

Bachelor Thesis

Decay of a light pseudo-scalar Higgs  
boson to the  $\Upsilon$ -meson and a photon

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## Abstract

This paper is concerned with calculating the branching ratio of the decay  $A_1 \rightarrow \Upsilon + \gamma$  of a pseudo-scalar Higgs boson, where  $\Upsilon$  is a bound vector meson state of the quarks  $b$  and  $\bar{b}$ . The results will then be applied to the phenomenology of the Next-to-Minimal Supersymmetric Standard Model in the case where  $A_1$  has a mass in the interval  $9.46 - 12 \text{ GeV}$ . The paper will review the concept of spontaneous symmetry breaking through a few simple examples and present the Higgs mechanism of the Standard Model. Following this is a general introduction to supersymmetric extensions of the Standard Model and a discussion of the Minimal Supersymmetric Standard Model and Next-to-Minimal Supersymmetric Standard Model. After a development of the formalism for treating decays to bound states the branching ratio for the decay  $A_1 \rightarrow \Upsilon + \gamma$  is computed at tree-level in the limit where the relative velocities of the  $b$ -quarks in  $\Upsilon$  are non-relativistic. For Higgs particle masses in the interval  $9.66 - 10.5 \text{ GeV}$  it is shown that the branching ratio  $\text{BR}(A_1 \rightarrow \Upsilon + \gamma)$  is large enough so that the decay channel could give an opportunity to detect the pseudo-scalar Higgs boson in that mass range.

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# 1 Introduction

Despite its unfortunate name, the Standard Model of particle physics is a very successful theory of the smallest scale physics, and it agrees with all data provided by accelerator experiments so far. It does however suffer from a number of theoretical shortcomings and leaves a lot unexplained. The most obvious such shortcomings are that it does not include gravity and does not in a natural way extend into the unexplored region between current physics at the electroweak scale and possible new physics at the Planck scale. This is a region extending over more than fifteen orders of magnitude, and it would be naïve not to expect some new phenomena to appear. There is also the problem of Grand Unification, which is an attempt to consolidate the three fundamental forces described by the Standard Model at some very high energy. This would happen if the running couplings of the forces came together there, but in this framework it seems to not really work out. Then there are things like the large number of free parameters in the theory, which at the moment are not calculable and whose values have to be put in by hand from experiment. These include fermion masses, electric charges, color factors and mixing angles, and to incorporate neutrino masses becomes even more *ad hoc*. At the other end of the spectrum, there is still no description of gravity, no satisfactory mechanism explaining the baryon asymmetry of the universe and there is no good candidate for cold dark matter. Finally it is worth to mention the hierarchy problem, which is concerned with large radiative corrections to the Higgs mass, and the fact that the Higgs boson so far has not been detected.

Many of these problems are solved by introducing supersymmetry as a fundamental symmetry of Nature, while others remain unsolved. This paper will give a general introduction to supersymmetric theories in chapter three followed by a look at the simplest and most common of these that are phenomenologically realistic. Before that comes a recapitulation of spontaneous symmetry breaking covering a few easy examples but also the Standard Model case. Then in chapter four a computation will be carried out in the framework of the Next-to-Minimal Supersymmetric Standard Model, of the decay of a possibly light pseudo-scalar Higgs boson  $A_1$  to an  $\Upsilon$ -meson and a photon. Finally in chapter five there will be a discussion relating the theoretically obtained results to possible experimental signals.

Readers familiar with spontaneous symmetry breaking and the Higgs sector of the Next-to-Minimal Supersymmetric Standard Model can if they wish skip the first two chapters and go directly to chapter four.

## 2 Spontaneous symmetry breaking

Probably the most intense research in particle physics today, if not in all of physics, is concerned with explaining, understanding and finding proof of the Higgs mechanism. This mechanism was introduced as a remedy to the obviously outrageous fact that without it all Standard Model particles are massless, and since then a ravaging search for the predicted Higgs boson has been going on. The very heart of the Higgs mechanism is the concept of spontaneous symmetry breaking, which is when the ground state solution of a theory breaks a symmetry apparent in the equations governing it. More technically, spontaneous symmetry breaking is when the Lagrangian of a theory is independent under certain symmetry transformations, but there exists a vacuum solution that is not. In this chapter this idea will be developed in stages, starting with some simple cases and ending with the Higgs mechanism of the Standard Model. At the end of the chapter the hierarchy problem will be presented as a motivation of the need to develop theories beyond the Standard Model. The material covered below is in large based on [1], [2] and [3].

### 2.1 The discrete case

To begin with, consider a real scalar field  $\phi(x)$  described by the potential<sup>1</sup>

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4$$

This potential has two minima for  $\phi \neq 0$  that can be found by solving the equation  $V' = 0$ , and they are given by  $\phi = \pm\mu/\lambda = v$ . Using the fact that the physics is invariant under addition of a constant to the potential, this can be rewritten as follows

$$V(\phi) = \frac{1}{4}\lambda^2(\phi^2 - v^2)^2$$

The system must be at one of the points  $\phi = \pm v$ , but choosing one spontaneously breaks its original symmetry. A thing to notice is that the systems vacuum state corresponds to a non-vanishing value of  $\phi$ , the field obtains a *vacuum expectation value* (vev).

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<sup>1</sup>A relativistic quantum theory is determined by the Lagrange function  $\mathcal{L} = K - V$ , where  $K$  gives the kinetic and  $V$  the potential energy terms of the fields. Since the kinetic terms only depends on the spin of the particle and thus always stays the same, it is sufficient here to study the potential.

## 2.2 $U(1)$ -symmetries

Now consider a complex scalar field  $\phi$  described by the potential

$$V(|\phi|) = \frac{1}{2}\lambda^2\left(|\phi|^2 - \frac{1}{2}v^2\right)^2$$

This potential is invariant under the transformation  $\phi \rightarrow \phi' = e^{i\alpha}\phi$  corresponding to a rotation in the complex plane. Since  $\alpha$  is a constant and thus independent of position the potential is said to possess a global  $U(1)$ -symmetry. A minimum is obtained for every  $\phi$  that fulfills  $|\phi| = v/\sqrt{2}$ , that is all points in the complex plane on the circle of radius  $v/\sqrt{2}$  centered at the origin. The system must be at such a point, call it  $v$ , which spontaneously breaks the symmetry, and once it is chosen the field can be expanded around it like

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \chi(x) + i\sigma(x))$$

where  $\chi$  is the component of the field perpendicular to the circle and  $\sigma$  the tangential component. Entering this into the potential gives

$$V = \frac{1}{2}\lambda^2 \left( \frac{1}{2}(v + \chi + i\sigma)^2 - \frac{1}{2}v^2 \right)^2 = \frac{1}{8}\lambda^2(\chi^2 + 2v\chi + \sigma^2)^2$$

By identifying the mass terms here it can be seen that the perpendicular component  $\chi$  of the field  $\phi$  has acquired a mass  $m_\chi = v\lambda$  while the tangential component  $\sigma$  remains massless.<sup>2</sup> This is easy to understand intuitively from the shape of the potential since moving in the tangential direction means going from one minimum to another, something that hardly would cost any energy. It is interesting however that this massless particle seems to be an effect of the spontaneous symmetry breaking, and in fact it is a general feature of all global symmetries broken this way. The massless particle is commonly referred to as a *Nambu-Goldstone boson*.

Now let  $\phi$  be a complex scalar like before, but instead described by the Lagrangian

$$\mathcal{L} = |D_\mu\phi|^2 - \frac{1}{2}\lambda \left( |\phi|^2 - \frac{v^2}{2} \right)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

This Lagrangian is invariant under the transformation  $\phi \rightarrow \phi' = e^{i\alpha(x)}\phi$  due to the introduction of the gauge field tensor  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  and the covariant derivative  $D_\mu = \partial_\mu - igA_\mu$ , where the gauge field transforms as  $A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu\alpha/g$  and  $g$  is the charge of the field. It is said to possess a local symmetry since  $\alpha$  is a function of position and

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<sup>2</sup>The mass term of a scalar field  $S$  is  $\frac{1}{2}m_S^2S^2$

can be different at different points in space. The field  $\phi$  can be written as  $\phi = \rho(x)e^{i\theta(x)}$  where  $\rho$  and  $\theta$  are both real, and without loss of generality  $\alpha$  can be chosen at each point in a way such that it cancels  $\theta$ . Then  $\phi$  will be real everywhere and can be expanded around the minimum  $v$  like so

$$\phi(x) = \rho(x) = \frac{1}{\sqrt{2}}(v + \chi(x))$$

When this is entered into the Lagrangian, leaving out the kinetic gauge terms, the following is obtained

$$\begin{aligned} \mathcal{L} &= |(\partial_\mu - ieA_\mu)\phi(x)|^2 - \frac{\lambda^2}{2} \left( \frac{1}{2}(v + \chi)^2 - \frac{v^2}{2} \right)^2 \\ &= \frac{1}{2}(\partial_\mu\chi(x))^2 + \frac{e^2}{2}A_\mu A_\nu (v^2 + 2v\chi(x) + \chi(x)^2) - \frac{\lambda^2}{8}[2v + \chi(x)]^2\chi(x)^2 \end{aligned}$$

Again identifying the mass terms it is apparent that the scalar field  $\chi$  as well as the gauge field  $A_\mu$  have acquired masses given by  $m_\chi = v\lambda$  and  $m_A = vg$ . In contrast to what happened in the case of a global symmetry, the massless Goldstone boson have disappeared and instead become the longitudinal part of the gauge field. It is said that the Goldstone boson is eaten by the gauge field or that the massless degree of freedom is gauged away. As a final remark to end this section, bear in mind that spontaneous breaking of a global symmetry introduces massless particles, while in the breaking of a local symmetry these degrees of freedom are used to give mass to gauge bosons. This way of giving mass to particles is what is called the *Higgs mechanism*.

### 2.3 Local $SU(2)$ -symmetry

Apart from the  $U(1)$ -symmetry that gives rise to the photon field, the Standard Model also incorporates a  $SU(2)$ -symmetry group responsible for the weak interactions. The  $SU(2)$ -group is different from the  $U(1)$  in that it distinguishes between helicities, i.e. it couples to left-handed but not to right-handed particles. The left-handed Standard model particles are put into so called  $SU(2)$ -doublets each containing two particles while the right-handed particles are said to be  $SU(2)$ -singlets. To understand spontaneous symmetry breaking of a local  $SU(2)$ -symmetry, consider a complex scalar doublet  $\Phi$  given by

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

and described by the Lagrangian

$$\mathcal{L} = |D_\mu \Phi|^2 - \frac{\lambda^2}{2} \left( |\Phi|^2 - \frac{v^2}{2} \right)^2 - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} \quad (1)$$

Here the covariant derivative is  $D_\mu = \partial_\mu - ig\tau_a W_\mu^a$  where  $\tau_a$  are the Pauli matrices,  $g$  the weak coupling constant and  $W_\mu^a$  the gauge fields. The magnitude of the field  $\Phi$  is  $|\Phi|^2 = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)$  and it is clear that the potential has a minimum when  $|\Phi|^2 = v^2/2$ . Using the gauge freedom of the  $SU(2)$ -group the field can be rotated to get  $\Phi = \phi_3$  at each point, which is called the *'t Hooft gauge*, and in the same way as before the field can be expanded around the chosen minimum

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

Using this in the kinetic part for the field  $\Phi$  in the Lagrangian gives

$$\begin{aligned} \mathcal{L}_{\text{Kin}} &\simeq \frac{g^2}{8} \left| \tau_a W_\mu^a \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2 = \frac{g^2}{8} \left| \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2 \\ &= \left(\frac{gv}{2}\right)^2 W_\mu^+ W_\mu^- + \frac{1}{2} \left(\frac{gv}{2}\right)^2 W_\mu^3 W_\mu^3 \end{aligned}$$

where  $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2)$ . In the same way as for the local  $U(1)$ -symmetry the gauge bosons  $W^\pm$  and  $W^3$  of the  $SU(2)$ -group acquire masses through the spontaneous breaking of the local symmetry, and these are given by  $m_{W^\pm} = m_{W^3} = gv/2$ . Had the symmetry instead been global its spontaneous breaking would have given rise to no less than three Goldstone bosons.

## 2.4 The Standard Model case

As a final example the Higgs mechanism of the Standard model will be studied. As mentioned in the section above the electroweak sector of the Standard Model incorporates both of the symmetry groups  $U(1)$  and  $SU(2)$ , which are combined to form the new group  $SU(2) \times U(1)$ . The complex scalar  $\Phi$  and the Lagrangian are as given in the section above with the covariant derivative

$$D_\mu = \partial_\mu - ig \frac{\tau_a}{2} W_\mu^a - ig' \frac{Y}{2} B_\mu$$

Here  $Y$  is the hypercharge of the particle and  $B_\mu$  the gauge boson of the  $U(1)$ -symmetry. Like in the previous section the gauge freedom of the symmetry group can be used to rotate the field in such a way that at all points the vacuum expectation value is in the  $\phi_3$  direction, and the field can be

expanded around the minimum in exactly the same way. Substituting into the Lagrangian and considering the kinetic part gives

$$\begin{aligned}
\mathcal{L}_{\text{Kin}} &\simeq \left| \left( -ig \frac{\tau^a}{2} W_\mu^a - ig' \frac{Y}{2} B_\mu \right) \Phi_0 \right|^2 \\
&= \frac{1}{8} \left| \begin{pmatrix} gW_\mu^3 + g'B_\mu & \sqrt{2}W_\mu^- \\ \sqrt{2}W_\mu^+ & -gW_\mu^3 + g'B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2 \\
&= \left( \frac{gv}{2} \right)^2 W_\mu^+ W_\mu^- + \frac{v^2}{8} \begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}
\end{aligned}$$

It is easy to see that the charged gauge bosons acquire mass, but in the neutral case what happens is a little more subtle. When the matrix above is diagonalized it can be demanded that one of its eigenstates remains massless while one eigenstate acquire a mass similar to the  $W^\pm$ -bosons; these are of course the photon and the  $Z$ -boson that are given by

$$A_\mu = \frac{gB_\mu + g'W_\mu^3}{\sqrt{g^2 + g'^2}} \quad Z_\mu = \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2 + g'^2}}$$

That the photon manages to remain massless despite the spontaneous breaking of the symmetries is because when this happens the original gauge group  $SU(2)_L \times U(1)_Y$  is reduced to the new group  $U(1)_{em}$ . The generator of this group is a particular linear combination of  $\tau$  and  $Y$  giving a symmetry operation of the vacuum and whose corresponding gauge field thus is massless. This is the electric charge generator given by  $Q = T^3 + Y/2$  where  $T^3$  is the generator of the third component of weak isospin. By looking at how the photon field couples to e.g. electrons and neutrinos it is possible to make a connection between the electroweak charges and the electric charge, and from this relation define the weak mixing angle  $\theta_W$  in the following way

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}} \quad \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

By working out the Lagrangian in detail and identifying the mass terms it can be shown that the Standard Model predicts the relation  $M_Z = M_W / \cos \theta_W$  between the masses of the  $Z$ - and  $W^\pm$ -bosons. This has been tested experimentally and has been found to hold to very high precision.

To give mass to the fermions of the theory one has to introduce so called Yukawa terms to the Lagrangian through which they couple to the Higgs field. The down-type quarks and the charged leptons acquire their mass through terms of the form  $-\lambda_f \bar{f} f H$ , while the up-type quarks and the neutrinos get mass from the conjugate  $H^*$  of the Higgs field through terms like

$\lambda_f \bar{f} f H^*$ . When the Higgs field acquire a vacuum expectation value  $H$  can be replaced by  $v + h(x)$ , which will lead to the terms  $-\lambda_f v \bar{f} f$  and  $-\lambda_f \bar{f} f h(x)$ . The first one of these is the one giving mass to the fermion field while the second one gives the coupling of the fermion to the Higgs particle.

## 2.5 The hierarchy problem

The Higgs field not only gives mass to the particles of the Standard Model, it also gives mass to itself. The part of the Lagrangian potential only involving the Higgs boson is

$$V = m_H^2 |H|^2 + \lambda |H|^4$$

as can be obtain by expanding Eq. 1 around the vacuum expectation value of the Higgs field. When the electroweak symmetry is broken the Higgs field acquires a vacuum expectation value  $v = \sqrt{-m_H^2/2\lambda}$  that has been measured, in processes involving the weak interaction, to be roughly  $174 \text{ GeV}$ . This fixes the probable mass squared of the Higgs boson around the order of  $(100)^2 (\text{GeV})^2$ . Unfortunately however this mass receives large quantum corrections from loop diagrams like the ones in Figure 1 below, pushing it unacceptable values. For fermions coupling to the Higgs boson like  $-\lambda_f H \bar{f} f$  the correction to the Higgs mass is given by [2]

$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2} \Lambda_{UV}^2 + \dots$$

where  $\Lambda_{UV}^2$  is an ultraviolet cutoff used to regulate loop integrals. The value of  $\Lambda$  is usually assumed to be around the Planck scale or at least at a much higher energy than the electroweak scale, since it is thought to indicate where new physics will appear. This will the give a correction to the tree-level mass about thirty orders of magnitude larger than the value obtained from experiments. For scalars coupling to the Higgs boson like  $-\lambda_S |H|^2 |S|^2$  the loop contribution to the mass is [2]

$$\Delta m_H^2 = \frac{|\lambda_S|^2}{16\pi^2} \Lambda_{UV}^2 + \dots$$

As can be seen this comes in with the opposite sign compared to the correction from the fermion, which opens for a possible solution of the problem presented in the next chapter.

It should be pointed out that the corrections to the Higgs mass not only depends on the ultraviolet cutoff, but also on the mass of the particle in the loop through the coupling  $\lambda = m/\sqrt{2}v$ . For the massive Standard Model particles, and especially for currently undiscovered particles of even

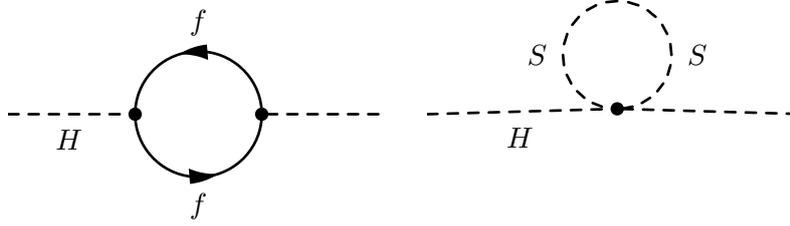


Figure 1: Diagrams contributing to the loop corrections of the Higgs mass

greater mass, this will lead to large corrections to the Higgs mass regardless of how  $\Lambda$  is chosen. Since this in turn will affect the vacuum expectation value  $v = \sqrt{2}\lambda_H m_H$  of the Higgs field it means that also the other Standard Model particles will receive corrections to their masses, though these will only depend on the mass logarithmically. The question arises why the Higgs mass is needed to be so small in comparison to the Planck scale when there seems to be a great amount of fine-tuning involved to keep the loop corrections from pushing it there. This is what is known as the *Hierarchy problem*.

## 3 Supersymmetry

The Standard Model of particle physics is an astonishing theory that explains all experimental data obtained so far at the large research facilities around the world. Nevertheless there are questions it cannot answer and things it cannot explain, which have led scientists to start exploring possible extensions of it. One such thing is the hierarchy problem presented at the end of the last chapter and another is the scale at which the electroweak symmetry is broken. These questions are both answered by supersymmetric extensions of the Standard Model, even if some of the answers lead to new questions. In this chapter the general theory of supersymmetric models will be presented and their common features pointed out, after which the two most developed theories will be studied in more detail. The chapter will end with a presentation of the Higgs sector of the Next-to-Minimal Supersymmetric Standard Model, which is what the rest of the paper will be concerned with. The material below is based on [2], [3], [4] and [6].

### 3.1 General concepts

#### 3.1.1 Notation

The main distinction between particles in the Standard Model is according to their spin. Particles with half-numbered spin are called fermions, behave according to Fermi-Dirac statistics and respect the Pauli exclusion principle; they make up the matter of the observable universe and are divided into quarks and leptons. Particles with whole-numbered spin are called bosons, behave according to Bose-Einstein statistics and do not respect the Pauli exclusion principle. These particles are thought to carry the currently known four fundamental forces, even though the particle mediating gravity is yet to be found. This category also includes the Higgs boson invoked in order to give mass to the other Standard Model particles.

In the Standard Model there is nothing connecting fermions to bosons, they are particles of completely different sorts. In some extensions of the Standard Model however, a new kind of symmetry is introduced relating fermions and bosons, and this is known as *supersymmetry*. Models incorporating this kind of symmetry are accordingly called *supersymmetric theories*. Basically, it is assumed that to each Standard Model particle there exists a *supersymmetric partner* or *superpartner* that is identical to the particle but differs by half a number in spin. The supersymmetric partners of fermions are called *sfermions* and have spin zero instead of spin one-half, while the supersymmetric partners of the gauge bosons are called *gauginos* and have

spin one-half instead of spin one. The names of the supersymmetric particles are obtained by putting *s-* in front of the name if it is the partner of a fermion and *-ino* after the name if it is the partner of a gauge boson. Thus e.g. the supersymmetric partner of the electron is called the *selectron*, the partner of the top quark is the *stop* and the photons partner is called the *photino*. Often the names *sleptons* and *squarks* are used to refer to the supersymmetric partners of leptons and quarks. The partner of the Higgs boson has spin one-half instead of spin zero and is called the *Higgsino*. All superpartners are denoted by putting a tilde on the letter normally used to denote the corresponding Standard Model particle, so e.g. the selectron is  $\tilde{e}$  and the Higgsino  $\tilde{H}$

The Standard Model particles and their supersymmetric partners are grouped together in what is usually referred to as a *supermultiplet* or a *superfield*. Each superfield contains one Standard Model particle and its supersymmetric partner, which can be obtained from each other by performing a supersymmetric transformation, the generators of which have to carry spin themselves in order to be able to change the spin of the particle they operate on. Superfields are denoted by hatted capital letters corresponding to the letters normally used to denote the Standard Model particles, and thus e.g. the superfield containing the electron and its superpartner is  $\hat{E}$  and that containing the Higgs boson and its partner is  $\hat{H}$ . The most common supermultiplets and their contents are shown in Table 1 below. It is customary to express the supersymmetric theories in terms only of left-handed fields, which is why all right-handed fields in the table are written with Hermitian or ordinary conjugates.

Notation	Spin zero	Spin one-half	Spin one
$\hat{U}$	$(\tilde{u}_L \quad \tilde{d}_L)$	$(u_L \quad d_L)$	
$\hat{u}$	$\tilde{u}_R^*$	$u_R^\dagger$	
$\hat{d}$	$\tilde{d}_R^*$	$d_R^\dagger$	
$\hat{L}$	$(\tilde{\nu} \quad \tilde{e}_L)$	$(\nu \quad e_L)$	
$\hat{e}$	$\tilde{e}_R^*$	$e_R^\dagger$	
$\hat{H}_u$	$(H_u^+ \quad H_u^0)$	$(\tilde{H}_u^+ \quad \tilde{H}_u^0)$	
$\hat{H}_d$	$(H_d^0 \quad H_d^-)$	$(\tilde{H}_d^0 \quad \tilde{H}_d^-)$	
		$\tilde{g}$	$g$
		$\tilde{W}^\pm \quad \tilde{W}^0$	$W^\pm \quad W^0$
		$\tilde{B}^0$	$B^0$

Table 1: Notation and content of the most common supermultiplets.

The Higgs sector of supersymmetric theories are different from the Standard Model in that it requires a minimum of two Higgs doublets. The easiest way to see this is by considering the Lagrangian of a theory, which will be done in the next section. Here it suffices to know that a supersymmetric Lagrangian must be an analytic function, and thus it cannot include both a Higgs field and its conjugate. In the Standard Model one of these are used to give mass to up-type quarks and the other to down-type quarks and fermions, but this is not possible once supersymmetry is introduced. To be able to give mass to all the particles in the theory it is thus necessary to introduce at least two Higgs fields.

Another and a bit more involved way of showing this, is to observe the fact that for a quantum field theory to be renormalizable there needs to be certain gauge anomaly cancellation. What is of relevance here is that the theory needs to fulfill the relation  $\text{Tr}(T_3^2 Y) = \text{Tr}(Y^3) = 0$  where  $T_3$  and  $Y$  are the third component of weak isospin and the hypercharge respectively, and where the traces are taken over all left-handed fermions in the theory. Now assume there was only one Higgs doublet. The Standard Model naturally fulfills this relation before the introduction of supersymmetric partners, and the only one of these that give a non-vanishing contribution is the Higgsino. This must be chosen to either have hypercharge  $\pm\frac{1}{2}$ , and no matter what is chosen it will always give rise to an anomaly causing the theory to break down. Hence there must be at least two Higgs doublets with hypercharges canceling each other, and if more are to be added they must always come in even numbers.

### 3.1.2 Solution of the hierarchy problem

In the last section of chapter two the hierarchy problem was introduced. This is one reason, maybe the most compelling, to study supersymmetric theories since it there gets an elegant solution. As was explained earlier, the core of the hierarchy problem is the unacceptably large corrections the mass of the Higgs boson acquires from loops. As a short recapitulation, the leading terms in these corrections arising from fermion and boson loops respectively are

$$\Delta m_H^2(\text{fermion}) = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{UV}^2 + \dots \quad \Delta m_H^2(\text{boson}) = \frac{|\lambda_S|^2}{16\pi^2}\Lambda_{UV}^2 + \dots$$

Observing the similarity between these two terms it is not a stretch to imagine that a symmetry relating fermions to bosons could be used to solve the problem, since if there to every Standard Model fermion existed two additional boson the terms would neatly cancel. This is exactly what the

supersymmetric theories introduce, and e.g. to every quark  $u$  there are two squarks  $\tilde{u}_L$  and  $\tilde{u}_R$ . Thus every supersymmetric theory automatically gives a solution to the hierarchy problem as long as the couplings  $\lambda_f$  and  $\lambda_S$  are the same. But this is guaranteed by supersymmetry since a particle and its superpartner are identical except regarding their spin, and the couplings are given by  $\lambda_f = m_f/v$  and  $\lambda_S = m_S/v$ .

### 3.2 Supersymmetry breaking terms

Since up until today no supersymmetric particles have been observed at any of the large collider facilities around the world it is clear that if supersymmetry is indeed a fundamental symmetry of Nature, it must be a broken symmetry. There is currently some discussion going on concerning exactly how the symmetry is broken, but regardless of the underlying mechanism it is simple to include supersymmetry breaking in computations. This is done by simply introducing a few extra terms in the Lagrangian that explicitly break the symmetry. There are however restrictions as to what kind of terms that may be introduced, which can be understood by again considering the hierarchy problem. For the mass correction terms to cancel it is necessary both that for each Standard Model fermion there exists two new bosons, and that the couplings of the Higgs boson to fermions and bosons are the same. These criteria are both fulfilled by exact supersymmetric theories, and thus it is necessary that even after supersymmetry breaking the relationship between the dimensionless couplings is maintained. This means that the only terms that can be allowed to be introduced in the Lagrangian must have couplings of positive mass dimension. These terms are usually called *soft supersymmetry breaking terms* and are included in the Lagrangian in the following way

$$\mathcal{L} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}$$

The soft supersymmetry breaking terms mainly consist of mass terms for the gauginos, sfermions and Higgs particles, and some Yukawa terms for Higgs-sfermion interactions. It can be shown, as is done e.g. in [2], that the Lagrangian and thus the whole supersymmetric theory is fully determined by specifying the particle content, gauge groups, supersymmetry breaking terms and an analytic function known as the *superpotential*. The superpotential contains all interaction terms, and from it the ordinary scalar potential can be obtained.

### 3.3 The Minimal Supersymmetric Standard Model

The two previous sections give an introduction to general supersymmetric theories and the features shared by these. In this section a specific theory known as the Minimal Supersymmetric Standard Model (MSSM) will be presented and it will be mentioned about what can be expected from its phenomenology. Its name derives from the fact that it is the simplest supersymmetric theory that incorporates the Standard Model of particle physics, while at the same time being internally consistent.

For a theory to be an extension of the Standard Model it needs to assume at least the same particle content as the former, and to be supersymmetric it also needs to assume the existence of supersymmetric partners to all Standard Model particles. From the arguments presented above it is clear that it has to include two distinct Higgs doublets in order for the superpotential to be analytic and the Higgs mechanism to give masses to all particles of the theory. The MSSM is the simplest theory that fulfill these criteria in that it assumes nothing except of what is absolutely needed. Basically it is the Standard Model augmented by an extra Higgs doublet and with supersymmetric partners.

To determine the kind of interactions allowed in the MSSM it is necessary to specify a superpotential. This is given by [2]

$$\mathcal{W}_{\text{MSSM}} = -\hat{y}_u \hat{U} \hat{H}_u - \hat{y}_d \hat{D} \hat{H}_d - \hat{y}_e \hat{L} \hat{H}_d + \mu \hat{H}_u \hat{H}_d$$

where the superfields are as given in Table 1 above and the Yukawa couplings  $\mathbf{y}$  are  $3 \times 3$ -matrices in family space. To see more easily what kind of interactions this implies it is useful to assume that the heaviest particle in each family dominates the Yukawa coupling matrices to such an extent that the other terms can be ignored. The superpotential can then be rewritten

$$\begin{aligned} \mathcal{W}_{\text{MSSM}} = & -y_t(\bar{t}tH_u^0 - \bar{t}bH_u^+) - y_b(\bar{b}tH_d^- - \bar{b}bH_d^0) \\ & - y_\tau(\bar{\tau}\nu_\tau H_d^- - \bar{\tau}\tau H_d^0) + \mu(H_u^+ H_d^- - H_u^0 H_d^0) \end{aligned}$$

where the bars represent the conjugates of the corresponding right-handed fields. From this it can be seen directly what kind of interactions each term gives rise to, where e.g. the first term describes the interaction of a top and antitop quark with a neutral Higgs boson. Since it turns out in the derivation of the superpotential [2] that the Yukawa couplings must be totally symmetric, this term not only implies the interaction already mentioned but also the ones obtained by substituting two arbitrary fields with their supersymmetric counterparts. Thus the same term in the equation above

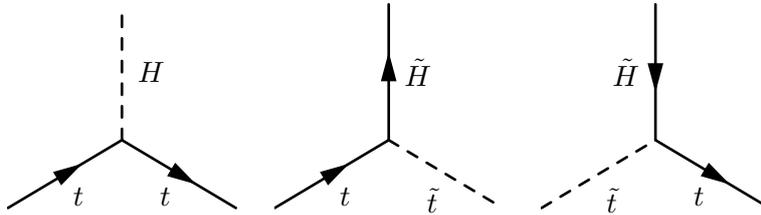


Figure 2: Example of interactions in the MSSM

also gives rise to a Higgsino-squark-quark and a Higgs-squark-squark vertex. Examples of these are shown in Figure 2.

To obtain a complete supersymmetric theory one also has to specify the soft supersymmetry breaking terms in the Lagrangian, which consists of mass and Yukawa terms. This is superfluous for the purposes here and will not be done explicitly, but it does however have to be mentioned that they include mass terms for the Higgs fields proportional to the supersymmetry breaking scale. Together with the Higgs mass term in the superpotential, these give rise to what is commonly referred to as the  $\mu$ -problem of the MSSM. The mass terms in the Lagrangian involving the Higgs and Higgsino fields can be written in the following way

$$\mathcal{L}_{\text{Higgsino}} = \mu(\tilde{H}_u^+ \tilde{H}_d^- - \tilde{H}_u^0 \tilde{H}_d^0)$$

$$\mathcal{L}_{\text{Higgs}} = |\mu|^2(|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2)$$

while the supersymmetry breaking terms involving Higgs masses are

$$\mathcal{L}_{\text{SoftHiggs}} = -m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d$$

Since the Higgs term is positive definite it cannot be the cause of the desired electroweak breaking which is why the soft supersymmetry breaking terms have to be added in the first place. Now from experimental searches at e.g. the Large Electron Positron collider the  $\mu$ -term has to be larger than around  $100 \text{ GeV}$  or otherwise some of the supersymmetric particles would have been discovered [4]. On the other hand, the term in the superpotential cannot dominate the term from the soft supersymmetry breaking Lagrangian or otherwise there can be no electroweak breaking as observed, and this means that  $\mu$  has to be smaller than the supersymmetry breaking scale. The natural thing to expect for the value of a parameter like  $\mu$  is that it is either zero or of an order around the Planck scale, but both of these options are ruled out by experiment and there are no physical reasons motivating why  $\mu$  should have value similar to the supersymmetry breaking scale. This problem has

been and is still the main reason to consider more complex supersymmetric theories, like the one presented in the next section.

By combining the terms from the superpotential and the supersymmetry breaking Lagrangian and expanding around the vacuum expectation values of  $H_u$  and  $H_d$ , the mass eigenstates of the MSSM Higgs sector can be found. These consist of two CP-even neutral scalars  $h$  and  $H$ , one CP-odd neutral scalar  $A$  and two charged scalars  $H^+$  and  $H^-$  [2]. The vacuum expectation values are usually denoted  $v_u$  and  $v_d$  and their ratio  $\tan\beta = v_u/v_d$  defines an often used mixing angle. The masses of  $H$ ,  $A$  and  $H^\pm$  are not restricted by any theoretical arguments, while the mass of  $h$  is bounded by  $m_h \lesssim 135 \text{ GeV}$  after including radiative corrections and assuming no particles included in the loop corrections have masses exceeding  $1 \text{ TeV}$ . Thus the MSSM predicts one possibly light Higgs scalar.

The neutral parts of the superpartners to  $H_u$  and  $H_d$  will mix with the neutral gaugino fields  $\tilde{B}^0$  and  $\tilde{W}^0$ , giving rise to four neutral particles known as neutralinos. The lightest of these could even be the lightest supersymmetric particle (LSP), and would in that case be an excellent candidate for the cold dark matter of the universe. But this can only happen if the LSP is stable and cannot decay to Standard Model particles, something which is guaranteed by introducing the conservation of a new quantum number known as *R-parity*. This is defined for each particle as

$$P_R = (-1)^{3(B-L)-2s}$$

where  $B$  is the baryon number,  $L$  the lepton number and  $s$  the spin of the particle. It is postulated to be multiplicatively conserved in such a way that an interaction is only allowed if it has the total R-parity  $P_R = +1$ . The introduction of this new quantum number might seem a bit *ad hoc*, but it is very well motivated phenomenologically by the lifetime of the proton. Without R-parity conservation this would only be a few seconds while the current experimental limits gives a lifetime around  $10^{32}$  years [2]. Standard Model particles all have  $P_R = +1$  and their supersymmetric partners  $P_R = -1$ , and the interaction vertices of the MSSM all contain an even number of supersymmetric particles. In such a theory conserving R-parity all decay chains including a particle with  $P_R = -1$  must end in the LSP, which is stable since it cannot decay to Standard Model particles. This is usually assumed about the MSSM.

### 3.4 The Next-to-Minimal Supersymmetric Standard Model

The  $\mu$ -problem of the Minimal Supersymmetric Standard Model (MSSM) presented in the previous section gives a reason for studying supersymmetric theories with a more complex Higgs sector than the minimal. In the simplest such theory, known as the Next-to-Minimal Supersymmetric Standard Model (NMSSM), it is assumed that the Higgs sector consists of two complex scalar doublets and one singlet. The superpotential of the NMSSM is obtained from the one of the MSSM by adding the following terms

$$\mathcal{W}_{\text{NMSSM}} = \mathcal{W}_{\text{MSSM}} + \lambda S H_u H_d + \frac{1}{3} \kappa S^3$$

where  $\lambda$  and  $\kappa$  are dimensionless couplings that need to fulfill  $\lambda, \kappa \lesssim 0.7$  in order for the theory to be perturbative up to the GUT-scale [5]. The new singlet field  $S$  introduced is the complex scalar component of a superfield  $\hat{S}$ . Apart from the Higgs sector the NMSSM makes the same assumptions regarding particle content as do the MSSM. It is based on the same gauge groups as the Standard Model and as a result includes the same gauge bosons, and it also assumes the same matter content as the former. Since the superpotential of the NMSSM only differs from the MSSM by a few Higgs terms, it naturally allows the same kind of interactions as the MSSM.

A solution of the  $\mu$ -problem is obtained by considering what happens when supersymmetry is broken in this theory. The supersymmetry breaking terms of the NMSSM will give rise to vacuum expectation values for  $S$ ,  $H_u$  and  $H_d$ , and using the gauge freedom of the theory to make appropriate phase rotations on the fields these can be made real and positive. Now from the vacuum expectation value of  $S$  there will arise an effective  $\mu$ -term for  $H_u H_d$  given by  $\lambda s$ . This will automatically be of the wanted order of magnitude since the vacuum expectation value of  $S$  is of the same order as the supersymmetry breaking scale. The beauty of this construction is that then the scale at which supersymmetry breaking happens is the only one in need of an explanation, and once this is done the right order of the electroweak breaking scale will follow naturally.

Adding a new field to the theory may not seem to alter much, but as it turns out this field will mix with other particles of the theory giving important changes to the phenomenology. The scalar component of  $\hat{S}$  mixes with the neutral scalar components of  $\hat{H}_u$  and  $\hat{H}_d$  giving rise to three CP-even and two CP-odd neutral scalar particles, and the fermionic component of  $\hat{S}$  mixes with the neutral fermionic superpartners of  $\hat{H}_u$ ,  $\hat{H}_d$  and the electroweak gauge bosons to give five neutralinos. Among the neutralions there is just like in the MSSM the possibility to have a light one, which may well

be the lightest supersymmetric particle. If R-parity is assumed to be present in the NMSSM as is customary, this would make an excellent candidate for the cold dark matter needed in cosmology to explain the strange behavior observed in the movement of galaxies.

In the Higgs sector the mixing alters the phenomenology. As mentioned above, the mixing of the singlet scalar field  $S$  with the neutral scalar components of  $\hat{H}_u$  and  $\hat{H}_d$  give rise to two pseudo-scalar fields usually called  $A$  and  $A_S$ , where  $A$  is the one corresponding to the pseudo-scalar particle in the MSSM. The mass eigenstates of the theory are found from these by diagonalizing the mass matrix in the basis  $(A, A_S)$ , and are given by

$$A_1 = A \cos \theta_A + A_s \sin \theta_A$$

$$A_2 = -A \sin \theta_A + A_s \cos \theta_A$$

where  $\theta_A$  is a mixing angle. From this process it follows that the physical pseudo-scalar particles have masses given by

$$m_{A_{1,2}}^2 = \frac{1}{2}(\mathcal{M}_{11}^2 + \mathcal{M}_{22}^2 \mp \Delta\mathcal{M}^2)$$

where  $\Delta\mathcal{M} = \sqrt{(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2)^2 + 4(\mathcal{M}_{12}^2)^2}$  and  $\mathcal{M}$  is the mentioned mass matrix that can be found in [7]. It has been shown in [6] that it is possible to chose the parameters of the NMSSM so that the mass of the lightest particle  $A_1$  is below  $10.5 \text{ GeV}$  without its couplings as compared to in the MSSM becoming to small. This value of the  $A_1$ -mass is also allowed experimentally, as can be seen in Figure 3 and Figure 4 taken from [7] and [8] respectively, which will be of great importance in the next chapter.

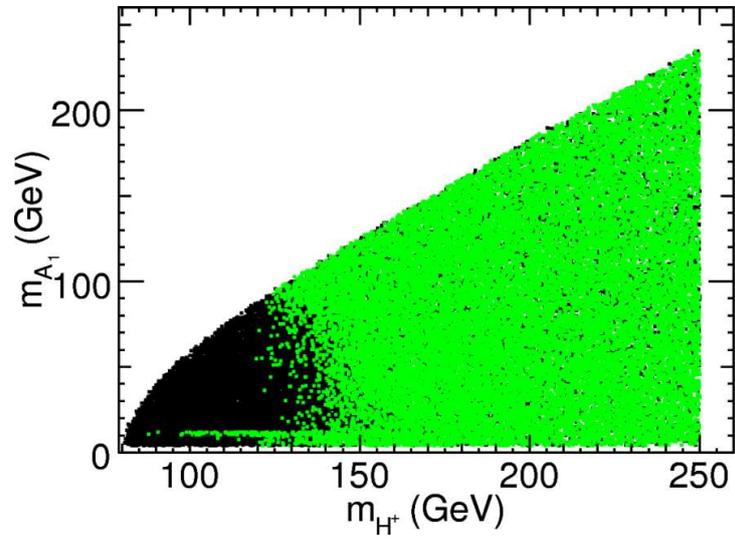


Figure 3: Black represents theoretically allowed values of the  $A_1$  mass and green represents values also allowed experimentally.

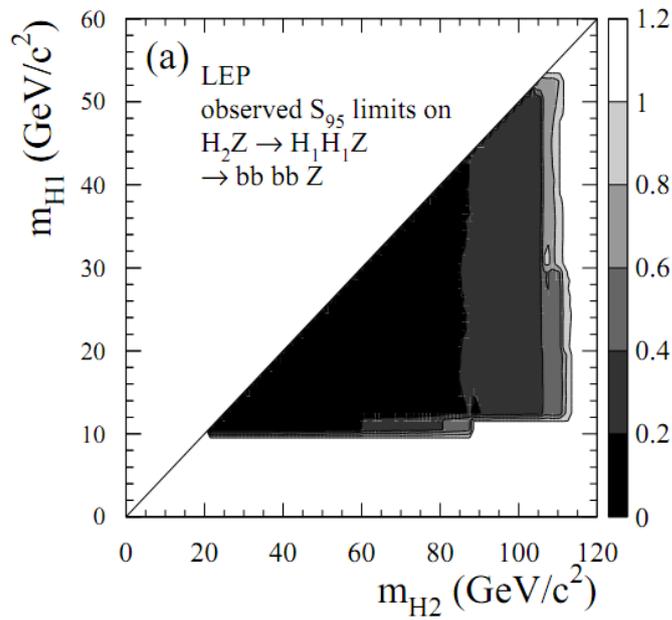


Figure 4: White represent allowed values of a light Higgs boson as obtained at LEP.

## 4 Decay of a light pseudo-scalar Higgs

After the introductions in chapters two and three to the concepts of spontaneous symmetry breaking and supersymmetric extensions of the Standard Model it is time to study the decay of interest in this paper, the decay of a light pseudo-scalar Higgs boson  $A_1$  to an  $\Upsilon$ -meson and a photon. The  $\Upsilon$  is a vector meson consisting of the quarks  $b$  and  $\bar{b}$ . To be able to work this out in detail, it is necessary to first treat the formalism for dealing with decays producing a bound state. This will be presented in the first section in reference to an example through which all important parts of the theory will be encountered. Following this is a few sections dedicated to working out the matrix elements both for the process in question and some interactions needed to work out the branching ratio of  $A_1 \rightarrow \Upsilon + \gamma$ , which will be done in the last section. The material in this chapter is in large based on [10] and [11].

### 4.1 Bound state formalism

In order to find the matrix element of a process involving a bound state, it is necessary to first compute the matrix element of the corresponding process in which all the particles are treated as free. This can then be convoluted with the wave function of the bound state in order to obtain the full matrix element. To make this more clear, consider the production of a bound muon state by collision of electrons and positrons. In the limit where the electrons are relativistic but the muons are not the colliding particles will each be of definite helicity, and since angular momentum must be preserved the initial spins will determine the spin of the final particle. If both the electron and the positron are assumed to have spin up it will thus lead to a muon state of spin one pointing up. The relevant free particle process are shown in Figure 5 below, and from this the matrix element is easily found to be

$$\mathcal{M}_0 = -2e^2$$

Now to treat the bound state it is easiest to perform the computations in its center of mass system, in which the relative position and momentum are given by

$$\vec{r} = r_+^{\vec{}} - r_-^{\vec{}} \quad \vec{k} = \frac{1}{2}(k_+^{\vec{}} - k_-^{\vec{}})$$

and the center of mass position and momentum are

$$\vec{R} = \frac{1}{2}(r_+^{\vec{}} + r_-^{\vec{}}) \quad \vec{k} = k_+^{\vec{}} + k_-^{\vec{}}$$

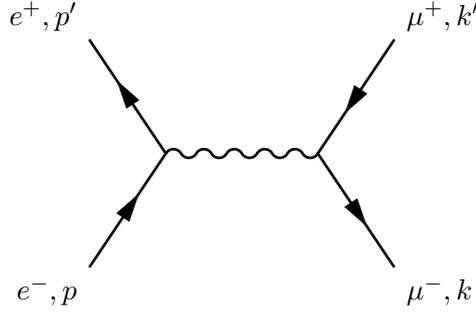


Figure 5: Muon production by electron-positron collision

What is left to find before computing the matrix element is the wave function  $\psi(\vec{r})$  of the bound state. This can be obtained by solving the non-relativistic Schrödinger equation, but this is left out since there is no need here for an explicit wave function. If however the wave function is assumed to be known, it is possible to find the matrix element by convoluting it with the free particle matrix element. The easiest way to do this turns out to be in momentum space, for which the wave function is found by the following Fourier transform

$$\tilde{\psi}(\vec{k}) = \int d^3x e^{i\vec{k}\cdot\vec{r}} \psi(\vec{r}) \quad \psi(\vec{r}) = \int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{r}} \tilde{\psi}(\vec{k}) \quad (2)$$

By the use of this the bound state can be expressed as

$$|\psi\rangle = \sqrt{2M} \int \frac{d^3k}{(2\pi)^3} \tilde{\psi}(\vec{k}) \frac{1}{\sqrt{2m}} \frac{1}{\sqrt{2m}} |\vec{k} \uparrow, -\vec{k} \uparrow\rangle$$

where the mass factors are for normalization in the conventions used in [10] and  $M = 2m$ . From this it is straightforward to write down the full matrix element, which is

$$\mathcal{M} = \sqrt{2M} \int \frac{d^3k}{(2\pi)^3} \tilde{\psi}(\vec{k})^* \frac{1}{\sqrt{2m}} \frac{1}{\sqrt{2m}} \mathcal{M}_0$$

where the conjugate on  $\psi$  comes from transforming the bound state from an initial state to a final. In this case, as in all cases where the free particle matrix element is independent of the momentum  $\vec{k}$ , the integral above is easily evaluated by using  $e^{i\vec{k}\cdot\vec{0}} = 1$  in the right part of Eq. 2 and substituting into the expression above. The production of a bound muon state is thus described by

$$\mathcal{M} = \sqrt{\frac{2}{M}} (-2e^2) \psi(0)^*$$

That the result is proportional to the wave function at the origin is a general feature of processes like this, as will be apparent below. This is not surprising

since the wave function takes as its argument the relative position of the particles, so its value at the origin is a measure of the probability that the particles will meet and form a bound state.

## 4.2 Matrix element for the decay $A_1 \longrightarrow \Upsilon + \gamma$

After going through the general formalism for the production of a bound state, it is time to consider the decay of a light pseudo-scalar Higgs boson  $A_1$  into an  $\Upsilon$ -meson and a photon. Following the formalism presented above it is necessary to first calculate the matrix element for the decay  $A_1 \longrightarrow \bar{b}b + \gamma$ , and then convolute this result with the wave function of the bound  $\Upsilon$ -state in order to find the full matrix element.

### 4.2.1 Free particle decay

For the process  $A_1 \longrightarrow \bar{b}b + \gamma$  the contributing Feynman diagrams are shown in Figure 6 below. Using the Feynman rules of Quantum Electro Dynamics (QED) as given in [10] and the coupling  $g_b = \frac{m_b}{\sqrt{2}v} \cos \theta_A \tan \beta$  for the  $A_1 \bar{b}b$ -vertex the matrix element can be written as follows

$$\begin{aligned} \mathcal{M}_0 &= \bar{u}(k) \left( i \frac{e}{3} \gamma^\mu \right) \frac{i(\not{k} + \not{p} + m_b)}{(k+p)^2 - m_b^2} (i g_b \gamma^5) v(k') \epsilon_\mu^*(p) \\ &+ \bar{u}(k) (i g_b \gamma^5) \frac{i(-\not{k}' - \not{p} + m_b)}{(-k' - p)^2 - m_b^2} \left( -i \frac{e}{3} \gamma^\mu \right) v(k') \epsilon_\mu^*(p) \end{aligned}$$

where the momenta are as given in the diagrams,  $m_b$  is the mass of the bottom quark and  $\epsilon(p)^*$  is the polarization vector of the photon. The mixing angle  $\tan \beta = \frac{v_u}{v_d}$  is the ratio of the vacuum expectation values for the Higgs fields  $H_u$  and  $H_d$ . Since  $k^2 = \bar{k}^2 = m_b^2$  and  $p^2 = 0$  the denominator simplifies, and evaluating the process in the center of mass system of the quarks the expression can be put into the following form

$$\begin{aligned} \mathcal{M}_0 &= \frac{i e g_b}{3} \bar{u}(k) \left( \frac{\gamma^\mu (\not{k} + \not{p} + m_b) \gamma^5}{2k \cdot p} + \frac{\gamma^5 (-\not{k}' - \not{p} + m_b) \gamma^\mu}{2\bar{k} \cdot p} \right) v(k') \epsilon_\mu^*(p) \\ &= \frac{i e g_b}{6 m_b |\vec{p}|} \bar{u}(k) [\not{\epsilon}^* (\not{k} + \not{p} + m_b) \gamma^5 + \gamma^5 (-\not{k}' - \not{p} + m_b) \not{\epsilon}^*] v(k') \end{aligned}$$

It is also assumed here that the four-vectors of the bottom quarks are dominated by their mass component so  $k \simeq k' \simeq m_b$ , an approximation that should be valid in the center of mass system. To further simplify the equation the matrix multiplications of the slashed momentums have to be

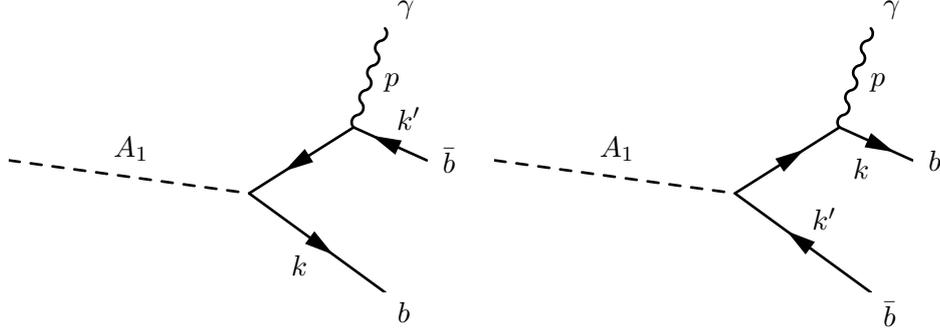


Figure 6: The two possible decays  $A_1 \longrightarrow \bar{b}b + \gamma$

carried out. The easiest and most clear way of doing this is to take one expression at a time and do this explicitly. The left term becomes<sup>3</sup>

$$\begin{aligned} (\not{k} + \not{p} + m_b)\gamma^5 &= \begin{pmatrix} m_b & \sigma(k+p) \\ \bar{\sigma}(k+p) & m_b \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -m_b & \sigma(k+p) \\ -\bar{\sigma}(k+p) & m_b \end{pmatrix}; \end{aligned}$$

$$\begin{aligned} \not{\epsilon}^*(\not{k} + \not{p} + m_b)\gamma^5 &= \begin{pmatrix} 0 & \sigma\epsilon^* \\ \bar{\sigma}\epsilon^* & 0 \end{pmatrix} \begin{pmatrix} -m_b & \sigma(k+p) \\ -\bar{\sigma}(k+p) & m_b \end{pmatrix} \\ &= \begin{pmatrix} -\sigma\epsilon^*\bar{\sigma}(k+p) & m_b\sigma\epsilon^* \\ -m_b\bar{\sigma}\epsilon^* & \bar{\sigma}\epsilon^*\sigma(k+p) \end{pmatrix} \end{aligned}$$

while the right term is

$$\begin{aligned} \gamma^5(-\not{k}' - \not{p} + m_b) &= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} m_b & -\sigma(k'+p) \\ -\bar{\sigma}(k'+p) & m_b \end{pmatrix} \\ &= \begin{pmatrix} -m_b & \sigma(k'+p) \\ -\bar{\sigma}(k'+p) & m_b \end{pmatrix}; \end{aligned}$$

$$\begin{aligned} \gamma^5(-\not{k}' - \not{p} + m_b)\not{\epsilon}^* &= \begin{pmatrix} -m_b & \sigma(k'+p) \\ -\bar{\sigma}(k'+p) & m_b \end{pmatrix} \begin{pmatrix} 0 & \sigma\epsilon^* \\ \bar{\sigma}\epsilon^* & 0 \end{pmatrix} \\ &= \begin{pmatrix} \sigma(k'+p)\bar{\sigma}\epsilon^* & -m_b\sigma\epsilon^* \\ m_b\bar{\sigma}\epsilon^* & -\bar{\sigma}(k'+p)\sigma\epsilon^* \end{pmatrix} \end{aligned}$$

<sup>3</sup>In the conventions used here and in [10]  $\sigma^\mu = (\mathbb{1}, \vec{\sigma})$  and  $\bar{\sigma}^\mu = (\mathbb{1}, -\vec{\sigma})$ , from which the  $\gamma$ -matrices are  $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$

Adding the two terms and simplifying the following is obtained, calling the combined matrix  $\mathcal{A}$

$$\begin{aligned}\mathcal{A} &= \not{\epsilon}^*(\not{k} + \not{p} + m_b)\gamma^5 + \gamma^5(-\not{k}' - \not{p} + m_b)\not{\epsilon}^* = \\ &= \begin{pmatrix} -\sigma\epsilon^*\bar{\sigma}(k+p) + \sigma(k'+p)\bar{\sigma}\epsilon^* & 0 \\ 0 & \bar{\sigma}\epsilon^*\sigma(k+p) - \bar{\sigma}(k'+p)\sigma\epsilon^* \end{pmatrix}\end{aligned}$$

Here it is convenient to write the expressions in the matrix in terms of their indices, for reasons that will be clear below. Concentrating on the  $2 \times 2$ -matrix that is the (1, 1) element of the matrix  $\mathcal{A}$  gives

$$\begin{aligned}\mathcal{A}_{11} &= -\sigma\epsilon^*\bar{\sigma}(k+p) + \sigma(k'+p)\bar{\sigma}\epsilon^* \\ &= -\sigma\epsilon^*\bar{\sigma}k + \sigma k'\bar{\sigma}\epsilon^* - \sigma\epsilon^*\bar{\sigma}p + \sigma p\bar{\sigma}\epsilon^* \\ &= -\epsilon_\mu^*k_\nu(\sigma^\mu\bar{\sigma}^\nu - \bar{\sigma}^\mu\sigma^\nu) - \epsilon_\mu^*p_\nu(\sigma^\mu\bar{\sigma}^\nu - \bar{\sigma}^\mu\sigma^\nu) \\ &= 2k\sigma\epsilon_i^*\sigma^i - 2p_0\epsilon_i^*\sigma^i + 2\epsilon_0^*p_i\sigma^i\end{aligned}$$

where it has been used that  $\sigma k' = \bar{\sigma}k$  which holds since  $k = (k^0, \vec{k})$ ,  $k' = (k^0, -\vec{k})$ ,  $\sigma = (\mathbb{1}, \vec{\sigma})$  and  $\bar{\sigma} = (\mathbb{1}, -\vec{\sigma})$ . The (2, 2) element can be simplified in a similar way to obtain the following expression for the matrix  $\mathcal{A}$

$$\begin{aligned}\mathcal{A} &= \begin{pmatrix} 2k\sigma\epsilon_i^*\sigma^i - 2p_0\epsilon_i^*\sigma + 2\epsilon_i^*p_i\sigma^i & 0 \\ 0 & -2k\sigma\epsilon_i^*\sigma^i - 2p_0\epsilon_i^*\sigma + 2\epsilon_i^*p_i\sigma^i \end{pmatrix} \\ &= 2k\sigma \begin{pmatrix} \vec{\epsilon} \cdot \vec{\sigma} & 0 \\ 0 & -\vec{\epsilon} \cdot \vec{\sigma} \end{pmatrix} - 2|\vec{p}| \begin{pmatrix} \vec{\epsilon} \cdot \vec{\sigma} & 0 \\ 0 & \vec{\epsilon} \cdot \vec{\sigma} \end{pmatrix} + 2\epsilon_0^* \begin{pmatrix} \vec{p} \cdot \vec{\sigma} & 0 \\ 0 & \vec{p} \cdot \vec{\sigma} \end{pmatrix}\end{aligned}$$

Now the matrix multiplications are done and it is time to consider the spinors of the matrix element. Since the decay considered is between particles of about the same mass, the kinetic energies involved will be small in relation to the rest mass and it is enough to consider the process in the non-relativistic limit. Here the spinors can be written explicitly as

$$\bar{u}(k) = u^\dagger\gamma^0 = \sqrt{2m_b} \begin{pmatrix} \xi \\ \xi \end{pmatrix} \quad v(k') = \sqrt{2m_b} \begin{pmatrix} \xi' \\ -\xi' \end{pmatrix}$$

Because of the different signs of the two-component spinors of the antiparticle, only the first of the three matrices above will survive when it is multiplied with the spinors. This gives a simple way of expressing the matrix element in terms of the two component spinors, and it turns out that it is only non-vanishing in the case where the spins are aligned. This is good since  $\Upsilon$  is a vector particle, and it was shown in the previous section that the spins of the initial particle determine the spin of the final ones. In the case where

both particles have spin up<sup>4</sup> and it is assumed that  $k\sigma = k_0\sigma^0 = m_b \cdot \mathbb{1}$  the matrix element becomes

$$\begin{aligned}\mathcal{M}_0 &= \frac{2ieg_b m_b}{3|p|} \xi^\dagger \vec{\epsilon} \cdot \vec{\sigma} \xi' = \frac{ieg_b m_\Upsilon}{3|p|} \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \epsilon_3^* & \epsilon_1^* - i\epsilon_2^* \\ \epsilon_1^* + i\epsilon_2^* & -\epsilon_3^* \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= \frac{2ieg_b m_\Upsilon^2}{3(m_A^2 - m_\Upsilon^2)} (\epsilon_1^* - i\epsilon_2^*)\end{aligned}$$

where  $m_A$  is the mass of  $A_1$  and the value  $|\vec{p}| = (m_A^2 - m_\Upsilon^2)/2m_\Upsilon$  has been obtained from the kinematics of the process in the center of mass system of  $\Upsilon$ . Here it is assumed that  $2m_b \simeq m_\Upsilon$ , an approximation that depends to some extent on what value is used for the bottom quark mass but whose error should not exceed the order of  $\sim 10\%$ . Also  $|\vec{p}|$  has been replaced with  $\frac{m_A^2 - m_\Upsilon^2}{2m_\Upsilon}$ , a value obtained from the kinematics of the process. Choosing instead the spins of both particles to be down, the matrix element will be as follows

$$\mathcal{M}_0 = \frac{2ieg_b m_\Upsilon^2}{3(m_A^2 - m_\Upsilon^2)} (\epsilon_1^* + i\epsilon_2^*)$$

#### 4.2.2 Decay into the bound state

Using the free matrix element calculated above, it is simple to do the following convolution to obtain the full matrix element

$$\begin{aligned}\mathcal{M} &= \sqrt{2m_\Upsilon} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2m_b}} \frac{1}{\sqrt{2m_b}} \tilde{\psi}(k)^* \mathcal{M}_0 \\ &= \sqrt{\frac{2}{m_\Upsilon}} \psi(0)^* \frac{2ieg_b m_\Upsilon^2}{3(m_A^2 - m_\Upsilon^2)} (\epsilon_1^* - i\epsilon_2^*)\end{aligned}$$

This process is shown in Figure 7 below. As a first step towards finding the branching ratio of the process the decay width needs to be computed, and the general formula for the partial decay width simplifies to the following in the case of a two-body decay in the center of mass frame

$$d\Gamma = \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{|\vec{p}|}{m^2} d\Omega$$

where  $m$  is the mass of the decaying particle and  $|\vec{p}|$  the magnitude of one of the outgoing momenta. From the kinematics in the center of mass frame of the  $A_1$ -boson it follows that this magnitude is fully determined and takes the

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<sup>4</sup>In the conventions used here the antiparticle spinor representing spin up is  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

value  $|\vec{p}| = (m_A^2 - m_\Upsilon^2)/2m_A$ , and that the decay is independent of angles. Thus the decay width can be obtained by a simple integration

$$\begin{aligned}
\Gamma &= \int \frac{d\Omega}{32\pi^2} |\mathcal{M}|^2 \left( \frac{m_A^2 - m_\Upsilon^2}{2m_A} \right) \left( \frac{1}{m_A^2} \right) \\
&= \int \frac{d\Omega}{32\pi^2} \frac{4e^2 g_b^2}{9m_\Upsilon} \left( \frac{m_\Upsilon^2}{m_A^2 - m_\Upsilon^2} \right)^2 \left( \frac{m_A^2 - m_\Upsilon^2}{2m_A} \right) \left( \frac{1}{m_A^2} \right) |\psi(0)|^2 |\epsilon_1^* - i\epsilon_2^*|^2 \\
&= \int \frac{d\Omega}{32\pi^2} \frac{4e^2 g_b^2}{9(m_A^2 - m_\Upsilon^2)} \frac{m_\Upsilon^3}{m_A^3} |\psi(0)|^2 |\epsilon_1^* - i\epsilon_2^*|^2 \\
&= \frac{e^2 g_b^2}{18\pi} \frac{1}{m_A^2 - m_\Upsilon^2} \frac{m_\Upsilon^3}{m_A^3} |\psi(0)|^2 |\epsilon_1^* - i\epsilon_2^*|^2
\end{aligned}$$

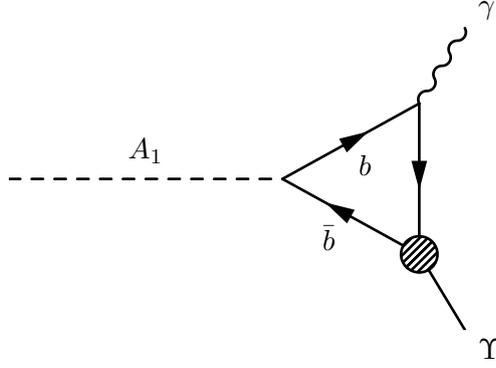


Figure 7: Decay of  $A_1$  to  $\Upsilon$  and  $\gamma$

It should be noticed here that the polarization vectors describe a circularly polarized photon, and that the particular expression above corresponds to left-handed polarization. It can thus be rewritten as  $\epsilon_1^* - i\epsilon_2^* = \epsilon_-^*$ , and in a similar way  $\epsilon_1^* + i\epsilon_2^* = \epsilon_+^*$ . The length of such a vector is  $|\epsilon_-^*| = |\epsilon_+^*| = \sqrt{2}$  so

$$\Gamma = \frac{e^2 g_b^2}{18\pi} \frac{1}{m_A^2 - m_\Upsilon^2} \frac{m_\Upsilon^3}{m_A^3} |\psi(0)|^2 |\epsilon_1^* - i\epsilon_2^*|^2 = \frac{e^2 g_b^2}{9\pi} \frac{1}{m_A^2 - m_\Upsilon^2} \frac{m_\Upsilon^3}{m_A^3} |\psi(0)|^2$$

This is only the expression for one of the possible spin states resulting from the decay, and to get a correct result also the other has to be added. The full decay width is thus

$$\Gamma = 2 \cdot \frac{e^2 g_b^2}{3\pi} \frac{1}{m_A^2 - m_\Upsilon^2} \frac{m_\Upsilon^3}{m_A^3} |\psi(0)|^2 = \frac{8\alpha g_b^2}{3(m_A^2 - m_\Upsilon^2)} \frac{m_\Upsilon^3}{m_A^3} |\psi(0)|^2 \quad (3)$$

where  $\alpha = e^2/4\pi$  is the fine structure constant of QED and a factor of three has been introduced to account for color.

### 4.3 Other dominant decays of $A_1$

After finding the decay width of  $A_1 \rightarrow \Upsilon + \gamma$  in the last section, the last missing piece before the branching ratio of the process can be computed is a determination of what other decay paths are available for the  $A_1$  boson and which ones that will dominate the decays. Since the Higgs boson considered in this paper is assumed to have a mass around  $9 - 12 GeV$ , i.e. precisely around the threshold where decay to a  $\bar{b}b$ -pair becomes possible, this decay should have the greatest width. The mass difference between the bottom and charm quark is about a factor four and the Higgs boson coupling goes as  $\lambda_f \propto \frac{m_f}{v}$ , which makes it plausible that the inclusion of decays to other quark pairs than  $\bar{b}b$  not will be relevant for the results. The only other decay that could then be interesting to consider is into the pair  $\tau^+\tau^-$ , which would also be easier to detect than a quark decay. In this section the matrix element of the two decays  $A_1 \rightarrow \bar{b}b$  and  $A_1 \rightarrow \tau^+\tau^-$  will be computed.

Since both of the decays under consideration have the same structure, the computation of the matrix elements will be very similar. It is therefore enough to evaluate it in one of the cases and then make the appropriate changes necessary to find the other. From the Feynman diagram in Figure 8 below the matrix element for decay into tauons is, using the value  $g_\tau = \frac{m_\tau}{\sqrt{2}v} \cos \theta_A \tan \beta$  for the  $A_1\tau^+\tau^-$  coupling

$$\mathcal{M} = \bar{u}(p')(ig_\tau\gamma^5)v(p)$$

with momenta as indicated in the diagram. This can be squared and re-grouped in order to obtain

$$\begin{aligned} |\mathcal{M}|^2 &= -g_\tau^2(\bar{u}(p')\gamma^5v(p))(\bar{v}(p)\gamma^5u(p')) \\ &= -g_\tau^2(u(p')\bar{u}(p')\gamma^5v(p)\bar{v}(p)\gamma^5) \end{aligned}$$

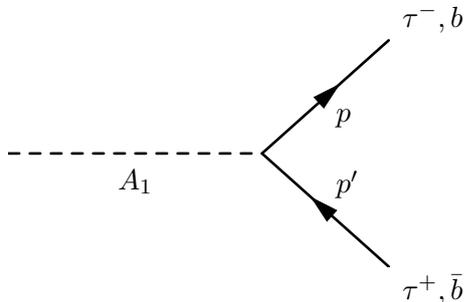


Figure 8: Decay of  $A_1$  into either  $\tau^+\tau^-$  or  $\bar{b}b$ .

Using the usual sum rules for Dirac spinors and the fact that the energy

of the tauons is half the mass of the Higgs boson, the calculation can be transformed into a trace computation and evaluated

$$\begin{aligned}
\sum_{1,2} |\mathcal{M}|^2 &= -g_\tau^2 (\not{p}' + m) \gamma^5 (\not{p} - m) \gamma^5 = -g_\tau^2 (\not{p}' + m) (-\not{p} - m) (\gamma^5)^2 \\
&= g_\tau^2 (\not{p}' + m) (\not{p} + m) = g_\tau^2 (p'_\mu p_\nu \text{Tr}(\gamma^\mu \gamma^\nu) + m^2) \\
&= g_\tau^2 (4p'_\mu p_\nu g^{\mu\nu} + m^2) = g_\tau^2 (p' \cdot p + m^2) \\
&= g_\tau^2 (E^2 + \vec{p}^2 + m^2) = 2g_\tau^2 E^2 = g_\tau^2 \frac{m_A^2}{2}
\end{aligned}$$

Up to this point the calculation is identical for the quarks except their different coupling to the Higgs boson. Since this is a two-particle decay like the one studied in the section above the same formula for the decay width can be used. This gives

$$\begin{aligned}
\Gamma &= \int \frac{d\Omega}{32\pi^2} |\mathcal{M}|^2 \frac{|\vec{p}|}{m_A^2} = \int \frac{d\Omega}{32\pi^2} \left( g_\tau^2 \frac{m_A^2}{2} \right) \frac{m_A}{2m_A^2} \left( 1 - \frac{4m_\tau^2}{m_A^2} \right)^{1/2} \\
&= g_\tau^2 \frac{m_A}{32\pi} \left( 1 - \frac{4m_\tau^2}{m_A^2} \right)^{1/2}
\end{aligned}$$

where the value  $|\vec{p}| = \frac{m_A}{2} \left( 1 - \frac{4m_\tau^2}{m_A^2} \right)^{1/2}$  has been obtained from the kinematics. The decay widths for the processes in Figure 8 are thus, after introducing a color factor the corresponding decay into quarks, the following

$$\Gamma(\tau^+ \tau^-) = g_\tau^2 \frac{m_A}{32\pi} \left( 1 - \frac{4m_\tau^2}{m_A^2} \right)^{1/2} \quad \Gamma(\bar{b}b) = g_b^2 \frac{3m_A}{32\pi} \left( 1 - \frac{4m_b^2}{m_A^2} \right)^{1/2} \quad (4)$$

#### 4.4 Branching ratio of $A_1 \rightarrow \Upsilon + \gamma$

The results of the above calculations can be used to find the branching ratio of  $A_1 \rightarrow \Upsilon + \gamma$ . The branching ratio gives the number of decays happening through a particular process in relation to the total number of decays, and is in this case defined as

$$\text{BR}(A_1 \rightarrow \Upsilon + \gamma) = \frac{\Gamma(A_1 \rightarrow \Upsilon + \gamma)}{\Gamma(A_1 \rightarrow \Upsilon + \gamma) + \Gamma(A_1 \rightarrow \bar{b}b) + \Gamma(A_1 \rightarrow \tau^+ \tau^-)}$$

To be studied below is what happens to this ratio as the mass of the Higgs boson runs through the interval  $9.46 - 12 \text{ GeV}$ , which is from where decay into an  $\Upsilon$ -meson becomes possible [9]. This value of this ratio will be quite different depending on if the decay  $A_1 \rightarrow 2B$  is allowed, which is the first way in which the bottom quarks can hadronize, since this will be strongly favored once the possibility is open. The mass of the  $B$ -meson according to

[9] is  $5.28 \text{ GeV}$  so this decay will be allowed once the mass of  $A_1$  reaches  $10.5 \text{ GeV}$ . The two cases above and below this threshold will therefore be considered separately.

Assume that  $A_1$  has a mass ranging between  $9.46$  and  $10.5 \text{ GeV}$  where the expression for the branching ratio simplifies to

$$\text{BR}(A_1 \rightarrow \Upsilon + \gamma) = \frac{\Gamma(A_1 \rightarrow \Upsilon + \gamma)}{\Gamma(A_1 \rightarrow \Upsilon + \gamma) + \Gamma(A_1 \rightarrow \tau^+ \tau^-)}$$

By use of Eq. 3 and Eq. 4 the explicit expressions for the decay widths can be introduced, after which the branching ratio becomes

$$\begin{aligned} \text{BR}(A_1 \rightarrow \Upsilon + \gamma) &= \frac{\Gamma(A_1 \rightarrow \Upsilon + \gamma)}{\Gamma(A_1 \rightarrow \Upsilon + \gamma) + \Gamma(A_1 \rightarrow \tau^+ \tau^-)} \\ &= \frac{\frac{8\alpha m_b^2 |\psi(0)|^2 m_\Upsilon^3}{3(m_A^2 - m_\Upsilon^2)} \frac{m_\Upsilon^3}{m_A^3}}{m_\tau^2 \frac{3m_A}{32\pi^2} \left(1 - \frac{4m_\tau^2}{m_A^2}\right)^{1/2} + \frac{8\alpha m_b^2 |\psi(0)|^2 m_\Upsilon^3}{3(m_A^2 - m_\Upsilon^2)} \frac{m_\Upsilon^3}{m_A^3}} \end{aligned}$$

The mass of  $\Upsilon$  according to [9] is  $9.46 \text{ GeV}$  so the only value missing before the ratio can be computed is that of  $|\psi(0)|^2$ . This is found from the measured value  $\Gamma(\Upsilon \rightarrow e^+ e^-) = 1.34 \text{ keV}$  taken from [9] and the corresponding expression for the decay width as calculated in [11]

$$\Gamma(\Upsilon \rightarrow e^+ e^-) = \frac{16\pi\alpha^2 Q_b^2}{m_\Upsilon^2} |\psi(0)|^2$$

where  $Q_b = 1/3$  is the magnitude of the charge of the bottom quark and  $\alpha = 1/137$ . Putting in the numbers gives  $|\psi(0)|^2 = 0.64 \text{ (GeV)}^3$ , and using this the results displayed in Figure 9 are obtained.

Now consider the case where  $A_1$  has a mass between  $10.5$  and  $12 \text{ GeV}$ . Using the same equations as before the branching ratio can be written explicitly as

$$\begin{aligned} \text{BR}(A_1 \rightarrow \Upsilon + \gamma) &= \frac{\Gamma(A_1 \rightarrow \Upsilon + \gamma)}{\Gamma(A_1 \rightarrow \Upsilon + \gamma) + \Gamma(A_1 \rightarrow \bar{b}b) + \Gamma(A_1 \rightarrow \tau^+ \tau^-)} \\ &= \frac{\frac{8\alpha m_b^2 |\psi(0)|^2 m_\Upsilon^3}{3(m_A^2 - m_\Upsilon^2)} \frac{m_\Upsilon^3}{m_A^3}}{m_\tau^2 \frac{3m_A}{32\pi^2} \left(1 - \frac{4m_\tau^2}{m_A^2}\right)^{1/2} + m_b^2 \frac{3m_A}{32\pi^2} \left(1 - \frac{4m_b^2}{m_A^2}\right)^{1/2} + \frac{8\alpha m_b^2 |\psi(0)|^2 m_\Upsilon^3}{3(m_A^2 - m_\Upsilon^2)} \frac{m_\Upsilon^3}{m_A^3}} \end{aligned}$$

This expression only differs from the previous one by the inclusion of a new term in the denominator, but also in this term all the factors are known. It is therefore easily computed using the same values for the masses and the wave function as before, and the results are given in Figure 10 below. As

can be seen in the figures and was mentioned before, the branching ratio once  $A_1$  is allowed to decay to  $B$ -mesons is much smaller than close to the  $\Upsilon$  mass. In Figure 11 the values of the branching ratio for the whole mass interval  $9.46 - 12 \text{ GeV}$  are shown.

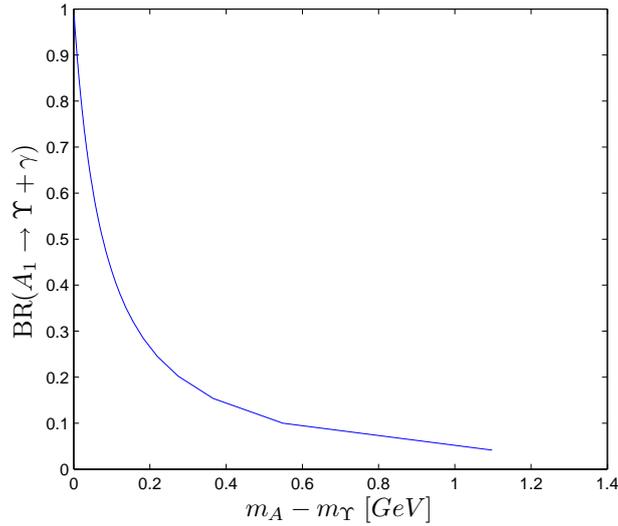


Figure 9: Branching ratio of  $A_1 \rightarrow \Upsilon + \gamma$  as a function of  $A_1$  mass

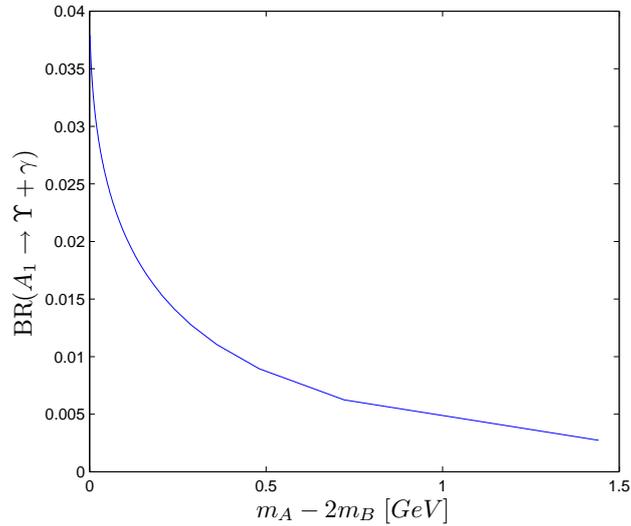


Figure 10: Branching ratio of  $A_1 \rightarrow \Upsilon + \gamma$  as a function of  $A_1$  mass

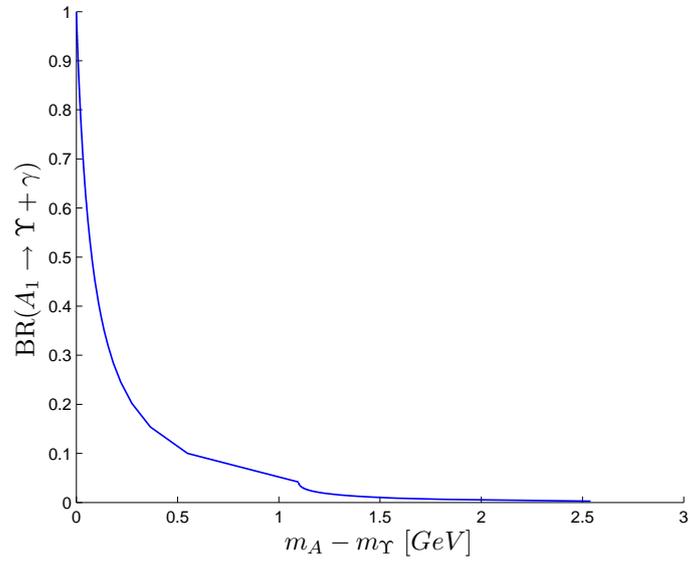


Figure 11: Branching ratio of  $A_1 \rightarrow \Upsilon + \gamma$  as a function of  $A_1$  mass

## 5 Summary and conclusions

The intention of this paper has been to investigate some of the phenomenological consequences of a pseudo-scalar Higgs particle in the mass interval  $9.46 - 12 \text{ GeV}$  within the framework of the Next-to-Minimal Supersymmetric Standard Model, and then to perform some calculations of a possible decay through which it in that case can be detected. For this it was first necessary to develop the concept of spontaneous symmetry breaking, and this was done in several steps. First a few easy examples were considered to understand the difference between the breaking of a global and local symmetry, how the first implies the existence of massless Nambu-Goldstone bosons and the second uses the degrees of freedom of these bosons to give mass to the gauge fields. Once this had been done the Higgs mechanism of the Standard Model was introduced, and it was explained how it can be used to give mass to the Standard Model particles. To finish of this chapter the hierarchy problem was presented as a motivation for studying extensions of the Standard Model.

The next piece needed to understand the work of this paper is the concept of supersymmetric theories, which was outlined in chapter three. First some general concepts like supersymmetric partners of particles and their names and notation were presented, followed by a short discussion of the Higgs sector of supersymmetric theories. It was then showed how these theories in a natural and elegant way solves the hierarchy problem, and some new concepts like the superpotential and supersymmetry breaking terms were introduced. Thereafter followed a review of the Minimal Supersymmetric Standard Model which is the simplest of all supersymmetric theories. Parts of its phenomenology like the Higgs sector and the Lightest Supersymmetric Particle were discussed in relation to the concept of R-parity conservation. Finally the Next-to-Minimal Supersymmetric Standard Model were considered and it was shown that both experimentally and theoretically a light pseudo-scalar Higgs boson is allowed in the mass interval  $9.46 - 12 \text{ GeV}$ .

The final chapter is dedicated to the computation of the branching ratio of the process  $A_1 \rightarrow \Upsilon + \gamma$ , where  $\Upsilon$  is a bound vector meson state consisting of the quarks  $b$  and  $\bar{b}$ . It began with the development of the formalism necessary to be able to treat decays to a bound state, through the consideration of production of a bound muon state by the collision of electrons and positrons. Following this the matrix element for the process  $A_1 \rightarrow \bar{b}b + \gamma$  shown in Figure 6 was calculated from which the matrix element and the decay width of  $A_1 \rightarrow \Upsilon + \gamma$  could be computed. After this other processes relevant for the branching ratio were considered and the corresponding decay

widths were obtained. The chapter then ended with the calculation of the branching ratio and a presentation of the results found in Figures 9, 10 and 11.

The first reflection regarding the results is that the branching ratio of the process  $A_1 \rightarrow \Upsilon + \gamma$  is quite small and thus the process will be difficult to detect once the decay  $A_1 \rightarrow \bar{B}^0 B^0$  is kinematically allowed. In the mass region after this becomes possible the branching ratio of the decay  $A_1 \rightarrow \Upsilon + \gamma$  has a maximum value of around 0.04 which is on the verge of being too low to be of interest experimentally. On the other hand, in the mass region around  $9.46 - 10 \text{ GeV}$  the branching ratio is much higher and especially interesting is what happens in the limit where  $m_{A_1} \rightarrow m_\Upsilon$ . As can be seen in Figure 9 the branching ratio grows very fast when the mass difference between the particles become smaller and approaches one in the limit. Thus if  $A_1$  were to have a mass close to that of the  $\Upsilon$ -meson this decay would be dominating.

Due to the kinematics of the process, the photon radiated will for a fixed mass of  $A_1$  have the energy

$$|\vec{p}| = \frac{m_A^2 - m_\Upsilon^2}{2m_A}$$

in the center of mass system of  $A_1$ . For the region  $m_A \simeq m_\Upsilon$  this value will in the limit approach zero and there is a physical reason restricting the lowest energy the photon can have. For the emission of a photon to be at all possible, its wave-length must be small enough to resolve the two quarks in the  $\Upsilon$ -meson or otherwise the photon will only see an uncharged particle to which it cannot couple. To resolve the quarks will be possible as long as the wave-length of the photon is smaller than the radius of  $\Upsilon$ , which is of the order  $1 \text{ fm}$ . The criterion commonly used for the resolution energy is  $E = \frac{hc}{\lambda}$  from which the value  $0.20 \text{ GeV}$  is obtained as the minimum photon energy. Below the energy  $9.66 \text{ GeV}$  the results obtained should thus be trusted, but from this energy and up until at least  $10.5 \text{ GeV}$  the branching ratio is still large enough that the decay could be detected.

Apart from the lightest  $\Upsilon$ -state already considered and that more correctly should be written  $\Upsilon(1S)$ , there are others of slightly larger mass corresponding to the first and second radial excitations of the particle. They are denoted  $\Upsilon(2S)$  and  $\Upsilon(3S)$  and have masses  $10.02 \text{ GeV}$  and  $10.36 \text{ GeV}$  respectively [9]. Since the only difference between these particles and  $\Upsilon(1S)$  is the mass, the discussions and calculations above can be applied to these particles as well. The result is that there are two additional mass regions where the  $A_1$ -boson could be detected, where the branching ratios

of  $A_1 \rightarrow \Upsilon(2S) + \gamma$  and  $A_1 \rightarrow \Upsilon(3S) + \gamma$  approaches unity. These regions are shown in Figure 12 below.

Since for all the  $\Upsilon$ -particles the branching ratio approaches one as the mass of  $A_1$  gets close to that of  $\Upsilon$ , these regions offer great possibilities of finding a pseudo-scalar Higgs boson. The photon radiated in the process will for a fixed mass of  $A_1$  have a specific energy that is easily calculable and thus easy to detect. The  $\Upsilon(1S)$ -meson decays with a branching ratio of about 0.025 to electrons, muons and taus, and the heavier  $\Upsilon$ -states decay to the lighter with a ratio around the ten percent level [9]. Since leptons are easy to detect these decays together with the known photon energy provides a nice experimental signature for a light pseudo-scalar Higgs boson. If there is such a particle in the mass interval  $9.46 - 10.5 \text{ GeV}$  this could be a good way of finding it. Also if one searches for  $A_1$  through other processes in this mass range it is important to consider these decays, since they will effect the branching ratios of these as well.

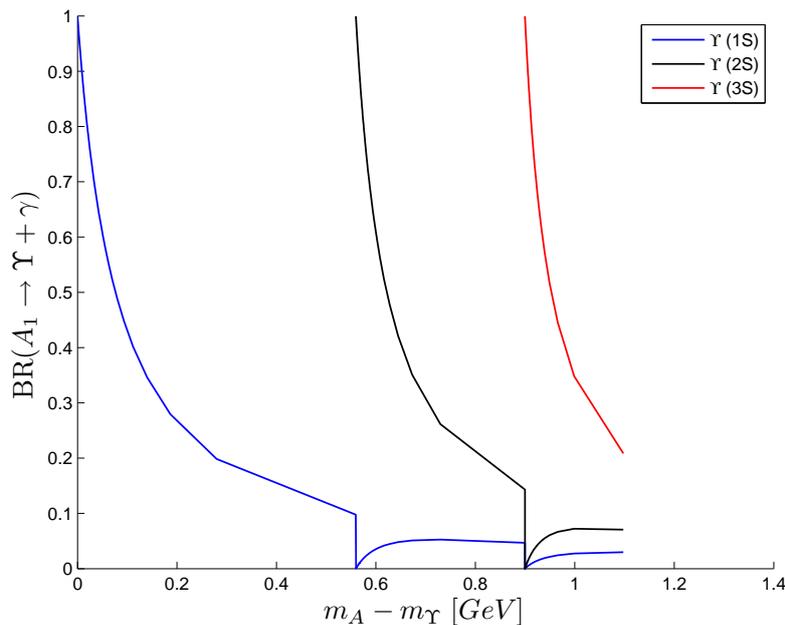


Figure 12: Branching ratios for the decays of  $A_1$  to the ground state and first two radial excitations of the  $\Upsilon$ -meson and a photon.

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