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Belief revision with revisable logics

- Trosrevision med reviderbar logik

Abstract

Belief revision is a theory of how an agent ought to change his or her beliefs in order to be considered rational. What's assumed is a formal language and a logical system which describe what consequences follow from an agent's beliefs. In addition, a list of postulates describe how the agent ought to do in order to be considered rational when revising his or her beliefs. This essay will try to integrate the logical system into the beliefs of the agent in order to make it as revisable as any other beliefs. The idea is based on Quine's idea that no statement is immune to revision and his idea of the web of belief. What will be looked upon is some ideas as to how to make this work and certain complications that these solutions give rise to.

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1 Introduction

In the last century, logic has flourished as a subject and many different branches have been created in order to explain different phenomenon within reasoning, language and mathematics. There are some basic laws of logic that are accepted or rejected rules depending on the purpose of the logic in question. For instance we have the law of non-contradiction which says that a proposition '*p'* and its negation '*not p'* cannot be true at the same time. An area of logic has rejected this rule to allow for contradictions since some sentences contain contradictions but are meaningful nonetheless to humans. This area is referred to as paraconsistent logics. Another example is intuitionistic logic which is often used in mathematics and pays little attention to the law of excluded middle. This law simply states that *'p'* or '*not p'* has to be true and when a proof that one or the other isn't true, one can simply derive that the other has to be true. The most common, however, is classical logic which accepts both of mentioned laws as well as a few other. It still stands though, that there are many versions and varieties to be found within logics depending on the original purpose of the logic.

Given all these variants of logic, a tempting thought is that logics are not solid structures immune to revision. The idea of revisability is to see something as subject to change should new information arise. To treat logics as revisable is to give it the same standard as any other theory invented to explain a certain phenomenon.

Revisability, then, is what is at heart of belief revision since it wants to show how a rational agent ought to change his or her beliefs in light of new evidence. What it does is suppose a logic that will show what inferences are valid for an agent. Given the beliefs of the same agent, we can then show what consequences these sentences have with the supposed logic.

The idea behind this essay is to not suppose a logic that stands outside the beliefs of an agent but instead have it as a part of the beliefs, making it as susceptible to revision as any other beliefs the agent has. This is based on the idea of Quine's web of belief, that all beliefs of an agent give support to one another. What will be done is to outline some of the problems that arise with such a theory.

1.1 Revisability

The position that everything is revisable in light of new evidence is perhaps best known to be held by the late Willard Van Orman Quine. There are two ideas of his that give rise to this claim, the first being the idea that our beliefs are entangled in a kind of web where beliefs give support to one another[1]. This view is a kind of coherentism which is the view that all beliefs are interconnected and stands as a whole rather than being based on some foundational beliefs which all beliefs are derived from. One could for instance look at the belief of a tree, having the belief "trees are plants" as well as "all living things reproduce". If we put the beliefs "plants are living things" and "trees reproduce" into this web we can see that they are connected to one another, the proposition "trees are plants" is connected to "plants are living things" while it in turn is connected to "all living things reproduce" to get the statement of "trees reproduce".

The second is the idea that since our beliefs are entangled in this web of belief the distinction between analytic and synthetic becomes watered down. The idea of analytic sentences is that they are true "come what may" but since our beliefs are entangled in a web of belief, any sentence may have this property. This makes Quine conclude that "[...] no statement is immune to revision".[2] In his book "Methods of Logic" he states a similar view by saying that "mathematical and logical laws themselves are not immune to revision if it is found that essential simplifications of our whole conceptual scheme will ensue." [3]

He is not alone in claiming that logic should be revisable, Susan Haack is a supporter of Quine but has another point of view as to why logics should have this property. Her suggestion is not that the truths of logic may be different from what they are but instead we may be mistaken about what the truths of logic are. To get the epistemological character of the discussion, she suggests that the question whether fallibilism extends to logic or not is more appropriate.[4]

Fallibilism is the idea that something may turn out to be false, for instance if a method is fallible it is liable to produce false results or if a person is cognitively fallible, he or she is liable to hold false beliefs. It is the idea of Haack that people are fallible since we have several examples of what people used to believe but turned out to be false, a flat earth, phlogiston etc. There are those who think that some beliefs are in fact infallible and are prepared to take logic in such a category.

What she does is discuss three ideas of how logic is seen as safe from theoretical fallibilism.

1 Quine, Willard Van Orman: "Two Dogmas of Empiricism" (1951) in A.P. Martinich(ed.) *The Philosophy of Language,* New york: Oxford University Press 2010, page 73

2 Ibid. page 73

3 Quine, Willard Van Orman: *Methods of logic* (4th edition), Cambridge: Harvard University Press 1950, page 3

4 Haack, Susan: *Philosophy of logics* Cambridge: Cambridge university press 1978, page 232

The first idea discussed is the idea of necessity, that logical laws are necessary. The argument is explained like this: “the laws of logic are necessary, that is to say, they couldn't be otherwise than true; so, since a logical law can't be false, one's belief in a logical law can't be mistaken, and so, is infallible”[5] An answer that is given to this argument is that fallible is here used as a predicate of propositions and not of persons. According to Haack, the thesis of 'proposition fallibilism' is uninteresting since it simply says that some propositions are possibly false. What is more interesting is to discuss the epistemological idea of 'agent fallibilism', an idea that agents may hold false beliefs. As an example she takes the idea of the law of excluded middle. If '$p \vee \neg p$' is necessary in the sense of proposition fallibilism we may still hold the belief that '$\neg(p \vee \neg p)$' and even yet, '$p \vee \neg p$' may not be necessary but we still hold the belief that it is.

Furthermore, she discusses the idea of logical laws as self-evident which seems to mean that a self-evident statement is obviously true. But this is no way near satisfactory as different agents hold different beliefs to be self-evident, if there's self-evident truths then we have no concise way of establishing that they are or we may hold beliefs to be self-evident and it may turn out that they are false.[6]

The third point that is taken up for discussion is that of analyticity, and analytic truths, logical laws here included, are manifest. Analyticity is understood as ' truth in virtue of it's meaning' which is not obvious what it means since there has to be an understanding of 'meaning' and also 'in virtue of'. But even if it were so, that analytic truths were formidable, then logical truths would be recognized as soon as they are understood but then again, there would also have to be a foolproof way to recognize that the logical truth is perfectly understood.[7]

Since agents are the ones holding beliefs of logic and are the creators of the theory, the three stated points leans towards being suspicious of the solid and manifest status of logics. There is no doubt that an agent may hold false beliefs even though a theory may still be sound. The fact of the matter is that we, as agents, sometimes treat theories as solid truths when we just have lack of counter-evidence for falsifying the theory.

The idea of 'agent fallibilism' is very interesting for a theory of belief revision with revisable logics since it treats all beliefs as fallible even though theories themselves may be necessarily true.

5 Ibid. page 233
6 Ibid. page 235-236
7 Ibid. page 236

1.2 Why AGM Theory

There are many kinds of belief revision theory and hence, we need to look upon what characteristics of the theory we want for our own. Belief Base Dynamics (BBD) and the AGM theory are two separate theories of belief revision which will be brought to discussion. While AGM is one of the most used in belief revision, BBD has a whole other approach to the nature of beliefs. The idea behind BBD is that beliefs of an individual is represented by a belief base *K* which is a set of sentences that acts as a foundation for other beliefs that are derived from the belief base.

Belief bases are seen as sets of sentences that are independent from other belief sets and are not closed under logical consequence, except in limiting cases. With inferences made on a belief set, call it *A*, we get the beliefs according to *A*, denoted *Cn(A)*. The consequences of *A*, however, are non-independent and not worth keeping for their own sake. In this way we have two sets of belief sentences, one is the intersection of the derived beliefs and the belief base *Cn(K)\K* and the other being the belief base *K*.[8]

Changes to one's belief is performed on the base which in turn changes the consequences that follows from these foundational beliefs. This basic idea of BBD gives it a more foundationalist approach which is the idea that our beliefs are grounded in some more basic belief. This stands in contrast to the AGM theory where beliefs are seen as more of a coherentist view.[9]

Given this general view of BBD and the earlier mentioned view of the web of belief, the more fitting approach is that of AGM since it goes hand in hand with a coherentist view of knowledge. According to the theory of Quine, sentences should, in theory, be equal in the sense of revision. Of course there is a discrepancy between what evidence supports a belief but in light of new evidence and other theories, these more entrenched sentences should be as susceptible for revision as any other.

2 Classic AGM theory

Belief revision was developed by Alchourrón, Gärdenfors and Makinson in the eighties as a theory that was set out to describe how one ought to change one's belief in a rational sense. The theory was given the name AGM from surnames of the authors. The basic idea is that a belief state is a set of sentences and in light of new information the belief state is either expanded, where a sentence *p* is

8 Hansson, Sven Ove, "Logic of Belief Revision", *The Stanford Encyclopedia of Philosophy (Fall 2011 Edition)*, part 5.1

9 Hansson, Sven Ove: *Belief Base Dynamics*, Uppsala: Lindbergs Grafiska HB 1991, page 25

simply added to the belief set *K*. Revised, when there already is a sentence that contradicts the new sentence that we want to add to *K*. Contracted, where no new information is added but some sentence *p* is needed to be removed from the belief set *K*.

AGM theory is based on the assumption of a logic that is supraclassical, a logic that can express at least as much as classical logic, and the definition of consequence satisfies the conditions for the tarskian operators.[10]

Inclusion: $A \subseteq Cn(A)$

Monotony: If $A \subseteq B$ then $Cn(A) \subseteq Cn(B)$

Iteration: $Cn(A) = Cn(Cn(A))$

The consequence relation, *Cn*, is a function from a set of sentences to another set of sentences. In this case it means that given a set *A*, *Cn(A)* is the set of logical consequences of *A* given the logic assumed. Formally, this can be expressed as $Cn(A) = \{ p \in L \mid A \vdash p \}$ where *L* is the formal language assumed. [11]

The usual connectives are assumed to be of a standard formal language using negation, disjunction, conjunction, implication and equivalence. As said earlier, the three forms of changing one's belief is either through expansion, revision or contraction. Starting with expansion, it's simply adding a sentence to the already existing belief state, as follows.

Expansion: $K+p = Cn(K \cup \{p\})$

This shows for just adding a new belief *p* to the original belief sets, when doing so will lead to no form of contradiction. Note that it's not only the sentence *p* that is added to *K* but also the consequences, this closure operation will be discussed further when examining the postulates that need to be satisfied for revision and contraction. When a contradiction occurs by adding a sentence *p* to the belief set *K* there are six postulates[12] that needs to be met in order for the agent to be seen as rational.

10 Hansson 2011, part 1.3

11 Enqvist, Sebastian: *Interrogative Belief Revision,* Lund: Media-Tryck 2011, page 12

12 The postulates of revision are discussed in Gärdenfors, Peter: *Knowledge in flux*, USA: Halliday Lithograph (1988) pages 52-55

Closure: $K^*p=Cn(K^*p)$

Closure is designed to make the agent believe in all sentences that follow from the sentences given the logical system already assumed, closing the set under it's logical consequence. This gives the notion of an ideal state of belief but can be seen as the beliefs the agent is committed to believe. Instead of having every agent logically omniscient, it shows what should be believed given the logical consequences of our beliefs.[13]

Success: $p \in K^*p$

Given any sentence *p*, if an agent revise it's belief sets with this sentence then it should lead the agent to believe the sentence *p*. The sentence *p* has to be an element of *K* revised by *p.*

Inclusion: $K^*p \subseteq K+p$

This states that whenever a revision by *p* is performed, the agent should not add sentences that are not part of the consequences of the sentence *K* is revised with. The set *K* revised by *p* is a proper subset of the set *K* expanded by *p.*

Vacuity: if $\neg p \notin K$ then $K^*p=K+p$

When revision is performed and there is no sentence in the already existing belief sets, the agent may simply add the sentence using expansion. If the sentence *not-p* is not an element in *K*, revision by *p* on *K* is then identical to *K* expanded by *p.*

Consistency: K^*p is consistent if p is consistent

The consistency postulate is a requirement that an agent belief sets should be consistent, which was the goal for belief revision in the first place, how to revise beliefs without holding contradictory beliefs.

Extensionality: if $\vdash p \leftrightarrow q$ then $K^*p=K^*q$

13 Hansson 2011, part 1.2

This is to capture the idea that what matters is the information contained in the sentence up for revision rather than how the information is presented. There are many ways of expressing the same belief using natural languages and by having extensionality, all sentences expressing the same belief can be shown as logically equivalent. This will result in a revision of the belief as such and not just a particular linguistic expression.

Another set of postulates is needed as well, to set the conditions for removing beliefs when faced with new information. These are called contraction postulates and the function is denoted by the symbol ÷. The postulates[14] are as follows

Closure: $K \div p = Cn(K \div p)$

Success: if $\nvdash p$ then $p \notin K \div p$

Inclusion: $K \div p \subseteq K$

Vacuity: if $p \notin K$ then $K \div p = K$

Extensionality: if $\vdash p \leftrightarrow q$ then $K \div p = K \div q$

Recovery: $K \subseteq (K \div p) + p$

These work in basically the same way as when using the revision postulates except for the recovery postulate which is to ensure that the principal of minimal change is followed. It tells us that when contracting *K* with a sentence *p* we remove just enough to remove *p* so that adding p again will fully restore *K*.[15]

Revision, contraction and expansion are all treated as functions from the set K to a new set that includes or does not include the new sentence depending on the action performed.

14 The postulates for contraction are thoroughly discussed in Gärdenfors 1988 pages 60-66

15 Ibid. page 62

3 Belief revision with revisable logics

I will begin with addressing the problems of the postulates with contraction since as we shall see, there is a postulate that is seen as necessary for the theory in order to make the logics revisable as they should be. The different postulates are looked upon in order.

Success postulate

In general, the basic idea with a belief revision theory with revisable logics is one that allows for many different kinds of logic and does not embark from a fixed logic system as AGM does. What we want to allow for is revision of the logical rules within the set of beliefs and hence, the first adjustment that we make is to substitute the success postulate of contraction with one we will call unrestricted success.

Unrestricted success: $p \notin K \div p$

This will allow, instead for just non-tautologies, that any sentence in the belief sets will be able to be removed from the belief set *K*. It is a most important postulate for this theory since it will make any belief sentence revisable and since we want to include logics into the belief set *K* these will be revisable as well.

Since this gives us what is sought for in the theory, the goal will be to review the other postulates in light of this postulate and attempt to defend it against the friction it may provide.

Closure postulate

With unrestricted success defined, a problem arises with the closure postulate. We want to keep the closure postulate since with coherentism, where beliefs are seen as interconnected in a direct or indirect way, an agent may not be seen as rational if this set isn't closed. There is, however, a serious problem when combined with unrestricted success. It gives us the following theorem:

Theorem 1: Given unrestricted success and closure with a monotonic logic, contracting a tautology *p* from *K* will result in a contradiction.

Proof: assume an arbitrary, monotonic logical system *L* such that $p \in Cn(\emptyset)$ where Ø is the empty set and a set of beliefs *K* which is closed under *L*. Using unrestricted success, $p \notin K \div p$, in this case we have a problem since Ø is a part of *K*. We would then have $p \in K$ since it is closed under *L*, *p* is then a part yet not a part of *K*. Q.E.D.

What is needed is a solution for the conflict between the closure and the unrestricted success postulates since we want the theory to include both of these. This paper will account for two different solutions, one treating modus ponens as non-revisable and have it close the belief set *K*. The other is a way of separating the inference rules from the basic beliefs into two different sets which will be referred to as the separation approach. Both of these solutions will be presented in turn.

3.1 Modus Ponens

Modus ponens is an inference rule which states that if *P* implies *Q* and *P* is true, then *Q* has to be true, formally it would look like $p \rightarrow q$ and *p*, therefore *q*. It gives the possibility to remove antecedents from arguments when it's known what is concluded from the inference. Given the earlier example with the tree as a living thing we can have these simple inferences be of the form of modus ponens. This allows the theory to be of a coherentist view as was wanted in the first place.

Closing the belief set K with modus ponens is an idea that treats the rule as unrevisable while other logic axioms as sentences in the belief set K. Given that a sentence *p* that implies *q* is an element in K, it would be considered irrational to not include *q* since we already hold the belief that *p* is true. Sentences cannot be inferred from the empty set using modus ponens as a closure operator since it does not contain any true sentences which solves the issue of theorem 1. Since all the logical axioms are now in the belief set *K* we can contract them as any ordinary sentence. The consequence can be defined as $\mathrm{Cn(K)}=_{\mathrm{df.}}\{\mathrm{p}\in\mathrm{K} \mid (\mathrm{p}\rightarrow\mathrm{q})\rightarrow\mathrm{q}\in\mathrm{K}\}$.

What is a problem for this approach is that it treats a rule of inference as non-revisable which undermine the goal set from the beginning. Even though modus ponens is a central thought in reasoning it may not be without problems. Vann McGee presents a few counterexamples to the rule which are illustrated as follows. An opinion poll taken before U.S.As presidential election in 1980 showed that the republican Ronald Reagan was in the clear lead over the democrat Jimmy Carter. In runaway third place there was another republican named John Anderson. The inference is then proposed as following:

1. If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

2. A Republican will win the election.

   Then by the rule of modus ponens we can infer the following

3. If it's not Reagan who wins, it will be Anderson.

The argument has the form of $p \rightarrow (q \rightarrow r)$ and p. It shows that there seem to be a problem using it in this form since the winner, if it's not Reagan, is Carter.[16] This example, shows that there may be a problem with modus ponens and to have it as a non-revisable law of consequence in this theory then seems as a problematic position to take.

3.2 Separation

The separation approach works in the way of having a belief state be divided into two different sets, one set, *K*, containing the beliefs and another set, *I*, containing the inference rules. As we saw with the modus ponens approach we landed on a solution that didn't treat all logical rules as revisable. In this approach we don't get that same problem since the change of belief takes one ordered pair of $\langle K,I \rangle$ to an ordered pair with the new, revised belief and inference set $\langle K',I' \rangle$. The change occurs in both sets and changes the logic the agent believes in.

In this way, the result from proof 1 is then dealt with in such a way that the new set will be closed under the new set of inference rules that won't lead to the result of *p* when we have contracted it from the set *K*.

The set of inference rules is treated as sequents. These are formalized statements of provability, expressions of the form $X \Rightarrow p$. Here *X* and *p* are both sets that are empty or non-empty and *X* entails *p*.[17]

Logical consequence: $p \in Cn_I(X) \Leftrightarrow X \Rightarrow p \in I$

Which says that *p* is an element of the closed set *X* if and only if the sequent $X \Rightarrow p$ is an element of *I*. As stated earlier, this approach does not have the drawback of treating any inference rule as non-revisable. However, there is a backside to this approach. By separating belief sentences and inference rules we get an obvious distinction between them. This is not an elegant solution since we want the logical laws to be treated as equal to ordinary belief sentences. The distinction allows for viewing inference rules as different from belief sentences.

16 Vann McGee: "A Counterexample to Modus Ponens" (1985) in *The Journal of Philosophy*, Vol. 82, No. 9 (Sep., 1985), page 464-465

17 Portoraro, Frederic, "Automated Reasoning", *The Stanford Encyclopedia of Philosophy (Summer 2011 Edition)*, part 2.2

4 The other postulates

Since there has been no focus on the other postulates, there will be no focus on solving these issues, instead there will be an outlining of the problems and how they fare with the different solutions for the closure/unrestricted success problem.

Vacuity

Seeing as the postulate says that *if* $p \notin K$ *then* $K \div p = K$, the general idea is that the set goes unchanged when p is not an element of K. This prevents unnecessary change of beliefs when not faced with a contradiction in the belief set. The same goes for the revision version of the postulate, *if* $\neg p \notin K$ t*hen* $K * p = K + p$, where the smallest possible change is just expanding K with p. These show no problems for the solutions earlier mentioned.

Inclusion

The postulate of inclusion, $K \div p \subseteq K$ tells us that contraction with p removes sentences from K, preventing sentences outside K to be added. One could say that the vacuity postulate is there to give a restriction against unnecessary change of the belief set, the inclusion postulate is there to make sure that the contraction is there to make a restriction against adding new sentences to K when the intention is to remove sentences. For both revision and contraction with the said solutions, we have no problems.

Recovery

Since revision, contraction and expansion takes the sentences in K over the inference rules in I from an ordered pair to a new ordered pair of the new sets of sentences and inference rules, the whole of the recovery postulate would look as follows when using the separation approach. This explanation of the postulate shows that the ordered pair $<K,I>$ is the initial state of K in recovery: $(K \div p)+p=<K,I>$. This is to explain the special case of tautologies, when the most drastic change in K is not p itself but rather the change of inference rules that allow for p to be concluded. This then allows for the whole set of inference rules to be recovered when adding the sentence p again.

Consistency

Consistency is another matter since we want the theory to allow many different logics to work with the belief sets. Seeing as paraconsistent logics want to deal with inconsistency by allowing for inconsistent propositions the original postulate doesn't work in our intended ways. What is a workable solution is for consistency to limit the amount of sentences that are allowed in *K*. Since there are presumably infinite sentences in a language, the restriction of having the belief set finite should do the trick.

Extensionality

There are some problems regarding this postulate. First, is assumes classical logic which stands in conflict with the initial idea of this essay in the first place. Second, there is a problem with having equivalence in a postulate all together with unrestricted success. Since any tautology is a consequence of the empty set and the empty set is always a part of a set we can show that any tautology is equivalent with any sentence whatsoever. This gives the conclusion of revising or contracting with one sentence would result in revising with every tautology.

Theorem 2: With unrestricted success and extensionality, contracting with one tautology would result in contracting with all tautologies.

Proof: Since a tautology is a consequence of the empty set, $t \in Cn(\emptyset)$, and the empty set is included in every set. We can show that any tautology is equivalent to any other and with unrestricted success which allow for contracting with tautologies we have the removal of all tautologies. Q.E.D.

I tried solving the problem by having extensionality depend on consequences of sentences. By seeing extensionality as if the consequences of a set of sentence were identical I arrived at the following postulate:

Revised Extensionality 1: If Cn(p)=Cn(q) then K*p=K*q

This gives rise to similar problems as with the original extensionality postulate since a tautology will follow from anything given any arbitrary inference rules. The consequences from any set will then be tautologies and contracting with any sentence results in contracting with all tautologies.

Against extensionality

One could argue for a removal of the extensionality postulate altogether from the theory using an argument from Quine. It's based on the idea that synonymity is impossible because of the definition of analyticity, truth in virtue of it's meaning. An example of this, which is often used, is that a bachelor is an unmarried man. The meaning of bachelor is in itself unmarried man but with a few examples from Quine we can see that this is not always the case. Conjoined expressions such as bachelor's buttons, which is a flower, have nothing to do with an unmarried man's buttons in the way it is understood in meaning. [18]

By having identical terms be substituted for one another we can arrive at some weird conclusions. Quine sees names as designators, which refer to an object in the real world. For example Cicero refers to a male, Roman politician as well as Tully refers to the same man. There is a discrepancy between designation where in a sentence a name is not purely designative but instead refers to some fact about the name. An example used is the sentence “Philip believes that Tegucigalpa is in Nicaragua.”. As some may know Tegucigalpa is the capital of Honduras where a substitution of terms would be “Philip believes that the capital of Honduras is in Nicaragua.”. An argument presented against this is that sentences that contain “... believes that...”, “...said that....” and so forth are not about designation but instead about relations between a person and a sentence.[19]

The occurrence of a sentence where the terms have been substituted in a belief set is however hard to see. Why would an agent like Philip believe the sentence “The capital of Honduras is in Nicaragua.” when this is so obviously false? The idea of closure when talking of designators can shed light on this situation. Having the belief set closed we can see that the identity relation solves this issue. Suppose that Philip come to believe the sentence “Tegucigalpa = the capital of Honduras.”. Since Philip wouldn't believe that a country's capital city is in another country a revision of the sentences would contract “Tegucigalpa is in Nicaragua.”

5 Summary

What has been done is a presentation of the general idea behind this essay with a short walk-through of revisability, AGM and some ideas of Quine regarding the web of belief. A basic requirement for the theory was proposed with the postulate of unrestricted success. Given the postulate, the friction between closure and unrestricted success was discussed and two solutions

18 Quine 1951, page 67

19 Quine, Willard Van Orman: “Notes on Existence and Necessity” (1943) in *The Journal of Philosophy*, Vol. 40, No. 5 (Mar. 4, 1943), page 115

were proposed to solve the issue. None of the solutions proved to be unproblematic and a more fitting solution could be found in order for the theory to work.

With that being said, the idea of a belief revision theory which treat logic as revisable is an interesting idea. There has been debate among supporters of different kinds of logic with proofs to support their claim. To step away from this debate and create a theory that allow for many kinds of logics could prove to be useful.

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