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Applying the Black-Litterman Model on the Swedish Stock Market

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ABSTRACT

This thesis investigates the applicability of the Black-Litterman portfolio allocation model on the Swedish stock market for the time period January 2008 to April 2012 and aims to bring clarity to the implementation process of the model. We also implement a new estimation approach for the confidence in analyst views. The portfolio of 36 companies used in the research was selected in order to benchmark against the Nordea Sverigefond. Also a Mean-Variance portfolio will be used for comparison purposes. Both a restricted and an unrestricted version of the Black-Litterman model are used in order to test differences in outcome when short selling is allowed. Results indicate that that the restricted model is more suitable than the unrestricted. However we find no significant differences in the performance of our benchmark portfolio and the Black-Litterman portfolio. We also find results that indicate longer positions in assets were the level of confidence is higher.

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Table of Contents

ABSTRACT	1
ACKNOWLEDGEMENT	2
1. INTRODUCTION	5
1.1 PROBLEM DISCUSSION	5
1.2 AIM AND PURPOSE.....	6
1.3 DISPOSITION	6
2. THEORETICAL FRAMEWORK.....	8
2.1 MARKOWITZ MEAN-VARIANCE MODEL	8
2.1.1 <i>Determining Optimal Portfolios</i>	9
2.2 THE BLACK-LITTERMAN MODEL	10
2.2.1 <i>Bayesian Theory</i>	10
2.2.2 <i>The Reference Model</i>	12
2.2.3 <i>The Black-Litterman Formula</i>	13
2.3 COMPONENTS OF THE BLACK-LITTERMAN	14
2.3.1 <i>View-Matrix and Expected Return-Matrix</i>	15
2.3.2 <i>Variance of views- Matrix</i>	15
2.3.3 <i>Weight-On-Views, Tau</i>	17
2.3.4 <i>Previous and ongoing Research</i>	18
2.3.5 <i>A Behavioral Perspective</i>	18
3. METHODOLOGY	21
3.1 ESTIMATION OF OPTIMAL PORTFOLIOS	22
3.2 POTENTIAL PITFALLS	24
3.3 THE BENCHMARK PORTFOLIO	24
4. DATA	26
5. EMPIRICAL RESULTS AND ANALYSIS.....	27
6 CONCLUSION	35
6.1 SUGGESTIONS FOR FURTHER RESEARCH	36

BIBLIOGRAPHY.....	37
APPENDIX	39

1. INTRODUCTION

Portfolio management is the art of decision-making using all available information to date in order to formulate a most likely scenario for the future while also balancing risk against performance. The early theoretical development of portfolio theory harks back to Harry Markowitz's article "Portfolio Selection" (1952) where he outlined the foundation for what is today known as the Mean-Variance (MV) theory. In this article, Markowitz postulates that investors are risk averse and that there is a tradeoff between risk and return. Markowitz's framework has since been further developed by scholars and one of the most influential contributions is the work by Robert Black and Bob Litterman (1991). The model proposed is known as the Black-Litterman (BL) model, which has in recent times come to achieve great recognition among portfolio managers worldwide.

While the MV portfolio optimization uses a complete set of expected returns the BL model allows the investor to use any number of market views on future returns and combines them with an equilibrium leading to optimal portfolio weights. In other words, the BL model tilts the portfolio weights towards the assets where the investor has specified views, thereby avoiding extreme allocations given by the MV model (Da Silva, Lee & Pornrojngkool, 2009). A notion of the model is that investors should take risk where they have views, and consequently take more risk where investors have stronger views (Bevan, Winkelmann, 1998). Despite the extensive literature in this field, few empirical studies have tested the applicability of the BL model on equity markets. This inadequacy can partly be explained by the obscurity of the model's qualitative side, meaning there is no clear framework on how to incorporate and quantify market views in a consistent manner.

1.1 PROBLEM DISCUSSION

Virtually all financial institutions face the problem of how to optimally allocate capital in portfolios and between different asset classes. Their decisions have to be made with respect to strengths and weaknesses, opportunities and threats, safety and growth, while balancing performance against the desired level of risk. This problem has initially been addressed by assuming risk aversion and that investors only care about risk and return. From that, mathematical models have been developed and argued to have brought clarity to parts of the problem with allocation of capital. These models, mainly Markowitz's MV model, have however been subject to heavy

criticism for being a theoretical solution that has no real impact in reality. As a result of these impractical features of Markowitz' model, scholars have opted to incorporate qualitative aspects in search for optimal portfolio allocations. Consequently, the BL model has emerged which allows investors to specify views on one or more assets in the portfolio. Since the views in this framework are highly subjective in nature and differ between investors, the problem is now more a question on how to consistently incorporate qualitative aspects of the model.

Furthermore, few empirical studies have been conducted within this area and do not fully provide an intuitive and consistent estimation approach. Bearing in mind the importance of portfolio theory for both institutional and private investors, it is not clear how the BL model performs in comparison with mutual funds and the market return when simply incorporating public information. Hence, is it possible for an investor to more effectively choose and construct a portfolio allocation strategy that yields higher returns while only having access to historical movements and subjective views about future excess returns?

1.2 AIM AND PURPOSE

The purpose of this thesis is to test the applicability of the Black-Litterman model and to clarify how the model can be used in practice. We aim to explore and bring clarity into the process of applying the BL model on the stock market and comparing the performance with a mutual fund but also with portfolios created under the MV framework. We also aim to develop a consistent estimation approach for the confidence matrix based only on public information.

We will limit our research to the Swedish stock market and the companies included in Nordea's Sverigefond. Furthermore, the time period will be restricted to the period from January 2007 to April 2012.

1.3 DISPOSITION

The disposition of this thesis is as follows: Chapter two will start by presenting the MV framework and BL model from a theoretical perspective. A comprehensive review of previous research will then be included in order to provide the reader with a clear overview of the leading academic literature on portfolio theory in general and the BL model in particular. The third chapter will, firstly, explain the general methodological approach we are using in order to successfully implement the

different models. Secondly, our approach will be motivated from an academic standpoint relating our estimations back to previous research. Thirdly, and finally, potential pitfalls with our chosen methodology will be brought forward and discussed. The fourth chapter will provide the reader with a detailing presentation of what type of data that is included, where it has been obtained, the time span it refers to and pertinent data transformations. The fifth chapter will give a presentation of the empirical analysis and results. This section presents our results from estimating our models but also from sensitivity analysis and significance tests. The results will then be interpreted and analyzed. Overall focus in this section will be to present and analyze characteristics of our model that is of value going forward. The sixth and final chapter will present the reader with a conclusion and a summary of our research and concluding remarks concerning our empirical results. Suggestions for future research will be included in this final chapter.

2. THEORETICAL FRAMEWORK

The following chapter will address and develop theoretical concepts within portfolio theory. Each model will be discussed separately and the section will conclude with discussing discussion of previous research.

2.1 MARKOWITZ MEAN-VARIANCE MODEL

Since the BL model was developed from the MV model, it is important to have a clear understanding of the Markowitz model. However, the purpose of this thesis is to use and implement the BL-model and we will therefore only give a shorter presentation of the MV model and its components. When Markowitz presented his article *Portfolio Selection* in 1952, he argued that the value of a security should be the net present value of all expected future returns. This was a mere modification of John Burr William's article from 1938, *Theory of Investment Value*, where he claimed that the value of the security should be the value of all future dividends. However, since dividends were a rather unfamiliar concept at that time, Markowitz used expected returns instead. In contrast to earlier models, Markowitz did not only consider the characteristics of individual securities, but also the covariance between securities in a portfolio. By doing this, a portfolio manager could receive a higher expected return given the same level of risk or lower level of risk but with the same expected return compared to models which did not consider the co-movements of the different securities included in a portfolio. In the model, risk is defined as the variance of an asset (Markowitz 1952).

Ever since Markowitz presented his model, it has been a cornerstone in modern portfolio theory up to date. What lead Black and Litterman (1992) to modify the model was some basic problems occurring when putting the model into practical use. These problems were first presented by Michaud in the article *The Markowitz Optimization Enigma: Is Optimize'd Optimal?* (1989). He points out four major aspects of consideration. Michaud means that the Markowitz model has a tendency to overweight securities with a large expected return and small variances and underweight those assets that have a low expected return and higher variances. These assets are also likely to have large estimation errors. The implication is that the MV-optimizers are actually estimation-error maximizing. He also discusses the impact of good and bad estimators, where the "normal" procedure is to use historical data to produce a sample mean which is then used as the expected return. According to Michaud this will have a great impact on the error-maximization problem. In his

article he also argues that the MV-model does ignore the liquidity aspect of an asset, or the asset's market capitalization. Smaller companies may have a large part of its market capitalization hold by a portfolio. This means that small changes in the portfolio might represent a significant amount of the value of the company. If the portion of the company that is purchased (sold) by the portfolio is significant the purchase (sale) price will fall (rise). A third aspect brought up by Michaud is the fact that the MV-optimizers do not differentiate between different levels of uncertainty associated with the inputs. The fourth, and perhaps one of the most major problems, is that the MV-model is very sensitive to changes in inputs, which will change the portfolio weights drastically and blames ill-conditioned covariance matrices. An ill-conditioned matrix is one constructed by historical data that is insufficient (Michaud 1989).

A problem discussed by Black and Litterman (1992) is the impact of constraints in the model. Often portfolio managers are imposed with short selling restrictions, which mean that they are not allowed to take a negative position in an asset. However, when one runs the model without constraints it often suggests large short positions/negative weights in several assets. The problem arising when adding constraints to the model is that the solution suggests zero weights in many assets and large positions in some assets, which leads to unreasonable large weights in some assets. One directly realizes the problem arising from a diversification point of view (Da Silva et.al, 2009).

2.1.1 DETERMINING OPTIMAL PORTFOLIOS

According to Markowitz (1952), investors are assumed to care about only expected return and risk. In order to derive attainable portfolios according to the MV framework we introduce the following optimization problems.

$$\begin{cases} \min: w^T \Sigma w \\ w^T \bar{r} = \bar{r} \end{cases} \quad (2.1)$$

$$\begin{cases} \max: w^T \bar{r} \\ w^T \Sigma w = \sigma^2 \end{cases} \quad (2.2)$$

Where w is the vector portfolio weights, Σ is the variance-covariance matrix of asset returns, σ^2 represents the portfolio variance and \bar{r} is the expected return vector (Drobtz, 2001).

Equation 2.1 minimizes portfolio variance with a certain level of expected return and equation 2.2 maximizes expected return given a certain level of risk measured as variance.

Solving equation 2.1 and 2.2 with the Lagrange method yields the formulae for optimal portfolio weights:

$$w^* = (\delta\Sigma)^{-1}\mu \quad (2.3)$$

Where δ is a risk aversion parameter and μ is the expected excess return vector (Drobetz, 2001).

2.2 THE BLACK-LITTERMAN MODEL

The Black-Litterman model was presented in a Goldman Sachs fixed income research paper in 1991 by Fisher Black and Robert Litterman. Many scholars have since then proposed expansions and estimation methods for the original model, most recognized is perhaps the work by Meucci (2005; 2006; 2008; 2010).

In realm of finance, the model presents a new approach on the classical portfolio selection problem, which is accomplished by incorporating views into the model. The market equilibrium portfolio is used as a starting point and the views then tilts weights towards assets where investors are bullish. Recent research has however shown that one does not have to use the equilibrium portfolio as a starting point but can instead use an investor's current portfolio or an index e.g. S&P 500 if U.S equities (Meucci, 2009). The views can be absolute or relative where an absolute view would be that an investor believes that the return from a certain asset will increase or decrease by a specific percentage. A relative view on the other hand would be the belief that an asset will perform in a certain way compared to another asset in the portfolio. Next, one has to assign confidence in the views, something that Black and Litterman did not provide any guidance on in their original paper. The literature has frequently explored this area and suggested a wide range of approaches to determine confidence in views, something that will be discussed in dept under section 2.3.2. BL then uses a Bayesian approach (see section 2.2.1) to combine the subjective views with the equilibrium expected returns to yield the BL expected return vector. Optimal weights are then determined by a MV optimizer. With this approach, investors should take more risk where they have views and even more risk where their views are stronger which intuitively makes perfect sense.

2.2.1 BAYESIAN THEORY

As previously discussed, the MV model suffers from extreme (corner) solutions partly caused by high sensitivity to movements in the variance-covariance matrix. The

Black-Litterman model helps overcome this problem and hence creates more stable portfolios. This is possible by combining investor subjective views with empirical data using Bayesian probability theory (Cass, Christodoulakis, 2002)

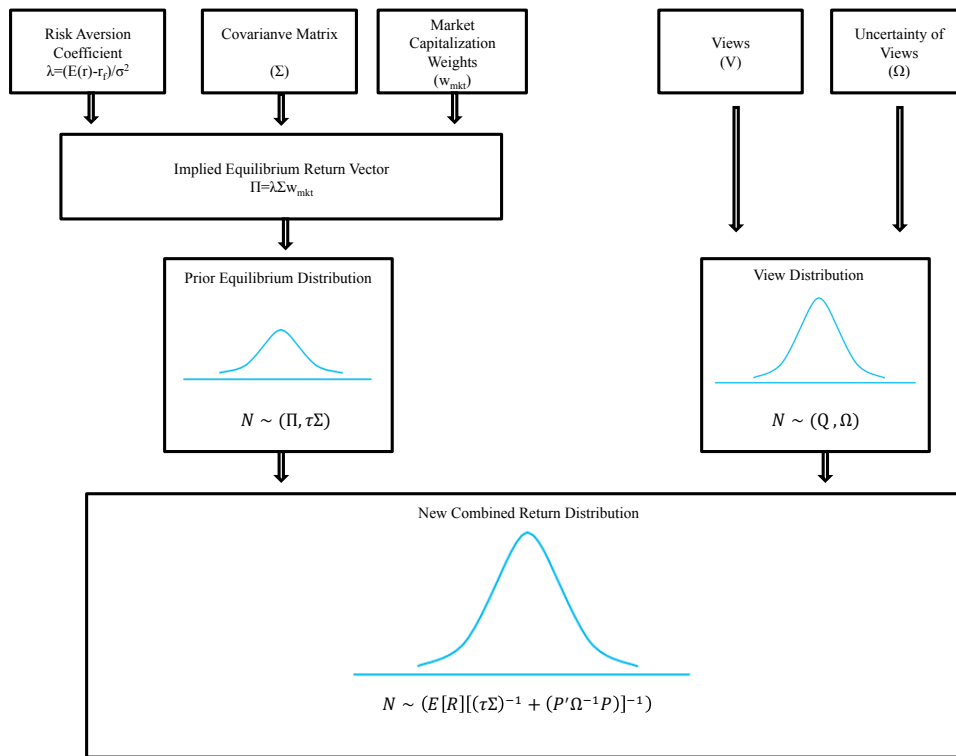
In other words the model provides a framework able to update subjective views on expected excess returns with equilibrium returns. This process makes it possible to tactically allocate funds accordingly. The Bayesian approach is the process by which the blending of views and expected returns is done. It follows the Bayes rule to compute the posterior probability, which means that one combines two antecedents, a prior probability and a likelihood function. Bayesian theory states the following:

$$P(H|E) = \frac{P(E|H)*P(H)}{P(E)} \quad (2.4)$$

$P(H)$ represents the prior information that is to be entered into a Bayesian model and is represented by a probability. The prior information is then updated by E which represents our sample data and is represented by the form of likelihood (Mankert 2006). Put differently, $P(H|E)$ is the conditional probability of H , given E and is interpreted by the probability of a correct view given the prior information available about historical returns. $P(E|H)$ is the conditional probability of E , given H (Guaranti 2006, p.32).

Figure 2.1 below is an illustrative example of how the new BL return distribution is obtained by the use of Bayesian theory. The prior equilibrium distributions are pooled with the views and uncertainty in views and the result is shown at the bottom of the figure as our new combined return distribution.

Figure 2.1 Graphic illustration of the Black-Litterman Return Distribution



(Idzorek 2004)

2.2.2 THE REFERENCE MODEL

The model upon which the BL model is built is called the reference model. The model aims to provide the investor with appropriate point of reference. According to Black and Litterman (1992) it is important for an investor to have a neutral reference in order to make use of the MV optimization model. They argue that investors could have views about different assets and have opinions about if they believe these assets are overvalued and undervalued. However, it is impossible to exactly state the expected excess return for every asset, and thus the equilibrium model can provide a good proxy.

The expected returns, which the BL-model aims to estimate, are assumed to be normally distributed with mean μ and variance Σ (Walters 2009). For clarification purposes we state the assumptions below.

Expected returns are normally distributed:

$$E(r) \sim N(\mu, \Sigma) \quad (2.5)$$

Distribution of the mean of returns (random variable):

$$\mu \sim N(\pi, \Sigma_\pi) \quad (2.6)$$

Where π is the estimate of the mean and Σ_π is the variance of the estimation from the mean return μ . The expression states that our estimate (π) varies around the actual value (μ) with a value of ε . The expression could therefore be rewritten into a more intuitive form:

$$\pi = \mu + \varepsilon \quad (2.7)$$

With equation 2.6 in mind and with the assumption that ε and μ are not correlated we can construct the expression.

$$\Sigma_r = \Sigma + \Sigma_\pi \quad (2.8)$$

Where Σ_r is the variance of π .

In a final step we construct the reference model for the BL model expected return as:

$$E(r) \sim N(\pi, \Sigma_r) \quad (2.9)$$

(He, Litterman, 1999)

2.2.3 THE BLACK-LITTERMAN FORMULA

In this part we will briefly describe the BL framework in a more technical fashion. We start with the combined return vector which has the following form (Meucci 2010):

$$\mu_{BL} = [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\pi + P^T\Omega^{-1}v] \quad (2.10)$$

Equation 2.9 is often referred to as the master formula of the BL model. Tau (τ) is the weight on views scalar and will be discussed more in detail in paragraph 2.12. Capital pi (Π) is the column vector with implied excess equilibrium returns and the sigma (Σ) stands for the variance-covariance matrix. P represents the “pick” matrix and is also given a more thorough discussion in section 2.3.1. The v stands for the column vector with the estimated returns for each view. A more detailed description can be also found in paragraph 2.3.1. The diagonal matrix with the variance of views is given by omega (Ω) and represents the uncertainty in the analyst views. According to Meucci (2008) equation (2.9) can be rewritten as

$$\mu_{BL} = \pi + \tau \Sigma P^T (\tau P \Sigma P^T + \Omega)^{-1} (v - P \Pi) \quad (2.11)$$

As can be observed, the BL expected return (μ_{BL}) starts from the equilibrium returns and adds on a component that is based on the weight-on-views (τ), the uncertainty in views (Ω) and the distance from the views to the implied excess return ($v - P \Pi$). The weight vector ($w_{msr(BL)}$) is obtained by using the estimated mean (Σ_{BL}^{μ}) and variance (Σ_{BL}). The formula for the weights can be written as:

$$w_{msr(BL)} = \left((\mu_{BL} - r_f) \Sigma_{BL}^{-1} \iota^T \right)^{-1} (\mu_{BL} - r_f) \Sigma_{BL}^{-1} \quad (2.12)$$

where ι in the above formulae represents a vector of ones. Σ_{BL} is given by:

$$\Sigma_{BL} = \Sigma + \Sigma_{BL}^{\mu} \quad (2.13)$$

and Σ_{BL}^{μ} in turn is given by:

$$\Sigma_{BL}^{\mu} = [(\tau \Sigma)^{-1} + (P^T \Omega^{-1} P)]^{-1} \quad (2.14)$$

Where as previously, P= ‘‘Pick’’ matrix, Ω =uncertainty in views and τ is the weight on views.

This method of estimating the BL model is obtained from Meucci (2010), there are other slightly different ways of modeling the posterior distribution and optimal weights but we will not go in to detail on those.

2.3 COMPONENTS OF THE BLACK-LITTERMAN

Although the implementation of views is one of the most crucial parts of the BL model, it has been one of the most difficult parts in our thesis. First of all one need to collect the views from a reliable source giving predictions about the future that can be considered reasonable. The second problem arises since we are doing an investigation based on historical data, hence we need historical views. The part of the views that probably caused the most cumbersome decisions was how to convert the analyst views into numbers and something that could be used in calculations of portfolio weights. We believe that there are several ways on how to approach this, however we have chosen the one way we believe will give the best forecast based on available information. The BL model requires the investor not only to estimate a positive, neutral or negative view, but also to specify the size or the relative performance of the specific view.

2.3.1 VIEW-MATRIX AND EXPECTED RETURN-MATRIX

The View-matrix (pick matrix) is a $k \times n$ matrix, which contains information about the different views and is denoted as P in the BL formula. Each column represents one company and each row represents a view. Since there are two ways in specifying the views, either absolute or relative, the sum of the rows will be one or zero, respectively. This can be illustrated by an example shown in the *Equation 2.15*. The first row sums to one which means that this is an absolute view and that asset two will have an excess return over the next period. The second row sums to zero and is a relative view where the investor believes that asset three will have a higher return than asset four over the next period. The third row is again an absolute view and represents the investor's view that asset seven also will have an excess return over the next period.

The Expected return-matrix (V), *Equation 2.16*, is a column vector containing the estimated returns for each view. Since the View-matrix just stated that the investor has some views of one or more specific assets, one also needs to specify the size of the views. The equations below are an illustrative example of the estimated returns of the views from the P matrix (Idzorek 2004).

Equation 2.15

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Equation 2.16

$$V = \begin{bmatrix} 8\% \\ 1,5\% \\ 4,5\% \end{bmatrix}$$

The investor believes that asset two will have an excess return of 8%, which is represented by a 1 in row one in the P -matrix. Asset three will have a 1.5% higher return than asset four, which is illustrated by a 1 for asset three and a -1 for asset four to represent the bullish and the bearish view, respectively. Asset seven will have an excess return of 4.5% and is represented by a one in row three in the P -matrix.

2.3.2 VARIANCE OF VIEWS- MATRIX

Ω is a $k \times k$ matrix and represents the uncertainty in the market views and is one of the most discussed areas within the BL model. Since we assume that views are uncorrelated, the matrix will be diagonal and only contain the variances, hence the covariances will be zero. The matrix is illustrated in *Equation 2.17*. The number of columns and rows represents the number of views. Black and Litterman do not

specify how to determine the uncertainty in views in their original paper but scholars have since then proposed different ways of determining Ω .

Equation 2.17

$$\Omega = \begin{bmatrix} \omega_1^2 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \omega_k^2 \end{bmatrix}$$

According to Idzorek (2004), and supported by Litterman (2003), Ω is the most abstract parameter of the BL model. Since the model forces an investor to determine a probability density function, according to Herold (2003), the model is only suitable for quantitative investors. Below follows a detailed overview of the most influential propositions of how to estimate Ω .

Idzorek (2004) discuss a number of different factors that, in addition to the variances of the view portfolios, have a great impact on the investor's confidence in a specific view e.g. historical accuracy of the model and the analyst that produced the view in question. If an investor had 100% confidence in each view means that one sets all the diagonal elements of Ω equal to zero. For those assets where the investor has a view, a 100% confidence will lead to a significant departure from the benchmark market capitalization weights.

$$E[R_{100\%}] = \Pi + \tau \Sigma P^T (P \tau P^T)^{-1} (Q - P \Pi) \quad (2.18)$$

Formula (2.18) above is the new combined return vector for the BL model under 100% confidence. We can now come up with three different weight vectors labeled \hat{w} , $w_{100\%}$ and w_{mkt} , representing the weights achieved by the new combined return vector, the new combined return vector with 100% confidence and the market cap weights, respectively. It is now possible to determine the implied level of confidence. First, one starts by calculating the weight differences between the new weight vectors and the market cap weights ($\hat{w} - w_{mkt}$) and ($w_{100\%} - w_{mkt}$). The last step is to divide those differences by the corresponding maximum weight difference ($w_{100\%} - w_{mkt}$).

Mankert (2006) introduces a confidence interval when calculating the diagonal elements ω_i^2 so that one standard deviation (about 2/3) of the cases results in a return of the specific view $= V_i \pm \omega_i$. This implies that investors have to specify a confidence interval around the expected return in order to arrive at the Omega matrix.

A slightly different approach is proposed by Idzorek (2004) who lets the investor assign a level of confidence in each view on a scale from 0-100%. He presents a fairly detailed step-by-step explanation of his approach and the main idea is to set Ω equal to zero (no uncertainty in views) and then tilt the weights based on the following formula:

$$Tilt_k \approx (w_{100\%} - w_{mkt}) * C_k \quad (2.19)$$

where $w_{100\%}$ is the weight vector when Ω is zero, C_k is the confidence in view k and w_{mkt} is the market capitalization weight vector. The optimal weight vector is then determined by

$$w_{k,\%} = w_{mkt} + Tilt_k \quad (2.20)$$

Meucci (2009) proposes an estimation method of Ω that is proportional to the covariance matrix of asset returns. To introduce uncertainty in the views Meucci includes the parameter “c” which leads to the following:

$$\Omega = \frac{1}{c} P \Sigma P^T \quad (2.21)$$

2.3.3 WEIGHT-ON-VIEWS, TAU

The majority of the existing literature seems to agree on that τ is the most confusing and cumbersome component of the BL model. Almost every paper uses its own value of τ , and some even ignores it. τ is a scalar and is often called the *weight-on-views*. In most available literature today τ takes on a value between 0 and 1. In their original paper from 1992, Black and Litterman set τ close to zero. Also Le (2000) sets τ close to zero, typically in a range from 0.01 to 0.05. Satchell and Scowcroft (2000) set τ to 1 while Blamont and Firoozye (2003) and Meucci (2008) interpret τ as 1 divided by the number of observations. The interpretation of Meucci’s approach is that the views will have a higher influence when the data is scarce and vice versa. A low value of τ indicates that the uncertainty in the mean of the distribution is less than the variance in the returns and means that the investor is confident in the estimates from CAPM (Drobtz, 2001).

2.3.4 PREVIOUS AND ONGOING RESEARCH

We have already touched upon previous researcher's estimation methods in terms of the different components of the model. However, there is ongoing research on optimal portfolio allocation and extensions to already existing models. One of the most recent contributions is from Cheung (2007) who augments the BL model by including additional sources of information in portfolio construction. Cheung claims that portfolio managers often include fundamental, technical, industrial and macroeconomic information when determining how to allocate capital. For example, a portfolio manager might have a bullish view on the oil price or the interest rate environment which could lead to positive views on whole industries. Therefore, it would be of great importance to include such information in the BL framework. This is done by using factor mimicking portfolios or factor models to help predict future stock returns and this way help gain accuracy within the model (Cheung, 2011).

2.3.5 A BEHAVIORAL PERSPECTIVE

As described earlier, the BL model uses a reference portfolio as a starting point, which is called the equilibrium portfolio. This is the portfolio against which portfolio/fund managers will be evaluated. In the Markowitz model, as within traditional theory of finance, investors are assumed to have a quadratic utility function (Figure. 2.2) (Mankert, 2004). Utility, and also the evaluation of an investment, is expressed in wealth and absolute terms, and the curve is concave which states that the marginal utility is decreasing with wealth. However, according to Tversky and Kahaneman (1984), when we consider a behavioral finance framework and evaluate the BL model the investor has, unlike in the traditional finance theory, a reference portfolio against which the investment is compared. What differs is that the performance of an investment is not expressed in absolute terms (wealth), but as gains and losses in relation to a point of reference. Compared to the traditional quadratic utility function, which is concave in the entire utility/wealth plain, the function will now be concave in the domain of gains and convex in the domain of losses (Figure 2.2).

Figure 2.2 Quadratic Utility Function

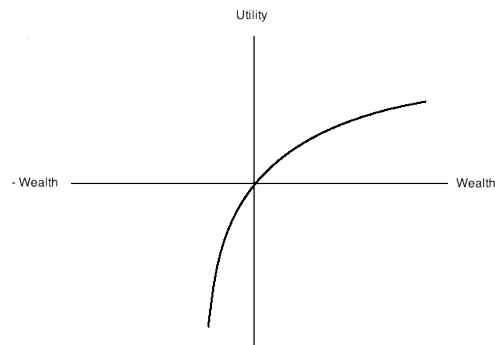
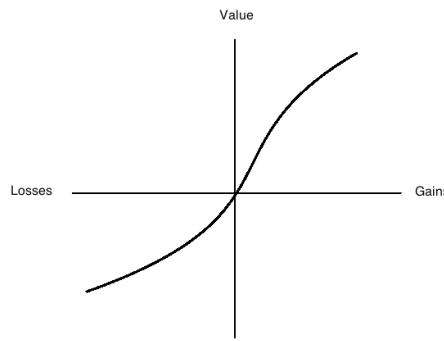


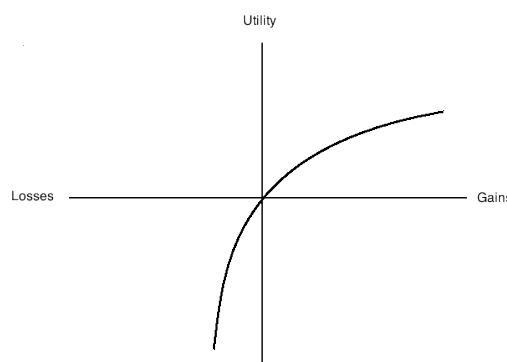
Figure 2.3 Behavioral Finance Utility Function



The interpretation is that an investor is risk averse in domain of gains but risk seeking in the domain of losses. According to the relatively theory, a portfolio manager that experiences a loss of 3% compared to the benchmark that had a loss of 4%, will still be content since he managed to outperform the benchmark. As explained earlier, the BL model has its origin in the Markowitz MV model. Based on this Mankert (2004), construct a utility function for the BL model that looks exactly as the Markowitz model, with the difference being the definition of the domains. The BL model becomes a combination of the Markowitz model and the value function where the utility function is defined as deviations from a predetermined point of reference. The function is illustrated in Figure 2.3. The concept can also be further supported by quoting Bevan & Winkelmann (1998).

A central feature of the Black-Litterman framework is the notion that investors should take risk where they have views, and correspondingly, they should take the most risk where they have the strongest views.

Figure 2.4 Black-Litterman Utility Function



The reason for the looks of the BL utility function in Figure 2.2, having a shape that differs from normal behavioral finance but having the same domains, is the concept of investor analyst views. The function does not represent investor loss aversion since that would be in conflict with the positions taken by an investor as suggested by the BL model and the investor views compared to the portfolio holdings in the reference portfolio.

3. METHODOLOGY

The following chapter aims at clarifying and motivating the empirical method used in this thesis. Various tests and sensitivity analysis performed will be discussed and explained in detail.

A Black-Litterman portfolio allocation strategy is applied on the Swedish stock market and is then compared with the returns from a MV portfolio and an equally weighted portfolio. To be able to test the returns obtained by the BL we have chosen to include the same assets in our portfolio as Nordea Banks's Sverigefond. The results will hence indicate whether the BL model can perform significantly better than Nordea's Sverigefond or not.

The test will be conducted with the use of weekly data going back to January 2007. A BL portfolio will be calculated for 2008 using one year of historical data and views from that point in time and the weights will be recalculated every year until 2012. That is to say, one additional year of data will be included for every time we re-weight our portfolio.

Investor views will be collected annually, starting in December 2007 to present (four years and three months). The views are collected from Dagens Industri stockwatch where the major financial institutions present recommendations about stocks in form of buy, sell and neutral. Also included in the recommendation is a 'target price' which will be used when determining the size of the expected return corresponding to our views. These target prices are set in relation to the spot price at the time of the recommendation in order to get the expected future return of our views.

The views are then assigned a value of 1, 0 or -1 depending on if the view recommends a buy, neutral or sell. The returns from the different funds will be collected on a weekly basis in order to achieve a significant number of observations. When a specific asset has no investor recommendation available there will be no view at that point in time. The portfolio weights will be updated with every new collection of views and the weights will be calculated using the returns from the previous year.

Since views are highly subjective in nature and different investors have different forecasts for the future we have chosen not to collect this information from one source/investor (e.g. financial institution) but rather to compile all available information from a vast number of financial institutions and structure our view for the

future provided by Dagens Industri. With this approach we hope to gain accuracy since we assume that an asset with 10 investors recommending “buy” is more likely to outperform the market than one asset with only one investor recommending “buy”.

3.1 ESTIMATION OF OPTIMAL PORTFOLIOS

When solving for optimal portfolios, one can solve the problem analytically or numerically. The Mean-Variance optimization problem is usually solved numerically through excel or any other algorithm. The Black-Litterman model however, can easily be solved analytically in Excel using the framework presented in section 2.2. The work by Meucci will be used throughout this thesis when estimating the BL model. Since we opt to compare the BL model with the MV model and Nordea Sverigefond (that is restricted from short selling), it falls naturally to impose short selling restrictions on both models. A problem that arises is that we need to solve the BL weights numerically because it is not possible to impose short selling restrictions in the analytical solution. This is conducted by estimating the following equation for the BL’s expected return vector sometimes called “The master formula”:

$$\mu_{BL} = [(\tau\Sigma)^{-1} + P^T\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\pi + P^T\Omega^{-1}v] \quad (3.1)$$

This is the expected return vector, which is used in the MV optimizer to solve for optimal weights. It is now possible to impose short selling restrictions by setting $w_i \geq 0$ for all i .

Optimal portfolios with no short selling in the MV framework can be written as:

(Elton, Gruber, Brown & Goetzmann, 2007)

$$\max \frac{\bar{R}_p - R_f}{\sigma_p} = SR_p \quad (3.2)$$

Subject to: (1) $\sum_{i=1}^N X_i$, (2) $X_i \geq 0$

In terms of the BL model, we can write 3.2 as:

$$\max \frac{w^T \mu_{BL}}{(w^T \Sigma w)^{1/2}} \quad (3.3)$$

This yields optimal BL weights and the solution to this problem is obtained by using the Solver-function in Excel. Similarly, the same procedure is used when calculating optimal MV weights but instead of using μ_{BL} in the numerator of 3.3, we use μ_{MV} .

When testing the statistical significance for the different return series, t-tests are used. The different hypothesis is specified in the following way:

$$\begin{cases} H_0: Return_{BL} = Return_{MV} \\ H_1: Return_{BL} \neq Return_{MV} \end{cases} \quad (3.4)$$

The null hypothesis is rejected if we receive a p-value lower than our significance level (Brooks, 2008).

Important aspects when determining estimation approach is how the views are specified. For example, we have in this paper collected views from market analysts who gives “buy, neutral or sell” recommendations. So, intuitively it would make sense to increase the confidence in a view if the number of analysts increases and not necessarily increase the expected return (V vector). The method proposed by Idzorek (2004) makes it possible to specify different confidence levels in different views which are an important advantage. The approach by Meucci (2009) only allows for one level of confidence for all views, the parameter he calls “c”. However, one can argue that it is not likely that investors can accurately determine if they are 35 or 30 percent confident in a view and then using Meucci’s approach will eliminate this ambiguity.

In this thesis, we have chosen a new estimation approach of Ω that enables us to specify different confidence in our views without having to specify percentage of confidence. Our aim is to decrease the variance dependent on how many market analysts that gave the same recommendation. For example, if three analysts suggested ‘buy’ for H&M in Dec. 2009 then our variance for this particular view is divided by 3 during the subsequent year. We can write this in the following way:

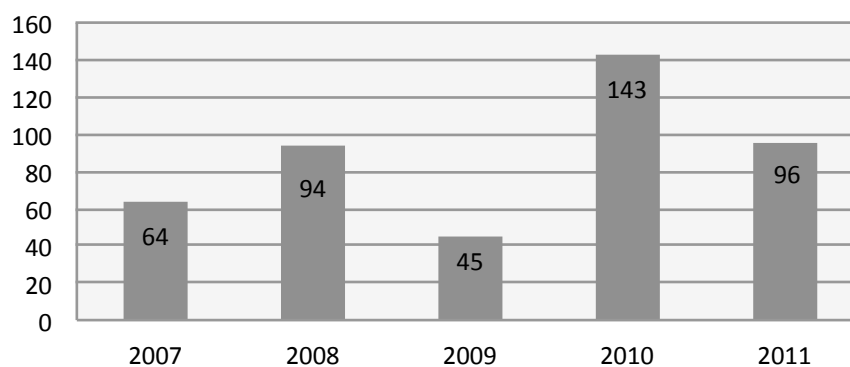
$$\omega_{k,t}^2 = \frac{\omega_{k,t}^2}{m_{k,t}} \quad (3.5)$$

Where k represents the different stocks in our portfolio that goes from 1 to 36 and m is the compiled number of analyst views. ω_k^2 is the variance of our view for asset k at time t.

3.2 POTENTIAL PITFALLS

During the process of collecting data for our study we noticed that the total number of views fluctuates widely over the course of the time period. Figure 3.1 is illustrating the total number of views for the different years. Since we incorporate a feature in the model that relates uncertainty in views to the actual number of views, this could be a potential source of bias. Moreover, one can also argue that since we compile all available views for each stock that we implicitly give each view the same weight. That is, regardless of which financial institution that gives a recommendation, same weight is given. Shefrin (2005) argues that potential problems with analyst recommendations are a conflict of interest resulting from agency problems. He points out that many financial institutions are doing business with companies they cover and hence have incentives not to give all that negative recommendations. This is not something we can adjust for but could be important to keep in mind when analyzing the results

Figure 3.1 Total number of views/year



3.3 THE BENCHMARK PORTFOLIO

In order to conduct our testing and practical implementation of the BL model we have chosen to use the Nordea Sverigefond as a basic starting-point and benchmark. The fund consists of 36 Swedish, mainly large cap companies, with a strong international presence. The companies are spread over all the major sectors/business areas represented on the market such as everyday commodity, energy, financials, healthcare, industrials, IT, power supply, materials, durable goods and telecom. This meets our aim to construct a portfolio that in a good way represents the Swedish stock market and gives a market wide and well-diversified approach without focusing on a specific branch or industry. All the companies in the fund are well recognized among

Swedish investors and represent a good mixture of long time Swedish industry tradition and entrepreneurial spirit. Many of the companies are popular among both private and institutional investors. We thus think the fund represents an average Swedish investor portfolio fairly well. As Swedish investors, in line with the home-bias theory and our own investment approach, we believe that the Nordea Sverigefond represents a composition that is of great interest for the average Swedish investor on a day-to-day activity. The fund has a riskiness of 7 on a scale where 7 are the highest. This means that the volatility is above 25% and thus should attract less risk averse investors that are prepared to accept large changes in the portfolio value. The fund has a five-year investment horizon, which can be considered to be medium-term. The medium term investment horizon allows us to test and compare the BL approach since we believe that five years is sufficient in order to observe differences in investment returns based on the portfolio weights. A crucial component in our choice of the Nordea Sverigefond was the need for analyst views. Since the BL model incorporates views about future developments, these need to be collected in a quantifiable way. As we are restricted in access to data, DagensIndustri stockwatch provide us with analyst views that cannot be found for international portfolios, thus significantly reducing our options of funds. However, based on the given discussion, we believe that the Nordea Sverigefond fulfills our requirements and will be sufficient in order to give results that will attract and be useful for Swedish investors with preferences towards the home market. A detailed description of the Nordea Sverigefond's specific portfolio can be found in Appendix 1.

4. DATA

The following chapter will detail the data type used in this thesis together with pertinent data transformations.

For our practical implementation of the BL model we have chosen to use same holdings as the Nordea Sverigefond since it contains a well-diversified portfolio of only Swedish stocks. The reason for choosing only Swedish stocks is due to the procedure of collecting analyst views. We collect our analyst views from DagensIndustri Stockwatch, which provides a convenient compilation of market views on particular Swedish stocks from the major investment banks and rating institutes. DagensIndustri Stockwatch also enables us to go back several years in time.

Data for the portfolio stocks is collected from Datastream and starts in December 2006. This enables us to estimate the 2008 portfolio based on one year historical data. We use the official closing price of each stock and weekly data. This leaves us with 52 observations which we consider adequate for the purpose of this thesis. The Nordea Sverigefond's actual return data was collected from Nordea.se. As the risk free rate of return we have used ten year Swedish T-bill rates collected from the Swedish Riksbanken webpage.

The reason for choosing this time period is that the Nordea Sverigefond has a long term investment horizon of about five years. Furthermore, we are also restricted from DI Stockwatch since analyst views cannot be found longer back in time. The time period is limited to 2007-01-01 to 2012-03-30. A longer time period could have been of interest but availability of investor views was limited which resulted in the aforementioned time period. We do not see this limited time period having any significant negative implications on this study since we do capture Nordea's long term investment horizon which is five years.

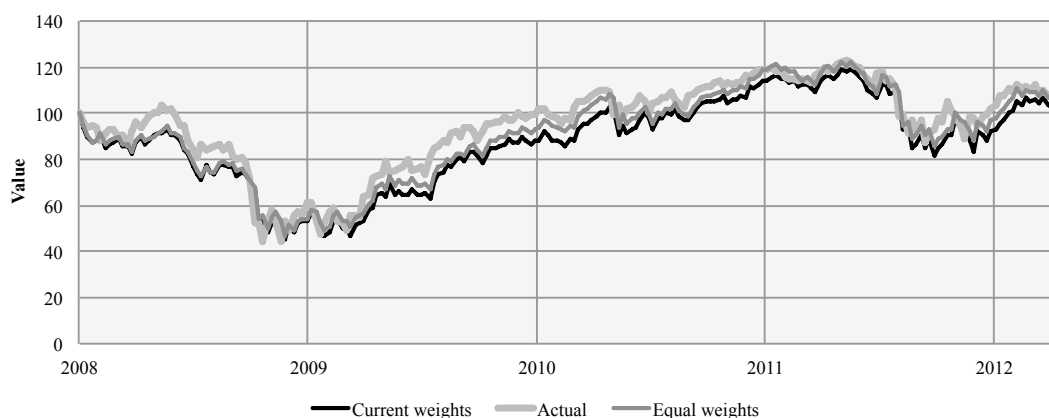
In order to transform our stock price data into returns we used logarithmic values. Furthermore, certain stocks have been subject to splits during recent time which will affect the ratio between analysts' target price and spot prices. For those individual cases we manually adjusted dependent on the magnitude of the split.

5. EMPIRICAL RESULTS AND ANALYSIS

The following chapter contains results from our empirical analysis and comparison of the Mean-Variance model, the Black-Litterman model and the Nordea Sverigefond.

In order to evaluate the performance of the BL-portfolio we will make several comparisons between different portfolio suggested by the MV model and three different versions of the Nordea Sverigefond. The reason for using three different portfolios of the Nordea Sverigefond is simply because we are not able to observe changes in the portfolio weights made over our chosen time period. Therefore we will construct one portfolio that have the same weights as the fund has today and derive the returns as if it have had the same weights over the entire observation period. One could argue that this is the portfolio that best reflects the weights during the observation period since the Nordea Sverigefond has a five-year investment horizon, thus covering the entire period of measure. For the second Nordea portfolio we will not have the possibility to provide any weights, but only to present the actual returns. For the third and last version we use a portfolio with equally weighted allocations. The results are shown in Graph 5.1 and in order to make the graphs more compatible we have transformed the data and use a starting point of hundred for all the portfolios.

Graph 5.1 Value of the Nordea Sverigefond using different weights

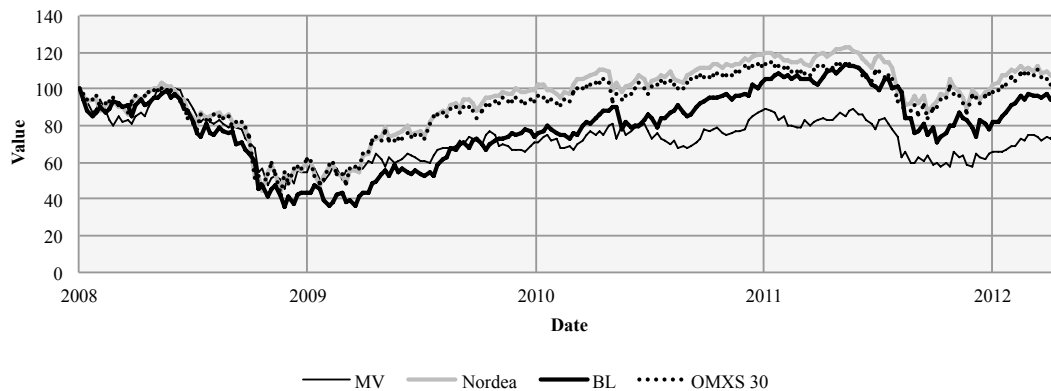


As can be seen from the graph, the three portfolios follow each other to a high extent which indicates that the portfolios are highly correlated. However, the actual Nordea return seems to be slightly higher during some periods.

In order to in dept analyze how the reallocation of the portfolio affects the excess return, we will look both at each year individually and the entire time period. Extreme positions in specific portfolios will be analyzed and discussed in order to give a

reasonable explanation to a specific outcome suggested by the model. As the reader will notice, year 2008 was characterized by instability and caused results that significantly deviated from other years in the observation period. Therefore we will include a discussion of what might have caused these deviations and if any model is regarded as more appropriate to use in such situations.

Graph 5.2 Optimal portfolio performance (short selling restrictions)



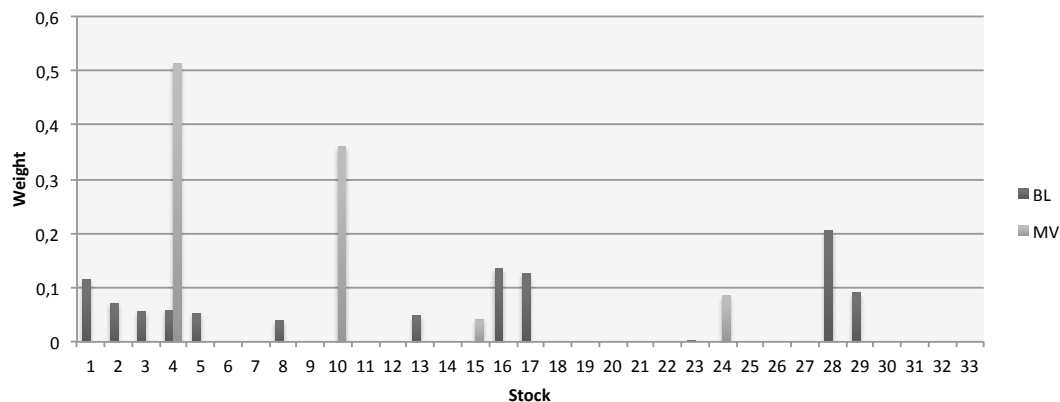
Graph 5.2 shows the performance of the three main portfolios included in our research, the Nordea fund, the MV portfolio and the BL portfolio, from January 2008 to April 2012. Short selling restrictions are imposed on the BL and MV models in order to be able to compare the models. These models will be referred to as the restricted BL and MV. It can be observed that the Nordea fund had the highest increase in value (9,57%) if we observe the entire time period. Both the MV portfolio and the BL portfolio experienced negative development in value, -24,6% and -10,9% respectively while the OMXS30 index increased by 4.05 %. By just studying the graphs, we can see that the portfolios follow each other fairly close until late 2008. The fall in 2008 seems to be consistent with and follow the rest of the world economy that experienced great financial turmoil after the bankruptcy of Lehman Brothers on September 15. Early 2009 was the starting point for what would turn out to be a period of strong recovery. During rest of 2009 and consistently during the entire 2010, the portfolio values increased and reached top levels in mid 2011. However, the summer of 2011 was the start for another storm in the world economy, mainly caused by great US deficits and anxiety about whether the US government would raise its debt ceiling. The turbulence caused a heavy drop in the market and hence the value of the funds. During the last year the funds have experienced a recovery and are now circulating around the same values that were observed at the beginning of our

observation period. Our calculations show that the average Sharpe ratio for the Nordea fund, the MV portfolio and the BL portfolio are 2,63, 0,64 and 1,93, respectively. This indicates that Nordea has the higher risk adjusted return followed by BL and MV. In addition, the MV portfolio carries a small number of assets (between 1 and 4), which further increases the exposure towards non-diversifiable risk making this portfolio less attractive for investors. Relating back to the section 2.3.5 about behavioral finance, it is hard to quantitatively evaluate the model. This can be linked to the subjective characteristics of the views, which makes it hard to determine if a specific outcome in comparison with a benchmark is due to the quality of the views or the model per se.

Graph 5.3-5.5 below outlays a more in dept presentation of how the BL and MV frameworks chose allocations and what determines the output.

Graph 5.3 2008 Weights Mean-Variance and Black-Litterman Portfolio

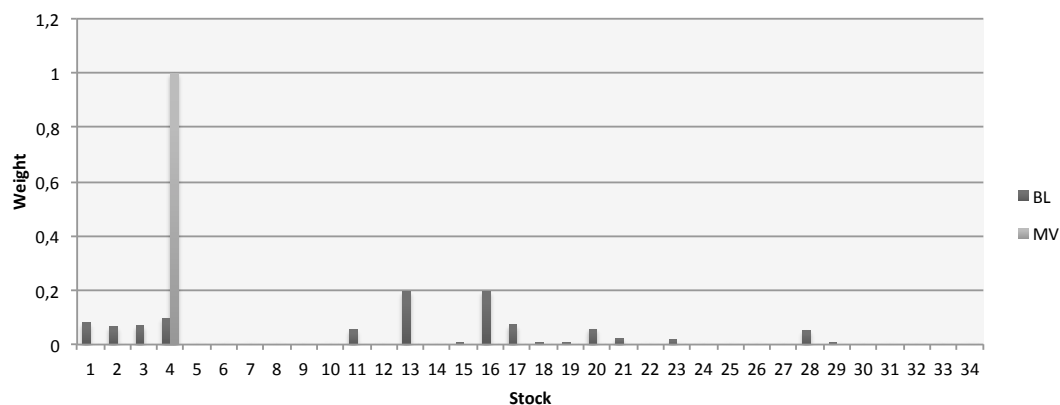
(Numerical solution and short selling restriction)



In Chapter 2 we discussed some potential drawbacks of the MV model when it comes to the construction of optimal portfolios. As is clearly evident from Graph 5.3 and in line with Black and Litterman (1992), when implementing short selling restrictions to the model the MV model takes extreme positions in a few numbers of stocks. For 2008 it chooses to put just over 50 percent in ABB and about 36 percent in Tele 2. For a rational investor this strategy will appear useless. It means putting almost all you assets in one basket, which does not correspond to the theory of risk diversification. The reason behind this extreme choice of asset allocation is that ABB had by far the highest mean return during 2007. Since the model base its decision on historical numbers, it will suggest investing heavily in ABB. Graph 5.2 also shows the optimal

portfolio weights suggested by the BL model obtained by a numerical solution. As can be observed the number of stocks included is higher than in the MV model, thus reduces the problem with extreme corner solutions and offers a more risk diversified asset allocation. Compared to the MV portfolio that chooses to invest heavily in the ABB stock the BL model suggests taking a long position in asset 28 (SSAB). The reason for this is that SSAB has the highest future expected return (μ) and has taken both historical returns and views into the considerations. However, after performing a t-Test we cannot reject the null-hypothesis that the returns from the MV and the BL portfolios are equal. Neither can we reject that that the BL and Nordea portfolio is equal, hence we cannot prove that the portfolios yield significant different returns. The correlation between the MV and BL portfolios is 0.62 and 0,91 between the BL and the Nordea portfolio.

Graph 5.4 2009 Mean-Variance Portfolio Weights

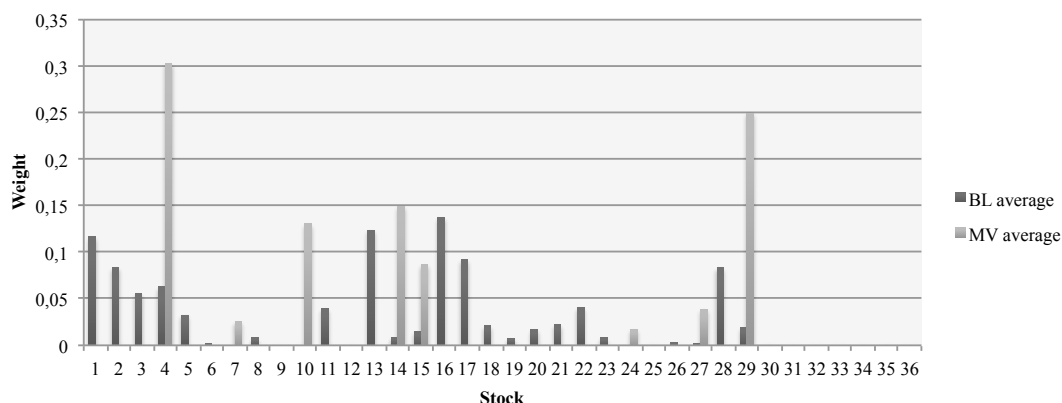


For 2009 the suggested portfolio weights suggested by the MV model are even more extreme than in previous year and it now suggests investing 100 percent of assets in asset 4 (ABB). The conclusion that can be made is that the MV-model suggests extreme corner solutions based only on historical returns while the BL model asset allocation is more evenly distributed and does not exceed more than 20 percent in one single asset.

Graph 5.4 presents the average weights from the MV and BL models for the entire period of study. Again, it can be seen that the MV model suggests much more extreme solutions than the BL model. The MV model clearly prefers two stocks (ABB and SSAB), which together make up 55 percent of the portfolio allocation. The

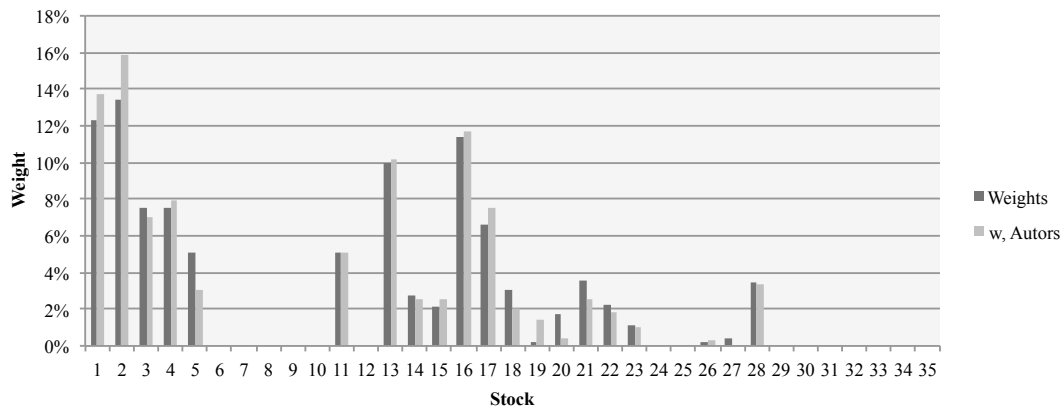
reason for this, as has already been touched upon earlier, is that these companies show the highest historical returns.

Graph 5.5 2008 – 2012 Average weights Mean-Variance and Black-Litterman

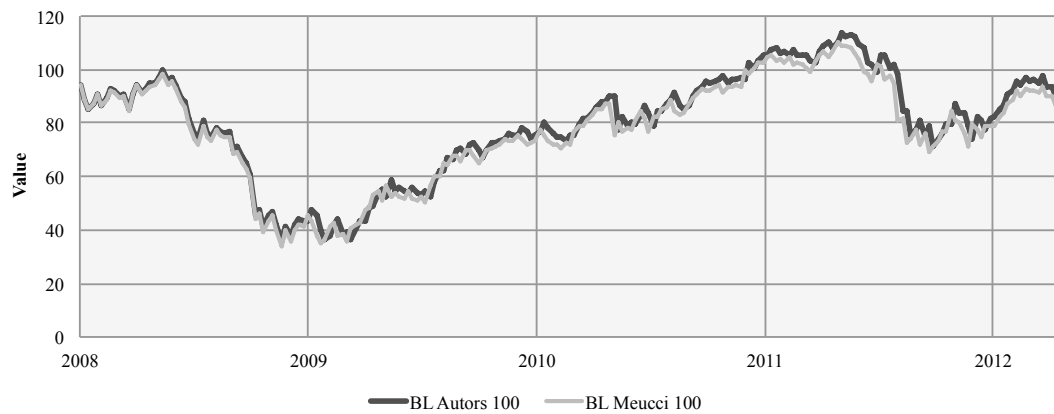


As discussed in section 3.3 we have chosen an estimation approach for determination of Ω (confidence on views) that does not require the investor to specify an exact percentage of confidence. We divide the specific variance of an asset with the numbers of analysts that gave the recommendation. We calculate the BL model both in accordance with Meucci (2010) and with our own approach of estimating uncertainty in views. The difference between the Meucci approach (where all views are assumed to have the same confidence) and our approach for 2010 can be observed in Graphs 5.6. It is possible to observe a clear pattern where the weights are more pronounced in assets where we have large number of views. Put differently, if we have a stronger and more confident view that an asset will yield a positive return, we will put more weight in this asset. The differences will have similar patterns for all the years but we choose only to show 2010 since this year had the largest number of views (see Figure 3.1). This is not surprising since the model will allow the investor to attach larger weights to assets that the investor feels more confident about. As illustrated by the graphs, the Meucci model suggests a smoother graph, avoiding more extreme positions. However, our positions are that it would be unjust to assign the same level of certainty to every view, hence causing skewedness of the weights. In the graph below (5.6) we show the returns of the portfolios using the Meucci method and our new method. Even though it is possible to see some differences along the time period, the two graphs seem to follow each other closely. In fact, the correlation between the graphs is 99%.

Graph 5.6 2010 Weights with different Omega (Ω)

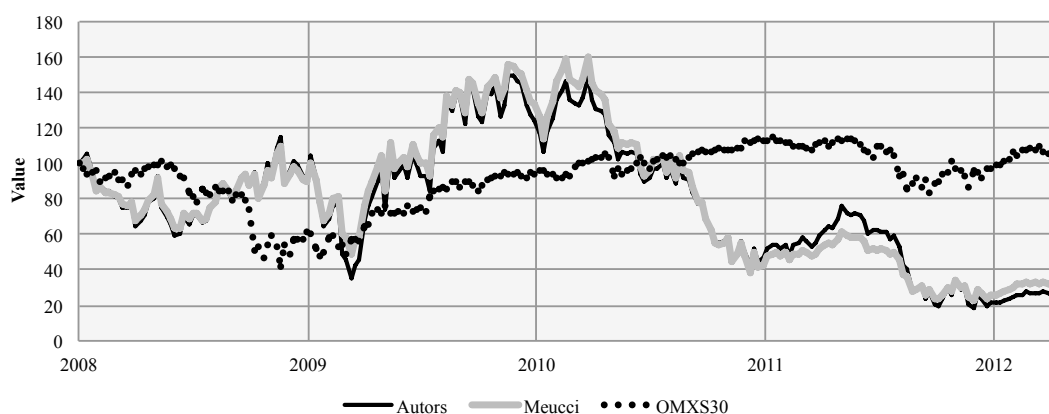


Graph 5.7 Value of portfolios using different Ω



Bearing the aforementioned results in mind, we will now expand our analysis of the models applicability by looking at the unrestricted BL model. However, applying an unrestricted MV model result in extreme weights that we consider non-applicable. For further information, see appendix. Estimating the BL model, using our approach regarding views, slightly outperforms Meucci (2010) when imposing short selling restrictions (Graph 5.7) but underperforms when solving analytically and not imposing any restrictions at all (Graph 5.8). As a result, we cannot say whether one approach is better or worse than the other.

Graph 5.8 Unconditional Black-Litterman (no short selling restrictions)



The above graph illustrates the performance of our unrestricted (analytical) BL model compared to OMXS30. In early 2012, both BL models have plummeted around 70-75% while the index is more or less on the same level as in the beginning of 2008. This is a result of large negative positions and “betting” on investor views. Up until early 2010 this works in favor of the model that performs relatively well considering the financial turmoil during 2008. After that point however, both BL models dramatically decreased. One can argue that the views collected for the second half is less accurate compared to the first half. One can also argue that the model got “lucky” in the first half in terms of betting on the right stocks. This can support the position that imposing no restrictions gives the model too much freedom in terms of allocating capital. In order to fully try and clarify the reasons behind the significant drop back in 2010 we have looked more closely at the specific views and weights that the BL model assumes during that specific year. What becomes obvious when studying the numbers more in detail is that the BL portfolio takes large short positions in four companies (Swedbank, Kinnevik, Autoliv, SKF). What is really surprising is that these four companies happen to show extremely good results for 2010 with an average return of 45 percent. Naturally, since the portfolio has taken short positions in these stocks, the value of the portfolio will be heavily negative affected. However, the model also made some good allocations during 2010, but these stocks really stand out and might be a part of the explanation to why we can see a significant drop that year while the OMXS model fluctuates around zero. Regardless of allocation within our portfolio, we will still carry non-diversifiable risk that we do not get paid for (Chhabra 2005). Consequently, when the model chooses optimal weights there are no safer assets with a low covariance towards equity markets available. Therefore, we

believe that an unrestricted BL model is not well suited in this context. When implementing a restricted BL on the other hand, weights are less aggressive and result in a more stable performance.

6 CONCLUSION

We have shown that the implementation process of the BL model is not without complications. The outcome depends on the interaction between several components and the determination accuracy of the same. The model also requires the investor to have a thorough understanding of the different components to be able to properly analyze and interpret the results. From our results we conclude that the restricted BL model shows better (although not significantly) performance than the MV model. Sharpe ratios also support these findings. In addition, the numbers of stocks included in each portfolio suggest that the BL model yields a superior diversification than the MV model. On the other hand we cannot conclude that our way of implementing the BL model will yield higher returns than what the Nordea fund has managed to do. However, this does not imply that the applicability of the model is inadequate in this context since part of the underperformance could be blamed on inaccurate views. We have also seen that the restricted version of the BL model helps to overcome the problems that come with the MV model. It clearly avoids the extreme corner solutions that are suggested by the MV approach in the conditional model and again provide the investor with a more diversified asset allocation. We also show that the BL model is fairly robust towards changes in confidence of views.

We can conclude that when applying an unrestricted model in this context, results drastically worsen. This is a result from more aggressive long and short positions in certain stocks. Consequently we have a more volatile and risky portfolio, more dependent on the performance of specific assets. Therefore, going from a restricted to an unrestricted model exacerbates the overall performance of the portfolio. We have also shown that the MV model yields extreme weights that are non-implementable in practice.

Our proposed estimation approach for incorporating views increases the applicability of the BL model and makes the subjective parts slightly more consistent. We have seen that when estimating the BL model, using our approach regarding views, slightly outperforms Meucci (2010) when imposing short selling restrictions but underperforms when solving analytically and not imposing any restrictions at all. This implies that we cannot conclude if one approach is more suited in this context than the other. We do conclude however, that the difference in terms of performance between the two approaches is dependent on the accuracy of investor views. This in turn

implies that the BL model might be better suited for solving optimal allocation problems including a wider range of assets. Put differently, investor views are less subject to large errors when subjectively forecasting indices or asset classes as opposed to single stocks.

6.1 SUGGESTIONS FOR FURTHER RESEARCH

Due to the fact that all assets included in our research are of one single asset class, which tend to have a high level of co-movement, it would be of interest to include additional asset classes to the portfolio e.g. fixed income. Expanding the magnitude of this study by including a wider range of stocks, longer or different time period and more geographically dispersed stock markets could also help increase the understanding of the inner workings of the BL model. Moreover, the BL model exclusively uses historical co-variances as a reference point when determining optimal weights, hence incorporating views directly in the co-variance matrix in order to construct a estimation of future co-variances would be a valuable and interesting topic for studies to come.

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APPENDIX

Table 1. General information, Nordea Sverigefond.

Nordea Sverigefond	
Company	weight %
Swedbank	8,94%
Volvo	7,53%
Ericsson	7,22%
ABB	7,02%
Sandvik	5,72%
Nordea	4,87%
HM	3,99%
Skanska	3,77%
Teliasonera	3,64%
Tele 2	2,93%
Stora Enso	2,93%
Astrazeneka	2,87%
SEB	2,86%
Electrolux	2,79%
Hexagon	2,44%
Kinnevik	2,37%
JM	2,24%
Autoliv	2,04%
Meda	1,97%
MTG	1,96%
Boliden	1,89%
Volvo	1,77%
Kungsleden	1,52%
Unibet	1,45%
Securitas	1,29%
Rezidor	1,27%
SKF	1,02%
SSAB A	0,92%
SSAB B	0,91%
ÅF B	0,55%
Nibe Industrier	0,54%
Alpcot Agro	0,38%
Connecta	0,37%
East Capital Explorer	0,36%
Sectra B	0,32%
Sigma B	0,31%
Etrion	0,01%
Sum	0,94980

General Info	
Fund size	7042 M SEK
Start of Trading	1978-01-11



Table 2. Statistical Inference, unrestricted models. One-tail and Two-tail displays p-values from the performed t-tests on the resulting return series. 5 % confidence level is used.

MV-Nordea Statistical Inference					
	2008	2009	2010	2011-2012	Whole period
One-tail	0,450	0,316	0,454	0,441	0,372
Two-tail	0,899	0,632	0,908	0,882	0,743
Variance MV	0,00387	0,002183	0,001462	0,002192	0,002413329
Variance Nordea	0,00652	0,003047	0,000626	0,001356	0,002772405
Mean MV	-0,0115	0,00491	0,00418	-0,00238	-0,00125505
Mean Nordea	-0,0095	0,009446	0,003368	-0,00125	0,000406005

BL-MV Statistical Inference					
	2008	2009	2010	2011-2012	Whole period
One-tail	0,164	0,321	0,282	0,496	0,432
Two-tail	0,327	0,641	0,563	0,992	0,863
Variance BL	0,0052	0,0035	0,0013	0,0018	0,0029
Variance MV	0,0039	0,0022	0,0006	0,0022	0,0024
Mean BL	-0,0162	0,0035	0,0066	-0,0025	-0,0005
Mean MV	-0,0115	0,0022	0,0034	-0,0024	-0,0013

BL-Nordea Statistical Inference					
	2008	2009	2010	2011-2012	Whole period
One-tail	0,339	0,464	0,282	0,424	0,428
Two-tail	0,678	0,928	0,563	0,847	0,857
Variance BL	0,005	0,003	0,001	0,002	0,003
Variance Nordea	0,007	0,003	0,001	0,001	0,003
Mean BL	-0,016	0,010	0,007	-0,002	-0,001
Mean Nordea	-0,009	0,009	0,003	-0,001	0,000

Table 3. Unrestricted Black-Litterman weights for 2010. Return shows the actual performance of the different assets during 2010.

2010 Unrestricted BL		
Stock	Weight	Return
Swedbank	-41%	32%
ABB	-26%	13%
H&M	71%	13%
Teliasonera	22%	3%
Tele2	30%	27%
AstraZeneka	35%	-8%
Kinnevik	-72%	28%
Autoliv	-36%	67%
Securitas	48%	12%
SKF	-25%	55%
Connecta	34%	24%
East capital explorer	90%	26%
Sectra	35%	-19%

Table 4. Performance measures for the different portfolios under short selling restrictions when solving the models numerically. Nr of shares indicates number of assets included in portfolios for different years.

	Black-Litterman				
	2008	2009	2010	2011	2012
Variance	0,001378	0,00295	0,003966	0,002677	0,002484
E[r]-rf	0,002147	0,005005	0,00227	0,004582	0,004634
Sharpe	0,057834	0,092161	0,036047	0,088546	0,092979
Nr of share:	11	16	14	19	21

	Mean Variance				
	2008	2009	2010	2011	2012
Variance	0,000862	0,003678	0,003198	0,002397	0,001783
E[r]-rf	0,006407	0,000294	0,003648	0,004153	0,002422
Sharpe	0,218166	0,004854	0,064502	0,084835	0,057359
Nr of share:	4	1	3	4	4

Table 5. Sharpe ratios measured on one year historical performance.

	Sharpe ratio		
	Nordea	Mean Variance	Black-Litterman
2008	-5,96	-7,91	-8,74
2009	13,26	4,93	11,85
2010	6,49	6,69	8,68
2011	-4,57	-5,58	-6,36
2012	3,95	5,08	4,21
Average	2,63	0,64	1,93

Table 6. Weights in unrestricted Mean-Variance portfolio (%)

	Weights unrestricted MV				
	2012	2011	2010	2009	2008
SWEDBANK 'A'	15617		1668340872	-12075516	-3203
VOLVO 'B'	153		-1067428146	3289921	-534
ERICSSON 'B'	-12134		-4880873982	-6982806	3944
ABB (OME)	19484		6536158353	11540338	35072
SANDVIK	6105		1525863354	-5804487	19427
NORDEA BANK	-5000		6406971312	14139261	13505
HENNES & MAURITZ 'B'	15962		4072669100	12147006	1857
SKANSKA 'B'	-474		888217316	21957094	-13117
TELIASONERA	-2639		2292887126	5790592	-4978
TELE2 'B'	10524		4002474986	6316789	8313
STORA ENSO 'R'	-12335		-6387452630	-6873701	-21712
ASTRAZENECA (OME)	-947		1523772216	-2237704	12113
SEB 'A'	-22817		-6240616181	593697	-10196
ELECTROLUX 'B'	-543		6096686453	117039	10327
HEXAGON 'B'	18100		2541863843	-587999	5381
KINNEVIK 'B'	6185		-1928140937	-2299959	-2283
JM	1743		4080324675	-11810798	16741
AUTOLIV SDB	-3405		-1652791989	-11474551	-7316
MEDA 'A'	-8666		-4163271894	-695685	-1781
MODERN TIMES GP.MTG 'B'	8148		933343754	-6993720	5978
BOLIDEN	13788		-57694089	-19015443	-17028
VOLVO 'A'	-2047		-2990126006	-5934355	-2818
KUNGSLEDEN	-6694		-1692897290	3609844	-7528
UNIBET GROUP SDB	6250		2998580205	1933777	8459
SECURITAS 'B'	-26472		-5025284472	13025668	-11428
REZIDOR HOTEL GROUP	-16018		-7578057343	-5750822	-9350
SKF 'B'	23364		3066923571	8217140	-11562
SSAB 'B'	-26844		307044782	10619444	533
AF 'B'	27968		8457189937	14596133	3335
NIBE INDUSTRIER 'B'	10646		-239019732	-319813	-6298
ALPCOT	-1684				
CONNECTA	11400		1264089224	12396251	-3580
EAST CAPITAL EXPLORER	-33507		-9152153903	-33140557	
SECTRA 'B'	1214		-2145953231	-778584	1846
SIGMA B	-9856		-3461639251	-7513492	-12117
ETRION	-4568				

The absence of numbers in 2011 is due to our inability to find convergence. Other absente numbers is due to lack of historical data.