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Hedging portfolio tail risk

An applied quantitative complement for fundamentally driven hedge funds

incorporating non-normal modeling and asset allocation optimization

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Chapter 1

Introduction

This chapter begins with a background and description of the problems leading up to the research conducted in the thesis. The purpose of the thesis will be defined, as well as its main contributions and demarcations.

1.1 Background

As financial investments by its nature are built on speculation and has a big part of uncertainty engraved in it, portfolio managers have always been concerned with the effects of their bets going wrong. Any portfolio manager knows that the market will not go favorably with ones bets/investments consistently over a long period of time. Portfolio managers also know there is a possibility of certain unfavorable events affecting their portfolio negatively. Events such as companies or countries defaulting on their loans or stock market crashes. The market drop of September 29, 2008 was a proof of just that. The Dow Jones Industrial Average plunged more than 7 percent and over 5 trillion USD of value was wiped out within a few hours (Guillén, 2009) . Although, no one can be certain of the severity and probability of these events, portfolio managers try to mitigigate their effect by diversifying their portfolios or buying insurance against the most fatal of events. That way, portfolio managers felt confident going into the 2007/2008 crisis, they could meet the future uncertainty well prepared. However, false assumptions of the future market behaviour and that classical hedging strategies would work as good as they have always done, led to the fact that portfolio managers went into the crisis virtually unprepared. Every financial bubble in history has been pre-seeded by a over confidence in the understanding of market behavior (Mandelbrot, 2004).

However, the protective measures portfolio mangers felt confident in showed inadequate. Although financial turbulence is not rare, they are assumed to be according to the most trusted and widespread models. Most financial breakdowns we have seen in the last decades should in fact never have happened according to those models. The probability of the September 29 plunge was one in a billion according to traditional financial risk models. Even if you would trade daily for 50,000,000 years such an event would not happen even once. Furthermore, on August 31, 1998 Dow Jones fell by 6.8 percent, because of a cash crunch in Russia, at the time the hottest emerging market in the world. Probability; one in 20 million. The plunge on August 31 was preceded by two other declines during the same month. Joint probability; one in 500 billion. A year earlier Dow Jones fell 7.7 percent in one day. Probability; one in 50 billion. The worst week during the dot-com bubble burst in July 2002. Probability, one in 4 trillion. On October 19, 1987 the worst trading day ever recorded the Dow Jones fell 29.2 percent. Probability, one in 10^{50}. Obviously, traditional market models are seriously flawed in terms of forecasting the frequencey of extreme events. Even worse, the assumed market dynamics in those events did not hold. Diversification of investments did not account for any protection as everything plummeted. Neither did buying insurance, since the companies issuing the insurance was the ones facing the biggest problems (Guillén, 2009). All this has resulted in more attention towards risk management, and rightly so, as two of the worst historically observed time periods have occurred since 2000 (see figure 1.1). Private investors, investment institutions and regulatory institutions are screaming for more modern and updated ways of conducting risk assessment. (Mandelbrot, 2004)

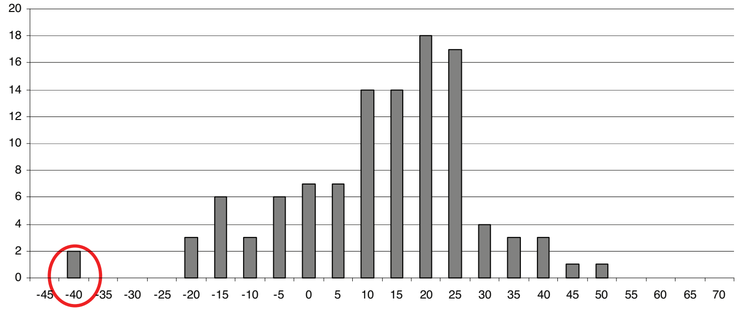

Figure 1.1: Distributions of annual returns (percent) on global equities 1900 - 2009. The two circled observations are the 2000/2002 and the 2008 downturns, both resulting in a 40 percent market decline. Source: Dimson, Marsh & Staunton (2002) Triumph of the Optimists, Morningstar.

Defining risk is hard and must in some sense answer to investors individual preferences. A common feature is that risk is expressed in some quantitative measure that requires a quantitative assumption of the probability of future markets outcomes. The aim of this thesis is to create a tool that can be used by, but not exclusively, large fundamental driven hedge funds. Fundamentally driven hedge funds are defined by their fundamentally driven investment philosophy, simply put; if you know the cause, you can forecast the event and manage that outcome (Mandelbrot, 2004). By large we mean hedge funds with thousands of different investments across a wide variety of markets and asset types. Henceforth, whenever hedge fund is mentioned it will refer to a large fundamentally driven hedge fund.

Making a quantitative guess about the future outcomes of a portfolio consisting of thousands of instruments, requires the creation of a model that includes characteristics for all instruments included. A fundamental approach to investment, focuses on investments that often are event driven and therefore can be hard to model. Modeling such portfolios can even lead to contra-productive behavior as investments are made based on the assumption of market abnormalities to revert. Nevertheless, modeling the joint distribution of the portfolio losses is required to get a comprehensive risk overview.

The investment philosophies of many fundamentally driven hedge funds requires a certain amount of patience, as some investment might go against the market from time to time despite being sound in the long run. However, as hedge funds need to provide quarterly and yearly reports to their clients, they must adapt to the investment goals of their clients in terms of time horizon. Therefore, hedge funds needs to be protected even for shorter time periods, despite the fact that their investments can be very long term. This was very apparent during the 2008/2009 crises when hedge funds' portfolios was not just hit hard, they also faced the threat of clients withdrawing their investments (Guillén, 2009). This emphasized the importance for hedge funds to worry about performing even during short periods of time, even though their investments might still be sound. Clients withdrawing their funds can potential cause significant problems, which is why hedge funds tries to mitigate this risk by extending the time between redemption periods and/or hedging individual investments or entire portfolios.

Hedge funds tend to be well aware of potential future bad events from a macro- and micro-economical perspective (i.e a China hard landing or Greece collapsing) and based on that knowledge invest in instruments going against those events. Although these investments and strategies are a core part of hedge funds operations, they tend be more alpha seeking and consequently more of an investment than a hedge. This thesis focuses on straight tail risk hedging, and how portfolio managers in general, and hedge funds in particular can benefit from applying strategies only aiming at mitigating their exposure to extreme events, meaning events classified as being part of the furthest left return percentiles.

In this thesis, a tool will be presented that has been created to help portfolio managers think of and take decisions related to mitigating tail risk. The tool is created to be used by hedge funds, however, it can be beneficial for any individual or organization that is facing tail risk hedging decisions. The tool will help portfolio managers look at their portfolio in a way that helps them evaluate whether or not certain hedging strategies or investments make sense from a tail risk perspective. In addition to providing portfolio managers support in making tail risk hedging decisions, the tool can identify the optimal tail hedge composition based on a pre-defined market view. This is not in any way an attempt at utilizing market mispricings, but to give the user support and serve as a compliment to other tail risk hedging approaches.

The presented tool is shown to provide fundamnetal support for tail risk hedging decisions. Applied to a fictitious hedge fund portfolio, the framework, evaluating more than 14,000 instruments, greatly outperformed a traditional tail risk hedging strategy.

1.2 Problem

Traditional risk assessment models typically originate from Markowitz's pioneering Modern Portfolio Theory (MPT) (Markowitz, 1952), where variance, or standard deviation, is the prime measure of risk. By only looking at the variance of a portfolio's return, the optimal investment strategy, or optimal asset allocation, for any given rate of return can easily be obtained by minimizing the variance for a set of assets. This simple portfolio theory has been utilized by a number of successful actors such as the Yale Endowment Fund (Yale Endowment Fund, 2009).

However, some of the major problems with conventional risk models, such as Markowitz's, stem from two correlated issues. First, the assumption of normality in asset returns, which constitutes a central pillar in MPT, has in recent years been broadly disregarded due to empirical evidence of the contrary. This can lead to significant underestimation of risk (Sheikh and Qiao, 2010) and originates from three distinct issues:

- Serial correlation: A key assumption in many traditional risk frameworks is that of independent and identically distributed asset returns. As Mandelbrot first showed in 1963, this does not hold true in many financial markets. As illustrated in figure 1.2, the empirical evidence of time varying volatility during the last five years is strong, and clear clusters of both high and low volatility can be identified. While both Engle (1982) and Bollerslev (1986) highlights the risks of disregarding auto-correlation, the return being correlated to previous returns, and heteroskedasticity, the presence of non-constant variance, few of the de-facto standard risk models account for these facts. Not considering serial correlation can lead to significant underestimation of portfolio risk (Sheikh and Qiao, 2010).

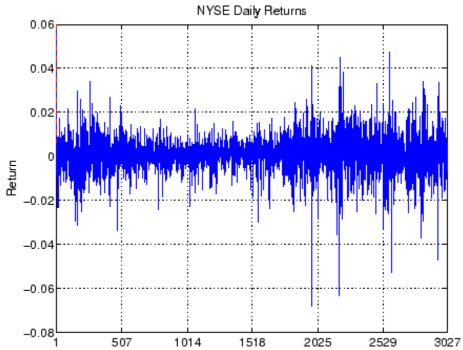

Figure 1.2: Volatility clustering for the NYSE Composite index.

- Fat left tails: Also first highlighted by Mandelbrot (1963), the major assumption behind asset models such as Markowitz's, that financial assets exhibits Gaussian distributed returns, does not always hold true. As illustrated in figure 1.3, the excess kurtosis, i.e. the variance being a result of fewer infrequent extreme events, leads to a more *peaked* return distribution than that of a Gaussian distribution. Additionally, many financial assets display significant skewness, for example, credit portfolios generally display a negative skewness which indicate a greater magnitude of extreme negative events due to the risk of defaults (Sheikh and Qiao, 2010), further diverging from the Gaussian distribution. Sticking with the assumption of Gaussian distributed returns, is one of the most severe reasons for underestimating portfolio risk (Heisman et al, 1998).

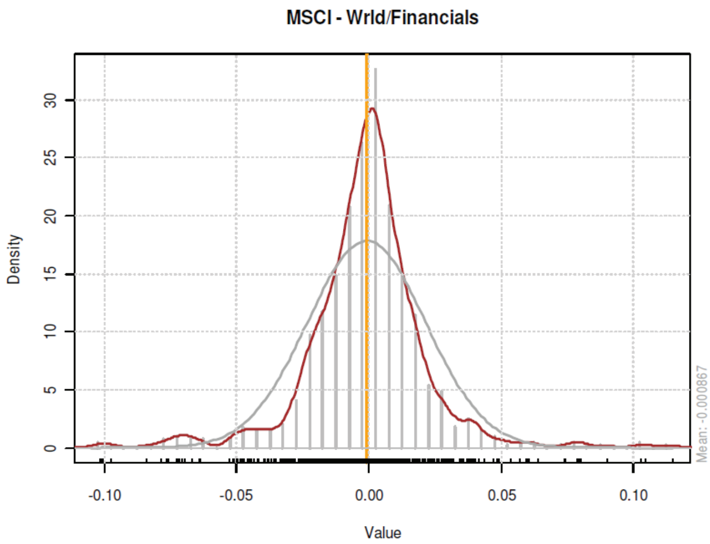

Figure 1.3: Empirically observed returns (red) for the MSCI World Financial index fitted using a normal distribution (grey) (Kjaer, 2010).

- Correlation breakdown: While traditional risk models based on mean-variance often assumes that return correlations between assets are linearly correlated, this does not always hold true. Assuming simple linear correlations, is the equivalent of modeling the joint distribution of asset returns as multivariate normally distributed. As Ang and Chen (2000) show, the correlation is significantly higher in times of distress, when the so called tail-dependency of asset returns can result from phenomena observed in behavioral economics such as bank runs, fire sales etc, totaly eradicating liquidity. This effect can lead to a substantial underestimation of risk as the benefits of diversification can be greatly exaggerated.

A second major weakness with conventional risk assessment relates to the fact that traditional risk measures such as standard deviation becomes inadequate (Sheikh and Qiao, 2010), this so for two reasons:

- First, the non normality issues described above leads to the fact that standard deviation as a symmetric risk measure fails to capture the dynamics of non-normal asset returns. Negative skewed return distributions are badly captured just because of their asymmetry, and return distributions with excess kurtosis hide the fact that a few infrequent extreme events are responsible for most of the variance. This means that two portfolios with the same standard deviation of their returns, can have significantly different risk profiles as the size of the worst potential losses can differ significantly. Using standard deviation may in fact inadvertently increase rather than decrease downside risk from a tail risk hedging perspective (Sheikh and Qiao, 2010).
  Further, the fact that tail risk hedging usually is conducted using non-linear instruments such as options, that has a far from normally distributed return structure, makes standard deviation inferior, as variance is what you seek in such an instrument.

- Secondly, as market agents treat losses and gains asymmetrically, as shown with the endowment effect (Kahneman, Knetsch and Thaler, 1990) , a risk measure as symmetrical as variance can result in a misguiding metric of risk as it does not distinguish between the effects of upward shocks from downside shocks. Additionally, this becomes increasingly important when looking at tail risk hedging, where the only focus is to protect the portfolio against major drops in value.

Furthermore, traditional risk assessment can, because of the inferior market assumption addressed above not produce accurate risk figures for periods of more than a few days. This, while the whole financial industry is in need of ways to assess risk for time periods up to a month or three months, even up to a year (Zumbach, 2007a).

Frequently used strategies mitigating tail risk has been to hedge individual positions, or specific risk factors the portfolio is particularly exposed to. Although this undoubtedly mitigates tail risk as intended, it tends to be cost ineffective and therefore suboptimal, since some risks might already be diversified away, and, hedging some risk factors might eliminate the need to hedge others (Bhansali, 2010). Consequently, when producing tail risk hedging strategies it is important to consider all positions' effect on each other as well as all evaluated hedging instruments' effect on the need for further hedging. Clearly, the complexity is substatial when contemplating a tail hedge for a portfolio consisting of thousands of instruments exposed to a tenfold of risk factors.

1.3 Purpose

The purpose of this thesis is to create a tool that can easily be used by portfolio managers in fundamentally driven hedge funds, to serve as a complement in the decision making behind the design of a hedging strategy aimed at mitigating portfolio tail risk based on risk reduction. The tool should be able to be used in the daily operations and must therefore be quick to use and easy to understand. It should at the same time be able to handle a broad set of asset classes including various types of hedging instruments as well as allowing for multiple lengths of forecasting horizons, where modeling must include contemporary knowledge about the characteristics of financial markets often excluded in classical risk assessment.

1.4 Contribution

The tool developed in this thesis[1] can be viewed as a valuable liaison between the fundamental and quantitative perspectives on portfolio risk. It includes contemporary quantitative models, better reflecting the severity, frequency and correlation of assets during extreme periods of stress. Extensive visualization capabilities, combined with numerous customization possibilities allows for fundamental adjustments of the quantitative framework, contributing to a transparency giving the portfolio manager the possibility to judge, as opposed to blindly trusting, the results. This facilitates the understanding and increased utility of a quantitative approach, enabling an enhanced knowledge about the market and the portfolio, as well as the effect of, or need for, risk interventions. In spite of the fact that contemporary financial theories has been around since the early 80s, their use as a complement to fundamental approaches has been limited. The presented tool provides an anticipated framework practically applicable for fundamentally driven portfolio managers.

1.5 Demarcations

The focus of this thesis is on hedging tail risk events, i.e. extreme events that inflict large drops in portfolio value that in nearly all cases can be categorized as systemic risk. Which implies distinctive market behaviour where all investors desires liquidity but nobody is willing to provide it. (Bhansali, 2010) Because of the focus on downside hedging after a certain portfolio loss, the thesis focuses only on hedging instruments and hedging strategies based on optionality; using vanilla options, options on futures, FX options and swaptions. As these instruments are liquid and flexible even in times of crisis, they are very useful for tail risk hedging (Bhansali,

[1] A brief description of the tool developed in this thesis, with notes on some of the practical implementation details, the different customization options available for a portfolio manager as well as a screenshot of the main application window is available in the appendix.

2010).

Further, with regard to the hedging instruments evaluated; only options with one year to maturity are included due to the yearly redemption system used by many hedge funds with fund withdrawals allowed once a year. Using hedging instruments with a single time-to-maturity also has the implication that the rolling of hedging instruments is excluded from the thesis.

Additionally, as the aim of this thesis is to develop a tool that is tailored towards use in hedge funds, and specifically fundamentally driven hedge funds, constriants assuring practicability has to be established. This has implications for the types of asset in the portfolio that is to be hedged, but also for the types and number of hedging instruments that the tool should be able to evaluate. Specifically, due to the difficulties to accurately model event-driven hedge fund strategies such as merger arbitrage, distressed debt etc., the evaluation only includes equity and credit assets in the unhedged portfolio. This, however, does not necessarily make the framework unsuitable for other asset classes or for investment vehicles other than hedge funds; the only modification required is the extension of the simulation framework in order to correctly model other types of assets.

Finally, in order to simplify the framework, we limit the hedging strategies evaluated by excluding the possibility of issuing hedging instruments such as options. The issuance of hedging instruments has several implications that further complicate the modeling as it requires constraints on capital requirements for short positions and it is deemed to be outside the scope of the thesis.

Chapter 2

Methodology

In this chapter, a description of the approach used creating the tool will be presented. Furthermore, the approach used to analyze the results and present them in a intuitive way for use by portfolio managers is discussed. A more comprehensive description with implementation details and theories corroborating the choices made are presented in chapter 3 through 5.

2.1 Approach

To determine the optimal hedging strategy, the first step is to find a suitable measure under which the optimality can be evaluated. The measure should be easy to compute, suitable for an optimization framework and reflect portfolio managers' risk preferences, especially in regards to protecting their portfolios in extreme events of stress. Portfolio managers must balance the potential of reducing risk with the cost of instruments and strategies involved, as well as the interaction between different investments and hedging strategies employed at the same time.

Evaluating portfolio tail risk and hedging strategies mitigating that risk requires a guess about the market's future behavior. This guess will henceforth be referred to as the market view. Naturally, a core part of the proposed framework is developing a market view that adequately reflects the future, but also and more importantly one that makes intuitive sense for portfolio managers. Creating an intuitive market view is essential since portfolio managers with a fundamental approach to investing live off their ability to predict the future development of markets, and to spot potential mispricings. Applying a non-intuitive market view is of little or no use for such portfolio managers and the most important function of the market view is therefore to provide an understanding of the severity and frequency, of infrequent but substantial negative events, as well as the dependence structure of instruments and market risk factors.

While developing the framework presented in the thesis, an essential feature has been that it should be able to be deployed as a stand alone tool; it should be able to be applied quickly and easily in different situations (i.e changes in time periods, market views or the portfolio and hedging instruments evaluated) and produce results directly comparable between changes in those situations. This has been important when testing the output on different historical periods of stress. But also to the understanding of the framework's sensitivity to changes in input assumptions, which is important to help portfolio managers understand and interpret the results.

2.2 Risk measure

As mentioned in section 1.2, asset returns can clearly not be assumed to be normally distributed, which brings about the problem of using standard deviation as risk measure. Another popular risk measure that better reflects investors' risk preference is *Value-at-Risk* (VaR), which is the upper percentile loss (i.e. the 95% VaR is the level of losses which should only be exceeded with a 5% probability). VaR has become popular because it is easy to use and it constitutes a good representation of large losses. VaR plays an important role and has become standard for major institutional investors as well as in financial regulation, for example when determining the capital cover ratio (Sheikh and Qiao, 2010). VaR is easy to compute when dealing with instruments were returns can be assumed to be normally distributed; however, for non-normal distributions, VaR has some undesirable mathematical properties such as lack of sub-additivity, i.e. VaR of a combination of instruments can potentially be greater than the sum of the individual instruments' VaR, which can discourage diversification. Furthermore, VaR is non-convex, which means that it has multiple local extrema and therefore makes it unsuitable to use in an optimization framework. When including instruments with optionality VaR will also treat two options with different strike prices (both below the VaR cutoff) the same, i.e. not fully accounting for the difference in downside risk reduction.

A risk measure closely related to VaR, *Conditional Value-at-Risk* (CVaR), has more attractive mathematical properties, although it has yet not gained popularity and become a standard risk measure in the finance industry. CVaR is defined as the weighted average of losses exceeding VaR, and is also known as *expected shortfall* or *expected tail loss* (Krokhmal, Palmquist and Uryasev, 2001). CVaR is proved to be a coherent risk measure, but more importantly it is convex, which is essential in an optimization framework since it guarantees that any minimum is also the global minimum (Pflug, 2000). In cases where the portfolio uncertainty is modeled by simulations, either historical or Monte Carlo, the optimization can be reduced to linear programming (Rockafeller and Uryasev, 2000). This allows optimization over all potential hedging instruments simultaneously, and is therefore suitable for very large portfolios with a large number of simulated scenarios, with relatively modest computational resources. (Uryasev,2000) (Rockafellar and Uryasev, 1999)

Furthermore, as tail risk hedging aims at providing protection in events of extreme stress, CVaR is more consistent with investor preferences (Sheikh and Qiao, 2010) . Also, as CVaR is an average rather than a maximum it will take into account the entire left tail which is preferable in a hedging framework (Kassam, Pangm, 2008). CVaR will hence be used as risk measure in this thesis and constitute the target for the optimization framework.

2.3 Process

The process under which the tool has been developed, as well as the structure for how the work is presented, is composed of three different parts: market view, optimization and analysis.

2.3.1 Market view

Fulfilling the purpose of the thesis, which is to create a tool tailored to the needs of a fundamentally driven hedge fund, the first step has been to create a portfolio that mimics a hedge fund as accurately as possible. This portfolio has been used throughout the thesis as a representation of the unhedged portfolio in the subsequent optimization and analysis steps. The portfolio is used as an example in order to demonstrate the analytical capabilities of the tool.

Secondly, a list of potential hedging instruments commonly used for tail risk hedging across different risk factors and markets has been compiled. The hedging instruments are evaluated as part of a suitable tail risk hedge and consists of instruments with different strike prices, different time-to-maturities with several important risk factors as underlying assets.

For both the fictitious portfolio and the list of potential hedging instruments, historical data were collected in order to construct the market view. Based on the gathered data, historical time series were used to calculate the price for all hedging instrument as well as all instruments included in the portfolio to obtain the current value of all components.

Based on the historical data, a Monte Carlo framework was employed in order to simulate independently drawn future outcomes for all assets.

To simulate future outcomes of the portfolio and hedging instruments, the probability of the future scenarios needs to be established. Important aspects to take into account establishing the probabilities, are the probability distribution of all hedging and portfolio instruments, the correlation between them, and how the future actually depends on the past. Contemporary knowledge about financial markets and methods to handle the issues mentioned in section 1.2 has been included in the simulation framework.

2.3.2 Optimization

The first step in the optimization part has been to qualitatively adjust the list of potential hedging instruments. This was done to ensure the practical applicability of the framework. All hedging instruments that did not constitute a practical option to constructing a tail risk hedge, based on a set of defined constraints, were removed.

Secondly, an optimization tool that is capable of optimizing using a large set of hedging instruments based on the chosen risk measure, CVaR, was created in order to find the unequivocal best composition among the available hedging instruments. The formulation of the optimization problem, reduced the problem to a classical asset allocation problem. For this, there are many available theories and methods, where one particularly suitable model has been applied.

To ensure an unequivocal best composition for the hedging instruments in terms of lowering CVaR and to ensure practicability, constraints have been introduced for the optimization framework. This led both to simplifying the model and reduce the computational requirements, but also to make it better match the preferences of a hedge fund.

2.3.3 Analysis

As the last thing investment professionals are interested in is a black box that yields a single answer, in terms of how much to invest and what to invest in, it is essential to establish a way of presenting the results that gives the portfolio manager support and works as a complement to the fundamental investment process. Because of this, the analysis in the thesis does not primarily focus on the actual results of applying the tool on the example portfolio, but is intended to present the analytical capabilities of the tool that helps the portfolio manager to analyze portfolio risk every time the framework is used.

A comprehensive set of graphs and tables are therefore presented, corresponding to the interests of a portfolio manager, with a description of the different options and parameters that can be specified depending on the purpose of the analysis. A qualitative discussion about the results is then presented, which aims at helping users of the tool to get a better understanding about how to think about the results and how to analyze them.

Chapter 3

Market view

This chapter will present a detailed description about how the market view is implemented. The models and methods used to develop the market view will be described, as will the Monte Carlo simulation framework details. Also, the chapter will describe the example portfolio, where a fictitious hedge fund is mimicked, as well as the hedging instruments included in the optimization.

3.1 Market forecasting

As mentioned in chapter 2, in order to successfully manage risk, a forecast of the future behavior of the market is required. Whether quantitative or fundamental, the only thing to base the forecast on is our knowledge of the past performance and dynamics of the market. Although commonly used by major international banks such as J.P. Morgan Chase and Société Générale to name a few, assuming future events will happen with the same frequency and severity as past observed events, is not always a good forecast. (Finger, 2006)

Although this assumption holds well in most cases, there are a few major flaws. First of all, historical returns are not at all independent, but highly dependent on the characteristics of previous events, which is why the probability of an event happening tomorrow is clearly not the same as it happening during the sample period. Secondly, and especially important to take into account when dealing with hedging tail risk, is that if we assume the future to mirror the past, no events will be worse than the ones observed during the sample period. Hence the quote, *"risk appears to be at its greatest when measures of it are at their lowest"* (Carney, 2009). This could potentially be solved by extending the sample period and including more historical data, and in that way capture more extreme events. However, this is both hard and can in some cases be fallacious.

First of all, there might not be relevant historical data for the different instruments as we go back in time. Secondly, for instruments with a time component, one needs to find similar instruments during the sample period with the same amount of time-to-maturity, or some other generic instrument resembling the actual position. This makes it complex, and the simplicity which is advantageous with the approach is lost. Thirdly, as was discussed in section 1.2, volatility changes over time and tends to cluster together. Consequently, including more history doesn't necessarily yield an improvement in forecasting the future (Finger, 2006).

A more robust approach at producing a market view would be to take into account everything we know about historical returns, fit that to a distribution, from which arbitrarily returns can be drawn through a Monte Carlo process (Finger, 2006). Furhter, achieving portfolio efficiency based on a more precise estimation of portfolio risk requires the incorporation of non-normal return distributions into the asset allocation and portfolio optimization process (Sheikh and Qiao, 2009). How the market view of the thesis is created through a Monte Carlo framework, incorporating non-normal return distributions, is discussed in section 3.5.

3.2 Portfolio

The example portfolio that is to be evaluated in the optimization framework has been modeled as a fictitious hedge fund portfolio. The fictitious portfolio have been designed to represent a common fundamentally driven hedge fund portfolio that focuses on equity and credit investments.

Asset type	Long (USD)	Short (USD)	Nbr of positions
Credit	150 000 000		143
Equity	100 000 000		50
Equity short		50 000 000	2
Total	250 000 000	50 000 000	195

Table 3.1: Holdings description for the fictitious hedge fund example portfolio.

The equity part of the portfolio has been constructed based on the composition of the Goldman Sachs VIP Index, an index based on 13F filings to the Securities Exchange Commission, designed to replicate the most common holdings by the largest hedge funds (Kostin et al, 2009). The credit part of the portfolio has been constructed after the Barclays Capital High-Yield ETF, as it is found to be a fair representation of the holdings of a hedge fund investing in high-yield credit.

The short equity part of the portfolio has been constructed to serve as a counterbalance to the portfolio's equity exposure and to bring down the net long ratio of the portfolio's equity side to better mimic a hedge fund. In this example, the short equity part is made up by equally sized positions in the S&P 500 and FTSE indices.

3.3 Hedging instruments

The hedging instruments to be evaluated as part of the optimal hedge has been chosen among four different asset types commonly used for the purpose of hedging tail risk for hedge funds. A summary of the more than 14 000 instruments included can be seen in table 3.3. These instruments have been selected without regard to any practical implementation issues. The practical constraints on the potential hedging instruments introduced for this purpose is explained in section 4.3.

3.4 Data

The data used to construct the simulation has been collected through Riskmetrics (RM), a company headquartered in New York, with an additional 19 offices world wide, employing over 1000 people and serving more than 2,300 institutions and 1,000 corporations with financial data. RM updates market data daily from over 85 markets and 4 million individual securities through primary data sources (exchanges) and third party data vendors (e.g Bloomberg, Reuters, etc).

Risk factor	Instrument type	Nbr
Long Gilt	Option on global long-term rates future	170
90D Sterling	Option on global short-term rates future	960
10 Year US	Option on global long-term rates future	1884
Euro bund	Option on global long-term rates future	270
3M Euribor	Option on global short-term rates future	944
WTI Crude Oil	Option on commodity future	1444
Natural Gas	Option on commodity future	1380
Euroyen	Option on global short-term rates future	306
Gold	Option on commodity future	558
Silver	Option on commodity future	1038
S&P Index	Equity option	1176
FTSE Index	Equity option	314
Euro STOXX 50 Index	Equity option	496
NASDAQ 100 Index	Equity option	752
Russell 2000 Index	Equity option	906
DAX Index	Equity option	560
NIKKEI 225 Index	Equity option	624
S&P ASX 200 Index	Equity option	308
HANG SENG Index	Equity option	352
Kospi 200 Index	Equity option	66
MSCI Brazil ETF	Equity option	300
Total		14 248

Table 3.3: Potential tail hedging instruments included in the evaluation

The library consist of approximately 10 years of historical data for over 750,000 time series including; equity, market indices, volatility surfaces, interest rate curves, break-even inflation curves, credit spreads, correlations, currencies and commodities, which all are used in pricing the assets included in this thesis (RiskMetrics, 2010).

3.4.1 Pricing

Comparing performance of securities over time and calculating the profit and loss for the over 4 million securities available in RM requires pricing functions converting the over 750 000 historical time series available in RM's library to individual prices for those securities. A list of the different types of securities and on what types of time series they are priced based on can be seen in table 3.5.

Instrument type	Time series
Credit	Risk free interest rate, generic interest rates based on rating, CDS spread curves and swap rates
Equity	Equity
Equity option	Equity, implied volatility surfaces and interest rates
Option on global long-term rates future	Interest rates, implied volatility surfaces, risk free interest rate, CDS spread curves and swap rates
Option on global short-term rates future	Implied volatility surfaces and interest rates
Option on commodity future	Commodity futures and interest rates

Table 3.5: Time series used to price different types of instruments.

In addition to the time series illustrated in table 3.5, foreign-exchange rates are used to price any security that is not directly valued in USD.

As can be seen in table 3.5, implied volatility is used to price most of the options included in this thesis. That implied volatility is treated as its own risk factor and simulated separately implies some very beneficial properties. The implied volatility surfaces are built based on several time series with specific delta and time to maturity. These time series are then interpolated, extrapolated and modified to construct the implied volatility surfaces. Modifications are made mainly to ensure no put/call parity violations. Because implied volatility encompass all necessary information for pricing options, calculating the price through Black & Scholes will therefore essentially result in the true price, which means that the correct price can be calculated for any option at any time. This facilitates historical comparisons and has enabled the stress tests that are shown in chapter 5.

3.5 Monte Carlo simulation

In section 3.5, three major aspects of non-normality in empirical return distributions was identified. Including these aspects creating a market view that coincide to what is known about financial returns, will be explained one after another in the following subsections.

3.5.1 Fat tails

As discussed in section 1.2, negative returns are observed in greater magnitude and with higher probability than implied by conventional market assumptions, especially the Gaussian distribution. However, assuming empirically observed returns as a predictor of future outcomes infers problems as discussed in the beginning of chapter 3. Therefore, it is essential when establishing a market view to assume a probability distribution (PDF) for the return residuals, and especially, the behavior of infrequent but substantial tail events (Zumbach, 2007b). As figure 3.1 shows, a single form of probability distribution can be enough to characterize all asset classes included in the thesis.

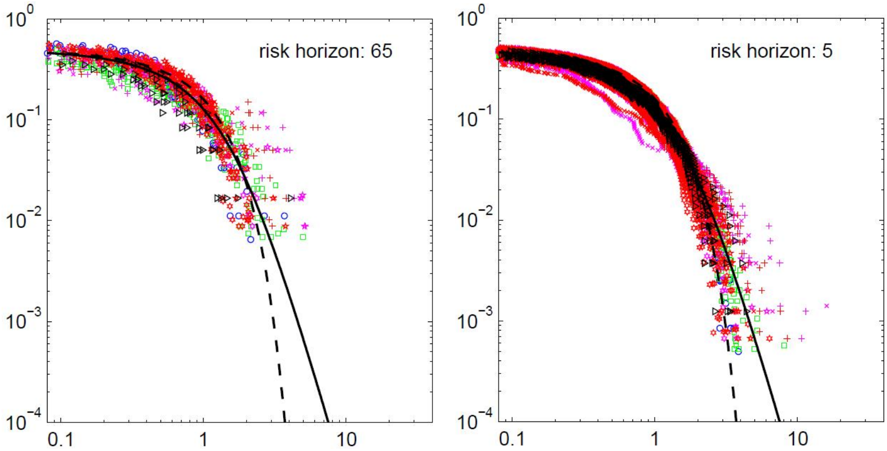

Figure 3.1: The cumulative probability distribution against the negative residuals for the left tail. The size of the negative normalized return residuals are shown on the x-axis, and the cumulative probability as it approaches zero is on the y-axis. Data are from 58 different time series for the G10 countries covering commodities, FX, stock indices and interest rates. The dashed line corresponds to a standard Gaussian distribution, and the solid line corresponds to a Student t-distribution.

As can be seen in figure 3.1, the standard Gaussian distribution constitutes a pretty good fit for all the plotted residuals at risk horizon 65 days. However, the solid line which corresponds to a Student t-distribution pose a slightly better fit, and a particularly better fit at the five day risk horizons, where it is obvious that the Gaussian distribution can be excluded as it misses the fat tails of the empirical data. However, with longer risk horizons the amount of data in the extreme tails diminishes, why the Gaussian distribution is almost as good an estimate. Furthermore, the Student t-distribution with five degrees of freedom and the Gaussian distribution converge and becomes in-differentiable for very large risk horizons (Zumbach, 2007a). Although the focus for this thesis has been on a risk horizon of three months (65 days), the tool must accurately be able to handle different risk horizons, why the Student t-distribution is chosen to constitute the probability distribution in this framework.

Determining the right amount of degrees of freedom for the Student t-distribution can be optimized as function over the length of the time horizon. However, there is no real benefit in using a more complex description of the PDF with regards to the time horizon. A Student t-distribution with five degrees of freedom provides a good fit for the entire PDF, tails included, for all risk horizons (Zumbach, 2007a). In practice, the number of degrees of freedom is independent of time horizon (Zumbach, 2007b).

3.5.2 Serial correlation

The issues of serial correlation is discussed in terms of volatility and return, and are presented separately.

3.5.2.1 Heteroskedasticity and volatility forecast

To draw random outcomes in a Monte Carlo simulation, the residuals are required to be approximately independent and identically distributed (MathWorks, 2010). However, this is clearly not the case as discussed in section 1.2. The dominant feature in financial time series is volatility serial correlation, and consequently to accomplish a good forecasts of future returns the single most important part is the forecasting model for the volatility. To capture and quantify the heteroskedasticity and volatility clustering, i.e. how historical volatility influence future volatility, historical returns are studied (Zumbach, 2007a).

The best volatility lag forecast is found to be given by a logarithmic decay of the lagged correlation of the historical returns (Zumbach, 2007a). Thus, the volatility process should ideally incorporate that characteristic. The volatility process used in this thesis is a *Long Memory Microscopic Linear ARCH* process. The process depends conveniently only on three parameters, which all makes intuitive sense.

The first two parameters are the end points of the span of historical returns the model takes into account. The reason why not all available data up until today are included, and the lower end point is set to a few days, is that giving strong weights to the most recent returns results in a lot of noise. The upper end point relates to the origin of heteroskedasticity which comes from the memory of traders and other market participants, which usually is a few months up to a year. The last parameter is the logarithmic decay factor which is estimated from the lagged correlation of the historical returns. Optimal overall values for the parameters are; a logarithmic decay factor of 6 years, and the sample span of historical data the model evaluates are between 4 and 512 days (Zumbach, 2007a).

There are two distinct improvements incorporated in this process, relative to traditional methods for lagged correlation, that induce very advantageous characteristics in terms of model simplicity and practical usability (Zumbach, 2007a).

The first improvement is the use of 10 days' realized volatility as volatility estimator instead of 1 day. Traditionally, under the normal assumption, volatility forecasts has been computed by using historical 1 day realized volatility, scaled up by $\sqrt{\triangle T}$ depending on time horizon. The lagged correlation of up to a month is proven to be a better volatility estimator than shorter horizons. Unfortunately, the number of independent samples diminishes with the length of the horizon. Studying the tradeoff between variance of the estimate and the sample size, 10 days has showed to be optimal among securities such as, commodities, foreign-exchange rates, stock indices, implied volatility and interest rates (Zumbach, 2007a).

The second improvement is the pooling of different time series. If the volatility memory is more than three months, the number of independent data points in 10 years of historical data is $\frac{120}{3} = 40$, which implies a large statistical error. To reduce this error, an average of the lagged correlations over independent time series is used. Studies of time series across securities such as commodities, foreign-exchange rates, stock indices, implied volatility and interest rates has shown that one decay factor can be used to successfully model all time series, which makes it possible to pool and take the average of the lagged correlation across a variety of time series (Zumbach, 2007a).

Not only has these improvements enabled the unadjusted methodology to be used across all security types included in the thesis, but studies has also shown that less than 0.5 percent of the performance of the volatility forecast can be gained by having the logarithmic decay factor as function of the risk horizon. The same is true for both the upper and the lower end points. Therefore the same method and the same parameters can be used for all security types and all risk horizons. Thanks to this consistency, long term horizons can also be dealt with, even though the amount of independent data is clearly insufficient to validate the forecasts using back testing, because of the few independent data points (Zumbach, 2007a).

3.5.2.2 Return correlation and drift

The lagged return correlations have to be zero, or close to zero, for frequently traded and liquid assets, according to the effective market hypothesis. However, as the asset returns are in average the risk free rate plus a risk premium, assets usually demonstrate a long term positive mean return. Forecasting returns is complex as lagged correlations often are at the noise level. To further complicate things, interest rates and implied volatility exhibits mean reverting characteristics, which implies negative short term correlation. Also, although short horizon empirical lagged return correlations are often below the statistical significance threshold, significant long term correlations are observed (Zumbach, 2007a).

Although lagged correlations can not be neglected, they are somewhat inaccurate and further research needs to be conducted to enable better forecasting models in this area. Nevertheless, a sum of a short term auto-regressive model based on two years of historical data, and a long term drift is used to forecast future returns. Fortunately, the inaccuracy of lagged return correlation is often negligible as the, by far, most dominant feature in financial time series is the volatility serial correlation (Zumbach, 2007a).

3.5.3 Tail correlation

As discussed in section 1.2, financial data is not characterized by linear correlations, and are instead showing joint extreme correlations. Traditional multivariate normal distributions based on linear correlations do not capture this property (Lindskog, 2000). In this thesis a ρ_{SSD} (Gnanadeskian and Kettenring, 1972) correlation estimator is used. The estimator is based on the assumption of an elliptical distribution. It is also very advantageous for large scale computations. (Gnanadeskian and Kettenring, 1972)

In section 3.4.1, implied volatility was showed to be used as one of the major risk factors in pricing options. And since all hedging instruments evaluated in this thesis are options of some kind, implied volatility is clearly essential for the accuracy of the proposed market view. Further, as this thesis aims at only minimizing the negative impact from the worst 5% scenarios, the tail correlation between implied volatility and the underlying instrument is of uttermost importance.

As can be see in figure 3.2, the correlation between implied volatility and the underlying asset is clearly not linear. The slope of the implied volatility becomes much steeper as the negative returns of the underlying asset becomes larger. This corresponds to the fact that when the returns are very negative, hedging becomes more expensive as implied volatility jumps.

3.5.4 Monte Carlo summary

Using the framework as described above, enables a very simple yet sophisticated tool for use in practical risk assessment. The method and its parameters can unadjusted be used for most security types over time

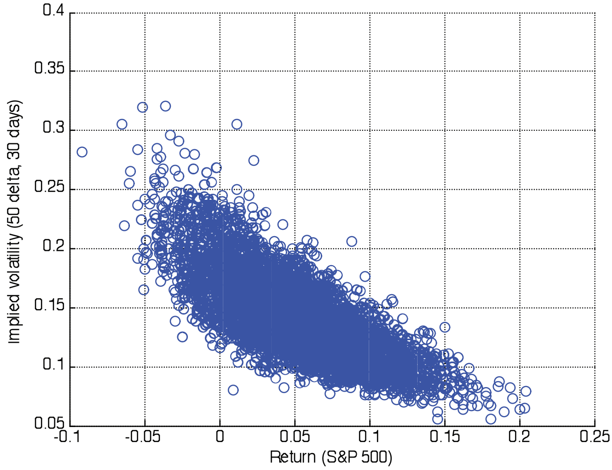

Figure 3.2: Simulated implied volatility plotted against simulated return of the S&P 500.

horizons between one day and up to more than one year without performance suffering. This simple and intuitive framework, taking into account much of what today is known about financial markets, is a much sought after element in the financial industry.

Chapter 4

Optimization

This chapter outlines the theoretical aspects and technical implementation details of the optimization model used in the tool developed for the thesis. The optimization model is described, as well as the constraints introduced in order to provide the practical setting for the framework and to customize it for use in a fundamentally driven hedge fund.

4.1 Risk and asset allocation

Knowledge of how risk changes with the composition of a portfolio is critical for effective risk management. Kassam and Pang (2008) suggests that the optimal hedging strategy can be identified by estimating the risk reduction per premium paid for individual hedging instruments, and then combining the best ones. However, the impact that the different instruments has on the aggregated risk profile when combined cannot be disregarded as it can lead to sub-optimal hedging solutions.

When using traditional parametric methods for calculating risk, changing the size of a single security in a portfolio will yield the trade risk profile (TRP) for that position (Mausser and Rosen, 1998). The TRP has one unique minimum, which can be seen as the optimal weight for that security in the portfolio. As the TRP captures the impact of small changes in the different securities through the risk measure gradient, the optimal hedge can be found when the incremental risk contribution of all hedging positions are zero. This is possible for example when using a delta-normal parametric VaR, which is coherent as demonstrated by Mausser and Rosen (1999).

Unfortunately, when using securities that cannot accurately be modeled using the Gaussian distribution, VaR has the undesirable mathematical property of not being coherent as discussed in section 2.2. Finding the optimal hedge must therefore be done by evaluating all possible combinations of hedging instruments simultaneously, which requires significant computational resources using a non-trivial number of positions for such an exhaustive search. In order to solve this problem, Uryasev and Rockafellar (1999) proposes an optimization model based on CVaR that is designed to handle the computational requirements associated with simultaneously adjusting the weights and evaluating all possible combinations of the hedging instruments.

4.2 Optimization model

The Uryasev (Uryasev and Rockafellar, 1999) approach to minimizing CVaR is a pure optimization model; defining an objective function that is designed to minimize CVaR with a confidence level β (e.g 95%), for a set of positions defined by the decision vector $\mathbf{x} = (x_1, ..., x_n)$ as the value for each of the n positions in the portfolio $\mathbf{x}$. By defining the loss function of the portfolio as $f(\mathbf{x}, \mathbf{y}_k) = \mathbf{x} - \mathbf{y}_k$, i.e. the dollar lost for the portfolio specified by $\mathbf{x}$, i.e. the difference between $\mathbf{x}$ and $\mathbf{y}_k$, where $k = 1 : q$. $\mathbf{y}_k$ is consequentely the realized value drawn from the Monte Carlo process for the total of the n positions in the portfolio $\mathbf{x}$. The probability for any of the realized values, $\mathbf{y}_k$, is consequently $\frac{1}{q}$ where q is the number of Monte Carlo simulations (e.g. 5000). Uryasev and Rockafellar defines the probability that the loss exceeds the threshold α:

$$\Psi(x,\alpha) = \frac{1}{q} \sum_{k=1}^{q} \mathcal{I}(f(\mathbf{x},\mathbf{y}_k))$$

where $\mathcal{I}(\chi)$ is the indicator function i.e. $\mathcal{I}(\chi) = \{ \begin{matrix} 1 & if & f(\mathbf{x},\mathbf{y}_k) \geq \alpha \\ 0 & & else \end{matrix}$. Given the probability of exceeding the threshold loss, Uryasev defines the VaR, $\alpha_\beta(\mathbf{x})$, and CVaR, $\phi_\beta(\mathbf{x})$, as:

$$\alpha_\beta(\mathbf{x}) = min\,\{\alpha \in \mathbb{R} : \Psi(\mathbf{x},\alpha) \leq 1-\beta\} \tag{4.1}$$

$$\phi_\beta(\mathbf{x}) = (1-\beta)^{-1} \times \frac{1}{q} \sum_{k=1}^{q} \mathcal{I}(f(\mathbf{x},\mathbf{y}_k)) \times f(\mathbf{x},\mathbf{y}_k) \tag{4.2}$$

The main contribution of the Uryasev model is that the optimization of CVaR for a portfolio can be reduced to the minimization of function 4.2 which can be written as:

$$F_\beta(\mathbf{x},\alpha) = \alpha + \frac{1}{N(1-\beta)} \sum_{k=1}^{q} max([f(\mathbf{x},\mathbf{y}_k) - \alpha]\,, 0) \tag{4.3}$$

where α in the optimal solution $\mathbf{x},\alpha$ of the problem is the portfolio VaR. This in turn means that, by minimizing the function $F_\beta(\mathbf{x},\alpha)$, we can find the optimal CVaR without first having to calculate VaR. This is can be done because $F_\beta(\mathbf{x},\alpha)$ is increasing from both directions when α is chosen so that either more or less than $N(1-\beta)$ scenarios for $f(\mathbf{x},\mathbf{y}_k)$ falls above α respectively. Furthermore, the function $F_\beta(\mathbf{x},\alpha)$ is shown to be convex in terms of both α and $\mathbf{x}$, which has the positive implication that the gradient of equation 4.3 can be numerically calculated and that the objective function therefore can be minimized using convex optimization run-times.

While the Uryasev model defines the losses and gains in terms of return, both on the portfolio and on the individual positions, we define them in absolute monetary values to allow for short investments while using the gross long value of the portfolio as capital base when calculating the return. Thus, the current values of the instruments is defined as the vector $\mathbf{x}$, and the Monte Carlo simulated values of the instruments at the risk horizon defined by the random vector $\mathbf{y}_k$.

This is in essence the core of the Uryasev model, that the CVaR of a portfolio can be optimized using the objective function $F_\beta(\mathbf{x}, \alpha)$, which significantly reduces the computational requirements for optimizing a portfolio consisting of a large number of instruments.

4.2.1 Hedging optimization

When applying the Uryasev model in this thesis, $\mathbf{x}$ and $\mathbf{y}_k$ in the loss function $f(\mathbf{x}, \mathbf{y}_k)$ consists of the core unhedged portfolio as well as all the hedging instruments evaluated. Since the unhedged portfolio is fixed and as the optimization relates to finding the best composition of the hedging instruments, only the weights (i.e the decision vector $\mathbf{x}$) for the hedging instruments are being varied, in search of the optimal hedging solution.

4.3 Constraints

In addition to the objective function, a number of constraints are constructed in order to customize the optimization model to the problem at hand. The following list of constraints is included in the model.

4.3.1 Hedging instruments

The initial list of potential hedging instruments to evaluate yielded 14.248 individual instruments as described above, but many of those are not suitable for building a tail risk hedge in an practical setting. To handle this, two sets of constraints are constructed to filter the input of the optimization function in order to better mimic the real world problem.

First, with regard to the purpose and the demarcations of the thesis, a one-year constraint is set on the time-to-maturity of the options to match the yearly redemption system:

$$0.9 < T_{ttm} < 1.1 \tag{4.4}$$

As stated, this is due to the goal of tail risk hedging to ensure the hedge fund's ability to pay its customers if needed, and to keep the customers from withdrawing their funds.

Secondly, to account for liquidity issues, to ensure that there will not be problems in sizing a hedge and to minimize transaction costs due to large spreads, a constraint is set on the delta of the evaluated hedging instruments. The hedging instruments are filtered based on delta:

$$0.3 < \Delta < 0.5 \tag{4.5}$$

where the values are chosen in order to best mimic the delta range of options that would be considered for a potential tail hedge in practice. Here, it's important to note that if there were no liquidity issues in the market, a lower delta range would be preferred. This is the case, as the delta value of an option often is seen as a proxy for the probability of the option ending up in the money (Reiss and Wystup, 2001). Thus, if a tail risk hedge were to be constructed from, for example put options, an appropriate delta range would be in the vicinity of the CVaR level being minimized (i.e. 5%), adjusted for the beta to that risk factor.

Combining the constraint on time-to-maturity and on delta, the initial list of over 14.000 instruments was filtered to a secondary list of 145 instruments, that is fed to the optimization framework.

4.3.2 Premium spent

The second area of constraints, is related to the risk budget allocated for tail risk hedging. As discussed above, the problem of finding the best tail hedge is formulated in this thesis such that considering we have a certain amount of funds to spend on hedging, how should it best be allocated? Here, the constraint is set so that the amount of premium spent on the hedge should be no more than one percent of the current market value of the portfolio:

Combined with the above constraint in time-to-maturity, and the exclusion of rolling hedging strategies, the premium spent will essentially equal the yearly risk budget for the portfolio.

4.3.3 Protection purchasing

The third set of constraints is one with the purpose of simplification. As stated in the demarcations, hedging strategies including the issuance of options are excluded due to the added complexity of capital requirements. Also, we would not like the optimization framework to exploit mispricings in our market view, but to focus on pure tail risk hedging. Thus, we introduce the constraint that we can only purchase hedging instruments for protection, not issue them:

$$x_i \geq 0 \tag{4.6}$$

That being said, we can still hedge long and short positions by buying call and put options. If issuance of hedging positions would be allowed, constraints on the size of each issued position (or the total for that matter) in terms of the notional amount could be employed. Another, more elaborate way of allowing the issuance of hedging instruments, would be to introduce constraints on the economic capital required to take a short position, potentially based on a sort of VaR measure for the position, such as those discussed in the Basel II regulations (see Bank for International Settlements, 2006).

4.3.4 Other constraints

In addition to minimizing the CVaR of the hedged portfolio given the constraints stated as above, other interesting options are available and worth mentioning. First, in the base implementation of the optimization framework in this thesis, the model only minimize CVaR, without regard to anything else than the constraints set in place as described above. However, one can easily imagine a situation where the portfolio risk can be significantly reduced, which is sought after, but to a very large cost in terms of a reduction in expected portfolio return. Secondly, the idea of having a fixed budget for tail risk hedging, and the constraint in premium spent for the hedge, might also not be the ideal way of approaching the problem depending on the conditions. Here, evaluating the risk reduction achieved with varying levels of premium spent can provide a better understanding of the risk-return performance of a tail hedge. More on this follows in section 5.4.

Chapter 5

Analysis

To emphasize the usability and the benefits with the presented tool, a practical example of how a portfolio manager would use it to analyze the result is presented in this chapter. Real hedging instruments, risk factors and stress test are included to exemplify how a portfolio manager realistically would think of them in order to establish a hedging strategy. A qualitative discussion around the output of the tool is presented. First, the risk of the unhedged portfolio is analyzed based on the Monte Carlo simulated scenarios. Secondly, the risk impact of the optimized hedge is compared to that of a standard hedge and the option of qualitatively adjusting the framework and its output is discussed.

5.1 Stress tests and risk factors

In addition to the Monte Carlo simulated scenarios used as a representation for the market view in this thesis, four separate stress tests are also included in the analysis. The stress tests are individual scenarios that have been constructed by examining historical data from stressful time periods due to certain events. In each of the individual stress tests, observed shocks on individual risk factors (such as the Dow Jones dropping 29% on Black Monday) have been used to calculate the effect of the stress test on the portfolio value as if it would happen today. The observed shocks are applied to the current levels of the risk factors, and the value of the portfolio is calculated as described in the pricing section using the factor model in order to estimate the total portfolio loss for each of the stress scenarios.

Using stress tests that actually have happened is found to be a good and intuitive way of evaluating portfolio risk and the effect of tail risk hedging as it is exactly the type of scenarios that the work in this thesis is designed to protect from (Hjort et al, 2010a). The stress tests are described in table 5.1.

Name	Description
Asian Crisis	Financial contagion of the entire Asian region in 1997.
Black Monday	International stock market crash of October 19, 1987.
Gulf War	US, Kuwait and Iraq war in the fall of 1990. Major implications on oil price, interest rates and international stock markets.
G8 Worst 1M	Worst 1M drop for each G8 primary equity index in 1997 - 2005

Table 5.1: Description of the four stress tests included in the analysis.

Additionally, a number of independent risk factors are included in the analysis and simulated in the same way as both the portfolio components and the individual hedging instruments used in the optimization. The risk factors are included in the analysis in order to provide an intuitive way of looking at different aspects of portfolio risk in relation to how certain stochastic variables develops within the assumed market view. For a fundamentally driven hedge fund, this is found to be especially important as it presents a way for the portfolio managers to get a better understanding of both portfolio risk, but also of the complex structure of the actual quantitative framework, in terms of important macro-economic variables that they are already comfortable thinking about.

A sample set of five risk factors is included in the analysis with three major equity indices and two major commodities; S&P 500, FTSE 100, Euro STOXX 50, NYMEX Gold, NYMEX Natural Gas and WTI Crude Oil. The sample set is included in order to exemplify how the risk factors can be used to improve the analysis, and is not suggested to be the best set as it is highly dependent on the purpose of the analysis and should be decided on a case by case basis.

5.2 Unhedged portfolio

Employing the simulation framework in order to estimate the risk of the unhedged portfolio yields a VaR and CVaR over a risk horizon of 3 months of 3.851% and 5.959% respectively. The expected return of the portfolio is estimated to be 1.053%. The portfolio's expected return, CVaR and VaR, and the losses related to the four stress tests are illustrated in figure 5.1 with the full return histogram of the unhedged portfolio.

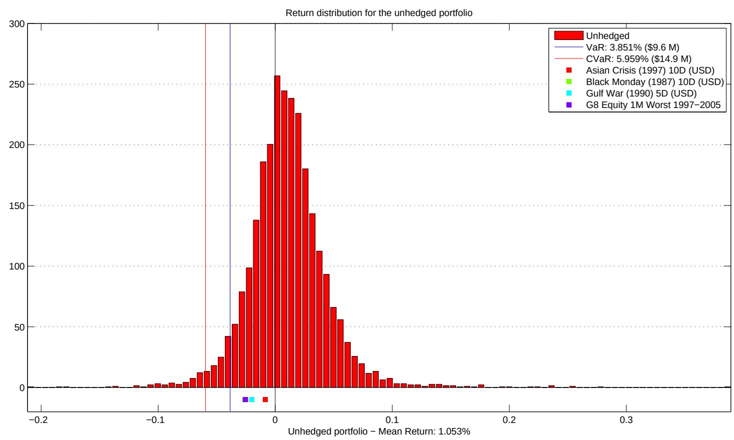

Figure 5.1: The return distribution for the unhedged portfolio (5,000 scenarios, 66 day risk horizon). The x-axis shows the total portfolio return while the y-axis shows the frequency represented by the histogram bin.

By breaking down each of the histogram bins in figure 5.1 by the individual portfolio parts, we can analyze the composition of the losses within each bin. As illustrated in figure 5.2, the breakdown of the bins are grouped by the three portfolio parts; *equity*, *equity short* and *credit*, in order to better provide an understanding about how each business area within the fictitious hedge fund contributes to the total portfolio risk, and which of those parts are driving the losses and gains respectively. The breakdown into three sub-portfolios is done to emphasize and exemplify benefits of the graph, not as a suggestion of how the breakdown should be done. This can be based on any available parameter, for example geographical market, credit rating, base currency, or even the portfolio manager responsible for the investment to name a few.

In figure 5.2, the return histogram of the unhedged portfolio from figure 5.1 is kept transparent in the background. Within each of the histogram bins, the mean return of all the scenarios in that bin is calculated, and each of the portfolio parts' share of that mean is visualized in color in the foreground. Here, it is important to note that while the magnitude of the portfolio losses appears to be significant in the flanks, the probability weighted losses related to those bins are small due to the small number of scenarios (often a single scenario in the tails) in those bin.

The breakdown provides a good visualization of how the diversification between the different portfolio parts perform, as is exemplified in both figure 5.2 and figure 5.3, where the strong (natural) inverse correlation between the *equity* and *equity short* parts is clearly shown for the example portfolio. As can also be seen in figure 5.2, the structure includes the breakdown of the four stress tests to the left for easy analysis of the stress tests provided.

Focusing on the tail risk of the portfolio, and looking at the tail of the return histogram for the unhedged portfolio, figure 5.3 illustrates the worst six percent of the simulated scenarios (300 scenarios). By visualizing the worst six percent of the scenarios, the 95% VaR and CVaR levels can be observed as the blue and red lines. Added to the figure is the number of scenarios that each bin represents (if less than ten). This lets the portfolio manager get a better understanding of the portfolio risk by

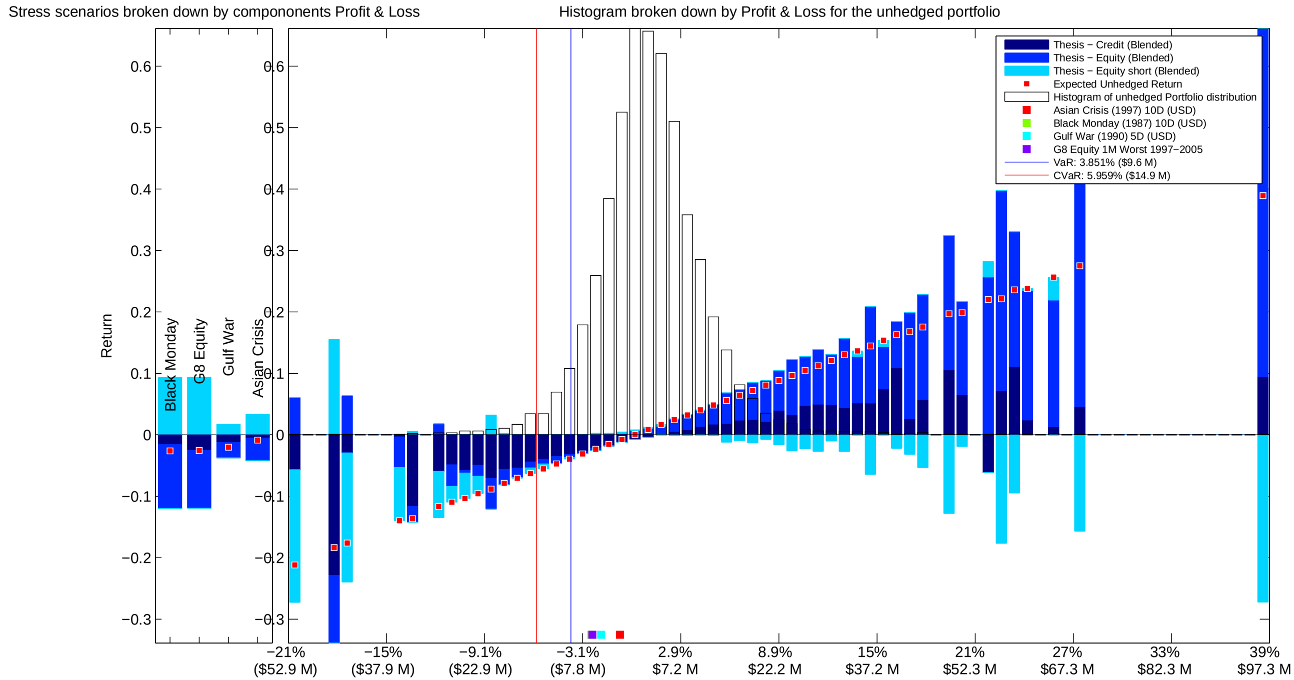

Figure 5.2: Breakdown of the return distribution for the unhedged portfolio (5,000 scenarios, 66 day risk horizon). Unhedged portfolio return is shown on both axes, where on the y-axis, the different portfolio parts' share of the unhedged return in each bin is visualized.

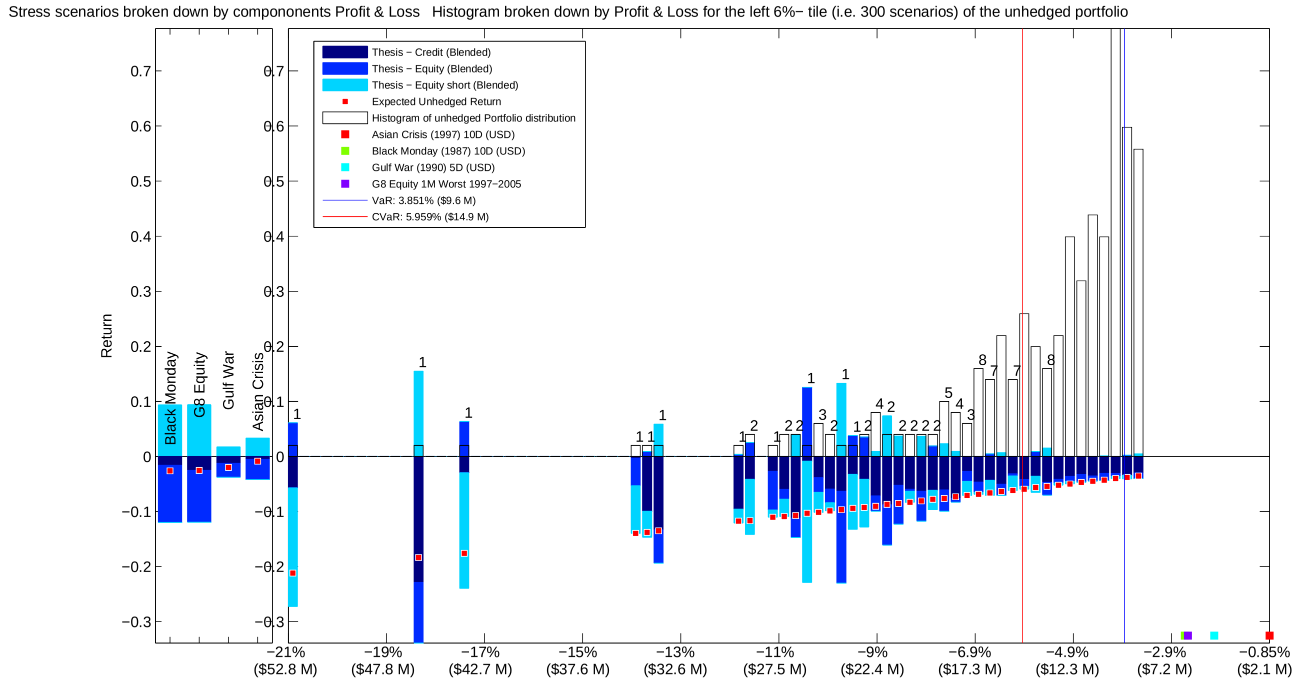

Figure 5.3: Breakdown (tail) of the return distribution for the unhedged portfolio (5000 scenarios, 66 day risk horizon). Unhedged portfolio return is showed on both axes, where on the y-axis, the different portfolio groups share of the unhedged return in each bin is visualized.

accounting for the fact that only a few scenarios can be responsible for a visually significant loss, directly relating the loss to the probability of it actually happening (number of scenarios in each bin, divided by the total number of simulated scenarios).

5.2.1 Risk factors

As discussed in section 5.1, independent risk factors can be used in the framework to favorably analyze different risk aspects in relation to major macro variables for example. In figure 5.4, this is illustrated by using the 30 worst simulated scenarios, where the S&P 500 Index lost the most in value, and analyzing what happened to the value of the different portfolio parts for those particular scenarios.

Here, for the example portfolio, it can be seen in figure 5.4 that the drop in the *equity* part of the portfolio is well mitigated by the *equity short* part. While this would be expected, the most interesting thing however, is the ability to discover the fact that in the scenarios where the S&P 500 Index exhibits the most significant drops, it is the credit part of the portfolio that is driving the largest losses. This is consistent with what we have seen in the last crisis, where companies defaulted on their payments resulting in huge downside losses.

Additionally, the four other risk factors (not being the risk factor visualized) are included in the bottom of figure 5.4. This is found to be a good way for a portfolio manager to visualize the structure of the quantitative framework; how different risk factors are modeled and the risk profile they generate. Furthermore, an understanding of how risk factors are correlated not just with the portfolio, but also between them, is essential when looking for ways to mitigate the effects of the worst drops in any risk factor a portfolio manager might be concerned with the most.

Given our market view, it is interesting to note that many of the worst case scenarios for the S&P 500 Index, is accompanied by a complete market melt-down in terms of all the other risk factors, where even gold,

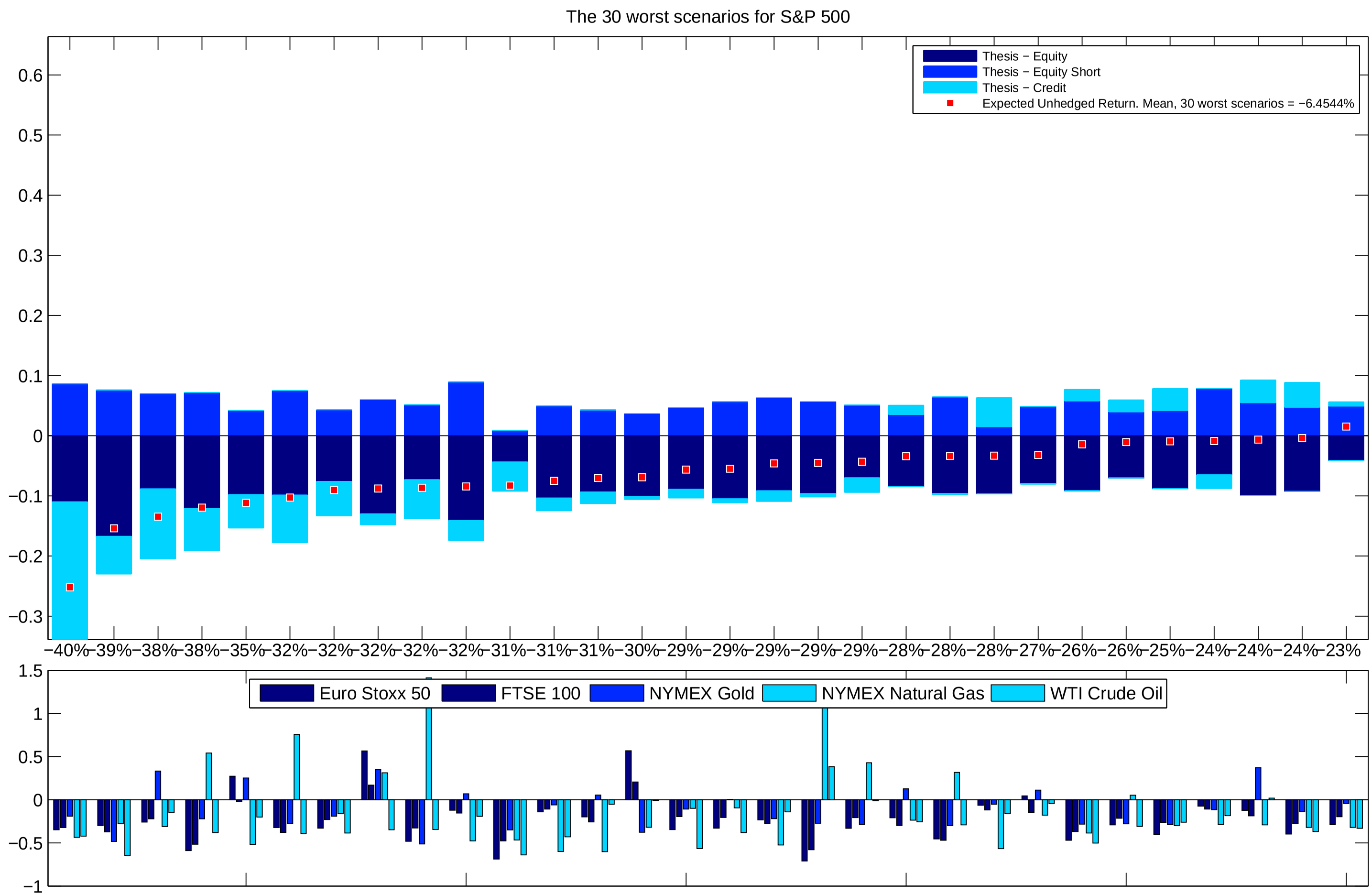

Figure 5.4: Worst case scenario analysis for the S&P 500 Index. The unhedged portfolio return is shown on the y-axis, where the different portfolio groups' share of the unhedged return in each scenario is visualized. The x-axis shows the drop for the actual risk factor (S&P 500). The bottom graph, shows the performance of five risk factors in each of the 30 worst scenarios for S&P 500.

which is generally though of being strongly counter-cyclical, is exhibiting large downward shocks correlated with the three major equity indices.

5.3 Hedged portfolio

Based on our assumed market view, we can use the framework in order to evaluate pre-defined hedging strategies, as well as optimized hedging strategies produced by the optimization part of the framework. The pre-defined hedge and the optimized hedge can be evaluated separately, but preferably evaluating the two approaches should be combined. For example by adjusting the optimized hedge based on fundamental aspects, re-importing it as a pre-defined hedge and evaluating the alterations based on its quantitative merits. Comparing and dissecting the two hedging strategies with regards to their effect on different parts of the tail, their performance during periods of stress and their correlating with specified risk factors, will undeniable lead to better decision making regarding tail hedging.

5.3.1 Pre-defined hedge

In the case of fundamentally driven hedge funds, the approaches and much of the work that goes into mitigating tail risk is based on deep and fundamental analysis of global macro-economical related risk factors such as consumer demand, international trade patterns, monetary policy, sovereign debt etc. This sort of analysis is often used to identify undervalued risk factors that can be used as hedges, and over-valued risk factors that needs to be hedged, and identifying factors or instruments negatively correlated with those risk factors (Bhansali, 2010).

By importing a hedging strategy that has been defined outside of the framework developed in this thesis, such as one that is the outcome of a fundamental risk analysis, we can use the framework to evaluate the strategy's quantitative merits based on our assumed market view. This can be thought of as an analytical complement, where the fundamental approach represents a more long term and strategical approach to risk management, while the quantitative framework developed in this thesis represents the tactical perspective, focused on managing short term vulnerabilities, as quantified over the risk horizon. For an interesting discussion about the strategical and tactical perspectives and integrated risk management, we refer to Kjaer (2010).

As an example, a standard hedge is constructed based on the simple idea of hedging tail risk in the equity markets, where put options are purchased at 75% at-the-money (ATM) in three major equity indices; S&P 500, FTSE 100 and Euro STOXX 50. The composition of the hedge and the impact that it has on the unhedged portfolio is illustrated in figure 5.5 while the simulated payoff of the hedging strategy is illustrated in figure 5.6 against the return distribution of the unhedged portfolio.

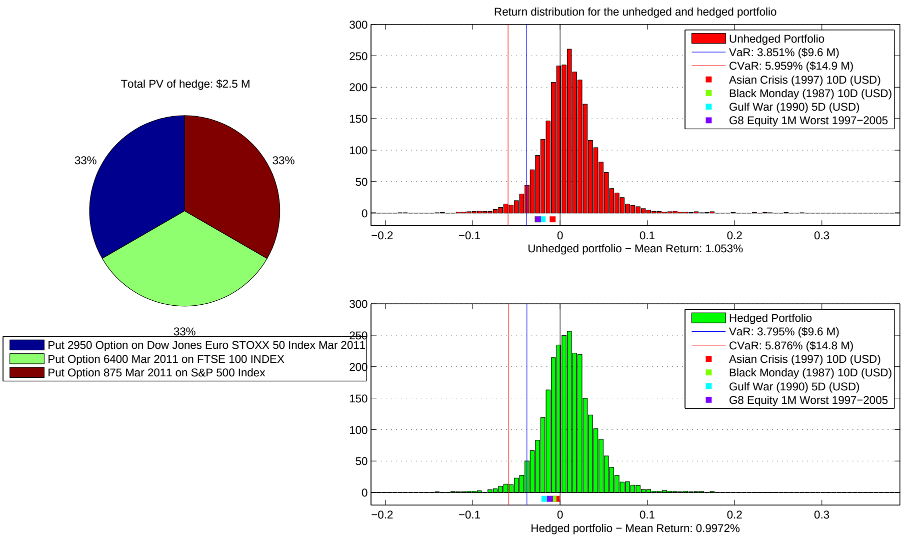

Figure 5.5: Standard hedge composition with its impact on the return distribution of the portfolio. The top and bottom histograms illustrates the unhedged and hedged portfolio respectively.

As can be seen in figure 5.6, the simulated payoff of the pre-defined hedge is clearly negatively correlated with the return of the unhedged

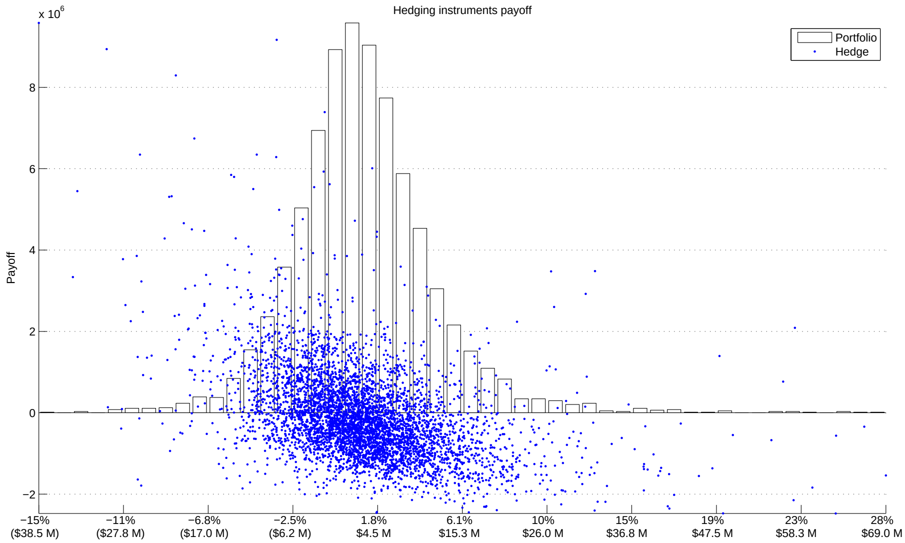

Figure 5.6: Simulated payoff for the pre-defined hedge against the return distribution of the unhedged portfolio. Each dot corresponds to the dollar return for the hedge in a single scenario.

portfolio, which would be expected from putting on a hedge based on 75% ATM put equity options, as both the portfolio equity and credit part are correlated to those three indices. However the absolute payoff of the hedge is not that large, in comparison to the cost of putting on the hedge (2.5 million USD premium), even in the out-most scenarios in the tail. Along the medium sized payoff scenarios in the center of the return histogram (which would be to the right of the VaR cut-off), this could imply that the hedge would be better constructed using further out-of-the-money options if tail risk is the focus.

If one or more risk factors are identified, based on fundamental analysis, to be a source of high risk due to identified overvaluation, it can be useful to analyze a hedge's performance based on a large drop in a specified risk factor, illustrated in figure 5.7. Here, a portfolio manager can analyze how the pre-defined hedge can mitigate risk in single risk factor, instead of on an aggregate portfolio level. This allows a user of the framework to target specific risk factors which are found to be important from a fundamental perspective, even though hedging in them might not reduce CVaR from the quantitative perspective, by adjusting the hedge composition and evaluating single risk factor issues. This is found to be especially useful for the tail risk problem outlined in this thesis, where a portfolio manager might want the portfolio to be partially hedged if a risk factor drops by 10%, but to be fully hedged when the same risk factor drops by 30%.

Additionally, if the pre-defined hedge that is to be analyzed is comprised by a number of separate hedging instruments, such as the three equity option positions in this example, it can be beneficially to explore the marginal risk impact of the different hedges on the portfolio. Figure 5.8 visualizes how the left tail moves to the right, while the VaR and CVaR decreases as the hedges are added one by one. Which scenarios and to what extent the different hedging instruments affect the tail is visualized. Most interesting to acknowledge however, is how each of the hedges performs conditioned that the other instruments already are in place. To accomplish this, the order in which the hedging instruments are marginally added needs to be changed. In figure 5.9 their order form figure 5.8 is reversed. Although, not extraordinarily, this example clearly shows how the risk reduction impact from the Euro STOXX 50 and FTSE 100 options clearly decreases conditioned that the S&P 500 option has already been put in place. This provides useful information for a portfolio manager, contemplating hedging several risk factors, in terms of the marginal cost of hedging.

The type of analysis presented in this subsection, supplying the framework developed with a pre-defined hedge, is found to be very beneficiary for a portfolio manager in order to get a quantitative complement to an existing fundamental approach, to evaluate a hedge. Given the assumed market view, the fundamentally constructed hedge is evaluated based on its quantitative merits and can be adjusted accordingly. If the hedge makes sense, from both a fundamental perspective, as well as from a quantitative perspective, chances are that the hedge will be effective

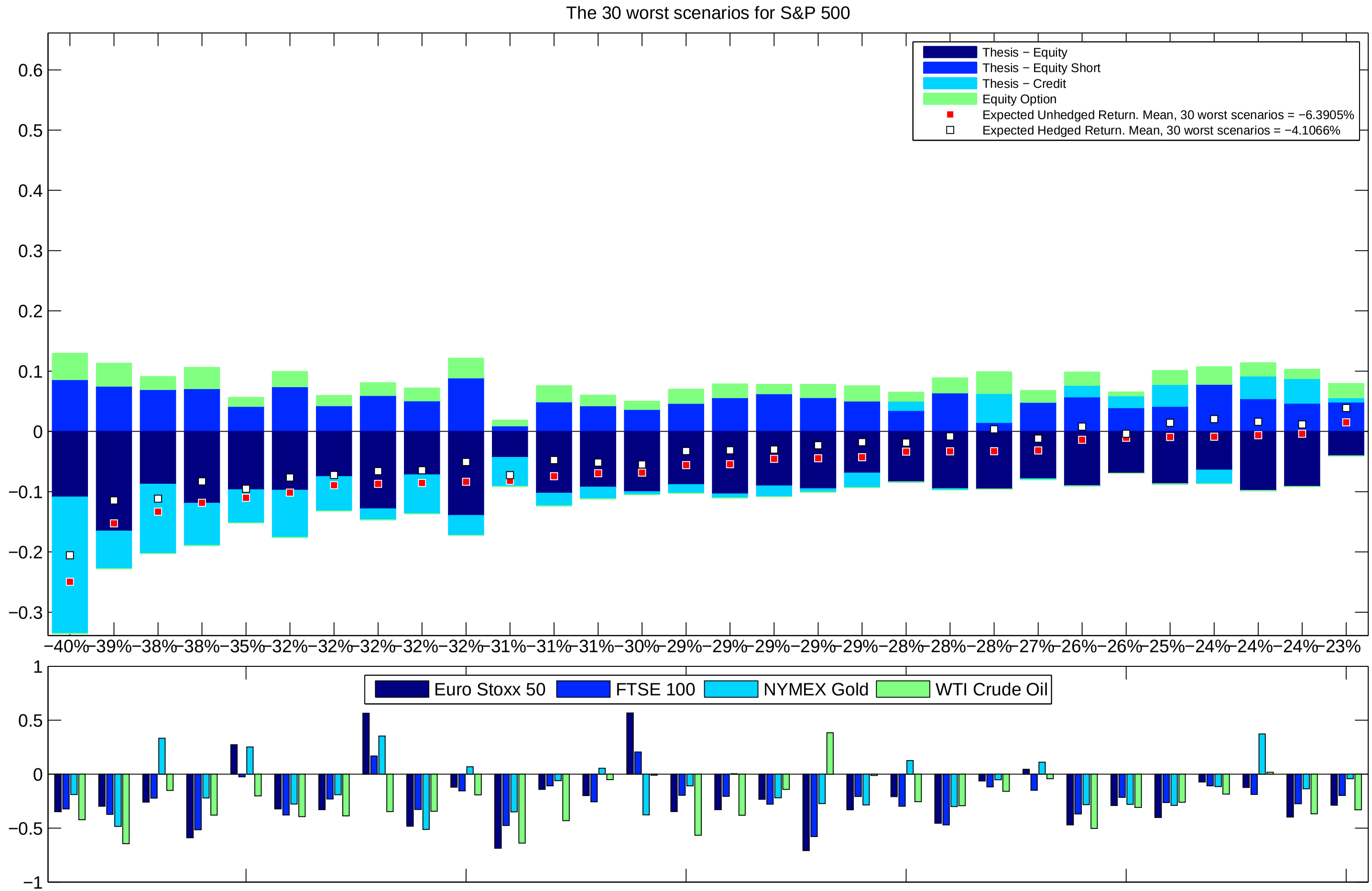

Figure 5.7: 30 worst scenarios for the S&P 500 Index for the pre-defined hedge. The hedged portfolio return is shown on the y-axis, where the different portfolio parts' shares of the hedged return in each scenario is visualized. The x-axis shows the drop for the risk factor (S&P 500). The bottom graph shows the performance of five risk factors in each of the 30 worst scenarios for S&P 500.

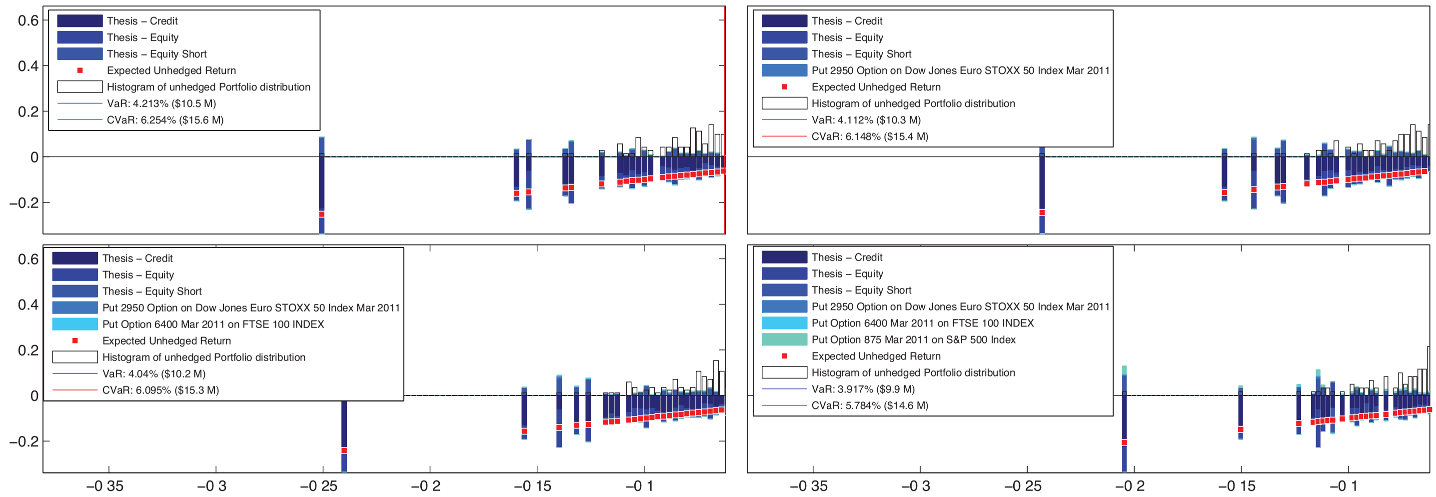

Figure 5.8: Marginal risk impact of the hedge components on the unhedged portfolio. Breakdown (tail) of the return distribution for the unhedged portfolio (5,000 scenarios, 66 day risk horizon). The unhedged portfolio return is shown on both axes, where on the y-axis, the different portfolio parts' share of the unhedged return in each bin is visualized.

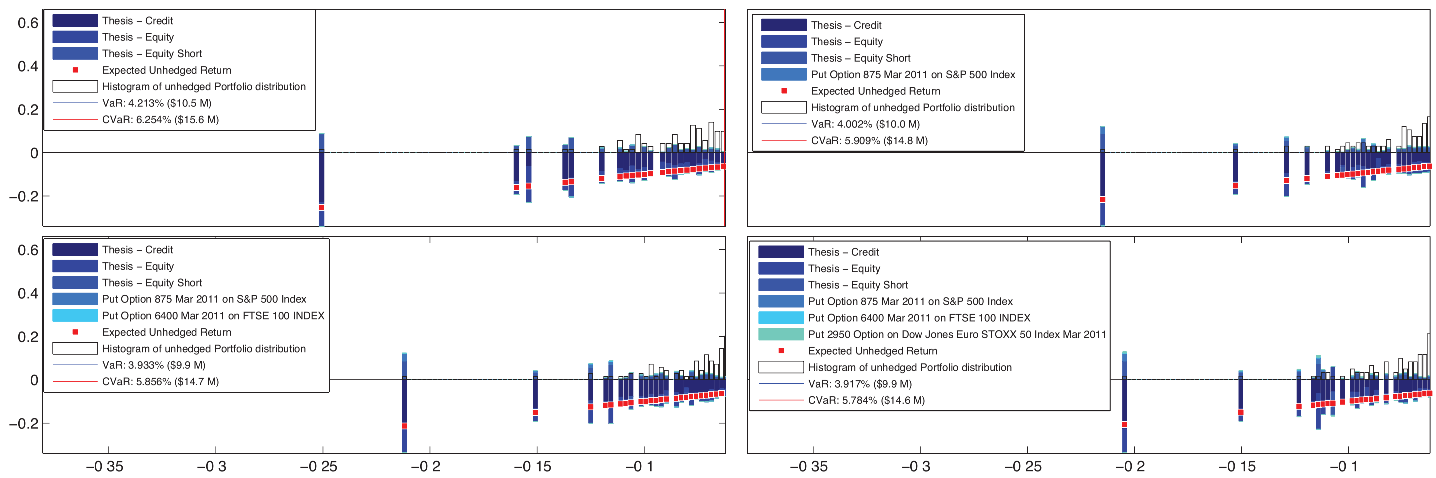

Figure 5.9: Marginal risk impact of the hedge components on the unhedged portfolio. Breakdown (tail) of the return distribution for the unhedged portfolio (5,000 scenarios, 66 day risk horizon). The unhedged portfolio return is shown on both axes, where on the y-axis, the different portfolio parts' share of the unhedged return in each bin is visualized.

both for mitigating long term macro-economical related risks, but also shorter term volatility driven vulnerabilities.

5.3.2 Optimized hedge

Applying the optimization framework to the example portfolio constructed in this thesis, an optimal hedging composition is identified based on the assumed market view. The composition of the optimized hedge and the impact it has on the unhedged portfolio is illustrated in figure 5.10 with a detailed description of the individual hedging instruments outlined in table 5.2.

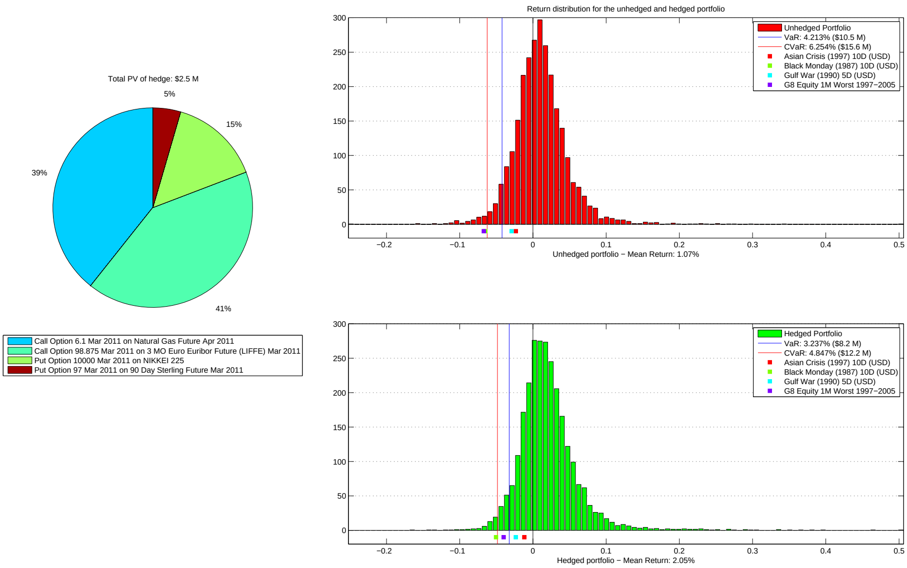

Figure 5.10: Optimized hedge composition with its impact on the return distribution of the portfolio.

As can be seen in figure 5.10, the optimized hedge effectively lowers CVaR from 6.254% (15.6 million USD) to 4.847% (12.2 million USD) by selecting options on equity indices, options on short term interest rate futures options on commodity futures. Comparing the simulated hedge payoff in figure 5.11 to that of the example pre-defined in figure 5.6, it

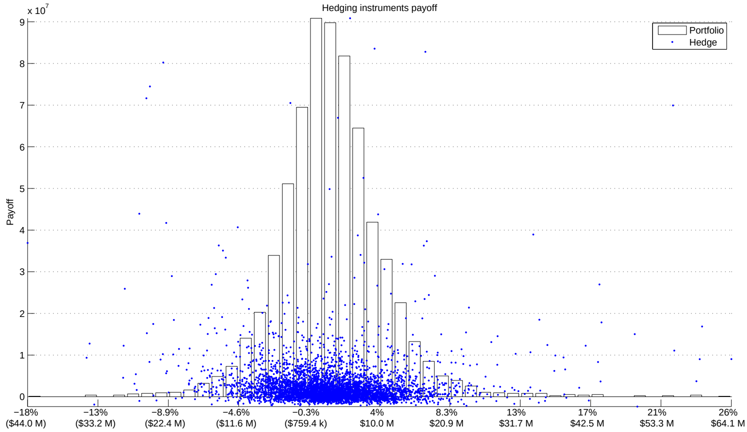

Figure 5.11: Optimized simulated hedge payoff against the return distribution of the unhedged portfolio.

Underlying	Type	Weight
Natural Gas future	Call option	39%
3MO Euribor future	Call option	41%
NIKKEI 225 Index	Put option	15%
90D Sterling future	Put option	5%

Table 5.2: Description of the optimal hedging strategy.

can be observed that while the payoff seems to be less correlated with the unhedged portfolio return, the payoff is an order of magnitude larger.

The same breakdown of the return histogram as illustrated in figure 5.2 can be performed for the optimized hedging strategy, and is shown in figure 5.12. Here, the payoff of the hedge from figure 5.11 is added to the return for the unhedged portfolio. This illustrates where the hedge is doing its job and where it's not, and it is clearly shown how the optimization framework effectively hedge the worst scenarios. Rearranging the scenarios based on the what happened after the optimized hedge is put in place, the same breakdown is visualized in figure 5.13. Comparing the two figures shows how the hedge brings in the left tail of the unhedged portfolio, eliminating the scenarios with the greatest losses and lowers the maximum loss over the risk horizon from 21% to 14%.

Unsurprisingly, the optimized hedge results in a greater reduction in CVaR when compared with the sample pre-defined hedge. The optimization framework will always yield the optimal hedging solution based on the assumed market view that is used to evaluate the pre-defined hedge. This makes comparing actual VaR, CVaR and returns to the pre-defined hedge somewhat irrelevant. However, what is more interesting is to look at how the optimal hedge is constructed, what the composition implies from a fundamental perspective, and whether or not it makes sense. In addition to the vindication of the chosen hedging instruments as put forward in section 1.5, figure 5.12 clearly shows how, with the use of options, a hedge mostly targeting the left tail can be created. This, as opposed to going short in any of the same risk factors, which has a clear cost in return as can be seen in figure 5.12 by looking at the *equity short* part.

A number of interesting things can be noted from the composition of the optimized hedging strategy in figure 5.10 and table 5.2.

First, the only equity based hedging position is one with put options on the NIKKEI 225 Index, relatively close to the money with a strike price at 92% of the current spot price. Hedging in Asia has as the time of the analysis been identified as favorable when looking at historical levels of implied volatility (Hjort et al, 2010b). Furthermore, not taking a position deeper out-of-the-money than 92% also implies a high portfolio beta towards the the Japanese market, where small movements of the NIKKEI 255 Index would correlate to very large movements in the unhedged portfolio value.

Secondly, the large position of call options on interest rate futures on the Euribor is the equivalent of a put option on the actual interest rate, which is also interesting. From a fundamental standpoint, this makes a lot of sense as the interest rates are very likely to come down in a time

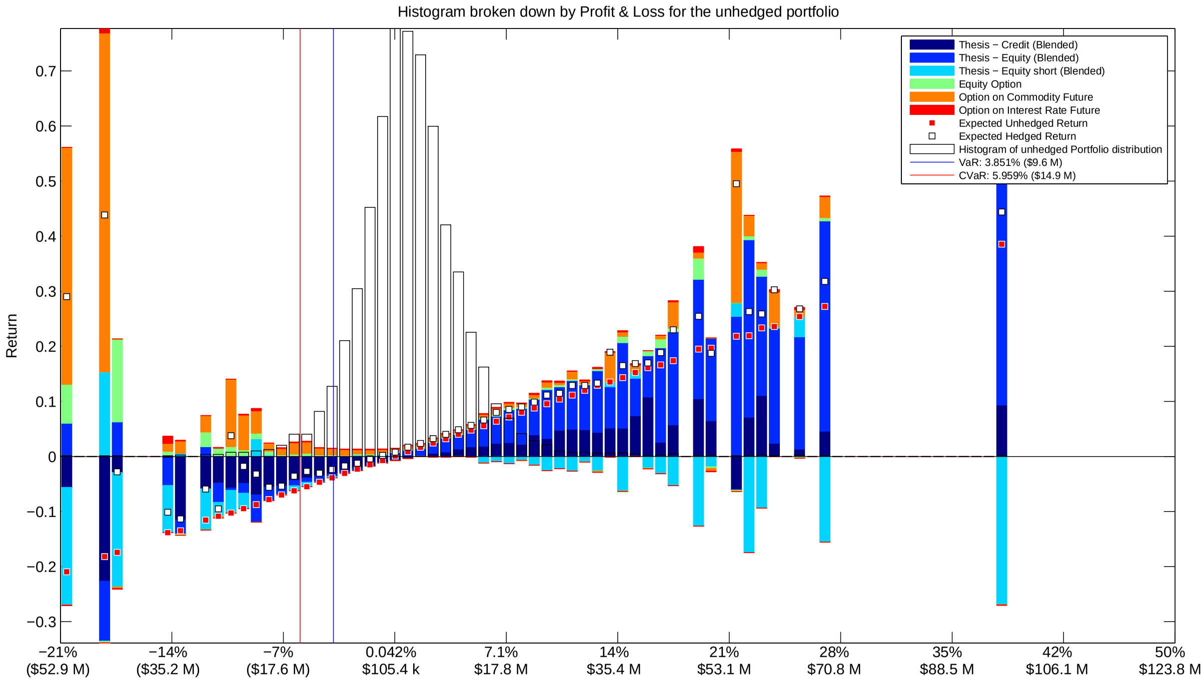

Figure 5.12: Breakdown of the return distribution for the unhedged portfolio (5,000 scenarios, 66 day risk horizon). Unhedged portfolio return is showed on both axes, where on the y-axis, the different portfolio groups', as well as the hedge's share of the unhedged return in each bin is visualized.

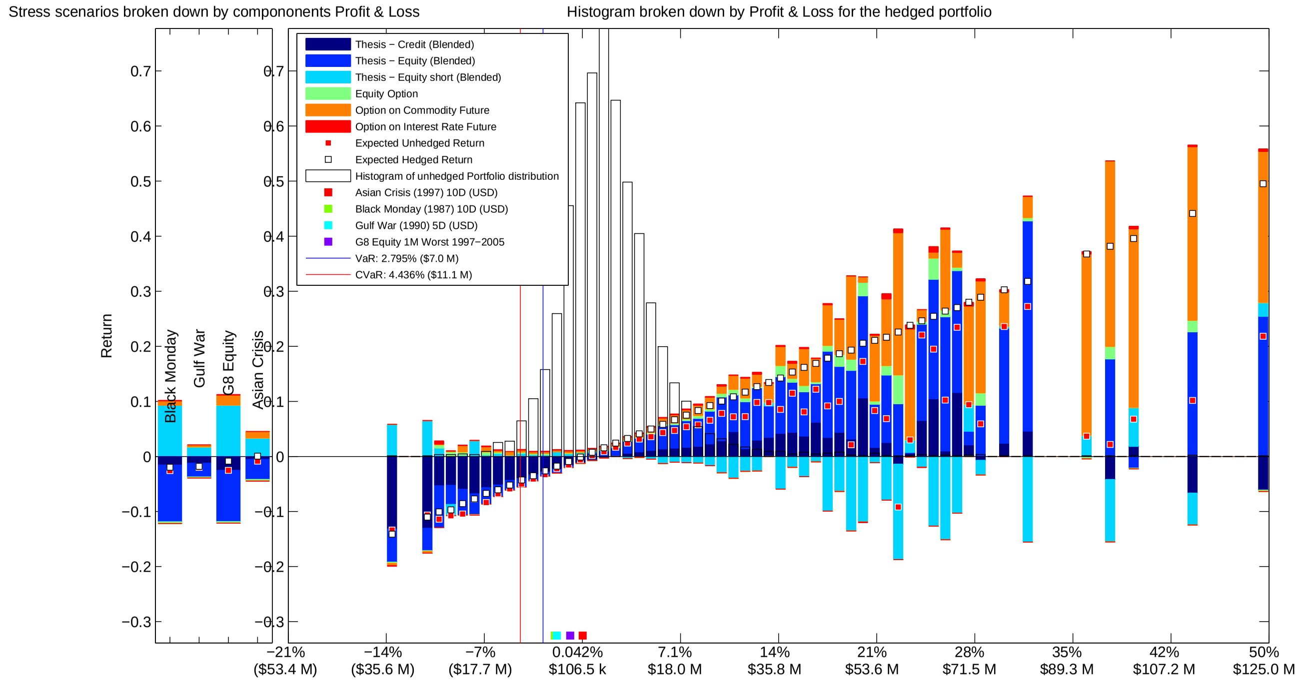

Figure 5.13: Breakdown of the return distribution for the hedged portfolio (5,000 scenarios, 66 day risk horizon). Hedged portfolio return is showed on both axes, where on the y-axis, the different portfolio groups', as well as the hedge's share of the hedged return in each bin is visualized.

of a large market downturn, as governments and central banks adjust monetary policies and prime lending rates in order to stimulate recovery.

While the first two positions make a lot of sense from a fundamental perspective, the other two positions are not quite that intuitive. For example, the call options on the natural gas futures might not be thought of as representing a good tail hedge based on the fundamental argument that the price for both natural gas, and other related energy commodities such as oil, would most likely decrease significantly as an effect of sharply dropping demand for energy in time of a global crisis.

The questionability of the natural gas position can be further emphasized by looking more closely at figure 5.12. Here, it can be seen that while the natural gas position has a significant payoff in the worst scenarios, the position affects the portfolio more or less across the board, raising the expected return of the portfolio considerably instead of just being focused on hedging tail risk. This, in conjunction with the fundamental argument against the natural gas position, could suggest that the market view fails to model the natural gas risk factor accurately. Reasons for this can potentially include either erroronous or noisy historical data, significal market abnormalities and events during the sampling period or some form of factor model breakdown.

5.3.2.1 Adjusting the optimized hedge

Given the optimized hedge composition and the conflicts of arguments between the quantitative framework and the fundamental perspective described above, it is found to be easy and intuitive to adjust the optimization inputs in order to balance the two perspectives. As an example, as the natural gas position makes little sense from the fundamental perspective, the portfolio manager has the option of either penalizing options on natural gas in the optimization framework, or completely removing the options from the input, in order to analyze how this effects the composition of optimal hedge and its hedging ability.

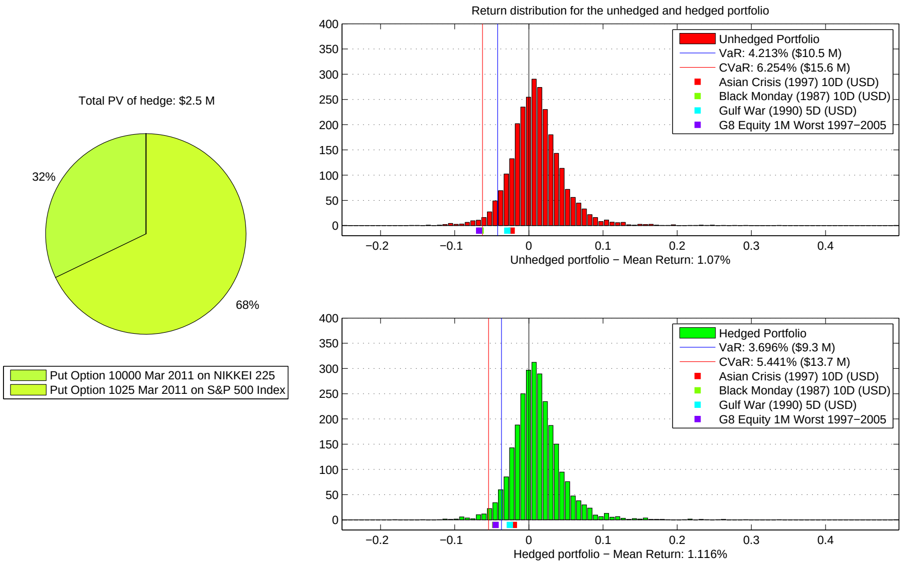

Figure 5.14: Adjusted optimized hedge composition and impact on the return histogram for the unhedged portfolio.

Completely removing all commodity options from the list of potential hedging instruments, and re-running the optimization, figure 5.14 illustrates the optimal hedge after adjusting the input. The result is a significantly higher CVaR, an increase from 4.847% to 5.441%, when compared to the unrestricted optimized hedge in figure 5.10.

Also, when comparing the breakdown of the impact the optimized hedge (figure 5.15) has on the return distribution of the unhedged portfolio, to that of the unadjusted optimized hedge (figure 5.12), it shows that the adjusted optimized hedge is much more focused on tail risk than that illustrated in figure 5.12, where the unadjusted optimized hedge lowers CVaR by raising the expected return to a greater extent. However, due to the issues discussed above about the irregularities with the natural gas position, it's likely that the large CVaR reduction of the unadjusted optimized hedge is not a realistic estimation.

Most interesting however, is the change in hedge composition when removing the ability to hedge with options on commodity futures. This

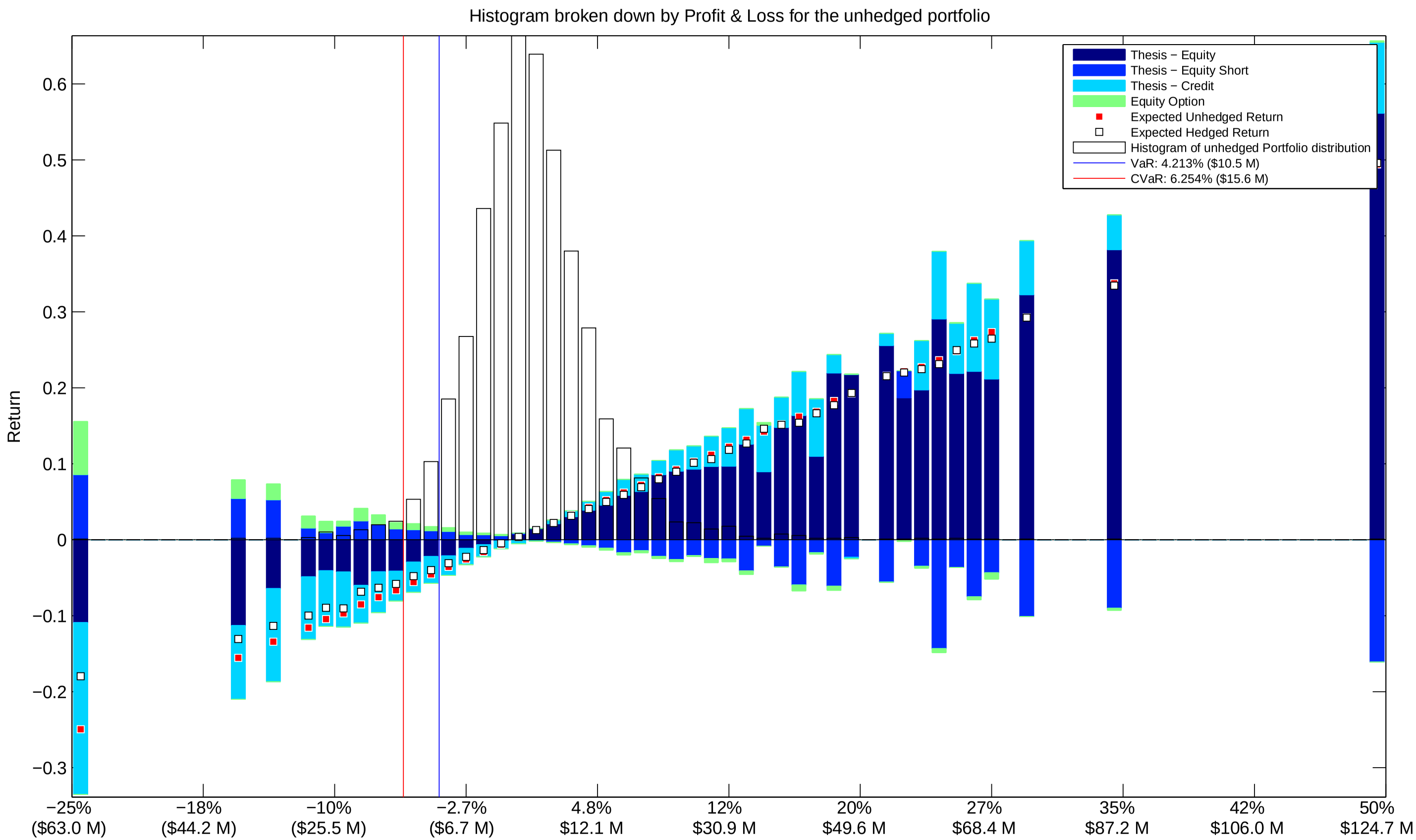

Figure 5.15: Breakdown of the return distribution for the unhedged portfolio (5,000 scenarios, 66 day risk horizon). Unhedged portfolio return is showed on both axes, where on the y-axis, the different portfolio groups', as well as the hedge's share of the unhedged return in each bin is visualized.

change not only removes the original position in natural gas, but also completely removes the two short-term rate positions, replacing them with a single short position on the S&P 500 Index. This highlights the importance of using an optimization framework which simultaneously optimizes over all possible hedging instruments and strike prices, as the different hedging positions has a strong impact on each other in a risk framework such as the one employed in this thesis, but also the importance of making sound fundamental adjustments as a user of the framework.

5.4 Efficient frontiers

As discussed in section 4.3.4, as well as highlighted in a follow-up paper by Uryasev et al (2001), the unconstrained optimization of a portfolio's CVaR might not always present the full picture. Portfolio managers are clearly concerned with more aspects than CVaR when contemplating a hedge. Although this framework finds the optimal composition for the hedge in terms of minimizing CVaR for the chosen amount of premium spent, portfolio managers might consider spending an other amount depending on the effect on CVaR reduction.

In the same way, similar to Markowitz's mean-variance framework, we can easily construct constraints on the required expected return for a portfolio in the CVaR optimization framework. This will produce the portfolio (or hedge) composition for any given level of expected return that has the lowest risk (Uryasev et al, 2001). By running the optimization framework iteratively, with varying constraints on expected portfolio return, an efficient frontier, famous from Markowitz's MPT model, can be plotted (see figure 5.4).

As can be deduced from the top plot in figure 5.4, any expected return can be acquired to the cost of increased risk. In the same way, the lower plot in the same figure shows that CVaR can be reduced to any level by spending more premium. As the top graph is ever increasing and the bottom is ever decreasing, there is no unequivocal answer to what is the best level of expected return or premium spent. It all depends on the individual portfolio manager's risk preferences. Clearly, the cost of lowering CVaR is different depending on were on the graphs in figure 5.16 you are located. Lowering CVaR from 4% to 3.5% clearly requires more premium than from 6% to 5.5%, in the same way lowering CVaR from 4.9% to 4.85% is significantly more expensive in terms of expected return than from 5.2% to 5.15%.

Beyond adjusting along the risk/return and risk/cost trade-off, further modifications of the methodology can be made to more accurately capture portfolio managers' risk preferences.

Changing the formulation of the optimization problem to finding the lowest cost for a specified reduction in CVaR, in terms of premium spent and expected return sacrificed, will essentially yield the same answer (Uryasev et al, 2001). Investors can that way satisfy requirements they might have on CVaR for capital coverage ratios or towards clients. Changing the formulation so that CVaR becomes a constraint instead of the objective function also enables a more customized controlling of the tail risk. One could potentially shape the left tail arbitrarily by having different constraints on CVaR 99%, CVaR 95%, CVaR 90% etc.

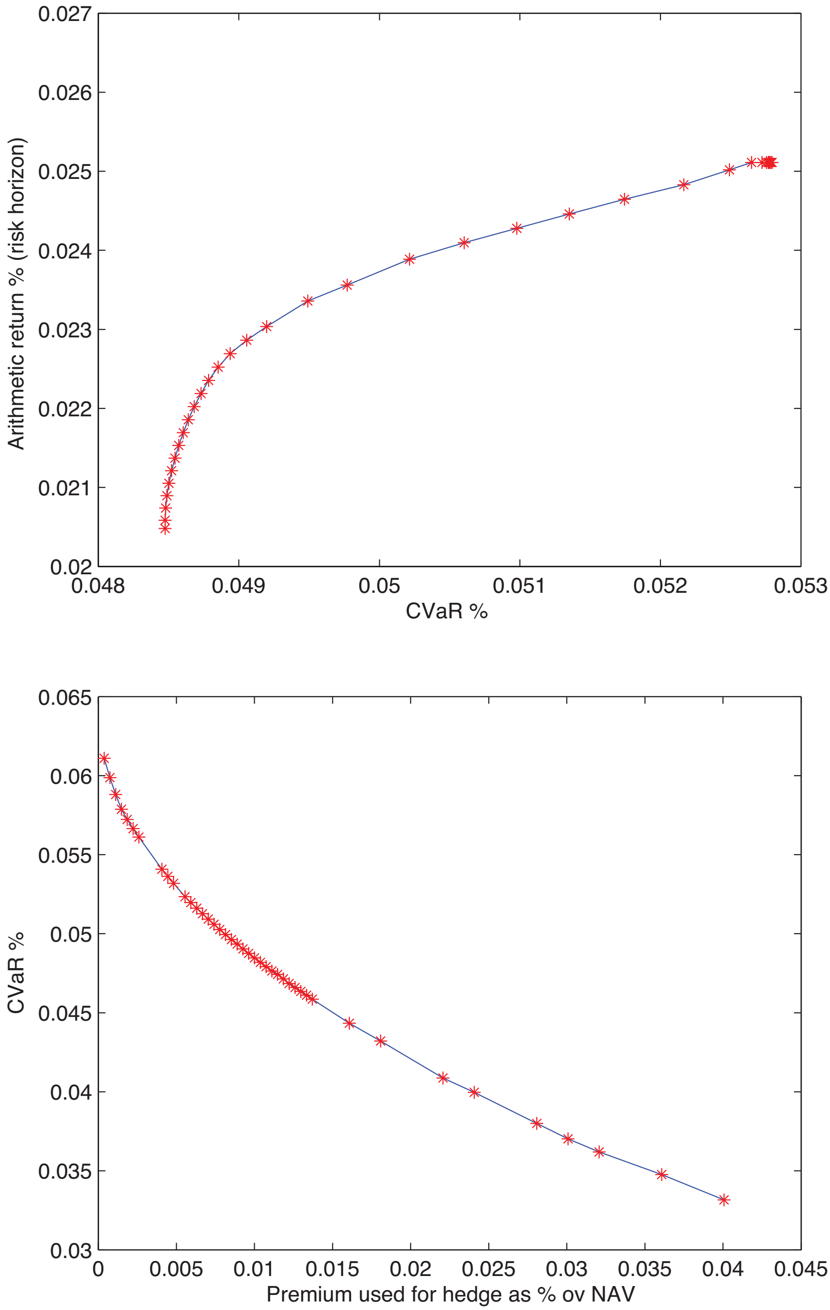

Figure 5.16: Efficient frontiers for the optimization framework; risk against return and risk against premium spent.

Chapter 6

Conclusions

This brief chapter will present a summary of the key take-aways from our work and talk about its shortcomings. Lastly, we list some areas that could provide interesting starting-points for further research within the area.

6.1 Key take-aways

Simulating and visualizing the unhedged portfolio has shown to give a good understanding of how the portfolio is diversified and how that diversification mitigates the worst negative losses. This can intuitively be illustrated through figures such as 5.2, and by marginally adding the different portfolio groups as in figure 5.8. Understanding these graphs enables insights into which risks are diversified, and which ones need to be hedged. Including stress tests, i.e. real historical periods of stress, gives the portfolio manager facts about the true performance in terms of portfolio correlation and losses. Including enough stress test enables comprehensive knowledge that can be translated to how the portfolio will perform and be correlated during an extreme scenario in the future. Essentially, this is what risk assessment is all about, whether applying a fundamental or quantitative approach.

Including risk factors that the portfolio manager feels comfortable with and can intuitively relate to the unhedged portfolio, can give an important picture of the portfolio's future performance in terms of the risk factors. If the portfolio manager has strong feelings about the performance of these risk factors the future dynamics of the unhedged portfolio can be better understood. In addition to predicting future portfolio outcomes from the included risk factors, they have the potential to be used as excellent stress tests, as they can serve as an alternative to historical time periods.

Having the option of importing a pre-defined hedge, quantitatively evaluate, reconstruct, and re-evaluate it, until the fundamental perspective converges to what can be deduced from the market view and the additional stress tests, can lead to a excellent hedging strategy. This strategy will surely be effective mitigating both long term strategic macroeconomic risks, but also shorter term volatility driven vulnerabilities. Evaluating and re-evaluating the fundamentally pre-defined hedge can easily and intuitively be done by comparing the marginal impact from changing the order in which different instruments are added, as well as feeding it to the optimization framework letting it allocate the optimal weights in term of lowering CVaR.

The numerous customization options included in the tool, enables the portfolio manager to present the portfolio in the market view in a way that makes sense and one that can help support individual investment philosophies. The transparency this contributes to induce an increased utility of the quantitative approach, enabling portfolio managers to be better prepared for unforeseeable and severe extreme events.

As stated in the background, tail risk hedging above all aims at answering to investors' short term goals and to mitigate the risk of investor redemption, and would be redundant only considering the investment perspective. Although this is very much true, financial institutions must attract and keep capital to do business, which is why managing tail risk should be and must be a core part of the daily business.

Lastly, when applying the tool on the example portfolio presented in the thesis, the optimization framework yields a CVaR reduction from 6.3% to 4.8%. This can, when put in contrast to that of a standard tail risk hedge, be considered to make sense from a fundamental perspective, and call for a reconstruction of the pre-defined hedge as it clearly underperforms, as it includes more or less redundant instruments in terms of marginal risk.

6.2 Methodology criticism

Historical data provide a guide about the future, but must be modified to recognize structural changes and compensate for anomalous periods. Quantitative measures have a difficulty incorporating factors such as market liquidity or the influence of significant, low-probability events (Yale, 2007). Assets with mean reverting characteristics such as volatility and interest rates can not quantitatively be modeled very accurately. The same applies to credit and other event driven assets. As these times series constitutes a key part of the presented market view, it is important to take the output with a grain of salt, and utilize it as a valuable complement rather than an investment philosophy.

As discussed in section 1.2, hedging in an ideal world is not necessary. Investors take rational decisions, have long term investment goals and are unaffected by psychological pressure in situations of extreme market stress. A world like that is clearly unrealistic and as stated, the presented methodology aims just at providing protection against extreme downside events. This implies that any such tail risk hedge is a bad long term investment, in terms of excess return. However, the optimal hedge presented in this thesis clearly enhances the overall performance of the hedged portfolio, which implies that the optimization framework utilizes mispricings in the constructed market view to lower the CVaR by seeking excess return. This further emphasizes the importance of fundamentally revising the optimal outcome and evaluating if the positions actually makes intuitive sense. In terms of tail hedging, we do however believe that the presented methodology brings important input to finding instruments that affect the worst scenarios most favorably.

Validating the market view assumptions is very hard. As the risk horizon lengthens, the number of independent sample periods diminishes. With 10 years of historical data there are only 40 independent residuals for quarterly risk horizons, and consequently only two sample periods should exceed the 95 percent VaR threshold. Creating any meaningful statistics validating the market view is therefore obviously quite difficult (Zumbach, 2007). Further, the limited number of Monte Carlo simulated scenarios used throughout the thesis does incur a potentially dangerous margin of error. While the number of scenarios has been limited to 5000 for practical reasons, using stastistical optimization methods such as the one employed should be provided with a greater number of scenarios as the margin of error is directly related to the sample size.

An additional issue that needs to be included when analyzing the results, is that the presented tool does not take into account any transaction costs or liquidity issues. These issues can have substantial influences to the practical applicability of the tool, as it decides which instruments can and should be evaluated as well as how often the tail hedge can be reevaluated and reconstructed. The issue with liquidity is party taken care of in the framework as we filter hedging instruments based on any arbitrarily constraint. However, including the option of penalizing those constraints instead of filtering, would result in a more dynamic model. More on this in section 6.3.

6.3 Further research

The primary area of improvements that would be suitable for further research is a more quantitative evaluation of how the framework presented in this thesis actually performs. Due to the problem of diminisshing historical samples as discussed in section 6.2, traditional backtesting is difficult, but the performance of the framework could potentially be evaluated against another systematic approach to tail risk hedging, using the few historical sample available. While this is mostly related to the performance of the market view, it's highly important for the credability of the framework in its entirety.

As the thesis is focused on the development of a practical tool to analyze tail risk, numerous areas of improvement related to the practical issues of the framework have been identified in order to further close the gap between the fundamental and quantitative perspectives to risk and to make it applicable in a practical setting:

First, as discussed in section 6.3, penalizing different aspects in the optimization function could help to better represent the real implications of hedging in certain instruments. Liquidity issues, transactions costs, operational limitations and capital requirements are all variables that are left unmanaged in the current version of the tool. These issues could be included as penalization functions within the optimization framework as an extra cost for selecting certain hedging instruments or on the actual composition of the hedge. This could potentially aid in the creation of an optimization model that better reflects how a tail risk hedging strategy is constructed based on real-world practical constraints. However, as these costs are often difficult to quantify, this requires significant research to correctly account for.

Secondly, a potential area of improvements is the inclusion of more complex hedging strategies. Currently, the framework does only allow for long hedging positions, but the inclusion of short positions with optionality (e.g. the issuance of options) could potentially be very valuable for portfolio managers depending on the structure of the unhedged portfolio and their risk preferences. Allowing for short positions would require the incorporation of constraints on how short a hedge could be, preferably on some sort of capital cover ratio base, in order to constrain the optimization.

Thirdly, a number of potential improvements is related to fundamentally adjusting the quantitative model. While the framework in its current incarnation has numerous options for a portfolio manager to influence the optimization and the visualization of hedges, the market view is to some extent less transparent and less customizable. Incorporating fundamental views on how different risk factors will develop over the near future, in terms of volatility, drift or correlation, can potentially improve the utility of the framework significantly. Adjusting the simulation framework that defines the market view is not an easy task however, as this must be presented in an intuitive way, yet complex enough to fully capture the dynamics of the market and the model.

Lastly, as the tool is deployed over time to optimize tail risk hedging, the issue of when to hedge and when to unwind the hedges arises. Should the hedge always be held to maturity? And what time-to-maturity should the hedges have? Using more sofisticated optimization models that includes multi-period hedging strategies, could potentially be incorporated into an optimization framework such as the one presented here and answer some of these questions, but requires significant research and validation efforts.

Appendix

Description of the tool

The tool developed in this thesis has been implemented using the mathematical software suite MATLAB, leveraging the statistical, financial and optimization toolboxes. It can be deployed to a portfolio manager as a stand-alone application and is easy to use for an end-user. The main application window is illustrated in figure 6.1.

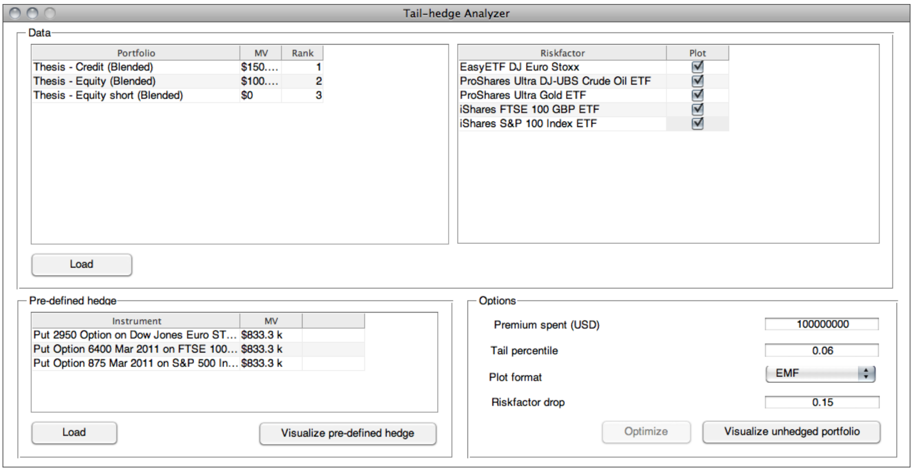

Figure 6.1: Main application window for the tool developed in the thesis.

Besides the main application described in this section, a number of additional support tools have been developed in order to connect to proprietary databases for pulling market data, to specify and model the portfolio, to crawl hedging instruments for the optimization etc.

The main application is visually composed of three sections: the data section at the top in which the user can import and load the different portfolio components and risk factors that are to be analyzed.

The bottom left section is related to the loading and visualization of hedges that are defined outside the optimization framework. Here, the user can load any type of hedge (or other component to be added to the portfolio) in order to use the visualization capabilities of the tool as exemplified in section 5.3.1.

Finally, the bottom right section allows the user to specify some of the parameters related to the main optimization. Most importantly, the premium that is to be spent on the optimized hedge is the primary variable, while the three other options are related to the visualization of the results. The other constraints related to the optimization, as discussed in section 4.3, can be specified in an external tool used to construct the list of potential hedging instruments.

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