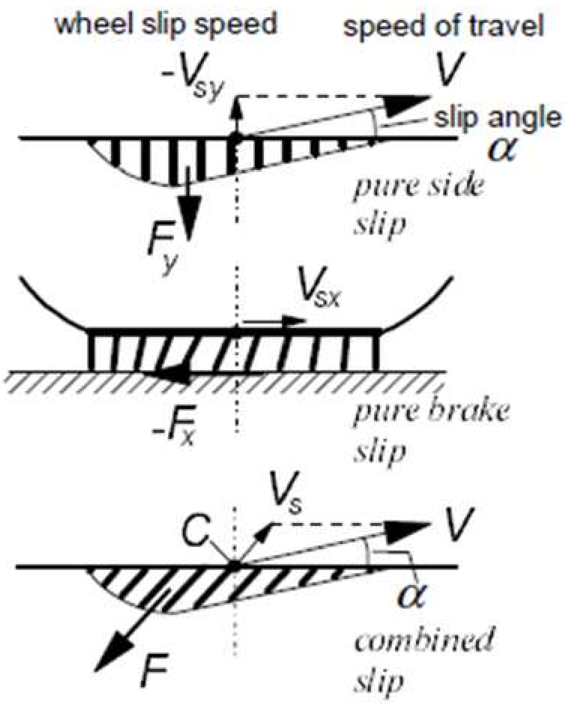

Figure 1.29: Combined slip-brush model[2]

In the upper illustration in figure 1.29 the "brush tyre" can be seen from above. It is here possible to see how the bristles will bend when only lateral force is affecting the tyre. The illustration in the middle show the wheel from the side where the bristles are bent backwards visualising braking. To illustrate the combination of these two a view from above is used again in the bottom illustration where the bristles are deformed to the side and back.

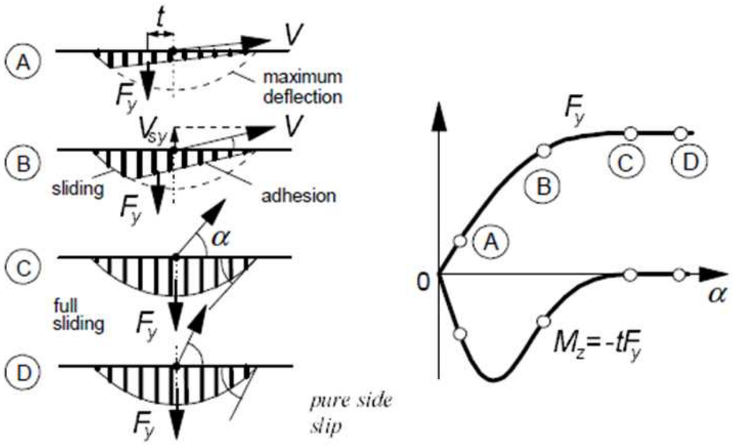

Figure 1.30: Pressure distrubution[2]

To further analyse the pure side slip condition the figure above can be used. The pressure distribution between the tyre and the road is assumed to have a shape of a parabola. In situation (A) in figure 1.30 do a very small slip angle occur and thereby not very much deformation. It can be seen that the deformation grows linearly from the front of the contact patch until the normal load is not large enough to support the lateral force. Behind this point where the normal load is not high enough the bristles slides over the road. The resulting lateral force will be placed a length "t" behind the centre of the contact patch which means that some aligning torque occur, also seen in the right hand figure.

In (B) the slip angle is increased, creating a larger deformation which gives higher lateral forces. Since the length "t" is now smaller the aligning torque has decreased although the lateral force is larger. The peak in aligning torque has been passed.

If the driver continue turning the steering wheel the slip angle will continue to increase and around point (C) the maximum lateral force is reached. If the maximum lateral force is reached, maximum deformation occurs and an increase in slip angle will only add sliding. In reality a loss in lateral force will occur if the slip angle is "too high", the curve shall point downwards after that the maximum lateral force is developed.

In the case of the brush model an increase in slip angle from point (C) to (D) will result in the same lateral force.

1.6.8.1 Combined slip with the Brush Model

When imagining a tyre as a row of bristles, a simple explanation of the models' combined slip calculations can be made. The approximation that friction and stiffness is the same in all direction enables us calculate the resulting friction force as the deflection times the stiffness. The resulting force will then act in the opposite direction to the deflection. Each bristle can with other words deflect in any direction.

With κ(slip ratio) and $tan(\alpha)$ two new theoretical slip values are developed, see formula 1.14 and 1.15.

$$\sigma_x = \frac{\kappa}{1+\kappa} \tag{1.14}$$

$$\sigma_y = \frac{tan(\alpha)}{1+\kappa} \tag{1.15}$$

These are the components in the following vector:

$$\sigma = \begin{pmatrix} \sigma_x \\ \sigma_y \end{pmatrix} \tag{1.16}$$

Total slip is calculated as hypotenuse of the x- and y-component.

$$\sigma = \sqrt{\sigma_x^2 + \sigma_y^2} \tag{1.17}$$

The technique is to calculate the total friction force and then divide it into a longitudinal and a lateral component through multiplying it with the fraction of each directions slip to the total slip. Since the total friction force is calculated differently depending on if adhesion or sliding occurs the maximum slip possible for adhesion must be evaluated. This is called σ_{sl} , formula 1.18, and is calculated with the composite model parameter θ, formula 1.19. c_p is lateral stiffness, a is half the contact length.

$$\sigma_{sl} = \frac{1}{\theta} \tag{1.18}$$

$$\theta = \theta_x = \theta_y = \frac{2}{3}\frac{c_p a^2}{\mu F_z} \tag{1.19}$$

To simplify the total force calculation a parameter λ, formula 1.20, can be calculated from σ and θ.

$$\lambda = 1 - \theta\sigma \tag{1.20}$$

For $\sigma \leq \sigma_{sl}$

$$F = \mu F_z(1-\lambda^3) = \mu F_z\{3\theta\sigma - 3(\theta\sigma)^2 + 3(\theta\sigma)^3\} \tag{1.21}$$

For $\sigma \geq \sigma_{sl}$

$$F = \mu F_z \tag{1.22}$$

When all this is done the longitudinal and lateral friction force can be calculated:

$$F_x = F\frac{\sigma_x}{\sigma} \tag{1.23}$$

$$F_y = F\frac{\sigma_y}{\sigma} \tag{1.24}$$

In vector format:

$$\mathbf{F} = F\frac{\sigma}{\sigma} \tag{1.25}$$

1.6.8.2 The Similarity Method

The Similarity method, [2], is a semi-empirical tyre model and the method is based on that the pure slip curves almost remain the same in shape when the tyre runs at conditions that are different from the reference condition. With reference condition means that the tyre has a given nominal load and has a camber angle equal to zero, is either free rolling or has a side slip equal to zero and has a given coefficient of friction. By a similar shape means that from the reference condition, by multiplying in a horizontal and vertical direction, a similar shape to that of the real curve is given

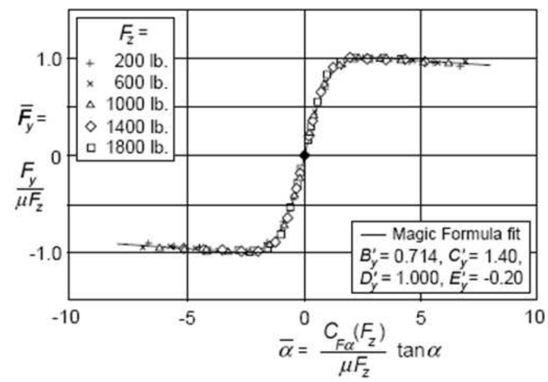

Figure 1.31: Utilized lateral force[2]

$$S_{H_y} = \frac{C_{F\gamma}(F_z)}{C_{F\alpha}(F_z)} \tag{1.26}$$

$$\alpha^* = \alpha + S_{H\gamma} \tag{1.27}$$

The Similarity Method[2] uses theoretical slip and other concepts adopted from the Brush Model[2]. The formulas below show the theoretical slip coefficients with a shifting α(slip angle) due to camber angles γ. Slip ratio is defined as κ.

$$\sigma_x = \frac{\kappa}{1+\kappa} \tag{1.28}$$

$$\sigma_y^* = \frac{tan(\alpha^*)}{1+\kappa} \tag{1.29}$$

and

$$\sigma^* = \sqrt{\sigma_x^2 + \sigma_y^2} \tag{1.30}$$

with

$$\alpha^* = \alpha + \frac{C_{F\gamma}(F_z)}{C_{F\alpha}(F_z)}\gamma \tag{1.31}$$

The theoretical slip produces the slope in the α-curves in the F_x vs F_y diagram, see in figure 1.22. The slip angle $\alpha*$ is compensated using the camber angle and the camber stiffness divided by the cornering stiffness. It should be noticed that the use of practical slip quantities might be enough to give good results. Since the general tyre has non-isotropic properties the pure longitudinal and lateral slip characteristics are not the same. Lateral and longitudinal forces can still be used but in the form of respective pure slip characteristics F_{yo} and F_{xo}. To use the theoretical slip quantities the pure slip characteristic must be available with σ_x and σ_y as abscissa.

If F_x is plotted versus σ_x an asymmetry is visible between the braking and driving side, this phenomenon can also be seen in the brush model.

Since camber has been accounted for, σ_y will be replaced by σ_y^* as defined in the formula above. With σ^* the force-components now become:

$$F_x = \frac{\sigma_x}{\sigma^*} F_{x0}(\sigma^*) \tag{1.32}$$

In the figure below the force created dependent on the pure slip characteristics is shown. What should be noticed in figure 1.31 is that with small slip the slip speed vector does not act opposite to the resulting force but for wheel lock it does.

For wheel lock and vanishing slip angle the side force should become zero, for the model this is not the case. This is due to α^* used in the formula for σ_y^*.

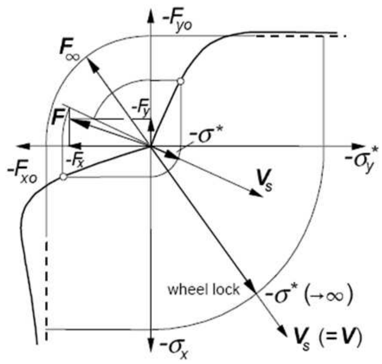

Figure 1.32: Resultning combined slip from pure slip characteristics[2]

After using the similarity expressions for pure slip characteristics the following formulas for combined slip are derived.

$$F_x = \frac{\sigma_x}{\sigma^*} \frac{\mu_x F_z}{\mu_{x0} F_{z0}} F_{x0}(\sigma^x_{eq}) \tag{1.33}$$

with the equivalent slip

$$\sigma^x_{eq} = \frac{C_{f\kappa}(F_z)}{C_{f\kappa 0}} \frac{\mu_{x0} F_{z0}}{\mu_x F_z} \sigma^* \tag{1.34}$$

and the side force:

$$F_y = \frac{\sigma_y}{\sigma^*} \frac{\mu_x F_z}{\mu_{y0} F_{z0}} F_{y0}(\sigma^y_{eq}) \tag{1.35}$$

with equivalent slip

$$\sigma^y_{eq} = \frac{C_{f\alpha}(F_z)}{C_{f\alpha 0}} \frac{\mu_{y0} F_{z0}}{\mu_y F_z} \sigma^* \tag{1.36}$$

Formulas taken from [2].

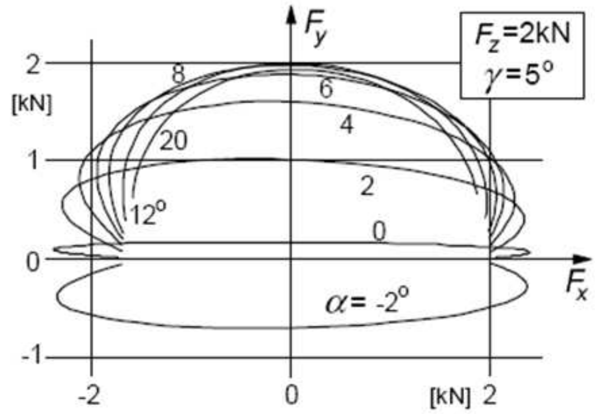

Figure 1.33: Longitudinal force versus lateral force[2]

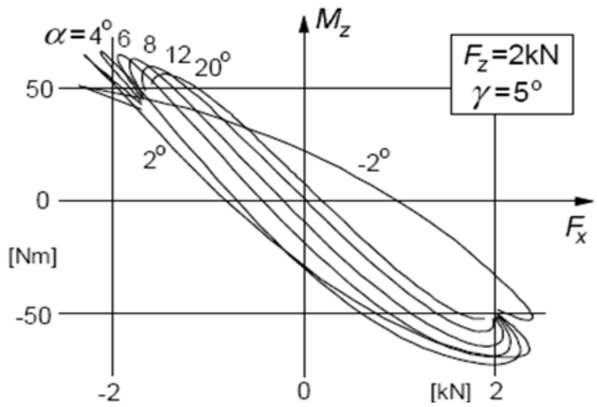

Figure 1.34: Aligning torque[2]

1.6.8.3 Magic Tyre Formula

The Magic Tyre Formula, [2], is a semi- empirical model since it is based upon measured data. It is used for vehicle simulation models and outputs tyre forces and torques. The formula was created by Pacejka.H.B. in collaboration with Volvo.

The general form reads:

$$y = Dsin[CarctanBx – E(Bx – arctanBx)] \tag{1.37}$$

with

$$Y(X) = y(x) + S_v \tag{1.38}$$

$$x = X + S_h \tag{1.39}$$

where

Y: output variable F_x, F_y or possibly M_z

X: input variable $tan(\alpha)$ or κ

and

B Stiffness factor

C shape factor

D peak side force value

E curvature factor

S_h horizontal shift

S_v vertical shift

$y(x)$ typically generates a curvature that passes through the origin, has a maximum and tends to a horizontal asymptote. The factors S_v and S_h makes it possible for the curve to have an offset from the origin. Slip angle α, longitudinal slip κ with the effect of load F_z and the camber angle γ are included in the parameters. The shape factor C is the coefficient that controls the limits of the range of the sine function; it thereby controls the shape of the curve. The product BCD corresponds to the slope at the origin. Since C and D are used to control other factors, B which is called the stiffness factor is left to control the slope at the origin. The factor E controls the curvature of the peak and its horizontal position.

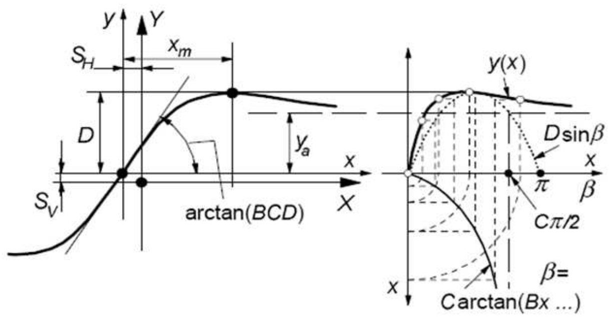

Figure 1.35: Shape factors in Magic Formula[2]

The shape factor C is computed through the equation below:

$$C = 1 \pm (1 - \frac{2}{\pi} arcsin(\frac{y_\alpha}{D})) \tag{1.40}$$

Using the shape factors B, C and the location x_m of the peak value E can be calculated.

$$E = \frac{Bx_m - tan\{\frac{\pi}{2C}\}}{Bx_m - arctan(Bx_{m)}} \tag{1.41}$$

Cornering stiffness, camber angles γ, nondimensional parameter p_3.

$$BCD_y = p_1 sin[2arctan(\frac{F_z}{p_2})] * (1 - p_3\gamma^2) \tag{1.42}$$

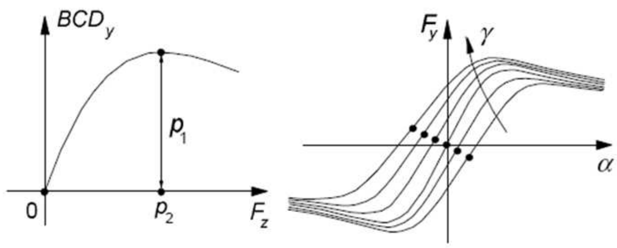

Figure 1.36: Cornering stiffness[2]

The aligning torque can be calculated through the side force and the pneumatic trail added with a small residual torque.

$$M_z = -t * F_y + M_{zr} \tag{1.43}$$

1.6.9 Tyre coordinate system

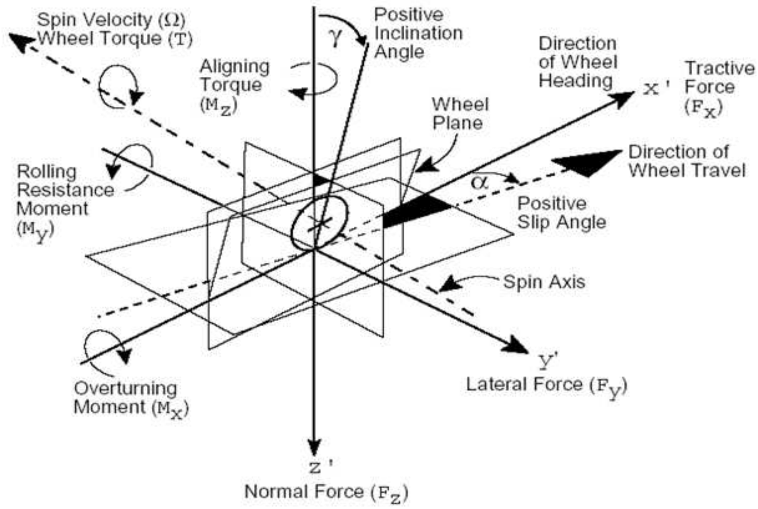

Figure 1.37: Tyre coordinate system[10]

α	Slip Angle. The angle formed between the direction of travel of the tyre contact patch center and the x'-axis. is positive if the wheel travel direction has a component in the +y'-direction. This produces a negative Lateral force (Fy). Note that the steer angle, or the vehicle attitude angle, plays no part in defining the slip angle.
γ	Inclination Angle. The angle formed between the x'-z' plane and the wheel plane. Inclination angle is positive if the wheel plane has a component lying in the +y'-direction.
F_x	Longitudinal Force. The x'-component of the resultant force acting on the tire by the road.
F_y	Lateral Force. The y'-component of the resultant force acting on the tyre by the road. Lateral Force may be produced by slip angle, inclination angle, conicity, plysteer, or any combination of the above.
F_z	Normal Force. The z'-component of the resultant force acting on the tire by the road. The direction of this force is up, but this nomenclature requires that Fz be negative whenever the tyre is in contact with the road, as the positive z'-axis is directed downward.
M_x	Overturning Moment. The moment of the forces at the contact patch acting on the tyre by the road with respect to the x'-axis.
M_y	Rolling Resistance Moment. The moment of the forces at the contact patch acting on the tyre by the road with respect to the y'-axis.
M_z	Aligning Torque. The moment of the forces at the contact patch acting on the tyre by the road with respect to the z'-axis.
Spin axis	The axis about which the wheel rotates. Perpendicular to the Wheel Plane, not necessarily about the y'-axis (only if inclination angle is zero).
spin vel Ω	The angular velocity of the wheel on which the tyre is mounted, about its spin axis.
T	Wheel Torque. The external torque applied from the vehicle about the spin axis of the wheel.
Vertical load	The normal reaction of the tyre on the road which is equal to the negative of Normal Force. This is always a positive quantity when the tyre is in contact with the road, otherwise it is zero.
Wheel plane	The central plane of the tyre and wheel, normal to the wheel spin axis.
$+x'$	Direction of wheel heading along ground. The intersection of the wheel plane and the road plane in the neighborhood of the tyre Axis System origin. This is not the same as the direction in which the wheel is traveling. If the tyre reverses its direction, the axis system flips 180 degrees about the z'-axis.
$+y'$	To the right along the ground, as viewed from behind a forward rolling tyre. Chosen to be Right-Hand Orthogonal to the definitions of x' and z'.
$+z'$	Perpendicular to the road in the neighborhood of the tyre Axis System origin with positive direction down. (If the road is flat and in the x-y plane, this is negative Global Z.)

2 YAW TORQUE

By shifting torque from one side to the other a LSD(Limited Slip Differential) can control the yaw torque applied to the car. The yaw torque is derived from the forces acting on the tyres and the distance to C.o.G(Centre of Gravity). With a shifting steering angle the lever arms will vary, by locking or opening the differential we can increase or decrease the yaw torque dependent on the drivers wish. This can be used to help the driver go faster through turns or have a yaw dampening effect to reduce accidents.

2.1 Yaw torque calculation

The yaw torque can be calculated by either dividing the forces into components acting in longitudinal and lateral directions and multiplying these with the fixed distances to the C.o.G or calculate the distances varying with steer angle.

The concept of these calculations comes from that the distances a and b, see figure 2.1, are considered to be constant which one could consider not to be entirely correct due to shifting position of C.o.G during roll or a heavy load in the trunk. This will not initially be taken into account.

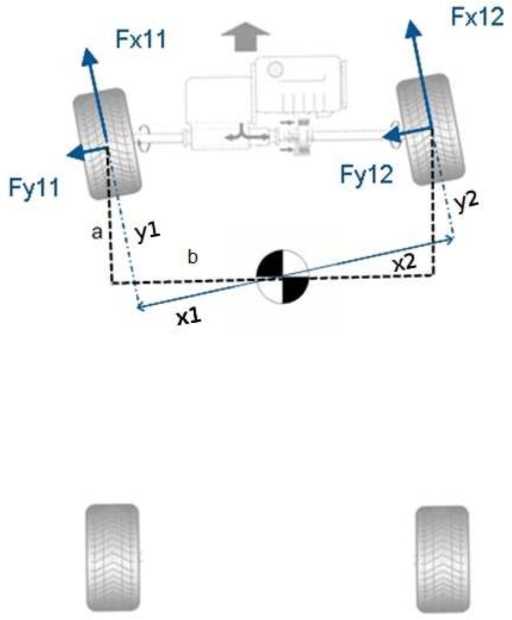

Figure 2.1: Forces and lever arms for the FWD vehicle[1]

The lever arms are calculated through the following equations, with wheel steering angle δ and the constant distances a and b as inputs, see figure 2.1.

$$x_1 = (\frac{tw}{2} - a * tan\delta)cos\delta \tag{2.1}$$

$$x_2 = (2\frac{tw}{2} - x_1) \tag{2.2}$$

$$y_1 = \frac{a}{cos\delta} + x_1 tan\delta \tag{2.3}$$

$$y_2 = \frac{a}{cos\delta} - x_2 tan\delta \tag{2.4}$$

The yaw torque is then calculated with a moment of equilibrium equation around the centre of gravity using the calculated lever arms and tyre forces.

$$I_{zz}\ddot{\psi}_{frontAxle} = x_2 F_{x12} + y_2 F_{y12} + y_1 F_{y11} - x_1 F_{x11} \tag{2.5}$$

The rear axle tyre forces are outside the scope of this thesis work since they are not directly affected by the control of the front axle LSD. To calculate the yaw torque the tyre forces needs to be derived.

3 YAW TORQUE USING “MAGIC FORMULA”

In the first phase of the project a far too advanced yaw torque was estimator built, to be used in the control software. The aim was to use advanced tyre models, in this case “Magic Formula”, to first be able to derive good estimations of parameters such as the tyres vertical load, the tyres longitudinal force, wheel radius, rolling resistance and more. Another reason was to be able to see how much various parameters affect the final result.

3.1 Description

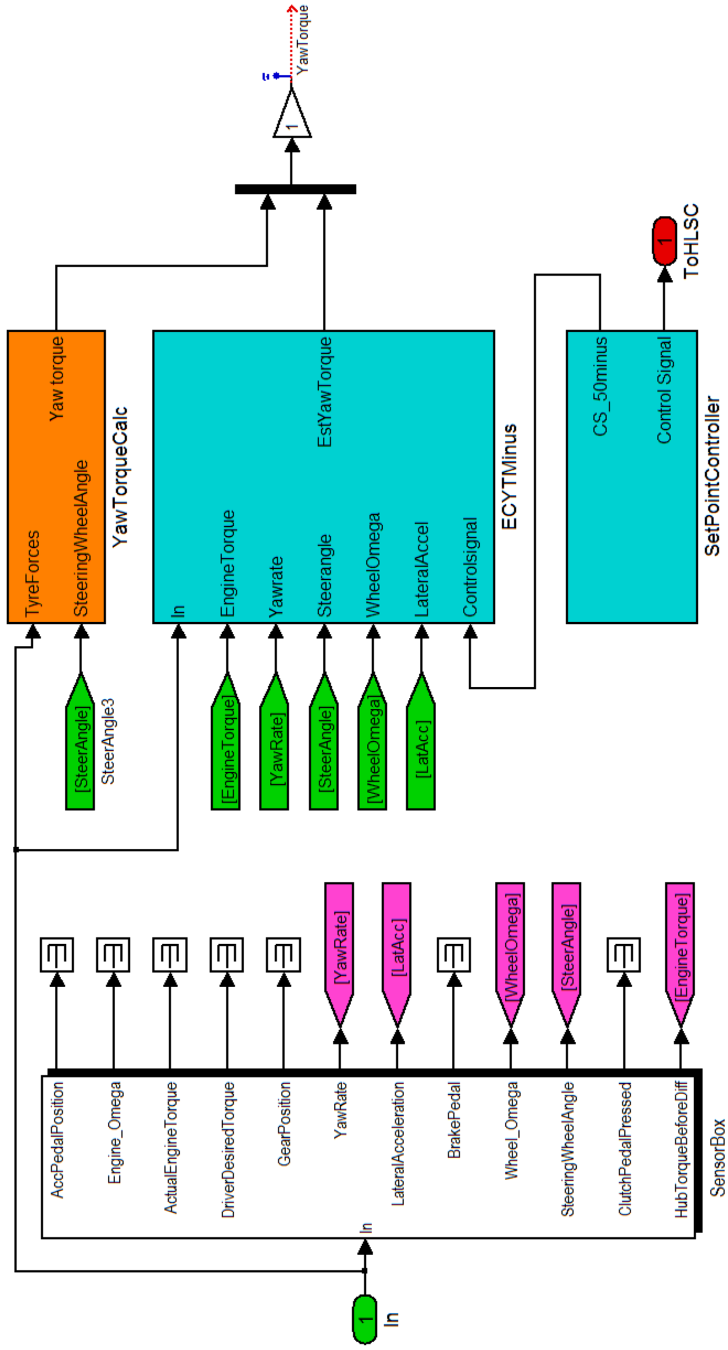

Figure 3.1: Simulink blocks

The yellow simulink block called *YawTorqueCalc* in figure *3.1,* calculates the yaw torque of the current state using the simulation models tyre forces. The required inputs to this block is the longitudinal and lateral tyre forces, steering wheel angel, steer ratio and distances to CoG according to formula 2.5. This yaw torque signal is used as a reference to compare our results against.

The gray block called *SetPointController* delivers the control signal to the clutch. This signal functions as a set point of how much torque should be transferred through the differential from the high speed wheel to the low speed wheel. In the figure 3.2 a set point of 100 Nm nominal torque with a pulse increasing the torque to 150 Nm every other second.

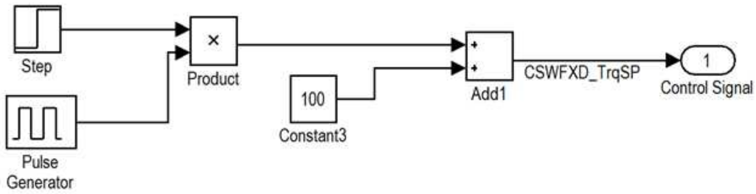

Figure 3.2: Simple control signal using a varying set point

3.2 Lateral force for each wheel

The tyre model “Magic Formula”[2] requires a number of input signals. In this first part of the project several of these signals will be taken directly from the simulation model. The signals that have been taken are; hub displacement, hub angles, hub velocities and the longitudinal forces to calculate the rolling resistance and transferred torque. Using these together with a number of measured signals the lateral forces of each front wheel can be derived.

The grey blocks depicted in figure 3.3 contains the tyre models written in c-code. As describe in the introduction, “Magic tyre formula” is a semi-empirical tyre model and therefore requires a lot of input parameters. In the model these are refered to as “typarr” and are simply a list of different parameters associated to this specific tyre. This is a negative aspect of using these types of tyre models, it requires a lot of specific tyre data which changes every time the tyres are changed.

In the yellow block the transferred torque is calculated using calculated rolling resistance, “real” longitudinal forces and the torque set point of the control software. When the LSD is over locked, the wheels have equal rotational velocity, it is hard to estimate how much torque is transferred through the LSD. The model estimates how much torque currently can be transferred to the ground. If the limited torque is smaller than the set point the limited torque controls the transferred torque. The sign of the torque decide which wheel to add torque and which to subtract. Half the gearbox output torque is then added which gives the hub torque.

The yellow block has been further described in chapter 2.1.

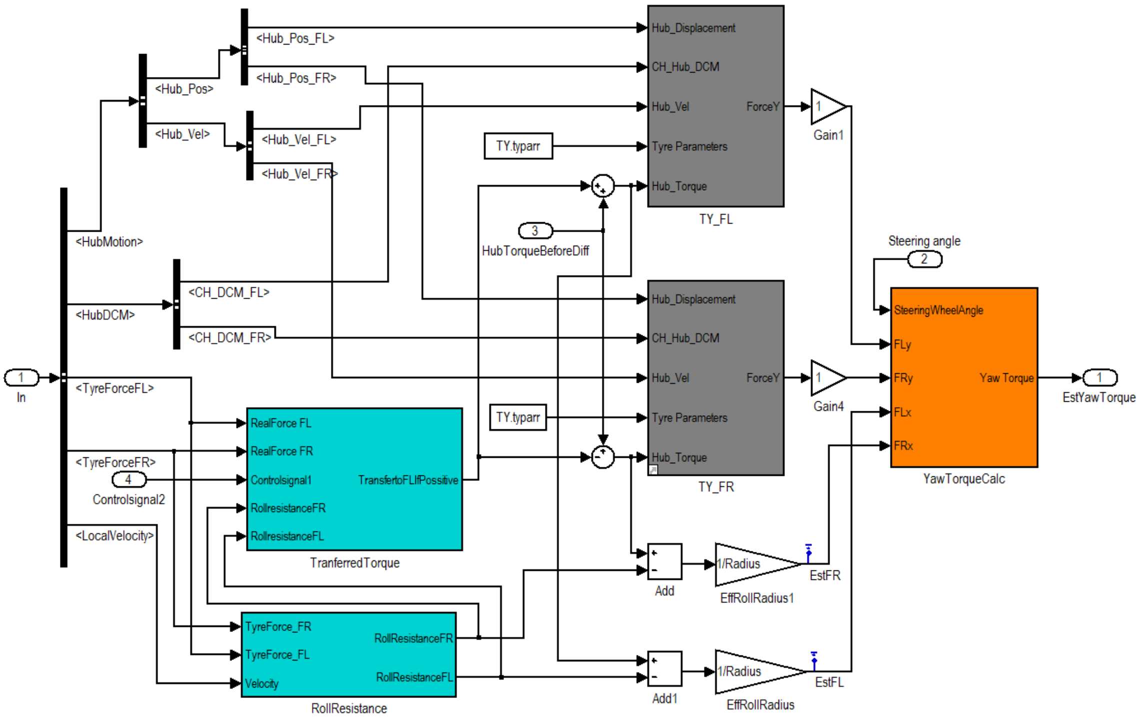

Figure 3.3: Yaw torque simulation blocks

3.3 Longitudinal force for each wheel

The radius that is used to derive the longitudinal force from the drive shaft torque and the rolling resistance is the effective rolling radius regarding to formula 3.2.

$$F_x * R_e = M_D - M_Y \tag{3.1}$$

M_Y is the roll resistance, M_D drive shaft torque(hub torque), F_x longitudinal force and R_e effective rolling radius.

Rearranged:

$$F_x = \frac{M_D - M_Y}{R_e} \tag{3.2}$$

$$M_y = -F_z * R_0 q_{sy1} * atan(\frac{V_r}{V_o}) + q_{sy2} * \frac{F_x}{F_{zo}} * \lambda_{M_y} \tag{3.3}$$

The roll resistance is always acting in the opposite direction to the longitudinal motion of the tyre. An approximation of the formula for roll resistance torque “Magic Formula”[2], formula 3.3, is used. Since $\lambda_{M_y} = 1$ and $q_{sy2} = 0$ for the tyre used is the formula shortened to:

$$M_y = -F_z * R_0 * q_{sy1} * atan(\frac{Vr}{V_0}) \tag{3.4}$$

F_z is the vertical load on the tyre, V_r is the rolling velocity, R_0 and V_0 is reference parameters for radius and velocity with the values 0.315 meters and 16 m/s.

$q_{sy1}$is also a tyre parameter and in this case 0.01.

Within the blue and green rectangle the longitudinal force on each front wheel is calculated using the formula explained above. The driveshaft torque subtracted with the rolling resistance gives the propelling torque of the vehicle. This is then divided with the wheel radius to get the longitudinal force. The driveshaft torque is evaluated from the delivered engine torque which is recalculated with the gear ratio and compensated for the LSD torque.

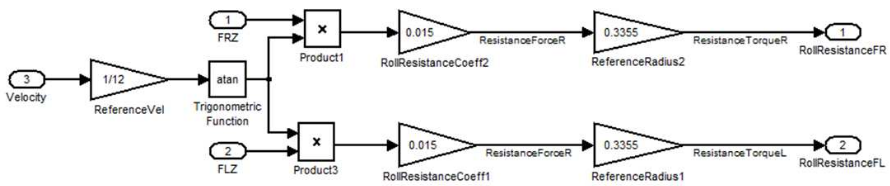

Figure 3.4: Estimation of roll resistance

Figure 3.4 illustrates how the rolling resistance is derived using formula 3.4. This is the contents of the blue block in figure 3.3.

3.4 Estimation of torque transferred through LSD

This part of the model is divided into three main parts depicted in figure 3.5, which corresponds to the yellow block in figure 3.3. Inside the red lines the torque difference between the left and right wheel is calculated. In this case are the "real" longitudinal tyre forces used but with a different tyre model these could be replaced.

Within the green lines the correct sign for the control signal/set point is chosen. This is done by looking at the sign on the torque difference signal. A hysteresis is also added to not encounter problems when the torque difference is close to zero. The output of the block is decided within the blue rectangle, it compares the values and outputs the smallest value, either the control signal or half the torque difference.

To get a stable system the control signal have to be a bit larger than the compared signal to become the output, this explaining the gain of 0.53 instead of 0.5 in the right bottom corner of the blue rectangle. This means that if the control signal is close to the calculated torque difference, the control signal is preferred to be used.

$$T_{diff} = (F_x r * r + M_{yFR} - F_{xf} * r - M_{yFL}) * 0.5 \tag{3.5}$$

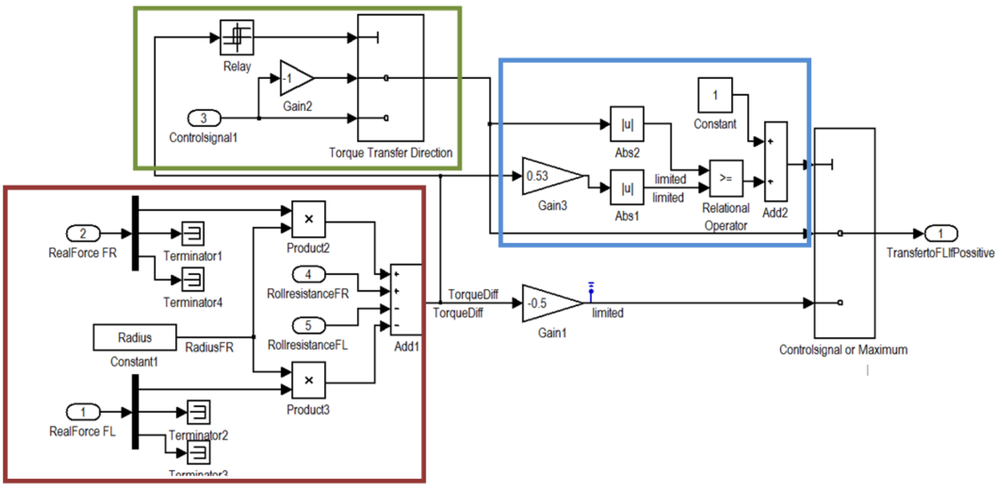

Figure 3.5: Calculation of torque through LSD

3.5 Wheel radius sensitivity test

To examine how sensitive the yaw torque estimation is to error in wheel radius a few different simulations has been executed. The radius for deriving the longitudinal force from the hub torque or reverse is the effective rolling radius as explained in section 1.6.2.

The radius affects the estimation in two ways. Firstly when the longitudinal tyre forces on the front wheels shall be derived from the propelling torques on the drive shafts. Secondly, when calculating the amount of torque transferred through the LSD during over locked behaviour. To demonstrate the simulations radius error-sensitivity, simulations have been conducted at both over locked behaviour and when varying the transferred torque.

In all of the simulations the vehicle has a start speed of 15 m/s and 100% throttle during the whole simulation.

3.5.1 Locked LSD

To start with the simulations have been done using a locked LSD.

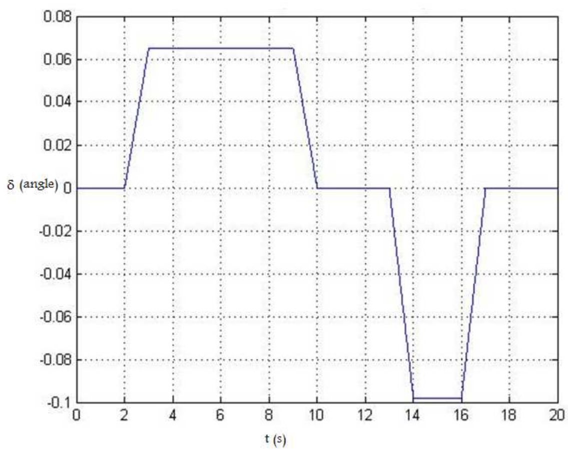

Figure 3.6: Steer angle in radians during the simulations

The change in steer angle during the first set of simulations can be seen in figure 3.6.

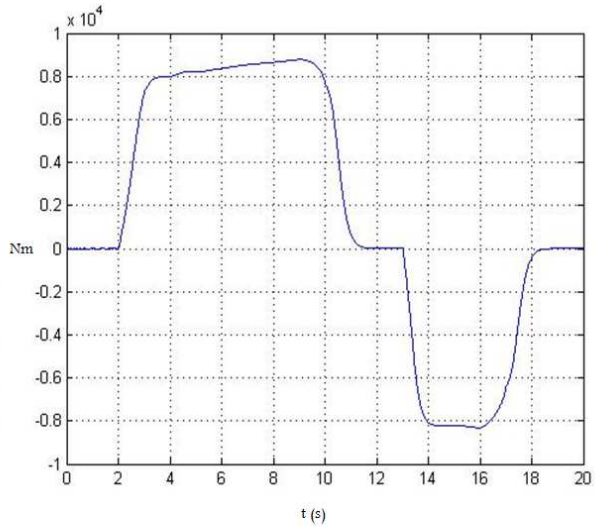

Figure 3.7: Graph over the contribution to yaw torque from the front wheels during the simulations. unit, Nm

Figure 3.7 illustrates how the front wheels contribute to yaw-torque. The maximum yaw torque contribution measured in this simulation is about 8900 Nm (including both longitudinal and lateral forces on the front tyres).

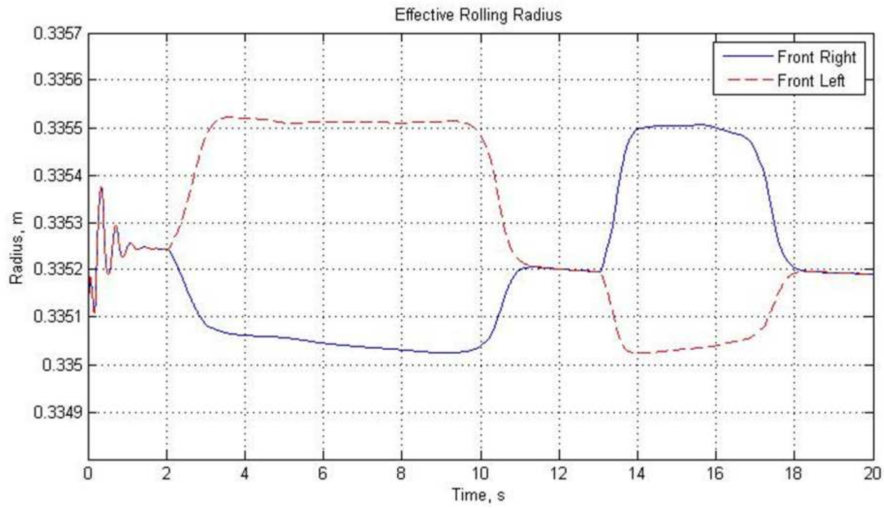

Figure 3.8: The variation of effective rolling radius over the simulation.

As seen in figure 3.8 the effective rolling radius varies between around 0.3350 m and 0.3355 m with a centre around 0.3352 m. Since the variation is very limited no effort is put in estimating the variations of the effective roll radii. With other words the radii are kept constant during the simulations.

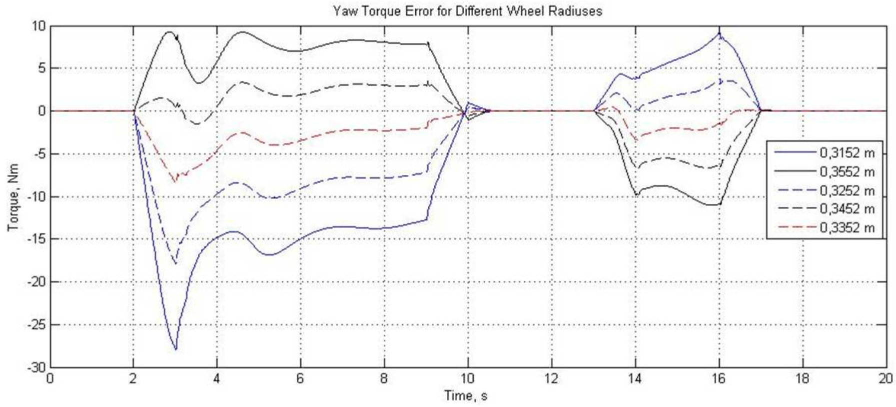

Figure 3.9: Yaw torque error for different effective wheel radii

Name	Radius	Maximum yaw-torque error*	Average yaw-torque error**
Test 1	0.3152 m	27.7 Nm (0.37%)	8.30 Nm (0.13%)
Test 2	0.3252 m	18.5 Nm (0.26%)	4.69 Nm (0.08%)
Test 3	0.3352 m	7.8 Nm (0.10%)	1.95 Nm (0.03%)
Test 4	0.3452 m	6.6 Nm (0.08%)	2.12 Nm (0.03%)
Test 5	0.3552 m	11.0 Nm (0.14%)	5.17 Nm (0.08%)

Table 3.1: Results of yaw torque sensitivity to radius error. * The percent values are calculated by dividing the error by the actual yaw torque contribution at the time. ** The average values are calculated over time from second 2 to second 18. The percent values are calculated by dividing the average error by the average yaw torque contribution.

Five simulations have been conducted using different wheel radii in the yaw torque calculations. Table 3.1 show that errors in effective rolling radius do not affect the yaw torque much when the LSD is locked. The maximum error recorded is 27.7 Nm which at the point was 0.37% of the real value and this with a 20 mm too small radius.

Notable is that the effect of a one centimetre error in radius only give an error of 0.03% in yaw torque. This is probably because too large radius compensate for other deviations in the model estimations such as hub torque. The simulation shows that the error due to this effect is very small and it shows better results when using a larger radius compare to smaller.

3.5.2 Variation in LSD torque transfer

A more common situation is when the LSD is not fully locked. This second set of simulations is conducted with alternating set points for the LSD during the simulations. The set point for the LSD is 100 Nm with a half second step to 150 Nm every other second. To dispose of transients in yaw torque these simulations are conducted using another sequence of change in steering wheel angle, see figure 3.6.

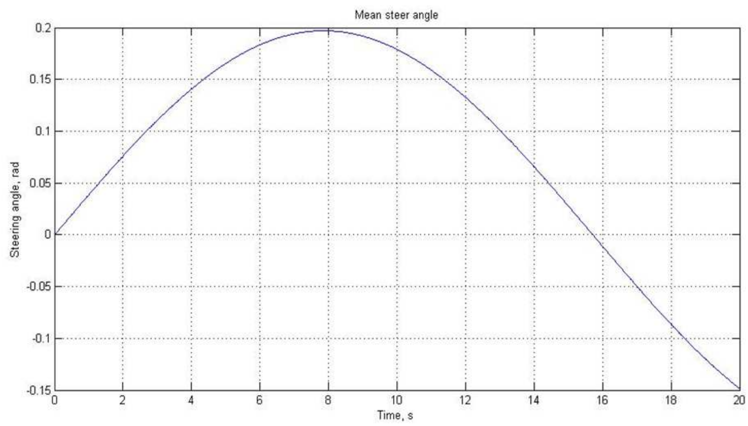

Figure 3.10: Steer angle in radians during the simulations

Exactly like in the simulations with a locked LSD the five simulations are conducted with the same variation in radius.

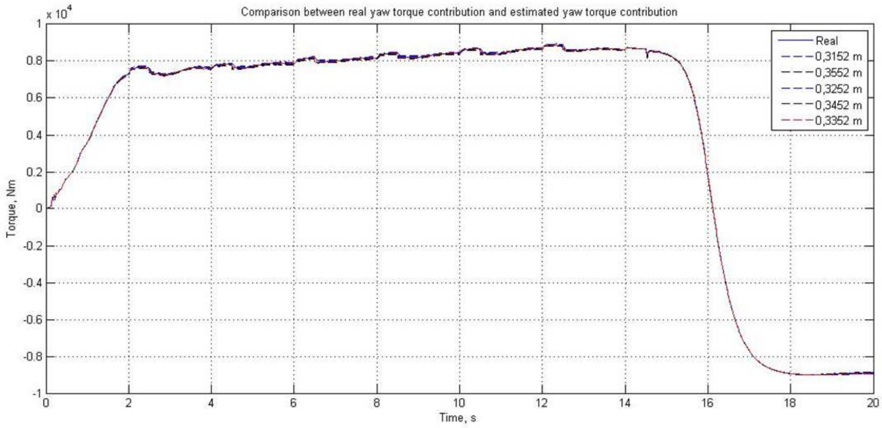

Figure 3.11: Comparison between real and estimated yaw torque contribution

Figure 3.11 visualize how the yaw torque contribution from the front wheels changes over time. It shows us that all the simulations give rather god results, all give the same characteristics.

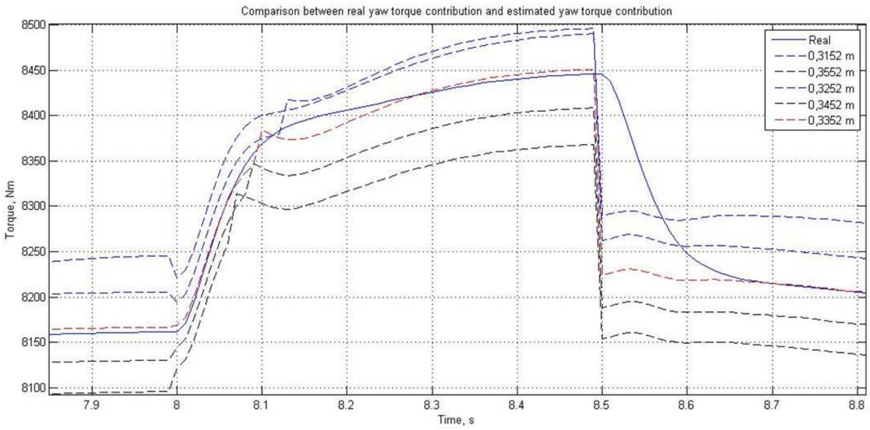

Figure 3.12: Comparison between real and estimated yaw torque contribution, close up

To further analyze the results figure 3.12 can be used. The blue solid line is the real yaw torque contribution and the red one corresponds to the simulation with correct effective radius for the tyre, kept constant. The black lines represent the long radii and the blue represent the short radii.

The yaw torque calculation has the same slope as the "real" yaw torque except an overshoot. At time 8.5 seconds in figure 3.12 the LSD the set point is decreased with 100 Nm. What happens in the simulation is that the actual yaw torque from the simulation model slowly decreases unlike the estimated yaw torque that rapidly changes. This is probably because of the tyres relaxations length and flexibility in shafts etc. which is not modeled for the estimated yaw torque. This creates a peak in yaw torque error which makes the maximum error less interesting. That is why only the average yaw torque error is shown in table 3.1. Most important to realize is that the change in yaw torque before and after the change in locking torque is almost the same for all simulations except from the one with radius 0.3152 m. Actually, the only thing that is interesting is the change in yaw torque with a change of 100 Nm in locking torque. The conclusion should be that the effect of wheel radii change does not affect the yaw torque enough to be included in the calculations.

Name	Radius	Average yaw-torque error*
Test 1	0.3152 m	43.1 Nm (0.57%)
Test 2	0.3252 m	34.1 Nm (0.44%)
Test 3	0.3352 m	11.1 Nm (0.1%)
Test 4	0.3452 m	29.7 Nm (0.39%)
Test 5	0.3552 m	56.6 Nm (0.74%)

Table 3.2: Results of yaw torque sensitivity to radius error. *The percentage values are derived by dividing the average error with the average yaw torque contribution.

The result of the test can be seen in table 3.1. An error of plus minus 10 mm will give an error of 0.3 % in the yaw torque calculation. This shows that an error in wheel radius do not affect the yaw torque enough to add an wheel radius estimation in the model. The value given from the tyre manufacturer is adequate to use in the yaw torque calculations.

4 STANDALONE YAW TORQUE CALCULATIONS

Using Magic Tyre Formula[2] requires a lot of computing power and inputs such as vehicle side velocity which is not an available signal. It is therefore not possible to use in the vehicles control software. The following model is a simpler but less accurate model. The inputs to this model is only measured signals from the CAN-bus or specific vehicle parameters.

4.1 Description

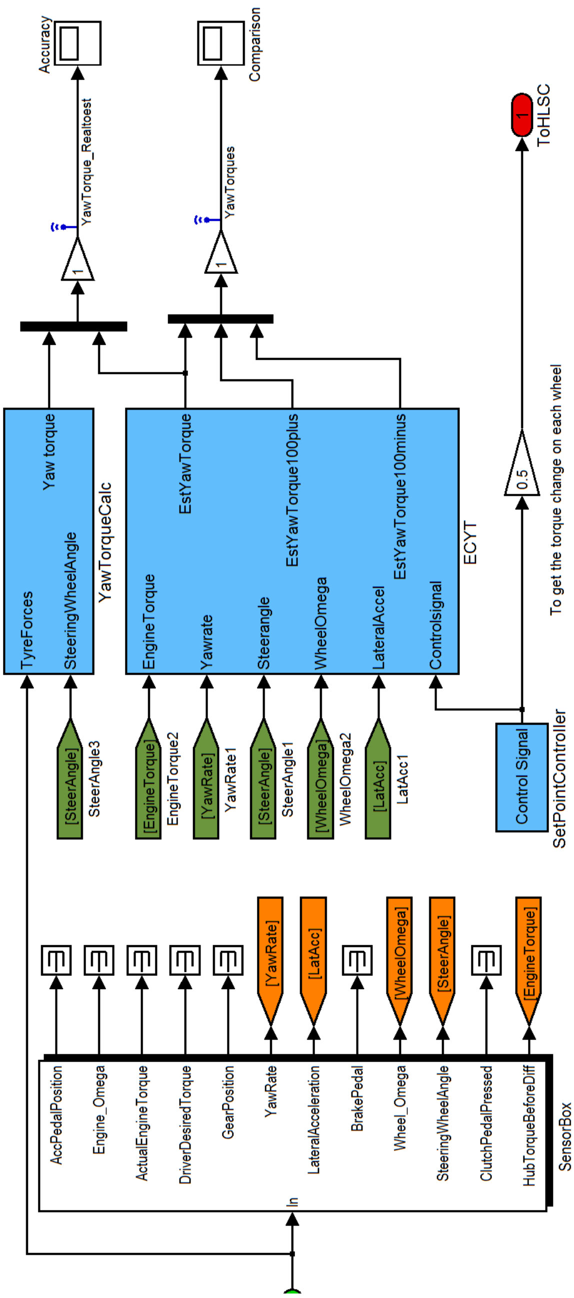

Figure 4.1: Yaw torque model

The Simulink model depicted above includes the main blocks of the software that estimates and calculates the yaw torque. The block "YawTorqueCalc" calculates the reference signal, the yaw torque derived from the "real" tyre forces. The large blue block includes all of the calculations and estimations to derive the yaw torque using the input signals shown in the picture and some specific vehicle parameters, these signals can be measured by incar sensors. The small blue block outputs the control signal, the torque set point for the differential.

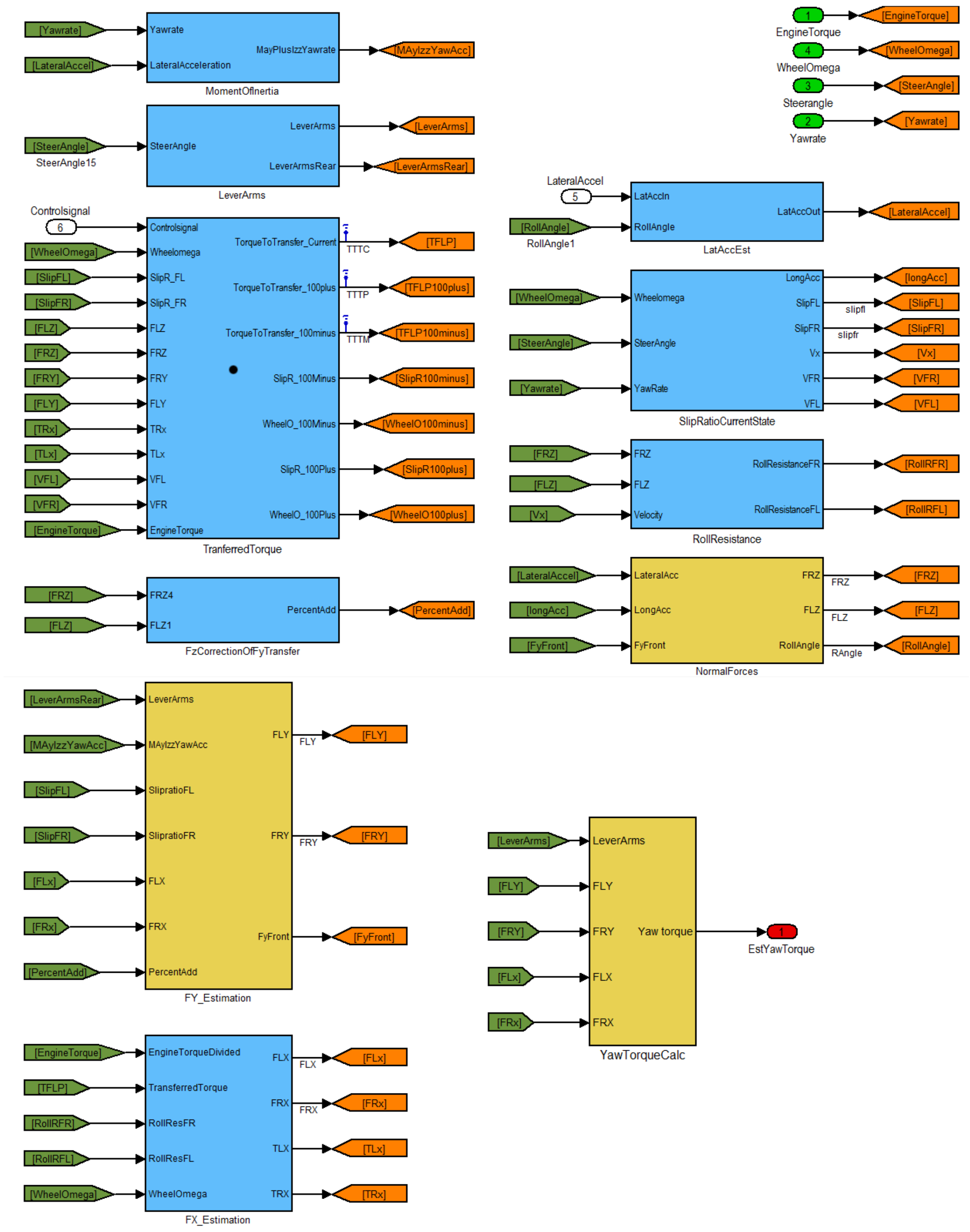

Figure 4.2: Yaw torque model

As mentioned earlier the estimator has been given a major improvement from the former model in chapter three. This model does not require variables as hub velocity, hub position or hub angles since the magic formula block has been replaced. The replacement is a simpler model that only require parameters that can be estimated or calculated from the signals that already exist as measured signals in most production vehicles.

The big orange block, called "FY_Estimation" seen in figure 4.2, contains the lateral force calculations. One support block that evaluates longitudinal slip-ratios(see equation 1.11) for the front wheels. Longitudinal velocity and acceleration are also created in this block that develops the normal forces between tyre and ground for each front tyre.

4.2 Normal forces

To get accurate results when calculating the longitudinal and lateral tyre forces, accurate normal forces are needed. In the following text two different methods are described.

4.2.1 Simple normal load calculation

To accurately calculate the forces acting on the tyres it is important to have good normal force estimation. One can easily calculate the total lateral and longitudinal forces acting on the vehicle, it is harder to estimate the distribution between the wheels.

The total lateral and longitudinal forces is calculated through respective acceleration multiplied with the total weight of the vehicle. These can be used to estimate the load transfer from right to left respective front to rear. This will give a rough estimation of the weight transfer.

Side to side normal load

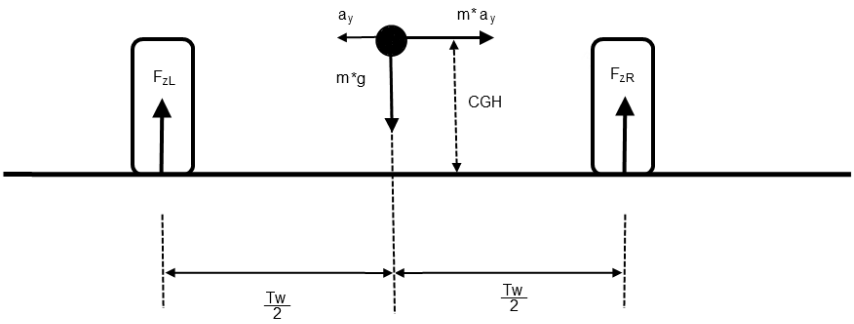

Figure 4.3: Simple representation of a vehicle seen from the front

$$F_{zR} + F_{zL} = mg \tag{4.1}$$

No dynamics have been included in formula 4.1 since no signal of the vertical acceleration exists.

$$F_{zR} * Tw = mg * \frac{Tw}{2} + m * a_y * CGH \tag{4.2}$$

As before has no acceleration of CoG in the vertical direction been included.

A further refinement would be to incorporate the unsprung mass and the influence of the front/rear anti roll bar balance. The unsprung mass accounts for wheels(including tyres and rims), wheel hubs and brakes and half the suspension mass. Having a varying CGH could improve the results further. This would require more computing power and a good estimation of the distance CGH.

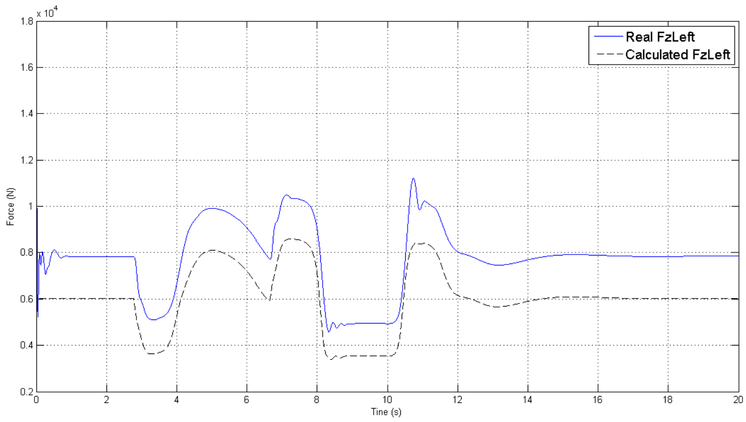

Figure 4.4: Normal force distribution right side, unsprung mass neglected

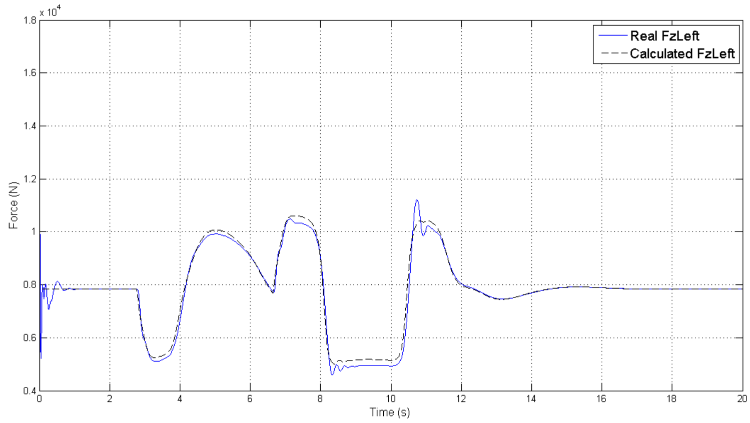

Figure 4.5: Normal force distribution right side, unsprung mass accounted for with constant CGH

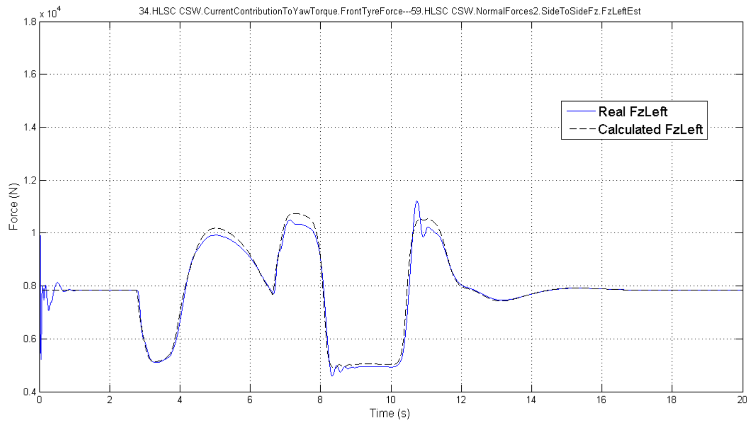

Figure 4.6: Normal force distribution right side, unsprung mass accounted for in CoG, constant CGH

A large stationary error arises if the unsprung mass is not included in the calculations like in figure 4.5. Figure 4.6 show the best estimation of the weight distribution. This estimation of the normal forces does not add the unsprung mass into the vehicles centre of gravity. The result in figure 4.5 only show a marginally better improvement compared to the the result in figure 4.6, where the unsprung mass has been added into the vehicles CoG.

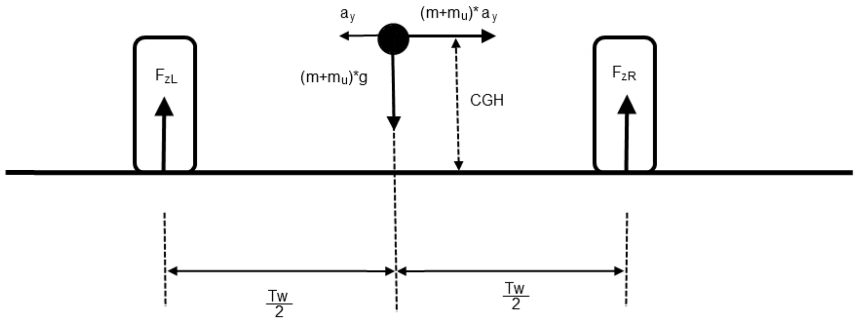

Figure 4.7: Simple representation of a vehicle seen from the front including unsprung mass

$$F_{zR} + F_{zL} = (m + m_{unsprung})g \tag{4.3}$$

$$F_{zR} * Tw = (m + m_u)g * \frac{Tw}{2} + (m + m_u) * a_y * CGH \tag{4.4}$$

Tests with a varying CGH only show very small improvements of the result. Since this is the case, this distance is set to constant.

Front to rear normal load

The next task is to estimate the distribution of the vehicles weight on each wheel. By using the longitudinal acceleration the front-to-rear load distribution can be derived using a simple moment of equilibrium.

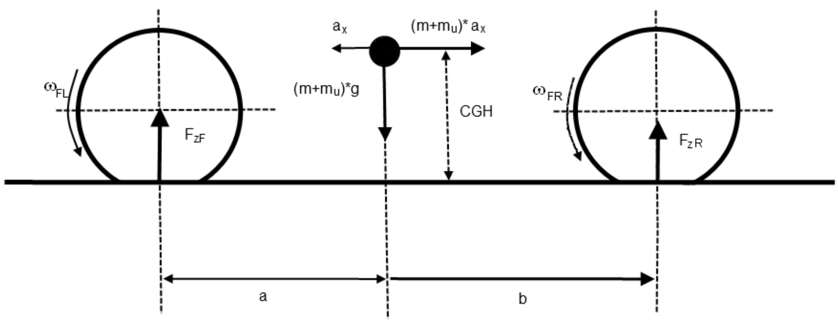

Figure 4.8: Simple representation of a vehicle seen from the side, including unsprung mass

$$F_{zFR} + F_{zFL} = (m + m_u)g \tag{4.5}$$

$$F_{zFR} * (a + b) = (m + m_u)g * b - (m + m_u) * a_x * CGH \tag{4.6}$$

When not including the anti roll bar balance of the vehicle a simple and fast distribution can be derived. By multipling the percent of load on the right side with the percent of load on the front axle the right-front tyre load can be derived.

$$\%F_{zRightSide} * \%F_{zFrontAxle} = \%F_{zRightFront} \tag{4.7}$$

$$\%F_{zFrontRight} * F_{zTot} = F_{zFrontRight} \tag{4.8}$$

This simple method works well and gives quite accurate results considering the approximations.

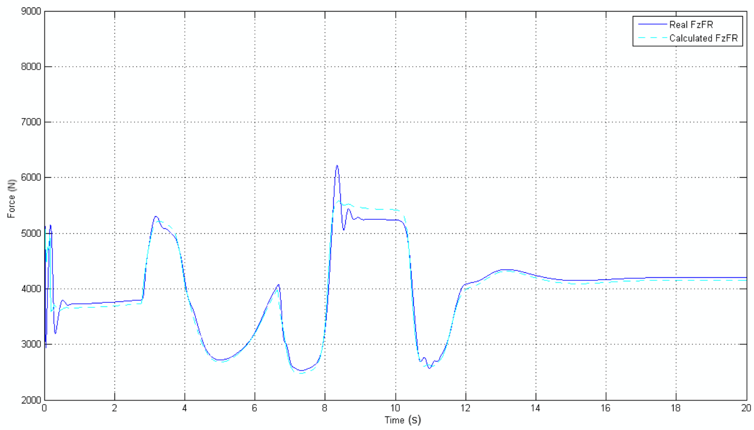

Figure 4.9: Real Fz force versus calculated Fz force, front right tyre

The result of the combined normal load calculations can be seen in figure 4.9. This simple calculation gives a good result but has a small stationary error. The result could be considered accurate enough for the rest of the estimations but to minimize the errors another method will be tested.

4.2.2 Normal load calculation using roll stiffness

Since the last normal load model was not accurate enough a second version was created. This model includes the roll stiffness of the vehicle and a calculated roll angle. The formula is build upon a moment of equilibrium for the front respective rear axle.

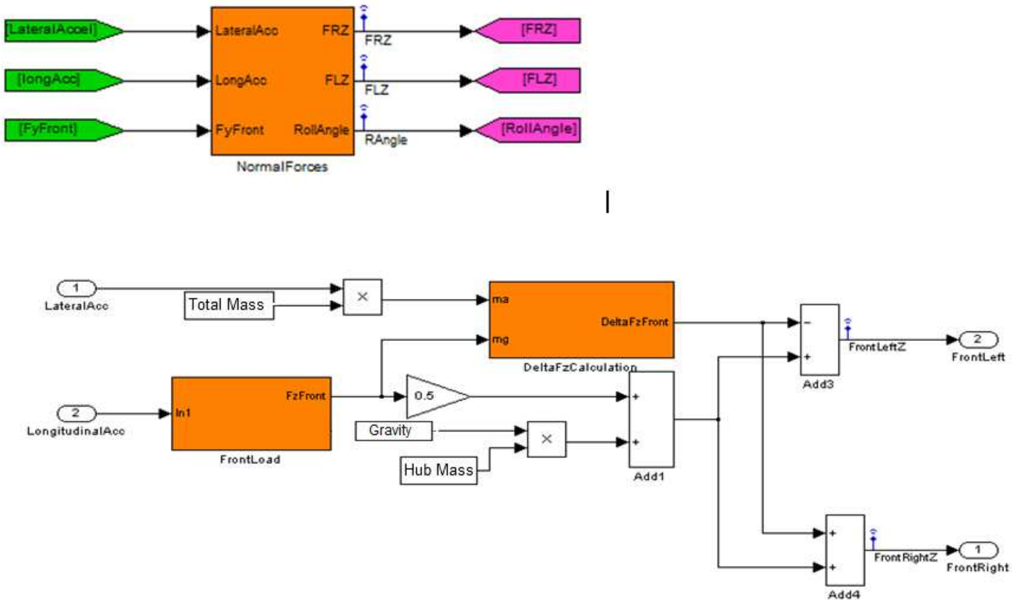

Figure 4.10: Normal force calculation in simulink

The formula calculates the load transfer between the front tyres, and the load transfer is then added or subtracted from the tyres calculated load dependent on the longitudinal acceleration.

The front load calculation uses formula 4.9, which include the longitudinal acceleration and a number of vehicle parameters. m described in formula 4.9 refers to the body mass not including the wheel mass.

$$F_{zFront} = \frac{m * g * b - m * a_x * CGH}{l} \tag{4.9}$$

The basic idea is that the load transfer is proportional to the centripetal force through a coefficient σ_i.

$$\Delta F_{zi} = \sigma_i m a_y \tag{4.10}$$

$$\sigma_i = \frac{1}{2s_i}\left(\frac{c_{\phi i}}{c_{\phi 1} + c_{\phi 2} - mgh'}h' + \frac{l - a_i}{l}h_i\right) \tag{4.11}$$

The distance s is half the track width, l is the wheelbase and a is the distance from C.o.G to respective axle. h is the height from the ground to the axle roll centre and h' is the distance from the roll centre to C.o.G. c is the roll stiffness for respective axle. Above equations taken from [2].

$$F_{zFrontRight} = \frac{F_{zFront}}{2} + m_{wheel} * g + \Delta F_z \tag{4.12}$$

As mentioned earlier the formula for the load transfer comes from a moment of equilibrium around the roll centre of the axle. Since this formula contains the centripetal force in every term, the formula has been simplified as a coefficient times the centripetal force.

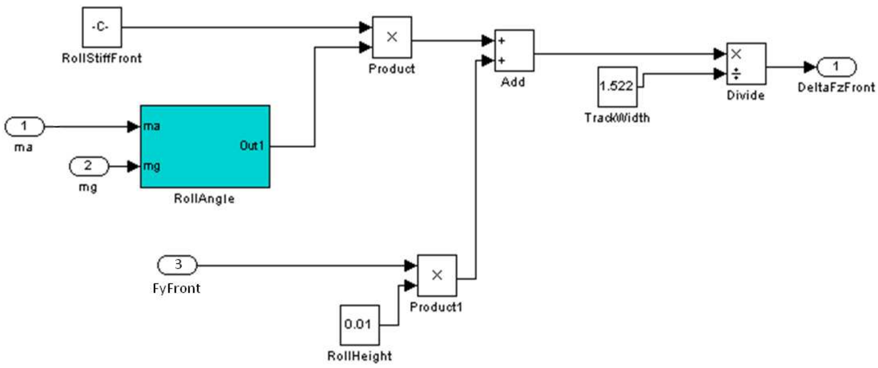

Figure 4.11: Load transfer calculations for the front axle

The Simulink block scheme in figure 4.11 calculates the load transfer through the front roll stiffness, a roll angle and the total lateral front axle force.

$$\theta = h_{CgToRollCenter} * \frac{ma_y}{c_{\phi Front} + c_{\phi Rear} - mg * h_{CgToRollCenter}} \tag{4.13}$$

The roll angle calculation is according to the formula above dependent on the total roll stiffness of the vehicle. There are different ways of calculating the roll stiffness, but no easy way. The several unknown parameters create a problem when using this method, to be able to use it some sort of verification is needed. If this method will be used, data from the vehicle manufacture is required or advanced measuring systems that can measure the total roll stiffness of the vehicle. The roll stiffness can be estimated and might give a result good enough, but for the best result vehicle data is preferred.

$$\Delta F_{zFront} = \frac{c_{\phi Front} * \theta + F_{yFront} * h_{rollheight}}{Tw} \tag{4.14}$$

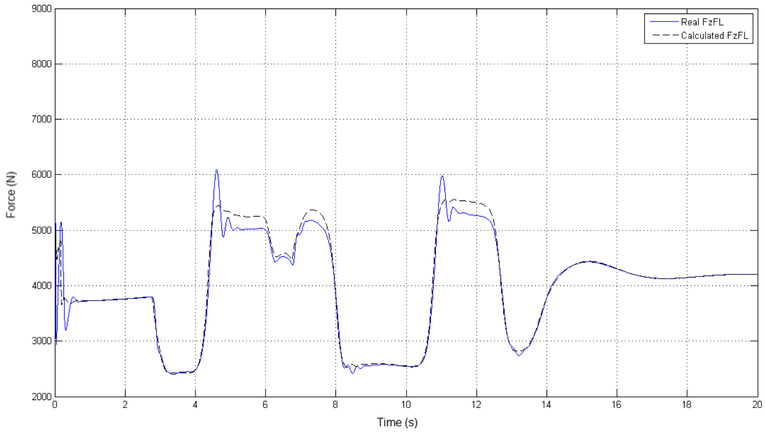

Figure 4.12: Real Fz versus calculated Fz

As seen in figure 4.12the stationary error found in the the last normal load estimation has almost disappeared. There still exist some deviations which could be decrease with further adjustments. This model of the normal load has been used in the yaw torque estimation because of the smaller stationary error and the faster response.

4.3 Slip ratio

To calculate the longitudinal slip the forward velocity of the tyre is required, this velocity has been estimated using the cars forward and side velocity, yaw rate and the steering angle. The steering angle together with yaw rate are signals taken directly from the CAN bus. The front and side velocities require some calculations and approximations.

4.3.1 Vehicle forward velocity

Since it is a front wheel drive vehicle the rear tyres are free rolling and is therefore appropriate to use when estimating the forward velocity of the vehicle i.e. they have no longitudinal slip except a small contribution from the rolling resistance. The velocity is calculated through the mean angular velocity multiplied with the radius of the tyres. A constant value of the tyre radius is chosen.

$$V_x = \frac{\omega_{RL} + \omega_{RR}}{2} * r_{wheel} \tag{4.15}$$

The formula above gives a good approximation of the vehicles forward velocity.

4.3.2 Vehicle side velocity

By using the single track bicycle model the formula below can be derived. To derive this formula a steady state approximation must be made. Since the side velocity has the most effect in low speeds this approximation can be considered adequate.

$$V_y = (\frac{b}{V_x} - \frac{m * V_x * Wdistrear}{C_{34}}) * a_y \tag{4.16}$$

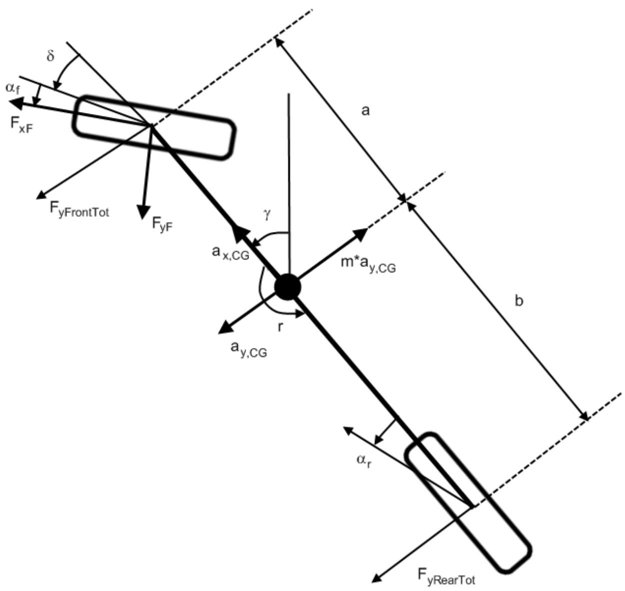

Figure 4.13: Bicycle model

As stated above formula 4.16 comes from the steady state approximation. The formula has been derived from[11]:

$$m * a_y = F_{y12} + F_{y34} \tag{4.17}$$

$$I_{zz} * \ddot{\psi} = F_{y12} * a - F_{y34} * b \tag{4.18}$$

Steady state

$$I_{zz} * \ddot{\psi} = 0 => F_{y12} * a = F_{y34} * b \tag{4.19}$$

$$m * a_y = m * \dot{\psi} * V_x = F_{y12} + F_{y34} \tag{4.20}$$

Combining formula 4.19 with formula 4.20

$$m * \dot{\psi} * V_x = F_{y34} * (\frac{L}{a}) \tag{4.21}$$

Linearized F_{y34}, small angle approximation

$$m * \dot{\psi} * V_x = -C_{34} * \alpha_{34} * (\frac{L}{a}) \tag{4.22}$$

$$\alpha_{34} = atan(\frac{V_{y34}}{V_x}) \tag{4.23}$$

Linearized α_{34}, small angle approximation:

$$atan(\frac{V_{y34}}{V_x}) = \frac{V_{y34}}{V_x} = \frac{V_y - b * \dot{\psi}}{V_x} \tag{4.24}$$

$$m * \dot{\psi} * V_x = -C_{34} * (\frac{V_y - b * \dot{\psi}}{V_x}) * (\frac{L}{a}) \tag{4.25}$$

$$V_y = b * \dot{\psi} - \frac{m * \dot{\psi} * V_x^2 * wdistRear}{C_{34}} \tag{4.26}$$

$$V_y = (\frac{b}{V_x} - \frac{m * V_x * wdisRear}{C_{34}}) * a_y \tag{4.27}$$

4.3.3 Wheel forward velocity estimation

The graph in figure 4.14 show the forward velocity of the front left wheel in the wheel rotational direction, both the estimated and the real. The graph only show a small error of margin.

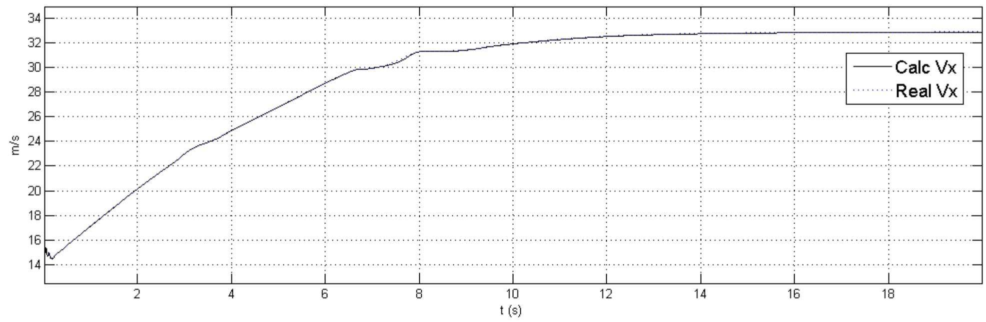

Figure 4.14: Real hub forward velocity versus calculated

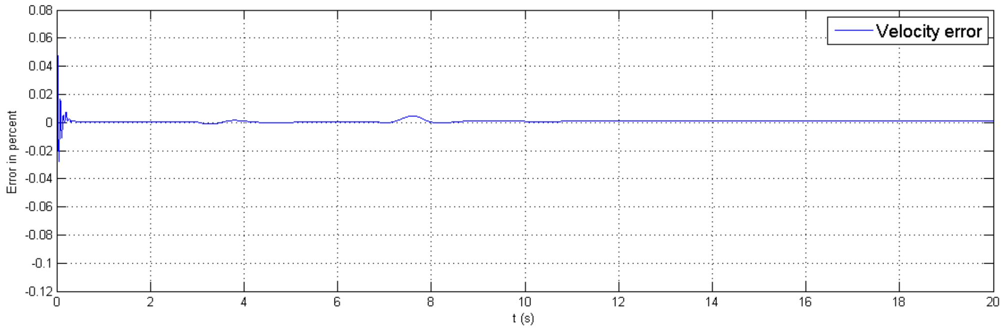

Figure 4.15: Velocity error in percent

When V_x and V_y has been calculated the hub forward velocities can be calculated using the formula below.

$$V_{xWheel} = (V_x \pm \frac{\dot{\psi} * Tw}{2}) * cos\delta + (V_y + \dot{\psi} * a) * sin\delta \tag{4.28}$$

The slip ratio is calculated through the formula below:

$$\kappa = -\frac{V_{x_{wheel}} - w_{wheel} * R_e}{V_{x_{wheel}}} \tag{4.29}$$

Where V_x is the wheels forward velocity, ω is the wheels angular rotation and R_e is the effective rolling radius.

Since the vehicles forward velocity is calculated using a constant effective rolling radius which is the same as used in the slip ratio calculation the effect of a changing radius will not affect the slip ratio much.

4.4 Lateral tyre forces

Since it is hard to estimate each front lateral tyre force independently with the limited amount of sensors given in most cars are the total front lateral tyre force estimated and then distributed to each tyre.

4.4.1 Total front lateral tyre force

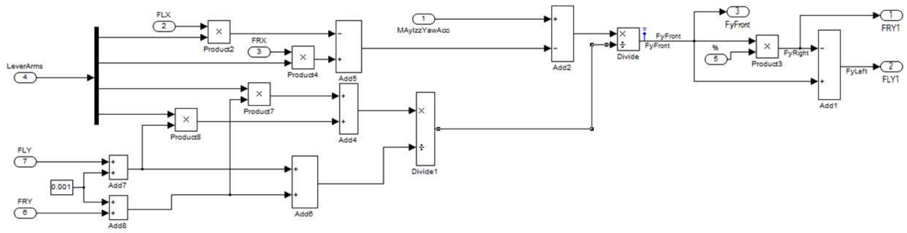

Figure 4.16: Total lateral force, simulink block

An approximation of the total front lateral load can be calculated using the bicycle model seen in figure 4.17. By using the lateral acceleration, vehicle mass, yaw acceleration(derivative of yaw rate) and the distances to CG the total lateral force "FyTotFront" can be derived through a simple moment of equilibrium around CG.

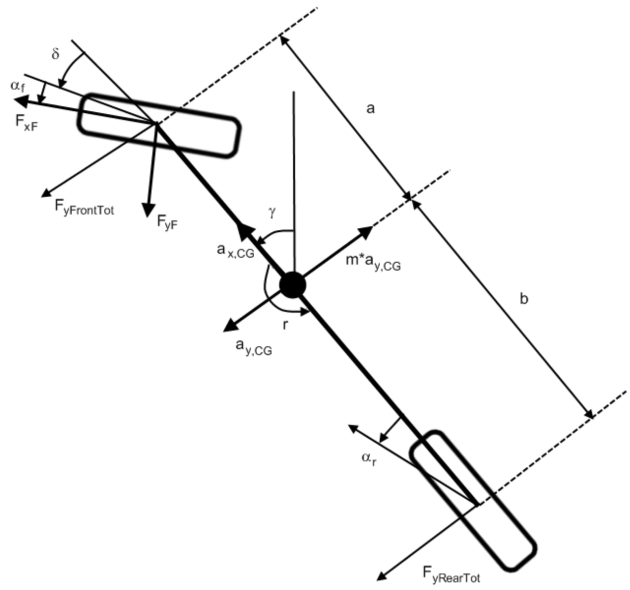

Figure 4.17: Bicycle model

$$m * a_y = F_{yFront} * cos\delta + F_{yRear} + F_{xFront} * sin\delta \tag{4.30}$$

$$I_{zz} * \ddot{\psi} = a * F_{yFront} * cos\delta + b * F_{yRear} + a * F_{xFront} * cos\delta \tag{4.31}$$

Combining formula 4.30 with formula 4.31 gives:

$$F_{yFront} * L * cos\delta = ma_y + b * F_{xFront} * cos\delta - a * F_{xFront} cos\delta + I_{zz} * \ddot{\psi} \tag{4.32}$$

$$F_{yFront} = \frac{ma_y + b * F_{xFront} * cos\delta - a * F_{xFront} cos\delta + I_{zz} * \ddot{\psi}}{L * cos\delta} \tag{4.33}$$

It should be noted that the lateral acceleration in the formulas above have been compensated for the influence of gravitational acceleration.

$$a_{ynew} = a_{yi}n - sin\phi * \mu \tag{4.34}$$

One drawback of using the bicycle is that the difference in both longitudinal and lateral tyre forces on the front wheels is neglected. For example; a torque over the LSD will create a yaw torque that is not considered in this model.

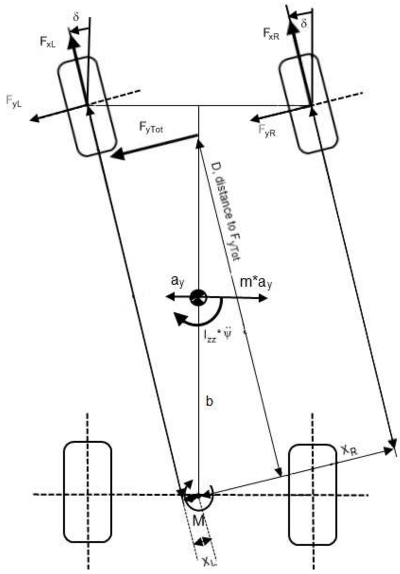

Figure 4.18: Two track vehicle model

To create a better approximation the bicycle model is exchanged for a two track model, see figure 4.18. Like in the previous case a moment of equilibrium equation around the centre of the rear axle is used, see equation 4.35 and 4.36.

$$m * a_y * b + I_{zz} * \ddot{\psi} + F_{xLeft} * X_L - F_{xRight} * X_R = F_{yTot} * D \tag{4.35}$$

$$F_{yTot} = \frac{M * a_y * b + I_{zz} * \ddot{\psi} + F_{xL} * X_L - F_{xR} * X_R}{D} \tag{4.36}$$

Variables as the longitudinal forces and the lever arms are already calculated and can be read about in section 3.3 and 2.1. Lateral acceleration is given from the CAN-bus but has to be compensated for the gravitational accelerations effect during roll and yaw acceleration can be derived from the yaw rate sensor. The lever arm to the resultant of the two lateral tyre forces is calculated in equation 4.37.

$$D = \frac{F_{yL} * X_L + F_{yR} * X_R}{F_{yL} + F_{yR}} \tag{4.37}$$

Since F_{yL} and F_{yR} are needed in equation 4.37, they will need to be taken from an earlier time-step.

4.4.2 Estimation of distribution of lateral force between right and left tyre

To estimate the distribution of the lateral force a number of models have been created. The distribution is dependent on the normal load of each tyre, the relationship between the normal loads. The lateral forces is also dependent on the slip ratio and slip angle.

For this have a number of techniques been tested some more promising than others. It has showed that the distribution is dependent on the difference between the wheels normal load (F_z), longitudinal force and longitudinal slip ratio. A function that takes all of these parts in account will be made but to start with will its components be described.

4.4.3 Lateral force dependent on normal load and longitudinal force

The foundation in this model is a nominal pure slip curve($F_{Y0}(\alpha)$) for this specific tyre. In other words data or a function is required that describes the lateral force of a tyre in a range of different slip angles. The pure slip curve will require a constant vertical force and friction coefficient and no longitudinal force. The friction coefficient and the vertical force should have nominal values.

Since the cornering stiffness is affected by variables as vertical force, friction coefficient, longitudinal force and camber change the slip angle in the model is corrected and named α_{eq}. A simplification of equation 4.40 is used where the affect of camber is neglected. Except from this the formulas used are as presented below, 4.38-4.42 [2]. The camber angle γ has been neglected and the variable n should be chosen between 2 and 8.

$$F_y = \phi_x \frac{\mu F_z}{\mu_0 F_{z0}} F_{y0}(\alpha_{eq}) \tag{4.38}$$

$$\phi_x = \frac{\sqrt{\mu^2 F_z^2 - F_x^2}}{\mu F_z} \tag{4.39}$$

$$\alpha_{eq} = \frac{1}{\phi_x} \frac{C_{F\alpha}(\mu, F_z, F_x)}{C_{F\alpha 0}} \frac{\mu_0 F_{z0}}{\mu F_z} (\alpha + \frac{C_{F\gamma}(\mu, F_z, F_x)}{C_{F\alpha}(\mu, F_z, F_x)} \gamma) \tag{4.40}$$

$$C_{F\alpha}(\mu, F_z, F_x) = \phi_{x\alpha} \{C_{F\alpha}(F_z) - \frac{1}{2}\mu F_z\} + \frac{1}{2}(\mu F_z - F_x) \tag{4.41}$$

$$\phi_{x\alpha} = \{1 - (\frac{F_x}{\mu F_z})^n\}^{1/n} \tag{4.42}$$

This model has two large drawbacks. The first is that the slip angle needs to be known and the second is that there is no correction for longitudinal slip ratio. The slip angle problem will be solved by approximating that the tyre acts linearly. Then the pure slip curve($F_{Y0}(\alpha)$) can be exchanged with a constant parameter, initial cornering stiffness. It will then be possible to take the lateral force for one tyre divided with the force for the other tyre and eliminate the slip angle seen in equation 4.43. The result is a good estimation of the distribution in lateral force between the front tyres when the tyres work in their linear range(low slip angles and slip ratios). Unfortunately will the final model have to be robust both with rather high slip angles and slip ratios.

$$\frac{F_{yL}}{F_{yR}} = \frac{C_{F\alpha L}}{C_{F\alpha R}}\frac{\alpha}{\alpha} \tag{4.43}$$

Another approach is to estimate each lateral force directly from the formulas above by doing an estimation of the slip angles. The problem is that slip angles is rather complex to estimate and that the lateral force which we search for needs to be an input.

To estimate each tyre´s slip angle is lateral force, normal force, slip ratio and friction coefficient needed. The estimation can be made with a conversion of Pacejkas Similarity Method[2] described in chapter 1.6.8.2. Instead of having slip angle as an input and lateral force as output is now lateral force the input and slip angle the result.

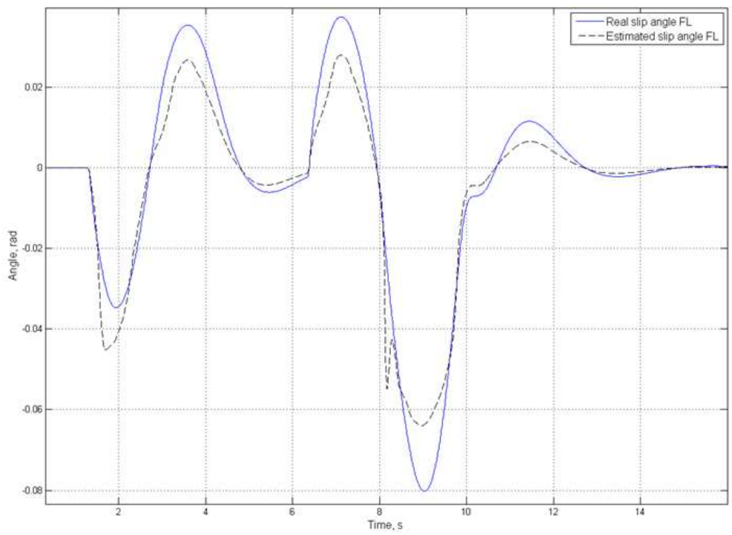

Figure 4.19: Slip angle estimation

In figure 4.19 is the result of this estimation shown were the Magic Formula[2] model´s lateral force is used as input. Since the result is both too high and too low in different parts of the simulation the tuning of parameters is not sufficient.

The same formula that is used for the slip angle estimation is also tested in its ground form were the lateral force is the output. Using the slip angle that is given in Haldex VehSim gives the result shown in figure 4.20.

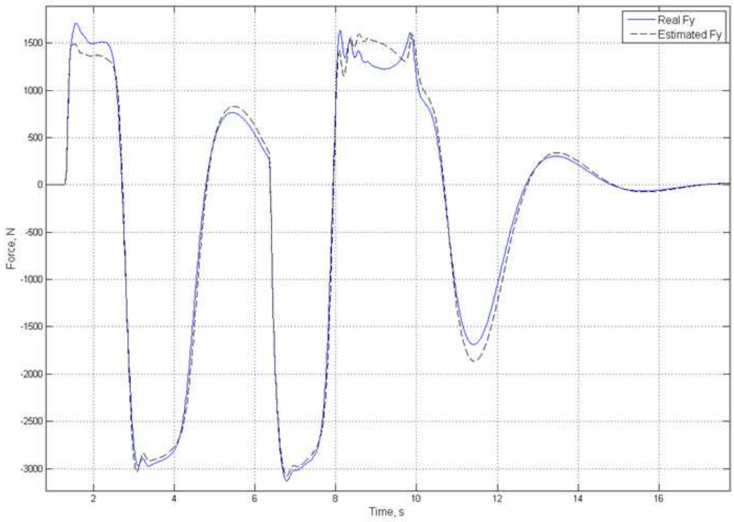

Figure 4.20: Fy using estimated slip angle

The conclusion is that lateral force and slip angle can be estimated rather simple and accurately. The key is to know the lateral force when estimating slip angle and the slip angle when estimating the lateral force. Because none of these are known the estimations are rather useless for this project.

4.4.4 Lateral force dependent on normal load and slip ratio

At small slip ratios the distribution between the tyres is dependent on the distribution of normal force between the tyres. Thereby the relationship between the slip angle and the lateral force can be used.

$$F_y = C\alpha * \alpha \tag{4.44}$$

Where C_α is the cornering stiffness of the tyre. This formula is only valid for the linear part of the tyres force to slip angle relationship.

By using the relation between the forces $\frac{F_{yR}}{F_{yL}}$ and approximating the slip angles to be equal for both front tyres, which it is except at larger slip angles, a force distribution can be derived without using the slip angle.

$$\frac{F_{yR}}{F_{yL}} = \frac{C_{\alpha L}}{C_{\alpha R}} \tag{4.45}$$

$$F_{yL} = \frac{F_{yL}}{F_{yL} + F_{yR}} * F_{yTot} = \frac{C_{\alpha L}}{C_{\alpha L} + C_{\alpha R}} * F_{yTot} \tag{4.46}$$

The cornering stiffness's below are taken from Pacejka[6]:

$$C_\alpha = c_{\phi 1} * c_{\phi 2} * sin(2 * atan(\frac{F_z}{Fzo * c_{\phi 1}})) \tag{4.47}$$

The cornering stiffness is dependent on the constants $c_{\phi 1}$ and $c_{\phi 2}$ that may be estimated through regression techniques, the nominal normal load and the actual normal load, this means that to get an accurate lateral force estimation we need to have good normal load estimation. The simulink block scheme for the distribution factor can be seen in figure 4.21.

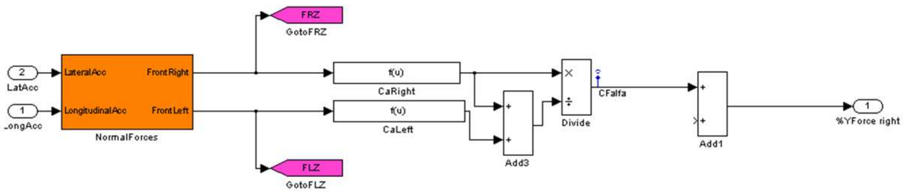

Figure 4.21: Distribution of the total lateral force, simulink block

The method gave an accurate result in areas with small slip ratio. The method was enhanced by also adding the influence of longitudinal slip on the left/right distribution of the cornering stiffness.

The idea is that low slip difference will have a small influence, and vice versa, on the lateral force distribution between the front tyres. This behaviour can be found in an e^x function, the function has been multiplied with a constant of 1.3 to get a good behaviour of the distribution seen in figure 4.22.

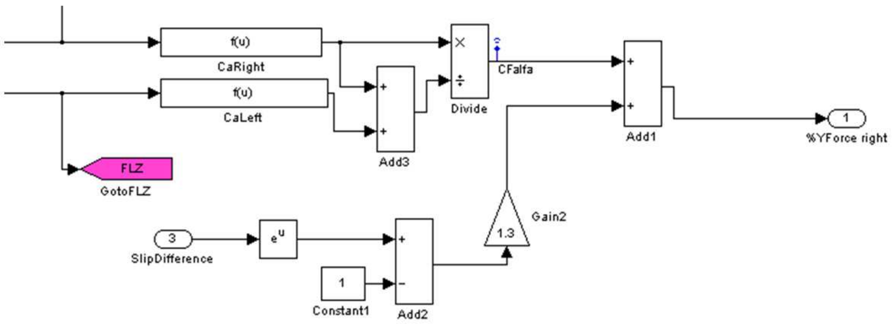

Figure 4.22: Distribution of lateral force with slip ratio included

This estimation works well when estimating the forces acting on the tyres since we have the total lateral force on the front tyres. The difference in slip ratio will tell us with the sign when one tyre starts to gain large slip ratios, and either increase or decrease the distribution factor. In figure 4.24 the slip ratio difference has not been accounted for. The error is larger in areas where the slip ratio difference is large. In figure 4.24 the slip ratio difference has been accounted for and the error is decreased.

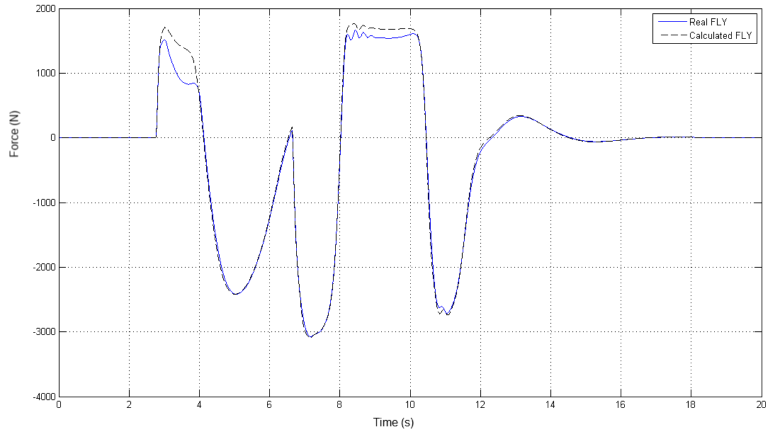

Figure 4.23: Lateral force for the left front wheel

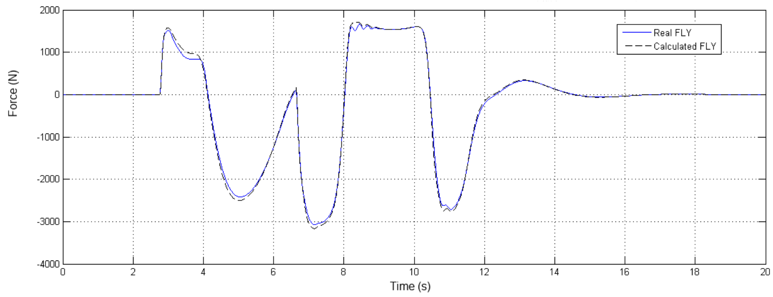

Figure 4.24: Lateral force for the left front wheel, slip ratio included in the calculation

5 TRANSFERRED TORQUE

Each one of the three different yaw-torque estimations will need a state telling what torque that currently is transferred through the LSD. This to be able to evaluate the two longitudinal tyre forces. This torque will most of the time be more or less the equal to the set point-signal, the set point signal plus 50Nm or the set point signal minus 50 Nm depending on which estimation. Unfortunately is this not always the case.

5.1 Over locked

A limit always exist describing how much torque can be transferred through the LSD without fully locking it. It is rather easy to know what torque is transferred when a speed difference between the front wheels exist, this since in the precense of relative motion induce that the fricion torque capacity in the clutch pack is fully saturated. The friction torque capacity is in its turn controlled by the known amount of clamping force applied by the LSD actuator. The problem arrives when the speed difference is zero(the LSD is locked), at this point it is not easy to decribe the transferred torque. To solve this another solution is created through estimating the longitudinal tyre forces, this to be able to tell how much torque is transferred. The reason to why the longitudinal force not always is estimated this way is that the estimation is rather rough. To get better accuracy the difference in longitudinal tyre force is estimated and converted to transferred torque before the engine torque is added. See how this is implemented in figure 5.1.

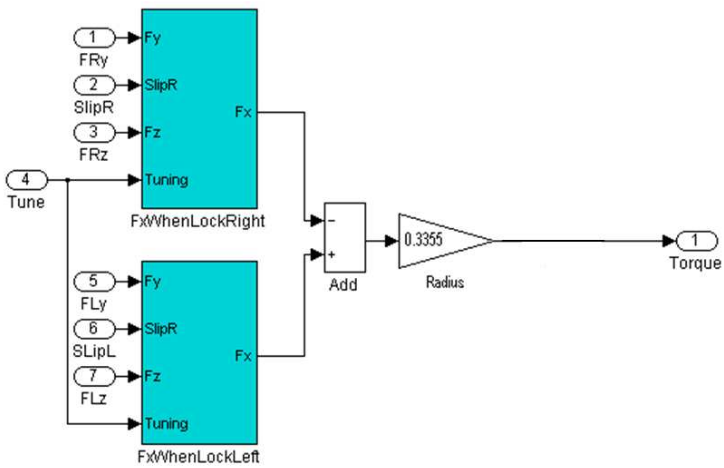

Figure 5.1: Simulink block for over locked

The upper simulink block estimates the longitudinal force on the right front wheel and the lower one of the left wheel. The difference of these is multiplied by the effective wheel radius to get the torque difference.

The principal of the longitudinal force estimators comes from Pacejkas way to estimate lateral tyre force from F_z, F_x, μ and slipangle, see chapter 4.2.2.1. [2]

Since the tyre behavior have similarities in lateral and longitudinal direction the approach is to convert this to derive F_x using F_z, F_y, μ and slipratio as inputs, se formula 5.1.

$$F_x = \phi_y * \frac{F_z}{F_{z0}} * \frac{\mu}{\mu_0} * F_{x0}(\kappa_{eq}) \tag{5.1}$$

$$F_{x0}(\kappa_{eq}) = \kappa * \frac{1}{\phi_y} * \frac{F_{z0}}{F_z} * \frac{\mu_0}{\mu} * \frac{C_{f\kappa}(F_z)}{C_{f\kappa 0}} \tag{5.2}$$

$$\phi_y = \sqrt{\frac{\mu^2 * F_z^2 - F_y^2}{\mu^2 * F_z^2}} \tag{5.3}$$

Simulations have been conducted to find what the longitudinal slip stiffness dependancies are. In the final results there were no big differences if the slip stiffness only depended on the F_z force or both F_z and F_y. This led to neglecting the F_y force to improve simplicity.

$F_{z0}$is chosen to 4000N because this was close to the medium value during acceleration in this simulation model.

To be able to use this method some tuning is required. A parameter that decide how much the result shall be affected of lateral and normal force is implemented like in formula 5.4.

$$F_x = \phi_y^{1+t} * (\frac{F_z}{F_{z0}})^{1-t} * \frac{\mu}{\mu_0} * F_{x0}(\kappa) \tag{5.4}$$

For the tyres in HdxVehSim the tuning parameter “t” is chosen to 0.4.

For a car equipt with torque sensors on the drive-shafts it is very easy to tune this parameter.

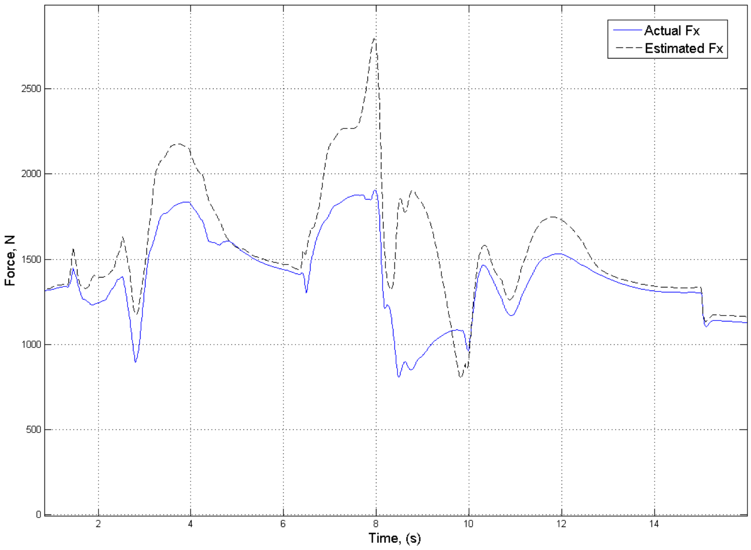

Figure 5.2: Fx estimation

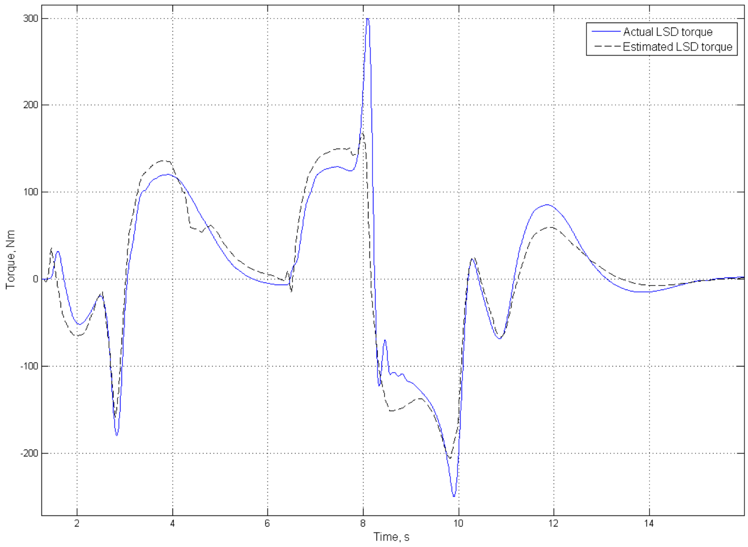

Figure 5.3: Torque estimation

In the case above the car is doing a lane change first to the left and then to the right. The torque set point is chosen very high so that the car is running with the LSD completely locked during the entire run.

Interesting is that this method is not very good in estimating each Fx independently, as can be seen in figure 5.2. But the difference in the front longitudinal force is estimated rather good, see figure 5.3.

5.2 Near over locked behaviour

One problem is if the LSD has less than 50 Nm left until it is fully locked. Then the transferred torque should not be the set point plus 50 Nm for the estimation with a higher set point then current. Instead, it should be the set point plus the amount that currently can be transferred until the LSD is locked. Two different techniques to calculate this, with some similarities, is tested and both of them will be presented in the following texts.

5.2.1 Technique one, using a controller to calculate the torque

This estimation method is based on the knowledge that when the clutch is locked the right and left wheel have equal angular velocities. The angular velocity can be calculated as a function of the slip ratio, wheel radius and wheel forward velocity.

$$\omega = \frac{\kappa * V_x + V_x}{R_e} \tag{5.5}$$

If the slip ratio can be calculated for each wheel the angular velocity for each wheel can be derived using the formula above. The angular velocities can the be compared to see if they are equal. When they differ from each other a controller can increase the LSD torque to the model and thereby change the slip ratio which will change the angular velocity for each wheel. The controller will do this until the angular velocities are equal and the torque output from the controller will then be the lock torque for each state.

5.2.2 Slip-ratio estimation

This method is based on a method from [2], it acts as a simplification of the combined slip equations in the Simularity method. The method uses the longitudinal force as an input instead of slip ratio etc.

The functions will only act as a crude approximation of the actual relationship. This method was originally created to derive the lateral forces using the longitudinal force as input. The idea is to do the opposite using the same method, deriving the longitudinal force using lateral force as input.

The original formula

$$F_y = \phi_x \frac{\mu F_z}{\mu_0 F_{z0}} F_{y0}(\alpha_{eq}) \tag{5.6}$$

Where

$$\phi_x = \frac{\sqrt{\mu^2 F_z^2 - F_x^2}}{\mu F_z} \tag{5.7}$$

and

$$\alpha_{eq} = \frac{1}{\phi_x} \frac{C_{f\alpha}(\mu, F_z, F_x)}{C_{f\alpha 0}} \frac{\mu_0 F_{z0}}{\mu F_z} \alpha \tag{5.8}$$

F_{y0} is a pure slip curve with α_{eq} as input. $C_{F\alpha}$ is the slip stiffness and how it depends on F_z, F_x and μ. ϕ_x is a coefficient that gives the potential of F_y force, how much F_y force that the tyre can generate. The same equations have been used except that F_y has been replaced with F_x and F_x has been replaced by F_y.

Using this method the equations now look like this:

$$F_x = \phi_y \frac{\mu F_z}{\mu_0 F_{z0}} F_{x0}(\kappa_{eq}) \tag{5.9}$$

Where

$$\phi_y = \frac{\sqrt{\mu^2 F_z^2 - F_y^2}}{\mu F_z} \tag{5.10}$$

and

$$\kappa_{eq} = \frac{1}{\phi_y} \frac{C_{f\alpha}(\mu, F_z, F_y)}{C_{f\alpha 0}} \frac{\mu_0 F_{z0}}{\mu F_z} \kappa \tag{5.11}$$

5.2.3 Mue

μ is tyre dependent and changes with normal load. The formula below show how μ depend on F_z and $\mu_{nominal}$[2].

$$\mu(F_z) = \mu_{nominal} * (\mu_{max} + \frac{F_z - F_{z0}}{F_{z0}} * -k) \tag{5.12}$$

μ_{max} and k are tyre parameters, specific for each tyre. μ is the friction coefficient at the current state. This coefficient is said to be known for this project, but still needs to be estimated.

5.2.4 Results of technique one

The result of this method is dissatisfactory. The slip ratio is correct as long as the slip angle is not too large. The idea of using this method was that we would not need to use the slip angle since there is no simple way of estimating this, see chapter 4.4.3. Since the slip ratio can not be estimated in a good way using this method the lock torque will not either give a good result.

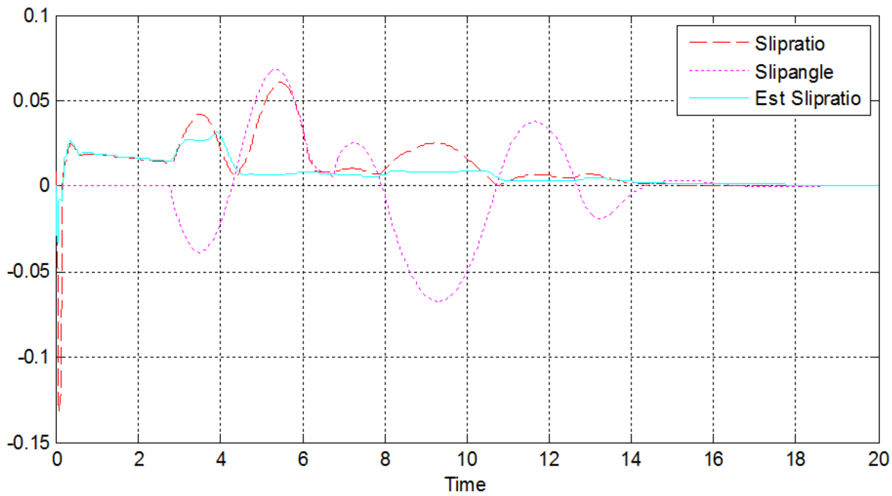

Figure 5.4: Slip ratio estimation

From the plot in figure 5.4 one easily notice that the estimation works fine when there is no slip angle. As soon as the slip angle increases the slip ratio estimation have an increasing error. Therefore is it not possible to use this method when estimating the slip ratio and LSD torque.

5.2.5 Using a controller to estimate the locking torque

To estimate the maximum possible LSD torque for the current state a controller was used. The estimated slip ratio was used to calculate the angular wheel velocity for each of the front wheels. In the formula for the slip ratio estimation a force is added or subtracted to the existing force which increases or decreases the angular velocity. The controller adds or subtracts forces that correspond to different LSD torques until the wheels angular velocities are equal. The added torque is then equal to the torque difference of the front wheels. This method requires good slip ratio estimation together with a good controller. One problem with this method is that the states are continuously changing which creates problems for the controller. This since the input to the controller changes with every time step. One way to control this problem is if the controller together with the slip ratio estimation has a faster sample rate compared to the rest of the calculations. This would allow the controller to adjust itself to the reference value using the current state.

A conclusion is that this method requires some tuning, for the slip ratio estimation, using the slip angle as a tuning variable and a rather fast controller.

5.2.6 Technique two, step by step testing of added LSD torque

To avoid the problem with a too slow controller, mentioned in technique one, this technique is based on step by step testing of how much more torque can be transferred through the LSD. Interesting is if it is possible to transfer 50 Nm more or not, how much more is it then possible to transfer?

Therefore it was first tested if it was possible to transfer 50 Nm more than the current lock torque. If it is not, 40 Nm is tested and so on. The largest torque that is possible to transfer is the output of the estimation.

A new approach to estimate the slipratio for a new amount of LSD torque is used in this method.

The same formulas that is used for overlocked behavior, chapter 5.1, is used here with the difference that the longitudinal force is known and the slip ratio, κ, is wanted. Since the longitudinal force is known through the driveshaft torque the formulas is modified to depend on the driveshaft torque instead of the longitudinal force.

The formulas are thereby converted as follows:

$$\kappa = \phi_y^{1+t} * (\frac{Fz}{Fz0})^{1-t} * \frac{\mu}{\mu_0} * \frac{C_{f\kappa 0}}{C_{f\kappa}(F_z)} \kappa_0(T) \tag{5.13}$$

$$\kappa_0(T) = T * \frac{1}{\phi_y} \frac{F_{z0}}{F_z} \frac{\mu_0}{\mu} \tag{5.14}$$

$$\phi_y = \frac{\sqrt{\mu^2 F_z^2 - F_y^2}}{\mu F_z} \tag{5.15}$$

As shown in previous chapter it is hard to get a good estimation of the slip ratio when the slip angle is too high. What happens is that the slip ratio is underrated and will thereby need modification.

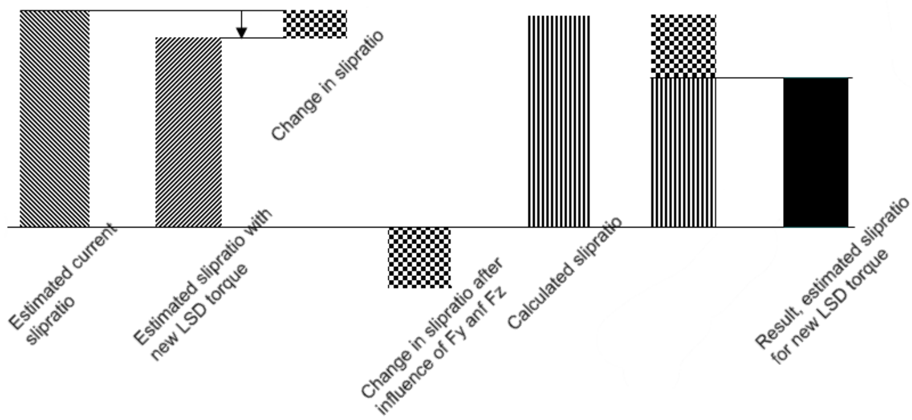

Figure 5.5: Slip ratio at a different LSD torque

In figure 5.5 the process of how the slip ratio for a new LSD torque is evaluated is depicted. As a start the slip ratio is estimated with the formulas above and the current LSD torque. Another slip ratio is also calculated using the formulas but now with the new LSD torque.

$$(\kappa_{Estimated,current})-(\kappa_{Estimated,newLSDtorque}) = \Delta\kappa \tag{5.16}$$

$$(\Delta\kappa_{compensated}) = (\Delta\kappa) * (F_y * t_1) * (\frac{F_{z0}}{F_z})^{t_2} \tag{5.17}$$

(Typical values on t_1 and t_2 is 0.001 respectively 1.6)

The difference between these two is then multiplied with the lateral force times a tuning factor and the relation F_{z0}/F_z to the power of the tuning factor t, see formula 5.17. This step is done to compensate the difference of variation in slip angle. Slip angle for each tyre is very hard to estimate but a adoption is done that slip angle is high when normal force is low and lateral force is high.

The simulations are made with a tyre model that works best in one region of tyre friction. Therefore it has not been tested which way the slip angle varies with mue. A variation do exist and need to be considered in future work.

The new delta between new and old slip ratio is then added to or subtracted from the slip ratio that is calculated from the wheels angular velocity and the hubs longitudinal velocity.

Using this method a rather stable amplitude of slip ratio is reached since just the offset between new and old slip ratio is estimated, the rest is calculated. This gives the opportunity to calculate the difference in wheel angular velocity for the front wheels with different LSD torques as input. In our model, which has no advanced LSD-model, a high LSD-torque will give that the wheel with the highest torque has the highest angular velocity. This is a physical impossibility in the real world but in the calculations it can be used to tell if a LSD-torque corresponds to a locked differential or not. This is tested for a number of different LSD-torques in the interesting range from 0 – 50 Nm and the output is highest value that does not lock the LSD.

5.3 Assembly of parts to LSD torque

The torque set point signal which is the input to the yaw torque estimation(current states) will either be the set point value or the value from the over locked estimation. A simple switch that moniters both these values outputs the lowest of these.

The case where the set point is 50 Nm below the actual set point works the same. The 50 Nm lower set point is compared to the over locked signal and the lowest of them will be the input.

It is a bit more complicated for the case where the set point is 50 Nm larger than the actual set point. Here the value from the estimation of how much more torque that currently can be transferred is added to the current LSD-torque signal. Just like in the other cases this signal is compared to the over locked signal and outputs the lowest.

6 RESULTS

Since the equiptment for measuring tyre forces on a real car is very expensive only comparisons have been made against the simulation model in Simulink. The tyre model, "Magic Formula"[2], that is used in the simulation model is very advanced and is accepted to be correct enough for the tolerances these estimations have.

During most of the validation work a double lane change driving case has been used. The "driver" in the simulation model is simple and only acts on an error in y-direction. In the first examples the blue line in figure 6.1 is the target, the axes units is meter. Notice that there are different scales on the y- and x-axis.

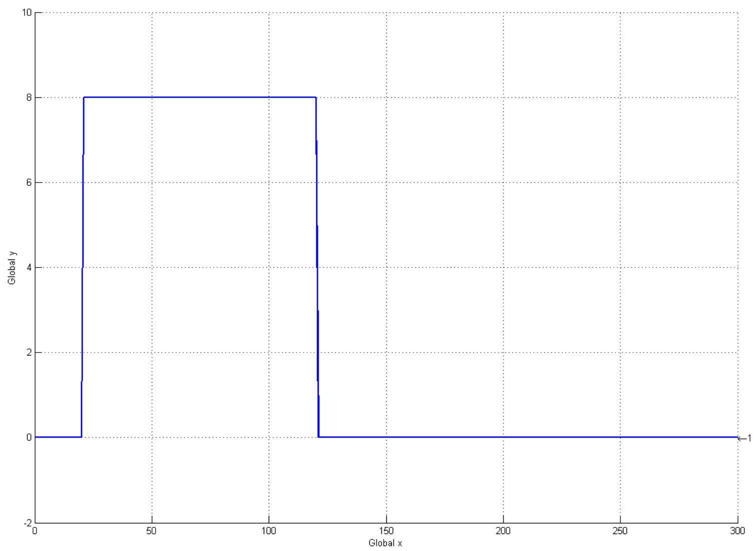

Figure 6.1: Driver path

During the drive cases in figure 6.2 and 6.3 the car has open throttle and a speed increasing from 50 km/h to 120 km/h. The set point for the LSD in figure 6.2 is constantly at 80 Nm and in figure 6.3 constantly at 0 Nm. It is hard to see which case that is the least accurate which mean that the estimated LSD-torque does not affect the result negatively in any severe way.

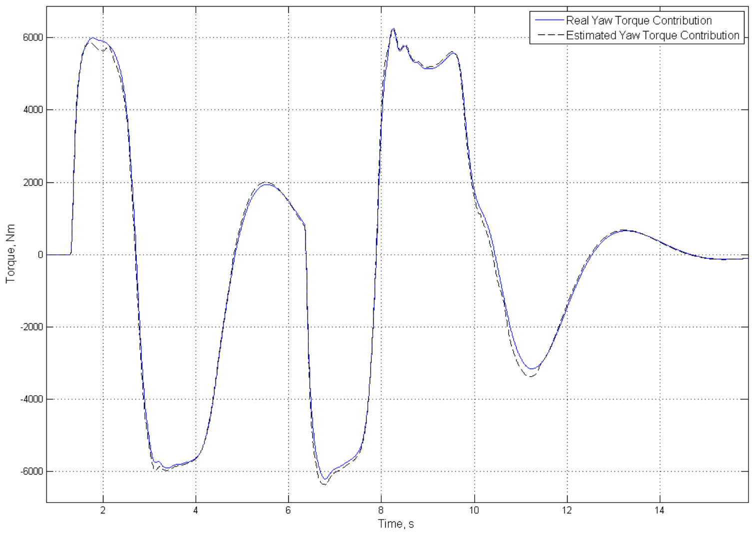

Figure 6.2: Comparison between estimated and real yaw torque contribution, Set point 80 Nm

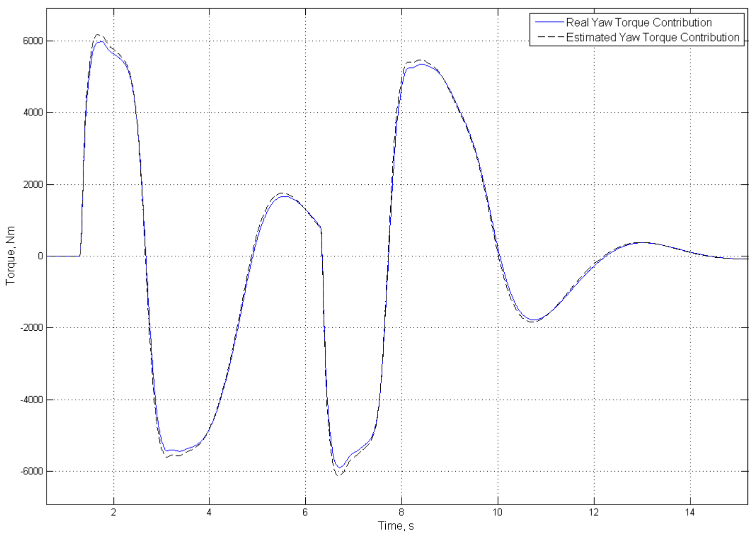

Figure 6.3: Comparison between estimated and real yaw torque contribution, Set point 0 Nm

The error of the estimations in the two graphs above is depicted in figure 6.6. Since the vehicle states will differ more and more during the simulations, because the driver will have to turn the steering wheel differently to get close to the path, does the amplitude on the curves in figure 6.4 differ. It can be seen that the peaks in figure 6.4 is around eight seconds were the yaw torque changes sign. When yaw torque changes sign is the LSD overlocked for a short time if a set point higher than zero is used like in figure 6.2. If the LSD is overlocked as described in chapter 5.1 hard to tell what torque that is transferred. That is an explanation to why the blue line has a larger peak than the black around eight seconds in figure 6.4. There is of course also a change of sign in yaw torque in the first lane change but then is the speed not so high which keeps the error small.

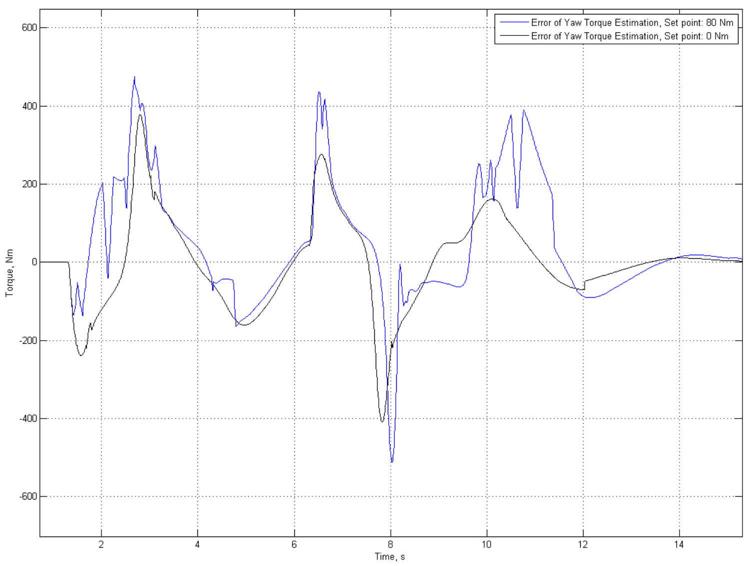

Figure 6.4: Errors of yaw torque estimations with different set points

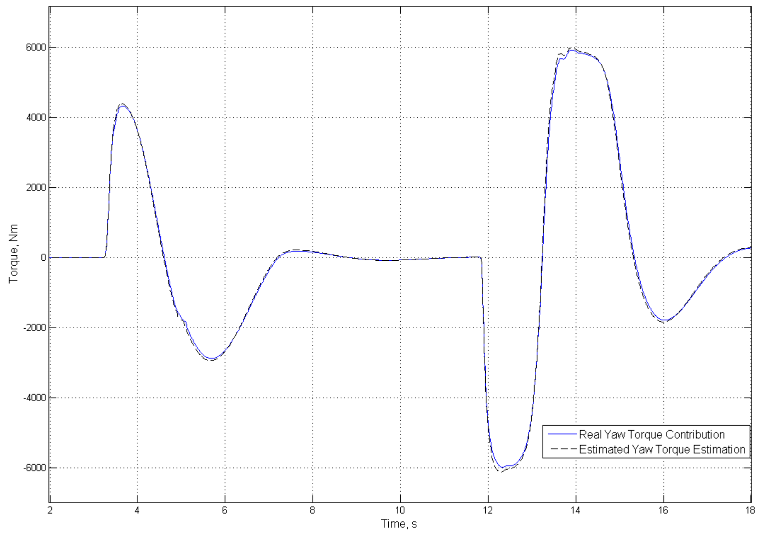

Figure 6.5: Comparison between estimated and real yaw torque contribution, Set point 80 Nm, low speed

The simulation in figure 6.5 is conducted using a lower initial velocity which gives a speed of 15 km/h increasing to 70 km/h. Just like before the throttle is open during the whole drive cycle.

In figure 6.6 the error of the estimation in figure 6.5 is compared to corresponding in figure 6.2. Since everything is slower in the later example is also the peak when the yaw torque changes sign smaller.

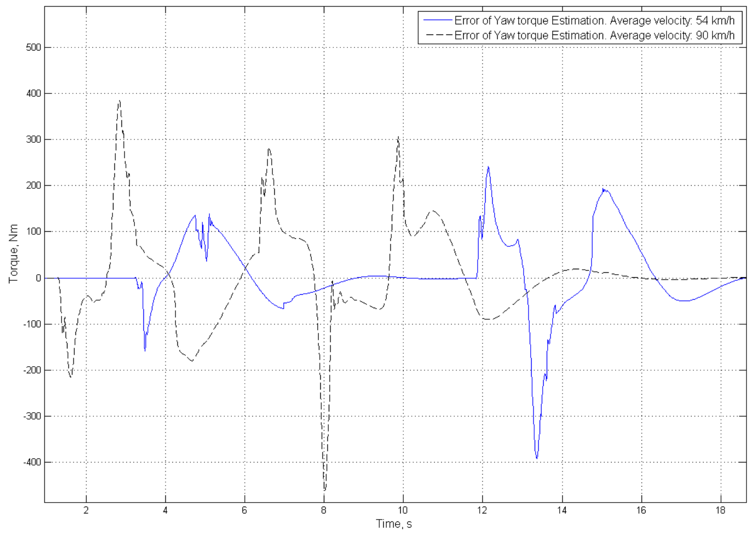

Figure 6.6: Errors of yaw torque estimations with different velocities

In figure 6.7 – 6.9 a drive cycle were three different yaw torques with different set points for the LSD are shown. The estimation of the cars current yaw torque is illustrated by the blue line. The red line show the yaw torque estimation for a lower LSD-set point and the black is for a higher LSD-set point. The model has a set point of 80 Nm which give the red lines estimation a set point on 30 Nm and the black 130 Nm.

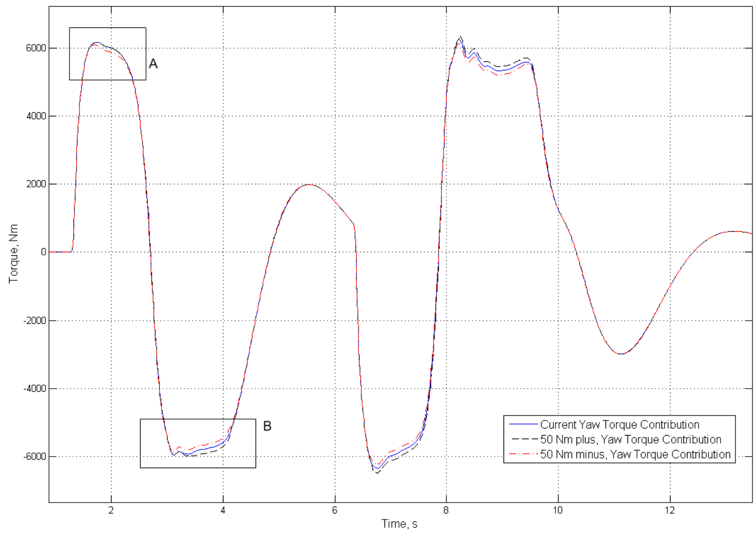

Figure 6.7: The three estimated yaw torque contributions

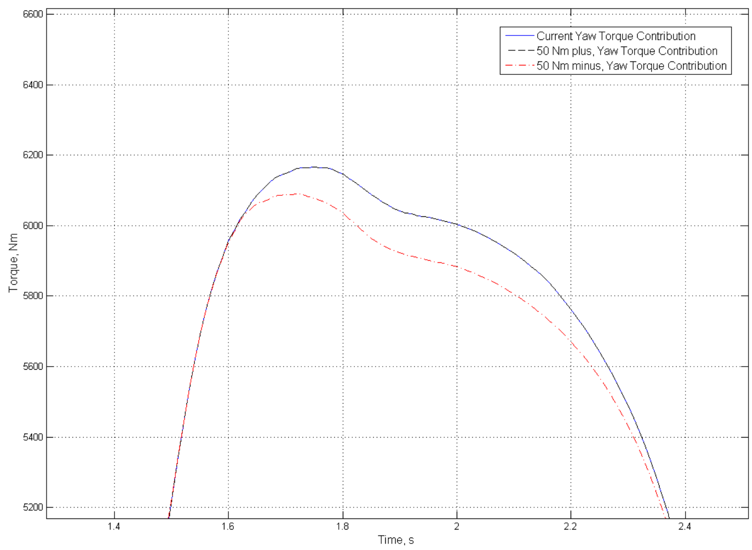

Figure 6.8: Part A from figure 6.7

Figure 6.8 is a close up from figure 6.7. It shows that the estimation with a higher set point estimate the same yaw torque as the estimation with the current LSD torque. This is because the LSD is over locked during the period and it is thereby not possible to affect the cars dynamics by increasing the set point.

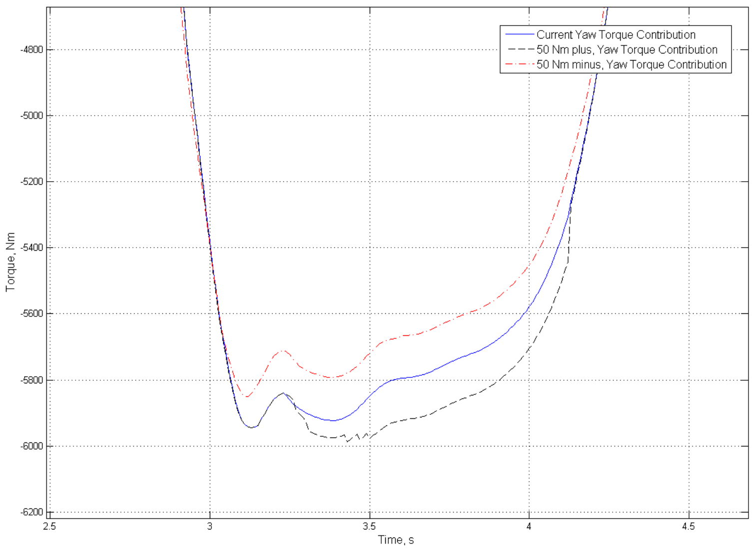

Figure 6.9: Part B from figure 6.7

Unlike in figure 6.8 the LSD is only over locked in the beginning of figure6.9. Lower LSD set point gives lower yaw torque and vice versa in this situation. In other words, lower LSD set point gives more understeer in this situation.

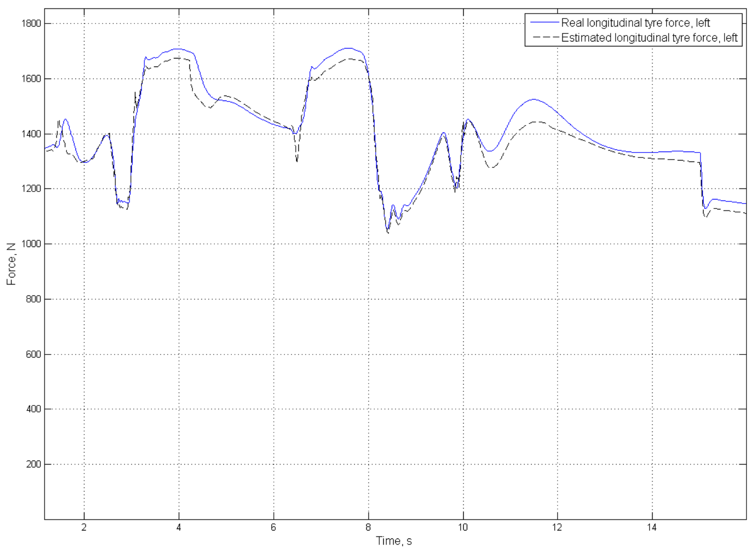

Figure 6.10: Real versus estimated Fx force, left tyre, 70Nm locking torque

To go a bit deeper in the estimations are also results of the longitudinal and lateral tyre forces shown. Like described earlier the peaks around eight seconds in figure 6.4 could come from a badly estimated LSD torque. Figure 6.10 – 6.13 strengthen this thought since rather large errors occur right after eight seconds in the figures for the longitudinal forces but not for the lateral forces.

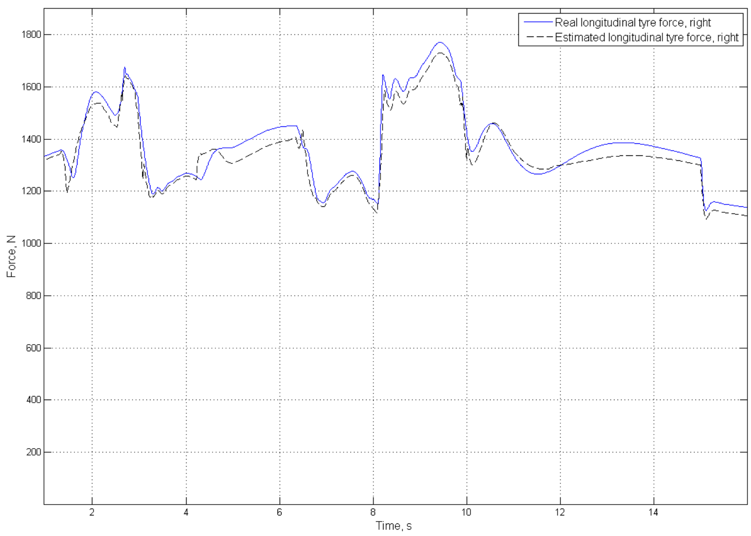

Figure 6.11: Real versus estimated Fx force, right tyre, 70Nm locking torque

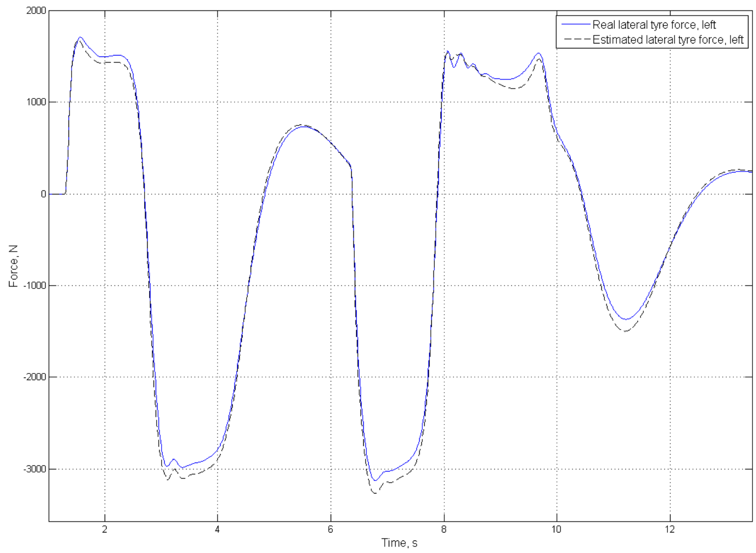

Figure 6.12: Real versus estimated Fy force, left tyre, 70Nm locking torque

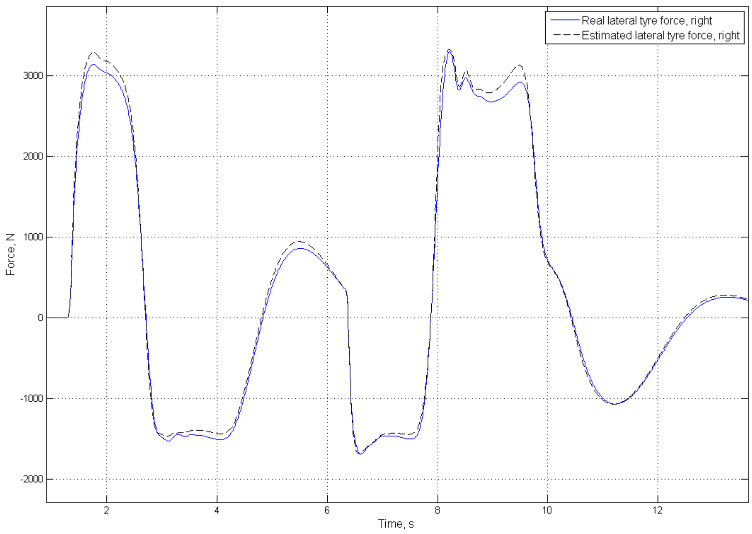

Figure 6.13: Real versus estimated Fy force, right tyre, 70Nm locking torque

7 DISCUSSION

Normal force calculations

As seen in section 4.2 the normal forces has been derived using two different approaches. The first method uses lateral and longitudinal acceleration to describe the weight transfer from side to side and front to rear. The approximations in this method is that the CoG is fixed in the vehicle and do not move with roll or pitch. Only the gravity is consider, no additional acceleration is considered in the vehicles vertical axis. The normal forces for each wheel is derived as described in equation 4.8, this method does not consider the roll stiffness between the front and rear axle. Even though this is not considered it gives an acceptable result.

The other normal force calculation is derived from the roll angle and the roll stiffness of respective axle. This method is more accurate but require more vehicle parameters, which can be hard to aquire.

Slip ratio

The slip ratio is derived from the angular velocity and forward velocity of the wheel according to formula 4.28. To calculate the forward velocity of the wheel the vehicles forward and side velocities are needed. The forward velocity is easily calculated through the rear wheels angular velocity, the side velocity is more complicated. If the drive case excessive oversteer is omitted it is possible to calculate the side velocity a steady state approximations is made. When driving without excessive oversteer (not considered in this thesis work) the side velocity has the most affect on the wheel forward velocity in lower velocities, the effect at higher velocities is negligible.

Total lateral force

The total lateral force has been calculated according to formula 4.31. The formula is based upon a two track model to describe how the lateral force changes with difference in tyre force. Lateral tyre force from an old time step is used in the calculation to determine the length of the lever arm to the resultant front lateral force. The total lateral force also depend on the longitudinal forces acting on the tyres.

Individual lateral forces

The total front lateral force is distributed between the front tyres, by using the normal forces of each tyre and the slip ratio. The dependency of normal forces acts in the linear region of the tyres and the difference in slip ratio between the tyres has then been added to compensate for the nonlinear part of the tyre. This is an approximation, which works fine for several handling situations. One situation which might create problems is when both tyres spin up. This might result in a small difference in slip ratio which results in a small difference in lateral forces between the wheels. This is correct but the total lateral force should be decreased to compensate for both wheels spinning up. This could be done using a compensation factor multiplied with the total lateral force.

Longitudinal force

The longitudinal forces are derived from the engine torque and the LSD torque with a rolling resistance model. This model demands the tuning of one parameter for each type of tyre(the coefficient of rolling resistance) and possibly the way the resistance changes with speed. In most situations the LSD torque can be taken from the current set point value. There are two times when this is not the case, if the LSD is over locked or if it is close to over locked. Both these signals are hard to estimate, but methods of doing this, rather good, have been developed. Both of them can be further developed, for example could some parts have more or less impact on the output.

We are open for alternative ways to switch between the three signals mentioned. If the "over locked" signal can be improved it could be possible to observe this signal and determine when to change direction on the LSD torque.

The affect of torque windup in the driveline is not considered which give errors in for example the lane change drive case. A model for the torque windup is recommended if the system not only should be used in steady state or close to steady state situations.

In the "over locked"-estimator one tuning parameter is available for the user. When changing this parameter the impact is either that the lateral force increase and the impact of normal load decrease or vice versa. The idea is to equip a test car with torque sensors on the drive shafts and then easily change this parameter until the estimation is as close to optimal as possible.

The same kind of tuning is used in the estimation of how much more torque that can be transferred. As described in section 5.2.6 is some kind of step by step method used. Better would be to have a controller which has the difference in wheel angular velocity as error and the LSD torque as output. The problem is that this controller has to be very fast so that it can decrease the error to an acceptable level before the states of the car has changed.

In chapter 6 several graphs from the same drive situation can be viewed. The drive case used is a double lane change which is a rather hard situation for these estimations since it includes changes in yaw direction and transients of the lateral forces. As seen in figure 6.4 the error of the yaw torque estimation is larger for the situation with a set point of 80 Nm. This means that the error in the transferred torque signal cause an error in the yaw torque estimation but the effect is limited and concentrated to the case when the LSD is over locked. Except from the situations with over locked LSD(at third and eight second) the error is low compared to the amplitude of the yaw torque. This is of course good and a proof of that the estimations are good but it is not critical to the result of the project. What is critical is that a change in yaw torque from a change in LSD set point is well estimated. So the idea is of course to use different LSD set points in the rather accurate yaw torque estimation model to achieve numbers on the change in yaw torque. This is done in the simulation software and the different yaw torques is shown in figure 6.7 – 6.9. So in the end the error in the yaw torque estimation, figure 6.4, is important but as already discussed the amplitude of the error is not as critical as the variation.

Figure 6.6 illustrates that the error in the yaw torque estimation gets smaller with lower velocity. This is not strange since the amplitude of the yaw torque also gets smaller with lower velocity in this case where the same lane change is made with the different velocities. There are neither that fast changes of yaw in low speed compared to high speed which give better accuracy in the LSD-torque estimation, thereby no high peak on the black signal in figure 6.6 .

7.1 Conclusion

The goals for this project were overall fulfilled. The estimations of the yaw torques with different torque set points are both accurate and not very complex. The most difficult parts of the project have been to estimate the lateral force on the front wheels and the signals describing the torque currently transferred through the LSD. A creative way of estimating the lateral forces was found where both simplicity and robustness was reached. The “transferred torque”-signals were also found using creative solution-techniques but was in the end the most complex part of the project. Depending on how the whole system is supposed to be used it could be possible to make approximations that would decrease the complexity. A conceivable approximation would be to always let the yaw torque estimation with the higher LSD set point, transfer 100 Nm more torque except when the LSD is over locked.

What definitely needs more focus, before using this in a vehicle software, is how different coefficients of friction affect the different estimations. This has not been done since a limitation for the project was that variation in friction coefficient could be ignored see chapter 1.2. Since the project did not include the work of validating the estimation model in car, it has not been investigated if the demand of computer power is low enough for a low-cost production ECU. What is sure is that the data which needs to be stored is small enough for use in a Haldex ECU. During the entire project the car specific parameters has been kept to a minimum so that the final software should could be used in different cars without too much tuning. Exactly as described in the project description, chapter 1.2, no focus have been put into the control of the LSD during braking. This since there is no reason to take the job away from the ABS-system. In the end we believe that this project can help to improve handling and safety for front wheel driven cars with active LSD.

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