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**Evaluation and Analysis of Value at Risk Methodologies
for Exchange Rate Risk in the Euro Market**

Master Thesis in Finance

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Abstract

The deteriorating European economic situation has suggested the necessity of risk management in the exchange rate of EUR for governments and corporations, but there is few researches studying in this field. In this thesis, by choosing USD/EUR, JPY/EUR and GBP/EUR as subjects, with a focus on the availability of different methods to the estimation of exchange rate risk to EUR, we aim to calculate the VaR of those three kinds of exchange rates and try to find the most accurate model to measure exchange rate risk in different environments and periods.

Based on the analysis of figures and the comparison among the results from different models, in this thesis, the conditional POT model and moving student t-distribution simulate the VaR for all exchange rates to EUR much better than others, whichever economic situations are. Besides this, no methods have significant improvement for the measurement of VaR when economic situation changes. Meanwhile, the empirical rule that higher confidence level may improve the accuracy of estimation of VaR gets proven, especially for EVT model.

Key words: Value at Risk, volatility clustering, exchange rate, volatility weighted historical simulation, normal distribution, student t-distribution, extreme value theory

This thesis is dedicated to our parents and teachers

for their precious love and support

throughout the whole life

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1. Introduction

As the global financial market changes rapidly, risk management has attracted immense attention. The continuous fluctuation makes the risk of exchange rate become one of the main risks that investors may undertake and manage, especially for those multinational corporations. For the managers of these corporations, the management of exchange rate risk is helpful to shelter corporate profits from the negative impact of exchange rate fluctuation and to get potential profit from exchange rate exposure management.

The serious economic crisis and recession at present in Europe also prove the necessity of risk assessment and measurement in the exchange rate of Euro. The depreciation of Euro which is caused by the crisis has a great negative influence on the economic situation of the Eurozone. Therefore the management of exchange rate risk is also significant for governments to forecast the loss, hedge the risk and get rid of the difficulty successfully.

Value-at-risk (VaR) and expected shortfall (ES) are considered as the core risk measurement of market risk. There are many methods to estimate VaR, such as historical simulation, parametric approaches and extreme value theory method, etc. In this thesis, volatility weighted historical simulation, parametric approaches and extreme value theory method would be chosen as three ways to calculate VaR. The analysis and comparison of VaR calculated by different methods will show the position of exchange rate risk of Euro and suggest which one is the most suitable to measure exchange rate risk under normal or bad economic situation respectively.

Three exchange rates which occur in this thesis are USD/EUR, GBP/EUR and JPY/EUR. While EUR is short for Euro, USD, GBP and JPY are short for US Dollar, British Pound and Japanese Yen. The same meanings for all the abbreviations exist in

this paper. The choice of these exchange rates is based on their world status, flexibility and fluctuation.

The discussion of the performances of different models in financial crisis and normal situations is the reason to choose two periods representing these two situations. The period from 01/10/2007 to 01/10/2011 is the crisis period and the period from 01/10/2002 to 01/10/2006 is the normal period.

The basic data for research is just the exchange rate; therefore the data source of the thesis is the database of the European Central Bank.

Even though the complicated performances of different risk measurement methods for different exchange rate make it hard to choose one specific model for different economic situation, the moving student t-distribution and conditional EVT models are considered as better options for investors to decrease the probability of underestimating the exchange risk of EUR under the background of financial crisis which increases the volatility of exchange rates, and using different methods and comparing the estimates are better for corporations and governments to measure the risk.

The findings in this thesis could be helpful to decipher the mechanisms that lead to risk management in Euro and thus can enhance the efficiency of the government's policy-making process to stimulate economic recovery and improve corporations' strategies to hedge or even benefit from the risk.

2. Literature review

The problem of risk management is an old one in statistics, economics and finance. As one measurement, value at risk is widely used in measuring the risk of loss on a specific portfolio of financial assets in financial mathematics and financial risk management.

Because of the publishing of J. P. Morgan's Risk Metrics document in 1994, VaR became one popular and widespread concept. The general recognition and use of VaR models has initiated considerable literatures including statistical descriptions of VaR and assessments of different models.

Kevin Dowd (2002)'s literature "Measuring at Risk" provides a brief overview of recent development in risk management and introduces the estimation of different measurements of risk including VaR. From this literature, the debate about the VaR recently is also mentioned. Some disadvantages like incoherent and silent about the size of loss reduce its applicability but since it is simple and convenient, it is still chosen as one main technology for risk management.

In the article of Angelidis and Degiannakis (2005), they investigate the accuracy of parametric, non-parametric and semi-parametric methods in estimating the VaR in three main markets, which includes stock exchanges, commodities and foreign exchange rates. Based on their results, our thesis will focus on the exchange rate risk of euro and try to find the accuracy of different models of VaR for exchange rate risk.

There are several similar articles discussing the use of VaR to measure the exchange rate risk of different currencies in different regions and countries. Wang, Wu, Chen, Zhou (2009)'s article analyzes the exchange rate risk of Chinese Yuan by using VaR and ES based on extreme value theory. Mazin A. M. and Al Janabi (2006) research the foreign exchange trading risk with VaR in the case of Moroccan market. The former

one indicates that the expected shortfall cannot improve the tail risk problem of VaR, and VaR values calculated by extreme value theory can measure the risk of exchange rate of JPY/CNY, EUR/CNY more accurately. The latter one demonstrates a constructive VaR based approach taking into account the adjustments for the illiquidity of long and short positions.

These literatures and articles provide us the theoretical basis and idea to choose the most suitable model to measure the exchange rate risk of euro under normal and crisis situation respectively.

The remainder of the paper is organized as follows. In the next section, an overview of different VaR models and methodologies is described, and then the characteristics of currency exchange market and the economic situation in Euro zone are illustrated. The results of the empirical tests are drawn in the final section along with conclusions and recommendations.

3. Methodology

In this section, we present the methods in mathematics for the calculation and test applied in this paper. As mentioned in the introduction, a mix of thoughts such as popularization, volatility, relative comprehension and so on is considered before the methods have been selected. The methods for the exchange rate risk calculation cover three aspects:

- Non-parametric models containing VWHS
- Parametric models with moving normal distribution and t-distribution
- EVT models

The results testing method applied here is called Kupiec test since it is the most popular method and easy to control. These methods are presented below in detail. Most of the equations come from the inspiration of the book published by Dowd (2002), see the detail in the section of reference. For those equations which are not the same as that in this book or quote from other literatures, some footnotes are made.

3.1 Definitions of VaR and ES

Value at Risk (VaR) is defined as the smallest loss " l " to make the probability of a future portfolio loss " L " which is larger than " l ", less than or equal to $1 - \alpha$. The mathematical equation defines VaR is shown below:

$$\text{VaR}_\alpha(L) = \min(l: \Pr(L > l) \leq 1 - \alpha) \quad (1)$$

Under the assumption of a continuous loss distribution, the equation can be rewritten as:

$$\Pr(L > \text{VaR}_\alpha(L)) = 1 - \alpha \quad (2)$$

Even though VaR is popular and has been applied in many studies, it still has

obvious drawbacks including fat tail and incoherency. Instead, Artzner et al. (1997) proposed the conception of expected shortfall (ES). Expected shortfall at confidence level α is the average VaR for confidence levels larger than or equal to α and the equation is:

$$ES_{\alpha}(L) = \frac{1}{1-\alpha} \int_{\alpha}^1 VaR_r(L) dx \quad (3)$$

Under the continuous distribution assumption, ES can be expressed as:

$$ES_{\alpha}(L) = E[L: L > VaR_{\alpha}(L)] \quad (4)$$

Since this thesis focuses on the application of VaR, the discussion about ES will not be mentioned too much.

3.2 Non-parametric approaches

Non-parametric approaches are methods which do not depend on restrictive parametric assumptions of the loss distribution such as normality. Instead it depends on the empirical loss distribution.

Basic historical simulation is the most basic and easiest one to calculate VaR and ES, which relies on the empirical loss distribution. According to the logic that approximately $(1 - \alpha)N$ losses larger than VaR are to be expected, $(1 - \alpha)N + 1$ largest loss should be considered as the estimate of $VaR_{\alpha}(L)$.

Volatility weighted historical simulation is another one non-parametric approach. It applies the basic historical simulation to a rescaled sample of T observed losses. Because of the phenomena of volatility clustering, the motivation of this method is to reflect volatility in the estimates of VaR and ES for the next holding period. The sample of new adjusted losses is shown:

$$\begin{aligned}
l_1^* &= \frac{\sigma_{T+1}}{\sigma_1} l_1 \\
l_2^* &= \frac{\sigma_{T+1}}{\sigma_2} l_2 \\
&\dots\dots \\
l_T^* &= \frac{\sigma_{T+1}}{\sigma_T} l_T
\end{aligned} \tag{5}$$

Where l_i^* denotes the rescaled exchange loss, l_i is the original loss. $\sigma_1, \sigma_2 \dots \sigma_T$ are the volatilities associated with the observed losses in the observation period, while σ_{T+1} is a forecast volatility in the next holding period. In this paper, the next holding period is the test period. In this way, volatility clustering has been considered into loss estimation, which means that every original loss in the observation period should be rescaled based on the first forecasted volatility in the next holding period. Through GARCH (1, 1) model or moving average volatility model (EWMA), σ_{T+1} can be calculated.

Then as in historical simulation, $(1 - \alpha)N + 1$ largest loss can be considered as the estimate of $\text{VaR}_\alpha(L)$.

3.3 Parametric approaches

The main idea behind parametric approaches is the assumption that the distribution of losses follows some specific distribution. In this thesis the assumption is that it follows a normal distribution or t-distribution.

Under the assumption of normal distributed losses:

$$\text{VaR}_\alpha(L) = \mu + \sigma_{T+1} z_\alpha \tag{6}$$

In which μ represents mean, σ_{T+1} is forecasted volatility under consideration of volatility clustering, and z_α denotes the α -quantile for the standard normal distribution.

Under the assumption of t-distributed losses:

$$\text{VaR}_\alpha(L) = \mu + \sqrt{\frac{v-2}{v}} \sigma_{T+1} t_{\alpha,v} \quad (7)$$

In which v denotes degrees of freedom.

In this paper, since the target is a single exchange rate instead of portfolio, covariance is not necessary to consider. Because of the character of exchange rate, the calculation of σ_{T+1} must take moving average volatility into account. To calculate the value of σ_{T+1} , λ needs to be estimated from the data at hand, but it is suggested in the Risk Metrics Technical Document that taking λ to be 0.94 for daily return data (Technical Document (1996, p. 97)).

3.4 Extreme value theory

As discussed by Lauridsen(2000), “the traditional methods used for estimation of VaR have various disadvantages as they are not aimed specifically at modeling the tails of the distribution of profits and losses; extreme value methods may prove valuable towards improving the current estimation methods.” EVT model is selected in this paper typically. The intuition behind EVT is extreme or large losses are more relevant for estimating VaR, therefore finding a distribution that fits these losses is one available way. Because of this idea, unconditional and conditional peaks over threshold (POT) models are chosen to calculate VaR.

One difficulty in the use of POT is choosing threshold, because there is one trade-off in the choice of threshold; for the underlying theory to go through a high threshold is required while for the estimation of the parameters a low threshold is necessary. More details about threshold will be discussed in the fourth part Empirical results.

Considering l_1, l_2, \dots, l_n as the series of loss, POT approach aims at modeling excess losses $L - \mu$, where L is a stochastic loss variable and μ is the predetermined threshold value. Given $L > \mu$, a cumulative density function for excess losses could be defined as¹

$$F_u(l) = \Pr(L - \mu \leq l | L > \mu) = \frac{F(l+\mu) - F(\mu)}{1 - F(\mu)} \quad (8)$$

Then the equation below can be derived²

$$F(l) = [1 - F(\mu)]F_u(l - \mu) + F(\mu) \quad (9)$$

Under the limit theorem by Pickands-Balkema-deHaan, which is posed by Pickands (1975), Balkema and de Haan (1974), the distribution of $F_u(l - \mu)$ as $\mu \rightarrow \infty$ is a generalized Pareto distribution:

$$G(l - \mu) = \begin{cases} 1 - (1 + \xi \frac{l - \mu}{\beta})^{-1/\xi} & \text{if } \xi \neq 0 \\ 1 - \exp\left(-\frac{l - \mu}{\beta}\right) & \text{if } \xi = 0 \end{cases} \quad (10)$$

In the model, three parameters μ , σ and ξ denote location parameter, scale parameter and shape parameter respectively. The shape parameter ξ governs the tail behavior of the distribution and when it is large, the tail is fat.

By derivation the unconditional POT estimate of VaR_α is:

$$\text{VaR}_\alpha = \mu + \frac{\beta}{\xi} \left[\left(\frac{N}{N_u} (1 - \alpha) \right)^{-\xi} - 1 \right], \text{ for } \xi \neq 0 \quad (11)$$

$$\text{VaR}_\alpha = \mu - \beta \ln \left(\frac{N}{N_u} (1 - \alpha) \right), \text{ for } \xi = 0 \quad (12)$$

¹Wang, Wu, Chen and Zhou(2010), eq.4

²Wang, Wu, Chen and Zhou(2010), eq.5

The difference of conditional POT and unconditional POT is that former is more dynamic by applying POT analysis to the standardized residuals from a GARCH or EWMA model. In this method q_α is the α -quantile of the generalized Pareto distribution (GPD) of the standardized residuals. Therefore,

$$\text{VaR}_\alpha = \mu + \sigma_{T+1}q_\alpha \quad (13)$$

In which μ is mean and σ_{T+1} is forecasted volatility. q_α can be obtained by applying the unconditional POT to the standardized residuals ($\varepsilon_T^* = \frac{l_1 - \bar{l}}{\sigma_T}$).

$$q_\alpha = \mu^* + \frac{\beta^*}{\xi^*} \left[\left(\frac{N}{N_{u^*}} (1 - \alpha) \right)^{-\xi^*} - 1 \right], \text{ for } \xi^* \neq 0 \quad (14)$$

$$q_\alpha = \mu^* - \beta^* \ln \left(\frac{N}{N_{u^*}} (1 - \alpha) \right), \text{ for } \xi^* = 0 \quad (15)$$

In which, the star-noted parameter values and threshold value are not to the original loss data; instead they are GPD parameters when POT is used to the standardized residuals.

For both unconditional and conditional POT, the calculation of β and ξ requires the application of Maximum Likelihood estimation. Taking logs of the respective probability density functions and summing over the m observations which larger than the threshold, it follows that the corresponding log-likelihood functions are:³

$$\log L(\beta, \xi) = -m \ln \beta - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^m \ln \left(1 + \xi \frac{l_\mu^i - \mu}{\beta} \right), \text{ for } \xi \neq 0 \quad (16)$$

$$\log L(\beta) = -m \ln \beta - \frac{1}{\beta} \sum_{i=1}^m (l_\mu^i - \mu), \text{ for } \xi = 0 \quad (17)$$

The ML estimates of β and ξ can be obtained by maximizing the log-likelihood functions with respect to the parameters.

³Wang, Wu, Chen and Zhou(2010), eq.14

3.5 Backtesting

This thesis applies Kupiec frequency test to test the validity of VaR models. The Kupiec test is the basic frequency test for a given estimator of VaR_α and it compares the actual or observed frequency of VaR violations with the expected or predicted frequency of VaR violations during some test period of observed data.

A VaR violation is defined as a loss larger than the VaR estimate. By coding a non-violation with 0 and a violation with 1, the number of VaR violation can be gotten. Since these codes which are 0 or 1 are Bernoulli variables, Kupiec test is a binomial test and the probability of observing x violations in a sample of N observations is:

$$\Pr(X = x) = \frac{N!}{x!(N-x)!} p^x (1 - p)^{N-x} \quad (18)$$

In which X is the number of violations and $p = 1 - \alpha$.

Applying the exact Kupiec test based on the binomial distribution in one-sided test, if $\Pr(X \geq x)$ or $\Pr(X \leq x)$ is less than the standard level for statistical tests, the underlying model will be rejected under the assumption that the actual frequency of violations is too large or too small respectively.

For a two-sided test to be implemented, the construction of a confidence interval for either the observed frequency of violations or the expected frequency of violations is required. The lower bound and the upper bound for the number of violations can be calculated by the cumulative probability equations:

$$\Pr(X \leq x) = \sum_{i=0}^x \frac{N!}{i!(N-i)!} p^i (1 - p)^{N-i} \quad (19)$$

By checking whether the actual frequency of violations is in the interval, the validity of the model can be found.

4. Empirical results

4.1 Data analysis

The three exchange rates of euro chosen to analyze in this thesis are USD/EUR, JPY/EUR and GBP/EUR. A new definition of daily logarithmic loss will be applied while the loss value is donated to be minus return. A detailed discussion is presented below in this section.

USD is the most important and prevailing universal currency. Its flexibility to EUR ensures the VaR results and tests on it clear and easy to analyze. Figure 4.1.a and Figure 4.1.b below suggest the daily logarithmic loss of USD/EUR in normal period and crisis period respectively. In normal period, the exchange rate loss of USD/EUR fluctuates between -1% and 1% except some extreme value. By contrast in crisis period, the exchange rate loss of USD/EUR is in the interval from -2% to 2% with some extreme value which exceeds 4%.

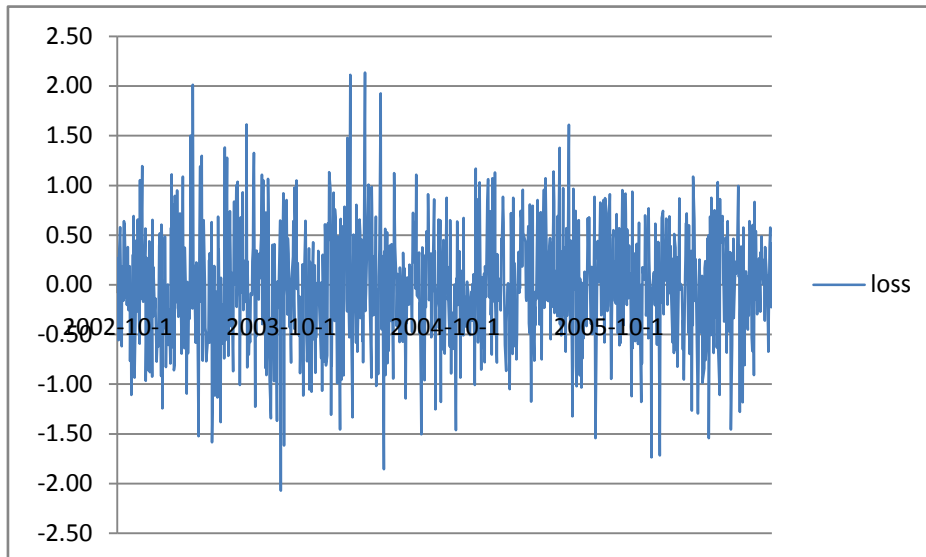


Figure 4.1.a Daily logarithmic Loss of USD/EUR in Normal Period

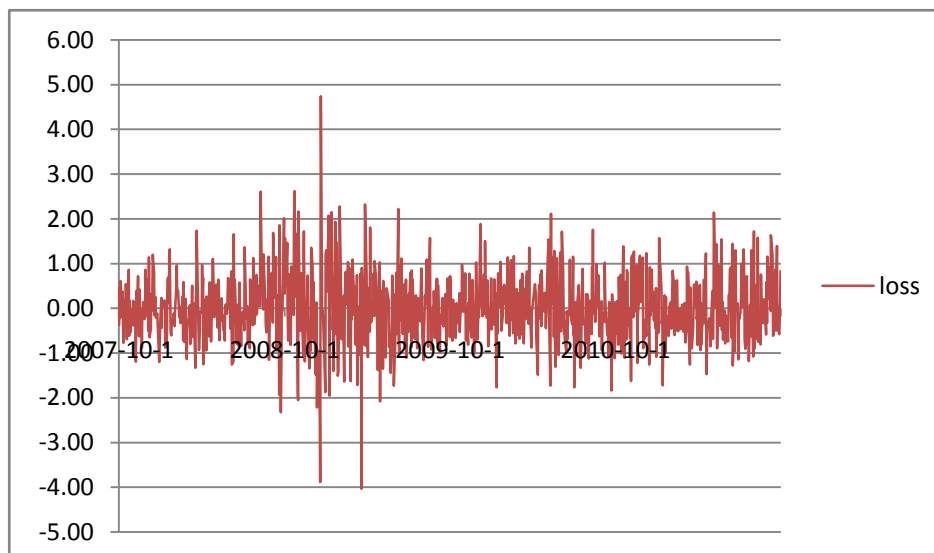


Figure 4.1.b Daily logarithmic Loss of USD/EUR in Crisis Period

Besides EUR, GBP is another important currency in Europe. Its obvious fluctuation to EUR makes it be taken into analysis. Two figures 4.2.a and 4.2.b below show that the exchange rate loss of GBP/EUR in crisis period is larger than that in normal period.

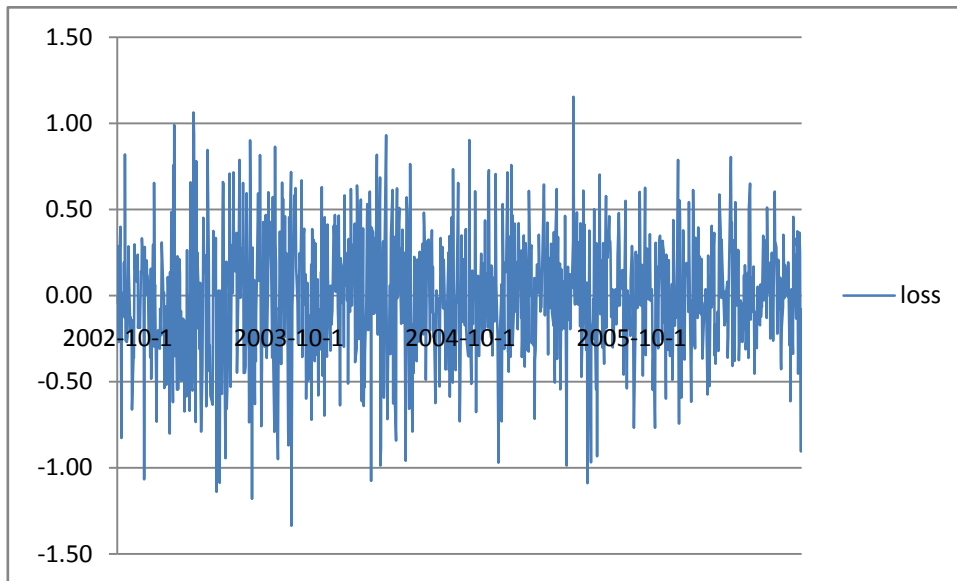


Figure 4.2.a Daily logarithmic Loss of GBP/EUR in Normal Period

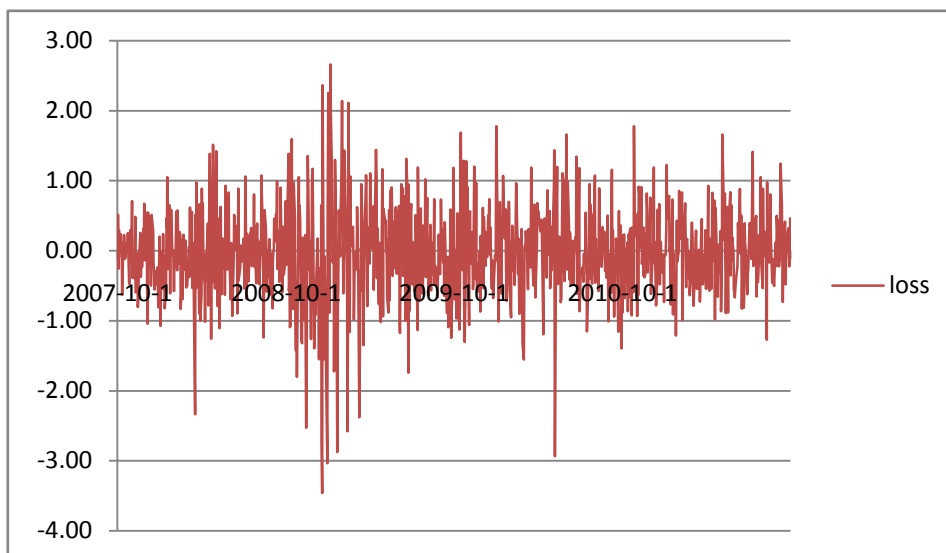


Figure 4.2.b Daily logarithmic Loss of GBP/EUR in Crisis Period

Since Asia is a region which has large influence on global economy, as the representation of currency in Asia, CNY (Chinese Yuan and JPY are both taken into account. However, compared to JPY, CNY is too fixed and does not have enough flexibility. Therefore, JPY/EUR is chosen as the third exchange rate under test in this thesis. The small daily loss of JPY/EUR in normal period and relatively large daily loss in crisis are shown in two figures 4.3.a and 4.3.b below.

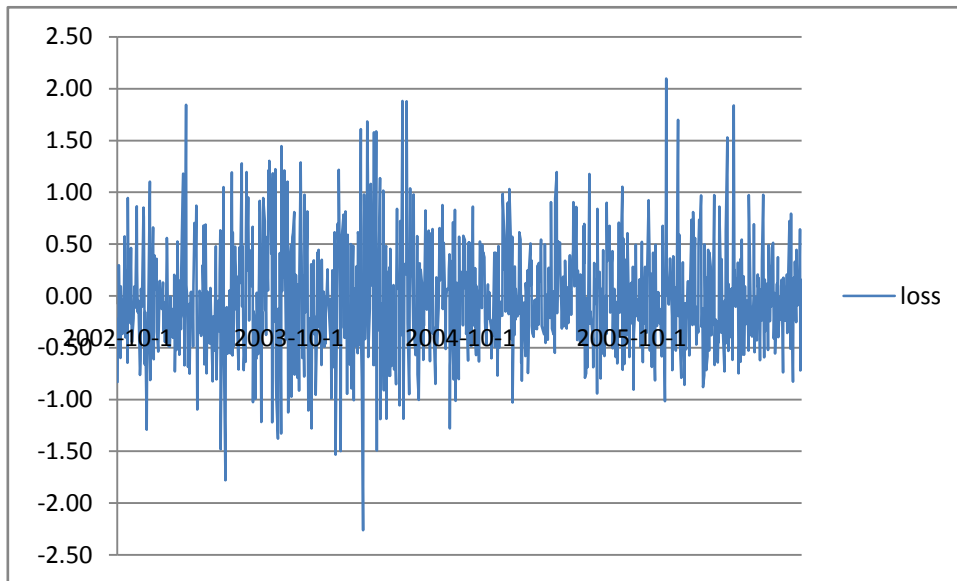


Figure 4.3.a Daily logarithmic Loss of JPY/EUR in Normal Period

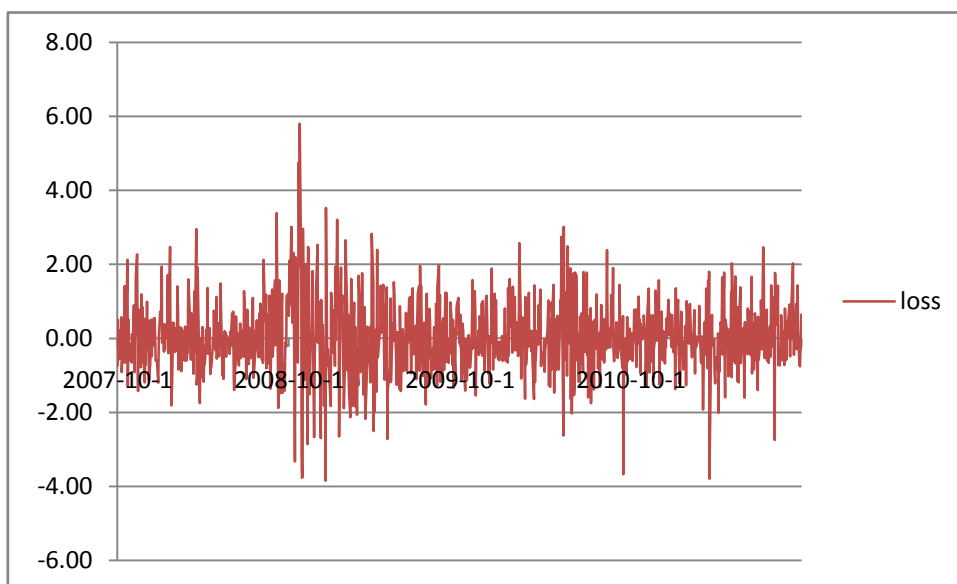


Figure 4.3.b Daily logarithmic Loss of JPY/EUR in Crisis Period

The 2.15 p.m. price of EUR exchange rate (reference rate) is selected in this thesis for analysis. The reference rates are usually updated by 3 p. m. CET. They are based on a regular daily concertation procedure between central banks across Europe and worldwide, which normally takes place at 2.15 p. m. CET. ⁴ In order to compare the

⁴See the detail description from <http://www.ecb.int/stats/exchange/eurofxref/html/index.en.html>

different performances of the different methods when estimating the exchange rate VaR, two individual periods are chosen for all the three exchange rate. The first period which is called normal period is from 1st Oct 2002 to 1st Oct 2006, 4 years in total. The second period which is called crisis period is from 1st Oct 2007 to 1st Oct 2011, also 4 years in total. Both of these two periods are divided into two parts: observation sample and test sample. Each period contains 1027 numbers. In normal period, the number of observation data is 771 and the number of test data is 256. In crisis period, the number of observation data is 768 and the number of test data is 259.

In our model, daily loss data is applied because of the requirement of VaR analysis. The daily loss of exchange rate can be defined as minus return. So firstly the price data is changed into return data. Then the daily logarithmic return R_t can be defined as

$$R_t = 100 * (\ln P_t - \ln P_{t-1}) \quad (20)$$

Where P_t is the close price of exchange rate at time t . The descriptive statistics of daily logarithmic loss are presented in Tables 3.1 and 3.2 below.

Table 4.1 Descriptive statistics of daily logarithmic Loss from three exchange rates of EUR in normal period (Loss=-return)

	USD/EUR	JPY/EUR	GBP/EUR
Mean	-0.0243	-0.0216	-0.0072
Median	-0.0246	-0.0589	0.0000
Maximum	2.1333	2.0959	1.1527
Minimum	-2.0725	-2.2620	-1.3360
Std. Dev.	0.5824	0.5357	0.3661
Skewness	0.0060	0.3293	-0.1973
Kurtosis	3.4834	4.0015	3.3269
Jarque-Bera	10.005	61.4820	11.2364
Observations	1027	1027	1027

Table 4.2 Descriptive statistics of daily logarithmic Loss from three exchange rates of EUR in crisis period (Loss=-return)

	USD/EUR	JPY/EUR	GBP/EUR
Mean	0.0048	0.0443	-0.0212
Median	-0.0205	-0.0179	-0.0227
Maximum	4.7354	5.7997	2.6573
Minimum	-4.0377	-3.8416	-3.4613
Std. Dev.	0.7687	1.0063	0.6626
Skewness	0.1472	0.2477	-0.3082
Kurtosis	5.8843	5.5297	5.6651
Jarque-Bera	359.6886	284.3489	320.1953
Observations	1027	1027	1027

Obviously, in the crisis period, all the three exchange rates have larger maximum value and minimum value than in the normal period, which indicates that the volatility in crisis period is much larger. Besides this, all the series have sharp peaks and heavy tails since the skewness values do not equal to zero and kurtosis value are all larger than 3. All the series are far away from normal distribution, especially in the crisis period.

4.2 Volatility Weighted Historical Simulation (VWHS)

Volatility weighted historical simulation (VWHS), as one of the most popular non-parametric methods when estimating VaR, relies on the real loss distribution without depending on some restrictive parametric assumptions. It is relatively easy to be estimated, explained and obtains some other advantages. Comparing to some other non-parametric methods such as basic historical simulation (HS) and age weighted historical simulation (AWHS), VWHS takes the volatility clustering into consideration directly and VaR larger than the largest loss in the original sample can be obtained. Volatility clustering means that “large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes.” (Mandelbrot, 1963). So it will directly influence the VaR value when estimating.

There are two alternatives when processing VWHS to estimate volatility. One is

the standard GARCH (1,1) model which is a popular approach for VWHS and easy to be estimated in Eviews. Another one is called exponentially weighted moving average volatility model (EWMA) which is applied in this paper. The estimation model is shown as below:⁵

$$\sigma_{T+1}^2 = \frac{1-\lambda}{1-\lambda^T} \sum_{t=1}^T \lambda^{T-t} \varepsilon_t^2 \quad (21)$$

Where λ is a fixed parameter and equals to 0.94 empirically, this particular value is the standard Risk Metrics value, which is easy to get and can be altered. σ_{T+1}^2 is the volatility of loss at time T+1, T is the total number of observed losses. ε_t is the unexpected losses at time t which can be defined as:

$$\varepsilon_t = l_t - \bar{l} \quad (22)$$

Where l_t is the real loss at time t, \bar{l} is the average loss of the sample of T observed losses.

Since when the sample T is reasonably large, $1 - \lambda^T \approx 1 - \lambda^{T-1}$, the equation (21) above can be rewritten:⁶

$$\sigma_{T+1}^2 \approx (1 - \lambda) \varepsilon_T^2 + \lambda \sigma_T^2 \quad (23)$$

Where $\lambda = 0.94$, the initial value ε_0^2 is often defined as 0, the initial volatility σ_0^2 is often defined as the average variance of the sample of T observed losses. In this way, all the volatility can be estimated.

Using VWHS method which is mentioned in the section of Methodology, the rescaled historical losses can be obtained. In this paper, the sample of test year (from

⁵Dowd(2002) p.313, eq.T10.6

⁶Dowd(2002) p.313, eq.T10.8

1st Oct 2005 to 1st Oct 2006 in the normal period and from 1st Oct 2010 to 1st Oct 2011 in the crisis period) is cut into twelve months; all the rescaled losses in every month are estimated separately. In detail, the holding period of the test sample is one month, which means that the forecasted volatility σ_{T+1} will be the first day standard deviation of every month in the test sample. For example, when estimating the volatility of October in 2005, the forecasted volatility is the first day standard deviation of October in 2005 while it should be the first day standard deviation of November in 2005 when we estimate the volatility of November in 2005.

By now, all the rescaled losses are estimated in the test sample by considering the volatility. By using the basic method of historical simulation mentioned in the Methodology section, the rescaled VaR values for both test samples in normal period and crisis period can be calculated. Not all the VaR values in this paper are present but they will be used for Kupiec test later.

4.3 Parametric simulation

Parametric method is another popular approach when estimating VaR, a distinctive feature of this method is using a suitable statistical distribution. In order to making use of this approach, what type of distribution the sample belongs to must be considered. Two possible distributions are taken into assumption: normal distribution and student t-distribution.

The equations mentioned in the Methodology section indicate that the key point in estimating VaR is selecting a distribution for the sample. Kurtosis (k) is a common measurement to decide whether a sample follows the normal distribution. If $k = 3$, it is hard to tell whether the distribution is a normal distribution; but $k \neq 3$ indicates that the distribution cannot be normal. In detail, $k > 3$ indicates a fact that the time series data has fat, heavy or long tails, which is common for economic time series data; $k < 3$ means that the tails are thinner than those for the normal distribution, and neither

normal distribution nor t-distribution will be suitable for the data. From the statistical description table 3.1 and 3.2 above, all the six Kurtosis values do not exactly equal to 3, typically in the crisis period, but it should be noticed that the statistical data above is based on a static and constant sample.

This constant sample would not be used to estimate the volatility to obtain the VaR values. The main problem with this sample to estimate the volatility is that the volatility clustering is not considered. Instead of this, a time varying volatility is applied in this paper, which means that to obtain the suitable volatility for the loss, a moving observation sample should be chosen, but the sample length is still constant. For example, in normal period, an observation sample contains 771 latest data will be chosen for an underestimated volatility; in crisis period, this sample will always contain 768 latest data. The volatility can be estimated by the same method EWMA which has been mentioned in section 4.2. Therefore, the Kurtosis values for a time varying volatility series are complex and difficult to estimate, whether the series follow the normal distribution or not cannot be told exactly. For this reason, VaR values are calculated under both normal distribution assumption and t-distribution assumption.

4.3.1 Normal distribution

Firstly, we assume that the observation sample follows the standard normal distribution. Considering volatility clustering, the VaR value for the normal distribution is easy to obtain by the formula below:

$$\text{VaR}_\alpha(L) = \mu + \sigma_{T+1}z_\alpha \quad (24)$$

Where μ represents the mean in the sample and σ_{T+1} is the volatility for the next daily loss (the latest daily loss out of sample) estimated by EWMA. Both mean and volatility keep moving, which has been discussed above. z_α denotes

the α -quantile for the standard normal distribution. α is the confidence level which equals to 99% or 95%. The VaR values are not presented here, but they will be used to test the performance of this method later.

4.3.2 t– distribution

If the distribution is not a normal distribution, it is assumed to follow the student t-distribution. Similar to the normal distribution, the VaR can be calculated by the equation:

$$\text{VaR}_\alpha(L) = \mu + \sqrt{\frac{v-2}{v}} \sigma_{T+1} t_{\alpha,v} \quad (25)$$

Where μ is the sample mean and σ_{T+1} is the volatility for the next daily loss, and both mean and volatility keep moving. $t_{\alpha,v}$ denotes the α -quantile for the t-distribution. v is a freedom parameter which is associated with Kurtosis (k). It is difficult to estimate and in general it requires an Maximum Likelihood estimation. In practice, for $v > 4$, there exists a mathematical relationship between v and k ; for $v \leq 4$, k does not exist:⁷

$$k = \frac{3(v-2)}{v-4} \rightarrow v = \frac{4k-6}{k-3} \quad (26)$$

Since the Kurtosis keeps changing due to the sample moving, v is also not constant. In this way, all the VaR which is assumed to follow t-distribution can be obtained. The confidence levels are $\alpha= 99\%$ and $\alpha= 95\%$. As in section 4.3.2 the VaR values are not presented here.

4.4 Extreme Value Theory

Extreme Value Theory (EVT) is another approach which is frequently used to

⁷Dowd(2002) p.82

estimate portfolio VaR, it examines the tail of the loss distribution based on the expectation that the loss exceeds VaR. The key point is to estimate based on extreme outcomes rather than all outcomes. Generally, a suitable large loss in the sample will be chosen in the subsequent analysis.

Due to the limitation of the traditional model in estimation, in this paper, a preferred method named Peaks Over Threshold (POT) model is applied. The mathematic method has been discussed briefly in the section of Methodology. An important definition is mentioned here—Threshold value. In the sense of the extreme value theory, all the losses in the sample which are larger than the threshold value are defined as “extreme value”. Since a specific threshold value will be estimated by using all the losses in the sample, the problem that more than one large losses exist in one sample will be solved, all the losses which are larger than the threshold value will be selected when estimating the parameters β and ξ .

The POT model measures the risk based on conditional and unconditional loss distributions. The differences between conditional and unconditional POT is that, in unconditional POT model, the losses in the sample are un-rescaled and remain as their actual value; in conditional POT model, the losses are rescaled, the volatility clustering is considered and estimated by GARCH or EWMA model (EWMA model is applied here), so the current market conditions are taken into account and the model becomes more “dynamic”. Meanwhile, when estimating VaR, the α -quantile of the GPD distribution of the standardized residuals q_α is used. The aspects below in this section will discuss the estimation and calculation of parameters and VaR values.

4.4.1 Selection of threshold value and estimation of parameter β and ξ

As discussed above, the threshold value means a lot to the POT model. The threshold value is difficult to define clearly and simply in the sample. In some previous literatures, such as Wang, Wu, Chen and Zhou (2010), the threshold value

was estimated by MEF and Hill plot. These two methods are discussed precisely in Beirlant J., Teugels J. & Vynckier P.’s Practical Analysis of extreme values (1996) and David Ruppert’s Statistics and Finance: an Introduction (2004) respectively. In this paper, an empirical constant percentage of the largest losses is selected. We rank the sample losses of each exchange rate and assume that 4% of the observations will be above the threshold value. 4% is considered quite well since it is not even low or high. It is a better trade-off since enough excess losses can be obtained but not too much to make reliable estimates. There are 771 sample observations in the normal period and 768 sample observations in the crisis period, so there will be 30 losses which are larger than the threshold value. The threshold values of the three exchange rates in two periods can be seen in table 4.3 and table 4.4 below:

Table 4.3 Threshold and number of excesses for exchange rate of EUR in the normal period

	USD/EUR		JPY/EUR		GBP/EUR	
	Uncond	Cond	Uncond	Cond	Uncond	Cond
Number of exceedance (4%)	30	30	30	30	30	30
Threshold(u)	1.0497	1.8733	1.0166	1.9993	0.6536	1.8236

Table 4.4 Threshold and number of excesses for exchange rate of EUR in the crisis period

	USD/EUR		JPY/EUR		GBP/EUR	
	Uncond	Cond	Uncond	Cond	Uncond	Cond
Number of exceedance (4%)	30	30	30	30	30	30
Threshold(u)	1.3521	1.8442	1.9466	1.9013	1.1793	1.7896

The percentage is constant, so the number of exceedance is also constant with the number of 30. The threshold value of the same exchange rate in unconditional and conditional situation is different, because the value in conditional situation has been rescaled. All the 30 losses which are larger than the threshold value in the sample are called “extreme value”.

The parameters β and ξ can be estimated by maximum-likelihood method which has been discussed in the section of Methodology after determining the threshold u . β denotes a scale parameter and ξ is the shape or tail parameter. There are two different situations when $\xi = 0$ or $\xi \neq 0$. The situation when $\xi > 0$ corresponding to the fat-tail return distribution seems to be more noticed by previous literatures, since it is common in financial return distribution, which indicates the distribution is a Frechet distribution; The situation when $\xi < 0$ indicates that the return distribution has thinner tail than normal distribution, it is unusual in financial data and the distribution is a Weibull distribution; $\xi = 0$ means that the distribution is a Gumbel distribution. These two different situations will both be discussed in this paper. See the results of parameter estimation in table 4.5 and 4.6 below:

Table 4.5 Results of parameters estimation in the normal period

USD/EUR				
	Uncond		Cond	
N/N _u	23.87		23.87	
β	0.280	0.183	0.502	0.637
ξ	0	0.393	0	-0.246*
ML	8.238	9.122	-9.337	-9.067
JPY/EUR				
	Uncond		Cond	
N/N _u	23.87		23.87	
β	0.288	0.363	0.454	0.532
ξ	0	-0.246*	0	-0.172*
ML	7.358	7.824	-6.299	-5.910
GBP/EUR				
	Uncond		Cond	
N/N _u	23.87		23.87	
β	0.144	0.177	0.399	0.391
ξ	0	-0.228*	0	0.019
ML	28.225	28.773	-2.420**	-2.413**

Note: The value with * represents a negative value which is different from the positive value in ξ estimation, while ** means that the two values in the same case (conditional or unconditional) are quite similar to each other, for example $2.420 \approx -2.413$. N denotes the total number of observations while N_u is the number of the observations exceeding the threshold value u.

Table 4.6 Results of parameters estimation in the crisis period

USD/EUR				
	Uncond		Cond	
N/N _u	23.87		23.87	
β	0.628	0.644	0.648	0.643
ξ	0	-0.027*	0	0.008
ML	-16.030**	-16.010**	-16.975**	-16.973**
JPY/EUR				
	Uncond		Cond	
N/N _u	23.87		23.87	
β	0.793	0.787	0.740	0.996
ξ	0	0.007	0	-0.331*
ML	-23.025**	-23.024**	-20.970	-19.952
GBP/EUR				
	Uncond		Cond	
N/N _u	23.87		23.87	
β	0.383	0.380	0.452	0.443
ξ	0	0.006	0	0.020
ML	-1.169**	-1.169**	-6.179**	-6.173**

Note: The value with * represents a negative value which is different from the positive value in ξ estimation, while ** means that the two values in the same case (conditional or unconditional) are quite similar to each other, for example $-23.025 \approx -23.024$. N denotes the total number of observations while N_u is the number of the observations exceeding the threshold value u.

From the results above, we can see that in some solutions, ξ involves a negative value, such as in conditional POT model of USD/EUR in normal period with $\xi = -0.246$, the negative values indicate thinner tail in return distribution which is uncommon in financial data, and it is statistically insignificant. Meanwhile, some maximum-likelihood values have small differences. For example, in condition POT model of JPY/EUR in crisis period, the maximum-likelihood value is -23.025 when $\xi = 0$ and it is -23.024 when $\xi = 0.007$. This simply means that ξ is not statistically different from zero, in onther words the distribution follows the Gumbel distribution. It is also easy to tell that the optimal ξ is very close to zero and the maximium–likelihood value does not increase very much compared to the model when $\xi = 0$, generally to say, the shape parameter ξ dose not improve the model very much. This analysis is equally applicable in other cases with the same situation.

4.4.2 Calculation of VaR

After the threshold value u , the parameters β and ξ have been estimated, the VaR value can be easily computed by using the equation (11) and (12) for unconditional model and equation (13) for conditional model. In this paper, all the values of VaR for each exchange rate of EUR in both normal and crisis period with the confidence level of 95% and 99% are estimated. Table 4.7 and 4.8 below show the value of VaR for unconditional POT model, since in unconditional POT model, the value of VaR is constant for all the losses in the test period.

Table 4.7 VaR based on unconditional POT model for each exchange rate in normal period

Confidence level	USD/EUR		JPY/EUR		GBP/EUR	
	$\xi = 0$	$\xi \neq 0$	$\xi = 0$	$\xi \neq 0$	$\xi = 0$	$\xi \neq 0$
95%	1.000	1.018	0.966	0.951	0.628	0.622
99%	1.450	1.402	1.429	1.454	0.859	0.870

Table 4.8 VaR based on unconditional POT model for each exchange rate in crisis period

Confidence level	USD/EUR		JPY/EUR		GBP/EUR	
	$\xi = 0$	$\xi \neq 0$	$\xi = 0$	$\xi \neq 0$	$\xi = 0$	$\xi \neq 0$
95%	1.241	1.238	1.806	1.807	1.112	1.112
99%	2.251	2.258	3.082	3.080	1.727	1.726

The unconditional POT model results show that, in the normal period, the losses of USD/EUR, JPY/EUR and GBP/EUR in the test sample will be less than 1.000, 0.966 and 0.628 when $\xi = 0$ and less than 1.018, 0.951 and 0.622 when $\xi \neq 0$ with a probability of 95%, this analysis is equally applicable in the confidence level of 99%. The situation is the same in the crisis period. The conditional POT model results cannot be presented in forms here, but we will interpret its performance in sections below and analyze it in the Kupiec test. Meanwhile, it is obvious to see that, in normal period, both the losses and the risk of USD/EUR and JPY/EUR are larger than those of GBP/EUR; in crisis period, the losses and the risk of JPY/EUR are larger than those of USD/EUR and GBP/EUR. Overall, the risk in crisis period is quite larger than that in normal period, it conforms the objective economic situations.

This information is helpful to commercial banks, enterprises and individual investors to make risk decisions.

4.5 Interpreting

The figures 4.1 to 4.3 represent the fluctuation of the different exchange rate daily loss, which might suggest that the daily loss for all the three exchange rate fluctuates rapidly. So the volatility clustering is required to be added in the estimating models.

In the method of VWHS, EWMA (exponentially weighted average volatility model) has been applied to forecast the volatility of the test period in order to rescale

the exchange rate loss, in this way, volatility clustering has been considered into this model. For parametric models which include normal distribution and t-distribution models, the volatility clustering has been also considered by using the same method EWMA.

EVT model is more complex relatively than other models applied in this paper. Both conditional and unconditional situations have been considered. A constant percentage threshold value which is 4% has been selected when estimating parameters β and ξ empirically. From Table 4.5 and 4.6, it is obvious to see that in some cases when $\xi \neq 0$, the value of ξ is quite closed to 0, such as 0.008 and 0.007 in Table 4.6. For these cases, the different ML (maximum likelihood) values between $\xi = 0$ or $\xi \neq 0$ are quite similar which indicates that ξ does not improve the POT model.

After processing the three different models which are commonly used in VaR estimation, the forecasted VaR values for the three exchange rates are calculated and have been shown graphically from Figure 4.4 to 4.9 in Appendix. It is hard to conclude which model simulates better from the performance in the figures. Therefore the assessment of different models and specific decisions should be achieved by applying the Kupeic test in the next section.

5. Backtesting

Kupiec frequency method is applied to test the validity of several VaR models chosen here. As known, Kupiec test is a common and practical method when estimating the performance of a VaR model, which is reported by Kupiec (1999). As discussed in section 3, the binomial version of the Kupiec test has been expanded mathematically. The key idea of this method is to calculate the probability that the observed violations X equal to or larger than the actual number of violations x ($\Pr(X \geq x)$) by assuming that the actual violations x equal to or larger than the expected number of VaR_α violations $(1 - \alpha)N$ ($x \geq (1 - \alpha)N$) where α is the confidence level for typical VaR and N is the number of observations in the test period. If the probability is less than the statistical significance level, the underlying model will be rejected. For example, within a confidence level of 95% for the Kupiec test, a standard level for statistical test is 5% (1-95%), if the probability calculated is less than 5%, the underlying model will be rejected. Alternatively, if the required statistical significance level has been decided, a critical value of observed violations X under the significance level is also easy to be calculated. By this way, an acceptance region for Kupiec test can be obtained, for those values which are out of the region will be rejected.

In this paper, the required confidence level for Kupiec test has been decided as 95% and 99%. So an acceptance region which is suitable for our requirements under different significant levels can be obtained (see the table 5.1). Additionally, one-sided test is applied in this paper.

Table 5.1 The acceptance region of Kupiec test

Confidence level	T=256	T=259
99%	$N < 7$	$N < 7$
95%	$6 < N < 21$	$6 < N < 21$

The value of T indicates the number of data in each test period, which means that the number of data in the test part of normal period is 256 while the number of data in the crisis period is 259. This paper applies the one sided test under the confidence level of 95% and 99%. So for the method that the failure number N (number of days that the real losses exceed the VaR value) falls into the acceptance region of Kupiec test indicates a good performance in simulation. Briefly, the underlying method simulates well and vice versa. In this way the different performances of the five selected methods can be compared in normal and crisis period.

5.1 Kupiec test results description

In this paper, we test five VaR estimation methods which are VWHS, parametric methods containing normal distribution and t distribution, EVT method containing unconditional and conditional aspects under the confidence level of 95% and 99%. The backtesting sample is the exchange rate of USD/EUR, GBP/EUR, and JPY/EUR from 1st Oct 2005 to 1st Oct 2006 in the normal period and from 1st Oct 2010 to 1st Oct 2011 in the crisis period. The backtesting results are shown in table 5.2 and table 5.3 below. The number in the table refers to the number of days which real losses exceed the daily VaR.

Table 5.2 Backtesting results in the normal period

		Non-parametric	Parametric		Semi-parametric			
					EVT			
					Unconditional		Conditional	
		VWHS	N-dist	t-dist	$\xi = 0$	$\xi \neq 0$	$\xi = 0$	$\xi \neq 0$
USD/EUR	0.99	0	0	0	0	0	0	0
	0.95	6*	9	9	2*	2*	8	8
JPY/EUR	0.99	4	7*	6	4	4	5	4
	0.95	16	17	18	8	8	12	12
GBP/EUR	0.99	5	4	3	0	0	3	3
	0.95	18	13	13	3*	4*	12	12

Note: The value with * represents that failure number of that case falls out of the acceptance region of Kupiec test in table 5.1

Table 5.2 presents the results of backtesting for the three exchange rates USD/EUR, JPY/EUR and GBP/EUR in the normal period by using three kinds of method—VWHS, Parametric and EVT under the confidence level 95% and 99%. Firstly, from the vertical results of the different methods, it is obvious to see that there are several cases that VaR values based on VWHS, normal distribution and unconditional POT model underestimate the exchange risk for all the three exchange rates. However, the results based on student t-distribution and conditional POT model seem to be good, they all pass the Kupiec test under the confidence level of 95% and 99%. It is hard to say whether ξ equals to zero or not influences the accuracy of the model very much. Secondly, from the perspective of exchange rate, there are three cases of USD/EUR underestimating the exchange risk, while there are one case of JPY/EUR and two cases of GBP/EUR showing the same situation. It is also hard to say which exchange rate risk is estimated better by using all the methods applied here. Among them, USD/EUR performs the worst relatively. Thirdly, from the perspective of confidence level, it seems that the higher the confidence level is the more accurate the model will be. Only one case of JPY/EUR under 99% confidence level underestimates the risk while 5 cases performance bad under 95% confidence level.

Overall, the performance of all the three exchange rates for all the methods in the normal period seem not far from satisfactory, while there are 6 cases underestimating the exchange risk.

Table 5.3 Backtesting results in crisis period

		Non-parametric	Parametric		Semi-parametric			
					EVT			
					Unconditional		Conditional	
		VWHS	N-dist	t-dist	$\xi = 0$	$\xi \neq 0$	$\xi = 0$	$\xi \neq 0$
USD/EUR	0.99	2	3	2	0	0	1	1
	0.95	18	16	18	14	14	15	15
JPY/EUR	0.99	4	3	2	0	0	1	1
	0.95	19	17	18	3*	3*	13	14
GBP/EUR	0.99	5	8*	5	1	1	6	6
	0.95	13	12	12	6*	6*	12	12

Note: The value with * represents that failure number of that case falls out of the acceptance region of Kupiec test in table 5.1

Table 5.3 presents the results of backing test for the three exchange rates USD/EUR, JPY/EUR and GBP/EUR in the crisis period by using three kinds of method—VWHS, Parametric and EVT model under the confidence level 95% and 99%. Firstly, from the perspective of methods, the situation seems to be similar with that in the normal period except VWHS. All the cases in three methods such as VWHS, student t-distribution and conditional POT model estimate the exchange risk well. Secondly, from the perspective of different exchange rates, the performance of USD/EUR seems to be better than that of other two exchange rates, all the cases of USD/EUR pass the test. Thirdly, from the perspective of confidence level, we obtain the same conclusion as that in the normal period, the higher the confidence level is the better the performance will be. Since there is one case under the confidence level of 99% underestimating the risk while there are 4 cases performing worse under 95% confidence level. Overall, there are 5 cases in the crisis period underestimating the

exchange rate risk which seems to be a little better than in the normal period. But we still cannot say that the methods what we select perform better when the economic situation goes into a “mire”.

5.2 Interpreting

From the discussions based on the results which are show in table 5.2 and table 5.3, conclusions from four aspects can be made.

Firstly, comparing the different performances of all the methods selected in the different periods, the results seem not so obvious. We cannot tell which method performs better either in the normal period or in the crisis period, since the overall results in table 5.2 and table 5.3 are similar and complex. This result is out of the previous expectation.

Secondly, by comparing all the results of different methods, it is obvious to see that the VaR values of the conditional POT model and the moving student t-distribution model all pass the Kupiec test both in the normal period and in the crisis period, these two methods seem better than other methods mentioned in our paper. This conclusion has overlap with Wang, Wu, Chen, Zhou (2009)’s article which proves EVT can measure the risk more accurate for the exchange rate of JPY/CNY and EUR/CNY. VWHS only performs well in the crisis period, so the result is relatively hard to analysis. The unconditional POT model is the worst choice in this paper either in the normal period or in the crisis period.

Thirdly, from the perspective of the two different confidence level 95% and 99%, all the results in the two tables show that a higher confidence level is helpful to improve the accuracy and decrease the probability of the method to underestimate the risk. Since only two cases in the two tables under 99% confidence level underestimate the risk while 9 cases exist under 95% confidence level.

Fourthly, the methods' performance of different exchange rate should be analyzed separately. For USD/EUR, the performances of all the methods obtain similar results under a higher confidence level of 99%. No matter in the normal period or in the crisis period, they all pass the Kupiec test and show no difference. But the situation changes when the confidence level decreases. The performances in the crisis period remain good while the results in the normal period are not satisfactory. For JPY/EUR, the results are complex and it is hard to tell how the confidence level, economic situation and different methods influence the performance. The conditional POT model and moving student t-distribution are relatively better choice when estimating the exchange risk. For GBP/EUR, under a higher confidence level 99%, only the moving normal distribution in the crisis period fails to pass the test while other methods perform well no matter what economic situations are. Under a lower confidence level 95%, only unconditional POT model results are unsatisfactory.

Additionally, no differences are found when ξ equals to zero or not, therefore it indicates that a nonzero ξ does not improve the model. Also from the results of parameter estimation before the POT model, most of the estimated ξ are very close to zero, which indicates that the parameter ξ dose not influence the model very much.

Consequently, the performances of the risk measurement methods selected in this paper for different exchange rate are complex, no matter what kind of economic situation there exists—normal or crisis, it is hard to make a conclusion in one word. However, for the three exchange rate of EUR, the moving student t-distribution and conditional POT model are relatively better choices if the investors or companies want to decrease the probability to underestimate the risk. The financial crisis makes the situations more complex since it will increase the volatility of the exchange rate and the risk. It is better to estimate the risk by using several different kinds of methods and make a comparison when the economic environment goes badly.

Comparing to those previous literatures mentioned in section 2, this paper focuses on the exchange rate risk which is one of the three main markets that the VaR methods are widely used in, whose idea is inspired by Angelidis and Degiannakis (2005). Another motivation is exchange market has been discussed less than stock market relatively. None of the literatures mentioned here such as Wang, Wu, Chen, Zhou (2009) and Mazin A. M. and Al Janabi (2006) focus on euro typically at the background of economic crisis. The results obtained in this paper is not as obvious as those in the paper reported by Wang, Wu, Chen, Zhou (2009), however, a similar result that the EVT model is relatively better when estimating exchange risk is obtained. Meanwhile, it is expected that the results in this paper may be changed by applying some adjusted models mentioned in the literature reported by Mazin A. M. and Al Janabi (2006).

There are still several limitations in our paper which may lead to existing bias of our results. Reasons are shown below: Firstly, the sample size of our data is not enough. There are almost 800 observations in the sample both for the normal period data and crisis period data, and less than 300 observations in the test period. It is not big enough to guarantee the accuracy of VaR. The results may change when the size increases. Secondly, the selection of the threshold value must be considered. In this paper, a fixed percentage of 4% has been determined since the threshold selection is a complex and full of controversy work, if the method to select the threshold is improved, different results may be obtained. Thirdly, the higher the confidence level is the better the EVT model will be. A confidence level which is higher than 99% may improve the performance of the POT model.

6. Conclusion

This paper aims to measure the exchange rate risk of euro by using different kinds of methods and compare their performances at different economic situations typically at the background of world financial crisis and European debt crisis.

In this paper, three main methods based on VaR which are non-parametric method including Volatility Weighted Historical Simulation (VWHS), parametric method including moving normal distribution and moving student t-distribution, Extreme Value Theory including unconditional and conditional POT models are adopted to measure the exchange rate risk of euro. Three exchange rates applied here are USD/EUR, JPY/EUR and GBP/EUR at the background of normal period and crisis period, in order to compare the different performances of these selected methods typically in different economic situations. These three exchange rates are selected due to their free volatility to euro, important influence to euro exchange market and some other typical features. Two different periods are selected and both are cut into two samples—observation sample and test sample. Each period contains 1027 numbers. In normal period, the number of observation data is 771 and the number of test data is 256. In the crisis period, the number of observation data is 768 and the number of test data is 259.

By applying the Kupiec test, the performances of several methods can be measured. From the results obtained, conclusions can be given from four aspects. Firstly, by comparing the results of the five methods, it is easy to find that the conditional POT model and moving student t-distribution simulate the VaR for all exchange rates much better than any other three methods, no matter what economic situations are. Secondly, we cannot find any obvious results to summarize the different performances under different economic situations; the result is complex whether the region economy goes well or bad. Thirdly, the results in this paper give

evidence again that the higher confidence level will increase the accuracy of VaR measurement models, typically for EVT model. In this paper, whether ξ equals to zero has little influence on the estimation of VaR in EVT models. Fourthly, from the perspective of exchange rate, all the methods perform well for USD/EUR under a higher confidence level but unsatisfactory under lower confidence level. Meanwhile, a complex result for the exchange rate of JPY/EUR and GBP/EUR exists. Although the more obvious results are not obtained like that in the paper reported by Wang, Wu, Chen, Zhou (2009), a similar conclusion can be made that the EVT model is relatively better than others. Meanwhile, exchange rate risk estimation process is quite complex and multiple use of different kinds of methods are appreciated.

Additionally, these results may be led by some limitations which exist in our data selection and methods application. For instance, the sample size may be not big enough since larger sample will be appreciated in any estimations. A higher confidence level also should be considered since it can increase the accuracy of the models. In the sample selection, it is hard to make an indeed definition that which period is normal or in crisis, even when the crisis began. These limitations may influence our test and results very much and the results may change when these limitations are improved. Inspired by Mazin A. M. and Al Janabi's research (2006), one constructive VaR which takes adjustments for other factors into account may improve the accuracy to measure the risk.

So far, specific suggestions should be given to those individual investors, enterprises or governments who pay attention to the world exchange market. A combination of several methods to estimate exchange risk will be an appreciated choice before making a decision for the exchange rate portfolio. A higher confidence level will help to increase the accuracy of the models. Finally, volatility clustering need to be under consideration, a moving volatility model may give more satisfactory results.

References

Acerbi, C. & Tasche D. (2001). Expected Shortfall: a Natural Coherent Alternative to Value at Risk. *Economic Notes*, Vol. 31 Issue 2 (2002), 379-388.

Angelidis, T. & Degiannakis, S. (2005). Modeling Risk for Long and Short Trading Positions. *Journal of Risk Finance*, Vol. 6 No. 3, 2005, 226-238.

Angelidis, T., Benos, A. & Degiannakis, S. (2004). The Use of GARCH Models in VaR Estimation. *Statistical Methodology*, no. 1, 105-128.

Artzner, P., Delbaen, F., Eber, J.M. and Heath, D. (1997). Thinking Coherently. *Risk*, 10, 1997, 68–71.

Balkema, A., & Laurens de Haan (1974). Residual Life Time at Great Age, *Annals of Probability*, 2, 792–804.

Bams, D., Lehnert, T. & Wolff, C. (2005). An Evaluation Framework for Alternative VaR-models. *Journal of International Money and Finance*, 24 (2005) 944-958.

Beirlant, J., Teugels, J. & Vynckier, P. (1996). *Practical Analysis of Extreme Values*. Leuven University Press, Leuven.

David Ruppert (2004). *Statistics and Finance: an Introduction*. Springer Press, New York, ISBN 0-387-20270-6.

European Central Bank (2012). Euro Foreign Exchange Reference Rates. Available from <http://www.ecb.int/stats/exchange/eurofxref/html/index.en.html> [Accessed 1 March 2012]

Hogg, R. & Klugman, S. (1984). *Loss Distributions*. Wiley, New York

J.P. Morgan (1996). *Risk Metrics Technical Document*. New York: Morgan Guaranty Trust Company.

Kevin, Dowd (2002). *Measuring Market Risk*. Chichester: John Wiley & Sons Ltd, ISBN 0-471-52174-4.

Kevin, Dowd (1999). Back Testing Market Risk Models. *ICBI conference*, Geneva.

Kupiec, P. H. (1995). Techniques for Verifying the Accuracy of Risk Measurement Methods. *Journal of Derivatives*, 3: 73–8

Kupiec, P. H. (1999). Stress Testing in a Value at Risk Framework. *Journal of Derivatives*, 6: 7–24.

Lauridsen, S. (2000). Estimation of Value at Risk by Extreme Value Methods. *Extremes* 3: 107 - 144.

Mandelbrot, B. (1963). The Variation of Certain Speculative Prices. *The Journal of Business*, Volume 36, Issue 4, 394-419.

Mazin, A. M. and Al Janabi (2006). Foreign-exchange Trading Risk Management with Value at Risk: Case Analysis of Moroccan Market. *The Journal of Risk Finance*, Vol. 7, No.3, 2006, 273-291.

Pickands, J. (1975). Statistical Inference Using Extreme Order Statistics, *Annals of Statistics*, 3, 119–131.

Pritsker, M. (1997). Evaluating Value at Risk Methodologies: Accuracy versus Computational Time. *Journal of Financial Services Research*, Vol. 12, 201-42.

Wang, Z., Wu, W., Chen, C., Zhou, Y. (2010). The Exchange Rate Risk of Chinese Yuan: Using VaR and ES based on Extreme Value Theory. *Journal of Applied Statistics*, Vol. 37, No. 2, February 2010, 265-282.

Zhan, Y. & Tian, H. (2000). The Application of Extreme Value Theory on Computing Value-at-Risk of Exchange Rates. *Journal of System Engineering*, Eng. 15, 44-53.

Appendix

1. Figures of exchange rate risk under 99% confidence level

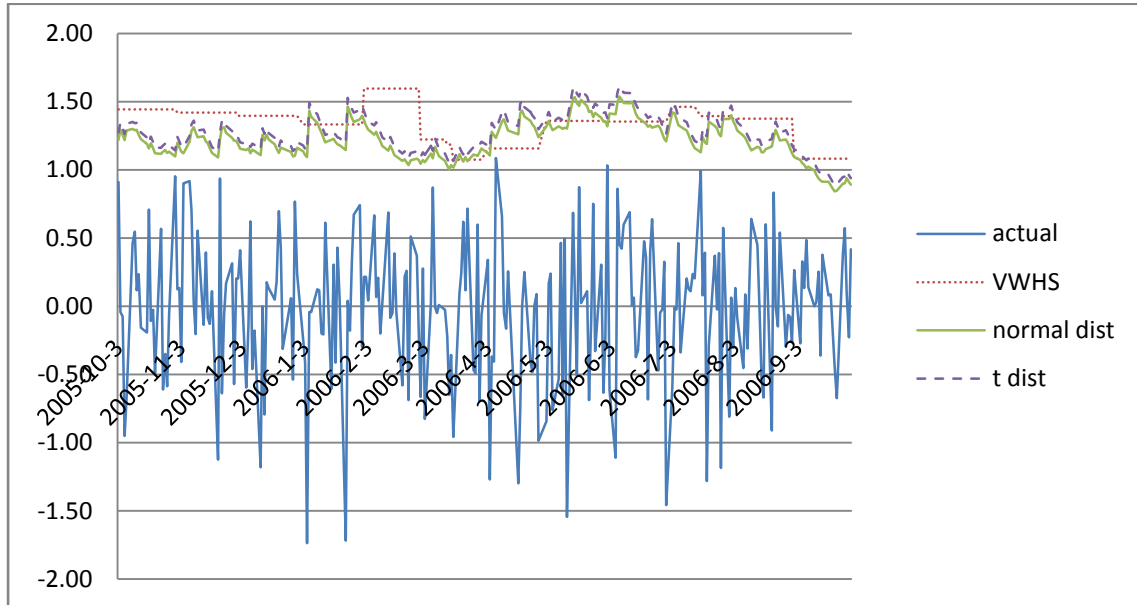


Figure 4.4.a Non-parametric and parametric VaR of USD/EUR for the test sample in the normal period with 99% confidence level

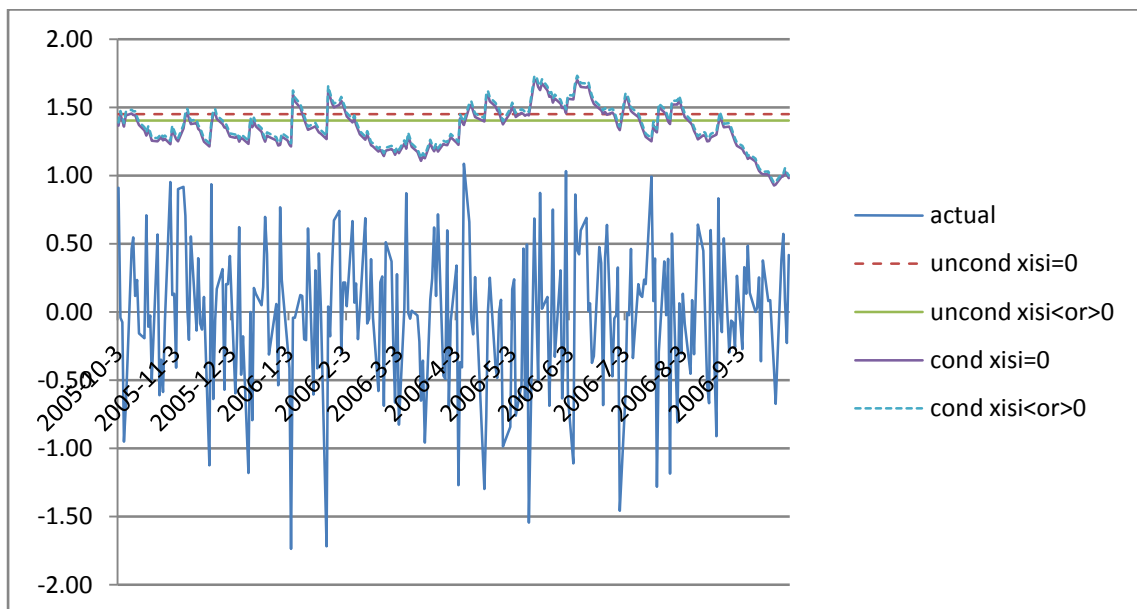


Figure 4.4.b Extreme value theory VaR of USD/EUR for the test sample in the normal period with 99% confidence level

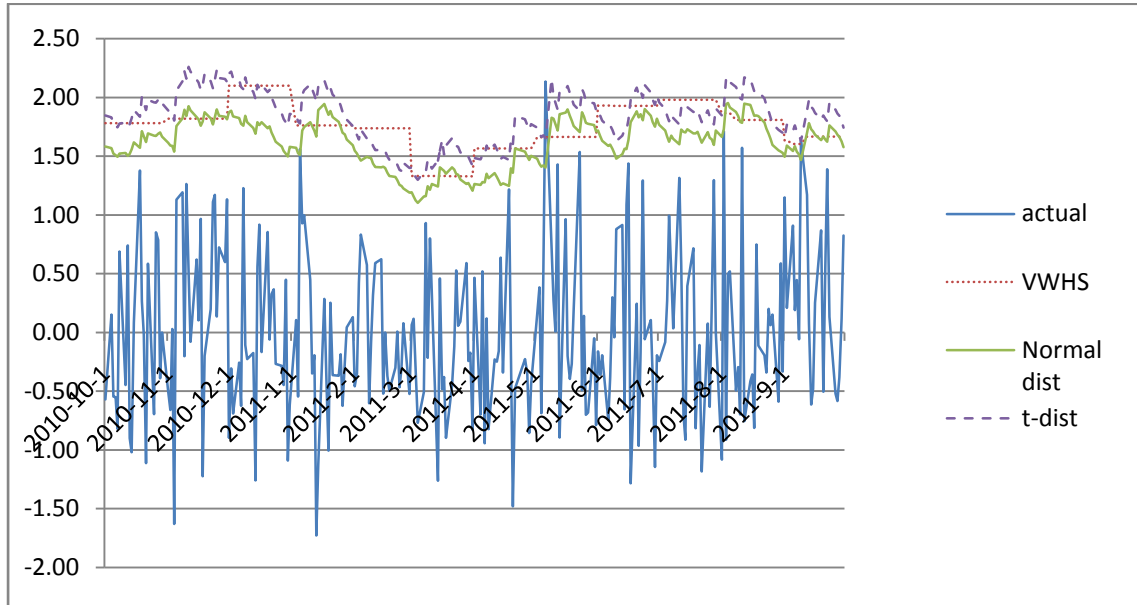


Figure 4.4.c Non-parametric and parametric VaR of USD/EUR for the test sample in the crisis period with 99% confidence level

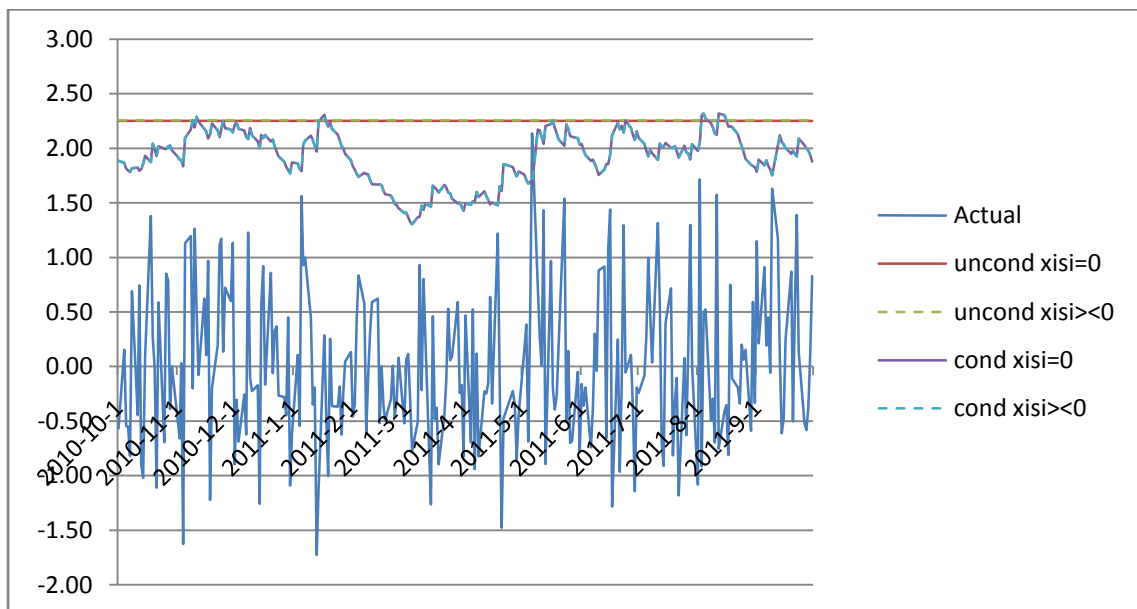


Figure 4.4.d Extreme value theory VaR of USD/EUR for the test sample in the crisis period with 99% confidence level

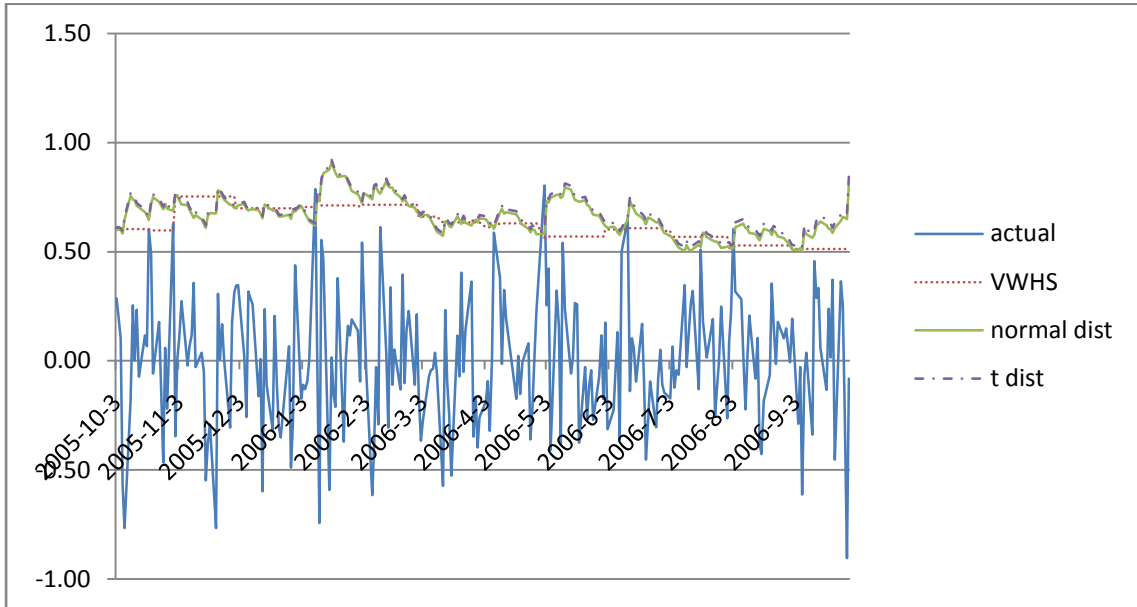


Figure 4.5.a Non-parametric and parametric VaR of GBP/EUR for the test sample in the normal period with 99% confidence level

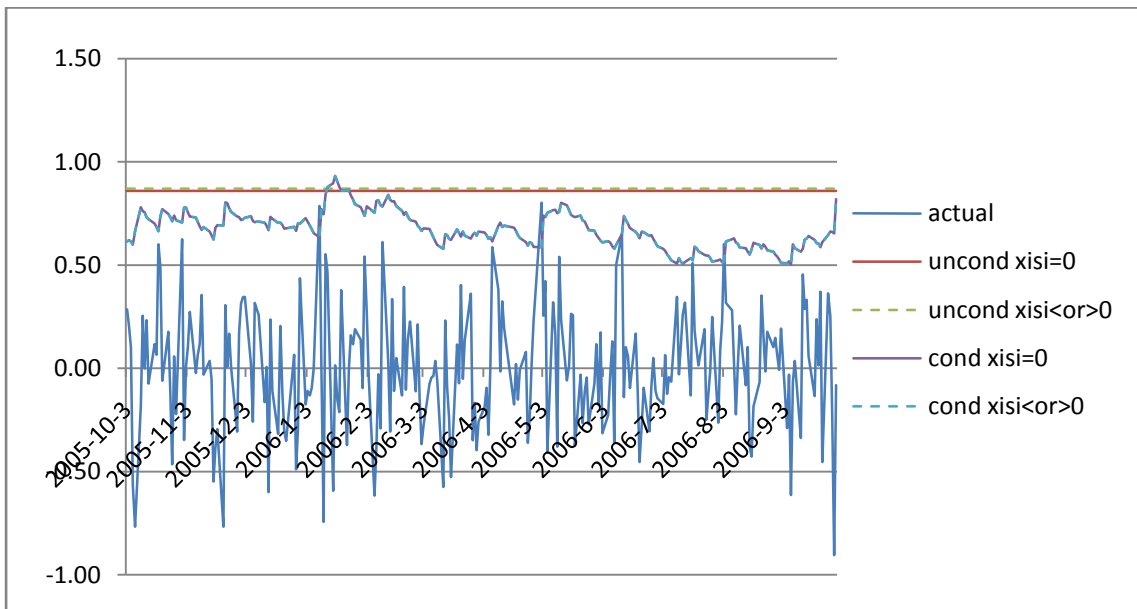


Figure 4.5.b Extreme value theory VaR of GBP/EUR for the test sample in the normal period with 99% confidence level

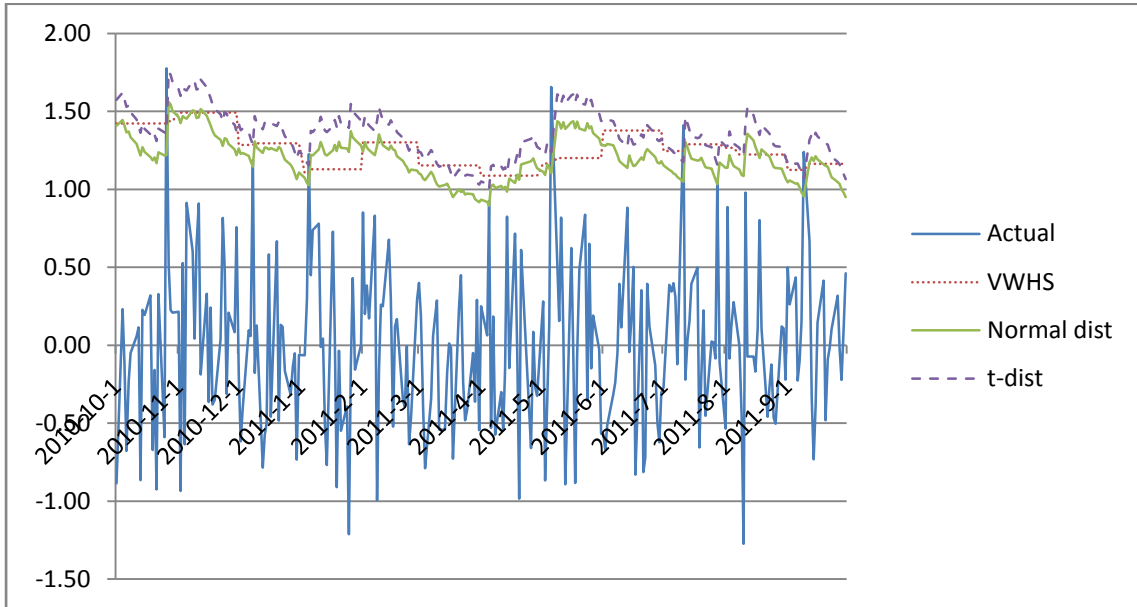


Figure 4.5.c Non-parametric and parametric VaR of GBP/EUR for the test sample in the crisis period with 99% confidence level

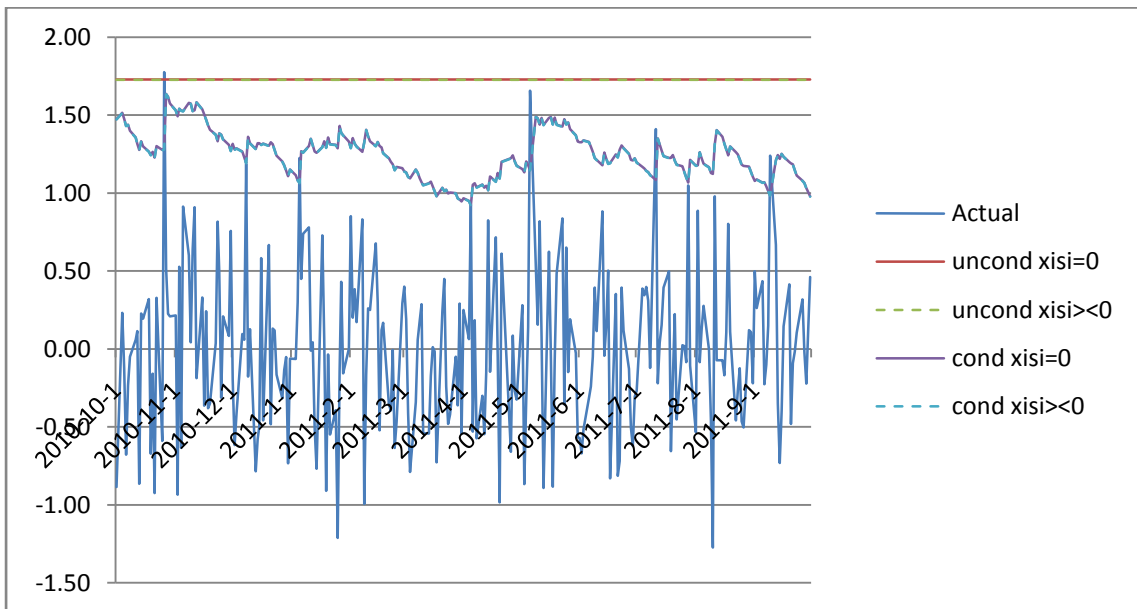


Figure 4.5.d Extreme value theory VaR of GBP/EUR for the test sample in the crisis period with 99% confidence level

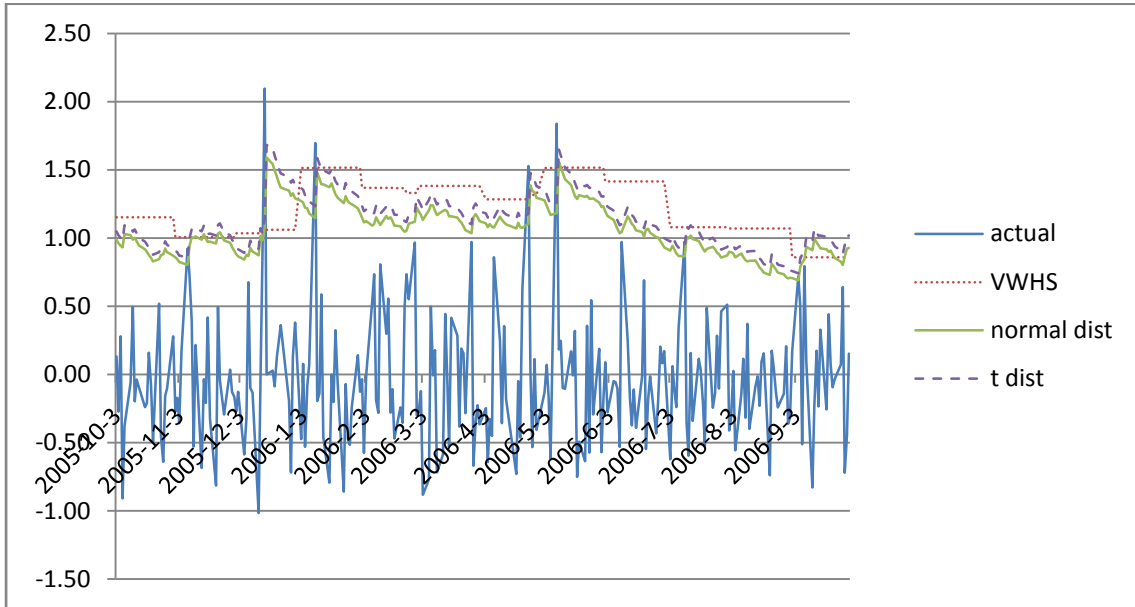


Figure 4.6.a Non-parametric and parametric VaR of JPY/EUR for the test sample in the normal period with 99% confidence level

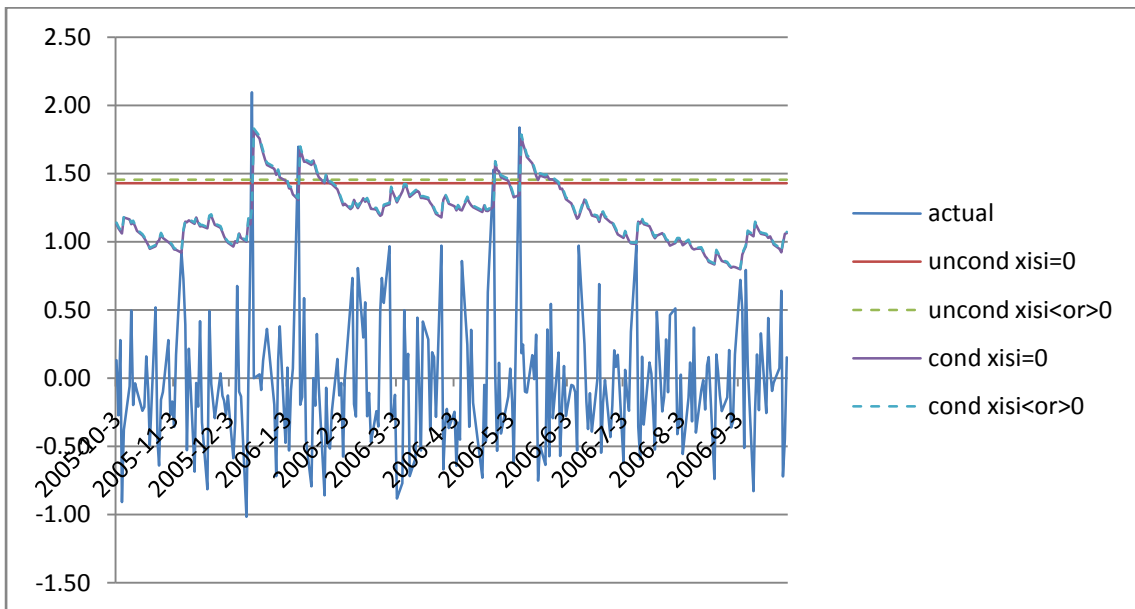


Figure 4.6.b Extreme value theory VaR of JPY/EUR for the test sample in the normal period with 99% confidence level

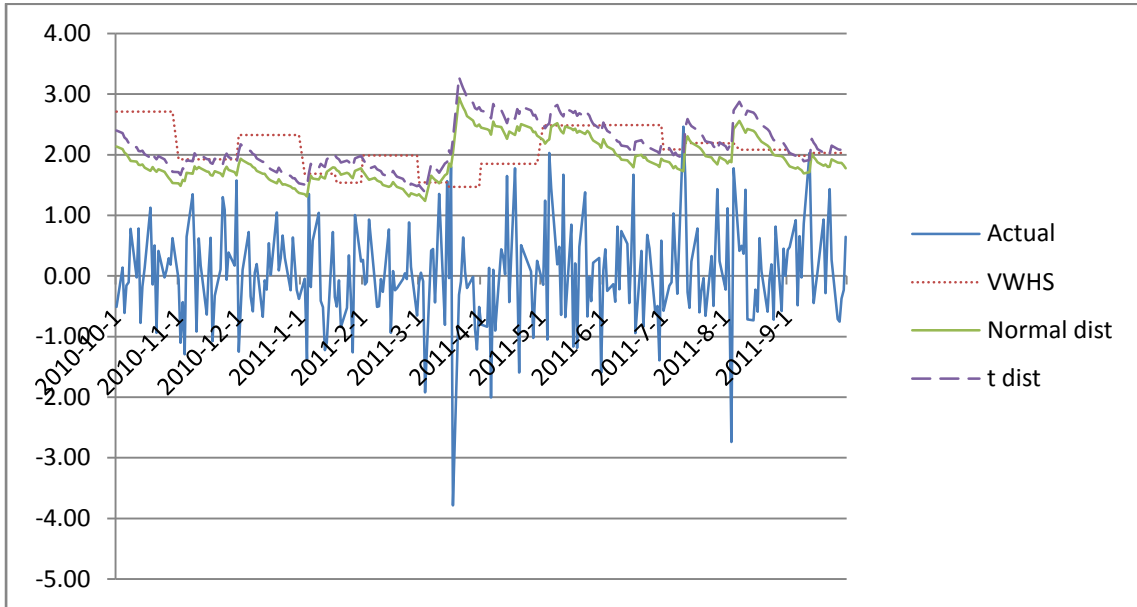


Figure 4.6.c Non-parametric and parametric VaR of JPY/EUR for the testsample in the crisis period with 99% confidence level

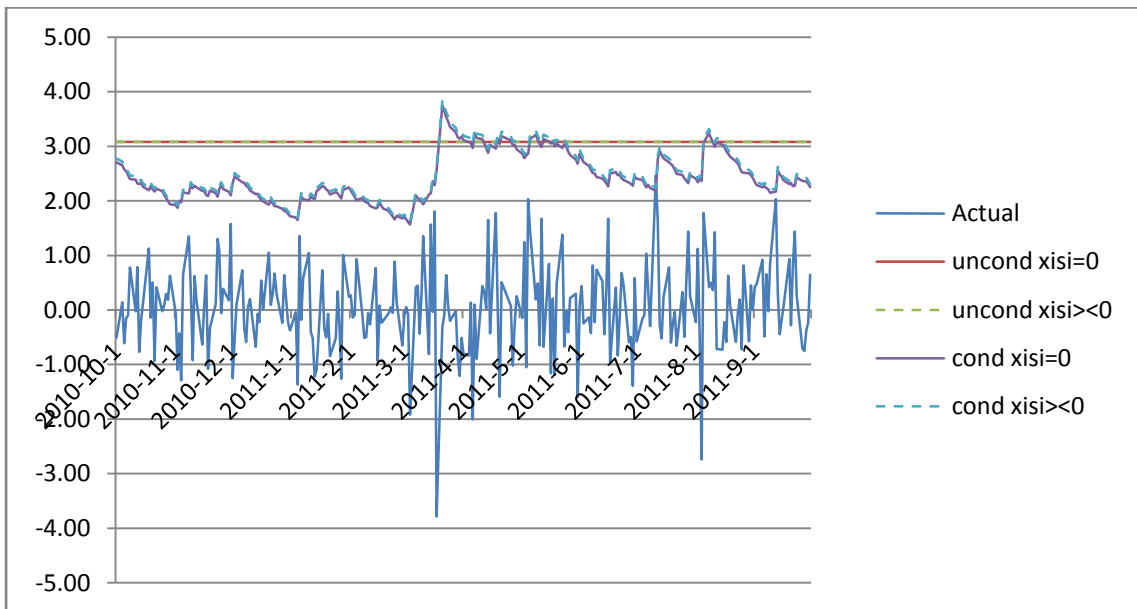


Figure 4.6.d Extreme value theory VaR of JPY/EUR for the test sample in the crisis period with 99% confidence level

2. Figures of exchange rate risk under 95% confidence level

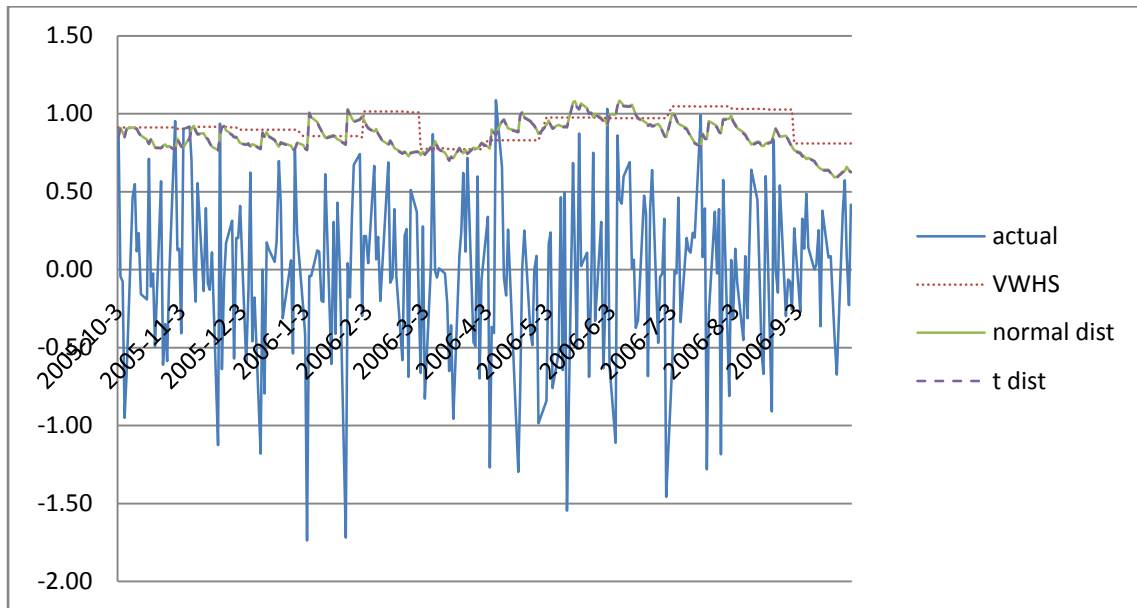


Figure 4.7.a Non-parametric and parametric VaR of USD/EUR for the test sample in the normal period with 95% confidence level

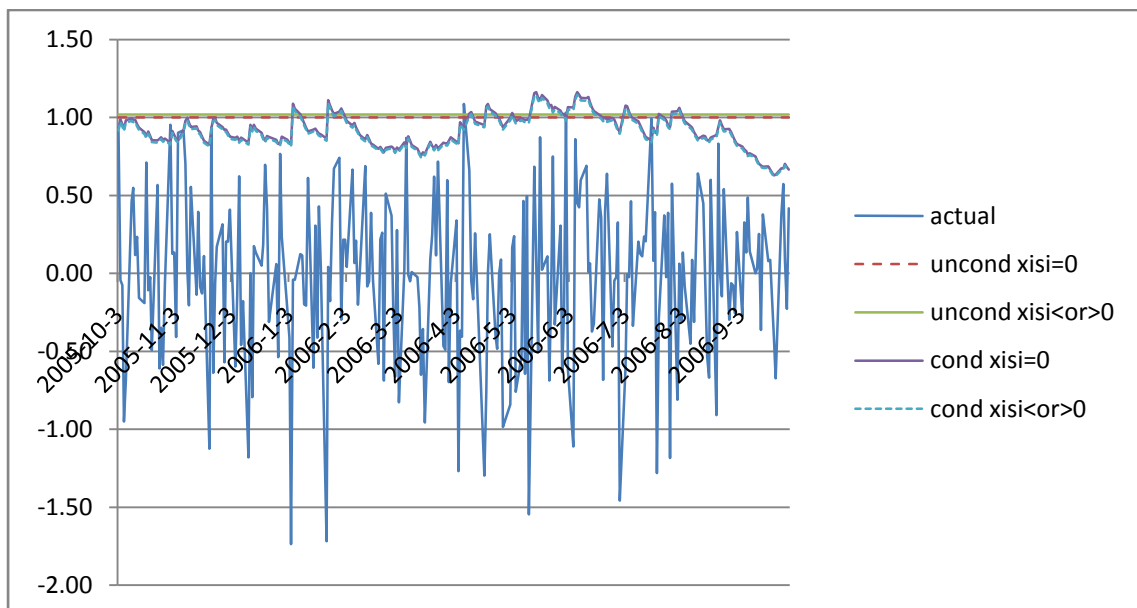


Figure 4.7.b Extreme value theory VaR of USD/EUR for the test sample in the normal period with 95% confidence level

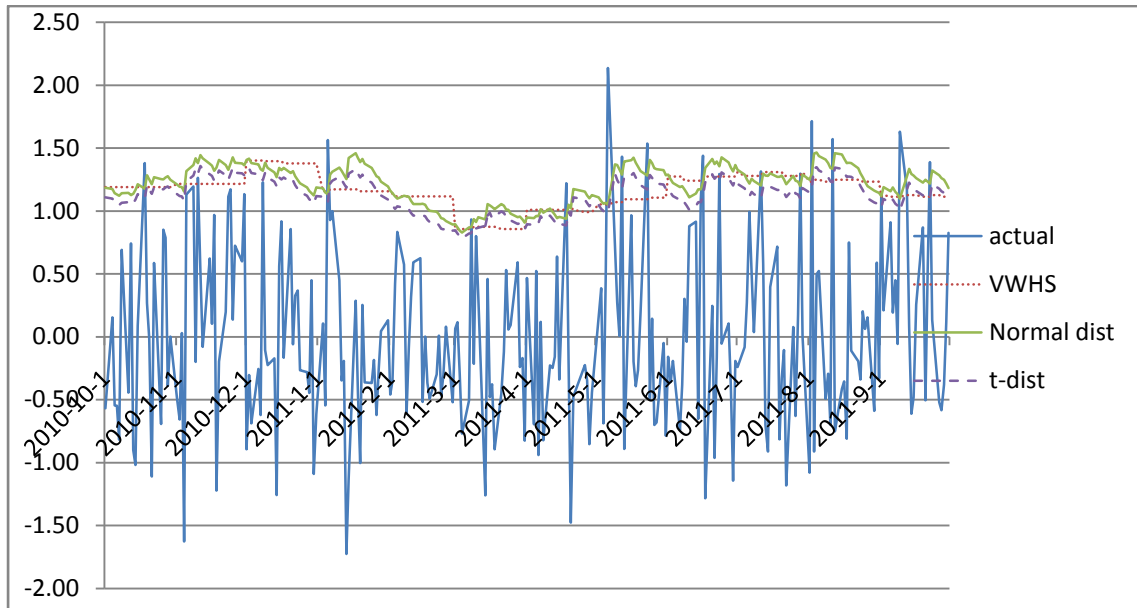


Figure 4.7.c Non-parametric and parametric VaR of USD/EUR for the test sample in the crisis period with 95% confidence level

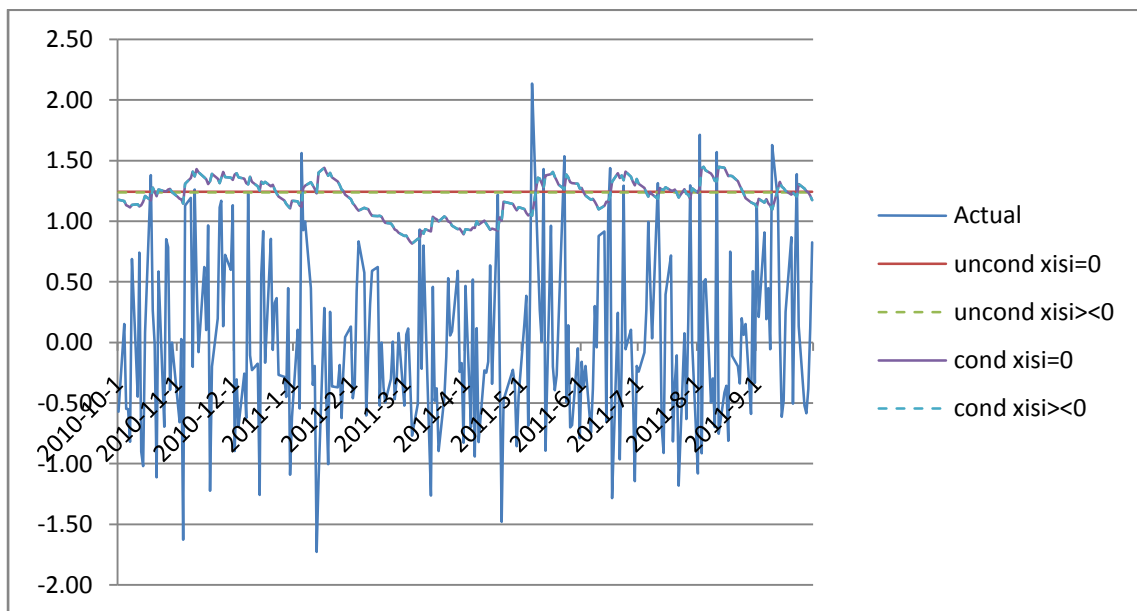


Figure 4.7.d Extreme value theory VaR of USD/EUR for the test sample in the crisis period with 95% confidence level

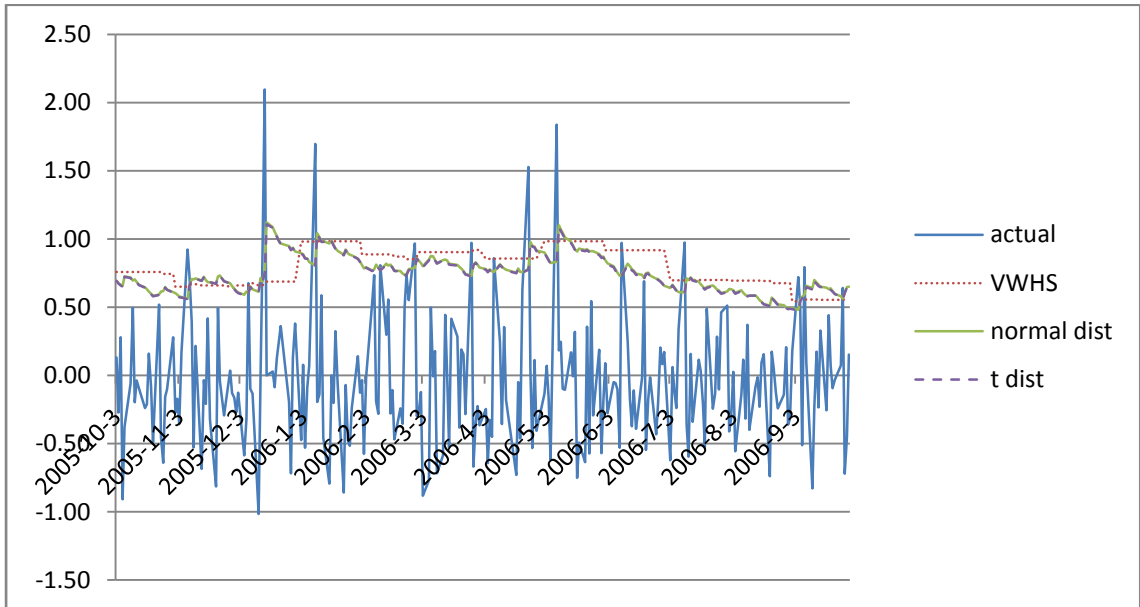


Figure 4.8.a Non-parametric and parametric VaR of JPY/EUR for the test sample in the normal period with 95% confidence level

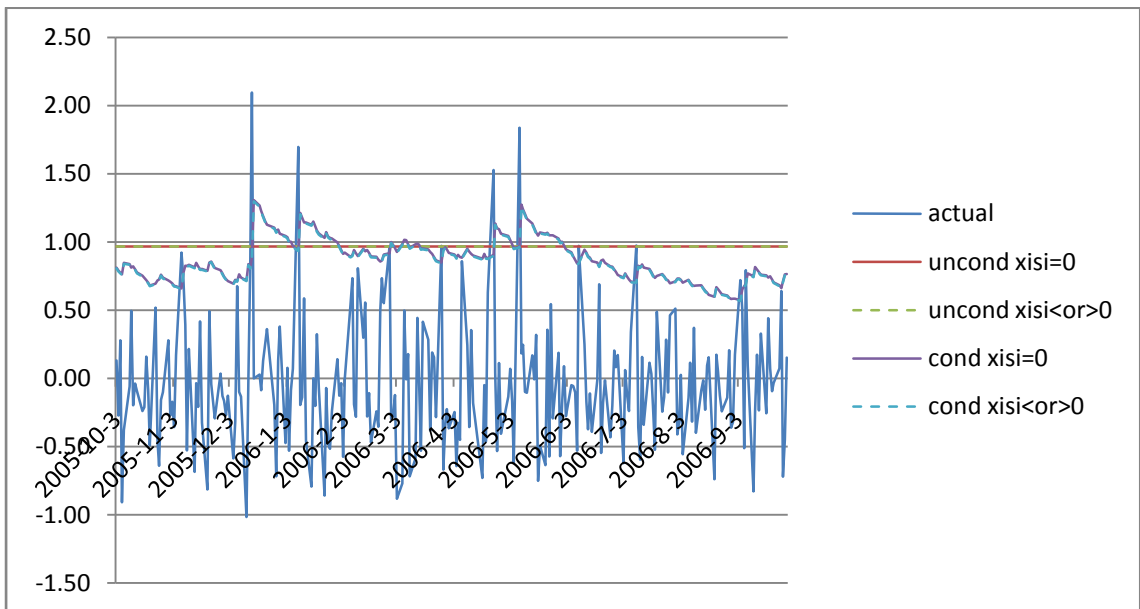


Figure 4.8.b Extreme value theory VaR of JPY/EUR for the test sample in the normal period with 95% confidence level

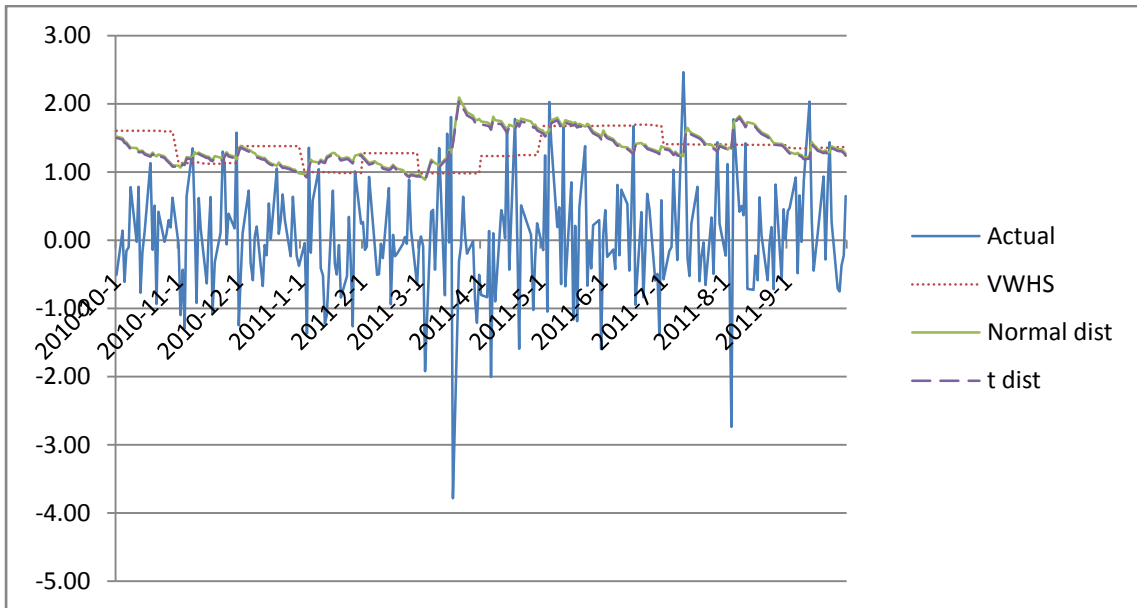


Figure 4.8.c Non-parametric and parametric VaR of JPY/EUR for the test sample in the crisis period with 95% confidence level

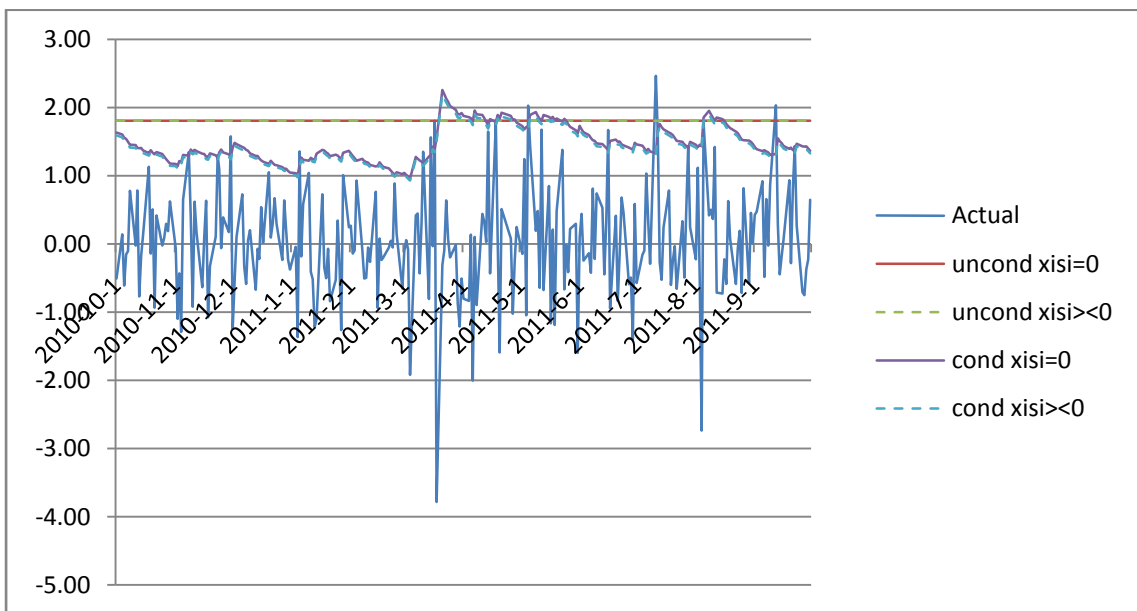


Figure 4.8.d Extreme value theory VaR of JPY/EUR for the test sample in the crisis period with 95% confidence level

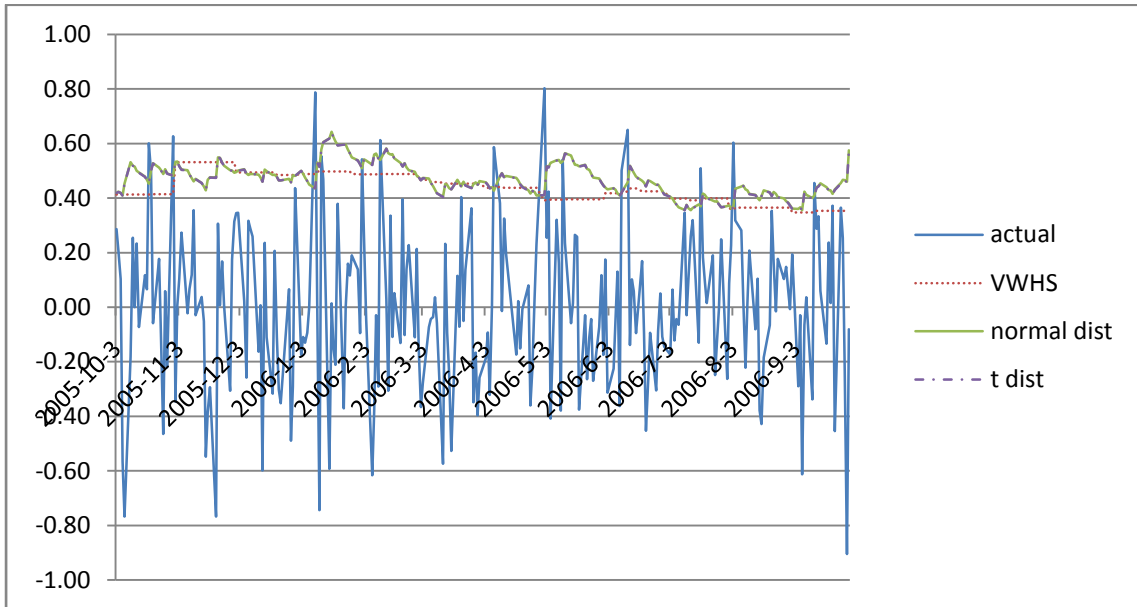


Figure 4.9.a Non-parametric and parametric VaR of GBP/EUR for the test sample in the normal period with 95% confidence level

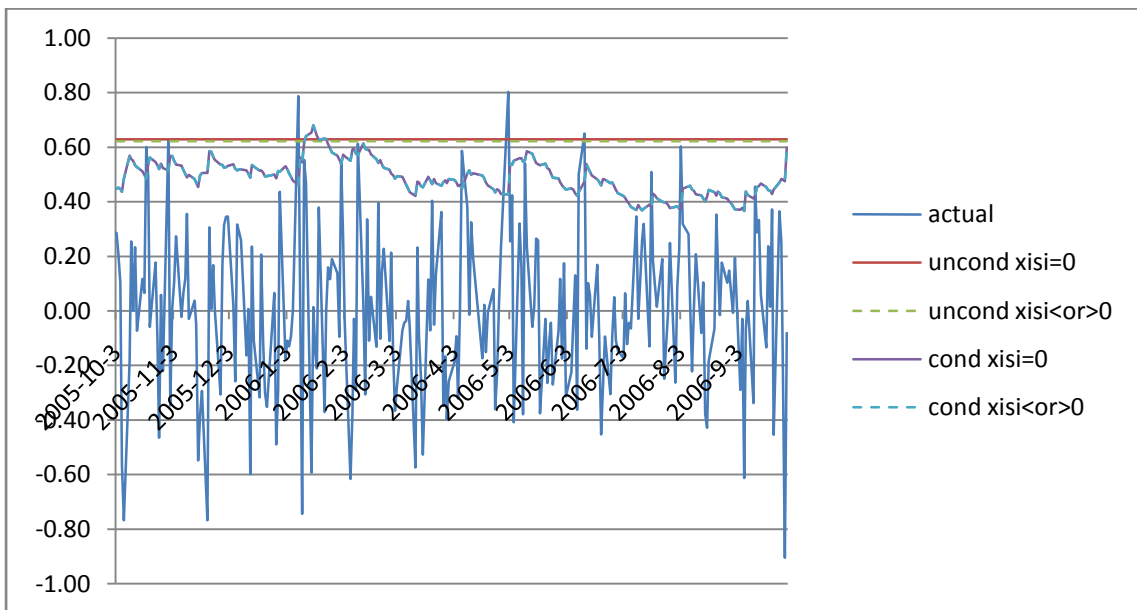


Figure 4.9.b Extreme value theory VaR of GBP/EUR for the test sample in the normal period with 95% confidence level

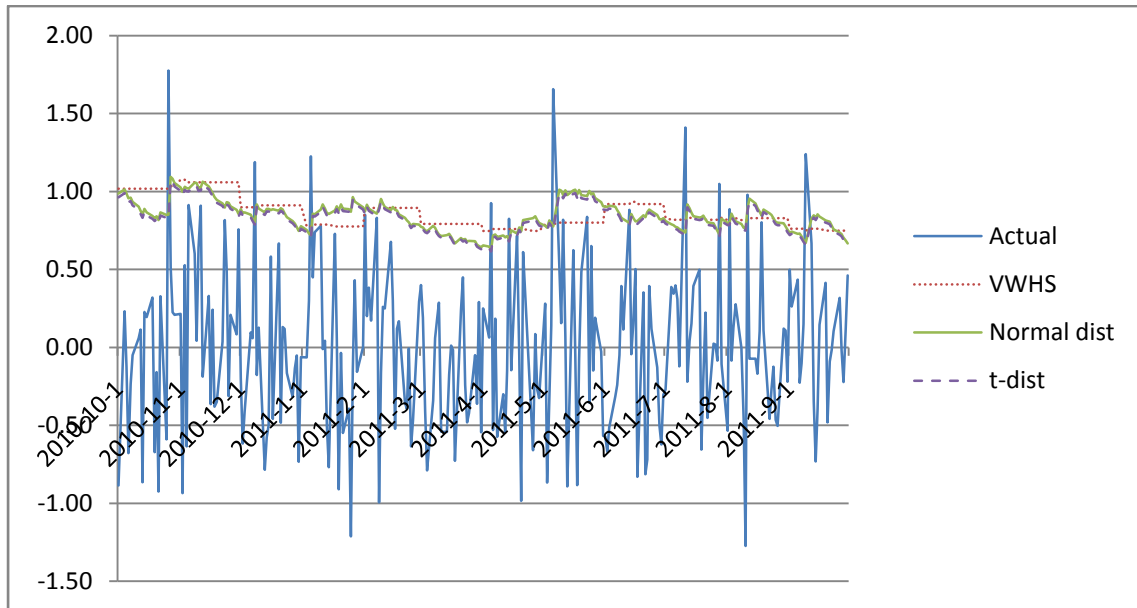


Figure 4.9.c Non-parametric and parametric VaR of GBP/EUR for the test sample in the crisis period with 95% confidence level

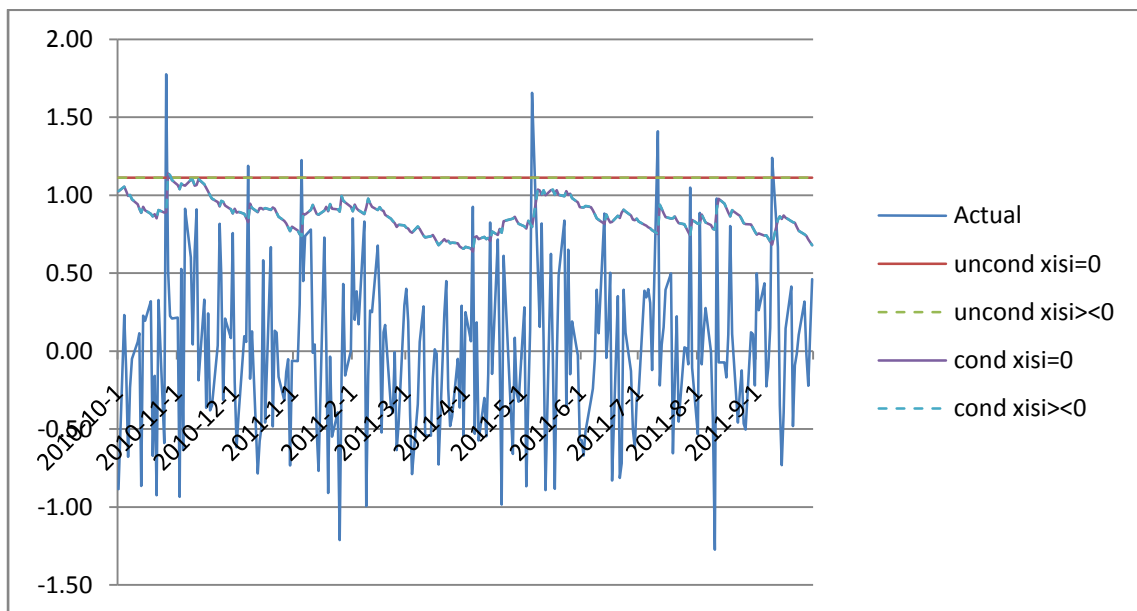


Figure 4.9.b Extreme value theory VaR of GBP/EUR for the test sample in the crisis period with 95% confidence level