School of Economics and Management
Department of Economics
Master Thesis, Spring 2012

Portfolio optimization: The Downside risk framework versus the Mean-Variance framework

Authors

Shubiao Ren	871128
Hulda Sigmundsdóttir	810115

Supervisor
Hans Byström

Abstract

The tradeoff between risk and return is a topic that most investors consider carefully before an investment decision is made. Markowitz's pioneer work on portfolio selection using the mean-variance framework has entailed a great extent of research in this field. One research is the Sharpe ratio, which is a measure of a financial performance using variance as a risk measure. The shortcoming of variance is that it puts equal weights on positive and negative returns. Investors' attitudes towards risk are different but in general investors are more concerned about the downside risk rather than the upside risk. This study has thus constructed a new ratio with similar interpretation as for the Sharpe ratio. The new ratio introduced referred to as the downside risk ratio, uses the downside risk measure expected shortfall as the risk measure instead of variance. To find out if there are any differences in asset allocation using different risk measure an actual performance of four stock market indexes will be evaluated using both the Sharpe ratio strategy and the downside risk ratio strategy. Optimal weights for both a portfolio with two indexes (total of six portfolios) and a portfolio containing all the indexes are found. Finally, both strategies the Sharpe ratio and the downside risk ratio will be assessed in terms of whether there are any differences in constructing a portfolio using either a variance or an expected shortfall as a risk measure.

Keywords: Mean-variance framework, downside risk, Sharpe ratio, downside risk ratio, variance, expected shortfall, portfolio optimization, asset allocation.

Table of contents

1. Introduction

With the development of financial markets throughout the years, the importance of risk management has increased, especially after the market failure in 2008. Globalization of financial markets, financial integration, technology improvement in trading systems and more complex derivative markets, result in new sources of risk. The growth in trading activity has made the environment more volatile, which exposes firms and investors to more financial risk. The expansion of complex financial structures calls for a better risk management where risk must be accurately identified and measured. Financial risk is the uncertainty of possible loss. There exist numerous financial risk measures and models for portfolio optimization, which is the topic of this thesis, and the choice of risk measure is important. The mean-variance framework and the downside risk framework of risk measure will be discussed and compared in terms of asset allocation using two investment strategies, the Sharpe ratio and the downside risk ratio.

Markowitz introduced the mean-variance theory of portfolio selection. He explains how mean-variance depends on investor's preference, when an investor selects a portfolio he might have to give up expected return to reduce the risk, or gain higher expected return by taking on more risk. The Sharpe ratio is based on the mean-variance theory and measures financial performance. It describes how well a return of an asset compensates the investor for the risk taken. The ratio is maximized with the highest ratio giving more return for the same risk. The mean-variance theory has been criticized since it uses variance as a risk measure and puts equal weights on positive and negative returns. The main concern for investors may not be the variance but the downside risk as the general assumption is that if a return is below their expected value investors will become more unsatisfied than if the return is above their expected value. Thus an alternative risk measure might be more convenient than variance. In this thesis a new ratio of fund performance is constructed, referred to as the downside risk ratio. This ratio combines the return over risk free rate and the downside risk measure expected shortfall. It is the same interpretation as for the Sharpe ratio but the variance is replaced by the expected shortfall as the risk measure.

In this paper, evaluation of four stock market indexes is performed using three selected indexes all from developed markets in different continentals (United States, United Kingdom and Hong Kong), and the forth one being in the largest developing market in the world (China). Because of multidimensional problems the indexes needed to be limited to four. In order to find out if there is any difference in asset allocation using different risk measures two strategies are set up, one according to the Sharpe ratio and the other according to the new constructed downside risk ratio. Optimal weights for both a portfolio with two indexes (total of six portfolios) and a portfolio containing all the indexes are then found for both the strategies. Finally, both the strategies will be assessed in terms of whether there are any differences in constructing a portfolio using variance or expected shortfall as a risk measure.

1.1 Problem discussion

In the theory of the mean-variance framework of a portfolio selection, variance is used as the risk measure. However, variance has been criticized as a risk measure as it equally weights the upside risk and the downside risk. In general, investors are more concerned about the downside risk as it results in losses while the upside risk results in unexpected profit. For investors deciding how to allocate assets a downside risk measure might be a better approach than using variance. Thus, the research question put forward is:

Are there any differences in asset allocation using the variance (Sharpe ratio) or the expected shortfall (the downside risk ratio) as the risk measure?

1.2 Purpose

In this paper the optimal portfolio selection is discussed under the framework of Sharpe ratio and the downside risk ratio that has been constructed. Performance of four stock market indexes will be evaluated using both the ratios as a measurement. Furthermore, optimal weights on different combinations of portfolios containing the indexes will be calculated. Finally, it will be examined if there are any differences on how investors should allocate their investment in regards to different risk measure.

1.3 Structure of the paper

The structure of the paper is following. Introduction chapter explains the background of the subject in hand. Chapter 2 reviews the theoretical background for understanding of the subject. Chapter 3 explains the methodology and in chapter 4 data is examined. Empirical results are presented and interpreted in chapter 5 and chapter 6 presents the conclusion.

2. Theoretical background

In financial theory, the tradeoff between risk and return has been a popular issue that many scholars have researched. Markowitz introduced variance as a risk measurement, but variance has been criticized as it emphasis on the upside return is equal to the downside risk. In general investors are more concerned about losses indicating that downside risk measures might better reflect investors' preferences than the variance. This chapter introduces financial risk measures and models that have been presented throughout the years. Furthermore, it introduces a new ratio, downside risk ratio which is attributing to the Sharpe ratio. The downside risk ratio is composed of the downside risk measure expected shortfall, since many downside risk measures exist we will introduce the Value at Risk and the expected shortfall and discuss its attractions and drawbacks.

2.1 The Mean-Variance Model

In 1952 Markowitz[1] made a pioneer work in finance that has resulted in further studies in the field of portfolio theory. He proposed the theory of the mean-variance framework of a portfolio selection since investors want to maximize expected return and minimize the variance (risk). Markowitz stressed that the variance cannot be eliminated by diversification as returns on securities are too intercorrelated. Investors' preferences are different when it comes to selecting an investment portfolio. There is a tradeoff between risk and reward, investors might have to give up expected return to reduce the variance or gain expected return by taking on some risk. Markowitz also discussed that it is not enough to invest in many securities, investment should as well be diversified across industries to obtain a lower covariance. The criticism of the mean-variance framework is that its assumptions are unrealistic. First, it assumes that returns are normally distributed which is not always the case. Second, for a quadratic utility function it assumes that investors prefer a portfolio with the minimum standard deviation for a given expected return.[2] The use of variance has as well been criticized as it is a symmetric risk measure

[1] Markowitz (1952)
[2] Rachev et al. (2007)

that put equal weights on positive and negative returns when the general assumption is that investors are more concerned about losses than dispersion of high returns.

2.1.1 The efficient frontier

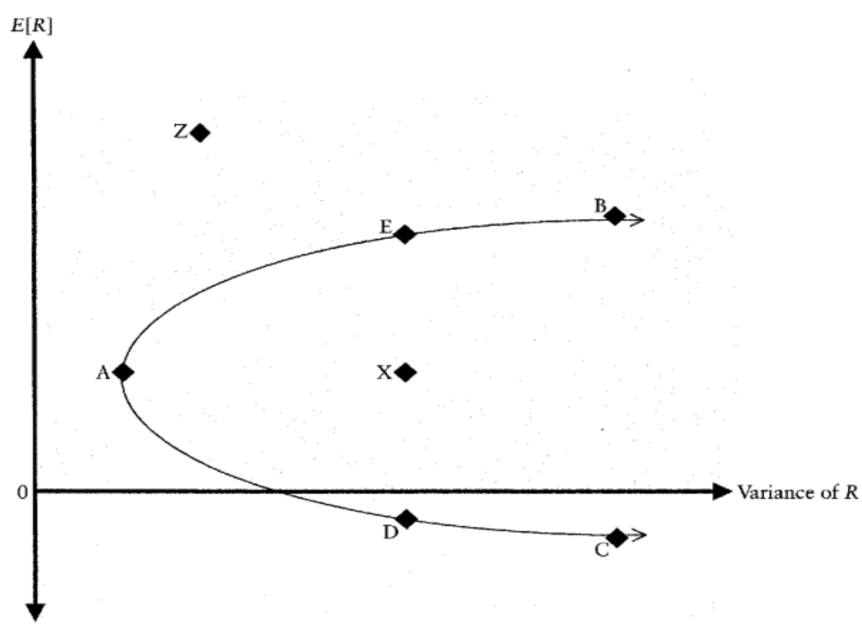
The efficient frontier is a frontier of efficient portfolios. A frontier portfolio is a portfolio that has the lowest variance of all portfolios with the same expected return. The efficient portfolio that has the lowest risk is the minimum variance portfolio (mvp). To construct the mean-variance efficient frontier a linear optimization problem is solved.

$$\begin{gathered} min_{(w_1,\ldots w_N)} \Sigma_{i=1}^{N} \Sigma_{j=1}^{N} w_i w_j \sigma_i \sigma_j \rho \\ s.t. \Sigma_{k=1}^{N} w_k = 1 \\ s.t. \Sigma_{q=1}^{N} w_q E(R_q) \geq \lambda \end{gathered} \tag{1}$$

The investor selects a set of portfolio weights to minimize the variance of any number of assets. The portfolio weights sum to one and the target return is λ.[3] Portfolio A is the minimum variance portfolio, see figure 1.

Figure 1. The mean-variance frontier.

Source: Culp (2001)



[3] Culp (2001) pp. 49

A rational investor makes a decision built on the mean and the variance of a distribution of portfolio returns and he will only select portfolios on the efficient frontier.[4] Only portfolios that are on the frontier are efficient portfolios, portfolios inside the frontier are inefficient and should not be held by utility maximizing investors.[5] Markowitz found out that only portfolios above the mvp were efficient. The most efficient portfolio is the one that achieves highest return for each amount of risk.

2.2 The Sharpe ratio

The Sharpe ratio is a measure of a financial performance and is based on the mean-variance framework. It is an intuitive measure that is easy to implement and provides a complete ranking of funds. It is a reward to risk ratio as it represents a fraction where a measure of reward is divided by a measure of risk. The higher the ratio is the better is the reward to risk.

$$SR = \frac{E[R]-Rf}{\sigma} \tag{2}$$

The Sharpe ratio only accounts for the first two moments of a distribution, the mean and the variance and is therefore only valid for either normally distributed returns or quadratic preferences. The Sharpe ratio is a meaningful measure of risk when risk can be sufficiently measured by the standard deviation. However, when the return distributions are non-normal the Sharpe ratio can lead to misleading conclusions and unsatisfactory paradoxes.

2.2.1 Standard deviation

Standard deviation is the square root of the variance, and is used to measure performance. It is a statistical measurement of dispersion around an average and describes how widely returns have varied over a certain period of time. Historical performances are used to predict the range of returns that is most likely to occur. A high standard deviation means that the predicted range of a performance is wide and it implies a greater volatility.[6]

---

[4] Bradley and Taqqu (2003)
[5] Culp (2001) pp. 49
[6] Unified trust (2007)

2.3 The downside risk ratio

It is generally accepted that rational investors should allocate their investment according to the reward to risk criterion like the Sharpe ratio introduces. However, for investors the main concern may not be the variance but the downside risk which may not be totally indicated by the variance. In this paper, the downside risk ratio is defined as:

$$R = \frac{E[R]-Rf}{ES} \tag{3}$$

E[R]: expected return of the portfolio
Rf: risk free rate
ES: expected shortfall

This ratio is also a reward to risk ratio and uses the expected shortfall as the measure of risk simlar to the Sharpe ratio. For investors who are more concerned about the downside risk than the variance it might be more applicable to maximize this ratio rather than the Sharpe ratio.

2.4 Portfolio risk

Portfolios are exposed to a wide variety of risks. There are several risk factors that can affect a portfolio such as liquidity risk, credit risk, counterparty risk, model risk and estimation risk. Here there will be a focus on market risk.

2.4.1 Market risk

Market risk is the risk of losses that results from movements in the level or volatility of market prices. It can be divided into absolute risk, which is measured in currency, and relative risk, which is measured relative to a benchmark index.[7] A risk factor is any market determined price, rate or index value that impacts the cash flows of an exposure. To determine how the risk factors affect the value of an exposure, five different types of market risks are presented. *Delta* is the risk that the value of an exposure will decline as the price or value of some underlying risk factor changes. For example, when interest rates rise, bond prices will fall. *Gamma* is the risk that delta will change if the value of an underlying risk factor changes. This risk is often named rate of change risk, as the amount of the price change of the bond depends on the level of interest rates. *Vega* is the

[7] Jorion (2007) pp. 22

risk that volatility changes of the underlying risk factor will influence a change in the value of an exposure. For long options, less volatility means that the option is less likely to be profitable, for short options it is the reverse. *Theta* measures the risk to certain exposures over time, for example, every day an insurance contract is unused it is one less day for the contract to become valuable. *Rho* is the risk that the interest rates will change which can impose unexpected losses.[8] Correlation risk is an unexpected change in the correlation of two factors that impact the value of a portfolio. As index depends on correlation between stocks it is highly related with the volatility of the stock market.

2.5 Downside risk measures and coherence

This thesis focuses on market risk with special attention on measures for downside risk. The risk measures Value at Risk and Expected Shortfall will be introduced. It has been stated that in order to manage risk in a good way, risk measure has to be coherent and four axioms must hold, therefore it is important to begin to explain coherence before introducing risk measures.

2.5.1 Coherent Risk Measures

Artzner et al. (1999) stated four axioms for risk measure and argued that these axioms should hold for any risk measure used to manage risk. He called the risk measures that satisfy the four axioms coherent. Below is an explanation of the four axioms:[9]

- Translation invariance. *For all* $X \in Ģ$ *and all real numbers* α,
  $\rho(X + \alpha \cdot r) = \rho(X) - \alpha$
  By adding the amount α to the position and invest carefully it reduces the overall risk of the position by α.
- Subadditivity. *For all* X_1 *and* $X_2 \in Ģ$, $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$
  For the benefit of diversification the risk of a portfolio made with two instruments may not be greater than the sum of an individual risk of these two instruments.
- Positive homogeneity. *For all* $\lambda \geq 0$ *and all* $X \in Ģ$, $\rho(\lambda X) = \lambda\, \rho(X)$
  If the size of a position directly influences risk then the consequences of lack of liquidity must be considered when the future net worth of a position is calculated.

[8] Culp (2001) pp. 16-17
[9] Artzner et al. (1999)

- Monotonicity. *For all X and Y Є Ģ with X ≤ Y, we have ρ(Y) ≤ ρ(X)*

  X is more risky if the future net loss X is greater than Y.

Ģ: set of all risks *α: initial amount*
X: random variable *Y: random variable*
r: strictly positive price *λ: the proportion of the portfolio*

2.5.2 Value at Risk

Value at Risk (VaR) is a downside risk predictor that measures the size of a loss in a change of an underlying risk factor such as assets or liabilities with a specified confidence interval within certain time period. With VaR the exposed amount and the likelihood of loss can be calculated. VaR is defined as:[10]

$$\Pr(x \leq VaR) = \int_{-\infty}^{VaR} f(x)dx = \alpha \tag{4}$$

α: confidence interval
f(x): distribution of exposures

The parameters' time horizon and confidence interval α must both be stated before estimating the VaR. Confidence interval is generally high or between 95% to 99% depending on the probability of loss occurring. The time horizon depends on how often a comparison between actual risk and tolerances are made and how quickly a firm can liquidate or hedge large losses. The time horizon can be from one day up to one year.[11] VaR has become attractive as a market risk measure for various reasons. First, it combines several sources of market risk into a single measure of potential change in value for a portfolio.[12] Second, VaR is a consistent measure of risk across different positions and risk factors and takes into account how different risk factors interact. Third, it is a holistic measure as it takes care of all driving risk factors.[13] On the other hand, VaR also has drawbacks; it only measures the percentiles of profit or loss distribution. It gives no information about the size of the loss if a tail event occurs (the loss can be much greater than the VaR indicates). Artzner et al. (1999) showed that VaR is not a coherent risk measure since it does not fulfill the axiom of subadditivity and thus it does not

[10] Culp (2001) pp. 342-343
[11] Culp (2001) pp. 343
[12] Johansson et al. (1999)
[13] Dowd (2005) p. 12

always encourage diversification. As VaR lacks the coherence it will not be used for the downside risk ratio as the measure of downside risk.

2.5.3 Expected Shortfall

The drawbacks and limitations of VaR have motivated many scholars to explore coherent risk measures. One alternative for VaR is expected shortfall (ES), which has some advantages over VaR. ES notifies what to expect when a VaR violation occurs and it is a more reliable risk measure during market turmoil. It is a coherent risk measure as it does not fail subadditivity by discouraging diversification and its estimates can be more accurate than the VaR's.[14] Subadditivity is essential in portfolio optimization because the convexity of the risk surface is minimized in the space of portfolios. If the surfaces are convex, the risk minimization process always finds a well-diversified optimal solution.[15] The ES measures the downside risk and is the conditional expectation of loss, given that the loss is beyond the VaR level and measures the size of the loss that is beyond the VaR level.[16] ES is the average of the worst 100(1 – α)% of losses.[17]

$$ES_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 q_p dp \qquad (5)$$

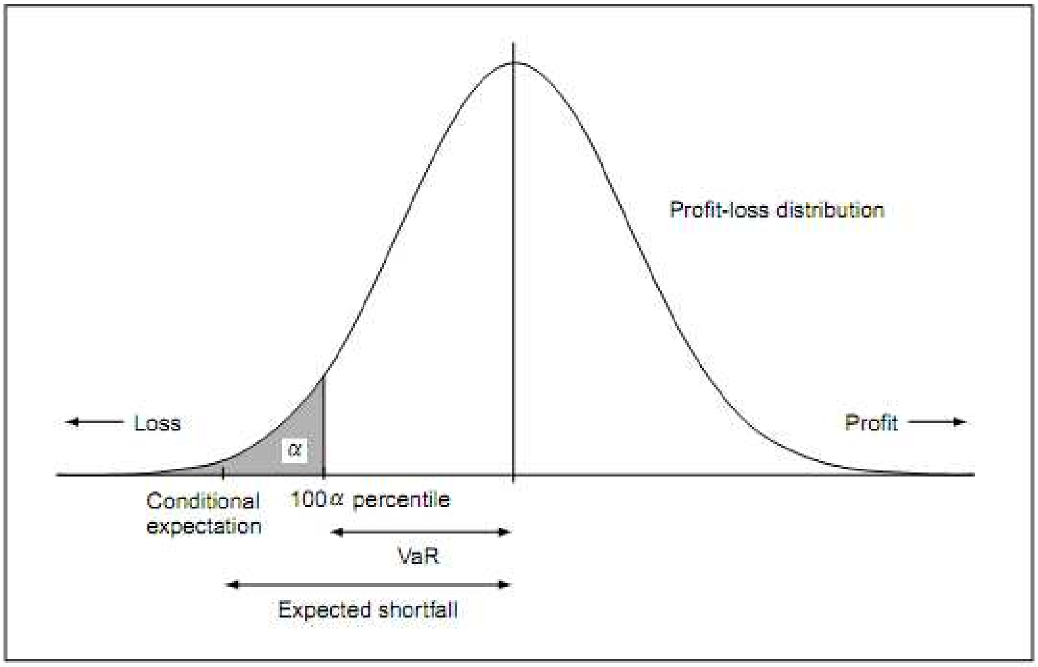
α: confidence interval
q_p: the p-quantile, p = α

---

[14] Angelidis and Degiannakis (2007)
[15] Acerbi and Tasche (2001)
[16] Yamai and Yoshiba (2002)
[17] Dowd (2005) pp. 35

Figure 2. Profit-Loss Distribution, VaR and Expected Shortfall.

Source: Yamai and Yoshiba (2002)



The ES has some drawbacks such as requiring more data for backtesting than VaR since the loss beyond the VaR level is irregular and it is more difficult to accurately estimate the average of the losses.[18] In a situation where the underlying distribution is fat-tailed, the estimation error is larger for the ES than the estimation error for the VaR. The reason being that because the ES estimate varies more since the probability of infrequent and large losses is high, this can be corrected by increasing the sample size.[19] The expected shortfall will be used as the downside risk measure for the downside risk ratio as it has advantages over VaR.

2.6 Portfolio theory and ES

There are some differences between portfolio theory and the expected shortfall even though the ES is a progression from earlier portfolio theory (PT):[20]

[18] Yamai and Yoshiba (2002)

[19] Yamai and Yoshiba (2005)

[20] Dowd (2005) pp. 11

- PT interprets risk using the standard deviation of the return while ES interprets risk in terms of the expected value of the loss (given that a VaR violation has occurred).

- ES is a more flexible measure of risk while PT assumes that returns are normally distributed. ES can however accommodate a wide range of possible distributions.

- PT is often limited to market risk while ES can be applied to broader fields of risks such as credit risk and liquidity risk.

2.7 Previous research

Minimizing the variance and maximizing the Sharpe ratio is a widely studied criteria, based on the first two portfolio moments. Scott and Horvath (1980) discuss a general set of positive preferences for odd moments, mean and skewness as well as aversion to even moments, variance and kurtosis.[21] In the literature for portfolio risk measurement, scholars seem to agree that in the presence of non-normal returns and non-quadratic utility functions, a downside risk measure should be preferred instead of portfolio variance.[22] Scholars have responded with numerous alternative performance measures, many of them use an interpretation of the Sharpe ratio as a reward to risk ratio and replace the standard deviation with an alternative risk measure. Zakamouline (2010) focuses on VaR as the risk measure as it can evaluate both downside risk, upside return potential and can be used for non-normal distributions.[23] Lower partial moment (LPM) was introduced by Bawa (1975), measuring different utility functions in terms of LPM of a return distribution. Furthermore, it assumes that investors have more than one utility function. With numbers of utility functions investors can be risk taking, risk neutral and risk averse. Grootveld and Hallerbach (1999) used LPM as their risk measure and found that the downside risk approach did produce on average slightly higher bond allocations than the mean-variance approach.[24]

[21] Scott and Horvath (1980)
[22] Boudt et al. (2008)
[23] Zakamouline (2010)
[24] Grootveld and Hallerback (1999)

3. Methodology

3.1 Core assumptions

When choosing the level of the ES calculation it does not matter if the expected shortfall of a portfolio (ES[p]) is calculated at a 5% level or at a 1% level. In this thesis the ES[p] will be calculated at a 5% level as it is the most common level used in practice. Additionally, it is assumed that the ES[p] is always negative due to two reasons. First, the mean value of the worst 5% returns for all the indexes is negative. Second, when the ES[p] is positive the best downside risk ratio will be the largest one but since the focus is more on empirical study in this paper, and all the ES[p] obtained from the results are negative, only the negative ES[p] will be discussed.

3.2 Construction of the ratios

In order to construct the Sharpe ratio and the downside risk ratio for a portfolio it is important to begin by discussing a situation of only two assets with the same period of historical returns. To find the optimal weights of an investment it is assumed that the proportion a% of the portfolio is invested in asset X with the time series return:

$$R[X_1], R[X_2], R[X_3], \ldots\ldots, R[X_i] \tag{6}$$

The proportion 1-a% is left to the other asset Y with the time series return:

$$R[Y_1], R[Y_2], R[Y_3], \ldots\ldots, R[Y_i] \tag{7}$$

Giving the time series return of the portfolio:

$$a*R[X_1]+(1-a)*R[Y_1],\ a*R[X_2],\ +(1-a)*R[Y_2],\ a*R[X_3]+ \\ (1-a)*R[Y_3], \ldots\ldots,\ a*R[X_i]+(1-a)*R[Y_i] \tag{8}$$

This equation can be used to generate the variance and the expected shortfall of the portfolio.

3.2.1 The Sharpe ratio

The Sharpe ratio measures the expected return per unit of risk, the higher the ratio, the more is the reward to risk. In order to find the best portfolio performance the ratio needs to be maximized. To composite the Sharpe ratio, historical returns are used to calculate returns in excess of the risk free rate of return. The mean of the excess returns are calculated as well as the standard deviation of the excess returns. The expected return of the portfolio is the mean value of the historical returns. These values are then placed in the Sharpe ratio formula:

$$SR = \frac{\mathrm{E[R]-Rf}}{\sigma} \tag{9}$$

E[R]: expected return of the portfolio
Rf: risk free rate
σ: variance

3.2.2 The downside risk ratio

To construct the downside risk ratio the same approach is used as in the Sharpe ratio, except that instead of using variance as the risk, measure expected shortfall is used. However, as the ES is negative for the sample used it is minimized to find the best performance. The formula:

$$\mathrm{R} = \frac{\mathrm{E[R]-RF}}{\mathrm{ES[P]}} \tag{10}$$

E[R]: expected return of the portfolio
Rf: risk free rate
ES[P]: expected shortfall of the portfolio

In order to compute the ES, historical simulation of the non-parametric approach is used. To refine and enhance accuracy, the bootstrapping method was used when historical simulation was applied in the empirical study. The expected return of the portfolio is obtained by calculating the average value of the time series returns.

3.2.2.1 Historical Simulation

Historical simulation (HS) is an estimation method that uses empirical distribution of historical returns to predict the future distribution of returns. HS can be applied to ES where ES is estimated as an average of quantile points of the empirical distribution beyond VaR.[25] By plotting the historical returns on a histogram the ES values can be observed. As an example, if there are 1,000 observations with the confidence interval at 95% then the VaR is the 51st highest loss value and the ES is the average of the 50 highest losses.[26]

[25] Inui and Kijima (2005)
[26] Dowd (2005) pp. 84

The advantages of HS are that it can contain any type of instruments as well as non-normal features. It is rather simple to implement and it accurately reflects the historical multivariate probability distribution. The approach has some drawbacks as it does not incorporate volatility updating.[27] Another drawback is the ghost effect that can appear when putting the same weight on all past observations in a sample, similar to the use of a rolling window. Rolling window throws an old observation out of the sample which can lead to a considerable change in the risk measure.[28] Furthermore the selection of the size of the window can be impeached if it is too short. Estimates can be sensitive to accidental outcomes from the recent past, however if the size of the window is long, past data is included and that might no longer describe the present situation.[29]

3.2.2.2 Bootstrapping

A refinement to the basic historical simulation is the bootstrap method. To enhance accuracy from the HS, bootstrapping can be used to estimate the expected shortfall by creating a distribution from the sample. Bootstrapping is connected to simulation, there is however a crucial difference of the data treatment. With simulation the data is built artificially while bootstrapping method uses the sample data to obtain a description of properties of empirical estimators. The procedure is then repeated with replacements from the actual data. The advantages of bootstrapping compared to simulation is that inference can be made without making strong distributional assumptions, since the distribution created is from the actual data. Bootstrapping looks at the variation of the statistics within a sample by estimating the sampling distribution.[30]

3.3 Finding the optimal asset allocation

In this section, it is demonstrated how to allocate assets by minimizing the downside risk ratio or maximizing the Sharpe ratio. First, it is demonstrated how to allocate assets in a portfolio with two assets and then by allocating assets in a portfolio with four assets.

[27] Hull (1998)
[28] Dowd (2005) pp. 100
[29] Goorbergh and Vlaar (1999)
[30] Brooks (2008) pp. 553-554

3.3.1 Asset allocation with two assets in a portfolio

3.3.1.1 The downside risk ratio

With the time series data of the portfolio return, expected shortfall of the portfolio ES[P] is computed. Only one unknown parameter, "a%", which is the proportion of the total investment allocated in asset X, will be in the representation of the ES[P]. The expected shortfall of a portfolio is then: ES[P] = f(a), stating that the ES[p] is a function with independent variable "a%". The downside risk ratio is then:

$$R = \frac{(E[P]-RF)}{ES[p]} \tag{11}$$

It can be written as R = g(a), indicating that the downside risk ratio is also a function of the independent variable "a%", different from function f(a).

For any rational investor, higher return and lower expected shortfall is favored. Since the downside risk ratio is negative, the minimum value or the maximum absolute value will be preferred. In order to find the optimal weights of an investment the optimal fraction "a%" is found, then the downside risk ratio is created as a function of the proportion invested into different assets and the proportion is solved by minimizing the ratio.

3.3.1.2 The Sharpe ratio

Similar to the method described in the previous section, the Sharpe ratio of the portfolio is a function of the independent variable "a%". By maximizing the ratio, the optimal weights can be found.

3.3.2 Asset allocation with more than two assets in a portfolio

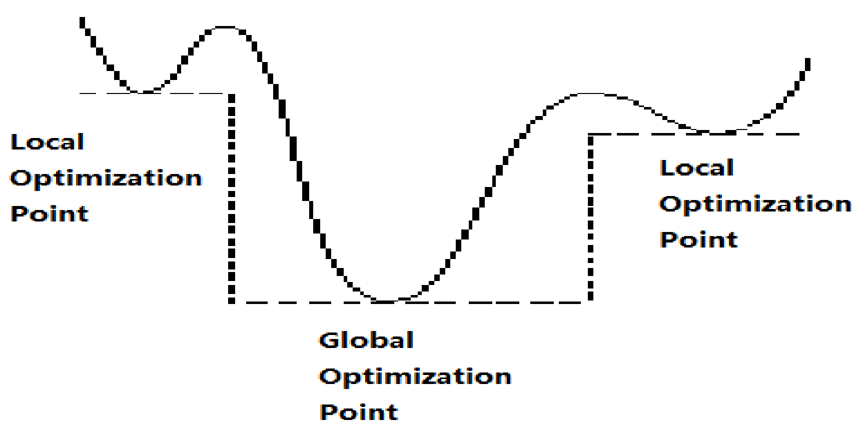
Previously, it has been explained how to construct an optimal portfolio with only two assets. Constructing an optimal portfolio with four assets is more complicated as it deals with a multidimensional problem.

3.3.2.1 The Nelder-Mead Method

The Nelder-Mead simplex algorithm is a popular method for multidimensional unconstrained optimization. It tries to minimize a scalar valued nonlinear function of several variables, by using only function values and without using derivative information.[31] Simplex is defined in one dimension as a line segment, in two dimensions it is a triangle, in three dimensions it is a tetrahedron and in four dimensions it is a pentachoron.[32] The Nelder-Mead algorithm maintains a simplex that is an approximation of the optimal point. The vertices are arranged in harmony to the objective function values and the algorithm tries to replace the worst vertex with a new point. The new point depends on the worst point and the center of the best vertices.[33]

One of the most common problems with the Nelder-Mead method is the possible confusion caused by the local optimization point. The optimization point indicates the local optimization point. Since the objective is to find the best weight in all possible combinations the global optimization point is the one that must be found. In figure 3 it can be noticed that a local optimum of a combinatorial optimization problem is a solution that is optimal (either maximal or minimal) within a neighboring set of solutions. It is in contrast to a global optimum, which is the optimal solution among all possible solutions.[34] However, several methods can be used to identify whether it is the global optimal point or the local optimal point, the one used here is comparing the optimization point generated by different starting points.

Figure 3. Local and global optimization point.



[31] Lagaris et al. (1998)
[32] Han and Neumann (2006)
[33] Baudin (2010)
[34] Liberti (2006)

4. Data

4.1 Presentation of data

The data for the indexes were obtained from appropriate stock exchanges attained from Yahoo Finance[35] except data for the Shanghai Index, which was obtained directly from the Shanghai Stock Exchange.[36] The four selected indexes in this thesis come from four different countries, China, Hong Kong, The United Kingdom and The United States. The last three countries all have developed markets and have an extended location in North America, Europe and Asia. The inclusion of China gives interesting aspects since it is the largest developing market in the world. The four indexes used are: Shanghai Composite Index (China), FTSE 100 (UK), S&P 500 (US) and Hang Seng Index (Hong Kong).

Along with the stock market data, risk free rates were acquired for all the selected indexes. One year deposit rate was selected as the risk free rate for China.[37] Hibor was selected for the Hong Kong market, Libor for the UK market and US base rate for the US market. All the data was obtained from DataStream except the one year deposit rate for China, that information was obtained from the People's Bank of China, which is the central bank of China.[38] The time weighted value was used if there was an adjustment on the rates during the year. As an example, a change in the risk free rate on the 1st of October 1992 from 3% to 4%, then the risk free rate in that year will be 3.25%.

4.1.1 Return Frequency and Period

The time period of the data set is 20 years from 1st of January 1992 to 31st of December 2011. This period is chosen because it is a relatively recent data with a long backward history. The indexes chosen are from different counties that have different public holidays. To make the returns consistent for all the indexes, common trading dates are used.

35 http://finance.yahoo.com/
36 http://www.sse.com.cn/en_us/ps/home.shtml
37 Han et al. (2005)
38 http://www.pbc.gov.cn/

4.1.2 Outliers

In the present study, the decision to not reject any abnormal returns was made since all fluctuations must be included. This is done because when analyzing the downside risk of returns, extreme values may represent an important part.

4.1.3 Return Calculation

Daily returns are obtained by calculating logarithmic returns of closing prices. Log return is chosen as its continuously compounded return is symmetric, while for example the arithmetic return is not as positive and negative percent arithmetic returns are not equal. Another advantage of the logarithmic returns is that the log return is more convenient to calculate total return for a certain period.

4.2 Data description

To make a better understanding of the final results, data statistics will be presented in this section. Table 1 shows the summary of the data statistics, further description graphs of the returns can be viewed in appendix 1.

Table 1. Data statistics.

	SH	HSI	FTSE 100	S&P 500
Mean	0.000458	0.000333	0.000184	0.000253
Standard deviation	0.000742	0.000351	0.00016	0.000164
Kurtosis	125.7507	14.29399	10.90096	12.79156
Skewness	5.044485	0.192782	-0.02389	-0.38948
ES	-0.05779	-0.04368	-0.02983	-0.03097
Variance of ES	2.130258e-05	1.225728e-05	6.636539e-06	5.251461e-06

ES: the mean of the expected shortfall

Jarque-Bera tests were performed to test for normality of the results for all the indexes. The results show that p-value = 0.000 for all the indexes. The null hypothesis that returns follow the normal distribution is rejected.

To enhance the accuracy of the results from the historical simulation, the following bootstrapping method was used when applying the historical simulation in the empirical study:

- Random selection of 1000 returns from the daily returns data.

- The expected shortfall is obtained by the mean value of the worst 50 returns, which represents the worst 5% returns.
- The above process is repeated 100,000 times.
- The distribution of the expected shortfall was found and the variance calculated.

The results are listed in appendix 2.

5. Empirical results

In this chapter the results will be presented and interpreted. Two investment strategies will be investigated together, the investment strategy minimizing the downside risk ratio and the investment strategy maximizing the Sharpe ratio. As explained in the preceding chapters, the time period of the data set is 20 years and the four indexes used to generate the portfolios are: Shanghai Composite Index (SH), FTSE 100, S&P 500 and Hang Seng Index (HSI).

5.1 Two ratios for four indexes

First, both the downside risk ratio and the Sharpe ratio are calculated for all the indexes. The 20 years' period is used as a single investment period and the expected shortfall and the variance are computed. The expected shortfall is defined as the mean value of the worst 5% returns. The indexes' returns are defined as the mean value of daily log return calculated from the four indexes. Furthermore, the mean value of the risk free rate for all the 20 years is used as the deducted variable in the two ratios calculation.

Table 2. The downside risk ratio and the Sharpe ratio for the indexes.

	SH	**HSI**	**FTSE 100**	**S&P 500**
The downside risk ratio	-0.0058177	-0.0052139	-0.0019263	-0.0053805
The Sharpe ratio	0.4533331	0.6479107	0.3597199	1.0141623

According to our ranking criteria, returns with higher Sharpe ratio or downside risk ratio represent a better performance. Table 3 informs the ranking of the indexes, the performance of FTSE 100 is highly behind the other indexes.

Table 3. The ranking of the indexes.

Rank	**Downside risk ratio**	**Rank**	**The Sharpe ratio**
1	SH	1	S&P 500
2	S&P 500	2	HIS
3	HIS	3	SH
4	FTSE 100	4	FTSE 100

5.2 The optimal weights for a portfolio with two assets

To find the optimal weights for a portfolio a hypothesized portfolio is constructed with two assets. Since there are 4 indexes in our sample, 6 different portfolios will be constructed. First, it is assumed that the proportion a% of the portfolio is invested in asset X with the time series return:

$$R[X_1], R[X_2], R[X_3],\ldots\ldots,R[X_i]. \tag{12}$$

The proportion 1-a% is left to the other asset Y with the time series return:

$$R[Y_1], R[Y_2], R[Y_3],\ldots\ldots,R[Y_i]. \tag{13}$$

Giving the time series return of the portfolio:

$$a*R[X_1]+(1-a)*R[Y_1],\ a*R[X_2],\ +(1-a)*R[Y_2],\ a*R[X_3]+ \\ (1-a)*R[Y_3],\ldots\ldots,a*R[X_i]+(1-a)*R[Y_i] \tag{14}$$

Equipped with the portfolio return, historical simulation is used to compute the expected shortfall for the portfolio, which can be expressed into: ES[P] = f(a), stating that ES[p] is a function with independent variable "a%". Since, E[p] can be obtained by calculating the mean value of the time series return and the risk free rate is known, the downside risk ratio is:

$$R= \frac{(E[P]-RF)}{ES[p]} \tag{15}$$

It can be written as R = g(a), indicating that the downside risk ratio is a function of the independent variable "a%", different from function f(a). To find the corresponding downside risk ratio R for every a%, "a" starts as zero then 100% was divided into 1000 pieces and a% was increased by one piece (0.001) at a time. Having constructed all the 1000 downside risk ratios it is possible to find the one point with the lowest downside risk ratio, which implies the best performance. The accuracy of the result is 0.1% since 100% was divided into 1000 pieces. Another way to search the best weights is by viewing the graph and observing the lowest point of the line. See figure 4 below.

Figure 4. The downside risk ratio weights.

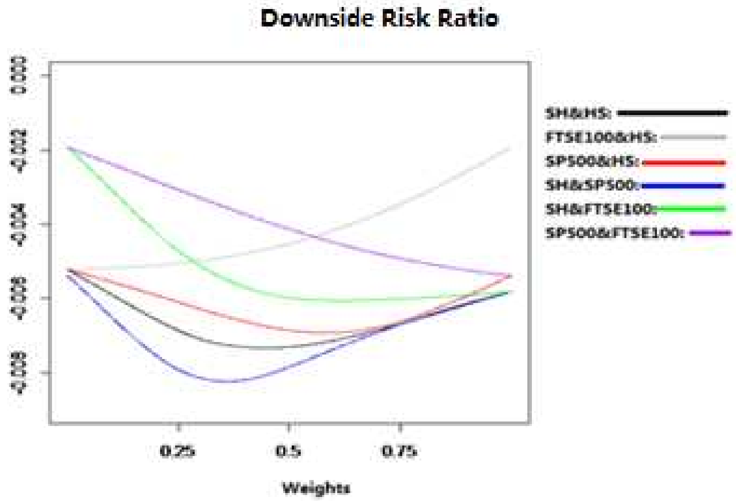


Table 4. The optimal weights for the downside risk ratio.

a 1-a	SH	HSI	FTSE 100	S&P 500
SH	*	0.448	0.616	0.36
HIS	0.552	*	1	0.415
FTSE 100	0.384	0	*	0
S&P 500	0.64	0.585	1	*

Table 4 summarizes the optimal weights for the indexes in the portfolio with two indexes. Take the 0.448 in the first row as an example, it means that in the portfolio combined with SH and HSI, the optimal weight is to put 44.8% of the total investment in the SH and invest the rest 55.2% into HSI. A higher weight indicates a better performance according to the downside risk ratio's measurement. Furthermore, the ranking is consistent with the result in table 3. No investment should be allocated in FTSE 100 when a portfolio is constructed with S&P 500 or HIS. It implies that the downside risk of the portfolio will not be diversified but the return will go down when adding investment in FTSE 100.

Likewise for the Sharpe ratio it can be represented as a function of weights a% and by repeating the same process as for the downside risk ratio the optimal weights can be obtained.

Figure 5. The Sharpe ratio weights.

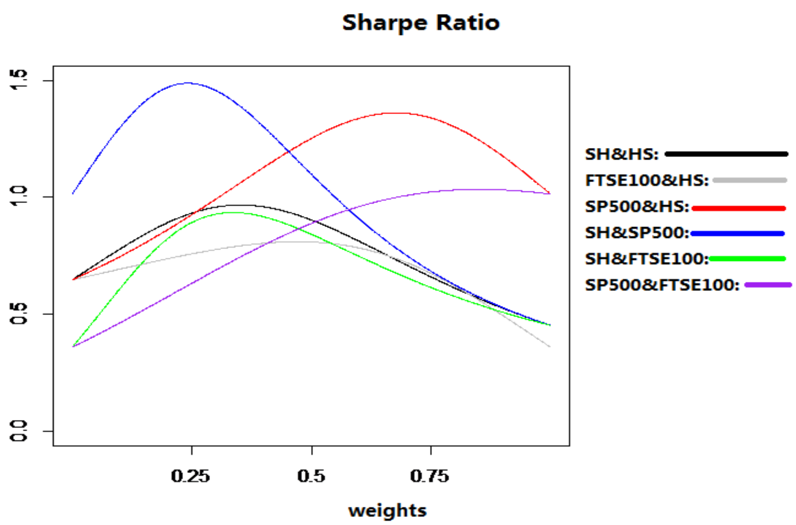


Table 5. The optimal weights for the Sharpe ratio.

a 1-a	SH	HSI	FTSE 100	S&P 500
SH	*	0.352	0.337	0.242
HIS	0.648	*	0.529	0.323
FTSE 100	0.663	0.471	*	0.149
S&P 500	0.758	0.677	0.851	*

When comparing the results in tables 4 and 5 it is apparent that there are the same trends in both the tables. For example, FTSE 100 has a quite low weight in a portfolio constructed with S&P 500. By selecting the strategy of minimizing the downside risk ratio, the FTSE 100 has a lower weight in a portfolio than in the strategy of maximizing the Sharpe ratio. It indicates that investors who care more about the downside risk should have less weight in that index or even invest nothing in FTSE 100 for their portfolio.

5.3 The optimal weights for a portfolio with more than two assets

Previously, a portfolio of two assets was constructed. Here a more complex level of portfolio construction will be presented with a more sophistical math solution. Theoretically, this new method can be used to construct a portfolio with even more assets available, however due to limited time and complexity a portfolio that includes all the four indexes will be constructed.

The Nelder-Mead Method is used to find the optimal weights for the indexes. First, the weight for different assets is assumed. For example, $W_1 W_1$ is the weight for SH, W_2 W_2 is the weight for HSI, W_3 is the weight for S&P 500, and 1- W_1-W_2-$W_3 W_1$ W_2 is the weight for FTSE 100. Second, a function to calculate the downside risk ratio or the Sharpe ratio is set up. Third, the constraints for the weights are:

$$0 \leq W_1 \geq 1$$
$$0 \leq W_2 \geq 1$$
$$0 \leq W_3 \geq 1$$
$$0 \leq W_1 + W_2 + W_3 \geq 1$$

The Nelder-Mead Method is then used to find the optimal weights. One concern of using the Nelder-Mead Method to find the optimal weights is the possible local optimal points. To avoid these points different start number were tested for W_1, W_2 and W_3. It is indicated by the method that if the optimal weights converge to the same number, it is the global optimal point.

Table 6. The optimal weights for the downside risk ratio.

SH	HSI	FTSE 100	S&P 500
26.24%	24.79%	0%	48.97%

Table 7. The optimal weights for the Sharpe ratio.

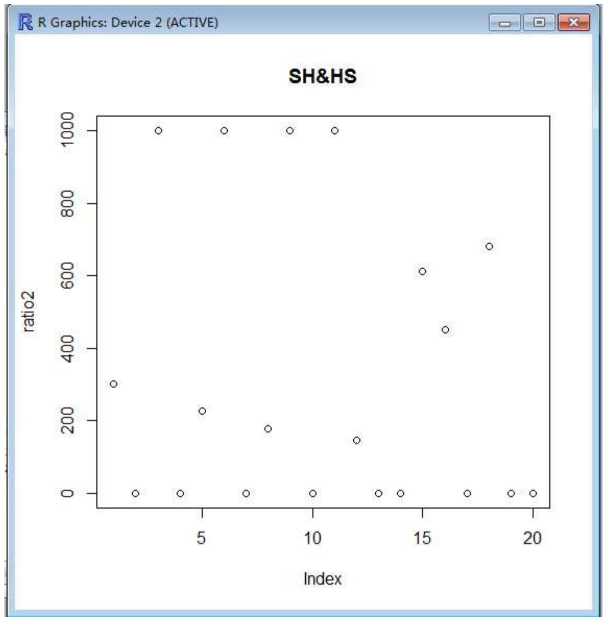
SH	HSI	FTSE 100	S&P 500
18.71%	22.59%	10.48%	48.22%

In line with previous results S&P 500 has the largest weight for both the strategies. The 20 years daily returns for S&P 500 have a relative high mean value and a relative smaller risk, it does not matter whether it is measured by the variance or the expected shortfall. However, for FTSE 100 it is suggested to put zero weight in the downside risk ratio strategy compared to 10.48% allocation in the Sharpe ratio strategy. This is consistent with the results in a portfolio constructed with two indexes. It implies that investors who are more concerned about the downside risk should allocate less investment in FTSE 100. If a portfolio includes only one index the downside risk of the portfolio will not be diversified but the return will decrease when adding investment in FTSE 100. When comparing the numbers in tables 6 and 7, the weights of S&P500 and HSI are quite stable while the weights of FTSE 100 and SH are more volatile. It suggests that FTSE 100 has more exposure to the downside risk than the variance and the variance of the SH returns have more upside volatility (good volatility) than on average.

5.4 Stabilities of the optimal weights

In this section the sensitivity of the optimal weights to different years is investigated. As before, 6 portfolios were constructed with all the four indexes and each portfolio consists of two indexes. Then yearly data was used to obtain the yearly optimal weights by minimizing the downside risk ratio. As can be noticed from figure 6, which shows the optimal weights for the portfolio containing the indexes SH and HIS, the weights vary significantly. Similar results for other portfolios are presented in appendix 3. The same procedure was repeated for the strategy of maximizing the Sharpe ratio the results are presented in appendix 4.

Figure 6. Optimal weights for Shanghai Composite Index and Hang Seng Index, minimizing the downside risk ratio.



The optimal weights are highly sensitive between years and as a result the forecast ability of the optimal weights can be questioned. Some reverse trend can be observed most of the time; a high weight in one year for a certain index implies a low weight in the next year for the same index. However, due to the limited time period of observed data the statistical significance cannot be proven.

6. Conclusion

Based on previous research and the empirical study results, investors who focus more on the downside risk can use the downside risk ratio as a measure to adjust the risk return. The downside risk ratio constructed in this paper excludes the good volatile which is in contrast to the Sharpe ratio computation. By minimizing the downside risk ratio, investors can obtain the best downside risk adjusted return, similar to the situation where the investors can obtain the best risk adjusted return by maximizing the Sharpe ratio. If the distribution of the assets is known by the investors, the methodology described before can be used to optimize the asset allocation according to investors' definition of risk. Investors that are more concerned about downside risk can select the strategy to minimize the downside risk ratio instead of the strategy maximizing the Sharpe ratio. According to the empirical results, the two different investment strategies suggest relatively different asset allocations. However, not surprisingly the historical results show that both the Sharpe ratio and the downside risk ratio are quite sensitive to sample data as they vary from period to period, implying that the forecasting ability of these optimal weights might be limited.

References

Angelidis, T. and Degiannakis, S., 2007. Econometric Modeling of Value-at-Risk. New Econometric Modeling Research.

Artzner, P., Delbaen, F., Eber, J.M. and Heath, D, 1999. Coherent Measures of Risk. *Mathematical Finance*, 9(3), pp. 203-228.

Baudin, M., 2010. *Nelder-Mead User's Manual*. Le Chesnay Cedex: Consortium Scilab.

Boudt, K., Peterson, B.G., and Carl, P. (2008). Hedge fund portfolio selection with modified expected shortfall. *MPRA Paper No. 7126*.

Bradley, B.O, and Taqqu, M.S. (2003). *Handbook of Heavy Tailed Distributions in Finance.* Chapter 2 Financial risk and heavy tails, pp. 35 – 103.

Brooks, C., 2008. *Indroductory econometrics for Finance.* 2nd edition. Cambridge: Cambridge University Press.

Culp, C.L., 2001. *The Risk Management process: Business Strategy and Tactics.* New York: John Wiley & Sons.

Dowd, K., 2005. *Measuring Market Risk*. 2nd edition. Chichester: John Wiley & Sons.

Goorbergh, R. and Vlaar, P., 1999. Value-at-Risk Analysis of Stock Returns Historical Simulation, Variance Techniques or Tail Index Estimation? *DNB Staff Reports*, 40, pp. 1-37.

Grootveld, H. and Hallerback, W., 1999. Variance vs downside risk: Is there really that much difference? *European Journal of Operational Research*, 114, pp. 304-319.

Han, L. and Neumann, M., 2006. Effect of dimensionality on the Nelder-Mead simplex method. *Optimization Methods and Software,* 21(1), pp. 1-16.

Han, R., Hu, W. and Lu, N., 2005. Choice Research of Non-risk Interest Rate while Fixing the Price in the Chinese Financial Assets. *Inquiry into economic problems*, 6, pp. 87-91.

Hull, J. and White, A., 1998. Incorporating volatility updating into the historical simulations method for Value-at-Risk. *Journal of Risk*, 1(Fall), pp. 5-19.

Inui, K. and Kijima, M., 2005. On the significance of expected shortfall as a coherent risk measure. *Journal of Banking & Finance*, 29, pp. 853-864.

Johansson, F., Seiler, M. J. and Tjarnberg, M., 1999. Measuring Downside Portfolio Risk. *Journal of Portfolio Management*, 26(1), pp. 96-106.

Jorion, P., 2007. *Value at Risk: The New Benchmark for Managing Financial Risk.* 3rd edition. New York: McGraw-Hill.

Lagaris, J.C, James, J.A, Wright, M.H. and Wright, P.E., 1998. Convergence Properties of the Nelder-Mead Simplex Method in Low Dimensions. *SIAM Journal on Optimization,* 9(1), pp. 112-147.

Liberti, L., 2006. Writing Global Optimization Software. In: Liberti, L. and Maculan, N., eds. 2006. *Global Optimization: From Theory to Implementation.* New York: Springer Science+Business Media, Inc.

Landsman, Z. and Valdez, E. 2005. Tail Conditional Expectation for Exponential Dispersion Models. *ASTIN Bulletin*, 35(1), pp.189-209.

Markowitz, H., 1952. Portfolio Selection. *The Journal of Finance*, 7(1), pp. 77-91.

Rachev, S., Jasic, T., Stoyanov, S. and Fabozzi, F.J., 2007. Momentum strategies based on reward-risk stock selection criteria. *Journal of Banking & Finance,* 31, pp. 2325-2346.

Scott, R.C. and Horvath, P.A., 1980. On the Direction of Preference for Moments of Higher Order than the Variance. *The Journal of Finance,* 35(4), pp. 915-919.

Shanghai Stock Exchange. [online] Available at: < http://www.sse.com.cn/sseportal/index/cn/i000001/intro.shtml> [Accessed 14 April 2012].

The People's Bank of China. [online] Available at: <http://www.pbc.gov.cn/publish/zhengcehuobisi/631/2011/20110406165642557972308/20110406165642557972308_.html> [Accessed 14 April 2012].

Unified Trust, 2007? *Asset allocation optimization using downside risk analysis*. [Leaflet] February 2007? Lexington: Unified Trust.

Yahoo Finance. [online] Available at: <http://finance.yahoo.com/q/hp?s=%5EFTSE+Historical+Prices> [Accessed 14 April 2012].

Yahoo Finance. [online] Available at: <http://finance.yahoo.com/q/hp?s=%5EGSPC+Historical+Prices> [Accessed 14 April 2012].

Yahoo Finance. [online] Available at: <http://finance.yahoo.com/q/hp?s=%5EHSI+Historical+Prices> [Accessed 14 April 2012].

Yamai, Y. and Yoshiba, T., 2002. On the Validity of Value-at-Risk: Comparative Analyses with Expected Shortfall. *Monetary and Economic Studies*, 20(1), pp. 57-85.

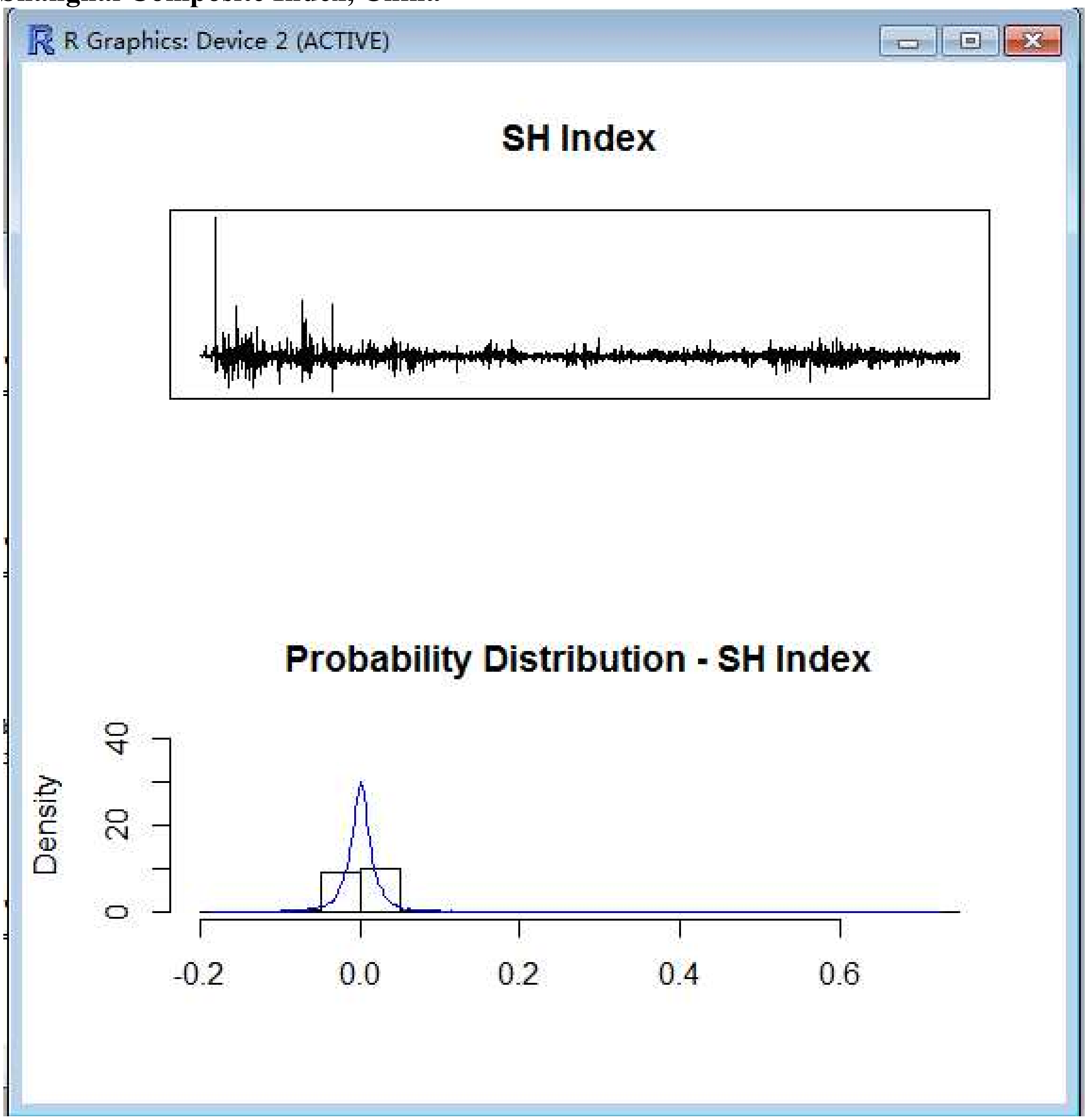
Yamai, Y. and Yoshiba, T., 2005. Value-at-risk versus expected shortfall: A practical perspective. *Journal of Banking & Finance*, 29(4), pp. 997-1015.

Zakamouline, V., 2010. On the Consistent Use of VaR in Portfolio Performance Evaluation: A Cautionary Note. *Journal of portfolio management,* 37(1), pp. 92-104.

Appendix

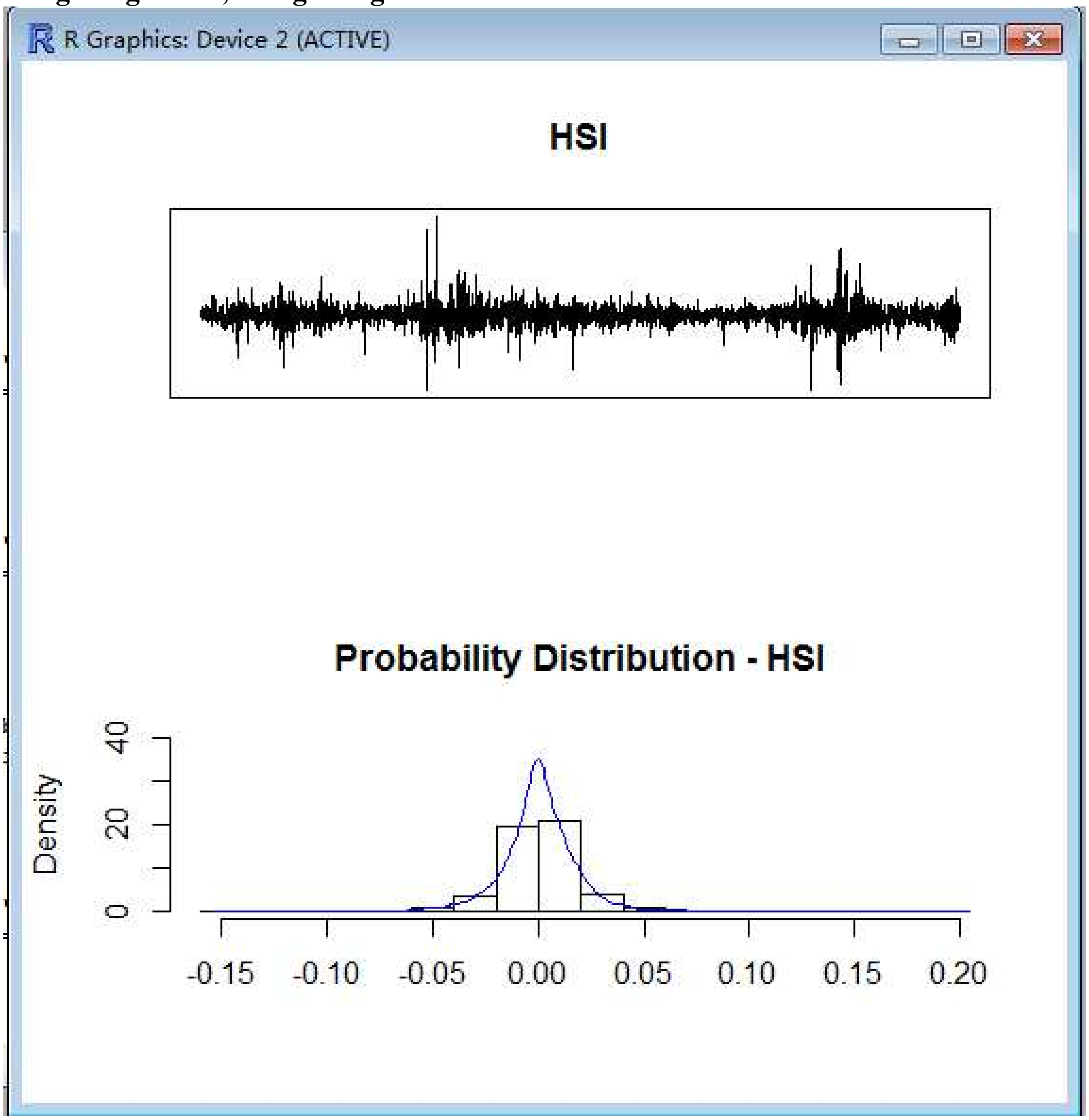
Appendix 1: Description of the returns for the indexes

Shanghai Composite Index, China



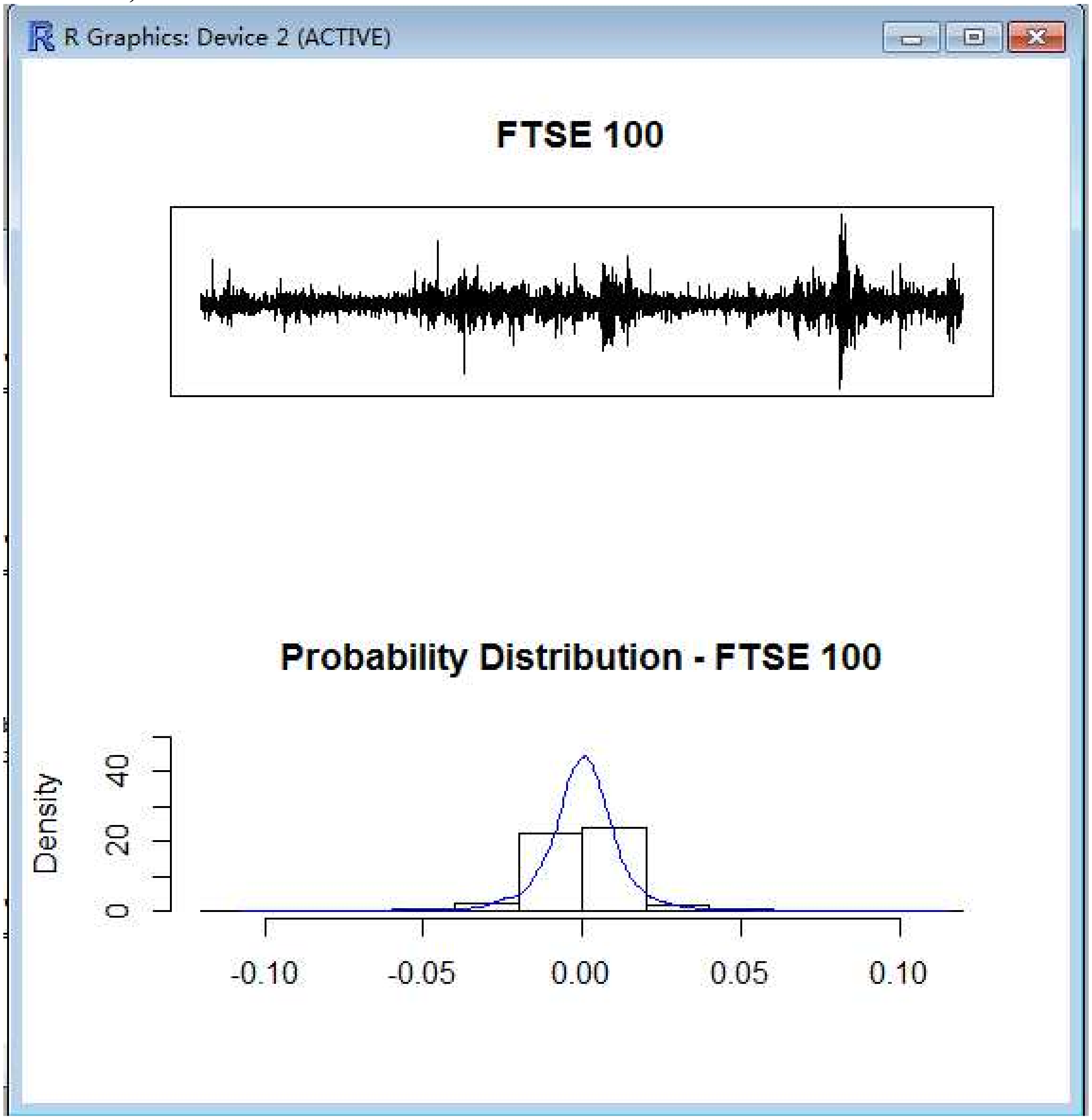
The graph above shows the daily log returns for the SH index for the observed time zone (1st of Jan 1992 to 31st of Dec 2011). The graph beneath shows the density distribution of the returns for the SH index.

Hang Seng Index, Hong Kong



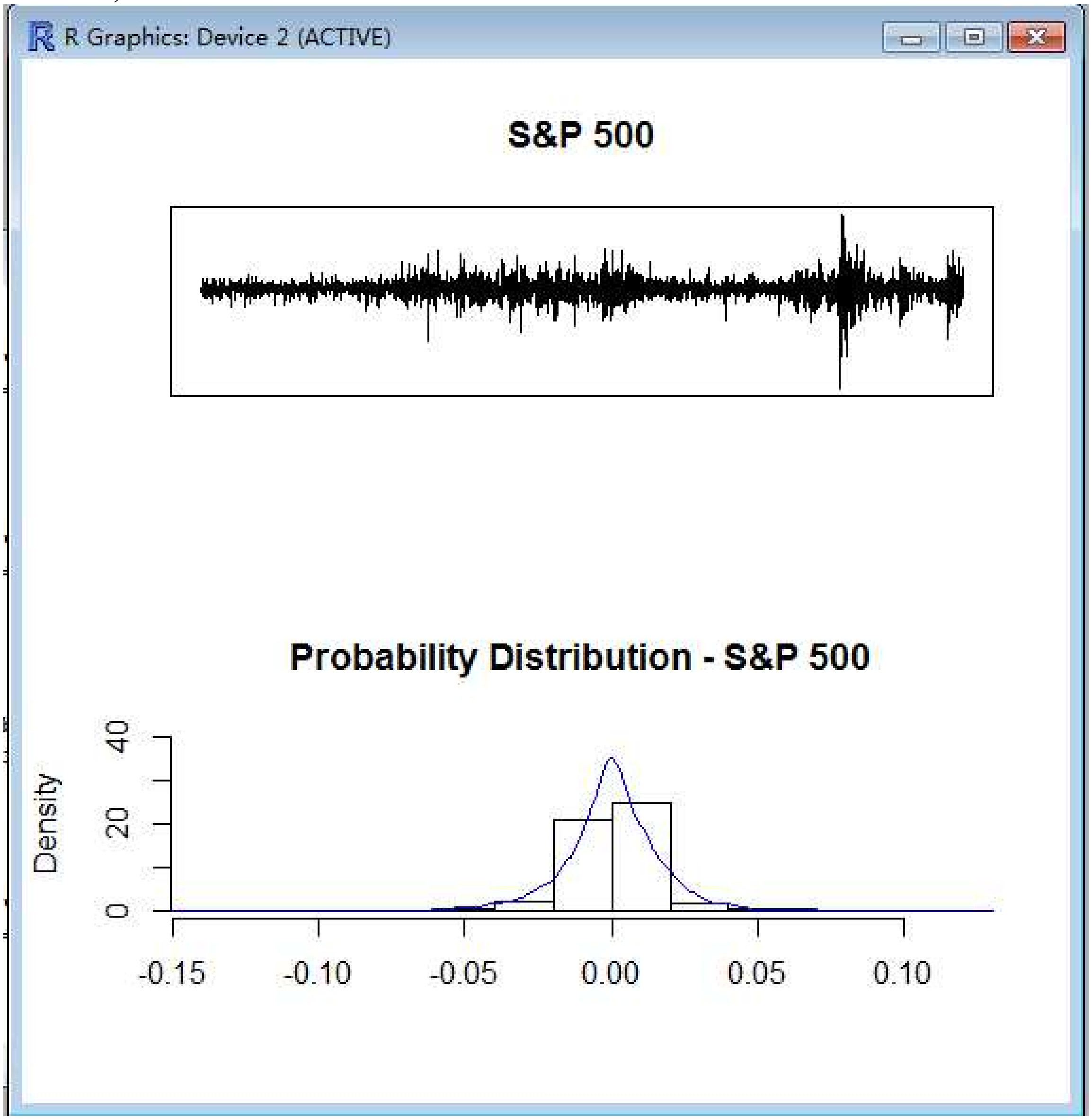
The graph above shows the daily log returns for the HSI for the observed time zone (1st of Jan 1992 to 31st of Dec 2011). The graph beneath shows the density distribution of the returns for the HSI.

FTSE 100, UK



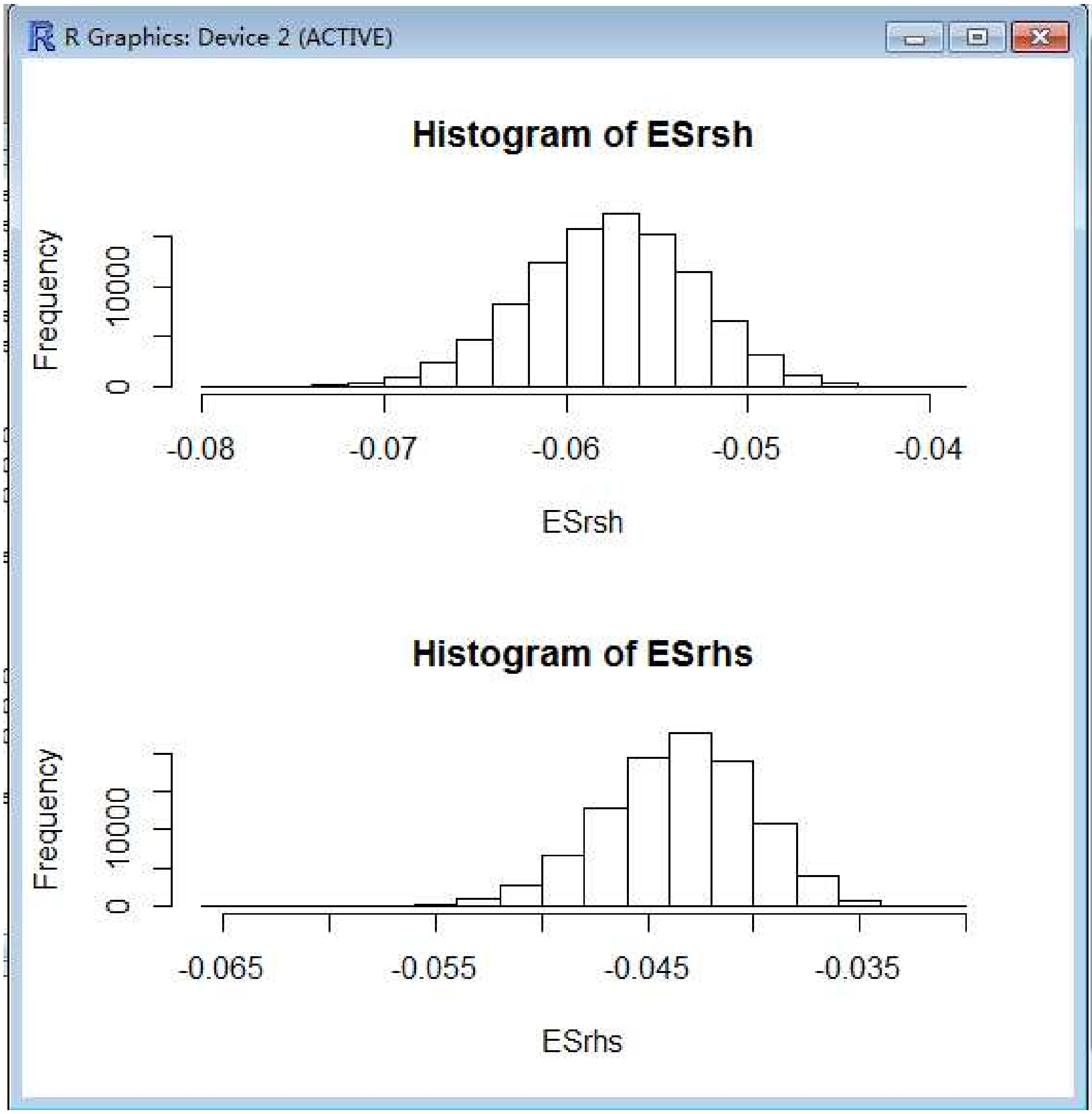
The graph above shows the daily log returns for the FTSE 100 for the observed time zone (1st of Jan 1992 to 31st of Dec 2011). The graph beneath shows the density distribution of the returns for the FTSE 100.

S&P 500, US

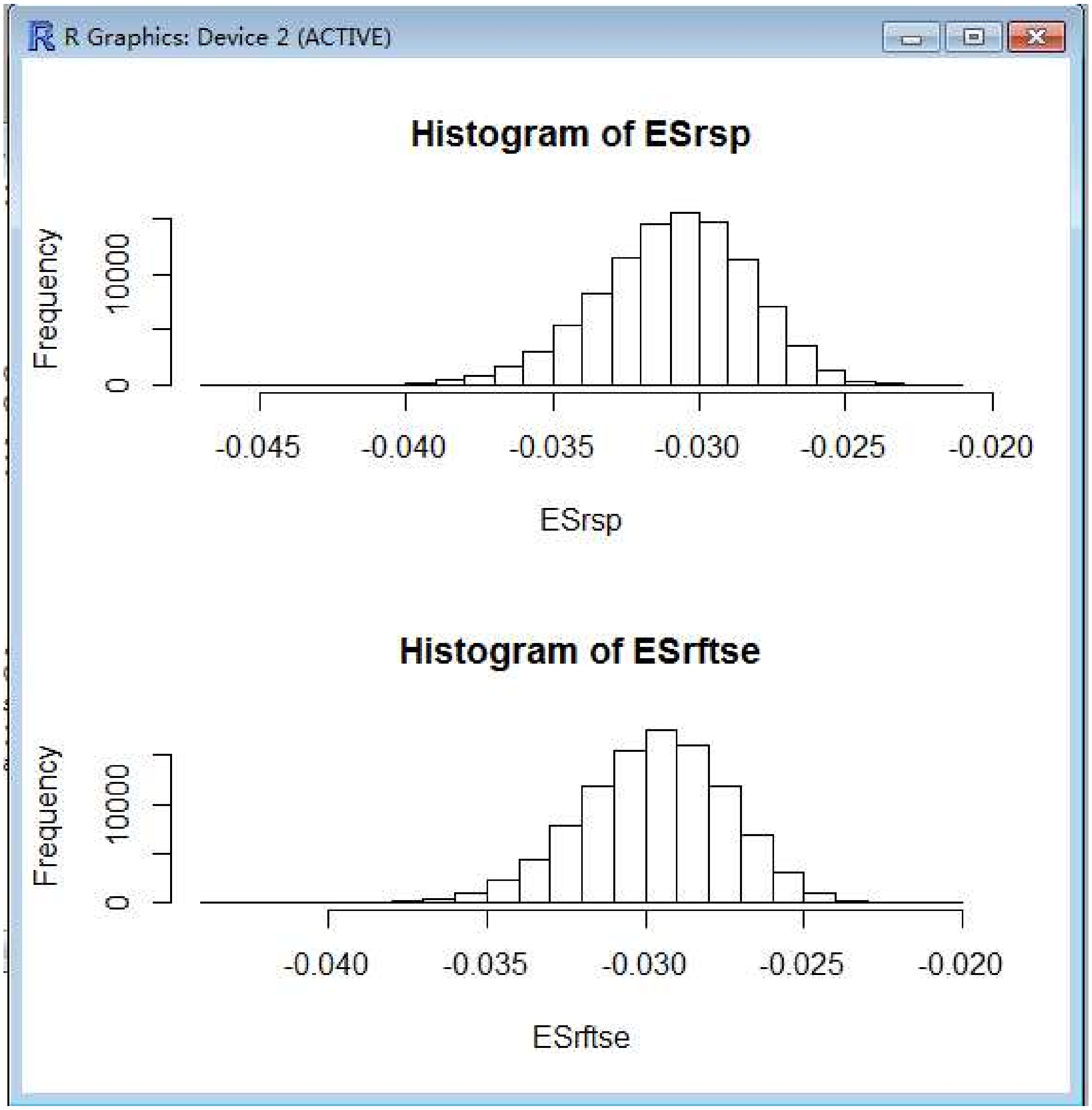


The graph above shows the daily log returns for the S&P 500 for the observed time zone (1^{st} of Jan 1992 to 31^{st} of Dec 2011). The graph beneath shows the density distribution of the returns for the S&P 500.

Appendix 2: Distribution of the ES (simulation results)

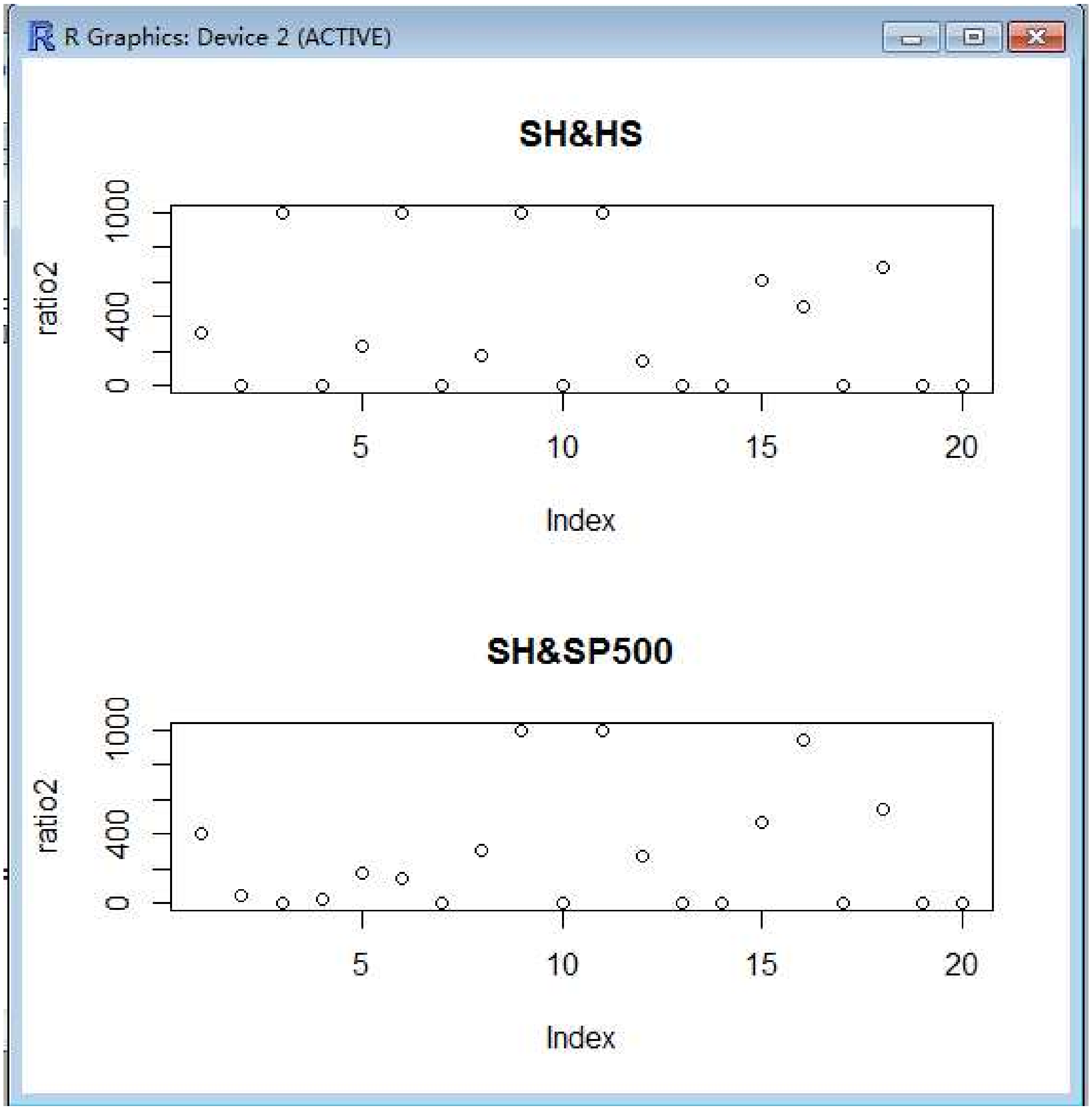


The graph above is the distribution of the bootstrapping results of the expected shortfall for the SH index, based on the 20 years' historical data. The graph beneath is the distribution of the bootstrapping results of the expected shortfall for the HSI, based on the 20 years' historical data.

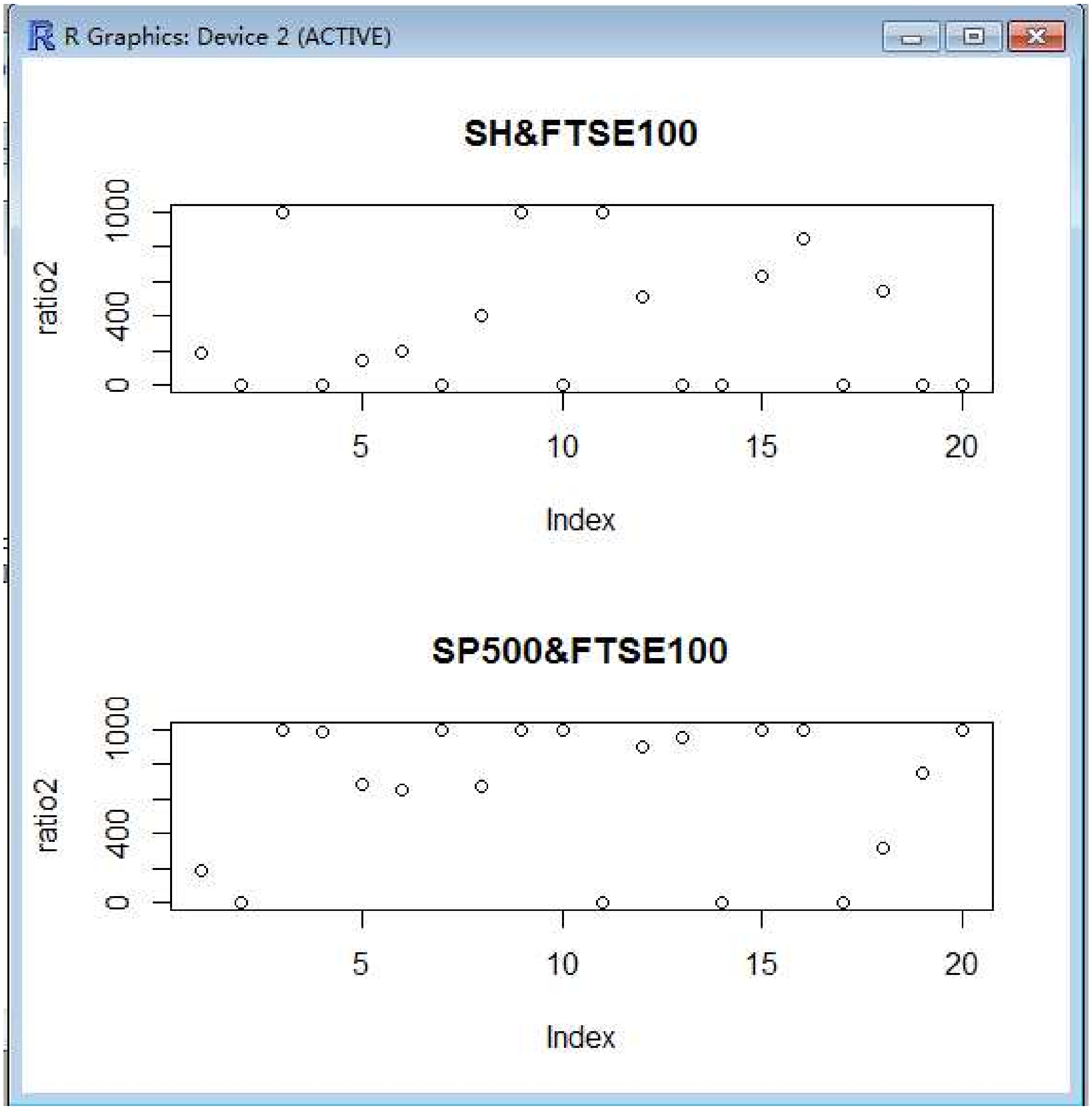


The graph above is the distribution of the bootstrapping results of the expected shortfall for the S&P 500, based on the 20 years' historical data. The graph beneath is the distribution of the bootstrapping results of the expected shortfall for the FTSE 100, based on the 20 years' historical data.

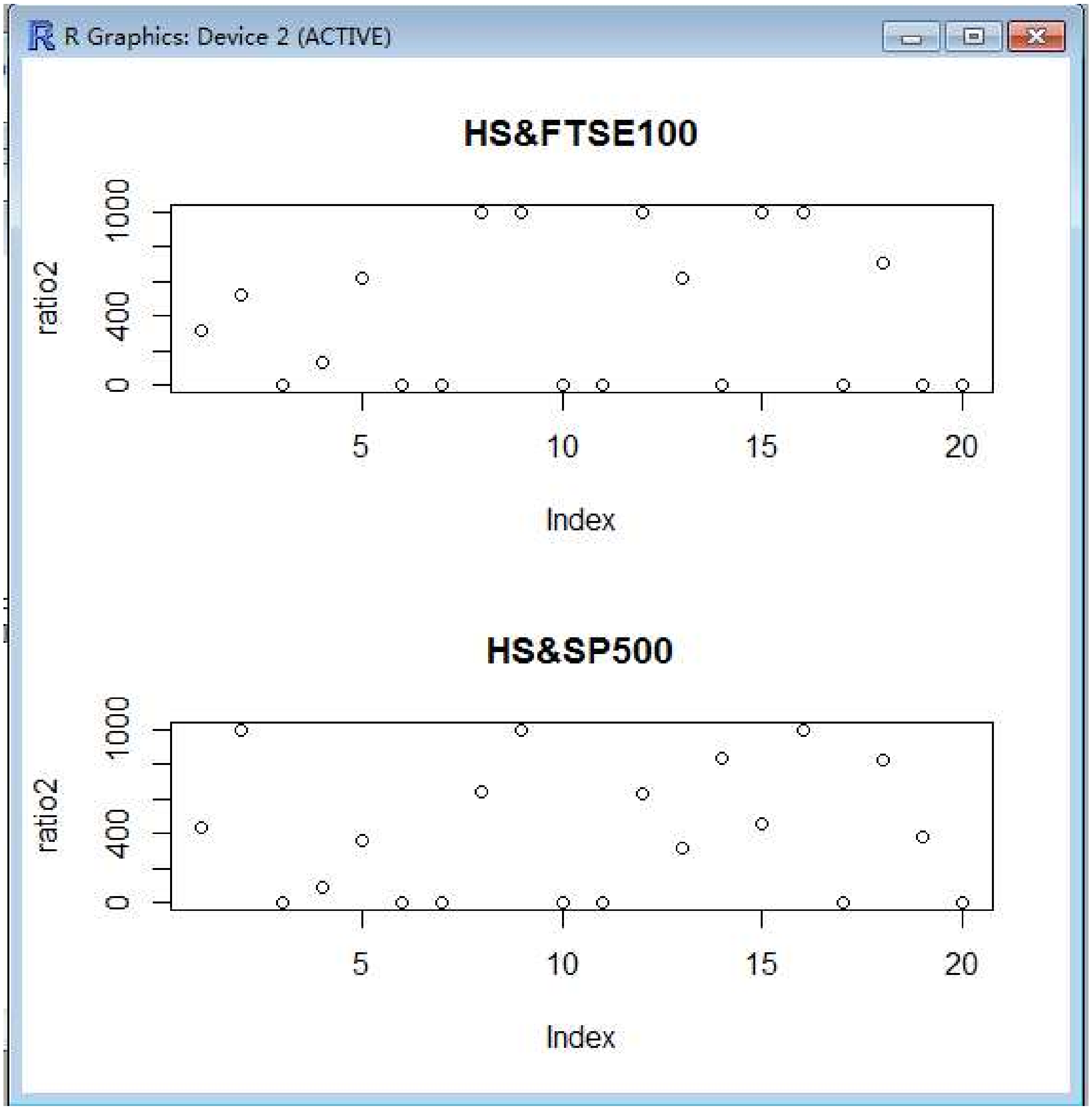
Appendix 3: Optimal weights for different years, the downside risk ratio



The graph above shows the optimal weights for the portfolio of the SH index and the HSI in the observed 20 years by minimizing the downside risk ratio. The graph beneath shows the optimal weights for the portfolio of the SH index and the S&P 500 in the observed 20 years by minimizing the downside risk ratio.

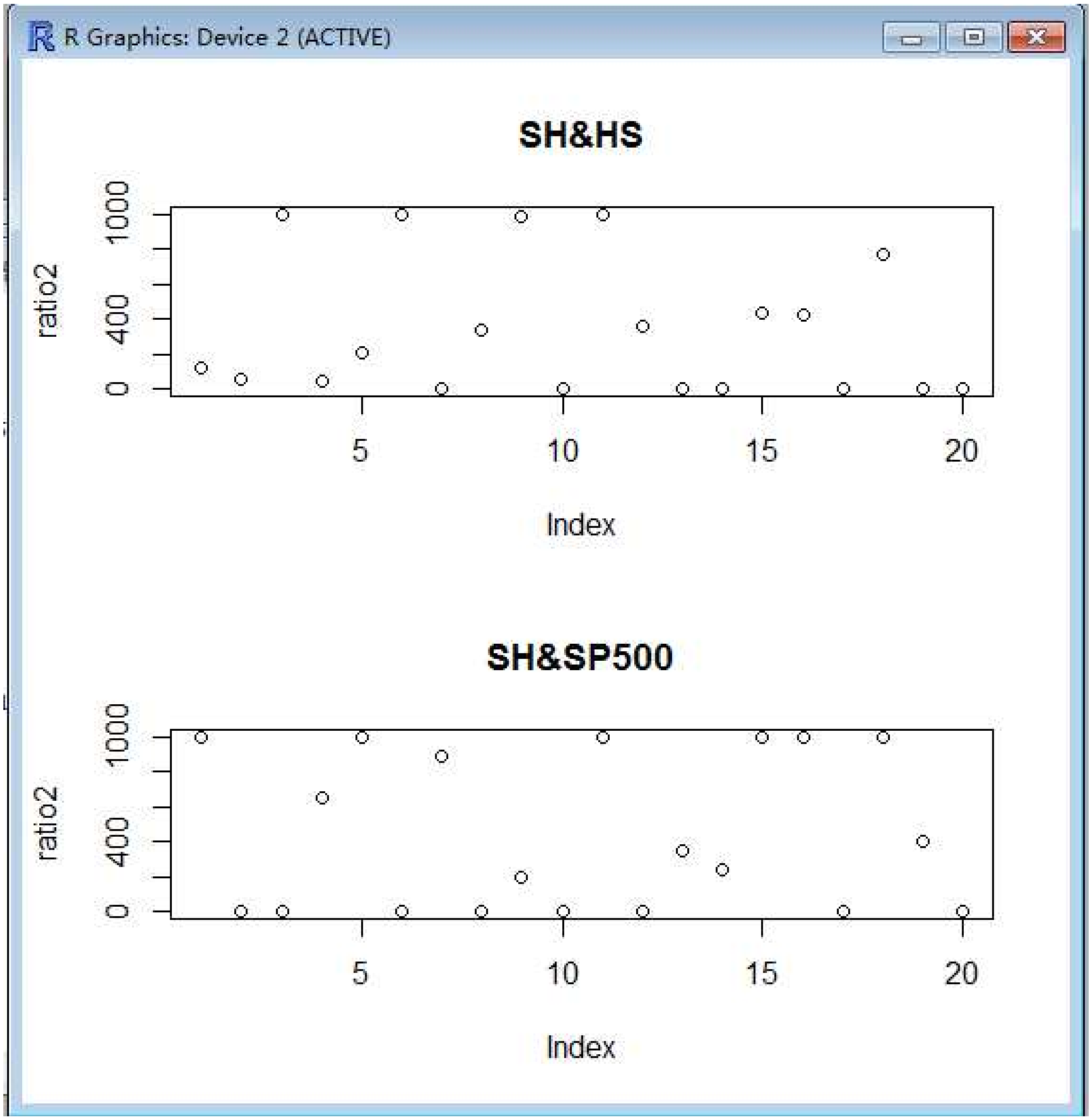


The graph above shows the optimal weights for the portfolio of the SH index and the FTSE 100 in the observed 20 years by minimizing the downside risk ratio. The graph beneath shows the optimal weights for the portfolio of the S&P 500 and the FTSE 100 in the observed 20 years by minimizing the downside risk ratio.

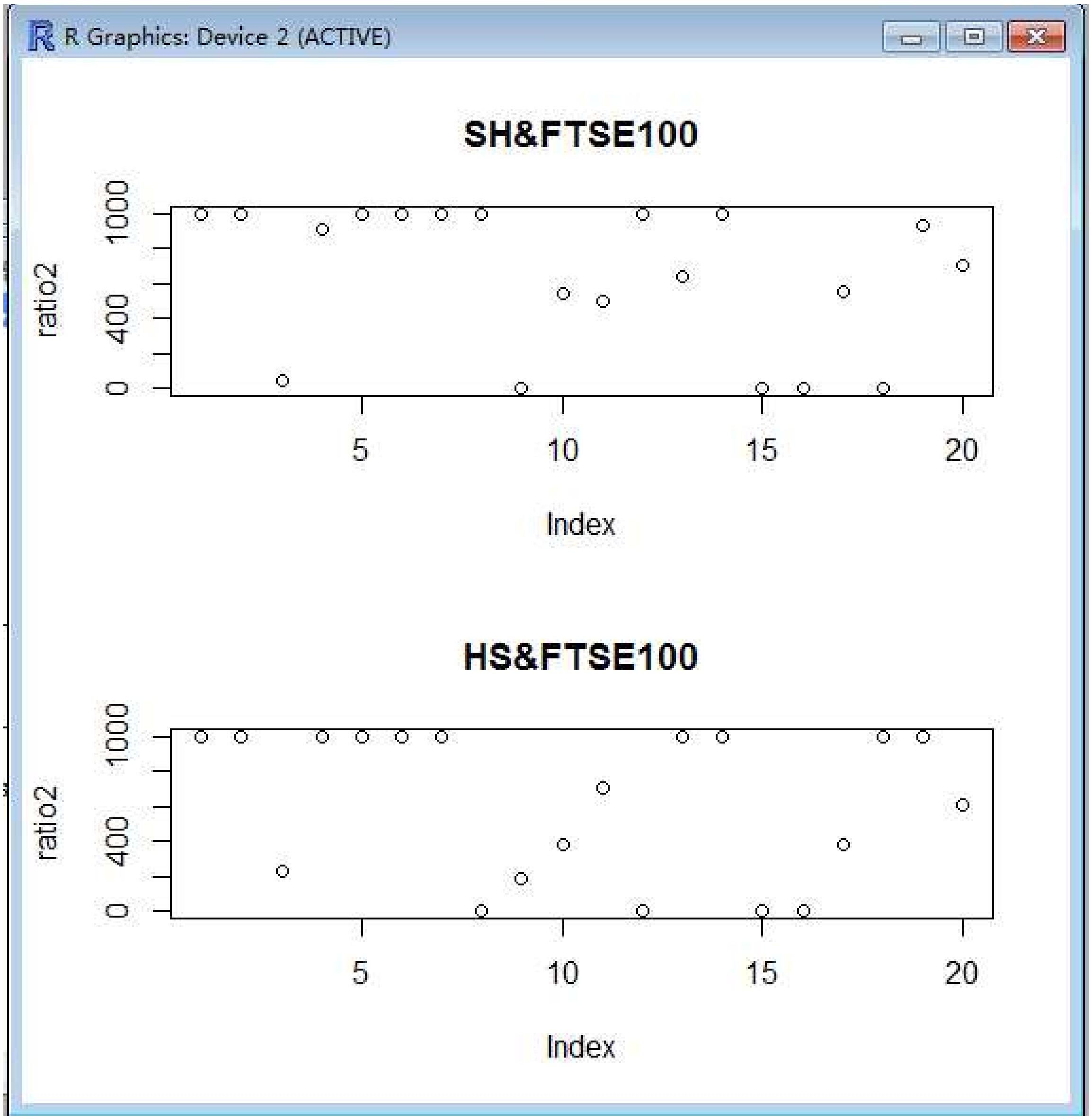


The graph above shows the optimal weights for the portfolio of the HSI and the FTSE 100 in the observed 20 years by minimizing the downside risk ratio. The graph beneath shows the optimal weights for the portfolio of the HSI and the S&P 500 in the observed 20 years by minimizing the downside risk ratio.

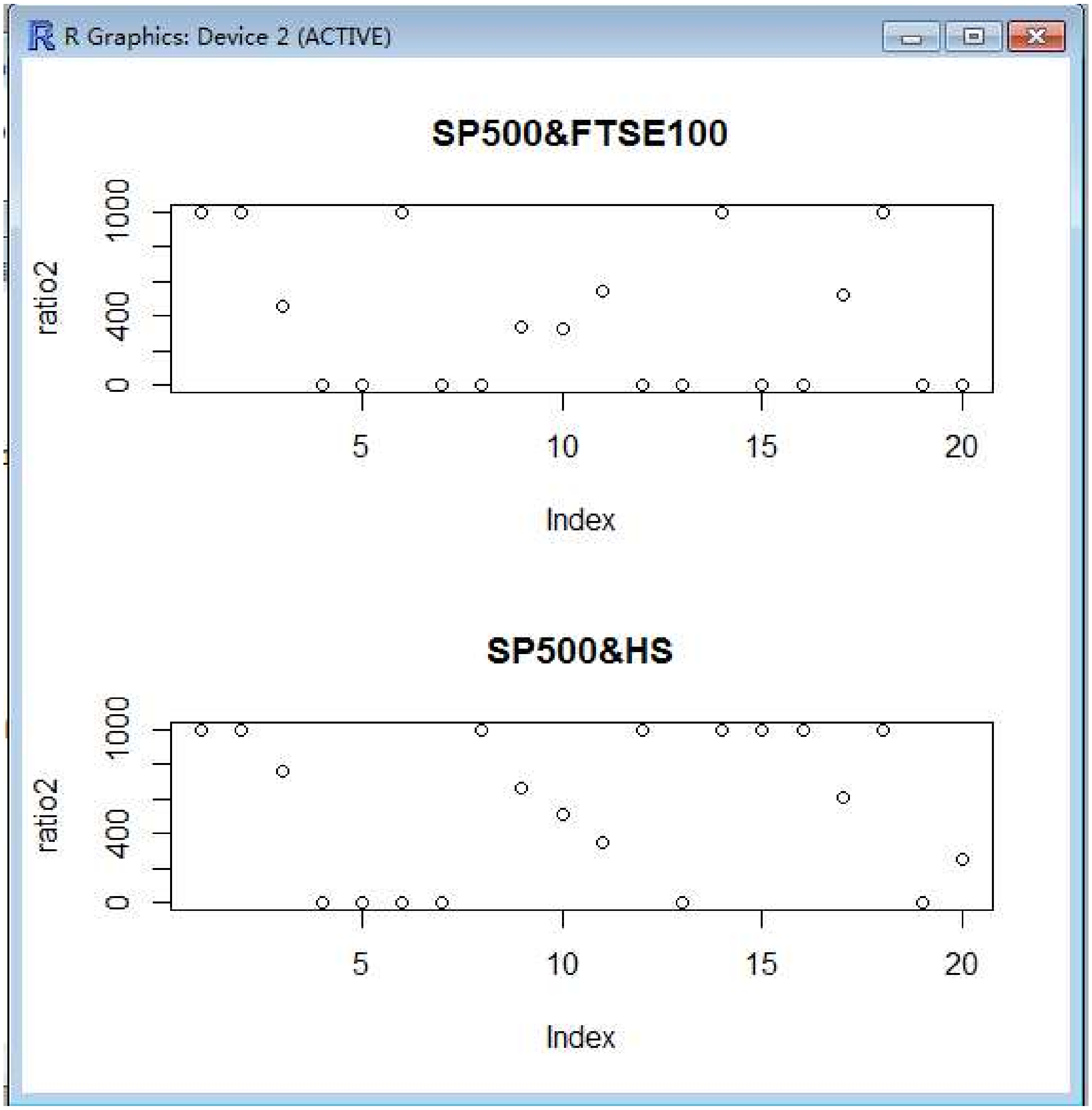
Appendix 4: Optimal weights for different years, the Sharpe ratio



The graph above shows the optimal weights for the portfolio of the SH index and the HSI in the observed 20 years by maximizing the Sharpe ratio. The graph beneath shows the optimal weights for the portfolio of the SH index and the S&P500 in the observed 20 years by maximizing the Sharpe ratio.



The graph above shows the optimal weights for the portfolio of the SH index and the FTSE 100 in the observed 20 years by maximizing the Sharpe ratio. The graph beneath shows the optimal weights for the portfolio of the HSI and the FTSE 100 in the observed 20 years by maximizing the Sharpe ratio.



The graph above shows the optimal weights for the portfolio of the S&P 500 and the FTSE 100 in the observed 20 years by maximizing the Sharpe ratio. The graph beneath shows the optimal weights for the portfolio of the S&P 500 and the HSI in the observed 20 years by maximizing the Sharpe ratio.

Appendix 5: Programming scripts

The folloing codes are programmed in VBA

Code 1 - Used to sort the data

```
Sub findtradingdays()

Dim firstdate, lastdate, interdate,
columndate As Date
Dim arr(20) As Integer

firstdate = "1 - 1 - 1992"
lastdate = "30-12-2011"

lastrow = Cells.Find("*", _
  SearchOrder:=xlByRows,
LookIn:=xlFormulas, _

SearchDirection:=xlPrevious).EntireRo
w.Row

n = DateDiff("yyyy", firstdate, lastdate)

For i = 1 To 20

interdate = DateAdd("yyyy", i, firstdate)
y = 2

   For j = y To lastrow

    columndate = Range("a" & j).Value
    If columndate > interdate Then

       Range("c" & i).Value = j - 1
       y = j - 3
       Exit For

    Else

    End If

   Next j


Next i

End Sub
```

Code 2 - Used to find the trading days in every year

```
Sub findtradingdays()

Dim firstdate, lastdate, interdate,
columndate As Date
Dim arr(20) As Integer

firstdate = "1 - 1 - 1991"
lastdate = "31-12-2011"

lastrow = Cells.Find("*", _
  SearchOrder:=xlByRows,
LookIn:=xlFormulas, _

SearchDirection:=xlPrevious).EntireRo
w.Row

n = DateDiff("yyyy", firstdate, lastdate)

For i = 1 To 20

interdate = DateAdd("yyyy", i, firstdate)
y = 3

   For j = y To lastrow

    columndate = Range("a" & j).Value
    If columndate > interdate Then

       Range("c" & i).Value = j - 3
       y = j - 3
       Exit For
    Else
    End If
   Next j
Next i

End Sub
```

The following codes are programmed in Rgui

Code 3 - Used to describe the data

```
data=read.csv("d://data.csv")
attach(data)
shreturn=diff(log(SH))
hsreturn=diff(log(HS))
ftsereturn=diff(log(FTSE))
spreturn=diff(log(SP))
nreturn=diff(log(N225))

#data description

par(mfrow=c(2,1))
plot(shreturn,
xaxt="n",yaxt="n",xlab="",ylab="",type
="l",main="SH Index")
hist(shreturn,freq=F,ylim=c(0,40),xlab="
",main="Kernel Density - SH Index")
lines(density(shreturn),col="blue")

plot(hsreturn,
xaxt="n",yaxt="n",xlab="",ylab="",type
="l",main="HK Index")
hist(hsreturn,freq=F,ylim=c(0,40),xlab="
",main="Kernel Density - HK Index")
lines(density(hsreturn),col="blue")

plot(ftsereturn,
xaxt="n",yaxt="n",xlab="",ylab="",type
="l",main="FTSE 100")
hist(ftsereturn,freq=F,ylim=c(0,40),xlab
="",main="Kernel Density - FTSE 100")
lines(density(ftsereturn),col="blue")

plot(spreturn,
xaxt="n",yaxt="n",xlab="",ylab="",type
="l",main="S&P 500")
hist(spreturn,freq=F,ylim=c(0,40),xlab="
",main="Kernel Density - S&P 500")
lines(density(hsreturn),col="blue")

plot(nreturn,
xaxt="n",yaxt="n",xlab="",ylab="",type
="l",main="N225")
hist(nreturn,freq=F,ylim=c(0,40),xlab=""
,main="Kernel Density - N225")
lines(density(hsreturn),col="blue")

#write a function to describe the statistic
of the return
myfunction1<-function(x){
mean<-mean(x)
var<-var(x)
kurtosis<-kurtosis(x)
skewness<-skewness(x)
result<-list(mean,var,kurtosis,skewness)
return(result)
}

myfunction1(shreturn)
myfunction1(hsreturn)
myfunction1(ftsereturn)
myfunction1(spreturn)
myfunction1(nreturn))
jarque.test(shreturn)
jarque.test(hsreturn)
jarque.test(spreturn)
jarque.test(ftsereturn)


par(mfrow=c(2,1))

plot(shreturn,
xaxt="n",yaxt="n",xlab="",ylab="",type
="l",main="SH Index")
hist(shreturn,freq=F,ylim=c(0,40),xlab="
",main="Kernel Density - SH Index")
lines(density(shreturn),col="blue")

par(mfrow=c(2,1))

plot(hsreturn,
xaxt="n",yaxt="n",xlab="",ylab="",type
="l",main="HK Index")
hist(hsreturn,freq=F,ylim=c(0,40),xlab="
",main="Kernel Density - HK Index")
```

```
lines(density(hsreturn),col="blue")
par(mfrow=c(2,1))

plot(ftsereturn,
xaxt="n",yaxt="n",xlab="",ylab="",type
="l",main="FTSE 100")
hist(ftsereturn,freq=F,ylim=c(0,40),xlab
="",main="Kernel Density - FTSE 100")
lines(density(ftsereturn),col="blue")

par(mfrow=c(2,1))

plot(spreturn,
xaxt="n",yaxt="n",xlab="",ylab="",type
="l",main="S&P 500")
hist(spreturn,freq=F,ylim=c(0,40),xlab="
",main="Kernel Density - S&P 500")
lines(density(hsreturn),col="blue")
```

Code 4 - Used to find the new ratio and Sharpe ratio for the four indexes.

```
##calculate the new ratio for the
portfolio
ESrsh=rep(0,100000)
for(i in 1:100000){
rsh=sample(sh,1000,replace=T)
rssh=sort(rsh)
ESrsh[i]=mean(rssh[1:50])
}
ssh=sort(sh)
shs=sort(hs)
sftse=sort(ftse)
ssp=sort(sp)


p=0.05
#the ratio used as the worst p%
op=p*T
ESsh<-mean(ssh[1:op])
EShs<-mean(shs[1:op])
ESftse<-mean(sftse[1:op])
ESsp<-mean(ssp[1:op])
list(ESsh,EShs,ESftse,ESsp)

meansh<-mean(sh)
meanhs<-mean(hs)
meanftse<-mean(ftse)
meansp<-mean(sp)


ratiosh<-(meansh-mean(rfsh))/ESsh
ratiohs<-(meanhs-mean(rfhs))/EShs
ratioftse<-(meanftse-
mean(rfftse))/ESftse
ratiosp<-(meansp-mean(rfsp))/ESsp


sratiosh<-(meansh-mean(rfsh))/var(sh)
sratiohs<-(meanhs-mean(rfhs))/var(hs)
sratioftse<-(meanftse-
mean(rfftse))/var(ftse)
sratiosp<-(meansp-mean(rfsp))/var(sp)
```

Code 5 - Used to find the optimal points for a portfolio with only 2 assets

```
#Draw the picture to find the where the
optimal point may located
#HS&SH
w=seq(0,1,0.001)
length(w)-1
ratio1<-matrix(0,length(w)-1,1)

preturn<-spreturn<-
matrix(0,length(sh),1)
for(i in 1:length(w)-1){
preturn<-(i/1000)*sh+(1-i/1000)*hs
sspreturn<-sort(preturn)
ESp<-mean(sspreturn[1:op])
meanp<-mean(preturn)
rfp<-(i/1000)*rfsh+(1-i/1000)*rfhs
ratio1[i]=(meanp-mean(rfp))/ESp
}
ratio1
plot(ratio1,main="New
Ratio",ylab="new
ratio",xlab="weights",xaxt="n",ylim=c(-
0.009,0),type="l")
which(ratio1==min(ratio1))

#HS&FTSE
w=seq(0,1,0.001)
length(w)-1
ratio1<-matrix(0,length(w)-1,1)

preturn<-spreturn<-
matrix(0,length(sh),1)
for(i in 1:length(w)-1){
preturn<-(i/1000)*ftse+(1-i/1000)*hs
sspreturn<-sort(preturn)
ESp<-mean(sspreturn[1:op])
meanp<-mean(preturn)
rfp<-(i/1000)*rfftse+(1-i/1000)*rfhs
ratio1[i]=(meanp-mean(rfp))/ESp
}
lines(ratio1,col="grey")
which(ratio1==min(ratio1))
```

```
#HS&SP
w=seq(0,1,0.001)
length(w)-1
ratio1<-matrix(0,length(w)-1,1)

preturn<-spreturn<-
matrix(0,length(sh),1)
for(i in 1:length(w)-1){
preturn<-(i/1000)*sp+(1-i/1000)*hs
sspreturn<-sort(preturn)
ESp<-mean(sspreturn[1:op])
meanp<-mean(preturn)
rfp<-(i/1000)*rfsp+(1-i/1000)*rfhs
ratio1[i]=(meanp-mean(rfp))/ESp
}
ratio1
lines(ratio1,col="red")
which(ratio1==min(ratio1))

#SH&SP
w=seq(0,1,0.001)
length(w)-1
ratio1<-matrix(0,length(w)-1,1)

preturn<-spreturn<-
matrix(0,length(sh),1)
for(i in 1:length(w)-1){
preturn<-(i/1000)*sh+(1-i/1000)*sp
sspreturn<-sort(preturn)
ESp<-mean(sspreturn[1:op])
meanp<-mean(preturn)
rfp<-(i/1000)*rfsh+(1-i/1000)*rfsp
ratio1[i]=(meanp-mean(rfp))/ESp
}
lines(ratio1,col="blue")
which(ratio1==min(ratio1))

#SH&FTSE
w=seq(0,1,0.001)
length(w)-1
ratio1<-matrix(0,length(w)-1,1)

preturn<-spreturn<-
matrix(0,length(sh),1)
for(i in 1:length(w)-1){
preturn<-(i/1000)*sh+(1-i/1000)*ftse
sspreturn<-sort(preturn)
ESp<-mean(sspreturn[1:op])
meanp<-mean(preturn)
rfp<-(i/1000)*rfsh+(1-i/1000)*rfftse
ratio1[i]=(meanp-mean(rfp))/ESp
}
lines(ratio1,col="green")
which(ratio1==min(ratio1))


#SP&FTSE
w=seq(0,1,0.001)
length(w)-1
ratio1<-matrix(0,length(w)-1,1)

preturn<-spreturn<-
matrix(0,length(sh),1)
for(i in 1:length(w)-1){
preturn<-(i/1000)*sp+(1-i/1000)*ftse
sspreturn<-sort(preturn)
ESp<-mean(sspreturn[1:op])
meanp<-mean(preturn)
rfp<-(i/1000)*rfsp+(1-i/1000)*rfftse
ratio1[i]=(meanp-mean(rfp))/ESp
}
lines(ratio1,col="purple")
which(ratio1==min(ratio1))

axis(1,at=250,"0.25")
axis(1,at=500,"0.5")
axis(1,at=750,"0.75")
```

Code 6 - Used to find the optimal weights for the portfolio with all 4 indexes

```
T=length(shreturn)
p=0.05
op=p*T
fun1<-function(w){
w1=w[1]
w2=w[2]
w3=w[3]
preturn=w1*shreturn+w2*hsreturn+w3*
spreturn+(1-w1-w2-w3)*ukreturn
sspreturn=sort(preturn)
ESp=mean(sspreturn[1:op])
meanp=mean(preturn)
rfp=w1*rfsh+w2*rfhs+w3*rfsp+(1-w1-
w2-w3)*rfuk
ratio=(meanp-mean(rfp))/ESp
return(ratio)
}
library(stats)
constrOptim(c(0.1,0.2,0.3),fun1,NULL,u
i=rbind(c(-1,-1,-
1),c(1,0,0),c(0,1,0),c(0,0,1),c(-
1,0,0),c(0,-1,0),c(0,0,-1)),ci=c(-1,0,0,0,-
1,-1,-1))

fun2<-function(w){
w1=w[1]
w2=w[2]
w3=w[3]
preturn=w1*shreturn+w2*hsreturn+w3*
spreturn+(1-w1-w2-w3)*ukreturn
varp=var(preturn)
meanp=mean(preturn)
rfp=w1*rfsh+w2*rfhs+w3*rfsp+(1-w1-
w2-w3)*rfuk
ratio=(meanp-mean(rfp))/varp
return(ratio)
}
library(stats)
constrOptim(c(0.1,0.2,0.3),fun2,NULL,u
i=rbind(c(-1,-1,-
1),c(1,0,0),c(0,1,0),c(0,0,1),c(-
1,0,0),c(0,-1,0),c(0,0,-1)),ci=c(-1,0,0,0,-
1,-1,-1))
```