The

Statistical Distribution

of

Irrationality

LUND UNIVERSITY

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Abstract

This paper investigates whether the shape of a probability distribution of a given problem has any predictable effect on the shape of the distribution of people's associated irrational choices. Focus is especially on the skewness of distributions. Based on previous research in psychology and behavioral economics, a theory of the causes of skewness of choice distributions is presented. Hypothesis tests on the effect of skewness of probability space in a card game on the skewness of associated choice distributions lead to the conclusion that the two are independent. This finding is consistent with the developed theory, suggesting that the pervasiveness and nature of ineffective heuristics are more important determinants of skewness in the distribution of choices. The results are applied to the valuation of goods in a free market.

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1. Introduction

Psychology and behavioral economics offer no clear-cut answers for what determines the distribution of irrationality, or rather too many potential answers. Behavioral economics has increased in popularity during the last few decades, but remains a messy field that can hardly be summarized sufficiently in any single paper. Nevertheless, a brief overview of the field is necessary to put the subject of this paper into a meaningful context.

The Allais paradox was a major breakthrough in what later would grow into behavioral economics, a new sub-field within economics taking greater account of the flaws and peculiarities of the human mind than what classical economics had previously done. By asking people which gambles they would choose in hypothetical situations, Allais (1953) reached the conclusion that people's behavior is inconsistent and in direct conflict with the independence assumption of the expected utility hypothesis. This assumption says that when choosing between two gambles, the inclusion of a third gamble fails to affect the preference order of the two preexisting gambles. Allais's suggestion that the assumption is unrealistic spurred an academic debate that is still ongoing today, six decades later. No consensus has been reached on the causes of the paradox (Weber 2005). Some critics, such as Kuilen & Wakker (2006), argue that the paradox is a product of an unnatural laboratory environment and that it has little relevance in a real market economy. Nevertheless, the Allais paradox is still researched and applied in modern social science. For example, Hess & Holthausen (1990) attempt to measure and quantify the phenomenon. Sergio, Dinorá & Raul (2013) find several factors that significantly correlate to susceptibility to the paradox. Furthermore, while the gambles in the standard version of the experiment involve money, Allais's paradox is also applicable to choices entirely absent of money, such as health choices (Oliver 2003).

That said, the Allais paradox – albeit an important phenomenon in behavioral economics – is only a tiny part of the field. Countless cognitive biases compete for the attention and as of 2012 there is no consensus about how to best classify and test cognitive and social phenomena relevant to economics. Some researchers, such as Falk & Heckman (2009), find an experimental approach more useful than flawed. Critics of experimental economics, such as Levitt & List (2007), argue that the experimental context is too far removed from actual markets for the conclusions to be applicable in a market context. To be sure, theories and models in behavioral economics are often controversial.

Some important cognitive biases in behavioral economics include the anchoring effect, the impact bias and the framing effect. The anchoring effect means that numerical estimates and valuation choices are significantly affected by irrelevant information such as a randomly generated number. The effect was first proposed by Tversky & Kahneman (1974) and later experiments confirmed the effect for actual money transactions (Ariely, Loewenstein & Prolec 2003). The impact bias refers to people's tendency to overestimate the duration of emotions that arise as a consequence of some hypothetical future event, such as a house burning down (Gilbert et al. 1998). The impact bias can thus cause mis-valuation of insurance among other goods and services. Framing is a broad concept in social science, but in behavioral economics it refers to a tendency to make choices based on the presentation of facts rather than on the facts them selves (Tversky & Kahneman 1981). Marketers routinely use framing to increase sales and profit.

In addition to the ubiquitous individually based cognitive biases, many social biases affect the behavior of market agents. In some cases – such as in an information cascade – the social bias can lead to behavior that is reasonable for each individual in a crowd, but irrational for the crowd as a whole (Holt & Anderson 1997). In other cases – such as when taking revenge – individuals are willing to pay to punish wrongdoers and enforce social norms, which at least from the perspective of classical economics seems irrational (Bechwati & Morrin 2003).

Seeing and understanding this chaotic field holistically can be difficult, and bringing structure to behavioral economics is, according to Fudenberg (2006), an important task for future researchers. Rather than building further on any particular idea in the field, this paper therefore investigates something that can be broadly applied to many phenomena, namely determinants of the statistical distribution of choices. Since behavioral economics already has produced ample evidence for the existence of irrationality, a natural extension of that idea is to define the statistical characteristics of that irrationality in more depth. The methods and the conclusions of this paper can be used in combination with any cognitive bias if the theories of behavioral economics can be unified in a quantitative model.

The lessons learned are applied to the statistical distribution of valuation decisions on a free market. Generally, a cognitive problem with the following necessary conditions is assumed in the analysis throughout the paper.

- Each subject choose among a set of several clearly defined options which exist either on an interval scale or on a ratio scale.
- There is an unambiguously optimal choice among the options.
- Subjects' choices are independent of each other.

The minimum requirement of several options on at least an interval scale enables more generalization than what would otherwise have been possible. The possibility to apply knowledge to decisions involving valuation of a good or service is especially important in economics. A minimum of an interval scale also allows the use of common statistics such as mean and variance and generally eases statistical analysis. The second necessary condition, that there must be an unambiguously optimal answer, must be true in order for irrationality to be quantified. This eases the mathematical analysis in this paper but is not crucial for the conclusions to hold true, as will be motivated in section 2.1. The third and last necessary condition ensures the absence of information cascades and other social effects, which are difficult to control.

Focus is especially on skewness, partly because it is easily applied and understood by statisticians and non-statisticians alike, and partly because it can be useful in microeconomics. The presence or absence of skewness in a crowd's valuation of goods can affect the effectiveness of market agents' strategies[1]. If realism is an ambition taken seriously, the stochastic nature of markets and people must be taken into account. The relationship between skewness of perception and skewness of choice is evaluated first by using a mathematical theoretical approach and then by conducting hypothesis tests on data from a card game experiment designed to emulate valuation decisions. The ambition is to determine whether a skewed probability distribution of value causes skewness in the distribution of associated choices.

1 Consider for example skewness' implications for the winner's curse in an auction or for a retailer whom sets fixed prices to maximize profits.

2. Theoretical Foundations

2.1. Defining Rationality

In order to avoid ambiguity and confusion in the mathematical analysis, a clear definition of rationality is stipulated. The maximization of economic profit and minimization of economic loss is assumed to be the ultimate goal of all market agents. Making utility and profit interchangeable is – for behavioral economics an unusual stance – that merely serves to make measurement of irrationality feasible. Rationality is, on the lead of Tsang (2008), here measured as computational power, allowing quantification of irrationality. According to this view, irrational choices are caused by flawed heuristics. Therefore, irrationality is defined and measured as choices that deviate from a utility maximizing option. Neither the usefulness of other perspectives, nor the unrealistic nature of this rigid definition is denied.

Performance based economical incentives for participants of experiments are unnecessary as it is unlikely to enhance performance in purely cognitive tasks. Indeed, the relationship may be the opposite (Gneezy & Rustichini 2000, Ariely et al. 2009). When economical incentives are absent, game participants are assumed to be motivated by the possibility of winning.

Expected profit and the probability of winning is used in this paper because given the right circumstances it can be measured and tested. Expected utility is probably more realistically defined as some risk adjusted present value function of brain activity, accounting for all emotions, thoughts and experiences perceived from the present moment until the end of life. Modern brain scanning technology enables measurement of various types of satisfaction and pain, but measuring long term well-being would be very expensive, if at all possible. For example, Fischbacher et al. (2004) used such technology and found that subjects experienced temporary well-being when punishing violations of social norms, challenging the classical view of rationality. In the end, the impracticality of such endeavors necessitates a more tractable approach.

The classical approach of measuring utility implicitly through revealed preferences is of little use when measuring irrationality. Since every economic agent is assumed to be rational, any choice would be rational by definition. Several other definitions of rationality have been suggested. Herrmann-Pillath (1994) for example, suggests a concept of *evolutionary rationality*, where the genetic and social nature of the human species are incorporated as important aspects.

While it is important to be aware of various views of rationality, it is not important to use a realistic or insightful definition of rationality when investigating the distribution of choices. For the purposes of this paper, such considerations are redundant. Regardless of which perspective is preferred in general and in particular circumstances, the distribution and skewness of choices will remain the same. Irrationality is not to be seen as a pejorative term, but rather as a reference to the observation that people's estimates and choices differ, even when everyone faces the same problem. Indeed, the title *“The Statistical Distribution of Choices”* might have better reflected the content of this paper.

2.2. Mathematical Notation & Method

Any choice that deviates from the option that maximizes the narrow form of utility defined in the previous section is considered irrational. Furthermore, different options can be ranked according to the degree of irrationality using a sequence $A=(\phi_1, \ldots, \phi_j, \ldots, \phi_k)$ where ϕ_1 is the expected utility maximizing choice, ϕ_j is the j^{th} best choice and ϕ_k is the choice that minimizes expected utility.

A finite or infinite set of options on an interval scale or ratio scale is defined as $S=\{y_1, \ldots, y_i, \ldots, y_k\}$ where y_i refers to the i^{th} option out of a total of k options, typically though not necessarily expressed in dollars. If the i^{th} option y_i is chosen, that choice is expressed as x_i. Each choice x is taken from a random variable defined as $X(\mu, \sigma^2)$ with iteration corresponding to the iteration in S. Expected utility of the i^{th} choice is then defined as some function of a that choice; $E[U](x_i)$.

To be sure, the order of the elements in $S=\{y_1, \ldots, y_k\}=\{x_1, \ldots, x_k\}$ is arbitrary and therefore irrelevant to the ranking of options. Although the cardinality of A is equal to the cardinality of S and each element in A corresponds to an element in S, the iteration of the set does not correspond to the iteration of the sequence. S merely defines the set of options while the sequence A ranks the options in that set according to the degree of irrationality.

For problems using a continuous scale, calculus must normally be used to find ϕ_1 and will likely also be useful in ranking the remaining options. For problems using a discrete scale, outcomes of all the different options are simply compared and chosen among accordingly, as in equation (1).

$$U(\phi_1) = max(\{U(X) : X \in S\}) \tag{1}$$

The remaining options can then be evaluated and ranked using a generalization of equation (1):

$$U(\phi_i) = max(\{U(X) : X \in S \setminus \{\phi_1, \ldots, \phi_{i-1}\}\}) \tag{2}$$

Both methods result in a sequence $A = (\phi_1, \ldots, \phi_k)$ where each element corresponds to an element in the unordered set S .

For the random variable X , population skewness is here defined in equation (3) where *N* is the population size. The sample estimator for the skewness of the population is defined in equation (4) where n is the sample size. There are other definitions and estimators, but the third standardized moment used here is the most widely used, and the different formulas for the sample estimators varies so little that when the sample is large, the difference between resulting estimates is very small.

$$\gamma = \frac{1}{N-1} \cdot E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] \tag{3}$$

$$G = \frac{n}{(n-1)(n-2)} \cdot \sum \left(\frac{x_i-\bar{x}}{s}\right)^3 \tag{4}$$

Wright & Herrington (2011) advise that bootstrap methods are preferred for estimating the standard error of the skewness for small samples, but also show that the traditional estimator is satisfactorily accurate for large samples. Thus, the standard error of γ is here estimated as in equation (5). Assuming that $G \sim N$ means that hypothesis tests easily can be conducted with two-tailed z-tests.

$$SE(G) = \sqrt{\frac{6(n-2)}{(n+1)(n+3)}} \cdot \frac{\sqrt{n(n-1)}}{n-2} \tag{5}$$

The analogous formulas also apply to the probability space of S . The mathematical notation introduced in this section will be used occasionally in the rest of this text to condense and clarify arguments. The skewness statistic is utilized in hypothesis tests on data from a card game experiment explained in greater detail in chapter 3.

2.3. Empirical Evidence of Skewness

Systematic errors in human decision making are old news for psychologists and have recently been subject to increased attention in economics. Using the mathematical notation developed in the last section, a significant amount of evidence suggests that $\phi_1 \neq \mu$ under certain conditions (e.g. Camerer, Loewenstein & Rabin 2004). The shape of the statistical distribution of X has received less attention. Micceri (1989) finds that human behavior in general rarely follows a normal distribution, but Krueger & Funder (2004) are concerned that social sciences suffer from a bias toward finding and reporting cognitive biases, which can lead to asymmetry in statistical distributions of samples. A closer look at empirical evidence confirms that perception and choice in a crowd not necessarily follow a normal distribution.

Bostian & Holt (2011) performed a classroom experiment where students were asked to participate in a sealed-bid auction of a jar full of marshmallows. Before the actual bidding started students were asked to guess the amount of marshmallows in the jar and the person with the closest guess earned a $5 reward. In this part of the experiment, there is no obvious asymmetry between underestimation and overestimation, but the data reveal a heavy positive skewness. This positive skewness was replicated in the subsequent auction for marshmallows, where transaction costs were minimized by allowing each marshmallow to be traded in for one cent.

On the other side of the Atlantic, more than a century earlier, Galton (1907) collected data from 787 individuals in a similar guessing game. Visitors at a cattle fair would buy lottery tickets, giving them the right to guess the weight of an ox upon slaughter. The guessing game was held annually and whoever made the best estimate was awarded a price. The data reveal that the distribution of guesses were negatively skewed.

In summary, the scattered evidence showing various shapes of distributions for estimates might suggest that the shape and skewness of the distribution depend on particulars of the experiment and its settings, that seemingly are considered so insignificant that they are not mentioned in writing. The finding that the volume of cylinders with a high height to circumference ratio are systematically overestimated, while the volume of cylinders with a low such ratio are systematically underestimated (Graham 1936), is one of several potential causes of the skewness in the distribution of Bostian's &

Holt's data. In the annual ox weight guessing contest, the weights of oxes from previous years might have established an anchor effect, leading to a skewed distribution. Without conducting rigorously controlled experiments, progress is unlikely to be made in determining the causes of skewness in choices.

In addition to convincing evidence of systematic cognitive bias and the presence of asymmetry in choice, 21st century economists have produced evidence that some people are generally more irrational than others. Delfabbro, Lahn & Grabosky (2006) find that adolescent problem gamblers are prone to irrational behavior. Suetens & Tyran (2011) find evidence for the gambler's fallacy for men, but not for women. Coates, Gurnell & Sanyai (2010) find that testosterone and cortisol levels can affect risk valuation and irrational behavior in financial markets, both in the short run and in the long run. They go on to speculate that because hormone levels correlate to age and gender, the latter variables may predict market performance. The exact causes of irrationality is continuously researched – but for the purposes of this paper – it is sufficient to know that cognitive bias exists in many different forms and that proneness to it differs among groups and individuals.

2.4. A Theory of the Cause of Skewness

An arbitrary good is assumed to have an unambiguous market value of $\$\,y_\delta$ that is not necessarily easily determined[2]. The market value is the price that it can be resold for, assuming the absence of transaction costs and flawless rationality of all market agents. Furthermore, willingness to pay (WTP) is defined as the maximum amount that a prospective buyer is willing to pay for the good while willingness to accept (WTA) is the corresponding concept for the seller. The optimal valuation and choice for both sides of the transaction is then given by equation (6). Without the narrow definition of rationality as profit maximization or loss minimization used here this would not necessarily be true.

$$x = \phi_1 = y_\delta \tag{6}$$

If rationality is the starting point, as is conventional in economics, there is no reason for any prospective buyer to deviate in any particular direction from optimality. Diminishing marginal utility of wealth would imply that an opportunity loss is preferred to an accounting loss, but if the trade is made

2 For illustrative purposes, one might imagine a financial asset or a jar full of coins.

on the margin, ignoring this effect is justified. Thus, in the absence of cognitive bias, it is easy to imagine that an individual's distribution of cognitive errors is approximately symmetrical. In reality however, cognitive biases are abundant.

An example of a pervasive cognitive bias is excessive loss aversion (Tversky & Kahneman 1991). While offering to pay less than the optimal amount would result in an opportunity loss for the buyer, paying more than the optimal amount would result in an accounting loss. Although this observation is largely irrelevant in traditional economics, the loss aversion bias suggests that accounting losses hurt more than opportunity losses. If all prospective buyers are affected equally much by such a cognitive bias, this new information can be incorporated into X and expressed mathematically as in equation (7) and (8).

$$E[X] < \phi_1 \tag{7}$$

$$\gamma = 0 \tag{8}$$

However, if a mere proportion of prospective buyers are affected by the cognitive bias while the remaining part of the group remains unaffected, the bias can cause asymmetry in the distribution of prospective buyers' choices. In general, acknowledging individual differences in proneness to cognitive bias allows for skewness in the distribution of a group's choices, in effect nullifying equation (8), regardless of the skewness of the distribution of each individual in that group.

This is not the end of the story however, because in addition to the loss aversion bias a wide array of cognitive biases can affect the skewness in either direction. Behavioral economics lacks efficient methods for explaining and accurately predicting skewness of irrational choice, and given the complexity of the human mind, such methods are unlikely to be developed in the near future. To be sure, such skewness can arise independently from irrationality as defined here, which only serves to decrease predictability further. Nevertheless, merely having a primitive framework for quantifying and aggregating cognitive biases leads to the conclusion that multiple effects can either reinforce each other or cancel each other out.

Although buyers rarely think in terms of probabilities explicitly, the fact that they implicitly and often intuitively recognize probability distributions can be incorporated into an expected utility function and expressed mathematically as in equation (9) where π is profit. Remember from section

2.1. that expected utility is assumed to be equal to expected profit. Consciously or not, a probability distribution for the set of options is estimated. Every option y_i signifies a potential valuation, that is accompanied by an estimated probability of that price being equal to the true value of the good; $P(y_i = y_\delta = \phi_1)$. WTP in equation (9) is the random variable being chosen by the buyer, more generally denoted by X . Since the highest possible profit is zero when opportunity cost is accounted for, the optimization problem derived from this equation in effect becomes a loss minimization problem.

$$E[U](WTP) = E[\pi(WTP)] = P(WTP \geq WTA) \cdot \sum_{i=0}^{\infty} [P(y_i = y_\delta) \cdot (y_i - WTP)] \tag{9}$$

The same reasoning applies to the supply side of the market, as expressed in equation (10) and indeed for any individual choice with or without money involved, as expressed in equation (11).

$$E[U](WTA) = E[\pi(WTA)] = P(WTP \geq WTA) \cdot \sum_{i=0}^{\infty} [P(y_i = y_\delta) \cdot (WTA - y_i)] \tag{10}$$

$$E[U](X) = \sum_{i=1}^{k} [P(y_i = \phi_1) \cdot (U(X))] \tag{11}$$

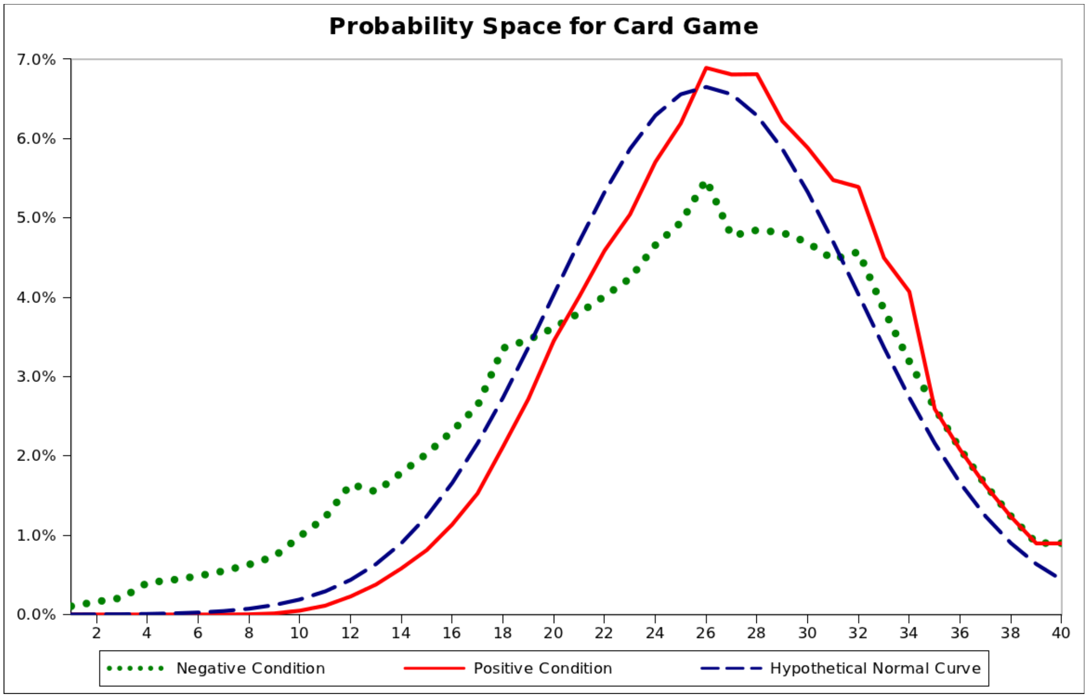
Since it is human to err, choice is a random variable which by definition has a statistical distribution – and taking the perspective of buyers as decision makers – only the probabilities in equation (9) are non-trivial and potentially subject to unrealistic estimation. A description of how this estimation is performed will not be attempted as it is a psychological question beyond the scope of this paper. Instead, the effects of underlying distributions are observed in a controlled experiment, where an estimated probability distribution is assumed to approximate an actual probability distribution of a card game, also referred to as the probability space of the card game.

3. Card Game Experiment

3.1. Method

The design of a simple solo card game allows for the probability space of the option set to be manipulated. In the game, the player is given four random cards from three standard decks of cards without jokers. Each card is assigned a value according to a table and then added together. The player is to guess the sum beforehand, such that the probability of winning is maximized[3].

A control group has a perfectly symmetrical probability space. In this condition the values assigned to the playing cards range from one to thirteen with equal probability for any integer value in that range. Since no obvious reason for bias exists in this group, the distribution of guesses is expected to be symmetrical. Two treatment groups introduce skewness to the probability space by assigning a value of ten to all face cards and a value of positive four and negative four for the two treatment groups respectively.



3 The exact instructions given to all participants are given in the appendix.

The positive condition differs from the negative condition only in the value of aces, and implicitly in the probability distribution. Although this may seem like a minor treatment, the implied change in the skewness of the probability distribution is large enough to be easily discernible with the naked eye, while avoiding a shift of the optimal choice. The negative treatment group faces a probability space that is more skewed than the one faced by the positive treatment group. Although the average outcome is lower for the negative treatment group than for the positive treatment group, the mode of the two treatment groups remain equal to each other.

If the sample distribution of choices differs significantly between the three groups, differences in the probability space must be the cause. While there are many measures of distribution, the definition of skewness introduced in section 2.2. as equation (3) is used to form the main hypothesis of this text, formalized in equations (12) through (15) where γ_c , γ_{t+} and γ_{t-} refer to the skewness of choices in the three groups. Since $n > 30$ by a large margin for every sub-sample, these two-tailed tests can be conducted using the common z-test.

$$H_0: \quad \gamma_c = \gamma_{t+} = \gamma_{t-} = 0 \tag{12}$$

$$H_1: \quad \gamma_c \neq 0 \tag{13}$$

$$H_1: \quad \gamma_{t+} \neq 0 \tag{14}$$

$$H_1: \quad \gamma_{t-} \neq 0 \tag{15}$$

A Kolmogorov-Smirnov test is used to determine whether the positive treatment group is distributed like the negative treatment group, formally as in (16) and (17). Further Kolmogorov-Smirnov tests are conducted on all three groups against a hypothetical normal distribution with $\mu = \bar{x}$ and $\sigma = s$ for each group, as in (18) and (19).

$$H_0: \; X_{t+} \sim X_{t-} \tag{16}$$

$$H_1: \; \neg\left(X_{t+} \sim X_{t-} \right) \tag{17}$$

$$H_0: \quad X \sim N\left(\bar{x}, s^2\right) \tag{18}$$

$$H_1: \quad \neg\left(X \sim N\left(\bar{x}, s^2\right)\right) \tag{19}$$

Although several more hypothesis tests are possible and potentially useful, their peripheral importance in this paper necessitates their omission.[4]

The experiment was first conducted in a classroom environment on undergraduate economics students, but because of the limited interest in participation, the collected data was disposed of. A new plan with less stringent demands on a controlled environment replaced the original sampling method. Randomly chosen people in Helsingborg and Malmö central stations were approached and asked to participate in a short simple guessing game. Those who volunteered were given instructions for the game in writing and orally in either English or Swedish depending on their preferences. Upon guessing a sum, the participants were asked to motivate their choice.

This somewhat informal data acquisition method could not guarantee that each participant received identical information. Although the written instructions were identical to everybody in a given group, the oral presentation of the task may have differed slightly. Nevertheless, the oral presentations consistently mirrored the written instructions. Furthermore, in such an environment, people are in widely different states of mind. Some respondents with a lot of time on their hands felt they could afford to think through the problem thoroughly for a minute or two and sometimes even discuss the purpose and potential implications of the experiment with the experimenter. Other volunteers were suspicious of the intentions of the experiment, stressed or preoccupied with more immediate and real problems. When hurrying off to a train departure, it might be more convenient to guess a number associated with the birthday of a family member than to calculate the most likely outcome of a card game without any tangible benefit. The tendency to perceive and use readily available information although it poorly reflects reality, is commonly referred to as the availability bias or the availability heuristic (Tversky & Kahneman 1973). Such considerations are relevant – not only because it can affect the average guess – but also because it can affect the shape of the distribution of choices. Thus, a laboratory environment where everybody is in the same situation and everybody receive the exact same instructions, might have been more appropriate for this experiment.

4 Examples of possible tests include but are not limited to rationality on average, kurtosis and χ^2-tests assuming different distributions. The interested reader may run any statistical test on the raw data provided in the appendix.

3.2. Results

The heuristics leading to decisions entirely determine the distribution of choices. The by far most common heuristic among participants was to imagine obtaining some cards in a way that was perceived to emulate a random card picking process, sum the values of those cards and guess that sum. Another commonly applied method was to – more or less explicitly and more or less successfully – calculate an average or optimal answer and guess according to that calculation. Intuitively choosing a number based on indistinct emotions was another common strategy. A small minority guessed their lucky numbers, entirely disregarding the likelihood of outcomes and another minority relied on perceptions of what kind of cards they usually obtain in card games, falling victim to the gambler's fallacy.

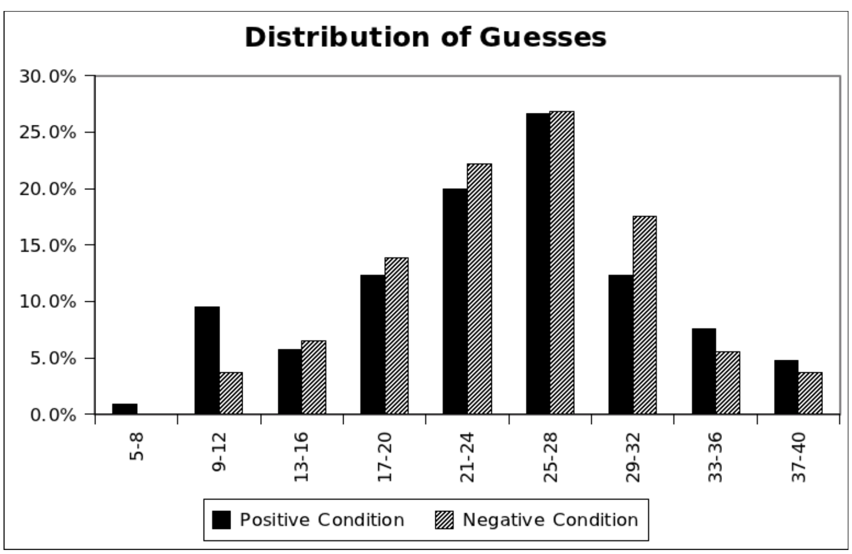
The null hypothesis in (12) cannot be rejected for any of the conditions on 5% significance level and thus, there is no evidence that skewness in the game's probability space has any effect on the skewness of the distribution of guesses. Histograms of the three samples give no obvious clues as to how the skewness of the distribution of guesses is affected by the skewness in the probability space.

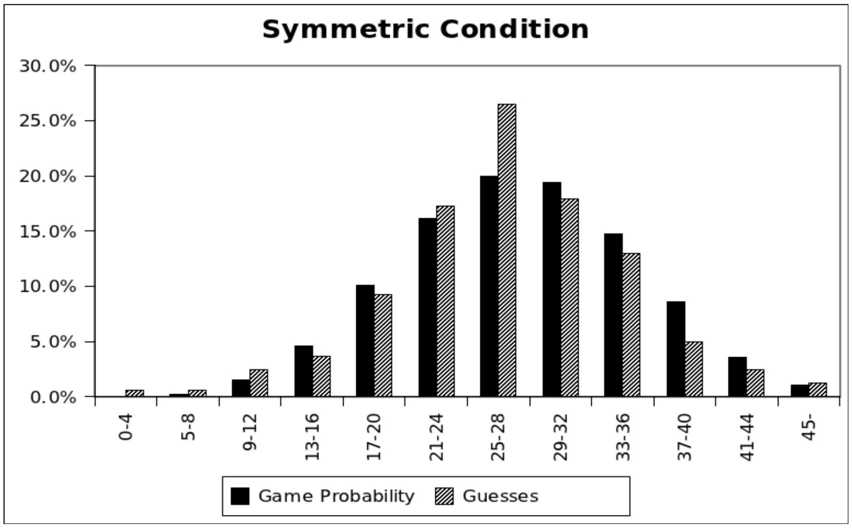
The Kolmogorov-Smirnov test conducted on the two treatment groups gives a p-value of more than 0.5 , meaning that the null hypothesis in (16) cannot be rejected. Using the same test, the null hypothesis in (18) cannot be rejected for any condition on a 5% significance level. Thus, the data indicate that increasing skewness in the game's probability space fails to significantly alter the distribution of the guesses and that the guesses remain roughly symmetrically distributed regardless of skewness in the probability space.

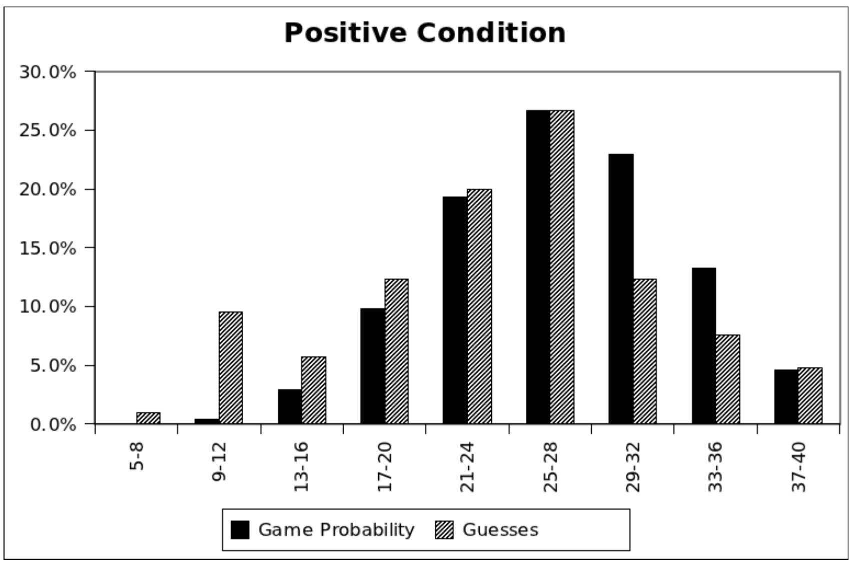
Unexpectedly, data from the control condition exhibit significantly positive kurtosis. This serves as a warning sign that guesses need not exactly follow a normal distribution. The ease with which an optimal guess can be calculated in the symmetrical version of the game might have caused the kurtosis. Also, with the definition of rationality used in this paper, the average guess is significantly ($\alpha = 5\%$) irrational in both the positive and the negative treatment group, contradicting predictions made by Surowiecki (2005) et al. that crowds are rational on average. Although not a central theme of this paper, this finding may serve as a reminder that the workings of the human mind are more obscure than what is commonly admitted in economics. Furthermore, although clearly skewed, the probability space of even the most heavily skewed version of the game, is bell-shaped and somewhat similar to a normal distribution. Considering this similarity, drawing inferences about the distribution of a population is far from trivial when the sample is as small as it is here.

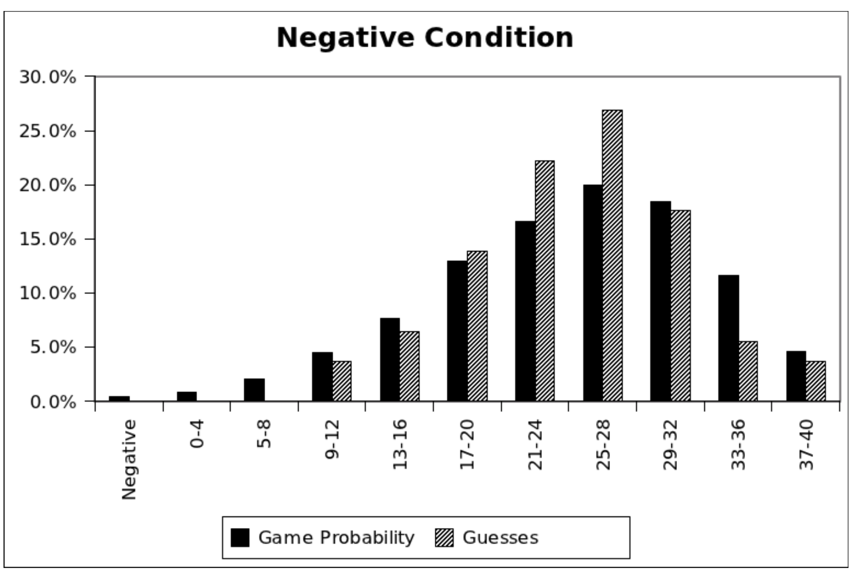
Results of Statistical Analysis of the Card Game Experiment

	Symmetrical	Positive	Negative
Parameters for Card Game			
Mode (Optimal Choice)	28	26	26
Median	28	22	12
Mean	28	27.08	24.62
Standard Deviation	7.48	5.67	7.99
Skewness	0	−0.11	−0.54
Sample Statistics			
Sample Size (n)	162	105	108
Median	27	25	25
Mean	27.07	24.35	24.81
Standard Deviation (s)	7.35	7.46	6.31
Standard Error of the Mean (SEM)	0.58	0.73	0.61
P-value (μ ≠ Optimal Choice)	10.64%	2.36%	4.93%
Skewness (G)	−0.13	−0.28	0.03
Standard Error of Skewness (SES)	0.19	0.24	0.23
P-value (γ ≠ 0)	49.65%	23.31%	91.27%
Kurtosis	0.51	−0.20	−0.24
Standard Error of Kurtosis	0.14	0.21	0.2
P-value (kurtosis ≠ 0)	0.03%	34.61%	24.41%
P-value (X~N KS-test)	76.49%	82.96%	93.90%




Symmetric Condition
30.0%
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
0-4
5-8
9-12
13-16
17-20
21-24
25-28
29-32
33-36
37-40
41-44
45-
Game Probability
Guesses


Positive Condition
30.0%
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
5-8
9-12
13-16
17-20
21-24
25-28
29-32
33-36
37-40
Game Probability
Guesses


Negative Condition
30.0%
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
Negative
0-4
5-8
9-12
13-16
17-20
21-24
25-28
29-32
33-36
37-40
Game Probability
Guesses

3.3. Generalization & Applicability

Surely, very few players seemed to understand the intentions of the experiment in which they participated, and how it connects to economics. The degree to which respondents realized that they were making an implicit valuation based on estimates of probability differed. This realization is not crucial because it is also absent in many market transactions. There is a risk however, that some players failed to comprehend the rules and the nature of the game itself, or refused to play by the rules[5]. A small number of cognitively challenged individuals can have a disproportionately large effect on the results of an experiment, while having a more limited effect on more practical problems in the real economy because of their limited financial power.

Assuming that all players did understand the game and played as instructed, the results can be applied to valuation problems in the real economy according to the mathematical framework presented in chapter 2. Although the market value of a good is best represented by an exact dollar amount, people's perception of the value can be presented in greater detail using a probability distribution. While a minority of market agents' perception of the market value can be spot on, the majority's best estimates are likely to deviate more or less from the market value.

As described in detail in section 2.4, the distribution of a market agent's random choice depends on the probability distribution of his estimate for the value of the good. Thus, the probability space of the card game can be seen as corresponding to a probability distribution of the estimated value of a good, and the distribution of guesses in the game as corresponding to a crowd's distribution of valuation decisions for that good. The distribution of WTP and WTA choices then depends on a probability distribution of estimated value for a good, in the same way that the distribution of guesses in the card game depends on the underlying probability space of the card game.

Although this transfer of patterns from one set of circumstances to another makes sense in theory, it is somewhat dubious in practice. Market transactions may be similar to the card game in some aspects but it can hardly be assumed that the thought processes underlying choices in the two are identical. Indeed, since heuristics entirely determine the distribution of choices, a continued focus on the thought processes that lead to decisions is worthwhile for behavioral economics.

5 One player insisted she perfectly understood the game and continued to guess 44 even though this outcome was impossible. Several players guessed single digit lucky numbers that was highly unlikely to occur. On a case to case basis, answers from some of the respondents not likely to have understood the game have been removed from the sample.

Notwithstanding this critique, applying the results of this paper to a market environment, suggests that a skewed probability distribution for perception of the value of a good not causes a skewed WTP or WTA distribution. Since many of the participants in the experiment exhibited cognitive bias, this conclusion is valid even in the presence of imperfect heuristics.

In conclusion, considering all theoretical and empirical evidence presented here, the distribution of economic choices depends more on the effectiveness of the heuristics used and on the ubiquitousness of these heuristics among market agents than it does on the distribution of an underlying probability distribution. The skewness of an underlying probability distribution seems to have no effect on the skewness of the distribution of associated choices.

4. Conclusion

The recent burst of research in behavioral economics has probably produced more questions than answers. The approach presented here is not so much a solution to any specific economic problem, as it is a potential new perspective for framing questions of irrationality in behavioral economics. When quantified with greater accuracy, all the heuristics and cognitive biases studied in behavioral economics can be used to predict the distribution of choices, in addition to aggregated choices on free markets.

The practical usefulness of estimating and predicting shapes of choice distributions is contingent on high quality data and detailed knowledge about human decision making processes. A tendency in experimental economics to record responses on a nominal scale rather than an interval or ratio scale constitutes a major difficulty in efforts to quantitatively model irrational behavior. The tractability of estimating and predicting shapes of statistical distributions of choice is dubious in 2012, but the prospect of continued research in behavioral economics, psychology, sociology and neuroeconomics coupled with improved data acquisition technology may make its implementation feasible and useful in a not too distant future.

Potential further experimental research on the distribution of irrational choices is likely to benefit economics more if the problems faced by participants better emulate a free market environment, allowing more direct interpretation of results. Examining the effects of various cognitive biases on the distribution of choices is a painstakingly long process. The development of an efficient taxonomy of cognitive biases might allow more focus in this process. Lastly, making raw data from previously conducted experiments readily accessible to fellow researchers, and perhaps even the public, would make the task more parsimonious than what it would be if experiments were designed and conducted for the sole purpose of investigating statistical distribution.

5. References

Allais, M. (1953). *Le comportement de l’homme rationnel devant le risque: critique des postulats et axiomes de l’école Américaine*. Econometrica 21 (4): 503–546

Anderson, Lisa R and Holt, Charles A. *Information Cascades in the Laboratory*. The American Economic Review, ISSN 0002-8282, 12/1997, Volume 87, Issue 5, pp. 847 – 862

Ariely, Dan; Loewenstein, George; Prelec, Drazen. *"Coherent Arbitrariness": Stable Demand Curves Without Stable Preferences*. The Quarterly Journal of Economics, ISSN 0033-5533, 02/2003, Volume 118, Issue 1, pp. 73 - 106

Ariely, Dan; Gneezy, Uri; Loewenstein, George; Mazar, Nina. *Large Stakes and Big Mistakes*. The Review of Economic Studies, ISSN 0034-6527, 04/2009, Volume 76, Issue 2, pp. 451 – 469

Bechwati, Nada Nasr and Morrin, Maureen. *Outraged Consumers: Getting Even at the Expense of Getting a Good Deal*. Journal of Consumer Psychology, ISSN 1057-7408, 2003, Volume 13, Issue 4, pp. 440 - 453

Bostian, AJ A; Holt, Charles A. *Veconlab Clicker Games: The Wisdom of Crowds and the Winner’s Curse*. Unpublished manuscript, 10/2011

Camerer, Colin, 1959; Loewenstein, George; Rabin, Matthew, 1963. *Advances in Behavioral Economics*. 2004, The roundtable series in behavioral economics, ISBN 9780691116822, 740 s.

Coates, John M; Gurnell, Mark; Sarnyai, Zoltan. *From molecule to market: steroid hormones and financial risk-taking*. Philosophical transactions of the Royal Society of London. Series B, Biological sciences, ISSN 0962-8436, 01/2010, Volume 365, Issue 1538, pp. 331 - 343

Delfabbro, Paul; Lahn, Julie; Grabosky, Peter. *It’s Not What You Know, but How You Use It: Statistical Knowledge and Adolescent Problem Gambling*. Journal of Gambling Studies, ISSN 1050-5350, 06/2006, Volume 22, Issue 2, pp. 179 - 193

Falk, Armin and Heckman, James J. *Lab Experiments Are a Major Source of Knowledge in the Social Sciences*. Science, ISSN 0036-8075, 10/2009, Volume 326, Issue 5952, pp. 535 - 538

Fischbacher, Urs; Treyer, Valerie; Schellhammer, Melanie; Schnyder, Ulrich; Buck, Alfred; Fehr, Ernst. *The Neural Basis of Altruistic Punishment*. Science, ISSN 0036-8075, 08/2004, Volume 305, Issue 5688, pp. 1254 - 1258

Fudenberg, Drew. *Advancing Beyond Advances in Behavioral Economics*. Journal of Economic Literature, ISSN 0022-0515, 09/2006, Volume 44, Issue 3, p. 2

Galton, Francis. *Vox Populi*. Nature, 03/1907, No 1949 Volume 75, pp. 450 – 451

Gilbert, Daniel T; Pinel, Elizabeth C; Wilson, Timothy D; Blumberg, Stephen J; Wheatley, Thalia P. *Immune Neglect: A Source of Durability Bias in Affective Forecasting*. Journal of Personality and Social Psychology, ISSN 0022-3514, 09/1998, Volume 75, Issue 3, pp. 617 - 638

Gneezy, Uri and Rustichini, Aldo. *Pay Enough or Don't Pay at All*. The Quarterly Journal of Economics, ISSN 0033-5533, 08/2000, Volume 115, Issue 3, pp. 791 - 810

Graham, J.L. *Illusory trends in the observations of volumes*. Journal of Experimental Psychology, ISSN 0022-1015, 1937, Volume 20, Issue 6, pp. 609 – 619

Herrmann-Pillath, Carsten. Evolutionary rationality, “Homo economicus,” and the foundations of social order. Journal of Social and Evolutionary Systems, ISSN1061-7361,1994, Volume 17, Issue 1, pp.41-69

Hess, James D and Holthausen, Duncan M. *Quantifying the Allais paradox*. Economics Letters, ISSN 0165-1765, 1990, Volume 34, Issue 1, pp. 21 - 25

Krueger, Joachim I and Funder, David C. *Towards a balanced social psychology: causes, consequences, and cures for the problem-seeking approach to social behavior and cognition*. The Behavioral and brain sciences, ISSN 0140-525X, 06/2004, Volume 27, Issue 3, p. 313

Kuilen, Gijs van de and Wakker, Peter P. *Learning in the Allais paradox*. Journal of Risk and Uncertainty, ISSN 0895-5646, 12/2006, Volume 33, Issue 3, pp. 155 – 164

Levitt, Steven D and List, John A. *What Do Laboratory Experiments Measuring Social Preferences Reveal about the Real World?* The Journal of Economic Perspectives, ISSN 0895-3309, 04/2007, Volume 21, Issue 2, pp. 153 - 174

Micceri, Theodore. *The Unicorn, The Normal Curve, and Other Improbable Creatures*. Psychological Bulletin, ISSN 0033-2909, 01/1989, Volume 105, Issue 1, pp. 156 – 166

Oliver, Adam. *A quantitative and qualitative test of the Allais paradox using health outcomes*. Journal of Economic Psychology, ISSN 0167-4870, 2003, Volume 24, Issue 1, pp. 35 - 48

Sergio Da Silva; Dinorá Baldo; Raul Matsushita. *Biological correlates of the Allais paradox*. Applied Economics, ISSN 0003-6846, 02/2013, Volume 45, Issue 5, p. 555

Suetens, Sigrid and Tyran, Jean-Robert. *The gambler's fallacy and gender*. Journal of Economic Behavior & Organization, ISSN 0167-2681, 06/2011

Surowiecki, James. *The wisdom of crowds: why the many are smarter than the few*. 2005, ISBN 0349116059, xxi, 295 s.

Tsang, Edward P. K. *Computational intelligence determines effective rationality*. International Journal of Automation and Computing, ISSN 1476-8186, 01/2008, Volume 5, Issue 1, pp. 63 - 66

Tversky, Amos and Kahneman, Daniel. *Availability: A heuristic for judging frequency and probability*. Cognitive Psychology, ISSN 0010-0285, 1973, Volume 5, Issue 2, pp. 207 - 232

Tversky, Amos and Kahneman, Daniel. *Judgment under Uncertainty: Heuristics and Biases*. Science, ISSN 0036-8075, 09/1974, Volume 185, Issue 4157, pp. 1124 - 1131

Tversky, Amos and Kahneman, Daniel. *The Framing of Decisions and the Psychology of Choice*. Science, ISSN 0036-8075, 01/1981, Volume 211, Issue 4481, pp. 453 - 458

Tversky, Amos and Kahneman, Daniel. *Loss Aversion in Riskless Choice: A Reference-Dependent Model*. The Quarterly Journal of Economics, ISSN 0033-5533, 11/1991, Volume 106, Issue 4, pp. 1039 – 1061

Weber, Bethany Jo. *Variations on the Allais paradox* 06/2005, ISBN 9780542412813, 106

Wright, Daniel B and Herrington, Joshua A. *Problematic standard errors and confidence intervals for skewness and kurtosis*. Behavior Research Methods, ISSN 1554-351X, 03/2011, Volume 43, Issue 1, pp. 8 - 17

6. Appendix

6.1. Raw Data

To import data, copy and paste the data below into a text file and save with the extension .csv. Most statistical software will be able to open the file.

Symmetric,27,26,24,26,18,25,29,27,25,25,12,18,22,28,27,16,23,30,23,23,33,24,30,30,35,38,33,29,28,2
7,30,21,30,34,30,19,35,15,42,17,4,18,21,23,26,24,26,30,32,28,25,19,8,32,31,21,41,22,25,32,28,34,26,3
0,20,26,20,24,32,32,29,25,20,34,25,19,21,27,40,25,35,23,24,22,32,41,40,31,23,25,31,17,33,26,24,28,2
0,27,32,28,35,34,28,37,29,34,15,31,33,22,35,36,28,18,28,22,35,26,30,25,28,23,22,32,22,15,10,25,30,3
4,35,22,33,42,20,15,27,27,37,40,28,37,45,24,25,30,48,20,36,28,12,26,28,23,22,28,40,16,32,35,32,9

Positive,40,28,31,20,19,34,15,32,35,27,19,23,30,27,22,19,20,26,26,27,40,23,9,10,14,12,22,20,34,32,21
,30,37,30,34,34,12,24,40,28,15,27,25,28,26,9,10,24,24,27,24,27,12,24,9,32,15,20,28,37,23,19,23,8,32,
27,33,18,26,23,31,27,28,35,27,30,28,17,25,24,20,24,28,15,23,36,28,25,18,10,24,16,26,24,22,29,28,30,
10,20,24,27,23,29,25,,,

Negative,20,34,15,34,28,24,25,25,28,32,25,21,10,21,19,25,16,25,23,30,19,23,21,15,17,34,23,21,25,40,
27,15,28,26,21,40,29,20,27,16,25,20,26,20,22,16,24,32,20,17,25,19,24,28,23,23,21,26,27,21,18,12,20,
37,33,12,32,27,22,30,14,32,30,27,32,30,27,32,26,27,31,27,18,22,30,22,25,21,37,35,32,32,32,25,22,21,
36,28,11,29,28,17,30,20,27,21,30,24,,

6.2. Card Game Instructions

The written instructions for all three conditions are presented on the next page.

Gissa Kortens Summa

LUNDS UNIVERSITET
Ekonomihögskolan

Kort	Poäng
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10
Knekt	11
Dam	12
Kung	13
Ess	1

Du får fyra slumpmässigt utvalda spelkort från tre kortlekar. Varje kort har ett värde enligt tabellen till vänster.
Det finns inga jokrar. Innan du tar upp och tittar på dina kort måste du gissa vad summan blir av dina fyra kort. Försök att gissa så att du maximerar chansen att vinna.

Gissa Kortens Summa

LUNDS UNIVERSITET
Ekonomihögskolan

Kort	Poäng
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10
Knekt	10
Dam	10
Kung	10
Ess	4

Du får fyra slumpmässigt utvalda spelkort från tre kortlekar. Varje kort har ett värde enligt tabellen till vänster.
Det finns inga jokrar. Innan du tar upp och tittar på dina kort måste du gissa vad summan blir av dina fyra kort. Försök att gissa så att du maximerar chansen att vinna.

Gissa Kortens Summa

LUNDS UNIVERSITET
Ekonomihögskolan

Kort	Poäng
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	10
Knekt	10
Dam	10
Kung	10
Ess	– 4

Du får fyra slumpmässigt utvalda spelkort från tre kortlekar. Varje kort har ett värde enligt tabellen till vänster.
Det finns inga jokrar. Innan du tar upp och tittar på dina kort måste du gissa vad summan blir av dina fyra kort. Försök att gissa så att du maximerar chansen att vinna.