Turbulent concentration of particles

Erik Dahlén

Lund Observatory
Lund University

2012-EXA64

Degree project of 15 higher education credits
May 2012

Lund Observatory
Box 43
SE-221 00 Lund
Sweden

1 Svensk sammanfattning (for English turn page)

Den bakomliggande teorin om planetbildning är att planeterna bildas samtidigt som stjärnan och därför av samma material som stjärnan. Därför måste dessa stoft- och gaspartiklar bindas samman till planeter. Teorin säger att detta görs i en disk runt stjärnan vilken uppstår vid stjärnbildning för att bevara rörelsemängdsmomentet och sådana diskar har observerats och skulle förklara varför alla planeter i vårt solsystem roterar i samma plan. Man kan dela in denna process i tre faser där olika krafter dominerar. Den första är när de enskilda particlarna binds samman av elektriska krafter till mikrometer stora partiklar. Den andra är när dessa mikrometer stora partiklar binds samman till kilometer stora objekt (planetesimal på engelska). Den tredje och sista är när dessa kilometer stora objekt binds samman av gravitation till tusen kilometer stora kroppar som sedan är så stora att de uppfyller definitionen som planet. Under denna process är det steg två som inte riktigt kan förklaras, eftersom både gravitationskraften och de elektriska krafterna är försumbara. En teori för att lösa detta är att turbulens i disken runt stjärnan skulle kunna ansamla så många partiklar på ett ställe att gravitationen skulle ta över och bilda objekt på över en kilometer i diameter.

Turbulens är en makroskopisk rörelse i en gas, eller vätska, som är kaotisk. Vid turbulens uppstår det virvlar som i den fria rymden har ett lågtryck i mitten av sig. Detta uppstår då gasen i virvlen känner en centrifugalkraft utåt så måste detta kompenserar av en tryckkraft inåt som enbart kan uppstå om virveln har ett lägre tryck i mittan än utanför. Om man skulle ha partiklar i en sådan turbulent gas kommer de att flyga ut ur virvlarna eftersom de också kommer känna centrifugalkraften men inte tryckkraften som gasen känner. Hur effektivt de lämnar virveln beror på hur väl de känner av gasens rörelse, vilket beror på hur väl sammankopplade de är.

Jag har därför försökt simulera en turbulent gas med partiklar enligt ovan beskrivning. Det har gjorts genom att använda Pencil Code för att numeriskt beräkna vad som händer med en gas och dess partiklar över tiden när man inför turbulens. Detta har gjorts genom att anta att gasen följer Navier-Stokes ekvation och att partiklarna är kopplade till gasen med en friktionstid. Dessa simuleringar visar att man med rätt friktionstid, vilken avgör hur väl gasen och partiklarna är kopplade, kan man få en mycket hög ökning av partikelkonsentrationen på vissa ställen. Simuleringarna visar även att upplösningen och gasens viskösitet inte har någon större betydelse för partikelansamlingarna.

2 Abstract

Turbulence which can increase the maximum particle density in some region is interesting for planet formation since the process for building up planets has some problem. Planets are formed by dust grains sticking together into planets during the star formation period and the major problem is the the growth from cm to km scale. During this phase both the microscopic molecular forces and the macroscopic gravity are negligible, therefore are there no well accepted theory for the entire planet formation phase. One idea is that turbulence could help in clustering particles together so that self gravity takes over and planetesimal can form.

The Pencil Code has been used to simulate how turbulence affects the particle density distribution for dust particles in a gas. In a 2D simulation one can find a peak of the particle density at some values of the friction time, the highest peak had 84 times higher maximum particle density than the mean particle density when averaged from time 0 to 4900. This indicate that turbulence could contribute to planetesimal formation. The simulation also shows that 2D has a inwards cascade that follows the $E(k) \propto k^{-3}$ line. The turbulence was artificially introduced with so called forced turbulence, which is when one add turbulence without thinking of its physical origin. If one stops the forced turbulence after a while one can see how the eddies in the turbulence dissipates and how the smallest eddies dissipates first since the viscosity affects them more.

3 Planet formation

The main idea is that planets are formed during the star formation period, which means that the planets are formed by the same gas cloud as the star (often called the protostar phase). The theory is that at a late stage of the star forming process the star will be surrounded with a disk, a so called protoplanetary disk. Such disks have been observed by spectroscopy of Orion HH 3 which showed that previous detected structure were protoplanetary disks (O'dell, 1993). This would explain why the planets in our solar system are all orbiting in the same plane. For this model to work individual dust grains must be able to stick together into planets. To make any sense of this we divide this process into three regimes where different forces dominate each phase which gives rise to different properties.

The first part we call the dust phase. In this phase the particles are μm in size. In this range the molecular forces dominate so that the particles can grow by sticking if they collide due to induced dielectric forces such as van der Walls (Dominik, 2006).

The second part is the rock phase, which is when the body is from $10cm$ to some hundred meters in diameter. In this regime neither gravity nor molecular forces are strong enough to make them grow and most collision will not end up in building any bigger bodies.

Planetesimals are the third and last phase before planets are formed, which is from km scale and $1000km$. In this phase gravity starts dominating and will be the force that brings bodies together in collisions. It is easy to see that this is a runaway situation where every collision will make it grow and the bigger the planetesimal is the stronger will the gravitational force be, therefore all growth beyond this size will also be gravitationally dominated.

The rock phase is the part with most problems in the theory. One of the problems is the radial drift time scale which is in the order of some hundred years (Philip, 2010). Radial drift, which is when particle in the protoplanetary disk drifts inward to the star, occur since the rocks are not well coupled to the gas and therefore do not move with the gas but with a higher speed (the gas is slowed down due to a pressure gradient force which the particle do not fell as strong as the gas). This higher speed makes the rock feel a headwind (when the rock hits new gas all the time because of its higher speed) which transfers some of the rocks angular momentum to the gas and hence the rock spiral inward to the star. This implies that it will only take a rock of this size some hundred of years to drift from the disk into the star, hence rocks has only a few hundred of years to form planetesimal.

There is more then one theory which explain how the rock phase can pass in the speed which it needs. Many of these theories are still debated and are under research. I have looked into the idea that eddies in turbulence could gather particles to high enough density for planetesimal formation to start in one place of the disk while some other parts gets low particle density. The main concept is that the particles feel a drag force from the gas when the gas swirls around which makes the particles moves out from the major eddies.

4 Turbulence

Turbulence is interesting, for a number of reason such as flow in tubes and mixing an inhomogeneous fluid into a homogenous fluid. Mixing can not be neglected when it is several orders of magnitude more efficient than random movements (Manneville, 2004). In this thesis the main interest lays in the creation of eddies and behavior of eddies, explained further down.

Fluids have typically three motions: the micro scale movement, laminar flows and turbulent flows. Micro scale movement is always considered being chaotic. Laminar flows occur when the fluid is moving in one direction smooth and at a constant rate. Turbulent flows are somewhere between chatic and laminar. Like laminar flows turbulent flows are also macroscopic in nature and move part of the fluid from one place to another, but like micro scale movement turbulent flows are characterized by chaotic movement and inhomogeneous velocity (both in direction and speed). Turbulent flows, like laminar flows, are created by a macroscopic force, e.g. by pressure gradient in a gas which can create a laminar flow when the gas moves into equilibrium or gravity pulling water downstream towards a stone which may cause turbulence in the water just after the stone (Frisch, 1995). When the force is no longer "active", which means that the force no longer adds any energy to the fluid, the turbulence will start decaying and eventually turns kinetic energy into thermal energy. Before the decay, the turbulence can have a period of development, which means that eddies could continue to build up like it does during the active phase (Manneville, 2004).

One way to measure whether a flow should be turbulent or not is by calculate the Reynolds number which is defined as

$$Re = lu/\nu \tag{1}$$

(Cuzzi, 2001), where l is the length scale of the system (the same as the diameter of the largest eddy), u the speed of the eddies compare to the background gas (often the same as the rotation speed of the largest eddy) and ν is the molecular viscosity. A larger Reynolds number implies an increased strength of the turbulence.

An eddy is a rotation, or swirling, in a fluid during turbulence. Eddies often occur after a fluid has hit an object, but they can occur everywhere in a turbulent flow. Eddies can both decay to smaller ones, as in figure 1, and build up to bigger ones. To decay into smaller ones can happen to all eddies and the characteristic decay time is given by l/u (Burden, 2008), also known as eddy turn over time, where l is the length scale and u the velocity scale. This process continues until the eddies become so small that the viscosity term in the Reynolds number starts dominate. To merge several smaller eddies into one bigger requires that the smaller ones are rotating in the same plane, hence turbulence in 3D are unlikely to merge into bigger eddies. Turbulence in 2D are likely to merge into bigger eddies, due to all eddies are in the same plane. Inside an eddy in free space will there always be a low-pressure, this because all rotation creates a centrifugal force and the only way to balance this out is by a pressure gradient which implies that there must be a higher pressure outside the eddy. In some system like on earth are there eddies with high pressure in the center of the eddy. In the atmosphere eddies are created by air going down from high altitude to low, or the other way around, and the rotation is created by the Coriolis force so the rotation change on the different side of the equator (eddies circle high pressure clockwise and low pressure anticlockwise on the northern hemisphere and vice versa) (Wallace, 2006).

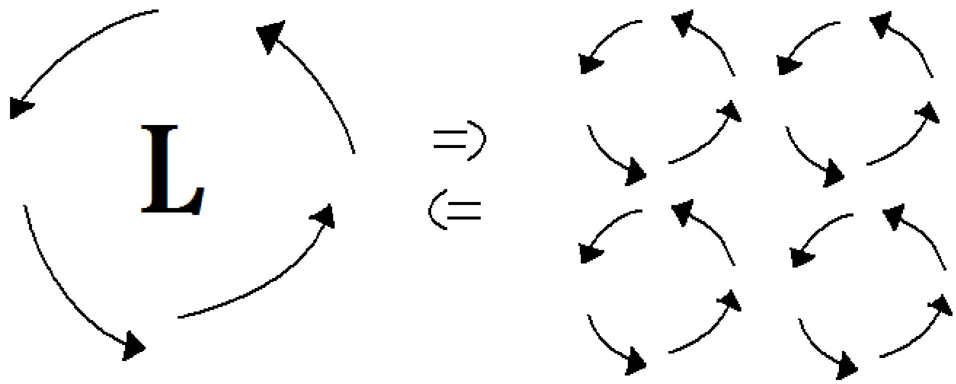


Figure 1: This illustrate that an eddy is a rotation of the gas, that an eddy in free space will always have a low pressure inside of it and the concept that eddies could cascade both inverse, the bottom arrow, and direct, the top arrow.

If we let l_0 be the length scale of the largest eddy, we can make a list of all the smaller length scales as $l_0/2, l_0/4, ...l_0/n$. We can then sum up all the energy for every eddy with the same length scale, and then plot them as a function of their wave number (which is related to the length scale as $k \sim 1/l$). This plot would look somewhat like figure 2. The energy containing range is where the largest eddies are present, this is limited because of the macroscopic property of the system that does not let the eddies grow bigger. Dissipation range is where the energy decreases because of the molecular viscosity is turning the turbulent energy into thermal energy. The dissipation range could have a spike in this plot which indicates that one of the smallest eddies have a lot of energy just before decaying. Between those two we have the inertial range, where the molecular viscosity of the fluid is negligible but the range of eddies is not bound by the size of the system. The inertial range is, if one plots the axis logarithmic, a constant slope which often has one of the two values -3 or $-5/3$ (Kraichnan, 1966).

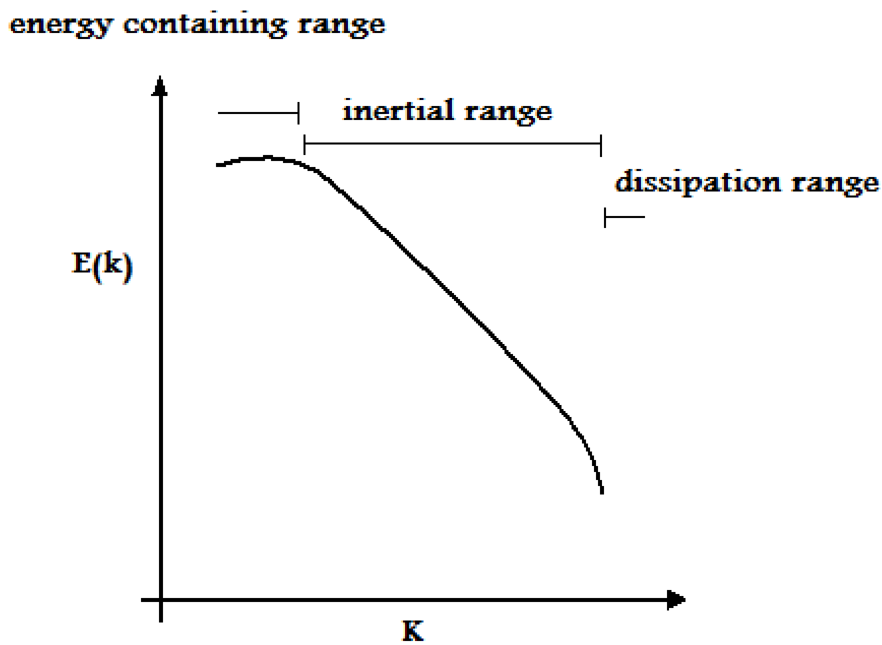


Figure 2: This is the shape of the spectrum of isotropic and homogeneous turbulence. It is an illustration like fig 7.4 in Instabilities, chaos and turbulence by Manneville. The x-axis is the wavenumber and the y-axis is the energy at that wavenumber, both axes are logarithmic.

The friction time τ of the particles is a constant, with dimension time, that tells how much the gas interacts with the particles. The following equation gives us the acceleration the particles gets from the gas.

$$\frac{dv}{dt} = \frac{1}{\tau}(u - v) \tag{2}$$

where v is the speed of the particle and u is the speed of the gas (Markiewicz, 1991). The friction time is dependent on the properties of the particle, and then mainly its size, due to different size are coupled to the gas different well.

The following equation is called the continuity equation and is used in simulation with fluids.

$$\frac{d\rho}{dt} + \bigtriangledown \rho u = 0 \tag{3}$$

(Elaine, 1987), where ρ is the density and u the velocity flow which is how much mass flows from one region to another at one time. This equation makes sure that the mass is conserved in the simulation. Simply this equation only allows the density to change in one region by the amount of mass passing into it and/or out from it to the adjacent regions.

Under the assumption that the gas is incompressible and the viscosity kept constant the gas would follow the Navier-Stokes equation,

$$\frac{d\mathbf{u}}{dt} + (\mathbf{u} \cdot \bigtriangledown)\mathbf{u} = -\bigtriangledown\frac{p}{\rho} + \nu\bigtriangledown^2\mathbf{u} + f \tag{4}$$

(Lundgren, 2003), where u is the velocity flow, p the pressure, ρ the density, ν the viscosity and f is the sum of the external forces upon the gas. If one wants to add a turbulent force into a gas governed by this equation one just give f a value, this is called forced turbulence since the force is added without any physical causes.

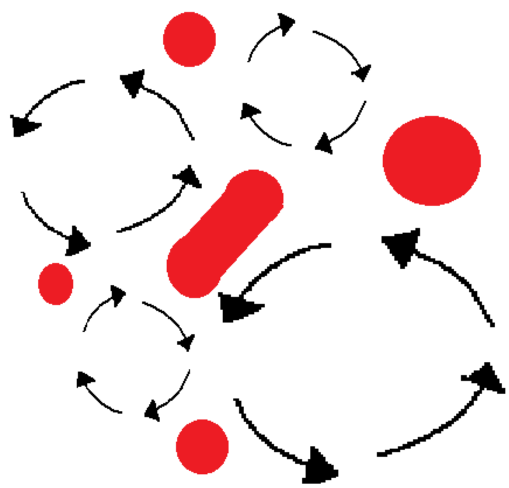


Figure 3: Inside a turbulent flow the eddies can vary in size as this picture. The red zones is what I call the "hot spots", which is between the major eddies.

I will introduced two terms which are not common used. The "hot spots" which is the regions between the eddies, look at figure 3. I also define the mean maximum particle density, in some figure captions called mmpd, to be the maximum particle density compared to the mean particle density and then averaged from start to end in one simulation.

5 The Pencil Code

I have used a code called Pencil Code, which is a code where the motions of gas and particle can be simulated. The code has preprogrammed functions which can be used for making all the simulations I have done. In all of the simulations there are no random elements, which imply that the simulations result is only dependent on the value of the initial state and the parameters.

The simulations were mainly focused on the gas itself, which has been approximated as a fluid governed by the Navies-Stoks equation and the continues equation. The gas was homogeneously distributed at the start of all simulations, with no velocity. Then we injected turbulence artificial, with so called forced turbulence, so we did not simulate whether the turbulence could be created or if it even could be present in protoplanetary disks. I also assumed that the gas were homogeneous and isotropic. Homogeneous means that the turbulence energy are the same everywhere and isotropic means that the eddies behave, in the sense of their motions, the same in all directions.

I have simulated both in 2D and 3D with grid sizes of 64, 128, 256 and 512. All simulations have three ghost cells outside the border of the grid. A 64 grid simulation has 70 gridpoints along the x-axis. If we say that the real grid cell number 1 to 64 then the ghost cell would be -2, -1, 0, 65, 66 and 67, where grid cell 0 is a copy of grid cell 64, -1 a copy of 63 and so on. This will give rise to a periodic phenomena where structure and motion can move from one side of the grid to the opposite side by travel through the border.

When I added the particles they only property I gave the particles were the friction time, which means that I did not assume anything about the size, shape and abundance. The particles were also at rest at the start of the simulation but started to move along with the gas according to equation 2. The particles did not interact with each other nor did they effect the gas. This implies that the particles could not hit one another, so bouncing, fragmentation or mass transfer were not simulated.

In the simulation I selected the value of the viscosity according to the following equation,

$$Re_M = u\delta x/\nu \tag{5}$$

where u is the velocity of the gas, δx the length of one grid cell and ν the viscosity. To make $Re_M \sim 1$ I avoid the simulation to become unphysical because the equation of motion is not valid for higher values, due to the simulation can not resolve the motion then. The simulation can not resolve an eddy which is smaller then one grid cell, and to increase the viscosity the smallest eddies will decay so that one is certain that all eddies which can not be resolved have decayed. If the viscosity is too low the speed can get into a runaway situation were the speed increases to such levels that the time steps become to small. One needs to decrease the time steps when the speed of the gas increases due to the property of one grid cell will then effect its neighbor grid cell faster, and than the time steps get to small the simulation crashes and one know that something unphysical has happened.

The velocity inside the simulation were of the unit Mach, which is define as $Mc = v/c_s$ where Mc is the Mach, v the speed and c_s the sound speed of the gas. The box has the size 2π. The time unit is derived from them from $time = length/speed$. To get any of the units to SI-units one need to make some assumption about the properties of the system.

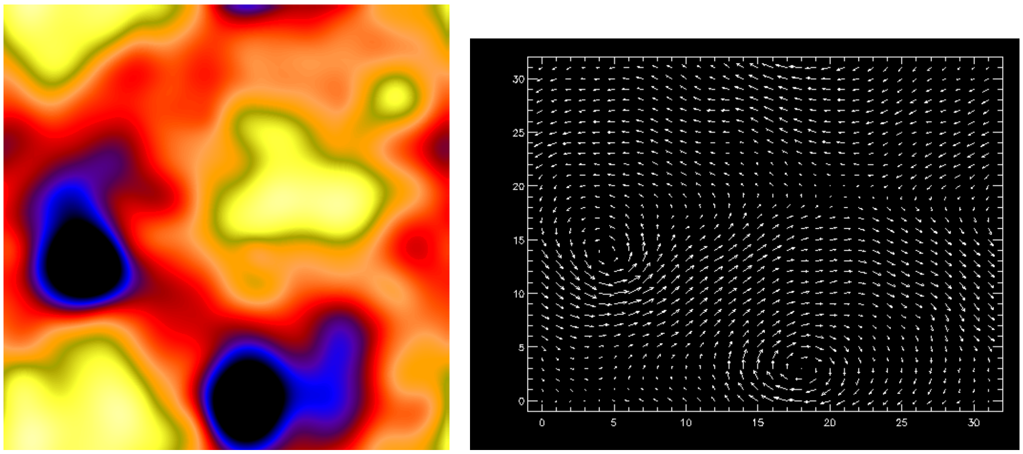
6 Simulation

Here I analyse and plot the result of the simulations. I have plotted everything with IDL, or Interactive Data Language. IDL is a programing language which is specialist on outputting and visualize data, and therefore used in some field of science such as astronomy. One can find a table of my simulations in the appendix.

6.1 2D

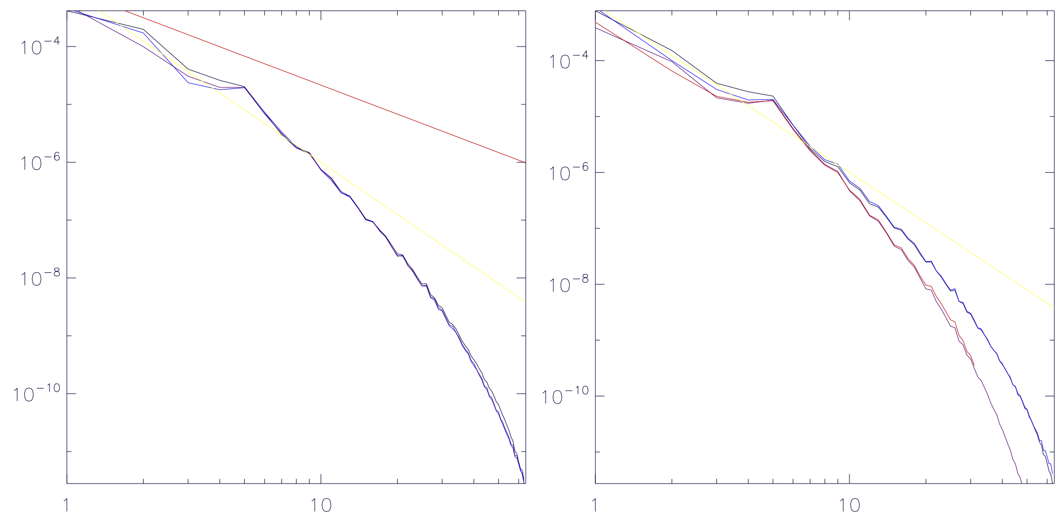
The 2D simulations can be used to compare the results from the 2D and 3D, and then find which properties they have in common and which they do not have in common.

The simulation for the figures in figure 4 had the grid size 128, viscosity $5 * 10^{-5}$ and forced turbulence 0.001, these picture were taken at time 3200. Figure 4a clearly shows how some regions become less dens and some more dens when one has added turbulence to an otherwise homogeneous gas. One can see two distinct regions with much less density and if one compares that to figure 4b one will find that the gas seems to orbiting around those regions. This is what one expect from a simulation where eddies would be able to merge into bigger eddies. The two different regions are orbiting at different directions, one clockwise and one anticlockwise, indicate that the eddies orbiting in one direction are merging with each other. One also see that the lowest speed is where the density is highest.



(a) Density (b) Velocity

Figure 4: These pictures shows how the gas are orbiting around the two low density regions, this indicate that we have two major eddies and therefore inwards cascade. That they orbits at different directions indicates that eddies merge with other eddies which have the same direction of rotation. In 4a orange is the mean density, black is $-3 \cdot 10^{-3}$ less than the mean density and white is $3 \cdot 10^{-3}$ more than the mean density. In 4b the arrows shows the velocity of the gas in on grid cell, to make it easier to spot the arrows have only one out of 16 arrows bin plotted here. This picture were taken from simulation 4 in the table in the appendix.



(a) Different grid size

(b) Different viscosity

Figure 5: The x-axis is the wavenumber and the y-axis is the energy, both axis are logarithmic. The left spectrum shows how the size of the system does not effect the result, at least between the size 128 and 512, and that spectrum seems to follow the $E(k) = k^{-3}$ line. The right spectrum shows how the viscosity is really important for the smaller eddies but not for the bigger ones, which it should since viscosity dissipate kinetic energy to thermal energy and this happen at smaller scales faster then big ones. (a) Black is the simulation 4, blue is the simulation 30, purple is the simulation 34, yellow is the line $E(k) = k^{-3}$ and red is the line $E(k) = k^{-5/3}$. (b) Black is the simulation 4, blue is the simulation 30, purple is the simulation 5, red is the simulation 1 and yellow is the line $E(k) = k^{-3}$. (Look for the simulations in the appendix)

In figure 5 the simulation had the following parameters. 5a the black spectrum has 128 grid, blue 256, purple 512, red is the line $E(k) = k^{-5/3}$, yellow is the line $E(k) = k^{-3}$. All of them had the viscosity $5 \cdot 10^{-5}$ and forced turbulence 0.001, and were averaged from the time 2400 to the time 2499. 5b the black spectrum has 128 grid with viscosity $5.0 \cdot 10^{-5}$, purple has 128 grid with viscosity 10.0^{-5}, blue has 256 grid with viscosity $5.0 \cdot 10^{-5}$, orange is the line $E(k) = k^{-3}$. All of them had the forced turbulence 0.001, and were averaged from the time 3400 to the time 3499. At the inertial range one can assume a linear relation between the wavenumber and the energy at that wavenumber, if one plot them with logarithmic axis. In figure 6 the simulations had the following parameters. The black has forced turbulence 0.002, blue has forced turbulence 0.001, purple has forced turbulence 0.0005. All of them had the grid size 128 and the viscosity $5 \cdot 10^{-5}$, and were averaged from the time 2400 to the time 2499. In figure 5a one can see that the -3 line is fitting rather good to the simulations. It is also obvious that the resolution does not matter if the rest of the variables are set to the same value. At figure 5b one see that the viscosity is a variable which matter and that a lower viscosity give higher value for bigger wavenumber, which means that the smaller eddies have more energy. It also gives a indication that the simulations come closer to the theoretical line with lower viscosity because it seems to move closer to the -3 line. Resolution gives a limit for how small eddies which could be resolved and that the red 64 grid line just end is a consequence of just that. The amount of energy per wavenumber should be proportional to the amount of forced turbulence and with more forced turbulence one expect higher energy, which could be seen in figure 6. It also shows that the shape of the spectrum does almost not change with different amount of forced turbulence.

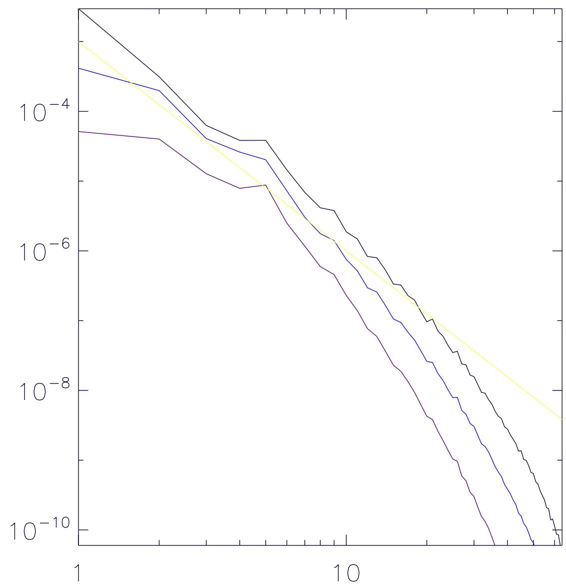
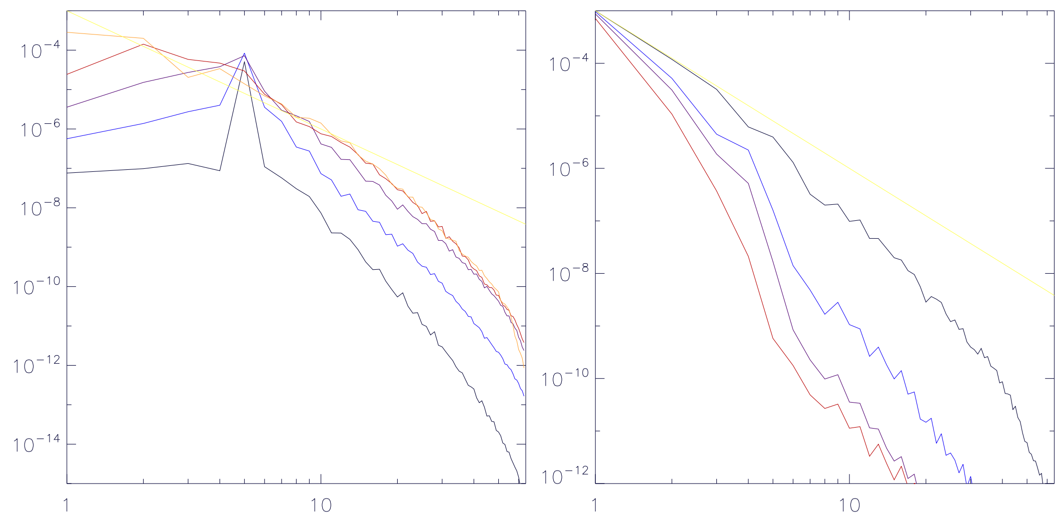


Figure 6: The x-axis is the wavenumber and the y-axis is the energy, both axis are logarithmic. This spectrum shows how the forced turbulence does not change the shape of the spectrum but change its position in y direction as it should since there is more energy present with higher forced turbulence. Black is the simulation 7, blue is the simulation 4, purple is the simulation 6 and yellow is the line $E(k) = k^{-3}$.(Look for the simulations in the appendix)

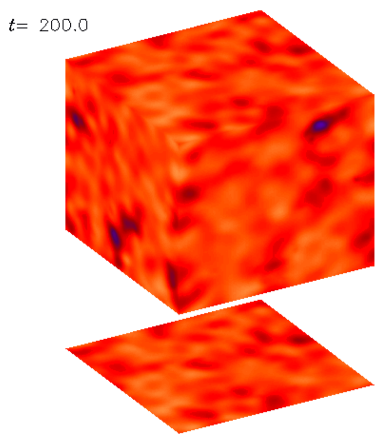


(a) Eddies build up (b) Eddies decay

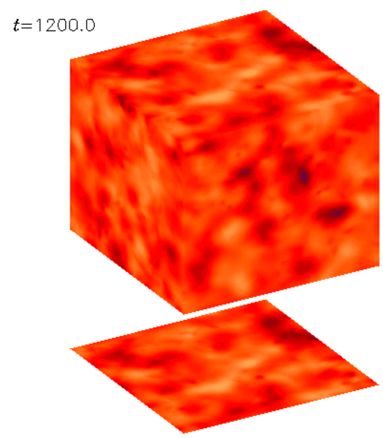
Figure 7: The x-axis is the wavenumber and the y-axis is the energy, both axis are logarithmic. The left spectrum shows how the shape of the spectrum forms with time and the right shows how the spectrum decays when the forced turbulence is removed. This indicate that the viscosity is more effective at higher wave numbers and dissipate them faster then the bigger ones, hence will large eddies live for long times ones they has formed. (a) This is simulation 4. (b) This is an simulation equal to simulation 4 but the force were stopped at the time 3765. (Look for the simulations in the appendix)

The simulation for the figures in figure 7 had the grid size 128, viscosity $5 \cdot 10^{-5}$ and forced turbulence 0.001. In figure 7a the black is at time 10, blue is at time 20, purple is at time 40, red is at time 80, orange at time 160 and yellow is the line $E(k) = k^{-3}$. In 7b the forced turbulence stops at time 3765. Black is at time 3765, blue is at time 4765, purple is at time 5765, red is at time 7765 and yellow is the line $E(k) = k^{-3}$. In figure 7a one can find at which k the turbulence are introduced in the simulations, the peak in the graph. One also see how the eddies both cascade direct and inverse as time pass. Smaller eddies have shorter lifetime and therefore one can expect that the energy for the bigger wavenumber to decays first when the forced turbulence stops, as one could see in figure 7b. This because of the viscosity are more effective at higher wavenumber, and therefore at smaller eddies. This will cause that large eddies will live very long even if the turbulence stops.

6.2 3D



(a) At time 200



(b) At time 1200

Figure 8: These pictures indicates that the 3D simulations do not have as much inverse cascade as the 2D, and already at time 200 does it not seems to evolve any more. Orange is the mean density, black is $-3 \cdot 10^{-3}$ less than the mean density and white is $3 \cdot 10^{-3}$ more than the mean density. The flat plane below the box is the bottom of the box lifted down. This is simulation 41. (Look for the simulations in the appendix)

The simulation for the figures in figure 8 had the grid size 128, viscosity $5 \cdot 10^{-5}$ and forced turbulence 0.001. In figure 9 the black spectrum had viscosity $5 \cdot 10^{-5}$, the blue had viscosity $5 \cdot 10^{-5}$, the purple had viscosity $5 \cdot 10^{-5}$ and yellow is the line $E(k) = k^{-3}$. All of them had the grid size 128 and force turbulence 0.001 and were averaged from the time 900 to the time 999. In 3D one does not expect the big inverse cascade as in 2D due to less merge between eddies. In figure 8 one can see that after 200 time frame the gas seems to stop evolve and even if one runs for another 1000 time steps one do not see any major merge as in 2D. This is what one can expect from a 3D simulation and in figure 9 one see that the energy spectrum does not form a inertial range as in 2D and that the forced turbulence just decay away. It also shows that it merely cascade direct as it decay and the absent of the rise of the bigger eddies. One can also see how higher viscosity are more efficient to kill of eddies and that smaller eddies are more effected by the viscosity.

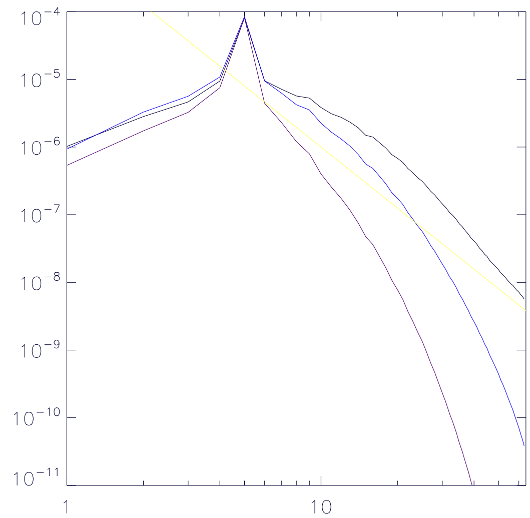
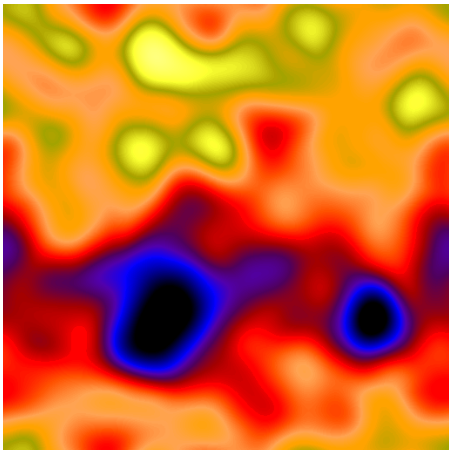
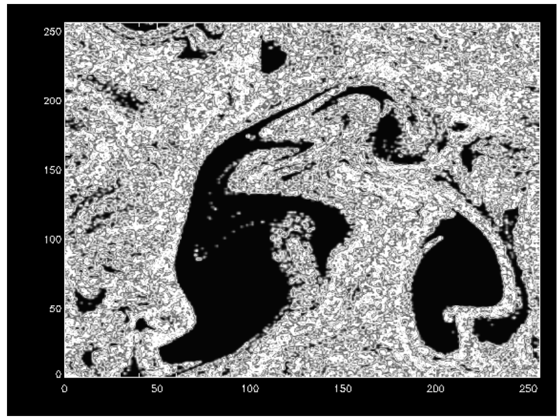


Figure 9: The x-axis is the wavenumber and the y-axis is the energy, both axis are logarithmic. This spectrum shows how the 3D simulation do not form a initial range and hence one can assume that there is mush less inwards cascade as in 2D. Black is simulation 41, blue is the simulation 42, purple is the simulation 43 and yellow is the line $E(k) = k^{-3}$. (Look for the simulations in the appendix)

6.3 Particles



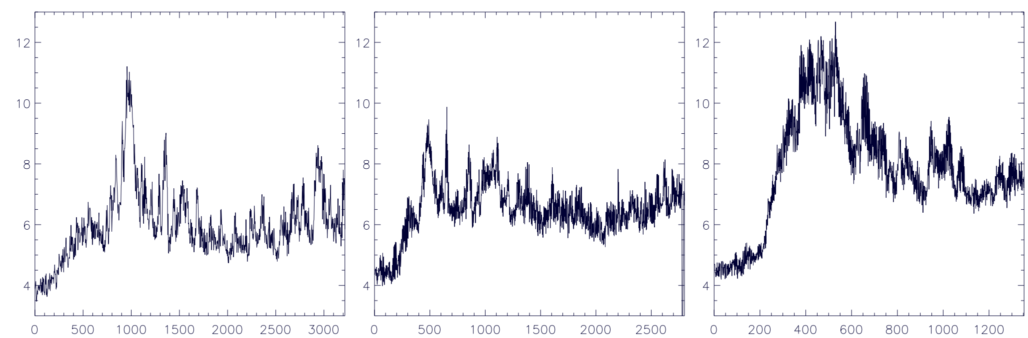
(a) Gas density



(b) Particle density

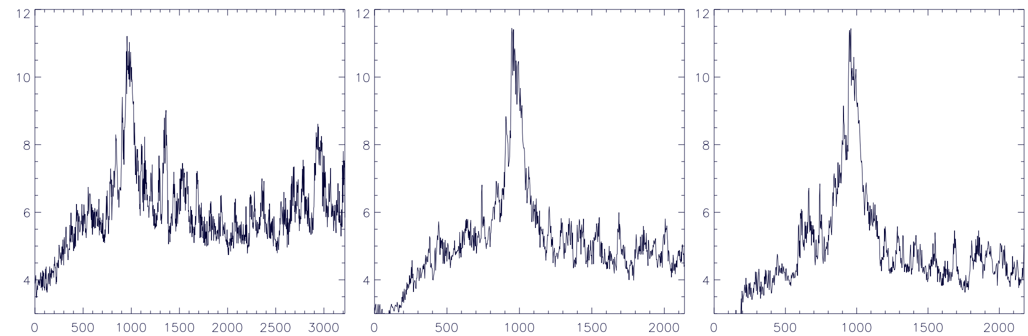
Figure 10: These pictures shows how the low pressure center has almost no particles, this because of the large eddies are throwing the particles out according to equation 5. (a) Orange is the mean density, black is $-3 \cdot 10^{-3}$ less than the mean density and white is $3 \cdot 10^{-3}$ more than the mean density. (b) Black means 0 particle density and pure white means 5 particle per grid cell. This is simulation 32. (Look for the simulations in the appendix)

The simulation for the figures in figure 10 were a 2D simulation and had the grid size 256, viscosity $5 \cdot 10^{-5}$, forced turbulence 0.001, friction time 1, 1 particle per grid cell as mean particle density and were taken at time 2300. The simulation for the figures in figure 11 were a 2D simulation and had the viscosity $5 \cdot 10^{-5}$, forced turbulence 0.001, friction time 1 and 1 particle per grid cell as mean particle density. 11a hade the gird size 128, 11b 256 and 11c had 512. In figure 10 one can easy see that the particle density is affected by the turbulence and that there is almost no particles in the center of the big eddies. Due to the strong difference of particle density between the low gas density region and the rest while there seems to be no particle density difference between the average gas density region and high gas density region one can assume that the effect are not caused by gas density difference. This shows that it is the major eddies that are causing this behavior and not the gas density profile. The eddies are causing this by producing a pressure gradient which must be equal to the centrifuge force that push out the particles. In figure 11 one can see how the maximum particle density has risen due to this effect. In all of them one can see how the maximum particle density begins at 4 times the mean density and rises to somewhere around 6 and 8 for the mean maximum particle density. One can also spot that with higher resolution the maximum tend to get somewhat higher, this is probably because there will be more merges between eddies in a simulation with higher resolution and this will create more "hot spots" which increase the mean maximum particle density by statistics.



(a) 128 (mmpd 6.03) (b) 256 (mmpd 6.50) (c) 512 (mmpd 7.81)

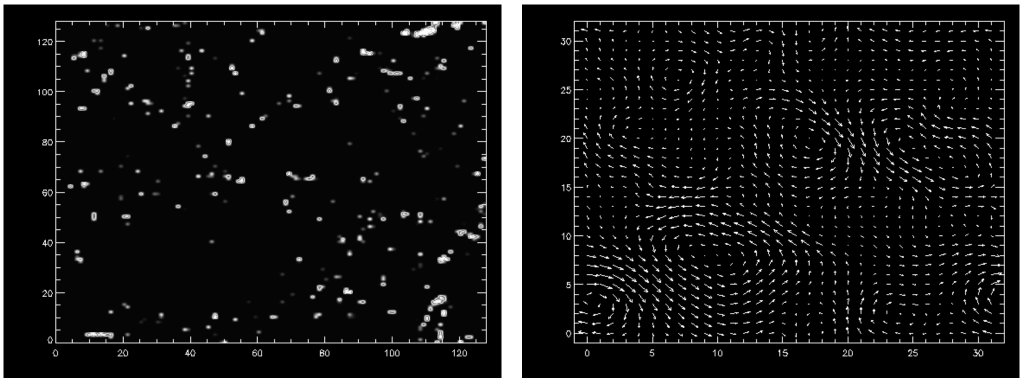
Figure 11: The x-axis is the time and the y-axis is how many times higher the maximum density is compare to the mean particle density density. Mmpd is the mean maximum particle density. With a higher resolution the maximum particle density slightly increases, this might be due to the increase of emerging eddies. (a) This is simulation 8. (b) This is simulation 32. (c) This is simulation 37. (Look for the simulations in the appendix)



(a) 1 particle (mmpd 6.03) (b) 2 particles (mmpd 5.12) (c) 3 particles (mmpd 4.79)

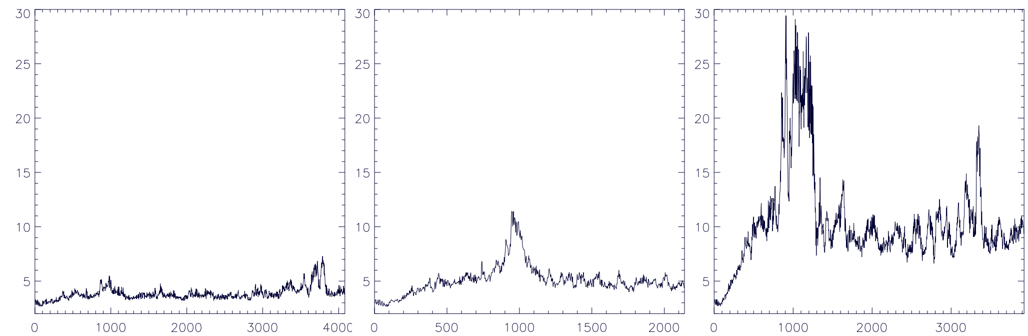
Figure 12: The x-axis is the time and the y-axis is how many times higher the maximum density is compare to the mean particle density density. Mmpd is the mean maximum particle density. With a higher amount of particles within the same resolution does slightly decrease the maximums and this is probably due to that the maximums do not increases as fast as the mean is increasing, because statistics will fine down the curve. (a) This is simulation 8. (b) This is simulation 15. (c) This is simulation 16. (Look for the simulations in the appendix)

The simulation for the figures in figure 12 were a 2D simulation and had the grid size 128, viscosity $5 \cdot 10^{-5}$, forced turbulence 0.001 and friction time 1. 12a had 1 particle per grid cell as mean particle density, 11b had 2 and 11c had 3 particles. The simulation for the figures in figure 13 were a 2D simulation and had the grid size 128, viscosity $5 \cdot 10^{-5}$, forced turbulence 0.001, friction time 1 and had 1 particle per grid cell as mean particle density. This pictures were taken at time 1000. In all of this three, in figure 12, one can spot that the average maximum density is slightly below 6 times the mean particle density even if the particle density is changed. This indicate that the mean density is not impotent for how much higher the maximum density regions become. The peak at about the time 1000 in all of them are probable due to something happening with the major eddies at that time and in all of this three simulations the gas behaves exactly the same because the particles do not effect the gas. In figure 13 one can find that the "hot spots" region is where one can find the increase of particle density.



(a) Particle density (b) Velocity

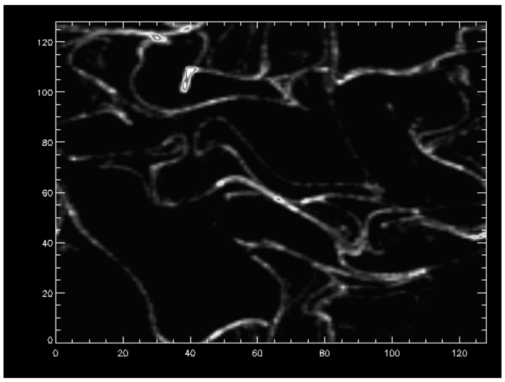
Figure 13: These pictures shows where the maximum peaks are compare to the eddies. One can find two regions which has a higher density at the top and bottom to the right hand side. (a) Black means 3 particle per grid cell or fewer and pure white means 5 particle per grid cell or more. (b) The arrows shows the velocity of the gas in on grid cell, to make it easier to spot the arrows have only one out of 16 arrows bin plotted here. This is simulation 8. (Look for the simulations in the appendix)



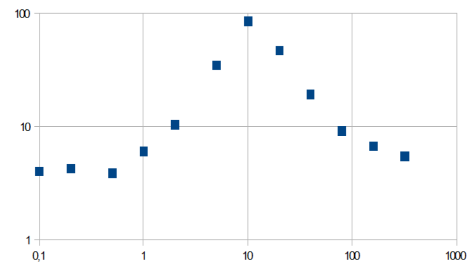
(a) friction time 0.5 (mmpd 3.87) (b) friction time 1 (mmpd 6.03) (c) friction time 2 (mmpd 10.46)

Figure 14: The x-axis is the time and the y-axis is how many times higher the maximum density is compare to the mean particle density density. Mmpd is the mean maximum particle density. The maximum particle density increases almost proportional with the friction time, which is not to strange when a higher friction time will leave larger regions empty in the simulations. (a) This is simulation 27. (b) This is simulation 8. (c) This is simulation 18. (Look for the simulations in the appendix)

The simulation for the figures in figure 14 were a 2D simulation and had the grid size 128, viscosity $5 \cdot 10^{-5}$, forced turbulence 0.001, and had 1 particle per grid cell as mean particle density. 14a had friction time 0.5, 14b 1 and 14c had 2. The simulation for the in figure 15a were a 2D simulation and had the grid size 128, viscosity $5 \cdot 10^{-5}$, forced turbulence 0.001, friction time 10, had 1 particle per grid cell as mean particle density and were taken at time 1000. Here, figure 14, one see that the maximum density increases if the friction time is increased, and that is what one might expect because it is the friction time that allows the particles to move and if they are coupled to the gas more they will leaves the major eddies faster which makes larger region of the simulation empty, as one can spot in figure 15a. It also shows how all the particles gather at the "hot spots". Figure 15b shows how the maximum particle density changes with the friction time, as one would expect does higher friction times give higher maximum particle density. The peak at around the friction time 10 has no good explanation but could be where the benefit of being coupled to the gas so the particles could leave the eddies matches the benefit of not being to well coupled to the gas so the particles leave the eddies with a to high speed for gathering in the "hot spots". The figure also shows that the random noise in this simulations is almost 4 in mean maximum particle density.



(a) Particle density

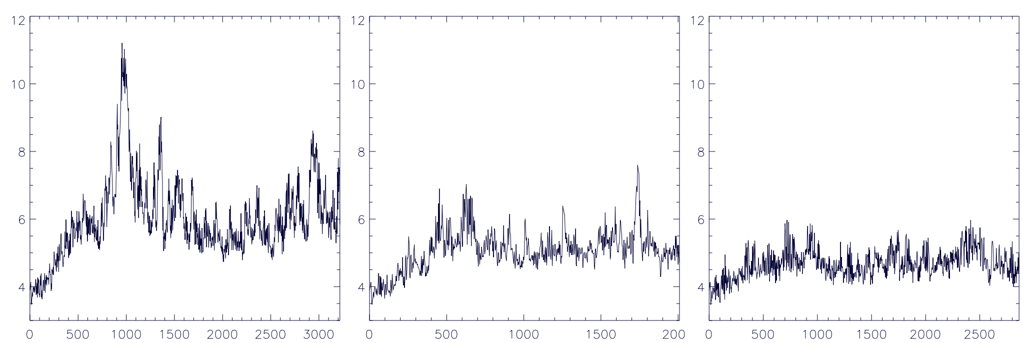


(b) Meanmax function

Figure 15: The left hand side plot shows how the big regions become empty when one increases the friction time and at the value 10 the particle are almost only distributed along the "hot spots". The right hand side plot shows how the maximum particle density changes with the friction time, one can assume a maximum somewhere around the friction time 10. (a) Black means 0 particle per grid cell and pure white means 100 particle per grid cell. (b) This is a plot of the average maximum particle density as a function of the friction time. x-axis is the friction time and y-axis is the mean maximum particle density, and both axis are logarithmic. This is simulation 21. (Look for the simulations in the appendix)

Table 1: These are the values for figure 15b rounded of to two digits accuracy.

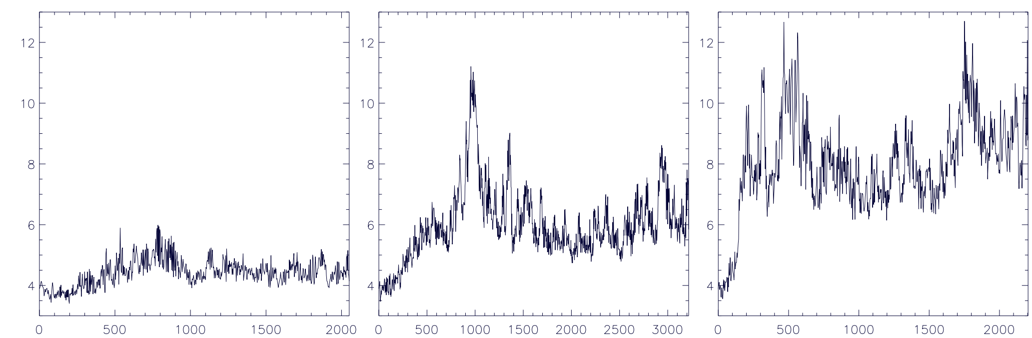
friction time	mean maximum particle density
0.1	4.3
0.2	3.9
0.5	4.0
1	6.0
2	10
5	35
10	84
20	47
40	19
80	9.2
160	6.7
320	5.5



(a) viscosity 5 (mmpd 6.03) (b) viscosity 10 (mmpd 5.08) (c) viscosity 20 (mmpd 4.67)

Figure 16: The x-axis is the time and the y-axis is how many times higher the maximum density is compare to the mean particle density density. Mmpd is the mean maximum particle density. An increase of viscosity slightly decreases the maximum density and this is probably due to viscosity kill of some of the small rapid motion by dissipate the small eddies. (a) This is simulation 8. (b) This is simulation 13. (c) This is simulation 14. (Look for the simulations in the appendix)

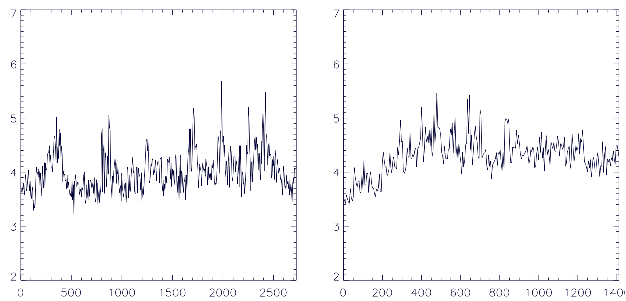
The simulation for the figures in figure 16 were a 2D simulation and had the grid size 128, forced turbulence 0.001, friction time 1 and had 1 particle per grid cell as mean particle density. 16a had the viscosity $5 \cdot 10^{-5}$, 16b 10^{-4} and 16c had $2 \cdot 10^{-4}$. The small chance of maximum density in figure 16 can be explained by less merge of smaller eddies, due to higher viscosity dissipates more smaller eddies e figure 5, which then creates fewer "hot spots", and with fewer "hot spots" there will be less chance of getting a very high maximum, due to statistics. But the major increase in the maximum particle density is produced by the major eddies and the viscosity are not changing energy spectrum for the large eddies and therefore will there still be an increase in maximum particle density.



(a) force 0.0005 (mmpd 4.42) (b) force 0.001 (mmpd 6.03) (c) force 0.002 (mmpd 8.01)

Figure 17: The x-axis is the time and the y-axis is how many times higher the maximum density is compare to the mean particle density density. Mmpd is the mean maximum particle density. An increase in the forced turbulence will result in an increase of the particle density, which is what one expect when one adds more turbulence into the simulation. (a) This is simulation 11. (b) This is simulation 15. (c) This is simulation 12. (Look for the simulations in the appendix)

The simulation for the figures in figure 17 were a 2D simulation and had the grid size 128, viscosity $5 * 10^{-5}$, friction time 1 and had 2 particle per grid cell as mean particle density. 17a had forced turbulence 0.0005, 17b 0.001 and 17c had 0.002. The increase of maximum particle density goes up with an increasing forced turbulence is quite straight forward, more turbulence will cause higher kinetic energy which can create motion to the particles so they could gather into higher concentration.



(a) 2D (mmpd 4.73) (b) 3D (mmpd 4.30)

Figure 18: The x-axis is the time and the y-axis is how many times higher the maximum density is compare to the mean particle density density. Mmpd is the mean maximum particle density. These has a really low maximum particle density so one should not trust them completely. But it seems that the 2D has somewhat higher maximum particle density, which one can expect due to the reduced amount of inverse cascade. (a) This is simulation 3. (b) This is simulation 40. (Look for the simulations in the appendix)

The simulation for the figures in figure 18 had the grid size 64, viscosity $2 \cdot 10^{-4}$, forced turbulence 0.001, friction time 2 and had 1 particle per grid cell as mean particle density. 18a were a 2D simulation and 18b were a 3D. In figure 18 one can see that the height of the maximum particle density is slightly higher in the 2D simulation, but it is not so clear that one can state it. A problem is that the 64 resolution is so small that the background noise is almost almost as big as the expected result, because lower resolution gives lower maximum density (see figure 11). The fact that the 3D simulation has 64 times as many particles as the 2D should not be a problem according to figure 12. The increased viscosity is also a problem according to figure 20, which state that higher viscosity gives slightly lower maximum particle density. All this factors working together are probably the reason why it is so hard to read out anything from this. But one can see a slight increase in the 2D simulation, which is what one would expect from this kind of simulation. The increase is an effect from the lack of inverse cascade in the 3D simulations compare to the 2D simulations.

7 Conclusion

When one changed the the grid size for the simulations but kept all other parameters fixed the spectrum did not change. This tell one that the resolution is not an important property of the eddies. If one changed the viscosity one would see that the small eddies gained much less energy, which indicate that the smaller eddies are stronger effected by the viscosity. It did also show how less viscosity seemed to make the inertial range move to smaller eddies and closer to the $E(k) = k^{-3}$ line. The amount of forced turbulence do not change the shape of the spectrum but an increase of turbulence give an increase kinetic energy and the spectrum is forming throw inverse and direct cascade. This shows how the forced turbulence is important for the amount of eddies that forms but not their properties. By looking at how the forced turbulence change the spectrum early in the simulations one can see that the 2D clearly has both inverse and direct cascade. When the forced turbulence stops one can find how the viscosity dissipate the small eddies much more effective than the large eddies, which will result in that large eddies will last much longer and would therefore be a better candidate for planetesimal formation than small eddies. The 3D simulation did not have any major inverse cascade which one can expect from a 3D simulation where the merge of eddies is much less likely. This resulted in a spectrum without any inertial range and therefore no major eddies swirling around for any long time, one could find that it only took until the time 200 before the 3D simulation stopped to evolve.

The particle simulation showed that the large eddies were effective for making large regions with low particle density, which makes the large eddies to a candidate for planetesimal formation. An increase viscosity did slightly reduce the mean maximum particle density, which can be explained by a little less of the smaller eddies which will give little fewer "hot spots". An increase of the resolution did slightly increase the mean maximum particle density, which can be explained by somewhat more "hot spots". An increase of particle number did slightly reduce the mean maximum particle density, which can be explained by a lower random noise. The only 3D simulation I made and its comparable 2D simulation did both have mean maximum particle density to close to the random noise to make any reliable statement. The only two variables which had any major effect on the increase of maximum particle density were the friction time and the amount of forced turbulence. That an increase of forced turbulence will give an increase of the mean maximum particle density is because it will have more energy along all sizes of eddies, which include the ones who creates the particle density to increase above the mean particle density. The big peak of mean maximum particle density around the friction time 10 is really an interesting result, this both shows that if turbulence are present in a protoplanetary disk the turbulence will contribute to get higher particle density which might create planetesimal and it also indicate that the most important property is the friction time.

Their are several aspect that would be interesting to look into if I had more time. The first and most obvious is the peak in the mean maximum particle density around the friction time 10. I would concentrate to look if I could find more behavior like this for different viscosity, grid size, forced turbulence and amount of particles. One could also try to find out whether any kind of 3D simulation could show any similar behavior.

8 References

Burden Tony, 2008, [http://www2.mech.kth.se/courses/5C1218/scales.pdf]

Cuzzi N. Jeffrey, Hogan C. Robert, Paque M. Julie, Dobrovolskis R. Anthony, 2001, Astrophysical Journal, vol. 546, p. 496-508

Dominik C., Blum J., Cuzzi J., Wurm G., 2006, [http://arxiv.org/pdf/astro-ph/0602617v1.pdf]

Elaine S.Oran, Jay P.Boris, 1987, Numerical simulations of reactive flows, fifth edition, p. 161

Frisch Uriel, 1995, Turbulence, p. 1

Kraichnan H. Robert, 1967, Physics of Fluids, Vol. 10, p.1417-1423

Lundgren T. S., 2003, [http://ctr.stanford.edu/ResBriefs03/lundgren.pdf]

Manneville Paul, 2004, Instabilities, chaos and turbulence, p. 271

Markiewicz, W. J., Mizuno, H., Voelk, H. J., 1991, Astronomy and Astrophysics, vol. 242, p. 286-289

O'dell, C. R., Wen, Z., Hu, X, 1993, Astrophysical Journal, vol. 410, no. 2, p. 696-700

Philip J. Armitage, 2010, Astrophysics of Planet Formation, Cambridge University Press, version 3

Wallace M john, Hobbs V Perer, 2006, atmospheric science an introductory survey, p. 13

9 Appendix

Table 2: All the simulation that I have done during my bachelor project. Where mpd stands for the mean particle density and τ stands for the friction time. All this simulation were made in 2D.

number	grid size	viscosity	forced turbulence	τ	mean particle density
1	64	10	0.001	-	-
2	64	20	0.001	1	1
3	64	20	0.001	2	1
4	128	5	0.001	-	-
5	128	10	0.001	-	-
6	128	5	0.0005	-	-
7	128	5	0.002	-	-
8	128	5	0.001	1	1
9	128	5	0.0005	1	1
10	128	5	0.002	1	1
11	128	5	0.0005	1	2
12	128	5	0.002	1	2
13	128	10	0.001	1	1
14	128	20	0.001	1	1
15	128	5	0.001	1	2
16	128	5	0.001	1	3
17	128	5	0.001	1	5
18	128	5	0.001	2	1
19	128	5	0.001	2	2
20	128	5	0.001	5	1
21	128	5	0.001	10	1
22	128	5	0.001	20	1
23	128	5	0.001	40	1
24	128	5	0.001	80	1
25	128	5	0.001	160	1
26	128	5	0.001	320	1
27	128	5	0.001	0.5	1
28	128	5	0.001	0.2	1
29	128	5	0.001	0.1	1

Table 3: All the simulation that I have done during my bachelor project. Where mpd stands for the mean particle density and τ stands for the friction time. The simulation $30-37$ were in 2D and $38-43$ were in 3D.

number	grid size	viscosity	forced turbulence	τ	mean particle density
30	256	5	0.001	-	-
31	256	2.5	0.001	-	-
32	256	5	0.001	1	1
33	256	2.5	0.001	1	1
34	512	5	0.001	-	-
35	512	2.5	0.001	-	-
36	512	1.25	0.001	-	-
37	512	5	0.001	1	1
38	64	20	0.001	-	-
39	64	20	0.001	1	1
40	64	20	0.001	2	1
41	128	5	0.001	-	-
42	128	10	0.001	-	-
43	128	20	0.001	-	-