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VaR for a portfolio of Swedish Index-bonds

- An empirical evaluation

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Abstract

Title: VaR for a portfolio of Swedish Index-bonds – An empirical evaluation

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Keywords: Value-at-Risk, Conditional Autoregressive Value-at-Risk, Age-weighted historical simulation, Volatility-weighted historical simulation, GARCH (1;1), Normal distribution, Student's t-distribution, Christoffersen's test

Purpose: The purpose of this paper is to empirically evaluate the performance of seven different methods that are used when estimating Value-at-Risk for a portfolio of Swedish index-bonds with different maturities. As a supplementary objective, the paper tries to determine the history that one needs to account for when calculating VaR.

Methodology: In order to calculate the VaR, a portfolio consisting of five Swedish index-bonds[1] is constructed. For this portfolio, one-day VaR-estimates are calculated using three different windows and seven methods; Basic historical simulation, Age-weighted historical simulation, Volatility-weighted historical simulation, Normal distribution, Student's t–distribution, Asymmetric slope and Symmetric absolute value. The methods are evaluated using the Kupiec's and Christoffersen's test for unconditional and conditional coverage.

Conclusion: The empirical results show that neither method is able to exactly capture the amount of acceptable exceedances. Though, it is evident that several of the estimated methods are accepted according to the Basel Rules. Also, it can be seen that the volatility-weighted approach is the most suitable method to use when calculating VaR for a portfolio of Swedish Index-bonds during the investigated period. The efficient window of historical observations is deemed to be equal to one year.

[1] The indexes are BMSD02Y, BMSD03Y, BMSD05Y, BMSD07Y an d BMSD10Y.

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1 Introduction

In the first part of this thesis, the background and problem discussion alongside with the purpose and delimitations are presented. The chapter ends with the structure that the thesis will follow in the following chapters.

1.1 Background

On a daily basis individuals, corporate entities and governments make decisions regarding different financial investments. These decisions are made upon some desired level of expected return or equivalent. The level of the expected return is dependent upon two factors; risk and uncertainty. The reason for why these concepts are so important is because they have an impact on the prospects of the market, but also on the necessary level of expected return of specific assets or portfolios. The intuition is that the higher risk of an asset, the higher expected return is demanded by investors. Uncertainty, for instance, tells us that the future is unknown, i.e. that we cannot with 100% certainty predict what is going to happen in the future. Though, it is possible to make some predictions, based on subjective assumptions, regarding the future. Risk on the other hand is intimately related to uncertainty, since one cannot know whether the outcome of a certain risky (non-risky) event will be positive (negative). Another fact regarding risk, is that one has to distinguish between different groups, since all risks associated with one certain group may not spill over to another and vice versa.

In the financial world, the concept of risk is distinguished in the following way, by Duffie and Pan (1997), Jorion (2001), Dowd (2005); business, strategic and financial risk. Business risks are those risks that a corporate entity is willing to take on in order to pursue value creating activities and create competitive advantages. The concept is closely related to the product market where the entity operates, but also to the exposure of macroeconomic risks. (Jorion, 2001) Strategic risks, on the other hand, are associated to the risks that the entity has no control over. An example on strategic risk is the political environment. The financial risk can be divided into several underlying subcategories. (Manganelli and Engle (2001); Jorion, 2001; Dowd, 2005) The subcategories of underlying risks are; market, credit, liquidity, operational and legal risk. Market risk is defined by Dowd (2005) and Jorion (2001) as the risk of a loss or a gain that is related to unexpected changes in the market prices of securities etc., or market rates related to changes in the interest and exchange rates. Credit risk, on the other hand, is defined as the risk of a loss stemming from that the counterpart cannot fulfill his (her) promised obligations (Manganelli and Engle, 2001). Liquidity risk, as defined by Jorion (2001), means that there is insufficient market participation. In other words, the undertaken transaction cannot be conducted at the prevailing market prices due to its size or risk. Operational risk is defined as the risk of a loss that arises as a consequence of the failure of internal systems or the persons that operate them. (Dowd, 2005) Finally, legal risk is the risk of transaction being unenforceable in law. In other words, the counterparty cannot pursue the transaction due to legal restrictions. (Jorion, 2001; Dowd, 2005)

As a consequence of the problem with the different risks, financial institutions, investors and organizations try to undertake actions that ease the exposure to risk. For instance, the undertaken actions can be an increased usage of risk management in order to reduce the risk of the asset(s) (portfolio(s)). As a result of the increased usage of risk management tools to control risks etc., the number of alternative instruments has increased. This evolvement has not always been positive since institutions, individuals and organizations has misunderstood and underestimated the inherent risk of certain instruments which has led to unforeseen consequences. (Linsmeir and Pearson, 1996; Dowd, 2005) The consequence of underestimating the risk of financial instruments has led to significant financial losses, which in some cases have led to bankruptcies and overall crisis situations. Though, one should not forget the impact of an overall financial turmoil on the riskiness of certain instruments.

The underestimation of the risk of certain instruments has been particularly apparent since the beginning of the 1990s and onward. Examples of known financial institutions and organizations that have made substantial losses are Metallgesellschaft, Orange County, Baring Bank, Long Term Capital Management and Société Générale. For instance, the $4,6 billion losses that LTCM anticipated in 1998 occurred as a consequence of the financial turmoil in Russia. This collapse could have become more widespread than it actually was due to the intervention and bailout by the Federal Reserve. (Edwards, 1999; Jorion, 2001; Dowd, 2005)

The direct consequence of these systematic failures of financial institutions and organizations has been the adaptation of different regulatory restrictions, such as the Basel Capital Accords. The first Basel Capital Accord was adopted in 1988. (Bank for International Settlements[2], 1988) The reason for why this capital accord was introduced was due to the need to secure an

[2] Henceforth denoted as BIS.

international convergence among supervisory regulations, which governed the capital adequacy of international banks, among others. Also, it was introduced in order to strengthen both the soundness and stability of the banking system and financial market. (BIS, 1988) The 1988 accord posits a framework for the minimum level of capital that financial institutions, particularly banks, are needed to uphold. The capital requirement level that institutions need to uphold is 8% of the risk-weighted assets, which consists of the total outstanding debt to individuals, corporate entities and other financial institutions. In other words, the risk-weighted amount is a measure of what the financial institutions stands to lose if the borrower cannot pay back its obligations. Thereby, the capital charge is a protection against unexpected changes in the market conditions. The amount that is equal to the capital requirements, and that financial institutions are needed to hold is called the capital charge. The only risk that was accounted for in the first capital accord was the credit risk. (BIS, 1988; Jorion, 2001) A problem with the first capital accord was that it was possible to undertake regulatory arbitrage, and thereby circumvent the requirements of 8% and posit lower capital requirement levels. (Jorion, 2001)

In 1996, an amendment to the first capital accord was provided by the Bank for International Settlements. This amendment considers the necessity to calculate the undertaken market risk. In other words, the amendment proposes that financial institutions must calculate the risk of losses in on- and off-balance-sheet positions that has arisen as a consequence of the movements in market prices of securities. In line with this, the amendment provided the acceptance of financial institutions to calculate and measure capital charges using internal methods, instead of more standardized methods. Although, the amendment posits that Value-at-Risk should be calculated on a daily basis using a 99% significance level, for a holding period of ten trading days. (BIS, 1996; Jorion, 2001)

The second capital accord was introduced and implemented on the 1st of February of 2007 in Sweden. (Finansinspektionen, 2007) The reason for why a second capital accord was implemented was due to the outdated capital accord of 1988. Especially, the old capital accord had not been able to account and cope for the development of the financial markets. Thereby, the new framework should account for more types of risks. (BIS, 2003) Also, it should encourage financial institutions to improve its risk assessment process. The second accord also included the amendment of 1996, which was left unchanged. (BIS, 2003)

The approval of BIS for financial institutions to use internal methods when calculating the market risk has led to a rapid development of the usage of VaR among practitioners and nowadays it is a standardized measure to calculate market risk. (Engle and Manganelli, 1999 and 2004; Khindanova and Rachev, 2000) The attractiveness of VaR is due to its simplicity, since it gives a single measure of how much we stand to lose on our investment during a specific time-period and given a certain confidence-level.

1.2 Problem discussion

As imposed by the amendment from 1996 and the second Basel Capital Accord from 2003, financial institutions and banks should calculate their risk of losses in on- and off-balance sheet positions stemming from movements in the market prices of different securities. These securities can have various forms, such as; equities, bonds, derivatives etc.

Since every security is affected by various price movements due to trading, one cannot leave out certain asset-classes and determine them to be risk-free, or at least less risky. As an example, government bonds is usually seen as an investment with low risk since the risk of the country to default is deemed to be low. Though, one should bear in mind that there exist bonds that have a higher risk than government bonds, for instance corporate bonds. Evidence provided from surveys by Mercer Asset Allocation has shown that bonds form the largest part of the investment portfolios of European pension funds. (www.mercer.com, 2010) Also, Mercer's survey implies that there is a continuous move from equities to both bonds and non-traditional investments. This finding is a continuation of the trend found in the surveys from 2009 by Mercer. The continuous move from equities to bonds can be a consequence of more unstable markets. Thereby, it can be seen that the risk-appetite among pension fund managers have changed. (www.mercer.com, 2009 and 2010) Though, it is not only pension funds' that are interested in investing in bonds, most financial firms offers some kind of investment in them[3].

Despite the attractiveness of bonds, evidence has shown that they can lose their value in the event of the default of the issuer, such as a government defaulting which Russia did in 1998. Thereby, it is questionable if one can see bonds as a "safe-haven" compared to other assets. Or should one see bonds as risky as other assets? First of all, one has to distinguish between

[3] A search on www.morningstar.se yields in a total of 920 funds investing in bonds. If one uses the Swedish counterpart, obligation, this yields in a total of 41 funds investing in bonds. Though, one has to bear in mind that the search includes all type of bonds, and not only investments in governmental bonds.

who is the issuer of the bond. I.e. which kind of rating does the issuer of the bond have? Secondly, is the bond used for trading purposes or is it going to be held until it matures? If the bond is held for trading purposes it is of interest to calculate how much one stands to lose in the following day, which VaR can be used for, since the bond is affected by price changes, as all other assets. Also, one should not forget that bond prices are affected by changes in the interest rate.

The Basel Accords nowadays allow banks and other financial institutions to use internal methods in order to calculate the daily VaR. The consequence of this lack of consensus regarding which type of method that financial institutions should use when calculating VaR has led to the emergence of a flood of applicable methods, one more technically advanced than the other. These methods can be divided into three different main categories, according to Manganelli and Engle (2001); parametric, non-parametric and semi-parametric methods.

The parametric method implies that assumptions are made regarding the distribution and behavior of asset (portfolio) returns. (Jorion, 2001; Manganelli and Engle, 2001; Dowd, 2005) The distributions that can be used are, for instance, normal-, Student's t- and log-normal distributions. Though, despite the attractiveness of parametric methods it is widely criticized, mostly due to underlying assumptions regarding return distributions. (Jorion, 2001; Dowd, 2005) Another approach that can be applied is the non-parametric one, where one uses rolling-windows of historical observations to calculate VaR. Therefore, one does not make any assumptions regarding the distribution of returns and thereby the returns are able to speak for themself. One of the most common non-parametric methods is Historical Simulation. (Engle and Manganelli, 1999; Dowd, 2005) The third branch is semi-parametric methods, such as Extreme Value Theory and Conditional Autoregressive Value-at-Risk[4] etc, incorporates two techniques namely a historical simulation and a parametric evaluation. (Khindanova and Rachev, 2000; Engle and Manganelli, 1999 and 2004; Manganelli and Engle, 2001) The reason for why one uses historical simulation is because it does not require an estimation of the distribution parameters. For instance, according to Manganelli and Engle (2001), CAViaR models the evolution of the quantile over time, instead of the whole distribution of returns.

The prior research that has been made on VaR is extensive, although there are few studies that consider the calculation of VaR for bonds. Notable studies where bonds have been used, as

[4] Henceforth denoted as CAViaR.

the underlying asset, have been provided by Vlaar (1999), Nieppola (2009) and Ottink (2009). The studies have some common features, namely that they consider methods from the non-parametric and parametric approach. Studies concerning CAViaR are somewhat limited. An overview of some of the previous research is presented in Fibozzi, Focardi, Fukushima, Huang, Lu and Yu (2008). The studies noted in Fibozzi et al. (2008) have the common feature that equities and equity indexes are used as the underlying data. Thereby, it is motivated to empirically evaluate the performance of CAViaR for other asset classes, such as bonds. As well, the limited empirical research on bonds, as a whole, for other methods further motivates the need for additional empirical studies using bonds as the underlying asset.

The above mentioned methods impose some consequences for financial institutions because they cannot be sure which method(s) is (are) the most useful to use in order to get the most accurate estimate of VaR using rolling window forecasts. Another fact that one has to account for is that some methods are more advanced than others. The question thereby is if it is worthwhile to use a more advanced and time-consuming method if the given results are the same as the ones given by a less advanced method. For instance, is it useful to use CAViaR at all if other methods produce the same result?

Since BIS allows firms to use whatever internal model to calculate VaR they want, there should be no restrictions on the model as long as it produces accurate estimates. Though, one should remember the criticism put forward by Beder (1995) and Marshall and Siegel (1996) that the results stemming from different methods can yield substantial differences in the estimates of VaR. Therefore, one should account for the underlying assumptions of the model. In other words, are the underlying assumptions made reasonable? The question is which method(s) is (are) the most useful in order to get the most accurate calculation of the VaR? And, not to forget, which is the most suitable window of included historical observations?

1.3 Purpose

The purpose of this paper is to empirically evaluate and compare some of the different traditional methods against the newly developed CAViaR model by Engle and Manganelli (1999 and so forth), which are used to calculate VaR. In order to calculate VaR, an equally weighted portfolio is constructed containing Swedish government Index-bonds[5] with 2, 3, 5, 7

[5] The indexes are BMSD02Y, BMSD03Y, BMSD05Y, BMSD07Y and BMSD10Y.

and 10 years to maturity. The calculation of VaR will be performed using a one-day rolling window forecast on both the 95 % and 99 % significance level. As a byproduct, these rolling windows will show how the estimated VaR have evolved during the estimated period. It should also be noted that the estimations are based upon three different sample lengths, which is done in order to see whether there are evidence regarding how often the estimation should be updated.

In order to evaluate the applied methods, Kupiec's (1995) unconditional coverage test and Christoffersen's (1998) conditional coverage test is performed. By using these tests it is possible to back-test the different methods ability to capture the market risk. In other words, it will be apparent which of the investigated method is the most suitable to use when calculating the VaR for portfolios consisting of Swedish Index-bonds? Also, this leads to the question whether it is more suitable to use a simpler method instead of a more time-consuming method?

1.4 Delimitations

For simplicity and time limitations, one equally weighted portfolio is constructed of five Swedish index-bonds. The composition of this portfolio is not deemed to be efficient or a mean-variance portfolio. The choice of not obtaining an optimal portfolio constructed of Index-bonds is because it is not in line with the purpose of this paper. Also, the study is limited solely to Swedish Index-bonds, due to home-bias. One should also note that the market risk is the only financial risk that is of interest and that the out-of-sample period is the only considered period.

Regarding the applied methods for calculating VaR, the paper is limited to the three main groups of approaches. As will be apparent later on, three non-parametric, two parametric and two semi-parametric methods are calculated.

1.5 Continued structure

The following structure will be applied in the paper. In the first chapter, an introduction to the problem, as well as an introduction of the necessary background is presented alongside with the purpose and the necessary delimitations. Thereafter the first chapter is followed by the theoretical background, presented in chapter two. Also, the advantages and disadvantages with the VaR are presented. In chapter three, the methodologies used are presented alongside a description of the data and its associated characteristics. In the fourth chapter the empirical investigation and its results are presented. In chapter five, these results are analyzed. In the sixth and last chapter, the conclusions are drawn from the preceding analysis.

2 Theoretical background

In the second chapter, the theoretical framework is presented. The following structure is applied; firstly Value at risk is defined, secondly the advantages as well as the disadvantages are presented, thirdly the different methods are presented. The chapter ends with an presentation of prior research.

2.1 Value-at-Risk

The concept of Value-at-Risk emerged in the late 1970s and the beginning of the 1980s as a way for financial institutions to calculate the risks that were associated with different instruments. The reason for this development was due to the increased awareness and necessity to account for the interactions between the risks of different instruments, but also due to the more complex nature of institutions. More simplified, the necessity of how to calculate risks increased. (Jorion, 2001; Dowd, 2005) In 1995, the investment bank JP Morgan made their RiskMetrics database, which were used internally to manage and measure risk, publicly available and thereby increased the knowledge of the measure. The database contained variances and covariances among a various number of asset classes, as well as securities. (Damodaran; Holton, 2002; Dowd, 2005) After the release of RiskMetrics things moved quickly forward. In 1996, the Bank for International Settlements responded to the criticism of the first capital accord by releasing an amendment stating that the undertaken market risks should also be calculated which could be done through the usage of an internal method. As a consequence, of these two events, the usage of VaR has expanded rapidly both in the ways of number of methods available but also in the usage among practitioners (Khindanova and Rachev, 2000; Damodaran).

Though, the question is what is Value-at-Risk? According to Jorion (2001) and Damodaran the definition of VaR is that it measures the potential loss in value of a risky asset or a portfolio over a defined period of time for a given confidence interval. Simplified this means that with X% probability we will not lose more than K SEK over the next Z days.

VaR is defined by the equation 1 and 2 below;

$$Pr[R_t < -VaR_t|\Omega_{t-1}] = \theta \qquad (1)$$

$$Pr[R_t > -VaR_t|\Omega_{t-1}] = 1 - \theta \qquad (2)$$

Where VaR_t is the VaR-value at time t, $1-\theta$ is the confidence interval and θ is defined as the probability that a loss greater than the VaR-value occurs given a certain time horizon. Ω_{t-1} denotes the information that is available at time t-1 and R_t is the return on the asset at time t. Thereby, it is necessary to determine which confidence level that should be used but also the holding period. Thus, the question is how one should determine these parameters. The confidence level is determined, according to Dowd (2005), upon which purpose of the risk measure. For instance, a high confidence level can be used to set the capital requirements. On the other hand, a low confidence level is used when back-testing the model in order to get the proportion of excess-loss observations. The holding period is usually a day or a month, though it is possible to vary it depending on the investment and/or the reporting horizon. (Dowd, 2005) As can be seen by equation 1 and 2, VaR will give an estimate of the lower quantile of the distribution, which implies that the distribution will have the following appearance;

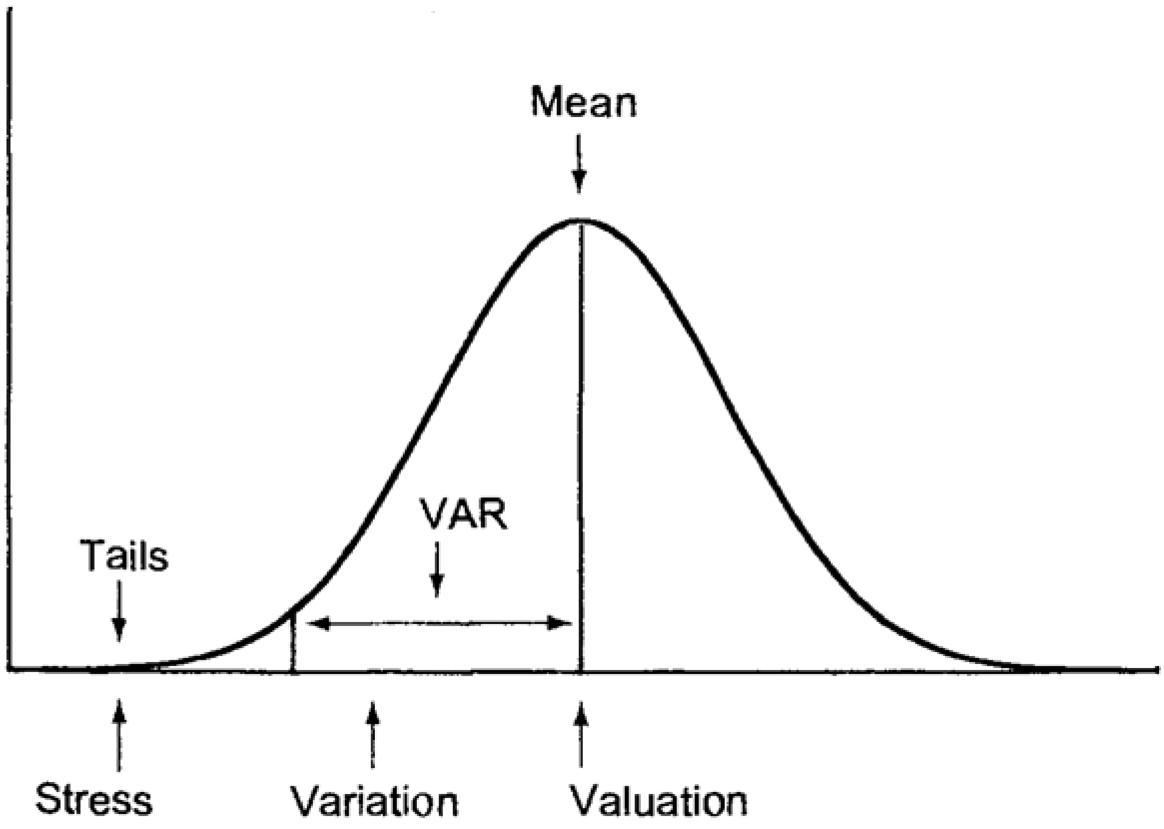

Figure 1: Value at risk (Jorion, 2001)

2.2 Advantages and disadvantages with VaR

During the years several advantages with the usage of VaR has been presented among both practitioners and researchers. (Dowd, 2005) Firstly, VaR is a consistent measure of risk, meaning that the measure can be applied on any asset class or portfolio. Secondly, VaR constitutes an overall single measure for a portfolio. Thirdly, it is a holistic measure which means that it takes all different risk factors at the same time into account. Fourthly, VaR provides information regarding the probabilities that are associated with specific loss amounts. This attractiveness implies that VaR is probabilistic. Fifthly, and lastly, it has a simple interpretation meaning that it is easy to account for and easy to understand, even for non-economists. (Linsmeir and Pearson, 1996; Dowd, 2005)

These advantages can be summarized into the following quotation used by Linsmeir and Pearson (p. 3, 1996);

"Subject to the simplifying assumptions used in its calculation, value at risk aggregates all of the risks in a portfolio into a single number suitable for use in the boardroom, reporting to regulators, or disclosure in an annual report. Once one crosses the hurdle of using a statistical measure, the concept of value at risk is straightforward to understand. It is simply a way to describe the magnitude of the likely losses on the portfolio."

Despite the attractiveness of VaR among practitioners and regulators, it has undergone some criticism during the years by scholars. A key issue in the criticism against VaR is that the estimates are too imprecise to be of much use. This criticism is supported by the empirical findings of Beder (1995), who found that different VaR methodologies can give vastly different VaR estimates. Other early criticizing research, based on Beder's approach, was provided by Marshall and Siegel in 1996. By constructing a test portfolio, that later was given to some software vendors using the RiskMetrics database, Marshall and Siegel were able to test the accuracy of the methodology. Since the same portfolio was given to all vendors, it would imply that the results should be the same. Despite this, differences in the results among the different vendors occurred. These differences were intimately related to the complexity of the asset class in question. Thereby, Marshall and Siegel were able to conclude that the variation among methods must be taken into consideration by practitioners, but also one has to model the systematic risk. (Marshall and Siegel, 1996)

Another criticism that has been put forward, concerns the extreme dependence on parameters, the underlying data, statistical assumptions and the methodology. This dependency implies that it is possible for the VaR estimates to be too inaccurate and thereby sway the user into a false sense of security where too big risks are taken than accounted for. In other words, the assumptions may be unrealistic and this may result in a misrepresentation of risk. (Beder, 1995; Daníelsson, 2002; Holton, 2002; Dowd, 2005) Also, VaR does not give any information regarding the size of the loss if a tail event occurs. It is not a coherent risk measure, since VaR does not satisfy the property of sub-additivity proposed by Artzner et al (1999). The property of sub-additivity implies that a diversified portfolio should contain less risk than holding each asset separately. (Artzner et al 1999; Dowd, 2005) Daníelsson (2002) criticizes VaR for being easy to manipulate and thereby a subject to moral hazard. The reason for why VaR is easy to manipulate is due to its reliance on only one quantile on the profit and loss distribution. Thereby, it is easy to manipulate the reported VaR using specially crafted trading strategies. Also, since VaR is measured on a 99% loss level, as proposed by Basel regulations, this implies that VaR is of little relevance to the probability of bankruptcy or financial crashes. (Daníelsson, 2002)

2.3 Methods to calculate VaR

In order to calculate VaR, one can choose between methodologies stemming from three different categories, namely; non-parametric, parametric and semi-parametric methods. Usually, the parametric and non-parametric methods are considered to be traditional methods whereas the semi-parametric methods are more newly developed. Despite the different methods, a general structure is used for calculating VaR, which contains three steps. The first step means that the historical return(s) of the asset(s) (portfolio(s)) is (are) calculated. In the second step, the distribution of portfolio returns is estimated and thirdly, VaR is computed for the asset(s) (portfolio(s)). The main difference between the different methodologies and approaches is regarding how one estimates the changes in the value of the asset(s) (portfolio(s)). (Manganelli and Engle, 2001)

2.3.1 Non-parametric approach

The non-parametric methods are characterized by not using any assumptions regarding the distribution of returns. Therefore, the essence of this approach is to let the observations speak for themselves as much as possible. The underlying assumption is that the near future will be sufficiently like the recent past. Due to this, it is possible to use data from the recent past in order to forecast the risks of the near future. (Dowd, 2005) The advantages with this category of methods are that it is easy to both use and interpret. It is not necessary to make any assumptions regarding the distribution and thereby it is possible to circumvent problems associated with skewness and kurtosis. It has been empirically proven that non-parametric approaches works quite well compared to other methods. Lastly, one should not forget that the non-parametric approaches produce results that are easy to both interpret and understand. (Jorion, 2004; Dowd, 2005) Despite the attractiveness associated with the available non-parametric approaches, the approaches are associated with some disadvantages. The first disadvantage that is stated in Dowd (2005) considers the fact that non-parametric methods tend to underestimate VaR when the considered period is less volatile. This is also true when the period is unusually volatile, although in that case the VaR will be overestimated. A second problem with the non-parametric approaches is that they have problems with shifts in the underlying data, which occur during the sample period. Usually, non-parametric approaches have the disadvantage that it takes time before a change is considered. In other words, it takes time for the method to reflect major events, such as sudden market turbulence. (Dowd; 2005, Damodaran)

2.3.1.1 Basic Historical Simulation

According to Linsmeir and Pearson (1996) and Damodaran, the historical simulation method is the simplest method to estimate VaR. This fact has led to its extensive use among practitioners. The reason for why the method is so easy to perform is because one does not need to estimate any distributions or parameters. The only thing that is necessary to have is historical return data, which constructs the actual distribution. By using the historical simulation method one assumes that trends of past price changes continues in the future. This fact implies that VaR is determined by the actual price movements. This fact imposes several constraints on the method as such that the past is not the future and that the method has problem with trends in the data due to the usage of equal weights. (Damodaran; Khindanova and Rachev; 2000)

In order to estimate the basic historical simulation method equation 3 is used;

$$VaR_t = \sum_{i=1}^{N} w_{i,t} R_{i,k} \qquad (3)$$

Where w_t denotes the weight given to each observation, where $k = 1, \ldots, t$. This implies that all weights are kept at their current values.

2.3.1.2 Modifications of the historical simulation method

During the years several modifications of the historical simulation has been made. Two of these modifications[6] are the age-weighted and the volatility-weighted historical simulation. (Dowd, 2005)

Age-weighted Historical Simulation

The Age-weighted historical simulation method was firstly developed by Boudoukh, Richardson and Whitelaw in 1998. The method implies that the observations are weighted according to their age. Instead of giving all observations the same weight, as the basic historical simulation method does, newer observations are given a higher weight than the oldest observations, using a decay factor. (Boudoukh et al., 1998)

Equation 4 is used, alongside equation 3;

$$w(i) = \frac{(1-\lambda)}{1-\lambda^n} \qquad (4)$$

Where λ denotes the decay factor, which is set to 0,999.

The attractions with the age-weighted historical simulation, as noted by Boudoukh et al (1998), Damodaran and Dowd (2005), are several. In comparison with the basic historical simulation, the method is able to adjust quickly to more volatile situations due to the usage of different weights for the observations. Another important fact is that the method helps to reduce the impact of events that are unlikely to recur and thereby it is able to reduce ghost effects. The impact of an observation will gradually diminish as the observation gets older and thereby is given a lower weight. This implies that the impact of a certain observation will decrease as time goes by. (Boudoukh et al; 1998, Damodaran, Dowd; 2005)

[6] Note that some authors classify the age- and volatility-weighted historical simulation as semi-parametric approaches. Though, the classification as non-parametric is provided by Dowd (2005), and this thesis follows his suggestion.

Volatility-weighted Historical Simulation

In 1998, Hull and White introduced their volatility-weighted historical simulation method. The idea behind the method is to update the return information considering the recent changes in volatility. In other words, volatility updating schemes are used in order for the returns to reflect to volatility changes.

In order to estimate the volatility-weighted historical simulation method equation 5 is used, alongside equation 3;

$$r^*_{t,i} = \left(\frac{\sigma_{T,i}}{\sigma_{t,i}}\right) r_{t,i} \qquad (5)$$

Where $r_{t,i}$ denotes the historical return on asset i, $\sigma_{t,i}$ is the historical GARCH forecast of the volatility of the return on asset *i* at day *t*. $\sigma_{T,i}$ denotes the most recent volatility forecast for asset *i* and $r^*_{t,i}$ denotes the volatility-adjusted return. The effect of this will be that the actual returns are increased or decreased depending on whether the current forecast of the volatility is greater or lower than the estimated volatility. (Hull and White, 1998)

As with the age-weighted method the volatility-weighted historical simulation method is associated with several advantages. For instance, the method takes volatility changes in consideration directly. Another advantage, according to Dowd (2005), is that the method produces risk estimates that are sensitive to the current volatility estimates and thereby incorporates additional information.

2.3.2 The parametric approach

As previously stated, non-parametric methods do not make any assumptions regarding the distribution of returns. The opposite is the parametric approach. Methods belonging to this approach do make assumptions regarding the distribution of returns. Examples on distributions that can be used when calculating VaR are normal-, Student's t- and log-normal distributions. (Dowd, 2005)

There are several advantages with using parametric approaches instead of for instance non-parametric ones. Firstly, parametric methods make use of additional information stemming from the distributions. Also, parametric methods provide straightforward VaR formulas which make these approaches easy to use. Despite the attractiveness of parametric approaches there are some disadvantages with the methods belonging to this group. First of all, parametric approaches are vulnerable to errors stemming from the distributions. This vulnerability increases if the used distribution does not fit the data. In other words, parametric approaches are subject to model risk, especially methods using normal distributions. (Khindanova and Rachev, 2000) A second problem that can occur stems from the quality of the parameter estimates. Therefore, Dowd (2006) draws two distinct conclusions, namely that it is necessary to have enough data in order to estimate the parameters and that the methods used are appropriate to the distributions used.

2.3.2.1 Normal distribution

A common assumption in statistics is the assumption regarding normally distributed returns. (Jorion, 2001; Dowd, 2005) As the appearance of Figure 2 shows, the normal distribution is bell-shaped with small tails which implies that the values converge around its mean and do not deviate too much.

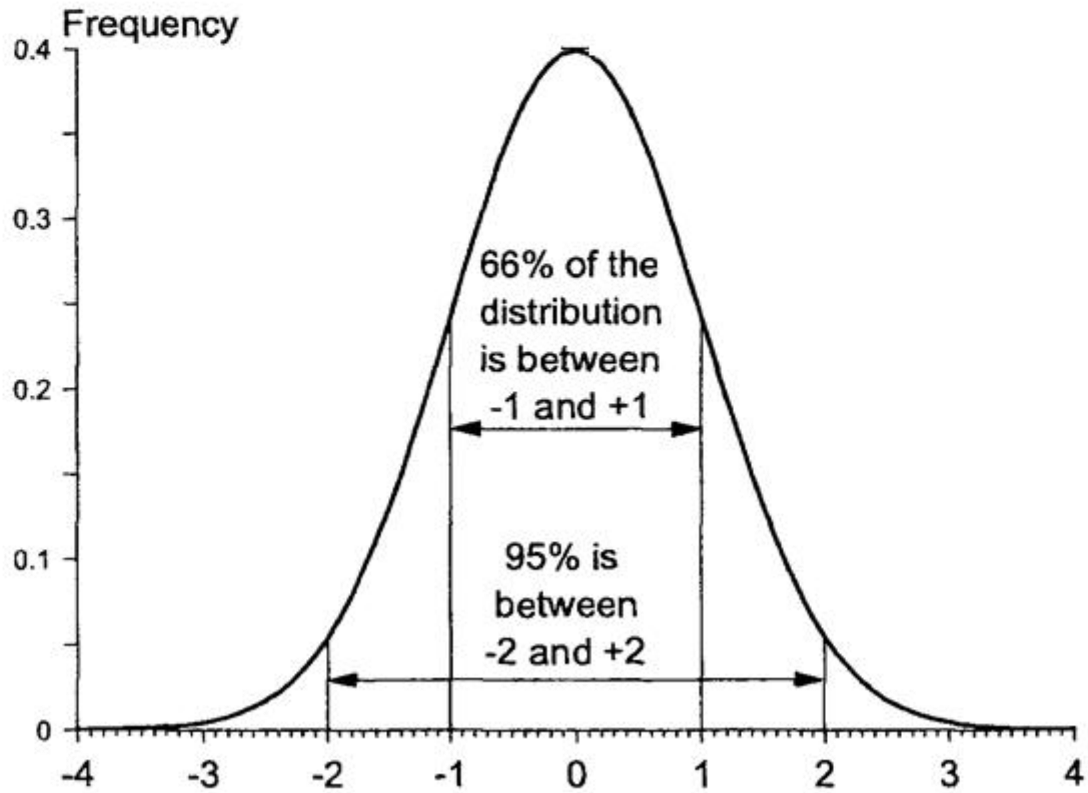

Figure 2: The normal distribution (Jorion, 2001)

In order to estimate the VaR using the normal distribution, equation 6 and 7 are used;

$$Z_\theta = \frac{-VaR - \mu_R}{\sigma_R} \qquad (6)$$

$$VaR = -\mu_R + \sigma_R Z_\theta;\ Z_\theta > 0 \qquad (7)$$

Where μ_R denotes the mean and σ_R denotes the standard deviation. The probability is given by the following equation;

$$Pr(R \leq -VaR) = Pr\left(\frac{R - \mu_R}{\sigma_R} \leq -\frac{VaR - \mu_R}{\sigma_R}\right) \sim N(0;1) = Z_\theta$$

The attractiveness of using an assumption regarding normal distribution is that it is easy to calculate since only two parameters are needed, namely the mean, μ, and the standard deviation, σ. Thereby, it is easy and straightforward to calculate VaR using this method. Despite its attractive features, there are some drawbacks with the model. First of all, financial data is rarely normally distributed thereby inducing fatter tails, implying that there exists kurtosis[7]. Also, there is a possibility that there exists skewness[8] in the returns. The consequence of this is that the risk of underestimating VaR increases. Secondly; the profit-loss scheme can take any value implying that there is a possibility of large losses. (Khindanova and Rachev, 2000; Damodaran; Dowd, 2005)

2.3.2.2 Student's t-distribution

The second parametric approach that is undertaken is the Student's t-distribution. By using the Student's t-distribution it is possible to circumvent the problem with kurtosis, which may impose problems to the method using the normal distribution.

In order to calculate VaR using the Student's t-distribution, equation 8 is used;

$$VaR = -\mu_R + t_\theta \sqrt{\frac{\nu - 2}{\nu}}\sigma_R \qquad (8)$$

Where the degrees of freedom, ν, is given by the following equation;

$$\nu = \frac{(4\kappa - 6)}{(\kappa - 3)}$$

[7] Where kurtosis denotes the degree of flatness of a distribution. (Jorion, 2001)
[8] Where skewness describes the departure from symmetry. (Jorion, 2001)

Where κ denotes kurtosis. The probability is given by the following equation;

$$Pr(R) = \frac{\Gamma((\nu_R + 1)/2)}{\Gamma(\nu_R/2)\sqrt{\nu_R}\pi\sigma_R}\left(1 + \frac{1}{\nu_R}\left(\frac{R - \mu_R}{\sigma_R}\right)^2\right)^{-(\nu_R+1)/2}$$

As previously stated, by applying the Student's t-distribution it is possible to circumvent and lessen the impact of excess kurtosis. Despite this there are some problems associated with the t-distribution. As the case with the normal distribution, the t-distribution fails to respect constraints on the maximum possible losses that can occur. Thereby, the risk of misleading estimates increases significantly. (Dowd; 2005)

2.4 Conditional autoregressive VaR (CAViaR)

The conditional autoregressive VaR was developed by Engle and Manganelli in 1999 and so forth, as a response to the drawbacks of the parametric and non-parametric methods. Especially, Engle and Manganelli noted that the previous models were not able to estimate VaR properly. CAViaR belongs to the semi-parametric branch of available approaches since it combines both parametric and non-parametric methodologies.

The idea behind CAViaR is to directly model the evolution of the quantile over time, instead of modeling the whole distribution of returns. (Engle and Manganelli, 1999; Kouretas and Zarangas, 2005) By doing this, it is possible to circumvent the problem with autocorrelation which is associated with financial data. Thereby, Engle and Manganelli proposed that one should use an autoregressive specification for the estimation. The main advantage with only modeling the quantile, instead of the whole distribution of returns, is that one does not have to make any assumptions regarding for instance normality. (Engle and Manganelli, 1999 and 2004; Manganelli and Engle, 2001; Kouretas and Zarangas, 2005; Allen and Singh, 2010)

A general version of the CAViaR can be specified as equation 9;

$$VaR_t = f(x_t, \beta_\theta) = \beta_0 + \sum_{i=1}^{p} \beta_i VaR_{t-1} + l(\beta_{p+1}, \dots, \beta_{p+q}; \Omega_{t-1}) \qquad (9)$$

Where $f(x_t, \beta_\theta)$ denotes the θ-quantile of the return distribution at time t, Ω_{t-1} denotes the information set that is available at time t and l denotes a finite number of lagged values of observations. Equation 9 can be simplified into the following first order model;

$$VaR_t = \beta_0 + \beta_1 VaR_{t-1} + l(\beta_2, y_{t-1}, VaR_{t-1})$$

Where the term $\beta_1 VaR_{t-1}$ ensures that VaR changes smoothly over time, i.e. it shows the autoregressive term. The term $l(\beta_2, y_{t-1}, VaR_{t-1})$ links the level of VaR_t to the level of y_t. In other words, it shows how much VaR should change based on the new information in y. This implies that when y_{t-1} becomes negative; one can expect that VaR will increase. The opposite is true, i.e. VaR declines, when y_{t-1} is positive. (Engle and Manganelli; 1999 and 2004; Kouretas and Zarangas, 2005)

2.4.1.1 Symmetric absolute value

The symmetric absolute value is estimated by equation 10, below;

$$VaR_t = \beta_0 + \beta_1 VaR_{t-1} + \beta_2 |y_{t-1}| \qquad (10)$$

Where $\beta_2 |y_{t-1}|$ models the absolute change of the return. This implies that the model responds symmetrically to past returns, and thereby the VaR should increase (decrease) when the last observation is negative (positive). It should be noted that the symmetric absolute value method is mean-reverting in the sense that the parameter for VaR_{t-1} is not constrained to be equal to 1. (Engle and Manganelli, 2004; Kouretas and Zarangas, 2005; Allen and Singh, 2010)

2.4.1.2 Asymmetric slope

Another specification that can be used when modeling the CAViaR is the asymmetric slope method. The equation for the asymmetric slope can be seen below;

$$VaR_t = \beta_0 + \beta_1 VaR_{t-1} + \beta_2 (y_{t-1})^+ - \beta_3 (y_{t-1})^- \qquad (11)$$

As can be noted in equation 11, above, the VaR at time *t* depends on the past value (*t-1*) as well as on past positive and negative returns. It is noted in Kouretas and Zarangas (2005) and Allen and Singh (2010), that the asymmetric slope model allows for an asymmetric response to past positive and negative returns and thereby the method is able to capture the asymmetric leverage effect. As the symmetric absolute value model, the asymmetric slope model is mean reverting. (Engle and Manganelli, 2004; Kouretas and Zarangas, 2005)

2.5 Forecasting volatilities

During the years, many researchers have focused on the question of how to forecast the conditional variance, σ_t^2. The reason for this attention is because one does not believe that basing the volatility forecasts on a historical moving average estimate provides accurate estimates. The concept of using moving average estimates are concerned with several shortcomings such as the assumption of constant volatility and that all events are given the same weight. Thereby, one does not give any regard to when the event happened. The first problem, that volatility is constant through time implies that differences arise as a consequence of sampling errors. The second problem implies that ghost effects can occur. In other words, a volatile event will influence the estimates after the conditions are set to normal, and the event has passed. (Jorion, 2001; Dowd, 2005)

As a result of the research, several methods concerned with conditional variance have emerged. One class of models is the exponentially weighted moving average volatility models, which uses moving average schemes with declining weights. The EWMA model gives the most recent observations a higher weight than more distant observations. By using this weighting scheme, the volatility is allowed to change over time in a stable way. (Dowd, 2005) Another class of models is the ones developed by Engle (1982) and Bollerslev (1986, where Bollerslevs work is a generalization of the work by Engle. This family of models is known as ARCH/GARCH-models.

2.5.1 GARCH models

The work on autoregressive conditional heteroscedasticity for time-varying volatility by Engle (1982) laid the founding stones for Bollerslevs generalized ARCH (GARCH) model. In the GARCH model the conditional variance considers both from autoregressive and moving average components. (Bollerslev, 1986 and 1992; Damodaran; Jorion, 2001; Dowd, 2005) The underlying assumption of the model is that the variance follows a somewhat predictable process. The conditional variance depends on the latest development, but also on the previous conditional variance. The conditional variance is model as equation 12;

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i u_{t-1}^2 + \sum_{i=1}^{p} \beta_j \sigma_{t-j}^2 \qquad (12)$$

Where ω, α_i and β are constants with the following restrictions;

$$\omega > 0;\ \alpha_i \geq 0, i = 1, \dots \dots, q; \beta_i \geq 0, i = 1, \dots \dots, p;\ \sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_j < 1$$

From this on, a GARCH-specification will be used in order to model the volatility of the portfolio, if nothing else is stated.

2.6 Prior research

During the years a substantial amount of research has been made in the area of VaR[9]. The result of this research has been several new and more advanced methods. Some of the earlier studies have been provided by Duffie and Pan (1997); Hendricks (1996); Damodaran; Khindanova and Rachev (2000). Duffie and Pans article from 1997 consists merely of a review of VaR and the issues associated with measuring the market risk. As stated in the article, no empirical evidence or survey is presented. Despite the fact that no empirical evidence is presented, Duffie and Pan provide a comprehensive and accessible overview of VaR. The characterization of an overview of the VaR literature can also be seen in Damodaran and Khindanova and Rachev (2000).

In his article "Value at Risk", Damodaran provides some necessary basic insights for the calculation of VaR. The article consists of several parts which gives a comprehensive description of the measure and the development of the measure. The last chapter consists of an evaluation of VaR and how well it performs against other risk assessment tools. Damodarans primary conclusion is that VaR is a step back instead of an advance in the thinking of risk. The reasons for this conclusion are several. For instance, it is not plausible to see VaR as the most optimal method for investment decisions since it only considers a slice of the downside risk. In other words, VaR do not consider all information from the whole distribution, just a small part of it. Damodaran is able to conclude that VaR is a natural measure of short-term risk for financial firms and that it performs adequately. For non-financial firms, on the other hand, VaR should be considered to be a secondary measure.

In 2000, Khindanova and Rachev provided some additional insights associated with the recent advances in the VaR-research. As Duffie and Pan (1997), Khindanova and Rachev do not provide any empirical evidence for the continued use of the different methods to calculate VaR. However, some useful conclusions are drawn. Examples on conclusions are that the

[9] A comprehensive summary and some insights about the most recent advances in VaR modeling can be found at www.gloriamundi.org.

existing methods, at that time, and their improvements do not provide a satisfactory evaluation of VaR. The key concerns that are put forward for future research is that more precise VaR-techniques are developed alongside adequate approximations for the distributional forms of returns.

In 1996, Hendricks provided some insights regarding the measurement of VaR. The article contains an empirical investigation of twelve approaches that can be used to calculate VaR, where the underlying data contains of 1000 randomly chosen foreign exchange portfolios collected between 1983 and 1994. Hendricks (1996) is able to conclude that the different approaches tend to produce risk estimates that do not differ greatly in average size. Though, historical simulation approaches yield larger risk measures on a 99th significance level compared to some parametric approaches. Also, Hendricks is able to conclude that there can exist substantial differences in the VaR estimation for the same portfolio on the same day. For instance, using a longer observation period tend to yield less variable results than shorter observation periods. It should also be noted that VaR estimates based on a 95th significance level is more reliable than those provided from a 99th significance level.

Empirical work on the performance of different VaR-techniques is nowadays widespread. Hendricks article marks somewhat of the beginning of empirical articles concerned with the performance of the VaR. Several articles have followed, one the more technical than the other. Articles that can be noted are for instance Boudoukh et al (1998), Hull and White (1998), Vlaar (1999), Ottink (2009) and Nieppola (2009).

Boudoukh et al (1998) developed the age-weighted historical simulation which can be considered to be a hybrid approach to calculate VaR. As stated in the article, the hybrid approach combines two approaches; exponential smoothing and historical simulation. Through this combination one is able to directly estimate the percentiles of the returns, using declining weights on past data. The empirical investigation of this hybrid approach is based on observations collected between 1991 to 1997 on four different assets; the USD/DM-exchange rate, the spot price for brent crude oil, the S&P 500 index and a Brady bond index. The results imply that the combination of exponential smoothing and historical simulation is a significant improvement of the estimation of VaR in comparison with not combining the two methods.

Hull and White’s article from 1998 also uses this hybrid approach where one combines volatility weighting with historical simulation. By using a volatility-weighting procedure one is able to incorporate the volatility changes that have occurred during the investigated period. The underlying observations are collected between 1988 and 1998, for twelve different exchange rates and stock indices. Empirically, the volatility-weighted method of Hull and White is compared to the age-weighted method by Boudoukh et al. The conclusions drawn by Hull and White is that the proposed method performs better than the age-weighted method in respect to currencies and that the method is a new, more effective way to calculate VaR. Though, the results for investments in stock indexes is somewhat mixed.

By constructing 25 different portfolios containing of different Dutch fixed-interest securities with different maturities, Vlaar (1999) investigated the accuracy of different VaR-models. The underlying data were collected between January 1980 and March 1997. The models that were investigated were the basic historical simulation, the variance-covariance and the, Monte Carlo simulation method. Vlaar's conclusions imply that the different models can produce different VaR-estimates. Therefore the question is, if it is necessary to use the model that calculates the most accurate VaR if the most accurate version leads to the higher capital requirements? Another conclusion is that the historical simulation requires a long history of observations. The variance-covariance method is deemed to be too naïve, since it is based on the normal distribution. Although, the VaR-estimates based on this method and the GARCH-specification for the variance, are correct. As an anti-pole to the assumption of the naïve normal distribution, the Student-t distribution is used. Though, the results from the estimates based on the t-distribution are much worse than the ones from the ones based on the normal distribution.

In 2009, Ottink provided an empirical investigation of historical VaR models using data on Italian floating rate government bonds collected between July 20, 2005 and June 12, 2009. The purpose of the study was to determine which historical simulation approach is the most appropriate to use when estimating the VaR for Italian floating rate government bonds. The investigated methods are the basic historical simulation method, the volatility- and age-weighted methods. Kupiec's and Christoffersen's test are applied in order to determine the accuracy of the models in question. The conclusions that Ottink drew is that the basic approach tends to underestimate the risk in the sample period due to the fact that is not able to adapt to changing market conditions. Also, Ottink is able to conclude that the age-weighted method performs significantly better than the basic method in terms of accuracy. Though, it should be noted that the volatility-weighted method performs the best in terms of accuracy. (Ottink, 2009)

Nieppolas study from 2009 has a focus on estimating the validity of certain specific VaR calculation software used at a Finnish institutional investor. The validity of the software is tested by using different back-testing procedures, such as the ones provided by Kupiec (1995), Christoffersen (1998) and Haas (2001). As a second objective, Nieppola tries to determine which tests are the most powerful and most accurate for the Finnish firm to use when estimating the validity of the models in practice. In other words, Nieppola tries to determine which back-testing method that should be used by the Finnish firm. The underlying data contains of daily observations for three different portfolios that the investor held between December 2007 and November 2008. One of these portfolios consists of government bonds, i.e. a fixed-income portfolio. The conclusions that Nieppola (2009) is able to draw is that either Kupiec or Christoffersen's back-testing procedures are good when estimating the accuracy of a particular VaR-model. Instead, the procedure by Haas should be used.

After the development of CAViaR by Engle and Manganelli (1999 and so forth), articles have emerged in order to provide the model with empirical support. A comprehensive overview of some of the empirical studies concerning CAViaR, can be seen in Fabozzi et al. from 2008. One of the first papers that used the CAViaR methodology was provided by Kouretas and Zarangas in 2005. Daily returns were collected between January 3, 1990 to November 30, 2004 for six US stocks listed on NYSE, six Greece blue-chip stocks listed on the Athens Stock Exchange and five general stock price indices. These three groups are estimated as portfolios. The sample consisted of 3261 observations per asset, where 2761 observations belonged to the in-sample and the last 500 observations to the out-of-sample. Kouretas and Zarangas applied four of the presented CAViaR specifications, namely; adaptive, symmetric absolute value, asymmetric slope and indirect GARCH (1;1). The results imply that the CAViaR methodology provides a very accurate estimation of VaR for companies listed on the NYSE. However, the results are not as satisfactory for the companies whose stocks are listed on the Athens Stock Exchange or the five price indices.

Fabozzi et al. (2008) provided the CAViaR methods with additional support. In order to test the accuracy of the CAViaR method in question, daily data between the 7th of November 1994 and the 30th of September 2008 were collected for six indexes constructed by K. French. Moving windows with 1000 days are used and the models are kept constant for 250 days before re-estimating the model. The empirical evidence suggests that time-varying CAViaR methods can perform better than other methods for predicting VaR. Although, it should be noted that this evidence is apparent when there exists spillover effects from one market to another. (Fabozzi et al., 2008)

In 2010, Allen and Singh empirically tested the performance of four different CAViaR methods using Australian stock market data. Daily returns were collected for two indices (ASX-200 and ASX-50 plus) and two stocks (NAB, ANZ) for the period September 1994 to September 2009. Due to the financial crisis, two periods are considered where one includes the crisis and the other excludes the crisis. The employed CAViaR methods (adaptive, symmetric absolute value, asymmetric slope and Indirect GARCH (1;1)) are compared with a GARCH (1;1) specification, RiskMetrics and a Skewed student APARCH (1;1) model. The findings imply that using methods from the CAViaR approach of Engle and Manganelli (2004) are more efficient than the other investigated methods. Especially, it is found that the results of the empirical investigation are in line with the results by Engle and Manganelli. Thereby, it is concluded that the CAViaR models applied works well on Australian data. However, it should be noted that this is true for the period which do not include the global financial crisis. The results from the sample considering the global financial crisis led to more extreme returns, and that an excessive number of violations are produced. The consequence of this is that the model does not appear to work when the underlying data is characterized by extreme volatility, and thereby there is a long way to go before such models are developed.

3 Methodology

In this chapter, the methodological approach is presented. The chapter starts with an introduction of the obtained data and its characteristics. Thereafter, the different methods are presented in detail. The chapter ends with a description over the two different back-testing procedures that will be applied in order to evaluate the VaR estimations.

3.1 Data description

Bond index data for five different maturities[10] (2, 3, 5, 7 and 10 years) was retrieved from the Datastream database for the period 1999-01-01 to 2010-12-31, where the period from 1999-01-01 to 2000-12-31 is used for the estimation of the first CAViaR parameters (2006). The latter part (from 2001-01-01 =>) is used for the estimation of VaR. There are several reasons for why this period is chosen. The first and most obvious reason is because the more observations give better and more stable estimates. Another fact that one has to bear in mind is that the investigated period contains financial crisis situations, as well as non-crisis situations. The estimation procedure has been divided into five estimation periods. Each period contain five years, out of which four years are over-lapping and the last year is added to the sample. This period is the in-sample-period. Following the in-sample-period is an out-of-sample-period containing one year, as suggested by BIS. (BIS, 1996) This out-of-sample-period is what is used in order to test the performance of the VaR-method in question. Table 1, below, shows the obtained periods for the portfolio;

	In-sample-period	Out-of-sample period
P1	1/1-01 - 31/12-05	1/1-06 - 31/12-06
P2	1/1-02 - 31/12-06	1/1-07 - 31/12-07
P3	1/1-03 - 31/12-07	1/1-08 - 31/12-08
P4	1/1-04 - 31/12-08	1/1-09 - 31/12-09
P5	1/1-05 - 31/12-09	1/1-10 - 31/12-10

Table 1: Considered periods

As can be seen by the table above, the out-of-sample-period is rolled forward one day at a time.

[10] The indexes are; BMSD02Y, BMSD03Y, BMSD05Y, BMSD07Y and BMSD10Y.

3.1.1 Transformation of the data

The second step after retrieving the data is to transform the obtained time series into daily continuous compounded returns using equation 13;

$$r_t = ln\left(\frac{P_t}{P_{t-1}}\right) \qquad (13)$$

Where;

r_t is the continuous compounded return at time *t*, P_t is the price of the index at time *t* and P_{t-1} is the price of the underlying index at time *t-1*.

Before transforming the data into daily continuous compounded returns the data was cleared from all Swedish holidays when there was no trading at all. Figure 3 and 4 in the appendix, show different returns for the five different maturities during the estimated period, as well as the log returns for the portfolio.

3.1.2 Descriptive statistics

In table 2, the descriptive statistics for the different periods, as well as the total period is presented. With the help of the descriptive statistics it is possible to deduct some conclusions regarding the appearance of the data. One of the more crucial things that can be concluded from the data is that it is possible to reject the assumption of normally distributed residuals. If the data is normally distributed the skewness of the data should be equal to zero. The kurtosis, on the other hand, should be equal to three. (Gujarati, 2003) The observed kurtosis and skewness, in this case, implies that the sample distribution does not follow a normal distribution since both the skewness and kurtosis are significantly different from zero and three, respectively.

Another way to test whether the residuals are normally distributed is to apply the Jarque-Bera[11] test. If the JB statistic is expected to be equal to zero it is not possible to reject the hypothesis of normally distributed residuals. (Gujarati, 2003) Though, this is not the case and therefore it is possible to reject this assumption. The rejection of normality can posit difficulties for methods relying on the assumption in question, especially for models stemming from the parametric approach. Especially, this may inflict the results stemming from the estimation of the normal VaR where one assumes normality distributed residuals.

[11] Henceforth denoted as JB.

	Total	Period 1	Period 2	Period 3	Period 4	Period 5
Mean	-7,84E-06	1,66E-05	8,73E-06	-3,20E-05	3,37E-05	-1,79E-05
Standard deviation	0,00188	0,001856	0,001741	0,001647	0,001737	0,001826
Skewness	-0,168643	-0,422795	-0,301263	-0,223282	0,072635	0,112177
Kurtosis	4,532674	3,982413	3,910752	4,311651	5,084856	5,270867
Jarque-Bera	257,7784	87,85826	62,40785	100,472	229,1236	272,7257
Probability	0	0	0	0	0	0
Sample	2512	1255	1256	1256	1259	1257

Table 2: Descriptive statistics

3.1.3 Forecasting horizon and interval

In this thesis, forecasts of one-day VaR are used. The reason for why using one-day forecasts instead of weekly, monthly or even yearly forecasts are because a longer horizon reduces the number of independent observations and thereby also the power of the tests. Consider the following example; using a monthly VaR horizon implies that there will only be 12 independent observations per year. Using a one-day VaR horizon will yield approximately 253 independent observations per year. In other words, a shorter horizon will thereby increase the power of the test. (Jorion; 2001) Also, the usage of a one-day horizon is proposed by the Basel Committee. (BIS, 1996 and 2003)

The length of the windows is a debated question, though no consistent answer is given to the question. Some empirical findings regarding this question can be found in Hendricks (1996) and Vlaar (1999). In order to estimate VaR properly, three sample lengths regarding the number of included historical observations are estimated for each VaR-method. The length ranges from one half-year of historical data up to two years of historical data. The reason for the usage of different windows for the estimation of VaR is to see whether there is any effect of including more or less than one year of historical observations. In other words, will the estimates be more effective than by using more or less historical data?

As noted above, three different sample lengths are considered; a half-, one- and two-years. The first window, which considers the inclusion of a half-year of historical observations in the estimation, contains between 127 and 130 observations. For example, when estimating the VaR-value for 2006-01-03 (which denotes the first day of trading during 2006), historical observations from 2005-07-01 to 2005-12-30 is used. The second window includes between 250 and 253 observations, which corresponds to one year. The third window, on the other hand, consists of between 501- 506 historical observations which correspond to two years. As said before, these windows are rolled forward one-day at a time until the value for 2010-12-30 is reached. Table 3 below, depicts the different windows.

Window	2001-2005	2002-2006	2003-2007	2004-2008	2005-2009
Estimation period	2006	2007	2008	2009	2010
0,5 year	130	128	127	128	129
1 year	253	251	250	252	251
2 years	506	504	501	502	503

Table 3: Sample periods

3.2 Applied VaR methods

The methods that are applied in this study stems from the three different approaches described in the theoretical part. A total of seven methods has been chosen, where two of the methods stems from the parametric approach, three from the non-parametric approach and two from the semi-parametric approach. Table 4 below depicts the applied methods, the approaches which they belong to, and the equations used to calculate the method. Table 4 also depicts how they results are presented in the empirical part of this thesis.

Method	Approach	Equation
Basic Historical simulation	Non-parametric	3
Age-weighted Historical Simulation	Non-parametric	3, 4
Volatility-weighted Historical Simulation	Non-parametric	3, 5, 12
Normal distribution	Parametric	7
Student's t-distribution	Parametric	8
Symmetric Absolute value	Semi-parametric	10
Asymmetric slope	Semi-parametric	11

Table 4: Methods, approaches and equations

3.3 Parameter estimation

In order to estimate the volatility-weighted historical simulation method, as well as the asymmetric slope and symmetric absolute value method, it is necessary to estimate the unknown parameters. The volatility parameters in the volatility-weighted method are estimated using a GARCH-specification. The unknown parameters for the two CAViaR specifications are estimated using a quantile regression.

3.3.1 GARCH

A GARCH (1;1)-specification is estimated, which implies that the conditional variance depends on both the past returns, as well as previous conditional variance. The reason for why a GARCH (1;1)-specification is used is due to the finding that additional lags do not give the model better explanatory power. In order to model this, the following equation for the conditional variance is used;

$$\mu_t = \sigma_t \varepsilon_t;\ \varepsilon_t \sim i.i.d$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \qquad (14)$$

Where it is assumed that;

$$\omega > 0;\ \alpha \geq 0;\ \beta \geq 0;\ (\alpha + \beta) < 1$$

As can be seen by the equation above μ_t depicts the mean equation including both a constant and a residual. σ_t^2, on the other hand, depicts the conditional variance. The conditional variance is dependent upon three elements, as depicted in equation 14. Firstly, σ_t^2, depends on the intercept, ω. Secondly, σ_t^2 depends on the information regarding past returns, $\alpha\varepsilon_{t-1}^2$. Thirdly, and lastly, σ_t^2 depends on the past conditional variance, depicted by $\beta\sigma_{t-1}^2$.

In order to make the volatility forecasts as reliable as possible, the GARCH-models have been estimated using rolling-windows containing five years of historical observations. These estimates are updated on a yearly basis as described by the different periods. For instance, in order to forecast the volatility for period 1, observations from 2001-01-01 to 2005-12-31 are used. The obtained parameters are then used to estimate the conditional variance of the next year. This procedure is employed for all periods and portfolios. The parameters, ω, α and β are estimated using maximum likelihood in Eviews.

3.3.2 Regression Quantiles

Since the parameters, β, in both the symmetric absolute value and asymmetric slope methods are unknown, it is necessary to estimate them. Instead of using the OLS-method for the parameter estimation, the quantile regression of Koenker and Basset (1978) is employed. The quantile regression is an extension of the OLS-method. (Allen and Singh, 2010)

Suppose a sample of random variables $y_1, \dots, y_t$, are available and distributed according to;

$$Pr(y_t < \tau|x_t) = F_y(\tau|x_t); t = 1,..,T$$

Where x_t is a (k,l) vector of regressors and the $x_t'\beta_\theta$ models the θ-quantile. The model can be written as;

$$y_t = x_t'\beta_\theta + u_{\theta t}$$

$$Quant_\theta(\varepsilon_t|x_t) = x_t'\beta_\theta$$

Where x_t is the p-vector of regressors and

$Quant_\theta(\varepsilon_t|x_t) = x_t'\beta_\theta$ is the θ-quantile of ε_t conditional on x_t. The θ^{th} regression quantile can be defined as any $\hat{\beta}$ which solves the problem;

$$min_\beta \frac{1}{T}\left\{\sum_{t:y_t \geq x_t'\beta} \theta|y_t - x_t'\beta| \middle| + \sum_{t:y_t < x_t'\beta} (1-\theta)|y_t - x_t'\beta|\right\}$$

As noted by Engle and Manganelli, the only required assumption is the specification of quantile process. This implies that it is not necessary to specify the whole distribution of error terms. (Kouretas and Zarangas, 2006)

The estimation of the quantile regression is performed in Eviews using the qreg-function, where five years of estimated historical VaR is used on both a 95%- and a 99%-significance level. It should be noted that the estimated VaR is based upon the different sample lengths. Thereby, this implies that the first estimated value during 2001 is estimated using data from 1999 to 2000 (if one considers two years of historical observations) and so on.

3.4 Back-testing procedures

In order to test the accuracy of a model, different back-testing procedures can be applied. In other words, the procedures are used in order to evaluate the performance of the method in place. Dowd (p. 321, 2005) defines a back-testing procedure as *"the application of quantitative methods to determine whether the forecasts of a VaR forecasting model are consistent with the assumptions on which the model is based, or to rank a group of such models against each other"*. This definition implies that back-testing procedures are used to verify that the actual losses are in line with the projected losses. (Jorion, 2001; Dowd, 2005)

Two of the more common tests that usually are applied by practitioners are the Kupiec unconditional coverage and the Christoffersen conditional coverage tests.

3.4.1 Kupiec's test on unconditional coverage

One of the more commonly used back-testing procedures is the one provided by Kupiec in 1995, which is based on the calculation of the failure rates. The main idea behind the procedure is to test whether the observed frequency of exceedances[12] are consistent with the chosen confidence level. An exceedance occurs when the estimated VaR is exceeded by the actual return. (Kupiec, 1995; Jorion, 2001; Dowd, 2005) If the frequency of losses is less than what is indicated by the chosen confidence level, the risk is deemed to be underestimated. The risk is overestimated if the frequency of losses is higher than the confidence level suggests. Under the null hypothesis that the model is correct it is assumed that the number of exceptions follow a binomial distribution. The probability of experiencing *x* or more exceptions are given by equation 15;

$$Prob(x|n,p) = \binom{n}{x} p^x (1-p)^{n-x} \qquad (15)$$

Where *x* models the number of loss observations exceeding VaR, *n* is the sample size and *p* defines the probability of a loss. The hypothesis tested is;

$$H_0: p = p^*$$

$$H_1: p \neq p^*$$

Where p^* denotes the observed failure rate and p denotes the failure rate suggested by the confidence level. (Kupiec, 1995; Dowd, 2005) In order to test the hypothesis of the model, equation 16 is used which depicts a likelihood ratio test.

$$LR_{UC} = -2ln[(1-p)^{n-x}p^x] + \ 2ln\left[\left(1-\frac{x}{n}\right)^{n-x}\left(\frac{x}{n}\right)^x\right] \qquad (16)$$

Under the null hypothesis, the likelihood ratio follows a χ^2-distribution with one degree of freedom. The critical value for the χ^2-distribution stems from both a 95% and 99% confidence-level which corresponds to 3,84 and 6,63. In other words, if the likelihood ratio exceeds 3,84 it is possible to reject the null hypothesis on a 5%-level.

[12] An exceedance occurs when the observed return is greater than the estimated VaR-value.

The Kupiec test has several attractions that have led to the acceptance among practitioners and scholars. Mostly this is due to the fact that the test is easy to perform and do not require much information. Despite the attractiveness of the test provided by Kupiec it has some pitfalls. Especially, the Kupiec only has a focus on the frequency of an occurring loss and thereby lose information regarding the time dynamics of the exceedances. Other problems that are associated with the Kupiec test is that it requires a long performance history in order to verify the VaR estimates. This implies that the test lacks power when estimating VaR for samples with a shorter performance history which can yield an underestimation of the potential loss amounts. (Kupiec, 1995; Dowd, 2005)

3.4.2 Christoffersen's test on conditional coverage

In order to circumvent the pitfalls with the Kupiec test, Christoffersen provided a conditional coverage test in 1998. The difference between this test and the Kupiec test, which is an unconditional coverage test, is that the Christoffersen test allows for time variation in the data. Also, the deviations are specified so that they must be serially independent. (Christoffersen, 1998; Jorion, 2001) The test can be divided into three parts, where the first part tests the likelihood ratio for the unconditional coverage, the second part tests the likelihood ratio of independence and the third part combines the two parts. The first part is not presented here due to its presentation in *"3.4.2 The Kupiec test on unconditional coverage"*.

The second part of the Christoffersen test, tests the independence among the exceedances. In other words, it is tested whether the exceedances occurs in clusters or not. I.e. do the exceedances occur dependently or independently of each other? In order to test for independency, several steps are performed. The first step is to specify whether any exceedances have occurred during the investigated period. (Christoffersen, 1998) If an exceedance has occurred, an indicator variable gets a value of 1, otherwise 0.

$$I_t = \begin{cases} 1 \; if \; violation \; occurs \\ 0 \; if \; no \; violation \; occurs \end{cases}$$

In the second step a transition probability matrix is formed, which has the following form;

$$\Pi = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix}$$

Where $\pi_{ij} = Pr(I_t = j|I_t = i)$. The probability of an exception tomorrow, is π_{01} if an exception occurs today. In the same manner, π_{11} models the probability of an exception occurring tomorrow, if an exception occurred today. $1 - \pi_{01}$ and $1 - \pi_{11}$ models the probability of a non-violation following a non-violation and the latter models the probability of a non-violation following a violation. (Christoffersen, 1998) In the last step, the likelihood ratio, that is used in order to test the independency, is estimated using equation 17;

$$LR_{ind} = -2ln\left[(1-\pi)^{(T00+T11)}\pi^{(T01+T11)}\right] + 2ln[(1-\pi_0)^{T00}\pi_0^{T01}(1-\pi_1)^{T10}\pi_1^{T11}] \quad (17)$$

Where $T_{ij}, i,j = 0,1$ is the number of observations following a j following an i. The independency test is χ^2-distributed with one degree of freedom, as the unconditional test. The probability, π_{01} and π_{11}, is estimated through the following equations;

$$\pi_{01} = \frac{T_{01}}{T_{00} + T_{01}} \; ; \; \pi_{11} = \frac{T_{11}}{T_{10} + T_{11}} ; \; \pi = \frac{T_{01} + T_{11}}{T_{00} + T_{10} + T_{01} + T_{11}}$$

The two parts can be combined into a conditional coverage test. The combined test-statistic for the conditional coverage is depicted in equation 18;

$$LR_{CC} = LR_{UC} + LR_{ind} \qquad (18)$$

Which is χ^2-distributed with two degrees of freedom. This implies that the Christoffersen test can be used to test both the coverage and the independence hypothesis at the same time. If the combined model fails to test both hypotheses, it is possible to test the models separately in order to see where the model failure arises. (Christoffersen, 1998) As in the Kupiec case, the model is rejected if the likelihood ratio exceeds 3,84 or 6,63 on a significance level of 5% and 1% respectively.

4 Empirical results

In the preceding part the empirical results are presented. The chapter is divided into four major parts. The first part describes the correlation among the underlying indexes, as well as the estimated parameters from both the GARCH (1;1)- and CAViaR-specifications. The second part considers the results from the non-parametric estimation. The third and fourth part considers the results from the parametric and semi-parametric estimations.

4.1 Historical correlations

In Table 5the historical correlation between the underlying assets are presented. It should be noted that this correlation is estimated for the whole period. As can be seen by the table, there exists a positive relationship between the different indexes. This positive relationship implies that when the asset value of one index increases, the other indices tend to follow. Although, this positive relationship of the assets may not always be positive since it implies that when the value of one asset falls the other tends to follow and thereby no diversification effect can be considered.

	2	*3*	*5*	*7*	*10*
2	1				
3	0,92702047	1			
5	0,86833978	0,94685	1		
7	0,81124489	0,899293	0,958988	1	
10	0,75517	0,840187	0,915056	0,967739	1

Table 5: Average correlation among the assets

It should be noted that the correlation may vary over time. If one uses another time-period the results may be different than the ones obtained here above. The relationship between the assets should still be positive.

4.2 Parameters

4.2.1 GARCH (1;1)

The considered specification for the GARCH-model is a GARCH (1;1). The reason for why this specification is chosen is because the (1;1) model produced better results than other GARCH-specifications. In other words, it is not useful to add any additional lags.

	P1	p-value	P2	p-value	P3	p-value	P4	p-value	P5	p-value
ω	5,56E-08	0.0485	2,36E-08	0.1400	1,73E-08	0.0115	1,67E-08	0.0423	2,05E-08	0.0123
α	0,039566	0.0001	0,033273	0.0001	0,023067	0.0000	0,035127	0.0000	0,038625	0.0000
β	0,944503	0.0000	0,958171	0.0000	0,969526	0.0000	0,959799	0.0000	0,954863	0.0000

Table 6: GARCH-parameters

Table 6 depicts the different coefficients that are used in order to estimate the volatility in the volatility-weighted historical simulation. As can be seen by Table 6, all estimated parameters are significant. The coefficients are estimated using five years of historical return observations, which are re-estimated for every period.

4.2.2 CAViaR

4.2.2.1 Symmetric absolute value

In order to estimate the VaR using the symmetric absolute value it is necessary to estimate the unknown parameters. The unknown parameters are estimated using a quantile regression in Eviews.

1%	P1	p-value	P2	p-value	P3	p-value	P4	p-value	P5	p-value
β0	-1,44E-05	0,0991	-8,48E-06	0,2818	-6,48E-06	0,2894	-5,74E-07	0,9513	-1,60E-06	0,8486
β1	0,996026	0	0,997425	0	9,98E-01	0,00E+00	9,99E-01	0,00E+00	0,99899	0
β2	-0,16175	0	-0,14627	0	-1,45E-01	0,00E+00	-1,72E-01	0,00E+00	-0,16854	0
5%	**P1**	**p-value**	**P2**	**p-value**	**P3**	**p-value**	**P4**	**p-value**	**P5**	**p-value**
β0	-8,11E-06	0,0719	-6,86E-06	0,11	-4,34E-06	0,1866	-2,05E-06	0,6256	-3,31E-06	0,2367
β1	0,996134	0	0,996729	0	9,97E-01	0,00E+00	9,98E-01	0,00E+00	0,997874	0
β2	-0,04837	0	-0,04376	0	-4,25E-02	0,00E+00	-4,13E-02	0,00E+00	-0,03619	0

Table 7: Parameters for the symmetric absolute value method, using a half-year window

1%	P1	p-value	P2	p-value	P3	p-value	P4	p-value	P5	p-value
β0	-7,45E-06	0,3347	-1,68E-06	0,7082	-4,44E-06	0,4017	-1,68E-06	0,6686	1,02E-06	0,8745
β1	0,997816	0	0,999259	0	9,98E-01	0,00E+00	9,99E-01	0,00E+00	0,99997	0
β2	-0,11683	0	-0,08905	0	-1,17E-01	0,00E+00	-8,95E-02	0,00E+00	-0,07053	0,0003
5%	**P1**	**p-value**	**P2**	**p-value**	**P3**	**p-value**	**P4**	**p-value**	**P5**	**p-value**
β0	5,52E-08	0,9887	-2,07E-06	0,5036	-5,87E-07	0,7833	-7,31E-07	0,7464	8,75E-07	0,5
β1	0,999299	0	0,998811	0	9,99E-01	0,00E+00	9,99E-01	0,00E+00	1,000093	0
β2	-0,03228	0	-0,02611	0	-2,37E-02	0,00E+00	-2,27E-02	0,00E+00	-0,01051	0

Table 8: Parameters for the symmetric absolute value method, using a one-year window

1%	P1	p-value	P2	p-value	P3	p-value	P4	p-value	P5	p-value
β0	1,13E-06	0,8614	1,08E-06	0,6736	6,30E-07	0,8163	1,42E-06	0,6464	-1,56E-06	0,7069
β1	1,000064	0	1,000138	0	1,00E+00	0,00E+00	1,00E+00	0,00E+00	0,999368	0
β2	-0,05008	0	-0,03553	0	-5,35E-02	0,00E+00	-5,52E-02	0,00E+00	-0,09023	0
5%	P1	p-value	P2	p-value	P3	p-value	P4	p-value	P5	p-value
β0	-3,36E-07	0,8527	-1,58E-07	0,9178	-3,70E-07	0,7099	1,84E-07	0,8316	-2,92E-07	0,8726
β1	0,999693	0	0,999747	0	1,00E+00	0,00E+00	1,00E+00	0,00E+00	0,999472	0
β2	-0,00908	0	-0,01004	0	-9,06E-03	0,00E+00	-9,34E-03	0,00E+00	-0,02317	0

Table 9: Parameters for the symmetric absolute value method, using a two-year window

As is evident by table 7, table 8 and table 9 the parameters are estimated for each window under consideration, as well as the chosen level of significance. The parameters are thereafter re-estimated for every year. As can be seen by the above tables, most parameters are significant.

4.2.2.2 Asymmetric slope

In order to estimate the asymmetric slope method, there is a necessity to calculate the unknown parameters. As with the symmetric absolute value, the parameters are estimated for every period, and thereby re-estimated once a year through a quantile regression. The obtained results are presented in the table 10 to 12 below.

1%	P1	p-value	P2	p-value	P3	p-value	P4	p-value	P5	p-value
β0	-0,00041	0	-0,00028	0	-5,26E-05	0	-0,00027	0,0053	-0,00024	0,0038
β1	0,938489	0	0,957487	0	0,99071	0	0,959218	0	0,966824	0
β2	0	1	0	1	-0,10303	0,0012	-0,01505	0,0184	-0,01901	0,004
β4	0,177689	0	0,18229	0	0,194051	0	0,252581	0	0,238339	0
5%	P1	p-value	P2	p-value	P3	p-value	P4	p-value	P5	p-value
β0	-0,00015	0	-4,78E-05	0,08	-4,08E-05	0,0095	-4,68E-05	0,0139	-1,13E-05	0
β1	0,966907	0	0,988917	0	0,99073	0	0,988614	0	0,997638	0
β2	0,003854	0,0296	-4,27E-18	1	-2,90E-15	1	-5,40E-16	1	-0,0002	0,2848
β4	0,026341	0,0001	0,025849	0	0,027749	0	0,026273	0	0,03332	0,0435

Table 10: Parameters for the asymmetric slope method, using a half-year window

1%	P1	p-value	P2	p-value	P3	p-value	P4	p-value	P5	p-value
β0	-0,00022	0	-2,41E-05	0	-5,27E-06	0,3281	-1,92E-06	0,7362	-1,01E-05	0,0203
β1	0,966184	0	0,995747	0	0,998789	0	0,999436	0	0,998377	0
β2	4,77E-18	1	-0,03216	0	-0,06565	0	-0,10071	0	-0,0826	0,0014
β4	0,200914	0	0,210555	0	0,211281	0	0,071525	0	0,048987	0
5%	P1	p-value	P2	p-value	P3	p-value	P4	p-value	P5	p-value
β0	-3,61E-05	0	-4,32E-06	0	-4,19E-05	0,0006	-3,40E-05	0,1319	-1,35E-05	0,2718
β1	0,99109	0	0,99893	0	0,989391	0	0,991395	0	0,996244	0
β2	0,00016	0,5764	1,06E-05	0,9118	-8,76E-05	0,6717	-8,28E-05	0,8203	0	1
β4	0,028275	0	0,013599	0,1401	0,007915	0,0007	0,014465	0	0,005761	0,56

Table 11: Parameters for the asymmetric slope method, using a one-year window

1%	P1	p-value	P2	p-value	P3	p-value	P4	p-value	P5	p-value
β0	-2,83E-18	1	1,09E-19	1	-4,20E-07	0,8664	-5,21E-07	0,8476	-0,00019	0,0484
β1	1	0	1	0	0,999872	0	0,999866	0	0,972028	0
β2	-6,34E-15	1	-5,06E-17	1	-0,0402	0	-0,03368	0,0001	-0,07551	0,0142
β4	0,047999	0	0,027327	0,0003	0,049021	0	0,043644	0	0,062126	0,0008
5%	P1	p-value	P2	p-value	P3	p-value	P4	p-value	P5	p-value
β0	-2,28E-18	1	-1,44E-18	1	6,61E-19	1	-2,86E-06	0	-3,20E-05	0,4866
β1	1	0	1	0	1	0	0,999251	0	0,992857	0
β2	2,41E-18	1	3,40E-19	1	9,82E-20	1	-1,12E-17	1	-0,00135	0,3909
β4	0,007436	0,3708	0,00512	0,2879	0,001183	0,0002	0,01036	0	0,013665	0,0067

Table 12: Parameters for the asymmetric slope method, using a two-year window

As can be seen by the tables above, most parameters are significant in all cases. Though, there is a tendency among the dummy-variables to be insignificant. This is deemed to be a minor problem since they only contain a positive (β2) or negative (β3) value, otherwise zero.

4.3 Estimation results[13]

4.3.1 Non-parametric methods

In Table 13 and Table 14 below, the results from both the 95%, as well as the 99%-level are presented for the non-parametric methods. The different columns and rows in the table depict the level of significance, the number of exceedance per method, as well as the different likelihood ratios that are obtained. Note that the highlighted bold numbers depicts that the method in question is rejected and that * implies that it has not been possible to estimate the likelihood ratio.

[13] Note that the calculations are based on $N(1-\theta)$, where θ denotes the level of significance.

4.3.1.1 Basic historical simulation

The results from the estimations based on the historical simulation on both the 95%- and 99%-level are depicted in Table 13 and Table 14. As can be seen, the different sample lengths yield a different number of exceedances during the investigated periods, which is true on both significance levels. Due to this fact, the different estimations yield different likelihood ratios and thereby it is possible to reject or accept the method(s). Though, it should be noted on average that it is not possible to reject either null hypothesis regarding the unconditional, independency or conditional coverage and thereby it is possible to accept the model. It should be noted that during period three the model is rejected in nearly all cases on both the 95%- and 99%-level. The only sample length that is accepted during period three is the one using one year of historical observations before re-estimating the model. Another fact that is necessary to consider is the likelihood-ratios using a 99%-level. In most cases, it is not possible to draw any distinct conclusions whether the acceptance or rejection of the method in question. The reason why, is because it is not possible to estimate the likelihood-ratios due to a low number of exceedances.

4.3.1.2 Age-weighted historical simulation

The results in Table 13 and Table 14, shows the estimated exceedances, as well as the estimated likelihood-ratios. As with the basic historical simulation, it is not possible to reject the model as such in all cases. On average, the model is accepted. The only period when the model is rejected is during period three when considering a 95%-significance level. Using a 99%-level, on the other hand depicts different results since it is not possible to reject the model. The only sample, in which the model is rejected, is the unconditional coverage when using a window of a half-year. Also, it is apparent that the model on the 99%-level fails to estimate the likelihood ratio for both the conditional coverage, as well as the independency due to a low number of exceedances. This implies, at least in this case that the possibility that an exceedance occurs simultaneously is low.

4.3.1.3 Volatility-weighted historical simulation

The results for the volatility-weighted simulation are depicted in row three in Table 13 and Table 14. It is apparent that the number of exceedances that has occurred during the estimated periods is slightly lower than the other non-parametric methods. As in previous methods the number of exceedances increases during period three, especially when using samples consisting of windows of two years of historical data. One should note that the volatility-weighted method is the only non-parametric method that is accepted using a 99%-level of significance during period three. Though, the method is rejected on the 95%-level. The problem with not being able to estimate the independency, as well as the conditional coverage is also apparent here. As in previous cases, the number of exceedances is too low.

		Basic historical simulation				Age-weighted historical simulation				Volatility-weighted historical simulation			
		Exceedances	**LR uncon.**	**LR ind.**	**LR cond.**	**Exceedances**	**LR uncon.**	**LR ind.**	**LR cond.**	**Exceedances**	**LR uncon.**	**LR ind.**	**LR cond.**
P1													
	0.5	11	0.2099	*	*	11	0.2099	0.4765	0.6864	10	0.5845	*	*
	1	13	0.0168	0.1557	0.1725	13	0.0168	2.0056	2.0224	11	0.2099	0.4765	0.6864
	2	12	0.0257	0.2917	0.3174	11	0.2099	0.4765	0.6864	11	0.2099	0.4765	0.6864
P2													
	0.5	17	1.5403	0.0252	1.5655	17	1.5403	0.0252	1.5655	16	0.9514	0.0006	0.9520
	1	18	2.2555	0.0843	2.3398	18	2.2555	0.0843	2.3398	18	2.2555	0.0843	2.3398
	2	11	0.1971	0.4720	0.6691	11	0.1971	0.4720	0.6691	10	0.5634	0.7108	1.2741
P3													
	0.5	19	2.9808	**6.9821**	**9.9629**	19	2.9808	**6.9821**	**9.9629**	17	1.4649	**5.4198**	**6.8848**
	1	22	**6.0972**	**10.8877**	**16.9849**	23	**7.3412**	**9.6763**	**17.0175**	21	**4.9529**	**8.4028**	**13.3557**
	2	27	**13.2396**	**8.5249**	**21.7646**	25	**10.1126**	**7.5353**	**17.6479**	26	**11.6332**	**6.5923**	**18.2255**
P4													
	0.5	7	3.0548	*	*	9	1.1677	1.0180	2.1856	5	6.1337	*	*
	1	10	0.5845	0.7160	1.3004	10	0.5845	0.7160	1.3004	9	1.1677	1.0180	2.1856
	2	18	2.2090	**7.9706**	**10.1796**	12	0.0257	2.5234	2.5491	17	1.5023	**5.3939**	**6.8963**
P5													
	0.5	17	1.4281	0.0211	1.4493	16	0.8647	*	*	16	0.8647	*	*
	1	20	**3.8501**	1.2391	5.0892	17	1.4281	2.5470	3.9751	18	2.1177	2.0487	4.1664
	2	5	**6.2588**	*	*	6	**4.5315**	*	*	6	**4.5315**	*	*

Table 13: Estimated results for the non-parametric approach using the 95%-level

		Basic Historical Simulation				Age-weighted historical simulation				Volatility-weighted historical simulation			
		Exceedances	**LR uncon.**	**LR ind.**	**LR cond.**	**Exceedances**	**LR uncon.**	**LR ind.**	**LR cond.**	**Exceedances**	**LR uncon.**	**LR ind.**	**LR cond.**
P1													
	0,5	3	0,0909	*	*	4	0,7570	*	*	2	0,1125	*	*
	1	4	0,7570	*	*	3	0,0909	*	*	2	0,1125	*	*
	2	3	0,0909	*	*	2	0,1125	*	*	2	0,1125	*	*
P2													
	0,5	4	0,7691	*	*	4	0,7691	*	*	4	0,7691	*	*
	1	4	0,7691	*	*	3	0,0949	*	*	3	0,0949	*	*
	2	3	0,0949	*	*	3	0,0949	*	*	3	0,0949	*	*
P3													
	0,5	8	**7,6442**	1,3999	9,0441	8	**7,6442**	1,3999	9,0441	5	1,9165	*	*
	1	7	5,4241	*	*	7	5,4241	*	*	6	3,4988	*	*
	2	10	**12,8331**	0,7211	**13,5542**	7	5,4241	1,8655	7,2896	7	5,4241	*	*
P4													
	0,5	2	0,1125	*	*	1	1,1886	*	*	1	1,1886	*	*
	1	3	0,0909	*	*	1	1,1886	*	*	1	1,1886	*	*
	2	3	0,0909	*	*	2	0,1125	*	*	3	0,0909	*	*
P5													
	0,5	4	0,7332	*	*	4	0,7332	*	*	3	0,0832	*	*
	1	3	0,0832	*	*	3	0,0832	*	*	2	0,1208	*	*
	2	1	1,2129	*	*	1	1,2129	*	*	1	1,2129	*	*

Table 14: Estimated results for the non-parametric approach using the 99%-level

4.3.2 Parametric methods

Table 15 shows the estimated VaR, on both the 95%- and 99%-level of significance, using the parametric methods. As in the non-parametric tables, the first column depicts the number of exceedances, the second, third and fourth column depicts the likeliohood ratio for the unconditional coverage, independency and conditional coverage. Note that the highlighted bold numbers depicts that the method in question is rejected and that * implies that it has not been possible to estimate the likelihood ratio.

4.3.2.1 Normal distribution

As can be seen in Table 15 the number of exceedances for both the 95%- and 99%-significance level differ with respect to the window used. As can be seen, the number of exceedances is fairly high considering a 95%-level, although still acceptable. As a consequence of this, the model is accepted in most periods, except period three and period five when using the two-year window. On the 99%-level it is possible to accept the method in all cases, except for period three. Though, it should be noted that it is possible to accept the method using the half-year window, during period three. As previous presented results for the non-parametric methods, it is not possible to estimate the likelihood ratio for the test of independency or conditional coverage.

4.3.2.2 Student's t-distribution

In the second part of Table 15, the estimated results for the Student's t-distribution are presented. The number of exceedances is slightly lower than the ones obtained when using the normal distribution. Thus, it is possible to see that the method can be rejected due to a low number of exceedances. This is apparent in period three, four and five on the 95%-level. Testing the unconditional coverage it is not possible to reject the method as such. On the 99%-level on the other hand, it is not possible to reject the method in neither period, due to the low number of exceedances.

	Normal distribution				Student's t-distribution				Normal distribution				Student's t-distribution			
	Exceedances 95%	**LR uncon.**	**LR ind.**	**LR cond.**	**Exceedances**	**LR uncon.**	**LR ind.**	**LR cond.**	**Exceedances** 99%	**LR uncon.**	**LR ind.**	**LR cond.**	**Exceedances**	**LR uncon.**	**LR ind.**	**LR cond.**
P1																
0,5	13	0,0168	0,1557	0,1725	8	1,9818	*	*	2	0,1125	*	*	1	1,1886	*	*
1	12	0,0257	0,2917	0,3174	8	1,9818	*	*	2	0,1125	*	*	1	1,1886	*	*
2	12	0,0257	0,2917	0,3174	7	3,0548	*	*	3	0,0909	*	*	0	*	*	*
P2																
0,5	12	0,0213	0,288	0,3093	7	3,0089	*	*	3	0,0949	*	*	2	0,1084	*	*
1	14	0,1827	0,062	0,2447	7	3,0089	1,852	4,8609	3	0,0949	*	*	2	0,1084	*	*
2	10	0,5634	0,7108	1,2741	6	**4,3687**	2,4304	**6,799**	3	0,0949	*	*	2	0,1084	*	*
P3																
0,5	19	2,9808	**6,9821**	**9,9629**	12	0,0305	**6,2321**	**6,2627**	7	5,4241	1,8655	7,2896	3	0,087	*	*
1	20	**3,9126**	**6,0542**	**9,9668**	14	0,1583	**4,4697**	4,628	9	**10,1232**	1,0238	**11,1469**	6	3,4988	*	*
2	24	**8,6808**	**8,5613**	**17,2421**	17	1,4649	**5,4198**	**6,8848**	12	**18,8604**	6,2321	**25,0925**	6	3,4988	*	*
P4																
0,5	7	3,0548	1,8588	4,9136	3	**10,8908**	*	*	3	0,0909	*	*	0	*	*	*
1	8	1,9818	1,3936	3,3754	5	**6,1337**	*	*	3	0,0909	*	*	1	1,1886	*	*
2	11	0,2099	0,4765	0,6864	7	3,0548	1,8588	4,9136	5	1,9366	*	*	3	0,0909	*	*
P5																
0,5	11	0,2365	*	*	5	**6,2588**	*	*	5	1,8966	*	*	2	0,1208	*	*
1	12	0,0357	*	*	5	**6,2588**	*	*	4	0,7332	*	*	1	1,2129	*	*
2	5	**6,2588**	*	*	2	**14,387**	*	*	1	1,2129	*	*	1	1,2129	*	*

Table 15: Estimated results for the parametric methods, on both the 95- and the 99%-level

4.3.3 CAViaR

The results from the two CAViaR-specifications are presented in Table 16. As previous results, the table is divided into four major columns for each method in question, no matter of which level of significance. The first column depicts the number of exceedances that are apparent during the evaluated periods. The second, third and fourth column depicts the likelihood-ratio for the unconditional coverage, independency and the conditional coverage. Note that the highlighted bold numbers depicts that the method in question is rejected and that * implies that it has not been possible to estimate the likelihood ratio.

4.3.3.1 Symmetric absolute value

By evaluating Table 16 it is possible to see that the symmetric slope method produces a high number of exceedances on both the 95%- and 99%-level. Due to the increased number of exceedances it is possible to reject the VaR-model in question in nearly all cases on the 95%-level. The period when the method cannot be rejected is during period one, two and five. The acceptable windows are one and two years. If one considers the 99%-level it is apparent that the number of exceedances increases significantly and thereby the method fails to accept the method. One fact that is apparent is that the estimation using a half-year window produces the “worst” number of exceedances. Although, compared to the other two windows it produces a lower number of exceedances during period three, than the other considered windows. This fact is also apparent on the 95%-level. It should be noted that the only two windows and periods that it is possible to accept the method is during period one and five, while using a one- and a two-year window.

4.3.3.2 Asymmetric slope

Compared to its counterpart, the results for the asymmetric slope show a fewer number exceedances on both the 95%- and 99%-level. The only periods that produces acceptable results, i.e. that it is possible to accept the method, is during period one, three and four. During period one the method is accepted when using a two year window. When using a half-year window, the method is accepted in period three and the method is accepted during period four using the one-year window. Considering the results from the 99%-level it is not possible to either reject or accept the method, due to the non-existent number of exceedances. The consequence of this is an underestimation of the risk of the assets.

		Symmetric absolute value				Asymmetric slope				Symmetric absolute value				Asymmetric slope			
		Exceptions 95%	**LR uncon.**	**LR ind.**	**LR cond.**	**Exceptions**	**LR uncon.**	**LR ind.**	**LR cond.**	**Exceptions** 99%	**LR uncon.**	**LR ind.**	**LR cond.**	**Exceptions**	**LR uncon.**	**LR ind.**	**LR cond.**
P1																	
	0,5	50	**69,5466**	0,6290	**70,1757**	0	*	*	*	92	**520,6906**	**33,3397**	**554,0303**	0	*	*	*
	1	14	0,1703	1,5624	1,7327	2	**14,2137**	*	*	6	3,527	*	*	0	*	*	*
	2	12	0,0257	0,2917	0,3174	7	3,0548	*	*	3	0,0909	*	*	0	*	*	*
P2																	
	0,5	39	**38,8244**	0,7965	**39,6209**	2	**14,1272**	*	*	79	**419,1453**	**27,4077**	**446,553**	0	*	*	*
	1	22	**6,2590**	0,6203	**6,8792**	2	**14,1272**	*	*	27	**82,005**	1,6175	**83,6225**	0	*	*	*
	2	16	0,9514	0,0006	0,9520	3	**10,8123**	*	*	8	**7,7336**	1,3873	**9,1208**	0	*	*	*
P3																	
	0,5	25	**10,1126**	**10,8048**	**20,9174**	7	3,101	*	*	13	**22,1445**	**9,6627**	**31,8071**	0	*	*	*
	1	94	**246,4935**	1,8501	**248,3437**	14	0,158	**13,283**	**13,441**	107	**644,8269**	0,8883	**645,7152**	0	*	*	*
	2	75	**160,6656**	**6,0693**	**166,7349**	27	**13,240**	**8,525**	**21,765**	103	**610,758**	0,8716	**611,6296**	0	*	*	*
P4																	
	0,5	30	**18,698**	1,834	**20,532**	4	**8,2556**	*	*	168	**1230,4076**	**57,9155**	**1288,3231**	0	*	*	*
	1	23	**7,430**	-1,119	**6,311**	8	1,9818	1,3936	3,3754	24	**67,3075**	0,6886	**67,9961**	0	*	*	*
	2	25	**10,219**	0,030	**10,250**	13	0,0168	**5,2750**	5,2918	15	**29,2916**	-4,4192	**24,8724**	0	*	*	*
P5																	
	0,5	27	**13,1160**	**8,5751**	**21,6911**	3	**11,0481**	*	*	12	**18,7831**	**10,0674**	**28,8506**	0	*	*	*
	1	5	**6,2588**	*	*	5	**6,2588**	*	*	1	1,2129	*	*	0	*	*	*
	2	8	2,0579	*	*	2	**14,3870**	*	*	1	1,2129	*	*	0	*	*	*

Table 16: Estimated results from the CAViaR-specifications

5 Analysis

In this part of the thesis, the analysis of the empirical results is presented. The results are analyzed with the help of the theoretical observations, as well as the prior research on the topic. The chapter is divided into three major parts which consider the different approaches. In the end of the chapter, a general analysis is presented where all approaches are considered.

5.1 Non-parametric methods

It is evident by Table 13 and Table 14 that there exist some differences between the different non-parametric methods. Especially, one needs to consider the number of exceedances that are produced using the different windows of historical observations for the estimation of VaR. These differences are not that surprising since the methods will adapt to changes faster (slower) depending on how long the estimation window is. For instance, this is apparent if one considers the results from period three in Table 13 and Table 14. The reason for this result is due to the fact that a shorter window is able to adapt to the changes in a higher pace than the longer counterpart, which can be considered to be more stable. This is also apparent if one investigates Figure 5- 10 in Appendix 8.2. Though, it should be noted that the methods fail to estimate the risk accurately during this period. As can be seen during period three there is a tendency among the methods to underestimate the actual risk, due to the high number of exceedances. If one considers the two-year window it can be noted that the effect of the turbulence spills over in the following period for both the basic and the volatility-weighted historical simulation method. This spillover effect is not apparent in the results concerning the age-weighted historical simulation, at least not in the period in question. Thereby, this method may be more appropriate to use, since the observations are given declining weights, compared to the other non-parametric methods. Although, one should note that this concerns a window of two-years and not the other two windows. For the other windows under consideration it can be seen that the volatility-weighted method produces more accurate results. The finding regarding the spillover effect corresponds to some of the empirical findings in Boudoukh et al. (1998), were the age-weighted approach is considered to be superior to the basic historical simulation method. It should be noted that this finding only is applicable for the period under and after a crisis when a long window is concerned.

On average, the volatility-weighted approach provides the most accurate results for all windows and periods, except the two-year window during period three and four. These results correspond to the findings provided by Hull and White (1998), as well as Ottink (2009), where it was stated that the volatility-weighted scheme performs better than its age-weighted counterpart and the basic historical simulation. If one compares the age-weighted approach with the volatility-weighted approach it can be seen that the results differ in some respect. This is especially true if one considers the fact that the volatility-weighted scheme is able to capture the volatility swings that occurred during the investigated period. The number of exceedances do not differ that much and therefore they mostly lead to the same conclusion whether it is possible to reject or accept the method under consideration.

5.2 Parametric methods

As depicted in the parametric tables, as well as Figure11- 14 in Appendix 8.3, the parametric methods provide somewhat stable estimates exceedances and the evolution of VaR during the investigated periods. It is apparent that the estimated VaR increased during period three and four, which may be a consequence of the financial turmoil during that period.

As described in the theoretical part of this thesis, it is shown that there are several problems associated with the normal distribution method. In this case, as presented in the descriptive statistics, the observations are not normally distributed during any period. Thereby, the question is whether or not the method suffers from this? As a consequence of not knowing the impact this has on the results it is evident than one needs to be cautious regarding the results from this method. Though, as the results show the student's t-distribution performs better since the method produces a lower number of exceedances than the normal distribution during the evaluated period. This is evident on both the 95%- and 99%-level. This finding is in contrast to what Vlaar found in 1999, where he was able to conclude that the student's t-distribution provides much worse VaR estimates than the normal distribution. Although, this may be a consequence of the fact that a different time-period is considered, but also due to the fact that Dutch bonds are considered. If ones consider the behavior in Figure 11- 14, Appendix 8.2 it is evident that the methods do not differ that much concerning the evolvement during the estimated period.

5.3 CAViaR

As is depicted in Table 16, the estimations based on the symmetric absolute value suffer from a large number of exceedances, and thereby the method tends to be rejected in almost every case on both the 95%- and 99%-level. These findings are somewhat contradictive during all periods. It makes sense that the number of exceedances increases during period three, but otherwise it does not. Although, the fact that the method as such is rejected on both the 95% and 99%-level is supported, to some extent, by empirical findings of Kouretas and Zarangas (2005) and Allen and Singh (2010). As can be seen by the results, a shorter window tends to overestimate the risk more severely than a longer one, leading to more volatile estimates as depicted in Figure 15- 18 in Appendix 8.4. Though, it should be noted that the half-year window performs “best” of all symmetric absolute value windows during period three which is deemed to be the most volatile period. Despite this, the method as such tends to overestimate the risk. Therefore, it seems that it is not necessary to use a window shorter than a year.

If one compares the symmetric absolute value with the asymmetric slope method it is evident that the asymmetric slope method produces more accurate results than its counterpart. Despite this, it is apparent that the model has problems to test the independency and conditional coverage hypothesis of Christoffersen (1998). Although, this may not come as a surprise since all methods have problems with these two tests due to a low number of exceedances. On the 99%-level it is clear that the asymmetric slope method severely underestimates the risk due to zero exceedances. Thereby, it is evident that the results differ from the ones obtained by Engle and Manganelli (1999, 2004), Koruetas and Zarangas (2005) and Allen and Singh (2010). This divergence is also evident when testing the symmetric method on both the 95%- and 99%-level, where the risk is severely overestimated. One fact that one has to consider when comparing the results with the ones from these three articles is that the DQ-test is not used for evaluation purposes. In other words, the focus of Engle and Manganellis (1999, 2004) DQ-test is to evaluate the extreme tails. This is in contrast to the scope of this study, since the performance is evaluated with respect to the number of exceedances, as proposed by the Basle Committee (1996). Due to this, a direct comparison of the results with previous studies is difficult to undertake. Another difference is the fact that bonds are considered, instead of equities or equity indices which may induce the risk of a departure from the original results. In other words, the CAViaR methods may have difficulties to properly estimate the VaR for bonds due to low volatility etc.

Despite these direct differences, there exist some common features regarding the out-of-sample tests. For instance, it is shown that there is a tendency for both the asymmetric slope method and the symmetric absolute value to show an excessive number of exceedances for VaR on both the 95%- and 99%-level. (Engle and Manganelli, 2004; Kouretas and Zarangas, 2005) This result is also apparent in this study, at least for the symmetric absolute value method.

5.4 General analysis

By comparing the different methods with each other, it is possible to see that there exist some differences among the evaluated methods. One of these differences is the number of exceedances that the methods produce during the evaluated period. As can be seen, most methods provide somewhat stable forecasts of the risk while others do not. The symmetric absolute value is unable to provide a stable forecast of risk during the evaluated period. This can be seen by the fact that the method in nearly all periods exaggerates the number of exceedances and thereby severely overestimates the risk of the considered portfolio. If one compared the symmetric absolute value with its CAViaR counterpart, the asymmetric slope method it is possible to see that the latter does a better job and greatly improves the estimation of the risk in question on the 95%-level. Although, on the 99%-level the asymmetric slope method underestimates the risk due to zero exceedances during the evaluated periods. Because of the inabilities of CAViaR to produce efficient and acceptable results, it is not the most optimal method to use when estimating the VaR for bonds. Despite this, the CAViaR proposes an attractive alternative that may work well on other asset classes other than bonds. Empirically, it has been shown that the CAViaR performs well when using equities or equity-indices as the underlying asset. (Fibozzi et al., 2008)

Using the other methods instead gives more acceptable results if one considers the number of exceedances. Therefore, it is possible (in most cases) to accept the methods in question, at least when performing Kupiec's unconditional coverage test. Usually, it is more difficult to both estimate and accept the latter parts of the Christoffersen test, at least when one tests on the 99%-level. Although, this problem may not be that severe since one can draw the conclusion that there do not exist any dependency between the exceedances. This implies that the exceedances occur independently of each other. However, it is possible to estimate both the independency and conditional coverage test on the 95%-level, since more exceedances are allowed. Another fact that one needs to consider is the fact that the portfolio in question is not deemed to be that volatile during the evaluated period. This stems from the fact that bonds constitutes the underlying asset of the portfolio. As can be seen in Figure 3 in the Appendix 8.1, the underlying bond indexes have not experienced severe swings that other assets did during the period, especially during 2008 and 2009. Despite the in-volatile behavior of the underlying indices, it is possible to see that the methods tried to resolve the increased risk during period three. Although, the evaluated methods have problems (which can be seen by the number of rejections) to properly estimate the risk, on the 95%-level, during that period. If one considers the 99 %-level, on the other hand, it is evident that several methods actually do a pretty good job to estimate risk properly during that period.

In order to choose the most accurate method, one has to consider how many exceedances are acceptable during a year. By using the 95%- and 99%-level, it implies that 12,5 and 2,5 exceedances are acceptable. As can be seen by the different methods, several of them are somewhat in line with these two numbers. Although, it is evident that the methods have problems to exactly estimate the number of exceedances according to both the 95%- and 99%-level. During certain periods it is evident that the methods in question greatly exaggerate or diminish the risk in the portfolio. This may impose problems for the different methods, especially regarding which method is preferable to the others. Thereby, some of the criticism by authors such as Beder (1995), Marshall and Siegel (1996) and Holton (2002) might very well be justified.

Despite the pitfall that more or less neither method is able to reach these two numbers, it is possible to draw some conclusions regarding which method is the one to use. The reason for this is because the methods in question, some of them, are acceptable according to the back-testing procedures. By not considering the above mentioned fact, the volatility-weighted historical simulation method is deemed to be the best performing method during the evaluated period. This can be seen by the fact that the method properly takes the changes in volatility into consideration and thereby is able to adapt to new circumstances. This conclusion is accepted on both the 95%- and 99%-level. Though, it should be noted that the student's t-distribution provides some attractive results during the estimated period and thereby the method can be considered to be a runner up.

The result of using three different sample windows will be some robustness when one considers the estimation. As is evident in Figure 5- 18 in Appendix 8.2- 8.4 there is no need to use a sample window that is longer than two years. This implies that the 1250-day window, proposed by Hendricks (1996), is too long. A too long window implies that the estimates are more stable, though the question will thereby be whether the method as such is able to adapt to new market conditions. As noted in Vlaar (1999) a sufficiently long window should be used in order to get the most accurate estimation of VaR. This implies, in line with the results and as depicted in Figure 5- 18 in Appendix 8.2- 8.4, that a window of one year is sufficient to use when estimating VaR. As a consequence of this, one will be able to capture both the swings that will be apparent in a half-year window as well as the stability that is provided by the two-year window.

6 Conclusions and future research

In this sixth and last chapter, the conclusions drawn from the analysis is presented alongside the suggestions for future research.

6.1 Conclusions

In this thesis, seven methods used in order to calculate VaR are compared and evaluated. The underlying data consists of a portfolio of Swedish index-bonds with different time to maturity which is collected between 1999-01-01 and 2010-12-31. The empirical results suggest that the applied methods differ in some respect, not only regarding the underlying assumptions. The purpose of this thesis is to investigate whether any of the applied methods are applicable when calculating VaR for an equally-weighted bond portfolio. As a supplementary purpose, the question regarding how many historical observations that should be included is investigated.

From the analysis in chapter five it is possible to see that the methods in question has problems regarding the number of acceptable exceedances at the 95%- and 99%-level. Thereby, it is possible to see that some of the criticism of Beder (1995), Marshall and Siegel (1996) and Holton (2002) is justified. Despite this, it is possible to see that most methods are accepted according to both the Kupiec and Christoffersen's test. This is true for most periods on the 95%-level. Though, the 99%-level posits some difficulties due to the low number of exceedances. The problem with a low number of exceedances is that the test of independency will be difficult to apply and thereby the conditional coverage test cannot be performed. Although, it should be noted that this may not be a problem since one can assume that the exceedances are not dependent upon each other. The most accurate method, for the period in question as well as the underlying asset portfolio is deemed to be the volatility-weighted historical simulation method. The reason for this choice is several; especially due to its ability to both capture the volatility of the asset but also since it is a simple and straightforward method. Another fact that one needs to consider is that the method produces stable estimates of VaR and a low number of exceedances on both the 95%- and 99%-level.

Regarding how many historical observations should be included in the sample it is evident that a one-year window is sufficient to use when estimating tomorrows VaR. This conclusion is based on the fact that the one-year window is possible to capture both the volatile behavior of the half-year window, as well as the stability of the two-year window.

6.2 Future research

It is evident from the current study that the CAViaR performed fairly bad when estimating VaR for the asset in question, as well as during the period. Despite the poor results of the CAViaR on bonds, it is evident that the CAViaR has performed rather well when estimating VaR for equities and equity-indices. Thereby, it is suggested that further studies which concerns the performance of CAViaR on other asset classes are made. Perhaps it would be interesting to see whether the CAViaR methods can produce accurate VaR-estimates for portfolios of different assets.

Also, it would be interesting to see more empirical evidence concerning the estimation of VaR for bonds. The reason for this is due to the fact of the fairly limited number of studies that concerns bonds.

7 References

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8 Appendix

8.1 Returns and log-returns

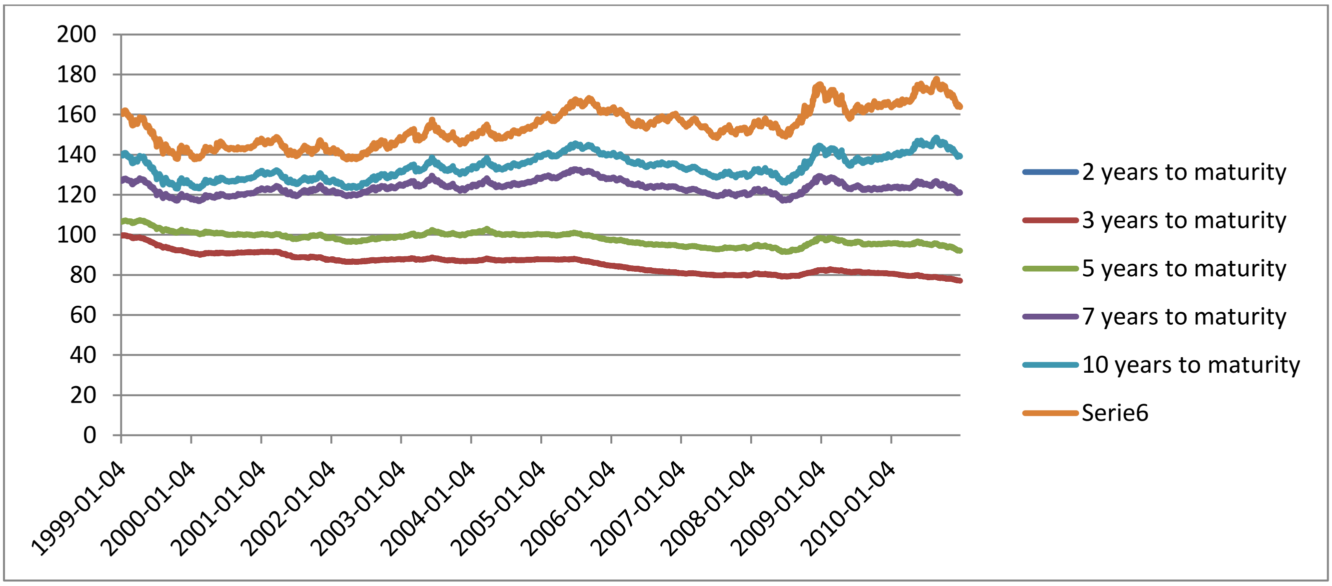

Figure 3: Returns for the underlying indices

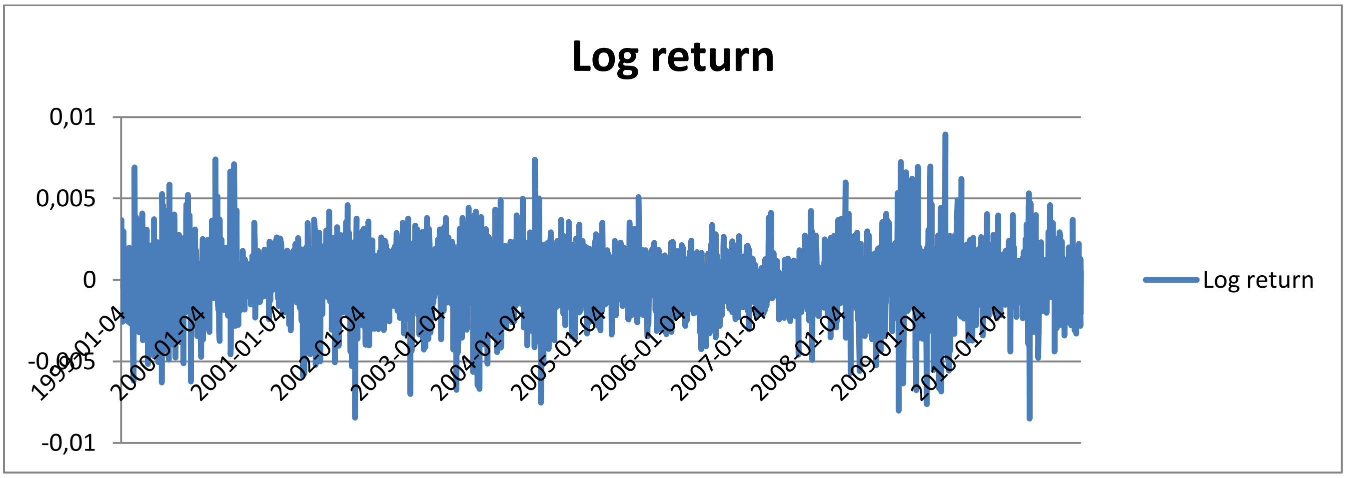

Figure 4: Log return for the portfolio of bonds

8.2 Non-parametric estimations of VaR

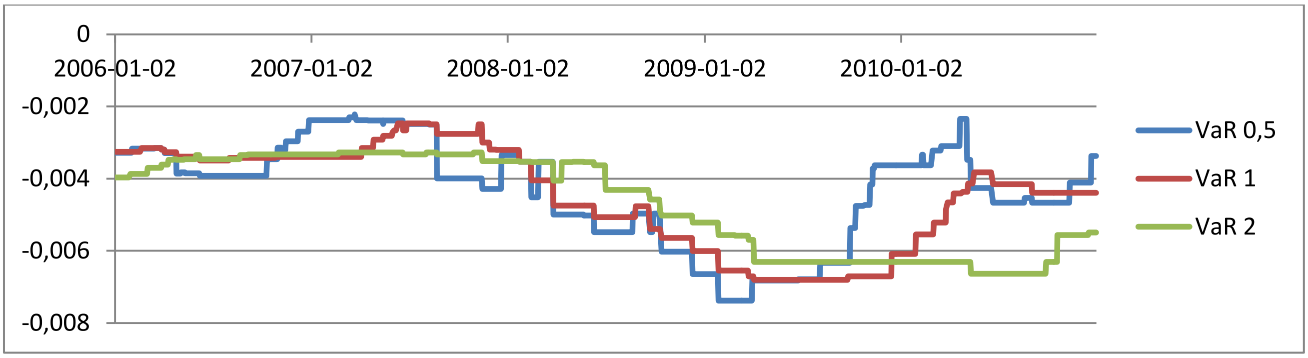

Figure 5: Basic historical simulation at the 99%-level

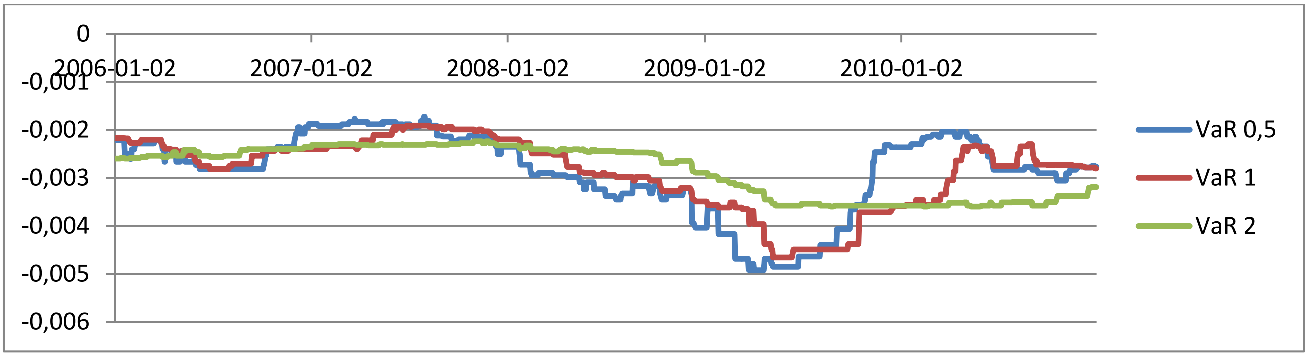

Figure 6: Basic historical simulation at the 95%-level

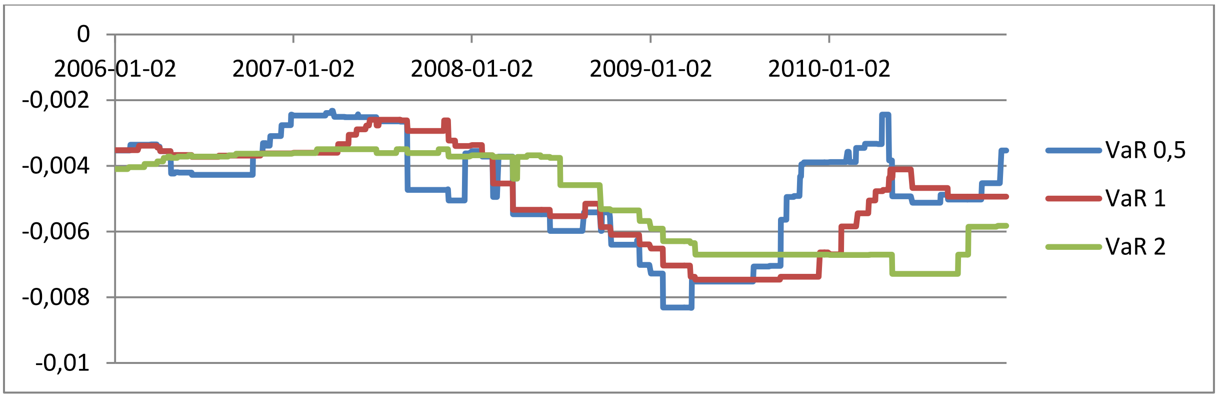

Figure 7: Volatility-weighted historical simulation at the 99%-level

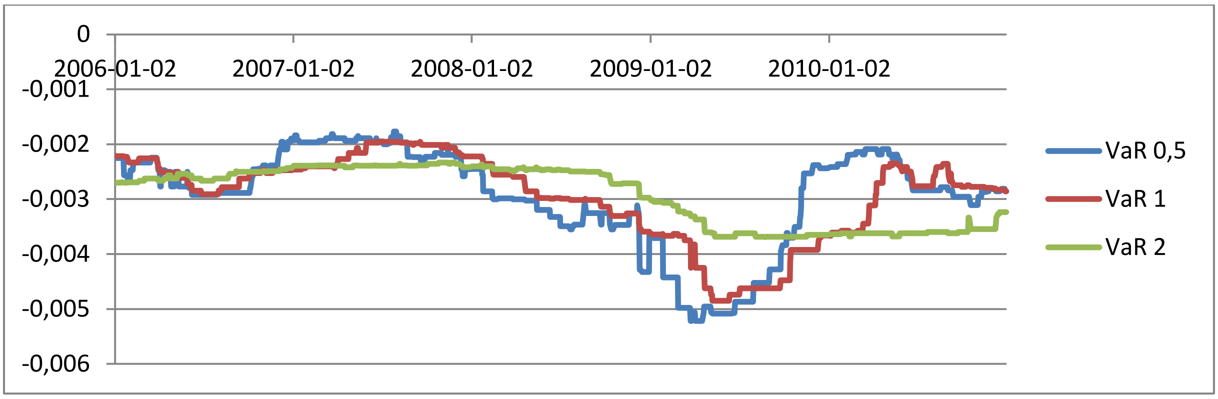

Figure 8: Volatility-weighted historical simulation at the 95%-level

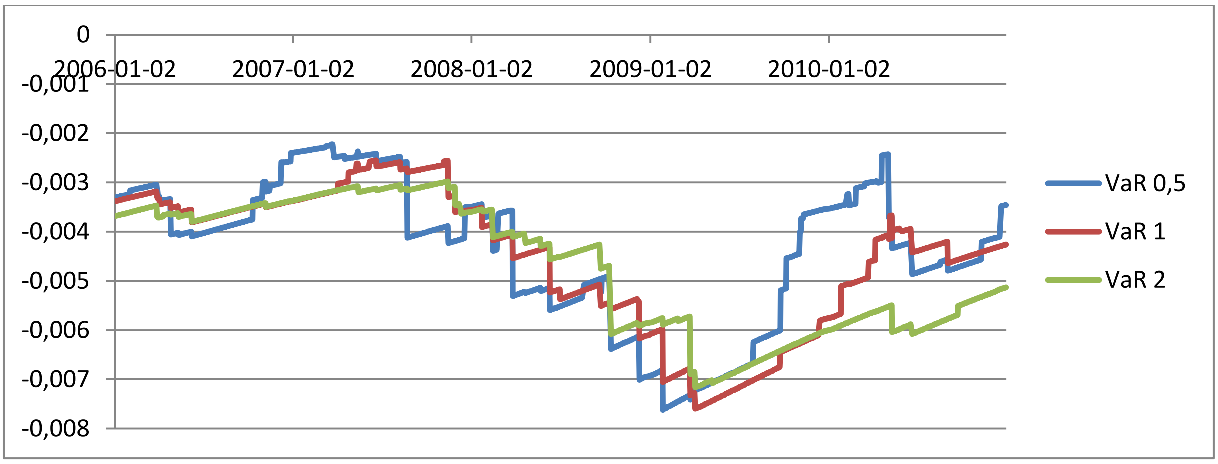

Figure 9: Age-weighted historical simulation at the 99%-level

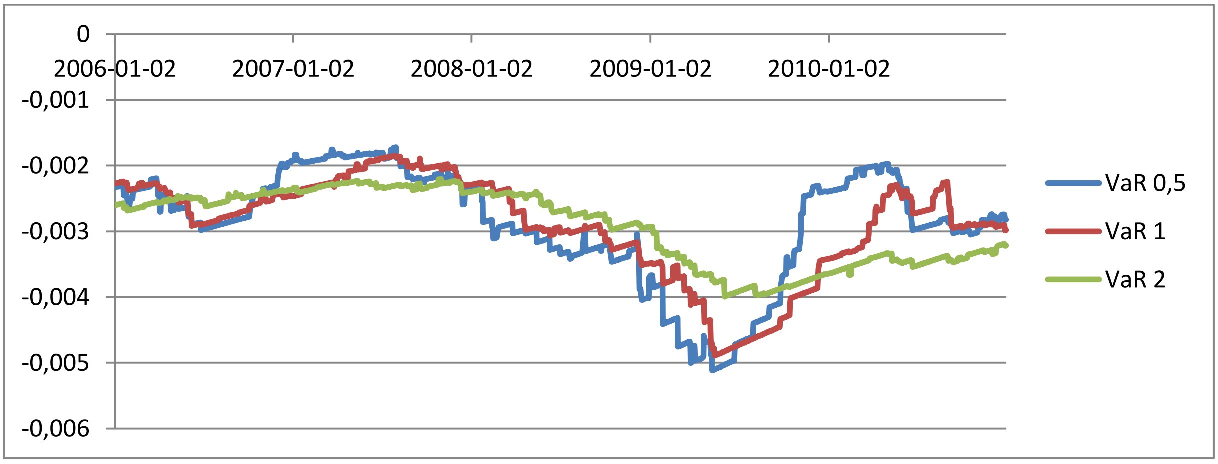

Figure 10: Age-weighted historical simulation at the 95%-level

8.3 Parametric estimations of VaR

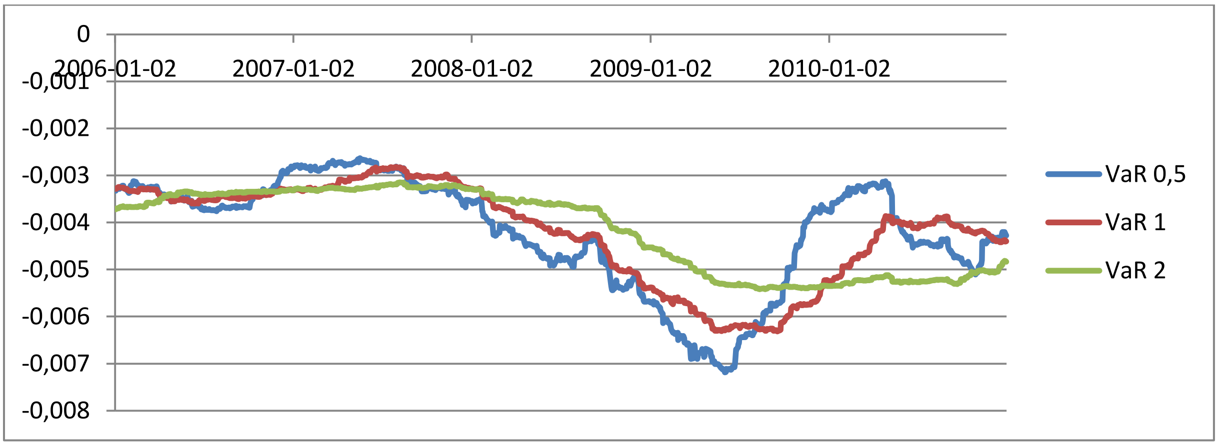

Figure 11: Normal distribution at the 99%-level

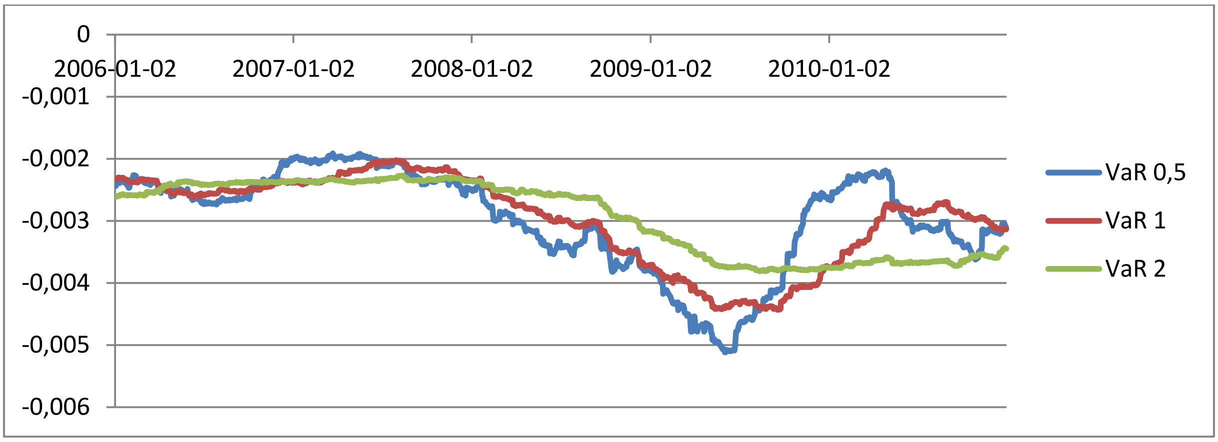

Figure 12: Normal distribution at the 95%-level

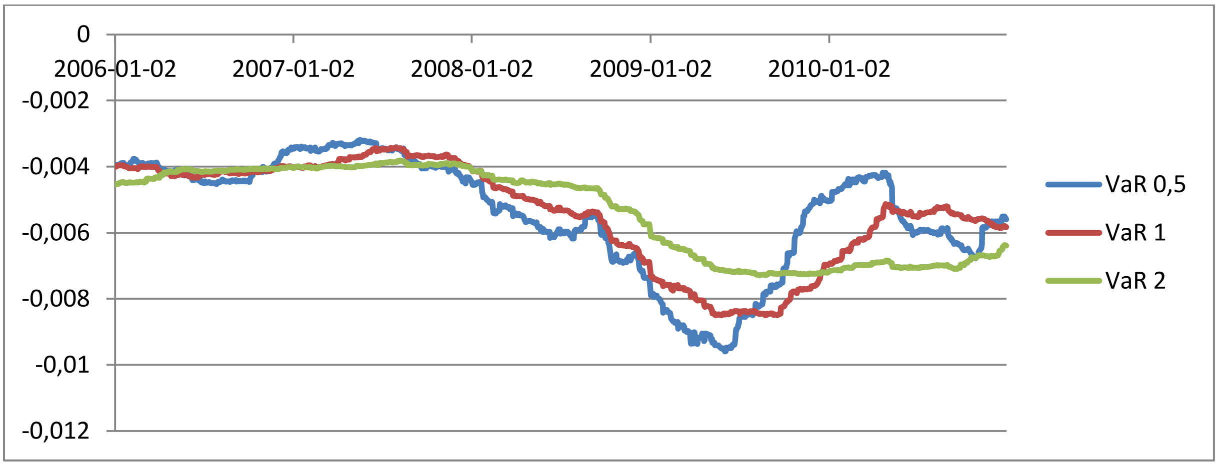

Figure 13: Student's t-distribution at the 99%-level

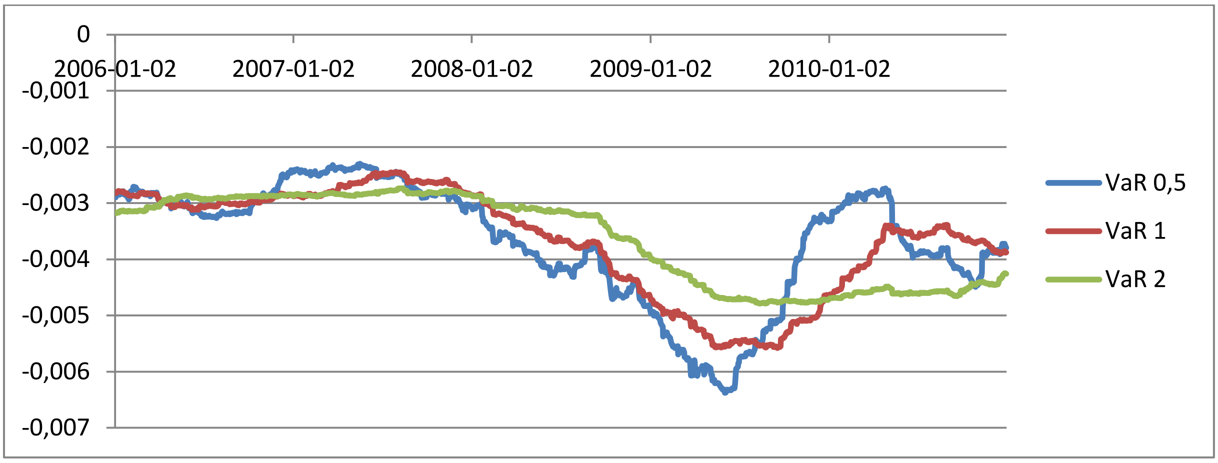

Figure 14: Student's t-distribution at the 95%-level

8.4 CAViaR estimations

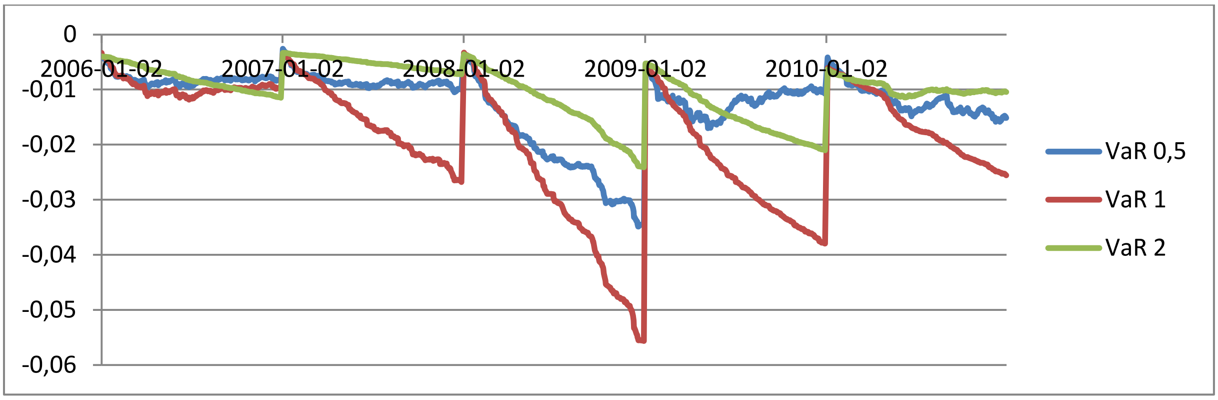

Figure 15: Asymmetric slope at the 99%-level

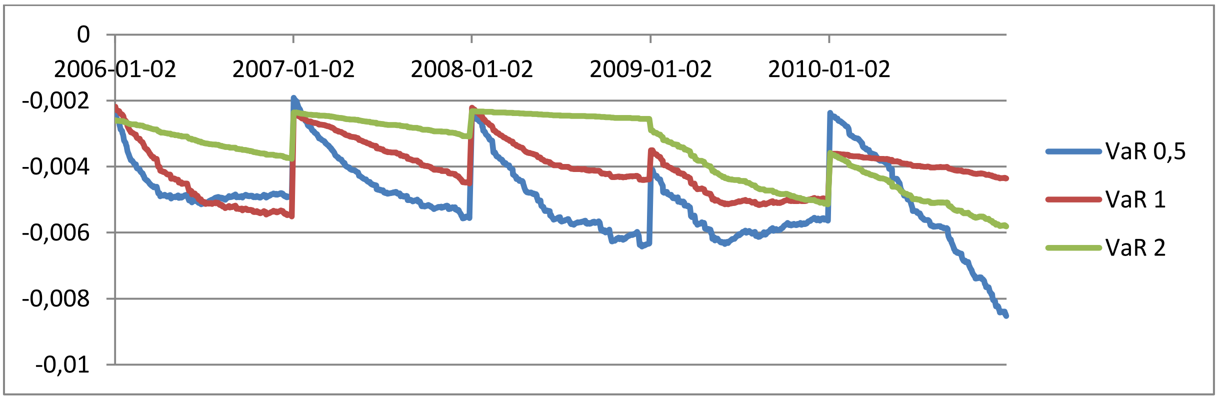

Figure 16: Asymmetric slope at the 95%-level

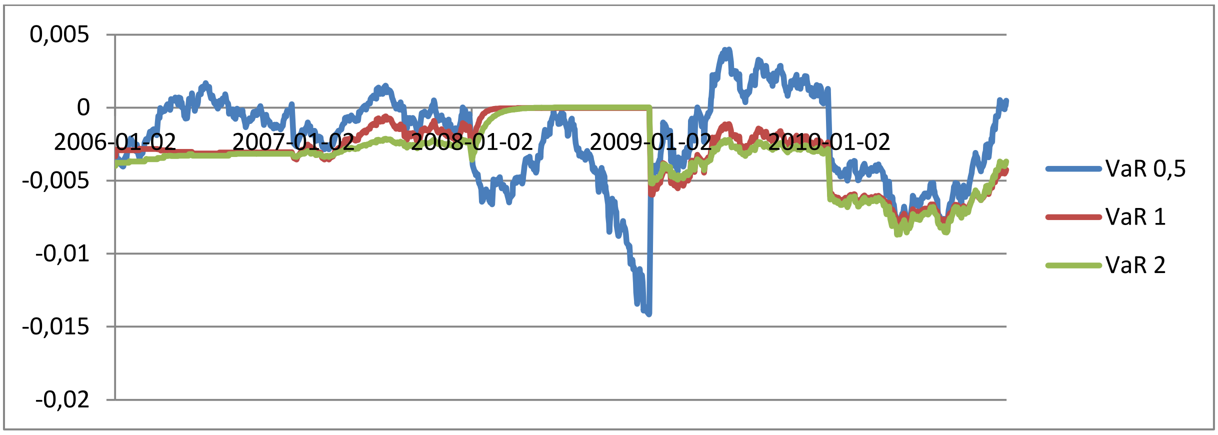

Figure 17: Symmetric absolute value at the 99%-level

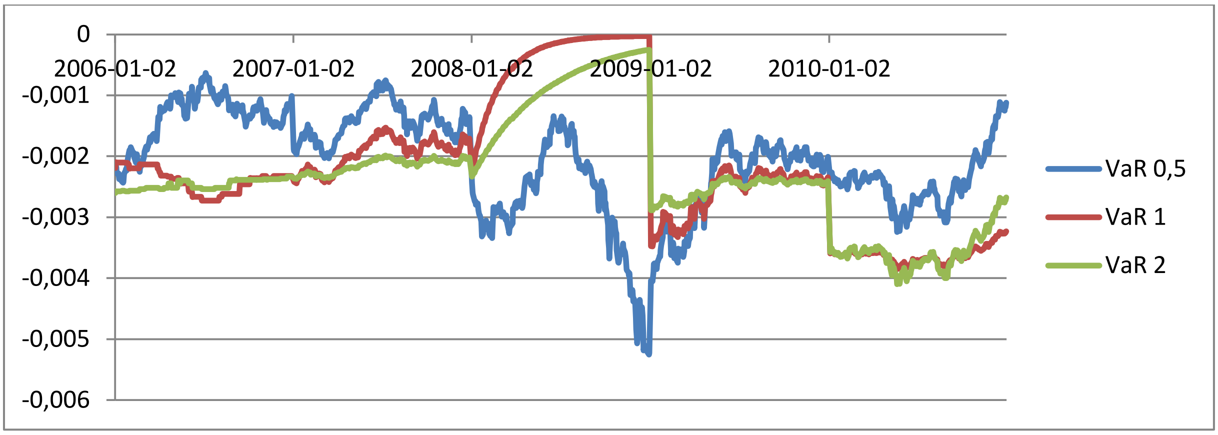

Figure 18: Symmetric absolute value at the 95%-level