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# Benefits of Multi-Echelon Inventory Control

A case study at Tetra Pak

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## ABSTRACT

This thesis studies a two-echelon distribution inventory system with a central depot which supplies  $N$  non-identical retailers with a single product. Stochastic customer demand occurs solely at the retailers. The purpose of the master thesis project is to evaluate the potential benefit of two methods in which the control of the inventory points in the inventory system is coordinated. The two models of coordinated control are benchmarked against a widely used single stage (uncoordinated) method.

Comparisons of the solutions produced by the methods are done using a discrete event simulation model based on real inventory data. The effectiveness of the models are evaluated in terms of the resulting total system cost, the average inventory levels, and different types of service levels.

The simulation results clearly indicate that the evaluated methods for coordinated control of inventories can offer considerable savings in reduced inventory levels and even, in many cases, at the same time increasing the fill rate. The reductions in total cost range 11-37% and an increase in fill rate accompanied by a reduction of the holding costs is observed in most cases. Especially the holding costs at the Central Warehouse can be reduced; when using a method for coordinated control the fill rate for orders from the retailers is 16-71% which corresponds to a reduction of the CW holding costs of 50-94%.

## FOREWORD

This master thesis is the concluding chapter of my journey to become a Master of Science in Industrial Engineering. The project has been done in collaboration with an ERP systems supplier who wishes to remain anonymous and Tetra Pak Technical Services who wanted to know more about coordinated control in multi-echelon inventory systems. For me, this has been a very rewarding journey, the analytical skills I have absorbed during my four years at Lund Institute of Technology has been tested to the maximum.

A would like to take this opportunity to thank my supervisor Johan Marklund at LTH, his help and support has been essential for this master thesis. I would also like to thank all of you that was always helpful and provided important input at the anonymous ERP systems supplier, you know who you are. A special thank goes to Jörgen Siversson at Tetra Pak for supplying the crucial inventory data. I am also grateful to Christian Howard for supplying me with his Extend model.

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# 1. INTRODUCTION

Supply Chain Management, the control of the material flow from suppliers of raw material to final customers, is today widely recognized as a crucial activity in most enterprises. The interest in coordinated control of multi-stage inventory systems in most supply chains is growing rapidly. A reason for this is that during the last two decades the research in this area has progressed substantially; many early models, which were rather restrictive in their assumptions, have been replaced by quite general ones. The new information technology has also greatly increased the possibilities for effective use of these models. Still, in practice today, there are very few examples of applications of the new research even though multi-echelon systems are common. This can be explained by the fact that the methods are quite difficult to understand and there are few illustrations of the benefits that coordinated control may bring in real cases. This master thesis aims to provide such an illustration by evaluating newly researched methods for coordinated inventory control using a simulation model based on real inventory data.

This study was initiated by an ERP system supplier who wishes to remain anonymous, and the inventory data required for the simulation models was acquired from Tetra Pak Technical Services. Tetra Pak Technical Services provide technical expertise, services and support in the areas of food processing and food packaging. They offer installation and start-up services at a new plant; customized solutions within maintenance, verification, and parts together with improvement services to enhance the performance; they also offer training services for operators, maintenance personnel and managers. The inventory system in question has its central warehouse in Lund and is handling spare parts for food processing and packaging machinery.

## 1.1. Background to inventory control in a multi-echelon system

A multi-stage, or multi-echelon, inventory system, consists of two or more inventory locations that are connected to each other. In a multi-echelon *distribution* system each location has as most one predecessor but an arbitrarily number of successors. The

complexities of managing inventory for such system increases significantly compared to the single-echelon case.

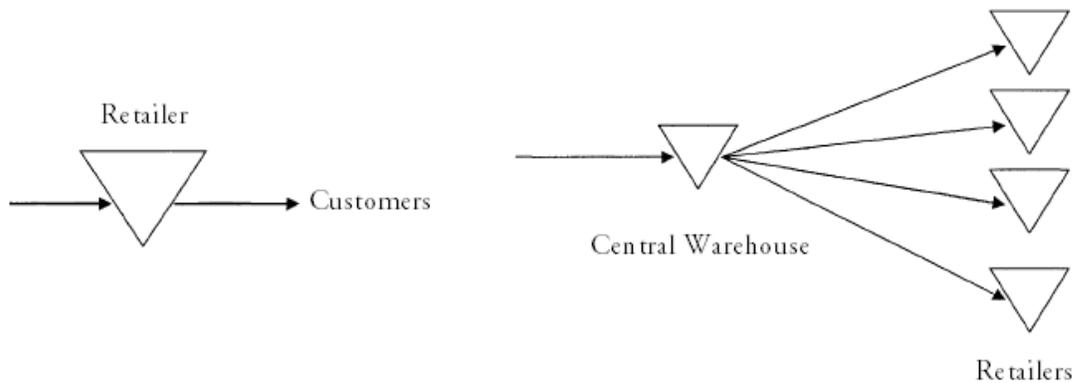


Figure 1.1 A single-echelon inventory system and a two-echelon distribution system.

Before indulging in the challenges of multi-echelon systems let us review the single-echelon problem, as seen in Figure 1.1. To control this inventory system one must make decisions about which items to stock; how much stock to keep on hand; how often to inspect the inventory; when to buy; how much to buy; controlling pilferage and damage; and managing shortages and back orders. This is what inventory control is about and the ultimate objective is to reduce the cost of the distribution, taking into account both costs associated with keeping products in stock, holding costs, and costs associated with not being able to satisfy customer demand directly, shortage costs. To narrow the scope of this master thesis the focus lies on one of these questions: When to buy? A common way to answer this question is to use the  $(R, Q)$  policy. This policy implies that a batch of  $Q$  units is ordered *when* the inventory position (stock on hand + outstanding orders – backorders) drops to, or below, the reorder point of  $R$  units. Therefore, to set a value for  $R$  is to answer the question of when to replenish. By assuming continually inspected inventories (the time between inspections is zero) the question on when to order is answered. The approaches for finding an optimum ordering policy is well known for the single echelon case (Axsäter, 2006), they depend on a series of factors:

- Demand forecast.
- Leadtime to the external supplier
- Service level goal

But how should you set the control variables in the multi-echelon case? Today, enterprises typically apply the single-echelon approach to each stock point in the system without explicitly considering how decisions at one stock point affects the others; we refer to this as an uncoordinated control of a multi-echelon system. Two questions arise:

- How much stock should be kept at the warehouse and at the different retailers?
- How do shortages at the warehouse affect the retailer leadtime?

The two questions are related, they have to do with the dynamics of the system; if you reduce the service level of the warehouse, the risk of shortages will increase and therefore the risk that an order from a retailer is delayed. As a result, the average retailer leadtime will increase. Since those dynamics are impossible to take into account when the coupled inventories are treated as independent there is no scientific way to determine the warehouse service level. The uncoordinated approach will thus result in suboptimal solutions often with excess inventory.

We now turn to the more advanced methods for control of multi-echelon inventory systems, which we refer to as methods for coordinated control. There exists no completely general method that works in all types of systems. The method depends on the characteristics of the inventory system in question, such as: The physical properties of the system, i.e., how the installations are coupled to each other, the size of the system, for example, larger systems with high demand generally require approximate methods, as exact methods tend to be computationally very demanding. Another aspect that governs the choice of model is whether the system is controlled centralized or decentralized. For an introduction on multi-echelon theories see Axsäter's Inventory Control (2006).

## **1.2. The studied inventory system and methods for coordinated control**

The considered inventory system is a two-level distribution system, which consists of one central warehouse to which an arbitrarily number of retailers,  $N$ , are connected, see Figure 1.1. Customer demand takes place at the retailers, which replenish their stock from the warehouse. The warehouse in turn places its orders with an outside supplier. The replenishment leadtime from the outside supplier is fixed. Stockouts at both echelons are

fully backordered. This means that in case of a stock-out the customer waits until the demand is fulfilled. Deliveries are made on a first-come first-serve basis – i.e. demand is satisfied in the order it arrives. Transportation times are constant but retailer leadtimes are stochastic due to possible shortages at the warehouse. All facilities use continuous review  $(R_i, Q_i)$ -policies. This means that when the local inventory position (stock on hand + outstanding orders – backorders) at the facility  $i$  decline to or below  $R_i$ , a replenishment order of  $Q_i$  units is placed. The objective is to optimize the reorder points  $R_i$ ,  $i=0, 1, 2 \dots N$  (0 corresponds to the central warehouse), the batch quantity and other inventory parameters are given and fixed. Also, indications are that to assume predetermined and fixed order quantities will only have a marginal effect on the expected cost, provided that the reorder cost is adjusted accordingly and that the given order quantities are not too far off, see for example Zheng (1992) and Axsäter (1996).

The two multi-echelon methods chosen to be evaluated in this study are both developed at the division of production management at Lund Institute of Technology. The first method, which will be denoted as Method 1 in the report, is the result from a series of three publications. The first article (Andersson et al. 1998) introduces an induced backorder cost at the warehouse, which will reflect the impact of the warehouse reorder level on the retailer leadtime, and ultimately the end-customer service level. A somewhat computationally demanding iterative procedure is used to find an optimal value for the induced backorder cost. The introduction of the induced backorder cost enables a decomposition of the multi-echelon system into a number of single echelon problems. The model is based on a simple approximation, in which the stochastic leadtimes perceived by the retailers are replaced by their correct averages. The method also assumes identical order quantities at the retailers and normally distributed end customer demand. The second article (Andersson & Marklund 2000) lifts the restriction of identical retailers. The third (Berling & Marklund 2006) uses the iterative procedure developed in the first article to build an intuitive understanding as to how the optimal induced backorder cost depends on the system parameters, and uses this knowledge to create simple closed-form estimates of the induced backorder, thus eliminating the computationally demanding iterative process in the earlier publications.

The second method is based on Axsäter (2003), it is denoted as Method 2 in the report. It uses normal approximations both for the retailer demand and the demand at the warehouse, i.e., orders from the retailers. The normal approximations make it possible to optimize even quite large systems effectively. The method also assumes that the mean and variance for the delay at the warehouse, due to shortages, is the same for all retailers. The general idea of the method is to (1) fit a normal distribution to the warehouse demand mean and variation. (2) Determine mean and variance for the retailer leadtime for each considered reorder level at the warehouse. (3) Determine each retailers optimum reorder point for each of the considered warehouse reorder points. This is done through a simple search procedure. (4) The optimum warehouse reorder level is determined by minimizing the total system cost, which is now only dependent on the warehouse reorder point (through step 3).

The uncoordinated approach that is evaluated in this project is an optimization method found in a widely used commercial application developed by the ERP company that initiated this study. This method is based on the standard techniques to optimize reorder points in single-echelon system for example described in Axsäter (2006).

### **1.3. Problem definition**

The purpose of this master thesis project is to investigate the performance of two models for coordinated control in a multi-echelon inventory system, and thereby illustrate the potential value of coordinated inventory control over the currently used uncoordinated method.

Comparisons of the solutions produced by the methods are done using a discrete event simulation model based on real inventory data. The effectiveness of the models are evaluated in terms of the resulting total system cost, the average inventory levels, and different types of service levels, in comparison with an uncoordinated approach.

## 1.4. Method

This project can be interpreted as a rather straightforward operations research study, “..a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control.” (Morse, 2003). Figure 1.2 shows the major phases of a typical operations research study.

- Phase 1. Formulating the problem.
- Phase 2. Constructing a mathematical model to represent the studied system.
- Phase 3. Deriving a solution from the model.
- Phase 4. Testing the model and the solution derived from it.
- Phase 5. Establish control over the solution
- Phase 6. Putting the solution to work: Implementation.

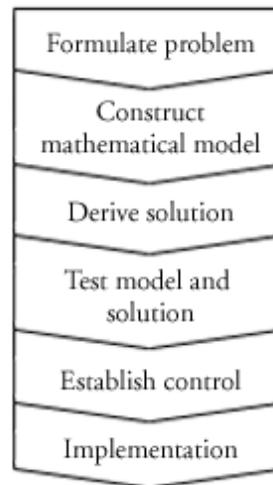


Figure 1.2 The six major phases in a typical operations research study.

In *phase 1* a well-defined statement of the problem together with appropriate objectives is developed. This phase is continually re-examined in the light of new insights obtained during the later process. *Phase 2* is in this project not about constructing a new mathematical model to represent the system analytically, but rather to find, read and understand the structure and assumptions of already developed models and to choose models for evaluation. The model of how to represent the inventory system in the simulation environment is constructed in this phase. The work of *phase 3*, to derive a solution from the mathematical model, is mostly done outside of this project. The majority of this project is carried out within *phase 4*, where the models' validity is tested.

This phase is divided into three steps.

1. *Select test cases* – Historical inventory data is analyzed to select a number of products that is to be used as test cases.

2. *Obtain and analyse in-data for simulation* – A complete set of parameters that define the inventory system for each test case is derived: leadtimes, batch sizes, demand patterns etc together with an optimal solution from each of the three compared inventory control models.
3. *Simulate and compare overall system costs* - Discrete event simulation is used for evaluating the performance of the alternative solutions produced by the studied methods. Total system costs and different types of service levels are estimated in these simulations.

*Phase 5*, to establish control over the solution, refers to installing a well-documented system for applying the model. This system would include the model, the solution procedure, and operating procedures for implementation. Since it is evident that the solution for the real problem remains valid only as long as the underlying models remains valid, a plan for detecting parameters and implement the model where it remains valid is part of the system for applying the model. The last phase, *phase 6*, implementation, will completely fall outside the scope of this project.

## 1.5. Outline of the report

For readers interested in the results of the study; in the effectiveness of models for coordinated control, turn to the conclusions in Chapter 5, or for a more detailed discussion about the results, to Chapter 4. Researchers interested in aspects of how the study was conducted, consult Chapter 3. In this chapter it is described how the simulation model was constructed and how in-data, in the form of parameters for the inventory system, was gathered. The analysis of the output data is also described. Turn to section 2.3 for an in-depth explanation of the methods for coordinated control evaluated in this study. The earlier sections of Chapter 2 provides a more general theoretical base from which the inventory system and the execution of the study can be understood, basic inventory theory is presented together with an introduction to simulation.

## 2. THEORETICAL FRAMEWORK

This chapter presents the theoretical base from which the inventory system and the evaluation of the multi-echelon optimization models can be understood. Section 2.1 starts with a number of definitions of concepts in inventory theory, then the theory used later in the report is provided in section 2.1.2-2.1.4; the Poisson process, the compound Poisson process and the gamma distribution. Expressions for optimizing reorder levels in a single-echelon inventory system are given in section 2.1.5. Section 2.2 and onwards focuses on inventory theory for the multi-echelon case. It begins with an overview of multi-echelon inventory systems, and then discusses traditional methods for control of this system and their strengths and weaknesses. Section 2.3 presents the theory for the two multi-echelon models we consider.

### 2.1. Inventory control theory

This section focuses on the theoretical framework for controlling single-echelon inventory systems.

#### 2.1.1. Concepts

This is a short introduction to the basic concepts used in inventory control theory, for a more in-depth treatment see for example the book *Inventory Control* (Axsäter 2006). To begin with, a few central concepts in inventory control are defined.

*Stock on hand* – The number of physical items found in the defined inventory facility.

*Backorder* – A record of a customer order that could not be fulfilled immediately and is waiting to be delivered.

*Outstanding orders* – Items ordered from the supplier that has not yet been delivered to the inventory location in question.

*Leadtime* – The time from the ordering decision until the ordered amount is available for demand on the shelves.

*Holding cost* – The cost for holding stock. The opportunity cost for capital tied up in inventory usually makes the dominant part of this cost. Other parts can be material handling, storage, damage and obsolescence, insurance and taxes. The holding cost per unit and time unit is often determined as a percentage of the unit value.

*Shortage costs* – Costs occur when demanded items cannot be delivered due to shortages. In some situations the customer agrees to wait, in other situations the customer chooses some other supplier.

*Service level goal* – In most cases the costs associated with shortages are difficult to estimate; therefore it is very common to replace them by a suitable service level constraint or target service level. In practice there are many ways to define a service level and measure service, the three most common are: (1) Probability of no stockout in an order cycle. (2) “fill rate” – fraction of demand that can be satisfied immediately from stock on hand. (3) “ready rate” – fraction of time with positive stock on hand.

*Inventory position* – Ordering decisions are based on the inventory position,

$$\text{inventory position} = \text{stock on hand} + \text{outstanding orders} - \text{backorders}.$$

*Inventory level* – Holding and shortage costs is based on the inventory level,

$$\text{inventory level} = \text{stock on hand} - \text{backorders}.$$

*Continuous review / Periodic review* – An inventory control system can be designed so that the inventory position is monitored continuously and decisions made as changes occur, this is called continuous review. An alternative, which is often motivated by the cost of continuous supervision, is to monitor the inventory position only at certain given points in time. In general, the intervals between these points are constant, this is called periodic review. Continuous review will reduce the needed safety stock since this stock only have to guard against demand variations during the leadtime. With periodic review the uncertainty

time is extended by the review period. Consider a review when no order is triggered. The next possible delivery is the leadtime plus one review period ahead.

### 2.1.2. Normally distributed demand

In practice the demand during a certain time interval is nearly always a nonnegative integer. However, if the demand is relatively large, it is more practical to use a continuous demand model as an approximation. The central limit theorem dictates that a sum of many independent random variables will have a distribution that is approximately normal (see, for example, Blom & Holmquist 1998). In many situations the demand comes from several independent customers, and it is therefore reasonable to let the demand be represented by a normal distribution. The normal distribution is also mathematically relatively easy to deal with. A drawback for an inventory modelling perspective is that with the normal distribution there is always a small probability for a negative demand. For a detailed discussion of various assumptions concerning demand distributions see Brown and Zipkin (1991). A stochastic variable  $X$  that is normally distributed with mean  $\mu$  and standard deviation  $\sigma$  is denoted  $X \in N(\mu, \sigma)$ . The normal distribution with mean  $\mu = 0$  and standard deviation  $\sigma = 1$  is called a standardized normal distribution and has the following density,

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad -\infty < x < \infty, \quad (2.1)$$

and cumulative distribution function,

$$\Phi(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du, \quad -\infty < x < \infty. \quad (2.2)$$

We can obtain the density of  $X \in N(\mu, \sigma)$  as  $(1/\sigma)\varphi((x - \mu)/\sigma)$  and the distribution function as  $\Phi((x - \mu)/\sigma)$  for other values of  $\mu$  and  $\sigma$ .

### 2.1.3. Poisson process

It is common in inventory theory to assume that the demand, i.e. the arrival of customers, follows a Poisson process. The well known memoryless property of the exponential distribution implies that the time until the next arrival is independent of when earlier arrivals have occurred. These results explain why the Poisson process often is a suitable demand model. Many systems, with large calling populations (many different customers), experience demand where customers arrive independently of each other and only demand

one unit at a time. The popularity of the process can also be explained by the fact that the memoryless property simplifies analytical calculations.

### Definition

Let  $k$  be the number of arrivals in the time interval  $(0, t)$ . A stochastic process  $X(t)$ ,  $t \geq 0$ , is a Poisson process if the following three conditions are met:

1. The process has independent increments, i.e. the number of arrivals in disjoint time intervals are independent.
2.  $P(\text{there are exactly one arrival in the interval } (t, t+h)) = \lambda h + o(h)$ .
3.  $P(\text{there are more than one arrivals in the interval } (t, t+h)) = o(h)$

The Poisson process is said to have the intensity  $\lambda$ , which is also known as the arrival rate. Two interesting results can be derived from these conditions. First, for a given  $t$ , the number of arrivals in the time interval  $(0, t)$  is Poisson distributed with mean and variance  $\lambda t$ , i.e. the probability distribution of  $X(t)$  is given by

$$P_X(x) = \frac{(\lambda t)^x}{x!} e^{-\lambda t} \quad (x = 0, 1, 2, \dots). \quad (2.3)$$

Second, the time between two arrivals is exponentially distributed with mean  $1/\lambda$ , i.e. the density function for the time between consecutive arrivals is given by

$$P(t) = \lambda e^{-\lambda t} \quad (t \geq 0). \quad (2.4)$$

Proofs of these results can be found in e.g. Blom (1998).

#### 2.1.4. Compound poisson demand

To make the simulations realistic a generalization of the poisson demand process is used when describing the demand in the simulation environment. The customers arrive according to a poisson process but the size of the customers' orders is stochastic. The quantity is assumed independent of other customer demands and of the distribution of the customer arrivals. The discrete distribution of the demand size is denoted the compounding distribution. Let

$f_j$  probability of demand size  $j$  ( $j=1, 2, \dots$ )

$f_j^k$  probability that  $k$  customers give the total demand  $j$

$D(t)$  stochastic demand in the time interval  $t$

We assume that there are no demands of size zero. This is no lack of generality, in both cases such demand processes can be replaced by an equivalent process without those characteristics. To determine  $D(t)$  we first note that  $f_0^0 = 1$ , and  $f_j^1 = f_j$ . Given  $f_j^1$  we can obtain the  $j$ -fold convolution of  $f_j, f_j^k$ , recursively as

$$f_j^k = \sum_{i=k-1}^{j-1} f_i^{k-1} f_{j-i}, \quad k = 2, 3, 4, \dots \quad (2.5)$$

Using (2.3) we then have

$$P(D(t) = j) = \sum_{k=0}^{\infty} \frac{(\lambda t)^k}{k!} e^{-\lambda t} f_j^k. \quad (2.6)$$

If  $K$  is the stochastic number of customers during a time unit,  $J$  the stochastic demand size of a single customer, and  $Z$  the stochastic demand during the time unit considered. Since  $K$  is generated through a poisson process  $E(K) = \text{Var}(K) = \lambda$ , and  $K$  and  $J$  are independent the mean and variance for the demand per time unit,  $\mu$ , is given through

$$\mu = E(Z) = E(K)E(J) = \lambda \sum_{j=1}^{\infty} j f_j. \quad (2.7)$$

$$\sigma^2 = \lambda E(J^2) = \lambda \sum_{j=1}^{\infty} j^2 f_j. \quad (2.8)$$

### 2.1.5. Gamma distribution

In the simulation environment a discrete distribution that is based on the gamma distribution is used as the compounding distribution in the compound poisson demand process. The gamma distribution, that is continuous, is converted to a discrete distribution by rounding the stochastic variable up to nearest integer (see section 3.2).

The gamma distribution is defined by the probability density function (pdf)  $f_X(x)$  and the cumulative distribution function (cdf)  $F_X(x)$

$$f_X(x) = \begin{cases} \frac{1}{a^p \Gamma(p)} \cdot x^{p-1} e^{-x/a} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \quad (2.9)$$

where  $p > 0$  and  $a > 0$ . A stochastic variable  $X$  that is gamma distributed can be denoted as:  $X \in \Gamma(p, a)$  and has mean  $E[X] = pa$  and variance  $\text{Var}[X] = pa^2$ . The name of the gamma distribution stems from the gamma function:

$$\Gamma(p) = \int_0^{\infty} x^{p-1} e^{-x} dx \quad (p > 0) \quad (2.10)$$

If  $p$  is an integer then, as can be found by repeated partial integrations of the integral above,  $\Gamma(p) = (p-1)!$ . If  $p=1$ , the gamma distribution becomes an exponential distribution (Blom, 1998).

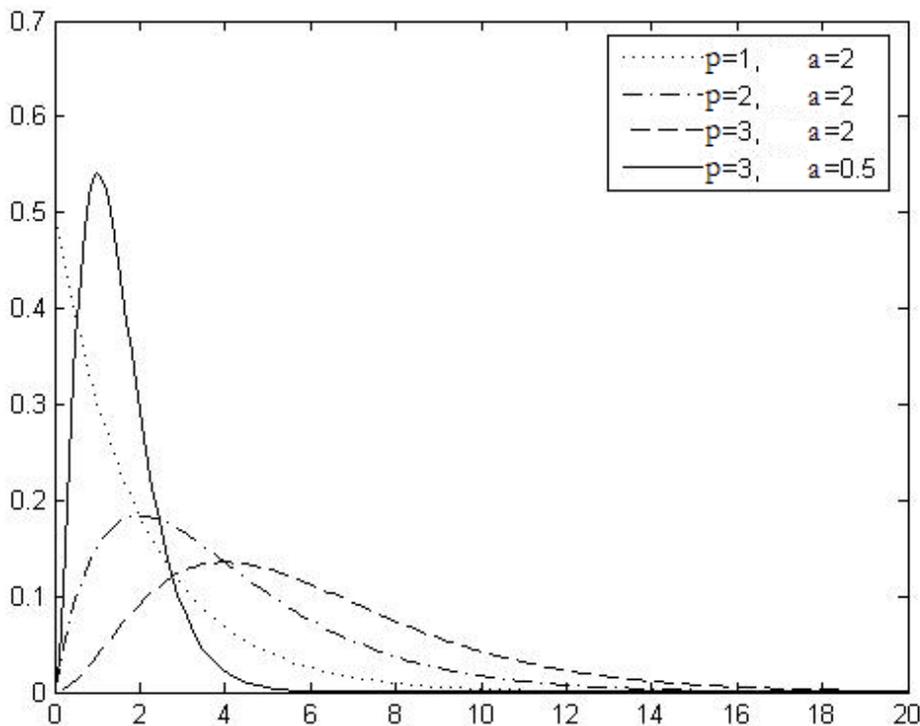


Figure 2.1 Probability density functions for a couple of gamma distributions.

### 2.1.6. Optimal reorder points for an continuous review (R, Q) policy

An ordering policy is the set of rules that control when and how much to order at an inventory location. The continuous review  $(R, Q)$  policy implies that the inventory position is inspected continually and that a batch of  $Q$  units is ordered at the instant that the inventory position reaches  $R$  units. The parameter  $R$  is referred to as the reorder point and  $Q$  as the ordering quantity or the batch quantity.

When implementing the  $(R, Q)$  policy, the objective is to determine the two parameters; the reorder point and the ordering quantity. How to determine ordering quantity lies outside the scope for this master thesis, this parameter will be treated as given. To determine an optimal reorder point we need to balance the need to minimize holding costs with the need to provide adequate service. This is either done through a prescribed service level goal, or by defining holding costs and shortage costs. We will assume that the demand is normally distributed. The process of finding an optimal reorder point is only discussed briefly here, for more details see Axsäter (2006) chapter 5, which this section follows quite closely.

Used notation:

IL	inventory level
$f(x)$	density function for the inventory level
$F(x)$	distribution function for the inventory level
$\varphi(x)$	The normal density function
$\Phi(x)$	The normal distribution function
$S_2$	Service level goal, fill rate
$S_3$	Service level goal, ready rate
$\mu$	Mean for the leadtime demand
$\sigma$	Standard deviation for the leadtime demand
$h$	Holding cost
$b$	Backorder cost

#### Using a service level constraint

In section 2.1.1, three different service level definitions are given. Here we will only consider the last two, fill rate ( $S_2$ ) and ready rate ( $S_3$ ). These are the constraints that is used

by Tetra Pak. Under the assumption of normally distributed demand  $S_2 = S_3$ , and the service level is the probability of positive stock. Using the so called loss function,

$$G(x) = \int_x^{\infty} (v-x)\varphi(v)dv = \varphi(x) - x(1-\Phi(x)), \quad (2.11)$$

the distribution function for the inventory level can be given by

$$\begin{aligned} F(x) &= \frac{1}{Q} \int_R^{R+Q} \left[ 1 - \Phi\left(\frac{u-x-\mu}{\sigma}\right) \right] = \frac{1}{Q} \int_R^{R+Q} \left[ -G'\left(\frac{u-x-u}{\sigma}\right) \right] du \\ &= \frac{\sigma}{Q} \left[ G\left(\frac{R-x-\mu}{\sigma}\right) - G\left(\frac{R+Q-x-\mu}{\sigma}\right) \right]. \end{aligned} \quad (2.12)$$

The service level can now be obtained from

$$S_2 = S_3 = 1 - F(0) = 1 - \frac{\sigma}{Q} \left[ G\left(\frac{R-\mu}{\sigma}\right) - G\left(\frac{R+Q-\mu}{\sigma}\right) \right]. \quad (2.13)$$

For a given service level we can search for the smallest R rendering the required service level.

### Using holding costs and shortage costs

We optimize the reorder point by minimizing the sum of expected holding and backorder costs. It is convenient to use the notation

$$(x)^+ = \max(x, 0) \text{ and } (x)^- = \max(-x, 0). \quad (2.14)$$

Note that  $x^+ - x^- = x$ . The expected cost per unit of time is given by

$$\begin{aligned} C &= hE(IL)^+ + bE(IL)^- = hE(IL) + (h+b)E(IL)^- \\ &= h(R+Q/2-\mu) + (h+b) \int_{-\infty}^0 F(x)dx \\ &= h(R+Q/2-\mu) + (h+b) \int_{-\infty}^0 \frac{1}{Q} \int_R^{R+Q} \left[ -G'\left(\frac{u-x-\mu}{\sigma}\right) \right] dudx \\ &= h(R+Q/2-\mu) + (h+b) \frac{\sigma}{Q} \int_R^{R+Q} G\left(\frac{u-\mu}{\sigma}\right) du. \end{aligned} \quad (2.15)$$

In the first line of (2.15) we use that the average inventory position is  $R + Q/2$  in the continuous case. Furthermore,

$$E(IL)^- = -\int_{-\infty}^0 uf(u)du = \int_{-\infty}^0 f(u) \left[ \int_u^0 dx \right] dx = \int_{-\infty}^0 \left[ \int_{-\infty}^x f(u)du \right] dx = \int_{-\infty}^0 F(x)dx \quad (2.16)$$

We now need to define the function  $H(x)$ ,

$$H(x) = \int_x^{\infty} G(v)dv = \frac{1}{2} \left[ (x^2 + 1)(1 - \Phi(x)) - x\phi(x) \right].$$

Note that  $H'(x) = -G(x)$ . We can now express the cost in (2.15) as

$$C = h(R + Q/2 - \mu) + (h + b) \frac{\sigma^2}{Q} \left[ H\left(\frac{R - \mu}{\sigma}\right) - H\left(\frac{R + Q - \mu}{\sigma}\right) \right]. \quad (2.17)$$

To find the optimal reorder point we now need to minimize (2.17) with respect to  $R$ . From (2.17) and (2.13) and the definition of  $H(x)$  we have

$$\begin{aligned} \frac{dC}{dR} &= h + (h + b) \frac{\sigma}{Q} \left[ G\left(\frac{R + Q - \mu}{\sigma}\right) - G\left(\frac{R - \mu}{\sigma}\right) \right] = h + (h + b)(S_2 - 1) \\ &= -b + (h + b)S_2 = -b + (h + b)S_3. \end{aligned} \quad (2.18)$$

Since  $C$  is a convex function of  $R$ , the optimal  $R$  is obtained for  $dC/dR = 0$ .

### Relationship between cost parameters and service level

In the optimal solution for (2.18) we have

$$S_2 = S_3 = \frac{b}{h + b}. \quad (2.19)$$

Since it is often the relation between holding costs and backorder cost that is of interest, the holding cost can be set to one. Then, if a service level,  $S_2$ , is given, (2.19) can be used to find a backorder cost. The backorder cost can in turn be used to find an optimal reorder level which will render the desired service level. In the case of compound Poisson demand this relation is only valid for the ready rate and it becomes

$$S_3(R^*) \leq \frac{b}{h + b} < S_3(R^* + 1). \quad (2.20)$$

(2.20) is also valid for the fill rate in case of pure Poisson demand, when each customer only orders one item.

## 2.2. Multi-echelon inventory systems

A multi-echelon inventory system consists of several inventory locations that are connected to each other. They appear in practice both when distributing products over large geographical areas and in production, where stocks of raw material, components, and finished products are coupled. The inventory control theory distinguishes between four main types of multi-echelon systems.

In the serial system each stock has at most one immediate successor and at most one predecessor, see Figure 2.2. For the pure distribution systems each stock point has at most one predecessor but an arbitrary number of successors. A stock point in an assembly system has an arbitrarily number of predecessors but at most one immediate successor. If each stock point can have an arbitrarily number of both predecessors and successors this system is called a general multi-echelon inventory system.

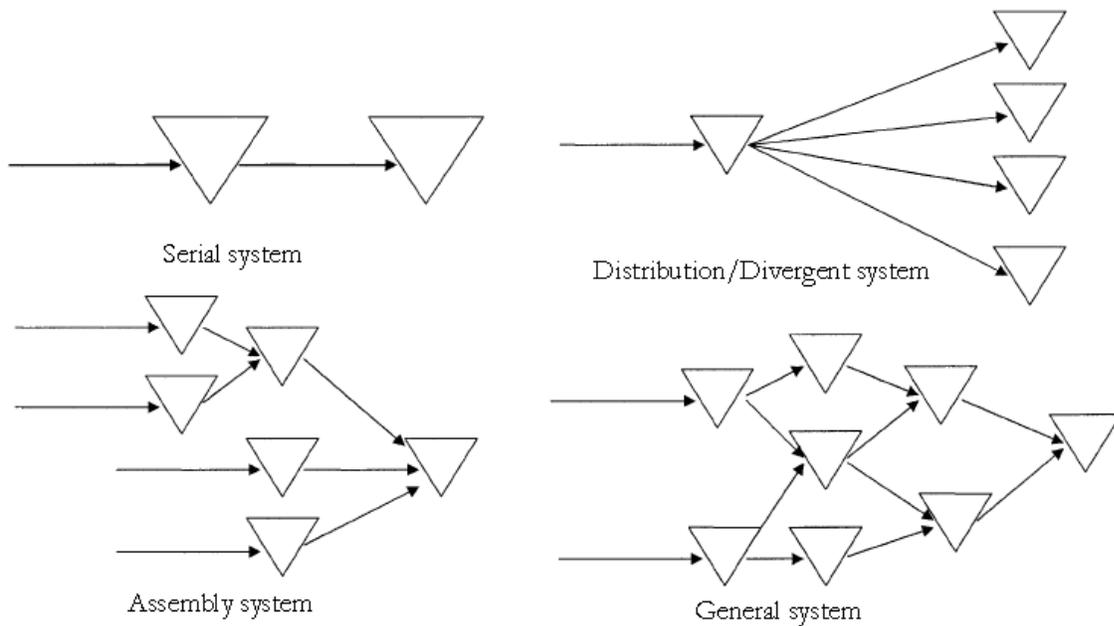


Figure 2.2 Upper left corner: a serial system. Upper right corner: a two-echelon distribution system. Lower left corner: a three-echelon assembly system. Lower right corner: a general multi-echelon system.

The different types of multi-echelon systems shown in Figure 2.2 are quite different in an inventory control context, and different multi-echelon models are required for controlling and evaluating them. Other characteristics of the inventory system also influence how it can be controlled.

- High / Low demand. Systems with high demand generally require approximate methods, as exact methods tend to be computationally very demanding.
- Repairable / Consumable items. Repairable items need to be transported upstream in the system.
- Centralized / Decentralized organization. Is each stock point included in single organization so that they can be regulated centrally or are administered by a multiple number of independent organizations.
- Lateral transshipments. Items can be transported between parallel stock points in order to avoid shortages.

I will now focus on the two-echelon distribution system, for consumable items. For simplicity, no lateral transshipments will be regarded. For a more detailed introduction of different types of multi-echelon systems and models see, for example, van Houtum et al (1996).

### 2.2.1. Uncoordinated control of multi-stage inventory systems

As previously mentioned we refer to an uncoordinated approach to control multi-stage inventory systems when single-echelon models are used to control stocks that are in fact connected and interdependent. Each stock point in the system is treated as independent with a set leadtime and service level, optimal reorder points is then determined using a forecast for the demand at this site.

The obvious strength of this method is its simplicity, multi-echelon methods are in general much more complex. The weakness is that it does not describe the real system in an adequate way and obvious interdependencies in the system are not taken into account, and as a result suboptimal solutions are produced.

In an uncoordinated approach two problems arise: (1) How we should determine the warehouse demand, this demand is in fact fully determined by the retailer demand and to do an extra forecast introduces an extra layer of error and a risk for bullwhip effect. (2) Since we assume that the purpose of the system is to serve the end-customer, the question is to deploy the safety stock in the system in the most efficient way to achieve this objective

in time. How should then the service level for the warehouse be set? The choice of service level goal at the warehouse will affect the leadtime to the retailer. With a low central inventory, orders from the retailers will often be delayed and, as a result, it may be necessary to increase the retailers reorder points. In an uncoordinated approach the last remark is not taken into account and the resulting solution will thus be suboptimal.

## 2.3. Coordinated multi-echelon inventory optimization methods

### 2.3.1. The considered multi-echelon models - preliminaries

The two considered methods deal with approximate optimization of the reorder points for continuous review  $(R, Q)$  policies in a two-echelon distribution inventory system. The batch quantities are assumed to be given. The stochastic customer demand occurs at the retailers. It is stationary with known mean and standard deviation, and is independent across retailers and over time. The retailers replenish their stock from the warehouse, and the warehouse in turn places its order with an outside supplier with supposedly infinite stock. Stockouts at both echelons are backordered and delivered on a first-come first-serve basis. All transportation times are constant but the retailer leadtimes are stochastic because of delays due to shortages at the warehouse. Also, to minimize the long run average system costs with respect to reorder points, the investigated methods use a cost structure with linear holding cost and shortage costs.

The following notation will be used for both methods.

- $N$  number of retailers
- $l_i$  constant transportation time for an order to arrive at retailer  $i$  from the warehouse
- $L_0$  constant leadtime for an order to arrive at the warehouse from the outside supplier
- $q$  largest common factor of  $Q_0, Q_1, Q_2, \dots, Q_N$ , i.e.,  $q$  is the largest positive integer such that all batch quantities are integer multiples of  $q$  ( $q$  may correspond to a package size or could be 1), this factor is referred to as a subbatch.
- $Q_i$  batch size at retailer  $i$ , expressed in units
- $Q_0$  batch size at the warehouse, expressed in units
- $b_i$  holding cost per unit and time unit at retailer  $i$
- $b_0$  holding cost per unit and time unit at the warehouse

$h_i$  shortage cost per unit and time unit at retailer  $i$   
 $R_i$  reorder point for retailer  $i$   
 $R_0$  warehouse reorder point expressed in units of  $q$   
 $C_i$  expected cost per time unit at retailer  $i$   
 $C_0$  expected warehouse cost per time unit  
 $C$  expected total system cost per unit of time

$\mu_i(t)$  mean of the demand at retailer  $i$  during time period  $t$   
 $\text{Var}_i(t)$  variance of the demand at retailer  $i$  during during time period  $t$   
 $\mu_i$  expected demand per time unit at retailer  $i$   
 $\mu_0$  expected demand per time unit at the warehouse  $\sum_i \mu_i$   
 $\text{Var}_i$  variance of the demand per time unit at retailer  $i$   
 $\text{Var}_0$  variance of the demand per time unit at the warehouse  
 $P_{i,k}(\mathbf{L}_0)$  probability for  $k$  orders at retailer  $i$  during the warehouse leadtime  
 $\mu_i^w(\mathbf{L}_0)$  average demand from retailer  $i$  at the warehouse during the warehouse leadtime  
 $\text{Var}_i^w(\mathbf{L}_0)$  variance of the demand from retailer  $i$  at the warehouse during the warehouse leadtime  
 $\mu^w(\mathbf{L}_0)$  average demand from the retailers at the warehouse during the warehouse leadtime  $= \sum_i \mu_i^w(\mathbf{L}_0)$   
 $\text{Var}^w(\mathbf{L}_0)$  variance of the demand from the retailers at the warehouse during the warehouse leadtime  $= \sum_i \text{Var}_i^w(\mathbf{L}_0)$   
 $D_0(\mathbf{L}_0)$  stochastic demand at warehouse during leadtime, expressed in units

Also, standard notations for the standard deviation is used,  $\sigma_i = (\text{Var}_i)^{1/2}$ .

### 2.3.2. Model 1: Heuristic coordination of a decentralized inventory systems using induced backorder costs

#### Introduction

The choice of a reorder level at the warehouse affects the retailers' leadtimes, which in turn influences the reorder level decisions at the retailers. To coordinate the control of the inventories in the distribution system this method introduces an induced backorder cost,  $\beta$ , which captures the impact that a reorder level decision at the warehouse has on the retailers. When an appropriate induced backorder cost is determined it is possible to optimize the reorder level at the warehouse. A relationship between the warehouse reorder level and the retailers' leadtimes can then be used to provide the lead-time estimate necessary for optimizing the reorder points at the retailers.

The method builds on the results from three research papers: The first (Andersson et al. 1998) presents an iterative procedure to find an induced backorder that minimizes the expected total system costs,  $\beta^*$ . In this paper the stochastic retailer leadtimes are replaced by their averages and all retailers are assumed to use a common batch quantity,  $Q_0$ . For general settings the iterative procedure is too computationally demanding in order to be used commercially. The second paper (Andersson et al. 2000) develops the model to allow for retailers to use individual ordering quantities. In the third paper (Berling & Marklund 2006) the iterative procedure in the first paper is used to build an understanding on how the optimal induced backorder cost depends on the parameters of the inventory system. This knowledge is then used to create closed form estimates which produce near-optimal induced backorder costs that will be denoted as  $\beta^*$ . An emphasis is placed on large systems with high demand and/or many retailers for which existing methods are not computationally feasible. The model assumes normally distributed demand and that, in case of shortages, orders are shipped when they can be delivered in full.

The optimization of the reorder level is performed in 6 steps: (1) The warehouse demand is determined (note that this is independent of the retailers' reorder points). (2) The system parameters are normalized. (3) The near-optimal induced backorder cost,  $\beta^*$ , is determined using the normalized system parameters. (4) A single-echelon method is used to determine

the optimal warehouse reorder point. (5) Given the optimized warehouse reorder level the stochastic retailer leadtime is determined and then replaced with its average. (6) Finally, the retailer reorder points can also be determined using ordinary single-echelon methods.

### Step 1: Warehouse leadtime demand mean and variance

$D_{0,q}(L_0)$  stochastic leadtime demand at the warehouse expressed in units of  $q$

Defining  $F(x)$  as the cumulative distribution function (cdf) of the associated gamma distribution (see 2.1.5), we use (2.21) to construct a probability mass function (pmf) to approximate the discrete warehouse lead time demand probabilities,  $P(D_{0,q}(L_0)=u)$  for  $u=0,1,\dots$

$$P(D_{0,q}(L_0) = u) \cong \begin{cases} F(0.5) & \text{for } u = 0 \\ F(u + 0.5) - F(u - 0.5) & \text{for } u = 1, 2, \dots \end{cases} \quad (2.21)$$

It is noteworthy that the mean and variance of the constructed pmf may deviate slightly from  $\mu^w(L_0)/q$  and  $\text{Var}^w(L_0)/q^2$ .

### Step 2: Normalize system parameters

To reduce the number of parameters that need to be varied independently Berling & Marklund (2006) propose that the system can be normalized with respect to  $l_i$ ,  $\mu_i$  and  $h_i$ . This means that a time unit is defined so that  $l_i$  is 1, a unit of demand is defined so that  $\mu_i$  is 100, and a monetary unit is defined so that  $h_i$  is 1. Figure 2.3 shows the relationship between original and normalized system parameters and corresponding  $\beta^*$  values. Figure 2.4 provides an illustrative example of how the transformation is performed, when  $q$  is 1.

Original system parameters	Normalized system parameters
$Q_i$	$Q_{i,n} = 100 Q_i / (\mu_i l_i q)$
$Q_0$	$Q_{0,n} = Q_0$
$h_i$	$h_{i,n} = 1$
$h_0$	$h_{0,n} = h_0 / h_i$
$p_i$	$p_{i,n} = p_i / h_i$
$L_0$	$L_{0,n} = L_0 / l_i$
$l_i$	$l_{i,n} = 1$
$\mu_i$	$\mu_{i,n} = 100$
$\sigma_i$	$\sigma_{i,n} = 100 \sigma_i / (\mu_i \sqrt{l_i})$
$\beta^* = \beta_i^* h_i$	$\beta_n^*$

Figure 2.3 Relationship between original and normalized system parameters and corresponding  $\beta$  values (source: Berling & Marklund 2006).

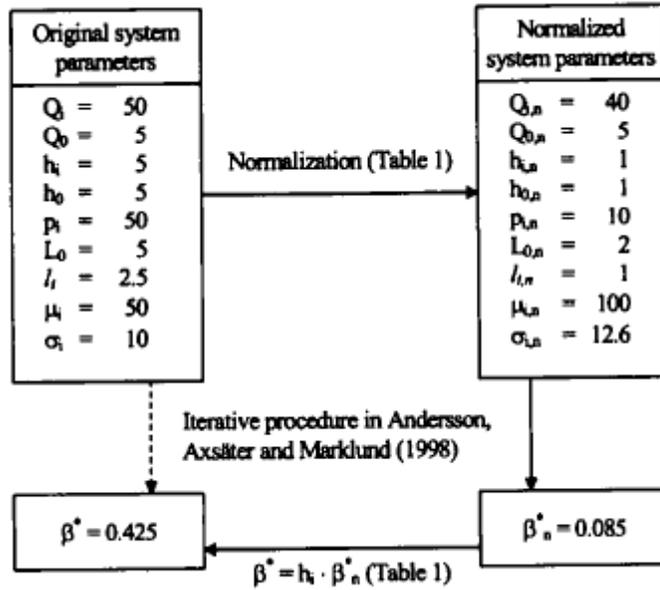


Figure 2.4 Example illustrating the normalization procedure (source: Berling & Marklund 2006).

### Step 3: Near-optimal induced backorder cost

The near-optimal induced backorder cost for orders from retailer  $i$  is denoted  $\beta_i^*$  and estimated through (2.22). Table A1 and Table A2 in Appendix A provide tabulated values for  $g(Q_{i,n}, p_{i,n})$  and  $k(Q_{i,n}, p_{i,n})$ . If the values of  $Q_i$  or  $p_i$  fall outside the tabulated range, use the estimated closed form expressions for  $g(Q_{i,n}, p_{i,n})$  and  $k(Q_{i,n}, p_{i,n})$  presented in Berling & Marklund (2006), see also Appendix A.

$$\beta_i^* = h_i \cdot g(Q_{i,n}, p_{i,n}) \cdot \sigma_{i,n}^{k(Q_{i,n}, p_{i,n})} \quad (2.22)$$

A single induced warehouse backorder cost is estimated as a weighted average with respect to the average demand rates.

$$\beta^* = \sum_{i=1}^N \frac{\mu_i}{\mu_0} \cdot \beta_i^*. \quad (2.23)$$

#### Step 4: Optimal warehouse reorder point

The objective is to find a near optimal reorder point to the original system by minimizing the expected holding and induced backorder costs at the central warehouse,  $C_0(\mathbf{R}_0)$ .

$$C_0(\mathbf{R}_0) = \frac{(h_0 + \beta^*)q}{Q_0} \left\{ \sum_{y=R_0+1}^{R_0+Q_0} \sum_{u=0}^y (y-u) P(D_{0,q}(L_0) = u) \right\} - \beta^* q \left( R_0 + \frac{Q_0 + 1}{2} - \frac{\mu^w(L_0)}{q} \right) \quad (2.24)$$

It is easy to show that, as  $\beta^* > 0$ ,  $C_0(\mathbf{R}_0) - C_0(\mathbf{R}_0 - 1)$  is increasing in  $\mathbf{R}_0$ , which implies that  $C_0(\mathbf{R}_0)$  is convex. Thus an optimal  $\mathbf{R}_0^*$  can therefore be found through a simple search

$$\mathbf{R}_0^* = \max\{\mathbf{R}_0 : C_0(\mathbf{R}_0) - C_0(\mathbf{R}_0 - 1) \leq 0\} \quad (2.25)$$

#### Step 5: Retailer leadtimes

Eq (2.26)-(2.28) provide an approach to approximate the average retailer leadtime proposed in Andersson & Marklund (2000). Numerical observations have shown that a large retailer in terms of  $\mu_i$  and  $Q_i$  tends to have longer mean leadtime,  $\bar{L}_i$ . In other words, a retailer which orders often and in large quantities have, relatively speaking, more units backordered and reserved at the warehouse than a retailer with low ordering frequency and small order quantities. This step is more thoroughly described in Andersson and Marklund (2000) section 5.2.

Define

$\bar{L}_i$	expected lead-time for an order to arrive at retailer $i$
$E(B_0)$	expected number of backorders at the warehouse in units of $q$

$E(B'_0)$  expected number of reserved subbatches at the warehouse  
 (ordered by a retailer at the central warehouse but not yet shipped)

$\bar{\Delta}_1 = E(B_0)q / \mu_0$  mean delivery delay due to stockouts at the warehouse

$\bar{\Delta}_2 = E(B'_0)q / \mu_0$  mean delivery delay due to reserved units waiting to be shipped

$\eta_i = \sum_{i=1}^N \mu_i(Q_i - q)$

$$E[B_0] = \frac{1}{Q_0} \sum_{y=R_0+1}^{R_0+Q_0} E_{D_{0,q}(L_0)}[(D_{0,q}(L_0) - y)^+] = \frac{1}{Q_0} \sum_{y=R_0+1}^{R_0+Q_0} \sum_{u=y}^{\infty} (u - y) P(D_{0,q}(L_0) = u) \quad (2.26)$$

$$\bar{L}_i = \bar{\Delta}_1 + \bar{\Delta}_1 \frac{\mu_i(Q_i - q) - (\eta_1 / N)}{\eta_1} + \bar{\Delta}_2 \frac{\mu_i(Q_i - q)}{\eta_1} + l_i \quad (2.27)$$

Where  $(x)^+ = \max(x, 0)$ . The first two terms in (2.27) represent an estimate of the mean delivery delay to retailer  $i$  due to stockouts at the warehouse and reserved units waiting to be shipped respectively. The estimate is found by differentiating the average delivery delay due to stockouts over all retailers based on the relative size of the retailer's relative deviation from the average retailer size, measured in terms of the product  $\mu_i(Q_i - Q)$ , as a basis for estimating the stockout delivery delay to that same retailer. The third term in (2.27) represents an estimate of the mean delivery delay to retailer  $i$  due to the existence of reserved units at the warehouse. The last term is the constant transportation time.

To get an explicit expression for  $E(B'_0)$  Andersson & Marklund suggests another approximation, inspired by an idea in Lee and Moynzadeh (1987). This approximation is based on the observation that reserved units only occur when a fraction of a retailer's order quantity is backordered. In (2.28)  $\bar{Q}$  can be interpreted as an average batch size. If we divide the number of backordered units  $B_0$  by  $\bar{Q}$  and the result is fractional, say 2.45, this indicates, due to the complete delivery policy at the warehouse, that the total number of delayed units is  $3\bar{Q}$  and subsequently that  $0.55\bar{Q}$  units are reserved at the warehouse.

Define

$\lceil x \rceil$  = the smallest integer  $\geq x$

$\lambda_0 = \sum_{i=1}^N \frac{\mu_i}{Q_i}$  = Demand intensity at the warehouse in number of retailer orders

$\bar{Q} = \sum_{i=1}^N \frac{\mu_i}{Q_i \lambda_0} \cdot Q_i = \sum_{i=1}^N \frac{\mu_i}{\lambda_0}$

$$E(B_0^r) = E\left(\left\lceil \frac{B_0}{\bar{Q}} \right\rceil \bar{Q}\right) - E(B_0) \quad (2.28)$$

### Step 6: Optimal retailer reorder points

With an estimate in place the retailer reorder points can now be determined in analogy with step 4 by minimizing (2.29) with respect to  $R_i$ .

$$C_i = h_0(R_i + Q_i / 2 - \mu_i) + (h_i + b_i) \frac{\sigma_i^2}{Q_0} \left[ H\left(\frac{R_i - \mu_i}{\sigma_i}\right) - H\left(\frac{R_i + Q_i - \mu_i}{\sigma_i}\right) \right] \quad (2.29)$$

### 2.3.3. Model 2: Approximate optimization of a two-echelon distribution inventory system

#### Introduction

The method originates with Axsäter (2003) and is based on normal approximations for both the retailer demand and the demand at the warehouse due to orders from the retailers. The approximations make it computationally feasible to optimize by successively searching for the set of reorder points that generates the least costs.

In short, the method can be described in six steps: (1) The warehouse demand mean and variance is determined. (2) For each feasible warehouse reorder point, the mean and variance for the retailer leadtime is determined. (3) Optimal retailer reorder levels for each feasible warehouse reorder level are determined and total costs for each solution is calculated. (4) The optimum warehouse reorder level is found by minimizing the total

costs. Both warehouse and retailer demand is assumed to follow normal distributions. An excel macro that executes this inventory optimization algorithm can be found at:  
<http://www.iml.lth.se/Sven/Approximate%20Optimization.xls>.

We define

$\mu_i^{\text{LT}}$  mean of the stochastic leadtime faced by retailer  $i$   
 $\text{Var}_i^{\text{LT}}$  variance of the stochastic leadtime faced by retailer  $i$

### Step 1: Warehouse leadtime demand mean and variance

First, the warehouse leadtime demand mean and variance,  $\mu^{\text{w}}(L_0)$  and  $\text{Var}^{\text{w}}(L_0)$  is determined. To do this the probability for  $k$  orders at retailer  $i$  during the warehouse leadtime,  $p_{i,k}(L_0)$ , is approximated by

$$\begin{aligned} p_{i,k}(L_0) &= \frac{1}{Q_i} \int_0^{Q_i} \left[ \Phi\left(\frac{x + kQ_i - \mu_i(L_0)}{\sigma_i(L_0)}\right) - \Phi\left(\frac{x + (k-1)Q_i - \mu_i(L_0)}{\sigma_i(L_0)}\right) \right] dx \\ &= \frac{\sigma_i(L_0)}{Q_i} \left[ G\left(\frac{(k-1)Q_i - \mu_i(L_0)}{\sigma_i(L_0)}\right) + G\left(\frac{(k+1)Q_i - \mu_i(L_0)}{\sigma_i(L_0)}\right) - 2G\left(\frac{kQ_i - \mu_i(L_0)}{\sigma_i(L_0)}\right) \right], \end{aligned} \quad (2.30)$$

where  $G(x)$  is defined by (2.11) in section 2.1.6. Refer to Axsäter (2003) or Andersson et al (1998) for details on how  $p_{i,k}(L_0)$  is derived.  $p_{i,k}(L_0)$  is determined for all  $k$  for which the probability is not too low,  $k$  is under normal conditions a small number. It follows that

$$\mu_i^{\text{w}}(L_0) = \mu_i(L_0) \quad \text{and} \quad (2.31)$$

$$\text{Var}_i^{\text{w}}(L_0) = \sum_{k=-\infty}^{\infty} (kQ_i - \mu_i^{\text{w}}(L_0))^2 p_{i,k}(L_0), \quad (2.32)$$

We now fit a normal distribution to the warehouse leadtime demand and determine its mean and variance,  $\mu^{\text{w}}(L_0)$  and  $\text{Var}^{\text{w}}(L_0)$ , by summing  $\mu_i^{\text{w}}(L_0)$  and  $\text{Var}_i^{\text{w}}(L_0)$  over  $i$ .

### Step 2: Mean and variance for retailer leadtimes

The mean and variance of the leadtime faced by the retailers are modelled as a function of the warehouse reorder point  $R_0$ , the delay is assumed to be equal for all retailers. Using

Little's formula the mean leadtime for the retailers are obtained from the expected number of backorders and the total demand,

$$\mu_i^{LT} = l_i + \frac{E(I_0)^-}{\sum_{i=1}^N \mu_i}. \quad (2.33)$$

The variance of the retailer leadtime can be expressed as

$$Var_i^{LT} = E(\delta^2) - (\mu_\delta)^2, \quad (2.34)$$

where  $\delta$  is the stochastic delay at the warehouse for retailer orders and  $\mu_\delta$  is its mean. The mean and variance of the retailer leadtime are calculated for each feasible warehouse reorder point  $R_0$ . For a complete derivation of these equations, see Axsäter (2003).

### Step 3: Optimize retailer reorder levels

A normal distribution is fitted to the retailer leadtime demand and  $\mu_i(LT)$  and  $Var_i(LT)$  determined through,

$$\mu_i(LT) = \mu_i^{LT} \mu_i \text{ and} \quad (2.35)$$

$$Var_i(LT) = Var_i \mu_i^{LT} + (\mu_i)^2 Var_i^{LT}. \quad (2.36)$$

Optimal retailer reorder points are calculated for each feasible warehouse reorder point by minimizing the retailer cost function,

$$C_i(R_0, R_i) = h_i E(I_i)^+ + p_i E(I_i)^-. \quad (2.37)$$

The formulas for the expected number of backorders and the expected stock level,  $E(I_i)^-$  and  $E(I_i)^+$  are omitted here (again, see Axsäter 2003). We denote the optimal reorder point for a given  $R_0$  by  $R_i^*(R_0)$ . Since  $C_i(R_0, R_i)$  is a convex function in  $R_i$  given  $R_0$  (see Axsäter 2000) the minimum is found using a computationally fast search procedure over  $R_i$ .

### Step 4: Optimal warehouse reorder level

In the last step the warehouse costs are calculated for each feasible warehouse reorder point using

$$C_0(R_0) = h_o E(I_0)^+, \quad (2.38)$$

where  $E(I_0)$  is the expected inventory level. Now the optimal warehouse reorder point,  $R_0$ , and the corresponding optimal retailer points,  $R_i^*(R_0)$ , are found by minimizing the total cost function

$$C(R_0) = C_0(R_0) + \sum_{i=1}^N C_i(R_0, R_i^*(R_0)). \quad (2.39)$$

To make optimization fast we disregard that  $C(R_0)$  is not necessarily convex and just look for a local minimum.

### 3. METHOD OF EVALUATION

This chapter explains how the performances of the optimization methods were investigated and compared. First a simulation model was constructed to represent the inventory system in the testing phase, see Section 3.1. The inventory data was analysed and three base cases selected, see Section 3.2. In-data to the simulation environment was supplied using the inventory data and output (reorder points) from the three optimization models, see Section 3.3. Lastly, the output data from the simulation was analysed to compare the optimization methods, Section 3.4.

#### 3.1. Simulation model

The simulation of the inventory system is performed using a discrete event simulation model in the simulation software Extend. A reason for using a high-level programming software like Extend is that considerable time can be saved in constructing the model. Also, the foundation for the simulation model was already available through another research project (Howard, C. 2007). The logic of a two-echelon inventory system, as described in Section 1.2, was implemented in the simulation model. See Appendix D for more information on the simulation environment.

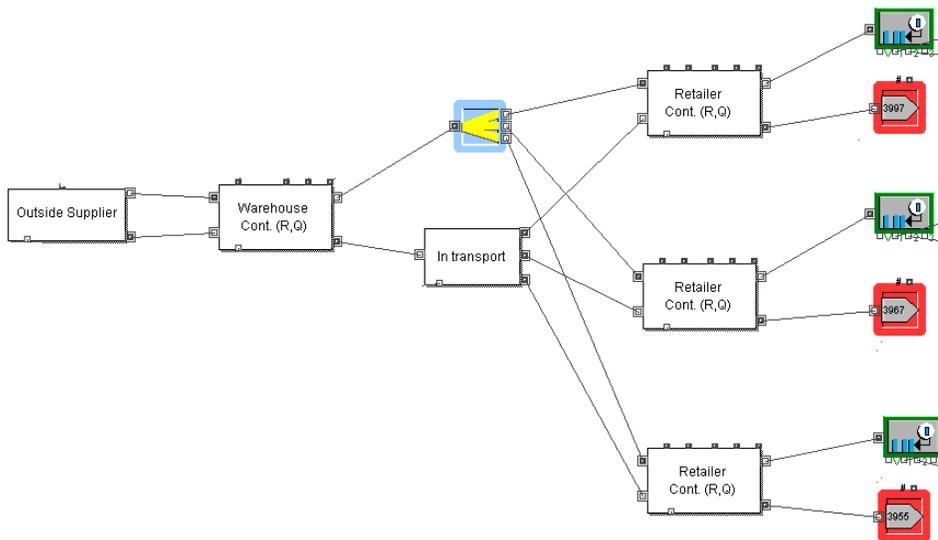


Figure 3.1. Graphical representation of the simulation model.

A graphical representation of the simulation model is presented in Figure 3.1, which is a screen dump from Extend. The rightmost blocks without the arrow generates customer

events with an intensity and order sizes according to a compound Poisson process. As described in Section 2.1.6 this process is often used to model demand data accurately, see this section for a more thorough discussion. The customer event is sent to the connected block labelled “Retailer Cont ( $R, Q$ )” which simulates a stock point governed by a continuous review ( $R, Q$ ) policy. The ordered items, represented by the customer events, which can be delivered immediately exits to the right entering to the block marked with an arrow, the ordered items that cannot be delivered are backordered. When the inventory position of the stock point reaches its reorder point an order of size  $Q$  is generated and sent to the block labelled “Warehouse Cont ( $R, Q$ )” which simulates a stock point in the same way as the retailer block. The warehouse block treats incoming orders the same way as the retailer block treats customer events. When an order (or part of an order) from a retailer is fulfilled, a replenishment event is triggered and sent to the “In transport” block, which delays the event for a set amount of time. After the event is delayed it is relayed to the retailer block, where the stock is replenished by the amount of units set by the ordering level of the retailer block. Order events triggered at the warehouse block is sent to the block labelled “Outside supplier”, where the event is delayed for a set amount of time and then sent back to the warehouse where the stock is replenished.

### 3.2. Selecting test cases

The inventory data supplied by Tetra Pak Technical Service AB in Lund was analysed to find adequate test cases. The data contained information on 80 products, characterized as spare parts, stocked in an inventory system that consisted of one central warehouse and 19 retailers. Few products were stocked at all 19 retailers. For each article and stock point the data included the following information:

- Order quantity
- Transportation time (time to transport unit from supplier to stock point)
- Target service level (fill rate)
- Achieved service level (fill rate)
- Date, time and size for orders during a period of three years

Tetra Pak views the distribution system as a part of the value chain and therefore increases the value of an item by 20% when it arrives at a retailer. The inventory data was analysed

statistically to provide an overview, tables and plots from this analysis is provided in Appendix B. An interesting observation is that for most products a large portion of the demand took place directly at the warehouse, a direct upstream demand. Three products were selected to provide in-data for the simulation environment, for a specification see Table 4.1 in Section 4.1, these items will be referred to as the base cases. The rationale behind this selection was primarily the fact that it's only relevant to evaluate the multi-echelon models in conditions they are designed for. We know that, in their current form, the methods are not designed for situations where the direct upstream demand is large or the total system demand low; in those cases other methods should be used. Therefore, only products where the total demand was more than five units/day and where the direct upstream demand comprised less than 60% of the total demand was considered. Within these limits the three products was selected to cover as much of the diversity in total inventory system demand, number of retailers, total system demand and mean size of the customer orders. All 80 products are plotted with respect to total demand and percentage of direct upstream demand in Appendix B.

### 3.3. Supply in-data for the simulation

The following parameters are needed to provide in-data to the simulation model:

- Transportation times from the outside supplier to the warehouse and from the warehouse to each retailer.
- Holding costs at all locations and shortage costs at all retailers.
- Ordering quantities for the warehouse and the retailers.
- Parameters for the compound Poisson process that generates the retailer demand.
- Reorder levels at the warehouse and each retailer.

The transportation times are readily available in the supplied inventory data and can be used directly. The need for holding costs and shortage costs arise since Method 1 and 2 are both, in their current form, so called backorder cost models. Since we are only interested in the relative performance of the optimization models, the retailer holding cost is normalized to 1 and, to accommodate for the fact that the value of the an item increases 20% when it reach the retailer, the warehouse holding cost is set to 0.84.

The shortage cost was derived from the target service level, that is available in the supplied inventory data, according to the process discussed in Section 2.1.6, Eq. (2.18) and (2.19). As noted in this section this method only produces accurate results for the fill rate when the demand is continuous or generated by a pure Poisson process. As the demand is generated by a compound Poisson process in the simulation model, the translation process will only be accurate when the service level is measured as a ready rate. This means that Method 1 and Method 2 is in fact optimizing the reorder level with regard to the ready rate (in the inventory data both the target and the achieved service level is measured with the fill rate in mind). Since the same is true for the uncoordinated method, as it assumes continuous demand, this does not constitute a problem when comparing the coordinated optimization models of the backorder type with the uncoordinated method which uses a service level constraint. However, there is an underlying problem of misalignment between the target service level and the service level used in the optimization of the reorder points for both the conventional single-echelon method and the investigated multi-echelon models.

Even though a parameter for the order quantity was explicitly available in the inventory data this value was not used when defining the test cases. The analysis of the inventory data showed that the actual size of the replenishment orders from the retailers did not follow the set order sizes. As the model assumes fixed order quantities the average of all replenishment orders from a retailer was used when defining the order quantities actually used in the test cases. The alternative, to fit the order quantities from the inventory data to a distribution and then treat them stochastically in the simulation model was disregarded for sake of simplicity. For the warehouse, the order quantity parameter available in the inventory data is used without modification since there is no data for the order placed with the outside supplier.

The arrival rate for the compound Poisson process that generates the retailer demand is determined as the mean number of customers arriving each day. The assumption of exponentially distributed interarrival times and gamma distributed customer order sizes was validated by series of goodness-of-fit tests; Kolmogorov-Smirnov test, Anderson-Darling test and Cramér-von Mises test. For a thorough discussion of these tests, see

D'Agostino and Stephens (1986). A discrete gamma distribution is fitted to the mean and variation of the customer order sizes and used as the compounding distribution. The gamma distribution was chosen because both its mean and variance can be varied through its parameters, it is non-negative, and it proved a good fit with the data. Arrival intensity, customer order sizes and demand data for the three test cases together with the resulting demand parameters is presented in Appendix C.

### **3.3.1. Virtual retailer**

As mention in section 3.2, a common feature in the analysed test data was that a large portion of the customer demand takes place directly at the warehouse. This is sometimes called direct upstream demand in the literature. The evaluated models for coordinated control are not specifically constructed to handle this. The way around this dilemma is to model the direct upstream demand as a separate retailer, a virtual retailer that is a separate stock reserved for direct customer demand at the warehouse (see Axsäter 2007). The order quantity for the virtual retailer is always 1 and the leadtime is set to 0. For the uncoordinated approach the reorder level is always set to -1. This means that each customer demand at the virtual retailer triggers a corresponding warehouse order of the same size. There will never be any stock on hand at the virtual retailer in the uncoordinated case. This is generally not true for a coordinated approach; in this case a reorder level is determined in the same way as for the other retailers.

### **3.3.2. Calculate reorder levels**

The last step when defining the test cases is to determine a set of reorder levels for each of the evaluated optimization methods. Reorder levels are calculated for Method 1 and Method 2 using software provided by Johan Marklund and Sven Axsäter respectively. For the uncoordinated method, the SCP method, the order levels were calculated using documentation received from the ERP company that initiated this project.

The simulated warehouse mean and variance is used as input when determining reorder levels for the SCP Method. In real conditions, an extra forecast with accompanied forecast error would have to be made at the warehouse. As this extra layer of error is eliminated by providing the method with an exact value for the warehouse demand this provides a slight

advantage for the SCP method. It should be noted that for the virtual retailer, the transportation time is naturally zero, but as the current form of method 1 cannot handle such transporting times a number close to zero is used instead, unfortunately the resulting reorder levels vary quite a bit depending on how close to zero this value is. The model was reported to have been adjusted and allow for transporting times of zero in the end of this project.

### 3.4. Output data analysis

Each test case was run for 400 000 time units (days) (200 000 time units for the sensitivity tests), excluding a warm-up period of 5000 time units. Observations of output variables were made every 40 000th time unit and then reset (every 20 000th time unit for the sensitivity tests). The number of observations in a simulation run was determined by minimizing the variance of the mean for the output variables; i.e. different time intervals was tested and the obtained variance was compared. The length of the complete simulation run was then set so that the variance of the mean was sufficiently low.

A set of output variables from the simulation was used to make comparisons between the evaluated models:

- Mean holding cost for the system per time unit.
- Mean total system cost (holding cost and shortage cost) for the system per time unit.
- Ready rate, defined as the fraction of time with a positive stock on hand.
- Fill rate, defined as the fraction of demand that can be satisfied immediately from stock on hand.

It could be argued that mean total system cost is the only output needed to make a satisfactory comparison of the models, however most corporations use different kinds of service levels, and not shortage costs, to measure inventory performance. The student's t-distribution was used to create confidence intervals for the output data and the t-test to make significance tests.

## 4. RESULTS AND DISCUSSION

In this chapter the results from the simulations are presented and discussed. First, results from the three base cases are presented and discussed in section 4.1. Direct upstream demand is excluded in section 4.2.1. The results from a number of sensitivity tests is presented and discussed in section 4.2.2 to 4.2.4. All discussed differences are significant at a level of 95%.

Four performance measurements are used in the analysis: Ready rate ( $S_3$ ), fill rate ( $S_2$ ), expected holding costs (HC) and total costs (TC). TC gives a balanced measurement of the performance and is easily comparable, it is the preferred choice of measurement. In practise  $S_2$  and HC are the most commonly used measurements. The ready rate is only included in the sensitivity tests, it is included because, as discussed in Section 3.3, the evaluated methods are in fact optimizing the reorder points with regard to meet the set ready rate.  $S_3 > S_2$  when the mean customer order is greater than one. The difference between  $S_2$  and  $S_3$  grows larger as the mean customer order size increases. When referring to averages for the retailer fill rate and ready rate in this section, it is the weighted average, with respect to the retailed demand. The relationship between  $S_2$  and TC is complicated by the fact that the shortage costs vary among the retailers; if  $S_2$  is increased at a retailer with a high shortage cost the TC will decrease more than if the retailer had a low shortage cost. HC and  $S_3$  have rather low variance in the simulations while the variances for TC and  $S_2$  are higher.

The following notations/abbreviations are used in this chapter.

CW	central warehouse.
VR	virtual retailer, a retailer that is created to model direct upstream demand at the central warehouse (see section 3.3.1).
$S_2$	average for the retailer fill rate, weighted with respect to demand.
$S_3$	average for the retailer ready rate, weighted with respect to demand.
HC	average holding costs for the system

TC	average total cost for the system (the sum of the expected holding costs and the expected shortage costs)
M1	Method 1: Heuristic coordination (see section 2.3.2)
M2	Method 2: Approximate optimization (see section 2.3.3)
SCP	A method for uncoordinated control used in a commercial system for inventory control.
N	Number of retailers.
L	Transportation time
Q	Ordering quantity or batch size

#### 4.1. Base cases

A detailed specification of the three base cases, T1-T3, is available in Table 4.1. There are a few things worth noticing: For all three test cases the majority of the total customer demand takes place in two of the retailers, of which the VR (see section 3.3.1) is one. The portion of demand at the VR is between 40 and 60%. The transportation times (L) vary considerably within each of the three test cases.

- Test Case T1: Consists of eight retailers and, relative to the other test cases, a high total mean demand. The customer order size is fairly low for all retailers, or, equivalently, the variance for the retailer demand is relatively low. This test case has a homogenous structure for the service level targets.
- Test Case T2: Consists of five retailers and a relatively high total mean demand. Since customer order sizes are generally very large, the variance for the retailer demand is high. This test case also has a mix of high and low service level targets.
- Test Case T3: Consists of six retailers and a relatively low total mean demand. Customer order sizes and retailer demand variance are generally low. This test case has a homogenous structure for the service level targets.

The service level targets for the retailers are generally high, for T1-T3 the weighted averages are 99.5%, 95.8% resp 98.5%. These distinctions are summarized in Table 4.2. More detailed specifications of the test cases are available in Appendix E.

	Loc	Transport. time L	Batch size Q	Target Serv. Level %	Dmnd per time unit		Opt. reorder points		
					Mean	Std dev	SCP	M 1	M 2
T1	CW	17	494	99.8	10.00	37.50	511	646	176
	VR	0	1	99.8	4.09	30.16	-1	29	117
	Re1	10	25	99.5	0.27	1.16	9	8	8
	Re2	9	40	99.0	0.32	1.12	6	6	6
	Re3	8	40	99.0	0.40	1.68	8	9	9
	Re4	14	15	98.0	0.33	1.48	12	12	12
	Re5	14	100	99.5	2.82	24.14	169	236	249
	Re6	30	15	99.0	0.29	1.05	18	18	18
	Re7	20	40	99.0	0.61	1.50	22	21	21
Re8	14	14	99.5	0.16	0.86	20	8	7	
T2	CW	45	154	98.5	11.04	44.88	1129	964	703
	VR	0	1	98.5	5.16	30.53	-1	69	145
	Re1	10	140	97.0	1.02	11.96	27	47	51
	Re2	9	80	90.0	1.47	11.27	27	30	35
	Re3	14	5	90.0	0.19	1.81	24	10	10
	Re4	5	140	94.0	3.08	24.22	54	67	78
	Re5	14	2	90.0	0.02	0.48	1	2	1
T3	CW	18	251	99.8	5.81	30.62	404	517	129
	VR	0	1	99.8	2.11	20.23	-1	16	79
	Re1	9	23	99.5	0.26	1.59	10	10	11
	Re2	8	25	90.0	0.10	0.83	12	-2	-2
	Re3	17	150	98.0	2.72	5.32	64	63	72
	Re4	30	32	99.5	0.17	1.02	15	14	13
	Re5	20	25	90.0	0.06	0.47	8	-2	-2
Re6	14	6	90.0	0.01	0.34	1	1	-1	

Table 4.1 Specifications of the base test cases T1, T2 and T3, including optimal reorder points for SCP, Method 1 and Method 2. The weighted averages for the target retailer service levels are 99.5% (T1), 95.8% (T2) resp 98.6% (T3). The time unit is number of days.

	T1	T2	T3
Number of retailer	8	5	6
Service level (weighted average)	High	Low	Medium
Total system demand	High	High	Low
Size of customer orders	Low	High	Low
Demand variance	Low	High	Low
Demand variation between retailers	High	High	High

Table 4.2 Summary of the differences between the base test cases.

The weighted average retailer fill rate, total holding costs, and total costs are presented for all three optimization methods in Table 4.3. In Table 4.4 only Method 1 and Method 2 is presented, and it is the relative difference of the performance measurements with respect

to the SCP method that is showed. Detailed results for each installation are available in Appendix E.

Method 1 demonstrates the lowest total cost for all three cases, with an average of 15% reduction in total costs compared to SCP (the current method used). But when analysing fill rate and holding costs the results are not that conclusive; for example, is the increase in T1 of the fill rate by 3.1% worth an increase of the holding costs by 21%? A similar question can be asked about T2.

The results for Method 1 in case T3 is rather peculiar; the fill rate is unchanged with holding costs increased by 12% but still the total costs is lowered by 15%. This can partly be explained from the discussion of the relationship between fill rate and total costs in the beginning of this chapter; the service level is increased in retailers with high shortage costs while it is decreased in retailer with low shortage cost. This leads to unchanged service levels overall and lower total costs, the increased holding costs takes place in the CW in this case. See Appendix E Figure E.3 for detailed results on this case.

Method 2 seriously underestimates the inventory needed in T1 and T3, only in T2, the test case where the variance is the largest, it can compete with the other methods. From the detailed results (appendix E) it is evident that it is the demand at the virtual retailer that is not fulfilled properly in Method 2.

Test Case	Retailer Fill rate %			Holding Costs			Total Costs		
	SCP	M 1	M 2	SCP	M 1	M 2	SCP	M 1	M 2
<b>T1</b>	89.9	93.0	89.1	848	1024	722	3241	2464	4059
<b>T2</b>	81.7	81.6	79.8	1004	834	708	1286	1212	1334
<b>T3</b>	96.2	96.2	91.0	566	634	372	1062	905	2243

Table 4.3 Statistical summary of simulation results for the base test cases, T1-T3.

Test Case	Retailer Fill rate %		Holding Costs %		Total Costs %	
	M 1	M 2	M 1	M 2	M 1	M 2
<b>T1</b>	3.1	-0.8	21	-15	-24	25
<b>T2</b>	-0.2	-1.9	-17	-29	-6	4
<b>T3</b>	0.1	-5.2	12	-34	-15	111

Table 4.4 Relative performance of Method 1 and 2 in comparison with the SCP method. For the fill rate this means the difference to the SCP result and for the holding costs and the total costs the difference in percent to the SCP result. Test cases T1-T3.

## 4.2. Sensitivity analysis

### 4.2.1. Eliminate direct upstream demand

The first modification of the base cases is to eliminate the direct upstream demand by removing the virtual retailers completely from the simulation model. Since this adjustment lowers the demand faced by the CW the batch quantity is also adjusted by the same proportion, see Table 4.5 for a complete specification. The weighted averages for the target service levels is 99.3%, 93.4% resp 98.2%. The optimal reorder points for the new test cases are presented in Table 4.5. Table 4.6 and Table 4.7 shows a statistical summary of the results. Test case S1\* is a small modification of test case S1, the modification has to do with the classification algorithm of demand patterns in the SCP system. By default the dominant retailer (the retailer with the highest demand) is assigned a special classification which results in a substantially lowered reorder point for the SCP method. In test case S1\* this classification is changed to the one ordinarily used, in which the SCP method determines order levels in a way that closely resemble the theory given in section 2.1.6. I advise readers who are not involved in the development of the SCP method to focus on test case S1\*.

In test case S1 the SCP method uses a special classification of the demand for the dominant retailer (the retailer with the highest demand) which results in a substantially lowered reorder point. In test case S1\* this classification is changed to the normal.

	Loc	Transport. time L	Batch size Q	Target Serv. Level %	Dmnd/day		Opt. reorder points		
					Mean	Std dev	SCP	M 1	M 2
S1	CW	17	202	99.8	5.766	25.849	343.2	3	88
	Re1	10	25	99.5	0.266	1.163	8.2	11.8	10
	Re2	9	40	99.0	0.323	1.124	5.9	9.5	7
	Re3	8	40	99.0	0.402	1.683	8.0	13.6	11
	Re4	14	15	98.0	0.327	1.481	11.8	15.6	13
	Re5	14	100	99.5	2.824	24.143	169.0	364.5	269
	Re6	30	15	99.0	0.291	1.053	18.0	20.9	19
	Re7	20	40	99.0	0.615	1.503	21.0	26.5	24
Re8	14	14	99.5	0.157	0.858	19.2	9.9	8	
S2	CW	45	81	98.5	6.014	33.701	768.0	316.0	386
	Re1	10	140	97.0	1.022	11.957	26.4	72.8	60
	Re2	9	80	90.0	1.465	11.273	26.7	53.8	45
	Re3	14	5	90.0	0.189	1.806	23.1	12.2	11
	Re4	5	140	94.0	3.080	24.218	53.3	159.7	105
	Re5	14	2	90.0	0.016	0.477	1.0	2.2	1
S3	CW	18	154	99.8	3.374	17.088	227.5	-66.0	26
	Re1	9	23	99.5	0.260	1.590	10.0	20.2	14
	Re2	8	25	90.0	0.102	0.829	11.9	0.7	-2
	Re3	17	150	98.0	2.719	5.320	63.8	196.9	98
	Re4	30	32	99.5	0.174	1.022	14.2	18.8	15
	Re5	20	25	90.0	0.061	0.470	7.9	0.0	-2
Re6	14	6	90.0	0.015	0.342	1.0	0.7	0	

Table 4.5 Optimal reorder points for test cases S1, S2 and S3. The weighted averages for the target service levels (fill rate) is 99.3%, 93.4% resp 98.2%.

Test Case	Retailer Ready rate %			Retailer Fill rate %			Holding Costs			Total Costs		
	SCP	M 1	M 2	SCP	M 1	M 2	SCP	M 1	M 2	SCP	M 1	M 2
<b>S1</b>	96.0	97.4	96.7	85.3	92.6	89.7	635	551	514	2470	1713	2046
<b>S1*</b>	97.1	97.4	96.7	89.3	92.6	89.7	702	551	514	1937	1713	2046
<b>S2</b>	94.6	93.6	93.2	67.2	77.5	72.9	751	540	521	987	875	859
<b>S3</b>	98.3	99.5	99.1	96.5	98.0	97.2	394	267	234	446	360	281

Table 4.6 Statistical summary of the simulation results for test cases S1, S2 and S3.

Test Case	Retailer Ready rate %		Retailer Fill rate %		Holding Costs %		Total Costs %	
	M 1	M 2	M 1	M 2	M 1	M 2	M 1	M 2
<b>S1</b>	1.4	0.7	7.3	4.5	-13	-19	-31	-17
<b>S1*</b>	0.3	-0.4	3.3	0.4	-22	-27	-12	6
<b>S2</b>	-0.9	-1.4	10.3	5.7	-28	-31	-11	-13
<b>S3</b>	1.2	0.7	1.5	0.7	-32	-40	-19	-37

Table 4.7 Relative performance of Method 1 and 2 in comparison with the SCP method. For the ready rate and the fill rate this means the difference to the SCP result and for the holding and the total costs the difference in percent to the SCP result. Test cases S1-S3.

When the direct upstream demand is removed both Method 1 and 2 performs better relative to the SCP method; in all cases they demonstrate that they can lower the holding cost and still increase the retailer fill rate. Method 2 has a lower service level and holding cost than Method 1 in all cases, and when it comes to total costs Method 1 performs best in test case S1 while Method 2 is best in S3.

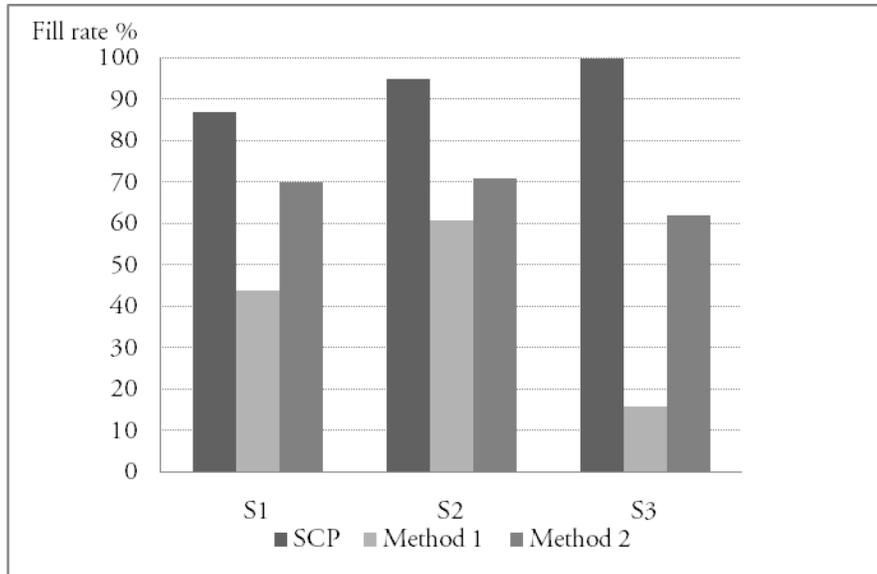


Figure 4.1 Warehouse fill rates for test cases S1-S3.

Figure 4.1 shows that the warehouse fill rate drops significantly when using a coordinated method (also, see the warehouse reorder points in Table 4.5). In test case S3 it is as low as 16% for Method 1 and at the same time generating a higher retailer fill rate at a lower cost. This effect is also seen in the following sensitivity tests, it seems that the coordinated methods pushes the inventory closer to the customer. However, in the base cases, this effect is not observed. The reason is that the large portion of direct demand in the base test cases makes it disadvantageous to lower the warehouse fill rate.

#### 4.2.2. Demand mean, transporting times and batch quantities

In test cases R0-R7 the retailer demand mean, the transportation times (including transporting time between the CW and the outside supplier) and the batch quantities of test case S1\* is successively varied. Each parameter has two levels (high and low) that are combined into eight test cases according to Table 4.8. It should be noted that the demand

mean is increased without affecting the variance. Results from the simulation are presented in Table 4.9 and Table 4.10.

	Demand Mean	Transportation Time	Batch Quantity
R0	x1	x1	x1
R1	x2	x1	x1
R2	x1	x0.5	x1
R3	x2	x0.5	x1
R4	x1	x1	x0.5
R5	x2	x1	x0.5
R6	x1	x0.5	x0.5
R7	x2	x0.5	x0.5

Table 4.8 Specification of the sensitivity test cases for analysis of demand mean, transportation time and batch quantity.

Test case	Retailer Ready rate %			Retailer Fill rate %			Holding Costs			Total Costs		
	SCP	M 1	M 2	SCP	M 1	M 2	SCP	M 1	M 2	SCP	M 1	M 2
<b>R0</b>	97.1	97.4	96.7	89.3	92.6	89.7	702	551	514	1937	1713	2046
<b>R1</b>	97.4	97.0	96.9	94.6	95.0	94.1	783	507	481	1451	1202	1399
<b>R2</b>	97.5	97.8	97.3	86.6	91.1	87.7	559	458	423	1534	1357	1591
<b>R3</b>	97.5	97.6	97.2	93.3	95.2	93.5	612	425	399	1193	982	1169
<b>R4</b>	96.8	97.1	96.4	88.9	91.7	88.9	635	483	452	1921	1775	2092
<b>R5</b>	97.0	96.6	96.3	94.1	94.3	93.2	736	440	419	1437	1272	1457
<b>R6</b>	97.0	97.5	96.7	84.9	89.8	85.7	477	381	349	1581	1446	1698
<b>R7</b>	97.1	97.1	96.5	92.4	94.0	92.1	537	349	326	1183	1042	1225

Table 4.9 Statistical summary of the simulation results for test cases R0-R7. The weighted average for the target retailer service levels (fill rate) is 99.3%.

Test case	Retailer Ready rate %		Retailer Fill rate %		Holding Costs %		Total Costs %	
	M 1	M 2	M 1	M 2	M 1	M 2	M 1	M 2
<b>R0</b>	0.3	-0.4	3.2	0.4	-21	-27	-12	6
<b>R1</b>	-0.4	-0.5	0.4	-0.5	-35	-38	-17	-4
<b>R2</b>	0.3	-0.2	4.5	1.0	-18	-24	-12	4
<b>R3</b>	0.1	-0.3	1.8	0.2	-30	-35	-18	-2
<b>R4</b>	0.2	-0.5	2.9	0.1	-24	-29	-8	9
<b>R5</b>	-0.5	-0.8	0.2	-0.8	-40	-43	-12	1
<b>R6</b>	0.5	-0.3	4.9	0.8	-20	-27	-9	7
<b>R7</b>	0.0	-0.6	1.5	-0.4	-35	-39	-12	4

Table 4.10 Relative performance of Method 1 and 2 in comparison with the SCP method. For the ready rate and the fill rate this means the difference to the SCP result and for the holding and the total costs the difference in percent to the SCP result. Test cases R0-R7.

In order to further investigate the results from these simulations Table 4.11 displays how the performance of the studied methods change when one parameter is varied and the other two kept fixed. The presented performance measurements are the difference between the high level and the low level of the varied parameters and they are calculated for each combination of levels for the other two parameters. The average change for all combinations of the other parameters is also determined. Taking the second row as an example: The first two letters in the row, L/H, report the levels for the two variables that are kept fixed, in this case the transportation time is at the low (L) level, that is half of the transportation times shown in Table 4.5, and the ordering quantity is at the high (H) level, that is the same ordering quantities as shown in Table 4.5. The fourth figure in the row, 5, tells us that the retailer fill rate is increased by 5% for the SCP method when: (a) The demand mean for all retailers (dmnd) is increased (doubled from the original level in test case S1\*). (b) The transportation times (L) are kept fixed at the low level (half the original level of test case S1\*). (c) The batch quantities (Q) are kept fixed at the high level (the original level of test case S1\*). Every fifth row is an arithmetic average of the four rows above.

	Retailer Ready rate %			Retailer Fill rate %			Holding Costs			Total Costs		
	SCP	M 1	M 2	SCP	M 1	M 2	SCP	M 1	M 2	SCP	M 1	M 2
Varying Dmnd												
Level for L / Q												
H / H	0.3	-0.4	0.1	5	2.2	4.1	12%	-8%	-6%	-24%	-29%	-30%
L / H	0	-0.2	-0.1	6.7	4.1	5.8	9%	-7%	-6%	-22%	-28%	-27%
H / L	0.2	-0.5	-0.1	5.2	2.6	4.3	16%	-9%	-7%	-25%	-28%	-30%
L / L	0.1	-0.4	-0.2	7.5	4.2	6.4	13%	-8%	-7%	-25%	-28%	-28%
Average	0.15	-0.38	-0.07	6.10	3.28	5.15	12%	-8%	-6%	-24%	-28%	-29%

Varying L												
Level for Dmnd / Q												
H / H	-0.1	-0.6	-0.3	1.3	-0.2	0.6	28%	19%	21%	22%	22%	20%
L / H	-0.4	-0.4	-0.5	3	1.7	2.3	25%	20%	21%	25%	24%	26%
H / L	-0.1	-0.5	-0.2	1.7	0.3	1.1	37%	26%	29%	21%	22%	19%
L / L	-0.2	-0.4	-0.3	4	1.9	3.2	33%	27%	30%	22%	23%	23%
Average	-0.20	-0.47	-0.32	2.50	0.93	1.80	31%	23%	25%	22%	23%	22%

Varying Q												
Level for Dmnd / L												
H / H	0.4	0.4	0.6	0.5	0.7	0.9	6%	15%	15%	1%	-6%	-4%
L / H	0.3	0.3	0.4	0.7	1.1	1.1	10%	14%	13%	0%	-5%	-4%
H / L	0.4	0.5	0.7	0.9	1.2	1.4	14%	22%	22%	1%	-6%	-5%
L / L	0.5	0.3	0.6	1.7	1.3	2	17%	20%	21%	-3%	-6%	-6%
Average	0.40	0.40	0.55	0.80	1.15	1.25	12%	18%	18%	0%	-6%	-5%

Table 4.11 Relative performance, one input parameter varied at a time. For the ready rate and the fill rate this means the difference to the SCP result and for the holding and the total costs the difference in percent to the SCP result. H is high level and L is low level.

A first observation is that when the demand increases, the performance (an increase of the fill rate and decrease of the holding costs and the total costs) of all three methods increases. For Method 1 and 2 the holding costs decreases even though the demand is increased. The increase of fill rate is larger while the transportation time is set at the lower level. The ready rate decreases on average 0.4% in Method 1 but is otherwise stable while the total costs decreases ~28% for Method 1 and 2, and ~24% for the SCP method. This increase in performance can be explained from the fact that all three methods assumes continuous leadtime demand and that the customer order sizes must be lowered in order to increase the demand mean without increasing the variance. When the customer order sizes are lowered the assumption of continuous leadtime demand becomes more appropriate. Relative to the SCP method there is an increase in performance for both

Method 1 and 2 when the demand increases for the fill rate, the holding costs and the total costs.

An increase in the transportation times,  $L$ , yields a reduction in the ready rate for all methods, with the largest reduction in Method 1. The fill rate increases for all methods and the holding cost increases substantially, with a larger increase when the batch size is low. The fill rate increase is largest for the SCP method and so is the increase in holding costs. Total costs increases around 22% for all methods. Relative to the SCP method there is an increase in performance for both Method 1 and 2 when the leadtime increases for the fill rate and the holding costs.

When increasing the batch size,  $Q$ , an increase of the ready rate is observed for all methods. The fill rate is also increased ( $\sim 1\%$ ) for all methods with no significant correlations with the other parameters. An increase is observed in the holding cost for all methods (18% for Method 1 and 2, and 12% for the SCP method), this increase is negatively correlated with both transportation time and demand mean. Despite this increase in holding cost the total cost still decreases for Method 1 and 2 ( $\sim 5\%$ ), no change is observed for the SCP method. Relative to the SCP method there is an increase in performance for both Method 1 and 2 when the batch size increases for the fill rate, the holding costs and the total costs.

Turning to the performance of Method 1 and Method 2 when compared to the performance of the SCP method (Table 4.10). The parameter with the most influence on the fill rate and the holding costs is the demand mean. When the demand mean is increased the relative fill rate is decreased somewhat and the holding costs are decreased substantially. The same effect, but smaller, is indicated when the leadtime is increased. This is not surprising since both these changes affects the leadtime demand. When increasing the batch quantity, the fill rate is stable but the relative holding costs is increased to some extent.

The conclusion from this test series is that overall the performance of both Method 1 and Method 2 is rather stable when varying demands, transportation times and batch quantities. The single variable that seems to have the greatest influence is the demand mean.

#### 4.2.3. Target service levels

The test series Q0-Q3 investigate how the target service level affects the optimization methods, the target service level of test case S1\* are modified according to Table 4.12. The dominant retailer is the retailer with the highest demand. This test series is designed to investigate the sensitivity to changes in the retailer service levels but also the sensitivity to homogeneous/heterogeneous retailer service levels. The warehouse target service level (only used by the SCP method) is set to 95% in all cases. Results from the simulation are presented in Table 4.13 and Table 4.14.

	Dominant Retailer	Other Retailers
Q0	92%	92%
Q1	99%	92%
Q2	92%	99%
Q3	99%	99%

Table 4.12 Specification of the service level targets in test cases Q0-Q3. The dominant retailer is the retailer with the highest demand.

Test case	Retailer Ready rate %			Retailer Fill rate %			Holding Costs			Total Costs		
	SCP	M 1	M 2	SCP	M 1	M 2	SCP	M 1	M 2	SCP	M 1	M 2
Q0	91.6	92.9	92.0	75.5	81.6	79.7	424	364	338	630	539	538
Q1	93.2	94.9	93.8	80.7	88.3	84.4	473	472	421	1612	1120	1348
Q2	93.4	95.0	94.2	78.3	85.1	83.0	455	411	393	716	631	635
Q3	95.1	97.1	96.5	83.6	91.8	89.0	503	520	488	1697	1217	1380

Table 4.13 Statistical summary of the simulation results for test cases Q0-Q4.

Test case	Retailer Ready rate %		Retailer Fill rate %		Holding Costs %		Total Costs %	
	M 1	M 2	M 1	M 2	M 1	M 2	M 1	M 2
<b>Q0</b>	1.2	0.3	6.2	4.2	-14	-20	-14	-15
<b>Q1</b>	1.7	0.6	7.6	3.7	-0	-11	-31	-16
<b>Q2</b>	1.6	0.8	6.8	4.7	-10	-14	-12	-11
<b>Q3</b>	2.1	1.4	8.2	5.3	3	-3	-28	-19

Table 4.14 Relative performance of Method 1 and 2 in comparison with the SCP method for test cases Q0-Q4.

An interesting observation is that when the average fill rate is low, Method 1 and 2 performs equally well in terms of total costs. In terms of holding costs and fill rate, Method 1 tends to keep a little more stock and thus achieve a higher fill rate. This observation could provide some insight on the grounds for the significant differences in performance between Method 1 and 2 in test cases S1-S3. Test case Q0 and Q3 have a homogenous structure for the target retailer service level while cases Q1 and Q2 have a heterogenic structure. In relative numbers there is no significant difference in performance, either for Method 1 or Method 2, when comparing these groups.

#### 4.2.4. Homogenous / Heterogenic demand structure

In a heterogenic demand structure there are large differences in the demand distributions across retailers, the opposite is true for a homogenous demand structure. One case, Test case P1 is a modification of S1\* (see section 4.1.2) where the demand distribution is equal for all retailers, the total demand remains the same, see Table 4.15. Performance is compared to S1\* in Table 4.16 and Table 4.17. The benefit of using coordinated methods drop considerably for test case P1; there is no significant difference statistically between any of the three methods in total costs. There is however still a large decrease in holding costs (-21%) accompanied by a small increase in retailer fill rate for the coordinated methods (the increase in fill rate is not significant for Method 2). Since the results are quite contradictory additional testing is needed to draw more definite conclusions.

	Loc	Transport. time L	Batch size Q	Target Serv. Level %	Dmnd per time unit		Opt. reorder points		
					Mean	Std dev	SCP	M 1	M 2
P1	CW	17	202	99.8	5.36	15.01	222	9	52
	Re1	10	25	99.5	0.65	4.13	33	42	37
	Re2	9	40	99.0	0.65	4.13	25	34	28
	Re3	8	40	99.0	0.65	4.13	23	32	27
	Re4	14	15	98.0	0.65	4.13	36	43	39
	Re5	14	100	99.5	0.65	4.13	33	42	35
	Re6	30	15	99.0	0.65	4.13	67	73	69
	Re7	20	40	99.0	0.65	4.13	45	51	47
	Re8	14	14	99.5	0.65	4.13	45	53	48

Table 4.15 Specifications for the test case P1, including optimal reorder points for SCP, method 1 and method 2. The weighted average for the target retailer service levels is 99.5%.

Test case	Retailer Ready rate %			Retailer Fill rate %			Holding Costs			Total Costs		
	SCP	M 1	M 2	SCP	M 1	M 2	SCP	M 1	M 2	SCP	M 1	M 2
S1*	97.1	97.4	96.7	89.3	92.6	89.7	702	551	514	1937	1713	2046
P1	97.6	97.3	97.3	88.5	89.6	88.8	582	462	461	1292	1313	1275

Table 4.16 Statistical summary of the simulation results for test cases S1 and P1.

Test case	Retailer Ready rate %		Retailer Fill rate %		Holding Costs %		Total Costs %	
	M 1	M 2	M 1	M 2	M 1	M 2	M 1	M 2
S1*	0.3	-0.4	3.2	0.4	-21	-27	-12	6
P1	-0.3	-0.3	1.1	0.3	-21	-21	2	-1

Table 4.17 Relative performance of Method 1 and 2 in comparison with the SCP method for test cases S1 and P1.

Location	SCP			Method 1			Method 2		
	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs
CW	94	298.6		44	42.2		70	96.3	
1	96	19.3	29.8	97	21.3	32.9	97	20.1	30.7
2	97	23.6	28.3	98	26.1	30.3	97	24.2	28.9
3	95	25.3	36.0	96	29.6	39.0	96	27.8	36.4
4	89	14.0	33.3	91	16.5	36.6	90	14.6	35.0
5	84	250.1	1419.3	90	349.7	1435.0	85	268.9	1723.0
6	95	17.2	28.0	96	19.2	28.8	96	17.9	28.1
7	97	28.8	38.5	98	32.1	40.3	98	31.1	38.8
8	100	25.3	25.7	96	14.8	27.6	94	13.2	28.3

Table 4.18 Detailed simulation results for test case S1\*.

	SCP			Method 1			Method 2		
Location	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs
CW	99	197.7		53	39.2		71	64.7	
1	88	40.2	159.0	90	46.6	190.7	89	43.0	178.8
2	87	40.2	105.8	89	46.3	130.5	87	41.9	123.4
3	87	38.7	93.1	89	44.9	119.9	88	41.4	109.2
4	83	35.9	92.9	84	40.4	108.1	84	37.8	101.4
5	93	74.5	163.3	94	79.2	186.6	93	74.2	183.2
6	91	57.0	157.4	91	60.4	167.9	91	57.8	165.6
7	91	53.6	132.7	91	56.7	145.9	91	54.3	140.5
8	89	44.2	189.6	90	48.7	224.6	89	46.1	207.9

Table 4.19 Detailed simulation results for test case P1.

## 5. CONCLUSIONS AND FUTURE RESEARCH

The objective of this thesis is to expose newly developed methods for coordinated multi-echelon control of inventory levels to real conditions and investigate the value of using those in comparison with a widely used single stage (uncoordinated) method.

The simulation results clearly indicate that the evaluated methods for coordinated control of inventories can offer considerable savings in reduced inventory levels and even, in many cases, at the same time increasing the fill rate. When the direct upstream demand is excluded the reductions in total cost range 11-37% and an increase in fill rate accompanied by a reduction of the holding costs is observed in most cases. Especially the holding costs at the Central Warehouse can be reduced, when using a method for coordinated control the fill rate for orders from the retailers is 16-71% which corresponds to a reduction of the CW holding costs of 50-94%. As expected, the benefits decrease when allowing for direct upstream demand. Particularly Method 2 has difficulties in this case with increases of total costs in all three cases, in one case as large as 111%. These results advise against using Method 2 in cases of direct upstream demand. Method 1, which still achieves reductions in total costs, seems to be more robust to large portions of direct upstream demand. A reason for the poorer performance could be that the direct upstream demand makes it disadvantageous to lower the CW fill rate.

Both Method 1 and Method 2 are quite robust to the changes in demand mean, leadtime and batch size. Method 1 performs better when the target service levels are low (in relative terms). Method 2 is more stable when target service levels are varied. The results are quite contradictory when it comes to the effect of a homogenous or heterogenic demand structure and additional testing is needed to draw general conclusions.

Apparent disadvantages of using coordinated control in a multi-echelon inventory system is that it requires more data in one location to make the calculations, and that the calculations is much more computationally demanding than for an uncoordinated method. Depending on the size and level of demand in the system the computation time range 4 - 9

seconds for Method 1 and 0.5 - 3 seconds for Method 2, on a computer equipped with an Intel Pentium M 1.8 GHz processor.

Regarding future research it would be valuable to map out the limitations of the methods more closely, to facilitate the process of applying the methods in situations they are verified to produce satisfactory results in. It would also be interesting to investigate if and how the methods can be adapted to handle direct upstream demand more effectively. If the methods for coordinated control would be implemented in a commercial system it would also be valuable to benchmark the computational requirements for the methods.

## 6. Appendix

### 6.1. Appendix A

Tabulated values used for  $g(p_{i,n}, Q_{i,n}) \cdot 10^4$  (Table A.1) and  $k(p_{i,n}, Q_{i,n})$  (Table A.2) used when determining reorder points in Method 1.

$Q_{i,n}$	$p_{i,n} = 1$	5	10	20	30	40	50	60	70	80	90	100	150	200	250
1	40.8	75.7	90.5	104	112	117	121	125	127	129	131	133	140	144	148
10	20.4	45.1	59.0	72.8	80.7	85.9	90.0	93.3	96.0	98.3	100	102	109	114	117
20	9.74	26.7	39.8	54.0	62.2	68.2	72.4	75.9	78.8	81.3	83.5	85.4	92.4	97.3	101
30	5.87	17.8	29.4	43.0	51.0	56.5	60.7	64.2	67.0	69.8	71.9	73.8	81.0	85.9	89.7
40	4.03	12.8	22.5	35.4	43.1	48.0	53.2	56.7	59.6	62.0	64.2	66.1	72.9	78.2	81.9
50	2.98	9.91	18.3	30.5	38.2	43.9	48.0	51.5	54.3	56.7	58.9	60.8	67.7	72.6	76.5
60	2.35	7.91	15.0	26.1	33.5	38.8	42.8	46.1	48.9	51.3	53.4	55.2	62.0	67.3	70.6
70	1.91	6.53	12.8	23.2	30.5	35.9	40.1	43.5	46.4	48.8	51.0	52.9	59.7	64.6	68.3
80	1.60	5.51	10.9	20.3	27.1	32.2	36.2	39.5	42.2	44.5	46.6	48.4	55.2	59.9	63.5
90	1.38	4.80	9.54	18.0	24.4	29.2	33.0	36.3	38.7	40.9	42.8	44.6	51.1	55.5	58.9
100	1.21	4.18	8.48	16.5	22.7	27.6	31.5	34.7	37.4	39.7	41.7	43.8	50.3	55.5	59.0
150	0.734	2.56	5.27	10.8	15.7	19.6	22.6	25.3	27.6	29.6	31.3	33.3	39.2	43.3	46.4
200	0.527	1.81	3.78	7.86	11.8	15.1	17.8	20.1	22.4	23.8	25.4	26.7	32.0	36.0	39.5
250	0.415	1.40	2.92	6.21	9.56	12.6	15.5	17.9	20.0	22.0	23.7	25.2	31.7	36.8	40.2
300	0.340	1.15	2.34	5.05	7.68	10.1	12.3	14.2	15.9	17.3	19.0	20.2	24.8	28.1	29.7
400	0.252	0.828	1.71	3.66	5.72	7.73	9.61	11.2	12.7	14.1	15.4	16.6	21.1	24.3	26.7
500	0.201	0.644	1.31	2.88	4.49	6.21	7.84	9.49	11.0	12.3	13.8	15.0	20.2	23.9	26.3

Table A.1. Tabulated values of  $g(p_{i,n}, Q_{i,n}) \cdot 10^4$ , i.e., if x is an entry in the table then  $g(p_{i,n}, Q_{i,n}) = x \cdot 10^{-4}$ .

$Q_{i,n}$	$p_{i,n} = 1$	5	10	20	30	40	50	60	70	80	90	100	150	200	250
1	0.943	0.957	0.961	0.965	0.966	0.967	0.968	0.969	0.969	0.970	0.970	0.970	0.971	0.972	0.973
10	1.163	1.119	1.094	1.076	1.066	1.062	1.059	1.056	1.054	1.053	1.051	1.050	1.046	1.043	1.042
20	1.381	1.271	1.207	1.158	1.138	1.124	1.117	1.111	1.107	1.103	1.099	1.097	1.088	1.084	1.079
30	1.519	1.383	1.287	1.217	1.189	1.172	1.162	1.153	1.147	1.141	1.136	1.132	1.119	1.111	1.106
40	1.613	1.469	1.357	1.264	1.228	1.205	1.194	1.183	1.175	1.168	1.163	1.158	1.145	1.134	1.128
50	1.686	1.533	1.410	1.299	1.255	1.228	1.213	1.200	1.193	1.185	1.179	1.174	1.159	1.148	1.141
60	1.736	1.584	1.458	1.337	1.287	1.257	1.240	1.226	1.216	1.207	1.200	1.194	1.177	1.163	1.158
70	1.779	1.630	1.491	1.363	1.306	1.271	1.251	1.235	1.223	1.214	1.206	1.199	1.183	1.170	1.161
80	1.814	1.666	1.529	1.392	1.333	1.296	1.272	1.255	1.242	1.235	1.227	1.220	1.197	1.183	1.174
90	1.838	1.691	1.556	1.419	1.355	1.316	1.291	1.271	1.265	1.254	1.246	1.238	1.214	1.200	1.190
100	1.861	1.721	1.582	1.437	1.370	1.327	1.300	1.280	1.266	1.254	1.244	1.235	1.214	1.196	1.186
150	1.931	1.803	1.673	1.520	1.441	1.390	1.365	1.341	1.323	1.309	1.297	1.285	1.257	1.240	1.228
200	1.963	1.852	1.725	1.579	1.492	1.437	1.400	1.379	1.355	1.349	1.336	1.325	1.291	1.270	1.250
250	1.974	1.880	1.764	1.626	1.533	1.471	1.425	1.393	1.369	1.354	1.338	1.325	1.276	1.244	1.227
300	1.987	1.896	1.796	1.650	1.567	1.506	1.464	1.432	1.408	1.389	1.371	1.359	1.318	1.294	1.300
400	1.995	1.926	1.829	1.698	1.608	1.543	1.494	1.468	1.439	1.417	1.398	1.384	1.333	1.306	1.287
500	1.997	1.948	1.865	1.733	1.655	1.587	1.536	1.493	1.461	1.435	1.409	1.390	1.323	1.287	1.278

Table A2 Tabulated values of  $k(p_{i,n}, Q_{i,n})$ .

## 6.2. Appendix B

Figure B.1 – B.4 presents results from the analysis of the historical demand data for all 80 products.

Mean actual ordered qty vs Arrival intensity among retailers

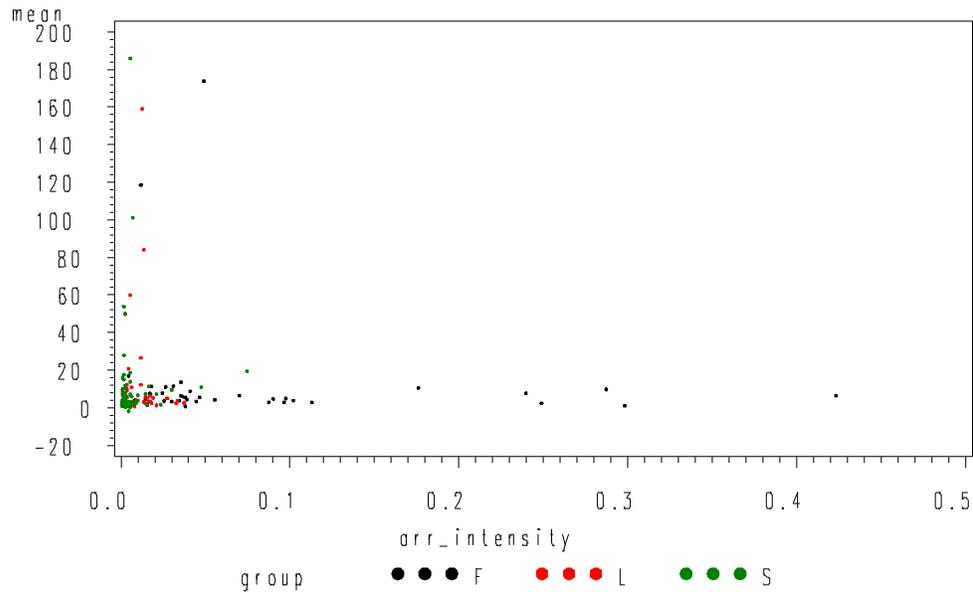


Figure B1 Mean actual ordered quantity vs arrival intensity (customers/day) among retailers.

Mean actual ordered qty vs Arrival intensity among retailers

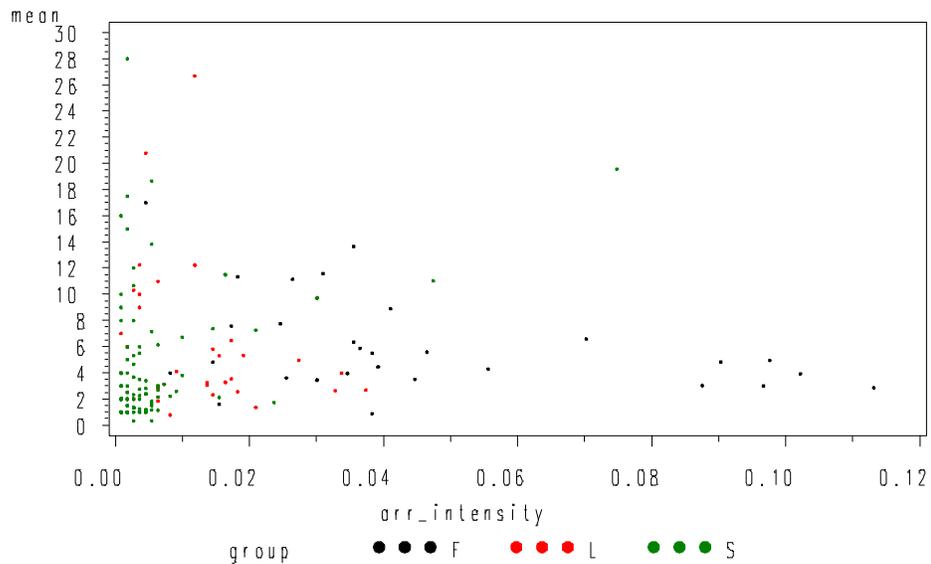


Figure B.2. Mean actual ordered quantity vs arrival intensity (customers/day) among retailers. Zoomed on both axes.

Mean actual ordered qty vs Std for actual ordered qty

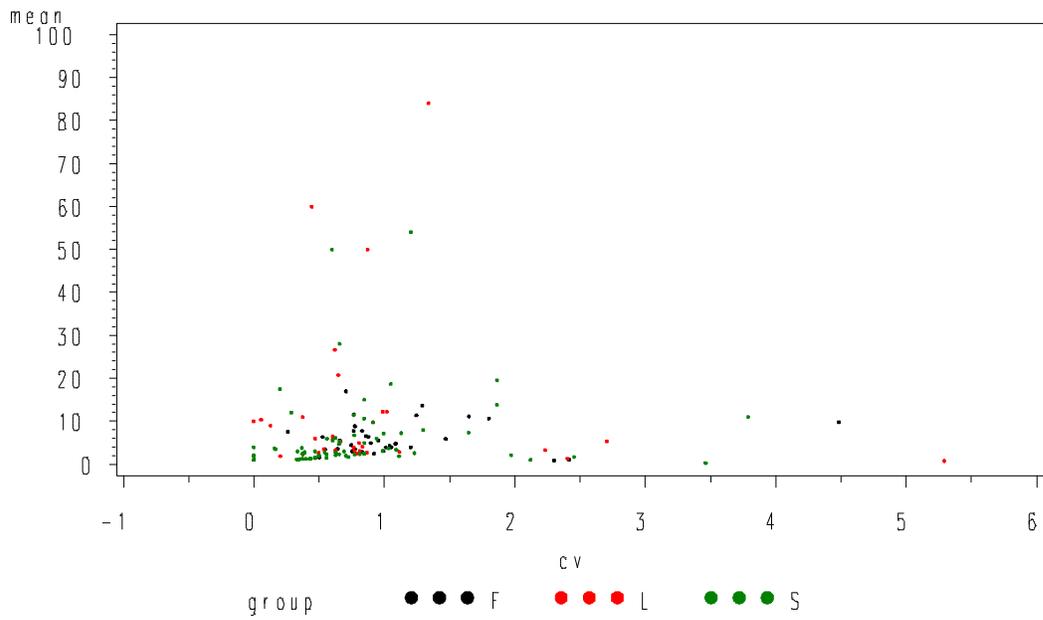


Figure B.3. Mean actual ordered quantity vs standard deviation for actual ordered quantity.

Total system demand vs percentage of demand at virtual retailer for each material

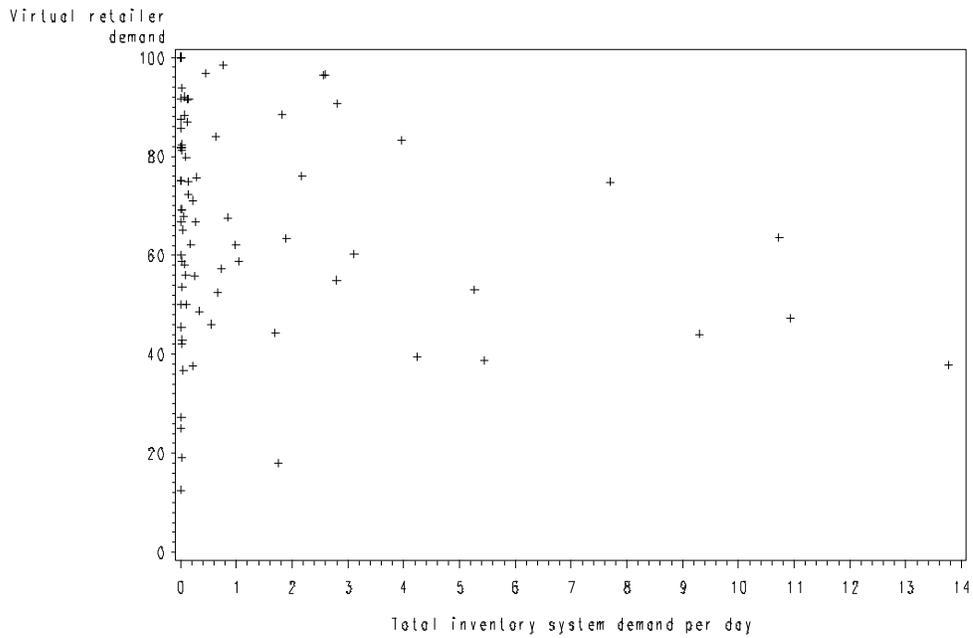


Figure B.4. Total system demand vs percentage of demand at virtual retailer.

### 6.3. Appendix C

Arrival intensity; mean and variation for the customer order size; mean and variation for the demand in units per day; and mean size for the replenishment order is derived using the historical demand data for the base cases, shown in Table C.1. In Table C.2 parameters for the compound Poisson process is presented.

Test case	Location	Arrival intensity	Order size		Demand (units/day)		Order qty mean
			Mean	Std	Mean	Std	
1	CW	0.10493	32.409	37.799	3.40055	16.0235	n/a
	VR	0.63777	6.412	37.294	4.08942	30.1551	n/a
	1	0.08759	3.031	3.007	0.26551	1.1632	26.667
	2	0.11314	2.855	2.371	0.32299	1.1240	38.111
	3	0.10219	3.938	3.987	0.40237	1.6829	38.462
	4	0.29836	1.095	2.646	0.32664	1.4813	15.176
	5	0.28741	9.825	44.059	2.82391	24.1434	99.545
	6	0.09672	3.009	2.274	0.29106	1.0532	15.417
2	7	0.24909	2.469	2.278	0.61496	1.5029	40.611
	8	0.04471	3.510	2.724	0.15693	0.8584	13.571
	CW	0.04471	100.102	103.053	4.47536	30.3038	n/a
	VR	0.17792	28.990	66.525	5.15785	30.5255	n/a
	1	0.03558	28.718	56.759	1.02190	11.9570	143.571
	2	0.07482	19.585	36.530	1.46533	11.2734	80.750
	3	0.01642	11.500	8.807	0.18887	1.8058	4.833
	4	0.05292	58.207	88.049	3.08029	24.2176	141.000
3	5	0.00182	8.500	7.778	0.01551	0.4766	n/a
	CW	0.04015	53.841	77.252	2.16150	18.8099	n/a
	VR	0.16515	12.773	48.242	2.10949	20.2292	n/a
	1	0.04653	5.588	5.334	0.26004	1.5901	22.941
	2	0.01916	5.333	3.540	0.10219	0.8293	25.250
	3	0.42336	6.422	5.638	2.71898	5.3204	151.500
	4	0.03923	4.442	3.326	0.17427	1.0216	32.143
	5	0.01734	3.526	1.896	0.06113	0.4702	25.000
6	0.00182	8.000	2.828	0.01460	0.3424	6.500	

Table C.1. Arrival intensity; mean and variation for the demand; and mean order quantity.

<b>Compound poisson process</b>				
<b>Test case</b>	<b>Location</b>	<b>Intensity</b>	<b>Gamma k</b>	<b>Gamma theta</b>
1	CW	0.10493	0.71261	44.777
	VR	0.63777	0.02513	235.256
	1	0.08759	0.70868	3.572
	2	0.11314	0.98652	2.387
	3	0.10219	0.74331	4.625
	4	0.29836	0.05054	11.769
	5	0.28741	0.04480	208.159
	6	0.09672	1.21772	2.061
2	7	0.24909	0.74674	2.637
	8	0.04471	1.22091	2.466
	CW	0.04471	0.93415	106.623
	VR	0.17792	0.18340	155.339
	1	0.03558	0.24716	114.168
	2	0.07482	0.27297	69.918
	3	0.01642	1.56011	7.051
	4	0.05292	0.42955	134.344
3	5	0.00182	1.05785	7.563
	CW	0.04015	0.47676	111.881
	VR	0.16515	0.06473	189.619
	1	0.04653	0.91012	5.591
	2	0.01916	1.86392	2.593
	3	0.42336	1.10345	5.367
	4	0.03923	1.40465	2.806
	5	0.01734	2.54653	1.188
6	0.00182	7.03125	1.067	

Table C.2. Parameters used in the simulation for the compound Poisson process.

## 6.4. Appendix E

Detailed simulation results for test case T1-T3 is presented in Table E.1 to Table E.3.

Results for test case S1, S1\*, S2 and S3 is presented in Table E.4 to Table E.7.

	SCP			Method 1			Method 2		
Location	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs
CW	96	498.0		98	608.0		80	231.4	
VR	96	11.6	735.0	98	25.1	348.9	89	94.8	2099.0
1	97	19.5	29.2	96	18.4	31.4	95	18.3	33.0
2	97	23.6	28.2	97	23.6	28.2	97	23.4	28.9
3	95	25.3	35.7	96	26.3	34.8	95	26.0	36.8
4	89	14.0	33.0	89	14.0	32.9	89	13.8	35.0
5	77	184.5	1790.5	85	249.1	1288.2	85	256.7	1494.4
6	95	17.2	27.9	95	17.2	27.8	95	17.0	29.1
7	97	28.8	38.2	97	28.9	38.1	97	28.4	40.1
8	100	25.3	25.6	95	13.4	25.9	93	12.3	30.9

Table E.1. Detailed results for test case T1.

	SCP			Method 1			Method 2		
Location	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs
CW	96	594.4		92	467.8		78	274.8	
VR	98	121.4	176.6	95	57.2	220.5	91	114.1	465.6
1	69	91.3	189.3	76	110.0	189.2	77	113.1	206.7
2	66	57.5	95.7	68	60.1	96.9	69	62.9	107.2
3	90	24.5	26.2	65	11.1	18.6	63	10.9	19.4
4	66	112.8	200.2	69	124.3	214.2	70	130.0	256.3
5	27	2.5	3.7	34	3.4	4.7	27	2.5	3.9

Table E.2. Detailed results for test case T2.

	SCP			Method 1			Method 2		
Location	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs
CW	97	364.2		99	458.5		82	141.4	
VR	96	14.2	457.3	98	14.2	224.8	84	64.1	1874.4
1	93	19.8	44.4	93	19.8	44.3	93	20.6	45.8
2	99	24.0	24.1	75	10.3	12.5	74	10.2	12.6
3	97	93.2	109.6	97	92.4	109.7	98	98.8	111.8
4	98	26.3	36.9	97	25.3	38.2	96	24.1	40.8
5	99	19.8	19.8	80	10.0	12.3	80	10.0	12.4
6	54	4.4	5.2	43	3.4	4.5	31	2.4	3.8

Table E.3. Detailed results for test case T3.

	SCP			Method 1			Method 2		
Location	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs
CW	94	298.5		44	42.2		70	96.3	
VR	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0
1	96	19.3	29.8	97	21.3	32.9	97	20.1	30.7
2	97	23.6	28.3	98	26.1	30.3	97	24.2	28.9
3	95	25.3	36.0	96	29.6	39.0	96	27.8	36.4
4	89	14.0	33.3	91	16.5	36.6	90	14.6	35.0
5	77	183.1	1951.6	90	349.7	1435.0	85	268.9	1723.0
6	95	17.2	28.0	96	19.2	28.8	96	17.9	28.1
7	97	28.8	38.5	98	32.1	40.3	98	31.1	38.8
8	100	25.3	25.7	96	14.8	27.6	94	13.2	28.3

Table E.4. Detailed results for test case S1.

	SCP			Method 1			Method 2		
Location	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs
CW	94	298.1		44	41.6		70	95.4	
1	96	19.3	30.6	97	21.4	33.8	96	20.1	31.3
2	97	23.5	28.3	98	26.1	30.0	97	24.2	28.7
3	95	25.3	36.2	96	29.5	39.2	96	27.8	36.4
4	89	14.0	33.0	91	16.4	36.8	90	14.5	35.3
5	85	250.1	1395.3	90	349.5	1405.9	85	268.9	1688.0
6	95	17.1	28.8	96	19.0	29.1	95	17.7	28.9
7	97	28.6	38.2	98	31.9	40.5	98	31.0	38.7
8	100	25.3	25.7	95	14.8	28.4	94	13.2	29.2

Table E.5. Detailed results for test case S1\*.

	SCP			Method 1			Method 2		
Location	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs
CW	95	463.0		61	121.3		71	167.0	
VR	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0
1	69	90.8	190.0	80	129.7	242.1	78	119.2	225.6
2	67	57.5	95.4	74	77.1	125.8	72	70.7	116.3
3	90	24.5	26.2	70	13.4	21.1	66	11.7	20.0
4	66	112.3	208.9	79	194.4	358.3	72	149.8	325.5
5	26	2.4	3.8	41	4.4	5.9	26	2.4	4.2

Table E.6. Detailed results for test case S2.

	SCP			Method 1			Method 2		
Location	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs	Fill rate %	Holding Costs	Total Costs
CW	100	206.0		16	11.6		62	52.9	
VR	0	0.0	0.0	0	0.0	0.0	0	0.0	0.0
1	93	19.8	44.1	93	25.9	84.3	95	23.1	45.1
2	99	24.0	24.1	79	11.6	15.9	74	10.0	12.9
3	97	93.5	109.5	100	175.4	176.1	99	109.2	113.9
4	98	26.3	36.9	96	27.1	52.5	97	25.8	39.4
5	99	19.8	19.8	83	10.9	13.6	79	9.9	12.5
6	54	4.4	5.2	52	4.2	6.3	42	3.4	4.7

Table E.7. Detailed results for test case S3.

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