# Construction of radiofrequency circuits for longer Coherence times in Rare-Earth Ions

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Bachelor of Science Thesis

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## Abstract

An approach to the construction of high-frequency radiocircuits for extending coherence-times in the Rare-Earth quantum computation scheme is presented, along with results from computer simulation of the radiofrequency circuit using a program called QUCS. The results were experimentally verified and showed good correspondence with theory and simulations. Problems to take into consideration when constructing high-frequency circuits are presented as well as possible solutions to them. A general background of the quantum-information field is also given as well as the necessary concepts needed in order to understand the background for the experiment.

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# Contents

# Populärvetenskaplig sammanfattning

Kortfattat kan en dator beskrivas som en räknemaskin; i en dator är enheten som utför beräkningarna processorn. I en kvantdator är det enskilda atomer som är ansvariga för beräkningarna. På atomnivå, mikrokosmos, råder andra lagar än i makrokosmos. Dessa lagar sammanfattas i vad som kallas för kvantmekanik. Ett kvantmekaniskt fenomen som utnyttjas i en kvantdator är superpositionsprincipen. Denna princip kan kortfattat beskrivas som en atoms förmåga att befinna sig i två tillstånd, 0 och 1, samtidigt. Detta kan utnyttjas för att utföra vissa beräkningar snabbare än i en klassisk dator.

Det finns många olika tekniker för att bygga en kvantdator. I detta projekt har den s.k sällsynta-jordartsmetall tekniken använts. Den går ut på att sällsynta jordartsmetaller, den grupp atomer som inte är så känsliga yttre påverkan, har dopats, dvs mixats, in i en kristall av ett annat ämne. Genom att en laserstråle av rätt färg riktas mot denna kristall kan tillstånden hos jordartsmetallen kontrolleras och bl.a sättas i nämnda superpositionstillstånd. För att förlänga tiden som jordartsmetallerna befinner sig i detta tillstånd, och därigenom den tid som finns för att utföra operationer och beräkningar, måste de ytterligare isoleras från påverkan av kristallen de sitter i. Detta kan göras genom att kristallen utsätts för ett magnetfält av rätt styrka och riktning. Ett sådant magnetfält skapas genom att en ledare, oftast en koppartråd, viras i cirkulära varv runt kristallen i vad som kallas en spole.

Ett statiskt magnetfält används för att splittra upp energinivåerna, de möjliga tillstånden som atomen kan befinna sig i. Ett skiftande magnetfält används för att driva övergångar mellan de möjliga tillstånden. Anledningen till att göra detta är för att hitta en styrka på magnetfältet där de sällsynta jordartsmetallerna påverkas minimalt av kristallen de sitter i.

I detta projekt har jag undersökt vad som krävs för att bygga en den spole med vilken övergångarna mellan energinivåerna drivs, genom att göra simuleringar i dator, och riktiga experiment, för att verifiera resultaten från dessa.

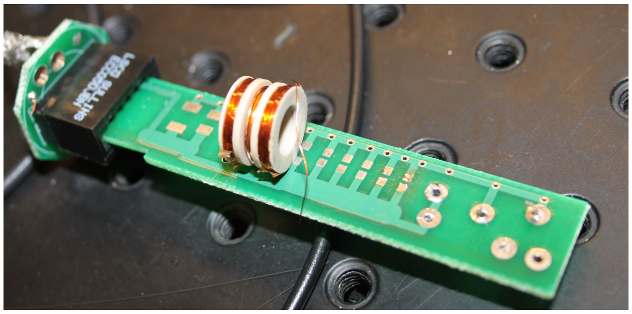

Figure 1: En av de kretsar som byggdes för att testa spolens kapacitet. Spolen i mitten är den som driver övergångarna, och de två spolarna på sidorna genererar ett statiskt magnetfält. Kretskortet är ritat av Mahmood Sabooni.

# 1 Introduction

This Bachelors Thesis is the result of a work with the Quantum Information group.

The focus of this project has been to construct a high-frequency radio circuit to put atoms/ions into coherent superposition states for longer times. To do this an alternating magnetic field had to be produced by feeding a coil with a signal at a frequency corresponding to the energy splitting between the energy-states in the ion of interest. In this case a signal of 100MHz was needed. The plan was to modify an existing coil-setup to accommodate the higher frequency.

When in comes to quantum-computing the coherence time of a state is one of the more important aspects since it determines how many operations that can be done, and also how long it is possible to store a quantum state before the phase is destroyed.

# 2 Theory

## 2.1 Quantum computation and atomic physics

In this part a small background of the quantum computation field is introduced along with an explanation of the neccessary concepts needed in order to understand the experiment.

### 2.1.1 Quantum computer background

Quantum Information is the general name of the field that is concerned with storing physical information in quantum systems; a branching of this is Quantum Computation. It is concerned with the task of manipulating single ions or ensembles of ions with the goal of creating a quantum computer. It was in 1982 that the nobel laureate Richard P. Feynman proposed the idea that the only way to fully simulate a quantum system was using a quantum computer and not a classical computer. [1] A classical computer uses transistors contained in the cpu to perform calculations whereas the quantum computer instead uses the quantum mechanical systems to perform the calculations. It took until 1994 when Peter Shor introduced Shors algorithm which showed that a certain types of calculations, such as the task of factorizing large numbers, could be performed with significant increase in performance on a quantum computer compared with a classical computer.[2] Furthermore quantum computation/information can be used to perform quantum cryptography/quantum key distribution, which lets the user send the keys to decode classically sent information without worrying that someone else gets hold of the decryption key; A technique which actually already has been used when handling the Switzerland election results.

In addition to this the importance of understanding and controlling quantum mechanical effects becomes more and more important as the manufacturing process of integrated circuits and cpu's are shrinking in order to house more transistors. In order to build faster and better computers a thorough understanding of the quantum world is needed.

There are many different ways of realising a quantum computer. After the introduction of Shor's algorithm a multitude of different methods to control atoms were introduced, some better than others. From physcial chemistry came the NMR-techniques, from solid state physics quantum dots and superconducting techniques, the well developed Trapped ion approach was one of the more successful since it built upon well established foundation of spectropscopy. In 2000 the IBM-engineer DiVincenzo set a number of criteria for a successful realization of a quantum computer. The criteria include the ability to set the system into a well defined initial state, the ability to do measurements on the states and the ability to scale the quantum system. It is also important to have a long coherence time, so that it is possible to do operations and manipulations before the pahse of the state is destroyed. Indeed, all of the criteria fit very well with the mentioned techniques for quantum computing. [3, 4]

The scheme used in this project is the Rare-earth quantum computer, where the rare-earth ions are put in different states corresponding to the 1 and 0 states of a classical transistor used in a modern central processing unit.

### 2.1.2 Quantum computer theory

The main component in quantum computers is the qubit, responsible for storing information, corresponding to the normal bit in a classical computer. Just as for the classical case the qubits have two possible states written the following way

$$|0\rangle \text{ and } |1\rangle \tag{1}$$

These states are often characterized by one ground-state and one excited state in an ion. The difference between classical bits and qubits of a quantum computer, is that the latter can take any value between $|0\rangle$ and $|1\rangle$. This is called a superposition or linear combination between the two possible states. Such a state is often written in the following manner

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{2}$$

Where $\alpha$ and $\beta$ are the probabilities of obtaining either of the two states when a measurement is made. They are defined so that the probability of the ion being in either of the two states is unity, $|\alpha|^2 + |\beta|^2 = 1$. The two states $|0\rangle$ and $|1\rangle$ are called base-states or base-vectors since all possible states can be represtented by a linear-combination of the two states. A good graphical representation of this is the unit sphere called the Bloch-Sphere, which shows the continuum of possible qubit states. To make the connection to the probabilities mentioned before more clear it is possible to rewrite relation (2) in terms of the angles in the Bloch-Sphere the following way.

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \exp(\mathrm{i}\phi)\sin\frac{\theta}{2}|1\rangle \tag{3}$$

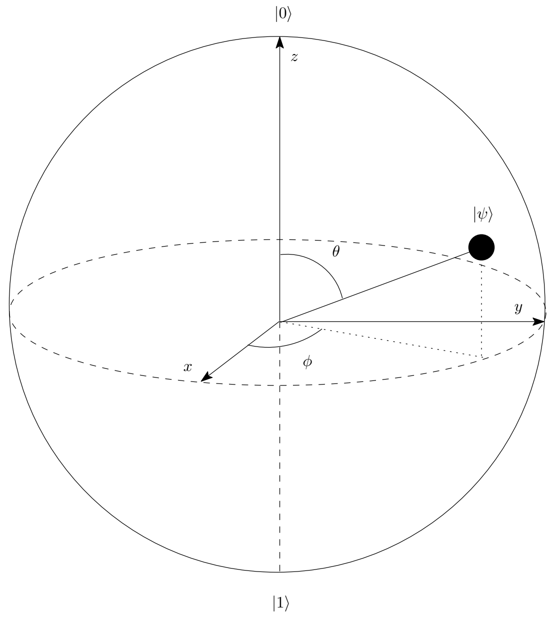

Figure 2: The Bloch-Sphere representation of the possible states of the qubits.

The superposition aspect is one of the many things that makes a quantum-computer interesting and distinguish it from the classical counterpart. By setting the state into a superposition before a computation two outcomes of the computation will be calculated at the same time. From one of the fundamental postulates of quantum mechanics it is known that a state will be thrown into one of the possible eigenvalues of the operator. In this case the operator is represented by the measurement done on the system to determine what state it is in, $|0\rangle$ or $|1\rangle$. The quantum-computer is therefore both deterministic and probabalistic. All the manipulations done until the readout are deterministic while in the readout phase is probabalistic as described above.

### 2.1.3 Orbitals and the manipulation of ions

The atomic model used today is more or less based on the Bohr-model introduced in the beginning of the 20th century. The Bohr model successfully explained the observations made by Rydberg when he studied the radiation emitted from atoms subject to a light source. The Bohr-model states that electrons orbit around a positively charged nucleus, similar to the way planets orbit around the sun. The atomic-orbits come in discrete energy levels described, and predicted, by quantum mechanics. The state of the electrons in a system is described by quantum numbers. Different sets of quantum numbers can be chosen depending on the type of problem or interaction under consideration. In LS-coupling scheme the total angular momentum and the total spin is coupled together to form the $J$-quantum number which give rise to finestructure. Since the nucleus consist of protons, which may be described as rotating point charges, they exhibit a nuclear magnetic moment $I$, which couple to the $J$ quantum number to form the resultant $F$ ( $F = J + I$). This is the hyperfinestructure. If no external field is present space is isotropic, and no splitting of the $F$-levels is seen. As soon as an external magnetic field is introduced the isotropy of space is destroyed and the magnetic sublevels have a preferred direction; this is seen as an energy level splitting and is called the Zeeman effect.

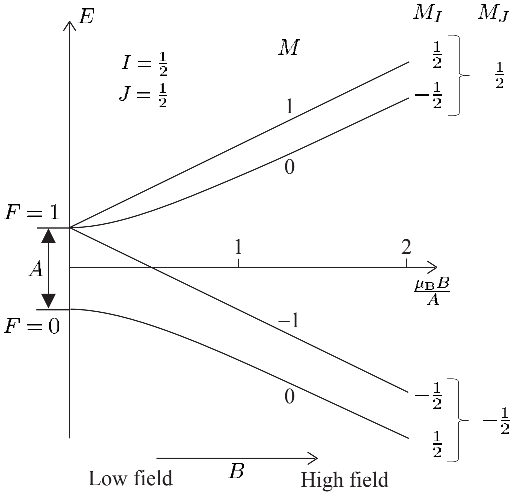

Figure 3: The effect of an external magnetic-field on the atom.[5]

When the atomic magnetic dipole moment is not perfectly aligned with the external magnetic field it will start to precess with a frequency $f$ measured in Hz. This frequency is proportional to the gyromagnetic ratio, $\gamma$, of the atom as well as the strength of the external field.

$$f = \frac{\gamma}{2\pi} B \tag{4}$$

The gyromagnetic ratio is a measure of the ratio between the magnetic dipole moment to the angular momentum of the atom.

### 2.1.4 How to move electrons between orbitals

When an electron is subject to a photon with energy equivalent to the splitting between two orbitals it will absorb the photon and be excited to the higher energy level. Most common is to use a coherent light source such as a laser to optically pump atoms from one state to another. This is true except for the case when the energy is small, as is the case with nuclear spin levels, radiofrequencies have to be used. By using the proper frequency it is possible to transfer between ground state hyperfine levels without having to take the detour to the excited state.

### 2.1.5 Rare-Earth Metals

The rare-earth metals were found late compared to many of the other elements because of their peculiar electronic structure. Instead of filled inner electronic shells with vacancies in the outer orbitals, they have filled outer shells and keep vacancies in inner orbitals closer to the nucleus. Therefore it was hard to chemically distinguish them from other elements with the same number of electrons in the outer shell. The electrons in the outer shell act as a shield, effectively minimizing the influence from the surrounding enviroment. This is very important, because the rare-earth element used for the computation is doped onto a host crystal. If all electrons were used for chemical bindings between the elements then it would be difficult, if not impossible, to do any operations at all. This is because the interaction with the surrounding environment determines the life and -coherence time of the created state, two very important aspects in Quantum-Computing. A longer coherence time makes it possible to do more operations on a state, as well as making it possible to send the state over greater distances, while still maintaining the coherence. An important thing to note is also that the operations done so far are carried out on an ensemble of ions, not single ones.

The crystal used also has to be cooled to low temperatures with either liquid nitrogen or liquid helium. There are several reasons for this, one of them being that the atoms have to be in the ground state, in room temperature they are spread out over several energy levels according to the Boltzmann-distribution. The most important reason is to minimizie the phonons which are present in high numbers when the crystal is at room temperature. The interaction with phonons will destroy the phase of the state and effectively shorten the coherence time.

In this experiment Europium is used because of the long coherence time. In this case the coherence time of the material is determined by spin-flips of the host material. The spin-flips of the surrounding atoms will cause small pertubations of the magnetic field, which in turn will destroy the coherence of the state. By applying first a static magnetic field in all three directions and then use a alternating mangetic-field to drive transitions between the hyperfine levels, it is possible to determine the magnetic field tensor and find a place where the derivative of the magnetic field is zero ($d\nu/dB = 0$). That the energy level splitting become indepenendent of the host ion spin-flips.

There are two Europium isotopes, $^{151}Eu$ and $^{153}Eu$ the latter used in this experiment. The splittings between the hyperfine ground state levels are 119MHz and 90MHz and the splittings in the excited state are 194MHz and 260MHz. The important ones are the hyperfine ground levels since those are the transitions we want to drive to determine the magnetic field tensor.

## 2.2 Theory for calculating magnetic-field requirements

In this part the calculations to estimate the power requirements are presented to produce the needed magnetic field as well as the values for the different capacitors. Most of the calculations were based on an internal paper for the Quantum-Information group written by Brian Julsgaard. To get a basic idea for the performance of the circuit some rough estimates were made. These were based on the known magnetic field tensor from another rare-earth element, namely Praseodymium, were a tuning range of the radiofrequency field of $\Delta\nu = 1 - 3MHz$ was used.[6] As a starting-point the same tuning range was used for the present rf-setup, so in order to find the transition frequency the range is $(90 \pm 3)MHz$ and $(119 \pm 3)MHz$.

### 2.2.1 Estimation of pulse time and magnetic field strength

To estimate the strength of the magnetic-field needed to transfer the population between the hyperfine groundstates a pulse-area of $\theta = 0.2\pi$ was used. The pulse-area is connected to the Rabi-frequency (that is how fast the state-vector rotates on the Bloch-Sphere fig. 2.1.2) and $\pi$ (the angle of the state-vector); if these are multiplied with each other a measure of how much the state-vector has moved during a certain time is achieved. The pulse-area can be expressed as $\theta = \Omega\tau$ where $\Omega$ is the Rabi-frequency and $\tau$ the pulse duration. In order to estimate the magnetic-field the pulse-area can also be written the following way

$$\theta = \gamma BT \tag{5}$$

Where $\gamma$ is the gyromagnetic ratio assumed to be $1kHz/G$, $T$ is the duration of the pulse and B the magnetic-field. In the Praesidium case the radio-frequency pulse was chirped (the change of frequency during a time-interval) across $\Delta\nu = 10MHz$ during a time of t = 50ms. The chirp-rate was then $r = 2 \cdot 10^8 Hz/s$. To get an estimate of the pulse-time the lifetime of a state is connected to the line-width, $\Gamma$, of a Lorentzian line-profile.

$$\Gamma\tau = 1/2\pi \tag{6}$$

In order to achieve a resolution equal to the line-width $\Gamma$ the following estimate was done, $t = \Gamma/r \geq \tau$. The shortest possible time of the pulse is equal to the life-time $t = \tau$. Using these esimates the following expression is obtained:

$$\tau = \frac{1}{\sqrt{2\pi r}} \tag{7}$$

Evaluating this a pulse-duration of $T = 30\mu s$ is realized. Using equation. 5 it is possible to get an estimate of the strength of the magnetic field to 20G (0.002T) [1].

### 2.2.2 Generating a magnetic field

To produce a magnetic field a coil/inductor is used. An inductor is basically a circularly wound wire with a current running trough. This current can be both direct (dc) to produce a static magnetic field, or alternating (ac) to produce a shifting magnetic field. In this project both types of inductors were used: The static ones to produce the Zeeman-shift and the alternating ones to drive the transition. The static magnetic field can be applied in an arbitrary direction in space using three perpendicularly oriented coil pairs in a typical Helmholtz setup. Two of the Helmholtz pairs are situated on the same holde as the radiofrequency coil, and the third is situated in the cryostat.

Each loop of wire around the coil is referred to as a turn. The more turns used the larger B-field is achieved according to the following expression: [7]

$$B_0 = \frac{\mu_0 N I}{2r} \tag{8}$$

where $N$ is the number of turns. Any current carrying wire experiences self-induction, that is, it opposes any change in voltage. A coil is wound in circular loops to increase the self-induction and produce a magnetic-field. The inductor/coil is an *electromagnet.* Inductance is measured in Henries [H =Wb/A] and denoted by $L$ which ranges in orders of magnitude based on the normal prefixes.

There are different type of inductors. All of them typically uses a circularly wound wire but inductors for different applications uses different core materials. Often seen on small circuit boards from, e.g, computers are circular iron cores wound with conducting copper-wire. In this project however an air-core inductor was used. Air-core refers to the fact that there is no core at all. There are a few reason for this, one of them beeing that the crystal is situated in the middle of the coil where the magnetic-field is produced, the other being that ferromagnetic cores demonstrate problems when exposed to high-frequency signals.

### 2.2.3 High-frequency effect on windings

When working with high-frequencies there are some additional effects to take into consideration compared with direct-current setups. The foremost is the skin effect: Any conductor subject to alternating magnetic fields will, due to the

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[1] As a reference the Earth magnetic-field is approximately 0.25 - 0.65G.

eddy currents, demonstrate an increased impedance. The skin effect becomes increasingly important as the frequency of the current increases, it is called the skin effect since most of the charge carriers will be located at the edge of the conductor, where the resistance is smaller. The skin depth is defined to be the place where the current density falls to 1/e of the value along the surface of the conductor. [8]

Another problem occuring when high frequencies are used is that the small distance between the inductor windings act as small capacitors, with the air between the conductors as the dielectric medium. A way to reduce this is to use less turns, to increase the distance between the inductors, since the capacitance is inversely proportional to the distance between the plates, or in this case, the wires.

$$C = \epsilon \frac{A}{d} \tag{9}$$

where $C$ is the capacitance, $\epsilon$ the dielectric medium, $A$ the area of the capacitor plates and d the distance between them.

A problem is that it is hard to predict how big this effect is since there are many different factors to take into account, such as the wire coating, the thickness of the wire, the core material used etc. This effect is often called parasitic or stray capacitance. There are articles were different methods to theoretically predict this, based on the electric field produced by conductors in different layers are introduced, but in the end a more experimental approach was taken rather than trying to simulate all these effects. [9, 10]

### 2.2.4 Impedance Matching

In AC-circuits it is important to match the incoming signal with the resonance signal of the circuit itself. Otherwise most of the power will be reflected. A good analogy is that of light incident on a material with a different refractive index. If the refractive index is completely mismatched most of the light will be reflected but if both the refractive indexes are more or less equal the light will instead be transmitted. There are a lot of different things to take into account in order to transmit the power to the circuit, the cable length, size of the circuit etc. In the case for this circuit the impedance matching was done experimentally using trim-capacitors.

Consider the following example;

As mentioned above the impedance matching is done with two capacitors, $C_1$ in series with an inductor which is parallel to another capacitor $C_2$.

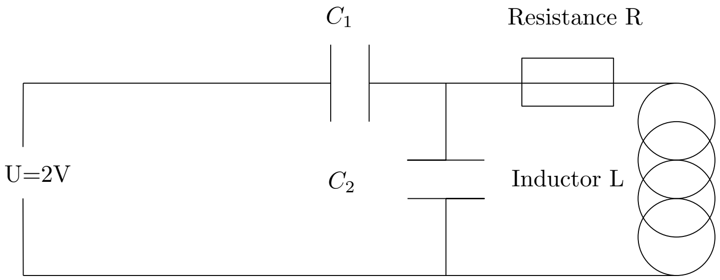

Figure 4: Example of impedance-matching with capacitors.

The impedance of such a circuit can be written as follows

$$Z = \frac{1}{i\omega C_1} + \frac{1}{i\omega C_2 + \frac{1}{i\omega L + R}} \tag{10}$$

It is then possible to find a frequency $\omega$ given by.

$$\omega = \frac{1}{\sqrt{LC_2}} \tag{11}$$

varying $C_1$ and $C_2$ so that Re(Z) $= 50\Omega$, that is, the circuit is impedance matched to the signal that is fed to the system.
For more info see [7].

### 2.2.5 Q-value

The resonance is seen as a dip in the frequency sweep resembling a lorentzian line-profile.The relation between the peak of the dip and the FWHM (full-width half-maximum) is used to put a number of the quality of the matching. This is called the Q-value.

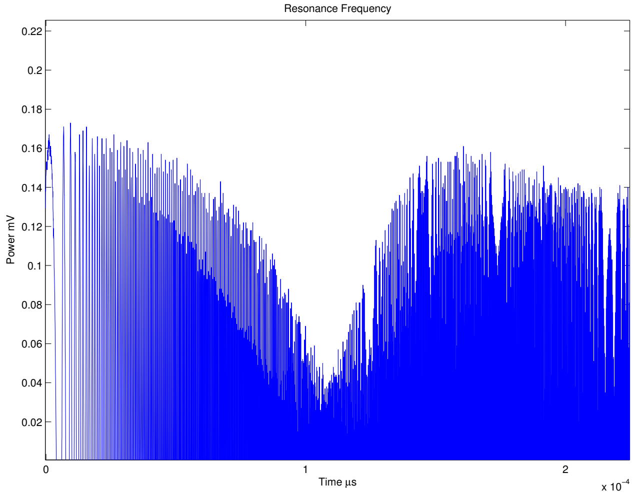

Figure 5: Picture of the resonance, the lorentzian profile is clearly seen.

$$Q = \frac{\omega_0}{\omega_{FWHM}} \tag{12}$$

A high Q-value means a very sharp line-profile whereas a lower Q-value implies a more spread out profile and that a larger frequency is covered. Depending on the situation this can be both good and bad. In the case of this project a high Q-value was not needed.

### 2.2.6 The dBm/dB Scale

It is very important to take the power requirements into consideration when working with electronic components. They cannot handle an infinite amount of power so care has to be taken in order to match their respective requirements. A thing that makes this a lot easier is to use a scale based on decibels rather than the normal volts/watts. The reason for the decibel-scale being easier is that instead of going trough the tedious process of converting between different units to be able to tell whether or not the selected component will qualify, it is possible to work this out with simple addition and subtraction. Another cause for using the decibel scale is of course that it is logarithmic, i.e, it describes both very large and very small numbers, something very common when working with power-units.

The power rating of the amplifiers used were given in dBm, which is the decibel-scale of power in watts, while the amplification/attenuation is given in dB which is a measure of the ratio between the incoming and outgoing signal. To convert watts to dBm the following relation can be used.

$$x = 10log_{10}(1000P), \tag{13}$$

where $P$ is the power in watts and $x$ the power in dBm.

The expression used to calculate the ratio between two different power-units $P_1, P_0$ is the following.

$$L_{dB} = 10log_{10}(\frac{P_1}{P_0}) \tag{14}$$

Where $L_{dB}$ is the change in dB.

Here is a small example to demonstrate how easy it is to use the dB/dBm scale rather than calculating directly:

> A signal have to be amplified and matched to a high-pass filter. The data-sheet of the amplifier specifies the gain (that is amplification) in dB since it is a ratio of the incoming and outgoing power. In this case the gain is 19.3dB, the signal-strength is 5dBm before the amplifier and since the gain is 19.3dB the outgoing power is 24.3dBm. This has to be matched to the high-pass filter which in this case has a maximum power-rating of 15dBm, the power is in this case too large. To remedy this an attenuator specified with -12dBm is used to lower the signal to 12.3dBm.
>
> Instead of working with different powers and ratios this problem was made very easy and it was possible to solve using only calculations in your head.

# 3 Calculations

In this section the calculations and estimations made to be able to construct the circuit are presented. The calculations were based on data from a kit[2] of 12 pre-built coils from a company called Coilcraft(`www.coilcraft.com`). Data from the pre-built coils were also used in simulations as well as when constructing test-circuits to be able to see if the results obtained theoretically corresponded to the real experiments.

## 3.1 Inductance

The inductance of the coil is important when constructing a rf-circuit since it determines the values of the other components as well as the magnetic-field produced by the coil at different currents. There are a lot of different formulas for calculating inductance that are more or less based on emperically obtained equations by Wheeler from the beginning of the 20th century [11]. When trying to determine the inductance of the real coil there were doubts as to how exact the different formulas were since they all produced different results with the same input. The solution was to use data from the pre-built Coilcraft inductors,

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[2] The name of the kit was 'Maxi Spring Air Core Inductors' Kit C319-2

since they had the inductance checked with hardware specifically designed to determine inductance with high precision. This provided a way to verify what formula to use when constructing the real coil.

The specifications from the following inductor was used to check the formulas.

- Model-Name: 132-12SMGL
- Turns (N): 12
- Inductance (L): 169nH
- Radius (r): $1.55 \cdot 10^{-3}$m
- Length (l): $7.98 \cdot 10^{-3}$m
- Wire-diameter (a): $0.6 \cdot 10^{-3}$m
- Self-Resonance frequency (SRF): 875MHz
- Q at 100MHz: 150

The different formulas used to check the accury of the calculation.

| Eq. | Formula | Result |
|---|---|---|
| 1 | $L = 4N^2\mu_0 r$ | $1.12\mu H$ |
| 2 | $L = 0.0050rN^2$ | $1.11\mu H$ |
| 3 | $L = N^2\mu_0 r[log(8r/a) - 7/4]$ | $358nH$ |
| 4 | $L = (r^2N^2)/(9r + 10l)$ | $145nH$ |
| 5 | $L = \mu_0(\frac{N^2A}{l})$ | $171nH$ |

Table 1: Eq. 1-3 are from [7], eq. 4 from [8] and eq. 5 from [12].

As is seen in the table equation 5 shows the best correspondence with the real inductance, and that was also the one used when determining the inductance of the real coil.

## 3.2 Resonance Frequency

To calculate at what point the resonance frequency is situated the following expression can be used.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{15}$$

Conversely, the capacitance can be obtained of the resonance-frequency and the inductance is known. When simulating inductors it is common to use a model where the stray-capacitances are modelled by a series resistance and a parallel capacitance. In such a model it is possible to use eq.15 to first calculate the capacitance, and then use the following relation to determine the series resistance. [7]

$$R = 2\pi \frac{f_0 L}{Q} \tag{16}$$

In section 3.1 the necessary data is specified and thus a model of the inductor can be completed using the following steps;

$$\begin{aligned} f_0 &= \frac{1}{2\pi\sqrt{LC}} && (17) \\ C &= \frac{1}{(f_0 2\pi)^2 L} && (18) \\ C &= \frac{1}{(875MHz \cdot 2\pi)^2 169nH} = 0.19pF && (19) \end{aligned}$$

Doing the same thing for eq. 16 the following resistance is obtained.

$$\begin{aligned} R &= 2\pi \frac{f_0 L}{Q} \\ R &= 2\pi \frac{875MHz \cdot 169nH}{150} = 6.63\Omega \end{aligned}$$

The model is shown below along with the values calculated here. In order to simulate another inductor the numbers used above are just exchanged to the numbers corresponding to the new inductor/coil. It is important to note that the self-resonance frequency used here (SRF) is the SRF for the inductor itself and not the whole circuit.

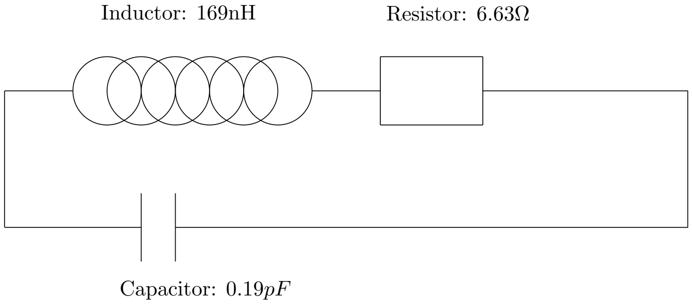

Figure 6: Common model for inductors when working with high-frequencies.

# 4 Experimental Setup

In this part the components used to do the experiment are presented.

## 4.1 The Signal Generator

To produce the radiofrequency responsible for moving the ions a signal-generator was needed. There were no signal-generators available with 100MHz frequency the decision to use a '*Standford Research System DS345*' (SRS) 15-30MHz signal-generator, along with frequency doublers was made. Besides producing the correct frequency, the instrument has to be able to produce sweeps between different frequencies; and also set markers at different points to know where in the (frequency) sweep a particularly interesting point is. The SRS was able to do all this and thus the choice fell on this device.

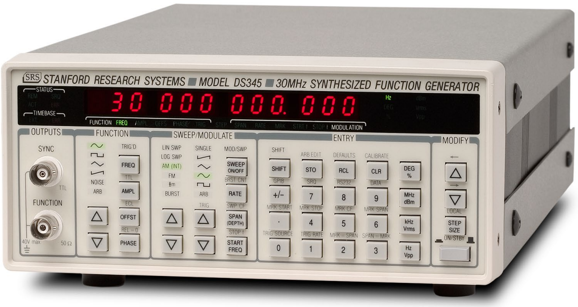

Figure 7: The SRS. (From the manual)

## 4.2 Frequency Doublers and Amplifiers

Frequency doublers are small devices which doubles the frequency sent in at the cost of attenuating the signal. In order to achieve 100MHz two frequency-doublers were used in series to first double the 25MHz signal to 50MHz and then double it again to 100MHz. After each frequency doubling device the power of the signal decreases because of the mentioned attenuation. To overcome this two amplifiers were used in the same chain as the frequency-doublers to amplify the signal. The first one was placed after the first doubler and the second one after the last/second doubler. This way it was possible to get a decent signal strength out, and if more power was needed an additional amplifier could just be placed at the end to increase it further.

Since the components used all have different power requirements and specifications, it is hard to exactly match the output power of one of the amplifiers with the input power of one of the doublers. This was made easier by adding attenuators, small devices put before or after the mentioned components to attenuate the signal. They come in a variety of values ranging from -1dBm to whatever number/attenuation needed.

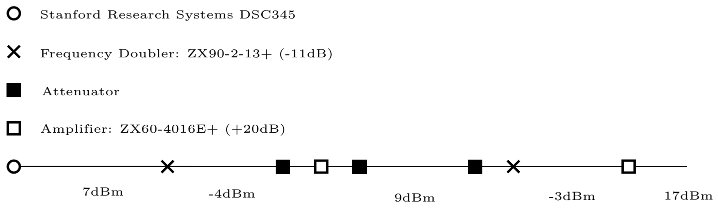

Figure 8: The signal-chain showing the signal strength in the different points. The black squares represent attenuators of different strengths.

## 4.3 Additional components

In addition to the already mentioned components a directional coupler and a high-pass filter was used. The directional coupler is a plug in component with three connections, one input, one output and a third connection named couple. By connecting one end to the circuit and the other to the input signal it is possible to monitor the reflected current via an oscilloscope connected to the couple input.

When using the frequency-doublers to achieve the correct frequency a problem was found. Not only was the doubled frequency sent on to the amplifier and in the end the oscilloscope, but the two lower frequency components of 25 and 50MHz were seen as well. The quality of the frequency doublers was not good enough and this affected the signal so that it became harder to detect and measure the resonance frequency. A way to solve this was to add a high-pass filter which cut all frequencies below 90MHz. This did not solve the problem completely but it made it somewhat easier to study the resonance frequency.

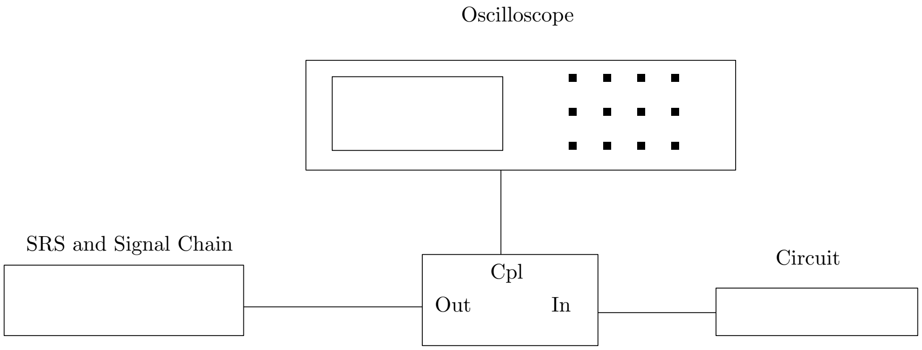

Figure 9: Picture of the whole setup with the SRS and signal chain connected to the directional-coupler, oscilloscope and circuit.

### 4.4 Oscilloscope

To study the behaviour of the circuit an oscilloscope was used. One input was used to connect the circuit to the oscilloscope and another was used for the trigger signal. The trigger signal is sent from the SRS to the oscilloscope to provide a stationary signal, if there were no trigger-pulse present the signal of interest would just drift across the screen making it impossible to draw any conclusions about the experiment.

In this case triggering was done with markers set on the SRS providing a start and stop point of a frequency interval of interest. To calculate the Q-value as described in section 2.2.5 the markers are used. For example, the mark-start position is set to $98MHz$ and the mark-stop is set to $102MHz$. Using the cursor function on the oscilloscope it is possible to measure the width of the marker in in terms of the x-axis, in this case time. Since the width of the mark-pulse is set to be $4MHz$ it is possible to measure the width of the pulse using the cursors to find out what $x$-value corresponds to what frequency. For instance $4MHz = 170\mu s \rightarrow 42.5\mu s/MHz$. With this information it is possible to measure both position and the FWHM of the resonance-frequency and calculate the Q-value.

## 5 Circuit Design and Simulations

The idea of the circuit-design was based on an earlier project where a 10MHz rf-circuit was built. The concept was to use two trim-capacitors, one in series and one in parallel with the inductor, to minimize the reflected current and impedance match the circuit.

To learn what kind of values that were appropriate for the inductor as well as the capacitors a small, open-source program, called QUCS was used (Quite-Universal-Circuit-Simulator, `http://qucs.sourceforge.net/`).

It is a program that is easy to understand yet powerful enough to provide information on how a circuit behaves when subject to different conditions. It is possible to set the working temperature for the different componenets as well as sweep parameters such as the capacitance and frequency. Discrete components can be dropped onto a grid and connected with each other to form a circuit. It is then possible to set values for the components and sweep between different frequencies just as in the real experiment. By placing probes to monitor both the current in the inductor and the reflected current it was possible to find out at what values (frequency, capacitance) the resonance was.

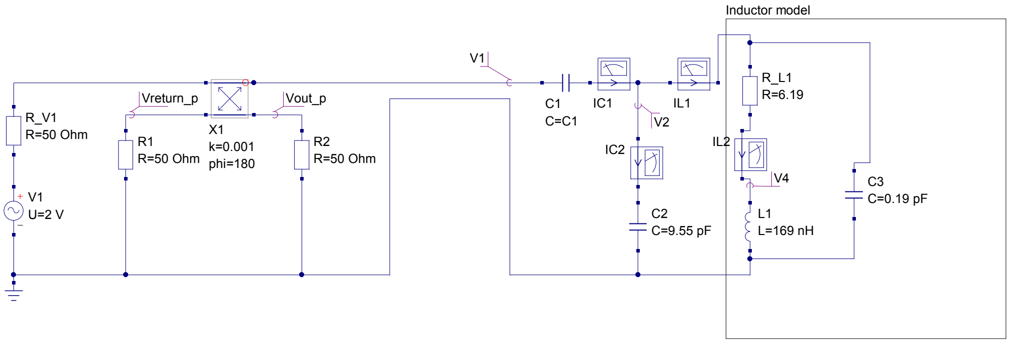

Figure 10: The circuit used in QUCS to simulate the resonance frequency for different inductors. The inductor model described in the earlier section is to the right, along with the probes to monitor voltage over, e.g, the inductor. The component to the right with 'X' is a directional coupler used to monitor reflected current. Resistances are used to set the resistance to 50Ω to mimic the resistance found in the laboratory equipment.

To have some idea of the start value for the parallel capacitance the same expression15 as in section 3.2 was used but instead of the self-resonance frequency (SRF) for the inductor, the desired SRF of the circuit was used as $f_0$, in this case $100MHz$. The series capacitance was obtained by noticing how the resonance-frequency of the circuit changed when sweeping over different values of the capacitance. It is important to note how the series and the parallel capacitance are connected; this was seen in the simulations as well as in the real experiment. If the parallel capacitance was increased a lower capacitance was needed in the series capacitor. The result was the same the other way around, using higher capacitance in the series a lower had to be used in the parallel case. In the end a balance had to be found between the two capacitors and the inductor.

## 5.1 Simulation 1: Inductance 206nH

The first simulation was made with a coil with the following properties.

- Part Number: 132-13SMGL
- Turns (N): 13
- Inductance (L): 206nH
- Self-Resonance Frequency (SRF): 800MHz
- Q at 100MHz: 140

The data from the inductor above was used to simulate the rf-circuit. The frequency sweep was between $1 - 120MHz$ and the capacitances were changed until a resonance was observed at the desired point of $100 MHz$.

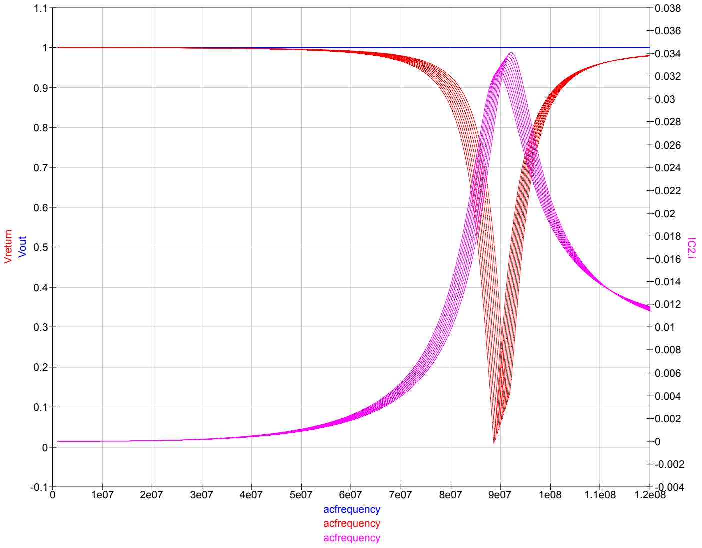

Figure 11: Plot of the resonance frequency with a 206nH coil, obtained with QUCS. The incoming current is blue, the reflected current red and the current trough the parallel capacitor pink.

In fig. 5.1 the parallel capacitance was calculated to be, $C_p = 12pF$ and the series capacitance $C_s = 5.22pF$. In order to get the resonance frequency to $100MHz$ the parallel capacitor was lowered to $C_p = 8pF$ and the series capacitor $C_s = 4pF$. This was a problem; if the circuit is dependent on capacitances of that size, they can easily be disturbed by stray-capacitancies in the circuit. A way to solve this was to reduce the inductance of the coil at the cost of using more current to create the magnetic field needed.

## 5.2 Simulation 2: Inductance 169nH

- Part-Number: 132-12SMGL
- Turns (N): 12
- Inductance (L): 169nH
- Self-Resonance Frequency (SRF): 875MHz
- Q at 100MHz: 150

The inductor was modelled as described in section 3.2 using the same values for the inductor as shown above. With the lower inductance it was possible to get a resonance frequency at $100MHz$ using higher capacitances. The series capacitor was set to $C_s = 5.22pF$ while the parallel capacitor was $C_p = 9.55pF$.

# 6 Construction of Test-circuits and Measurements

With help from the simulations it was possible to determine what values the capacitances should have as well as the inductor. In this case the inductor from section 5.2 was chosen.

The first try was building a test-circuit on a simple breadboard with hole-mounted trim-capacitors in the range of $1.8 - 22pF$ using the design mentioned earlier. A problem was that the hole-mounted capacitors covered the whole capacitance-range by turning a trim-capacitor 180 degrees, thus it was hard to achieve good precision.

Below is a picture of the resonance frequency as seen on the oscilloscope. Unfortunaly it was not possible to include the marker-pulse in the same plot because of different scales on the $y$-axis.

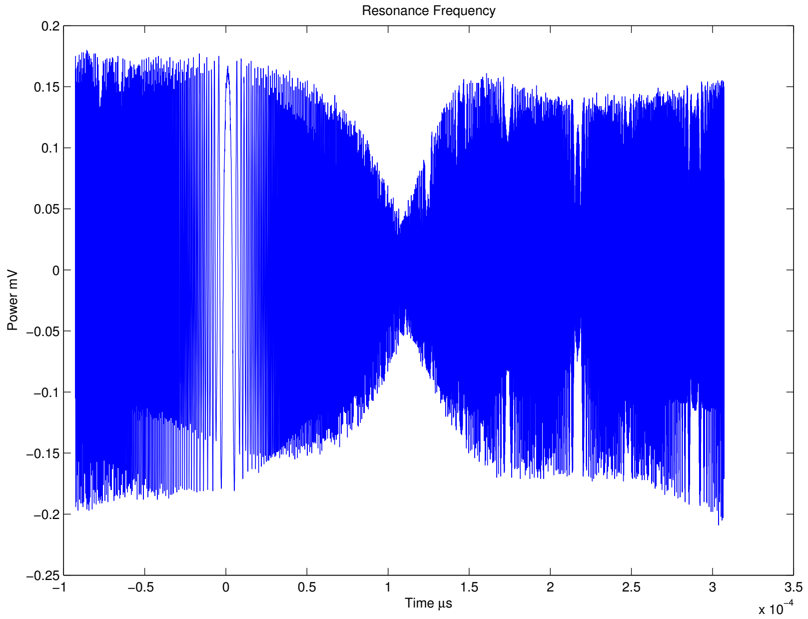

Figure 12: The resonance frequency as seen on the oscilloscope.

## 6.1 Hole-mounted Circuit

Despite the lack of precision turning it was still possible to detect a resonance frequency. Using the techniques mentioned in section 4.4 it was possible to determine the position of the resonance frequency and by visually noticing how much the capacitors were turned estimations of the capacitance could be obtained.

| Inductance | 169nH |
|---|---|
| $C_p$ | 15pF |
| $C_s$ | 4.4pF |
| Resonance-Frequency | 97MHz |

Because of the difficulties in setting the capacitances the hole-mounted setup had to be abandoned in favor for a surface-mounted pcb circuit.

## 6.2 PCB-Circuit

The pcb used to build this circuit was constructed by Mahmood Sabooni for the 10MHz rf-circuit. Everything worked fine using this board but when working with high-frequencies it is important to keep everything as small as possible to avoid uncessary resistances and stray capacitance. For the real circuit space limitations in the cryostat, used to cool the crystal, had to be taken into account as well as being able to fit the coil. To use the 100MHz coil in a real experiment a smaller pcb had to be constructed.

Surface-mounted components come in smaller values than the hole-mounted, for example, the trim-capacitors used in the hole-mounted case covered a range of $1.8 - 22pF$ in a 180 degree turn. The one used in the surface-mounted case covered a range of $0.8pF - 8pF$ in 18-turns. In this case turning the surface-mounted capacitor did not give any visual feedback as to what the capacitance was, to determine the capacitance I had to estimate how many turns I did from one of the end-points to the resonance-frequency of interest. From the simulations it was seen that more capacitance than the trim-capacitor could produce was needed in the parallel case, thus a fixed capacitor of 6.8pF was soldered to the pcb, yielding a minimum parallel capacitance of $0.8pF + 6.8pF = 7.6pF$.

There are tools out there called LCR-meters specifically designed to measure inductance, capacitance and resistance. In this case the values for both inductance and capacitance were very small and a machine capable of measuring these values would be very expensive, so the method with just counting the number of turns was used.

The plan was to test the circuit at six different frequencies, from $100Mhz$ to $130MHz$ in steps of 10MHz, since the inductance and capacitancies were chosen to have the resonance around $100MHz$ it was only possible to produce a good result at three of the steps, $90, 100, 110MHz$.

As I experienced during testing, that rf-circuits are VERY sensitive when it comes to cables moving etc. Just by touching the circuit the resonance frequency could move several MHz up or down. This could also lead to the effect that a really good resonance and Q-value was achieved while holding the circuit just to disappear as soon as it was placed on a table -the capacitance of the circuit changed by touching it.

### 6.2.1 Measurment 1 90MHz

| Marker-pulse width | 4MHz |
|---|---|
| Resonance-Frequency, $\omega_0$ | 89.7MHz |
| FWHM, $\omega_{FWHM}$ | 1.86MHz |
| $C_s$ | 3.6pF |
| $C_p$ | 10.4pF |
| Q | 48.2 |
| Reflection (voltage) | 0.46 |
| Reflection (power) | 0.211 |

The reflection is calculated by comparing the heigth of the peak-position with the height of the part of the signal were the reflection largest.

### 6.2.2 Measurment 2 100MHz

| Marker-pulse width | 4MHz |
|---|---|
| Resonance-Frequency, $\omega_0$ | 99.6MHz |
| FWHM, $\omega_{FWHM}$ | 1.309MHz |
| $C_s$ | 1.6pF |
| $C_p$ | 9pF |
| Q | 76 |
| Reflection (voltage) | 0.35 |
| Reflection (power) | 0.122 |

### 6.2.3 Measurment 3 110MHz

| Marker-pulse width | 4MHz |
|---|---|
| Resonance-Frequency, $\omega_0$ | 109MHz |
| FWHM, $\omega_{FWHM}$ | 1.396MHz |
| $C_s$ | 0.8pF |
| $C_p$ | 8.2pF |
| Q | 78.8 |
| Reflection (voltage) | 0.30 |
| Reflection (power) | 0.09 |

### 6.2.4 Measurement 4 Liquid Nitrogen

To see how cooling the circuit affected the resonance frequency the whole circuit was put into liquid nitrogen. The most prominent effect was that the resonance frequency was down-shifted which implied that a bit more capacitance was needed in order to bring it up to the right frequency. The procedure was to first set the circuit off-resonance, and then put it into liquid nitrogen, which put the circuit into resonance again because of the mentioned change.

| Marker-pulse width | 1.16MHz |
|---|---|
| Resonance-Frequency, $\omega_0$ | 99.16MHz |
| FWHM, $\omega_{FWHM}$ | 0.97MHz |
| $C_s$ | 1pF |
| $C_p$ | 7pF |
| Q | 102 |
| Reflection (voltage) | 0.12 |
| Reflection (power) | 0.014 |

In the case of this experiment it can be seen that the reflected current is a lot smaller than in room-tempertaure. The Q-value is also higher. I think these results are expected since lower temperature means lower overall resistance in the circuit. The series trim-capacitor was aslo desoldered and replaced by a fixed 1pF capacitor, since it was easier to control the circuit with just one variable parameter.

When cooled the circuit was even more sensitive than before, just by touching the cable the resonance frequency was shifted and/or destroyed.

# 7 Constructing the Real Circuit

What was learned from the test-circuits was that a low inductance is needed in order to bring the capacitances to more reasonable values. The bigger circuit board used for the test-circuit was also unnecessary since very few fixed capacitors were needed in order to reach the capacitance region of interest.

In the previous project where the same coil setup had been used it had been wound with 10 turns, using a coil-diameter of $6mm$, and a length of $2.5mm$. This produced the following inductance using the formula producing best results from section 3.1.

$$L = \mu_0(\frac{10^2\pi 3 \cdot 10^{-3}}{2.5 \cdot 10^{-3}}) = 1.42\mu H \tag{20}$$

The inductance for this coil is ten times higher then the one used on the test circuits. This implies using the same coil for a $100MHz$ setup would require ten times smaller capacitances, which would not be possible. A solution to this would be to make the coil longer to decrease the inductance, as expression (5) in section 3.1 describes, but since the length is fixed this is not possible. Instead it was decided to lower the number of turns in order to bring the inductance down. As seen in eq. 8 in section 2.2.2 the magnetic-field produced is proportional to the number of turns and also the current, using less turns means more current in order to yield the same field.

Putting in the proper numbers into expression 20 with different turns the following was obtained.

| Turns | Inductance |
|---|---|
| 7 | $694nH$ |
| 5 | $355nH$ |
| 3 | $127nH$ |

With the information from the test-circuits the conclusion is that 3 turns is the proper choice. A lower inductance means that higher capacitances can be used and the stray capacitances affecting the circuit is less of a problem. The downside is that 3 turns require more current to produce the same magnetic field and thus increasing the power dissipation in the cryostat.

## 7.1 Measurements

Two tests were made, one in room temperature and one in liquid-nitrogen.

### 7.1.1 Room Temperature

| Marker-Width | 4MHz |
|---|---|
| Resonance-Frequency, $\omega_0$ | 106.8MHz |
| FWHM, $\omega_{FWHM}$ | 2.14MHz |
| $C_s$ | 2.4pF |
| $C_p$ | 12.7pF |
| Q | 49.9 |
| Reflection (voltage) | 0.35 |
| Reflection (power) | 0.122 |

### 7.1.2 Liquid Nitrogen

| Marker-Width | 4MHz |
|---|---|
| Resonance-Frequency, $\omega_0$ | 110.47MHz |
| FWHM, $\omega_{FWHM}$ | 1.52MHz |
| $C_s$ | 0.8pF |
| $C_p$ | 13.5pF |
| Q | 72.67 |
| Reflection (voltage) | 0.54 |
| Reflection (power) | 0.291 |

As seen from these results the Q-value increases a lot when the circuit is cooled , one of the reasons for this is that the resonance in this case was located at a higher frequency which implies a higher Q-value. The same pattern for the series capacitance $C_s$ is also seen as for the test-circuit. When the resonance frequency is moved to higher frequencies the series capacitance has to be lowered.

## 8 Discussion

The aim of this project was to construct a high-frequency radio circuit operating at 100MHz. However, in the beginning not much was known as to how the circuit would behave when subject to this frequency. As noted in the text there are a lot of things to think about when working with higher frequencies compared with lower ones. I experienced the circuit was really sensitive to touching. This effect was amplified when the circuit was cooled and thus it is very important to set everything up as rigid as possible. For the real circuit I think it is important to construct it such that it is possible to control the capacitances without touching the circuit itself -it must be mounted in a proper way.

Another aspect worth noticing is that the capacitance for the series capacitor is very very low, especially when the frequency is increased. This makes it very sensitive to stray capacitances in the circuit. The only way to achieve a lower series capacitance is to have a lower inductance. The ideal case would be to rewind the existing coil setup where the coil for the static magnetic field was situated, but as it seems now this will be very hard. To decrease inductance one can take several steps, by looking at expression (5) in secion 3.1 it is seen that the inductance is inversely proportional to the length of the coil. A longer coil would be an option but this could come at the expense of losing the homogenity of the created magnetic field.

In order not to lose homogenity another approach could be using a smaller area $A$, but this means that another crystal has to be manufactured.

Clearly there are a lot of things to think about when constructing a coil like this. For my part the conclusion is that it may not be possible to use the existing coil but a new one has to be constructed, it is important to really take the frequency that is going to be used into consideration since this affects a lot of different parameters in the circuit. A more general thought on how to construct things is that sometimes I think it is easier to start building as soon as possible rather than theoretically predicting the exact behaviour of the circuit. As soon as you start building you bump into problems that have to be solved etc, so in time you learn what the most important aspects of the circuit are.

As for the quantum-computing field it still has a very long way to go before realizing something that outshines the classical counterpart. However, by coming up with new clever ways of controlling and manipulating quantum states we learn more about the quantum world and how it works -knowledge which not only is important for computer manufacturing, as the size of the processors and chips decreases, but also for constructing more effective solar cells and also recently, learn more about the photosynthesis and the quantum-processes that control it. [13]

# 9 Acknowledgements

I would like to thank my supervisor Stefan Kröll for patiently answering all my questions about this project as well as Mahmood Sabooni for helping out with the many practical details, and also Lars Rippe for helping me get started with the actual constructing/building of the circuits. I would also like to thank the entire Quantum Information group for all the interesting discussions during lunches and coffee-breaks.

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