Credit Valuation Adjustment

Risk Capital Charge under Basel III

Johanna Carlsson and Anna Silén

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Anna Silén och Johanna Carlsson

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SAMMANFATTNING

This study explores the calculations behind Credit Valuation Adjustment (CVA) Risk Capital Charge under the Basel III framework and gives an example of how one can structure a model for the underlying CVA computations. Losses that can be attributed to CVA played an important role in the most recent financial crisis and is likely to continue being a highly influential component in the risk quantifying process in the financial sector. Our work shows that the quantification of CVA is far from trivial and requires both qualitative data input as well as a mathematically correct interpretation. By constructing our own model for CVA calculation based on the Basel III framework, a deeper knowledge of the complexity has been obtained, and as a demonstration a supporting case study of the CVA for a set of counterparties has been made. The conclusion of this work is that, to make a complete model for CVA calculation, one needs to carefully verify both input and choice of underlying models, as well as optimising the programming environment to ease the execution.
Key Words: Credit Valuation Adjustment, CVA Capital Risk Charge, Basel III, Counterparty Credit Risk, Interest Rate Swap, Over the Counter Derivatives, Cox Ingersoll Ross, Monte Carlo Simulation

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2 Preface

This thesis started as a proposal from Quantitative Advisory Services (QAS) at Ernst & Young (EY) in Copenhagen. It was suggested that we work with Counterparty Credit Risk (CCR) and more specifically develop a model for Credit Valuation Adjustment (CVA). Even though our previous experience in the area were quite limited we found the proposal very interesting. With an ongoing discussion about Credit Risk in general and the recent financial crisis still in mind, we felt really curious to take on the task. During our work with the thesis we have had a great support from the EY QAS team and we would like to give a special thanks to our supervisor Jim Gustafsson who has been encouraging and supportive through the entire process. We would also like to thank David Christopherson, Karin Gambe and Markus Wahlgren who have given us important input and comments on our work.

Last but not least, we would like to thank Magnus Wiktorsson and Erik Lindström at The Division of Mathematical Statistics at the Faculty of Engineering at Lund University for important and rewarding discussions regarding our work.

Anna Silén and Johanna Carlsson

Copenhagen
May 15 2012

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3 Acronyms

BCBS - Basel Committee on Banking Supervision
CEBS - Committee of European Banking Supervisors
CCR - Counterparty Credit Risk
CIR - Cox Ingersoll Ross
CVA - Credit Valuation Adjustment
EAD - Exposure At Default
EE - Expected Exposure
EL - Expected Loss
EPE - Expected Positive Exposure
ES - Expected Shortfall
EY - Ernst & Young
FRA - Forward Rate Agreement
FX - Foreign eXchange
ICAAP - Internal Capital Adequacy Assessment Process
IFRS - International Financial Reporting Standards
IMA - Internal Models Approach
IMM - Internal Model Method
IOSCO - The International Organization of Securities Commissions
IRB - Internal Ratings Based
ISDA - International Swaps and Derivatives Association
LGD - Loss Given Default
LIBOR - London Inter-Bank Offer Rate
MtM - Mark-to-Market
OTC - Over the Counter
PD - Probability of Default
PFE - Potential Future Exposure
RWA - Risk Weighted Assets
SFT - Securities Financial Transactions
S&P - Standard & Poors
VaR - Value at Risk
VBA - Visual Basic for Applications

4 Introduction

4.1 Background

The severity of the recent financial crisis, with its beginning in 2007, could to some extent be explained by the excessive on-and off-balance sheet leverage that the banking sectors of many countries built up. The effect of this behaviour was a gradual decrease of the level and quality of the capital base. On top of this several banks were holding insufficient liquidity buffers which made the banking system unable to absorb the systemic trading and credit losses that followed. The market lost confidence in both solvency and liquidity during the most severe episodes of the crisis and the weaknesses in the banking sector transmitted to the rest of the financial system.[21] For large players in the financial markets it is important to have a clear view of what risks they might encounter. There are many factors that affect market movements and recognizing these patterns is important. When creating models replicating the real world there is always a risk that necessary assumptions and simplifications will skew the result. Since 1930 the Bank of International Settlements (BIS) has served central banks towards monetary and financial stability [13]. BIS has also been working for international cooperation and been acting as a bank for central banks. The Basel Committee on Banking Supervision (BCBS) is one of four committees based at the BIS. The mission of the BCBS is to strengthen banking supervisory frameworks, encourage further development in the risk management area and to support improvements of standards in financial reporting. Basel II is a publication regarding capital adequacy produced by the Basel Committee. [14] An extension of Basel II is under progress and will lead to Basel III. One of the new topics treated is the Credit Valuation Adjustment (CVA) capital charge which deals with potential mark-to-market losses on CVA.

4.2 Research Questions

This research will be focusing on the following questions:

- What is CVA and CVA Risk Capital Charge under Basel III?
- How is CVA related to CVA Risk Capital Charge?
- How can one model CVA?

4.3 Purpose

The purpose is to construct a model for CVA. The model is to be used when calculating CVA risk capital charge under Basel III, and will deliver input to an IMM-bank's internal VaR-model. The CVA model should thus calculate CVA as described in Basel III. The model will serve as an example of CVA calculation, with a setup for a plain vanilla swap. Construction of the model should enable future extension to other types of derivatives. The model should be able to handle different counterparties, and should hence have counterparty specific inputs. Visual Basic for Applications (VBA) will be used for implementation. The VBA model should be user friendly and provide a clear presentation of results. In order to develop the CVA model a deeper knowledge of VBA, Counterparty Credit Risk under the Basel III framework, and especially knowledge of the new capital requirement, CVA Risk Capital Charge, needs to be acquired.

4.4 Sources of Information

The research has been based on books, articles and technical reports about CVA and its components exposure, probability of default and loss given default. A CVA training course, held by Ernst & Young in Amsterdam was also attended. Further, literature about derivatives and derivative pricing has been studied. The Basel II and Basel III framework have been an important source of information. Information available on-line and books about VBA has been used in the process of learning the programming language.

4.5 Data

In the CVA-model, CDS-data is needed for estimation of probability of default and yield data is needed for constructing discounting factors. The CDS data for Swedish companies quoted in basis points used. The maturities of the CDSs are 6 months, 1 year, 2 years, 3 years, 4 years, 5 years, 10 years, 20 years and 30 years. The source of the CDS-data is Thomson Reuters, Datastream. The yields of German government bonds, taken from Bloomberg[5], with maturities 3-months, 6-months, 1-year, 2-years, 3-years, 4-years, 5-years, 6-years, 7-years, 8-years, 9-years, 10-years, 15-years, 20-years and 30-years are used as euro risk free discount factors.

4.6 Limitations

This study is limited to calculating CVA for plain vanilla swaps. It is assumed that there are no collateral- or netting- agreement with the counterparty. The formula used for the CVA calculation is the one specified under Advanced CVA Risk Capital Charge in Basel III. The CVA calculation is thus designed for banks with IMM approval and Specific Interest Rate Risk VaR-model approval for bonds. The full CVA capital charge will not be calculated since the bank's own VaR model should be used. The CVA calculated will serve as input to the VaR-model.The developed CVA model will be able to handle one counterparty and one contract at a time. Furthermore, it is assumed that the model input is accurate. Lastly, the LGD_{MKT} will not be extracted from market data, but given a value based on reasonable assumptions.

4.7 Outline

The thesis starts with a theoretical background to give the reader a basic knowledge of counterparty credit risk and other concepts that are needed to understand credit valuation adjustment and its constituents. In the next chapter CVA is described in more detail. It is followed by a chapter about the new Basel III framework and its guidelines when it comes to capital requirements and counterparty credit risk. The CVA Capital Charge under Basel III, and how it is calculated, is specified in the next chapter. The following two chapters regard the mathematical and technical parts of the model. Then the CVA-model built using VBA is presented. Its structure and features are explained in more detail in Appendix. As a part of the result the optimal number of terms in the CVA formula is analysed. A case study of CVA-values for some Swedish companies is also made to demonstrate the model. This is finally followed by discussion and conclusion.

5 Theoretical Background

5.1 Counterparty risk and connected risk types

In the financial market counterparty credit risk is a major risk factor that cannot, and must not be ignored. Credit risk is the risk that a counterparty deviates from the initial contract by not making payments or not fulfilling other contractual obligations. It might depend on unwillingness or that the counterparty is unable to make a transaction.[20] Credit risk between derivative counterparties is generally defined as counterparty risk. In financial risk it might be considered as a simple risk type but since the recent credit crisis of 2007 onwards and the failures of large prestigious institutions, counterparty risk has been upgraded to be considered as the key financial risk. Historically, many financial institutions limited their counterparty risk by only trading with the most sound counterparties. However, there are more ways to mitigate counterparty risk than avoiding the riskiest actors. There are a number of different risk types to be considered. We have market risk, liquidity risk, operational risk and systemic risk. Thus, dealing with counterparty risk implies acknowledgment of all financial risks and how they interact. When trading with counterparties it is important to consider all present risks.

5.1.1 Market risk

Short-term volatility in market prices is the source of market risk. Exposure to movement of underlying variables such as stock prices, interest rates foreign exchange rates, commodity prices or credit spreads is possible sources of market risk. Counterparty risk represents a combination of credit risk and market risk with a counterparty at time of credit quality decay.[20]

5.1.2 Liquidity risk

Liquidity risk in an asset context is the risk that an asset for some reason is non executable at market price. The size of the position or liquidity of an underlying asset could be the explanation. Liquidity risk from a funding perspective refers to instability of fund payments that might force an early liquidation of assets with losses as a consequence. Collateralised counterparty risk could be exposed to liquidity risk if some credit event forces an early sale.[20]

5.1.3 Operational risk

Operational risk is hard to quantify since it includes human errors, system failure or other external disturbances that are difficult to forecast. Insufficiently developed or badly calibrated models, fraud or legal risk (in the sense of no possibility to enforce legal commitments) are other sources. When creating models replicating the real world there is always a risk that necessary assumptions and simplifications will skew the result. Dealing with counterparty risk, netting or collateralisation policies and practices themselves may introduce operational risks.[20]

5.1.4 Systemic risk

Systemic risk in the world of finance refers to the potential failure of an institution which starts a flow of instability and default among many other linked institutions. These events threaten the general stability of the financial markets. With intermediaries present with the aim of concentrating counterparty risk in the set of market participants the resulting effect is an increased systemic risk which needs to be addressed.[20]

5.2 Risk mitigation

Netting, collateralisation and hedging are all included in the set of risk mitigation and can be used to reduce counterparty risk but with an additional operational cost. One way of decreasing counterparty credit risk with the cost of an increased systemic risk is that major counterparties act as intermediaries. Other financial risks that appear with the presence of intermediaries are operational risk and liquidity risk. Thus,dealing with counterparty risk implies appreciation of all financial risks and how they interact.[20] When trading with counterparties it is important to consider all present risks.

5.2.1 Netting and Collateral

Credit exposure to a specific counterparty does not solely arise from just one transaction. Given a wide set of scenarios some of the transactions will perhaps contribute in a positive way, and others may give a negative contribution to the calculated exposure. By adding these exposures to the same counterparty, a reduction in overall exposure is possible. This is illustrated in figure 1. Although legal agreement will play an important role in the ability to net these positive and negative exposures against each other. An additional challenge when dealing with netting agreements is the requirement of scenario consistency. When pricing contracts with opposite contribution to the overall exposure it is crucial to base the calculations on the same scenario for each type of contract. With this in mind, it is understandable that the software layout has an impact on the computational complexity of the exposure calculations.[9] Calculating all trades individually does not capture possible benefits from trades with values of opposite signs. The impact of netting sets has to be aggregated through every individual transaction. By using Monte Carlo approach it is possible to quantify exposure profiles due to netting agreement in an effective manner. However, as always when using Monte Carlo simulation it is important to use a large number of scenarios to capture as many different outcomes as possible to get a accurate representation of how netting agreement may change the profile of the exposure curves.[20]

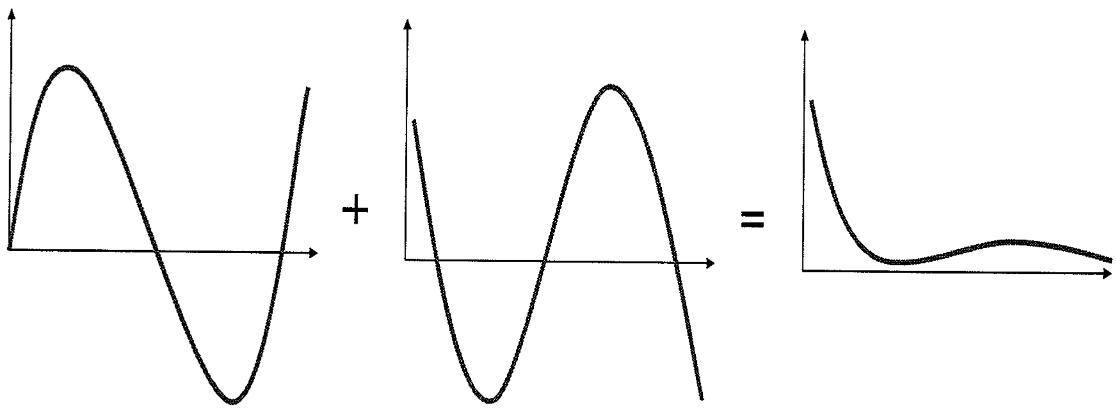

Figure 1: Illustration of netting effects.

For the last two decades collateral management has been a big part of counterparty risk control. It all started in the 1980s when taking collateral against credit exposure were first introduced. In the beginning there were no legal standards and the majority of the calculations were made manually. From 1990 onwards, dealing with derivatives exposure became common. To start with the typical collateral was cash or government securities. A few years later the first International Swaps and Derivatives Association (ISDA) document started the standardisation process. Different events since then have resulted in an increased monitoring of credit controls as well as interest in different risk mitigating arrangements, such as collateralisation has been noted. When dealing with credit exposure on an individual counterparty level one needs to consider the limitations of notional amount, and/or generate demand for offsetting positions so that the overall exposure decreases due to netting effects. Although netting agreements may reduce counterparty risk some limiting trading actions may remain. Collateral enables less credit-worthy counterparties to take a more prominent role in the market. Derivatives collateral is fundamentally different in the characteristics compared to physical assets as security for debts. The ability to realise the value of the collateral asset is dependent on the bankruptcy process and releasing securities is not completely trivial. Since 2003 there has been a substantial growth in collateral use. And by this date the level of collateralisation among OTC derivatives exposures is approximately 50%[20]. With products of higher sensitivity to counterparty risk, this proportion increases. Keeping in mind that netting agreement reduces exposure and introduction of collaterals lower the exposure further, the overall effect of risk mitigation is significant.

5.3 Over the Counter Derivatives

Over the counter (OTC) derivatives are traded between two parties in contrast to derivatives that are traded on exchanges. OTC contracts can vary in structure depending on needs and preferences, while contracts at an exchange are standardised. An exchange has the advantage of being liquid and the credit risk of a single counterparty is negligible because of the many members of the exchange. For OTC-derivatives counterparty credit risk is prominent and needs to be managed.

There are many different types of OTC derivatives. Some common derivatives are forward rate agreements, swaps, credit default swaps, and options.

5.3.1 Forward Rate Agreement

A forward rate agreement (FRA) is a contract between two counterparties that can be used to fix an interest rate, or a currency exchange rate, during a future time period. The contract is based directly on the interest rate or exchange rate, and market movements decide which of the counterparties benefit from the contract. At maturity one payment is made. The least fortunate party will pay the difference between the fixed rate and the reference rate multiplied by the notional amout of the agreement.

In an FRA, the following is specified:

- fixed rate
- floating rate (reference rate)
- start date
- maturity
- notional amount

An FRA can be used to manage interest rate or exchange rate risk. The buyer of a contract is hedged against an increase in interest rate or exchange rate, and the seller is hedged against a decline.

5.3.2 Swap

An interest rate swap is an agreement where two counterparties exchange one stream of cash flows against an other stream,where cash flows could be in the same currency or in different currencies. In a swap one party pays a fix rate while the other pays a floating. The floating rate is typically based on LIBOR or similar. Market changes of the interest rate during time to maturity decides which party benefits from the contract. If the cash flows are in different currencies the market changes of the exchanges rates also plays an important role. A swap contract could be entered for numerous reasons such as speculation, perceived arbitrage possibilities or risk management. Companies typically enter swap contracts to control interest and exchange rate risk.[2]

In a swap contract between party A and B, see figure 2, the following is specified:

- fixed rate (swap rate)

- floating rate
- maturity
- payment dates for A
- payment dates for B
- notional amount

The notional amount is used for calculating the cash flows and is not actually exchanged. In general only the net cash flow is paid.

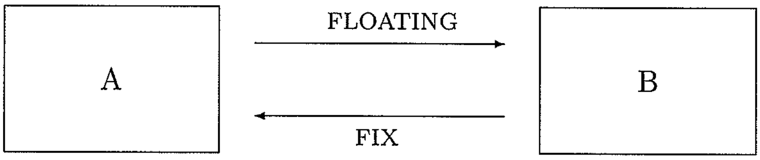

Figure 2: Interest rate swap

5.3.3 Credit Default Swap

A credit default swap (CDS) is an agreement where the seller of the contract will compensate the buyer in case of a credit event, such as delay of payment or default, of a specific counterparty, loan or sovereign, see figure 3. Usually the buyer of the CDS wants to hedge credit exposure of the underlying entity, but it is also possible to enter CDS contract just speculatively.The buyer of the contract pay a premium (CDS spread) to the seller. If the reference entity defaults the seller will compensate buyer for the credit loss. Typically the compensation will be the notional times (1-recovery rate). If the reference entity doesn't default the seller pays nothing. The CDS-spread depends on the probability of default.

In a CDS-contract, the following is specified:

- reference entity
- premium (CDS-spread)
- payment dates for premium

- maturity
- notional amount

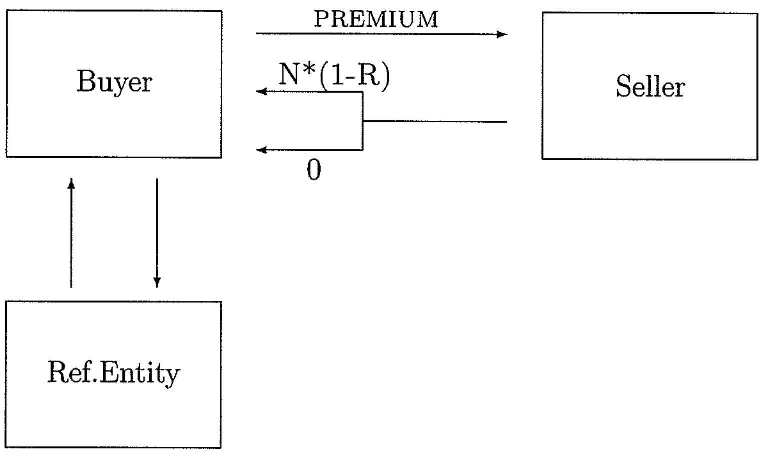

Figure 3: CDS

5.3.4 Options

An OTC-option is similar to an FRA in the way that it is an agreement of a future interest rate or exchange rate, with a certain start date and maturity. In contrast to the FRA an option is not obligatory. At maturity the owner of the contract simply decides if he/she wants to exercise it or not. Some common type of options are: interest rate options, currency rate options, and swaptions. A swaption gives the holder the right to enter into a swap contract.

5.4 Development of OTC-Market

The market of OTC derivatives has grown significantly during the last ten years as illustrated in figure 4. The global total notional amount of all OTC derivatives outstanding was \$707.6 trillion at mid-year 2011 compared to \$99.8 trillion at mid-year 2001. A split of the OTC derivatives in product types as of first half 2011, show that interest rate derivatives account for the greatest part, 87%, of notional outstanding, see figure 5. The second largest product is foreign exchange rate derivatives with 9% of the total notional outstanding, followed by credit default swaps [1] with 6%[6]. During the last decade the market share of the different derivatives have remained fairly constant. As expected an increase in the outstanding notional of credit default swaps can be seen around the financial crisis in 2007.

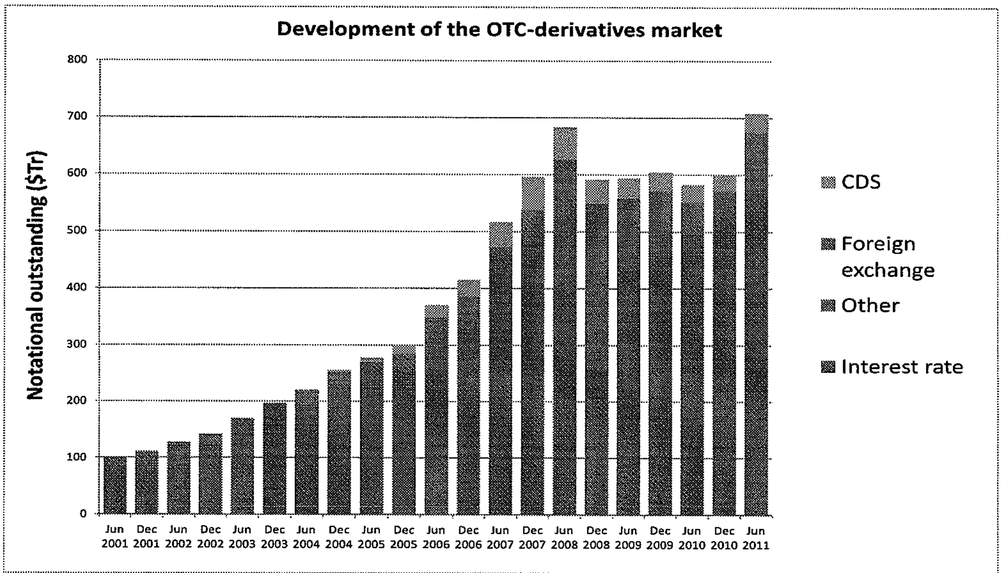

Figure 4: The total outstanding notional of OTC derivatives in trillion UD-dollar at end-June 2001 to end-June 2011. It is evident that the OTC-market has grown significantly during the last decade, mostly due to interest rate products, exchange products and in later years also CDS products [18][19][27][26][3][16][28][6]

Based on the OTC-market growth during the last 10 years it is likely that the OTC-market will continue to grow. As the market becomes larger it

[1]Bank of International Settlements only provide CDS data from 2004 and onwards

will probably be subjected to regulations which would slow down the growth somewhat, but the fact remains; the OTC-market has become more important and will remain so within the foreseeable future. It is therefore vital to understand and manage counterparty credit risk. One important part of CCR-management is to calculate CVA in order generate a fair price for the OTC-contract.

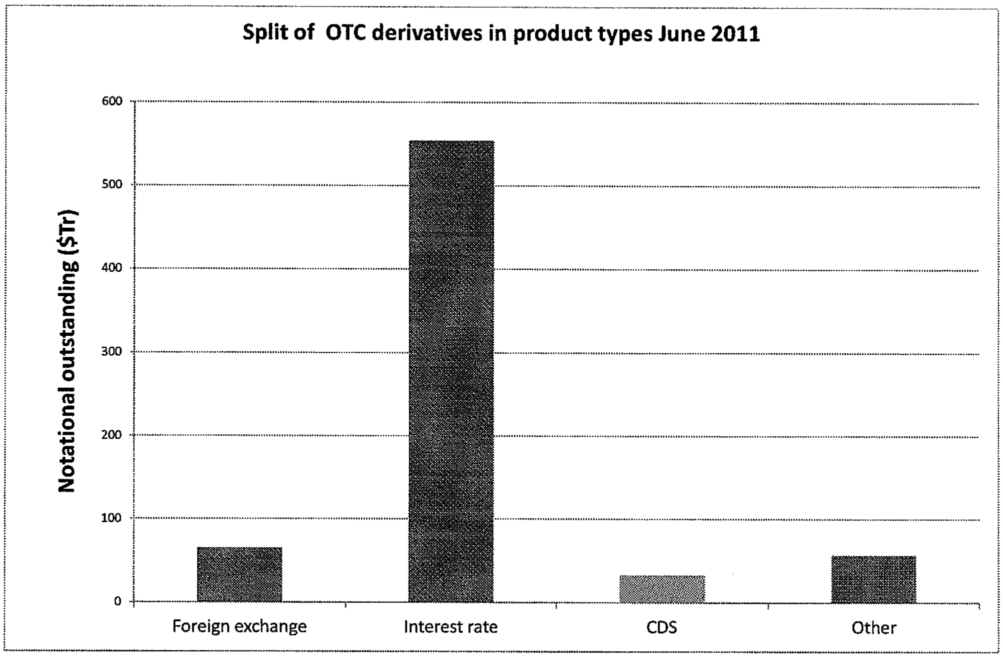

Figure 5: A split of the OTC derivatives in product types as of first half 2011. Interest rate derivatives clearly account for the largest part of the total notional amount outstanding [6]

5.5 Basel Committee on Banking Supervision

The Basel committee established in 1974 has the intention of formulating supervisory standards and guidelines and recommends that authorities will implement them in their own system through detailed arrangements.This approach makes it possible to get an overview of the different national systems since it strives towards convergence. Although the Committee provide guidance they have no legal force. In 1988 a capital measurement system, Basel Capital Accord, was introduced by the Committee. The framework stated a minimum capital standard of eight percent. After this initial publication a revised framework has been published, commonly known as Basel II, consisting of three pillars. These are minimum capital requirements aimed at refining the 1988 rules, supervisory review of internal assessment processes and capital adequacy within an institution. The third pillar is for improvement of market discipline by effective use of disclosure as support to supervisory engagements.[15]

5.6 Introduction to CVA and CVA Capital Charge

The standard practice for several years has been to evaluate derivatives portfolios using mark-to-market without taking account of the credit quality. For large actors on the financial market it is important to have a clear view of what risks they might encounter. There are many different factors that affect market movements and recognizing these patterns is important when quantifying risks. Common risks, such as already mentioned above, are market risk, liquidity risk, operational risk and credit risk. By using internal ratings-based approaches and models regarding credit risk under the Basel framework, banks can quantify and determine their capital requirements. With more sophisticated models a better estimate of the required capital will be given and the amount of capital can be optimized further. Credit Valuation Adjustment is describing the price of the risk of a counterparty defaulting. With the Basel III framework an additional capital charge to cover potential mark-to-market losses on the expected counterparty risk, or CVA losses, must be incorporated. How to calculate the additional CVA charges depends on the bank's internal method of calculating capital charge for counterparty credit risk and specific interest rate risk VaR models. The derivation of CVA and its uncertainty due to simplifications is important to manage for accurate results.

6 CVA

Being a participant at the OTC derivatives market a firm is exposed to the risk of its counterparty defaulting and therefore counterparty credit risk. CVA is a quantitative measure and will give an idea of the extent of this risk. In other words the CVA value gives an indication of the cost of compensation for the potential loss of a defaulting counterparty. Lately the importance of CVA has increased and the managing of CVA is becoming a widely discussed topic. Below the formula for calculating CVA is presented [20].

$$CVA(t,T) \approx (1-\delta)\sum_{j=1}^{m}\int_{t}^{T} B(t_j)EE(t_j)q(t_{j-1},t_j)$$

CVA consists of three important components; loss given default, expressed in the formula above as $(1-\delta)$, where delta denotes the expected recovery fraction, expected exposure $(EE(t_j))$ and probability of default $(q(t_{j-1},t_j))$. Additionally $B(t_j)$ is a discount factor. The above formula is a simplification of a more theoretical explanation of the CVA calculation. The main idea mathematically, is to integrate over the period from inception to maturity i.e. the time period where a default can occur.

The more advanced formula of CVA is given below [20].

$$CVA(t,T) = -(1-\delta)\left[\int_{t}^{T} B(t,u)EE(u,T)dS(t,u)\right]$$

Where δ is the expected recovery fraction, thus (1-δ) denotes the loss given default, B(t,u) is a discount factor, $EE(u,T)$ is the expected exposure and S(t,u) denotes the probability of no default during the length of the contract. For full derivation of these formulas see Gregory([20]). The time window of interest is from date of inception of the contract until maturity of the contract. Knowing that a default can occur at any time during the length of the contract, CVA is calculated as an integral over the designated interval. This integral can then be simplified further by using a approximating sum, provided a sufficient number of terms is used. Gregory ([20]) suggest at least 12 terms in the summation for an accurate result. However, it is important that the components in the formula, especially exposure, are derived in an accurate way so that uncertainty is minimized before calculation of CVA. The result of this simplification is the first presented formula in this section.

6.1 Exposure

Counterparty credit exposure, or simply exposure, is the amount a company could lose if its counterparty defaults[9]. The exposure depends on the mark-to-market (MtM) value of the OTC-contract,which needs to be replaced in case of a default. The MtM can be either positive or negative depending on the value of the underlying asset. An important aspect when it comes to exposure is that there is an asymmetry of potential losses. If the MtM value is positive at default, the counterparty will be unable to make future commitments, and the loss will be MtM-value. If the MtM-value is negative at default the company will still owe its counterparty due to legal obligations. The position is hence unchanged and there is no gain from the default. Consequently the company loses if the MtM is positive and does not gain anything if it is negative. Exposure can therefore be defined as[20]:

$$Exposure = max(0, V_t)$$

where V_t is the MtM-value of the contract.

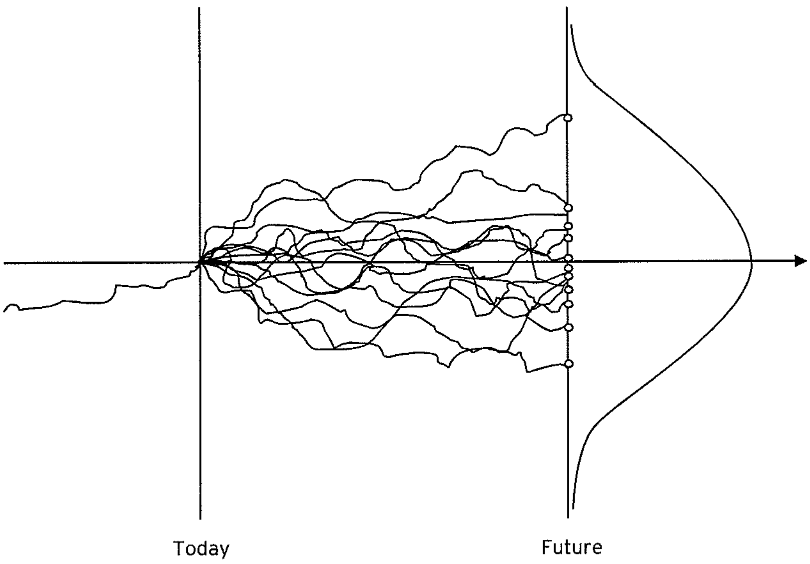

Figure 6: Future scenarios of contract value are simulated and some statistical measure is then used to quantify the exposure.

Exposure can be calculated by simulating future scenarios of the MtM-values and then using some statistical measure to quantify the exposure at a given point in time, see fig 6. Calculation of exposure is in general quite complex since a default can occur at any time. Depending on contract type, this complexity can become very computationally intensive.

6.1.1 Statistical Measures of Exposure

There are many ways to quantify exposure and unfortunately slightly different definitions of these metrics. The measures below are those defined by the Basel Committee on Banking Supervision[22], and are probably the most widely used.

Expected Exposure Expected exposure (EE), see figure 8 represents the expected loss if the counterparty defaults, based on a zero recovery rate. It is the average of the exposure given the different scenarios:

$$EE_t = \frac{1}{N}\sum_{i=1}^{N} max(0, V_{ti})$$

where $i = 1, \ldots N$ is the scenario. Since EE is the average of the positive MtM-values,it is always larger than the average of the MtM-values [2],see figure 7.

Potential Future Exposure Potential future exposure (PFE) is defined as the worst possible exposure with reference to a certain confidence level, see figure 7. It is the same definition as the traditional measure value at risk (VaR), but with the difference that the time horizon is in general longer for PFE. VaR usually have a horizon of 10 days, while PFE could have a horizon of years, depending on the maturity of the contract. The confidence level used is normally 99%. In that case the worst possible exposure will be lower than PFE with 99% certainty.

[2]or equal to MtM, if MtM is very low.

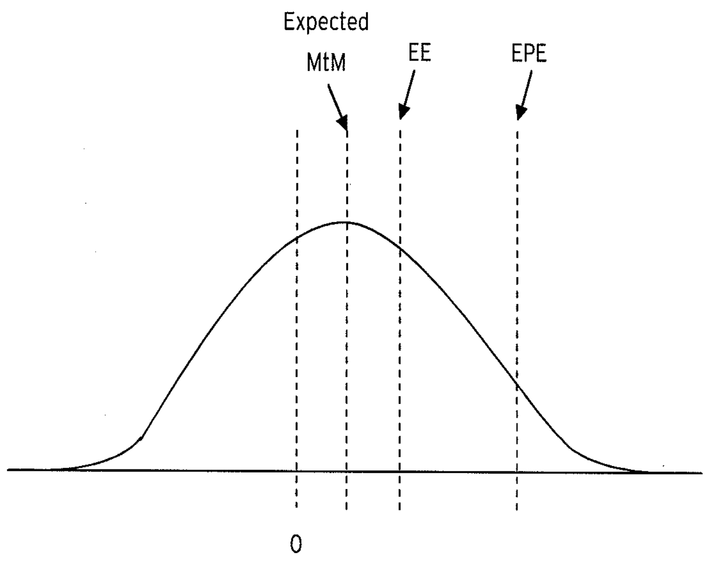

Figure 7: Illustration of Potential Future Exposure, Expected Exposure and Expected MtM. Only positive MtM-values are seen as exposure.

Expected Positive Exposure Calculating EE for values of t ranging from zero to maturity give an exposure profile. Expected positive exposure (EPE) is defined as the time average of the EE_t:

$$EPE = \frac{1}{M}\sum_{t=1}^{M} max(0, EE_t)$$

where $t = 1, \ldots M$ is the time. Note that both EE_t and PFE_t are probability-weighted averages for a point in time, while EPE is a time-weighted average of expected exposure over the whole contract length, see figure 8.

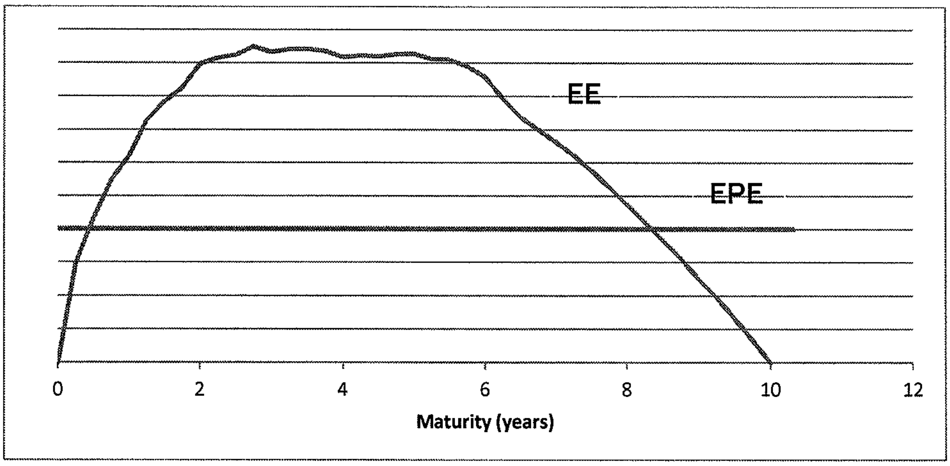

Figure 8: EPE is the time-weighted average of expected exposure over the whole contract length.Expected exposure is $max(0, v_t)$

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6.2 Probability of Default

Probability of default (PD) is the likelihood of a default during a specified period of time. In a financial setting PD is an estimate of the likelihood that a financial institution is incapable of or unwilling to fulfil its debt obligations. PD depends on the risk characteristics of the counterparty but also on macroeconomic factors. In an economical recession there is a higher probability of default generally across counterparties, but it is also a question of how well the specific counterparty copes with the economical down turn.

PD is an important factor in modern credit risk modelling. The accuracy of the default probability will determine the quality of the whole credit risk model. It is generally difficult to estimate probability of default. One problem is that defaults are rare events and hence there is a lack of proper statistical data. For companies with high creditworthiness it is more difficult since the rare event of default make the volatility of the few observed defaults high. A usual banking practice is to derive PD by using some sort of qualitative mapping where different factors, that are not necessarily numerical, are quantified and weighted together.[12]

External ratings of companies and sovereigns can also give an indication of default probability. There are three major rating agencies, Standard & Poor (S&P), Moody´s and Fitch[12]. They grade obligators according to their creditworthiness and their estimates are well acknowledged and accepted in the financial sector. S&P communicate their ratings expressed as letter grades which give a relative level of credit risk. They have a rating scale between AAA and D, where a company with rating D is far more likely to default than one with AAA[1]. Moody´s and Fitch have similar rating scales. Credit ratings are just opinions about credit risk and does not guarantee credit quality or future credit risk. Despite this the use of external ratings has become very popular, probably due to its simplicity.

6.3 Loss Given Default

In case of default it is essential to have a clue in which range the recovered value of the defaulting contract will be. A high recovery fraction will decrease the total value of CVA and a lower recovery will lead to a higher value of CVA. Since a default is a single event, it is difficult to collect enough data to make a good quantitative estimation of the LGD, especially when dealing with new products or portfolios of low default. With not enough data available other methods of determining LGD need to be considered. With limited access to objective methods more subjective approaches can be helpful for the assessment. Examples of this are expert judgments, comparison with similar products or different scenario techniques. Depending on the characteristics of the available data, quantitative methods allows explicit or implicit estimation of the LGD. An explicit method called market LGD approach uses market prices of bonds shortly after default and comparing these prices with their par value. By discounting all recoveries and costs observed after default the value of the defaulted entity is determined and then compared with the exposure at the actual time of default. With this information the LGD is then extracted. Even though there is no information available that enables direct computation of LGD, implicit methods with approaches of extracting relevant information may be applied an give an estimation of LGD.[12] The size of LGD varies a lot depending on the method used for estimation. Theoretically LGD can vary between 0 %, in case of full recovery and in the worst case 100% when nothing can be saved. By looking at the probability distribution of recoveries between 1970-2003 for all bonds and loans with data from Moody's an average of recovery rate of approximately 40% can be observed, which gives a LGD of 60%[25].

7 Basel III

In 2010-2011 the Basel Committee on Banking Supervision agreed on a third global regulatory standard for banks, Basel III. It was developed in order to strengthen global capital and liquidity rules and thereby create a more resilient banking sector. The reform incorporates lessons from the financial crisis that began in 2007. One of the main reasons for the severity of the crisis was that the banking sectors of many countries had built up excessive on- and off-balance sheet leverage. Gradual erosion of the level and quality of capital base and also insufficient liquidity buffers amplified the crisis. Basel III introduces a number of reforms with the aim of preventing severe financial crises. For individual banking institutions there is a greater focus on having capital buffers sufficiently large for coping with periods of stress. There is also a focus on macro-level stresses and cyclical risks that can build up across the banking sector.[21]

7.1 Capital Requirements

The Basel Accord requires banks to hold a certain amount of assets to ensure that they have enough capital to sustain their operation even in periods of stress. This amount is referred to as regulatory capital or capital requirement. Under Basel II and Basel III the amount of capital held, needs to be at least 8% of risk weighted assets (RWA),[21] [23]:

$$\frac{Total\ capital}{RWA} > 8\%$$

Risk weighted assets is the sum of the banks different assets weighted according to risk. For example, a mortgage would have a lower risk weight than a loan without collateral, and equity would have a risk weight of zero since it is risk free. Risk weighted assets are based on three types of risks; market risk, operational risk and credit risk. The capital requirements force the bank into a sound relation between capital and risky positions. It is a non-static measure that can be used by banks of different size and risk profile.

In Basel III there are additional requirements on the quality of the regulatory capital. The capital can be split into two sub groups; Tier 1 and Tier 2. Tier 1 is the bank's core capital and consists of common equity and retained earnings, and other issued instruments. Tier 2 consists of supplementary capital that does not qualify as Tier 1 capital. To increase the quality of the regulatory capital BIS suggests that Tier 1 capital should be more than 6%

of RWA. Furthermore the sub group Common Equity Tier 1 should be larger than 4,5% of RWA. The new capital requirements will be phased in during 2013-2015.[21]

7.2 Counterparty Credit Risk under Basel III

The Basel III counterparty credit risk reform will become active in January 2013. The total counterparty credit risk capital charge, which is a part of risk weighted assets, is the sum of default risk capital charge and CVA-capital charge.

$$Total\ CCR\ Capital\ Charge =$$
$$Default\ Risk\ Capital\ Charge + CVA\ Risk\ Capital\ Charge$$

The Default risk capital charge covers the risk of counterparty default. It is based on counterparty exposure and more specifically the metric Exposure at Default (EAD). The Default Risk Capital Charge is obtained by multiplying the EAD with a risk weight. BIS specifies four different methods for determining the EAD of OTC derivatives:

- Original Exposure Method
- Current Exposure method
- Standardised Method
- Internal Model Method

The methods are presented in ascending order of risk sensitivity. The less sensitive models generally give a larger capital requirement which provides an incentive for banks to move toward the more sensitive methods. There are also two different methods for calculating the risk weight; the internal ratings based approach (IRB) and the standardised approach. IRB is based on internal credit ratings and the specialised approach uses ratings from external sources.

CVA-capital charge, ie. the market risk capital charge of changes in CVA due to changes in credit worthiness of the counterparty, was not a part of the Total CCR capital charge in Basel II. During the financial crisis it was noted that roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses, and only about one third were due to actual defaults[24]. It was therefore decided that the CVA capital charge should be added to the Default Risk Capital Charge. The CVA-capital charge will be described in detail in the following section, CVA capital Charge under Basel III. The Basel Committee of Banking Supervision estimates that the total capital requirements for counterparty credit risk under Basel III will double the amount required under Basel II[24].

8 CVA Capital Charge under Basel III

8.1 CVA Capital Charge

The CVA Capital charge was added to the total CCR capital charge to account for the risk of market-value losses due to a worsening in the counterparty's credit quality. The CVA Capital Charge has a one year risk horizon. Depending on the banks approved method for calculating capital charges for counterparty default risk and specific interest rate risk, there are two different ways to calculate the CVA capital charge. Banks with IMM approval for counterparty credit risk and approval to use the market risk internal models approach for the specific interest-rate risk of bonds, should calculate the Advanced CVA risk capital charge. All other banks should calculate the Standardised CVA risk capital charge.[21]

8.2 Advanced CVA Risk Capital Charge

Under this approach the bank uses its own Specific Interest Rate Risk VaR model for bonds to calculate a VaR on CVA, by modelling changes in the counterparties' CDS-spreads. The VaR model is restricted to changes in the counterparties' credit spreads and will not model the sensitivity of CVA to changes in other market factors. The VaR is based on the aggregated CVAs of all OTC derivatives counterparties. The CVA capital charge is constructed as the sum of a stressed and a non-stressed VaR component. The non-stressed VaR is based on current parameter calibrations for Expected Exposure. The stressed VaR component is based on stressed calibration for Expected Exposure, meaning that the calibration should be based on a data period of at least three years and it should cover a full range of economic conditions, such as a full business cycle. The credit spread should in this case be calibrated on the most severe one-year-period contained in the three-year-period for the exposure calibration. When calculating the value at risk a 99th percentile, one-tailed confidence interval is used. The risk horizon for the VaR should be one year as opposed to 10 days which is used under the market risk framework.[21]

No matter which formula the bank uses to calculate the value of CVA, the CVA capital charge calculation must be based on the following formula for each counterparty:

$$CVA = (LGD_{MKT}) \sum_{i=1}^{T} \left(\frac{EE_{i-1} * D_{i-1} + EE_i * D_i}{2} \right) *$$

$$Max\left[0; exp\left(\frac{-s_{i-1} * t_{i-1}}{LGD_{MKT}}\right) - exp\left(\frac{-s_i * t_i}{LGD_{MKT}}\right)\right]$$

Where,

- t_i is the time of the i-th revaluation time bucket, starting from $t_0 = 0$.
- t_T is the longest contractual maturity of the set of contracts with the counterparty.
- s_i is the credit spread (CDS-spread) of the counterparty at time t_i.
- LGD_{MKT} is the loss given default of the counterparty, based on the spread of a market instrument of the counterparty. This must be a market assessment and not an internal estimate.
- EE_i is the expected exposure to the counterparty at time t_i,
- D_i is the default risk-free discount factor at time t_i, where $D_0 = 1$.

The second factor in the sum,

$$Max\left[0; exp\left(\frac{-s_{i-1} * t_{i-1}}{LGD_{MKT}}\right) - exp\left(\frac{s_i * t_i}{LGD_{MKT}}\right)\right]$$

is an approximation of the market implied marginal probability of a default occurring between times t_{i-1} and t_i. For derivation of the probability of default estimation see Hull and White [17].

The capital requirements for counterparty credit risk, using the advanced CVA risk capital charge, is summarised in figure 9

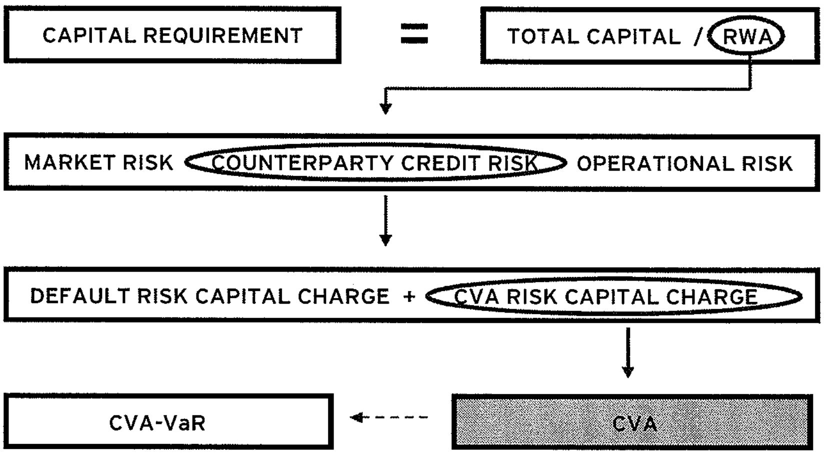

Figure 9: Illustration of capital requirements for counterparty credit risk and how it is connected to CVA.

8.3 Standardised CVA Risk Capital Charge

The standardised approach is simpler and more straight forward than the advanced approach. The CVA capital risk charge is given directly by a formula where the exposure of different counterparties are weighted together using external risk weights. The risk horizon is one year and the weighted exposure is scaled by 2.33 to get a value that is comparable to the 99 percentile VaR in the advanced approach.[21]

9 Interest Rate Simulation using CIR

The model used for the short rate simulation is a Cox-Ingersoll-Ross (CIR) model. It was chosen because it is a good trade off between simplicity and performance. It is a rather simple model that still catches the main features of interest rate movements.

9.1 The CIR-Model

Under the martingale measure Q it has the following dynamics[10]:

$$dr = \kappa(\theta - r)dt + \sigma\sqrt{r}dW$$

where κ, θ and σ are positive constants and W is a one-dimensional Wiener process. The randomly moving interest rate is pulled towards the long term mean, θ. The speed of the adjustment is determined by κ. The CIR-model is popular because of its analytical tractability and the fact that it always returns positive values of r[7]. If $2\kappa\theta \geq \sigma^2$ the interest rate is also guaranteed to be non-zero[10]. A simulation of interest rate using the CIR model is seen in figure 10 below:

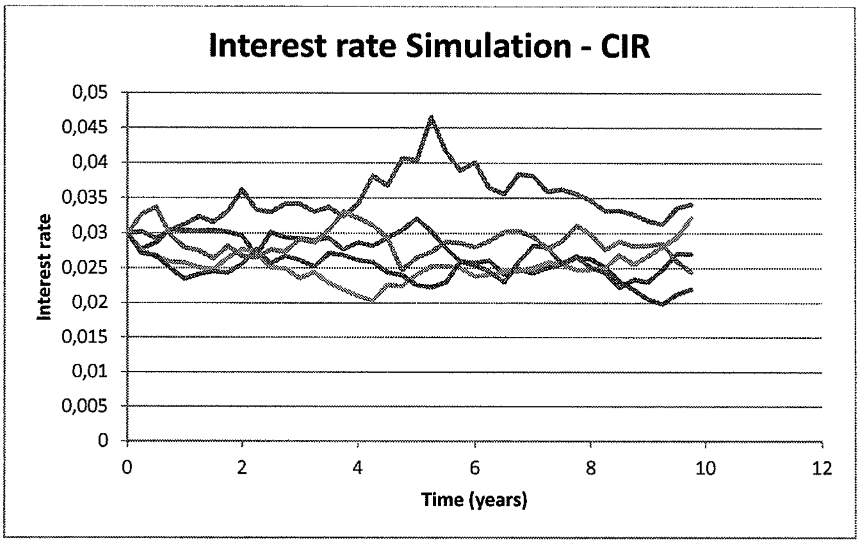

Figure 10: Interest rate simulation using the CIR-model with $\kappa = 0.1$, $\theta = 0.03$ and $\sigma = 0.02$. The figure show 5 different scenarios.

9.2 Exact Simulation of CIR-Model

The interest rate is simulated exactly using an algorithm described by Broadie ([8]).If the interest follows a CIR-process with parameters κ, θ and σ, the distribution of r_t given r_u for u less than t is given by a noncentral chi-squared distribution times a scaling factor:

$$r_t = c_t \chi_d'^2(\lambda)$$

where c_t denotes the scaling factor, λ the non centrality parameter and d the degrees of freedom.

$$\lambda = \frac{4\kappa e^{-\kappa(t-u)}}{\sigma^2(1-e^{-\kappa(t-u)})} r_u$$

$$d = \frac{4\theta\kappa}{\sigma^2}$$

$$c_t = \frac{\sigma^2(1-e^{-\kappa(t-u)})}{4\kappa}$$

It is thus possible to sample r_t given r_u exactly if you can sample from a non-central chi-squared distribution.

If d>1, a non central chi-squared random variable can be represented by a sum of a non central chi-squared random variable with one degree of freedom, and an ordinary chi-squared random variable with d-1 degrees of freedom:

$$\chi_d'^2(\lambda) = \chi_1'^2(\lambda) + \chi_{d-1}^2$$

$\chi_1'^2(\lambda)$ can be generated using a standard normal random variable, Z:

$$\chi_1'^2(\lambda) = (Z + \sqrt{(\lambda)})^2$$

For d 1, the generation of r_t is thus reduced to sampling from an independent normal distribution and an ordinary chi-squared using the following set-up:

$$r_t = c_t((Z + \sqrt{(\lambda)})^2 + \chi_{d-1}^2)$$

If the nonzero restriction on the CIR-parameters, $2\kappa\theta > \sigma^2$ holds, d is always larger than 1. The above representation can thus be used for the interest rate simulation.

9.3 Calculation of bond prices

The CIR-model give the following term structure for the price at t of a zero coupon bond with maturity T[4]:

$$p(t,T) = A(t,T)e^{-B(t,T)r(t)}$$

where:

$$B(t,T) = \frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + \kappa)(e^{\gamma(T-t)} - 1) + 2\gamma}$$

$$A(t,T) = [\frac{2\gamma e^{(\gamma+\kappa)((T-t)/2)}}{(\gamma + \kappa)(e^{\gamma(T-t)} - 1) + 2\gamma}]^{\frac{2\kappa\theta}{\sigma^2}}$$

$$\gamma = \sqrt{\kappa^2 + 2\sigma^2}$$

10 Pricing of Vanilla Swaps

The calculation of exposure depends on how the value of the underlying contract changes over time to maturity. The contract of interest in this study is a vanilla swap. The price of an interest rate swap at t<T_0 is given by[4]:

$$\Pi_t = Kp(t, T_0) - K\sum_{i=1}^{n} d_i p(t, T_i)$$

Where,

$$d_i = R\delta, i = 1, \dots, n-1$$

$$d_n = 1 + R\delta$$

K is the notional amount and $p(t, T_{i-1})$ denotes the price (at time t) of a bond with maturity at T_{i-1}.

The swap rate R is given by:

$$R = \frac{p(0,T_0) - p(0,T_n)}{\delta \sum_{i=1}^{n} p(0,T_i)}$$

11 Results

To illustrate how CVA can be computed a model in Visual Basic for Applications has been constructed. The formula for calculating CVA, specified in chapter 8.2 Advanced CVA risk capital charge, has been implemented during the development of the model. Initially the intention was to have a highly dynamic model adjustable for a selection of products with different characteristics. However, during the implementation of the model some limitations to the flexibility were imposed to ease the programming complexity. The CVA is calculated for vanilla swaps. A major part of the implementation has been to simulate the exposure. It is based on an underlying Monte Carlo simulation of the interest rate, using the Cox-Ingersoll-Ross model. The discount factors in the CVA formula are based on a linear interpolation of government bond yield data. CDS spreads are also linearly interpolated. Loss given default is given as input. See Appendix for a more detailed description of the CVA model.

11.1 Optimal CVA Grid Structure

As an illustration of the accuracy of the approximation sum in the Basel III CVA formula, a number of CVA calculations for different counterparties were performed. To exclude error effects due to number of Monte Carlo scenarios, one simulation with 2000 scenarios were run and the generated exposure from that simulation as well as the interpolated yield, were used as input for all calculations through the set of counterparties. With the firm specific CDS spread the CVA calculations were made several times with different exposure frequency. The result of this investigation for contract length of 10 years is presented in figure 11 and for 30 year in figure 12.

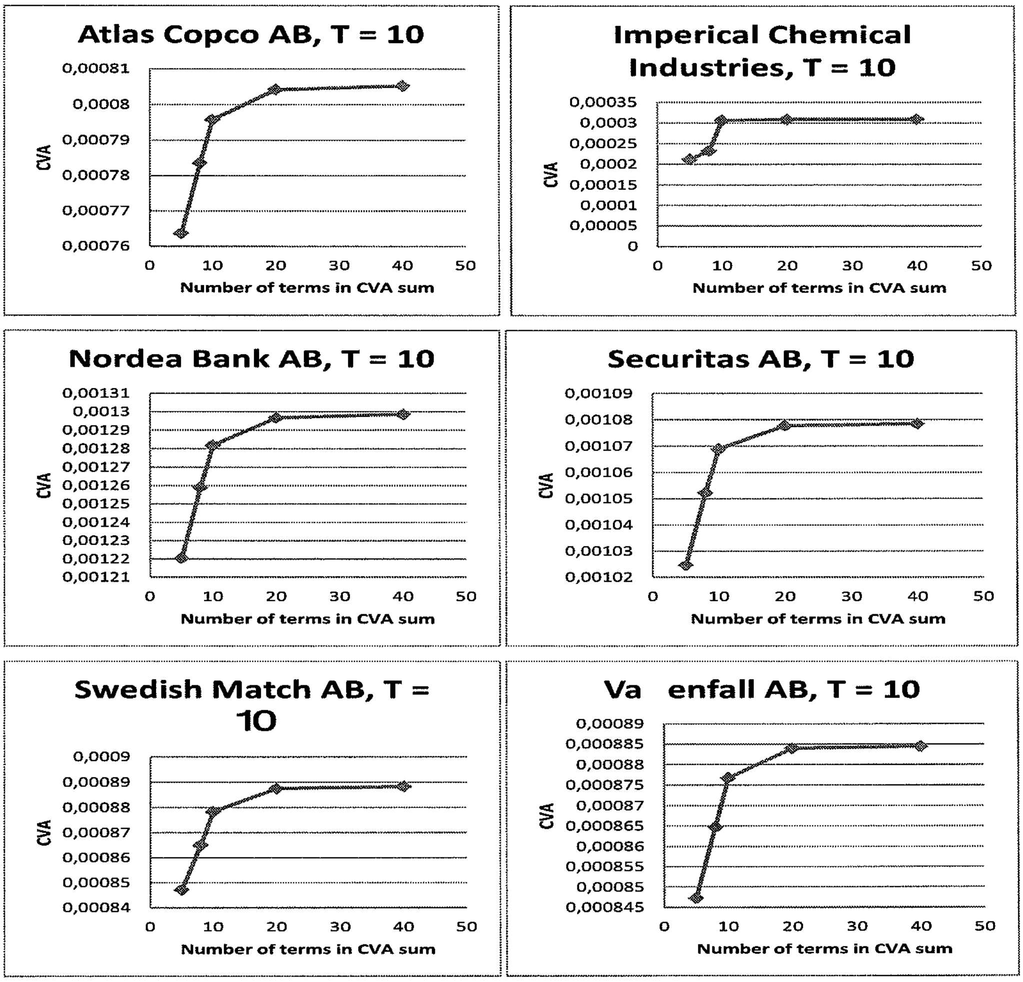

Figure 11: Analysis of optimal number of terms in CVA sum for swap contract with 10 year to maturity.

For longer contracts it is noted that the number of terms in the CVA sum is less important as long as the data input is reliable. When fixing the exposure and yield curves it is the shape of the curve from CDS spreads that becomes interesting. Since this data is used for the estimation of the probability of default it has a significant impact on the resulting CVA value. If the CDS curves are very volatile the sample frequency needs to capture even odd shapes of CDSs. Based on this illustration of the impact of the number of terms in the CVA sum a minimum number of 20 terms is necessary for an adequate accuracy. In other words this means a sample frequency of semiannual measurements for a swap contract with 10 year to maturity.

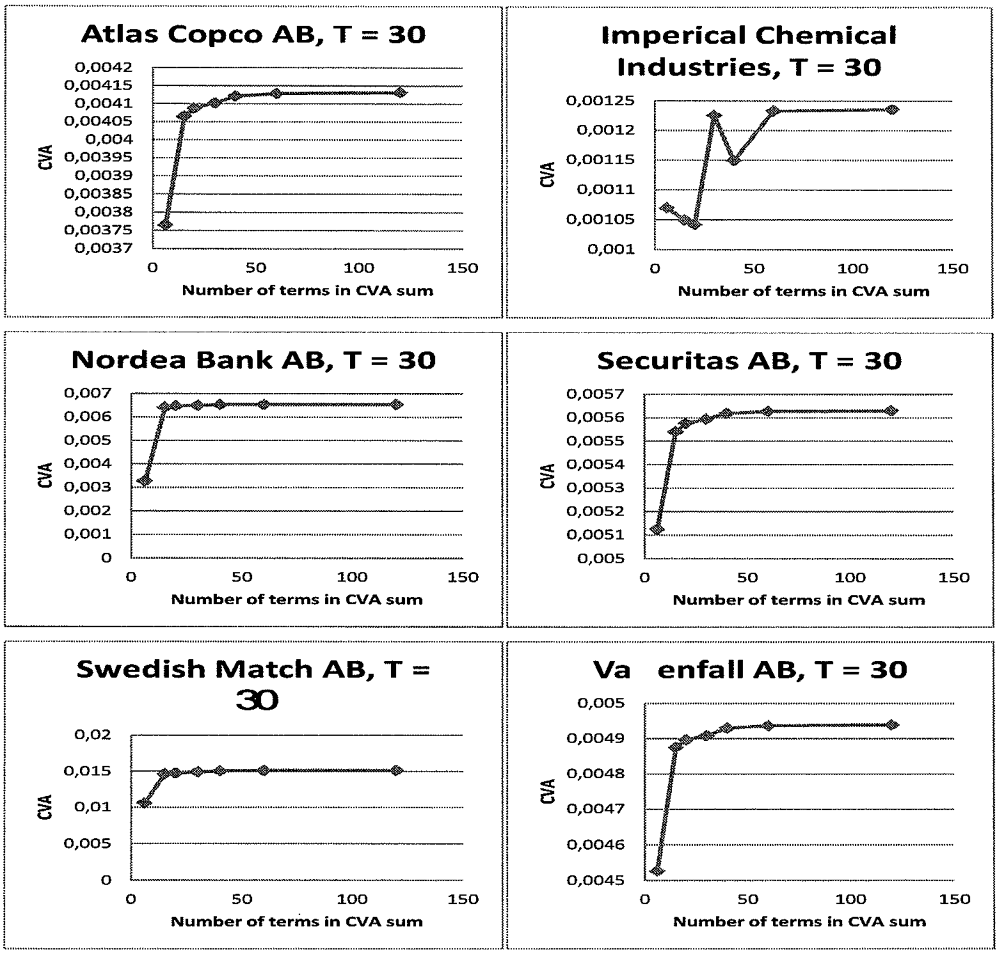

Figure 12: Analysis of optimal number of terms in CVA sum for swap contract with 30 year to maturity.

11.2 Case study: CVA for some Swedish Companies

As a demonstration of the CVA-model, CVA is calculated for a vanilla swap with the following five Swedish companies as counterparties:

- Atlas Copco AB
- Nordea Bank AB
- Securitas AB
- Swedish Match AB

- Vattenfall AB

The specific companies were chosen because they represent different industries and had proper CDS data sets available. The CDSs are quoted in basis points (euro). German government bond yields quoted in percent are chosen to represent a European risk free rate. The CVA model is run with the input values described in table 1. The par swap rate is used.

Table 1: Input CVA-model

Parameter	Input value
Maturity	10
Payment freq.	0.25
Exposure freq.	0.5
Swap rate	0.03
Nbr of MC	2000
κ	0.1
θ	0.03
σ	0.02
r_0	0.03

CVA is based on data from 9th of May, 2012. The yield of the German government bonds can be found in table 2.

Table 2: Yield of German government bonds

Maturity	3 m	6 m	1 y	2 y	3 y	4 y	5 y	6 y	7 y	8 y	9 y	10 y	15 y	20 y	30 y
Yield	0.09	0.06	0.07	0.09	0.19	0.32	0.55	0.79	0.98	1.16	1.33	1.52	2.07	2.25	2.20

The CDS-data for the five companies are presented in table 3.

Table 3: CDS-spread in basis points

Maturity	6 m	1 y	2 y	3 y	4 y	5 y	10 y	20 y	30 y
Atlas Copco	21.69	27.2	38.04	48.33	57.18	65.24	80.26	78.19	78.52
Nordea	55.17	85.10	92.59	117.40	126.86	148.74	166.23	167.82	167.97
Securitas	25.80	34.5	52.61	70.55	86.31	95.64	120.30	120.4	120.45
Swedish Match	14.59	29.30	34.31	53.95	61.33	73.29	89.11	93.18	93.17
Vattenfall	13.73	18.45	31.81	45.05	58.12	71.09	94.68	94.65	94.63

The CVA-model give the following results:

Table 4: CVA in percent on notional

Company	CVA
Atlas Copco	0.0580%
Nordea	0.0586%
Securitas	0.0612%
Swedish Match	0.0579%
Vattenfall	0.0574%

This case study gives an indication of the size of CVA. At first glance it may appear as a diminishingly small amount but taking into consideration the fact that the notional of swap contracts can be as large as billions of euros, the CVA should not be negligible.

11.2.1 Error Estimation

To get an idea of the CVA volatility an error analysis was made. In this analysis CDS data from Atlas Copco was used. The CVA-model was run 500 times using the same parameter values as in the case study above. The sample variance was then calculated and a 95% confidence interval was constructed. CVA is presented both as a percentage of notional and as actual amount of notional of one billion euro, see figure 5.

Table 5: Confidence interval of CVA for Atlas Copco

	Lower Bound	Mean	Upper Bound
CVA percentage of notional	0.0562%	0.0589%	0.0617%
CVA for notional of one billion euro	562000	589000	617000

With support from this result it is concluded that the CVA-value is within an reasonable error range. The main source of the volatility is the expected exposure estimation, with its underlying interest rate simulation. One part of this error is the Monte Carlo error that arises from running the EE-simulation a limited number of times. The MC-error can be decreased by increasing the number of simulations. Approximately, to reduce the error by half the number of scenarios need to be quadrupled. The error from the exposure estimation is scaled by the other components in the CVA-formula, LGD, estimation of PD and discount factors.

Other sources of error that will become evident when recalculating CVA over time is the variation of the input data, and especially the CDS data, since it can vary a lot. There is of course also a discrepancy between the market movements in real life and the modelled ones.

12 Discussion and Conclusion

The model developed gives an overview of the characteristics of calculating CVA under the Basel III framework. It is a straightforward model which provides the user with graphical output from the simulation as well as mathematical results of CVA. Due to the complexity of the programming structure, the CVA calculation is quite heavy to execute. Further optimisation of the programming environment would ease the execution of the model. By evaluating the exposure with a less dense frequency the complexity of the calculation will significantly decrease. However, too sparse measurements will worsen the accuracy of the approximating sum so it is a trade off between an eased simulation and the accuracy in the CVA formula. An other very important parameter is the number of Monte Carlo scenarios. An optimal number would be infinity, but since this is out of reach a more modest setting is required. With only 2000 MC scenarios run, which was found as a limiting number for our computational capacity, it is hard to conclude the error range despite the commonly known relationship that the MC error is proportional to one divided by the square root of the number of simulations. Even though the model gives some insight in calculations of CVA a significantly larger number of scenarios would have been needed to make a statistically accurate estimation of the CVA value.

A major part of the calculation of CVA depends on the exposure which is based on an underlying interest rate simulation. In this study a one factor CIR-model was used. During trials to calibrate the interest rate model, using a Kalman filter, we came to the conclusion that the one factor model does not fully capture the market movement very well. No convergence of the CIR-parameters was to be found even though the Kalman filter did work well for simulated data. Since the calibration was unsuccessful the CIR-parameters were set to reasonable values with guidance from Csajková[11]. To capture interest rate movements for time horizons as long as 30 years a two factor model with a second factor describing movements of long term mean, would probably be a better approach.

One can also discuss the rigidity of the CVA-formula for the advanced CVA risk capital charge, specified in Basel III. First of all the structure of the formula might be too basic from a theoretical point of view. The approach of taking the sum over time of $LGD * PD * EE$ is sufficient for pricing CVA at a point in time, but the approximation of , especially, PD might not be completely accurate. Since the PD estimation is based on CDS-data it requires good data quality. A problem with the new CDS-market is that it is not fully liquid and the data might thus be unreliable. In the case study made of CVA for Swedish companies this was evident. If no CDS-data is available Basel III suggests that CDS proxy data from the same industry should be used. The risk of that approach is of course that the specific counterparty could be riskier than the industry average. So the overall quality of the CDS data that is obtainable today is definitely something to worry about.

Additionally the number of terms in the CVA sum is not specified in Basel III and as shown previously the number of terms is important. Basel does not specify exactly how the expected exposure should be calculated which leaves a lot of freedom for the banks to use their own models for simulating movement of the underlying assets. Depending on what models used, the characteristics and volatility of the exposure will differ. The LGD_{MKT} should be extracted from the market and depending on calibration method this factor is also highly volatile. This LGD is of great importance since it in one sense scales the CVA-value. An error in the LGD_{MKT} has thereby a large effect.

Even though the CVA-formula specified in Basel is a simplification, it serves its purpose of quantifying the capital requirement. It is important to have a general and international standard for measurement of market risk and default risk, they are then at least comparable even though the accuracy of the calculations can be discussed. It is though a trade off between having too high a capital requirement that perhaps will damp the world economy and having an excessively optimistic view on requirements that will jeopardise the robustness of the financial sector.

But the fact remains, since there is a great insecurity in PD, LGD and EE it is extremely difficult to get an accurate CVA value. Lessons from the last financial crisis shows that CVA is important and needs to be managed. Losses that can be referred to CVA had an important role in the last financial crisis and therefore it must be taken seriously. The best thing one can do is to constantly try to develop better and more refined models used for the different parts of CVA.

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13 Appendix

13.1 CVA model

The following sections give a brief presentation of the constructed VBA-model for CVA.

13.1.1 Input/output

In the setting of the model, a number of input values have been used. To enable as much flexibility as possible for the user, some of the inputs are changeable to customize the calculation each time the model is run. Figure 13 shows the input sheet in the Excel model. This sheet contains all input used in the calculations as well as a start button for the entire simulation. In this view the output, actual CVA value, also will be given.

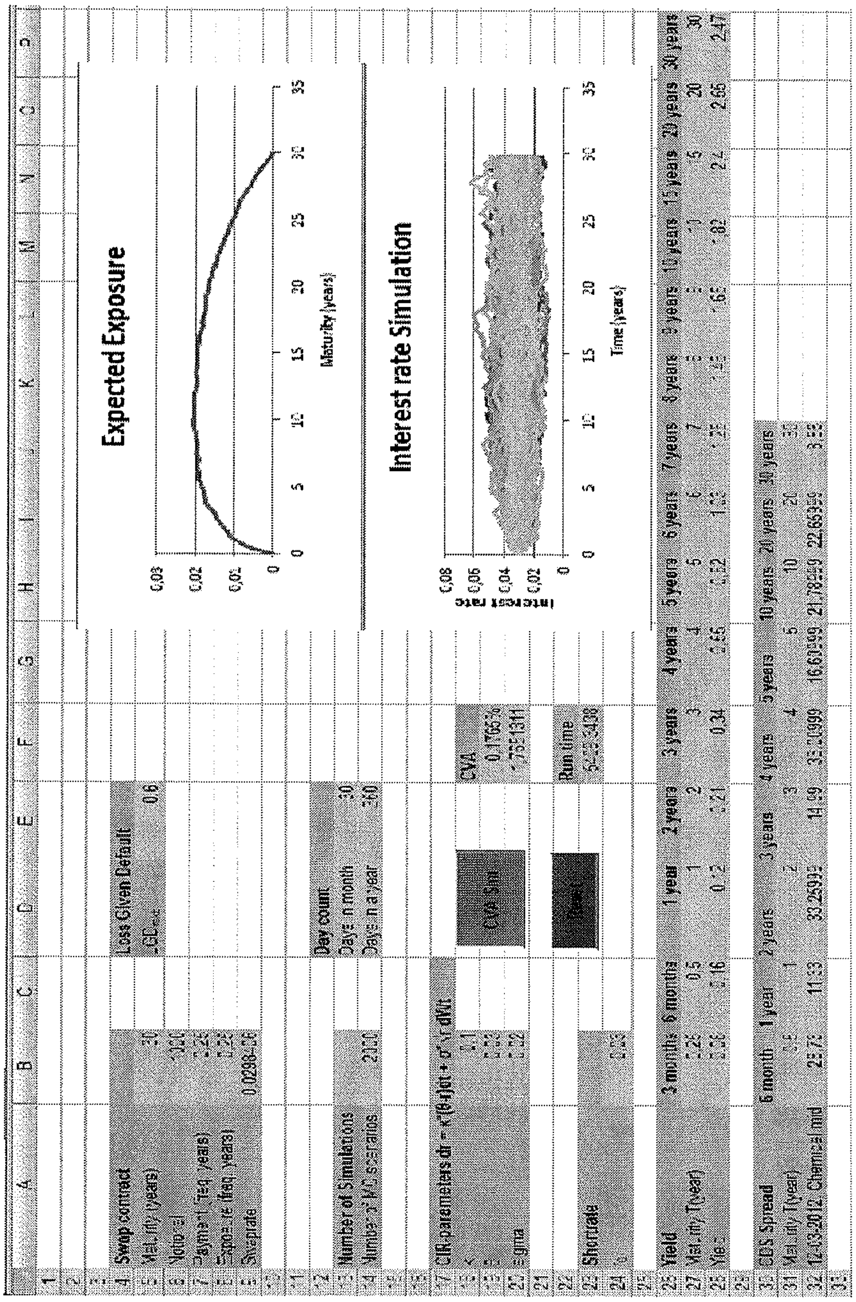


Figure 13: Screen capture of the setting of the CVA model

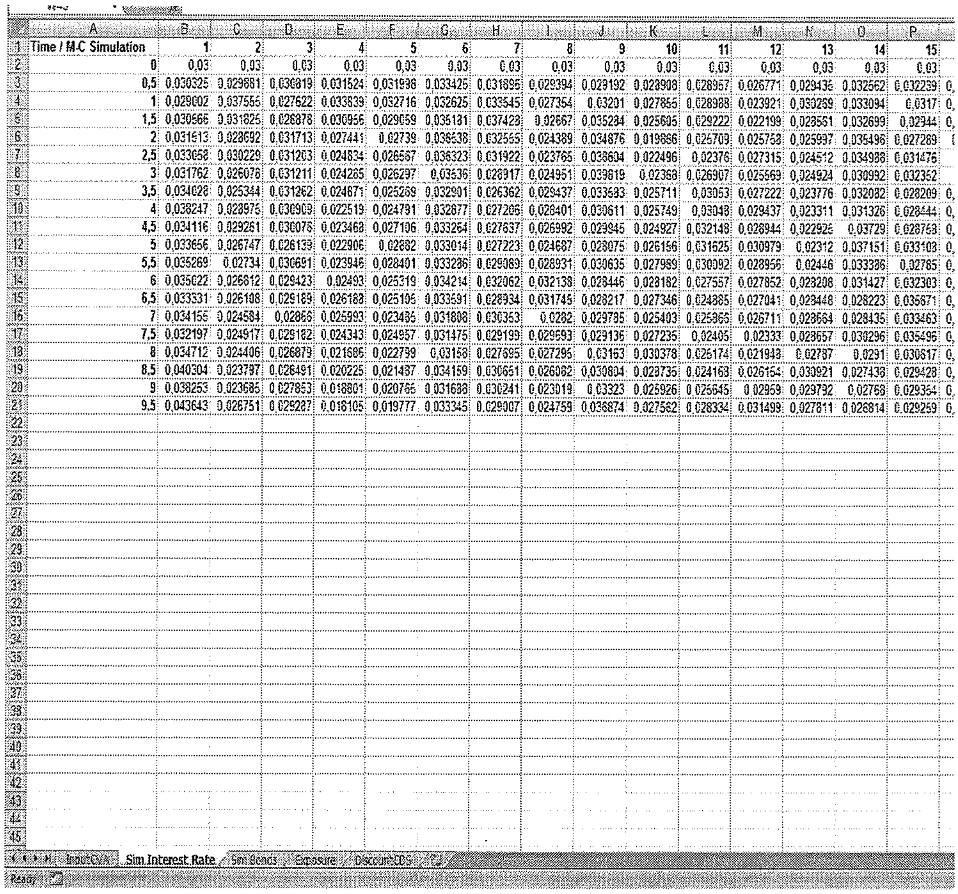
The input information concerning the type of contract, cell reference of the range A4 to B9, allows some flexibility in its values. The length of the contract in cell B5 is entered by the user and is to be written in units of year. The actual notional value is to be written in cell B6 and the swap rate in cell B9. The Exposure frequency in cell B8 is changeable as long as the frequency is expressed in terms of quarters on a yearly base and not less than payment frequency in cell B7. The exposure frequency in cell B8 is an important parameter for the total number of terms in the final summation of the CVA. Depending on the length of the contract the exposure frequency must be high enough to obtain accurate result. A more detailed explanation of the accuracy of the sum regarding total number of terms will be given in a later section.

The CIR-parameters in cells B18 to B21 and r_0 in cell B24 are assumed to be calibrated pre modeling. Loss Given Default (LGD) in cell E5 is used directly in the final summation in the CVA calculation. This value is optional for the user depending on which method that has been used for estimation of the LGD. In the simulation of the model an LGD of 60% has been used. Further, cell B14 refers to the number of Monte Carlo scenarios that will be performed per run. In a perfect model the number of Monte Carlo simulations should tend to infinity but this is impossible to implement in reality. During this study a couple of thousands MC-simulations has been possible to execute but as the number of simulations increases much over 2000 the performance of the computer becomes important. The simulation engine has a complex structured code and quite small changes in the number of MC-scenarios will have a significant impact in the execution of the model.

The model has a day count, as given in cells E13 and E14, with a year consisting of 360 days and a month equal to 30 days. This simplification is chosen to ease the logic in the VBA-code and is commonly used. To keep track of how the complexity in the model increases with larger number of MC-scenarios the simulation run is measured and given as output in cell F23. Further inputs in the model are given in the range A26:P28 which are spreads from CDSs with different time to maturity and in the range A30:J32 as yields based on government bonds. The calculated CVA-value is given as a percentage as output in cell F19, and in cell F20 as an actual amount with actual notional value.

13.1.2 Sim. Interest Rate

In the sheet named Sim Interest Rate the simulated rate paths will be printed for every run of the model. These prints will act as input for the calculation of the bond prices through time to maturity, see figure 14.



Time / M-C Simulation	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0,03	0,03	0,03	0,03	0,03	0,03	0,03	0,03	0,03	0,03	0,03	0,03	0,03	0,03	0,03
0,5	0,030325	0,029881	0,030819	0,031524	0,031938	0,033425	0,031895	0,029394	0,029192	0,028908	0,028957	0,026771	0,029436	0,032562	0,032239
1	0,029002	0,037555	0,027622	0,033839	0,032716	0,032625	0,033545	0,027354	0,03201	0,027855	0,028988	0,023921	0,030259	0,033094	0,0317
1,5	0,030566	0,031825	0,026878	0,030956	0,029059	0,035181	0,037428	0,02667	0,035284	0,025605	0,029222	0,022199	0,028561	0,032699	0,02944
2	0,031613	0,028692	0,031713	0,027441	0,02739	0,036538	0,032565	0,024389	0,034876	0,019866	0,025709	0,025758	0,025997	0,035496	0,027289
2,5	0,033658	0,030229	0,031203	0,024834	0,026567	0,036323	0,031922	0,023765	0,038604	0,022496	0,02376	0,027315	0,024512	0,034988	0,031476
3	0,031762	0,026078	0,031211	0,024265	0,026297	0,03536	0,028917	0,024951	0,039819	0,02368	0,026907	0,025569	0,024924	0,030992	0,032352
3,5	0,034028	0,025344	0,031262	0,024871	0,025269	0,032901	0,026362	0,029437	0,033683	0,025711	0,03053	0,027222	0,023776	0,032082	0,028209
4	0,038247	0,028976	0,030909	0,022519	0,024791	0,032877	0,027206	0,028401	0,030611	0,025749	0,03048	0,029437	0,023311	0,031326	0,028444
4,5	0,034116	0,029261	0,030076	0,023468	0,027106	0,033264	0,027637	0,026992	0,029945	0,024927	0,032148	0,028944	0,022926	0,03729	0,028763
5	0,033656	0,026747	0,026139	0,022906	0,02882	0,033014	0,027223	0,024687	0,028075	0,026156	0,031625	0,030979	0,02312	0,037151	0,033103
5,5	0,035269	0,02734	0,030691	0,023946	0,028401	0,033286	0,029089	0,028931	0,030635	0,027989	0,030092	0,028956	0,02446	0,033386	0,02785
6	0,035022	0,026612	0,029423	0,02493	0,025319	0,034214	0,032062	0,032138	0,028446	0,028182	0,027557	0,027852	0,028208	0,031427	0,032303
6,5	0,033331	0,026108	0,029189	0,026188	0,025105	0,033591	0,028934	0,031745	0,028217	0,027346	0,024885	0,027041	0,028448	0,028223	0,035671
7	0,034155	0,024584	0,02866	0,025993	0,023485	0,031808	0,030353	0,0282	0,029785	0,025403	0,025865	0,026711	0,028664	0,028435	0,033463
7,5	0,032197	0,024917	0,029182	0,024343	0,024957	0,031475	0,029199	0,029693	0,029136	0,027235	0,02405	0,02333	0,028657	0,030296	0,035496
8	0,034712	0,024406	0,026879	0,021686	0,022799	0,03158	0,027695	0,027295	0,03163	0,030378	0,026174	0,021948	0,02787	0,0291	0,030617
8,5	0,040304	0,023797	0,026491	0,020225	0,021437	0,034159	0,030651	0,026062	0,030804	0,028735	0,024163	0,026154	0,030921	0,027438	0,029428
9	0,038253	0,023685	0,027863	0,018801	0,020766	0,031688	0,030241	0,023019	0,03323	0,025926	0,025645	0,02959	0,029792	0,02768	0,029354
9,5	0,043643	0,026751	0,029287	0,018105	0,019777	0,033345	0,029007	0,024759	0,036874	0,027562	0,028334	0,031499	0,027811	0,026814	0,029259

Figure 14: Screen capture of simulated interest rate.

13.1.3 Exposure

The Exposure sheet visualises the exposure for every simulation and a calculation of the average in the bottom of the sheet. The average exposure is recalculated for each scenario and an updated chart is drawn in the Input/output sheet for every new Monte Carlo scenario, see figure 15.

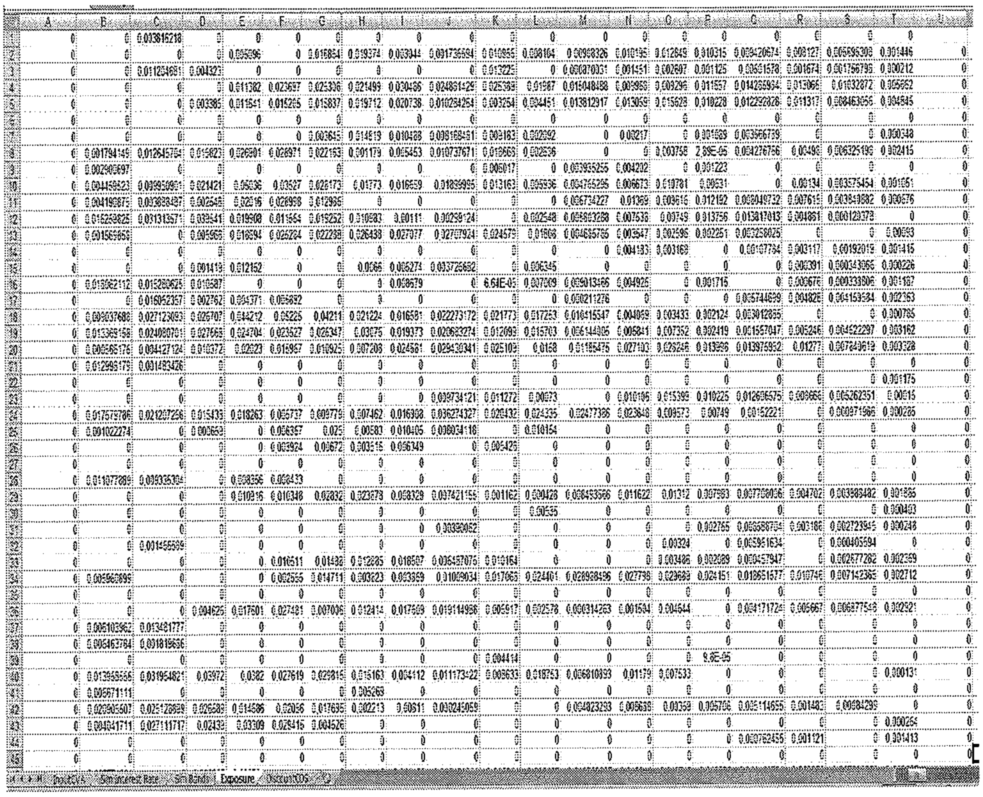


Figure 15: Screen capture of exposure values from simulation.

13.1.4 Sim. Bonds

In this sheet the calculated bond prices from the simulated interest rate are shown in a triangular structure for each Monte Carlo scenario. The structure eases the computational complexity since only future cash flows need to be discounted. The remaining cash flows to maturity through every payment date is in the very left column and at the bottom of the triangular structure one see the time to maturity expressed in the same density as the exposure frequency, see figure 16.

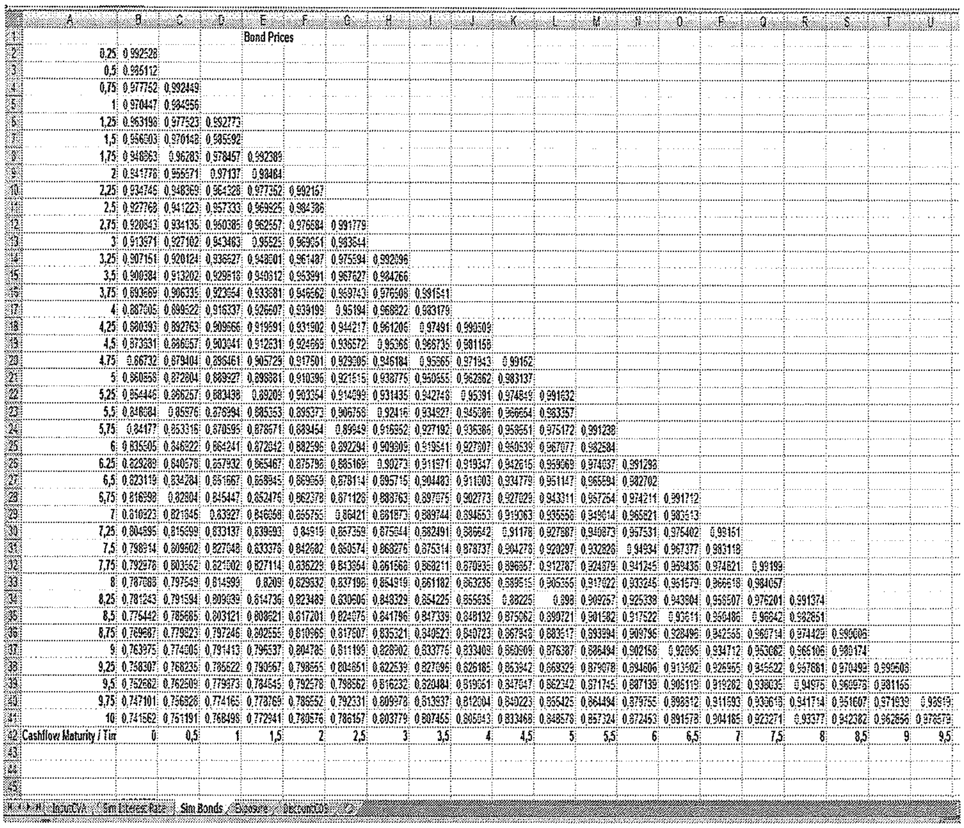
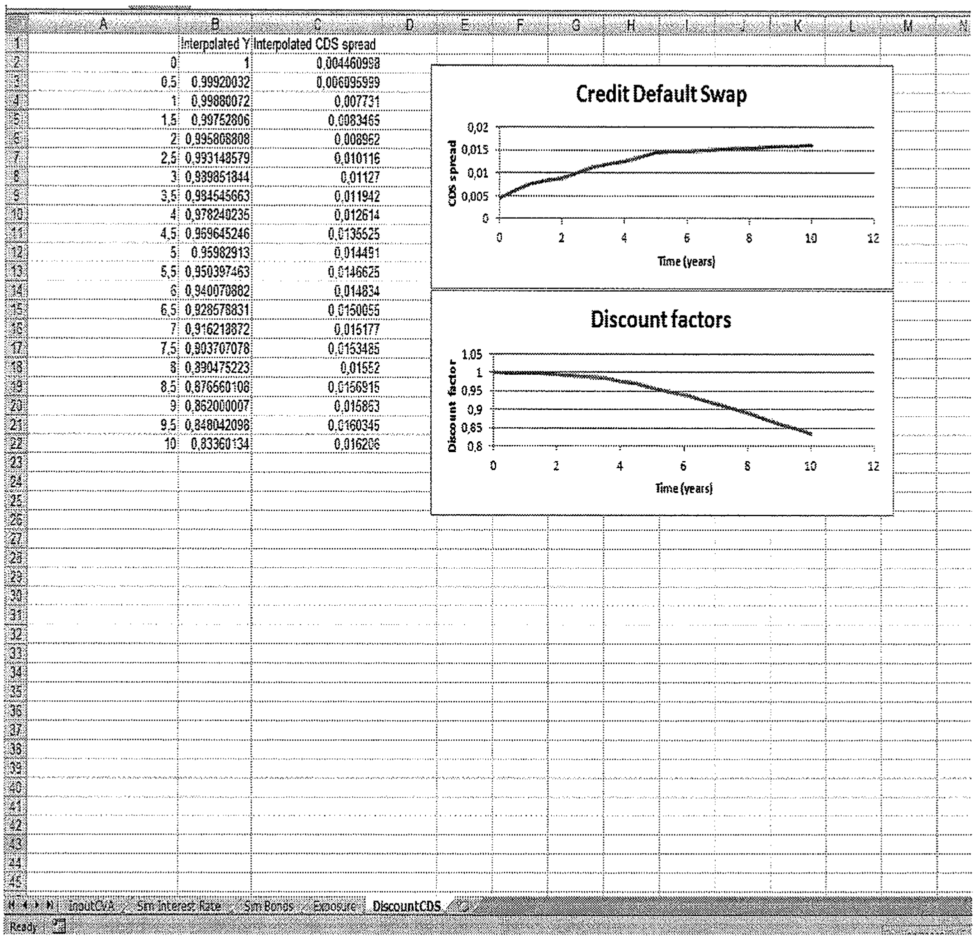


Figure 16: Screen capture of the triangular structure of bond prices.

13.1.5 DiscountCDS

The last sheet in the CVA model contains linearly interpolated CDS spreads and discount factors from the yield data and CDS data in the input sheet. These numbers are used in the final calculation of the CVA value. To ease the interpretation of the data the interpolation is also presented graphically, see figure 17.



A	B	C
	Interpolated Y	Interpolated CDS spread
0	1	0,004460998
0,5	0,99920032	0,006095999
1	0,99880072	0,007731
1,5	0,99752806	0,0083465
2	0,995808808	0,008962
2,5	0,993148579	0,010116
3	0,989851844	0,01127
3,5	0,984545663	0,011942
4	0,978240235	0,012614
4,5	0,969645246	0,0135525
5	0,95982913	0,014491
5,5	0,950397463	0,0146625
6	0,940070882	0,014834
6,5	0,928578831	0,0150055
7	0,916218872	0,015177
7,5	0,903707078	0,0153485
8	0,890475223	0,01552
8,5	0,876560108	0,0156915
9	0,862000007	0,015863
9,5	0,848042098	0,0160345
10	0,83360134	0,016206

Figure 17: Screen capture of interpolated discount factors and CDSs.

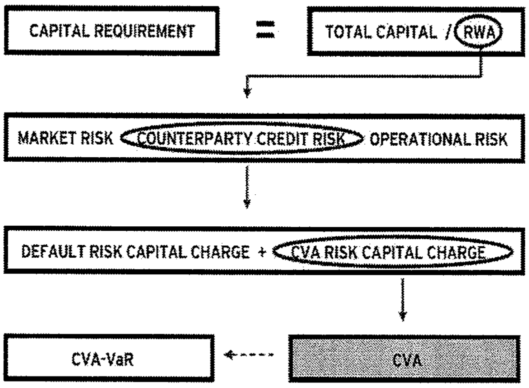
Credit Valuation Adjustment Risk Capital Charge enligt Basel III

Oroliga tider på finansmarknaden

Ett problem hos flertalet banker som uppdagades under den senaste finanskrisen runt 2007-2008 var att de inte hade tillräckligt stora kapitalreserver. Likviditeten var således inte god nog för att hantera de stora kreditförluster som blev ett faktum. Oroligheten inom banksektorn spred sig sedan vidare till att omfatta hela finansmarknaden. Som ett led i att förhindra framtida kriser av samma karaktär och omfattning pågår en vidareutveckling, Basel III, av det befintliga ramverk för finansiell stabilitet inom banksektorn, som går under namnet Basel II. Dessa rekommendationer ges ut av the Basel Committee of Banking Supervision. De verkar interanationellt för bankers finansiella stabilitet i ett längre perspektiv.

Krav på ökat kapitalbehov

En viktig del i rekommendationerna från Basel är att bankerna ska hålla en viss mängd kapital för att upprätthålla en tillräckligt hög likviditetsnivå. Denna kapitalreserv grundar sig på en kvot mellan det totala kapitalet och bankens riskkapital. Bankens riskkapital kategoriseras i förhållande till tre olika risktyper. Dessa är

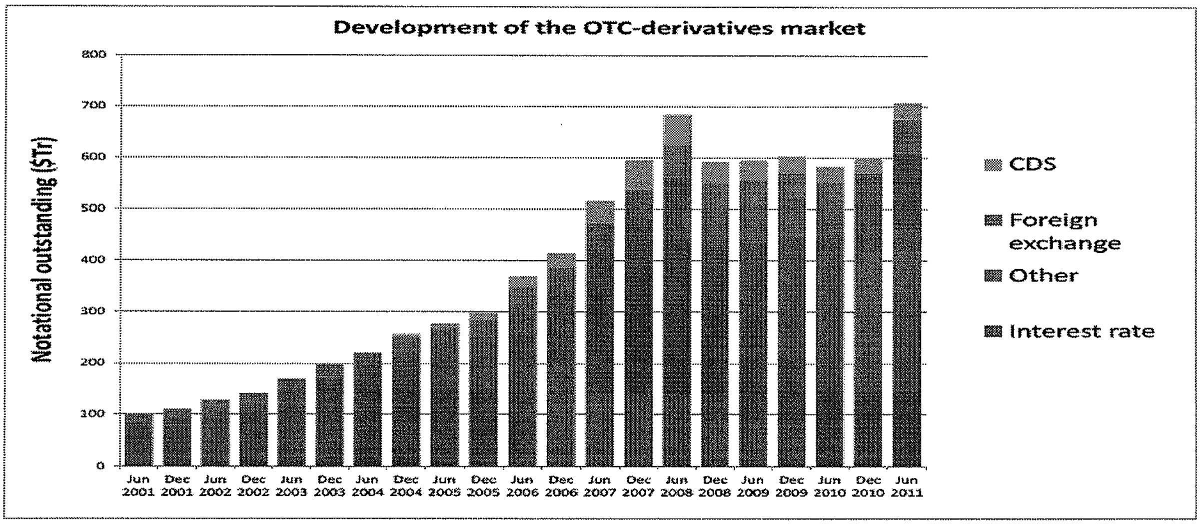


Figur 1. Schematisk bild av CVA Risk Capital Charge och CVA.

operativ risk, marknadsrisk samt kreditrisk. En viktig del i det nya Basel III är att det inom kreditriskkapitalreserven adderats en post som avser risken för marknadsändring av Credit Valuation Adjustment (CVA). I Basel benämns denna post som CVA Risk Capital Charge. CVA är priset för risken att en motpart i en finansiell överrenskommelse inte uppfyller sina åtaganden.

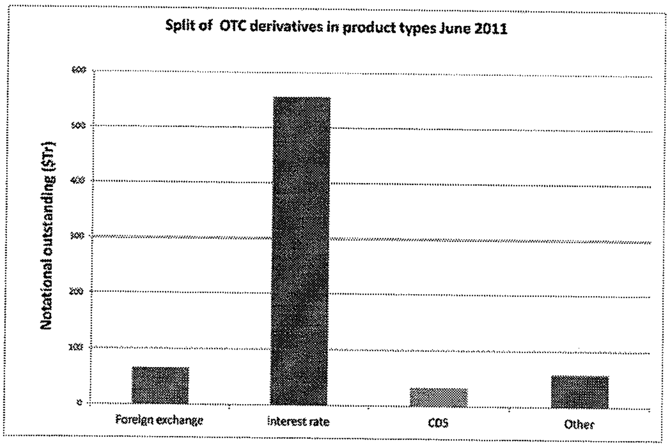
OTC-kontrakt

Over the Counter (OTC-) kontrakt handlas inte på



Figur 2. OTC-marknadens utveckling mellan 2001-2011.

en börs utan skräddarsys för motparterna vid varje köptillfälle. Detta ger upphov till kreditrisk eftersom man är helt beroende av att ens motpart uppfyller sin del av kontraktet. OTC-marknaden har under den senaste tioårsperioden uppvisat stark tillväxt och således fått en ökad betydelse. Det totala utställda beloppet i juni 2011 var ungefär 708 biljoner dollar, dvs 708 tusen miljarder, att jämför med 99.8 biljoner 10 år tidigare, vilket kan ses i figur 2. Den största ökningen kan härröras till räntekontrakt, vilket kan ses i figur 3 där OTC-kontrakt för juni 2011 är indelade efter kontraktstyp.



Figur 3. OTC-kontrakt uppdelade efter typ, juni 2011.

Modellering av CVA

För att räkna ut kapitalbehovet för CVA-risken behövs först en formel för att räkna ut CVA. I rekommendationerna från Basel III finns följande formel föreslagen:

$$CVA = \sum_{t=1}^{T} EE_t * PD_t * LGD$$

De komponenter som bygger upp CVA-värdet är EE (Expected Exposure), eller förväntad exponering som beskriver det förväntade värdet av det utestående kontraktet vid en viss tidpunkt under kontraktstiden. PD (Probability of Default), eller sannolikhet för konkurs, beskriver hur trolig en konkurs är under ett givet tidsintervall. Den tredje komponenten LGD (Loss Given Default) talar om hur stor andel som kan förväntas tillbaka vid det administrativa efterspelet av en konkurs. CVA kan således beskrivas som en produkt av förväntad exponering, sannolikheten för konkurs samt förväntad återbetalning, summerad över kontraktslängden.

En viktig del av CVA-beräkningen är hur de tre komponenterna tas fram. Med hjälp av simulering av möjliga scenarier kan framtida marknadsrörelser simuleras. Efter att ha modellerat CVA, har betydande volatilitet på CVA observerats. Modellen bygger på förväntad exponering, med en underliggande räntesimulering, sannolikheten för konkurs baserat på data från Credit Default Swaps (CDSs) samt LGD satt till ett fixt värde. Stor betydelse för tillförlitligheten på det beräknade CVA-värdet har antalet scenarier som modellen bygger på. En försvårande faktor vid simulering är att det blir mycket beräkningsintensivt vilket begränsar antalet körbara scenarier. Därmed kvarstår en betydande felkälla som skulle kunna minskas vid en optimering av modellen samt bättre datorprestanda.

CVA Risk Capital Charge

För att bedöma risken att CVA förändras över tiden, används en så kallad Value at Risk-modell. Resultatet av denna analys talar om maximal storlek på möjlig CVA-förlust med 99% procents säkerhet. Denna bedömning som ligger till grund för CVA Risk Capital Charge görs med en riskhorisont på ett år.

Slutsats

Det är i allmänhet mycket svårt att bygga en tilförlitlig modell för CVA. Detta beror på att de tre ingående komponenterna i sig är mycket osäkra och svåruppskattade. De kan dessutom förändra sig mycket från dag till dag. Vidare ställs stora krav på datorkapacitet då beräkningarna som utförs är mycket intensiva. Trots detta är CVA och dess volatilitet i förhållande till marknaden viktig att uppmärksamma. Att optimera och förbättra de modeller som finns idag är därför att rekommendera.

Anna Silén och Johanna Carlsson

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