Inquiries into the absence of extended globular clusters in the Milky Way Galaxy

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Abstract

Thirteen stellar objects known as extended globular clusters were found in the Halo of the Andromeda Galaxy. Three of these extended clusters have quantitative properties which are subsequently used to determine whether clusters of extended size could survive in the orbits of the Milky Way Galaxy. The question of whether extended clusters could survive in the orbits of the Milky Way, is interesting to explore because Astronomers hypothesize that the existence of these rare objects is not restricted to M31. However, none have been found in the Milky Way to date; and it is the purpose of this report to begin to investigate whether this fact is indeed curious or whether there is a plausible explanation for why they are absent. Two major factors affecting the likelihood of extended clusters surviving in the Milky Way Galaxy were investigated; namely, Tidal effects on the extended clusters in the orbits of the Milky Way; and the comparative cluster distribution of both Galaxies. The resulting data reveal that the cluster distribution in M31 favors the existence of extended globular clusters more than that of the Milky Way Galaxy; and that tidal shredding would play a prominent role in removing most of the extended clusters from the Milky Way, if they were present in the first place; however, despite the latter, a small fraction of extended clusters could still survive despite the tidal forces present in these orbits.

Contents

1 Introduction

The central theme of the present report can best be described as an attempt to investigate the possible activity leading to the lack of objects, known as extended globular clusters, present in the Milky Way Galaxy, today, where the use of the term “extended” is understood to be synonymous with a globular cluster with an unusually large half-light radius, R_h (Huxor, 2005). The absence of these unusual clusters is useful to investigate, since Astronomers suspect that there are other Galaxies, in addition to M31, in which they reside.

The present report is based largely on the work of two publications, the 2005 paper by A.P. Huxor et al. in which 3 extended globular clusters were found in the Halo of M31 (see Figure 1 (Huxor, 2005) for details), as well as the 2006 paper by C. Allen et al. in which orbits of 48 of the 150 globular clusters of the Milky Way are computed using available space velocity data and their corresponding tidal radii, computed. Using the parameters of the extended globular clusters provided by Huxor (2005), combined with the orbital information provided by Allen (2006), the present report seeks to answer the question of whether extended clusters could have existed in the Milky Way at one time but have been subject to tidal disruption, to such a degree that the present clusters are no longer extended, but rather, classified as regular globular clusters. Placing the extended globular clusters in these known orbits is one way of investigating the tidal effects on clusters of “extended” dimension. Further, the result of such a test provides valuable information about some of the possible conditions under which globular clusters of extended size could possibly exist. There is currently no orbital information available for the remaining globular clusters of the Milky Way, however, physical data; such as magnitude and globular cluster coordinates provided by the Harris Catalogue (1996), is used in combination with information on the known orbits (Allen, 2006), to predict the tidal radii of the remaining Milky Way globular clusters.

Hence, the major objectives of this report are to:

- determine whether extended clusters would survive in the known orbits of the Milky Way
- predict the orbits of the full set of Milky Way clusters based on information available from the Harris Catalogue (1996) as well as the known orbits of Allen (2006)
- determine whether extended clusters can survive in the estimated orbits
- relaxing the definition of “extended globular cluster” to include extended clusters that have undergone some degree of tidal shredding and see how many additional extended clusters survive in the Milky Way orbits
- compare the distribution of clusters in M31 and Milky Way Galaxies
- determine whether Milky Way clusters could be considered a representative subset of those found in M31

- determine whether the cluster distribution in the Milky Way can explain the lack of extended clusters observed
- determine how many extended globular clusters one would expect to see in the Milky Way if the same ratio of extended to regular globular clusters existed in the Milky Way as in M31
- determine the conditions under which one could expect to see extended clusters in the Milky Way

The nomenclature found throughout this report is as follows: "Allen 48" is used to denote the 48 orbits computed in Allen (2006), the orbits for which full orbital information is known, "estimated orbits" is used to denote the *orbits* of the 102 Milky Way globular clusters for which orbital information is not available but is *inferred*, and "Non Allen clusters" is used to denote the *globular clusters* for which orbital information is not available.

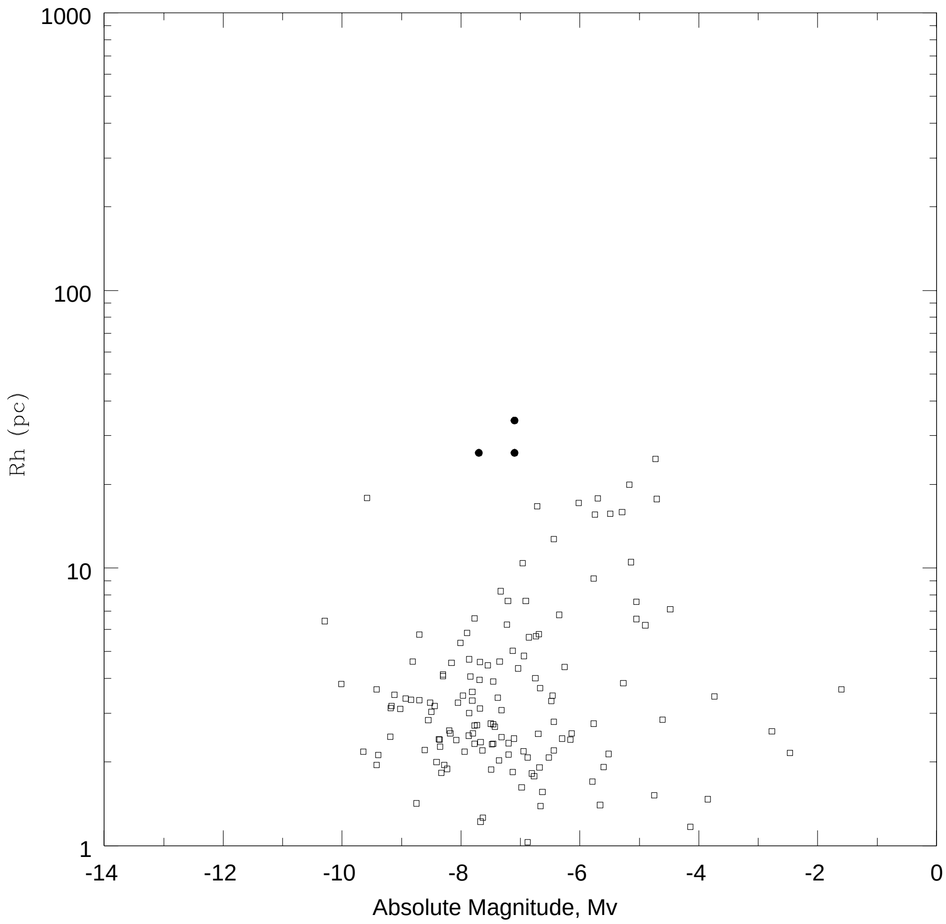


Figure 1: Plot of log R_h against M_v of Milky Way globular clusters (squares) and M31 extended clusters (filled circles). Positions of extended clusters indicate globular cluster luminosities comparable to those in the Milky Way, but significantly larger half light radii (Huxor, 2005).

1.1 Summary of Huxor (2005)

A globular cluster is a dense group of stars ($\sim 10^5 - 10^6$), bound by gravity and spherical in shape. Globular clusters reside in the Halo of a Galaxy, orbit the Galactic Center and typically resemble the image in Figure 2, with the highest concentration of stars located near the center, within a region known as the Core Radius. The Half Light Radius is the radius of the globular cluster containing half of its total luminosity. In 2005, three extended globular clusters were discovered in the Halo of M31 (Huxor, 2005). These clusters were so named because they have the same characteristics (stellar population and spherical shape) as regular globular clusters, but a different distribution of the stars of which they are composed. The density of stars toward the center is lower and in particular, the half light radii of these objects are found to be much larger.

Figure 2: The globular cluster pictured here is NGC 6093, one of the 150 globular clusters found in the Milky Way Galaxy. Image Credit: The Hubble Heritage Team (AURA/STScI/NASA).

Images of the 3 extended globular clusters in M31 were taken in projection. The resulting images of the spherically shaped clusters resemble discs of varying luminous density, from center to edge. The disc is subdivided into numerous, finite, concentric rings radiating from its center. The intensity of light in each unit ring is determined by counting the number of illuminated pixels registered by the CCD camera taking the image, the pixel count being proportional to the amount of light emitted by the cluster, which is proportional to the density of stars emitting the light detected. Hence, the cluster's Surface Flux Density Profile can be determined, flux density being; the power of radiation per unit area.

Physical parameters of the extended clusters, such as core and tidal radii, r_c and R_t, were determined by fitting an Empirical King Profile to the Surface Flux Density Profile mentioned earlier and adjusting the values of r_c and R_t to make the fit.

1.1.1 King Profile

The Empirical King Profile is an empirical law describing the surface density distributions of globular clusters, galactic clusters and Sculptor-type dwarf elliptical galaxies (King, 1962); surface density distribution being; the measured brightness of the object's surface as a function of radius.

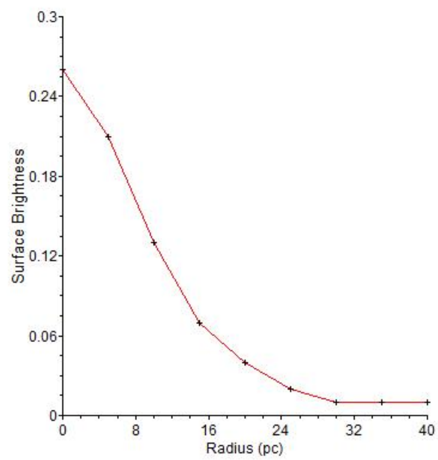


Figure 3: An example of the King Profile fitting procedure, whereby cluster parameters are adjusted in order to obtain the most ideal fit to the observed surface flux density profile.

The King Profile embodies the characteristics of a cluster's density distribution and hence, its surface brightness, at its minimum and maximum limits, and is defined in the following manner (King, 1962):

$$\Sigma(r) = \Sigma_o \left(\frac{1}{(1 + (\frac{r}{r_c})^2)^{\frac{1}{2}}} - \frac{1}{(1 + (\frac{r_t}{r_c})^2)^{\frac{1}{2}}} \right)^2 \tag{1}$$

where Σ_o is the central surface density (surface brightness at cluster center), r is the radius from the cluster center, $\mathrm{r_c}$ is the core radius, the radius at which most of the enclosed stellar distribution is more or less uniform, and $\mathrm{R_t}$ is the tidal or limiting radius which is set by external forces acting on the globular cluster. The physical tidal radii of the three extended clusters of Huxor (2005) are determined using this fitting procedure.

1.2 Summary of Allen (2006)

Orbits for 48 known clusters in the Milky Way are computed from initial conditions (initial radius, obtained via magnitude measurements and initial velocity measurements) relative to the Milky Way Galactic Center. Initial velocity is a combination of the cluster's radial velocity (obtained via Doppler measurements) and its transverse velocity or proper motion (obtained by measuring the cluster's motion across the sky for a given time interval).

The simulations of Allen (2004), are computed using the potentials and mass distributions of Allen and Santillan's (1991) axisymmetric model and Pichardo's (2004) nonaxisymmetric galactic bar model. The adopted parameters of the model are as follows: R_o = 8.5 kpc with circular velocity, $V(R_o) = 220$ km s^{-1}, total mass of 9×10^{11} M$_\odot$, $\rho_o = 0.15$ M$_\odot$ pc^3, A = 12.95 km s^{-1} kpc and B = -12.93 km s^{-1} kpc where R_o is the Sun's position relative to the Galactic centre, ρ_o is the local mass density and A and B are Oort rotation constants. Theoretical and Observed Tidal Radii are tabulated for the purposes of illustrating the effect of the bar on the dynamics of the clusters.

1.2.1 Axisymmetric model of Allen and Santillan (2004)

The axisymmetric model of Allen and Santillan consists of a bulge and a flattened disc of the Miyamoto and Nagai (1975) form and a massive spherical halo extending to a radius of 100 kpc. The potential in each component of the Galaxy (Bulge, Disc, Halo) is in units of 100 km^2 s^{-2}.

The Bulge has a potential and corresponding density of the following form:

$$\Phi_1(\tilde{\omega}, z) = \frac{-M_1}{(\tilde{\omega}^2 + z^2 + {b_1}^2)^{\frac{1}{2}}} \tag{2}$$

$$\rho_1(\tilde{\omega}, z) = \frac{3{b_1}^2 M_1}{4\pi(\tilde{\omega}^2 + z^2 + {b_1}^2)^{\frac{5}{2}}} \tag{3}$$

where $R^2 = \tilde{\omega}^2 + z^2$, $\tilde{\omega} = \sqrt{x^2 + y^2}$, $M_1 = 606.0$ and $b_1 = 0.3873$. M_1 indicates the relative importance of the central component, b_1 is the scale length and the total mass of the central component is $1.41 \times 10^{10} M_\odot$.

The Disc has a potential and corresponding density of the following form:

$$\Phi_2(\tilde{\omega}, z) = \frac{-M_2}{\left(\tilde{\omega}^2 + [a_2 + (z^2 + {b_2}^2)^{\frac{1}{2}}]^2\right)^{\frac{1}{2}}} \tag{4}$$

$$\rho_2(\tilde{\omega}, z) = \left(\frac{{b_2}^2 M_2}{4\pi}\right) \left(\frac{a_2\tilde{\omega}^2 + [a_2 + 3(z^2 + {b_2}^2)^{\frac{1}{2}}][a_2 + (z^2 + {b_2}^2)^{\frac{1}{2}}]^2}{\left(\tilde{\omega}^2 + [a_2 + (z^2 + {b_2}^2)^{\frac{1}{2}}]^2\right)^{\frac{5}{2}} (z^2 + {b_2}^2)^{\frac{3}{2}}}\right) \tag{5}$$

where $M_2 = 3690.0$ and is related to the relative importance of the disc component, $a_2 = 5.3178$ and $b_2 = 0.2500$ and the total mass of the central component is $8.56 \times 10^{10} M_\odot$

The Halo has a potential and corresponding density of the following form:

$$\Phi_3(R) = -\left(\frac{M(R)}{R}\right) - \left(\frac{M_3}{1.02a_3}\right)\left[-\frac{1.02}{1+(\frac{R}{a_3})^{1.02}} + \ln\left(1+\left(\frac{R}{a_3}\right)^{1.02}\right)\right]_R^{100} \tag{6}$$

where $M_3 = 4615.0$ and $a_3 = 12.0$, $M(R) = \left(\frac{(M_3)Ra_3{}^{2.02}}{1+(\frac{R}{a_3})^{1.02}}\right)$ and the total mass of the Halo out to 100 kpc is $8.002 \times 10^{11} M_\odot$.

The total potential of all three components is the sum total of the three potentials listed.

1.2.2 Nonaxisymmetric model of Pichardo et al (1991)

The Nonaxisymmetric model of Pichardo (1991) consists of a superposition of four ellipsoids which produce the apparent "boxy" edge on brightness profile of the Milky Way Galactic bar (Pichardo, 1991). The gravitational potential of the galactic bar is superimposed on the axisymmetric mass distribution of Allen and Santillan (1991) and 70% of the bulge mass is replaced with a galactic bar. The purpose of including the bar, is to examine whether the kinematics of the globular clusters are altered by the presence of a bar when compared to the axisymmetric model. The latter is true only if the resulting gravitational potential includes orbits actually reaching the bar region. The modelled galactic bar is an inhomogeneous ellipsoid that extends to a galactocentric distance of 3.13 kpc, has axial ratios of 1.7 : 0.64 : 0.44 and the following mass distribution:

$$\begin{aligned} \rho(R_\mathrm{s}) &= \mathrm{sech}^2(\mathrm{R_s}) & R_\mathrm{s} \leq R_{\mathrm{end_s}} \quad (7) \\ &= \rho_o \mathrm{sech}^2(\mathrm{R_s}) e^{-\left(\frac{R_s - R_{\mathrm{end_s}}}{h_{\mathrm{end_s}}}\right)^2} & R_s \geq R_{\mathrm{end_s}} \quad (8) \end{aligned}$$

where $R_\mathrm{s} = \left(\frac{x^2}{a_x{}^2} + \frac{y^2}{a_y{}^2} + \frac{z^2}{a_z{}^2}\right)^{1/2}$ and $h_{\mathrm{end_s}} = \frac{h_{\mathrm{end}}}{a_x}$ and h_{end} is the z component of angular momentum of the inner region.

The potential and force field are obtained at any point $\vec{r} = (x, y, z)$ in the Cartesian System of the principal axes of the bar using standard potential theory (Kellogg, 1953).

Newton's Law of Gravitation applies to idealized point masses; and since such a treatment of individual point masses in a Galaxy can become cumbersome, it is more convenient in the context of Galactic Potentials, to consider a field as having a gravitational potential. The notion of a gravitational field is defined in terms of the gravitational acceleration that a test mass would experience as a function of its position in the vector field. Both the axisymmetric and nonaxisymmetric potentials used by Allen (2006) to compute the orbits of the Milky Way clusters, provide the potentials of each Galactic component as well as their corresponding density distributions. Hence, the corresponding force acting at each position can be determined by taking the gradient of the given potential; which yields the the gravitational acceleration for a given position in the field times the total mass of that component of the Galaxy. Dividing the latter by the total mass and and multiplying this acceleration with the corresponding mass at the same position in space, provides the appropriate force acting at each position in each Galactic component.

2 Theory

2.1 Magnitudes Scale

The term "magnitude" is used in reference to the measure of the brightness of stars. It is a negative logarithmic scale, meaning that brighter objects have more negative magnitudes. Apparent magnitude, is the brightness of an object as it would appear to an observer on earth. Absolute magnitude is the brightness an object would have at a distance of 10 pc from earth.

The magnitude difference of two objects, $\triangle m$ is related to the brightness ratio of the two objects by the following:

$$\triangle m = m_2 - m_1 = -2.5 \log \left(\frac{L_2}{L_1} \right) \tag{9}$$

where m_2 and m_1 are the apparent magnitudes and L_2 and L_1 are the luminosities of object 2 and object 1, respectively. For the purposes of this discussion, the absolute magnitude will be denoted, M_v such that:

$$M_v = m - 2.5 \log \left(\frac{d}{10} \right)^2 \tag{10}$$

where d is the distance of the object in question and 10 refers to a distance of 10 pc. It is evident from (10) that $M_v = m$ when $d = 10$ pc.

Further, when the mass to light ratio of a stellar object is assumed to be constant, as is the case in this report; the ratio of the masses is equivalent to the ratio of the luminosities:

$$\frac{M_1}{M_2} = \frac{L_1}{L_2} = 100^{\frac{(M_{v2} - M_{v1})}{5}} \tag{11}$$

where M_1, M_2, L_1, L_2, M_{v1} and M_{v2} are the masses, luminosities and absolute magnitudes of object 1 and 2 respectively.

Note: a magnitude difference of 5 corresponds to a factor of 100 in brightness which is apparent when the above exponent assumes a value of 1.

In the context of extended clusters in Milky Way orbits, the relationship between a cluster's absolute magnitude and its corresponding mass is used extensively in order to calculate its tidal radius.

2.2 Tidal Radius

Since globular clusters are not solid objects, they are especially subject to the influence of tidal forces acting on the individual stars making up the cluster. In particular, the stars on the cluster's exterior, least bound by the gravitational forces produced by the cluster's core, are vulnerable to tidal forces; capable of removing these stars; thereby reducing the cluster's overall size.

In the context of globular clusters residing in Galaxies, one is concerned with the interaction between three masses; the force felt by a point mass, m, in a neighborhood of the collective cluster mass, m_c and the gravitational force exerted by a massive Galactic Center, M, a much further distance away from the other two masses of interest. Tidal forces are of particular relevance because the objects of interest, namely, globular clusters, are sitting in a very massive Galaxy, and hence, are subject to very strong forces that can tidally shred the globular cluster at critical distances. The tidal radius is the maximal distance in which a mass element placed in the neighborhood of a point mass, m_c, placed at a much further distance to another point mass, M, can remain suspended in space due to the equal and opposite forces acting on m.

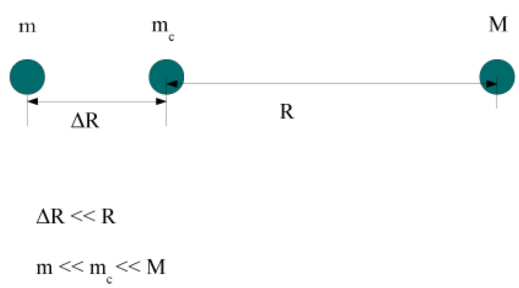


Figure 4: A point mass, m, in the vicinity of a cluster mass, m_c, residing in a very massive Galaxy, represented by the point mass, M, located at the Galactic Center.

At this critical distance, the gravitational force due to m_c and the tidal force on the mass element are exactly balanced. Hence, the mass element would not remain in the vicinity of m_c at any distance greater than the tidal radius or put another way, the mass would be tidally shredded from the collective body, m_c at any greater distance. The tidal radius is also referred to as the cluster's physically observed radius. For clarity, all calculated tidal radii referred to in the body of the report refer to the theoretical tidal radius derived here and not the physical tidal radius, unless otherwise stated. Consider a point mass, m, in the neighborhood of a collective mass, m_c, separated by a distance, $\triangle R$ such that $m << m_c$. If m_c is located a distance, R, from the Galactic Center, M, such that $\triangle R << R$, then the forces acting on m are:

$$|\vec{F}_{m_c,m}| = \frac{Gmm_c}{(\triangle R)^2}$$

$$|\vec{F}_{M,m}| = \frac{GmM}{(R \pm \triangle R)^2} = \frac{GmM}{R^2(1 \pm (\frac{\triangle R}{R}))^2}$$

Expanding the latter term using the McLaurin Series, one obtains:

$$\frac{GmM}{R^2} \times \left(1 \mp 2\left(\frac{\triangle R}{R}\right) \pm 3\left(\frac{\triangle R}{R}\right)^2 \mp 4\left(\frac{\triangle R}{R}\right)^3 + ...\right)$$

$$|\vec{F}_{M,m}| = \frac{GmM}{R^2} \pm \frac{2GmM}{R^2}\left(\frac{\triangle R}{R}\right) = \frac{GmM}{R^2} \pm \frac{2GmM\triangle R}{R^3}$$

where $\left|\frac{2GMm\triangle R}{R^3}\right|$ is the tidal force on mass element, m. When this tidal force is in dynamic equilibrium with the gravitational force attracting m to m_c, the following holds:

$$|\vec{F}_{tidal}| = |\vec{F}_{m_c,m}| \Leftrightarrow \left|\frac{2GMm\triangle R}{R^3}\right| = \left|\frac{Gmm_c}{(\triangle R)^2}\right| \Rightarrow (\triangle R)^3 = \frac{m_c R^3}{2M}$$

hence,

$$\triangle R = \sqrt[3]{\frac{m_c}{2M}}R \tag{12}$$

which defines the tidal radius.

2.3 Half Light Radius

As discussed previously, the core radius is the radius of a globular cluster in which the density of stars, and hence, surface brightness, remains more or less uniform. A cluster's luminosity steadily decreases with increasing distance from the cluster's core. The Half-Light Radius is an intermediate distance between the cluster's core and physical tidal radius, which contains half of the cluster's total luminosity.

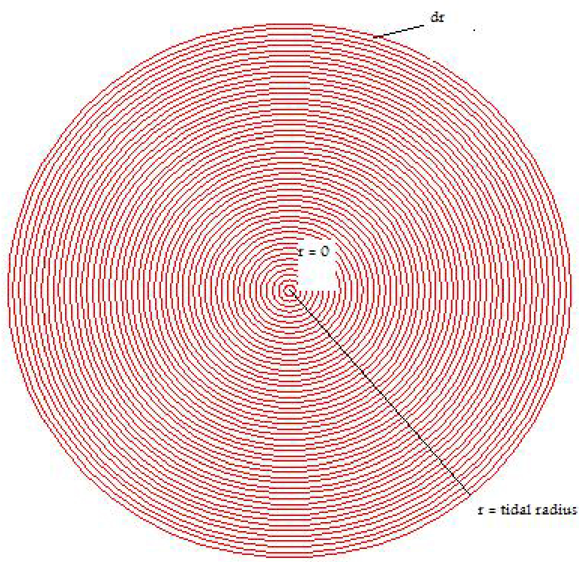


Figure 5: A schematic representation of what an extended cluster would look like in projection, subdivided into infinitesimal concentric rings radiating from its center. Integrating the surface brightness of each ring (dA = 2πrdr) from center to edge (r = 0 to r = tidal radius), gives the cluster's total luminosity.

By integrating the flux density distribution of the globular cluster over the entire disc shaped surface, one obtains the total luminosity of the object. This is achieved by integrating the Empirical King Profile (King, 1962), with previously determined values of r_c and r_t:

$$\Sigma(r) = \Sigma_o \left(\frac{1}{(1 + (\frac{r}{r_c})^2)^{\frac{1}{2}}} - \frac{1}{(1 + (\frac{r_t}{r_c})^2)^{\frac{1}{2}}} \right)^2 \tag{13}$$

with respect to r to obtain the cluster's total luminosity (King, 1962):

$$L(x) = \pi {r_c}^2 \Sigma_o \left(\ln(1 + x) - 4 \left(\frac{(1 + x)^{\frac{1}{2}} - 1}{(1 + x_t)^{\frac{1}{2}}} \right) + \frac{x}{1 + x_t} \right) \tag{14}$$

where L(x) is the cluster's total luminosity obtained from the projected image, $x = \left(\frac{r}{r_c}\right)^2$, $x_t = \left(\frac{r_t}{r_c}\right)^2$ and r runs from 0 to r_t. The radius corresponding to half the light contained within the globular cluster, the Half Light Radius, is obtained by observing the radius from the cluster center at which its integrated flux density or total luminosity drops by a factor of 0.5.

2.4 Radial Velocity

The radial component of an object's velocity is defined as:

$$\vec{\mathrm{v}}_{\mathrm{rad}} = \frac{\mathrm{dr}}{\mathrm{dt}}\ \hat{\mathrm{r}} = \mathrm{v}_{\mathrm{rad}}\ \hat{\mathrm{r}} \tag{15}$$

where $\hat{\mathrm{r}}$ is the normalized unit vector directed along the line of sight. An object's radial velocity is obtained by making use of Doppler shifts of spectral lines. The relationship between a star's radial velocity and its associated Doppler shift is:

$$\frac{\triangle\lambda}{\lambda_{rest}} = \frac{\mathrm{v}_{\mathrm{rad}}}{c} \tag{16}$$

where $\triangle\lambda$ is the wavelength shift associated with the star's motion, λ_{rest}, is the wavelength

of the spectral line corresponding to the object at rest and $c = 3.0 \times 10^8 \mathrm{m\ s}^{-1}$. The Doppler shift determines whether a given object is moving toward or away from an observer and is obtained by observing the wavelength shift associated with the star's motion. Since an absorption line in the spectrum of a given star in motion should be coincident with the spectrum of the star at rest, the resulting wavelength shift observed, is due solely to the motion of the star in the line of sight; where the rest wavelength, λ_{rest}, is determined from the laboratory measurements of emission lines, corresponding to the discrete wavelengths and energies of electronic transitions.

Positive velocities indicate a star moving away from the point of observation. The resulting wavelength of such a star will appear to be redshifted. Negative velocities indicate a star moving towards the point of observation; the corresponding wavelength in such a case will be blueshifted. The radial velocity is the velocity component most easily obtained for most objects and is one of the velocity components making up the full space velocity of a stellar object. As mentioned earlier, a star or star cluster's full space velocity is one of the parameters required in order to compute its orbit.

2.5 Proper Motion

As mentioned in 2.4, the radial velocity (motion of object along the line of sight) is only one component making up an object's space velocity, the other component, is the tangential velocity, which is an object's motion perpendicular to the line of sight. The object's angular or proper motion, $\frac{d\theta}{dt}$, is required in order to calculate the tangential component of the object's motion and is measured by observing the object's angular motion across the sky per unit time ("/yr). Since the objective is to obtain the tangential velocity of the object, which is a linear velocity, the angular velocity must be converted into a linear velocity by making use of the following relationship:

$$\mathrm{v_{tang}} = \mathrm{r} \times \frac{\mathrm{d}\theta}{\mathrm{dt}} \tag{17}$$

where r is the distance from the point of observation to the stellar object, and $\frac{d\theta}{dt}$ is the object's proper motion measured in arcseconds (photons per year). The arcsecond measure can be made unitless by converting to radians in the following manner:

$$1 \text{ arcsec} = \frac{1}{3600} \times 1^{\circ} \tag{18}$$

therefore 1 arcsec in radians is:

$$\frac{1^{\circ}}{3600} \times \frac{2\pi}{360^{\circ}} \approx 4.85 \times 10^{-6}\mathrm{rad} \tag{19}$$

Once the proper motion is corrected to the appropriate units using the above relation (19), the tangential velocity (17) can be determined. The complete space velocity of the object can subsequently be determined in the following manner:

$$\mathrm{v_{tot}} = (\mathrm{v_{tang}}^2 + \mathrm{v_{rad}}^2)^{\frac{1}{2}} \tag{20}$$

2.6 Barred Potentials in Galaxies

Although there is strong evidence that a Galactic bar indeed exists at the center of the Milky Way Galaxy (Blitz, 1991), it is not yet obvious what role it plays in triggering the activity observed within the Galaxy (Friedli, 1993). Hence, it is useful to include the potential associated with a Galactic bar in modeling methods in order to better understand these processes; particularly in cases where unbarred and barred potentials can be compared.

A Bar is a linear, rotating, triaxial stellar structure crossing the disc of a Galaxy (Blitz, 1991). The presence of a rotating bar in a Galactic disc is believed to have accumulated galactic matter over a long period of time, resulting from the gravitational pull induced by the bar's rotation (Gerhard, 1996). The barred potential is the mechanism for slow evolutionary changes taking place within the Galaxy and therefore gives insight into the origin of the Galaxy's formation (Gerhard, 1996). Further, orbits of stellar objects near the centre of the disc of a barred galaxy are more affected than ones further out from the centre of the bar; which is useful to know when investigating the orbits of clusters affected by the presence of a nonaxisymmetric potential induced by a Galactic bar.

2.7 Formulations for Estimating Tidal Radii

King's formula is the tidal radius of a star at the moment of perigalactic passage such that the star is pulled neither toward or away from the globular cluster of mass $\mathrm{M_c}$, to which it belongs, sitting in a galactic force field, with galactocentric distance r_{min}, on an orbit of eccentricity, e, and is defined as (King, 1962):

$$r_k = \left(\frac{M_c}{M_g(3+e)}\right)^{1/3} r_{min} \tag{21}$$

where M_g is the galactic mass producing the force felt by the cluster at the perigalactic point and M_c is the cluster mass. Note that (21) is a variant of the tidal radius previously presented (12), and differs in that it is the limiting tidal radius of the cluster at the perigalactic point of its orbit rather than the instantaneous value at the cluster's present location (12). An alternative formulation for the tidal radius of a cluster in a general Galactic field can be obtained in which at a certain position on the line joining the cluster and the Galactic center, a test particle with velocity $v' = 0$, measured in the non-inertial reference frame at the center of the cluster, has an acceleration a' in this frame with no component along this line and is defined as (Allen, 2006):

$$r^{\star} = \left(\frac{GM_c}{\left(\frac{\partial F_{x'}}{\partial x'}\right)_{r'=0} + \dot{\theta}^2 + \dot{\Psi}^2 \sin^2\theta}\right)^{\frac{1}{3}} \tag{22}$$

where M_c is the cluster mass, $G = 6.673 \times 10^{-11}$N m^2 kg^{-2}, $\left(\frac{\partial F_{x'}}{\partial x'}\right)$ is the differential force per unit mass or differential acceleration of the test particle at a fixed position along x'. The angles, θ and Ψ are the angular and spherical coordinates of the cluster in an inertial Galactic frame and $\dot{\theta}^2 + \dot{\Psi}^2 \sin^2\theta$ is the component of the noninertial acceleration in the $\hat{x}'$ direction, where $\hat{x}'$ points toward the Galactic centre.

2.8 Mass to Light Ratio

Where there is gravity, there is mass. Hence, in the context of globular clusters, there must be enough matter inside the cluster to bind it sufficiently so that it manages not to disperse while rapidly rotating within the Galaxy. However, the mass of the visually luminous matter detected by Astronomers does not encompass the mass required to gravitationally bind clusters. Calculations indicate that a typical cluster requires roughly 10 times the mass of detectable matter in order to remain bound in orbit (Kaufmann, 1999). The latter has lead Astronomers to distinguish between the type of matter of which Galaxies are composed; namely, visible or luminous mass and nonluminous mass, known as dark matter.

The Mass to Light Ratio is the ratio of an object's mass; to its luminosity, contained within its spatial volume. This ratio is 1 for solar or sun-like objects, however, this value is higher for stellar objects, such as clusters, since the emitted radiation is not entirely detectable in correspondence with the known mass of the star cluster volume. A typical mass to light ratio for clusters of stars is $\sim 2 - 3$. Hence, the latter estimate is useful in determining the mass of luminous objects such as globular clusters. The working hypothesis of this report is that the mass to light ratio of extended globular clusters is the same as that of regular globular clusters.

The Mass to Light Ratio is defined as (Peebles, 1993):

$$\frac{L_x}{L_\odot} \times \Upsilon_x = \frac{M_x}{M_\odot} \tag{23}$$

using the relationship between magnitude and luminosity, the above relation can be rearranged to obtain the following:

$$\Upsilon_x = \frac{\frac{M_x}{M_\odot}}{\frac{L_x}{L_\odot}} = 10^{0.4(M_{vx} - M_{v\odot})} \frac{M_x}{M_\odot} \tag{24}$$

where M_x is the stellar object's mass, $M_\odot$ is the solar mass, L_x is the luminosity of the stellar object, $L_\odot$ is the solar luminosity, M_{vx} is the absolute magnitude of the stellar object in the v band and $M_{v\odot}$ is the absolute magnitude of the sun in the v band.

Hence, the mass of the luminous object in question is simply:

$$M = \Upsilon_x \times 10^{-0.4M_{vx} - M_{v\odot}} \times M_\odot \qquad (M_\odot = 1)$$

3 Method

Computations performed in this project:

3.1 Extended Globular Clusters in Allen 48 Orbits
Since no extended globular clusters have been observed in the Milky Way, it is desirable to determine the tidal radii that extended clusters would have if placed in the orbits of the Milky Way Galaxy. This was achieved by substituting the extended cluster masses in the 48 known Milky Way Orbits (Allen, 2006) and subsequently comparing the result to the physical tidal radii of the extended clusters found in M31 (Huxor, 2005). The main purpose of this test was to determine whether any extended clusters could in fact survive in some of the Milky Way Orbits; and if so, how many.

3.2 Extended Globular Clusters in Estimated Orbits
Since only 48 of the orbits of the Milky Way Galaxy are currently known, and it most desirable to determine the tidal radii of the extended clusters (Huxor, 2005) in all of the Milky Way Orbits; the orbits for which full orbital information is unavailable, were estimated. This was achieved by using the tidal radii of the known Allen 48 Orbits (Allen, 2005) and establishing a relationship between tidal radius and distance from the Milky Way Galactic Center. Since the latter scales linearly with the former, this relationship was used to predict the tidal radii of the 98 other Milky Way Orbits. Once this was achieved, the extended clusters were once again substituted, this time, into the estimated orbits; and their tidal radii, computed.

3.3 Milky Way at orientation of M31
Since it was useful to compare the globular cluster distribution in M31 to that of the Milky Way, it was necessary to view the Milky Way clusters from the same perspective that one observes M31; in order to achieve this. Hence, the clusters of the Milky Way were placed at a distance of 780 kpc from the point of observation and transformed to the orientation of M31.

3.1 Extended Globular Clusters in Allen 48 Orbits

Computation of tidal radii for the known Milky Way orbits using extended cluster masses, was performed in order to determine the number of extended globular clusters that could survive in the orbits of the Milky Way.

Recall from Section 2.2, the definition of the tidal radius, $\triangle R$:

$$\triangle R \equiv R_{tidalMilkyWay} = \sqrt[3]{\frac{m_c}{2M}} R$$

where m_c is the mass of the Milky Way cluster, M is the mass of the Milky Way Galactic Centre, R is the distance from the globular cluster to the Milky Way Galactic Centre. The idea is to replace m_c in the above formula with m_{ext}, the mass of the extended globular cluster found in M31. However, masses of extended globular clusters are not given, absolute magnitudes of Milky Way clusters (Harris Catalogue) and extended globular clusters (Huxor, 2005) are provided. Hence, it is possible to determine m_{ext} by the following method:

Ratio of the Luminosities is equivalent to the Ratio of the Masses:

$$\frac{L_1}{L_2} = \frac{M_1}{M_2}$$

hence,

$$\frac{L_{ext}}{L_{Harris}} = \frac{m_{ext}}{m_{Harris}} = 10^x$$

and

$$M_{vext} - M_{vHarris} = -2.5 \log\left(\frac{L_{ext}}{L_{Harris}}\right) = -2.5 \log\left(\frac{m_{ext}}{m_{Harris}}\right)$$

therefore,

$$\left(\frac{M_{vext} - M_{vHarris}}{-2.5}\right) = x$$

where M_{vext} is the absolute magnitude of the extended globular cluster, $M_{vHarris}$ is the absolute magnitude of the Milky Way globular cluster found in the Harris Catalogue, L_{ext} is the apparent luminosity of the extended cluster, L_{Harris} is the luminosity of the Milky Way cluster, m_{ext} is the mass of the extended globular cluster and m_{Harris} is the mass of the Milky Way cluster. Hence, the following relation:

$$\left(\frac{m_{ext}}{m_{Harris}}\right) = 10^{\left(\frac{M_{vext}-M_{vHarris}}{-2.5}\right)}$$

where $m_{Harris} \equiv m_c$

since, $R_{tidalMilkyWay} \propto \left(\frac{m_c}{M}\right)^{\frac{1}{3}}$,

it follows that,

$$R_{tidalext} = \left(\frac{m_{ext}}{m_c}\right)^{\frac{1}{3}} \times \left(\frac{m_c}{2M}\right)^{\frac{1}{3}} \times R$$

equivalently,

$$R_{tidalext} = \left(10^{\frac{(M_{vext}-M_{vHarris})}{-2.5}}\right)^{1/3} \times (R_{tidalMilkyWay}) \tag{25}$$

where $R_{tidalMilkyWay}$ is the tidal radius reported for the 48 clusters of Allen (2005) and (25) is used to compute the tidal radii of the extended globular clusters in the known Milky Way orbits.

3.2 Extended Globular Clusters in Estimated Orbits

In addition to the Allen 48, there are 102 other Milky Way clusters for which orbital information is not known, tidal radii were estimated for 98 of these 102 globular clusters, since 4 of the 102 Milky Way clusters do not have available magnitude measurements required for the estimates. The sample calculation below is performed using $r^{\star}$ (22), for convenience, however, the estimated tidal radii are calculated in the following manner for both formulations of the tidal radius in Section 2.7:

A linear relationship was found between the theoretical tidal radii of the 48 clusters for which orbital information is known as a function of distance from the galactic centre.

The tidal radius, $R_{tidalMilkyWay}$, is first normalized to $10^6 M_{\odot}$:

$$R_{tidalMilkyWay} = r^{\star} \times \left(\frac{10^6 M_{\odot}}{M_c}\right)^{1/3}$$

where M_c, in this case, denotes the cluster mass, expressed in solar masses
Recall from (26) that the mass to light ratio is:

$$\Upsilon_x = \frac{\frac{M_c}{M_{\odot}}}{\frac{L_x}{L_{\odot}}} = 10^{0.4(M_{vc} - M_{v\odot})} \frac{M_c}{M_{\odot}}$$

Assuming a mass to light ratio, Υ_x, of 2, the cluster mass is:

$$M_c = 2 \times 10^{-0.4(M_v - M_{v\odot})} M_{\odot}$$

and since $M_{\odot} = 1$, $M_{v\odot} = 4.83$ in the v-band and $M_v \equiv M_{vHarris}$ (absolute magnitudes for the 98 Non-Allen clusters are taken from the Harris Catalogue), the cluster mass, M_c, can be expressed in the following way:

$$M_c = 2 \times 10^{0.4(4.83 - M_{vHarris})}$$

plotting $R_{tidalMilkyWay} = r^{\star}\left(\frac{10^6 M_{\odot}}{M_c}\right)$ as a function of distance, $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$ in kpc, from the Milky Way galactic centre and fitting the distribution linearly, results in a relationship between tidal radius and distance from the Galactic centre that is subsequently used for predictive purposes.

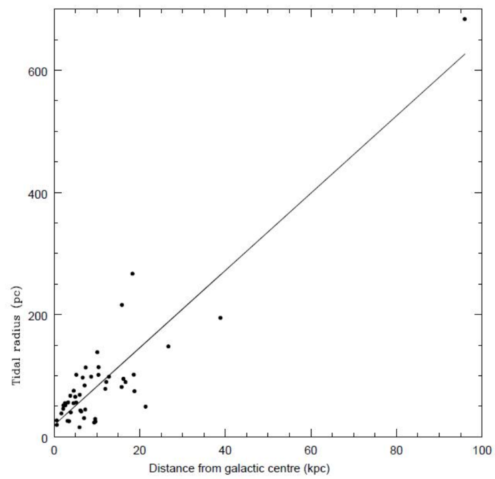


Figure 6: Tidal Radius of known orbits (Allen 48) in pc as a function of distance from the galactic center (kpc) for the General Galactic Field Model, $r^{\star}$, renormalized to 10^6 solar masses. $r_{t,avg} = 6.33$ (pc/kpc)$\times$ d + 19.23 (pc), $R^2 = 0.83$, where d is the distance from the Galactic Center in kpc and $r_{t,avg}$ is the tidal radius obtained via weighted linear regression.

Therefore, the new estimated tidal radius can be written as:

$$R_{tidalest} = (slope \times r + intercept) \times \left(10^{\frac{(M_{vext} - M_{vHarris})}{-2.5}}\right)^{\frac{1}{3}} \times M_c^{\frac{1}{3}} \times 10^{-2}$$

where the slope and intercept are acquired via the linear fitting procedure described earlier for the original Allen data set, r is the distance from the cluster to the galactic centre in kpc and is a required field in the calculation. M_c, 10^{-2} and $M_{\odot}$ are included to rescale the expression appropriately.

hence,

$$R_{tidalest} = r^{\star}_{avg} \times \left(\frac{10^6 M_{\odot}}{M_g}\right)^{\frac{1}{3}} \times \left(\frac{M_{ext}}{M_c}\right)^{\frac{1}{3}} \times M_c^{\frac{1}{3}} \times 10^{-2}$$

where $r^{\star}_{avg}$ is the average tidal radius obtained through linear regression and M_g is the galactic mass.

The latter is equivalent to:

$$R_{tidalest} = \alpha \times dist \times \left(\frac{M_c}{2M_g}\right)^{\frac{1}{3}} \times 10^2 \times \left(\frac{M_{\odot}}{M_g}\right)^{\frac{1}{3}} \times \left(\frac{M_{ext}}{M_c}\right)^{\frac{1}{3}} \times M_c^{\frac{1}{3}} \times 10^{-2}$$

where α is the numerical scaling factor of the calculated slope (neglecting units) and dist is the distance from the cluster to the galactic centre in (kpc) which is equivalent to:

$$R_{tidalest} = \alpha \times dist \times \left(\frac{M_{ext}}{2M_g}\right)^{\frac{1}{3}} \tag{26}$$

as required.

(26) is used to calculate all estimated tidal radii for the 98 Non Allen clusters.

3.3 Milky Way at Orientation of M31

For comparative purposes, the Milky Way Galaxy is tilted and rotated to the orientation of M31 as it is seen from earth. The three Cartesian spatial coordinates of the 150 Milky Way globular clusters from the Harris catalogue are transformed where the distance components, x, y, z are in kiloparsecs, in a Sun-centered coordinate system; with x pointing toward the Milky Way Galactic center, y, in the direction of Galactic rotation and z, towards the North Galactic Pole. The spatial coordinates are corrected so that the positions of interest are relative to the Milky Way galactic center which is located 8 kpc from the sun in the $\hat{x}$ direction. Since the sun and Galactic centre are always colinear, the adjusted x distance component with respect to the Galactic centre, X, is simply the x distance component found in Harris, - 8 kpc along the line joining these points. The other distance coordinates, y and z are unchanged by correcting to a galactocenteric coordinate system. Hence, the $(\mathrm{X}, \mathrm{y}, \mathrm{z})$ distance coordinates of the Harris Catalogue are tilted and projected into a plane and subsequently rotated.

According to Henderson (1979), M31 has an inclination of somewhere between 74° and 79°, and the apparent major axis has a position angle of 38°. An intermediate inclination in Henderson's right-handed coordinate system of 77° was assumed; in which the Z axis is directed toward the viewer and the normal to the plane of M31 defined as the inclination angle (Henderson, 1979). In this report, the inclination is defined as the angle between the Z axis and the plane of M31, rather than the normal to the plane. Further, the position angle of M31 is defined as the angle directed from the X axis to the North on the sky and is taken to be 38° (Henderson, 1979). In this report, the position angle is defined as the angle directed from the -Y axis to the North on the sky in Henderson's coordinate system; which results in a position angle equal to 52°; consistent with Kaaret's value measured from North to East (Kaaret, 2002).

Hence, the 2D projection of the (X, y, z) plane inclined at an angle, $\phi = 13°$ (Henderson, 1979) gives the result:

$x_{tilt} = X$

and

$y_{tilt} = y \sin\phi + z \cos\phi$

where x_{tilt} and y_{tilt} are the coordinates of the projected image.

(x_{tilt}, y_{tilt}) is then rotated by an angle, $\theta = 52°$ (Kaaret, 2002) in the following manner:

$$\begin{pmatrix} X_{trans} \\ Y_{trans} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_{tilt} \\ y_{tilt} \end{pmatrix}$$

where X_{trans} and Y_{trans} are the transformed coordinates of the Milky Way globular clusters

hence,

$$X_{trans} = x_{tilt}\cos(52°) - y_{tilt}\sin(52°) \tag{27}$$

$$Y_{trans} = x_{tilt}\sin(52°) + y_{tilt}\cos(52°) \tag{28}$$

(27) and (28) are used to compute the Milky Way globular cluster coordinates as they would be seen at the orientation of M31.

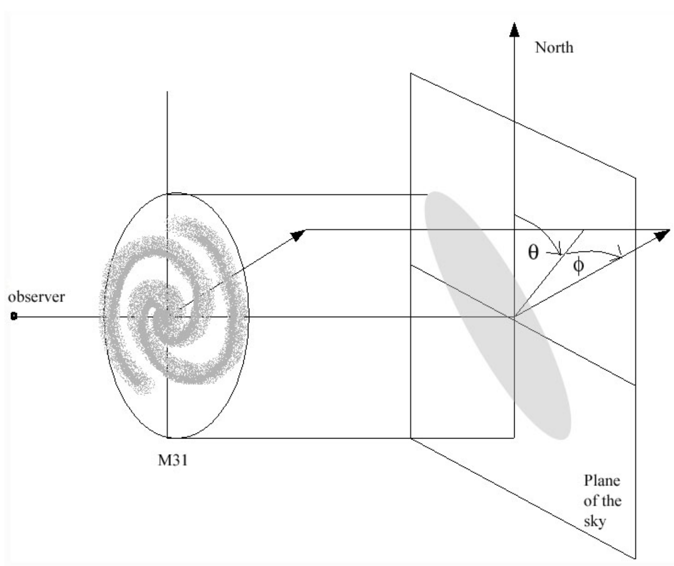


Figure 7: Orientation of M31 relative to the Milky Way.

4 Results

The following figures and tables are meant to illustrate the differences between the cluster distributions of the Milky Way and M31 Galaxies; as well as present the results of substituting the extended globular clusters into the orbits of the Milky Way. The corresponding cluster statistics; namely, the number of surviving extended globular clusters and expected number of extended clusters in the Milky Way Galaxy are also presented.

Figures 8(a) and 8(b) illustrate the result of substituting the extended globular cluster with absolute magnitude, -7.3, in the known (Allen 48) orbits of the Milky Way. The purpose of such a test is to determine what fraction of the extended clusters could be accommodated in these orbits; with the result being that a small minority of extended clusters could be accommodated. Plots 8(a) and 8(b) are included for the purposes of comparing the effects of unbarred and barred potentials; and illustrate that the extended globular clusters are rather unaffected by the presence of a barred Galactic potential, since the distributions of tidal radii do not differ significantly between the two plots.

Figures 9(a) and 9(b) illustrate the result of substituting the extended globular cluster with absolute magnitude, -7.3, in the estimated orbits of the Milky Way. In this case, more extended clusters can be accommodated in the orbits, however, the majority of them are not capable of surviving, as in the previous case. It is also evident from Figures 9(a) and 9(b) that there is no observable difference between the tidal radii distributions of the axisymmetric and nonaxisymmetric potentials. The latter can be explained by the fact that the modelled galactic bar only extends to a galactocentric distance of 3.13 kpc and since most of the clusters are located at a greater distance from the galactic centre, the majority of them are unaffected by the presence of the bar.

Figure 10 illustrates the projected cluster distribution in M31, relative to the Galactic Center. It is evident from the figure that while most of the clusters are located closer to the Galactic Center, there exist a number of clusters further out from this region. In particular, the extended globular clusters are located at greater distances from the Galactic Center than the regular globular clusters of M31.

Figure 11 illustrates the projected cluster distribution of the Milky Way, relative to the Galactic Center; transformed to the orientation at which one observes M31, from our position in the Milky Way Galaxy. From this figure, it is evident that the majority of the globular clusters are located closer in to the Galactic Center.

When one compares Figures 10 and 11, it is evident that fewer globular clusters exist further out from the Milky Way Galactic Center than those of M31. In addition, the extended globular clusters found in M31 are located further out from the M31 Galactic Center. Since clusters located further out from the Galactic Center are capable of maintaining larger tidal radii; the latter indicates that the cluster distribution in M31 favors the existence of extended clusters; more than that of the Milky Way Galaxy.

Figure 12 is the Cumulative Fraction Distribution of the globular clusters of M31 and the Milky Way, superimposed. It quantitatively supports that a much greater fraction of globular clusters in M31 are located further out from the Galactic Center than those of the Milky Way.

Table 1 lists the number of surviving extended globular clusters in the Milky Way Galaxy for the various extended cluster magnitudes and potentials computed using the King Formula, r_k (King, 1962). The results indicate that as many as 9 and as few as 6 extended globular clusters are expected to survive in the known (Allen 48) and estimated orbits.

Table 2 lists the number of surviving extended globular clusters in the Milky Way Galaxy for the various extended cluster magnitudes and potentials computed using the alternative formulation for estimating the tidal radius, $r^\star$ (Allen, 2006). The results indicate that as many as 9 and as few as 5 extended globular clusters are expected to survive in the known (Allen 48) and estimated orbits.

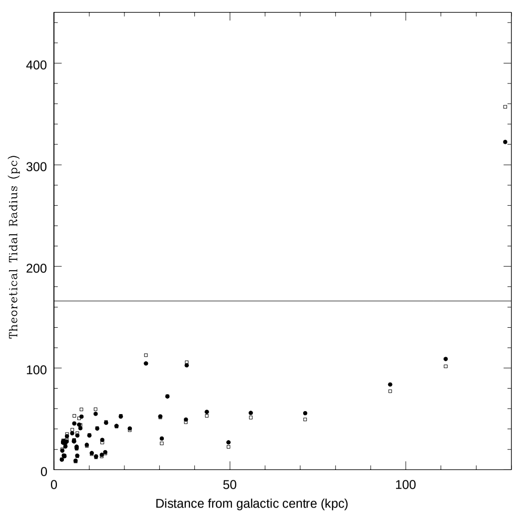
Tables 1 and 2 indicate that in the most conservative estimate of expected extended clusters in the Milky Way Galaxy, one would expect to see at least 5 extended globular clusters, given the cluster population size and distribution.

Tables 3, 4 and 5 list the results of an experiment to determine the number of surviving extended globular clusters which have experienced tidal shredding. The number of surviving extended globular clusters with computed tidal radii in the range of the half light radius, R_h and the physical tidal radius, R_t, of Huxor (2005) are counted in order to determine how many could survive, having experienced varying degrees of tidal shredding.

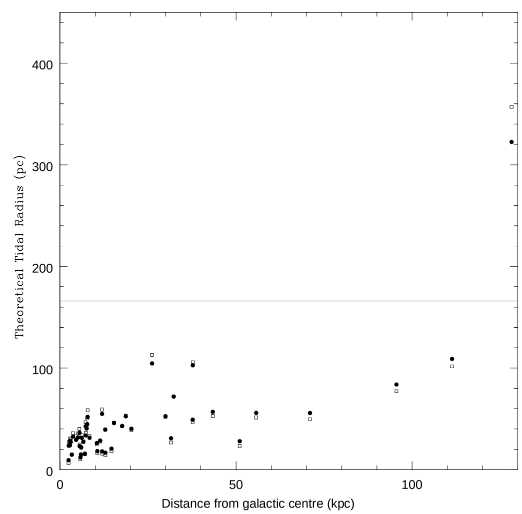
Table 6 lists the total number of extended globular clusters one would expect to see in the Milky Way if the definition of extended cluster was "relaxed", such that it included extended clusters having experienced varying degrees of tidal shredding. The results indicate that a large number of extended clusters do survive in the Milky Way orbits with computed tidal radii with values less than but close to the values of the physical tidal radii reported in Huxor (2005). In fact, as many as 17 and as few as 7 extended clusters are expected to survive according to this criteria; and since a total of 13 extended globular clusters were found in M31, the stated values are comparable. Hence, it is indeed possible for the same number of extended clusters found in M31; to survive in the orbits of the Milky Way, provided that the tidal shredding experienced by the clusters is minimal ($\approx$ 30 pc lost).

Table 7 lists the categories and corresponding fractions of clusters found in M31. Of the known globular clusters in M31, only 3% classify as extended. However, given the fact that more than half of the total clusters found, classify as candidate globular clusters; it is a possibility that more extended globular clusters do in fact exist, but have yet to be identified. It seems reasonable to hypothesize that more extended globular clusters exist in M31 given the fact that the known globular cluster distribution favors the existence of extended clusters, unless there are other factors affecting the survival of extended globular clusters which have yet to be explored.

Table 8 lists the number of extended globular clusters one would expect to see in the Milky Way if the same fraction of extended clusters existed in the Milky Way as in M31. Interestingly, despite the fact that extended globular clusters are incredibly rare in M31 (3%), one would still expect to see 4 extended clusters in the Milky Way Galaxy, which makes the fact that none are found; rather unusual.

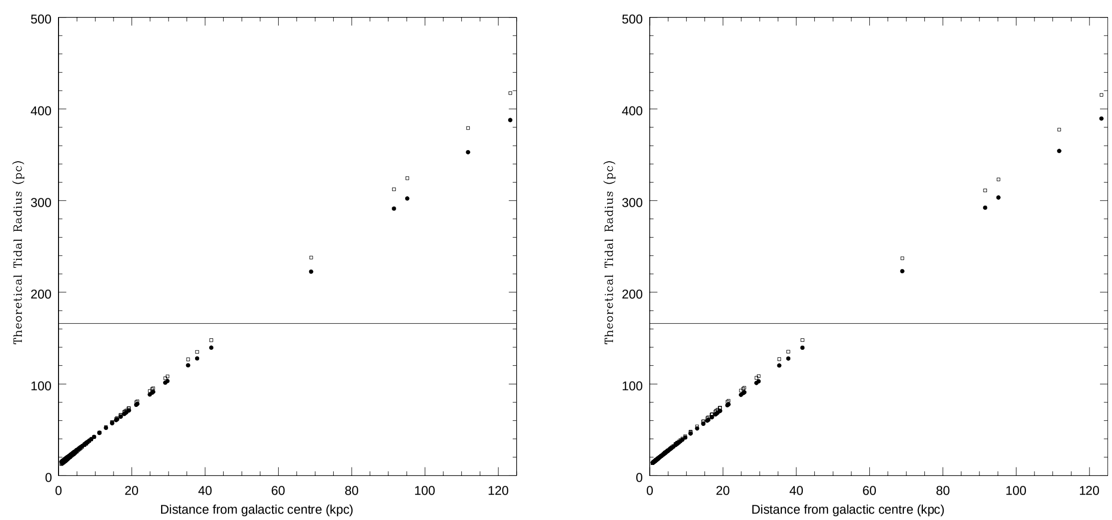


(a)



(b)

Figure 8: Theoretical Tidal Radii (pc) as a function of Distance from the Galactic Centre (kpc) for the known orbits (Allen 48) of the Milky Way, computed for $M_v = -7.3$ for (a) axisymmetric and (b) nonaxisymmetric potentials with horizontal lines indicating the physical tidal radius (166 pc) of the extended cluster (M31) of the same absolute magnitude, for comparative purposes. Squares and filled circles indicate the tidal radius computed for the alternative approximate formula (Allen, 2006), $r^{\star}$, and the King formula (King, 1962), r_k, respectively. Note: The computed radii represent the tidal radii the extended cluster of M31 would have if placed the known orbits of the Milky Way. An overwhelming majority of said known orbits cannot accommodate globular clusters of 166 pc or greater and no obvious difference between the plots of the two potentials is evident.



(a) (b)

Figure 9: Estimated Theoretical Tidal Radii (pc) as a function of distance from the Galactic Centre (kpc) for the orbits for which Milky Way orbital information is not available, computed for $M_v = -7.3$ for (a) axisymmetric and (b) nonaxisymmetric potentials, with horizontal lines indicating the physical tidal radius (166 pc) of the extended clusters (M31) of the same absolute magnitude, for comparative purposes. Squares and filled circles indicate the tidal radius computed for the alternative approximate formula (Allen, 2006), $r^{\star}$, and the King formula (King, 1962), r_k, respectively. Note: The computed radii represent the tidal radii the extended cluster of M31 would have if placed the estimated orbits of the Milky Way. No obvious difference between the plots of the two potentials is evident, as before.

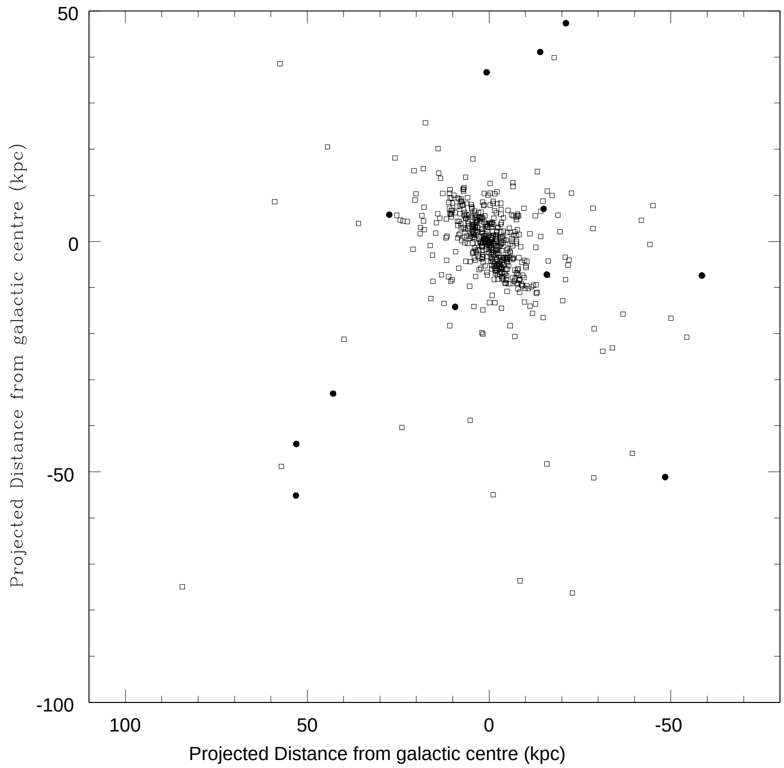


Figure 10: M31 globular cluster distribution, in projection (kpc scale), relative to the Galactic Center, where squares and filled circles represent globular clusters and extended globular clusters, respectively. Note: The 13 extended globular clusters found (Mackey, 2006); are a considerable distance from the M31 Galactic Centre.

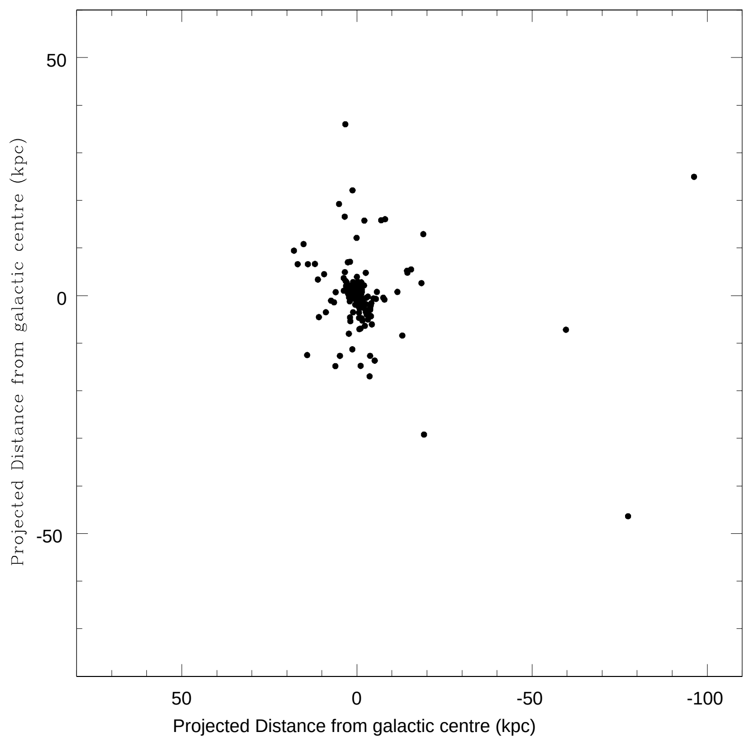


Figure 11: Milky Way galaxy coordinates transformed to the orientation of M31, in projection (kpc scale), relative to the Galactic Center. Milky Way globular cluster distribution seen here is what one would see if the Milky Way was viewed from the perspective from which one currently observes M31. Note: Clusters appearing close in proximity may actually be a considerable distance from one another since they are projected in an arbitrary plane.

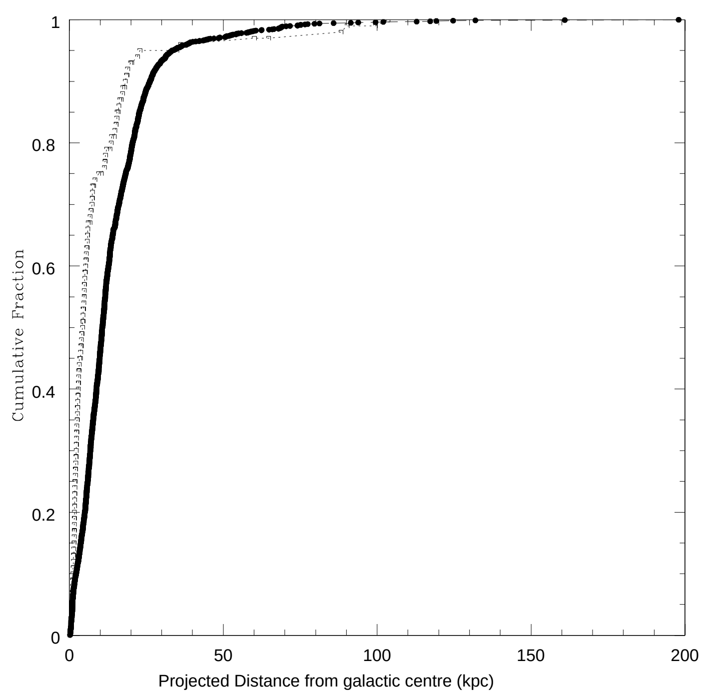


Figure 12: Cumulative Fraction Distribution of Milky Way and M31 clusters, superimposed, as a function of projected distance from the galactic centre (kpc). Squares represent Milky Way globular clusters and filled circles, M31 regular and extended globular clusters. The distributions illustrate the disimilarity between the globular cluster population distribution of both galaxies. In particular, an overwhelming majority of the Milky Way globular clusters are concentrated closer to their galactic centre than are those of M31. Note: Reoriented Milky Way globular clusters are compared to the M31 clusters in this image. Further, globular clusters are far more numerous in M31. By inspection, a distance of $\sim$10 kpc from the galactic centre comprises $\sim$0.80 of the Milky Way globular cluster population but only $\sim$0.55 of the M31 population.

Potential; Extended globular cluster properties	Known: $r_k > R_t$	Estimated: $r_k > R_t$	Number of surviving extended clusters
ASP; $M_v = -7.3$ $R_h = 34$ pc $R_t = 166$ pc	0.02	0.05	6
NASP; $M_v = -7.3$ $R_h = 34$ pc $R_t = 166$ pc	0.02	0.05	6
ASP; $M_v = -7.8$ $R_h = 26$ pc $R_t = 132$ pc	0.02	0.08	9
NASP; $M_v = -7.8$ $R_h = 26$ pc $R_t = 132$ pc	0.02	0.08	9
ASP; $M_v = -7.2$ $R_h = 26$ pc $R_t = 140$ pc	0.02	0.06	7
NASP; $M_v = -7.2$ $R_h = 26$ pc $R_t = 140$ pc	0.02	0.05	6

Table 1: Fraction of extended globular clusters with computed tidal radii greater than the observed tidal radii, R_t, of the extended clusters of Huxor (2005), computed in both the known Allen 48 orbits of (Allen, 2006) and the estimated orbits. The fractions listed pertain to calculations using King's Formula (1962), r_k. R_h is the extended cluster's half light radius (Huxor, 2005). The potential and extended cluster parameters are listed in the leftmost column where the abbreviations, ASP and NASP are used to denote the Axisymmetric and Nonaxisymmetric Potentials, respectively. In addition, the corresponding number of surviving extended clusters are listed in the last column. The latter was computed using a weighted average of the fractions listed and the cluster population in each set (Allen 48 and Non Allen clusters).

Potential; Extended globular cluster properties	Known: $r^\star > R_t$	Estimated: $r^\star > R_t$	Number of surviving extended clusters
ASP; $M_v = -7.3$ $R_h = 34$pc $R_t = 166$pc	0.02	0.05	6
NASP; $M_v = -7.3$ $R_h = 34$pc $R_t = 166$pc	0.02	0.04	5
ASP; $M_v = -7.8$ $R_h = 26$pc $R_t = 132$pc	0.02	0.08	9
NASP; $M_v = -7.8$ $R_h = 26$pc $R_t = 132$pc	0.02	0.08	9
ASP; $M_v = -7.2$ $R_h = 26$pc $R_t = 140$pc	0.02	0.06	7
NASP; $M_v = -7.2$ $R_h = 26$pc $R_t = 140$pc	0.02	0.05	6

Table 2: Fraction of extended globular clusters with computed tidal radii greater than the observed tidal radii, R_t, of the extended clusters of Huxor (2005), computed in both the known Allen 48 orbits (Allen, 2006) and the estimated orbits. The fractions listed pertain to calculations using the alternative formulation for estimating the tidal radius (Allen, 2006), $r^\star$. R_h is the extended cluster's half light radius (Huxor, 2005). The potential and extended cluster parameters are listed in the leftmost column where the abbreviations, ASP and NASP are used to denote the Axisymmetric and Nonaxisymmetric Potentials, respectively. In addition, the corresponding number of surviving extended clusters are listed in the last column. The latter was computed using a weighted average of the fractions listed and the cluster population in each set (Allen 48 and Non Allen clusters).

Potential; Extended globular cluster properties; tidal radius formulation	tidal $R_h \leq$ radius $\leq 2R_h$ (34pc)	tidal $2R_h \leq$ radius $\leq 3R_h$ (34pc)	tidal $3R_h \leq$ radius $\leq 4R_h$ (34pc)	tidal $4R_h \leq$ radius $\leq R_t$ (30pc)
ASP; $M_v = -7.3$ $R_h = 34$ pc $R_t = 166$ pc r_k	49	6	4	1
NASP; $M_v = -7.3$ $R_h = 34$ pc $R_t = 166$ pc r_k	46	6	4	1
ASP; $M_v = -7.3$ $R_h = 34$ pc $R_t = 166$ pc $r^\star$	54	7	4	3
NASP; $M_v = -7.3$ $R_h = 34$ pc $R_t = 166$ pc $r^\star$	52	7	4	3

Table 3: This table lists the number of extended clusters that would survive in the orbits of the Milky Way with computed tidal radii in the range between the half light radius, R_h, and the physical tidal radius, R_t of Huxor (2005), with the range divided in multiples of R_h. The corresponding tidal radii are computed using the King Formula (1962), r_k, and the alternative formulation (Allen, 2006), $r^\star$. The R_h value is listed in the leftmost column for the given potential alongside the extended cluster parameters. The abbreviations, ASP and NASP are used to denote the Axisymmetric and Nonaxisymmetric Potentials, respectively. The purpose of this test was to relax the definition of "extended cluster" to include extended clusters that have undergone some degree of tidal shredding and observe how many additional clusters survive.

Potential; Extended globular cluster properties; tidal radius formulation	tidal $R_h \leq$ radius $\leq 2R_h$ (26pc)	tidal $2R_h \leq$ radius $\leq 3R_h$ (26pc)	tidal $3R_h \leq$ radius $\leq 4R_h$ (26pc)	tidal $4R_h \leq$ radius $\leq R_t$ (28pc)
ASP; $M_v = -7.8$ $R_h = 26$ pc $R_t = 132$ pc r_k	71	19	8	4
NASP; $M_v = -7.8$ $R_h = 26$ pc $R_t = 132$ pc r_k	70	19	8	6
ASP; $M_v = -7.8$ $R_h = 26$ pc $R_t = 132$ pc $r^\star$	70	20	8	8
NASP; $M_v = -7.8$ $R_h = 26$ pc $R_t = 132$ pc $r^\star$	68	19	7	6

Table 4: This table lists the number of extended clusters that would survive in the orbits of the Milky Way with computed tidal radii in the range between the half light radius, R_h, and the physical tidal radius, R_t of Huxor (2005), with the range divided in multiples of R_h. The corresponding tidal radii are computed using the King Formula (1962), r_k, and the alternative formulation (Allen, 2006), $r^\star$. The R_h value is listed in the leftmost column for the given potential alongside the extended cluster parameters. The abbreviations, ASP and NASP are used to denote the Axisymmetric and Nonaxisymmetric Potentials, respectively. The purpose of this test was to relax the definition of "extended cluster" to include extended clusters that have undergone some degree of tidal shredding and observe how many additional clusters survive.

Potential; Extended globular cluster properties; tidal radius formulation	tidal $R_h \leq$ radius $\leq 2R_h$ (26pc)	tidal $2R_h \leq$ radius $\leq 3R_h$ (26pc)	tidal $3R_h \leq$ radius $\leq 4R_h$ (26pc)	tidal $4R_h \leq$ radius $\leq R_t$ (36pc)
ASP; $M_v = -7.2$ $R_h = 26$ pc $R_t = 140$ pc r_k	33	12	6	2
NASP; $M_v = -7.2$ $R_h = 26$ pc $R_t = 140$ pc r_k	40	14	8	5
ASP; $M_v = -7.2$ $R_h = 26$ pc $R_t = 140$ pc $r^\star$	34	10	5	4
NASP; $M_v = -7.2$ $R_h = 26$ pc $R_t = 140$ pc $r^\star$	39	8	6	7

Table 5: This table lists the number of extended clusters that would survive in the orbits of the Milky Way with computed tidal radii in the range between the half light radius, R_h, and the physical tidal radius, R_t of Huxor (2005), with the range divided in multiples of R_h. The corresponding tidal radii are computed using the King Formula (1962), r_k, and the alternative formulation (Allen, 2006), $r^\star$. The R_h value is listed in the leftmost column for the given potential alongside the extended cluster parameters. The abbreviations, ASP and NASP are used to denote the Axisymmetric and Nonaxisymmetric Potentials, respectively. The purpose of this test was to relax the definition of "extended cluster" to include extended clusters that have undergone some degree of tidal shredding and observe how many additional clusters survive.

Potential; Extended globular cluster properties; tidal radius formulation	tidal radius $\geq 2\mathrm{R_h}$	tidal radius $\geq 3\mathrm{R_h}$	tidal radius $\geq 4\mathrm{R_h}$
ASP; $\mathrm{M_v} = -7.3$ $\mathrm{R_h} = 34$ pc $\mathrm{R_t} = 166$ pc r_k; $r^\star$	17; 20	11; 13	7; 9
NASP; $\mathrm{M_v} = -7.2$ $\mathrm{R_h} = 34$ pc $\mathrm{R_t} = 166$ pc r_k; $r^\star$	17; 19	11; 12	7; 8
ASP; $\mathrm{M_v} = -7.8$ $\mathrm{R_h} = 26$ pc $\mathrm{R_t} = 132$ pc r_k; $r^\star$	40; 45	21; 25	13; 17
NASP; $\mathrm{M_v} = -7.8$ $\mathrm{R_h} = 34$ pc $\mathrm{R_t} = 132$ pc r_k; $r^\star$	42; 41	23; 22	15; 15
ASP; $\mathrm{M_v} = -7.2$ $\mathrm{R_h} = 26$ pc $\mathrm{R_t} = 140$ pc r_k; $r^\star$	27; 26	15; 16	9; 11
NASP; $\mathrm{M_v} = -7.2$ $\mathrm{R_h} = 26$ pc $\mathrm{R_t} = 140$ pc r_k; $r^\star$	33; 27	19; 19	11; 13

Table 6: Number of total surviving extended globular clusters with computed tidal radii greater than multiples of the half light radius, $\mathrm{R_h}$ of Huxor (2005). This table lists the total number of extended clusters one would expect to see if the number of extended clusters in the Milky Way included those having undergone varying degrees of tidal shredding. The values are listed alongside one other for the King formula (King, 1962), $\mathrm{r_k}$, and the alternative formulation of the tidal radius (Allen, 2006), $\mathrm{r}^\star$, respectively. The abbreviations, ASP and NASP are used to denote the Axisymmetric and Nonaxisymmetric Potentials, respectively.

Cluster Classification	Number of clusters	Total number of clusters (candidate globular clusters included)	Total number of clusters (candidate globular clusters removed)	Fraction of total clusters (candidate globular clusters included)	Fraction of total clusters (candidate globular clusters removed)
Globular Cluster	509	1571	522	0.32	0.97
Candidate Globular Cluster	1049	1571	522	0.67	0
Extended Globular Cluster	13	1571	522	0.01	0.03

Table 7: Globular cluster statistics for M31: Extended cluster fraction of total M31 globular cluster population (including candidate globular clusters), Extended cluster fraction of total M31 globular cluster population with candidate globular clusters removed and Candidate globular cluster fraction of total globular cluster population, respectively.

Ratio of extended globular clusters to regular globular clusters observed in M31	Number of expected extended globular clusters in the Milky Way
13 : 509	4

Table 8: Number of extended globular clusters one would expect to see in the Milky Way Galaxy if the same ratio of extended to regular globular clusters existed in the Milky Way as in M31, assuming a Milky Way globular cluster population of 150.

5 Discussion

In this section, the main questions regarding the plausibility of extended globular clusters existing in the Milky Way Galaxy; found in the Introduction of the report, are revisited.

Since no extended globular clusters have been found in the Milky Way Galaxy to date, it was necessary to investigate what factors favor or disfavor the existence of extended clusters in the first place. More specifically, the influence of tidal shredding and cluster proximity to the Galactic Center were considered. It is not known whether extended clusters have existed in the Milky Way Galaxy at one time; since none exist presently. Hence, it was meaningful to test the conditions under which extended globular clusters could survive; by placing them in the orbits of the Milky Way Galaxy. The result of this test was that an overwhelming majority of the extended globular clusters were not able to survive in the known (Allen 48) and estimated orbits of the Milky Way, but a small minority of the clusters could. All of the cases, where extended clusters were substituted in the orbits of the Milky Way, resulted in non zero numbers of surviving extended clusters. In fact, a lower limit of 5 extended globular clusters could be accommodated in the Milky Way, under the current conditions. The latter indicates that tidal shredding in the orbits of the Milky Way could have played a significant role in reducing the number of extended globular clusters in the Milky Way; if they indeed existed, however, it is not a legitimate mechanism for removing all of them. It is therefore a curious fact that no extended clusters exist in the Milky Way at all.

In addition, it was necessary to investigate the effect of tidal shredding on the extended clusters to a further extent; and determine whether it is a reasonable method of removing them from the Milky Way. By considering the subset of clusters with computed tidal radii in the region between the half light radius and the physical tidal radius, reported in Huxor (2005), it was possible to determine how many of these clusters could still survive. The results reveal that if one defines an extended cluster to include extended clusters having computed tidal radii approximately 30 pc smaller than those reported in Huxor (2005), then as many as 17 and as few as 7 extended globular clusters would be expected to survive in orbits of the Milky Way. Since a total of 13 extended clusters were discovered in M31, it would not be unreasonable to expect to see the same number of extended clusters in the Milky Way as are currently observed in M31.

By comparing the distribution of clusters in M31 and Milky Way galaxies, it became apparent that the cluster distributions in the two Galaxies differed significantly and that the cluster distribution in the Milky Way could not be considered representative of the distribution found in M31. In addition, since the M31 clusters are located further out from the Milky Way Galactic Center, it would appear that the cluster distribution in the Milky Way disfavors the existence of extended globular clusters. Hence, it would appear as that the cluster distribution in the Milky Way could begin to explain the lack of extended clusters observed. However, all of the calculations performed, indicate that one would expect at least 5 extended globular clusters to survive in the orbits of the Milky Way. Consequently, the Milky Way cluster distribution may not favor the survival of very many extended clusters, but fails to explain why none are seen at all. Furthermore, if one considers the Milky Way to have the same ratio of extended clusters as M31, one would expect to see 4 extended clusters in the Milky Way. This estimate does not consider cluster distribution, however, when cluster distribution is considered, the lower limit of surviving extended clusters is 5. Hence, none of the tests performed indicate that one should observe no extended clusters in the Milky Way Galaxy.

When considering the conditions under which extended clusters could exist in the Milky Way, it is apparent that the current conditions in the Milky Way could support the existence of extended clusters already. The fact that none are actually observed, suggests that either no extended clusters existed in the Milky Way in the first place or perhaps that there were extended clusters in the Galaxy, but that they were somehow removed by factors that have yet to be explored.

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