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MILP Modelling of Production-related Disturbances

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MILP Modelling of Production-related Disturbances (Modellering av produktionsrelaterade störningar (med hjälp av linjärprogrammering))

Abstract
Rising cost awareness within the process industry has emphasized the need for better scheduling and modelling of manufacturing sites. In collaboration with the Process Industrial Centre at Lund University and Perstorp AB, this study aims to model utility disturbances in manufacturing sites. Utilities are services or equipment necessary for the production in one or several of the areas that a site consists of. The goal of the modelling is to minimise economic losses related to utility disturbances.

Manufacturing sites can be mapped as area networks with interdependent areas, buffer tanks and utilities that have defined capabilities. In this study a general method for modelling area networks as mixed integer linear programs (MILP) have been presented. If the utility disturbances length and scope are unknown it is possible to use stochastic programming in the model, which would enable minimisation of losses by optimising over a number of scenarios, or guesses, of how a disturbance will unfold.

To demonstrate how a site can be represented by a MILP-model, an area network corresponding to a part of the Perstorp AB Stenungsund site has been modelled. The main result,from optimising the modelled site are production rates for the areas, which would minimise the economic losses from a specific disturbance.

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Contents

1

Introduction

1.1 Background

Historically the forest industry, iron ore mining and hydroelectric power have been of great importance for Sweden. From these roots the Swedish process industry has developed into a high-tech industry, which produces refined products. [IVA, 2006] highlights the process industry sectors: Pulp and paper, chemicals and plastics, pharmaceutics, mining, iron/ steel and food processing. High productivity, high level of automation and export orientation are characteristics for this industry in Sweden.

Challenges for the Process Industry

Recently the industrialisation of China and Eastern Europe have imposed pressure on the industry to reduce costs and innovate faster to stay competitive. One way to stay ahead of competition is development of more sophisticated information systems for control and supervision of the manufacturing processes [IVA, 2006]. In a more recent report about the challenges in enterprise-wide optimisation, the same pressure from the global marketplace in relation to the U.S process industry is identified [Grossmann and Furman, 2009]. Even though enterprise-wide optimisation involves optimising all aspects of the business to reduce costs, the scheduling and modelling of manufacturing facilities are identified as a major focus of this trend [Grossmann and Furman, 2009].

Process Industrial Centre at Lund University

This study stems from a problem related to the utility disturbance management research-project within the Process Industrial Centre at Lund University (PIC-LU). Within this centre, the aim is to conduct research and develop competence in process automation and control in collaboration with the Swedish process industry. From the Faculty of Engineering at Lund University the departments of Chemical Engineering and Automatic Control participate in PIC-LU. The PIC-LU centre is founded by the Swedish process industry and the Foundation of Strategic Research (SSF).

Perstorp AB

Perstorp AB is an international company in the process industry focused on speciality chemicals. Perstorp has manufacturing sites in eleven countries [Perstorp AB, 2010]. Perstorp AB manufactures paints and coatings, materials, adhesives, feed and food, synthetic lubricants, formalin technology and catalysts fuels. The Perstorp AB production site in Stenungsund has been used as an example site in this study. In the Stenungsund site Perstorp AB produces biofuels and plasticisers. The biofuel may be used for blending into fossil diesel or used as 100% renewable fuel. Plasticisers are used by many industries like automotive, construction consumer products and cable & wires. Currently the growth of the oxo (biodegradable plastic) market has been driving the expansion of the Stenungsund site [Perstorp AB, 2010].

1.2 Problem Description

How to operate a site within the process industry is typically known when the site is running without disturbances. But when equipment that delivers services for the production fails, the production may stop or be reduced. Examples of required services, referred to as utilities, are electricity, steam and cooling-water. Unpredictable failures force the site's operators to make decisions for minimising the losses. However, knowing how to respond to utility-disturbances becomes increasingly hard since sites can be divided into a network of several areas. Each area needs one or several utilities in order to be able to produce and each utility has a different likelihood, length and reason for failure.

Aim of the Study

If the operators at, for example the Stenungsund site, could experiment with a model of the site, they would be better equipped to take the appropriate measures when operating the site. The model needs to be able to simulate failure in utilities and their affect on the production.

The next step would be to find the optimal operating trajectories for the model during a disturbance, and finally a method for modelling uncertainty is needed to model the fact that the length and scope of a disturbance may not be known in advance. To summarise, this study aims to:

1. Build a model capable of representing an area network with utilities that can affect the production in one or several areas.
2. Find a method for optimising the production in the areas that minimises the revenue loss during a disturbance in one of the utilities.
3. Introduce uncertainty so the conclusions drawn from the model mimics the fact that operators seldom have perfect information about disturbances in advance.

Focus and Limitations

This report is focused on finding and verifying that the solution to the posed problems are suitable for solving problems related to utility disturbances. So far the utility disturbance management at the site level does not fit well into any research area, which makes it difficult to find relevant work to expand upon. Therefore, this study will have to start more or less from scratch, and through trial and error and collaboration with the industry find a solution that can satisfy the aims.

The study will be limited to looking for solutions in the fields of modelling, operations research and production & operations analysis.

2

Theory

2.1 Hierarchical Structure of Enterprises

Enterprises that conduct discrete or continuous manufacturing usually have their assets used for production divided into a hierarchical structure. At the top is the enterprise level, followed by one or several sites. A site is generally defined by its geographic location or what is a logic distinction depending on its production [ISA-95, 2009].

For this study, the focus will be on the area level that resides within the site. Areas are, just as sites, defined by geography (proximity) or based on what they produce. An important characteristic of areas are that they generally have well defined manufacturing capabilities and capacities which makes them good building blocks within a model.

Flowcharts and Dependencies

Flowcharts can be used to illustrate the flow of products or material through the enterprise, plants, areas or units [Lindholm, 2011].

Here it is appropriate to construct flowcharts displaying the flow of material between areas. The buffer tanks available for storage between areas will also be shown as separate blocks in the flowchart. But physically the buffers may be found inside an area, or at the other side of the factory, which means the physical layout of the plant may not resemble the flowchart. An example flowchart of the Stenungsund site is displayed in Figure 2.1.

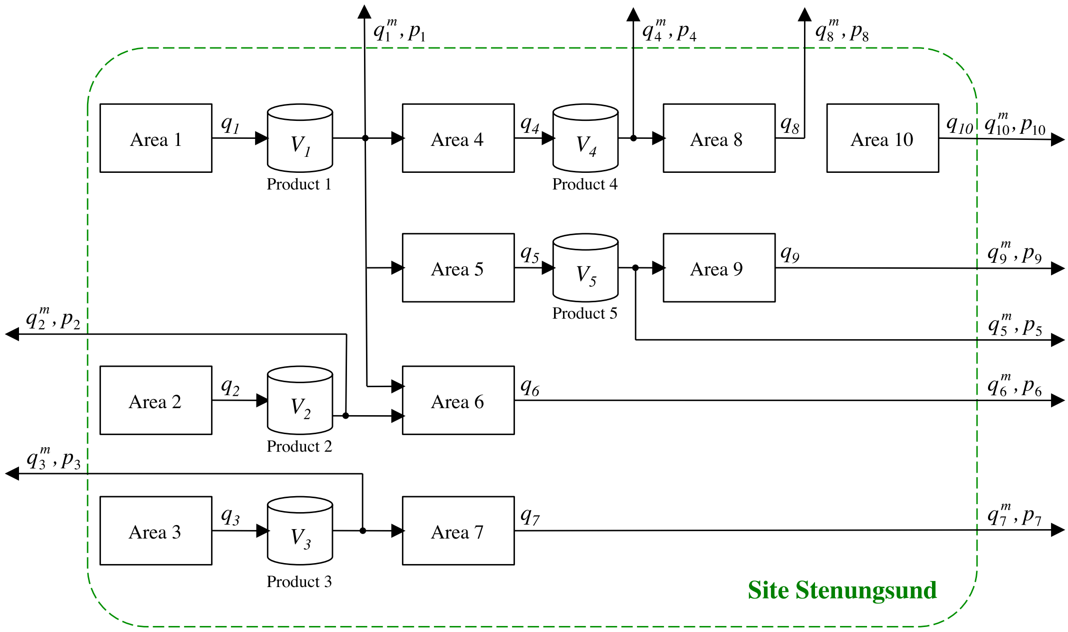

Figure 2.1: Flowchart of the Stenungsund site

Storage Units

Of several lower-level elements that may be found within an area, the storage zone, which in turn contains storage units, is worth mentioning here.

Storage areas contain buffer tanks also known as intermediate storage vessels, holding tanks, surge drums, inventories etc. Buffer tanks are of interest here because their use in between areas can allow independent operation of areas upstream and downstream from the buffer tank. This may be necessary if for example there is a temporary shut-down of the areas feeding the buffer tank [Faanes and Skogestad, 2003].

2.2 Utilities

In the process industry the support processes that are necessary for production, but are not part of the end product, are called utilities [Lindholm, 2011]. In the [ISA-95, 2009] standard these processes are defined as material, and they may also be referred to as services. Examples of utilities are steam, fuel, electricity, cooling water, raw water, compressed air, nitrogen, and refrigerated coolants [Brennan, 1998].

Steam is used for heating or providing energy and can be needed in distillation or reaction processes, and cooling water may be needed for the distillation phase or for cooling the reaction processes [Lindholm, 2011].

Disturbances in Utilities

Normally utilities do not affect the production unless some disturbance interrupt the service, or pushes the utility properties over a limit beyond which the production is affected. For example cooling water could have a temperature limit over which the production is affected negatively. When the utilities operate outside their limits a disturbance is said to have occurred [Lindholm, 2011].

This is illustrated in an example from [Lindholm, 2011], where the temperature limit for cooling water is examined by plotting maximum production against cooling water temperature. The graph for one area at an industrial site is displayed in Figure 2.2, and has the suggested limit (27°C) represented by the red dashed line.

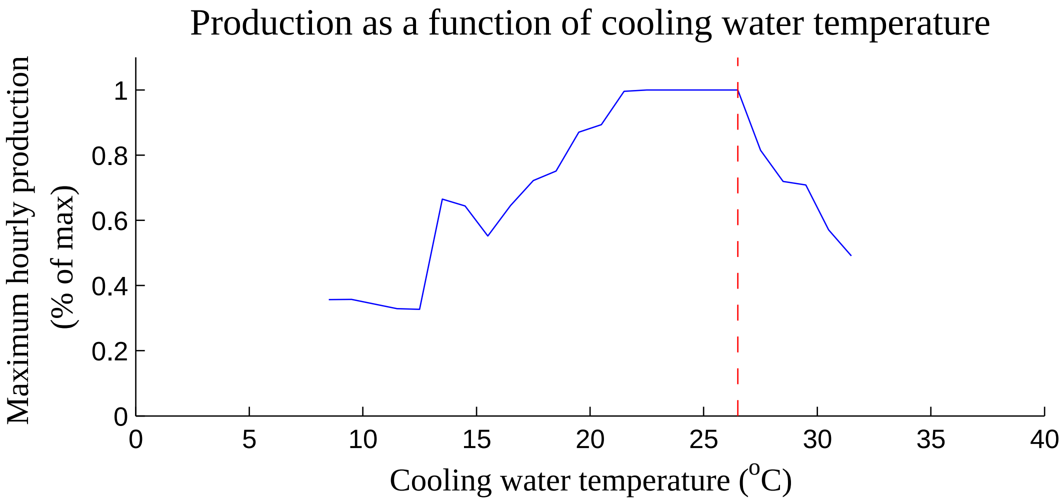

Figure 2.2: Production as a function of cooling water temperature

The effect of a utility disturbance that rises the temperature of the cooling water will only affect the areas production if it pushes the temperature over the limit. In the [Lindholm, 2011] example, cooling water temperatures above this limit reduces the areas maximum production in a somewhat linear way, but not all areas necessarily respond in such an ideal way. Some may respond in nonlinear ways, and the production may also have been changed for reasons unrelated to utility disturbances [Lindholm, 2011].

In discussions with industry representatives it has been noted that some areas may be prioritised during utility disturbances at the expense of other areas.

Start-up Costs

Areas within a manufacturing plant are usually considered hard to start. An example of equipment with this characteristic are distillation columns. Start-up of distillation columns usually takes long time, during which energy and raw material is wasted since the initial products do not meet specification requirements. [Wozny and Li, 2004].

This has also been pointed out by industry representatives stating that preferably the flow of material through an area should not go under a critical level, else the production cannot continue, and the products are of poor quality and have to be discarded.

Because of this behaviour, modelled areas are here considered to have a lower limit of operation. This means areas may only be stopped or has to produce at or above the lower limit. If it is necessary to stop an area, the start-up procedure that follows will be represented by a penalising start-up cost.

2.3 Performance and Economic Indicators

Evaluating the performance of areas or sites in economic terms requires the knowledge of some economic indicators, which are introduced here (in alphabetic order).

Availability dictates how large part (normally in percent) of the total time of a period that the facility have been operational. It might be more convenient to use downtime, which is measured not as a fraction but a length of time (hours, year, etc.) that the facility has been stopped [Forsman, 2005].

Contribution margin for each sold product is the selling price minus the variable costs. It is called contribution because this margin have to contribute to paying off the fixed costs and hopefully even provide a profit [Forsman, 2005].

Fixed costs are costs that are independent of how large quantities that are produced. Some good examples are property rents, salaries and depreciation of value in production equipment [Forsman, 2005].

Holding cost is commonly used in inventory theory, and represents all the costs associated with holding inventory. This could include costs for binding capital, space, insurance, protection and taxes attributed to storage [Hillier and Lieberman, 2001].

Market limited production plants have spare capacity since the market will not buy everything the plant can produce. External factors such as price and demand structure in the market will affect this limit [Forsman, 2005].

Plant utilisation is how much the facility has produced during a time interval divided by its production capacity during that interval. Or more precisely: *Availability* $\times$ *production rate* $\div$ *production capacity* [Forsman, 2005].

Production capacity is the physical upper limit for of how much a facility can produce per time period [Forsman, 2005].

Production rate is the amount produced in the facility per unit of time [Forsman, 2005].

Variable costs are the costs that can be directly attributed to the production of another unit or amount of the product. This can for example be costs of raw material that goes into the product or energy used for the production process [Forsman, 2005].

2.4 Linear Programming (LP)

Developed in the mid-20th century linear programming (LP) has been widely used as a tool for allocating limited resources among competing activities in an optimal way. In LP, the problem is expressed as a mathematical model of linear functions. The decisions are represented by a number of decision variables that are included in the objective function, which then are to be maximised or minimised. The objective function is restricted by a number of constraints including non-negativity constraints [Hillier and Lieberman, 2001]. If x are our decision variables, then our LP problem can be stated as (2.1).

$$\min c^T x \text{ subject to } \begin{cases} Ax = b \\ x \geq 0 \end{cases} \tag{2.1}$$

Where $x \in \mathbb{R}^{n\times 1}$, $b \in \mathbb{R}^{m\times 1}$, $c \in \mathbb{R}^{n\times 1}$ and $A \in \mathbb{R}^{m\times n}$

If a solution exists that satisfies all constraints, it is called a feasible solution, otherwise the solution is said to be infeasible. Optimal solutions are feasible solutions that generate the best value of the objective function, which could be a maximising or minimising objective. If the optimal solution grows to infinity, the problem is called unbounded [Goemans, 1994].

Mixed-integer Linear Programming (MILP)

If all decision variables are constrained to only take on integer values, the problem is called an integer program (IP). When both integer and continuous decision variables are allowed the problem is called a mixed-integer program (MIP) [Smith and Taşkin, 2007]. If all of the functions in a MIP are linear it is called a mixed-integer linear program (MILP) [Grossmann *et al.*, 1999].

IP and MIP expansions of (2.1) is stated similarly but with an additional condition that some or all variables x_i may only be integer [Smith and Taşkin, 2007].

By constraining some decision variables to only take integer values IP or MIP models may better represent real systems since integrality of quantities, if-then statements, enforce at least k out of p restrictions and non-linear product terms can be modelled [Goemans, 1994].

Robustness of the Solution to a LP or MIP

If there are two decision variables and two constraints, in addition to $x_1 \geq 0$, $x_2 \geq 0$, then a LP problem, originally from [Böiers, 2010], can be illustrated as in Figure 2.3. In this plot the feasible set S is shown in grey, and the axes of the figure are the decision variables x_1 and x_1. The objective function f decides the slope of the bold straight line, and grows in the direction indicated by the perpendicular arrow. Any optimal solution minimising f must then be found in one of the points A, B or C. Which point that is optimal depends on the slope of f and the constraints. Since f and the constraints are linear there may be times when small changes made in f or the constraints make the optimal solution move from one point to another [Böiers, 2010].

The behaviour just described can be a problem in this study since the optimal solutions will represent decisions for how to operate a site during utility disturbances. If the solution is not robust big changes in the production plans may be suggested in response to small changes in the model input, or the solution may not be optimal for very long.

Solving MIP Problems

Solving MIP problems can be done with dedicated LP optimisation codes or by extending the 'linprog' function in Matlab. In this study an open-source optimisation code called LPSolve will be used. This program solves LP and MIP problems with the revised simplex method and the branch-and-bound method for MIP [Thorncraft *et al.*, 2006].

Computational speed is not a key issue in this study, but it is good practice to understand the basics of solving your own models. If larger systems or longer evaluations will be of interest it might be good to have knowledge of how modelling practices affect the solving speed.

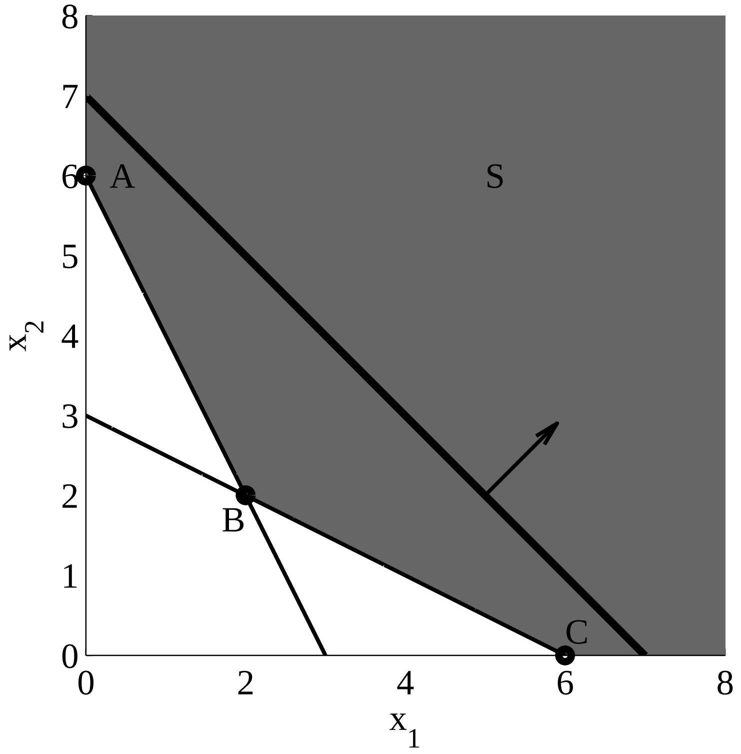

Figure 2.3: Illustration of a LP example

To illustrate the branch-and-bond algorithm, consider an example from [Smith and Taşkin, 2007]:

$$
\begin{aligned}
\min\ & 4x_1 + 6x_2 \\
\text{s.t. }\ & 2x_1 + 2x_2 \geq 5, \\
& x_1 - x_2 \leq 1, \\
& x_1, x_2 \geq 0 \text{ and integer.}
\end{aligned}
\tag{2.2}
$$

If the problem is relaxed to allow non-integers then the feasible region of the problem will be the grey area in Figure 2.4. For the relaxed problem a minimum objective function value of 11.5 is found in the point (1.75, 0.75). But for the original MIP problem this solution is infeasible since x_1 and x_2 are not integers. The feasible solution to the MIP problem must be equal to or larger than the solution to the relaxed problem, which thus provides a lower bound for integer solutions. If the problem then is split into two sub problems as in Figure 2.5 one of the infeasible regions can be excluded and for each of the two new areas the best relaxed solution can be searched for.

The procedure above is called branching, and the areas in Figure 2.5 are branched further until the optimal relaxed solution for an area is found to be feasible. The algorithm will also stop branching an area when its relaxed solution is larger than any other area's feasible solution.

Since the branch-and-bound algorithm requires solving many relaxed problems and compare the solutions it can be computational intensive. It is thus wise to use as few integer variables as possible when setting up large MIP problems.

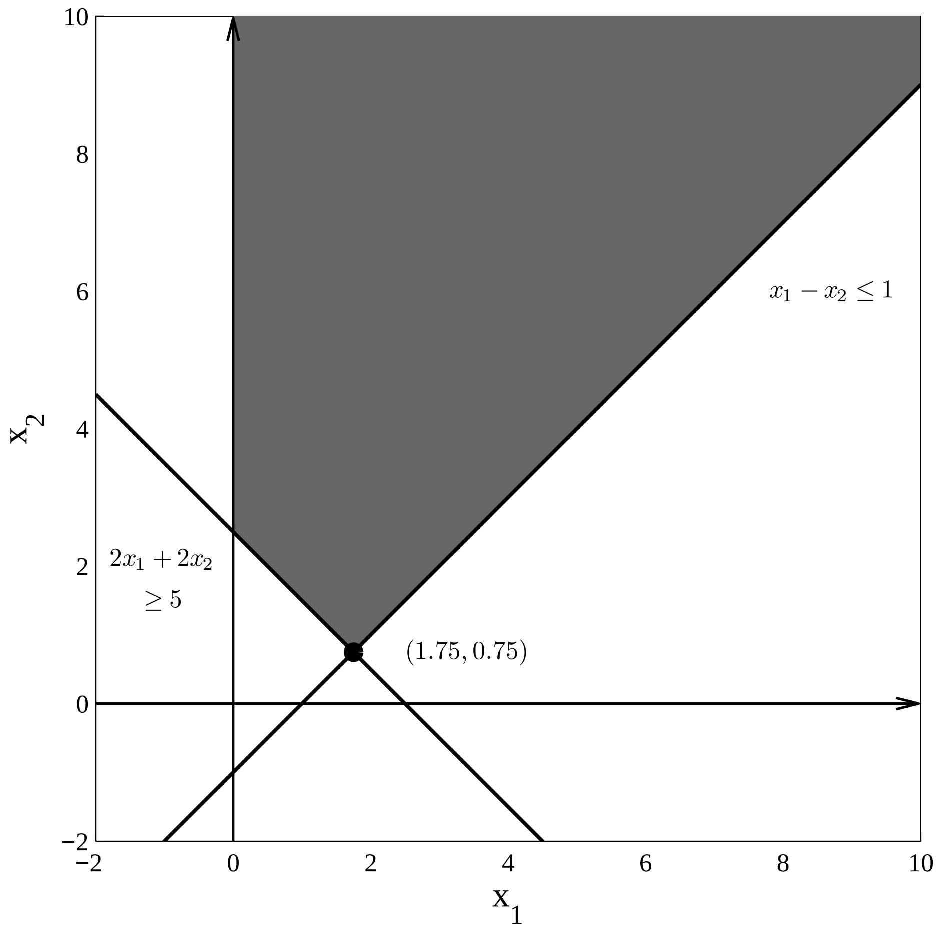

Figure 2.4: Feasible region of the LP relaxation

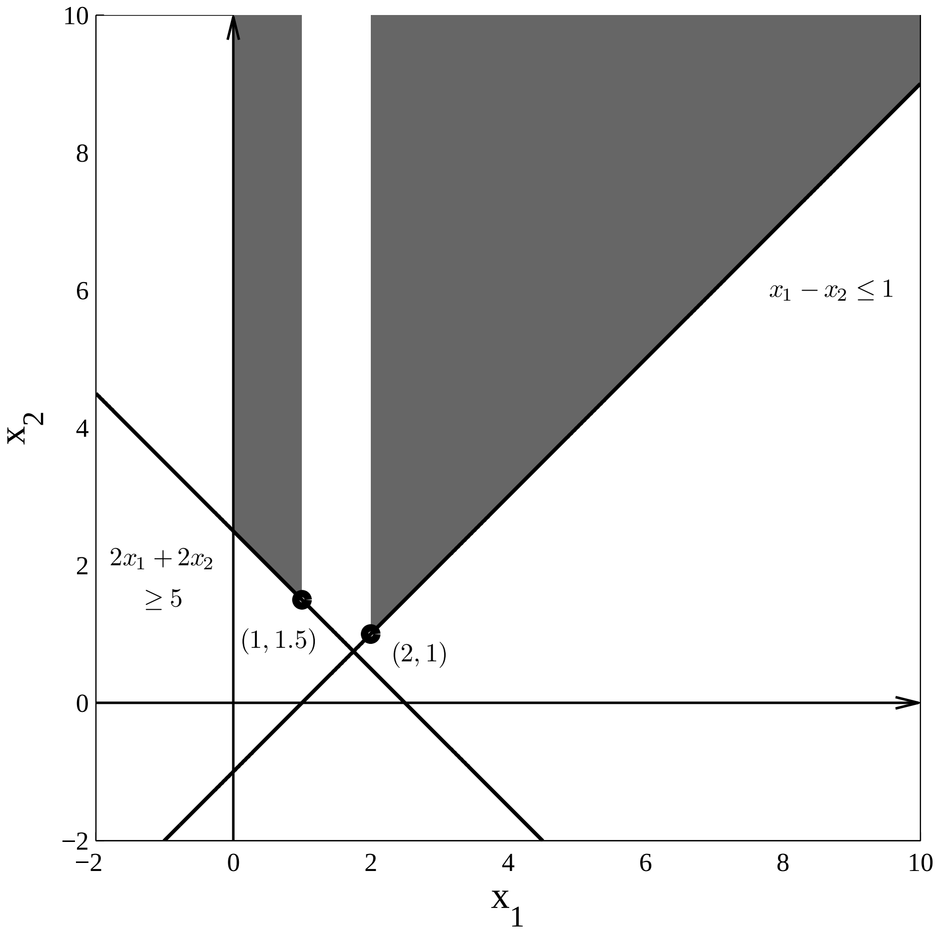

Figure 2.5: Feasible regions for the subproblems

2.5 Stochastic Programming

When solving LP or MIP problems it is assumed we have full information of the constraints. If some constraints represent future limitations that may be uncertain stochastic programming can be used to optimise the present decisions in relation to the uncertainty of future events.

Farmer's Example

An illustrative example, adapted from [Smith and Taşkin, 2007], is farmer allocating land to crops without knowledge of how weather conditions will affect future crop yields. Data for an example with two types of crops: wheat and corn is presented in Table 2.1.

	Wheat	Corn
Normal yield (Ton/acre)	2.5	3
High yield (Ton/acre)	3	4
Low yield (Ton/acre)	2	2
Planting cost ($ /acre)	150	230
Selling price ($ /Ton)	170	150
Purchase price ($ /Ton)	238	210
Minimum requirement (Ton)	200	240
Total Available land: 500 acres		

Table 2.1: Data for farmer's problem

By using the data in Table 2.1 a LP model for the farmer's example can be expressed as in (2.3), where x are the crop's yield, y the purchased amount of crops and w the sold amount of crops. The objective function in (2.3) tries to minimise the cost. The first constraints is a limit for total available land followed by constraints for the minimum required crops the farmer needs to have left after harvesting and any purchases or selling of crops.

$$\begin{aligned} \min\ & 150x_1 + 230x_2 + 238y_1 + 210y_2 - 170w_1 - 150w_2 \\ \text{s.t.}\ & x_1 + x_2 \leq 500, \\ & 2.5x_1 + y_1 - w_1 \geq 200, \\ & 3x_2 + y_2 - w_2 \geq 240, \\ & x, y, w \geq 0. \end{aligned} \tag{2.3}$$

In stochastic programming, initial decisions that is taken without full information are called first-stage decisions. Decisions made later, when full information is available, are called second-stage decisions [Smith and Taşkin, 2007]. For the farmer land allocation in springtime is first-stage decisions, and buying or selling of crops come autumn is second-stage decisions.

Assuming the random variables affecting the crop yields are correlated, then the farmer can try to maximise the expected profit of his first-stage decision by introducing scenarios for the low and high crop yields listed in Table 2.1. The LP model from (2.3) can now be expanded to consider first and second-stage decisions with two scenarios of equal probability. In (2.4) this is done by splitting the crop's yield variables y and the purchased amount variables w into one set of variables for each scenario and then add them to the objective function according to the probability of the scenarios. The high and low yields from Table 2.1 are the new coefficients in front of the x variables in the constraints.

$$\begin{aligned}
\text{Min } 150x_1 + 230x_2 + \frac{1}{2}(238y_{11} + 210y_{21} - 170w_{11} - 150w_{21}) \\
+ \frac{1}{2}(238y_{12} + 210y22 - 170w_{12} - 150w_{22}) \\
\text{s.t. } x_1 + x_2 \leq 500, \\
3x_1 + y_{11} - w_{11} \geq 200, \\
2x_1 + y_{12} - w_{12} \geq 200 \\
4x_2 + y_{21} - w_{21} \geq 240, \\
2x_2 + y_{22} - w_{22} \geq 240, \\
x, y, w \geq 0.
\end{aligned} \tag{2.4}$$

General Formulation

A general formulation of a two-stage stochastic program can be expressed as in (2.5). E_ξ is mathematical expectancy with respect to the scenario vector ξ where coefficients that are changed with the scenarios is stored. The optimal second-stage decisions are represented by $Q(x, \xi)$ [Smith and Taşkin, 2007].

$$\min c^T x + E_\xi Q(x, \xi) \text{ subject to } \begin{cases} Ax = b \\ x \geq 0 \end{cases} \tag{2.5}$$

2.6 Time Representation

For the basic case, the objective is evaluated over just one period of time during which only one decision is necessary. However, it might be of interest to take different actions depending on fluctuations of model properties and constraints over time.

Therefore, a discrete time representation is introduced as in [Kondili *et al.*, 1993], by which the period of interest is divided into several intervals of equal length. Changes to the model, and actions upon any decision variable are only allowed to take place between the intervals $t = t_1, t_2, t_3, ..., t_H$. Note that t_n is a referral to the discrete time interval from t_n to the beginning of the next discrete time interval t_{n+1}.

The length of the intervals depends on what properties that are being investigated. How many periods to be used is not only determined by the properties of the system that is investigated, but also by how computational intense each evaluation of an interval is.

3

MILP Model for Area Networks

This chapter builds on the mathematical formulation for short-term scheduling of batch operations presented in [Kondili *et al.*, 1993]. The original MILP problem introduced in Subsection 2.4 is adapted to fit area networks affected by utility disturbances.

The model suggested in this chapter can be illustrated as in Figure 3.1. Input to the MILP model are vectors $X(t_0)$ and $U(t)$, where $X(t_0)$ defines the initial states of the model and $U(t)$ describes the production capacity of the utilities at each t. The result from the optimisation, $X^*(t)$, is the optimal states for each t, and f_{X^*} is the result given by the objective function as in (2.1) such that $f_{X^*} = c^T X^*$.

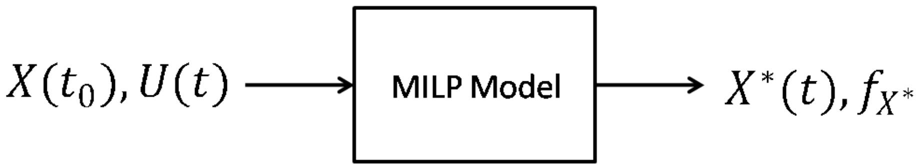

Figure 3.1: Input output representation of the MILP model

3.1 Constraints

For the model to be in the form of (2.1), constraints from [Kondili *et al.*, 1993] are adapted to represent physical limitations of the actual system. Simplifications have to be made, and it is assumed areas furthest upstream in the network have unlimited supply of raw material, and that the site is not market limited. It is necessary to have an upper limit for the amount of product each area can produce, the amount of available utilities, and how much the buffer tanks can hold at any given period. As discussed in Section 2.2, most areas can only operate at a minimum speed or they have to be shut down. This has to be represented in the model as well.

In order to represent the capacity limitations, and the option to stop an area, the following parameters are introduced:

$q_i(t)$	= Production rate for area i in period t;
q_i^{max}	= Production capacity for area i;
q_i^{min}	= Production rate under which area i has to be stopped;
$W_i(t)$	= Binary parameter that is 1 if area i is running at t, else it is 0.

The parameter $W_i(t)$ was used in [Kondili *et al.*, 1993] to handle allocation constraints, and here it will be used to handle stops and start-ups.

If area i is not run at period t then $W_i(t) = 0$ and no material can be processed by the area, which means the amount of material that starts being processed at area i at period t is constrained by (3.1).

$$W_i(t)q_i^{min} \leq q_i(t) \leq W_i(t)q_i^{max} \tag{3.1}$$

Next constraint connects the areas and buffer tanks by defining the material balances and uses the parameters:

$P_i(t)$	= Amount of product i being sold at period t;
$I_i(t)$	= Amount of material held in buffer tank i at period t;
I_i^{max}	= Maximum volume that buffer tank i can hold;
I_i^{min}	= Minimum allowed volume in buffer tank i.

If C_i is a set of the areas that feed buffer tank i, and $\bar{C}_i$ is a set of areas which receives material directly from the buffer tank, and P_i is the flow of the product from the buffer tank to the market, then the material balance of the system for each t has to satisfy:

$$I_i(t) = I_i(t-1) + \sum_{k \in C_i} q_k(t) - \sum_{k \in \bar{C}_i} q_k(t) - P_i(t) \tag{3.2}$$

$$I_i^{min} \leq I_i(t) \leq I_i^{max} \tag{3.3}$$

There are several ways to represent how utilities affect the production for areas. A general formulation used in [Kondili *et al.*, 1993] requires the parameters:

β_{pi}	= Utility p required for each output from area i;
$u_p(t)$	= Total demand for utility p at period t;
u_p^{max}	= Production capacity for utility p;
$U_p(t)$	= Availability for utility p at period t;

Some utilities can be seen as production units that produce a product or service that some areas are dependent on. Then β_{pi} can be used to have areas more or less dependent on utility p. The parameter u_p^{max} allows $U_p(t)$ to be between 0 and 1. Let C_p be a set of the areas that uses utility p. Then the limitations for utility p are given by:

$$u_p(t) = \sum_{k \in C_p} \beta_{pi} q_k(t) \tag{3.4}$$

$$0 \leq u_p(t) \leq U_p(t) u_p^{max} \tag{3.5}$$

This means that there is a linear relation between utility assignment to an area, β_{pi}, and the production in the area, $q_i(t)$.

If any area is stopped and then restarted, a start-up cost should be imposed on the objective function. Therefore, a constraint is needed, stating that if $t_1 < t_2$ and:

(i) No task is performed at area i at t_1 and;

(ii) Any task is to be performed there at t_2

then a start-up has to be made. To represent this, an additional parameter and constraint is needed:

$S_i(t)$ is an integer that is 1 if $W_i(t_2) = 1$ and $W_i(t_1) = 0$, else it is 0.

With the constraint expressed as:

$$W_i(t_2) - W_i(t_1) \leq S_i(t_1) \leq 1 \text{ , where } t_1 < t_2 \tag{3.6}$$

This constraint limits $S_i(t_1)$ in such a way that it must be 1 if $W_i(t_1) = 0$ and $W_i(t_2) = 1$. Therefore, a stop cost for area i, signalled by the parameter $S_i(t_1)$ taking the value 1, will only be imposed if area i has been stopped at period t_1, and then started at any later period t_2. For practical reasons, a start-up cost will also be imposed if a stopped area remains stopped at the final period of the evaluation.

3.2 Objective Function

Next step in the model formulation is to define the objective function, that will be maximised or minimised. Here a revenue maximising objective function will be used, but other functions could just as readily be used with the same constraints. Parameters needed to express this objective function are:

m_i = Contribution margin for product i;
s_j = Start-up cost for area j;
h_k = Holding cost per time unit for buffer tank k;
H = Number of periods in the evaluation.

With decision parameters taken from the constraints the objective function can be expressed as:

$$\text{Max} \sum_{t=1}^{H} \left(\sum_i m_i P_i(t) - \sum_j s_j S_j(t) - \sum_k h_k I_k(t) \right) \tag{3.7}$$

Equation (3.7) states that for every sold quantity of product i, we earn its margin m_i. The profit is then decreased by any costs from start-ups that might have been necessary, and the holding cost for buffer tanks I_k also decreases the profit.

3.3 Implementation

A MILP model can be implemented directly in Matlab by extending the inherit function called 'linprog' with the use of the branch-and-bound algorithm, or a dedicated MILP solver software can be used. In this study, the model was implemented in Matlab. But for solving the optimisation problem, an external open source solver package called LPSolve was used [Thorncraft *et al.*, 2006]. This solver was called from within the Matlab environment by the use of a plug-in which guaranteed efficient transfer of information between the programs.

4

Stochastic Programming MILP Model for Area Networks

In this chapter, the MILP model for area networks is expanded so it can be used with scenarios, as in Section 2.5. Scenarios is a way to make the model factor in uncertainty when searching for an optimal solution. This expanded model is here called a Stochastic Programming MILP (SP-MILP) model.

The major changes from the MILP model are that the SP-MILP model has the additional inputs $M(t)$ and $P(M)$, as seen in Figure 4.1. The $M(t)$ vector contains estimations or guesses of future utility availabilities, here called scenarios. The estimated probabilities for each of the scenarios to be realised is found in $P(M)$. The other inputs $X(t_0)$ and $U(t)$ stays the same as in Chapter 3. Output from the SP-MILP model will as before be $X^*(t)$ and f_{X^*}.

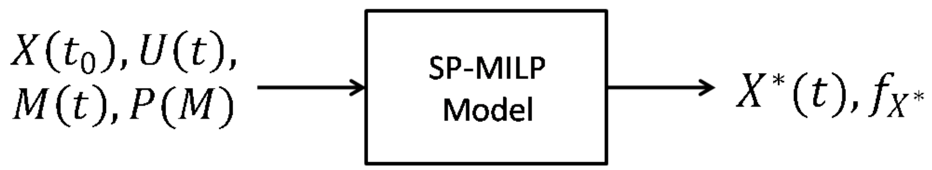

Figure 4.1: Input output representation of the SP-MILP model

4.1 Scenarios

The scenario-vector $M(t) = [M_1(t), M_2(t), M_3(t), ..., M_q(t)]$ contains one vector, of same length as t, for each of the q scenarios. $M(t)$ could just as well be expressed as a matrix, but in this study they are implemented as a row vectors, and therefore described as such.

Each of the scenarios also have a probability for being realised, which are found in the probability vector $P(M) = [P(M_1), P(M_2), P(M_3), ..., P(M_q)]$. Since the scenarios are mutually exclusive, $\sum P(M)$ has to be 1. In this study the SP-MILP model only allows scenarios for one property, which here will be a utility. If scenarios for more properties are to be included $M(t)$ should be expanded.

It will not be possible to revise the scenarios while running the model. This is notable because it would be a better representation of the real system if the scenarios could be updated, e.g. representing an operator that receives additional information about the disturbance as time progresses, and takes actions accordingly.

Later it will be shown that it would be simple to amend the system to take into account scenario updates, but scenario gathering and handling is not the focus of this study, and it is sufficient to demonstrate that scenarios may be used to mitigate uncertainty.

4.2 First and Second-stage Decisions

First and second-stage decisions, introduced in Section 2.5, are used here to tie the model to the actual limits caused by a disturbance.

Because of the discrete time representation, our first-stage decisions are said to be the decision variables for the nearest time period, that is between t_0 and t_1. From that follows the condition that we must have full information from t_0 to t_1.

The second stage decisions will be divided into a number of scenarios, $M_1,\ M_2,\ ...\ , M_q$, with known probabilities, $P(M_1),\ P(M_2),\ ...\ , P(M_q)$. In the current model, the scenarios are expected future trajectories, and may not be changed during the optimisation.

4.3 Revised Constraints

Since utilities are of interest here, the changes needed in the constraints to include the scenarios will only affect the utility constraints. With scenarios the utility constraints from (3.5) now has the form:

$$
\begin{gathered}
0 \leq u_p(t_0) \leq U_p(t_0) u_p^{max} \\
\text{and,} \\
0 \leq u_{p_1}(t) \leq M_1(t) u_p^{max} \\
0 \leq u_{p_2}(t) \leq M_2(t) u_p^{max} \\
0 \leq u_{p_3}(t) \leq M_3(t) u_p^{max} \\
\vdots \\
0 \leq u_{p_q}(t) \leq M_q(t) u_p^{max}
\end{gathered}
\tag{4.1}
$$

It is necessary to use one constraint for the first-stage decision, at t_0, and q constraints (one for each scenario) when $t > t_0$.

4.4 Revised Objective Function

The objective function (3.7) must be expanded to accommodate the scenarios. This will be done by removing the time steps following t_0 from the part of the objective function representing the first-stage decision, that is the first row in (4.2). Following the first row in (4.2) is the objective functions for the scenarios evaluated over the subsequent time periods $t > t_0$.

$$
\begin{aligned}
\text{Max } & \sum_{i=1}^{h} m_i P_i(t_0) - \sum_{j=1}^{n} s_j S_j(t_0) - \sum_{k=1}^{n} h_k I_k(t_0) \\
+P(M_1) & \sum_{t=t_0+1}^{H} \left(\sum_{i=1}^{h} m_{i,1} P_{i,1}(t) - \sum_{j=1}^{n} s_{j,1} S_{j,1}(t) - \sum_{k=1}^{n} h_{k,1} I_{k,1}(t) \right) \\
+P(M_2) & \sum_{t=t_0+1}^{H} \left(\sum_{i=1}^{h} m_{i,2} P_{i,2}(t) - \sum_{j=1}^{n} s_{j,2} S_{j,2}(t) - \sum_{k=1}^{n} h_{k,2} I_{k,2}(t) \right) \\
& \vdots \\
+P(M_q) & \sum_{t=t_0+1}^{H} \left(\sum_{i=1}^{h} m_{i,q} P_{i,q}(t) - \sum_{j=1}^{n} s_{j,q} S_{j,q}(t) - \sum_{k=1}^{n} h_{k,q} I_{k,q}(t) \right)
\end{aligned}
\tag{4.2}
$$

Now (4.2) will make sure that the first-stage decisions will not be too costly when considering the possible future scenarios. Because decisions taken now for a small gain, that may cause big losses in one of the scenarios, will be less profitable.

4.5 Implementation

If the SP-MILP model is run incrementally with a fixed H, then at each time interval t_n a first stage decision will be taken based on future scenarios. Practically this can be done in a for-loop, with the result and the decisions for each time step saved in a vector. This differ from only running the SP-MILP model once from t_0 to t_H, which assumes our scenarios are the exact future trajectories for the utility availabilities. Since that is not equivalent to a solution with full information of $U(t)$, it might lead to situations where the suggested solution is infeasible because the availability $U(t)$ is only considered in t_0. Instead, the first stage decisions at t_0 will be taken with full information but these decisions will be limited by the actual capacity of the utilities since $U(t)$ will be known for the next time step.

To summarise, solving of the SP-MILP model when $t = t_n$ follows a simple heuristic rule:

(i) Solve the model for t_n with regards to $U(t_n)$, which is known, and the future scenarios where $t > t_n$ and $U(t)$ unknown;

(ii) Apply the decisions for the interval t_n to t_{n+1}, suggested by (i);

(iii) Let time propagate from t_n to t_{n+1} where $U(t_{n+1})$ is known;

(iv) Start from (i) with t_{n+1} instead of t_n.

5

Example Model

5.1 Flowchart

In order to demonstrate how a site within the process industry can be represented by a SP-MILP model, a part of the Stenungsund site is modelled. A flowchart of the Stenungsund site, with the smaller example highlighted, is displayed in Figure 5.1. A smaller system is chosen in order to limit the number of variables, and keep the results easy to grasp.

The example site consists of three areas combined in a network, where the product from area one (A_1) is delivered to area two (A_2) and area three (A_3). The output from A_1 could also be stored in the buffer tank (I), or sold as product 1 (P_1). Production in A_2 and A_3 are equivalent with the delivered amount of products (P_2 and P_3). Also note that the example site is not market limited.

5.2 Time Representation

A discrete time representation will be used within the model. The length of each step t_i for this model is five minutes. The model will be evaluated in 12 steps making the length of the simulation 60 minutes.

Past time will be shown in the graphs as negative time just to give a reference of what has preceded the disturbance. The model starts evaluating from $t = 0$, so the past time reference points is added only in the plots.

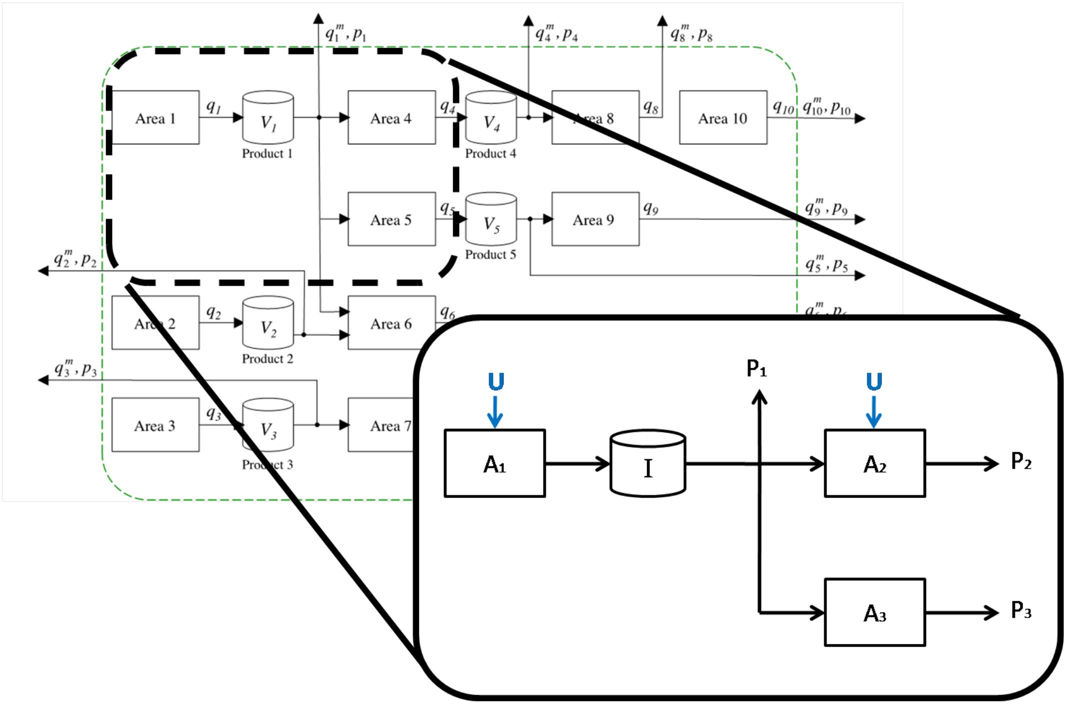

Figure 5.1: Flowchart of the example model in relation to the Stenungsund site

5.3 Utilities

Only one utility (U), steam, will be modelled in this example. The production rate of the areas A_1 and A_2 are affected by U.

5.4 Disturbances

If the historical disturbance data for steam are analysed, some typical disturbances might be found. However in this study the focus is not to find accurate representations for different disturbance types but to exemplify how disturbances can be introduced into the model. For that purpose it is enough to arbitrarily choose two types of somewhat random disturbances affecting the availability:

(i) **Short disturbance**: Affects the availability more but during a shorter time;

(ii) **Long disturbance**: Affect the availability less but are more prolonged;

The short and long disturbances have the property that they are uniformly random within an interval of 25% (availability percent) from their respective scenarios, which are introduced later. For the example model, the disturbances are assumed to be equally likely to occur, and it is known that no other disturbances will occur in the system at the same time. This is important when later the probabilities for the scenarios are to be defined.

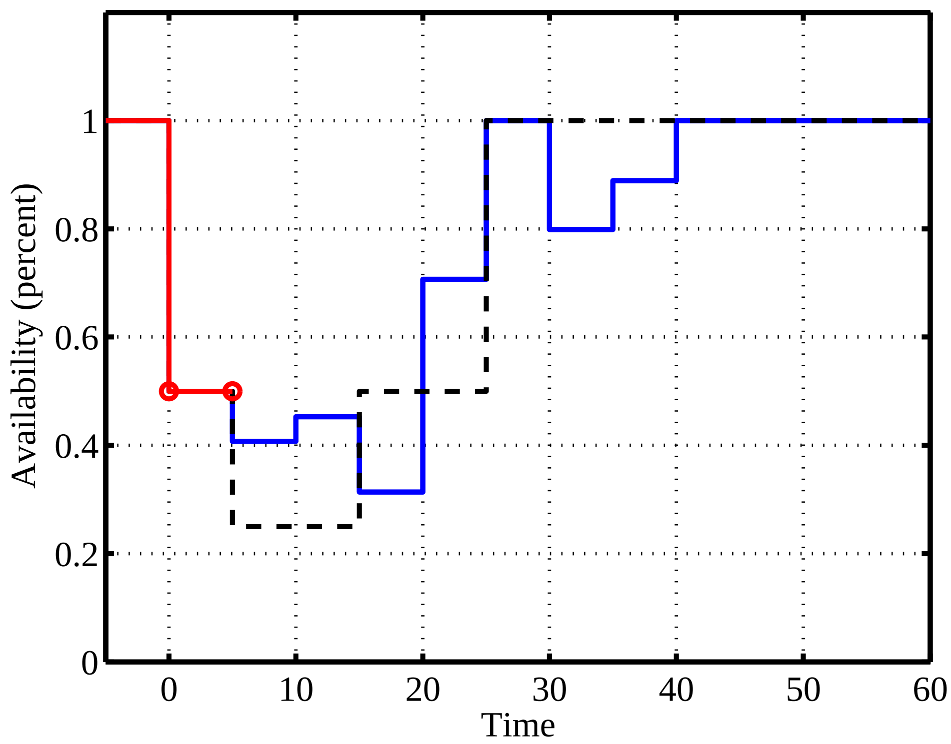

Figure 5.2: Availability for steam during the short disturbance

The disturbances are displayed in Figures 5.2 and 5.3. In these plots, the respective scenarios is plotted as black dashed lines, and an example of a disturbance (one for each type) as blue lines. For both types of disturbances, the current status at time $t = 0$ is that the availability has dropped to 50%. As discussed before, the next period must also be known, which in the figures is the line between the two circular markers. After this, the availability of $U(t)$ follows a trajectory that is random within the limits defined by the two disturbance types. The red line from $t = -5$ to $t = 0$ show the availability that has preceded the disturbance.

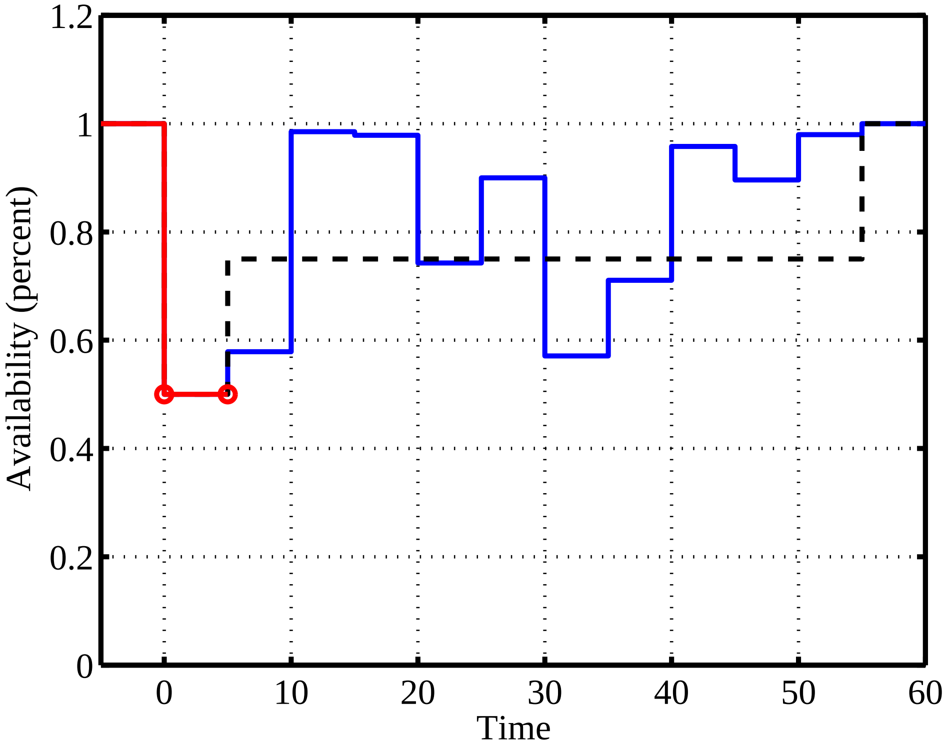

Figure 5.3: Availability for steam during the long disturbance

5.5 Scenarios

Since only two disturbances may occur it is possible to introduce two scenarios and optimise over those as in Section 2.5. The two scenarios used (M_1 and M_2) will be estimates or guesses how a disturbance will behave. The two scenarios are:

(i) **Short scenario** where the availability is expected to drop more but regain faster than for the;

(ii) **Long scenario** which is expected to last longer but have less effect on U's availability.

The scenarios are plotted as black dashed lines in Figures 5.2 and 5.3.

Assuming probabilities for the disturbances are equal, then the probability vector becomes $P(M) = [1/2, 1/2]$. This means each of the two scenarios will be equally likely to describe a disturbance well (but not exactly because of the randomness in the disturbances).

5.6 Decision Variables

For this example site, four decision variables can be identified which are:

$q_1(t)$ = Production rate for A_1 in period t;
$q_2(t)$ = Production rate for A_2 in period t;
$q_3(t)$ = Production rate for A_3 in period t;
$P_1(t)$ = Amount of P_1 being sold in period t.

How much the buffer tank I contains is decided by the production in A_1 and the outflows from I to P_1, A_2 or A_3.

5.7 Parameters

Before describing the constraints, some parameters have to be defined for this example:

I^{max} = Maximum volume allowed in the buffer tank;
h = Holding cost per time unit for the buffer tank;

u^{max} = Production capacity for U;
β_1 = Utility required for each output from area A_1;
β_2 = Utility required for each output from area A_2;

m_1 = Contribution margin for P_1;
m_2 = Contribution margin for P_2;
m_3 = Contribution margin for P_3;

s_1 = Start-up cost for A_1;
s_2 = Start-up cost for A_2;
s_3 = Start-up cost for A_3;

q_1^{max} = Production capacity for A_1;
q_2^{max} = Production capacity for A_2;
q_3^{max} = Production capacity for A_3;

q_1^{min} = Production rate under which A_1 has to be stopped;
q_2^{min} = Production rate under which A_2 has to be stopped;
q_3^{min} = Production rate under which A_3 has to be stopped;

Values of the parameters are shown in Table 5.1. Note that these values are only chosen to demonstrate the model.

I^{max}	$= 100$	m_1	$= 2$	q_1^{max}	$= 30$
h	$= 1$	m_2	$= 4$	q_2^{max}	$= 15$
		m_3	$= 6$	q_3^{max}	$= 10$
u^{max}	$= 100$	s_1	$= 100$	q_1^{min}	$= 10$
β_1	$= 3$	s_2	$= 200$	q_2^{min}	$= 5$
β_2	$= 2$	s_3	$= 300$	q_3^{min}	$= 5$

Table 5.1: The parameter values

5.8 Initial States

Aside from the parameters the model needs some initial states for the variables: $I(t_0)$, $W_1(t_0)$, $W_2(t_0)$, $W_3(t_0)$ and $U(t_0)$. Being able to alter the initial values for these parameters gives the option to, for example, have an area stopped before the optimisation starts. In this example all areas will be running i.e. $W_i(t_0) = 1$, the utility will be at 50%, $U(t_0) = 0.5$ and the initial buffer tank level, $I(t_0)$, will be 5.

5.9 Constraints

The constraints from the general SP-MILP formulation are adapted to the example system. Some of these constraints are shown here for the example system. Most important is the mass balance of the example, stated as:

$$I(t) = I(t-1) + q_1(t) - q_2(t) - q_3(t) - P_1(t) \tag{5.1}$$

$$0 \leq I(t) \leq I^{max} \tag{5.2}$$

which states that the volume in I at time t is dependent on how much it contained at time $t-1$, and on the production in the areas and sold P_1 at time t.

The utility constraint for $u(t)$ becomes:

$$u(t) = \beta_1 q_1(t) + \beta_2 q_2(t) \tag{5.3}$$

$$0 \leq u(t) \leq U(t) u^{max} \tag{5.4}$$

Where the availability $U(t)$ is an input to the model (see Figure 4.1) and u^{max} is a parameter.

5.10 Objective Function

If the objective function (3.7) from the general MILP formulation is used here, without considering scenarios, it becomes:

$$\text{Max} \sum_{t=1}^{H} \Big(m_1 P_1(t) + m_2 P_2(t) + m_3 P_3(t) - s_1 S_1(t) - s_2 S_2(t) - s_3 S_3(t) - hI(t) \Big) \quad (5.5)$$

In (5.5), no scenarios are considered, and note that the summation is for the complete time period from 1 to H. If the two scenarios are used then the full SP-MILP objective function (4.2) adapted to this example will have the form:

$$\begin{aligned}
&\text{Max } m_1 P_1(t_0) + m_2 P_2(t_0) + m_3 P_3(t_0) \\
&\qquad -s_1 S_1(t_0) - s_2 S_2(t_0) - s_3 S_3(t_0) - hI(t_0) \\
&+P(M_1) \textstyle\sum_{t=t_0}^{H} \Big(m_1 P_1(t) + m_2 P_2(t) + m_3 P_3(t) \\
&\qquad -s_1 S_1(t) - s_2 S_2(t) - s_3 S_3(t) - hI(t) \Big) \qquad (5.6) \\
&+P(M_2) \textstyle\sum_{t=t_0}^{H} \Big(m_1 P_1(t) + m_2 P_2(t) + m_3 P_3(t) \\
&\qquad -s_1 S_1(t) - s_2 S_2(t) - s_3 S_3(t) - hI(t) \Big)
\end{aligned}$$

In (5.6) the objective function for $t > t_0$ is split into a part for each scenario M_1 and M_2. The weight of the scenarios in (5.6) will depend on their probabilities $P(M_1)$ and $P(M_2)$.

5.11 Operating Strategies

First the model will be run with no disturbance to give a reference point. Then the model is run with full information about how the disturbance affects the future availability for the utility. If a disturbance's future behaviour is unknown a guess can be made about which scenario that fits the disturbance best. Therefore at each run the result from guessing the right and wrong disturbance will be shown. Another alternative is to use both scenarios with their (known) probabilities, which might give a better overall result.

These strategies will be tested for six short disturbances, six long disturbances and the results are presented in the next chapter.

6

Results from Example Model

In this chapter the results from running the example model in Chapter 5 is presented for three cases. The first case shows how the model operates when there are no disturbances, which means the utility availability $U(t)$ is 100%. Case two will display how the strategies affect the profit when the disturbance is of the short type. Finally in the third case the profit from the strategies will be shown for when the disturbance is of the long type.

6.1 Plot Preliminary

Each of the following plot figures are divided into five sub-plots. All of the plots have time on the x-axis from past-time $t = -5$ to the end time $t = 60$.

The Utility plot displays the availability of U over time as the thick blue line. When applicable this plot will also show one or both of the scenarios as: dotted green line for the short scenario M_1, and dashed red line for the long scenario M_2.

Next plot displays the production rate $q_1(t)$ for A_1 as the thick blue line. The minimum production rate q_1^{min} is graphed as a dashed black line. Since production rate in A_1 is not equivalent to sold P_1 this is also plotted here as the thin red line with its y-label to the right in red.

The third and fourth plots display the production rate for the areas as the thick blue lines. The minimum production rate is also plotted as a dashed black line but since P_2 and P_3 are equivalent to the respective area's production they are not plotted.

The last plot displays the level of the buffer tank I.

6.2 Case One - No disturbance

This case could be called the steady state of the system, with the exception of the emptying of the buffer tank at $t = 0$. The model choose to empty the tank since no disturbance will occur, and there is a holding cost for keeping material in the buffer tank. As expected A_3 is prioritised over A_2, which in turn is prioritised over A_1.

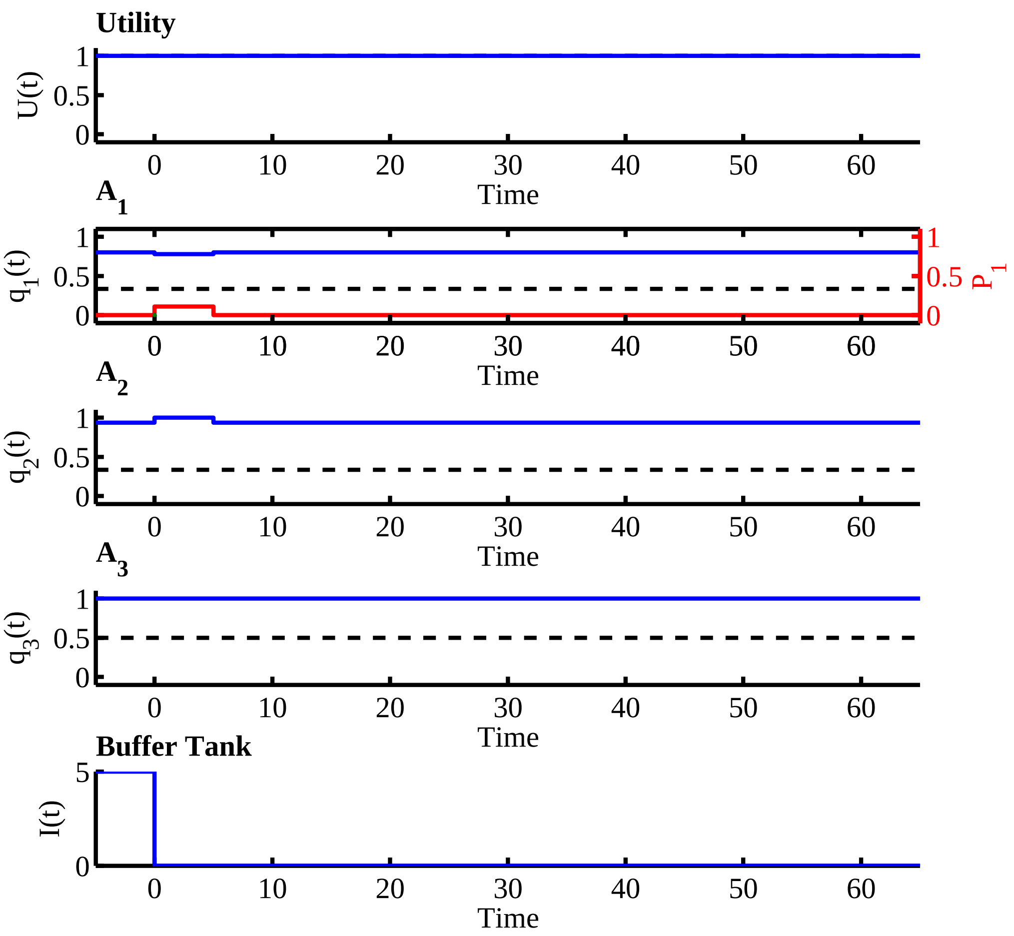

Figure 6.1: Optimal solution for the example model when no disturbances occur

6.3 Case Two - Short Disturbance

Short Disturbance with Full Information

Since the disturbance trajectory is known in advance, the model adds fifteen volume units to the initial buffer in the first time period ($t = 0$ to $t = 5$). This material is then used during the simulation to keep A_3 running as much as possible. At $t = 20$ the model choose to shut down A_1 and accept a start-up cost for that area.

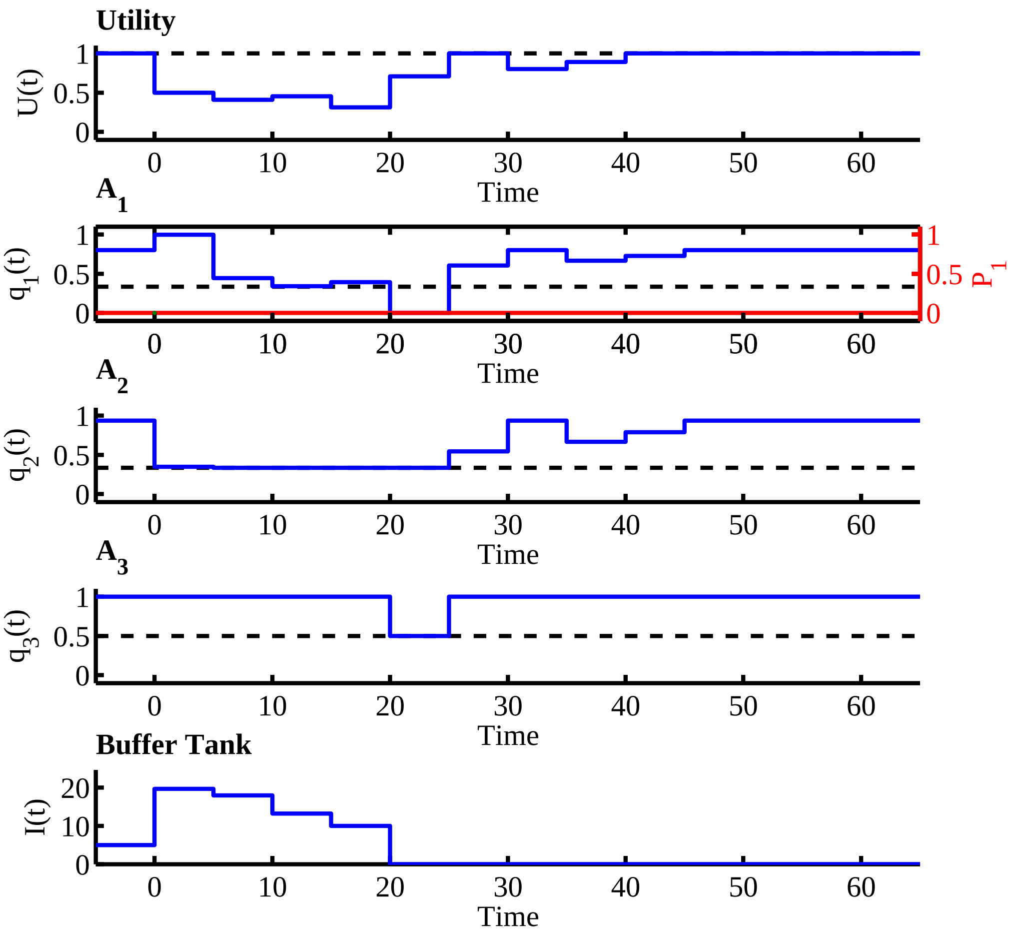

Figure 6.2: Optimal solution for the example model during a short disturbance with full information

Short Disturbance with Short Disturbance Scenario

The model now anticipates the disturbance to follow the green dotted line in Figure 6.3 below. In the same way as with full information this makes the model saving up buffer in the beginning of the simulation which then is used to prioritise A_3.

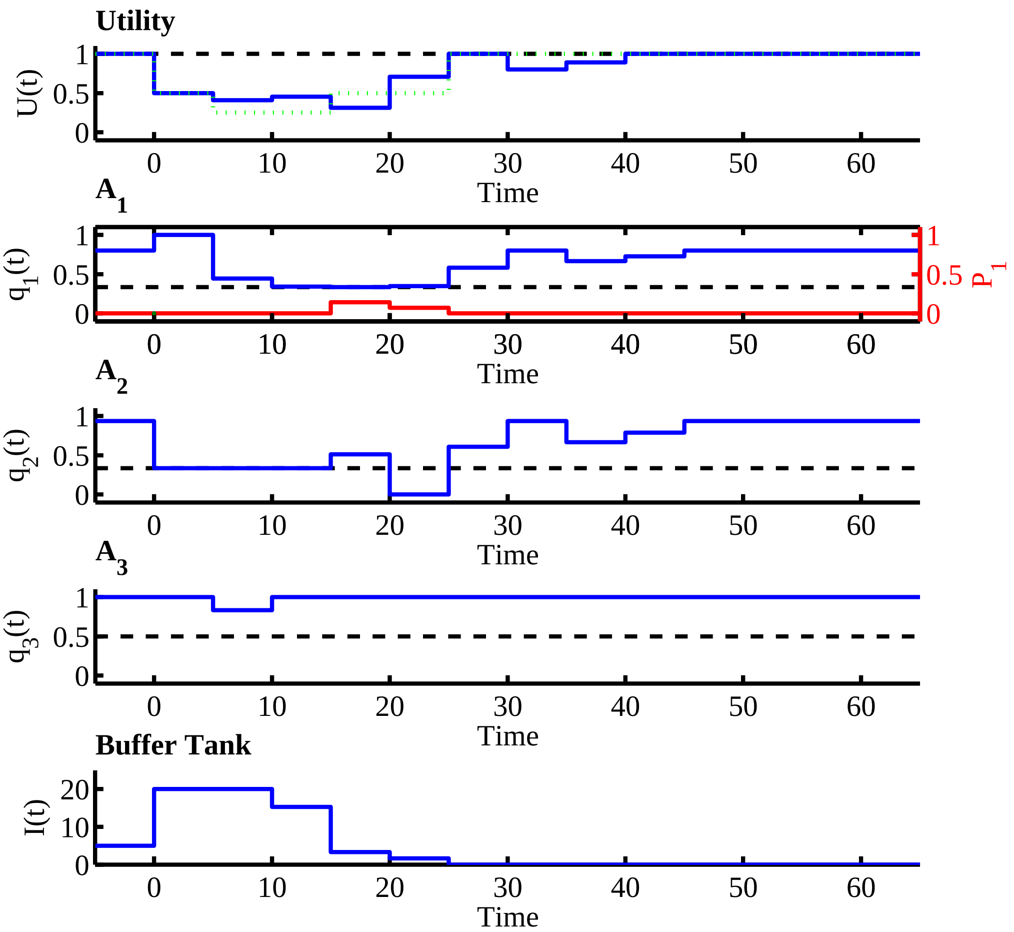

Figure 6.3: Optimal solution for the example model during a short disturbance when using the short disturbance scenario

Short Disturbance with Long Disturbance Scenario

In this simulation the model anticipates a long disturbance trajectory, which is plotted in Figure 6.4 as the red dashed line. This makes the model believe the utility availability will jump back up to 75% after $t = 5$ and the result are an unwillingness to keep material in the buffer tank. With no excess buffer the production in both A_3 and A_2 falls notably lower than before.

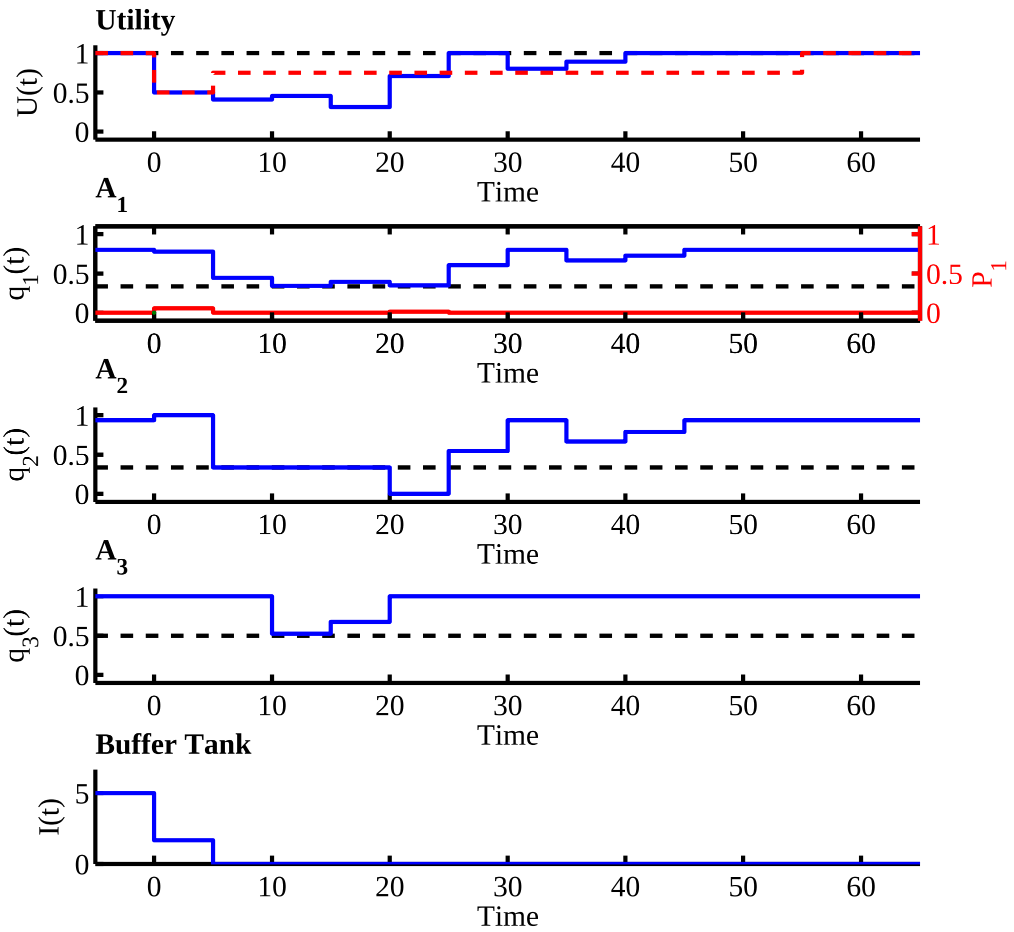

Figure 6.4: Optimal solution for the example model during a short disturbance when using the long disturbance scenario

Short Disturbance with Both Disturbance Scenarios

Here the full objective function (5.6) are used, which means the model anticipates that the disturbance could just as likely be of any type. In response to this the model choose to keep material in the buffer because the cost of having to slow down A_3 outweighs the cost of holding material in the buffer tank. Also note that after $t = 24$ the model still anticipates that the availability might drop in accordance with the long disturbance as indicated by the red dashed line. This uncertainty might be the reason for why A_2 is not started and P_1 sold instead.

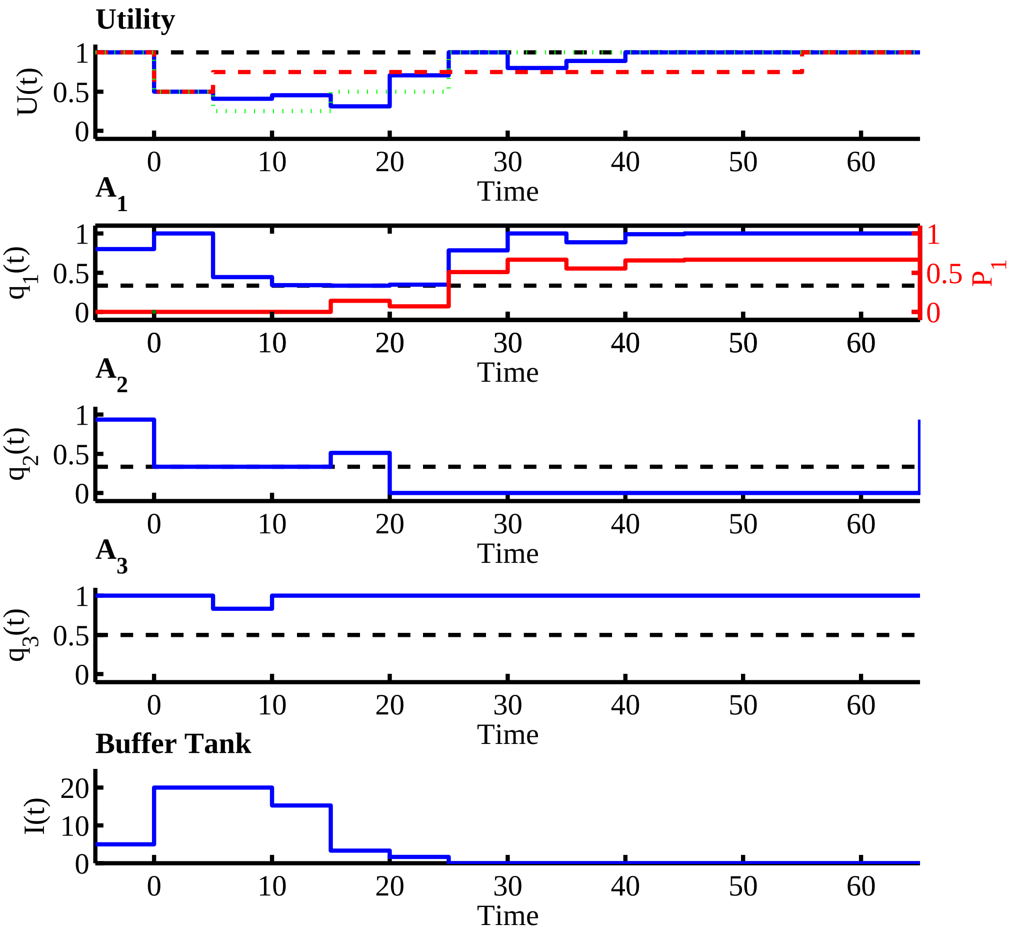

Figure 6.5: Optimal solution for the example model during a short disturbance when using both disturbance scenarios

6.4 Case Three - Long Disturbance

Long Disturbance with Full Information

Figure 6.6 indicates that even though the system is experiencing a disturbance the optimal response is running A_3 at its maximum capacity and not holding any buffer.

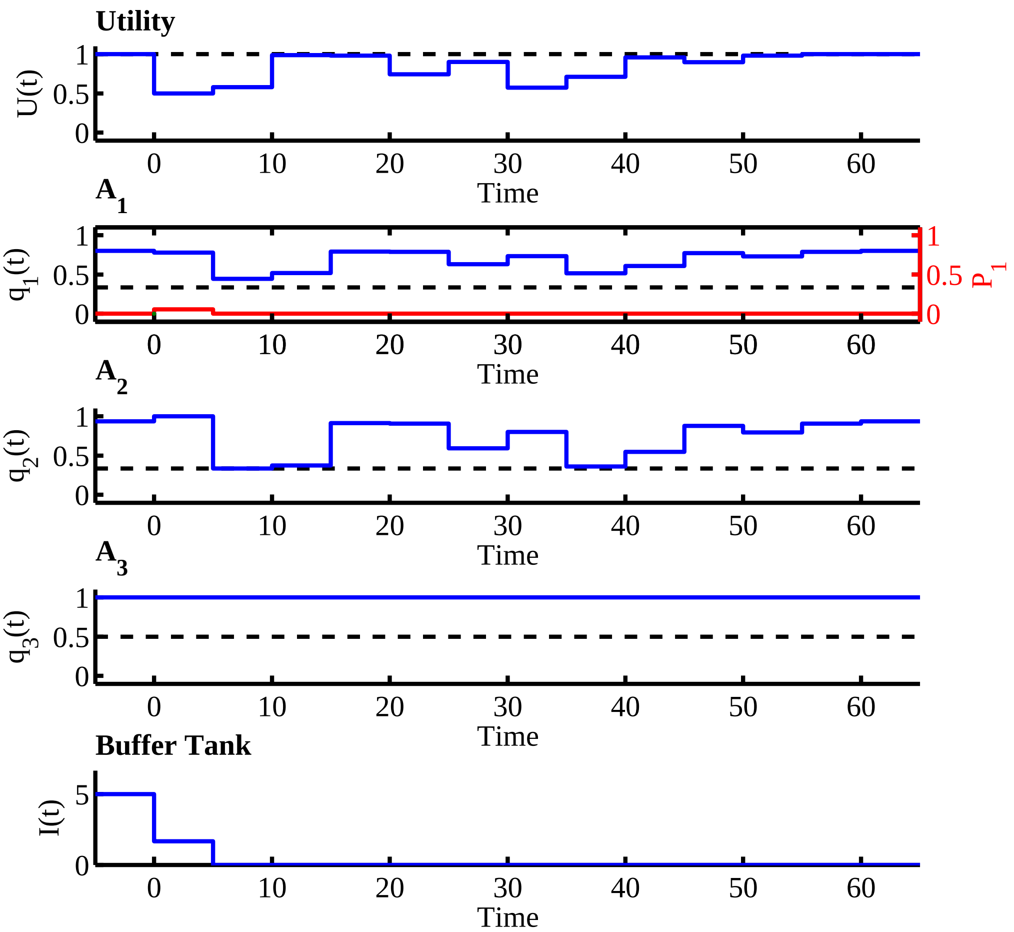

Figure 6.6: Optimal solution for the example model during a long disturbance with full information

Long Disturbance with Short Disturbance Scenario

Believing the disturbance is of the short type makes the model store material in the buffer tank in the beginning of the simulation. But as time progresses the buffer tank is emptied.

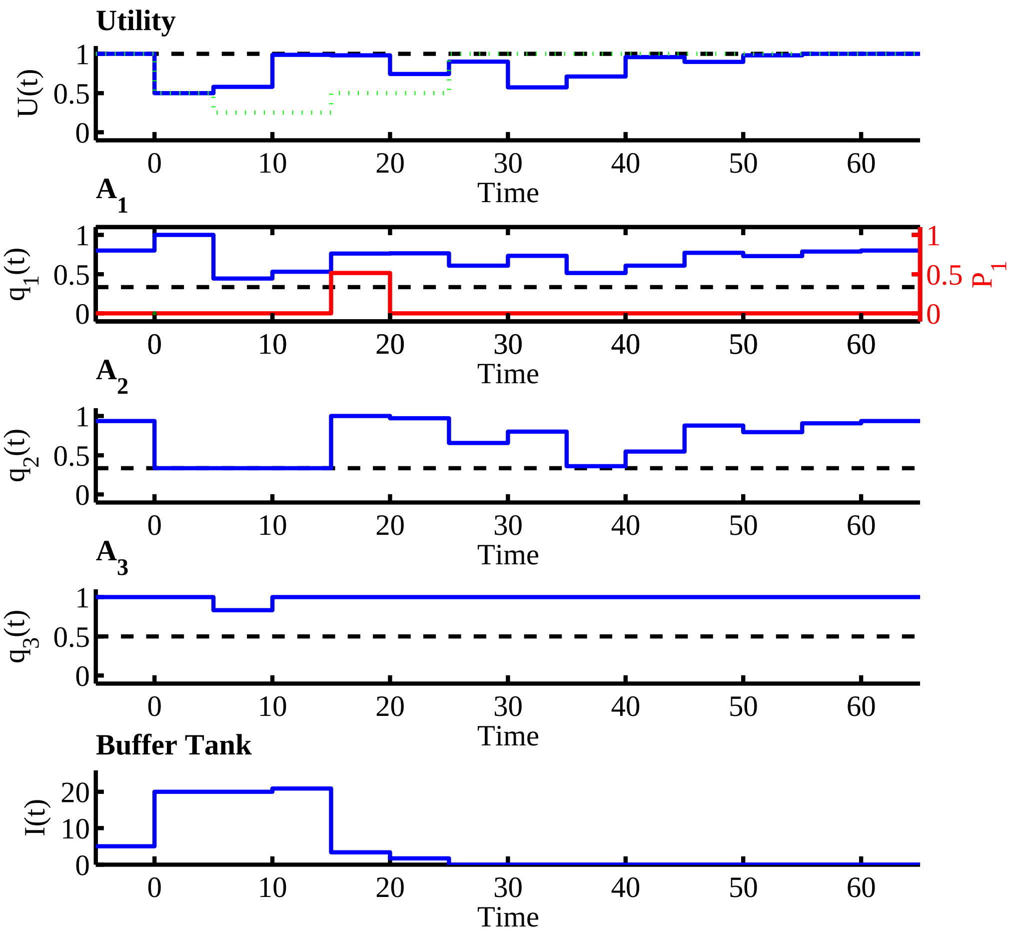

Figure 6.7: Optimal solution for the example model during a long disturbance when using the short disturbance scenario

Long Disturbance with Long Disturbance Scenario

In this case the areas are run almost identical as when the model had full information.

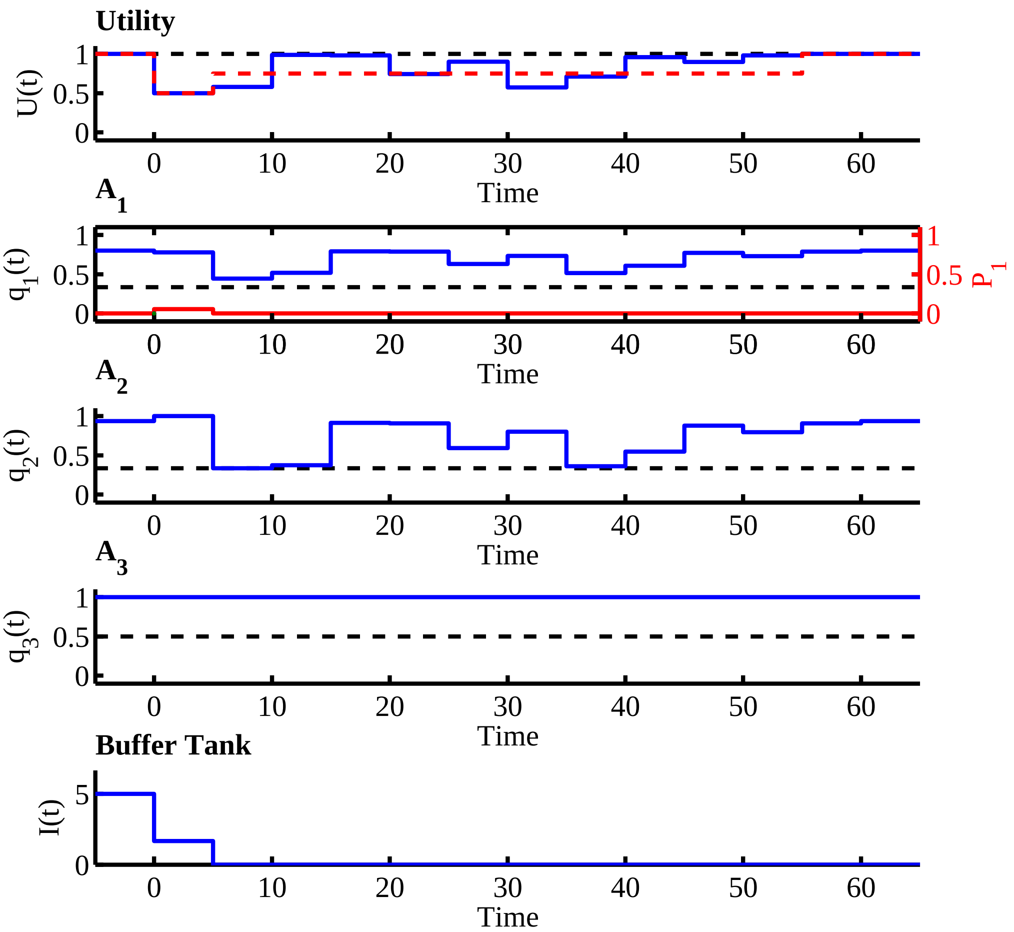

Figure 6.8: Optimal solution for the example model during a long disturbance when using the long disturbance scenario

Long Disturbance with Both Disturbance Scenarios

When using both disturbance scenarios the model uses the buffer tank to make sure no area has to be stopped due to shortage of material. This has been done before when anticipating only the short disturbance. The model behaves like this even though the model does not know which type of disturbance that are acting on the system.

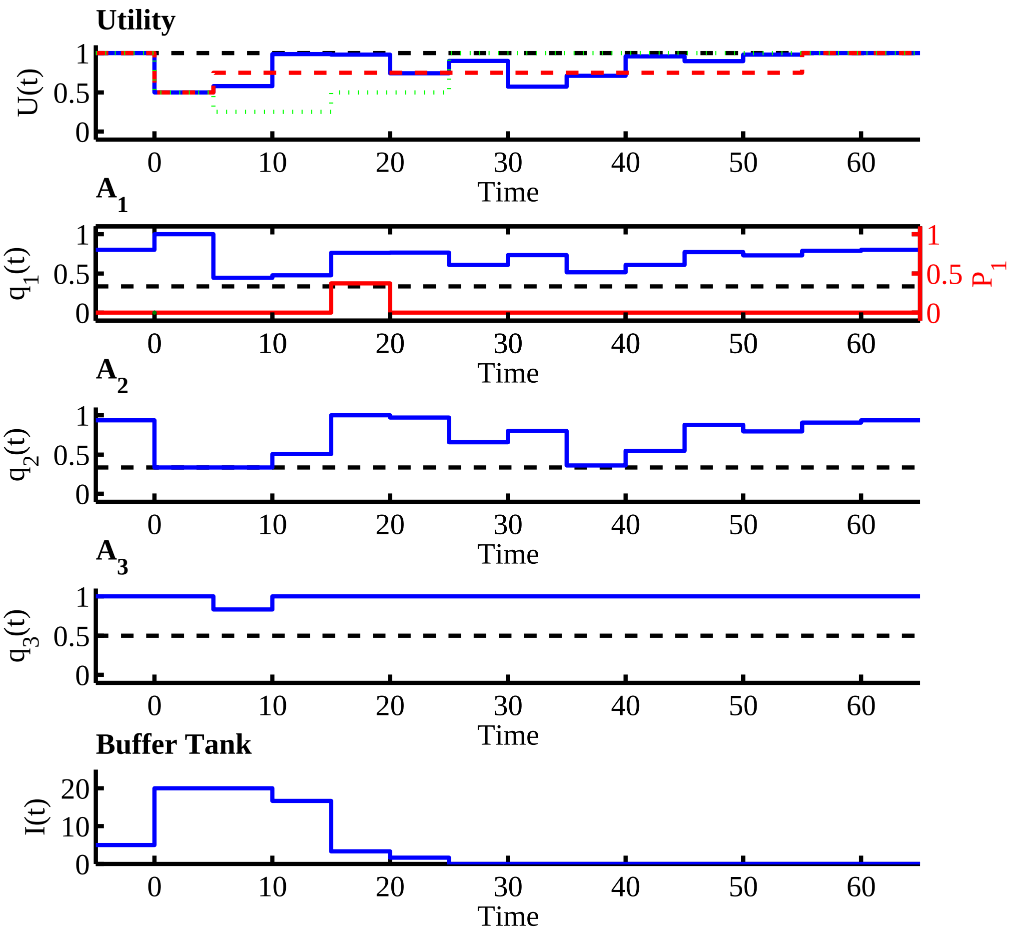

Figure 6.9: Optimal solution for the example model during a long disturbance when using both disturbance scenarios

6.5 Objective Function Values

Figure 6.10 and Figure 6.11 shows the result from running the model for six different disturbances of each type. The hight of the bars show the expected profit from the four different strategies. The first strategy is estimating that the disturbance is of the long type from Section 5.4. The second strategy is formed by estimating the disturbance is of the short type. In the third strategy no guess is made but instead two scenarios is used as in (5.6). Finally the expected profit from full knowledge of the disturbances is indicated by the last bars in Figure 6.10 and Figure 6.11.

Table 6.1 and Table 6.2 shows the objective function's values (rounded of to the nearest integer) that are graphed in Figure 6.10 and Figure 6.11. The tables also includes the averages for the different strategies for comparison.

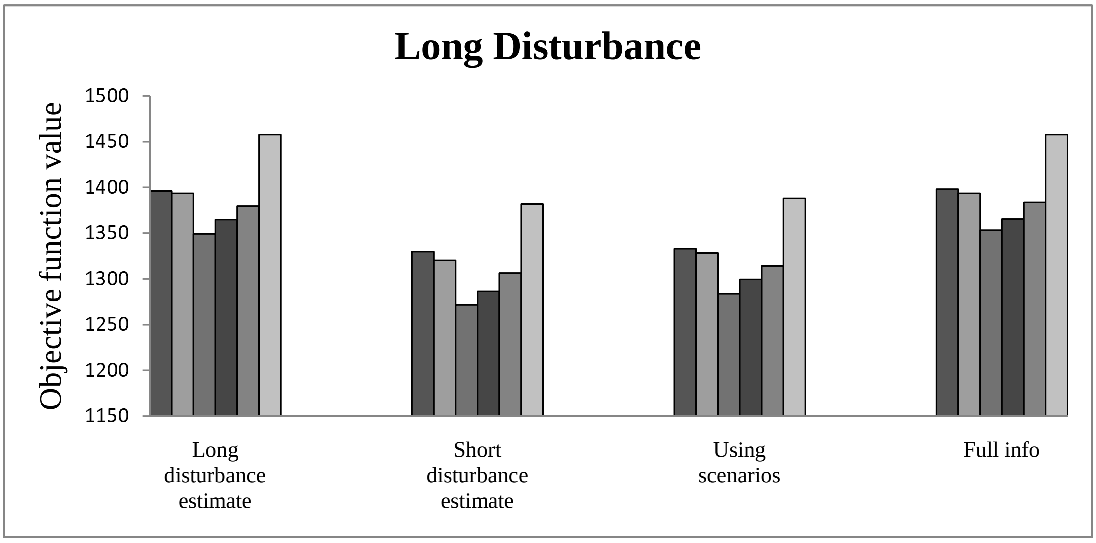

Figure 6.10: Graphed results from six optimisations using long disturbances as model inputs

Long Disturbance				
	Long disturbance estimate	Short disturbance estimate	Using scenarios	Full info
Run one	1396	1330	1333	1398
Run two	1394	1320	1328	1394
Run three	1349	1272	1284	1353
Run four	1365	1286	1299	1365
Run five	1380	1306	1314	1384
Run six	1458	1382	1388	1458
Average	1390	1316	1324	1392

Table 6.1: Detailed results showing the objective function value from six optimisations using long disturbances as model inputs

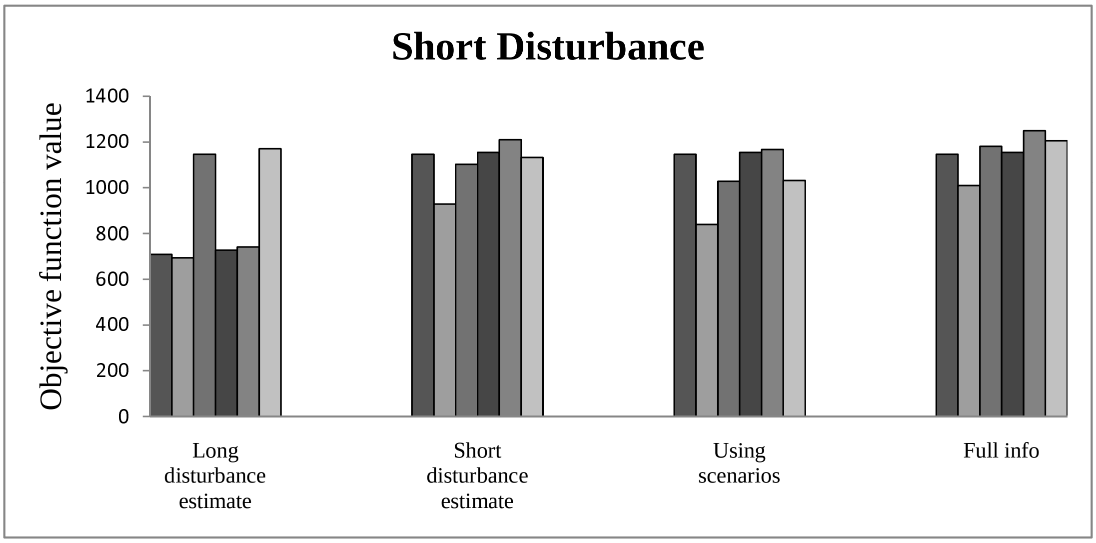

Figure 6.11: Graphed results from six optimisations using short disturbances as model inputs

Short Disturbance				
	Long disturbance estimate	Short disturbance estimate	Using scenarios	Full info
Run one	709	1146	1146	1146
Run two	694	929	840	1010
Run three	1146	1103	1029	1182
Run four	728	1155	1155	1155
Run five	741	1210	1167	1250
Run six	1170	1133	1032	1206
Average	865	1113	1061	1158

Table 6.2: Detailed results showing the objective function value from six optimisations using short disturbances as model inputs

7

Discussion of Results from Example Model

The results from the example model gave some expected answers and some surprises. Expected behaviour was the unwillingness to shut down areas because of the start-up cost. But it was also expected that the model would prioritise the most profitable area in this case, A_3.

Because of the low buffer holding cost h, the model could be expected to use the buffer tank strategically. Good examples of this are when area two is stopped. In this case, it is a good idea to keep it stopped and fill up the buffer tank instead. Later the stopped area can, when started, "catch up" by using the buffer tank during the final phase of the simulation. Having material in the buffer tank also reduces the risk that A_2 or A_3 have to be stopped later in the simulation.

The high value of the objective function in run three and six (see Table 6.2), when using a short disturbance estimate for a long disturbance, was a surprise. Using both scenarios was only slightly better than guessing all disturbances are of the short type.

8

Conclusions and Future Work

8.1 Conclusions

In this study a general method for modelling of product-related disturbances within process industry manufacturing plants has been introduced.

First, a mixed integer linear programming (MILP) model for area networks was constructed. The model aimed to maximise the simulated plant's profit for a period of time. In this model a site was represented by linked areas with defined properties, such as production capacity. A key property of the model was that the production capacity could be affected by the availability of services known as utilities. The areas could also be separated by intermediate buffer tanks, which would allow independent operation of otherwise dependent areas. By allowing the use of integers in the constraints, it was possible to model costs for stopping areas which is an issue for process industry manufacturing sites.

A limitation of the MILP model was that any utility disturbance trajectory had to be known in advance. To remedy this stochastical programming (SP) was used to introduce uncertainty. This extended, so called SP-MILP model, was able to optimise over a number of scenarios, which represented guesses as to how a disturbance would unfold.

Constraints and objective functions for both models were implemented in Matlab, and solved by using an open source software called LPSolve which communicated with Matlab.

To exemplify the SP-MILP model a part of the Perstorp AB Stenungsund site was modelled. This example consisted of a small network of three areas, a buffer tank and one utility affecting two of the areas. For this example, the objective was to maximise profit. Two disturbances with some variation in its trajectories was used with corresponding scenarios to simulate uncertainty. The profit from running the model with one or two scenarios was compared to the optimal way of running the model, that is with full information about the disturbance trajectory.

From running the example model it could be concluded that initial parameters play a key role in the model's behaviour and result. For example see the results from run three and six in Table 6.2. In these two cases the profit for using the long disturbance estimate for a short disturbance was higher than correctly predict and use a short disturbance estimate for a short disturbance. This is surprising, and the reason for this could be that the interplay between parameters makes the wrong strategy pay off in these cases.

8.2 Future Work

The project demonstrates a method that can be used to simulate different disturbance management scenarios of a production site. The method is based on Stochastic Programming Mixed-integer Linear programming (SP-MILP). Only some properties that are found in industrial sites have been modelled in the general SP-MILP model, introduced in this study. More properties or better representation of already modelled properties could be included in future models. For the result to be useful the parameters used in the models needs to be taken from an industrial site, and then the models need to be validated.

When experimenting with early versions of the model it was found that if a future disturbance was known, the optimisation suggested that storing material in the buffer tank early would be profitable. This is not surprising, but it gives an indication that a MILP model might also be used to shed some light over safety stock levels in the buffer tanks.

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