GRAZING INCIDENCE MONOCHROMATOR

P. Bengtsson

Lund Reports on Atomic Physics

LRAP-122

(1991)

GRAZING INCIDENCE MONOCHROMATOR

Abstract

Different methods to obtain accurate absolute wavelengths from the experimentally observed peak positions are investigated.

Introduction

Wavelength shorter than about 300 Å cannot be detected in a normal incidence set-up because of the low reflectivity of the grating. In the study of highly ionized atoms, that often emits light below 250 Å, the use of a grazing incidence geometry is a necessity.

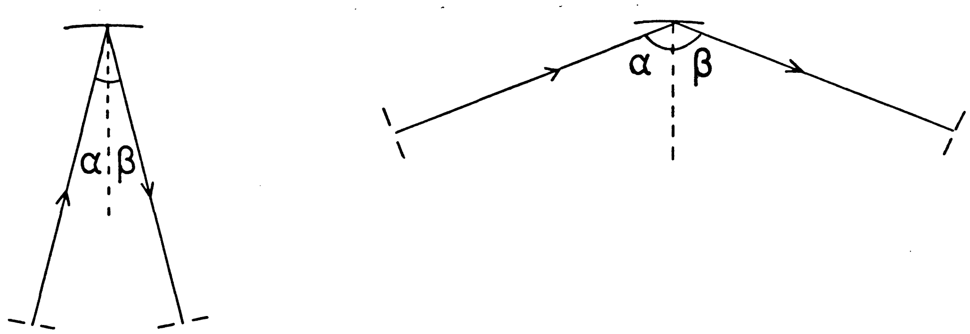

Fig 1. The normal and the grazing incidence set-ups.

In the grazing incidence set-up the incident angle α is close to 90°. The reason is that the reflectivity at short wavelengths increases dramatically when the incident angle increases as shown in Fig 2. The grazing set-up is mostly used in the range 0.5 Å to 250 Å. More about this can be read in reference [1].

Our monochromator is a Minuteman model 310-G, equipped with four gratings with 133.6, 600, 1200 and 2400 grooves per mm, respectively. The first grating is useful for calibration purposes since the spectral range extends up to 2600 Å. The Hg-line at 2536 Å can then be used for focus adjustments.

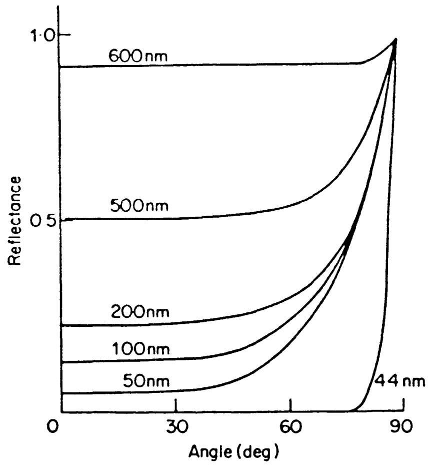

Fig 2. The reflectivity as a function of wavelength. (From "Diffraction gratings" by M.C.Hutley (1982) [1]).

The exit slit is moved along the Rowland circle and both the entrance and the exit slits are perpendicular to the circle. Light detection is normally done by a channeltron.

In the following chapters different methods to obtain accurate absolute wavelengths from the experimentally observed peak positions are investigated.

The grating equation and the dispersion

The investigation starts with the grating equation illustrated in Fig 3.

$$\Delta S = d(\sin\alpha - \sin\beta) = m\lambda \qquad m=1,2,3.... \qquad (1)$$

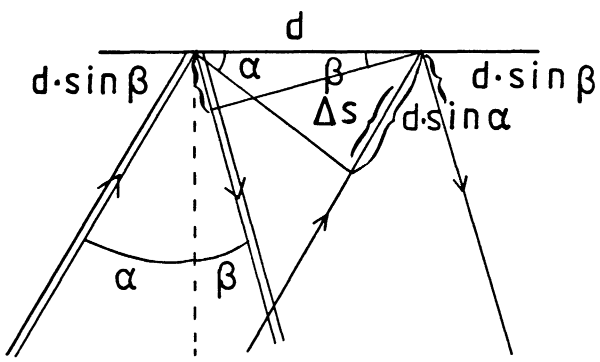

Fig 3. For constructive interference the distance difference ΔS has to be $m\lambda$.

In our set-up the chordal-length, X, is measured. The relation between the diffraction angle β and the chordal-length X is easily seen from Fig 4 to be $X/2=(R/2)*\cos\beta$.

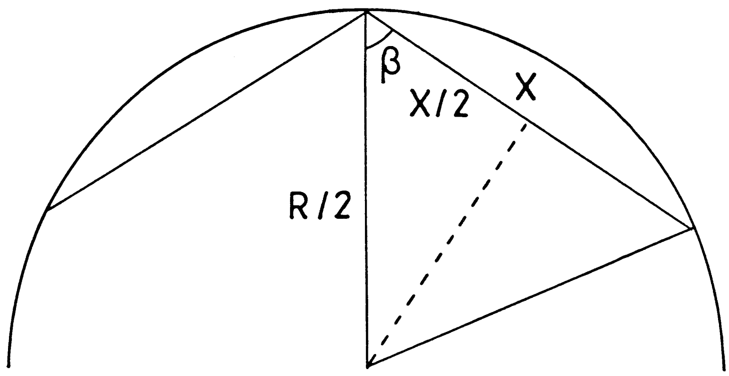

Fig 4. The chordal length X is measured.

Using $\sin\beta = \sqrt{1-\cos^2\beta} = \sqrt{1-(X/R)^2}$, the grating equation can be written as

$$m\lambda = d(\sin\alpha - \sqrt{1-(X/R)^2}) \qquad (2)$$

This relation is illustrated in Fig 5.

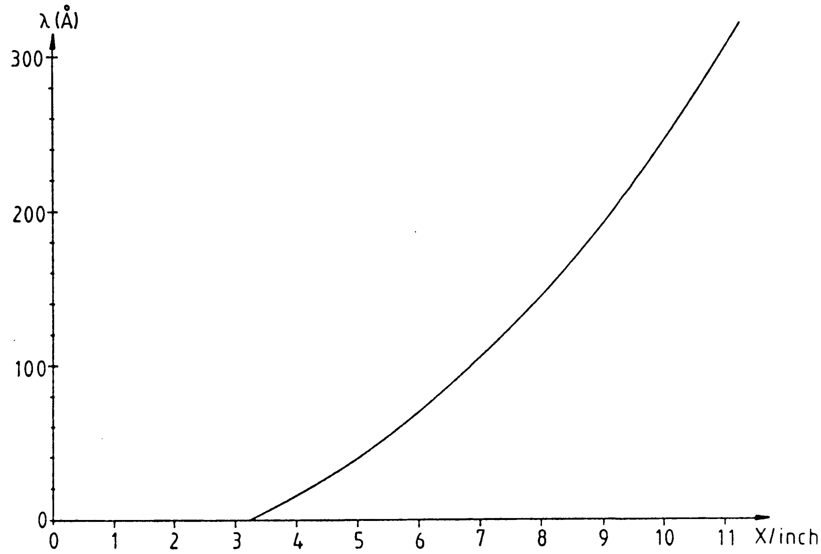

Fig 5. Wavelength as a function of chordal length for $\alpha = 85.3°$, $R = 39.323$ inches, and a grating with 1200 grooves per mm.

The dispersion as a function of λ is an interesting parameter in the grazing incidence case, because it is far from constant. Expressing the dispersion in terms of measurable parameters will help getting correct wavelength determinations. In our case the chordal length is measured, and the dispersion is unit wavelength per unit chordal length. Before looking at the dispersion in the grazing set-up, we will study the dispersion in the normal incidence case. Here the starting point is (1). For small β and α (1) will become $\lambda=d\beta$. The dispersion is taken to $\frac{d\lambda}{d\beta}$, which is a constant.

In the grazing case it is suitable to take the $\frac{d\lambda}{dX}$ as the dispersion and get the expression in (3), which depends on X.

$$\frac{d\lambda}{dX} = \frac{d}{R\sqrt{(R/X)^2-1}} \qquad (3)$$

This relation is close to linear, since a MacLaurin expansion will give a linear term and the next term in magnitude will be in the order of X^3/R^3. The dispersion can also be expressed as a function of wavelength.

$$\frac{d\lambda}{dX} = \frac{d}{R\sqrt{\frac{1}{1-(\sin\alpha-\frac{\lambda}{d})^2} - 1}} \qquad (4)$$

The dispersion relations are shown in Fig 6 and Fig 7.

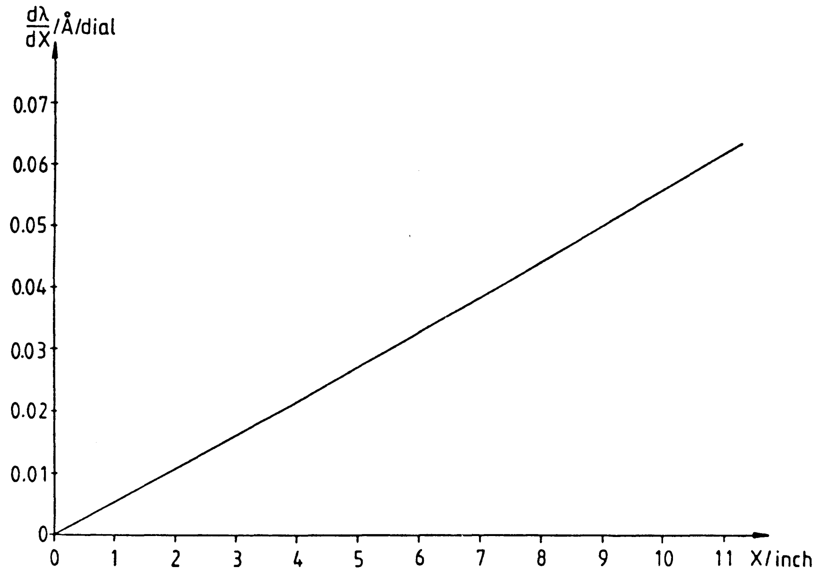

Fig 6. The dispersion as a function of chordal length for α = 85.3° and R = 39.323 inches.

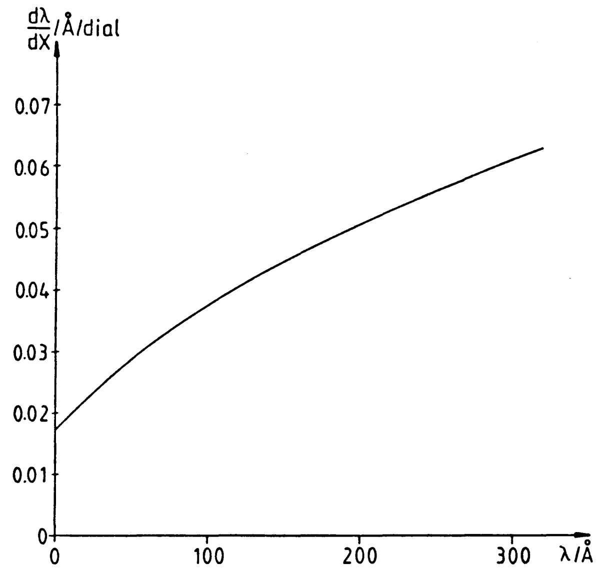

Fig 7. The dispersion as a function of wavelength for α = 85.3° and R = 39.323 inches.

It is now important to determine how the linewidth varies with the wavelength. The linewidth is given in our approximation by the difference between λ_1 and λ_2, see Fig 8. Here S stands for the width of the exit slit. We assume that the light is reflected in a pointlike grating. In this approximation the width of the entrance slit has no influence on the linewidth.

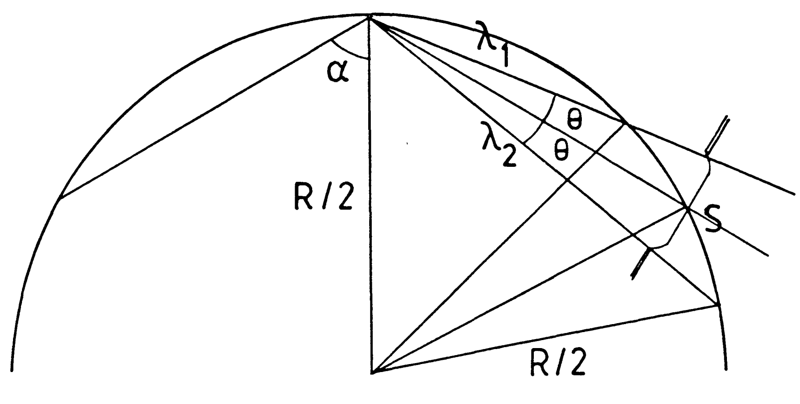

Fig 8. The linewidth is found by taking the difference of the wavelengths λ_1 and λ_2.

The linewidth will then be

$$\Delta\lambda = d(\sin(\beta+\theta)-\sin(\beta-\theta)) = 2d\cos\beta\sin\theta = 2\frac{dX}{R}\sin\theta.$$

Here $\sin\theta$ can be expressed in terms of S as follows

$$\Delta\lambda = 2\frac{dX}{R}\frac{1}{\sqrt{1+(2X/S)^2}}$$

which for small S is nearly constant in terms of wavelength, but not in terms of chordal length see Fig 9. For the 1200 grooves per mm grating the width for a 50 μ slit is about 0.42 Å, which has been verified experimentaly.

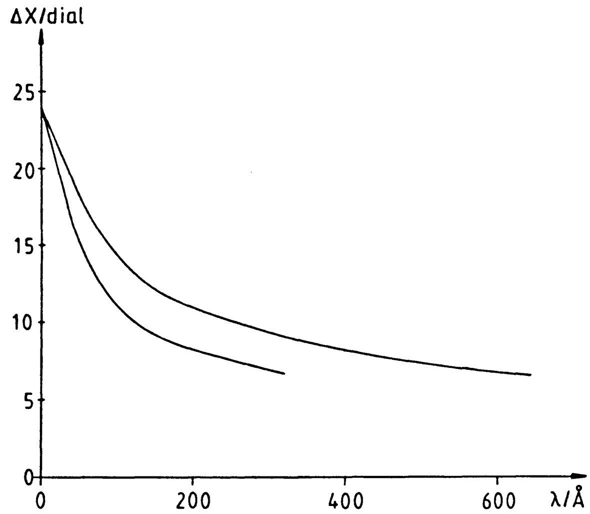

Fig 9. The theoretical linewidth, in terms of chordal length as a function of wavelength for α = 85.3°, R = 39.323 inches and slitwidth S = 50 μm. Data are shown for both a 1200 and a 600 grooves per mm grating. The exit slit can not be moved to chordal length longer than 11.25 inches, which means that the 1200-grating is restricted to below 320 Å (the lowest curve).

Experimentally observed linewidths in a fluorine spectrum between 80 and 300 Å are shown in Fig 10. As can be seen the FWHM of these lines is constant at about 0.49 Å in the 75 - 255 Å interval, and then increases to 0.6 Å.

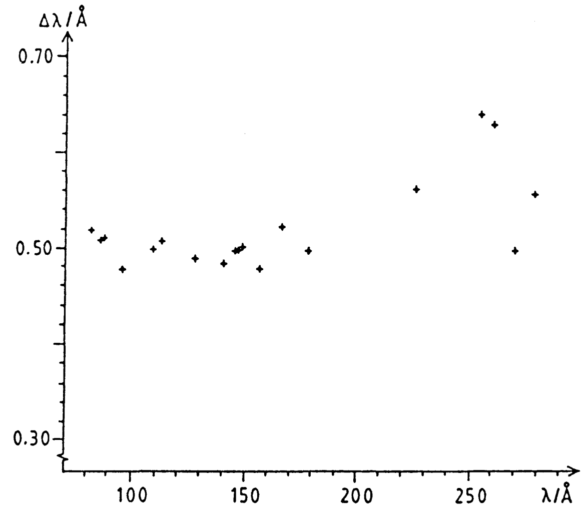

Fig 10. Experimental studies of line width using a beam of F ions.

Definition of parameters and derivation of wavelength expressions

The rather simple expression (2) is valid only for an ideal set-up. In practice several deviations from the ideal case must be considered.

1) An off-set in the chordal length. That is the dial setting is different from $R\cos\alpha$ in the zero order.
2) A displacement of the exit slit track in radial direction, Δr. The track is supposed to be on the Rowland circle of the grating, but the circle defined by the track could be off along the line through the centre of the Rowland circle and the centre of the grating.
3) A tilt of the track. By a tilt we mean that the track is turned an angle θ around the centre of the grating.
4) The mechanical track has a radius of curvature that differs from that of the Rowland circle.
5) A displacement and a tilt at the same time.
6) A displacement, a tilt and a different radius at the same time.

To start we will define a number of variables used in the following text.

α,	the angle of incidence.
R,	the radius of the Rowland circle.
R_m,	the radius of the track on which the exit slit is moving.
$X_{dial} = 2R_m \cos\beta_{dial}$,	this is the displayed setting of the chordal length, that is the distance between the centre of the grating and the exit slit.
$X_{real} = R\cos\beta_{real}$,	this is the real chordal length.
$\Delta X = X_{dial} - X_{real}$,	ΔX will be refered to as the off-set.

In the first case, shown in Fig 11, it is quite simple to derive an expression. X_{real} is given by $X_{dial}-\Delta X$ so the expression will be the one in (5).

$$\lambda = d(\sin\alpha - \sqrt{1-(\frac{X_{dial}-\Delta X}{R})^2}) \qquad (5)$$

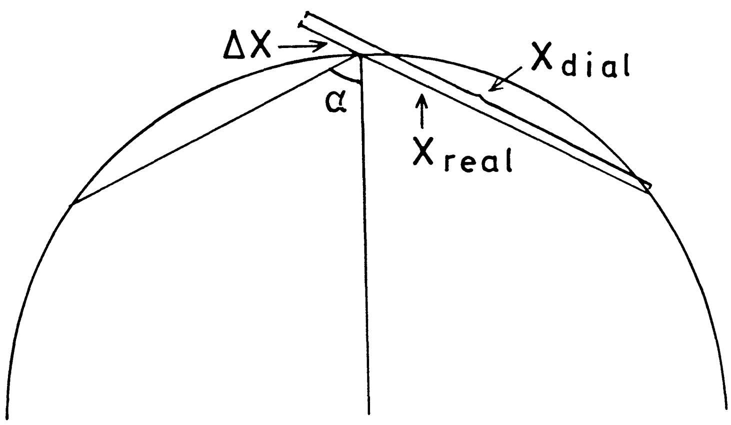

Fig 11. If the dial setting is different from the theoretical $R\cos\alpha$ in the zero-order, we have to subtract ΔX from X_{dial} to get X_{real}. This occur when the dial has not been properly set in absolute distance.

The treatment of the second case, illustrated in Fig 12, becomes more difficult. But using the sine theorem, $\sin\beta_{real}$ can be determined.

$$\frac{\sin(\beta_{real}-\beta_{dial})}{\Delta r} = \frac{\sin(\pi-\beta_{real})}{X_{dial}} \Rightarrow$$

$$\frac{\Delta r}{X_{dial}} = \frac{\sin\beta_{real}\cos\beta_{dial} - \sin\beta_{dial}\cos\beta_{real}}{\sin\beta_{real}} \Rightarrow$$

$$\cot\beta_{real} = (\cos\beta_{dial} - \frac{\Delta r}{X_{dial}})\frac{1}{\sin\beta_{dial}} = (\frac{X_{dial}}{R} - \frac{\Delta r}{X_{dial}})\frac{1}{\sqrt{1-(X_{dial}/R)^2}}$$

$\cot\beta_{real}$ can be rewritten in terms of $\sin\beta_{real}$

$$\cot\beta_{real} = \frac{\sqrt{1-\sin^2\beta_{real}}}{\sin\beta_{real}} = \sqrt{\frac{1}{\sin^2\beta_{real}} - 1} \Rightarrow \sin\beta_{real} = \sqrt{\frac{1}{1+\cot^2\beta_{real}}}$$

Using (1) we will get (6).

$$\lambda = d(\sin\alpha - \frac{1}{\sqrt{1+(\frac{X_{dial}}{R} - \frac{\Delta r}{X_{dial}})^2 \frac{1}{1-(X_{dial}/R)^2}}}) \qquad (6)$$

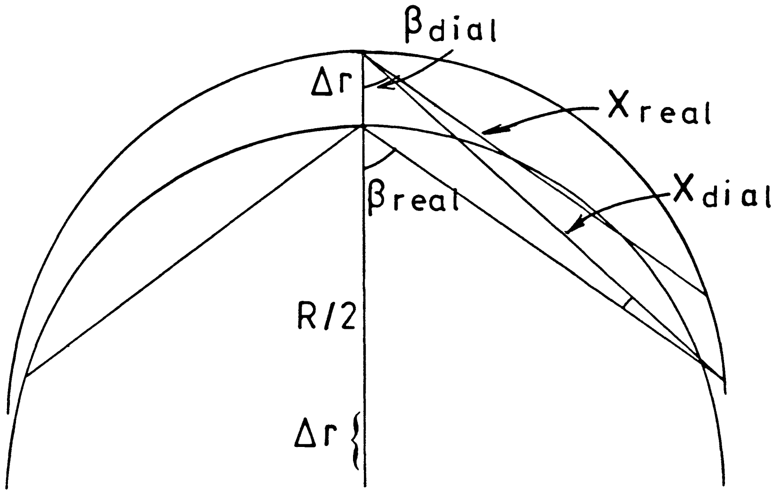

Fig 12. The grating is moved in radial direction, versus the track of the exit slit.

Note that if Δr is put to zero, X_{dial} will be equal to X_{real}, and (6) can then easily be shown to be equivalent to (5).

To treat a tilt of the track, which is demonstrated in Fig 13, it is only necessary to add an angle θ to β_{dial} to get β_{real}. That is $\sin\beta_{real} = \sin(\beta_{dial}+\theta)$. The relation $X_{dial} = R\cos\beta_{dial}$ will give $\sin\beta_{real} = \sqrt{1-(X_{dial}/R)^2}*\cos\theta+X_{dial}/R*\sin\theta$. Putting this expression into (1) the result will be (7).

$$\lambda = d(\sin\alpha-(\sqrt{1-(X_{dial}/R)^2}*\cos\theta+X_{dial}/R*\sin\theta)) \qquad (7)$$

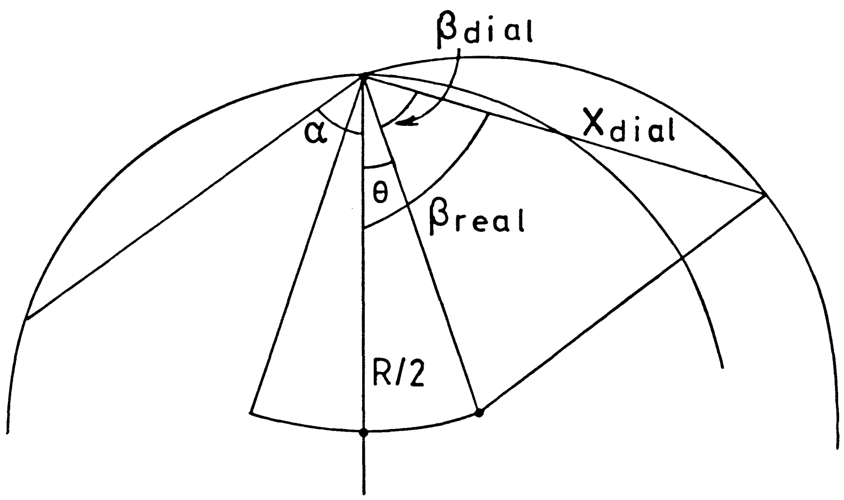

Fig 13. A tilt of the track around the axis through the centre of the grating.

The next task is to deal with the radius of curvature of the track see Fig 14. Here the real angle and the angle corresponding to the dial setting are the same. The chordal length is then related to the real angle as $X_{dial}=2R_m\cos\beta_{real}$, that is R is replaced by $2R_m$ in (2) see (8).

$$\lambda = d(\sin\alpha-\sqrt{1-(X_{dial}/2R_m)^2}) \qquad (8)$$

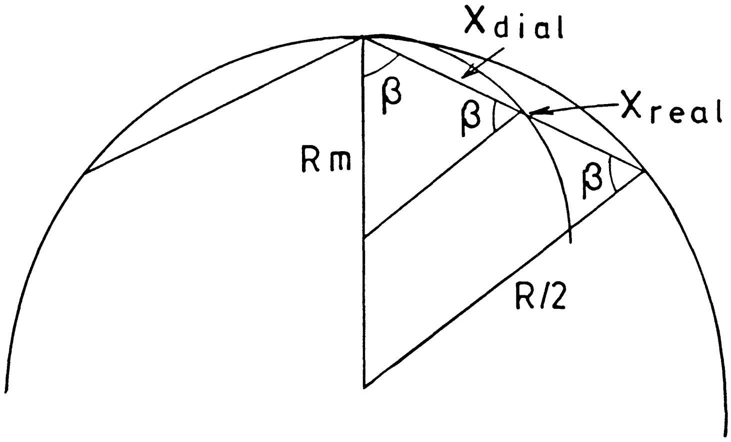

Fig 14. The case of the bending radius of the track different from the one of the Rowland circle is considered.

The combination of a tilt and a displacement illustrated in Fig 15, is now not so difficult to treat. Noticing the triangle with the two sides Δr and X_{dial}, then it is obvious that:

$$\frac{\sin(\beta_{real}-\theta-\beta_{dial})}{\Delta r} = \frac{\sin(\pi-\beta_{real})}{X_{dial}}$$

From this an expression for $\cot\beta_{real}$ is obtained:

$$\cot\beta_{real}=(\cos(\beta_{dial}+\theta)-\frac{\Delta r}{X_{dial}})\frac{1}{\sin(\beta_{dial}+\theta)}$$

The relation $X_{dial}= R\cos\beta_{dial}$ is known and $\cot\beta$ can be rewritten in terms of $\sin\beta$.

$$\sin\beta_{real}=\frac{1}{\sqrt{1+\cot^2\beta_{real}}}=\frac{(X_{dial}/R)\sin\theta+\sqrt{1-(X_{dial}/R)^2}\cos\theta}{\sqrt{1+(\Delta r/X_{dial})^2-2\Delta r(\cos\theta/R-\sin\theta\sqrt{1-(X_{dial}/R)^2}/X_{dial})}}$$

The final expression is given in (9).

$$\lambda = d\left(\sin\alpha - \frac{(X_{dial}/R)\sin\theta + \sqrt{1-(X_{dial}/R)^2}\cos\theta}{\sqrt{1+(\Delta r/X_{dial})^2 - 2\Delta r(\cos\theta/R - \sin\theta\sqrt{1-(X_{dial}/R)^2}/X_{dial})}}\right) \qquad (9)$$

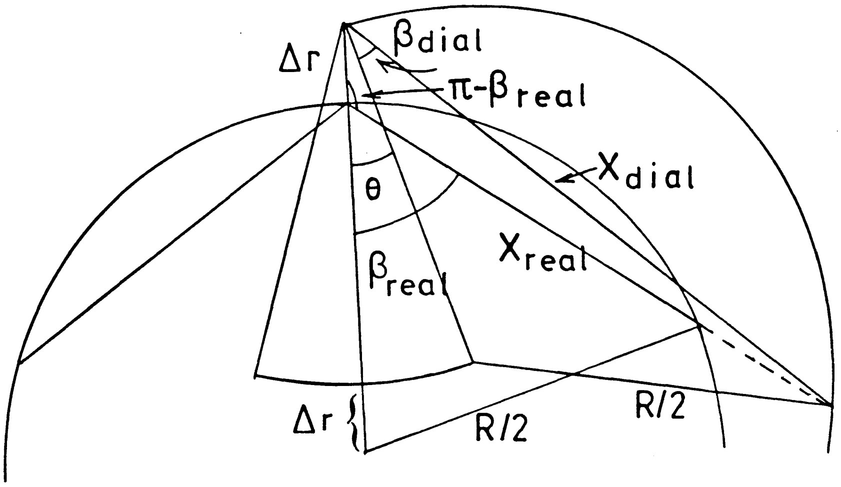

Fig 15. Both a displacement and tilt of the track are applied.

Knowing (9) the last case, with a different radius of curvature shown in Fig 16, is simple. It involves replacement of R with $2R_m$.

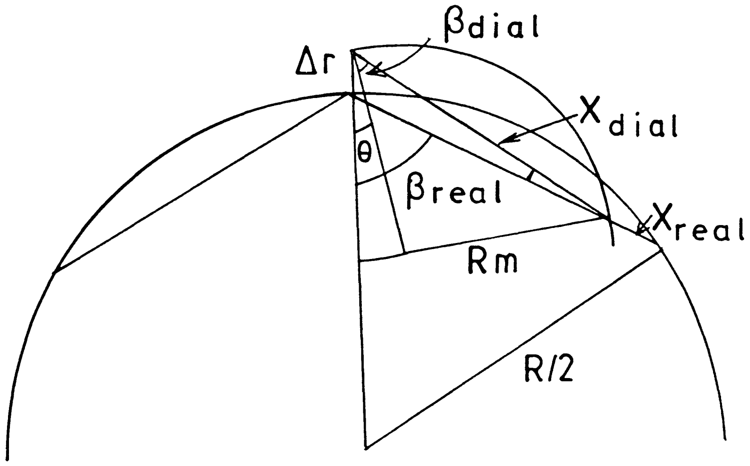

Fig 16. Displacement, tilt and a different radius of curvature applied.

$$\lambda = d\left(\sin\alpha - \frac{(X_{dial}/2R_m)\sin\theta+\sqrt{1-(X_{dial}/2R_m)^2}\,\cos\theta}{\sqrt{1+(\Delta r/X_{dial})^2-2\Delta r(\cos\theta/2R_m-\sin\theta\sqrt{1-(X_{dial}/2R_m)^2}/X_{dial})}}\right) \quad (10)$$

If now θ is inserted (7) this equation will be equivalent to (2). The same operation could be done to θ in (9), ending up in (6), or if $\Delta r=0$ in (9), that would be the same as using (7).

In this chapter a set of parameters have been defined, which can be useful to correct the wavelength scale, when making spectral studies. The next issue is to find out how big the effect is for each parameter, and that will be done in the next chapter.

The wavelength dependence on the parameters

To find out how sensitive the wavelength scale is to the parameters defined in the previous chapter, equation (10) is used and the derivatives with respect to the different parameters are computed. A table of the values for a 1200 grooves per mm grating with an incident angle of 85.3° is presented. The ΔX, Δr and θ are taken to zero and R_m to half of the radius of the Rowland circle. The values are given both for the zero order where X_{dial} is 3.226 inches and for 100 Å where X_{dial} then is 6.874 inches.

TABLE

λ	0	100	(Å)
$\frac{d\lambda}{d\alpha} \simeq$	11.9	11.9	(Å/degree)
$\frac{d\lambda}{dR_m} \simeq$	-0.112	-0.516	(Å/mm)
$\frac{d\lambda}{d\Delta r} \simeq$	-8.31	-8.21	(Å/mm)
$\frac{d\lambda}{d\theta} \simeq$	-11.9	-25.4	(Å/degree)
$\frac{d\lambda}{d\Delta X} \simeq$	-0.342	-0.739	(Å/mm)
	-0.0087	-0.0188	(Å/dial)

In a calibration experiment about 40 reference lines in fluorine were used to check the parameters and the model. The parameters were shown to differ very little from the expected, except for α, which should be 86° according to specification, but was found to be 85.3°. This difference shifted the whole spectra 8.5 Å. The incident angle α is depending on the refocusing, which is made to correct for the fact that we use a moving light source. The instrument is used in a beam-foil experiment and different ions and energies will in this type of experiment have different velocities and will then require different refocusing. This effect on the incident angle is in the order of 0.1°, which will give a shift of about 1 Å for the 1200 grooves grating.

Discussion

The aim of this work was to find physical parameters to explain the relation between the displayed chordal length and the wavelength. Of the parameters investigated, the incident angle α, the tilt angle θ and the displacement Δr have shown to be especially important.

A computer program has been developed to give a wavelength list out of a list of peak positions. Intensity is collected versus stepmotor positions. The stepmotor has 100 steps on a dial, and here a dial is equal to 0.001 inch. The stepmotor positions are then easily converted into a chordal length in a suitable unit. The program is called GRAZING. It needs a number of reference lines to fit the above mentioned parameters. Once the parameters are fitted, the program can read the peak position list and give the corresponding wavelengths. In the program it is possible to choose how many of the mentioned parameters that are to be fitted, letting the rest be fix.

Our hope is that this type of description of the wavelength scale will give a more correct wavelength list than an ordinary polynomial fit-function.

Summary

A number of possible parameters in a grazing incidence set-up have been defined. Those parameters describe the slits position relative the Rowland circle defined by the grating. The expressions of the wavelength as a function of these parameters has been presented. The sensitivity of each parameter has been presented in formulas and for a specified grating at two wavelengths, 0 Å and 100 Å.

References

[1] M.C.Hutley, "Diffraction gratings" (1982)