Effect of Effort on Self-Image: The Case of Monotonically Increasing Self-Image Functions

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Abstract

In this paper, the model of moral motivation as developed by Brekke et al (2003) is analysed with the new assumption that self-image is an increasing function of effort. While the effects of increased efficiency and new information on optimal effort levels are largely similar, different results are obtained when individuals are faced with responsibility and economic incentives. Following Brekke et al, this paper uses the example of *dugnad*, whereby members of an organisation can choose to voluntarily do practical work for the organisation. Participation is sustainable as a Nash equilibrium even when it is not considered morally ideal. Two specific examples are used to demonstrate the different behaviours produced when varying the steepness of the self-image functions. All results adhere to previously established theories on responsibility and crowding out.

Keywords: Self-image, Volunteerism, Economic incentives, Crowding out

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1 Introduction

Many activities that occur in society, despite contributing to a common good, are unrewarded. Examples are working voluntarily, recycling and donating money to charities. It is difficult to explain these actions only in terms of the material benefits accrued to the doer, if indeed any. Recent economic research has therefore focused on the non-material and psychological rewards derived from voluntary contribution to public good.

One of these psychological rewards from giving is a better self-image. Individuals want to be generous with their time and money because this virtue is associated with being, and implicitly makes them, a better person. The act of being generous increases the individual's image of himself. When the individual also knows that he is perceived to be good by others, his self-image increases further. The more effort in terms of time or money he donates, the more generous he becomes – in the eyes of both his peers and himself – and hence the higher his self-image is. This points to the rationality of a self-image function which is monotonically increasing in effort level.

An often-quoted paper on self-image as the motivation behind a moral action is "An Economic Model of Moral Motivation" by Brekke et al (2003). The authors built the model in two steps: firstly, an individual finds the morally ideal effort level by maximizing total social welfare assuming everyone else acted like him. In the next stage, he finds the optimal actual effort level by maximizing his own utility, taking into account that the more effort exerted the higher his self-image is, but the lower his leisure time will be. However, in their model self-image is assumed to be of the shape of an inverted-U: increasing up until effort is morally ideal, and decreasing when effort exceeds the morally ideal level.

This paper will challenge the self-image assumption made by Brekke et al by applying a monotonically increasing self-image function to their model. The effects of increased efficiency and new information will be analysed, yielding generally similar results. The conclusions drawn from investigating individual actions when faced with responsibility and economic incentives, however, are different. Most significantly, a monotonically increasing self-image function gives rise to individual participation in voluntary activities even when non-participation is morally ideal.

The remainder of this paper will be organized as follows: Section 2 presents a review of the literature on self-image, both in economic models and in society, as the motivation behind volunteering. Section 3 will describe the model as developed by Brekke et al. In Section 4 this paper will depart from the assumption of an inverted-U-shaped self-image function, and analyse the model in terms of a general monotonically increasing function. Two examples of such functions are given in Section 5, with varying results. These will be discussed in Section 6, while Section 7 concludes.

2 Literature Review

Altruism, while initially difficult to explain using neoclassical economic theory,[1] has in more recent years been extensively studied as the main motivation behind seemingly selfless or pro-social acts.[2] In a seminal paper by Andreoni (1990), it has been shown that pure altruism alone cannot be the only reason for examples such as the incomplete crowd-out of private provision of public good by government, the fact that total provision of public good is dependent on the distribution of income among donors, and the non-neutrality of subsidies. Andreoni developed a model of impure altruism in which individuals may derive some private benefit from giving, a so-called 'warm glow of giving', which increases their utility.

While pure altruism represents an individual's intrinsic motivation, that is, "the value of giving per se represented by private preferences for others' well-being" (Ariely et al, 2009 p. 544), impure altruism may represent both extrinsic motivation in material reward from giving or image motivation. Image motivation, or signalling motivation, is defined by Ariely et al as "an individual's tendency to be motivated partly by others' perceptions", capturing "the rule of opinion in utility, i.e., the desire to be liked and respected by others and by one's self" (p. 544).

In conducting his laboratory experiment, Ariely et al concluded that image motivation is the major driver behind pro-social activities. People want to be seen as doing good, but not if they appear to be taking some reward for it, as it decreases their intrinsic motivation as perceived by others. The effectiveness of monetary incentives depend on their visibility: material reward is more attractive when the pro-social activity takes place in a private

[1] See for example Sen (1977) and Sugden (1984).
[2] See Piliavin and Charng (1990).

setting, so that any public giving is perceived as a result of pure altruism. It is this concern for social esteem, and hence self-image, that will be the focus of this paper.

A recent study by Carpenter and Myers (2010) on volunteer fire-fighters and non-volunteer community members also confirmed this finding: "the decision to volunteer is positively correlated with altruism as well as with concern for social reputation or 'image'" (p. 911). The presence of external reward has a negative effect on the likelihood to volunteer, due to image concerns, supporting the Crowding Theory (Frey and Oberholzer-Gee, 1997) that external incentives may crowd out image motivation.

Another study confirming both the image motivation behind pro-social behaviour and the Crowding Theory is a paper by Ellingsen and Johannesson (2008), whereby an individual agent's utility depends positively on how he is perceived by the principal, and in turn how he himself perceives the principal. If the principal is seen as trusting, he is held in higher regard by the agent than if he displays a lack of trust.

A concrete example that will be discussed more extensively in Section 5 is volunteer tourism, which has been gaining in popularity in the past decade. Brown (2005) reported that the number of organisations responsible for this activity has grown significantly from 75 in 1987 to 275 in 2003. In addition, US Bureau of Labor Statistics figures indicate that 3.5 million people are engaged in some form of civic, political, professional or international volunteering in 2011, an almost 12% increase from 2007, and making up 5.4% of the total number of volunteers.

Sin (2009) studied the motivations behind a group of volunteer tourists, and similarly found that the motivation of self-image - 'I want to see if I can do this' - plays a large part when individuals decide to volunteer. The author emphasises that "it is important to realize upfront that many volunteer tourists are typically more interested in fulfilling objectives relating to the 'self'" (p. 497). Brown (2005) also found that volunteerism is driven by both altruism and self-interest across all demographic categories, and that "the motivations of the volunteer vacationers appear to be conforming to some degree to volunteering motives in general in such aspects as 'personal fulfilment', 'identity enhancement and self-expression'" (p. 492).

All these contexts in which self-image plays a major role require a model that can serve as a framework for economic analysis. While Andreoni (1990) has been instrumental in developing the model for impure altruism, his model cannot explain the Crowding Theory. When there is a fee for non-participation, this implies a subsidy for participation, which by the Crowding Theory should reduce participation. However the impure altruism model predicts increased participation.

An alternative model has been provided by Brekke et al (2003). In their paper, the authors seek to explain pro-social behaviours by analysing the economics of moral motivation. In doing so, they have created a model that not only serves as a framework for the formal analysis of individual choice given moral incentives, but is also consistent with the Crowding Theory. This important paper has since been popularly cited in many other studies of social co-operations.[3]

This paper will provide a modification to one of the key assumptions in Brekke et al's model of moral motivation. The model assumes an inverted-U-shaped self-image function, in which self-image initially increases with effort level, reaches a maximum when the effort level is morally optimal, and decreases when effort exceeds the morally optimal level. However, it is more reasonable to argue that self-image should be monotonically increasing in effort level, as supported by the following papers studying the components of self-image.

In their 2000 paper, Akerlof and Kranton model utility as a function of identity, which can be interpreted as self-image. This is inevitably linked to what is perceived to be the ideal norms in society; the morally ideal action which affects how an individual perceives his action compared to others: "Since an individual's self-concept may be formed by seeing oneself through the eyes of others [Gleitman 1996, p.343], these gains and losses may also depend on how others interpret i's actions" (p. 719). Moreover, to the extent that it is possible to exceed society's ideal, identity and hence self-image should be increasing in effort. "In the case of a category with low social status, a person j may gain when own characteristics are far from the ideal" (p. 719).

Santos-Pinto and Sobel (2005) found that positive self-image is increasing in "the ease of the task, the number of different skills needed for the task, and the variability of production

[3] 201 citations on Google Scholar as at 12-04-2012.

technologies in the population" (p.1386). Therefore if an individual is at the morally ideal effort level, going beyond that by making the morally ideal effort level easier to achieve would result in higher self-image. These results, together with Akerlof and Kranton's findings, provide further justification for the modification to the self-image function so that it is monotonically increasing in effort level.

3 The Model

In analysing the consequences of a monotonically increasing self-image function, this paper will make use of the model presented in Brekke et al (2003). In that paper, the model is first explained, then an analysis is made on the effects of increased efficiency and new information, and finally the authors investigate individuals' decision-making when faced with responsibility and economic incentives. This model is presented below, however the full descriptions and derivations of Brekke et al's conclusions using the inverted-U-shaped self-image function is left to the Appendix.

Brekke et al's model (2003) is based on the individual answering the following two questions:

1. What is the morally ideal effort level?

2. What is my own ideal effort level?

In answering the first question, the individual considers the consequences for social welfare if everybody acted like him, and finds that optimal effort level which maximises social welfare – the morally optimal effort level. For the second question, the individual considers the trade-off between better self-image and less leisure time to choose his optimal effort level that maximises his own utility. Self-image in Brekke et al is determined by how close one's effort level is to the morally ideal one.

Following Brekke et al (2003), the economy consists of N identical individuals with the following increasing and strictly quasi-concave utility function:

$$U_i = u(x_i, l_i, G, I_i) \qquad (1)$$

where x_i is i's consumption of private goods, l_i is i's leisure, G is public good, and I_i is i's self-image.

The individual's time constraint is

$$l_i + e_i = T \tag{2}$$

where e_i is i's effort level in contributing to the increased supply of the public good, as measured in units of time. T is i's exogenous total time available, having accounted for i's exogenous labour supply.

Public good supply is described by

$$G = G_p + \sum_i g_i \tag{3}$$

where G_p is public provision and g_i is private provision which can be expressed as

$$g_i = \gamma(e_i, \theta) \tag{4}$$

θ is an exogenous efficiency parameter. It is also assumed that $\gamma(0, \theta) = 0,\ \gamma_e > 0,\ \gamma_{ee} < 0,\ \gamma_\theta > 0,\ \gamma_{e\theta} > 0.$

Self-image is determined by the relationship between an individual's own effort level and his morally ideal effort level. This is assumed to be maximised at the morally ideal effort level e_i^*. Brekke et al thus specify the following self-image function:

$$I_i = f(e_i, e_i^*) = -a(e_i - e_i^*)^2, \qquad a > 0 \tag{5}$$

such that a global maximum of 0 is attained at $e_i = e_i^*$. Hence self-image is increasing in effort level when it is less than ideal, and decreasing when effort level is greater than ideal. Brekke et al also experimented with a more general self-image function of the same inverted-U-shape, but concluded that it gives little further insight into the problem compared to equation (5), which simplifies calculations.[4]

To find the morally ideal effort level e^*, in the first stage the individual maximises the social welfare function below with respect to e_i:

[4] See footnote 6 in Brekke et al (2003).

$$W = \sum_i u_i(x_i, l_i, G, I_i) \tag{6}$$

As all individuals are assumed to be identical, substituting $l_i = T - e_i,\ G = G_p + \sum_i \gamma(e_i, \theta)$ and $I_i = -a(e_i - e_i^*)^2$ into (6) and differentiating with respect to e_i yields

$$\frac{\partial W}{\partial e_i} = N\left[-u_l + u_G N \gamma_{e_i} + u_I\left(-2a(e_i - e_i^*)\right)\right]$$

which equals zero at the maximum where, by definition, $e_i = e_i^* = e$ also holds. The first order condition above then simplifies to

$$u_l = N u_G \gamma_e \tag{7}$$

This expression implicitly gives the morally ideal effort level, and states that at that optimum, the marginal utility of leisure is equal to the marginal social benefit of the public good produced by the marginal effort.

In the next stage, given the morally ideal effort level and the effort level of all other individuals in the economy, the individual now maximises his own utility with respect to the choice variable of his own effort level:

$$\max_{e_i} u_i\,(x_i, l_i, G, I_i)$$

This leads to the optimal own effort level e_i, which is implicitly given by

$$u_l = u_G \gamma_e + u_I\left(-2a(e_i - e_i^*)\right) \tag{8}$$

From equations (7) and (8), one can see that individual utility maximisation cannot produce the socially optimal effort level. This is shown by setting $e_i = e_i^*$ in (8), and equating (7) and (8). Unless $u_G = 0$ or $\gamma_e = 0$, the resulting solution is $N = 1$, which is inconsistent with the assumption of an economy with many identical individuals. Brekke et al attribute this result to the property of their self-image function: individuals will only contribute to the point where the marginal benefit of effort, given by an increase in public good supply and self-image, is equal to the marginal cost, in terms of lost leisure (and lower self-image when effort is more than optimal). Since the marginal increase in self-image at the optimum effort level is zero, own effort level will be below optimal for positive marginal effect on self-

image. Hence this model results in the underprovision of public good. However, in Section 4 this result will be generalised to show that such underprovision will occur also for other self-image functions which are monotonically increasing.

4 A General Monotonically Increasing Self-Image Function

This paper will now depart from Brekke et al's assumption of an inverted-U-shaped self-image function. While Brekke et al's model has successfully explained certain individual behaviours when faced with responsibility and economic incentives (see the *dugnad* example in the Appendix), there are reasons to investigate their conclusions with the more general case of a monotonically increasing self-image function. As explained in the literature review, volunteer tourism is a common example of individuals who are highly motivated by self-image. It is therefore reasonable to model this as an increasing function of their effort level.

Let the self-image function be

$$I_i = f(e_i, e_i^*) \tag{9}$$

where $I_e > 0, I_{ee} < 0$, and $I_{e^*} < 0$. The function is thus monotonically increasing in effort level at a decreasing rate, and decreasing in morally ideal effort level (as an increase in e^* makes e relatively smaller, thus decreasing self-image). Let the function also be normalized such that $I_i = 0$ when $e_i = e_i^*$.

Let the same utility function as in Brekke et al (2003) be used, that is:

$$U = u(x, l) + v(G) + I \tag{10}$$

where both u and v are increasing and concave.

As described in Section 3, the individual in the first stage maximizes total social welfare assuming every member of the society exerts the morally ideal effort. This leads to the first-order condition

$$\frac{\partial W}{\partial e_i} = N\left(-u_l + N v_G \gamma_{e_i} + I_{e_i}\right) = 0$$

$$u_l = N v_G \gamma_{e_i} + I_{e_i} \tag{11}$$

This states that at the optimum, the marginal utility of leisure is equal to the marginal social benefit of the public good plus the marginal effect of effort on self-image.

In the second stage, the individual maximizes his own utility taking the morally ideal effort (implicitly calculated in equation (11)) as given. The first-order condition is then

$$\frac{\partial U}{\partial e_i} = -u_l + v_G \gamma_{e_i} + I_{e_i} = 0$$

or simply

$$u_l = v_G \gamma_{e_i} + I_{e_i} \tag{12}$$

Hence at the optimal actual effort level, the marginal utility of leisure is equal to the marginal individual benefit of the public good plus the marginal utility of self-image.

Again, comparing equations (11) and (12) shows that individual utility maximisation cannot produce the socially optimal effort level unless $N = 1$. As it turns out, this is the case regardless of the self-image function chosen. The marginal benefit of actual effort from the individual's point of view is never as high as the marginal benefit when all individuals exert their morally ideal effort – an opposite case of tragedy of the commons.

4.1 Effects of Increased Efficiency and New Information

This section will now proceed with the analysis of increased efficiency and new information. While the results obtained here do not deviate very much from Brekke et al's, major differences will be found in the later section on responsibility and economic incentives.

Suppose there is an increase in efficiency, given by an exogenous upward shift in the parameter θ. Differentiating equation (11) above with respect to θ yields the effect of increased efficiency on the optimal effort level:

$$\frac{\partial}{\partial \theta} u_l\big(x, T - e(\theta)\big) = \frac{\partial}{\partial \theta}\Big[N v_G \Big(G_p + N\gamma(e(\theta), \theta)\Big)\gamma_e(e(\theta), \theta) + I_e\big(e(\theta), e^*(\theta)\big)\Big]$$

$$-u_{ll} e_\theta = N\big(v_{GG}(N\gamma_e e_\theta + N\gamma_\theta)\gamma_e + v_G(\gamma_{ee} e_\theta + \gamma_{e\theta})\big) + I_{ee} e_\theta + I_{ee^*} e_\theta^*$$

Solving for e_θ^* and simplifying gives

$$e_\theta^* = N\frac{v_G\gamma_{e\theta} + Nv_{GG}\gamma_e\gamma_\theta}{-u_{ll} - N^2v_{GG}\gamma_e^2 - Nv_G\gamma_{ee} - I_{ee} - I_{ee^*}} \quad (13)$$

The first term in the numerator represents the total increase in public good from the increase in marginal effort level, which is positive. The second term represents the decreasing marginal utility from the increased supply of public good through increased efficiency. Since v_{GG} is strictly negative, the sign of the numerator is ambiguous. Hence, the overall effect of increased efficiency on the optimal effort level can be either positive or negative.

To analyse the effect of increased efficiency on individual ideal contribution $g^* = \gamma(e^*, \theta)$, differentiate this expression with respect to θ:

$$g_\theta = \gamma_e e_\theta + \gamma_\theta$$

Substituting the expression from (13) into e_θ above and simplifying,

$$g_\theta^* = \frac{Nv_G\gamma_{e\theta}\gamma_e + \gamma_\theta(-u_{ll} - Nv_G\gamma_{ee} - I_{ee} - I_{ee^*})}{-u_{ll} - N^2v_{GG}\gamma_e^2 - Nv_G\gamma_{ee} - I_{ee} - I_{ee^*}} \quad (14)$$

Without specifying the sign of I_{ee^*}, the effect of θ on individual ideal contribution is also ambiguous.

The effect of increased efficiency on actual effort is given by differentiating equation (12) above with respect to θ. Additionally, Brekke et al made the assumption that individuals are unable to perceive the change in public good provision from a change in own actual effort, hence $v_G\gamma_e = 0$. Therefore equation (12) becomes

$$u_l = I_{e_i} \quad (15)$$

and individuals increase their effort level up until the marginal benefit in terms of self-image exactly offsets the marginal cost of lost leisure, without taking into account the improvement in utility due to increased supply of public good. Differentiating with respect to θ,

$$e_\theta = \frac{I_{ee^*}e_\theta^*}{-u_{ll} - I_{ee}} \quad (16)$$

While the denominator is positive, the sign of e_θ^* is unknown. Hence the effect of θ on e is also ambiguous.

Brekke et al also analysed the case when an increase in θ is interpreted as an increase in the perceived effect of private effort such that both e_θ^* and $e_\theta > 0$. Differentiating U with respect to θ and substituting the Nash equilibrium condition (12) for optimal actual effort yields

$$U_\theta = v_G\gamma_\theta + (N-1)v_G(\gamma_e e_\theta + \gamma_\theta) + I_{e^*}e_\theta^* \qquad (17)$$

With e_θ^* and$e_\theta > 0$, an increase in perceived effect of private effort raises both morally ideal and actual effort levels, leading to an increase in G and hence individual utility U. The first two terms on the right hand side of (17) are therefore positive. The last term is always negative since the self-image function is decreasing in e^* - an increase in the morally ideal effort level makes the current actual effort level relatively small, impacting negatively on self-image. Hence the overall effect is again ambiguous and may be negative if the increase in θ leads to such a high increase in morally ideal effort that the actual effort level is so small in comparison, reducing self-image.

4.2 Responsibility and Economic Incentives

Brekke et al use the example of '*dugnad*', a Norwegian tradition whereby members of an organization participate voluntarily in practical work for the organization once or twice a year, to investigate the effort level of individuals when faced with responsibilities and economic incentives. Three cases will be analysed: first when there is no fee for non-participation, second when the fee for non-participation fully covers the cost to the organization of replacing the labour of those who do not participate, and third when the fee for non-participation is merely symbolic and does not cover the cost of non-participation. In each case, two morally ideal effort levels are possible: either $e^* = 1$ for participation or $e^* = 0$ for non-participation. Due to the normalization of the self-image function, $I(0,1) < 0$ and $I(1,0) > 0$.

4.2.1 No fee for non-participation

For participation ($e^* = 1$) to be the morally ideal action, the following condition derived from the utility function in (10) must hold:

$$u(m, T-1) + v(Ng^p) > u(m, T) + v(0) \quad (18)$$

where m is monetary income (which may be different from consumption x in case of a fee) and g^p is the individual contribution when participating (zero otherwise).[5] This condition is fulfilled when the benefits of participation from higher provision of the public good outweigh the loss of leisure time.

Given (18) holds, for an individual to choose to participate ($e_i = 1$), the benefits of his extra labour (compared to not participating and only getting $(N-1)g^p$ worth of public good provision) and reaching higher self-image must be greater than the lost leisure time. This is true whenever the following holds:

$$u(m, T-1) + v(Ng^p) > u(m, T) + v\big((N-1)g^p\big) + I(0,1) \quad (19)$$

Similarly, individual non-participation can also be supported as a Nash equilibrium whenever

$$u(m, T) + v(0) + I(0,1) > u(m, T-1) + v(g^p) \quad (20)$$

That is, whenever the benefit of extra leisure time is worth more to the individual than the extra public good provided by his own labour and a better self-image. Due to the concavity of v, (19) and (20) cannot hold simultaneously and therefore there is a unique Nash equilibrium.[6]

If non-participation ($e^* = 0$) is instead the morally ideal action, the opposite of (18) will hold:

$$u(m, T-1) + v(Ng^p) < u(m, T) + v(0) \quad (21)$$

In this case, individuals may still participate if they derive high enough utility from self-image:

$$u(m, T-1) + v(Ng^p) + I(1,0) > u(m, T) + v\big((N-1)g^p\big) \quad (22)$$

Otherwise, non-participation will be the Nash equilibrium, implying

[5] That is, $\gamma(e, \theta) = \begin{cases} g^p, & e = 1 \\ 0, & e = 0 \end{cases}$. γ is no longer a continuous function of e, and $\gamma_e > 0$ is no longer assumed, such that overprovision is now possible.

[6] See Appendix for proof.

$$u(m,T) + v(0) > u(m,T-1) + v(g^p) + I(1,0) \quad (23)$$

Again, due to the concavity of v, only one of (22) or (23) will hold.

4.2.2 Non-participation fee fully covers cost

Assume that the contribution of each individual g^p can be bought for a cost of c in the market. If a fee t is introduced that fully covers the cost of non-participation, then $t = c$. The total public good provided is therefore fixed at Ng^p, regardless of participation, since any labour lost through non-participation can be replaced in the market at no cost to the organization.

Participation is morally ideal if the monetary income not paid as fee is more valuable than the lost leisure time:

$$u(m,T-1) + v(Ng^p) > u(m-c,T) + v(Ng^p) \quad (24)$$

Given $e^* = 1$, individuals choose to participate if

$$u(m,T-1) + v(Ng^p) > u(m-c,T) + v(Ng^p) + I(0,1) \quad (25)$$

which is always true given (24) since $I(0,1) < 0$. Therefore individuals will always participate if it is the morally ideal action.

Non-participation will be morally ideal if the extra leisure time is considered more valuable than the fee paid:

$$u(m-c,T) + v(Ng^p) > u(m,T-1) + v(Ng^p) \quad (26)$$

Given $e^* = 0$, individuals will participate if

$$u(m,T-1) + v(Ng^p) + I(1,0) > u(m-c,T) + v(Ng^p) \quad (27)$$

However, individuals will choose non-participation if

$$u(m-c,T) + v(Ng^p) > u(m,T-1) + v(Ng^p) + I(1,0) \quad (28)$$

Again, both (27) and (28) can hold, but not together. Hence there will be a unique Nash equilibrium. Similar to results obtained by Brekke et al, the imposition of the fee has the possibility of reducing participation as both (19) and (28) can hold. In the former, there is no non-participation fee so the responsibility for public good provision is perceived to be the

individuals', while in the latter individuals can 'shift' the responsibility for the public good to the hands of the authority by paying the fee.

4.2.3 Non-participation fee is only symbolic

Now assume that the fee for non-participation is only symbolic and does not cover the cost of market provision of labour, such that $t < c$. Therefore non-participation *will* affect the supply of public good.

$e^* = 1$ will be morally ideal if

$$u(m, T-1) + v(Ng^p) > u(m-t, T) + v\left(\frac{Ng^p t}{c}\right) \tag{29}$$

That is, the monetary income t plus the higher amount of public good is worth more than the extra leisure time.

In this case, individuals will choose to participate when

$$u(m, T-1) + v(Ng^p) > u(m-t, T) + v\left((N-1)g^p + \frac{g^p t}{c}\right) + I(0,1) \tag{30}$$

Brekke et al make the assumption that $u(m-t, T) + v\left(\frac{g^p t}{c}\right)$ is decreasing with t, such that an individual having no public good with utility $u(m, T) + v(0)$ will never choose to spend t and buy g^p units of public good individually in the market. In the case where $e^* = 1$ both when there is no fee and when the fee is symbolic, the right-hand-side of (30) is less than the right-hand-side of (19), making (30) more likely to hold, and thus the symbolic fee weakly increases participation.

However individuals will choose not to participate when

$$u(m-t, T) + v\left(\frac{Ng^p t}{c}\right) + I(0,1) > u(m, T-1) + v\left(g^p + \frac{(N-1)g^p t}{c}\right) \tag{31}$$

Again, by the concavity of v, only one of (30) or (31) can hold and there is a unique Nash equilibrium.

If leisure is worth more than the fee and the extra provision of public good, non-participation will be morally ideal and the following condition will hold:

$$u(m-t,T)+v\left(\frac{Ng^p t}{c}\right) > u(m,T-1)+v(Ng^p) \quad (32)$$

Individuals will participate if

$$u(m,T-1)+v(Ng^p)+I(1,0) > u(m-t,T)+v\left((N-1)g^p+\frac{g^p t}{c}\right) \quad (33)$$

However, individuals will choose not to participate if the following instead holds:

$$u(m-t,T)+v\left(\frac{Ng^p t}{c}\right) > u(m,T-1)+v\left(g^p+\frac{(N-1)g^p t}{c}\right)+I(1,0) \quad (34)$$

Again, due to the concavity of v, only one of (33) and (34) will hold and therefore there will be a unique Nash equilibrium.

These results will be discussed more extensively in Section 6. What is interesting to note is that in each case where $e^* = 0$, it is still possible to sustain $e_i = 1$ regardless of fees. This is in contrary to Brekke et al's results, given the shape of their self-image function which punishes effort beyond the morally optimal level. Brekke et al also make the point that it is important whether the fee is perceived to be sufficient or symbolic in determining participation, as a sufficient fee may decrease participation while a symbolic fee may weakly increase it. The results obtained here with a monotonically increasing self-image function are consistent with this claim. The presence and characteristic of the fee also determine individuals' perception of responsibility. No fee or a symbolic fee makes the individual feel responsible to contribute to the public good, so that he will choose to participate in the work, but he will choose not to when the responsibility is perceived to be in the hands of the authority, which is the case if the fee is sufficient.

5 Two Examples of Monotonically Increasing Self-Image Functions

The effects of effort on self-image will now be investigated using two different self-image functions: the square root and logarithmic functions specifically. The aim is to illustrate the behavioral differences that can emerge even within the framework of monotonically increasing self-image functions.

5.1 The Square Root Function

Consider the following self-image function:

$$I_i = f(e_i, e_i^*) = \sqrt{\mathrm{e_i}} - \sqrt{\mathrm{e_i^*}} \tag{35}$$

This function is also monotonically increasing, and is equal to zero when actual effort is at the morally ideal level.

The optimal value of e^* is given by the first-order-condition from maximization of social welfare. Substituting the above self-image function into equation (11) and setting $e_i = e_i^* = e$ then leads to

$$u_l = N v_G \gamma_{e_i} + \frac{1}{2\sqrt{e}} \tag{36}$$

Given this value of e^*, maximization of individual utility gives the optimal actual effort. Substituting the square root self-image function into (12) yields

$$u_l = v_G \gamma_{e_i} + \frac{1}{2\sqrt{e_i}} \tag{37}$$

As equations (36) and (37) show, setting $e_i = e_i^* = e$ when individual utility is maximized will again lead to the underprovision of public good when $N > 1$. This result confirms the earlier observation that individual utility maximisation cannot produce the socially optimal effort level unless $N = 1$, regardless of the self-image function chosen.

5.1.1 Effects of increased efficiency and new information

The effect of an increase in θ can be found by differentiating equation (36) with respect to θ:

$$e_\theta^* = N \frac{v_G \gamma_{e\theta} + N v_{GG} \gamma_e \gamma_\theta}{-u_{ll} - N^2 v_{GG} \gamma_e^2 - N v_G \gamma_{ee} + \frac{1}{4e^{\frac{3}{2}}}} \tag{38}$$

The denominator is positive while there are opposing signs in the numerator. The increased supply of public good from increased efficiency will have a positive effect on morally ideal effort, however it also brings about a decrease in marginal utility. Hence the overall effect is ambiguous.

The effect of increased efficiency on individual ideal contribution is however positive. To see this, substitute the expression for e_θ^* above into $g_\theta = \gamma_e e_\theta + \gamma_\theta$ to get

$$g_\theta^* = \frac{Nv_G\gamma_{e\theta}\gamma_e + \gamma_\theta\left(-u_{ll} - Nv_G\gamma_{ee} + \frac{1}{4e^{\frac{3}{2}}}\right)}{-u_{ll} - N^2v_{GG}\gamma_e^2 - Nv_G\gamma_{ee} + \frac{1}{4e^{\frac{3}{2}}}} \tag{39}$$

which is positive.

The effect of θ on actual effort is calculated in the same way as in Section 4. Differentiating (37) with respect to θ and setting $v_G\gamma_e = 0$ gives

$$e_\theta\left(-\frac{1}{4e^{\frac{3}{2}}} + u_{ll}\right) = 0 \tag{40}$$

Since $u_{ll} < 0$ and $\frac{1}{4e^{\frac{3}{2}}} > 0$, the only solution is $e_\theta = 0$. Therefore, with the square root function, actual effort is independent of the efficiency parameter. This is because the effect of effort on self-image is independent of the morally ideal effort level – that is, $I_{ee^*} = 0$. When individuals have reached actual effort level that maximises their utility, that effort level will be maintained regardless of any change in the morally ideal effort due to the efficiency parameter.

If θ is now interpreted as the increase in the perceived and unobservable effect of private effort, assuming $e_\theta^* > 0$, the effect of θ on utility is then

$$U_\theta = u_l * (-e_\theta) + Nv_G(\gamma_e e_\theta + \gamma_\theta) + \frac{1}{2\sqrt{e}}e_\theta - \frac{1}{2\sqrt{e^*}}e^*{}_\theta$$

Since $e_\theta = 0$ from above,

$$U_\theta = Nv_G\gamma_\theta - \frac{1}{2\sqrt{e^*}}e^*{}_\theta \tag{41}$$

The first term represents the effect of efficiency on the supply of public good, which is positive. The second term is however negative due to the property of a monotonically increasing self-image function, which is decreasing in morally ideal effort level. Hence the overall effect is ambiguous.

5.1.2 Responsibility and economic incentives

In this section, individuals' decision to participate ($e = 1$) or not participate ($e = 0$) will again be analysed using the same method as in the previous section, considering three cases whereby no fee, a sufficient fee, and a symbolic fee is imposed for non-participation respectively.

With no fee for non-participation, collective participation is the morally ideal action ($e^* = 1$) when

$$u(m, T-1) + v(Ng^p) > u(m, T) + v(0) \tag{42}$$

Individuals will choose to participate when the benefit of higher supply of public good and self-image is worth more than leisure:

$$u(m, T-1) + v(Ng^p) > u(m, T) + v\big((N-1)g^p\big) - 1 \tag{43}$$

Otherwise, the following will hold and non-participation will be the Nash equilibrium:

$$u(m, T) + v(0) - 1 > u(m, T-1) + v(g^p) \tag{44}$$

By the concavity of v, only one of (43) or (44) will hold and there will be a unique Nash equilibrium.

Collective non-participation ($e^* = 0$) is chosen if leisure is worth more than the potential supply of public good:

$$u(m, T) + v(0) > u(m, T-1) + v(Ng^p) \tag{45}$$

In this case, individuals will participate if

$$u(m, T-1) + v(Ng^p) + 1 > u(m, T) + v\big((N-1)g^p\big) \tag{46}$$

Non-participation will be a Nash equilibrium if

$$u(m, T) + v(0) > u(m, T-1) + v(g^p) + 1 \tag{47}$$

Again, due to the concavity of the v-function, only one of (46) or (47) will hold.

When a fee t is imposed, which is exactly sufficient to cover the cost of non-participation c, collective participation will be the morally ideal action if

$$u(m, T-1) + v(Ng^p) > u(m-c, T) + v(Ng^p) \quad (48)$$

That is, when the fee is seen as too expensive to pay for leisure.

The actual action chosen by individuals will be $e = 1$ if

$$u(m, T-1) + v(Ng^p) > u(m-c, T) + v(Ng^p) - 1 \quad (49)$$

which will always hold given equation (48) holds. Hence individuals will always choose to participate if the fee is high enough. Note also the contrast with the case where there is no fee above, where non-participation is still a possible Nash equilibrium (equation (44)). Here, individuals value their leisure time more than self-image and the extra supply of public good. However when the fee is imposed and the morally ideal action is to participate, individuals value their monetary income and self-image more than their leisure time.

If it is collectively ideal to pay for market-supplied labour rather than to spend one's leisure time volunteering, $e^* = 0$ and

$$u(m-c, T) + v(Ng^p) > u(m, T-1) + v(Ng^p) \quad (50)$$

Individuals will choose participation as long as the increase in self-image is high enough that

$$u(m, T-1) + v(Ng^p) + 1 > u(m-c, T) + v(Ng^p) \quad (51)$$

Otherwise, non-participation will be the Nash equilibrium:

$$u(m-c, T) + v(Ng^p) > u(m, T-1) + v(Ng^p) + 1 \quad (52)$$

It is possible that the project is socially desirable ($e^* = 1$ without the fee) but individuals would rather pay for it than volunteer their time if given the option ($e = 0$ with the fee). The fee may even reduce participation if the cost is seen as small relative to the corresponding increase in self-mage from participation.

The third case involves a fee that is only symbolic and does not cover the cost of non-participation. Here, it is morally ideal to participate when keeping the extra monetary income and higher supply of public good is worth more than the loss of leisure:

$$u(m, T-1) + v(Ng^p) > u(m-t, T) + v\left(\frac{Ng^p t}{c}\right) \tag{53}$$

Individuals will find it optimal to do the morally ideal action and participate if

$$u(m, T-1) + v(Ng^p) > u(m-t, T) + v\left((N-1)g^p + \frac{g^p t}{c}\right) - 1 \tag{54}$$

Assuming further that $u(m-t, T) + v\left(\frac{g^p t}{c}\right)$ is decreasing with t as before, the right-hand-side of (54) is less than the right-hand-side of (43). Hence when $e^* = 1$ holds both without the fee and when the fee is symbolic, the introduction of the fee again weakly increases participation.

It is still possible for individuals not to participate when the fee is symbolic if the following holds:

$$u(m-t, T) + v\left(\frac{Ng^p t}{c}\right) - 1 > u(m, T-1) + v\left(g^p + \frac{(N-1)g^p t}{c}\right) \tag{55}$$

Due to the concavity of v, only one of (54) and (55) will hold as a Nash equilibrium.

When leisure is valued more than the fee and public good supply, it will be morally ideal not to participate:

$$u(m-t, T) + v\left(\frac{Ng^p t}{c}\right) > u(m, T-1) + v(Ng^p) \tag{56}$$

Given the monotonically increasing self-image function, individuals will still participate if the increase in self-image is high enough to compensate for this loss of leisure:

$$u(m, T-1) + v(Ng^p) + 1 > u(m-t, T) + v\left((N-1)g^p + \frac{g^p t}{c}\right) \tag{57}$$

However, individuals will conform to the morally ideal action and not participate if

$$u(m-t, T) + v\left(\frac{Ng^p t}{c}\right) > u(m, T-1) + v\left(g^p + \frac{(N-1)g^p t}{c}\right) + 1 \tag{58}$$

There will be a unique Nash equilibrium as only one of (57) and (58) will hold due to the concavity of v.

5.2 The Logarithmic Function

Consider now the extreme case of a steeply increasing self-image function. Self-image is now assumed to be

$$I_i = f(e_i, e_i^*) = \ln(1 + \varepsilon + e_i - e_i^*) \tag{59}$$

where ε is small. The function has been normalized such that $I_i \approx 0$ when $e_i = e_i^*$, which will play a role so that the function has a lower bound when analysing responsibility and economic incentives in a later section.

To find the morally ideal effort level, the individual starts by maximizing total social welfare assuming everyone else acted like him. This leads to the first-order condition

$$u_l = N v_G \gamma_{e_i} + \frac{1}{1 + \varepsilon + e_i - e_i^*}$$

Since at the maximum actual effort is morally ideal, setting $e_i = e_i^*$ leads to

$$u_l = N v_G \gamma_{e_i} + \frac{1}{1 + \varepsilon} \tag{60}$$

In the next stage, the individual maximizes his own utility given the morally ideal effort level above. The first-order condition is then

$$u_l = v_G \gamma_{e_i} + \frac{1}{1 + \varepsilon + e_i - e_i^*} \tag{61}$$

Again, comparing equations (60) and (61) shows that individual utility maximisation cannot produce the socially optimal effort level unless $N = 1$, confirming the result obtained in Section 4.

5.2.1 Effects of increased efficiency and new information

To find the effect of an increase in the efficiency parameter θ on the morally ideal effort level, differentiate equation (60) with respect to θ. Solving for e_θ^* gives:

$$e_\theta^* = N \frac{v_G \gamma_{e\theta} + N v_{GG} \gamma_e \gamma_\theta}{-u_{ll} - N^2 v_{GG} \gamma_e^2 - N v_G \gamma_{ee}} \tag{62}$$

Again, while the denominator is positive, the numerator contains both positive and negative terms. Hence the overall effect is ambiguous.

However, individual ideal contribution $g^* = \gamma(e^*, \theta)$ is always increasing with θ as the negative terms are cancelled out:

$$g_\theta^* = \frac{N v_G \gamma_{e\theta} \gamma_e + \gamma_\theta(-u_{ll} - N v_G \gamma_{ee})}{-u_{ll} - N^2 v_{GG} \gamma_e^2 - N v_G \gamma_{ee}} \tag{63}$$

The effect of a change in θ on actual effort is found by differentiating equation (61) above with respect to θ, with the previous assumption that $v_G \gamma_e = 0$, where individuals are unable to perceive the impact of their actual effort on public good provision. Hence,

$$\frac{\partial}{\partial \theta} u_l\big(x, T - e(\theta)\big) = \frac{\partial}{\partial \theta} \frac{1}{1 + \varepsilon + e_i(\theta) - e_i^*(\theta)}$$

$$e_\theta = \frac{e_\theta^*}{1 - u_{ll}(1 + \varepsilon + e - e^*)^2} \tag{64}$$

Although the denominator is positive, the sign of e_θ^* can be both positive and negative. Therefore the overall sign of e_θ is ambiguous.

If the increase in θ is interpreted as an increase in the perceived effect of private effort, such that both e_θ^* and $e_\theta > 0$, the effect on individual utility is then

$$U_\theta = v_G \gamma_\theta + (N - 1) v_G(\gamma_e e_\theta + \gamma_\theta) - \frac{e_\theta^*}{1 + \varepsilon + e - e^*} \tag{65}$$

The first two terms are positive due to the fact that both e_θ^* and $e_\theta > 0$, so an increase in θ will lead to an increase in G which increases utility. The last term is simply $e_\theta^* * \frac{dI}{de^*}$, which is always negative. Hence the overall effect is ambiguous.

5.2.2 Responsibility and economic incentives

When no fee is introduced, collective participation $e^* = 1$ is possible when

$$u(m, T - 1) + v(N g^p) > u(m, T) + v(0) \tag{66}$$

This is achieved when the benefits of higher supply of public good outweigh the loss of leisure time.

If equation (66) holds, individuals will choose to participate when

$$u(m, T-1) + v(Ng^p) > u(m, T) + v\big((N-1)g^p\big) + \ln(\varepsilon) \tag{67}$$

which will always hold, as $\ln \varepsilon$ would be a large negative number. In contrast, non-participation at the individual level will not hold since

$$u(m, T) + v(0) + \ln \varepsilon \ll u(m, T-1) + v(g^p) \tag{68}$$

Therefore with this self-image function, if the morally ideal action is to participate, the shame or guilt from non-participation is always large enough to induce individuals to always participate.

Collective non-participation is ideal when leisure is valued more than the provision of public good from the effort of all individuals:

$$u(m, T) + v(0) > u(m, T-1) + v(Ng^p) \tag{69}$$

In this case, individuals will only participate if

$$u(m, T-1) + v(Ng^p) + \ln(2+\varepsilon) > u(m, T) + v\big((N-1)g^p\big) \tag{70}$$

That is, whenever the extra supply of public good plus the self-image benefit exceeding the morally ideal (at which $I \approx 0$) are enough to compensate them for the loss of leisure.

However individuals will not participate if

$$u(m, T) + v(0) > u(m, T-1) + v(g^p) + \ln(2+\varepsilon) \tag{71}$$

Again, due to the concavity of the v-function, only one of (70) or (71) can hold and hence a unique Nash equilibrium is guaranteed.

When the fee for participation sufficiently covers the cost of getting a market replacement, the morally ideal effort level is still $e^* = 1$ if the fee is considered more expensive than the cost of leisure:

$$u(m, T-1) + v(Ng^p) > u(m-c, T) + v(Ng^p) \tag{72}$$

In this case, individuals will participate whenever

$$u(m, T-1) + v(Ng^p) > u(m-c, T) + v(Ng^p) + \ln \varepsilon \tag{73}$$

Again, this will always hold since $\ln\ \varepsilon$ is a large negative number. Therefore, individuals will always choose to participate given it is seen as the morally ideal action.

If on the other hand $e^* = 0$, the following will hold:

$$u(m-c,T) + v(Ng^p) > u(m,T-1) + v(Ng^p) \tag{74}$$

Here, leisure is considered to be worth more than the non-participation fee. Individuals will then only choose to participate if the increase in self-image from doing more than morally ideal is enough to overturn the inequality:

$$u(m,T-1) + v(Ng^p) + \ln(2+\varepsilon) > u(m-c,T) + v(Ng^p) \tag{75}$$

If that increase in self-image is not high enough to compensate for the loss of leisure, the following will hold:

$$u(m-c,T) + v(Ng^p) > u(m,T-1) + v(Ng^p) + \ln(2+\varepsilon) \tag{76}$$

and individuals will not participate. Both cases are possible and whichever action will be taken by the individual will depend on how he values leisure relative to the increase in self-image. Again, if equations (66) and (76) hold together, the project is socially desirable but market provision is preferred by individuals who choose to pay the fee rather than volunteer. In addition, if individuals' actual action without the fee was to participate, the introduction of the fee has the possibility of reducing participation even with the monotonically increasing self-image function.

Consider now the case when the fee t is symbolic and is less than the cost of replacing the labour with market supply c. The morally ideal effort level will be participation when the increased supply of public good and the money saved is enough to compensate for the loss of leisure:

$$u(m,T-1) + v(Ng^p) > u(m-t,T) + v\left(\frac{Ng^p t}{c}\right) \tag{77}$$

Individuals will then always choose to participate since

$$u(m, T-1) + v(Ng^p) > u(m-t, T) + v\left((N-1)g^p + \frac{g^p t}{c}\right) + \ln \varepsilon \qquad (78)$$

always holds. With the assumption that $u(m-t, T) + v\left(\frac{g^p t}{c}\right)$ is decreasing with t as before, the right-hand-side of (78) is less than the right-hand-side of (67). Hence when $e^* = 1$ holds both without the fee and when the fee is symbolic, the introduction of the fee weakly increases participation.

However, individuals will collectively choose not to participate if the loss of leisure hurts more than the fee and the reduced supply of public good:

$$u(m-t, T) + v\left(\frac{Ng^p t}{c}\right) > u(m, T-1) + v(Ng^p) \qquad (79)$$

In that case, individuals will participate if by doing so he can increase his self-image by a sufficiently high amount to outweigh the loss of leisure:

$$u(m, T-1) + v(Ng^p) + \ln(2+\varepsilon) > u(m-t, T) + v\left((N-1)g^p + \frac{g^p t}{c}\right) \qquad (80)$$

Contrary to the case with inverted-U-shaped self-image function, participation can still be supported as a Nash equilibrium (both equations (79) and (80) can hold together) as the increase in self-image from participation may be high enough.

Otherwise, the Nash equilibrium will be non-participation, and the following will hold:

$$u(m-t, T) + v\left(\frac{Ng^p t}{c}\right) > u(m, T-1) + v\left(g^p + \frac{(N-1)g^p t}{c}\right) + \ln(2+\varepsilon) \qquad (81)$$

This cannot hold together with (80) due to the concavity of v, and hence there is a unique Nash equilibrium.

Note that in all three cases, whenever the morally ideal action is to participate, individuals will always participate (contrary to the square root function and Brekke et al's inverted-U-shaped function where non-participation may be supported as a Nash equilibrium, when there is no fee or the fee is symbolic). This is due to the steepness of the logarithmic function when effort is less than morally ideal. Self-image is reduced to such a large extent

through non-participation, that individuals are compelled to participate. This represents an extreme case for individuals that are highly motivated by self-image.

However, a similarity to the square root function can be found in that even in cases where $e^* = 0$, participation is always a possibility. These results will be analysed in more detail in the next section.

6 Discussion

The results from the previous sections compared to those obtained by Brekke et al are now summarized in the following table:

I_i:	$-a(e_i - e_i^*)^2$	$f(e_i, e_i^*),\ I_e > 0$	$\ln(1 + \varepsilon + e_i - e_i^*)$	$\sqrt{e_i} - \sqrt{e_i^*}$
Increase in θ				
Effect on e_i^*	Ambiguous	Ambiguous	Ambiguous	Ambiguous
Effect on e_i	Ambiguous	Ambiguous	Ambiguous	Zero
Effect on g^*	Positive	Ambiguous	Positive	Positive
Effect on U[7]	Ambiguous	Ambiguous	Ambiguous	Ambiguous
Responsibility and Economic Incentives				
No fee for non-participation				
Case 1: $e^* = 1$	$e = 1$ or 0	$e = 1$ or 0	$e = 1$	$e = 1$ or 0
Case 2: $e^* = 0$	$e = 0$	$e = 1$ or 0	$e = 1$ or 0	$e = 1$ or 0
Sufficient non-participation fee $t = c$, fully covers costs				
Case 1: $e^* = 1$	$e = 1$	$e = 1$	$e = 1$	$e = 1$
Case 2: $e^* = 0$	$e = 0$	$e = 1$ or 0	$e = 1$ or 0	$e = 1$ or 0
Symbolic non participation fee $t < c$, does not cover costs				
Case 1: $e^* = 1$	$e = 1$ or 0	$e = 1$ or 0	$e = 1$	$e = 1$ or 0
Case 2: $e^* = 0$	$e = 0$	$e = 1$ or 0	$e = 1$ or 0	$e = 1$ or 0

Table 6-1: Summary of results

When comparing the effect of an increase in the efficiency parameter θ across all cases, the results are largely similar. e_θ^* is always ambiguous in sign due to the positive effect from the increase in public good and the negative effect from the corresponding decrease in marginal

[7] Assuming $e_\theta^*, e_\theta > 0$.

utility. Hence when e_θ is calculated, its sign will also depend on e_θ^* and is therefore ambiguous. An exception is in the case of the square root self-image function, whereby the optimal effort level for the individual is independent of the morally ideal effort level. This is arguably an unrealistic model, since it is reasonable for individuals to consider the morally ideal effort level when choosing their own contribution. When the effect of θ on individual ideal contribution g^* is analysed, the negative effect is cancelled out and the result is always positive for the specific functions analysed.

The effect of θ on U is also ambiguous regardless of the self-image function chosen, due to the positive effect of increased total supply of public good and the negative effect from the variation in e^* and the resulting effect on I. However this negative effect is caused by different reasons depending on the self-image function: in Brekke et al's case, I_{e^*} is always negative due to the less than ideal contribution achieved in utility maximization for the individual. In contrast, when self-image is monotonically increasing in effort level, an increase in the morally ideal effort makes the current effort level smaller in comparison, thus reducing self-image and impacting negatively on utility. This is regardless of the level of e relative to e^*.

While the effects of modifying the self-image function on the analysis of θ above are minimal, the changes are more evident when analysing responsibility and economic incentives with and without fees.

The most significant result is obtained when non-participation is seen as morally ideal. This deviation stems from the fact that self-image is a benefit that is derived by the individual himself, without accruing to the rest of society. When self-image is decreasing in effort beyond e^*, it is impossible to induce participation because any extra effort is 'punished' by the negative effect on self-image and therefore utility. For many individuals, this might not necessarily be the case. An individual who is highly driven by self-image, for example, will reasonably experience a higher self-image when he donates more of his time or money to a cause. If self-image is instead monotonically increasing in effort level, participation can still be sustained as a Nash equilibrium even if it is not morally ideal. This is arguably a fruitful modification on the self-image function, supported by economic examples whereby a

project is not viable to participate voluntarily in, and yet individuals still contribute to it in order to improve their self-image.

When there is no fee for non-participation, this paper will proceed with the morally ideal action of $e^* = 1$.[8] That is, the project is socially desirable. Suppose a fee is imposed that is sufficient to cover the cost of non-participation. Then individuals are effectively given the choice of either paying the fee or participating in the project – both options will result in the same amount of public good provision. If leisure is socially valued more than the non-participation fee, the morally ideal option would be non-participation, $e^* = 0$. With a monotonically increasing self-image function, it is possible that individuals derive enough utility from participation (through the increase in self-image) to deviate from the morally optimal action.

When the fee for non-participation is symbolic, individuals trade off the benefits of paying a less than sufficient fee and keeping their leisure time versus the less than full provision of public good. If everyone would rather pay the small symbolic fee and have a smaller supply of public good than lose their leisure time, then the morally optimal option would also be non-participation. However when self-image is considered, it can induce participation at the individual level.

A very common example is that of volunteer tourism, in which participants from developed countries travel to less developed countries in order to perform tasks to help the local community, such as building houses, schools or other infrastructure. The volunteers are not paid for their effort, and often they may even have to pay an organizer to take part in the project. It is non-debatable that the project is socially desirable as it helps improve the local facilities. An individual therefore has two choices: either pay the non-participation fee (in effect donating the amount of money that would have been sufficient to replace his labour cost, or donating only part of it as a symbolic fee) or volunteering his own time and effort. It would be morally ideal not to participate and pay the fee instead, given the fact that the value of labour from a developed country is much higher than that in a less developed country. With the amount of money paid corresponding to the individual's labour as valued

[8] If $e^* = 0$ with no non-participation fee, the project is socially undesirable or unfeasible. This case occurs very rarely in practice, and requires a steeply increasing self-image function, but has been included here for completeness.

in his developed home country, the local community can hire a much bigger labour force than just one person. This would often be the case as well even if only a symbolic fee is paid. However many people still choose to volunteer in these projects, potentially deriving higher self-image and giving support to the monotonically increasing self-image function.

When comparing the logarithmic and the square root self-image functions, the results slightly differ in the cases of no fee or a symbolic fee for non-participation. If the project is socially desirable that the morally ideal action is to participate, the logarithmic self-image function predicts individual participation with certainty while the square root function may still result in non-participation. This is due to the steepness of the logarithmic self-image function when $e = 0$, indicating non-participation. Self-image would be significantly lowered that it would be better for the individual to participate. Using the square root function, choosing not to participate only incurs a small reduction in self-image which may still be preferred if leisure is valued highly. In all cases where $e^* = 1$, the results from the square root self-image function closely resemble those obtained by Brekke et al.

For a socially desirable project with no fee for non-participation, individuals have no market-based alternative to their own effort. Hence when the morally ideal action is to participate, it is more reasonable to predict participation by individuals. However, the opposite often holds in practice. An example is recycling rubbish correctly, which by nature is a socially desirable activity. There is no control by the authority or fines imposed for non-compliance, and yet some individuals do recycle, while others do not. This supports the square root function over the logarithmic one.

Voting is an example when a symbolic fee for non-participation is imposed in certain countries, in the form of a fine. It is in the best interest of the society that every member votes regardless of the fee, because the non-participation fine no matter how high is only ever symbolic – it can never replace the actual vote of a member. Therefore $e^* = 1$. However it is also possible for individual equilibrium action to be $e = 0$, as evident in the voter turnout rate in Australia where compulsory voting still results in around 5% of voters

choosing non-participation.[9] Hence both examples support the square root function, and even also the more general function, over the more extreme logarithmic function.

Whether individuals participate or not in different cases can also be dependent on whether the responsibility for public good provision is perceived to lie in the hands of the authority imposing the fee or on the individual. As analysed in Brekke et al (2003), with the inverted-U-shaped self-image function, a sufficient fee shifts the responsibility to the authority, thus possibly reducing participation. A symbolic fee on the other hand may have the effect of increasing participation. This is in accordance with Frey's Crowding Theory (1997), whereby the sufficient fee for non-participation implies a subsidy for participation, which reduces the perceived intrinsic motive. Similarly for the general case of a monotonically increasing self-image function, a sufficient fee may result in lowering participation, while a symbolic fee makes participation more likely. Hence altering the self-image function does not cause results to deviate from previously established theories on responsibility and crowding out.

Despite the results obtained which seem to favour the square root over the logarithmic function, it is impossible to choose the correct self-image function out of the two specific examples, or even to pinpoint the most accurate representation out of all monotonically increasing functions, due to bounded rationality. Individuals may act as though they are maximizing a certain utility (and implicitly self-image) function, but in reality individuals do not actually commit to this process. The two functions given as examples in this paper only serve to make the point that different behaviors can be observed even though both self-image functions are monotonically increasing. Perhaps a certain percentage of society can be modeled using one function, and the rest the other, or there might even be another monotonically increasing self-image function that is more appropriate depending on the size of $I(0,1)$ and $I(1,0)$. The extent to which individuals are really driven by self-image would be a possible topic of further research into the subject. However, making the self-image function monotonically increasing in effort has certainly been a worthwhile modification to the model by Brekke et al.

7 Conclusion

[9] Australia Electoral Commission 2001 Federal Election Voter Turnout By Division.

The modification to Brekke et al's self-image function has produced some important results. While the effects of increased efficiency and new information are ambiguous and thus largely unchanged, individual choice of actions differs under the two self-image functions. Most significantly, when self-image is monotonically increasing in effort level it is possible to sustain individual participation even when it is not morally ideal. This is arguably an important contribution to the economic literature on self-image, as it explains the common occurrence of volunteer tourism.

When deciding the specific self-image function to be used, one needs to decide which is appropriate based on the size of $I(0,1)$ and $I(1,0)$, that is the extent to which an individual is motivated by his self-image. The logarithmic function has a large negative value for $I(0,1)$, and therefore always predicts participation when it is morally ideal. This represents the extreme case of an individual who is highly motivated by self-image. A more realistic case is represented by the square root function, whereby non-participation is still a possibility even if participation is morally ideal. This is supported by examples such as voting and recycling. However, the logarithmic and square root functions are only two examples and a direction of further research would therefore be in capturing individual behavior in the most accurate representation.

The results obtained here confirm previously established theories on responsibility and crowding out. A fee on non-participation can be seen as a subsidy for participation, and the introduction of the fee thus reduces participation. The responsibility for public good provision is then perceived to lie in the hands of the authority, and individuals will choose to pay the non-participation fee instead of participating. With a symbolic fee, individuals take responsibility for the public good provision and participation is increased. The modification to Brekke et al's model of moral motivation in this aspect is thus valid.

Despite its shortcomings, it is hoped that this paper shall contribute to the economic literature by improving the model used for analysing moral motivation through the more realistic use of a monotonically increasing self-image function.

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9 Appendix

9.1 Proof of Uniqueness of Nash Equilibrium Solution from Concavity of v

Using equations (19) and (20) as an example, it will now be proven by appealing to the concavity of v that they cannot hold together, and hence that there is a unique Nash equilibrium solution for individual choice of action.

Equation (19) represents the case where individuals choose to participate:

$$u(m, T-1) + v(Ng^p) > u(m, T) + v\big((N-1)g^p\big) + I(0,1)$$

Therefore,

$$u(m, T) - u(m, T-1) + I(0,1) < v(Ng^p) - v\big((N-1)g^p\big) \qquad \text{(A1)}$$

Similarly, equation (20) represents the case where individuals choose not to participate:

$$u(m, T) + v(0) + I(0,1) > u(m, T-1) + v(g^p)$$

Simplifying gives

$$u(m, T) - u(m, T-1) + I(0,1) > v(g^p) - v(0) \qquad \text{(A2)}$$

If both equations hold together, (A1) and (A2) can be combined to give

$$v(g^p) - v(0) < v(Ng^p) - v\big((N-1)g^p\big) \qquad \text{(A3)}$$

which contradicts the concavity of v.

Therefore, only one of (19) and (20) can hold and hence there is a unique Nash equilibrium.

All other proofs referring to the concavity of v are done in a similar way, and are left to the reader as exercise.■

9.2 The Inverted-U-Shaped Self-Image Function

In this section, the analysis done in Brekke et al (2003) regarding the effects of increased efficiency, new information and responsibility and economic incentives will be presented for the reader's convenience.

For simplification, Brekke et al have used the following additive utility function:

$$U = u(x, l) + v(G) + I \tag{A4}$$

where both u and v are increasing and concave.

9.2.1 Effects of increased efficiency and new information

Let effort be a function of the efficiency parameter, $e = e(\theta)$. Using the above utility function, the socially optimal effort level is now given by substituting the above utility function into equation (7) in the model in Section 3. This leads to

$$u_l\big(x, T - e(\theta)\big) = N v_G \Big(G_p + N\gamma(e(\theta), \theta)\Big)\gamma_e(e(\theta), \theta) \tag{A5}$$

Additionally, the actual effort level is now given by applying the utility function (A4) in equation (8) which becomes

$$u_l\big(x, T - e(\theta)\big) = v_G \Big(G_p + N\gamma(e(\theta), \theta)\Big)\gamma_e(e(\theta), \theta) + \big(-2a(e_i - e_i^*)\big) \tag{A6}$$

Suppose there is an increase in efficiency, given by an exogenous upward shift in the parameter θ. Differentiating equation (A5) above with respect to θ yields the effect of increased efficiency on the optimal effort level:

$$e_\theta^* = N\frac{v_G\gamma_{e\theta} + N v_{GG}\gamma_e\gamma_\theta}{-u_{ll} - N^2 v_{GG}\gamma_e^2 - N v_G\gamma_{ee}} \tag{A7}$$

Since u_{ll}, v_{GG} and γ_{ee} are all strictly negative, the denominator is positive. The first term in the numerator represents the total increase in public good from the increase in marginal effort level, which is positive. The second term represents the decreasing marginal utility from the increased supply of public good through increased efficiency. Since $v_{GG} < 0$, the overall effect of increased efficiency on the optimal effort level is ambiguous.

To analyse the effect of increased efficiency on individual ideal contribution $g^* = \gamma(e^*, \theta)$, substitute the expression from (A7) into $g_\theta = \gamma_e e_\theta + \gamma_\theta$ and simplify to get

$$g_\theta^* = \frac{N v_G\gamma_{e\theta}\gamma_e + \gamma_\theta(-u_{ll} - N v_G\gamma_{ee})}{-u_{ll} - N^2 v_{GG}\gamma_e^2 - N v_G\gamma_{ee}} \tag{A8}$$

which is positive. Hence individual ideal contribution is always increasing in efficiency, regardless of its effect on morally ideal effort.

The effect of increased efficiency on actual effort is given by differentiating equation (A6) above with respect to θ. Brekke et al also made the assumption that individuals are unable to perceive the change in public good provision from a change in own actual effort, hence $v_G\gamma_e = 0$. Therefore equation (A6) becomes

$$u_l = -2a(e - e^*)$$

Differentiating with respect to θ gives

$$u_{ll} * (-e_\theta) = -2ae_\theta + 2ae_\theta^*$$

$$e_\theta = \frac{2ae_\theta^*}{-u_{ll} + 2a} \qquad \text{(A9)}$$

While the denominator is positive, the numerator contains e_θ^* which is ambiguous in sign. The only thing that can be said is that if the efficiency parameter θ is exogenously given, then both e_θ^* and therefore e_θ equal zero, and increased efficiency has no effect on equilibrium ideal and actual effort.

When an increase in θ is interpreted as an increase in the perceived effect of private effort, both e_θ^* and $e_\theta > 0$. The effect on individual utility is then

$$U_\theta = u_l * (-e_\theta) + Nv_G(\gamma_e e_\theta + \gamma_\theta) - 2a(e - e^*)(e_\theta - e_\theta^*)$$

Substituting the Nash equilibrium condition (A6) for optimal actual effort yields

$$U_\theta = v_G\gamma_\theta + (N-1)v_G(\gamma_e e_\theta + \gamma_\theta) + 2a(e - e^*)e_\theta^* \qquad \text{(A10)}$$

With e_θ^* and$e_\theta > 0$, an increase in perceived effect of private effort raises both morally ideal and actual effort levels, leading to an increase in G and hence individual utility U. The first two terms on the right hand side of (A10) are therefore positive. However, given individual utility maximisation cannot produce the socially optimal effort level (see Section 3), setting $e < e^*$ makes the last term negative and the overall effect is ambiguous.

Note that an increase in the efficiency parameter θ could also lead to a decrease in utility: with $e_\theta^* > 0$, the resulting increase in morally ideal effort e^* increases the size of the deviation from actual effort. With the inverted-U-shaped self-image function utility will thus decrease.

9.2.2 Responsibility and economic incentives

Using the example of the *dugnad*, consider firstly the case where there is no fee for non-participation.

For participation ($e^* = 1$) to be the morally ideal action, the following condition must hold:

$$u(m, T-1) + v(Ng^p) > u(m, T) + v(0) \quad \text{(A11)}$$

For an individual to choose to participate ($e_i = 1$), the benefits of his extra labour and reaching maximum self-image must be greater than the lost leisure time. This is true whenever the following holds:

$$u(m, T-1) + v(Ng^p) > u(m, T) + v\big((N-1)g^p\big) - a \quad \text{(A12)}$$

Similarly, individual non-participation can also be supported as a Nash equilibrium whenever

$$u(m, T) + v(0) - a > u(m, T-1) + v(g^p) \quad \text{(A13)}$$

which occurs whenever the benefit of extra leisure time is worth more to the individual than the extra public good provided by his own labour and a better self-image. Due to the concavity of v, (A12) and (A13) cannot hold simultaneously and therefore there is a unique Nash equilibrium.

If non-participation ($e^* = 0$) is instead the morally ideal action, the opposite of (A11) will hold:

$$u(m, T-1) + v(Ng^p) < u(m, T) + v(0) \quad \text{(A14)}$$

In this case, individual participation is optimal when

$$u(m, T-1) + v(Ng^p) - a > u(m, T) + v\big((N-1)g^p\big) \quad \text{(A15)}$$

Clearly, this cannot hold together with (A14). Hence, if non-participation is morally ideal, it is optimal for individuals not to participate as well and the following condition will hold:

$$u(m, T) + v(0) > u(m, T-1) + v(g^p) - a \quad \text{(A16)}$$

That is, the extra leisure time and better self-image from non-participating are always worth more than the public good provided by his own effort.

When the non-participation fee t is sufficient and equals c, participation is morally ideal if the monetary income not paid as fee is more valuable than the lost leisure time:

$$u(m, T-1) + v(Ng^p) > u(m-c, T) + v(Ng^p) \quad \text{(A17)}$$

Given $e^* = 1$, individuals choose to participate if

$$u(m, T-1) + v(Ng^p) > u(m-c, T) + v(Ng^p) - a \quad \text{(A18)}$$

which is always true given (A17). Therefore individuals will always choose to participate if it is morally ideal.

Non-participation will be morally ideal if the extra leisure time is considered more valuable than the fee paid:

$$u(m-c, T) + v(Ng^p) > u(m, T-1) + v(Ng^p) \quad \text{(A19)}$$

Given $e^* = 0$, individuals will not participate if

$$u(m-c, T) + v(Ng^p) > u(m, T-1) + v(Ng^p) - a \quad \text{(A20)}$$

Clearly, this is always true (and the opposite will not hold) and therefore $e_i = 0$. If $e^* = 1$ and individuals choose to participate ($e_i = 1$) when there is no fee, and $e^* = 0$ when the fee is imposed, equation (A20) shows that the fee has the undesirable effect of reducing participation.

Now assume that the fee for non-participation t is only symbolic. $e^* = 1$ will be morally ideal if

$$u(m, T-1) + v(Ng^p) > u(m-t, T) + v\left(\frac{Ng^p t}{c}\right) \quad \text{(A21)}$$

That is, the monetary income t plus the higher amount of public good is worth more than the extra leisure time.

In this case, individuals will choose to participate when

$$u(m,T-1)+v(Ng^p)>u(m-t,T)+v\left((N-1)g^p+\frac{g^pt}{c}\right)-a \qquad \text{(A22)}$$

With the assumption that $u(m-t,T)+v\left(\frac{g^pt}{c}\right)$ is decreasing with t, the right-hand-side of (A22) is less than the right-hand-side of (A12), making (A22) more likely to hold, and thus the symbolic fee weakly increases participation.

However individuals will choose not to participate when

$$u(m-t,T)+v\left(\frac{Ng^pt}{c}\right)-a>u(m,T-1)+v\left(g^p+\frac{(N-1)g^pt}{c}\right) \qquad \text{(A23)}$$

Again, by the concavity of v, only one of (A22) or (A23) can hold and there is a unique Nash equilibrium.

Assuming non-participation is the morally ideal strategy, the following condition will hold:

$$u(m-t,T)+v\left(\frac{Ng^pt}{c}\right)>u(m,T-1)+v(Ng^p) \qquad \text{(A24)}$$

Individuals will participate if

$$u(m,T-1)+v(Ng^p)-a>u(m-t,T)+v\left((N-1)g^p+\frac{g^pt}{c}\right) \qquad \text{(A25)}$$

which is inconsistent with (A24) since $t<c$. Hence individuals will choose not to participate, and the following will hold:

$$u(m-t,T)+v\left(\frac{Ng^pt}{c}\right)>u(m,T-1)+v\left(g^p+\frac{(N-1)g^pt}{c}\right)-a \qquad \text{(A26)}$$

These results have been summarized for comparison purposes in Table 6-1.