Lund University
Department of Economics

Alternative Mechanisms For School Choice

- The Case of Malmö Stad

Author:
Jim Ingebretsen Carlson
Master's thesis

Supervisor:
Tommy Andersson
June 4, 2012

Abstract

This paper investigates the possible gains from implementing an alternative student assignment mechanism in Malmö stad. The analysis is based on the school choice literature and data from Malmö stad. The overall result tells us that the current system for assigning students in Malmö stad is associated with problems. A Gale-Shapley Student Optimal Mechanism customized for Malmö stad would be the appropriate alternative for solving these problems. This further implies that Malmö stad and similar Municipalities in Sweden ought to revise their current student assignment systems and consider an alternative mechanism.

Kewords: School choice, Student assignment mechanisms, Malmö stad

Contents

1 Introduction

The possibility for a student to freely choose which school he/she wishes to attend has been a possibility in Swedish elementary schools since the late 1980s (Dahlstedt, 2007, p. 24). Today are numerous municipalities in Sweden taking students' preferences into account when assigning them to schools. The free choice of school has been criticized by, amongst others, Skolverket who believes that the free choice is correlated with segregation. Moreover, Skolverket suggests that the free choice widens the gap between the well and the badly performing schools (Skolverket, 2012).

Despite the wide implementation of a free elementary school choice in Swedish municipalities, has there been little discussion in Sweden, regarding how to effectively assign students to schools. The main issue in school choice is how to assign students to schools given the students' preferences, the schools' rankings of the students and the schools' capacities. Abdulkadiroğlu & Sönmez (2003) were the first to address the school choice problem, with focus on how to find a good student-school match using a student assignment mechanism.

There are three properties which are desired for school assignment mechanisms, these are efficiency, stability and strategy-proofness. A student assignment mechanism which is efficient selects a matching which cannot be pareto improved upon, from the view point of the students. A stable matching is a matching where no student prefers another school to the one which he/she is currently matched with and that school prefers to assign the student a seat. Finally, a strategy-proof mechanism, is a mechanism which makes it a weakly dominant strategy for students to represent their true preferences.

Abdulkadiroğlu & Sönmez (2003) propose two student assignment mechanisms, the Gale-Shapley Student Optimal Mechanism and the Top Trading Cycles Mechanism, in order to solve a school choice problem. Both these two mechanisms are strategy-proof but while the Gale-Shapley Student Optimal Mechanism selects a stable matching will the Top Trading Cycles Mechanism select an efficient matching.

These theoretical results have had impact on how students are assigned to schools, since they are practically implementable. In New York City and Boston are currently two different versions of the Gale-Shapley Student Optimal Mechanism in use.

The main purpose of this thesis is to evaluate what effects a practical implementation of a student assignment mechanism would have in Malmö stad. Malmö stad has been chosen since it is Sweden's third largest municipality with 27078 students currently studying in elementary school and because the system is associated with problems regarding the student-school assignment. The thesis will focus on the school choice problem for students in elementary schools which includes students in grades 1 to 9. In order to conduct the evaluation will the thesis concentrate on the following three questions:

1. Are there any problems associated with the current system used in Malmö stad?
2. Can a student assignment mechanism solve these problems?
3. Which student assignment mechanism would be appropriate for Malmö stad?

The main finding in this thesis is that the system in Malmö stad is associated with problems regarding transparency, efficiency, stability and segregation. The optimal choice of mechanism, for solving these problems would be a Gale-Shapley Student Optimal Mechanism accompanied by adequate information. Furthermore, controlled choice could be incorporated into the mechanism in order to reduce segregation across schools. A limitation of the analysis is that data is scarce in some important aspects, however is this not affecting the theoretical results by much.

The thesis is organized in the following way: Section 2 gives a theoretical overview on school choice as well as presenting the two implemented mechanisms in New York City and Boston. Section 3 presents the current system in Malmö stad. Problems with the current system and solutions to these are discussed in section 4. Finally, is a conclusion presented in section 5.

2 Theoretical Overview

2.1 A School Choice Problem

According to Abdulkadiroğlu & Sönmez (2003, p.733) is a school choice problem consisting of:

> A number of students, each of whom should be assigned a seat at one of a number of schools. Each school has a maximum capacity but there is no shortage of the total seats. Each student has strict preferences over all schools, and each school has a strict priority ordering of all students. Here priorities do not represent preferences but they are imposed by state or local law.

Formally, a school choice problem consists of the following (Pathak & Sönmez, 2008, p. 1638):

- A finite set of students $I = \{i_1, \ldots, i_n\}$
- A finite set of schools $S = \{s_1, \ldots, s_n\}$
- A capacity vector $q = (q_{s_1}, \ldots, q_{s_n})$
- A list of strict student preferences $P_I = (P_{i_1}, \ldots, P_{i_n})$
- A list of strict school priorities $\pi = (\pi_{s_1}, \ldots, \pi_{s_m})$

It is assumed that $q_s \geq 1\ \forall s \in S$ (Erdil & Ergin, 2008, p. 674) and that the list of students' preferences as well as the list of schools' priorities are complete, transitive and antisymmetric relations (Kesten, 2010, p. 1305). For any student $i \in I, P_i$ is a strict preference relation over $S \cup \{i\}$, where $s_n P_i i$ means that school n is acceptable for student i, which is equivalent to that student i strictly prefers school n rather than not being assigned a seat at any school at all (Pathak & Sönmez, 2008, p. 1638)

(Abdulkadiroğlu *et al.*, 2009, p. 1957). Furthermore, let $s_1 R_i s_2$ denote that either $s_1 P_i s_2$ or $s_1 = s_2$ (Abdulkadiroğlu *et al.*, 2009, p. 1957), which hence is the analogous "at least as good as" relation from P_i (Pathak & Sönmez, 2008, p. 1638). For any school $s \in S, \pi_s$ is a mapping from $\{1, \ldots, n\} \longrightarrow \{i_1, \ldots, i_n\}$, which generates a list of priorities for school s of students where $\pi_s(1)$ is the student with the highest priority, $\pi_s(2)$ is the student with the second highest priority etc (Pathak & Sönmez, 2008, p. 1638).

The outcome of a school choice problem is called a *matching* which is a correspondence $\mu : I \cup S \longrightarrow S \cup I$, such that:

- $\mu(i) \in S \cup \{i\}, \qquad \forall i \in I$, and
- $\mid \mu^{-1}(s) \mid \leq q_s, \qquad \forall s \in S.$

Where $\mu(i)$ is the assignment of agent i under matching μ. The interpretation of these criteria is that each student is either assigned a seat at a school or remains unassigned, and that the number of students matched to a certain school cannot exceed the number of seats available at the school (Pathak & Sönmez, 2008, p. 1639), (Abdulkadiroğlu *et al.*, 2009, p. 1957).

A matching μ *Pareto dominates* another matching υ if: $\mu(i) R_i \upsilon(i), \forall i \in I$, and $\mu(i) P_i \upsilon(i)$, for some $i \in I$. A matching is *efficient* if it is not Pareto dominated by any other matching. Moreover, an *individual rational* matching is μ such that it matches all $x \in I \cup S$ with a student/school acceptable to x. If, $s P_i \mu(i)$, is a matching μ said to be *blocked* by the student-school pair (i,s) if either: $[\mid \mu(s) \mid < q_s$ and $i \succ_s s]$, or $[i \succ_s i'$ for some $i' \in \mu(s)]$. Hence, given that student i prefer school s to the school he/she is currently matched with under $\mu(i)$, will the matching $\mu(i)$ be blocked by school s if: There are either still unfilled seats at school s and s prefers to assign a seat to student i rather than leaving it unfilled, or if school s prefers student i to another student i' with whom school s is currently matched with (Abdulkadiroğlu *et al.*, 2009, p. 1957). Restated, would a matching like $\mu(i)$ give rise to *justified envy* (Abdulkadiroğlu & Sönmez, 2003, p. 735). Lastly, a matching μ is known to be *stable* if it is individual rational and is not blocked by any student-school pair (i,s) (Abdulkadiroğlu *et al.*, 2009, p. 1957).

In order to solve a school choice problem is a student assignment mechanism implemented which systematically selects a matching for a given problem. A mechanism which is designed in such a way that students have to reveal their preferences and selects a matching based on these preferences and schools' priorities is a *direct mechanism* (Abdulkadiroğlu & Sönmez, 2003, p. 733). Formally, a direct mechanism φ is a mapping from every $(P_I, \pi) \longrightarrow \mu$ (Abdulkadiroğlu *et al.*, 2009, p. 1957). Furthermore, is a *Pareto-efficient student assignment mechanism*, a mechanism which selects a Pareto-efficient matching (Abdulkadiroğlu & Sönmez, 2003, p. 733).

Another desirable property of student assignment mechanisms is strategy-proofness. A student assignment mechanism is *strategy-proof* if no student can stand to gain from misrepresenting his/her true preferences (Abdulkadiroğlu & Sönmez, 2003, p. 733). Under a strategy-proof student assignment mechanism, revealing their true preferences will be a dominant strategy for all students . Formally, a mechanism φ is *dominant strategy incentive compatible* for $i \in I$ if for every (P_I, π) and every P_i': $\varphi_i(P_I;\pi) R_i \varphi_i(P_i', P_{-i};\pi)$. Hence, let $P_i \in P_I$ be agent i's true preferences, and P_i' be agent i's preferences when misrepresenting. Given that all other students and schools do not misrepresent their preferences/priorities will the mechanism φ make student i weakly better off representing his/her true preferences compared to if he/she would misrepresent. Finally, a student assignment mechanism is strategy-proof if it is dominant strategy incentive compatible $\forall i \in I$ (Abdulkadiroğlu *et al.*, 2009, 1957-1958).

2.2 Two Student Assignment Mechanisms

Abdulkadiroğlu & Sönmez (2003) propose two mechanisms for solving a school choice problem: *The Gale-Shapley Student Optimal Mechanism* and the *Top Trading Cycles Mechanism*. These two mechanism will select different matchings for a given school choice problem. Generally, The Gale-Shapley Student Optimal Mechanism focuses on stability while the Top Trading Cycles Mechanism gives priority to efficiency. The following two sections will give a thorough review of these two mechanisms.

2.2.1 The Gale-Shapley Student Optimal Mechanism

A school choice problem has striking resemblance to the college admissions problem first considered by Gale & Shapley (1962). The major difference between the two problems is that colleges themselves have preferences over students, whereas schools are viewed as only to be consumed by students (Abdulkadiroğlu & Sönmez, 2003, p. 733). This is reflected by schools having "priorities" rather than "preferences". Abdulkadiroğlu & Sönmez (2003, p. 735) suggest that schools' priorities are to be interpreted as preferences, which allows for the use of the *Deferred Acceptance Algorithm* which was proposed by Gale & Shapley (1962) as a mechanism for solving the college admissions problem, in order to solve a school choice problem. The algorithm works in the following way:

Algorithm 1. *The Deferred Acceptance Algorithm*

Step 1: *Students propose to their most preferred school. Students are then, one at a time, assigned seats in accordance with the schools' lists of priorities. Students who are not assigned a seat are rejected.*

Step k: *In general, each student that was rejected in the previous step, proposes to the school which is next on his/her list of preferences. The schools will then choose which students to assign a seat, taking the new proposers together with the proposers which they have already given seats in the previous steps. The schools do so by once again assigning seats to the proposers, one at a time, following their list of priorities. Any rejected proposer is left unassigned.*

The algorithm stops when no proposers are rejected and each student is assigned his/her final assignment (Abdulkadiroğlu & Sönmez, 2003, p. 735).

Gale and Shapley (Gale & Shapley, 1962, p. 14) have proved the following appealing property

Proposition 1. *The unique stable matching selected by the deferred acceptance algorithm Pareto dominates any other stable matching, from the viewpoint of the proposers.*

The core of the proof lies in that the algorithm will reject students from a seat at a school if the seat is not *possible* for the students. A seat is possible for a student if the assignment is *stable*, which is connected to *justified envy* and that schools will *block* any unstable matching. Consider students $i_2, \ldots, i_n \in I$ who have higher priorities at a school s than i_1. The algorithm will then reject i_1 from a seat at s and it must hence be proven that s is impossible for i_1. Consider the opposite, that i_1 is assigned a seat at school s. This would mean that at least one of the students $i_2, \ldots, i_n$, say i_2 will be given a seat at another school at which he/she is worse off. This however, will be an unstable matching since school s prefers i_2 to i_1 and i_2 prefers school s to his/her current matching. Hence will i_2 suffer from justified envy and the matching will be blocked by school s. School s is thereby impossible for i_1 and the algorithm will reject any student a seat at a school which is not possible for the student in any stable matching. The matching selected by the Deferred Acceptance Algorithm will hence pareto dominate any other stable matching (Gale & Shapley, 1962, p. 14).

Proposition 1 holds true when preferences and priorities are strict. The unique stable matching which is selected by the Deferred Acceptance Algorithm, when students are proposing is known as the *student optimal stable matching* and the mechanism generating this matching is called the *student optimal stable mechanism* (**SOSM**) (Erdil & Ergin, 2008, p. 670)

Later, the following property of the Deferred Acceptance Algorithm was proven: (Roth, 1982, p. 623):

Proposition 2. *The Gale-Shapley student optimal stable mechanism is strategy-proof.*

In order to see that this holds true consider if student i_1 misrepresents his preferences by P'_{i_1} where $P'_{i_1} \neq P_{i_1}$ and P_{i_1} is student i_1's true preferences. The matching $\varphi_i(P_I; \pi)$, when i_1 reveals P_{i_1} is called x and the matching $\varphi_i(P'_{i_1}, P_{-i_1}; \pi)$, when i_1 misrepresents by P'_{i_1} is called y. Roth (1982, p. 624-626) shows that no misrepresentation by i_1 can make him better off, but in fact a misrepresentation leads simply to that $x = y$. Consider first the case where i_1 reports P_1 and makes a match at step t giving x. Now consider $i_n \in I\backslash_{\{i_1\}}$ who also makes a match at step t in x, it must then be that i_n is matched with the same school under the different matchings, $x_{i_n} = y_{i_n}$. This holds true since say i_n was matched with s_1 in x, this would then mean that i_n was the only one proposing to s_1 at step t. At matching y no student is worse off than at matching x, since every other student except i_1 truthfully reports their preferences. This implies that either did i_1 propose to s_1 in x

and was rejected in which case s_1 prefer i_n to i_1, or does i_1 prefer the school with which he/she is match to in x to s_1 in which case he/she is worse off by misrepresenting. This will hence lead to that i_1 has no incentives to misrepresent and $x = y$. By the same reasoning Roth shows that $x = y$ holds $\forall i \in I$ that are matched at any step prior to step t and the Deferred Acceptance Algorithm is strategy-proof (Roth, 1982, p. 624-626).

In spite of these attracting features, is there one short coming with the SOSM. There is a trade-off between stability and Pareto efficiency when using the SOSM (Abdulkadiroğlu & Sönmez, 2003, p. 736). A Pareto efficient matching does not have to be stable and since proposition 1 only refers to *stable matchings* does not the optimal stable matching necessarily coincide with the Pareto efficient matching. Hence, the complete elimination of justified envy, which is part of what defines a stable matching, may yield a matching which is not Pareto efficient (Abdulkadiroğlu & Sönmez, 2003, p. 736).

2.2.2 The Top Trading Cycles Mechanism

If efficiency is of higher priority than stability, ought another mechanism called the Top Trading Cycles Mechanism (**TTC**), be used to solve a school problem. Implementing the TTC requires a different interpretation of school priorities. Assume that $i_1 = \pi_s(1)$ and $i_2 = \pi_s(2)$, using the SOSM will this be interpreted as i_1 being assigned a seat at school s before i_2. When using the TTC on the other hand, will i_1 not automatically be assigned a seat before i_2, instead does this represent that i_1 has a better opportunity of getting in to school s than i_2 (Abdulkadiroğlu & Sönmez, 2003, p. 736). The SOSM gives high importance to that students are given seats based on the schools' priority orderings. While the TTC gives the students the opportunity to trade priorities among themselves in order to get a seat which they prefer. Priorities, in this case, are merely seen as a way to distribute the students among the vacant seats (Abdulkadiroğlu *et al.*, 2006, p. 10). The TTC is Pareto efficient, however, does it not completely eliminate justified envy (Abdulkadiroğlu & Sönmez, 2003, p. 736).

The TTC mechanism is a direct mechanism and the algorithm finds a matching in the following way (Abdulkadiroğlu & Sönmez, 2003, p. 736-737):

Algorithm 2. *The Top Trading Cycles Algorithm*

Step 1: *First, a counter is to be assigned to every school which makes it possible to keep track of how many remaining seats there are at the different schools. In step 1, the counters equals the capacities of the school. Second, every student points at their most preferred school and every school points at the student with the highest priority at that school. A cycle is then formed of the students and schools who are pointing at each other in a way such that if students trade priorities pareto improvements will occur. The cycle* $(s_1, i_1, s_2, \ldots, s_k, i_k)$ *would be interpreted as* s_1 *pointing to* i_1*,* i_1 *pointing to* s_2*,...,* s_k *pointing to* i_k *and* i_k *pointing to* s_1*. Since the number of students and schools are finite, this procedure will result in at least one cycle, but each student and each school can only be part of one cycle. After the cycle(s) has been determined every student that belongs to* *a cycle is assigned a seat at the school which he/she is pointing to and is then removed. Finally, the counter of each school in a cycle is reduced by one and if there are no remaining seats, the school is removed as well. The counters for all other schools are left unchanged.*

Step k: *In general, every remaining student points at his/her most preferred school of those still remaining, and each school points towards the student with the highest priority of the students still remaining. At least one cycle is formed. The students who forms part of a cycle are assigned a seat at the school they are pointing at and are removed. Each school who belongs to a cycle has its counter reduced by one and if there are no remaining seats, the school is removed as well. The counters for all other schools are left unchanged.*

The algorithm stops when all students have been assigned a seat at a school.
It has been proven that the TTC is efficient (Abdulkadiroğlu & Sönmez, 2003, p. 737):

Proposition 3. *The Top Trading Cycles mechanism is efficient.*

This holds true since if a student i_1 is removed at step 1, he/she is given his most preferred choice and cannot be better off. If a student i_2 is removed at step 2 he/she has been given his/her most preferred choice out of those still remaining. Since preferences are assumed to be strict can only i_2 be better off by making someone who was removed at step 1 worse off by taking his/her place. The TTC is hence pareto efficient since a student cannot, at any step, be better off without making anyone else who was removed at an earlier stage worse off (Abdulkadiroğlu & Sönmez, 2003, p. 744). The TTC is also strategy-proof (Abdulkadiroğlu & Sönmez, 2003, p. 738).

Proposition 4. *The Top Trading Cycles mechanism is strategy-proof.*

Strategy proofness can be seen since if a student i is removed at step k the student would like to misrepresent in order to be removed prior to step k. But the cycles in the steps prior to k are independent of what preferences the student reports. This is so since the other students and schools are reporting the same preferences/priorities as before and there is hence no school in the cycles which is pointing at student i. This leads to that the cycles are the same as before. Henceforth, the same students will be assigned seats as before and there is no way that misrepresentation leads to a better outcome for the student. It can only lead to that the student is worse off than under truthful preference revelation (Abdulkadiroğlu & Sönmez, 2003, p. 738).

While the SOSM selects a unique stable matching which pareto dominates all other stable matchings. The TTC selects the pareto efficient matching. Since both are strategy-proof, truthful revelation of preferences will be a dominant strategy for all students.

2.3 Controlled Choice

Since the 1960s has the positive correlation between a student's family's socioeconomic status (SES)[1] and the student's achievement in school been known (Caldas & Bankston III, 1997, p. 269). This has been confirmed for Swedish students as well by Skolverket which in a recent study concludes that parent's level of education and the child's social background influences the child's results in school (Skolverket, 2009, p. 112). Caldas & Bankston III (1997, p. 275) further confirms that a student's achievement in school tends to increase, when the student is attending a school where the other children come from a high SES background.

A contemporary study on the situation in Malmö stad concludes that SES measured as parents' level of education highly influences the students' scores. Bunar (2012, p. 21-22) shows that the students in Västra Innerstaden have the highest academic achievement while the students in Rosengård have the lowest. Furthermore is the parents' level of education highest in Västra Innerstaden and lowest in Rosengård. Moreover are 20 % of the children immigrants in Västra Innerstaden while 91 % are immigrants in Rosengård (Bunar, 2012, p. 25).

If decision makers in Malmö stad would want to make schools more equal, while at the same time allowing parents to choose at which school their child will attend, an ethnic balance could be incorporated into the SOSM or TTC. Different quotas for different groups could be incorporated in the mechanism allowing for e. g. 20 % (or whichever share the decision makers feel is appropriate) of the seats at schools in Malmö stad to be reserved for immigrants . In the school choice literature this is known as *controlled choice*

2.3.1 The Gale-Shapley Student Optimal Mechanism with Controlled Choice

Controlled choice is rather easily incorporated into the SOSM. Every student now belongs to a certain *type* and there are at least two different kind of types. Furthermore is every school given *type-specific quotas* (Abdulkadiroğlu & Sönmez, 2003, p. 739). To the formal definition of a school choice problem discussed on page 4 we can now add the following:

- A type space $T = \{\tau_1, \ldots, \tau_k\}$
- A type function $\tau : I \longrightarrow T$
- A vector of type specific quotas for every school s, $q_s^T = (q_s^{\tau_1}, \ldots, q_s^{\tau_k})$

Such that: $q_s^\tau \leq q_s$, $\forall s \in S$ and $\forall \tau \in T$ and $\sum_{\tau \in T} q_s^\tau \geq q_s$.

These constraints are known as the *controlled choice constraints* (Abdulkadiroğlu, 2005, p. 543).

[1] Measures of SES are usually education, income, occupation or a combination of the three (Winkleby *et al.*, 1992, p. 816)

These constraints imply that at every school, the quota of a certain type at one school has to be less or equal to the total quota at that school. And that the sum of all quotas at one school has to be larger or equal than the total quota at the school. When these constraints are perfectly rigid the SOSM can simply be applied separately to each type in order to assign the available seats for the students of each type. If the constraints are flexible, the following algorithm is to be used:

Algorithm 3. *The Gale-Shapley Student Optimal Algorithm with Controlled Choice*

Step 1: *Every student proposes to his/her most preferred school. Students are then, one at a time, assigned seats according to the schools' lists of priorities. Once a type-specific quota is filled are all remaining students of that type rejected and the algorithm continues only with the students of the remaining types. Any student not assigned a seat is rejected.*

Step k: *In general, each student that was rejected in the previous step proposes to the school which is next on his/her list of preferences. The schools will then choose which students to assign a seat, taking the new proposers together with the proposers which they have assigned seats to in the previous steps. When a type-specific quota is filled all remaining students of that type are rejected and the algorithm continues with the students of the remaining types. Any student not assigned a seat is rejected.*

The algorithm stops when all students have been assigned a seat a school (Abdulkadiroğlu & Sönmez, 2003, p. 739).

The modified mechanism satisfies *weak stability* (Abdulkadiroğlu, 2005, p. 10). A matching μ is weakly stable if it does not violate the type-specific quotas and there exists no student-school pair (i, s) such that, $sP_i\mu(i)$ and either:

1. $\mid \{i \in I : \tau(i) = \tau_1\} \mid < q_s^{\tau_1}$ and $\mid \mu(s) \mid < q_s$ or
2. $\mid \{i \in I : \tau(i) = \tau_1\} \mid < q_s^{\tau_1}$ and $i \succ_s i'$ for some $i' \in \mu(s)$
3. $\mid \{i \in I : \tau(i) = \tau_1\} \mid = q_s^{\tau_1}$ and $i \succ_s i'$ for some $i' \in \mu(s) : \tau(i) = \tau(i') = \tau_1$

A matching μ is hence weakly stable if: There does not exist a student i who prefer school s to his/her current matching and either: (1) The type-specific quota for student i is not filled at school s and there are still available seats at school s. (2) If the type-specific quota is not filled at school s and school s prefers student i to student i' with whom it is currently matched (regardless of student $i's$ type). Lastly, (3) if the type-specific quota for student i is filled and school s prefers student i to student i' with whom it is currently matched and the two students belong to the same type (Ergin & Sönmez, 2006, p. 229). This is the analogous definition for a matching to be *blocked* as discussed for the original case at page 5. Since this is satisfied, the Gale-Shapley Student Optimal Mechanism with Controlled Choice completely eliminates *justified envy* between students of the same type. Moreover is the Gale-Shapley Student Optimal Mechanism with Controlled Choice strategy-proof (Abdulkadiroğlu, 2005, p. 544).

Proposition 5. *The Gale-Shapley Student Optimal Mechanism with Controlled Choice is strategy-proof.*

If this version of the SOSM would be implemented in Malmö stad students would probably only be divided into two different kind of types, these could for example be: Immigrants (I) and non-immigrants (NI). If only two types of students are to be considered the following version of the SOSM can be used:

Consider a school s with quotas q_s^I and q_s^{NI} where $q \geq q_s^I$, $q \geq q_s^{NI}$ and $q_s^I + q_s^{NI} \geq q$, s is then seen as three different schools, s^1, s^2 and s^3. Where s^1 has $q - q^{NI}$ seats reserved for type I students, s^2 has $q - q^I$ seats reserved for type IN students and s^3 has $q - q^I - q^{IN}$ reserved seats for students regardless of type. The priority list for s^1 is simply constructed as for the original case with the difference that type NI-students are removed from the list. The NI-students are hence unacceptable at school s^1. The priority list or s^2 is constructed in the same way as for s^1 with the difference that the I-students are not acceptable. Finally is the priority list for s^3 the same as in the original case for s (Abdulkadiroğlu & Sönmez, 2003, p. 740).

This mechanism does the following:

1. $s = s^1 + s^2 + s^3$, $\forall s \in S$ as explained above.

2. $s^1 \succ_i s^2 \succ_i s^3$, $\forall i \in I$ and $\forall s \in S$

3. if $s \succ_i t$ then $s^1 \succ_i s^2 \succ_i s^3 \succ_i t^1 \succ_i t^2 \succ_i t^3$

4. Selects the student optimal stable matching (Abdulkadiroğlu & Sönmez, 2003, p. 740)

2.3.2 The Top Trading Cycles Mechanism with Controlled Choice

Incorporating controlled choice into the TTC is neither very difficult nor changes the algorithm by much. When the controlled choice constraints are flexible each school are given one more counter for each type of student and the following version of the TTC is run:

Algorithm 4. *The Top Trading Cycles Algorithm with Controlled Choice*

Step 1: *Each type-specific counter is set equal to the given quota of that type and each school specific counter is set to the capacity of the school. Each student points to their most preferred school and each school points to the student with highest priority. A cycle of students and schools are formed such that students can change seats in order to pareto improve their outcome. Each counter is reduced by one and each of the type-specific counters are reduced by one for those students who are given a seat. The other counters remain unchanged and if a school specific counter equals zero, the associated school is removed. If there are unassigned students the algorithm continues with the next step.*

Step k: *In general, Students point to their most preferred school of those who remain and has at least one vacant seat for the students' type. Schools then point at the students who have highest priority and at least one cycle is formed this way. The students can then change seats in order to pareto improve their outcome. Each counter is reduced by one and each of the type-specific counters are reduced by one for those students who are given a seat. The other counters remain unchanged and if a school specific counter equals zero, the associated school is removed. If there are unassigned students the algorithm will continue with the next step.*

Due to that the TTC with controlled choice has to meet the controlled choice constraints is it *constrained efficient*. This implies that the TTC with type-specific quotas is less efficient than the TTC without type-specific quotas.

Proposition 6. *The Top Trading Cycles Mechanism with Type-specific Quotas is constrained efficient.*

Moreover is the TTC with type-specific quotas strategy-proof (Abdulkadiroğlu & Sönmez, 2003, p. 740-741).

Proposition 7. *The Top Trading Cycles Mechanism with Type-specific Quotas is strategy-proof.*

Implementing one of these two mechanisms could have impact in making schools in Malmö stad more equal and could be a tool for making immigrants in Malmö stad more integrated.

2.4 Two implemented Gale-Shapley Student Optimal Mechanisms

Between the years 2003-2004 was a Gale-Shapley student optimal mechanism designed for solving the school choice problem in New York City (NYC) (Abdulkadiroğlu *et al.*, 2005b, p. 364). One year later in July 2005 another version of the SOSM was implemented in Boston (Abdulkadiroğlu *et al.*, 2006, p. 2). In this section is a review given of the two mechanisms highlighting the main reasons for their implementation.

2.4.1 The Gale-Shapley Student Optimal Mechanism in NYC

A SOSM was designed for matching over 90 000 entering students to high school each year in NYC. The mechanism showed positive results in the first year it was used. Only 3000 students were matched with a school which was not stated on their list of preferences, compared to the 30 000 students the year before (Abdulkadiroğlu *et al.*, 2005b, p. 364).

In the old system, students applying for high school programs were given the opportunity to make a preference list of five programs (only a bit more than 50 % of the students listed five programs). This preference list was then sent to the schools stated on the list and the schools proceeded differently depending on the type of school. Unscreened schools assigned their seats by lottery, whereas screened schools could make individual evaluations of all students. Zoned schools gave priority to students who lived close to the school and Educational option could make individual evaluations of students for half of their seats. In addition to this, seven schools known as specialized schools used entrance exams in order to assign seats at their schools (Abdulkadiroğlu *et al.*, 2005b, p. 364). Once the above was taken into account by the different types of schools could the schools accept, reject or put applicants on a waiting list. Students were then given decisions from the different schools, to which they could accept only one offer and be put on a waiting list for one other school. Once the students had responded were new offers made from schools which still had vacant seats. This procedure was repeated three times and if a student was not assigned a school in these three steps he/she was usually matched with the appropriate zoned school (Abdulkadiroğlu *et al.*, 2005b, p. 365).

There were three major problems associated with the old mechanism. First of all, was it suffering from congestion since the market did not clear and 30 000 students were assigned a school to which they had not stated any preference for. Secondly, had the parents (and students) to think strategically when stating their list of preferences. Third, schools were also strategic in such a way that they were not reporting their true total capacity of available seats to the central administration (Abdulkadiroğlu *et al.*, 2005b, p. 365). The old mechanism did hence not seem to be strategy-proof.

When the new mechanism was designed, the NYC school market was seen as two-sided. The reason for this was that schools also acted strategically and that some schools had different preferences over students with low scores. Since the market was assumed to be two-sided was a Gale-Shapley Deferred Acceptance Algorithm implemented in order to get a stable assignment as described at page 6. Since the students' welfare was of priority, was the mechanism chosen to be student-proposing rather than school-proposing. Furthermore, it would give incentives for students to represent their true preferences (Abdulkadiroğlu *et al.*, 2005b, p. 365-366).

For unscreened schools, list of priorities were generated randomly whereas each half of the educational option programs were divided into three different programs and priorities were based on the 16/68/16[2] reading score distribution.

There were two aspects of the mechanism that gave incentives for students to misrepresent. First did the educational option programmes automatically assign seats to the students with the top 2% reading scores if they listed the school as their top choice. Secondly was the list of preferences limited to a number of 12 schools (Abdulkadiroğlu *et al.*, 2005b, p. 366). Furthermore, one aspect of the new design could make the matching unstable. The decision makers in NYC wanted students that were admitted to specialized schools also to be given an offer from a nonspecialized school. Due to this, a first round of the algorithm was run with all the students which gave the students who

[2]The Educational option programs had to admit half of their students based on the results of a standardized English Language Arts exam. 16 % of the seats were to be assigned to the top performers, 68 % to middle performers and 16 % to low performers (Abdulkadiroğlu *et al.*, 2005b, p. 364)

were given a seat at a specialized school the choice between that school and a non-specialized school. After these students had chosen which school to attend, they were removed and the algorithm ran a second round. In this round the remaining students were given information about their assignment. Separating the mechanism into two rounds could make some students suffer from justified envy. If a student i has the following preferences: $s_{nonspecialized\ school\ 1} \succ_i s_{specialized\ school} \succ_i s_{nonspecialized\ school\ 2}$ Then if i is given a choice in the first round of choosing between $s_{specialized\ school}$ and $s_{nonspecialized\ school\ 2}$ (since he is ranked low at $s_{nonspecialized\ school\ 1}$) he/she will accept a seat at $s_{specialized\ school}$ and be removed. This would create instability if a sufficient number of students in the first round rejected the offer from $s_{nonspecialized\ school\ 1}$ such that i could actually have been given a seat at $s_{nonspecialized\ school\ 1}$.This would require the algorithm to only run one round such that i would not be removed. This problem is however limited if all students given an offer from a specialized school are ranked high at all schools' list of priorities (Abdulkadiroğlu *et al.*, 2005b, p. 366).

Finally, the students who had not been given a seat at any school after the second round were asked to make a new preference list of 12 schools, choosing between the ones who had yet empty seats. School priorities were not updated and students were given the a random priority, which applied at all schools. After the third round, the remaining students were assigned seats at different schools administratively (Abdulkadiroğlu *et al.*, 2005b, p. 366).

Despite some problems of information and implementation was the new mechanism in NYC a success. The number of unassigned students was 10 % compared to the year before. 20 000 more students were given a choice from their first list of preferences and 3000 more students were assigned a seat at a school which was top-five at their list of preferences. Furthermore, an additional 7600 students were assigned a seat based on the students' list of preferences, with the old mechanism, these students were unmatched. Much of these positive results were due to that students now ranked 12 schools instead of five and that students did not receive several offers from different schools (Abdulkadiroğlu *et al.*, 2005b, p. 366-367). Furthermore, in the years 2004-2007 more students have been given their first, second or third choice respectively than in the old mechanism in 2003 (Abdulkadiroğlu *et al.*, 2009, p. 1967).

The mechanism seems to work better each passing year and there may be several reasons for this. First of all do the schools' incentives to not report their true capacity, seem to have vanished with the new mechanism (which was expected by theory). Over the years school have capacities increased in total and especially the popular schools have been reporting higher capacities than before. Secondly, the schools have ranked many more students over the years. Prior to the new mechanism did many schools only rank the students who listed their school as their most preferred alternative. Since the information about the students' preference lists are not available to the schools under the new mechanism they have to rank many more students in order to not have any vacant seats after the algorithm has run. Moreover, the informational aspects have been highlighted. Many high school fairs have been arranged in NYC since the new mechanism has been in use and student guidance counselors now have more knowledge on how the new system works (Abdulkadiroğlu *et al.*, 2009, p. 1968).

2.4.2 The Gale-Shapley Student Optimal Mechanism in Boston

In Boston the school choice problem consisted of matching students to four different entry grades: Kindergarten, 1, 6 and 9. In 2004 were there approximately: 4800 students entering kindergarten, 4000 first grade, 4300 grade three and 4000 grade 9. Parents were asked to make a list of preferences of at least three (but no more than five (Abdulkadiroğlu *et al.*, 2006, p. 4)) schools and depending on which entry grade the student was at, the school options differed. For students applying for schools at grade 1 and 6, could only schools in their resident zone be applied for plus five other schools which were open to all students independent of where they lived. The students applying for high schools could choose between 18 unscreened schools, 13 screened schools and 5 schools which were not part of the centralized system (Abdulkadiroğlu *et al.*, 2005a, p. 368).

In Boston are the schools divided in three different zones and priorities to half of the students at most schools are determined in the following way:

1. Guaranteed priority: If the student is already studying at the school (in a lower grade) he/she is guaranteed a seat at the school.
2. Sibling-walk priority: If the Student is living within the walk zone of the school and has a sibling studying at the school.
3. Sibling priority: If the Student has a sibling studying at the school and is living outside of the walk zone of the school.
4. Walk zone priority: If the Student is living within the walk zone of the school.
5. Other students living within the walk zone.
6. Ties within each category are broken by a random lottery.

The other half of the students are assigned regardless of where in Boston they are living following: 1) Guaranteed priority 2) Sibling priority 3) Random lottery for ties (Abdulkadiroğlu *et al.*, 2006, p. 4).

The mechanism used in Boston prior to 2005 is known in the school choice literature as *the Boston mechanism* (Abdulkadiroğlu *et al.*, 2006, p. 2). The Boston mechanism is a *priority matching mechanism* in the sense that it tries to give all the students their first choice (Abdulkadiroğlu *et al.*, 2005a, p. 368). The Boston algorithm resembles to algorithm 1 on page 6 with the difference that students who are given a seat at a step say k is not considered together with the new proposers at the steps following k (Abdulkadiroğlu *et al.*, 2005a, p. 370). Hence, once a student is assigned a seat a school he/she keeps it even though there might be students with higher priority proposing to the school at a later step. This leads to that students have to act strategically when stating their preferences. Consider a student i who states his/her true preferences which are:

$P_i = s_1 \succ_i s_2$ if student i is not admitted to s_1 in step 1, then once proposing to s_2 this school might not have any vacant seats left. Even if student i has the highest priority among all students in Boston at s_2 i. e $\pi_{s_2}(1) = i$ he/she does not get a seat at the school. This will lead to that student i will be assigned a seat at a school which is less preferred to both s_1 and s_2. It would hence be better for i to misrepresent his preferences by: $P_i' = s_2 \succ_i s_1$ in order to be better off (i. e. at least get a seat at s_2). The Boston mechanism is hence not strategy proof (Abdulkadiroğlu *et al.*, 2006, p. 6).

The Boston school committee voted in July 2005 to replace the Boston mechanism with a mechanism which did not require families to game the system (Pathak & Sönmez, 2008, p. 1636). The main problem identified by the Boston school committee was that families who had time and resources to learn the system could game the system and by doing so they hurt families who did not game the system. The Boston school committee wanted the new mechanism to be strategy proof and they identified further arguments for implementing such a mechanism. First of all would the new system be transparent since the incentives to misrepresent preferences would disappear. Second would the new mechanism probably yield a more efficient match than the Boston mechanism. Third, by true preference revelation the Boston school committee could distinguish the popular schools and would be able to measure the effects of policy changes more accurately (Abdulkadiroğlu *et al.*, 2006, p. 24).

One of the main differences between the system in Boston and the one in NYC was the way school priorities were determined. In NYC the schools themselves made lists of priorities for the students. In Boston on the other hand the priorities were determined by the local laws discussed at page 15. The incentives for schools to game the system which existed in NYC did hence not seem to be present in Boston (Abdulkadiroğlu *et al.*, 2005a, p. 370).

The Boston school Committee considered replacing the Boston mechanism with either the SOSM or the TTC (since both are strategy proof). The final choice fell on implementing the SOSM. The arguments were that some priorities such as sibling priority were of higher importance than efficiency and was not something which was ought to be traded between students. Moreover did the Boston school committee regard the TTC as more complicated to explain to the parents which could make it less transparent and the trading feature could be a way for parents to game the system (Abdulkadiroğlu *et al.*, 2006, p. 25-26). Once the SOSM was implemented, families were urged to list as many schools as possible or at least six, in order to be assigned a seat at a school on their list of preferences (Abdulkadiroğlu *et al.*, 2006, p. 27).

A number of papers have been written analysing the positive effects that stem from a change in student assignment mechanism from the Boston mechanism to the SOSM (or the TTC) by characterizing behavior in the different Nash equilibria (see Ergin & Sönmez, 2006; Pathak & Sönmez, 2008; Chen & Sömnez, 2006).

Chen & Sömnez (2006) construct a designed and a random school environment in order to test what the outcomes of a change from the Boston mechanism to the SOSM or TTC would be. They find first of all, that the SOSM and TTC make more students state their true preferences (Chen & Sömnez, 2006, p. 212). This experimental result is also theoretically confirmed by Pathak & Sönmez (2008) who concludes that students who game the system (sophisticated players) are better off at the expense of students who state their preferences truthfully under the Boston mechanism. Furthermore, sophisticated players have to coordinate their actions in order to reach the pareto dominant Nash equilibria in the Boston mechanism (Pathak & Sönmez, 2008, p.1642-1643). There are hence incentives for students to become sophisticated. Moreover, are students better off if they game the system compared to if they state their true preferences in the Boston mechanism (Pathak & Sönmez, 2008, p.1646). Hence, by replacing a priority mechanism such as the Boston mechanism with the SOSM or TTC will students be given incentives to state their preferences truthfully. Another important result, confirmed both experimentally and theoretically is that the SOSM and TTC both produces more efficient outcomes than the Boston mechanism (Ergin & Sönmez, 2006, p. 235), (Chen & Sömnez, 2006, p. 216).

Following these results, the Boston School Committee seem to have made a correct choice by replacing the Boston mechanism with the SOSM since many problems have been solved.

2.5 Theoretical developments

Since Abdulkadiroğlu and Sönmez published their paper in 2003, some aspects of the mechanisms have been discussed. This section will highlight two aspects, random tie-breaking and incentives in large markets.

2.5.1 Random tie-breaking in the Gale-Shapley student optimal mechanism

The general problem with tie-breaking is that schools seem to have strict priorities over students for which they actually are indifferent, and this might lead to that an inefficient matching is selected. When schools are indifferent between students the tie is usually broken by a random lottery, which has been the case in both NYC and Boston. The SOSM will then find a stable matching based on student preferences and school priorities. Since some priorities are determined randomly will this stability be artificial (Abdulkadiroğlu *et al.*, 2009, p 1956). In order to improve efficiency when random tie-breaking is used Erdil & Ergin (2008) have developed the *stable improvement cycles mechanism*. The general idea of the stable improvements cycle is similar to the TTC since students are allowed to change seats in order to make pareto improvements. The main difference is that the stable improvement cycles mechanism preserves stability. The following example might clarify the difference:

Consider four students $\{i_1, i_2, i_3, i_4\} \in I$ and three schools $\{s_A, s_B, s_C\} \in S$. Suppose the TTC is run and a cycle is formed in step 1 such that: $\{s_A, i_1, s_B, i_2, s_C, i_3\}$ then students would trade schools and i_1 would be matched with s_B. Consider however if i_4 ranks s_B as his/her top choice and has the priority $\pi_{s_B}(1) = i_4$, then justified envy arises and the matching is not stable. This instability stems from that i_4 does not have top priority at school s_A and does hence not form part of the cycle. The stable improvement cycles solves this by requiring that in order to be part of a cycle must a student i have the top priority of all students desiring the school s. In this example i_4 would belong to the cycle rather than i_1 since he/she has higher priority at school s_B (Erdil & Ergin, 2008, p. 672).

For the formal definition let $sP_i\mu(i)$ mean that student i *desires* school s to the school with which he/she is currently matched with. Moreover, D_s denotes the set of students which desires school s and have top priorities at school s. Furthermore, let $\mu(i_{n+1})$ denote the school which is most preferred by student i_n. A stable improvements cycle can then be defined as the following:

Definition 1. *Stable improvements cycle*

In a stable improvements cycle there exists a set of students: $\{i_1, \ldots, i_n\} = I_c \subseteq I$ *such that:*

1. $\mu(i_n) \in S$,
2. $\mu(i_{n+1})P_{i_n}\mu(i_n)$ *and*
3. $i_n \in D_{\mu(i_{n+1})}$
4. *The above holds for all* $i_n \in I_c$

The interpretation of this is first of all that every student should be given a seat at a school. Secondly, every student should desire another school compared to the one they are currently matched with. Finally, all students should belong to the list of top-priorities issued by the school they prefer. If this is satisfied, a stable improvements cycle can be created and a new matching μ' can be defined as:

$$\mu'(j) = \begin{cases} \mu(j) & \text{if } j \notin I_c, \\ \mu(i_{n+1}) & \text{if } j = i_n. \end{cases}$$

The interpretation of $\mu'(j)$ is that students are matched with the same school as before if they are not in the stable improvements cycle and if they are, they will be matched with a, for them, more preferred school which hence makes them better off. Important features of the matching μ' is that it will still be stable and it will also pareto dominate the matching μ (Erdil & Ergin, 2008, p. 675). Furthermore, the following proposition has been proven:

Proposition 8. *If* P_I *and* π *are fixed and* μ *is a stable matching. A stable improvements cycle is then allowed if* φ *is any other stable matching that pareto dominates* μ.

The interesting with this proposition is that in order to know if pareto improvements are possible for a stable matching is it sufficient to find a stable improvements cycle (Erdil & Ergin, 2008, p. 675-676).

Based on this result can an additional feature be added to the Deferred Acceptance Algorithm in the following way.

Algorithm 5. *The Stable Improvements Cycles Algorithm*

Step 1: *The deferred acceptance algorithm is run and a matching* μ^1 *is selected. Ties that occur, based on the schools' list of priorities, are broken in the way decision makers see fit.*

Step k:

1. *Given the matching* μ^{k-1}*, if a student* i *is matched to* s_1 *i. e* $\mu^{k-1}(i) = s_1$ *and* $i \in D_{s_2}$*, let school* s_1 *point to school* s_2*.*

2. *If there exists any cycles i. e* s_1 *and* s_2 *are both pointing at each other, choose one cycle. Next is a student chosen such that* $\mu^{k-1}(i) = s_1$ *and* $i \in D_{s_2}$*, and a stable improvements cycle is conducted allowing for* μ^k *to be selected. Once this is done the algorithm continues at step k+1(1). If no cycle is found* μ^{k-1} *will be the selected matching.*

In the stable improvements algorithm are, the tie-breaking rule in step 1, the cycles and the students in step k, to be chosen by the decision makers. Furthermore, the stable improvements cycles algorithm is computationally easy to carry out which is an advantage. An option for using the stable improvements cycles algorithm would be to run the Deferred Acceptance Algorithm for every possible tie-breaking procedure. This would however, require a lot of time (Erdil & Ergin, 2008, p. 676). Erdil and Ergin confirms that pareto improving upon a matching selected by the SOSM might harm strategy proofness (Erdil & Ergin, 2008, p. 683).

How efficiency gains might affect strategy proofness is something that has been further explored by Abdulkadiroğlu *et al.* (2009), who apply different tie-breaking procedures to the NYC school high school match. When designing the NYC high school match there was a discussion on how to break ties in order for the outcome to be fair. Decision makers in NYC preferred a multiple tie breaking rule (MTR) which was to base schools' list of priorities on a random number generated for each student at each different program. A single tie breaking rule (STR), which assigns a random number to each student which is to be used at every program, was suggested since students are usually better off when such a tie-breaker is applied. The final decision fell however on the multiple tie-breaker (Abdulkadiroğlu *et al.*, 2009, p. 1961).

Abdulkadiroğlu *et al.* (2009) have compared the performance of the Deferred Acceptance Algorithm using a STR, a MTR and the stable improvements cycles algorithm applied to the matching first selected by the STR. For the STR they apply 250 tie breaking rules, which are uniformly drawn, to the preferences submitted by the NYC students in the year 2006-2007. These numbers are then compared with the case of the MTR, where 250 tie breaking rules that are specific to the different schools, are uniformly drawn and applied to the students' preferences. From these simulations a STR clearly outperforms a MTR since more students are given their top choice. Furthermore do the simulations from the STR with the stable improvements cycles algorithm suggest that even more students would be given their first choice than if the stable improvements cycles were not performed. These facts made the decision makers in NYC to change the MTR to a STR (Abdulkadiroğlu *et al.*, 2009, p. 1961-1962).

One provided result is that a matching cannot be a student optimal stable matching if the matching is generated using a MTR and the same matching cannot be generated using a STR. More importantly is the following proposed (Abdulkadiroğlu *et al.*, 2009, p. 1962):

Proposition 9. *For a matching μ which is selected using any tie-breaking rule and the deferred acceptance algorithm, there does not exist any other matching φ that pareto dominates μ and is still strategy-proof.*

To see this it is is important to first establish that the same set of students will be matched in the two different matchings i. e $\mid \varphi(S) \mid = \mid \mu(S) \mid$ Consider the matching φ which pareto dominates μ. A student which is matched in μ will then also be matched in φ , $\mid \varphi(S) \mid \geq \mid \mu(S) \mid$. If this was not true μ would not be individual rational since a student matched in μ but not in φ would have the preferences: $\varphi(i) = iP_i\mu(i)$. This would imply that μ assigns i to a school, which is not acceptable for him/her, this in turn, is a contradiction.

If there are more students matched in φ than in μ, i. e $\mid \varphi(S) \mid > \mid \mu(S) \mid$, μ would not be stable. This is due to that there would exist a school $s \in S$ and a student $i \in I$ such that $\varphi(i) = s \neq \mu(i)$ which implies that in the matching μ a seat is left open even though i is acceptable for s. Moreover, since φ dominates μ is student i preferring to be assigned to s rather than to remain unassigned, this would hence make μ unstable. All this brought together implies that all students that are assigned a seat in μ are also assigned a seat in φ, $\mid \varphi(S) \mid = \mid \mu(S) \mid$.

Now to see that φ is not strategy-proof we call $x_i = \mu(P_I, \pi)$ the assignment given to i in μ and $y_i = \varphi(P_I, \pi)$ the assignment given to i in φ. Since φ pareto dominates μ is $y_iP_ix_i$. Consider the case where i misrepresent his/her preferences by $P'_I = (P'_i, P_{-i})$, where y_i is the only school on the list. Since μ is strategy proof, will i be worse off misrepresenting and will hence not be assigned a seat at any school, i. e $\mu(P'_I, R_S) = i$. Since $\mid \varphi(S) \mid = \mid \mu(S) \mid$, will student i also be unassigned in the matching φ, i. e $\varphi(P'_I, \pi) = i$. Suppose now that P'_I are student $i's$ true preferences, i could then misrepresent by submitting P_I in order to be better off such that: $\varphi(P_I, \pi) = y_i$ instead of being unassigned. Since y_iP_ii, will pareto improving a strategy-proof mechanism give incentives for students to misrepresent and the associated pareto improved matching is hence not strategy-proof (Abdulkadiroğlu *et al.*, 2009, p. 1963).

Proposition 9 states clearly that there is a trade-off between efficiency and strategy-proofness. Furthermore, it implies that the TTC cannot pareto dominate the SOSM with a STR, since it is strategy-proof, in fact neither one dominates the other (Abdulkadiroğlu *et al.*, 2009, p. 1963). When tie-breaking is present is it best to use a STR and as shown above, the inefficiency from random tie-breaking can only be improved by not making the mechanism strategy-proof. In the case when random tie-breaking is a severe problem a thorough examination of the consequences of pareto improving the matching should be made. When designing a new mechanism strategy-proofness and the transparency that comes with it are usually of high importance. This suggests that the inefficiency associated with random tie-breaking is a cost that might be worth to pay.

2.5.2 Strategy proofness in large markets

From proposition 2 it is known that the SOSM is strategy proof from the viewpoint of the proposers. Since the mechanisms that have been implemented in NYC and Boston are student proposing, have students had no incentives to misrepresent their preferences. The schools on the other hand can successfully manipulate the SOSM, but this seems rarely to be the case. Kojima & Pathak (2009) have investigated why the SOSM seem to work well in practice even though theory suggests it might not. There are generally two ways a school can manipulate the SOSM, first by misrepresenting its priorities and secondly by withholding capacity (Kojima & Pathak, 2009, p. 613). As discussed on page 13, did both these problems exist prior to that the SOSM was implemented in NYC. These problems did however vanish when the SOSM was implemented.

Kojima and Pathak construct random markets by randomly generating preferences for each student. A random market is formally: $\tilde{\Gamma} = (S, I, \pi_s, k, D)$, where D is a probability distribution on S and k is a positive integer which denotes the length of the students' preference lists. Moreover is a sequence of random markets defined as $(\tilde{\Gamma}^1, \tilde{\Gamma}^2, \ldots)$ where $(S^n, I^n, \pi_{s^n}, k^n, D^n)$ would be a random market and $\mid S^n \mid = n$ the number of schools in the market. A sequence of random markets is assumed to be *regular* if there exists two positive integers k and $\bar{q}$ such that:

1. $k^n = k$, $\forall n$,
2. $q_s \leq \bar{q}$ for $s \in S^n$ and $\forall n$,
3. $\mid I^n \mid \leq \bar{q}n$, $\forall n$ and
4. $\forall n$ and $s \in S^n$, every $i \in I$ is acceptable to s.

First (1) assumes that the length of students' preference lists is constant, regardless of how many schools are in the market. Second, (2) says that the quotas at each school have to be bounded. Third, (3) requires the number of seats at schools to grow at least as fast as the number of students in the market. Finally (4) states that every student is acceptable to every school (Kojima & Pathak, 2009, p. 615-616).

Now let $\alpha(n)$ be the expected number of schools who can manipulate the SOSM when the other schools act truthful. The following result will then hold true in markets of complete information where a sequence of random markets is regular:

Proposition 10. *The proportion of schools who can manipulate the SOSM, $\alpha(n)/n$ goes to zero as the number of schools goes to infinity.*

When the number of schools in the market is large it becomes less profitable for schools to manipulate the SOSM (Kojima & Pathak, 2009, p. 616). Furthermore, a useful definition of *thickness* is provided:

Definition 2. *A sequence of random markets is sufficiently thick if there exists a $T \in \mathbb{R}$ such that:*

$$E[\mid V_T(n) \mid] \to \infty$$

as $n \to \infty$

$V_T(n)$ is a set where schools belong who are popular enough and have vacant seats. Formally,

$$V_T(n) = \{s \in S^n \mid \max_{s' \in S}\{P^n_{s'}\}/P^n_s \leq T, \sharp\{s \in S^n \mid sP_i i\} < q_s\}$$

The ratio $\{P^n_{s'}\}/P^n_s$ is a ratio of popularity between two schools. The condition $\{P^n_{s'}\}/P^n_s \leq T$ ensures that when a market grows larger, the ratio of popularity will not grow without bound, this implies that there are many schools in the market who are popular. Hence, a market which is sufficiently thick is a market which, as it grows larger, the number of popular vacant seats also grows larger. In other words will unmatched students in a sufficiently thick market find a seat at another school acceptable for them. Having defined thickness, another interesting proposition can be presented:

Proposition 11. *If a random sequence of markets is regular and sufficiently thick. For $\epsilon > 0$ and n_0, telling the truth will be a ϵ-Nash equilibrium for the schools in any market with more schools than n_0 and that belongs to the sequence .*

What these two propositions imply is that schools will not misrepresent their priorities nor withhold school capacities in large markets who are sufficiently thick (Kojima & Pathak, 2009, p. 620-622). These results add more reasons to adopt a SOSM since it gives conditions where the SOSM does not give incentives for the schools as well as for the students to game the system.

3 School choice in Malmö stad

In Malmö stad are there 98 elementary schools divided in 10 different zones. Out of these 98 schools are 70 public, 23 private and 5 specialized for children with special needs (Malmö stad, n.d.b). Furthermore, there were 27078 students studying in Malmö stad in September 2011 (see table 1).

The main difference between the private and the public schools, in a school choice perspective, is how they rank students. Hence, the list of priorities looks different for public schools compared to private. For public schools, according to Swedish law[3], should a student be given a seat at the school he/she wishes to attend. However, If giving the student a seat at a school causes another student to be left with no seat at the school closest to his/her home, the wish of the first student should be ignored. Moreover, the municipality, in this case Malmö stad may disregard a student's wish if:

1. The student's wish imposes organizational and economic difficulties for the municipality, or if

[3]Skollagen, Chapter 10, 30§

2. it is necessary regarding other students' safety and their study environment (Sveriges Riksdag, n.d., p. 42-43).

The central idea is that a student living close to a school has higher priority at that school than a student living further away. Furthermore, (1) could be used in order to deny a student's request if the seats at a school are filled. In Malmö stad public schools gives priority to students in the following way:

1. To the students living closest to the school
2. To the students who applied first to the school

Moreover, the system is decentralized in the sense that all school manages their queue by themselves (Wramell, 2012). For private schools, the rule is different and Swedish law[4] states that if there are not enough seats for all students applying, should the selection be based in a way that is approved by Statens skolinspektion (Sveriges Riksdag, n.d., p. 44). This gives the schools more freedom when selecting their students compared to the public schools. The customary priority ordering approved by Statens skolinspektion is the following:

1. The date when the student applied for the school
2. If the student has any siblings currently attending the school.
3. If the student is already studying at the school (in a lower grade) he/she is given priority at the school when applying for a seat in a higher grade.

This may however vary depending on the school (Skolverket, n.d.). As public schools, are the private schools managing their own queues (Malmö stad, n.d.c).

One year prior to that students are enrolled in school, are they sent a letter stating which public school they have been assigned to. This assignment is based on in which of the 10 zones the student lives in and which school within the zone, that is closest to the students' home. If the students want to change to another public school are they required to return a letter stating which schools they wish to attend. In this letter the students can state as many schools as they wish and once having applied will students be put in a queue as described above. If a student wish to attend a private school, must he/she apply to the school separately. Students must however obtain information about which schools that have vacant seats by themselves. Furthermore, the information given by Malmö stad about the different schools regarding quality etc. is very limited which further complicates the students' choices.

The private schools assign students to seats according to their list of priorities and students who are not given a seat have to look elsewhere for a vacant seat. For public schools an administrative process tries to assign every student to his/her most preferred choice without causing another student to be left without a seat at the school close to

[4] Skollagen, Chapter 10, 36§

his/her home. This gives first of all rise to an interchange of students amongst the schools within the same zone (since these students by law have priority at the schools). Secondly, students from other zones are given seats based on the list of priorities discussed above. The administrative process requires a lot of working hours and since the system is decentralized and the schools do not know how the students rank their school can the process be very time consuming. Due to the lengthy process are students sometimes given their assignment in late May (Johansson, 2012). This seems natural however since the schools do not want to assign seats to too many nor too few students at their schools.

Malmö stad does not collect data on which schools are the most popular nor the flow of students within the different zones. There does however exist data on the flow of students between the different zones and how many students from each zone that attend private schools. This data is presented in table 1 in the appendix. It can first of all be noted that 1603 students or 5.6 % of the students in Malmö stad choose to study in another municipality. Furthermore, are 4111 students or 14,4 % studying at a private school. In total have 8913 students or 31.3 % chosen a school in a zone where they are not living. From this data a total of 13024 students or 45.7 % have chosen either a private school or a school outside of the zone which they are living in. Either way are these students not happy with the school they have been assigned to by Malmö stad. As mentioned above does this data not include the number of students who prefers another school within their zone, nor the number of students who are denied a seat at a school. The actual number of students preferring another school than the one they are being assigned to by Malmö stad is hence higher than 13024.

It is however possible, from available data, to determine which of the zones that are the most popular. According to table 2 is Västra Innerstaden the most popular zone with 598 students followed by Centrum, 234 students and Husie, with 191 students. The least popular zones are Rosengård with a negative flow of 660 students, Hyllie with 215 students and Södra Innerstaden with 168 students.

As discussed on page 9 is Västra Innerstaden the zone which has the highest academic performance. This would explain why Västra Innerstaden has the most popular and Rosengård the least popular schools in Malmö stad. It is however interesting to note that most students living elsewhere but studying in Västra Innerstaden are from Limhamn-Bunkeflo which is another area with high SES and is the second best performing zone in Malmö stad, while only a few students from Rosengård are studying in Västra Innerstaden. The reason for this can of course depend on factors such as distance to school. However do these facts seem to suggest that the school choice system used in Malmö stad further deepens the problem of segregation.

The main conclusion that can be drawn from this data is that the willingness, by students, to change school requires a well-functioning system for assigning the students to the vacant seats at the schools in Malmö stad. This is mainly important for the reason that students should be assigned the best school possible given their preferences and school capacities.

By comparing the system in Malmö stad to the mechanisms used in NYC and Boston prior to the implementation of the SOSM, some differences and similarities can be seen. First of all, the system in Boston was centralized since regardless of which zone the student was living in the assignment of seats were handled by a centralized mechanism, which is not the case in Malmö stad. Secondly, students in both NYC and Boston were required to present a list of strict preferences (true or not true) in order for the mechanisms to work. In Malmö stad, no students are submitting a ranking of the schools they would like to attend. The students who prefer other schools compared to the one they have been assigned to by Malmö stad can however express this preference. The lists of preferences for students are hence not complete nor does Malmö stad collect this vital information, which is a foundation for a well functioning system. Third, the list of priorities for schools in Malmö stad is most similar to the one used in Boston since students are given priorities to schools depending on where they live, which is determined by law. Furthermore, even though parents in Malmö stad cannot game the system as in Boston seems there to be a similar problem with well-informed (sophisticated) parents and less informed parents.

Regarding tie-breaking, ought the system in Malmö stad to not yield major problems since schools' list of priorities are determined by date of application. The risk of two (or more) students applying at the same date for the same school ought to be low. This could however lead to a problem since the well informed parents will apply for popular schools when their children are very young. Thus will parents who are less informed or who have recently moved to Malmö stad suffer since they will not apply for their desired school in time to get a high priority.

Schools misrepresenting their preferences or withholding capacities ought not be a problem in Malmö stad for two reasons. First of all are the schools, both public and private, managing their own queues and second are the priorities at schools determined by law or approved by Skolinspektionen. Moreover, schools in Malmö stad cannot screen students as was the case for some programs in NYC. There seems hence not be any incentives for schools to game the system, and if they would, they would be breaking the law. However, the notion of thickness is important for a well-functioning school market in Malmö stad. Malmö stad is a municipality with a growing population and the need for more seats in schools has become a problem. The most urgent problem at the moment is for children who are to attend kindergarten. Between the year 2010 and 2011 did the number of children in ages 1-5 grow by 6 % and there has been reports on long queues and the need for more seats in the kindergarten (Malmö stad, 2011, p. 7), (Skånska Dagbladet, 2010), (Metro, 2011). Furthermore, the number of students attending school rose by 27 % in the years 1996 - 2010 and Malmö stad is investigating the possibility of a reorganization to meet the future needs (Malmö stad, n.d.a). Not only does a sufficiently thick school market in Malmö stad require more seats to meet the needs of the growing population. It also requires the seats and hence the schools to be popular enough in order for students to be content with their assignment. If this problem is not solved will the current system probably further limit the free choice of school. This stems from that there will be many students applying for and living close to the limited number of popular schools. This could lead to that the popular schools become overcrowded and there will be few seats left for students actively applying for the popular schools.

However, if the public schools will fail to satisfy the need for popular seats could private schools probably fill this void.

4 A Student Assignment Mechanism for Malmö stad

If a student assignment mechanism was to be designed for Malmö stad would it be important to look at the problems which are existing in the current system and see how a new mechanism might be able to solve these problems. The system used in Malmö stad is associated with a number of problems.

A basis for a well-functioning school choice market is that every agent knows how the system works, key to this is of course available information. The system in Malmö stad seems however to be complicated and not very transparent, especially for the students and parents. The decentralized system with schools managing their own queues instead of Malmö stad managing them all together gives rise to a lot of uncertainty for the students and parents. Since the students are assigned administratively at every school, is there a possibility that a student will be offered a seat at all schools he/she has stated on his/her list but at different times. This in turn implies that parents have to be in constant contact with the schools to know where in the process the different schools are. Moreover, the risk for someone in this extensive process to commit an error ought to be higher than for a computerized algorithm.

The system requires parents to find information themselves of which schools are popular and which priority the child has. This might be a time consuming process, especially if the student is applying to many schools and could lead to an informational disadvantage for the parents who has not got the time for doing all the required research. The information about the schools is also very limited especially when it comes to comparing the performance or other measures of qualities for the schools. This is a crucial short coming of the current system since parents and students cannot base their choices on solid information. An easy solution would be to gather all the information in a web portal available for the parents and students. Furthermore, school fairs could be arranged to further inform the parents and students about their different options.

The major problem with the system in Malmö stad is similar to one of the problems in NYC where a lot of students were assigned a school for which they had not stated any preference. An appealing aspect of the mechanisms discussed in section 2 is that they are direct mechanisms. The mechanisms require students to reveal their preferences which is the basis for selecting a student optimal matching. The system used by Malmö stad does not urge all students to reveal their preferences which makes the matching process harder and this information could be used in making more accurate policy decisions. There will hence be a high risk that the current system assigns a student to a seat which he/she has not stated any preference for.

This is connected to the important properties of stability and efficiency. Since the system in Malmö stad is decentralized and does not base the matching on all students' submitted list of preferences, is the risk that the matching will not be stable nor efficient high. If a student for example applies to two public schools and is accepted to both of them will it be important that both schools communicate this to the student in time, and to each other. Consider a student: $i \in I$ and two schools: $s_1, s_2 \in S$ who has the following preferences $s_1 P_i s_2$ where s_2 accepts i in January whereas s_1 accepts i in May. Since i does not know that he/she will be accepted to s_1 later on he/she might accept s_2:s offer which would lead to an unstable matching.

Most importantly will the matching probably not be stable nor efficient since no student is submitting a strict list of preferences. Since the matching selected by the current system does not take all students' preferences into account will a stable or efficient matching be almost impossible to select. Since the system in Malmö stad does not extract nor uses this vital information, will a lot of students be given an assignment which they have not stated any preference for. Moreover, since no student reveals a strict list of preferences can the system in Malmö stad not be said to be strategy-proof. For a mechanism to be strategy proof has there to be no gain from misrepresenting your preferences. Since the students in Malmö stad do not reveal any preferences in the first place can not strategy-proofness be determined.

In the current system, many students are not revealing any preferences. However, since parents and students do not apply to other schools than the one they have been assigned to does not mean that there are no other schools in Malmö stad which they prefer to their assignment. There can be a number of reasons why parents/students do not express preferences for other schools where information about the system, the schools and school choice in general might be one.

An aspect of the current system worth discussing is if the list of priorities for the schools should be based on date of application and where you live. While date of application almost eliminates the problems associated with random tie-breaking, is it a feature which gives well informed parents an advantage over less informed parents. If parents knows how the system works, is the optimal strategy (given that moving is not an option) to apply the day their child is born, to every school which they prefer to the one which their child will be assigned to, once entering school. This will hence negatively affect the parents who do not know the system or who have recently moved to Malmö stad. If all (or a sufficient amount of) parents knows how the system works and uses this strategy will the list of priorities be based on the date when the children were born, which does not seem very fair. If the possible future problem of a not sufficiently thick market is added to this, parents would probably be applying for schools at an early stage. Furthermore, priorities based on where you live seem to worsen the segregation across schools. Rich families (i. e with high SES) will be able to afford to move to the zones where the well-performing and popular schools are, where they would get a high priority at these schools. Families from a low SES background will however not have this opportunity and their children will hence get a lower priority simply because they do not have the means to move to the popular areas. This does not seem fair nor gives an opportunity for everybody to choose their school freely. One solution to this is of course to redefine the list of priorities another would be to incorporate a controlled choice version of the mechanism. Controlled choice would give students from lower SES backgrounds a higher chance of being assigned a seat at a popular school.

Available data does not show how comprising the school choice problem is in Malmö stad. This is due to that students are not submitting a strict list of preferences nor is data collected on how many students which are denied a seat at a school to which they apply. However, the large number of students who actively apply for other schools than the one they have been assigned to, suggest that a well-functioning system is needed.

If a new mechanism would be designed for Malmö stad would it first of all centralize the system. Instead of the queues being managed by the separate schools would Malmö stad handle all students and schools together. Furthermore, the mechanism should assign students to both private and public schools in the same step, in order to avoid problems with unstable matchings similar to the ones discussed for the NYC high school match. This change would further not be compatible with date of application as one of the basis for schools' list of priorities since parents would apply for schools at the same time and to Malmö stad, not to the schools separately. Hence, the schools' list of priorities would have to be based on other criteria which the decision makers in Malmö stad believe are appropriate, where looking at for example Boston could be of help. If a random tie-breaker has to be used should it be a single tie-breaking rule applied for all schools.

Strategy proofness is an important property when designing a new mechanism. The main reason for this is that a strategy-proof mechanism makes the school choice very simple for families and schools. If a strategy proof mechanism would be implemented in Malmö stad such as the SOSM or the TTC would students only be required to submit a strict list of preferences over all schools. Since the mechanisms are strategy proof is this exactly what they will do. Parents will hence save a lot of time and would be less uncertain regarding how the assignment process works, given that Malmö stad will provide all the necessary information. Furthermore, communicating this information would reduce the gap between well and less informed parents. Moreover, the students' list of preferences ought to contain as many schools as possible for the market to clear when using the mechanism. By demanding students to submit a long list of preferences, Malmö stad will obtain very important information which it does not have today. From this information it would be possible to see which are the most/least popular schools and effects of policy decisions could easily be measured. Furthermore, this information does seem vital in satisfying the future need for popular vacant seats in Malmö stad in order to assure that the school market is sufficiently thick.

When choosing between the two strategy-proof mechanisms, SOSM and TTC is it a question of if stability or efficiency should be given priority. Since it is imposed by Swedish law that children who live close to a school should be given priority at that school, does it seem safe to assume that this priority ought not be traded between students. This would hence suggest that the SOSM is the optimal choice of mechanism for Malmö stad. In addition to the positive aspects discussed above would the SOSM generate a student optimal stable matching where no student would suffer from justified envy.

Implementing the SOSM will not only make school choice simpler for families, it will also make it simpler for Malmö stad. Since the deferred acceptance algorithm is computerized and computationally easy to carry out will the process of matching all students with all schools probably not take more than a couple of hours. This would hence save Malmö stad a lot of time and money since the problem does not need to be solved administratively at each school. Furthermore, families will be sent a letter with their final assignment which cannot be improved upon. There is hence no student who will receive his/her assignment in May, and the associated problems with justified envy and the risk of schools assigning too few/many students vanishes.

5 Conclusion

Looking at students' choices of elementary schools confirms that a well-functioning student assignment mechanism is required for assuring that students are given a good student-school match. The current system in Malmö stad does not yield an optimal match however and is unnecessarily complicated for the agents acting within it. Families are suffering due to the lack of transparency and information which creates uncertainty and obligates them to spend too much time learning how the system works. Schools are managing their own queues which makes the assignment process complicated and requires time and money. Furthermore, the current system is not designed to be a direct mechanism and the associated information which is not obtained makes an efficient or stable matching almost impossible. Moreover, segregation across schools is a problem with the current system and the basis for the schools' list of priorities does not seem fair in all aspects.

The current situation in Malmö stad suggests that a Gale-Shapley Student Optimal Mechanism would solve many of the current problems. In addition to change the system is information key for a well-functioning mechanism and controlled choice is an option for making schools less segregated.

These findings suggest that Malmö stad and similar municipalities in Sweden ought to revise their student assignment systems in order to assure that students are given a, which they consider, good match. Furthermore, more resources ought to be spent on spreading information of the schools and on how the system works.

For further research would it be important to determine how comprising the efficiency/stability problem is in Malmö stad. This would however require that data is collected on how many children are applying to all the different schools in Malmö stad and how many students are denied a seat at their preferred school(s). More interestingly would research be, estimating the effects of a possible implementation of the SOSM in Malmö stad. This would however in addition require the students' list of strict preferences over schools, which most easily is extracted using a strategy-proof mechanism. While writing this thesis, is Malmö stad undergoing a reorganization of the educational organization. Once the reorganization is realized would perhaps this analysis have to be revised if any important aspects of the school choice assignment system have been changed.

6 Appendix

Table 1: Flow of students between zones in Malmö stad.

Zones	CE	SI	VI	LB	HY	FO	OX	RO	HU	KI	OM	U	Prod
CE	1684	197	42	9	17	40	3	107	14	69	16	1	2199
SI	81	1286	6	6	25	47	2	120	13	14	11	0	1611
VI	51	31	1290	318	248	36	2	18	3	5	5	1	2008
LB	13	16	27	3920	179	25	8	20	5	3	14	12	4242
HY	15	52	15	33	1834	183	10	12	22	9	51	1	2237
FO	48	144	11	11	75	3384	22	184	15	6	28	3	3931
OX	4	4	2	7	2	11	1321	18	5	0	14	4	1392
RO	5	12	2	1	7	17	4	2085	5	9	3	3	2153
HU	21	7	4	1	4	23	5	215	1713	6	6	0	2005
KI	26	19	5	7	9	25	7	28	13	1040	10	0	1189
PS	319	330	329	828	497	432	62	562	497	227	18	10	4111
OM	122	180	108	174	188	300	67	335	34	95			1603
TOT	2389	2278	1841	5315	3085	4523	1513	3704	2339	1483	176	35	28470

Read horizontally are the students divided into the zone they are living in, vertically in the zone they are studying in. CE stands for Centrum, SI = Södra Innerstaden, VI = Västra Innerstaden, LB = Limhamn-Bunkeflo, HY = Hyllie, FO = Fosie, OX = Oxie, RO = Rosengård, HU = Husie, KI = Kirseberg, OM = Other Municipality, U = Unspecified, Prod = Total number of students studying in the zone, PS = Private School, TOT = Total number of students living in the zone.

Table 2: Net flow of students between zones in Malmö stad.

Zones	CE	SI	VI	LB	HY	FO	OX	RO	HU	KI
CE		-116	9	4	-2	8	1	-102	7	-43
SI	116		25	10	27	97	2	-108	-6	5
VI	-9	-25		-291	-233	-25	0	-16	1	0
LB	-4	-10	291		-146	-14	-1	-19	-4	4
HY	2	-27	233	146		-108	-8	-5	-18	0
FO	-8	-97	25	14	108		-11	-167	8	19
OX	-1	-2	0	1	8	11		-14	0	7
RO	102	108	16	19	5	167	14		210	19
HU	-7	6	-1	4	18	-8	0	-210		7
KI	43	-5	0	-4	0	-19	-7	-19	-7	
TOT	234	-168	598	-97	-215	109	-10	-660	191	18

References

Abdulkadiroğlu, Atila. 2005. College admissions with affirmative action. *International Journal of Game Theory*, **33(4)**, 535–549.

Abdulkadiroğlu, Atila, & Sönmez, Tayfun. 2003. School Choice: A Mechanism Design Approach. *The American Economic Review*, **93(3)**, 729–747.

Abdulkadiroğlu, Atila, Pathak, Parag A., Roth, Alvin E., & Sönmez, Tayfun. 2005a. The Boston Public School Match. *The American Economic Review*, **95(2)**, 368–371.

Abdulkadiroğlu, Atila, Pathak, Parag A., & Roth, Alvin E. 2005b. The New York City High School. *The American Economic Review*, **95(2)**, 364–367.

Abdulkadiroğlu, Atila, Pathak, Parag A., Roth, Alvin E., & Sönmez, Tayfun. 2006. Changing the Boston School Choice Mechanism. *NBER Working paper No. 11965.*

Abdulkadiroğlu, Atila, Pathak, Parag A., & Roth, Alvin E. 2009. Strategy-proofness versus Efficiency in Matching with Indifferences: Redesigning the NYC High School Match. *American Economic Review*, **99(5)**, 1954–1978.

Bunar, Nihad. 2012 (12/01). *Skolan och staden, forskningsperspektiv på integration och skolrelaterade klyftor i den moderna staden.*

Caldas, Stephen J., & Bankston III, Carl. 1997. Effect of school population socioeconomic status on individual academic achievement. *Journal of Educational Research*, **90(5)**, 269–277.

Chen, Yan, & Sömnez, Tayfun. 2006. School choice: an experimental study. *Journal of Economic Theory*, **127(1)**, 202–231.

Dahlstedt, Magnus. 2007. I val(o)frihetens spår: Segregation, differentiering och två decennier av skolreformer. *Pedagogisk Forskning i Sverige*, **12(1)**, 20–38.

Erdil, Aytek, & Ergin, Haluk. 2008. What's the Matter with Tie-Breaking? Improving Efficiency in School Choice. *The American Economic Review*, **98(3)**, 669–689.

Ergin, Haluk, & Sönmez, Tayfun. 2006. Games of school choice under the Boston mechanism. *Journal of Public Economics*, **90(1)**, 215–237.

Gale, David, & Shapley, Lloyd S. 1962. College Admissions and the Stability of Marriage. *American Mathematical Monthly*, **69(1)**, 9–15.

Johansson, Anders. 2012 (04/05). *Interview.* Malmö stad.

Kesten, Onur. 2010. School choice with consent. *The Quarterly Journal of Economics*, **125(3)**, 1297–1348.

Kojima, Fuhito, & Pathak, Parag A. 2009. Incentives and Stability in Large Two-Sided Matching Markets. *The American Economic Review*, **99(3)**, 608–627.

Malmö stad. *Framtida skolorganisation ska vara tydlig och målfokuserad.*

Malmö stad. *Grundskola.*

Malmö stad. *Nyinflyttad med barn i skolålder.*

Malmö stad. 2011 (20/01). *Aktuellt om: Malmös befolkningsutveckling 2010.*

Metro. 2011. Här finns lediga förskoleplatser i Malmö. 23/02.

Pathak, Parag A., & Sönmez, Tayfun. 2008. Leveling the playing field: Sincere and sophisticated players in the Boston mechanism. *The American Economic Review*, **98(4)**, 1636–1652.

Roth, Albin E. 1982. The Economics of Matching: Stability and Incentives. *Mathematics of Operations Research*, **7(4)**, 617–628.

Skånska Dagbladet. 2010. Mer än tusen förskoleplatser behövs. 17/12.

Skolverket. *Skolverkets upplysningstjänst, quoted, 26/4 2012.*

Skolverket. 2009. *Vad påverkar resultaten i svensk grundskola? Kunskapsöversikt om betydelsen av olika faktorer.*

Skolverket. 2012 (04/05). *Skolan blir allt mindre likvärdig, pressmeddelande.*

Sveriges Riksdag. *Skollag (2010:800)*.

Winkleby, Marilyn A., Jatulis, Darius E., Frank, Erica, & Fortmann, Stephen P. 1992. Socioeconomic status and health: how education, income, and occupation contribute to risk factors for cardiovascular disease. *American Journal of Public Health*, **82(6)**, 816–820.

Wramell, Kerstin. 2012 (04/05). *Interview*. Malmö stad.