LUND UNIVERSITY
School of Economics and Management

Where the rainbow ends…

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BUSM65 Degree project in Finance II VT11
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Abstract

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Keywords: Rainbow option, Volatility forecasting, Correlation forecasting, GARCH, CCC-GARCH, DCC-GARCH

Purpose: The purpose of this thesis is to evaluate which model for forecasting the variance covariance matrix is the most accurate. This is important because the more accurate forecast the more correct pricing of derivatives with several underlying instruments.

Methodology: Using currency data of EUR/USD and GBP/USD five different variance covariance matrix forecasting models are used to price simulated currency rainbow option. The models are then evaluated by simulated trading between five agents using their own forecasting model which the one with the best forecasting model should have accumulated the higher profit. Statistical evaluation is also used to determine the most superior variance covariance matrix forecasting model.

Results: The results shows that for the whole sample period the naïve forecasting model is the most superior when looking at the accumulated trading profit, followed by CCC-GARCH. According to the statistical loss evaluation DCC-GARCH is the superior model followed by CCC-GARCH.

Terminology

ARCH Autoregressive Conditional Heteroscedasticity

CCC Constant Conditional Correlation

DCC Dynamic Conditional Correlation

GARCH Generalized Autoregressive Conditional Heteroscedasticity

MAE Mean absolute error

MSE Mean squared error

Rainbow option An option with several underlying instruments

RMSE Root mean squared error

Two-colored rainbow option An option with two underlying instruments

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1.0 Introduction

The concept of option pricing has mostly been based on a single asset as the underlying instrument. This has been discussed frequently since the introduction of the Black-Scholes (1973) formula. The field of derivatives has grown tremendously since then and the idea has been extended to options whose payoff depends on performance of several underlying assets. To price these kind of derivatives not only the volatility has to be taken into consider, but also the co-movement between the assets. The risk in multi-asset options can be measured with strength using the correlation, which is the linear relationship between assets. To price or hedge an option with several underlying assets as stocks, currencies, commodities etc. can have implications. A simple type of a multi-asset option is a basket option or, as in this thesis, a two-colored rainbow option i.e. an option with two underlying instruments. This thesis will use the concept of correlation to discuss option pricing and volatility.

The options correlation input, as its volatility, is the only variable that is unknown in the Black-Scholes formula and hence, it has to be predicted. The implied volatility is often discussed as the main player in pricing the option, but it will be seen that in the case of multi-asset options, the implied correlation also plays a main role. To be able to forecast the correlation is not just important in option pricing. Markowitz (1952) discussed this using multiple assets within a portfolio. He discussed that diversification within the portfolio can lower the exposure of individual assets. Using Markowitz (1952) finding, one can lower the risk in a basket option by diversifying using multiple assets.

In addition to the implied volatility, which includes both forward and backward looking information, the implied correlation adds additional information in a multi-asset world. Since there is no liquid market for multi-asset options, the options correlation has to be decided through other methods. To measure the correlation in a proper way is not an easy task since documentation shows that correlation is changing over time and increases during market crisis.

1.1 Problem discussion

This thesis evaluates different methods of forecasting the variance covariance matrix. The major problem expected is to achieve a “clean” estimate of the variance covariance matrix. Therefore we will evaluate the variance covariance matrix forecasts in a rainbow option setting where different forecasting models will price one simulated rainbow option. Similar tests have been done on equities and indices but none has been done with newer models for variance covariance matrix forecasting like Dynamic Conditional Correlation (DCC) GARCH in this kind of setting and with exchange rates as underlying instruments.

There is a problem when modeling variance covariance matrix because it is time dependent and not constant. Since the derivatives market is constantly developing with new exotic options it is of great importance to be able to determine how the underlying assets are co-varying. Since this can be estimated by different models, it is crucial to know which one is the superior. There is different ways to determine this i.e. the economic point of view and the statistical. It is not possible to observe the true volatility and hence, it is not possible to observe the true variance covariance matrix.

1.2 Purpose

The purpose of this thesis is to evaluate which model for forecasting the variance covariance matrix is the most accurate. This is important because the more accurate forecast the more correct pricing of derivatives with several underlying instruments.

1.3 Delimitations

We limit the thesis to include pricing of rainbow options with two underlying instrument, so called two-colored rainbow options. As underlying instrument we choose to use exchange rates of the US dollar against the British Pound and the Euro. Further on we investigate five different variance covariance matrix forecasting models each represented by an agent on a simulated closed option market. The chosen forecasting models are naïve historical, 20-day moving average, exponential moving average, Constant Conditional Correlation (CCC) GARCH and Dynamic Conditional Correlation (DCC) GARCH.

1.4 Outline

Section 1 presents the problem behind variance covariance estimation leading up to the purpose of the thesis and the specific delimitations. The introductory part is follow by a presentation of the essential theories behind the study in section 2. Beside previous studies an intuitive presentation of the Black-Scholes (1973) formula is presented with the Margrabe (1978) extension to a multi-asset framework. The models used in the study are also presented.

Section 3 treats the methodology with a motivation of the data, GARCH estimation and the statistical evaluation method. In addition, a thoroughly presentation of the economic evaluation method is described. Section 4 presents the results and describes some intuition behind it. Both the economical and statistic results are shown in this section and the differences between them are discussed in section 5. The latter section also presents the major analysis of the differences between the models and the fundamentals behind the results.

Section 6 concludes the thesis and includes a proposition of further research in the variance covariance estimation field. Section 7 is a presentation of the sources and section 8 is the Appendix containing the programming used for estimating the DCC-GARCH.

2.0 Previous studies

Even though the subject for predicting the variance covariance matrix is constantly developing and is an up-to-date topic, the knowledge reaches back a few decades. Margrabe (1978) derives an extension of the Black-Scholes (1973) formula, where the paper addresses the issue of pricing an option with two, or more underlying assets. In recent years, this topic has progressed with e.g. Bollerslev, Engle and Wooldridge (1988) measurement of the conditional variance covariance matrix using a Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model with constant conditional correlation to test its validity using nominal exchange rates. In comparison to the previous mention research, Bollerslev (1990) finds that the multivariate GARCH reduces the computational complexity. In line with this thesis, Bollerslev and Engle (1993) examine exchange rates, where they find co-persistence in the series using a multivariate linear GARCH.

A more recent paper by Engle (2002) tests how well different forecasting models are able to estimate the variance covariance matrix. The paper finds that the different dynamic conditional correlation models are most accurate in their estimation in most of the cases. The study examines the correlation using different data series such as the correlation between Dow Jones Industrial Average and NASDAQ for ten years of daily data, daily correlation between stocks and bonds and also, examines daily correlations between currencies and major historical events.

Different methods are used when evaluating which of the forecasting models that estimates the variance covariance matrix most accurate. Engle (1993) for example uses mean absolute error (MAE), autocorrelation and Value-at-Risk. Another approach, which is an extension of Engle (1993), presented by Gibson and Boyer (1998), uses options to determine which of the five tested model that produces the most accurate estimates. Using a matrix of joint asset returns, a forecast of the variance covariance matrix are used to generate a trading strategy for a package of simulated options. An extension by Byström (2001) uses a portfolio of two-colored currency rainbow options to evaluate relative performance of different forecasting models.

3.0 Theory

3.1 Theory of option pricing

To be able to price a call option using Black-Scholes formula with two underlying assets, the formula has to be re-written. An intuitive explanation is given below starting with the pricing of a plain vanilla call option following Black and Scholes (1973).

Assume a contingent claim with an underlying asset which pays no dividends. The underlying asset is in this case a stock, but can be change to represent e.g. a series of exchange rates. Assume that the underlying asset follows a diffusion process and is a Wiener process:

There is also a risk free rate with the following process

Now, consider a European call option on the stock with maturity T, exercise price X, price C and with date t value . Since the maturity date , the value at maturity, and initial condition, is given by:

Applying *Ito's Lemma* and inserting the stock dynamics yields:

The call option has the same source of risk as the underlying stock, reflected by the Brownian motion, .

The objective is to determine the price of the option such that the market is free from arbitrage. This is done by forming a portfolio of call options and stock which replicates the bond.

By collecting and terms:

Setting the following restrictions and solve gives the risk free portfolio weights i.e. Black-Scholes take a long position in the stock and a short position in the option and form a riskless investment.

For the market to be free from arbitrage, the risk free portfolio should equal the risk free rate:

Substituting in the above given risk free weights and re-arranging yields the Black-Scholes partial differential equation:

The call option value must satisfy the above equation given its initial condition:

The solution to the expression can be solved by using the Feynman-Kač method:

As can be seen by the derivation of the formula to the pricing equation, the expected return on the stock is not included in the pricing and this comes from the no arbitrage argument. The market participants may disagree on the expected returns, but they will agree on the price and the price on the call is uniquely determined by the prices on the other assets by no arbitrage.

Now, consider the case with two underlying assets i.e. a rainbow option to exchange one risky asset for another. The following model is an extension to the plain vanilla Black-Scholes pricing formula and has the same underlying assumptions as previous. The asset is assumed to follow the same diffusion process as before but now containing two assets i.e. . The correlation in this case is given by . The initial condition in this case includes two assets and time with the option to switch on asset for the other. The extension follows Margrabe (1978):

The option will be exercised at if the value of asset one is greater than asset two in the case of a call option i.e. is the strike. If not exercised, the option will yield nothing. As in the previous case, Black-Scholes eliminates all the risk by putting up portfolio restrictions and solving for the risk free weights.

Further, Margrabe (1978) restricts the options value by the fact that it must be at least 0 but not more than if asset one and two are worth at least zero:

The option buyer can hedge its position and by the hedge to eliminate the risk, the return is also eliminated. The pricing formula is linear homogeneous in and and gives the hedger's investment (see Margrabe, 1978 for proof).

The value of the hedged position is hence zero. Using Black-Scholes derivation and stochastic calculus the return on the option is given by:

Given the stochastic calculus and the hedger's investment, the Black-Scholes partial differential equation is given by:

As in the derivation with one asset, the value if the two-colored rainbow call option must satisfy its initial condition. The solution to the equation gives:

The two-colored rainbow option presented is called an outperformance option. This type of option is preferred by many since it both has an analytical solution as presented above, but also can decide which forecasting model that performs best.

3.2 Correlation

The use of historical data to compute the linear relationship between two assets is called historical or statistical correlation. This can also be called the realized correlation and is the analogy of the realized volatility (see Bunjaku and Näsholm (2010) for discussion of realized volatility). In the case of two assets, as a two-colored rainbow option, the correlation between the two underlying assets is given by:

The covariance between the two assets is expressed in relation to their respective variance. are the means of the respective series. (Bouzoubaa and Osseiran, 2010)

From the equation above it is shown that the correlation is expressed from the covariance between the two series. The two expressions are not that different since both describe the degree of similarity between the two series. The difference is thought that correlation is rearranged to be easier to interpret and it is known that the correlation should lie between minus one and one depending on if there is a negative or positive similarity. This can be shown by first assuming the following:

From expanding the square the correlation boundaries can be shown that:

The other equality can be proven by changing the sign:

And hence, the coefficient of correlation is proven to be between:

When using the correlation as a variable one has to understand that it might vary through time. It has been shown that the correlation between assets tend to increase during globally turbulent markets. This gives rise to estimation difficulties since the normal, or log normal distribution used under multivariate models does not model the risk relationship completely. The use of the correlation to reduce the aggregated portfolio risk by diversification could be erased due to increased correlation under turbulent markets. The use of historical data to forecast the correlation could hence be underestimated during time when the volatility on the individual asset and the correlation increases. (Jorion, 2007)

3.3 Forecasting models

3.3.1 Naïve historical forecast

The first forecasting model and perhaps the most straightforward forecasting method are to use the naïve historical sample variance and covariance of all the past data to predict the variance and covariance one time step ahead. It implicitly assumes conditional and unconditional moments are equal.

3.3.2 Equally weighted moving average forecast

The naïve historical forecasting technique can be narrowed to a 20-day moving average forecasting model which only uses the last 20 days' returns.

3.3.3 Exponentially weighted moving average forecast

It is common known that volatility is often clustering over time and therefore it could make sense to assign higher weight to more recent observations. The exponentially weighted moving average forecast (EWMA) model uses a smothering parameter λ to make this possible. The method of RiskMetrics™ put λ=0.94 as well as assuming the unconditional mean = 0. We truncate the past data to 250 observations.

3.3.4 GARCH

The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model is a generalization of the ARCH model developed by Engle (1982). Below we show the general form of the model.

A more parsimonious GARCH model is the univariate GARCH(1,1) model. In GARCH (1,1) the conditional variance matrix is calculated from a long-run average variance rate and from the lagged terms of and .

Campbell et al. (1997) states that the major advantage of this model is that it incorporates recent volatility and return shocks in a higher grade than older volatility and this is consistent with empirical findings of volatility clustering effects. This can be interpreted as a high indicates that the volatility is persistent and a high means that volatility is thorny and reacts fast to market changes. One major critique is that the history not necessarily has to repeats itself in the future; therefore relying on the historical distribution might lead us to wrong estimates. If we set we get the following more common expression.

The standard GARCH(1,1) model assumes our return series to be stationary and have a long-run variance to convert towards. This is due to the restrictions below.

3.3.5 CCC-GARCH

When considering covariation of variables, like assets in a portfolio, an extension of the univariate GARCH model to the multivariate case is necessary. The Constant Conditional Correlation (CCC) model was introduced by Bollerslev (1990) which estimates the conditional covariance matrix indirectly by estimating the conditional correlation matrix . The conditional correlation are elements in the matrix and are assumed to be constant, i.e. , while the conditional variances are varying. is a diagonal matrix of time varying standard variation from univariate GARCH-processes. The expression for h are typically thought of as univariate GARCH models. With positive conditional variances the resulting variance covariance matrix will always be positive definite. The variance covariance matrix is hence determined only by the time varying conditional variance forecasts.

3.3.6 DCC-GARCH

In 2001 Engle (2001) presented Dynamic Conditional Correlation (DCC) GARCH which is a generalized version of CCC-GARCH and a parsimonious way to estimate multivariate GARCH models. As in the CCC model this generalization is a nonlinear combination of univariate GARCH models with a two-step methodology to estimate the parameters. In the first step, the conditional variance is estimated via univariate GARCH model for respectively return series. In the second step the parameters for the conditional correlation given the parameters from the first step are estimated. The DCC model also includes conditions that make the variance covariance matrix positive definite at all points in time and the process covariance stationary.

This model has been found to predict significant dependence in the covariance of asset returns in a portfolio. To recognize this feature through a multivariate model would generate a more reliable model than separate univariate GARCH models. The DCC model differs from the CCC model only in allowing *R* to be time varying giving the expression below.

Where is the N×N symmetric positive definite conditional correlation matrix. These equations will produce a variance covariance matrix at each point in time. The dynamic covariance follows a GARCH(1,1) process described below where is the unconditional correlation of standardized disturbances and .

3.5 Maximum Likelihood

The estimation of the parameters is done by using the method of Maximum Likelihood, where the log-likelihood function to be maximized are shown below. The method of Maximum Likelihood selects values of the model parameters that produce a distribution that gives the observed data the greatest probability. This method is preferred when considering non linear models over ordinary least square (OLS) even though the approximately will give the same estimates. (Verbeek, 2008)

3.6 Statistical evaluation models

This section follow theory chapter of Bunjaku and Näsholm (2010). We can never observe volatility and the use of an imperfect volatility approximation could lead to undesirable outcomes when evaluating different volatility forecasting models. A good evaluation model is robust against the presence of noise. The impact from a few extreme outcomes may lead to a large influence on forecast evaluation and comparison tests. One solution is to employ a more robust forecasting loss function that is less sensitive to extreme. According to Patton (2006), a robust loss is not only a function that is robust to noise in the approximation but also to an expected loss ranking of between two volatility forecasts is the same. This means that for the model to be robust against noise, the true conditional variance should be the optimal forecast. Tests conducted by Patton (2006) indicate that the only evaluation model that is robust, according to this criterion, is the mean squared error (MSE).

According to Vilhelmsson (2006), MSE as a loss function is sensitive to outliers. Instead, the mean absolute error (MAE) is preferred by Vilhelmsson (2006) in the sense that it is more robust against outliers.

Both Gibson and Boyer (1998) as well as Byström (2001) uses root mean squared error (RMSE) and for comparative reasons we also use this measure to evaluate the forecasts.

3.7 Statistical topics

3.7.1 Autocorrelation

The residuals of an adequate model e.g. GARCH(1,1) should be approximately white noise. For a white noise series the autocorrelation is zero i.e. it has no memory. The Ljung-Box portmanteau test statistic, shown below, checks the overall acceptability of the autocorrelation in the residuals. The Q statistic is Chi-squared distributed and hence a high Q statistic, the lesser risk for having autocorrelation present in a given model. (Verbeek, 2008)

3.7.2 Normality

Financial time series data is often leptokurtic which means that the data has fatter tails than the normal distribution. A normal distribution is mesokurtic when it has a kurtosis equal to 3. Testing the data for normality can be done by a Jarque-Bera test. The Jarque-Bera test evaluates the third moment (skewness) and the fourth moment (kurtosis). The formula for the

Chi-squared Jarque-Bera statistic can be shown below. The null hypothesis states that distribution of the series is mesokurtic i.e. normally distributed. Hence a rejection is done if the residuals from the model is skewed or leptokurtic. (Verbeek, 2008)

4.0 Methodology

4.1 Data

The currency data used is daily, at noon New York time, log returns of the US dollar vis-à-vis the Euro (EUR) and the British Pound (GBP), from 2 January 2002 to 26 October 2010. The choice of data follows the methodology of Byström's (2001) extension of Gibson and Boyer (1998) which uses an in-sample of 500 observations and an out-of-sample period of 1800 observation. This yields a total number of 2300 days observations. The advantage of this sample length is that ARCH-type models often need a large amount of data to produce robust estimations. This circumvents the problem of negative parameter estimates and exploding models. The reason for using exchange rates is due the small problem of market microstructure effects i.e. high liquidity leads to tight bid-ask spreads and low noise. Also, the most traded rainbow options are currency options which make the analysis more realistic. The risk free rate is approximated by the 3-month US Treasury bill in line with previous mentioned studies. The data is second hand data and is retrieved from Datastream. All data testing and estimation are done in EViews and processed in Excel.

4.2 Deciding the appropriate lag length of GARCH

Since this thesis covers GARCH-type models, it is appropriate to decide how the model should be expressed. To decide this, different formulations of the ARCH-type models are tested. The both series are estimated through a mean equation of an autoregressive type with one lag. This type of mean equation has been shown to explain financial data well and hence it is also used here. Different kinds of formulation of the variance equation are then tested to decide which one explains the exchange rate series most accurate. The one with lowest information criteria is the most accurate. Further, the models are tested for serial correlation i.e. whether the model still suffers from serial correlation that is not captured by the model. The appropriate lag length is hard to decide since including too many lags gives low power in detecting serial correlation. Using three different lengths is a proper way to do it since the lag length stretches both from few lags to a high number of lags giving a broad view of the properties of autocorrelation in the model (Byström, 2001).

4.3 Option trading

Gibson and Boyer's (1998) methodology evaluate variance covariance matrix forecasts by their ability to correctly price different types of two-colored rainbow options i.e. options with two underlying assets. Following Byström's (2001) extension of Gibson and Boyer's (1998) methodology of pricing two-colored rainbow options with equally weighted exchange rates as underlying instruments, the Outperformance option, is created. The main reason for choosing this particular rainbow option is that among all options depending on more than one asset, the Outperformance option is one of few with an analytical solution for the option price, found by Margrabe (1978). By using simulated rainbow options priced with the Margrabe formula derived in the theory chapter, and not financial data, a more pure impact of the variance covariance matrix is created.

The setup and the idea is that each agent makes a forecast of the variance covariance matrix at $t-1$ using that agents particular forecasting model i.e. there are five agents on the simulated market. Then each agent uses their variance covariance matrix forecast to price a 1-day at-the-money Outperformance option at time $t-1$ to exchange one currency for a certain amount of another currency at time t, both expressed in US dollar. Agents trade the option among themselves at $t-1$. Each agent trades the option with each of the other four agents. An agent, who finds another agent's option to be underpriced, buys that option and vice versa. If an agent makes a relatively high forecast of the variance covariance matrix then the option she is pricing will be relatively high and hence she will be buying the other agents option because she finds them underpriced. In this way each agent trades four options each day. Each agent's 'bank account' is credited (if a seller) or debited (if a buyer) with the mean of the two traders' options prices because it is on the mean they will make the trade.

The balance of the bank accounts earns one day's interest at the risk free rate. Depending on if the agent has bought or sold options at time $t-1$ the payoff realized at time t are again credited or debited the agents bank account by this amount. Going from $t-1$ to t the balances of each agent's bank account have changed an amount equal to that day's profit (positive or negative). The last day in the test period the agent with the 'best' forecasts should have made the highest accumulated profit measured in US dollars. The option trading simulated is a closed market and a zero sum game which means that the aggregated profits earned by the agents will be zero each day.

4.4 Statistical evaluation method

Using the specified formulas in the theory, the error in the forecast is evaluated. The “true” sample variance is said to be best approximated using the realized volatility. The realized volatility is proven to converge to the conditional variance as the data frequency increases (Patton, 2006). In this case though, high frequency data is not available and the realized volatility is approximated using the realized range estimator introduced by Parkinson (1980). The calculations are from here straight forward using the formulas in chapter 3.6.

5.0 Empirical results

5.1 Descriptive statistics

The table below shows which kind of ARCH-type models that was tested. The information criterion is to be used when evaluating the ARCH-type models. In the case of EUR/USD, the only model that has significant parameters is the GARCH(1,1) model, which indicates that the usage of that kind of specification is to be used when forecasting correlation matrices in this case. That implies that the use of information criterion is excessive in this case. In the case of GBP/USD the result are ambiguous. The two different tests indicate of different GARCH-type model. SBIC is said to be a better indicator than AIC since AIC has an asymptotic bias but also, the usage of a more parsimonious model is recommended in a lot of the financial literature (Figlewski, 1997). The GARCH(1,1) model is hence also to be used when predicting the correlation for this exchange rate.

Table 1. Deciding appropriate lag length for ARCH-type models

EUR/USD	AIC	SBIC	GBP/USD	AIC	SBIC
ARCH(1)	-7.272	-7.262	ARCH(1)*	-7.282	-7.272
ARCH(2)	-7.311	-7.298	ARCH(2)*	-7.338	-7.326
ARCH(3)	-7.321	-7.306	ARCH(3)*	-7.361	-7.346
GARCH(1,1)*	***-7.430***	***-7.417***	**GARCH(1,1)***	**-7.510**	***-7.497***
GARCH(1,2)	-7.432	-7.417	GARCH(1,2)*	*-7.511*	-7.496
GARCH(2,1)	-7.433	-7.418	GARCH(2,1)	-7.511	-7.496
GARCH(2,2)	-7.433	-7.415	GARCH(2,2)	-7.512	-7.494

* significant parameters

* significant parameters

The log return data show both skewness and excess kurtosis in the respective series. The Ljung-Box test for both the return and squared the return series shows that the autocorrelation is explained by the model. Since the residuals do not depend on its own lags, the results indicate that the chosen model for the variance equation is correctly specified.

Table 2. Sample statistics on continuous compounded returns

Statistics	EUR/USD	GBP/USD
No. observations	2300	2300
Mean (%)	0.010	0.012
S.D. (% on yearly basis)	15.889	15.867
Skewness	0.026	0.109
Kurtosis	3.59	3.72
Jarque-Bera	33.07	53.57
Ljung-Box		
Q(6)	2.56	2.47
Q(12)	12.72	15.91
Q(18)	23.37	21.85
Ljung-Box (squared returns)		
Q(6)	15.47	10.61
Q(12)	18.05	14.99
Q(18)	28.20	16.75

5.2 GARCH parameter output

Formula 1. CCC-GARCH

$GARCH_1$ (1,1)
$GARCH_2$ (1,1)

Formula 2. DCC-GARCH

$GARCH_1$ (1,1)
$GARCH_2$ (1,1)

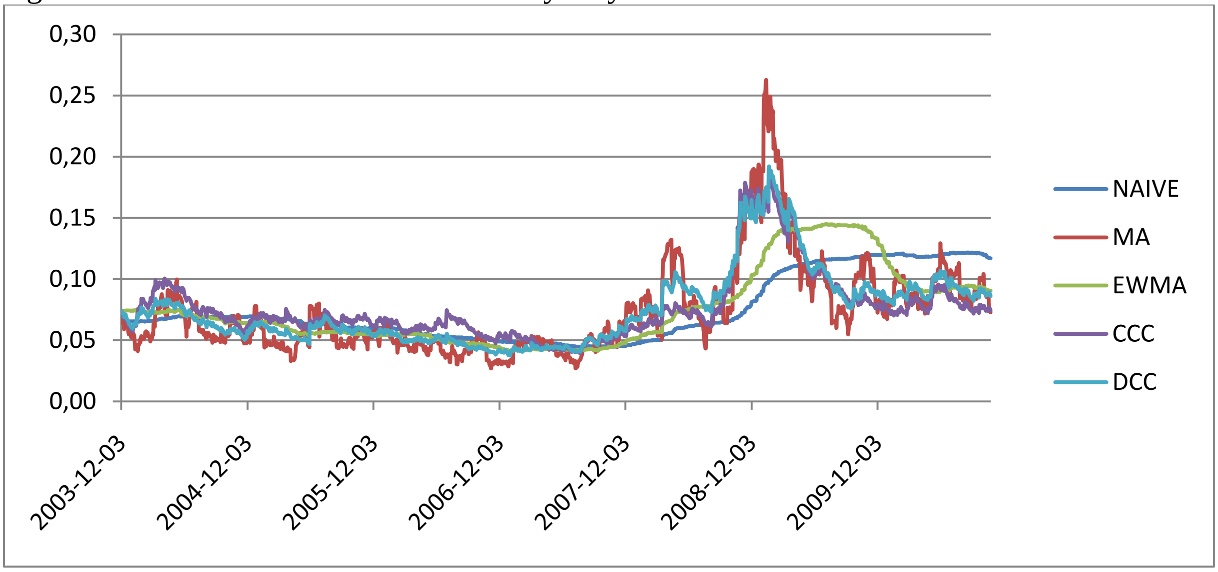
DCC-GARCH

As discussed previously, a GARCH model where the parameter estimates sums to one lead to non-stationarity. As can be seen in Formula 1 and 2, the parameter estimates are below one, but close. The parameter estimates presented show a high persistence in the GARCH models meaning that the volatility today is close to the volatility in the period before. This result is in line with much of the GARCH research and implies that the model works well under periods where the volatility does not change too much from one period to the next. Under more fluctuation periods though, the model are not able to capture large changes. Since the variance covariance matrix is estimated from the GARCH models, the matrix will also be persistent.

5.3 Option trading evaluation

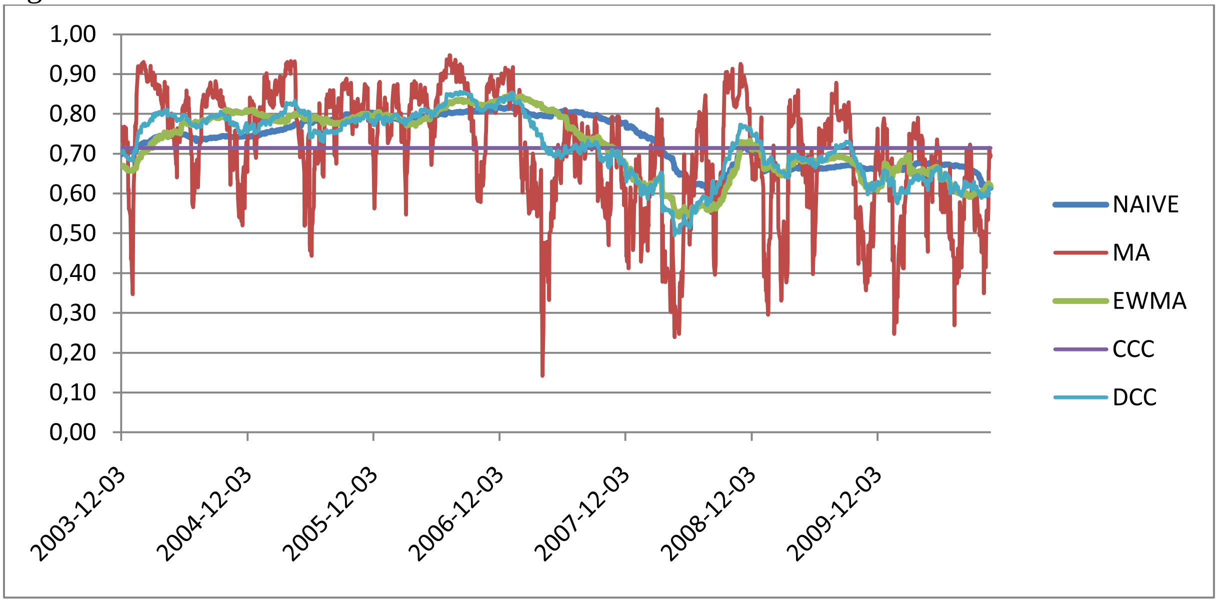
The figure below shows the estimated portfolio standard deviation of the two exchange rates estimated by the different models. We observe two periods, where the first period is a tranquil period followed by a more fluctuating. The fluctuating period can be explained by the financial crisis starting 2008. As can be seen, the naïve model and EWMA reacts more slowly to fluctuating market climate. The 20 day moving average, CCC and DCC models adjusts faster to the market climate.

Figure 1. Estimated standard deviation on yearly basis for the different model



The figure below shows the phenomena of time varying correlation. As we can see, the constant conditional correlation is as the name suggest, constant. This is explained from the models assumption that the conditional covariance matrix is determined solely by the conditional variances. The 20 day moving average estimation has the highest peaks in the fluctuations, which can be explained by the short in-sample period letting the model fluctuate. On the contrary, the naïve model with long in-sample estimation fluctuate the least. Also, we can observe a shift in fluctuation moving into the period of financial crisis. This can be explained by that the crisis hit the GBP harder than the EUR because the British economy suffered more from the crisis than the aggregated EURO zone.

Figure 2. Estimated correlation for EUR/GBP for the different models



The option price presented below in Figure 3 is counterintuitive since the option prices decreases even though the variances increases and correlation decreases under the second period. This implies that the option price should increase. The counterintuitive result can be explained by the decreasing option payoffs. The difference in the strike and the exercise price affects the payoff, leading to a decrease in the options price. The change in payoff offsets the impact of the covariance matrix. Also, as can be seen in figure 4, the GBP depreciates heavily making the spread between the call options strike and exercise price to tighten. This makes the options payoff to decrease. Due to the small differences we also present Figure 5 with the difference in the models to the naïve model as a benchmark. This highlights the fact that the price differ during high volatility periods. Since the moving average has the shortest in-sample, it reacts strongest to changes in volatility relative the other models.

Figure 3. Simulated option prices for the different models

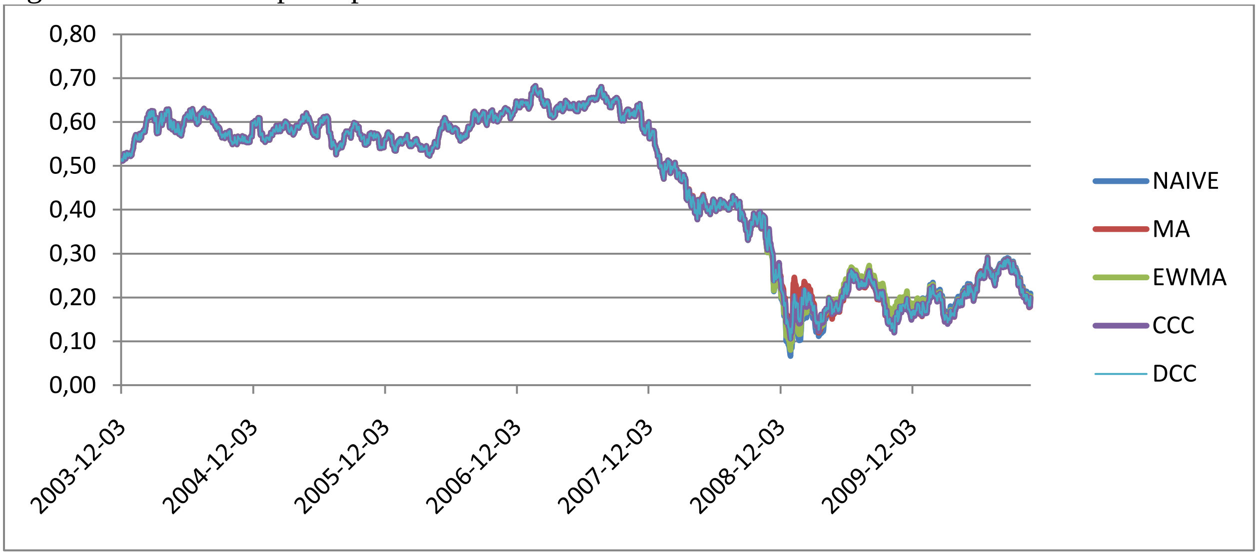


Figure 4. Evolution of the currency rates

Figure 5. Simulated option prices in difference with the naïve forecasting model as a benchmark

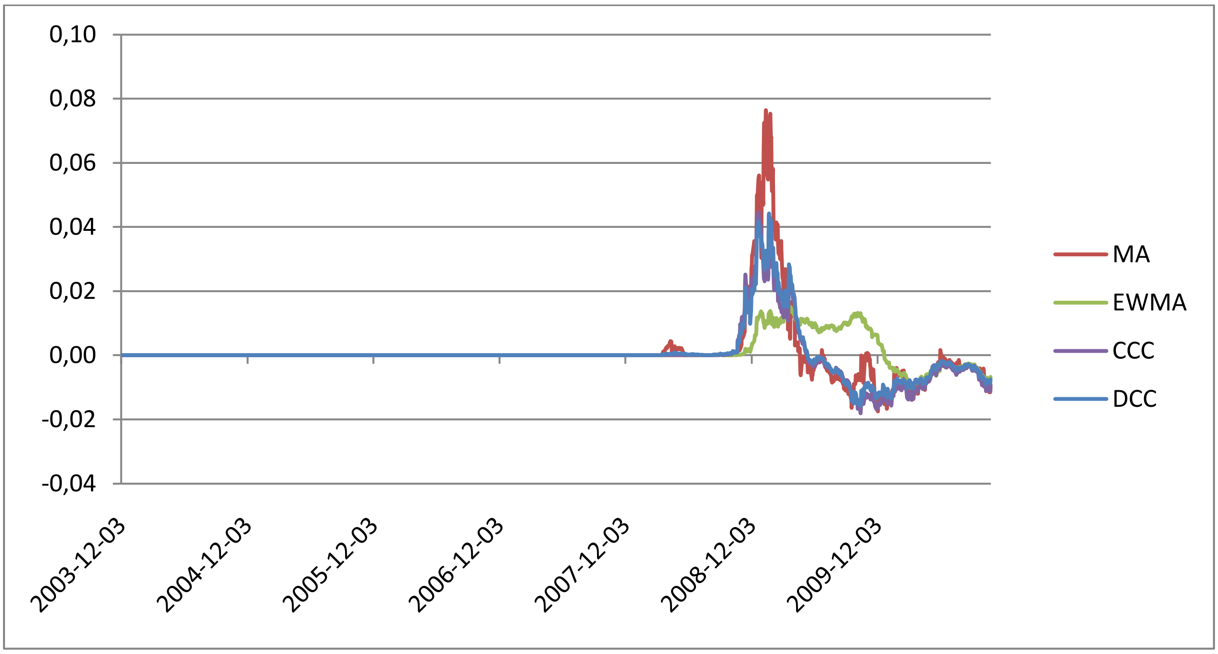
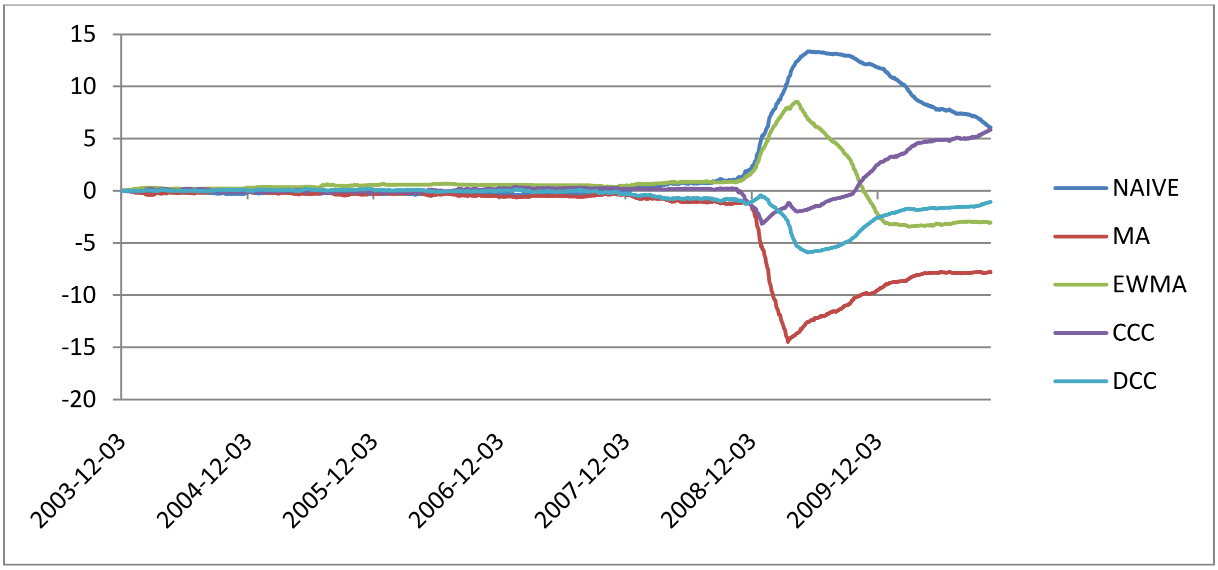


Figure 6 below shows the result for the different trading strategies based on the agents specific models. The interpretation at a first glance is straight forward where the model with the highest end of sample accumulated profit is the model that yields the best economic estimation. The figure shows that the naïve forecast is the best model to price a two colored rainbow option followed by the CCC-, and DCC-GARCH. The EWMA model yields a higher profit than the 20 days moving average yet both of the models are beaten by the previous mentioned. Judging from Figure 6 it is ambiguous to decide which model performs best overall because the high volatility period has great impact on the trading profits. Hence, the time period is divided arbitrary into two periods one containing a tranquil period and one with higher volatility.

Figure 6. Accumulated USD profits for the different trading agents using simulated currency rainbow options



In Figure 7 below we observe that EWMA generate the highest accumulated profit during the tranquil period. Questionable is if the naïve model is superior the CCC-GARCH since the accumulated profit is relatively lower during a long period of time. Therefore it is of interest to look at Figure 8 where the naïve is clearly outperforming the other models for a long period of time and converge towards CCC-GARCH in the end. Also, EWMA is clearly no longer the model that produces the highest accumulated profits. Overall we can see that DCC-GARCH, the most advanced model, has a mediocre economic performance on exchange rates relative the other models.

Figure 7. Accumulated USD profits for the different trading agents using simulated currency rainbow options for the tranquil period

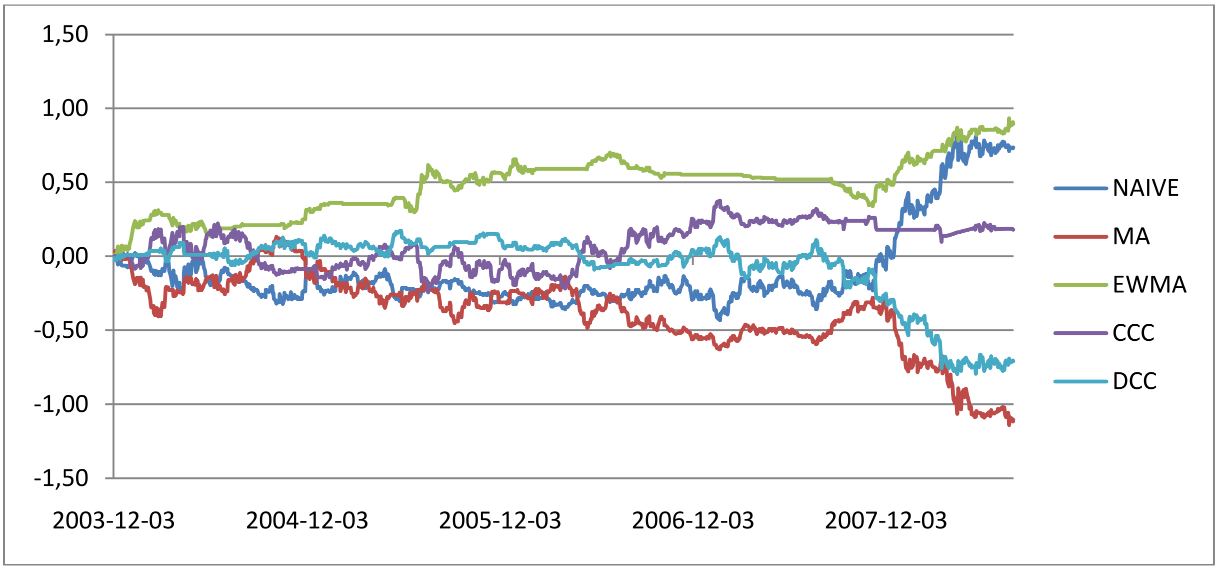
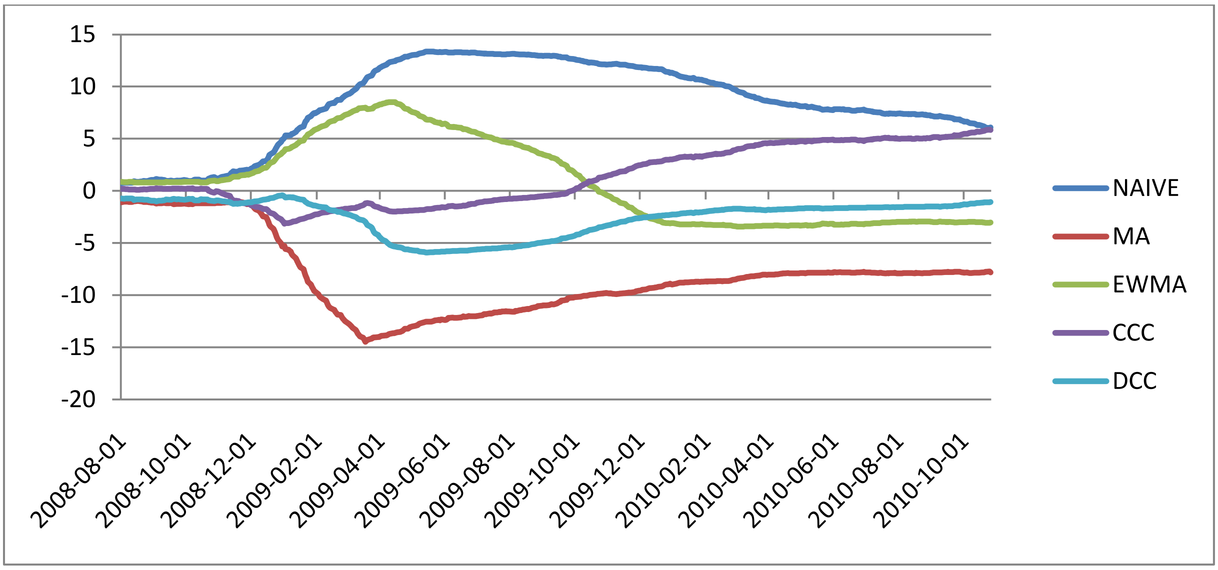


Figure 8. Accumulated USD profits for the different trading agents using simulated currency rainbow options for the volatile period



5.4 Statistical evaluation

Following Figure 6, Table 3 shows a positive mean daily profit for the naïve and CCC-GARCH models. Of course, the mean daily profit sums to zero since it is a closed simulated market. The model with highest mean daily profit is also the model with highest accumulated profit. The ranking of the mean daily profit is the same as in the accumulated for the entire sample.

Table 3. Mean daily profit and standard deviation from trading options based on the five models

	NAIVE	MA	EWMA	CCC	DCC
Mean daily profit	0.0034	-0.0043	-0.0017	0.0032	-0.0006
S.D.	0.0403	0.0458	0.0338	0.0288	0.0258

Below are the calculated statistical loss functions where the bold points out the lowest and best estimation, followed the underlined as the second best and the italics as third. Every number is multiplied with 10000. As can be seen, the DCC-GARCH is the one with lowest errors with five bold estimates out of nine. According to the loss functions, the CCC is one of the best models in respect to the error terms followed the 20 days moving average and an ambiguous result for the others.

Table 4. Mean squared error for the variance and covariance forecast

	NAIVE	MA	EWMA	CCC	DCC
Var. EUR	0. 00329	*0. 00296*	0. 00338	**0. 00218**	<u>0. 00217</u>
Var. GBP	0. 00565	*0. 00409*	0. 00512	<u>0. 00360</u>	**0. 00356**
Cov. EUR/GBP	**0.00297**	*0.00309*	<u>0.00308</u>	0.00314	0.00312

Table 5. Mean absolute error for the variance and covariance forecast

	NAIVE	MA	EWMA	CCC	DCC
Var. EUR	0.00332	*0.00302*	0.00335	**0.00261**	<u>0.00261</u>
Var. GBP	0.00342	*0.00283*	0.00319	<u>0.00272</u>	**0.00270**
Cov. EUR/GBP	0.00406	*0.00401*	0.00406	<u>0.00397</u>	**0.00396**

Table 6. Root mean squared error for the variance and covariance forecast

	NAIVE	MA	EWMA	CCC	DCC
Var. EUR	0.00574	*0.00544*	0.00581	<u>0.00467</u>	**0.00466**
Var. GBP	0.00752	*0.00639*	0.00716	<u>0.00600</u>	**0.00596**
Cov. EUR/GBP	**0.00545**	0.00561	<u>0.00555</u>	0.00560	*0.00558*

6.0 Discussion

Based on our sample we can see that the GARCH models show greater persistency in the volatility. This impacts the variance covariance matrix which in turn affects the option pricing. Clearly an accurate estimation model of the variance covariance matrix is of great importance when acting on a derivative market. The evidence shows that the more persistent models such as the naïve, CCC-, and DCC-GARCH models are better to capture the current volatility in fluctuating market climate and hence, it prices the option more accurately. This phenomenon does not seem to be as important during more tranquil periods where the results are more ambiguous.

To get a more rigorous evaluation of the variance covariance matrix forecasts we found it important to investigate both the economic loss via simulated trading and the statistical loss functions. It is obvious that the DCC-GARCH, followed by the CCC-GARCH is the best performing models evaluated by statistical loss functions. The reason for DCC-GARCH to be best performing statistically is due to that it forecasts the least error on average. This does not mean that it is the best model in an economic loss evaluation. It can be explained by that the DCC-GARCH estimates with least error on average but is systematically wrong in the estimated period. The systematically bias is explained by the fact that it overestimates the correlation, which in an option pricing framework means less uncertainty. Remember that low uncertainty yields lower option price and hence, the option is systematically underpriced. This shows the importance to do an economic evaluation where the DCC-GARCH may statistically be the best, but produces mediocre accumulated profits. On the contrary, the CCC-GARCH is found to perform well both statistically and economically. This can be explained by the sampling and estimation. The model parameters incorporate information about the whole sample i.e. a tranquil and fluctuation period. This implies that the CCC-GARCH neither overestimates, nor underestimates the variance covariance matrix leading to a stable economic evaluation.

Comparing the simpler models with the GARCH-type models it is shown that the latter does not turn out to be superior just because they are more advanced. This can be because the assumptions about the conditional variance are too strong and the well known fact that they perform worse on currency data than e.g. equities due to the lack of strong ARCH-effects. DCC-GARCH also assumes the conditional correlation to follow the same dynamic structure which affects the portfolio standard deviation used as input in the option pricing formula.

Previous research show that the more advanced models in general produce better estimations. This can be seen in e.g. Gibson and Boyer (1998) where the GARCH based models outperform the simpler models like naïve and EWMA. Remember that Gibson and Boyer (1998) have a sample and estimation period reaching from 1976 until 1997 and cleans the data from financial crisis. They also use equity indices which are more likely to incorporate ARCH-effects. Our results during the tranquil period are in line with Byström (2001) where they finding of EWMA to be the best performing model are analogous.

As discussed previously the two-colored rainbow options are sensitive to the entire variance covariance matrix and the ability to predict the matrix is shown to be crucial in an option pricing market. An agent with the ability to make better predictions on the variance covariance matrix has an opportunity for greater trading profits.

7.0 Inference

7.1 Conclusion

This thesis discusses variance covariance evaluation considering both bivariate systems and less complicated models. The purpose was to evaluate which of these models that forecasts the variance covariance matrix most accurate evaluating both using economical and statistical methods. This is interesting in e.g. risk management, portfolio optimization and also, pricing of derivatives with several underlying assets.

The main results shows that for the whole sample period the naïve forecasting model is the most superior according to the accumulated trading profit, followed by CCC-GARCH. According to the statistical loss evaluation DCC-GARCH is the superior model followed by CCC-GARCH. For the tranquil period in our sample EWMA shows steady accumulated profit above the other models but is outperformed by the naïve forecasting model when entering into the more volatile period.

7.2 Further studies

Our study has been conducted on two major exchange rates, the US dollar vis-à-vis Euro and British pound. It would be of interest to investigate additional exchange rates to see if our results still hold. One could also extend the complexity of the rainbow option to depend on more than two underlying instruments. This would yield a larger variance covariance matrix and the efficiency of the forecasting models would be further stressed. Rainbow options with other underlying instruments like equities or commodities could be evaluated in the same simulated trading framework and investigation of whether economic loss or statistical loss functions are the superior evaluation method. It could then be interesting to test the models under different distribution assumptions e.g. the Copula distribution and considering asymmetric effects in the data using e.g. asymmetric GARCH models. It would also be interesting to compare the results of the variance covariance estimation with the implied correlation gather from rainbow option data. We are certain that future research will be done in the area and our hope is that a more gradate view on statistical loss functions is undertaken and economic loss methods becomes more common.

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8.2 Electronic sources

Datastream 2011-04-13

9.0 Appendix

9.1 DCC-GARCH EViews code

```
sample s1 2/03/2002 10/26/2010

scalar pi=3.14159


series y1=usd_eur
series y2=usd_gbp

equation eq_y1.arch(1,1,m=1000,h) y1 c
equation eq_y2.arch(1,1,m=1000,h) y2 c

eq_y1.makeresids(s) z1
eq_y2.makeresids(s) z2

eq_y1.makegarch() garch1
eq_y2.makegarch() garch2

scalar var_z1=@var(z1)
scalar var_z2=@var(z2)
scalar cov_z1z2=@cov(z1,z2)
scalar corr12=@cor(z1,z2)

series var_z1t=var_z1
series var_z2t=var_z2
series cov_z1tz2t=cov_z1z2

coef(2) T

logl dcc
dcc.append @logl logl
```

```
dcc.append var_z1t=@nan(1-T(1)-T(2)+T(1)*(z1(-1)^2)+T(2)*var_z1t(-1),1)
dcc.append var_z2t=@nan(1-T(1)-T(2)+T(1)*(z2(-1)^2)+T(2)*var_z2t(-1),1)
dcc.append cov_z1tz2t=@nan((1-T(1)-T(2))*corr12+T(1)*z1(-1)*z2(-1)+T(2)*cov_z1tz2t(-
1),1)
dcc.append pen=(var_z1t<0)+(var_z2t<0)

dcc.append rho12=cov_z1tz2t/@sqrt(@abs(var_z1t*var_z2t))

dcc.append detrRt=(1-(rho12^2))
dcc.append detrDt=@sqrt(garch1*garch2)
dcc.append pen=pen+(detrRt<0)
dcc.append detrRt=@abs(detrRt)

dcc.append logl=(-1/2)*(2*log(2*pi)+log(detrRt)+(z1^2+z2^2-2*rho12*z1*z2)/detrRt)-
10*pen

smpl s1
dcc.ml(showopts, m=500, c=1e-5)

show dcc.output
graph corr.line rho12
show corr
```