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Pricing Temperature Weather Derivatives

1st Yr. Master Thesis

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Abstract

The key aim of the current paper is to analyse the plausibility of a pricing model for temperature weather derivatives. The historical data are studied in order to propose a stochastic process that describes temperature dynamics in three Swedish cities. The prices of the contracts in an incomplete market of weather derivatives are obtained using a constant positive market price of risk of a benchmark temperature derivative. Numerical examples of prices of contracts are shown using Monte-Carlo simulations and an approximation formula. The precision of the approximation formula is scrutinized depending on the changes in strike, market price of risk, risk-free rate, mean temperature, speed of mean-reversion and volatility. Moreover, theoretical prices of temperature options for two Swedish cities, which are not represented on the weather derivative market, are proposed.

Key words: Weather derivatives, commodities, temperature stochastic modelling, Monte-Carlo simulations, market price of risk

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INTRODUCTION

Weather derivatives are relatively young financial instruments that allow retail, insurance, construction, entertaining, agricultural and other types of companies to protect themselves from weather fluctuations.

In a short period of time there has already been created a sound background for weather hedging. A significant number of studies dedicated to analysing underlying weather processes, constructing and pricing weather derivatives have been made (see, for example, Jewson & Brix, 2005, Barrieu & El Karoui, 2002; Brockett et al., 2010; Cao & Wei, 2004; Benth et al., 2007). However, each of them left a range of unexplained questions and directions for further research. Moreover, the main region of interest of the aforementioned studies is the United States of America, while the European market remains rather underdeveloped, and, therefore, relatively understudied.

Despite the solid research made in the area of weather derivatives, there is still no commonly accepted and widely used approach for practitioners to price these financial instruments, which makes the topic extremely relevant both theoretically and practically. Therefore, the purpose of the current research is to analyse the pricing framework for temperature derivatives in terms of three Swedish cities. The acceptability of the valuation approach proposed by Alaton et al. (2002) is tested exploiting new data. The criteria are that the model is convenient to implement and provides satisfactory fit to the data. In order to achieve the established goals, firstly, temperature evolution is analysed and modelled. Stochastic modelling is used to determine temperature dynamics. Pricing is performed using an approximation formula as well as Monte-Carlo simulations. Despite the fact that Monte-Carlo simulations might contain an error, it is believed that after making a vast number of simulations it would reflect the reality relatively plausibly. That is the reason why the results provided by this method are taken as a proxy for actual option prices, which were unavailable due to the extremely low volumes of the weather derivatives traded for the analysed region. Later, the results given by the approximation formula are compared with those given by Monte-Carlo simulations. Furthermore, it is studied how the change of strike, risk-free rate, mean temperature, speed of mean-reversion, volatility and market price of risk influence price levels and precision of the approximation.

The Swedish cities to be analysed are Stockholm, which is among the traded European cities on the Chicago Mercantile Exchange (CME), Gothenburg and Malmö. The mentioned cities were chosen based on their population sizes¹: since the aim of the analysed derivatives is to protect, for instance, against low demand in the energy sector, it is prudent to account for highly populated cities, with relevant electricity consumption. Despite the fact that neither Gothenburg nor Malmö is among the European cities for which CME offers weather derivatives, they both have a full potential to become such due to level of development of their electricity market. Therefore, the current paper not only studies the temperature process in these cities, but also proposes possible prices of weather derivatives for them. The temperature processes in the three Swedish cities are compared and correlations between them are scrutinized, thus, identifying the region specific characteristics to be incorporated as the features of the new derivatives.

It is found in the process of temperature modelling that temperature characteristics are similar to those previously detected by Cao and Wei (2004), Benth et al. (2007), Alaton et al. (2002) etc., indicating seasonality patterns in variance and mean-reversion. It was concluded that the approximation formula for option pricing previously used by Alaton et al. (2002) is plausible and its precision improves with the model parameters converging to their most probable levels. Yet accountancy for diverse values of the market price of risk for different months and cities is suggested as it has a significant influence on the option prices.

The research in the paper is presented in the following way. The first part is an introduction to weather derivatives, their development history, relevance, and characteristics. The second one provides a summary of the previous studies made in the area, their contribution and limitations in regards to the topic. The third part of the current work presents the data and their characteristics. The fourth and the fifth parts are dedicated to methodology used in the analysis. The overall results and conclusions are explained in the penultimate and the last section, respectively.

¹ Stockholm, Gothenburg and Malmö are the first, the second and the third biggest cities in Sweden respectively

1. WEATHER DERIVATIVES: SHORT HISTORY, MARKET, AND TECHNICAL PARTICULARITIES

Weather derivative is a derivative security that allows an investor to hedge against the undesirable weather state (The Free Dictionary, 2012), i.e. it gives weather dependent industries a possibility to protect themselves against unpredictable natural changes and losses to the business those can cause.

The weather derivative's history starts back in 1997 when they were used in the U.S.A for the first time (the first weather derivative swap was negotiated between two power companies: Enron and Koch) (Hamisultane, 2007). The market under question developed rapidly, the derivatives starting being traded on a regulated market (Chicago Mercantile Exchange) in 1999. Currently the derivative securities having weather processes as the underlying traded on CME are derivatives on temperature, frost, hurricanes, snowfall, and rainfall (CME Group, 2012). By their type specification these are weather swaps, put/call options, and option collars (Campbell & Diebold, 2005).

The American market remains the biggest for this type of derivatives; however, they began to be more widely used in Europe as well. Thus, the temperature derivatives are traded in 18 US, 9 European, and 21 Japanese cities, (Benth, et al. 2007) as well as for Canadian and Australian cities. The European cities represented on CME are: Amsterdam, Barcelona, Paris, Rome, Madrid, Berlin, Essen, London, and Stockholm. The only currencies available for trading the weather derivatives are the American dollar for the US market, and the British pound for Europe. Until recently in the U.S.A. there have been made 3000 deals worth \$ 5.5 billion compared to 100 deals at a value of 30 million pounds in Europe. The main reason of smaller representativeness in Europe is lower liquidity (Alaton et al., 2002), thus, limiting the derivatives' utility to speculative purpose rather than effective hedging.

From the theoretical point of view weather derivatives have a tremendous utility, considerably minimizing the possible losses from weather variation. According to previous studies, one-seventh of the American market depends on the weather state (Challis, 1999; Hanley, 1999). As it was previously stated, there is a range of industries that are directly related to weather behaviour: entertainment, tourism, agriculture, energy, retail et cetera.

For a better understanding, a fully assembled for winter sports tourist complex is assumed. All the profits of the considered business strongly depend on whether there is or there is not snowy weather during the coming tourist season. Thus, in order to protect their profit, management can choose to go short in a weather derivative contract, with snow as an underlying. In case there is no snow, the complex would cover its losses from the derivative position, and vice versa in case of a snowy weather. A similar approach to dealing with unexpected weather conditions by using weather derivatives can be applied to the other mentioned sectors.

By their nature weather derivatives are very close to insurance contracts. However, there are some important differences that make the weather derivatives more attractive. Firstly, it is not necessary to document the damage. Secondly, it is possible to hedge even against the favourable state of the weather (Cambell & Diebold, 2005). Therefore, it is possible to obtain a pay-off in both unfavourable and favourable states.

The most developed and the most used weather derivatives are those with temperature as underlying. According to Cao and Wei (2004) temperature-related deals accounted for 80% of the weather derivatives turnover².

1.1 Weather hedging in the energy sector: temperature derivatives

Among the major highly temperature dependent sectors, as it was mentioned above, is the energy sector. There was found a strong correlation between the temperature levels and electricity consumption. Analysing these variables for the state of Illinois, U.S.A., Cao and Wei (2003) found the value of the R^2 being equal to 0.9416 after having regressed the monthly delivery of natural gas against the monthly average temperature for the region considered.

Basically, as it was explained above, the risk in the energy sector can be divided in two parts: price related risk and volume risk. Weather derivatives account for the volume related risk (thus supplying utility in hedging against low demand for electricity, if referring to the external environment of an energy business, and against low supply in case of companies developing alternative energy production, more specifically, related to wind farms production).

² Comparing to the 4,165 traded contracts in 2002, according to the CME volume reports, temperature derivatives achieved the limit of 18, 295 as a total for 2011 (HDD contracts) (see CME Group, 2012, official website).

Providing arguments for temperature hedging in the energy industry, the explanation comes from the temperature considered to maintain the thermal comfort for an average person. In the U.S.A. this temperature level is considered to be 65°F, or 18°C; the same assumption is applied for the European markets. However, determinants of the thermal comfort are related to both human body and environment's characteristics, such as humidity, atmospheric pressure, etc. (Auliciems & Szokolay, 2007), therefore, a deeper and more detailed research related to the choice of the optimal temperature level would be prudent, as these characteristics can vary depending on the region.

Thus, 18°C being considered as the comfort level, it is, consequently, assumed that a temperature level below 18°C will lead to heating of the dwelling, or other areas in use, and a temperature exceeding the critical value will make an individual use air conditioning.

The above mentioned logic stands at the foundation of the temperature based contracts: HDD, CDD, and CAT.

HDDs, coming from the Heating Degree Days, are characteristic for the winter periods (from November to March), and account for the cumulated days when temperature is below the critical level of 18°C – Heating Degree Days. The daily HDD is calculated as:

$$HDD_t \equiv \max \{18^\circ C - y, 0\}, \quad (1.1)$$

where y is the average observed temperature level (Cao & Wei, 2004), calculated as the mean value of the daily maximum and minimum:

$$y = \frac{T_i^{max} + T_i^{min}}{2} \quad (1.2)$$

CDD (Cooling Degree Days) counts the cumulated days when the temperature exceeded 18°C level, and is, respectively, calculated as:

$$CDD_t \equiv \max \{y - 18^\circ C, 0\}. \quad (1.3)$$

These contracts cover the summer season – the months from April to September. However, due to the fact that CDD contracts are not traded on CME for European cities, they are not the subject of investigation of the current paper.

The CAT index represents the accumulated average temperature over a specified period. On the Chicago Mercantile Exchange it is used for the summer season contracts for Europe and Canada (CME Group, 2012).

The main financial contracts for temperature are temperature futures, options and swaps. Temperature options generally have the same trading principals as any other option. Thus, there are two main types of contracts: put and call. The value of the pay-out depends on the strike level and the tick size – the amount of money that a holder of a call (put) option receives for each degree-day above (below) the strike level over the determined period (Alaton et al., 2002).

The main elements of a temperature option are: a) the underlying variable; b) the accumulation period; c) a specific weather station reporting temperature values for a specific city; d) the tick size; e) the strike; f) specification of the contract type (put/call); g) the maximum pay-out, if applicable (Cao & Wei, 2004; Alaton et al., 2002).

Let K denote the strike, α denote the tick and let the contract period consist of n days, then a pay-out formula of an HDD call option is

$$\mathcal{X} = \alpha \max \{ H_n - K, 0 \}, \quad (1.4)$$

where H_n is the number of HDDs for the chosen period. The pay-outs for analogous contracts such as HDD puts and CDD calls/puts are obtained using the same logic (Alaton et al., 2002).

Weather swaps suppose two parties, one paying a fixed price, and the other one paying a variable price. Weather swaps have just one earlier specified date, when they can be used. Therefore, they can be seen as a kind of weather forwards (Alaton et al., 2002).

On the biggest market for the weather derivatives, CME, weather derivatives are privately negotiated via voice broker as block trades and cleared through CME clearing. The minimal size of block trades is 20 contracts. In order to determine the value of an HDD contract, the HDD value is multiplied by 20£ (the tick size), the multiplier for each weather index point (CME Group, 2012). In the current paper, however, this multiplier is assumed to be equal to one in the sake of simplicity, as its value does not have a critical influence on the model specification.

$$\sum \text{HDD} \times 20 = \text{value of month's contract} \quad (1.5)$$

2. PREVIOUS RESEARCH

In a relatively short period since the introduction of weather derivatives a significant number of studies dedicated to studying underlying weather processes, constructing and pricing weather derivatives have been made.

Dornier and Querel (2000) study weather futures prices based on temperature indices using Ornstein-Uhlenbeck stochastic process with constant variance to forecast temperature behaviour in Chicago, U.S.A. Later papers take into consideration such temperature feature as variation in the variance.

Alaton et al. (2002) modify Ornstein-Uhlenbeck model to allow for monthly variation in the variance when modelling temperature evolution for Bromma Airport, Stockholm, Sweden. Seasonality, a positive assumed-to-be-linear trend in the data and its mean-reverting property are taken into account. The mean-reverting parameter is estimated using the martingale estimation functions method. Unique prices of weather contracts in an incomplete market are obtained using the market price of risk, which is assumed to be constant. Numerical examples of prices of some contracts are presented, using an approximation formula as well as Monte-Carlo simulations. Alaton et al. (2002) rely up to a certain extent on visual results to draw conclusions about data characteristics and acceptability of the model.

Extended Ornstein-Uhlenbeck model is further used by Zhu et al. (2012) for modelling daily temperature in a dry region of China (Jinan climate station) in order to price 'drought option' via stochastic simulation. The drought option price is estimated using such techniques as Historical Burn Analysis, Index Value Simulation, and Stochastic Simulation. It is found that the latter method produces results that have the lowest variance. The current paper will, therefore, use stochastic simulation in pricing weather derivatives.

Campbell and Diebold (2005) reveal conditional mean dynamics as well as conditional variance dynamics in daily average temperature and observe seasonality in the autocorrelation function for the squared residuals. The seasonal volatility component was modelled using Fourier series and the cyclical volatility was approximated using an approximated generalized autoregressive conditional heteroscedasticity (GARCH) process. Both seasonal and cyclical components are revealed to be significant. Moreover, the authors conclude that strong seasonal

volatility, which appears to be higher during winter months, indicates that correct pricing of weather derivatives may be crucially dependent on the season covered by the contract.

Mraoua and Bari (2005) suggest Vasicek mean-reverting model with mean-reverting stochastic volatility. Temperature swap contract is priced using the Euler approximation formula as well as Monte-Carlo simulations. Principal Component Analysis is used to fill the missing temperature data.

Benth et al. (2007) use continuous-time autoregressive model driven by a Wiener process with seasonal standard deviation for the temperature in Stockholm, Sweden, developing the fractional Ornstein-Uhlenbeck model used in Brody et al. (2002). The proposed model has number of lags, p equal to three. The weather derivatives traded at CME are analysed. An explicit futures price dynamics for the different futures are derived and the prices of options are analysed as well as their associated hedging strategies. The authors conclude that the seasonality feature plays a crucial role in the formation of prices, both on weather futures and options.

Davis (2001) proposed to use the 'marginal substitution value' technique to price temperature derivatives such as swaps and options based on HDD index.

Cao and Wei (2004) propose and implement a valuation framework for temperature derivatives and study the significance of the market price of weather risk by generalizing the Lucas model of 1978 (Lucas, 1978) to include the weather as another fundamental source of uncertainty in the economy. The reason why the same approach for determination of the market price of risk is not satisfactory for the current paper is explained later in the fifth section. Daily temperature is modelled by incorporating seasonal cycles and uneven variation throughout the year. It is also asked whether the market price of weather risk is a significant factor in valuation of weather derivatives. It is found that the risk premium can represent a significant part of the temperature derivative's price evaluated with risk aversion and aggregate dividend process parameters conforming to empirical reality. The revealed pattern is that the temperature variation is larger in the HDD season than in the CDD season.

Hardle and Cabrera (2011) argue that high order AR is enough to capture temperature process.

Platen and West (2005) price weather derivatives using a benchmark approach taking the world stock index as the numeraire such that all benchmarked derivative price processes are martingales. The fair price of particular weather derivatives (the Wine Producer example) are

derived using historical and Gaussian residuals. In order for a reader to create a more structured understanding Table 1.1 is created.

In the light of everything mentioned it is possible to distinguish several revealed features of temperature data that a successful model should take into account:

- The model should capture the seasonal cyclical patterns
- The daily variations should around average temperature
- The model should incorporate autoregressive property
- The extent of variation must be bigger in winter and smaller in summer
- Small warming trend (possibly, due to the global warming effect)

(Cao & Wei, 2004)

Table 1.1 Previous research concerning weather derivatives

Author(s)	Year	Data	Model	Pricing derivatives
Dornier & Querel	2000	Chicago, U.S.	Ornstein-Uhlenbeck stochastic process, constant variance	-
Davis	2001		marginal substitution value technique	swap, option
Alaton et al.	2002	Stockholm, Sweden	Ornstein-Uhlenbeck, monthly variation in the variance	HDD option
Campbell & Diebold	2005	U.S. cities	seasonal AR conditional heteroskedastic process	-
Mraoua & Bari	2005	Casablanca, Morocco	mean-reverting model with mean-reverting stochastic σ	swap
Benth et al.	2007	Stockholm, Sweden	continuous-time AR model driven by a Wiener process with seasonal σ	futures, option
Platen & West	2005	Sydney, Australia	benchmark approach	option

While taking into consideration the results obtained by the previous studies, the current paper mostly relies on the approach of Alaton et al. (2002) when modelling temperature and pricing derivatives. The presented model captures important temperature features, and is relatively easy to be used by practitioners. When valuing the weather derivatives, it takes into consideration the market price of risk, although with simplifications, assuming it being constant. The contribution of the current study is analysing the model in terms of the new data and speculation about the possible option values for the two aforementioned, currently untraded on the weather derivative market, Swedish cities that, however, have a full potential to become traded. Moreover, the option price sensitivity and the proposed model's accuracy are tested in regards to changes in the model's parameters.

3. DATA CHARACTERISTICS

Before proceeding to the methodology description and the model specification, a data analysis and argumentation for the chosen approach is presented.

3.1 Temperature data series

The temperature modelling is based on the mean daily temperature observations (average temperature) registered at the Bromma airport in Stockholm, and local meteorological stations in Malmö and Gothenburg (NOAA Satellite and Information Service, 2012). The analysed period consists of 35 years starting in January 1978 until April 2012 for Stockholm, and starting from January 2000 until April 2012 for Gothenburg and Malmö. For simplicity in calculations and results interpretation the 29th of February of every leap year was omitted.

Data was obtained in Fahrenheit degrees; therefore, a transformation in Celsius degrees was applied:

$$T_c = \frac{5}{9}(T_f - 32), \quad (3.1)$$

where T_c is the temperature in degrees Celsius, and T_f is the temperature in degrees Fahrenheit.

The first step in pricing the derivatives is the modelling of the underlying. In the analysed case the underlying is temperature, therefore, temperature modelling, i.e. finding the stochastic process that characterizes temperature variations for the regions of interest, is the aim.

Analysing the time series of historical temperature observations, an evident cyclical variation is detected, which is to be expected due to the seasonal patterns in temperature variations. Figure 3.1 presents the plot of the average temperature observations over the examined period. The X and Y-axis show dates and Celsius degrees respectively.

The graphical representation also indicates an observable trend in temperature over the considered period, though not significantly high. The sign of the trend mentioned, however, cannot be clearly identified from a graphical analysis, though, it is anticipated to be slightly positive, which could be explained by the global warming effect that has already been previously

mentioned in other studies (Alaton et al., 2002; Cao & Wei, 2004; Diamond, 2011). Alaton et al. (2002) also proposed the urban heating effect³ as a possible explanation.

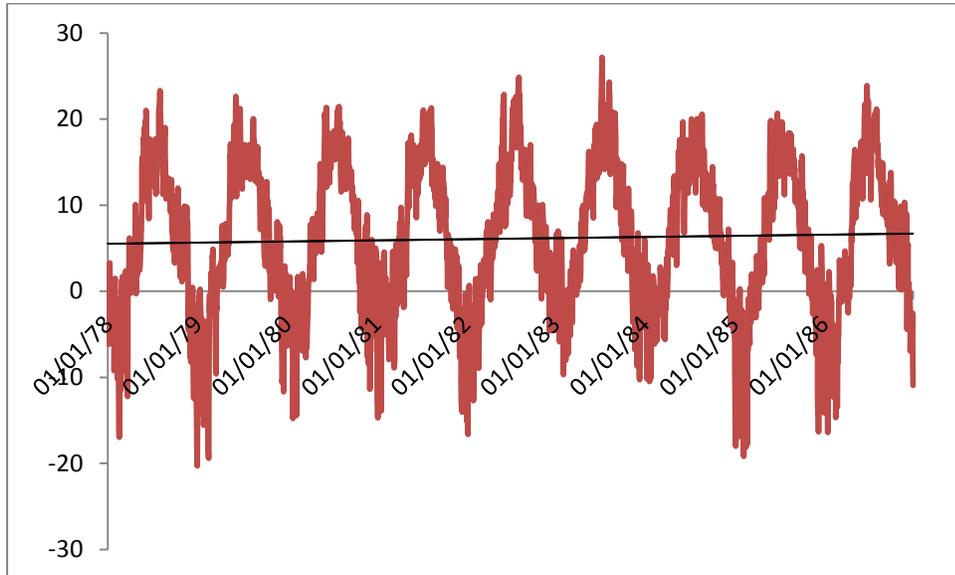


Figure 3.1 Daily mean temperatures. Stockholm, Sweden. 01.01.1978-31.12.1986, Celsius degrees

The aforementioned relationship's nature expressing the increasing daily temperatures was tested, thus, different trend specifications (linear, logarithmic and polynomial) were examined. The obtained figures did not show any significant differences between the mentioned specifications, therefore, a linear trend is assumed in the further analysis in the temperature modelling.

As it was previously mentioned, a high cyclical variation can be observed in Figure 3.1. The most obvious explanation of this feature of the data is the seasonality effect, which is a logical explanation, when taking into consideration that the studied data series represent temperature, which normally varies with seasons.

In order to express seasonality and the exact temperature characteristics, which tend to be accentuated in a certain period of the year, an analysis of the standard deviation behaviour of separate days depending on the months was implemented.

³ Urban areas are known to warm up due to increased area of thermal mass such as concrete buildings, pavements, etc. The increased thermal mass results in increasing temperatures with time since not all of the heat is released. This is known as the **urban heat island (UHI) effect** (Global Warming Science, 2012).

Thus, the standard deviation of each of the 30/31 days depending on the months over the whole considered period considered was calculated.

Figure 3.2 reflects the values of the standard deviations of the first days of each month for Stockholm's temperature. The X and Y-axis show dates and volatility values respectively. It can be observed that the winter period is characterized by higher volatility, while the summer months reflect a much lower volatility, the minimum point being achieved in September. A similar behaviour characterises other days of each month; the graphical representation, as well as the numerical results for day i of month j standard deviation, can be found in Appendix 1.

Cao and Wei (2004) had similar results when analysing the American market: they also obtained a higher volatility for the Heating Degree Days period, however, they registered the lowest standard deviation in July. This observation adds relevance to the studied topic, indicating different temperature behaviour depending on the geographical region; therefore, it may imply an altered approach to be necessary when analysing the European countries.

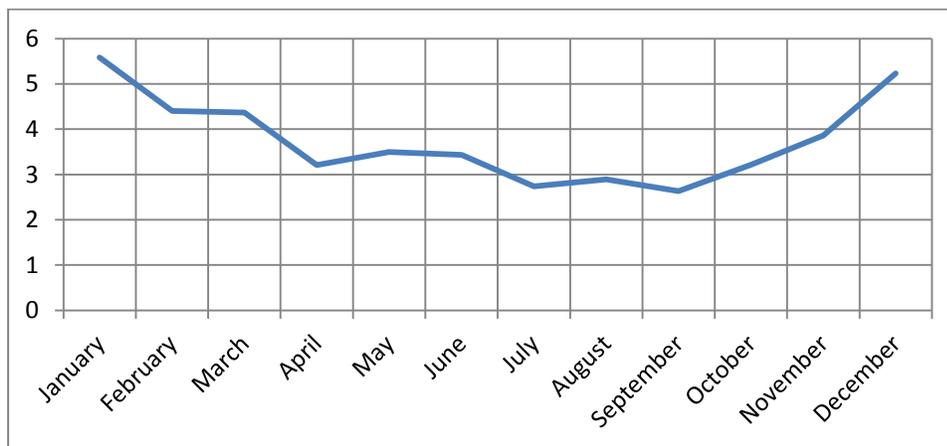


Figure 3.2 Standard deviation of day one over the months of the year, Stockholm, 01.01.1978-01.04.2012

In addition, an important observation related to the temperature variance is that it was detected to vary across but not within a month. This fact will be further taken into consideration when modelling temperature and pricing options.

Due to the seasonality patterns discussed earlier, in order to test the data for normality, the deseasonalized series were obtained. These values were tested for normal distribution. The results are reflected in Figure 3.3.

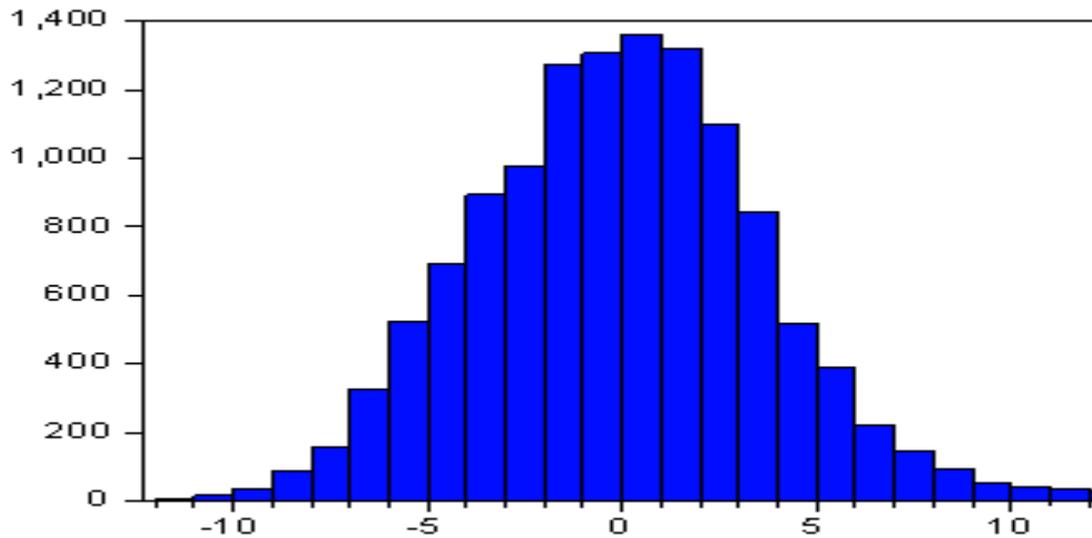


Figure 3.3 Deseasonalized temperature series, Stockholm, 01.01.1978-01.04.2012

Although the data do not show a strict normal distribution, visually their representation looks very close to the theoretical distribution considered. Therefore, later in the work the normal distribution assumption will be accepted.

3.2 HDD index related data

HDD index with its respective cumulated values were obtained from the official website of Chicago Mercantile Exchange (CME Group, 2012). Data frequency used for these series is daily, accounting for months of the years.

However, the period analysed is much shorter compared to the temperature related one due to the data unavailability. It starts from January 2007 and ends in April 2012. From the statistical point of view the period considered is, nevertheless, significant, consisting of 1884 observations.

4. MODELLING TEMPERATURE EVOLUTION

A particular feature of the weather derivatives is that their underlying (weather) is untradeable. Consequently, it is impossible to construct a self-financing strategy that could duplicate the underlying asset (Mraoua, 2005). In order to proceed to pricing the weather derivatives, it is needed to form a comprehensive opinion about the temperature dynamics throughout the year and to find a model to forecast temperature evolution. Therefore, the aim of this section is to find the stochastic process that describes the temperature behaviour (see Alaton et al., 2002).

It is expected to experience a strong influence of the temperature historical information. As it was mentioned before, Figure 3.1 suggests that the model should reflect the mean-reverting property of daily temperature. Additionally, a small positive trend is anticipated. A significant seasonal variation in the temperature should be captured as well. Moreover, due to the fact that temperatures are not deterministic, some stochastic noise should be added to the model.

4.1 Deterministic model for the mean temperature

Let t designate time measured in days, and $t = 1$ stand for the 1st of January, ... , and $t = 365$ mean the 31st of December (the 29th of February of leap years are omitted). Then seasonal variation could be modelled using the following sine-function (Alaton et al., 2002)

$$\sin(\theta t + \gamma), \quad (4.1)$$

where $\theta = 2\pi/365$. A phase angle γ is introduced, because the lowest and highest temperature values of the year do not coincide with January 1 and July 1 respectively. According to the previous observations, the trend is assumed to be linear. Consequently, the mean temperature T_t^m could be modelled as following:

$$T_t^m = \rho_1 + \rho_2 t + \rho_3 \sin(\theta t + \gamma). \quad (4.2)$$

The parameters $\rho_1, \rho_2, \rho_3, \gamma$ need to be estimated.

4.2 Stochastic noise

In order to make the model more accurate, stochastic noise has to be added to the deterministic part of it. As the normal distribution is assumed, the Wiener process is chosen to obtain the aim.

The Wiener process is a particular case of the Markov process with a zero mean, and variance equal to one (Hull, 2002). Over a small period of time δt the change δz can be expressed as:

$$\delta z = \epsilon \sqrt{\delta t}, \quad (4.3)$$

where ϵ is the normally distributed random variable ($\varphi(0,1)$).

The variable δz has a zero mean and the standard deviation equal to $\sqrt{\delta t}$, implying that z follows a Markov process.

Considering an increase of the variable z (which is representing the H_n in this case) over a longer period of time T , denoted as $[z(T) - z(0)]$, the mean and the standard deviation become:

$$\text{Mean of } [z(T) - z(0)] = 0 \quad (4.4)$$

$$\text{Variance of } [z(T) - z(0)] = N \delta t = T \quad (4.5)$$

$$\text{Standard deviation of } [z(T) - z(0)] = \sqrt{T} \quad (4.6)$$

The inter-period variation $[z(T)-z(0)]$ was assumed to be the sum of smaller variations, δt , thus

$$[z(T)-z(0)] = \sum_{i=1}^N \epsilon_i \sqrt{\delta t} \quad (4.7)$$

where ϵ_i are uncorrelated normally distributed ($\varphi(0,1)$) random drawings from $i=1...N$ (Hull, 2002).

Despite the fact that the quadratic variation is not constant within a year, it is approximately constant within each month (see Figure 3.2). Consequently,

$$\sigma_t = \begin{cases} \sigma_1 \text{ in January} \\ \dots \\ \sigma_{12} \text{ in December} \end{cases} \quad (4.8)$$

and σ_i is a positive constant, $\forall i$. Therefore, the stochastic noise is represented by $(\sigma_t W_t, t \geq 0)$.

4.3 Mean-reversion

Common sense suggests that temperature does not constantly increase each day for a long period and tends to return to its mean level. Therefore, the model for describing temperature evolution should incorporate the mean-reverting property. As it is mentioned above, the daily temperature differences are assumed to be normally distributed. Consequently, the temperature dynamics would follow a Brownian Motion. A Vasicek process with mean-reversion is used to model the temperature evolution. Temperature can be modelled by a stochastic process solution of the following stochastic differential equation (see Alaton et al., 2002)

$$dT_t = \alpha(T_t^m - T_t)dt + \sigma_t dW_t, \quad (4.9)$$

where $\alpha \in \mathbb{R}$ indicates the speed of mean-reversion; T_t^m is the mean temperature to which the process reverts; σ_t is the volatility; dW_t is the Wiener process.

The solution to (4.9) is an Ornstein-Uhlenbeck process. However, Doenier & Queruel (2000) proved that (4.9) does not return to T_t^m in a long-term period. This problem can be mitigated by adding the term

$$\frac{dT_t^m}{dt} = \rho_2 + \theta\rho_3 \cos(\theta t + \gamma) \quad (4.10)$$

to the drift in (4.9). This term will adjust to the drift so that the long run mean of the stochastic differential equation will be T_t^m .

Starting at $T_s = l$, the following model for the temperature is obtained:

$$dT_t = \left[\frac{dT_t^m}{dt} + \alpha(T_t^m - T_t) \right] dt + \sigma_t dW_t, \quad t > s, \quad (4.11)$$

and its solution is

$$T_t = (l - T_s^m)e^{-\alpha(t-s)} + T_t^m + \int_s^t e^{-\alpha(t-\tau)} \sigma_\tau dW_\tau, \quad (4.12)$$

The equation 4.12 shows that the temperature at future time, T_t , equals to the sum of

- the deseasonalized temperature of today, s , where today's deviation from the normal level is decaying exponentially;
- the future mean temperature, T_t^m , that is defined by (4.2);

- the stochastic noise that might accumulate between current and future dates (s and t respectively) and has the expected value of zero.

4.4 Estimation of the parameters

In the estimation process there was used the temperature data from Bromma Airport, Stockholm, Sweden, from 1.01.1978 to 1.04.2012 and from January 2000 to April 2012 from for both Malmö and Gothenburg.

4.4.1 Estimation of the mean temperature parameters

T_t^m from (4.2) can be expressed as :

$$T_t^m = \rho_1 + \rho_2 t + \rho_3 [\sin(\theta t) \cos(\gamma) + \cos(\theta t) \sin(\gamma)]. \quad (4.13)$$

The parameters can be rewritten

$$\begin{cases} \rho_1 = \beta_1 \\ \rho_2 = \beta_2 \\ \rho_3 = \sqrt{\beta_3^2 + \beta_4^2} \\ \gamma = \arctan\left(\frac{\beta_3}{\beta_4}\right) - \pi \end{cases} \quad (4.14)$$

and then T_t^m follows

$$T_t^m = \beta_1 + \beta_2 t + \beta_3 \sin(\theta t) + \beta_4(\theta t). \quad (4.15)$$

The method of Ordinary Least Squares (OLS) was applied to the series of historical data. After analysing the historical temperature data for the three chosen Swedish cities, the obtained numerical values were inserted in (4.2) suggesting the following function for the mean temperature in Stockholm, Gothenburg and Malmö, respectively.

$$T_t^m(\text{Stockholm}) = 7.33 - 1.44 * 10^{-3}t - 10.51 * \sin(0.017 * t + 1.21) \quad (4.16)$$

$$T_t^m(\text{Gothenburg}) = 7.59 - 0.66 * 10^{-3}t - 9.16 * \sin(0.017 * t + 1.18) \quad (4.17)$$

$$T_t^m(\text{Malmö}) = 8.50 - 0.93 * 10^{-3}t - 8.77 * \sin(0.017 * t + 1.12) \quad (4.18)$$

The regressions ran in Eviews are shown in Appendix 2. The obtained R-squared values indicate that in all the three cases relatively high proportion (about 80%) of the mean temperature is explained by the proposed model. The amplitude of the sine-function in the Stockholm's equation is approximately -10° C. It could be interpreted so that the difference between a usual winter and summer day is about 20° C. Malmö reflects the least amplitude among the three cities and the highest starting temperature. It is possible to conclude that the more southern a city is situated, the warmer and the milder climate it has.

Unlike the other estimates, the trend is statistically insignificant for Malmö and Gothenburg (p-values exceeding 0.18 and 0.38 respectively), nonetheless, as expected. The influence of the trend is supposed to be noticeable in the long term perspective. Surprisingly, however, it turned out to be negative, not positive, for all the investigated cities, which implicates that in the long-run the mean temperature in the analysed regions was slightly decreasing. Yet this discovery does not necessarily contradict the global warming theory as the scrutinised period only includes a part of the twentieth and a part of the twenty first centuries and does not take into account earlier epochs. Despite the statistical insignificance of the trend parameter, it was decided not to exclude it from the model due to its contribution to the increase of the model's accuracy in the long-term perspective.

4.4.2 Estimation of the volatility

As it was stated above, this paper shares the assumption of Alaton et al. (2002) that the temperature volatility is constant within a certain month, however, varies throughout the year. Each month k includes N_k days. The outcomes of the observed temperatures during the month k are denoted by T_j , $j = 1, \dots, N_k$. The volatility estimator is based on the quadratic variation of T_t .

$$\hat{\sigma}_k^2 = \frac{1}{N_k-2} \sum_{j=1}^{N_k} (T_j - \alpha T_{j-1}^m - (1 - \alpha)T_{j-1})^2 \quad (4.19)$$

However, in order to obtain the sigma values, the alpha parameter estimation is required and, therefore, described below.

4.4.3 Estimation of the speed of mean-reversion

Alaton et al. (2002) argue that an unbiased and efficient estimator of α , $\hat{\alpha}_n$, is the zero of the following martingale function

$$G_n(\alpha) = \sum_{i=1}^n \frac{\dot{b}(T_{i-1}; \alpha)}{\sigma_{i-1}^2} (T_i - E[T_i | T_{i-1}]), \quad (4.20)$$

where $\dot{b}(T_i; \alpha)$ is the derivative of the drift term

$$b(T_t; \alpha) = \frac{dT_t^m}{dt} + \alpha(T_t^m - T_t) \quad (4.21)$$

with respect to α :

$$\dot{b}(T_i; \alpha) = \partial b / \partial \alpha \quad (4.22)$$

It follows from (4.12) for $\forall t \geq s$ that

$$T_t = (T_s - T_s^m)e^{-\alpha(t-s)} + T_t^m + \int_s^t e^{-\alpha(t-\tau)} \sigma_\tau dW_\tau, \quad (4.23)$$

that leads to

$$E[T_i | T_{i-1}] = (T_{i-1} - T_{i-1}^m)e^{-\alpha} + T_i^m, \quad (4.24)$$

where T_t^m is defined by (4.2).

Consequently,

$$G_n(\alpha) = \sum_{i=1}^n \frac{T_{i-1}^m - T_{i-1}}{\sigma_{i-1}^2} (T_i - [T_{i-1} - T_{i-1}^m]e^{-\alpha} - T_i^m) \quad (4.25)$$

Solving (4.25) presents the unique zero of the aforementioned martingale function (4.20)

$$\hat{\alpha}_n = -\log\left(\frac{\sum_{i=1}^n Y_{i-1} [T_i - T_i^m]}{\sum_{i=1}^n Y_{i-1} [T_{i-1} - T_{i-1}^m]}\right), \quad (4.26)$$

where

$$Y_{i-1} \equiv \frac{T_{i-1}^m - T_{i-1}}{\sigma_{i-1}^2}, \quad i = 1, 2, \dots, n. \quad (4.27)$$

Inserting numerical values into equation (4.19), estimations of σ were obtained for different months and cities as well as respective speeds of mean-reversion. The calculations were performed using the Solver function in Excel. The results are presented in Table 4.1 More visually intelligible presentation of volatility behaviour through time is presented in the Appendix 3.

Earlier studies (Cao & Wei 2004) discovered higher temperature volatility during winter periods. The results derived by the current study confirm it. Moreover, it could be expected that more northern (consequently, colder) cities would have higher volatility. The mean volatility of Stockholm is, as anticipated, slightly higher than those of Gothenburg and Malmö. The latter has the lowest volatility among the three cities and the highest speed of mean-reversion. This fact was also awaited, because, as it was mentioned earlier, Malmö temperatures have the lowest amplitude, therefore, this city has the mildest climate and the lowest temperature risk among the other studied cities.

Table 4.1 Estimates of the volatility and the speed of mean-reversion for the temperature in Stockholm, Gothenburg and Malmö

Month	Stockholm		Gothenburg		Malmö	
	sigma	alpha	sigma	alpha	sigma	alpha
January	3.28	0.21	2.70	0.20	2.65	0.26
February	2.97	0.18	2.43	0.13	2.31	0.21
March	2.36	0.21	2.16	0.20	1.95	0.17
April	2.29	0.25	2.16	0.31	1.89	0.25
May	2.40	0.25	2.17	0.21	1.85	0.32
June	2.20	0.28	2.05	0.18	1.83	0.22
July	1.90	0.23	1.90	0.36	1.69	0.22
August	1.82	0.22	1.89	0.46	1.83	0.43
September	2.01	0.27	1.89	0.31	1.89	0.31
October	2.33	0.27	2.02	0.15	2.08	0.17
November	2.70	0.28	2.57	0.25	2.44	0.27
December	3.40	0.28	2.80	0.20	2.52	0.26
Mean	2.47	0.24	2.23	0.25	2.08	0.26

After having obtained all the parameters it is possible to simulate trajectories of Ornstein-Uhlenbeck process. Figure 4.2 reflects the obtained simulation results, where the X-axis and Y-axis show days of the year and Celsius degrees respectively.

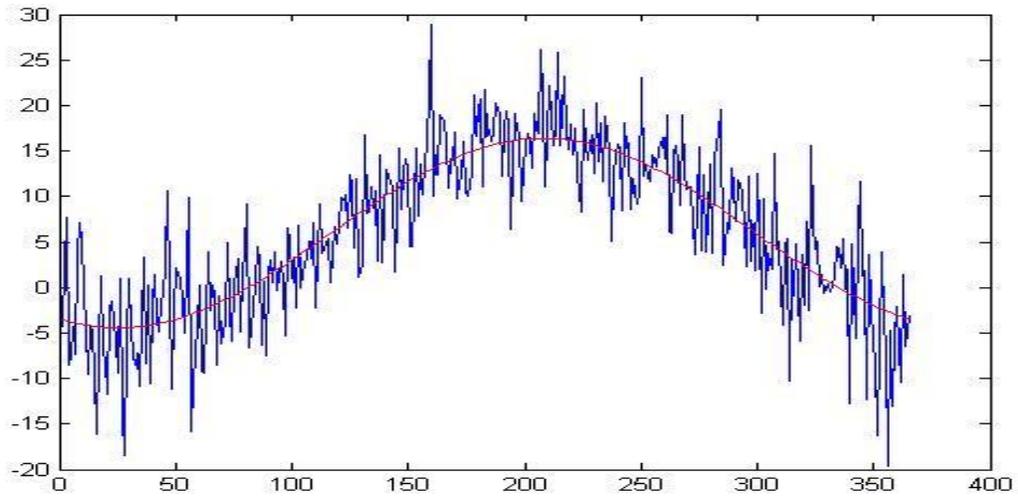


Figure 4.1 One trajectory of Ornstein-Uhlenbeck process to be used in temperature modelling, one-year length

It can be observed that the temperature values obtained by simulations (Figure 4.1) have a very similar distribution to the real-time temperature (Figure 3.1).

5. PRICING TEMPERATURE DERIVATIVES

Currently there is no formula, widely used and accepted by practitioners, for pricing the temperature derivatives. Therefore, the task of determining the satisfying pricing method is of a high relevance. Due to the fact that the daily temperature behaviour is a stochastic process examined in the fourth section of the current paper, the simulated daily temperatures are the base of the pricing of the temperature derivatives. Two methods are under this paper's scrutiny in order to price weather derivatives: an analytical and numerical one.

Weather derivatives market is not liquid enough (though speedily developing and expanding). Furthermore, it is not complete, because the underlying is not tradable. That is the reason why the market price of risk λ should be considered when valuing the weather contracts. Cao and Wei (2004) show that the market price associated with the temperature is significant and represents a considerable portion of the derivative's price. They also demonstrate that it varies over time, and the assumption of it being constant does not have an empirical argumentation. However, a time varying and consumption dependent market price of risk λ cannot be considered in the current study. The main reason is that Cao and Wei (2004) use the aggregate dividend in λ determination, i.e. the aggregate consumption for the U.S.A. Although it is prudent to use it when analysing the US market, it would not be properly motivated to use the US consumption for analysing the Swedish market or to combine Swedish consumption with derivatives traded on the US market in US dollars for the analysis. When weather derivatives are introduced on Swedish commodity market, it will be possible, though, to study market price of risk for weather derivatives for Sweden.

Consequently, it is assumed in this paper that the market price of risk is constant, despite its actual variation in time and probable changes in sign. Thus, the proposed model is further from the reality, however, less cumbersome and more easily applied in practice. Additionally, in equilibrium the market price of risk is anticipated to be common for regions with homogenous weather risk. Ergo this paper assumes equal market price of risk for Stockholm, Malmö and Gothenburg when pricing and proposing possible temperature derivatives.

Moreover, it is anticipated that there is a given risk-free asset with a constant interest rate and a contract that for each degree Celsius gives a pay-off of one unit currency (1 SEK) (see

Alaton et al., 2002). Under the martingale measure \mathbb{Q} , characterized by the market price of risk λ , the price process (T_t) can be specified:

$$dT_t = \left[\frac{dT_t^m}{dt} + \alpha(T_t^m - T_t) - \lambda \sigma_t \right] dt + \sigma_t dV_t, \quad (5.1)$$

where $(V_t, t \geq 0)$ is a \mathbb{Q} -Wiener process.

The derivative contract is expressed as the present value of the expected future pay-offs (prices), therefore, it can be expressed as the discounted prices under the martingale measure \mathbb{Q} , (Alaton et al., 2002). The first step in model specification is, therefore, the calculation of the expected value and variance of T_t .

Due to the fact that Girsanov process changes just the drift term, the variance of T_t is the same for both measures.

$$Var[T_t | \mathcal{F}_s] = \int_s^t \sigma_u^2 e^{-2\alpha(t-u)} du \quad (5.2)$$

Additionally, following from equation (4.12) the expected value is equal to:

$$E^{\mathbb{P}}[T_t | \mathcal{F}_s] = (T_s - T_s^m)e^{-\alpha(t-s)} + T_t^m, \quad (5.3)$$

dropping out the drift term $\int_s^t e^{-\alpha(t-\tau)} \sigma_\tau dW_\tau$.

Accounting for 5.1 though, the expression becomes:

$$E^{\mathbb{Q}}[T_t | \mathcal{F}_s] = E^{\mathbb{P}}[T_t | \mathcal{F}_s] - \int_s^t \lambda \sigma_u e^{-\alpha(t-u)} du \quad (5.4)$$

For the integrals with a constant volatility (single months) it is obtained:

$$E^{\mathbb{Q}}[T_t | \mathcal{F}_s] = E^{\mathbb{P}}[T_t | \mathcal{F}_s] - \lambda \frac{\sigma_i}{\alpha} (1 - e^{-\alpha(t-s)}) \quad (5.5)$$

Respectively, the variance is:

$$Var[T_t | \mathcal{F}_s] = \frac{\sigma_i^2}{2\alpha} (1 - e^{-2\alpha(t-s)}). \quad (5.6)$$

The covariance of the temperature between two days is calculated for the later use, for $0 \leq s \leq t \leq u$:

$$Cov[T_t, T_u | \mathcal{F}_s] = e^{-\alpha(u-t)} Var[T_t | \mathcal{F}_s] \quad (5.7)$$

It is denoted that t_1 and t_n are the first and the last days of a month. The process starts at some time s , which is in the month before interval $[t_1, t_n]$. Thus, in order to compute the variance and the expected value of T_t , the integrals in (5.2) and (5.4) are split in two integrals where σ is constant and the following variance and the expected value are obtained:

$$E^{\mathbb{Q}}[T_t|\mathcal{F}_s] = E^{\mathbb{P}}[T_t|\mathcal{F}_s] - \frac{\lambda}{\alpha} (\sigma_i - \sigma_j)(e^{-\alpha(t-t_1)}) + \frac{\lambda\sigma_i}{\alpha} e^{-\alpha(t-s)} - \frac{\lambda\sigma_j}{\alpha} \quad (5.8)$$

$$Var[T_t|\mathcal{F}_s] = \frac{1}{2\alpha} (\sigma_i^2 - \sigma_j^2) e^{-2\alpha(t-t_1)} - \frac{\sigma_i^2}{2\alpha} e^{-2\alpha(t-s)} + \frac{\sigma_j^2}{2\alpha} \quad (5.9)$$

The equations (5.8) and (5.9) calculate the discounted expected value and the variance of T_t under martingale measure \mathbb{Q} for two periods with different volatilities and can be generalized in order to obtain those for larger intervals. However, due to the fact that the current paper prices options within one-month frame, it uses the formulas (5.5) and (5.6) for further calculations.

5.1 Pricing a Heating Degree Day (HDD) option

As it was claimed earlier in the first section, usually, temperature derivatives are based on heating degree days (HDD) and cooling degree days (CDD). The aim of the current section is to reflect how a standard HDD option is priced. The pay-off of the standard HDD call (see section 1.1) is as follows

$$X = \alpha \max \{ H_n - K, 0 \}, \quad (5.10)$$

where $\alpha = 1$ SEK/HDD and

$$H_n = \sum_{i=1}^n \max \{ 18 - T_{t_i}, 0 \}. \quad (5.11)$$

If the underlying process is supposed to be log-normally distributed, there is no exact analytic formula to price such an option that is widely used. The analysed process is, however, assumed to be normally distributed. The obstacle to finding a proper pricing formula is a maximum function, therefore, an approximation is made. It is known that under Wiener process \mathbb{Q} conditional on information at time s ,

$$T_t \sim N(\mu_t, v_t), \quad (5.12)$$

where μ_t and v_t are defined via (5.8) and (5.9) respectively.

The period of one month prediction was chosen for option pricing, according to the specific standards of the option contracts traded on CME (CME Group, 2012). HDD option contracts are traded for seven consecutive months of the winter season, thus the strike level indicated in them represents the cumulative heating degree days for the specified month. Thus, for example, for November (the first traded month) the prediction period considered is 30 days, starting from the value of 0, considered to be the respective value of the cumulated heating degree days at the beginning of the contract period.

The pay-out of a weather option is determined by the accumulation of HDDs during a chosen period. In Sweden during winter the probability of $\max\{18 - T_{t_i}, 0\} = 0$ is insignificant, consequently, for the easiness of the distribution determination, it could be stated that

$$H_n = 18n - \sum_{i=1}^n T_{t_i}. \quad (5.13)$$

(see Alaton et al., 2002). Due to the fact that it is known that T_{t_i} , $i = 1, \dots, n$ are the samples of a Gaussian Ornstein-Uhlenbeck process, the vector $(T_{t_1}, T_{t_2}, \dots, T_{t_n})$ is Gaussian. The sum in (5.13) is a linear combination of the elements in the abovementioned vector, therefore, H_n is also Gaussian. New structure of H_n allows calculation of the first and the second moments. For $t < t_1$,

$$E^{\mathbb{Q}}[H_n | \mathcal{F}_t] = E^{\mathbb{Q}}[18n - \sum_{i=1}^n T_{t_i} | \mathcal{F}_t] = 18n - \sum_{i=1}^n E^{\mathbb{Q}}[T_{t_i} | \mathcal{F}_t] \quad (5.14)$$

and

$$Var[H_n | \mathcal{F}_t] = \sum_{i=1}^n Var[T_{t_i} | \mathcal{F}_t] + 2 \sum_{i < j} Cov[T_{t_i}, T_{t_j} | \mathcal{F}_t] \quad (5.15)$$

$E^{\mathbb{Q}}[T_{t_i} | \mathcal{F}_t]$ used in (5.14), and $Var[T_{t_i} | \mathcal{F}_t]$ and $Cov[T_{t_i} | \mathcal{F}_t]$ used in (5.15) are calculated by the equations (5.5), (5.6) and (5.7) respectively. Later, after the necessary calculations (see Alaton et al. 2002), it was obtained:

$$E^{\mathbb{Q}}[H_n | \mathcal{F}_t] = \eta_n \quad \text{and} \quad Var[H_n | \mathcal{F}_t] = \sigma_n^2 \quad (5.16)$$

Therefore, H_n is $N(\eta_n, \sigma_n^2)$ distributed. The equation (5.16) follows the same logic as (5.12). Consequently, the price of interest at $t < t_1$ (5.10) is

$$c(t) = e^{-r(t_n - t)} E^{\mathbb{Q}}[\max\{H_n - K, 0\} | \mathcal{F}_t]$$

$$\begin{aligned}
&= e^{-r(t_n-t)} \int_K^{\infty} (x - K) f_{H_n}(x) dx \\
&= e^{-r(t_n-t)} \left((\eta_n - K) \Phi(-\alpha_n) + \frac{\sigma_n}{\sqrt{2\pi}} e^{-\frac{\alpha_n^2}{2}} \right), \tag{5.17}
\end{aligned}$$

where $\alpha_n = (K - \eta_n) / \sigma_n$ and Φ stands for the cumulative distribution function for the standard normal distribution (5.18).

$$\Phi = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{-\alpha_n} e^{-\frac{(-\alpha_n-\mu)^2}{2\sigma^2}} d\alpha_n \tag{5.18}$$

Similarly, a formula for an HDD put option can be derived. It is known that

$$Y = \max \{ K - H_n, 0 \}. \tag{5.19}$$

It follows that the price is

$$\begin{aligned}
p(t) &= e^{-r(t_n-t)} E^{\mathbb{Q}} [\max\{ K - H_n, 0 \} | \mathcal{F}_t] \\
&= e^{-r(t_n-t)} \int_0^K (K - x) f_{H_n}(x) dx \\
&= e^{-r(t_n-t)} \left((K - \eta_n) \left(\Phi(-\alpha_n) - \Phi \left[-\frac{\eta_n}{\sigma_n} \right] \right) + \frac{\sigma_n}{\sqrt{2\pi}} \left(e^{-\frac{\alpha_n^2}{2}} - e^{-1/2 \left(\frac{\eta_n}{\sigma_n} \right)^2} \right) \right). \tag{5.20}
\end{aligned}$$

The obtained formulas for call and put option pricing (5.17) and (5.20) respectively, however, are relevant only for the heating season (usually from November to March). Applying these formulas during the cooling season would demand additional restrictions, as the probability of $\max \{ 18 - T_t, 0 \} = 0$ is significantly high (see Figure 5.1).

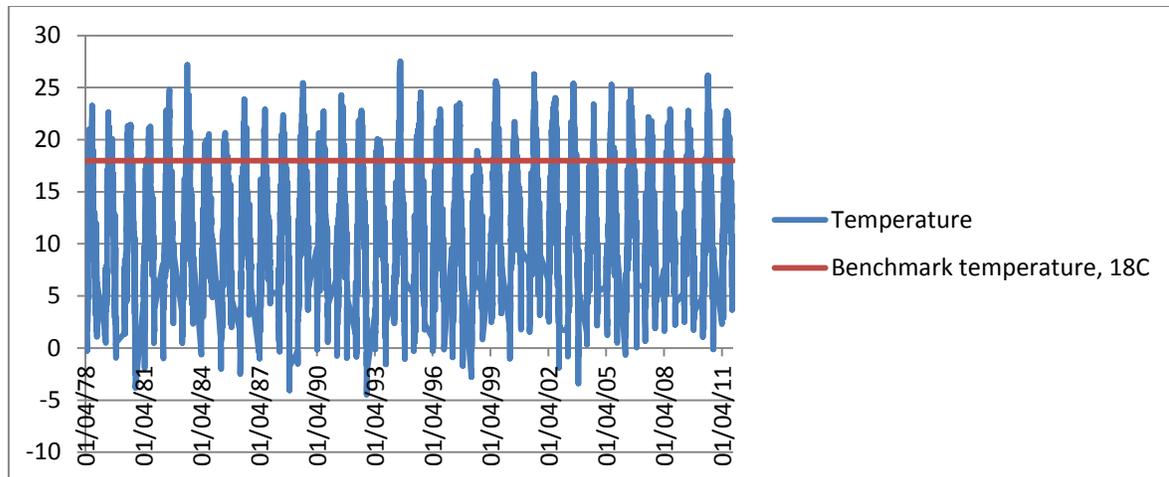


Figure 5.1 Historical temperatures for the summer season in comparison to the reference temperature of 18°C

This paper does not provide an explicit formula for a capped option. Despite the fact that regularly an option has a cap of a maximum pay-out to mitigate the risk of extreme weather conditions, such an option could be constructed from two options without an upper pay-out limit. A strategy with a long position in an option with a lower strike level and a short position in an option with a higher strike level provides a pay-out function of a capped option.

The call option price consists of two parts, representing the intrinsic value, which is the actual pay-out of the derivative, $\max(K - H_n; 0)$, and the time value expressed as the premium for waiting the period of the contract, which is e^{-rt} , where r is the risk-free rate, and t is the contract period. Based on the main components, characteristic for a simple vanilla option, the option's price is supposed to be sensitive to the variation of one of the above mentioned components. Table 5.1 presents the common reaction of the call prices to the variation in one of the critical components.

These influences are checked later through a sensitivity analysis. Thus, it is expected, for example, that for months with higher volatility the option prices are higher, as well as when obtaining a higher value for the cumulated heating degree days (the underlying considered, H_n) in the case of a call option or when raising the risk-free rate.

Table 5.1 Summary of the effect on the price of a stock option of increasing one variable while keeping all other fixed.

Variable	European Call	European Put	American Call	American Put
Current underlying's price	+	-	+	-
Strike price	-	+	-	+
Time to expiration	?	?	+	+
Volatility	+	+	+	+
Risk-free rate	+	-	+	-

Source: Hull (2002) "Options, Futures and other Derivatives"

Additionally, Hull (2002, p. 289) claims that options written on the futures rather than on spot tend to have higher prices. This is explained by the longer settlement period for the futures, thus, implying a higher cost characterising the time value mentioned earlier. This situation is, however, not expected to be observed in the case of temperature futures, due to the impossibility to store the underlying, thus, transforming them automatically in spot prices, just at a future time.

5.2 Monte-Carlo simulations

The method of Monte-Carlo simulations is used to calculate numerically the expected pay-off of a weather option, i.e. the value $E[g(X(t))]$, where X is a solution of a chosen stochastic differential equation and g is a chosen function. This method does not require any assumptions on the historical data, which offers high flexibility when pricing the contracts. The approximation follows from

$$E[g(X(t))] \approx \frac{1}{N} \sum_{i=1}^N g(\bar{X}(t, \omega_i)), \quad (5.21),$$

where \bar{X} is an approximation of X and it has to be used due to the unavailability of the exact solution for X (Alaton et al., 2002). A significant number of trajectories of the process are simulated and the expected value with the arithmetic average is approximated. One possibility to simulate the temperature behaviour for a chosen period of time is to start the simulation today and use today's observed value as initial one. The other way is to start the simulation in future time near the first day of the period of interest. The former way could be used in order to price

contracts that start relatively soon from the current date. The latter approach is more convenient for pricing the contracts starting long time ahead. The intuition behind it is that today's temperature is usually insignificantly affected by its high-order lags. The method is applied via MatLab software.

5.3 Incorporation of weather forecasts

The pricing mechanisms described earlier did not take into account neither short-term nor long-term meteorological forecasts. However, it is possible to alter the mean temperature formula coefficients in order to reflect real-life weather expectations and make the results more accurate. Meteorologists are able to give long-term forecasts providing information whether the anticipated temperatures are going to be higher or lower than normal. In the former case, it is possible to increase the parameter ρ_1 of the equation (4.13) that would result in the rise of the mean temperature and, consequently, in decrease of the value H_n for the period (Alaton et al., 2002). Furthermore, it is possible to change volatility σ , or the amplitude ρ_3 .

The section 6.1 presents numerical examples of the option price sensitivity to the changes in the mean parameter ρ_1 .

6. RESULTS

The final step of the given research is the actual implementation of the described methodology. Therefore, call options written on temperature futures were priced using the two methods: the approximation formula, which takes into consideration all the assumptions made, and Monte-Carlo simulations, which is to represent the actual call prices. After running a large number of simulations, the obtained value is supposed to convert eventually to its real level. In the current case a number of 40 000 simulations were conducted.

As it was stated earlier, Monte-Carlo simulations of a call price are based on the temperature simulations that are the underlying in the simulated call prices. Thus, the main components of the option are yet to be identified in order to determine the pay-off and the respective derivative's price. These are: the risk-free rate, the strike level, the contract period, and the starting point in the predictions.

Despite the fact that this work focuses on the Swedish market, it was decided to consider German 10-year bond yield as a proxy for the risk-free rate. The main reason of this choice was the high credibility and liquidity of the German market, which can be seen as representative for the European area. Thus, when considering the German bonds as the risk premium associated with the time factor of the futures and option contracts (Flasza et al., 2011), this alternative has a theoretical argumentation (Koller et al., 2010).

In the strike level determination, it was decided not to assume it is as a constant value for the contracts for the considered periods, since there are significant differences between the cumulated monthly HDDs over the seasons. Conventionally, the strike level of an option is set between zero and a standard deviation above (in case of a call option) or below (in case of a put option) the expected index value (Kung, 2007). Therefore, it was chosen to refer to historical average of the cumulated HDDs for the period of interest.

One month was chosen as a contract maturity, in order to match the market reality, considering the fact that one-month contracts are traded on the CME. Hence, the first days of each month were chosen as the starting point in the pricing process. The prices of the HDD calls for seven months of the winter period were calculated.

In order to simulate temperature behaviour under the risk neutral measure \mathbb{Q} , the market price of risk λ has to be determined. The estimate of λ can be found through studying market

prices of actual contracts and finding what value of λ used in the model results in the most plausible derivative prices. Yet temperature derivatives market for Swedish cities is underdeveloped. That is the reason why the benchmark derivative proposed by Alaton et al. (2002) was used to determine the market price of risk. The benchmark derivative was an HDD call option for Stockholm, February and its corresponding market price of risk equalled 0.08.

Due to the fact that the aforementioned benchmark derivative is priced in the Swedish currency, the options studied in the current paper are priced in SEK as well.

Table 6.1 represents the results for two of the considered months for Stockholm obtained by using the approximation formula (P_A) and by applying the Monte-Carlo simulation (P_{MC}).

Table 6.1 Call prices and contracts description, Stockholm, 2011

Stockholm							
Month	Strike	Index	Type	P_{MC}	P_A	Difference	λ
January	602	HDD	Call	80.00	78.27	2.17%	0.08
February	574	HDD	Call	21.00	20.90	0.47%	0.08

The results obtained by the simulations and the approximation formula were found to be slightly different. The highest precision of the approximation formula was detected for two winter months (January and February), while the differences between the results given by the Monte-Carlo simulations and the approximation formula tended to increase for the autumn and the spring periods.

This observation, however, does not necessarily indicate implausibility of the analysed approximation formula. The chosen market price of risk (λ) could be the reason of unrealistically high prices obtained for those periods. λ parameter was assumed to be equal to 0.08, referring to the benchmark derivative. However, according to the previous studies, it is not constant, not always positive, significant, and contract specific (Cao & Wei. 2004; Hardle & Cabrera. 2011). Therefore, there is a high probability of the value 0.08 being dissatisfactory market price of risk for spring and autumn which may cause doubtfulness of the results for the respective months.

Previous studies give an incentive to assume negative price of market risk during several spring and autumn months. For example, Hardle and Cabrera (2011) found that the market price of risk for the winter months is considerably higher than for the warmer seasons, moreover, a

negative market price of risk was registered. Therefore, slightly negative market price of risk for March and November was tested and produced more probable option prices.

This leads to the conclusion and a proposal for a further research accounting for a season-dependending and contract specific market price of risk and risk premium.

Due to lack of data it was impossible to elaborate the current research taking into consideration non-constant market price of risk. However, due to the fact that in the recent years a significant improvement in the European weather derivatives market was observed, it would be possible to conduct such a study in the near future.

6.1 Sensitivity analysis

In addition to the aforementioned calculations, a sensitivity analysis was conducted in order to test the theoretical background described earlier (Hull, 2002) and its applicability to the weather derivatives. The sensitivity analysis was implemented in terms of an HDD call option for January 2011 for Stockholm.

Firstly, the price sensitivity and the approximation accuracy sensitivity to the changes in the strike level were checked. The strike was changed in the array from 500 to 640 degrees with the step equal to 20. The risk-free rate was taken constant and equalled to 1.53%, which is close to the current level of the German ten-year bond yield, an earlier mentioned proxy for the risk-free rate. It was noticed that a strike rise by 20 degrees corresponded to a decrease in the price by roughly 20 SEK. The results are consistent with the theoretical background stating that the price of an option is considered to be the present value of the expected pay-off (the amount by which the underlying temperature level exceeds the strike), and, therefore, it is anticipated that the call option becomes less valuable when the strike increases (Hull, 2002).

The precision of the approximation formula rose with the strike approaching its most proper value, 602 degrees, at which point the minimum difference between the results of the two pricing methods was detected and equalled 2.63%. The investigation of the price and the accuracy sensitivity to the changes in the strike level is shown in Figure 6.1, where the ordinate axis reflects the option prices and the axis of abscissas displays the various strikes.

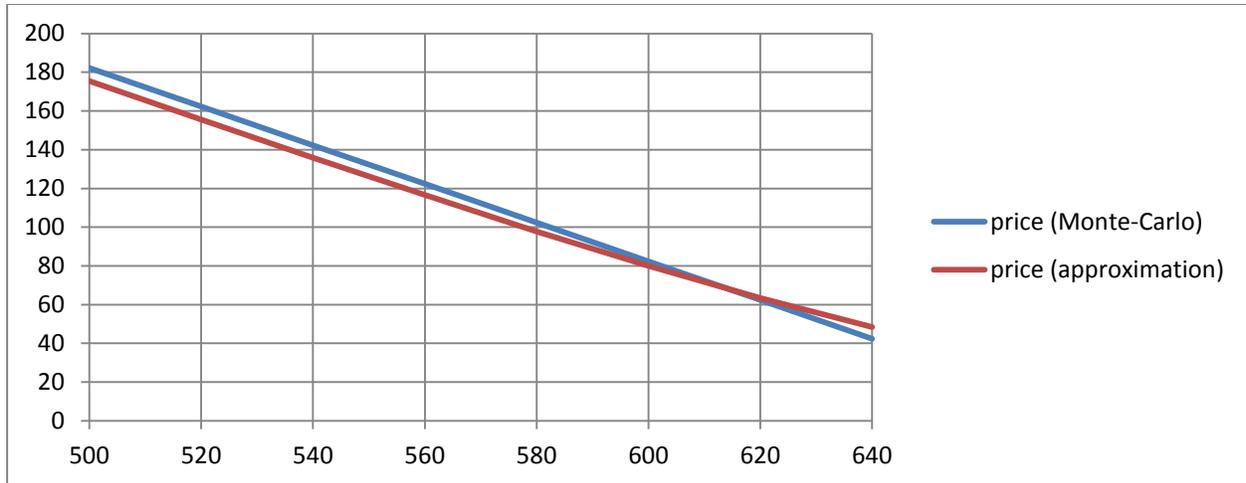


Figure 6.1 The option price and the approximation accuracy sensitivity to the changes in strike

The further the strike level diverges from the expected value of the cumulated HDDs, the more evident is the discrepancy between the results of Monte-Carlo simulation and the approximation formula proposed by Alaton et al. (2002). Moreover, if the taken strike is lower than the expected value of cumulated HDDs, the approximation formula slightly undervalues the call option price. On the contrary, when the taken strike is higher than the expected value of the cumulated HDDs, the approximation formula overprices the call option.

Secondly, the price sensitivity and the approximation precision depending on the changes in the risk-free rate were analysed. In order to define the range of possible risk-free rates, rough maximum and minimum of the German ten-year bond yields during the last year were taken (3.50% and 1.42% respectively). However, the corresponding changes made an insignificant impact on the price of the studied call. The results are reflected in Table 6.2, where the column P_{MC} shows the results of Monte-Carlo simulations in SEK, R_f stands for risk-free rate and P_A corresponds to the prices in SEK given by the approximation formula.

Table 6.2 The option price and the approximation accuracy sensitivity to the changes in the risk-free rate

Strike	P_{MC}	R_f	P_A	Difference
602	80.25	3.50%	78.14	2.63%
602	80.38	1.53%	78.27	2.63%
602	80.39	1.42%	78.28	2.63%

It is obvious that the price and the accuracy sensitivity to the changes of the risk-free rate within a one-year frame are minor and could be ignored.

Thirdly, the impact of the changes in the market price of risk λ on the call option prices given by the Monte-Carlo method and the approximation formula was inspected. As it was stated earlier, in order to price options, this paper used the benchmark derivative proposed by Alaton et al. (2002) with the corresponding market price of risk λ equal to 0.08. However, the constant market price of risk is only used as a simplifying assumption and does not reflect the full reality (as it was explained above). Therefore, it is essential to study its impact on the option prices. Again, the closest match between the two methods was observed when the market price of risk was converging to its most plausible value determined by Alaton et al. (2002). Several examples of the conducted calculations are to be found in Table 6.3.

Table 6.3 Influence of changes in market price of risk on option prices

Strike	P_{MC}	R_f	λ	P_A	Difference
602	97.01	1.53%	0.12	90.80	6.39%
602	80.38	1.53%	0.08	78.27	2.63%
602	63.76	1.53%	0.04	69.48	-8.98%
602	47.13	1.53%	0.00	55.34	-17.42%

Negative market price of risk for the winter months results in unrealistically low option prices.

Moreover, in order to illustrate how it is possible to incorporate actual weather forecasts into the model and how it would affect the call option prices and the approximation accuracy, Table 6.4 is presented. For instance, if the expected mean temperature in January is going to be lower than usually, it is needed to decrease the mean parameter ρ_1 from the equation (4.2).

Table 6.4 Option price and approximation accuracy sensitivity to changes in the mean parameter ρ_1

Strike	P_{MC}	R_f	ρ_1	P_A	Difference
602	142.30	1.53%	5.33	126.68	10.98%
602	111.34	1.53%	6.33	101.62	8.73%
602	80.38	1.53%	7.33	78.27	2.63%
602	49.42	1.53%	8.33	56.73	-14.79%

The strong influence of the expected mean temperature on the call option price is obvious. Logically, the lower is the mean parameter ρ_1 , the higher is the call price obtained both by Monte-Carlo simulations and by the approximation formula. The increase in the mean parameter results in the rise of the mean temperature and, consequently, in the decrease of H_n and the fall of

the option pay-off (see eq. (5.13)), thus, reducing the option price. The precision of the approximation improves with the parameter ρ_1 converging to its historical value 7.33.

Furthermore, the price sensitivity and the approximation precision according to the changes in the speed of mean-reversion parameter α and to the standard deviation were inspected. The results are shown in Tables 6.5 and 6.6 respectively.

Table 6.5 The option price and the approximation accuracy sensitivity to the changes in the mean-reversion parameter α

Strike	P_{MC}	R_f	α	P_A	Difference
602	90.75	1.53%	0.18	86.26	4.94%
602	86.80	1.53%	0.20	83.29	4.04%
602	83.54	1.53%	0.22	80.73	3.36%
602	80.38	1.53%	0.24	78.27	2.63%
602	78.49	1.53%	0.26	76.41	2.65%
602	76.50	1.53%	0.28	74.70	2.35%

Table 6.6 The option price and the approximation accuracy sensitivity to the changes in the standard deviation

Strike	P_{MC}	R_f	σ	P_A	Difference
602	71.05	1.53%	2.35	70.89	-0,16%
602	73.21	1.53%	2.61	72.90	-0,14%
602	76.14	1.53%	2.87	75.07	-1,24%
602	79.36	1.53%	3.14	77.17	-2,37%

It is observed that the higher alpha parameter is the lower is the option price. An opposite reaction of the price is identified in regards to the standard deviation variation: the call option price increases as sigma rises. The former relationship can be explained by the meaning of the speed of mean-reversion parameter, which accounts for preventing the temperature to increase or decrease by significantly high values comparing to the average historical temperature levels. The speed of mean-reversion and volatility are inversely related and show negative correlation. The lower the standard deviation is, the faster the temperature converges to its long-run mean. According to the theoretical literature and the previous empirical results on this topic, a higher volatility would lead to a higher price of the call option (Hull, 2002; Copeland & Antikarov, 2003). Higher volatility implies higher probability that the option will end in-the-money. On the contrary, a higher level of mean-reversion would lead to lower option prices, which is, in fact, detected. The precision of the approximation rises as the speed of mean-reversion increases. The

approximation accuracy sensitivity to the changes in volatility analysis shows that the better fit corresponds to the low values of volatility.

6.2 Possible weather options for Gothenburg and Malmö

In the perspective of the future development of the European weather derivative market, the final part of this paper is dedicated to proposition of possible weather derivatives for two other Swedish cities: Malmö and Gothenburg. Table 6.7 presents the HDD call option prices for the two cities as well as the previously given results for Stockholm.

Table 6.7 Call prices and the contracts description, Stockholm, Gothenburg and Malmö, 2011

Month	σ	P_{MC}	Strike	P_A	Difference	λ
Stockholm						
January	3.28	80.00	602	78.27	-2.21%	0.08
February	2.97	21.00	574	20.90	-0.48%	0.08
Gothenburg						
January	2.70	44.88	583	44.03	-1.92%	0.08
February	2.43	164.06	400	132.21	-24.29%	0.08
Malmö						
January	2.65	42.02	547	46.99	10.58%	0.08
February	2.31	174.05	360	142.07	-22.51%	0.08

The option's components such as the risk-free-rate and the market price of risk were chosen the same as for Stockholm. As it was previously stated, the assumption about the identical market price of risk is based on the empirical evidence given by Hardle and Cabrera (2011), suggesting that for regions with homogeneous weather risk there is a common market price of weather risk. The strike levels were chosen individually for the each city according to the expected level of HDDs for the corresponding months.

Despite the strong correlation levels between the cities' temperatures (the respective coefficients of correlation exceeded 0.9; see Table 3A.1, Appendix 3), the strikes and, consequently, the option prices determined for Stockholm, Malmö and Gothenburg differ substantially. Moreover, the call option prices obtained for Malmö and Gothenburg in February seem questionable due to their relatively high values. Furthermore, the precision of the

approximation formula is fairly low during this period. Among the possible reasons could be the fact that the chosen market price of risk is not appropriate for the other cities. The market price of risk equal to 0.08 was determined by the abovementioned benchmark derivative for Stockholm. However, temperature level and volatility differences in the three cities might mean that the weather risk in these regions is not homogenous, even despite the fact that the cities are relatively close geographically. Consequently, it would follow that the market price of risk in the chosen cities might not be identical. That is the reason why it was decided to analyse theoretical call option prices in terms of Gothenburg in February with other values of the market price of risk. The results are presented in Table 6.8.

Table 6.8 HDD call option prices, February 2011, Gothenburg

Gothenburg						
Month	σ	P_{MC}	Strike	P_A	Difference	λ
February	2.43	164.06	400	132.21	-24.29%	0.08
February	2.43	104.81	400	95.18	-10.21%	-0.08
February	2.43	75.18	400	76.62	1.88%	-0.16

The accuracy of the precision formula increases and the call prices seem more reasonable when the market price of risk decreases and becomes negative. It would be premature to draw firm conclusions about the levels of the market price of weather risk in Gothenburg or Malmö without analysing actual option prices. However, it could only be done when the cities become tradable on the weather derivatives market.

CONCLUSION

Weather derivatives are comparatively newly established financial instruments that allow companies from a range of industries to protect themselves from risks connected with weather fluctuations. The biggest weather derivatives market is concentrated in the United States of America, where this type of financial instruments appeared for the first time. The respective European market is currently actively developing; however, its illiquidity poses a considerable drawback.

Referring to the potential of the European weather derivatives market, this paper focuses on the pricing of the call options on temperature for Stockholm, Sweden. Moreover, it speculates about possible option prices for two other Swedish cities such as Gothenburg and Malmö. The aim of the paper was to scrutinize the weather derivatives pricing method proposed by Alaton et al. (2002) and study option price sensitivity to the changes in strike, market price of risk, mean temperature, speed of mean-reversion, standard deviation and risk-free rate.

In order to achieve the formulated objectives several steps were made. Firstly, temperature dynamics in the chosen regions was analysed, revealing such temperature features as seasonality, mean-reversion and volatility that changes throughout a year, thus, reasserting the results of previous studies. Secondly, these features were taken into account and a way to model daily temperature behaviour was tested and proved to be satisfactory. Thirdly, HDD call options were priced using Monte-Carlo simulations (the obtained results were considered to be a proxy to the real-life prices) and the approximation formula. Due to the fact that temperature is non-tradable, the option prices in an incomplete market are derived using the positive market price of risk proposed by Alaton et al. (2002) for the benchmark weather derivative for Stockholm.

It was shown how it is possible to incorporate weather forecasts into the model and what a strong influence on prices has the choice of the temperature mean parameter. The lower is the chosen mean parameter, the higher is the expected pay-off and, consequently, the higher is the HDD call price. The precision of the approximation improves with the mean parameter ρ_1 congregating to its historical value. The call option price was proved to be positively dependent on volatility and inversely related to the speed of mean-reversion. These results are consistent with the previous studies.

The changes in the risk-free rate did not lead to significant fluctuations of the HDD call option prices.

Moreover, it was revealed that the precision of the approximation increases when the strike and the market price of risk converge to their most plausible values. It was concluded that the market price of risk of the chosen benchmark derivative might not apply for some spring and autumn months (such as, for example, March and November), which is most likely caused by the fact that in reality the market price of risk corresponding to these months could be negative. Furthermore, unrealistic values of this parameter most probably would result in implausible HDD call option prices.

In addition, the current paper speculated about the probable HDD call option prices for Malmö and Gothenburg. Despite the fact that initially it was proposed to use the same market price of risk as for the Stockholm benchmark derivative (based on the assumption that the three cities might have homogenous weather risk), later it was suggested that the market price of risk in Malmö and Gothenburg might differ from the one in Stockholm not only in level but also in sign. However, this assumption can only be proved or refuted when there exists a liquid weather derivative market for the chosen cities.

In the light of everything mentioned, it is recommended to choose the model parameters utterly carefully when pricing weather derivatives.

Since weather derivative pricing is based on the temperature modelling, the proposed pricing method could be upgraded, for example, by presenting a more sophisticated way to account for the driving noise process or by incorporating more elaborated volatility patterns. Nonetheless, the suggested stochastic temperature formula proved to be a satisfactory fit to the data and can be used by practitioners without further modifications in order to avoid cumbersomeness. A meaningful contribution would be to scrutinize the market price of risk characterizing Swedish market and its seasonal variation. However, it is only possible as the weather derivative market evolves.

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APPENDICES

Appendix 1. Day i of month j standard deviation

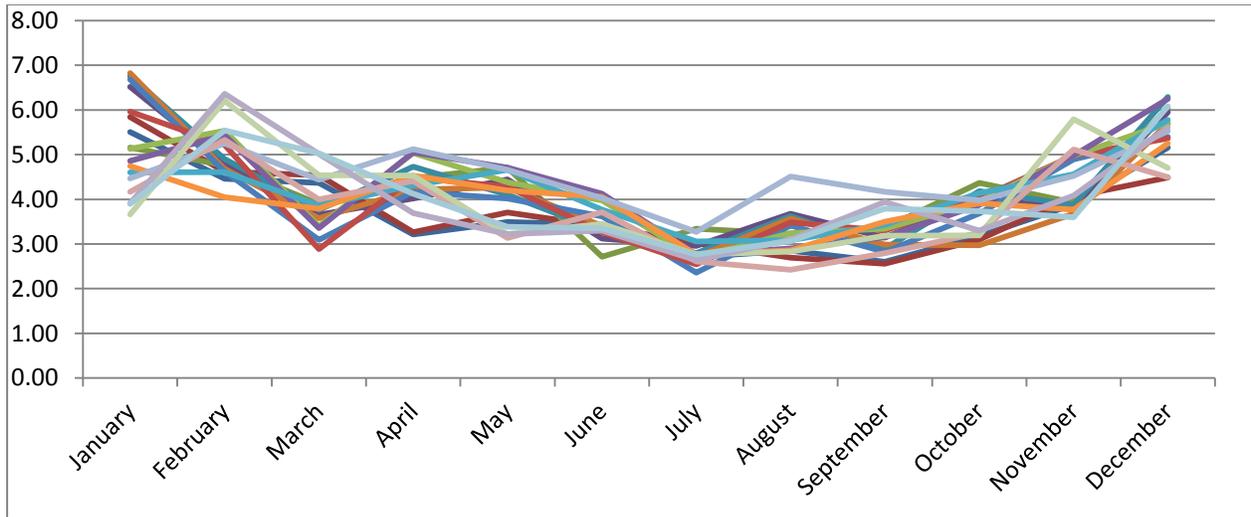


Figure reflects the standard deviation of each day i (from one to twenty eight/thirty/thirty one) across twelve months, each line representing one day. The scope of the graph is to observe the seasonal patterns in standard deviation. It is seen that the lowest values are characteristic for July, while the highest levels of variation are observed for winter months January and December.

Appendix 2 Estimation of the mean temperature

Appendix 2.1 Estimation of the mean temperature for Stockholm

Dependent Variable: X

Method: Least Squares

Date: 04/11/12 Time: 10:55

Sample: 1 12502

Included observations: 12502

Convergence achieved after 8 iterations

$X=C(1)+C(2)*T+C(3)*\text{SIN}(0.0172*T+C(4))$

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	7.328993	0.100480	72.93968	0.0000
C(2)	-0.001437	0.000518	-2.775877	0.0055
C(3)	-10.51268	0.052837	-198.9634	0.0000
C(4)	1.208245	0.007069	170.9285	0.0000
R-squared	0.789560	Mean dependent var	7.012579	
Adjusted R-squared	0.789510	S.D. dependent var	8.329374	
S.E. of regression	3.821453	Akaike info criterion	5.519458	
Sum squared resid	182514.5	Schwarz criterion	5.521837	
Log likelihood	-34498.13	Hannan-Quinn criter.	5.520254	
F-statistic	15630.63	Durbin-Watson stat	0.398176	
Prob(F-statistic)	0.000000			

Appendix 2.2 Estimation of the mean temperature for Gothenburg

Dependent Variable: X

Method: Least Squares

Date: 05/07/12 Time: 14:46

Sample: 1/01/2000 4/01/2012

Included observations: 4474

Convergence achieved after 8 iterations

$X=C(1)+C(2)*T+C(3)*\text{SIN}(0.017214*T+C(4))$

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	7.586243	0.151348	50.12440	0.0000
C(2)	-0.000660	0.000782	-0.844199	0.3986
C(3)	-9.158298	0.081538	-112.3199	0.0000
C(4)	1.178490	0.012219	96.44465	0.0000
R-squared	0.776752	Mean dependent var	7.310647	
Adjusted R-squared	0.776603	S.D. dependent var	7.353243	
S.E. of regression	3.475506	Akaike info criterion	5.330251	
Sum squared resid	53993.75	Schwarz criterion	5.335978	
Log likelihood	-11919.77	Hannan-Quinn criter.	5.332269	
F-statistic	5184.205	Durbin-Watson stat	0.365048	
Prob(F-statistic)	0.000000			

Appendix 2.3 Estimation of the mean temperature for Malmö

Dependent Variable: X

Method: Least Squares

Date: 05/07/12 Time: 14:55

Sample: 1/01/2000 4/01/2012

Included observations: 4474

Convergence achieved after 9 iterations

$X=C(1)+C(2)*T+C(3)*\text{SIN}(0.017214*T+C(4))$

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	8.497010	0.135469	62.72284	0.0000
C(2)	-0.000925	0.000700	-1.321930	0.1863
C(3)	-8.769323	0.074901	-117.0784	0.0000
C(4)	1.121933	0.011260	99.64035	0.0000
R-squared	0.798741	Mean dependent var	8.176700	
Adjusted R-squared	0.798606	S.D. dependent var	6.931981	
S.E. of regression	3.110862	Akaike info criterion	5.108571	
Sum squared resid	43258.27	Schwarz criterion	5.114298	
Log likelihood	-11423.87	Hannan-Quinn criter.	5.110589	
F-statistic	5913.393	Durbin-Watson stat	0.406205	
Prob(F-statistic)	0.000000			

Appendix 3. Monthly temperature volatility in Stockholm, Gothenburg and Malmö

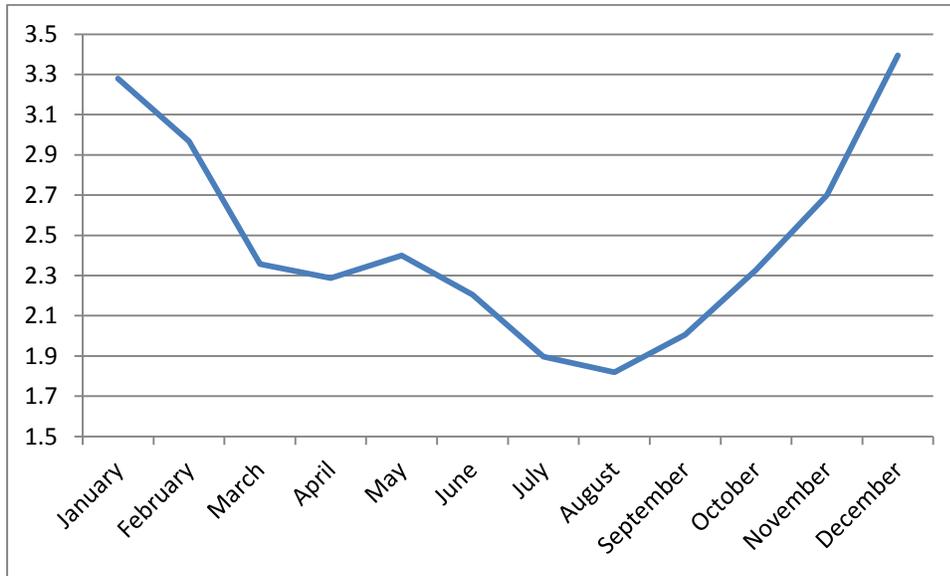


Figure 3A.1 Monthly temperature volatility in Stockholm

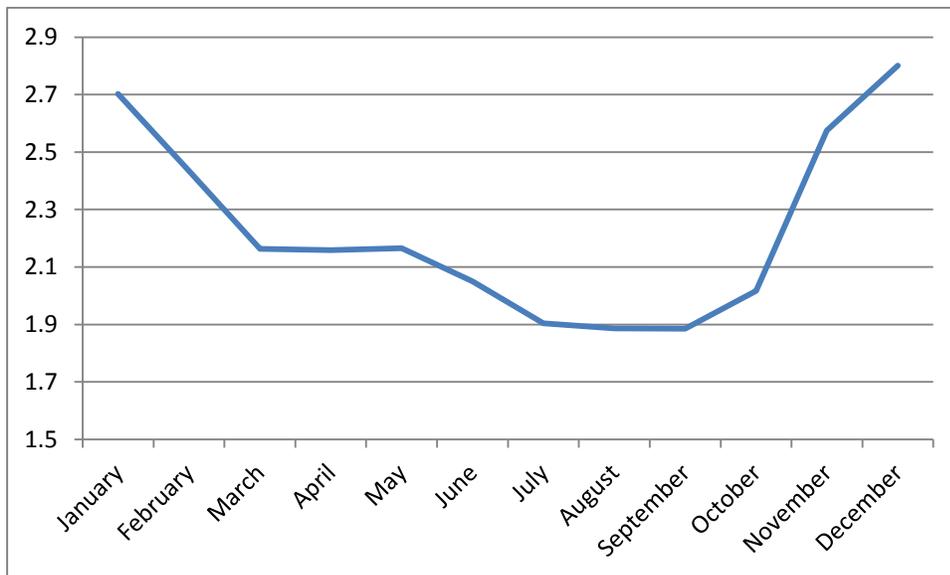


Figure 3A.2 Monthly temperature volatility in Gothenburg

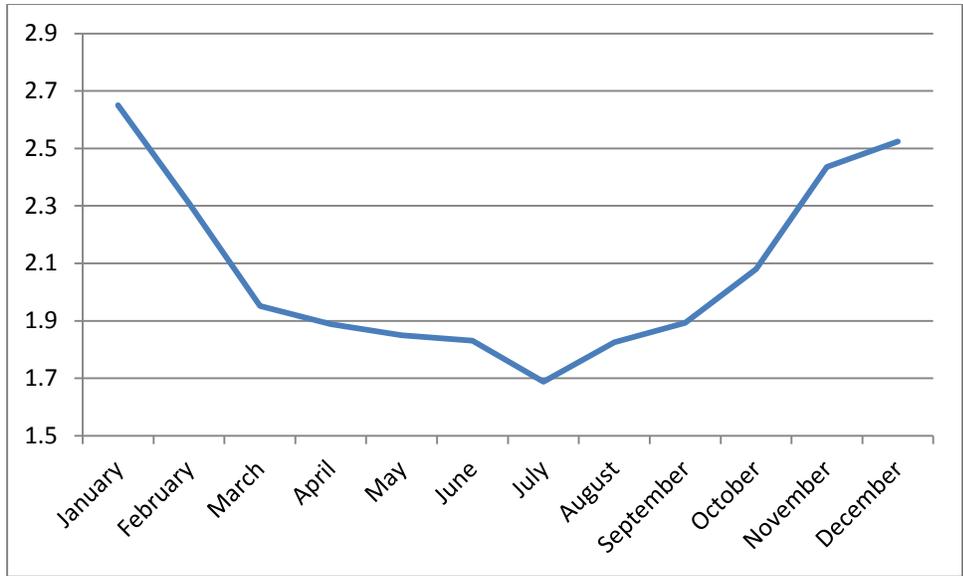


Figure 3A.3 Monthly temperature volatility in Malmö

Table 3A.1 Correlation matrix between the temperature levels in Stockholm, Malmö and Gothenburg

	Gothenburg	Malmö	Stockholm
Gothenburg	1		
Malmö	0,939158	1	
Stockholm	0,926377	0,901809	1