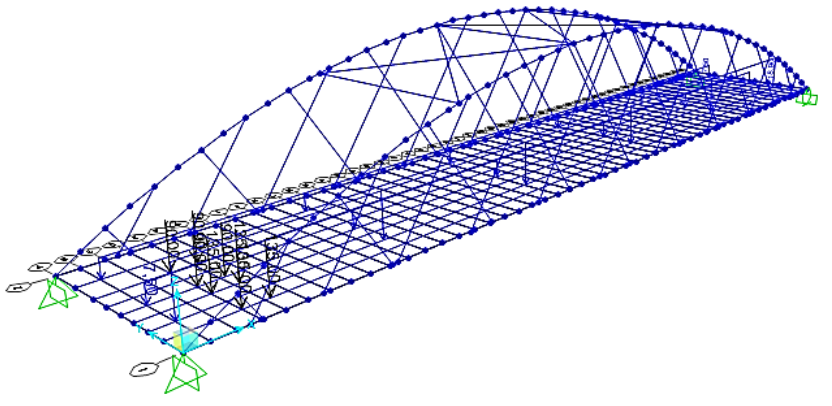
A feasibility study of a network arch bridge with glulam arches



Erik Nylander

Avdelningen för Konstruktionsteknik
Lunds Tekniska Högskola
Lunds Universitet, 2012

Rapport TVBK - 5211

Avdelningen för Konstruktionsteknik
Lunds Tekniska Högskola
Box 118
221 00 LUND

Division of Structural Engineering
Faculty of Engineering, LTH
P.O. Box 118
S-221 00 LUND
Sweden

A feasibility study of a network arch bridge with glulam arches

En rimlighetsanalys av en nätverks bro med limträ bågar

Erik Nylander
2012

Rapport TVBK-5211
ISSN 0349-4969
ISRN: LUTVDG/TVBK-12/5211+103p

Examensarbete
Handledare: Roberto Crocetti
April 2012

Abstract

The drive of using more renewable material has increased the interest in timber bridges. This makes it more interesting in looking in to new ways of constructing timber bridges. Most timber bridges for road traffic are using an arch with a hanging deck suspended from the arch using vertical hangers. This master thesis has looked at the possibility of using a system (developed for steel arch bridges) that is using inclined hangers, “the network arch”. This system has proven to greatly lessen the bending moment in the arch for steel arch bridges.

A parametric study has been performed to decide the influence of the different parameters of the bridge. The study was made using a finite element program to calculate different influence lines. These influence lines have then been the basis of the study. The study showed that by changing the stiffness of the arch and hangers the influence lines for the designing bending moment could be changed and the point for the critical section could be moved.

The study has also showed that the problem with relaxing hangers was bigger than anticipated. Hanger relaxation is a problem for network arches and is due to the fact that a partial load of the bridge will deform the arch in a sideways movement. This movement will decrease the distance between the nodes for some of the hangers making them “to long” to take any load.

The study was made on a hanger constellation using the same angle for all of the hangers. It was proven that the problem with relaxation was biggest close to the supports. This proved that another hanger constellation using a constant or parabolic change of hanger inclination would probably lessen the risk of relaxing hangers.

The feasibility of the system was then tested by designing a 50 m long and 10 m wide network arch bridge with a hanger inclination of 55 degrees. A reference bridge with a more conventional design with vertical hangers has also been designed.

This design showed that by using the network arch system the bending moment that was achieved was almost nine times smaller and the cross-section of the arch was almost half the size. The lessening of the bending moment also had a great impact on the tension perpendicular to the grain of the arch, a load that is a big problem for arches especially when using the Eurocode design standards.

Introductory remarks

Here I would like to thank all the people that have helped me throughout this master thesis. Without the help and support of you this would not have been possible.

I would like to thank Per Tveit and Anders Asp for the tips and help during the thesis.

A special thank goes out to my supervisor Roberto Crocetti, professor at LTH, who has always taken the time to discuss the problems that I have encountered and answer all of my questions.

A special thank also goes to Jan Holdhus, Technical Consultant at EDR Medeso AS, who supplied me with the software SAP2000 v.15. He has also answered my questions about why the program doesn’t work the way I wanted it to. Without his help with the software this thesis would never have been made.

Lund April 2012

Erik Nylander

Table of Contents

1 Introduction

1.1 Background

Timber is becoming an increasingly more popular material due to its good strength to weight ratio, being a relatively cheap and a renewable material. Timber has mainly been used in smaller buildings, houses and pedestrian bridges. However, increasing knowledge of manufacturing techniques for glue laminated timber (glulam) has made it possible to use timber in several bigger timber constructions, such as multistory buildings and bridges for road traffic.

The change of designcode, from BKR to EKS 5, in 2011 decreased the material properties of construction timber and glulam, which can be a big blow to these bigger timber constructions.

Most bigger timber bridges built today use some kind of arch construction. This is due to the fact that timber has its best strength capabilities in compression and that it is fairly cheap and easy to construct the arched shape.

There are three different kinds of arch bridges. You can have underlying arches supporting the bridge deck (figure 1); you can have a hanging deck supported by arches and hangers (figure 2) or something in between.

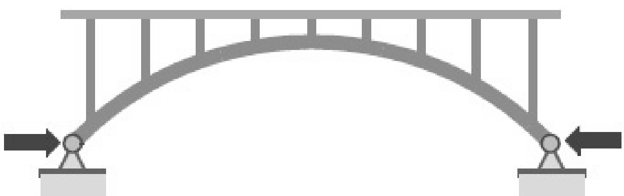


Figure 1 – A two hinged arch bridge supported by underlying arches (Bell, 2010)

The advantage of using underlying arches is that you can use several smaller arches besides each other, but it requires higher demands on the abutments.

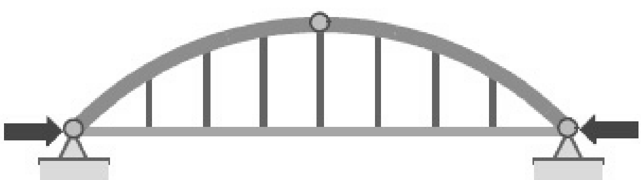


Figure 2 – A three hinged arch bridge with a hanging deck (Bell, 2010)

The arch bridge with a hanging deck on the other hand can be fitted with a tension rod between the ends of the arch which takes care of the horizontal forces conducted by the arch bridge. This lessens the demands on the abutments.

The most commonly used design for timber bridges is the arch bridge with a hanging deck where steel cables or rods are used as hangers, like figure 2.

Per Tveit, professor emeritus at Agders University in Norway, has developed a system for arch bridges with a hanging deck were the hangers are inclined and cross each other, “The Network Arch”, see figure 3.
His design is developed for two hinge steel bridges and is, according to himself best used for a length between 80 m to 170 m, (Tveit, 2011).

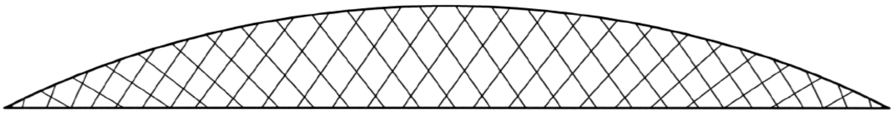


Figure 3 - A network arch bridge with a hanging deck (Tveit 2011)

One of the biggest differences between bridges and other types of buildings is that bridges are subjected to large moving point loads, (in this case traffic loads). These loads will subject the bridge to large bending moments that (due to the shape of the arch) can induce large tension perpendicular to the grain. Since timber is particularly sensitive to these types of stresses this is a big problem.

An arch with tilted hangers has been proven to lessen the moment in the arch which will lessen the tension perpendicular to the grain and will give you a smaller cross-section. It has also been proven to lessen the deflection of the bridge, (Tveit, 2011).

1.2 Purpose / Aim

The purpose of this master thesis is to determine the feasibility of the use of Network Arch design for glue laminated timber bridges with a length of 50 m according to the Eurocode standards.
The purpose is also to determine the importance of the different parametrical values such as the importance of the stiffness of the deck, arch and hangers in a network arch design.

1.3 Method

The parametric study part of this master thesis is realized by studying a series of influence lines that are constructed with the help of a finite element program for three different hanger constellations. The parametric study is made using 2D- models of the bridge.

The parametric study is the foundation for a design of a “Network arch bridge”. To evaluate the success of the design a reference bridge with straight hangers will also be designed.
All of the designs will follow the Eurocode standards (EKS 5) and Trafikverkets implementation for bridges (TRVK Bro 11).

2 Theory

2.1 Glue Laminated Timber - Glulam

The technique of glue laminating timber has made timber constructions much more versatile. The timber elements are no longer limited to the size of the tree which has made timber into a material not just used in smaller projects.

The technique was developed in Germany in the late 1800. The first Swedish glulam beam was constructed in Töreboda in 1912. But it took until the 1960s before the production started to grow. Today we have a production in Sweden of about 200 000 m^3 per year. Of this half is used in Sweden and half is exported. The use of glulam in Sweden has grown by more than 300 % since the 1990s, (Carling, 2008).

There are several advantages with the use of glulam in construction, (Carling, 2008):

- An appealing esthetic
- High strength to weight ratio which enables a big span.
- A high fire resistance.
- Good insulation capacity.
- Low self-weight which lessens the cost of transportation and assembly.
- Good durability in corrosive environments.
- Flexible production enabling different shapes like curved beams.
- Renewable material

2.1.1 Production of glulam

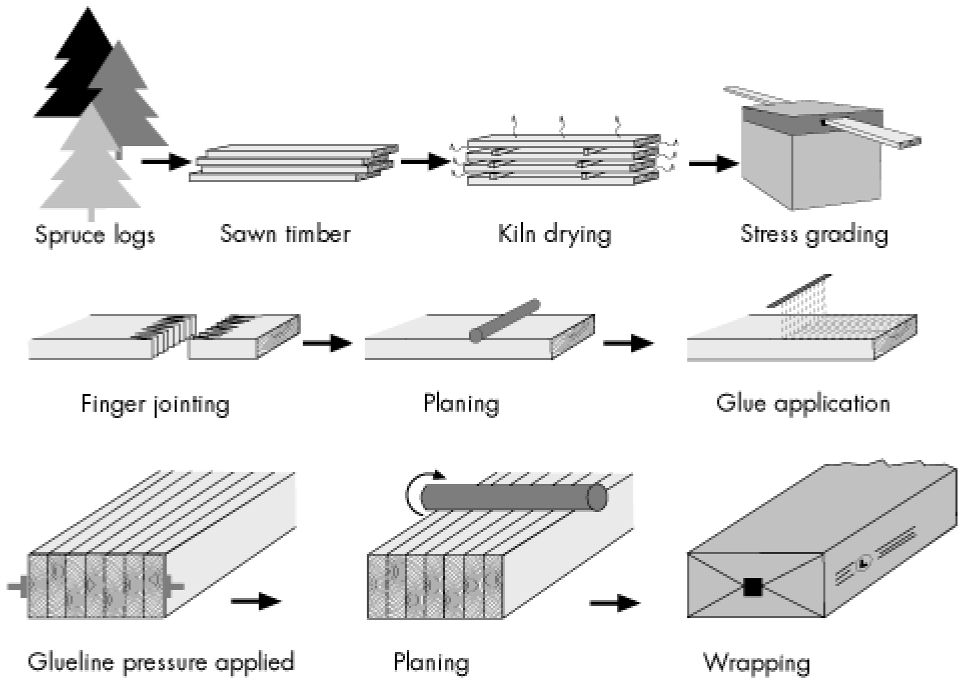


Figure 4 - A schematic figure of the production of a glue laminated timber beam. (Carling, 2008)

Glue laminated timber is produced by gluing several pieces of sawn timber together. By finger jointing together several pieces lengthwise you can get a much longer glulam beam than you could get a normal timber beam. Gluing the finger joints together and making sure that no joints share the same cross-section ensures the strength of the beam. The production sequence is shown in figure 4 above. The limiting factor is the size of the production factory. For example Moelven Töreboda has a maximum length of production of 30-32 m and a max height of the completed beam at 2 m. The normal thickness of each of the lamella is normally 45 mm and the maximum width of the beam is 215 mm. The beam can be made wider by gluing several beams together besides each other. To get a different shape the lamella are glued together and bent in to the desired shape. The beam is pressured together in this shape until the glue has hardened, (Carling, 2008).

2.2 Arch Bridges

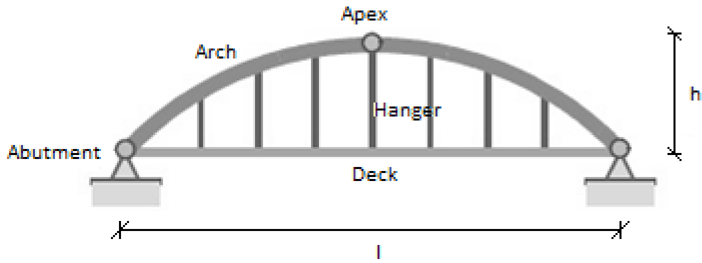


Figure 5 - A tied three hinged arch bridge with a hanging deck

Arch bridges are bridges that are supported by an arch either under or over the bridge deck, see figure 1 and 2.

There are three types of arches: Zero hinged, two hinged and three hinged.
The zero hinged arches are rigidly connected to the abutments, see figure 6. This is the most stable type of arch with the best bending moment distribution, but it is also very sensitive to ground settlements and has high demands on the abutments. The rigid connections between the arch and the ground can also be hard to guarantee. Most zero hinged arches are built in concrete.

Figure 6 - Zero hinged arch

The bending moment distribution in the two hinged arch (figure 7) is not as good as the zero hinged arch but the arch is less sensitive to ground settlements and is easier to construct since it doesn't need the rigid connections between the arch and the abutments. Most two hinged arches are constructed in steel.

Figure 7 - Two hinged arch

The three hinged arch (figure 8) has got the worst bending moment distribution and will be subjected to the largest deflections. On the other hand it is almost insensitive to ground settlements. Most three hinged arches are constructed in timber. Because it is hard to get timber elements that will span the whole length of the arch this is a good way to use the top hinge to join them together without a troublesome connection.

Figure 8 - Three hinged arch

If the arch is subjected to a constant load, than the arch-shape can be designed so that the only forces in the arch will be axial forces. This is regardless to the number of hinges; they will all work just as well.

But since the loading on a bridge is varying it is not possible to design an arch to just be subjected to axial force. Most arches have a parabolic or circular form. If the height to the length ratio (h/l) according to figure 5 is small (less than 0,2) ; the circular and parabolic design will yield similar results. (Lectures by Crocetti, 2011) and (Lorentsen & Sundquist, 1995).

2.3 Network Arch

The biggest difference between a building and a bridge is the loading configuration. In a building most loads are seen as stationary and the building will be designed for these well-known stationary loads. A bridge on the other hand has its critical loads as dynamic loads (traffic loads) this means that in the critical load case the loads can be put anywhere on the bridge. This creates the problem of partial loading.

An arch bridge subjected to a uniformly distributed load works well with normal straight hangers (figure 9). The hangers will distribute the load to the arch and the structure will be subjected to mostly axial forces.

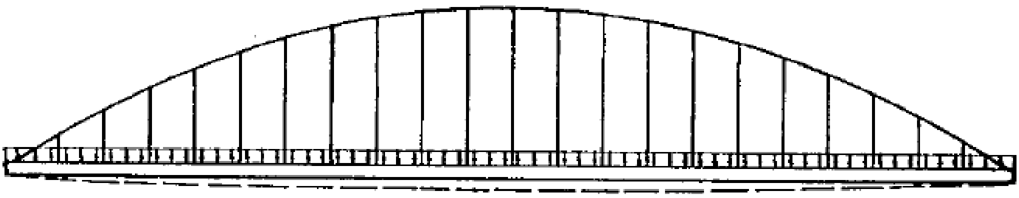


Figure 9 - Arch bridge with straight hangers subjected to a uniformly distibuted load. The arch is subjected to mostly axial forces. The deflection is shown by the dotted line. (Tveit, 2011)

The problem for arch bridges with straight hangers is when the bridge is subjected to a partial loading. This load will make one side of the bridge deflect more than the other creating an uneven deflection of the arch which increases the deflection of the bridge and induces a sideways motion of the arch (see figure 10).

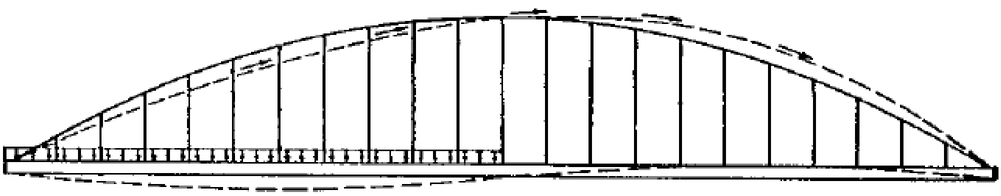


Figure 10 - Arch bridge with straight hangers subjected to a partial load. The deflection is shown by the dotted line. The partial load induces a sideways motion of the arch. (Tveit, 2011)

To accommodate this problem it was suggested to build a bridge were the hangers were inclined to resist the sideways movement of the arch, (figure 11). This type of bridge is called a Nielsen Bridge after the designer Octavius F. Nielsen who built around 60 of this type of bridges in Sweden between the two world wars.

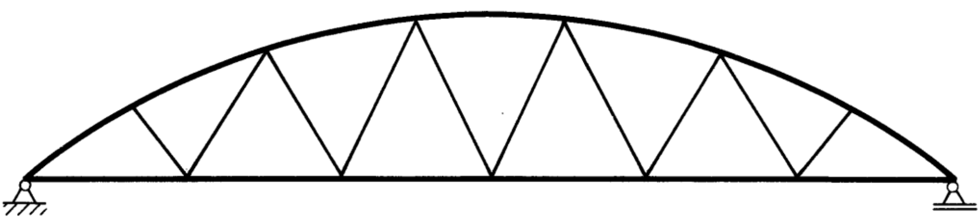


Figure 11 - Nielsen Bridge. A bridge built with inclined hangers to resist the sideways deflection due to partial loading. (Tveit, 2011)

This bridge was built to have some of its hangers to relax when subjected to a large partial load. A way to decrease the risk of relaxation was to increase the distance between the nodal points but this would also increase the loading in each of the points. Nielsen realized that by putting in a second set of hangers (figure 12) he could increase the distance between the nodal points for each set without increasing the loading in the nodal points. Even if he designed and patented the first arch bridge with crossing hangers it was never built.

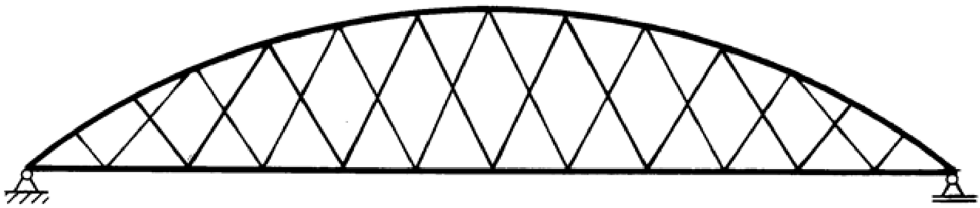


Figure 12 - An arch bridge with two overlapping sets of hangers crossing each other. Two sets of hangers decreases the risk of relaxation without increasing the distance between the nodal points. (Tveit, 2011)

Per Tveit, professor emeritus at Agders University in Norway, wrote in 1955 his master thesis on arch bridges with three or more sets of inclined hangers, The Network Arch, which led to a lifelong dedication. He has devoted his entire career to the development of the bridge type and is still very active in his quest to make the network arch known throughout the industry.

The definition of a Network arch is according to Tveit:"A network arch bridge is an arch bridge where some of the hangers cross other hangers at least twice." (Tveit, 2011).

After 50 years of development Tveit has come up with some suggestions to design the optimal network bridge:
The deck should be made out of concrete to give the necessary dead load to lessen the risk of relaxing hangers. The tie should be imbedded in the deck in the form of prestressing cables; this will decrease the tendency for the concrete to crack and thus increase the durability.
The arch should be made as part of a circle; this will give a more evenly distributed bending moment in the cords and will make the production easier.
The upper node of the hangers should be placed equidistantly along the arch and the hangers should not be merged in the nodal points; this gives a better support in the in plane buckling and evens out the bending moment. This gives an appearance that look like figure 3. All hangers should have the same cross-section. The maximum bending in the deck will be found in the middle of the slab between the arches. If the deck is wider than 10 m transverse prestressing cables should be considered. (Tveit, 2011)

Tveit has only worked with two hinged steel arches and says that the optimal network arch bridge is obtained using universal columns as arches. But he also says that the use of arches made up of box-sections will increase the number of hangers that will be allowed to relax.

Tveit has designed two Network arch bridges that have been built, both in Norway: the Steikjer bridge and the Bolstadstraumen bridge both built in 1963. They are both still in great working condition.

2.4 Influence Lines

The use of influence lines is a method mainly used in bridge design to determine the load placement of moving loads to achieve a critical load condition. The influence line is a graph that shows the influence of a load placement for a specific point.

The easiest way to achieve an influence line is to move an arbitrary point-load along the path that you’re interested in finding the load-placement for and calculate the effect for the point that you’re interested in for a number of distances along this path. Then put your values in a graph with the distance from the left support on the x-axis and the effect on the y-axis. For easier structures it is possible to derive an equation for the influence line, but for more complex structures calculations have to be made by a computer based finite element program.

Influence lines aren’t just used to determine a critical load condition; they can also be used to determine a critical point by comparing influence lines from several points with each other.

2.4.1 Example - Influence Line

To clarify how to achieve an influence line and the importance of an influence line an example has been constructed. The example will be using a normal 50 m long, three hinged arch bridge with a hanging deck, (figure 13).

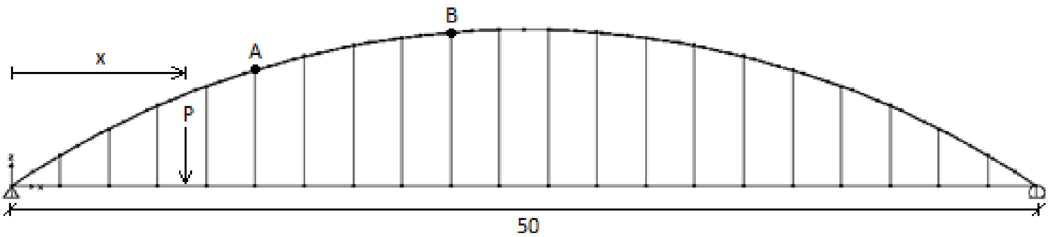


Figure 13 - A 50 m long three hinged arch bridge with a hanging deck and straight hangers. The point-load P is moved along the deck and the effect in point A and B is calculated for every meter.

The maximum moment for a normal arch is situated around the ¼ - point, (Lorentsen & Sundquist, 1995). In this example that is around 12,5 m from the support and has been named point A. The calculations have been made using the finite element program SAP2000 v.15 and have been made for every meter of the deck. The point-load P has been chosen as 1 kN. The data have been compiled using an excel graph, (figure 14).

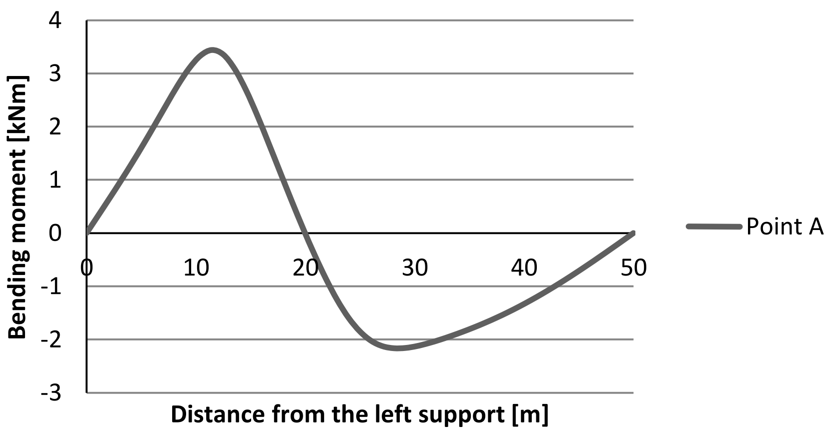


Figure 14 - The influence line for bending moment for point A using a 1 kN point-load.

This graph shows that positive bending moment in point A is achieved for loads placed on the first 20 m from the left support. This means that a greater positive bending moment is achieved by loading only the first 20 m of the bridge than would have been achieved if the whole bridge was loaded. Influence lines can also be used to achieve an approximate bending moment in point A for any given loads.

A 10 kN point-load 10 m from the left support would give you a bending moment in point A of about 30 kNm. Looking at the influence line at 10 m we can see that our original 1 kN point-load gave an effect of approximately 3 kNm and a 10 times bigger point load will give us a 10 time bigger result. To achieve the bending moment due to a distributed load you multiply the area under the graph with the distributed load.

Traffic loading uses a big point-load and a distributed load that can be distributed in any way. To achieve the maximum positive bending moment in this example we would put the point-load at approximately 12 m from the left support and the distributed load between 0 – 20 m from the left support.

Influence lines can also be used to compare different points with each other to for example find the most critical point. In this example we will look at which point A or B is the critical point for bending moment. According to the theory of arches it should be point A.

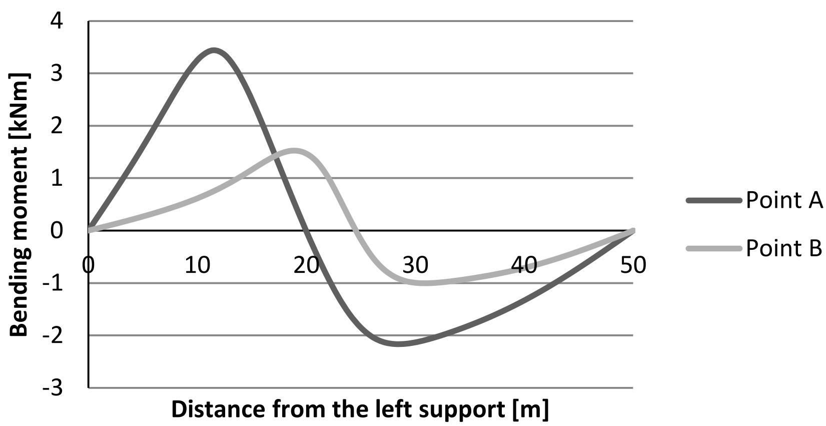


Figure 15 - The influence line for bending moment for point A and B using a 1 kN point-load.

Figure 15 shows, just as the theory suggested, that point A is the critical point for bending moment. This is certain since it has both a higher max-value and a bigger area under the graph.

2.5 Hanger relaxation

One of the biggest problems with arch bridges with inclined hangers is hanger relaxation. This results in some of the hangers being unable to take any loads.

Even though the inclined hangers restrain the movement of the arch due to partial loading it will not completely prevent it. This small sideways movement of the arch will cause the distance between the nodes for a hanger inclined “in the opposite direction” to decrease, see figure 16. The decreased distance between the nodes makes the hanger “to long” to be able to take any loads.

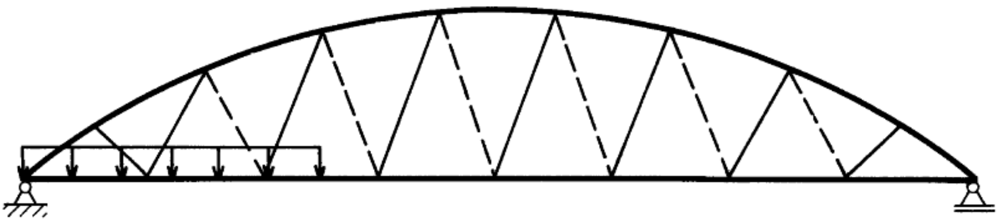
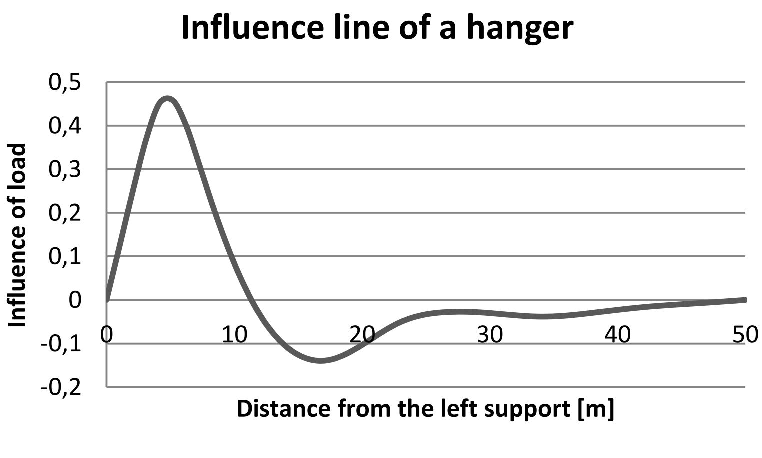


Figure 16 - Relaxation of hangers due to partial loading. The dotted hangers may relax.

According to Tveit, the risk of relaxation is directly linked to the angle of the hangers. The more horizontal the hangers are the smaller the bending moment in the arch will be but it will increase the risk of relaxation. In other words small inclination of the hanger results in great bending moment and low risk of relaxation (strait hangers), big inclination of the hangers results in small bending moment in the arch but a greater risk of hanger relaxations.

To lessen the risk of relaxation one can use bigger permanent loads. The permanent loads will act as a prestress for the hangers lessening the risk for relaxation. Unfortunately it is not just the permanent load that decides the risk of relaxation. The risk of relaxation is decided by the permanent load compared with the partial load. If you have a big partial load this will increase the risk of relaxation. This can be shown by looking at this influence line for an arbitrary hanger, (figure 17).



Graph 1 - A load on the positive part of the graph (approximately 0-10 m) will cause tension in the hanger while loading on the negative side of the hanger will cause relaxation. It is the combined load that decides if the hangers relax or not. The scale on the y-axis indicates the magnitude of the influence.

2.5.1 Problems with relaxation

The problems with relaxing hangers are, (Tveit, 2011):

- Since the relaxed hanger is unable to take any loads an increased load will occur on surrounding hangers.
- If the hangers are not merged into the same nodes the relaxation of a hanger will increase the distance between supporting hanger nodes for the deck.
- An increased risk for problems related to fatigue for the hanger connections.
- Esthetically not pleasing if the hangers relax and start to bend due to the relaxation.

3 SAP2000 v.15

3.1 Theory

Finite element programs are used to analyze complex structures. They use the relation in stiffness to determine the distribution of forces in the object. This means that you can put a load on a finite model and it will provide you with the bending moment, axial force, etcetera.

SAP2000 is a general purpose structural finite element program using a SAP engine. SAP was first developed in 1975. SAP2000 is a made to be able to analyze most structural cases from a simple small 2D static frame analysis to a large complex 3D nonlinear dynamic analysis.

For this master thesis the most interesting aspect of this program (except to provide force – reactions) is its capability to perform an influence line calculation and buckling analysis.

SAP2000 is using a P-delta effect analysis (second order analysis) when analyzing nonlinear cases.

To calculate influence lines the program uses two different methods. The first is a built-in method of constructing influence lines. This can only be used to construct linear influence lines. To construct nonlinear influence lines it uses a time histology method, where it moves a load in steps over the bridge over a determined time and calculates a nonlinear equation for each step. This is a very time-consuming method.

3.2 Limitations

The program also has a couple of limitations. The first one is its ability to define different materials. SAP2000 has got predefined materials like steel and concrete but it doesn't have any timber. When defining a new material you can only define the modulus of elasticity and not its strength capabilities.

When using influence lines the program only has the ability to follow a simple frame as a path and not to use an area as a path. This limits you to only calculate 2D influence lines.

The program can't calculate the fact that cables or frames can't take any compressive force in a static linear model. To take this into consideration you have to use a static nonlinear model.

4 Assumptions and Limitations

Since it would be impossible to take all different parameters into consideration and still do a comparison between the network arch and a normal arch bridge some assumptions and limitations had to be made.

The assumptions on the bridge appearance have been made according to some of the recommendations made by Tveit.

The length of the bridge has been chosen to be 50 m and the width has been chosen to be 10 m. This means that no transverse prestressing is needed.

The arch will be made as part of a circle. Tveit recommends the height to length ratio of the arch be between 0,10 – 0,18 but preferably between 0,15 – 0,16. (Tveit, 2011). To get a feasible radius that can be produced and ease the calculations later on the following calculation is performed:

$$\frac{h}{l} = 0{,}15\,, l = 50 \ \rightarrow h = 7{,}5 \rightarrow R \approx 45\,m.$$

$$R = \frac{l^2 + 4h^2}{8h}, \qquad R = 45, \qquad l = 50 \rightarrow h = 7{,}5834\,m.$$

$$\frac{h}{l} = \frac{7{,}5834}{50} = 0{,}1517, \qquad 0{,}15 < 0{,}1517 < 0{,}16 \rightarrow OK!$$

The radius of the arch is chosen to be 45 m. (Annotations according to figure 5).

This thesis will concentrate on the design of the arch and the hangers. This means the deck will not be designed. Since the deck mainly carries in the transverse plan a similar deck can be found that is already designed in other reports. The deck chosen in this report is inspired by the decks designed in (Tveit, 2011), (Brunn & Schanack, 2003) and (Teich & Wendelin, 2001). The deck uses prestressed edge beams as the ties of the arch. The stiffness of the prestressed edge beam is assumed to emulate the decks capacity to distribute the loads in the longitudinal direction. An illustration of the deck can be seen in appendix A.

The hangers have been chosen to be steel rods. The Swedish steel distributor Pretec has been contacted to make the rods more realistic. The hangers have been chosen to be ASDO hangers, a new hanger system developed by Pretec. Pretec has provided design limitations and steel details which can be seen at (Pretec, 2011). The hangers will be assumed to not be able to take any compression forces.

There are several different ways to set the hangers. You can use the same angle for each hanger or use a constant or parabolic change of the hanger's angles. The most commonly used hanger constellation and the most esthetically pleasing is according to Tveit to have the same angle on all the hangers. The hangers have therefore been chosen to have the same angle. This also makes it easier to compare the difference between different hanger angles.

The hangers will be placed equidistantly along the arch. The distance between the hangers should according to Tveit be between 2,5 – 4 m, depending on the length of the bridge. The number of hangers has to be an even number to make sure the bridge will be balanced. Since the bridge in this thesis is a relatively short bridge compared with the bridges designed by Tveit a shorter distance between the hangers was chosen. This led to the number of hangers used to be 20, which means the distance between the hangers is 2,52 m.

To determine the importance of the hanger inclination three different hanger angles will be looked into: 45, 55 and 60 degrees. The angle is the angle between the hanger and the deck where 90° is a normal straight hanger and 0 ° is a horizontal hanger. The angle can't be bigger than 60° because it then no longer applies to Tviet's definition of a network arch bridge.

The material in the arches, deck and hangers will not be changed during this study.
The arch will be made up of glulam (GL32c), because it is a stronger standard glulam that most manufacturers can produce.
The deck will be made up by C40/50 to follow the recommendation of Tveit and be as similar to the decks in (Tveit, 2011), (Brunn & Schanack, 2003) and (Teich & Wendelin, 2001) as possible.
The hangers will be made up of steel 520-S which is the steel recommended by Pretec and the standard steel for the ASDO hanger system.

The arch will for practical reasons be made as a three hinged arch. Since the full length of the arch is approximately 53 m it is not possible to be produced as a single element. It would also be a problem to transport the arch if you could produce it. The arch will because of those reasons be produced and transported in halves. To achieve a rigid connection between the arch halves on site (to make it in to a two hinged arch) is a difficult procedure and it is hard to foresee the behavior of the connection. For this reason the arch will be hinged in the apex and will work as a three hinged arch.

The only loads that will be considered in this study are the self-weight of the bridge and traffic loading. This study will not consider wind-loading because it does not have a large impact on the arches or the hangers, (Crocetti, 2012). Since snow-loading can't operate at the same time as traffic loading it will not be considered either.

5 Codes and regulations

During this thesis the Eurocodes with Swedish applications will be applied.
The following parts will be used:

EC0: Eurocode 0, Basis of structural design (SS-EN 1990)

EC1-1-7: Eurocode 1, Part 1 - 7, Accidental action (SS-EN 1991-1-7)

EC1-2: Eurocode 1, Part 2, Traffic loads on bridges, (SS-EN 1991-2)

EC2-1-1: Eurocode 2, Part 1-1, Design of concrete structures (SS-EN 1992-1-1)

EC3-1-1: Eurocode 3, Part 1-1, Design of steel structures (SS-EN 1993-1-1)

EC5-1-1: Eurocode 5, Part 1-1, Design of timber structures (SS-EN 1995-1-1)

EC5-2: Eurocode 5, Part 2, Design of timber structures - Bridges (SS-EN 1995-2)

Trafikverkets implementations for bridges (TVRK Bro 11) will also be used.

6 Material

6.1 Glulam

The arches will be made up of GL32c.

$$X_d = k_{mod} \cdot \frac{X_k}{\gamma_M} \tag{1}$$

$\gamma_M = 1{,}25$ according to table 2.3 in SS-EN 1995.

Climate class has been decided to class 3.

$k_{mod} = 0{,}7$ according to table 3.1 in SS-EN 1995.

$$k_h = min \begin{cases} \left(\frac{600}{h}\right)^{0,1} \\ 1{,}1 \end{cases} \tag{2}$$

Since the height of the arches will never be less than 600 mm. $k_h = 1$.

Table 1 – Characteristic and design values for GL32c, according to SS-EN 1995

Glulam GL32c	Characteristic value [MPa]	Design value [MPa]
Bending	$f_{mk} = 32$	$f_{md} = 17{,}92$
Tension parallel to the fibers	$f_{tk} = 19{,}5$	$f_{td} = 10{,}92$
Tension perpendicular to the fibers	$f_{t90k} = 0{,}45$	$f_{t90d} = 0{,}252$
Compression parallel to the fibers	$f_{ck} = 26{,}5$	$f_{cd} = 14{,}84$
Compression perpendicular to the fibers	$f_{c90k} = 3$	$f_{c90k} = 0{,}168$
Shearing	$f_{vk} = 3{,}2$	$f_{vd} = 1{,}792$
Modulus of elasticity	$E_{0,05} = 11100$	
Weight	4 kN/m^3	

6.2 Concrete

The concrete deck will be made up of C40/50 concrete.

Compressive strength: $$f_{cd} = \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_C} \tag{3}$$

Tensile strength: $$f_{ctd} = \alpha_{ct} \cdot \frac{f_{ctk0,05}}{\gamma_C} \tag{4}$$

$\alpha_{cc} = \alpha_{ct} = 1, \gamma_C = 1{,}5$ according to (Isaksson & Mårtensson, 2008).

Table 2 - Characteristic and design values for C40/50, according to SS-EN 1992

C40/50	[MPa]
Characteristic value, compressive	$f_{ck} = 40$
Characteristic value, compressive, (cube)	$f_{ck,cube} = 50$
Characteristic value, tensile	$f_{ctk0,05} = 2{,}5$
Design value, compressive	$f_{cd} = 26{,}67$
Design value, tensile	$f_{ctd} = 1{,}67$
Modulus of elasticity	$E_{0,05} = 35000$
Weight	24 kN/m^3

6.3 Steel

The hangers will be made out of steel rods according to the ASDO tension rod system.

The technical data of the steel has been gained by Hagemann, engineer and technical advisor for the ASDO tension rod system.

$N_{Rd,\ Tension\ Rod}$ =	**min {A · $f_{y,k}/\gamma_{M1}$; 0.9 · A_S · $f_{u,k}/\gamma_{M2}$}**
A =	minimum cross section of the unthreaded part of the tension rod
A_S =	cross section of the threaded part of the tension rod
$f_{y,k}$ =	characteristic value of the yield strength of the tension rod material according to $R_{p0,2}$ given in Annex 2
$f_{u,k}$ =	characteristic value of the tensile strength of the tension rod material according to R_m given in Annex 2

Figure 17 – The design calculation for the ASDO tension rods. An extract from the ETA (European Technical Approval) for the ASDO system.

The specification of the ASDO 520-S steel also known as S550 steel is according to (Hagemann, 2012):

Table 3 – Yield stress and tensile strength of the ASDO 520-S steel according to Hagemann.

Thread size / nominal diameter	Yield stress ($R_{0,2}$) [MPa]	Tensile strength (R) [MPa]
$\leq$ **M85 / ø85**	$f_y = 540$	$f_u = 700$
$>$ ***M*85 / ø85**	$f_y = 520$	$f_u = 700$

Modulus of elasticity, $E_{0,05} = 210\ 000\ MPa$

The modulus of elasticity will not decrease due to the angle of the hangers according to (Asp, 2012).

$$E_{eq} = \frac{E}{1+\left(\frac{\gamma l^2}{12\sigma^3}\right)E} \approx E \qquad (5)$$

According to Pretec's calculations in (Pretec, 2011) the design strength of the hangers available in the ASDO system between 30 – 60 mm is as follows:

Table 4 - The design strength of the hangers available in the ASDO system between 30-60 mm.

Dimension [mm]	Design strength [kN]
M30 / ø30	303
M36 / ø36	441
M42 / ø42	605
M45 / ø45	705
M48 / ø48	795
M52 / ø52	950
M56 / ø56	1096
M60 / ø60	1276

7 Load cases and loads

7.1 Load Cases

The load case that has been used is defined in SS-EN 1990. Two different load cases were looked into for the ultimate limit state, Load case 6.10a and 6.10b.

Load case 6.10a:

$$E_d = \gamma_d 1{,}35G + \sum \gamma_d 1{,}5\psi_{0,i} Q_{0,i} \tag{6}$$

Load case 6.10b:

$$E_d = \gamma_d 1{,}2G + \gamma_d 1{,}5Q_1 + \sum \gamma_d 1{,}5\psi_{0,i} Q_{0,i} \tag{7}$$

For the cases with accidental loading the load cases for exceptional design situations has been used (Load case 6.11a).

Load case 6.11a:

$$E_d = \gamma_d G + \gamma_d A + \sum \gamma_d \psi_{0,i} Q_{0,i} \tag{8}$$

7.2 Loads

The arches will be subjected to permanent and traffic loading. No other loads are relevant in this study.

7.2.1 Permanent loads

The decks permanent load consists of the self-weight of the concrete deck, a 50 mm thick layer of asphalt, safety railings and hangers.

The cross-section of the concrete deck has been calculated using the area of the deck shown in Appendix A.
$A_{deck} = 3{,}35\ m^2$

The weight of the safety railings and the hangers is assumed to be 2 kN/m per side of the bridge.
$G_{railing} = 2\ kN/m$

The asphalt is assumed to just cover the 8 m wide carriageway.
$A_{Asphalt} = 0{,}05 \cdot 8 = 0{,}4\ m^2$

$$G = A_{deck} \cdot \gamma_{concrete} + A_{Asphalt} \cdot \gamma_{Asphalt} + 2 \cdot G_{railing} \tag{9}$$

$$G = 3{,}35 \cdot 24 + 0{,}4 \cdot 23 + 2 \cdot 2 = 93{,}6\ kN/m \tag{10}$$

The permanent loads will be distributed equally to both arches.

$$G = \frac{93{,}6}{2} = 46{,}8\ kN/m\ per\ arch \tag{11}$$

7.2.2 Traffic loading

In the traffic loading the only consideration has been taken to load model 1 according to SS-EN 1991-2. Load model 1 comprises of two parts: one point-load part to simulate a heavy vehicle and one distributed part to simulate busy traffic.

The carriageway is divided into smaller notional lanes with a width of 3 m, and one for the remaining width according to table 4.1 in SS-EN 1991-2. In this study we have an 8 m wide carriageway that will be divided into two 3 m and one 2 m traffic lane, (figure 18).

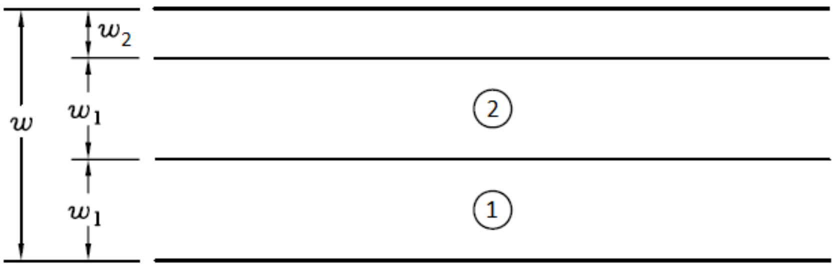


Figure 18 - The carriageway is divided into smaller notional lanes of 3 m in width and one for the remaining width. In this study there will be 2 notional lanes and the remaining width is 2 m.

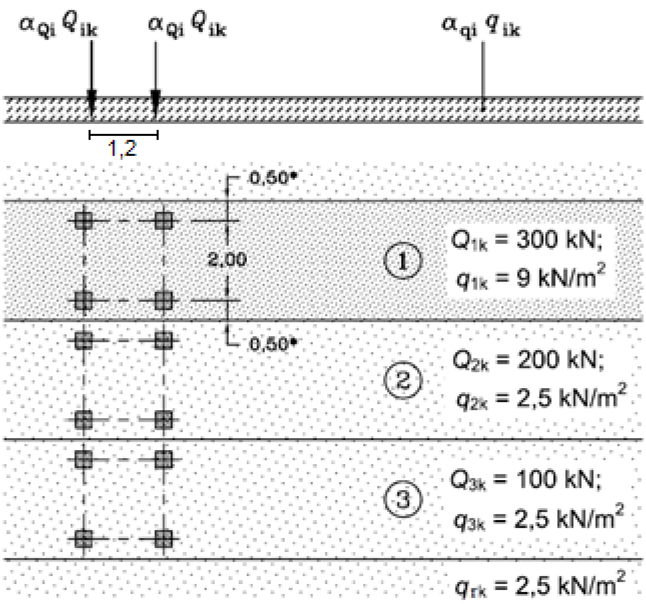


Figure 19 - Load placement according to traffic loading load model 1. The distance between the axel loads is 1,2 m. According to figure 4.2a in SS-EN 1991-2.

Table 5 - Conversion factors for the traffic loads recommended in the Swedish national annex

Conversion factors	
α_{Q1}	0,9
α_{Q2}	0,9
α_{q1}	0,7
α_{q2}	1,0
α_{qr}	1,0

In this study only two notional lanes and the remaining area are used. The characteristic loads are multiplied with the conversion factors. The axel loads are divided into wheel loads.

Table 6 - Design loads according to load model 1 and the Swedish national annex

	Lane 1	Lane 2	Remaining area
Axel load	270 kN	180 kN	-
Wheel load	135 kN	90 kN	-
Distributed load	6,3 kN/m	2,5 kN/m	2,5 kN/m

The loads have then been calculated to decide the girder distribution of the loads. This will give us the maximum loaded arch (the designing arch) which will be used in the 2D-models. For full calculations see equations 46 - 47 in section 10.1.

The arch that was subjected to the maximum load due to axel loads was subjected to two point loads 1,2 m apart of 286,6 kN each. Since the full length of the bridge is 50 m this distance apart is very small. To simplify the calculations and the modeling these loads are assumed as one point-load of 573,2 kN.

For the distributed load the girder was subjected to a distributed load of 19 kN/m.

Table 7 - The designing loads due to traffic load model 1 for the maximum loaded side used during the 2D-analysis.

Design loads for 2D-analysis	
Point load (Q)	573,2 kN
Distributed load (q)	19 kN/m
Permanent load (G)	46,8 kN/m

8 Parametric Study

In this section of the thesis the parameters of the constructed network arch will be studied. The main parameters are the stiffness of the arch, hangers and deck and the inclination of the hangers.

The tests will be performed on four different sets of hangers: three inclined with an inclination of 45, 55 and 60 degrees, and one with vertical hangers as a reference, (figures 20 - 23). The hanger inclination is the angle between the hanger and the deck.

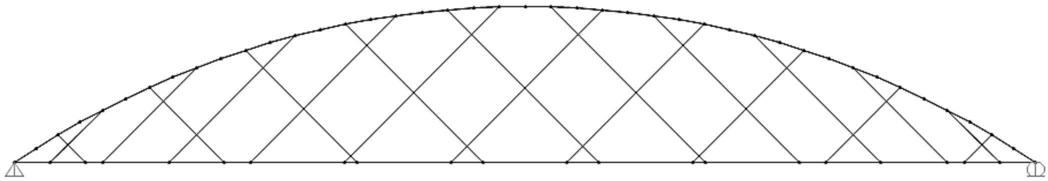


Figure 20 - A three hinged network arch with a hanger inclination of 45 degrees used in the parametric study. (For a more detailed picture see appendix B.)

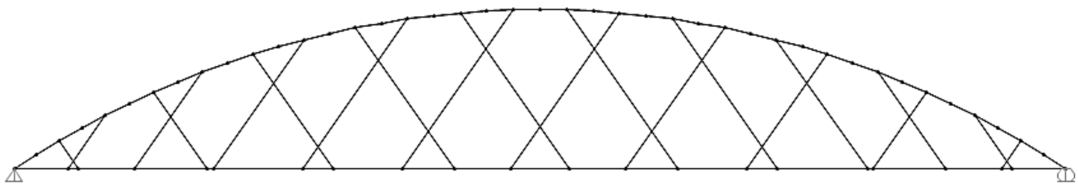


Figure 21 - A three hinged network arch with a hanger inclination of 55 degrees used in the parametric study. (For a more detailed picture see appendix C.)

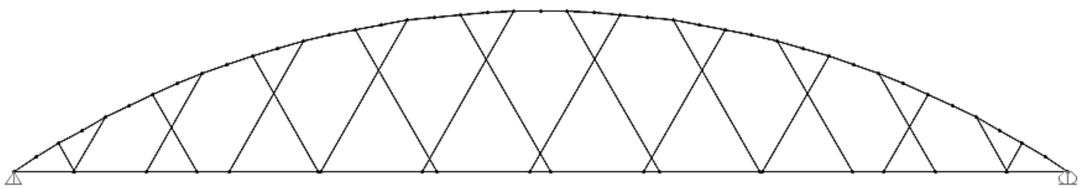


Figure 22 - A three hinged network arch with a hanger inclination of 60 degrees used in the parametric study. (For a more detailed picture see appendix D.)

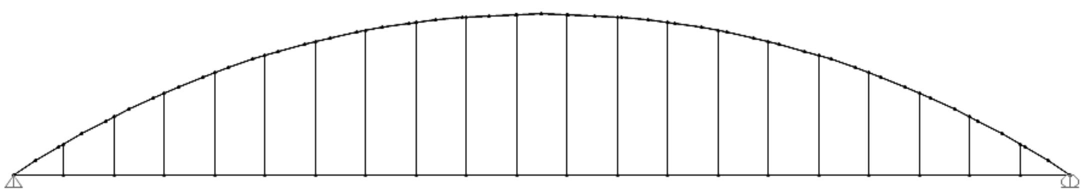


Figure 23 - A three hinged arch bridge with straight hangers used as a reference during this study. (For a more detailed picture see appendix E.)

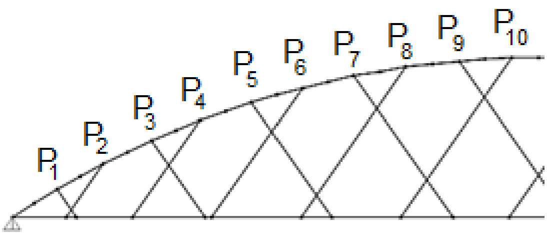


Figure 24 - The annotations used will follow this model, Px is the point of the arch where a hanger connects. Hangers will be similarly named after their connection point (Hx where x is the conection point).

The annotations used will follow figure 24 regardless of the angle of the hangars, the reference bridge will also follow this pattern.

The tests will be done with the use of influence lines using a finite element program called SAP2000. This program can only use influence lines in 2D-modeling so the study will be done using a 2D-model.

The theoretical influence line for a three hinged arch is described by the formula below.

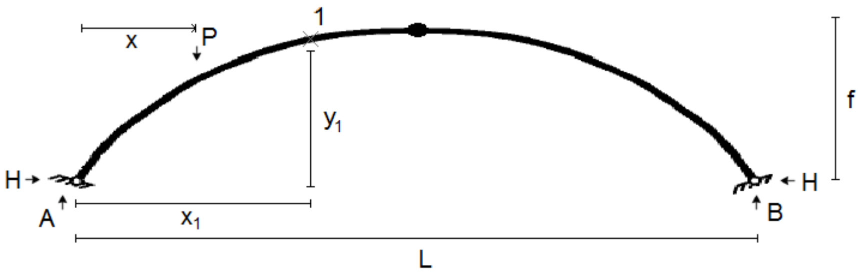


Figure 25 - Setup for calculation of the theoretical influence line.

Moment around A:

$$H = \frac{1}{2}P \cdot \frac{x}{f} \tag{12}$$

$$B \cdot L = P \cdot x \rightarrow B = \frac{P \cdot x}{L} \tag{13}$$

$$A = P - B \tag{14}$$

$$M_1 = B \cdot (L - x_1) - H \cdot y_1 \qquad if\ x \leq x_1 \tag{15}$$

$$M_1 = A \cdot x_1 - H \cdot y_1 \qquad if\ x \geq x_1 \tag{16}$$

Theoretical influence lines (figure 26 – 27) have been calculated with the use of Matlab. The shape of the arch is the shape that has been used during the study, L = 50 m and f = 7,5834 m. One calculation has been made for every mm of the arch (50 000 calculations) to achieve an accurate result. In the calculations the load P was set to 1.

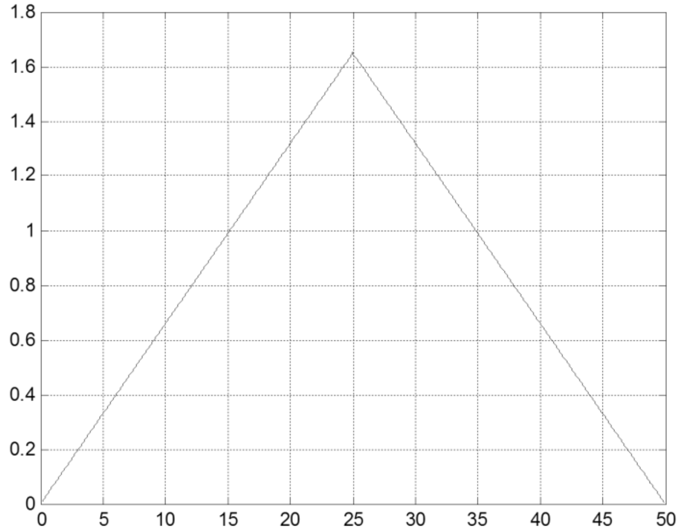


Figure 26 - Influence line for the H-force of a three hinged arch.

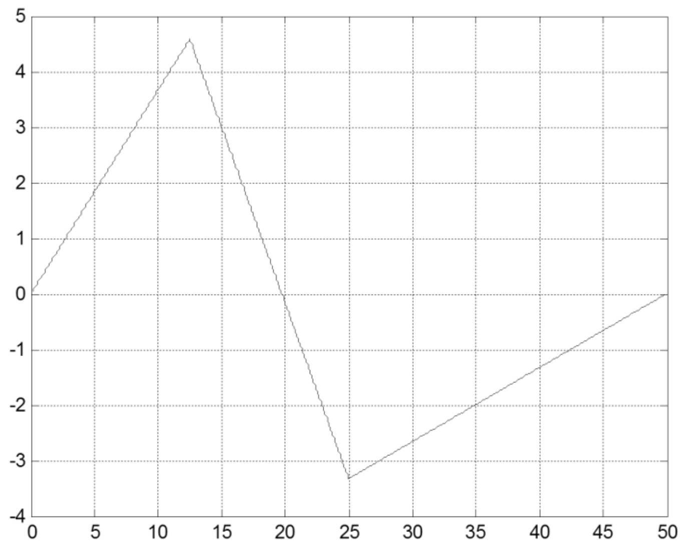


Figure 27 - Influence line for the bending moment in the ¼-point of a three hinged arch.

An influence line has been assembled for the reference bridge with straight hangers for the critical point for bending moment (the ¼ -point) and can be seen in figure 28. This is done to prove that it follows the theoretical influence lines.

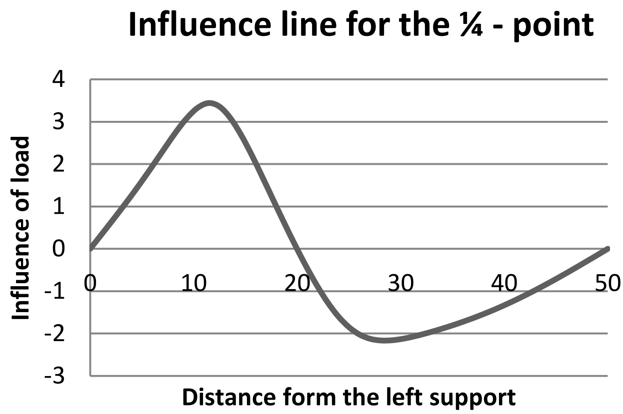


Figure 28 - Influence line for critical bending moment for the reference bridge with straight hangers. Compared with graph 5, it shows great similarities between the influence lines.

By comparing figure 27 and 28 it is clear that the influence line of a bridge with straight hangers is similar the theoretical one. The difference in appearance is due to the smoothing effect of the deck. Since the loads in the bridge are placed on the deck and not the arch directly, the stiffness of the deck will distribute the loads over several hangers, thus smoothing out the loads.

8.1 Linear or nonlinear influence lines

To be able to do this study of the network arch bridges we have to start by making an assumption. Since it is impossible to construct influence lines where a random hanger relaxes we have to assume that the bridge will behave like if no hanger will relax. This will be proven by designing two influence lines. One where the hangers relax at will due to the single point load in the calculation of the influence line (a nonlinear calculation, see figure 29). This calculation does not take into consideration the positive prestressing effect of the permanent loads. In the other the hangers are assumed to be able to take compressive forces just like if they were prestressed (a linear calculation, see figure 30).

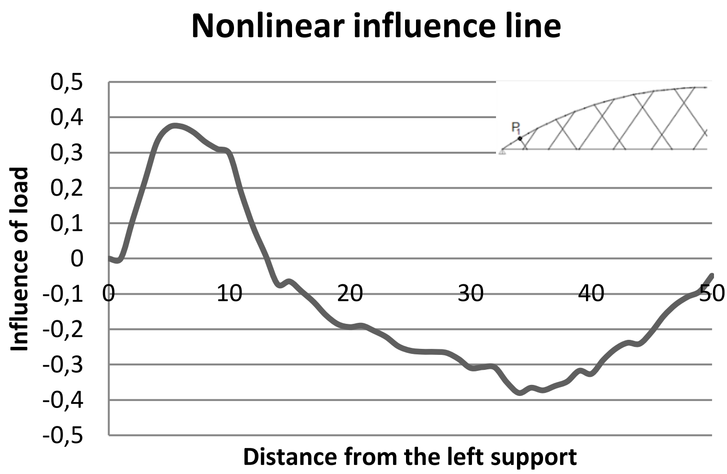


Figure 29 - A nonlinear influence line for the bending moment in P1 in the arch of a network arch. Showing the influence line if we assume that all hangers can relax due to the point load used to calculate the influence line, (no positive prestress due to the permanent loads).

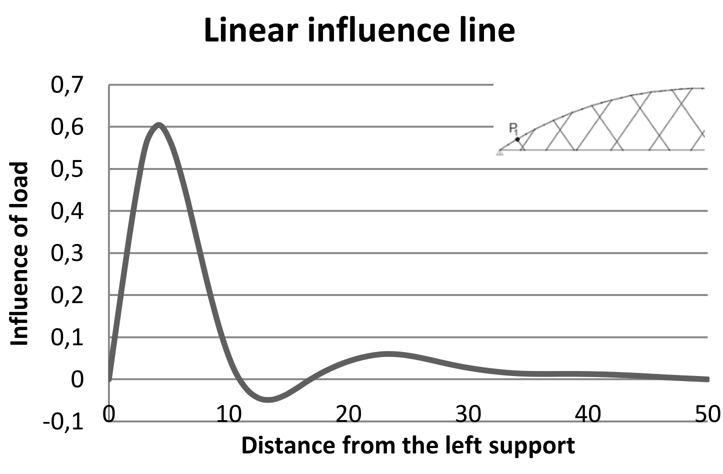


Figure 30 - A linear influence line for the bending moment in P1 in the arch of a network arch. Showing the influence line if we assume that no hangers will relax due to the point load used to calculate the influence line, (the positive prestress due to the permanent loads is big enough to ensure that no hangers relax).

These influence lines have been calculated using the model for a network arch with 45 degrees inclination of the hangers. The influence line is for bending moment in the arch where the first hanger is connected. Similar studies have been made for the 55 and 60 degrees network arches showing the same result.

To decide which of the influence lines will give the most correct result compared with the reality a small test was done. By loading the SAP2000 model according to the two different influence lines with the goal to get the biggest positive bending moment, the results could be compared and the load placement that gave the biggest moment was the one that best reflected the reality. To accommodate for all of the structures complex behavior the load calculation is done using a nonlinear calculation.

The results showed that the linear influence line gave the best result. In his study (Tveit, 2011) Tveit argues for the use of this principle as well.

To prove the study, the same method was used on the reference bridge. Since this bridge should not have a problem with relaxing hangers the two influence lines should be very similar to each other.

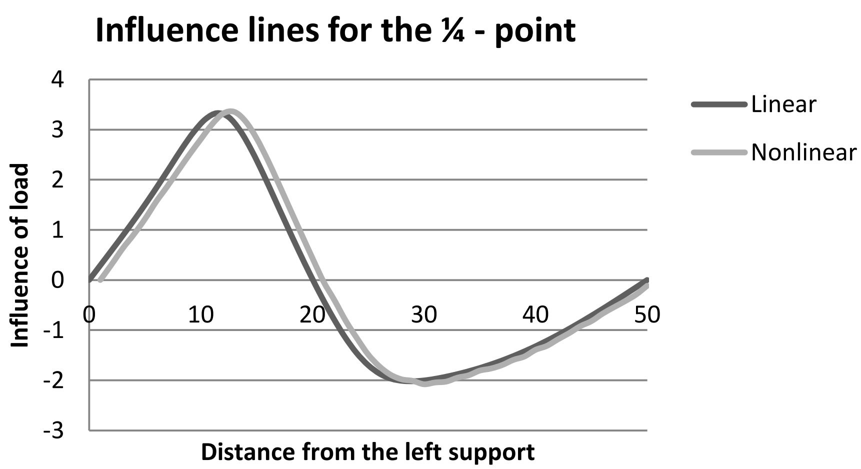


Figure 31 - Difference between linear and nonlinear influence lines for an arch bridge with vertical hangers. (The small difference is probably due to the different calculation methods).

As predicted figure 31 shows that the influence line of a bridge with vertical hangers does not differ due to the linear or nonlinear method of calculation.

In the future of this study all influence lines will be calculated using a linear calculation. When calculating the effects of loads on the actual bridge a nonlinear model will be used. This is done to take in to consideration the second order effects and the non-compressive capabilities of the hangers.

8.2 Critical section and effect of hanger stiffness

The next step in the study was to decide the critical point for the maximum bending moment. This study proved to be interlinked with the study of the stiffness of the hangers.

By studying the bending moment distribution of the arch when subjected to the permanent loads and comparing it to the distribution for the reference bridge it could be decided that the critical section probably would not be at the same place.

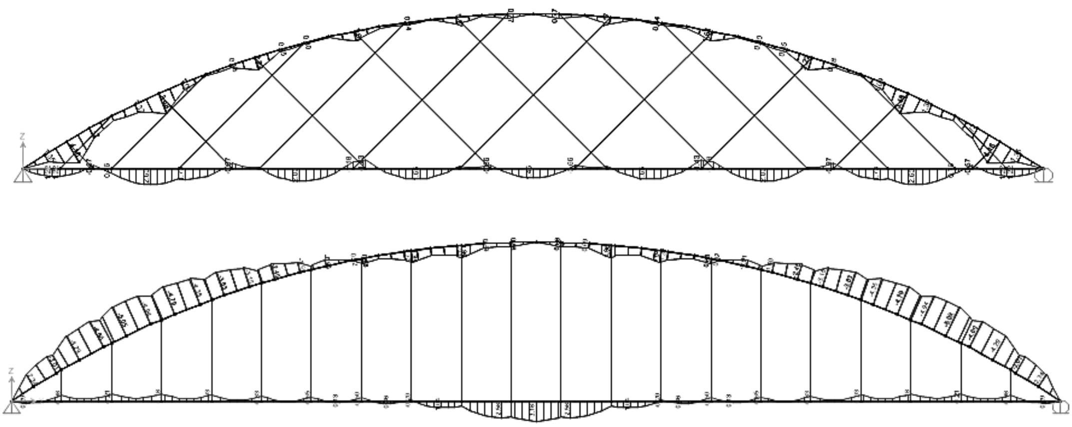


Figure 32 - The moment distribution in the network arch and the reference bridge when subjected to a unifomly distributed load.

As seen in figure 32 the critical section for bending moment for the network arch would probably be at the connection with one of the hangers. Influence lines were constructed for each of the connection points. The influence lines were collected in two groups; one group for the hangers inclined towards the middle of the bridge (P_1, P_3, P_5 and P_7) and one group for the hangers inclined in the other direction (P_2, P_4 and P_6). This was done to make it easier to compare them among themselves.

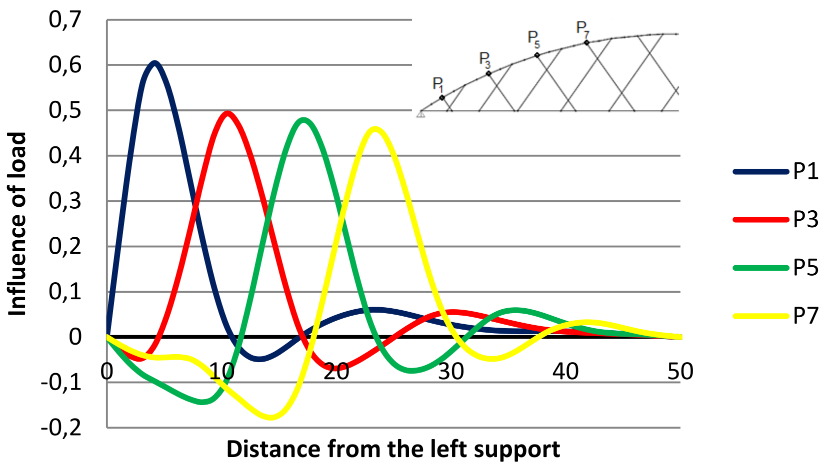


Figure 33 - Comparison between influence lines to decide the critical section of the arch for bending moment.

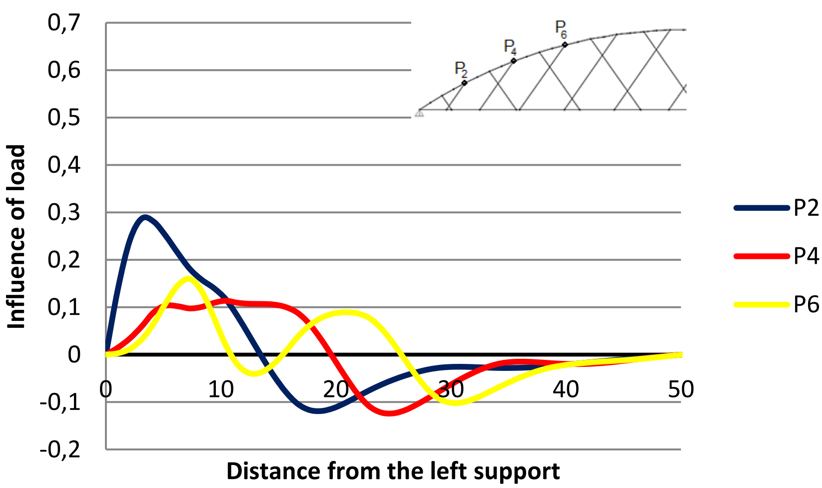


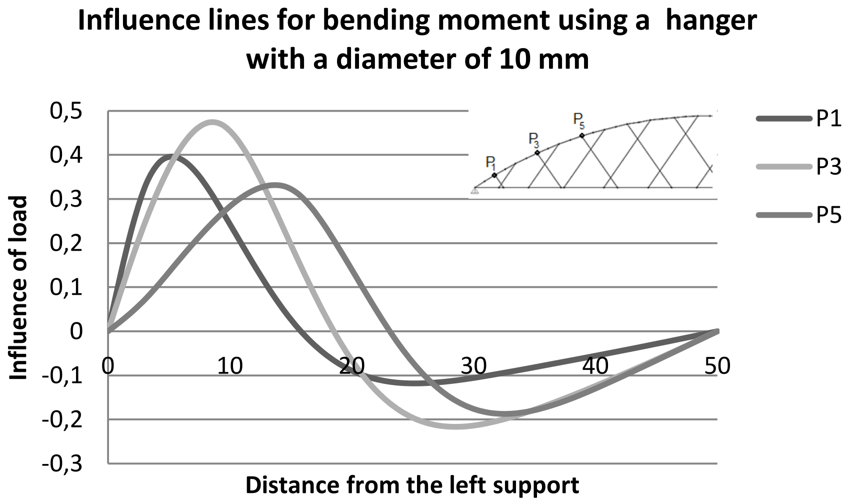
Figure 34 - Comparison between influence lines to decide the critical section of the arch for bending moment.

Figure 33 and 34 shows that points where the hangers are inclined towards the supports (P_2, P_4 and P_6) are not interesting in the search for a critical section since the influence from these points are about half as big compared to the other points. This is not hard to understand looking at figure 32.

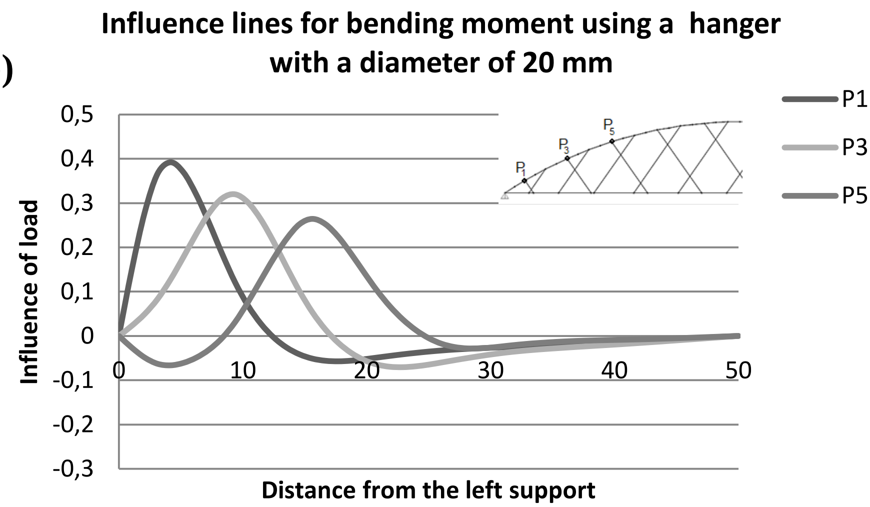
According to the finding in figure 33 the critical section for network arches is where the first hanger is connected to the arch.

When studying the importance of the hangar stiffness it first appeared to have a pure beneficial effect to the bending moment. But after more in depth studying it was clear that this wasn't the only effect. The stiffness of the hangers also defined the position of the critical section. This can be seen in these next five graphs (figure 35, a - e) showing the effect of the influence line in point P_1, P_3 and P_5 when the stiffness of the hangers are changed. To change the stiffness of the hangers the area has been manipulated, since the stiffness equals to the area times the modulus of elasticity this will change the stiffness.

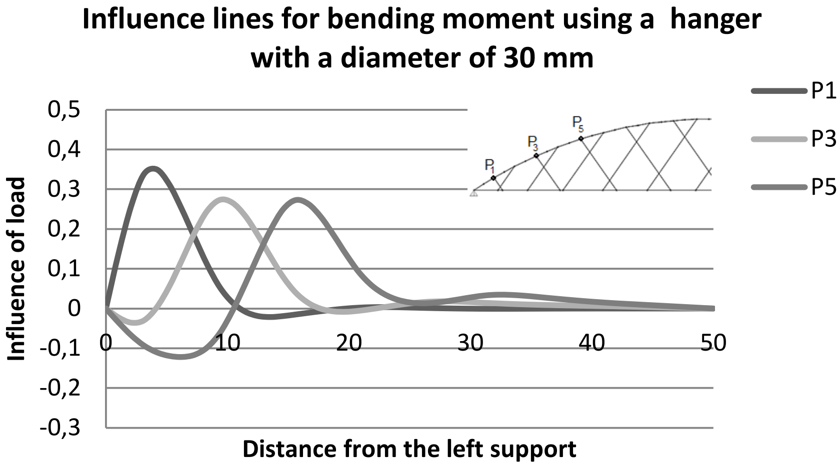
a)



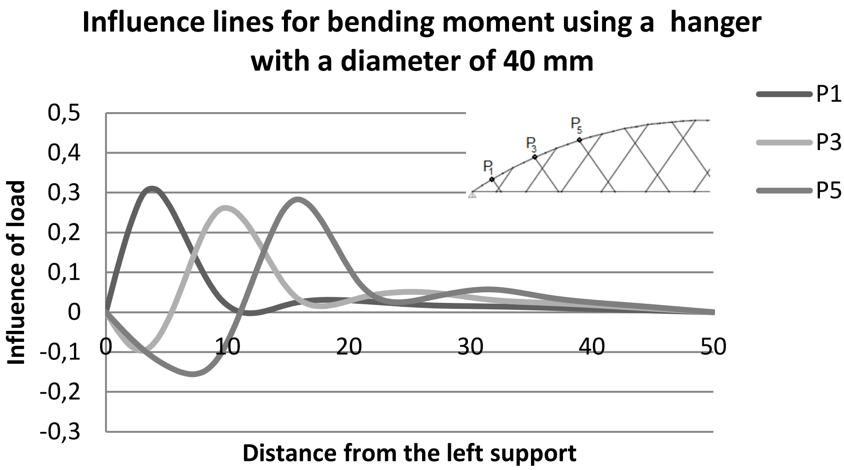
b)



c)



d)



e)

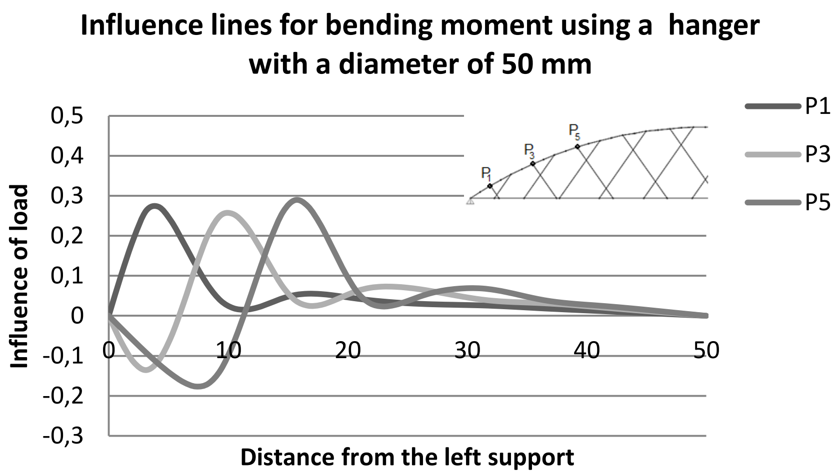


Figure 35 – These graphs shows the effect on the influence lines if the hangar stiffness is changed.

As seen in the graphs above (figure 35, a - e), if the stiffness is very low the influence lines will take on an appearance similar to the theoretical influence line and the critical section will shift.

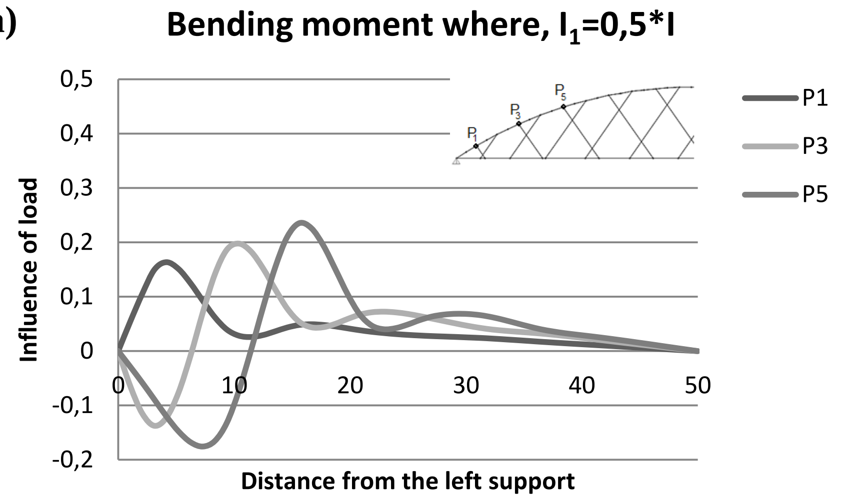
The stiffness of the hangers determines the position of the critical section as well as the magnitude of the bending moment. This proves that it is a mistake to use high strength cables when constructing network arches since the high strength will decrease the area of the hanger and decrease the stiffness.

In the reference bridge a change of the stiffness of the hangars showed no effect for the bending moment in the arch.

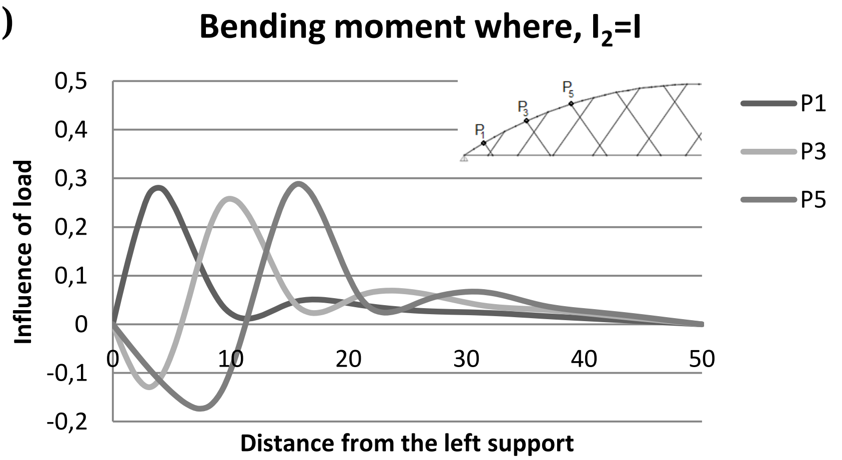
8.3 Effect of arch stiffness

To test the effect of the arch stiffness on the bending moment the bending stiffness had to be manipulated. SAP2000 has the ability to single out and manipulate a single parameter without changing the model. Since the bending stiffness is proportional to the moment of inertia (I), the stiffness was manipulated through a change of the moment of inertia. These graphs show the change of the influence lines due to a change of the bending stiffness of the arch for the network arch.

a)



b)



c)

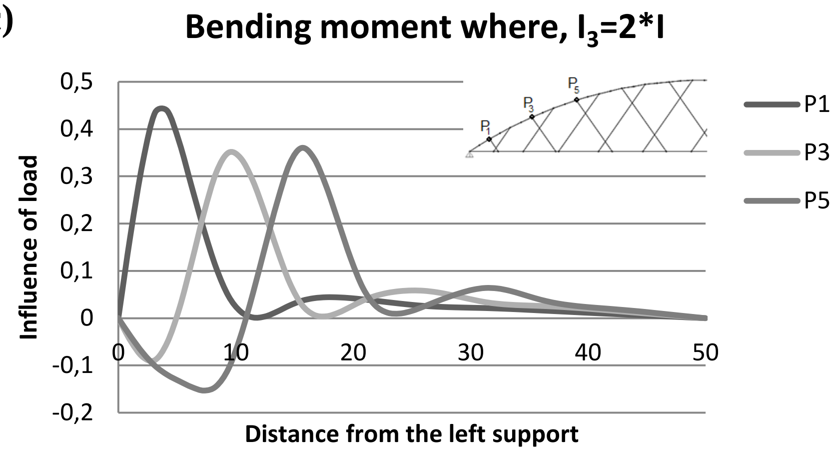


Figure 36 - Graphs a - c shows the change of the bending moment due to a change of the arch stiffness. The arch stiffness has been changed by manipulating the moment of inertia. A higher arch stiffness equals to a bigger induced bending moment.

Figure 36, a - c above show that the change of the arch stiffness will not only change the size of the bending moment but may also change the critical section.

To make the differences even clearer a graph was constructed where the influence lines for point 1 (P1) from figure 36 were compiled (see figure 37).

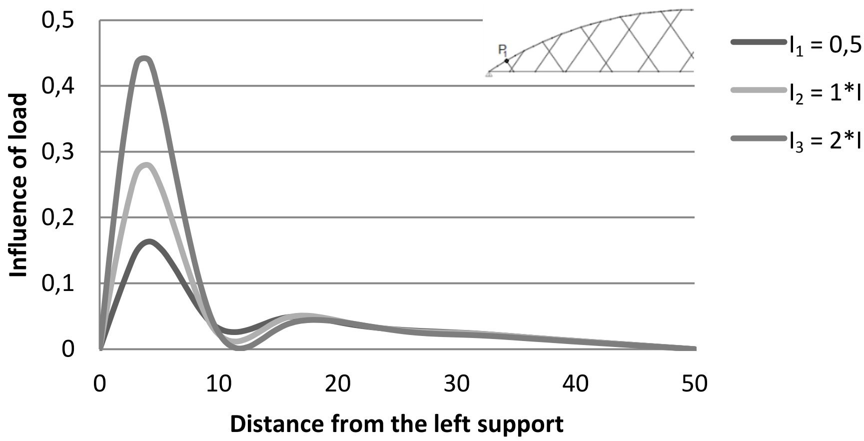


Figure 37 - Influence line for bending in P1 for the network arch if the bending stiffness of the arch is changed. (The arch stiffness has been changed by manipulating the moment of inertia.)

Further tests going up to ten times the stiffness proved that the change of arch stiffness will not change the actual shape of the influence lines like the change of the hanger stiffness did. This proves that these changes have nothing to do with the difference between the arch and hanger stiffness. The changes are individual effects.

Similar tests were performed for the reference bridge. The changes did not change the position of the critical section for the bridge but yield similar results to the increase of bending moment due to an increase of the arch bending stiffness (see figure 38).

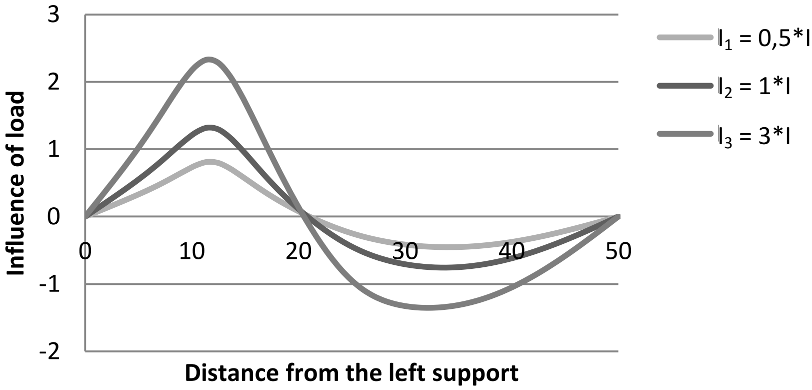


Figure 38 - Influence line for bending in the ¼ - point for the reference bridge if the bending stiffness of the arch is changed. (The arch stiffness has been changed by manipulating the moment of inertia.)

8.4 Effect of deck stiffness

The effect of the deck stiffness on the bending moment has been studied. As predicted, the stiffness of the deck yields similar results for both the network arch and the reference bridge, even though the impact is much greater for the reference bridge. It does not change the position of the critical section or the shape of the influence line. The stiffness of the deck simply determines the distribution of the loads to the hangers. A stiffer deck will distribute the load to a greater number of hangers and by doing so distribute the loads over the arch lessening the bending moment.

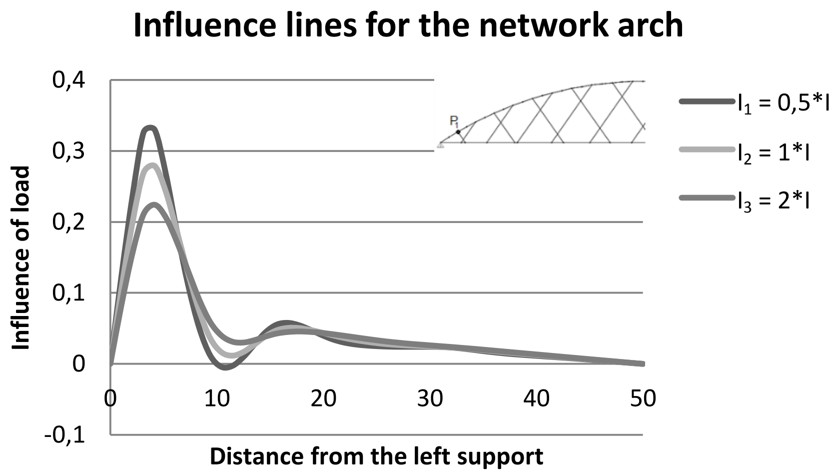


Figure 39 - Influence lines for the network arch for point 1 (P1) showing the effect of the bending moment due to a change in the deck stiffness. (The deck stiffness has been changed by manipulating the moment of inertia.)

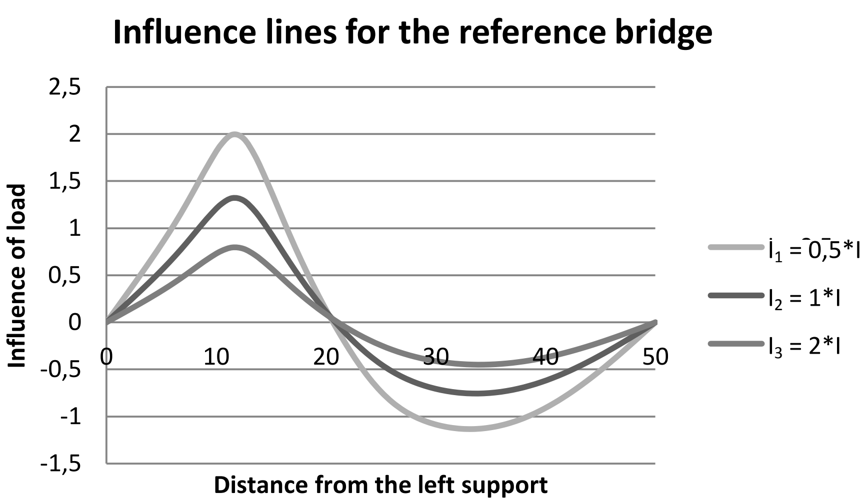


Figure 40 - Influence lines for the reference bridge showing the effect of the bending moment due to a change in the deck stiffness. (The deck stiffness has been changed by manipulating the moment of inertia.)

8.5 Effect of hanger inclination

According to the theory, the network arch with the smallest inclination (45 degrees) should have the smallest bending moment. Influence lines have been constructed for each of the hanger inclinations (see figures 41 – 43).

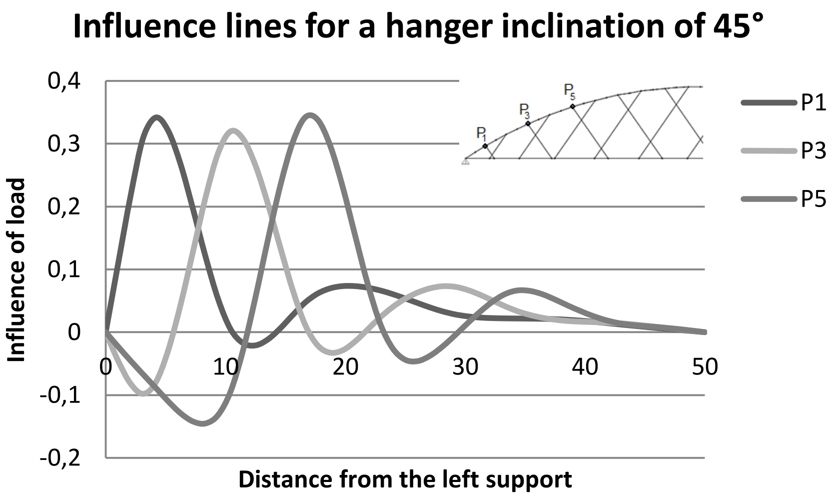


Figure 41 - Influence lines for bending moment for a network arch with a hanger inclination of 45 degrees.

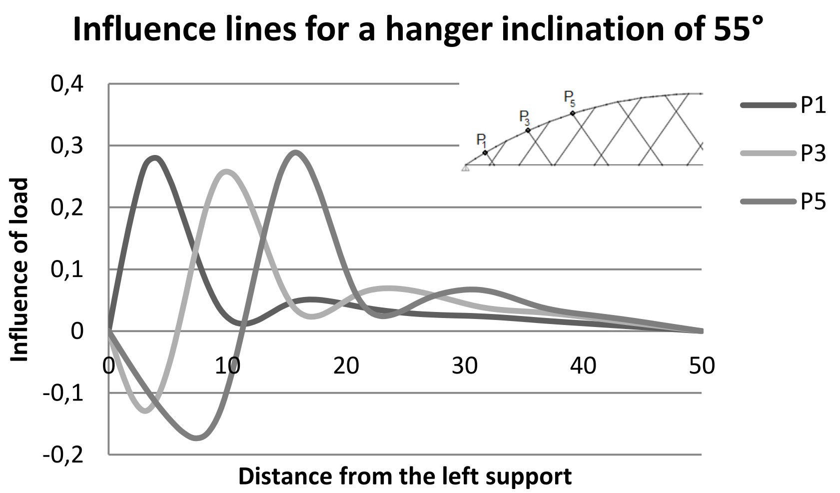


Figure 42 - Influence lines for bending moment for a network arch with a hanger inclination of 55 degrees.

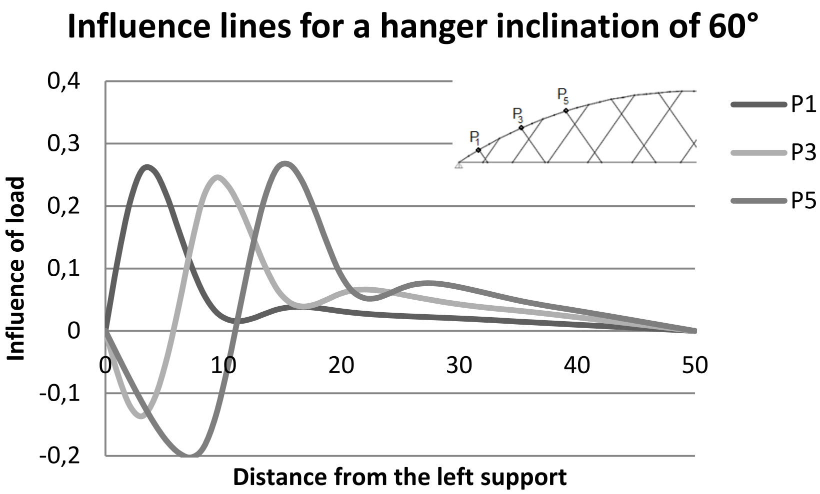


Figure 43 - Influence lines for bending moment for a network arch with a hanger inclination of 60 degrees.

By looking in the graphs above (figure 41 - 43) we can tell that the inclination does not change the critical section or the shape of the influence line. Though it seems to change the magnitude of the bending moment in the arch just as suspected. To get a better look at the change of the bending moment a graph has been constructed compiling the influence lines for P1 for the different hanger inclinations (see figure 44).

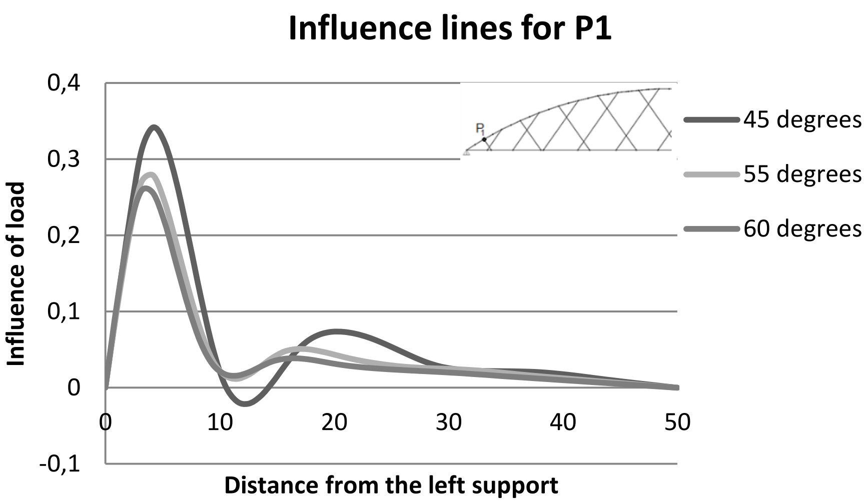


Figure 44 - influence lines for bending moment in P1 for the three different network arches.

The graph clearly shows that the network arch with 45 degree inclination of the hangers has the highest bending moment. This is in direct contrast to the prediction made by Tveit. No clear reason for this behavior has been detected.

8.6 Relaxation

To determine the risk of hanger relaxation each of the network arches were loaded according to the load placement that would be most prone to relaxation. The hangers most likely to relax are the hangers close to the edges of the bridge and inclined towards the supports (H_2, H_4, H_6 and H_8). This can be seen by studying the influence line for H_2 for the network arch with an inclination of 55 degrees (see figure 45).

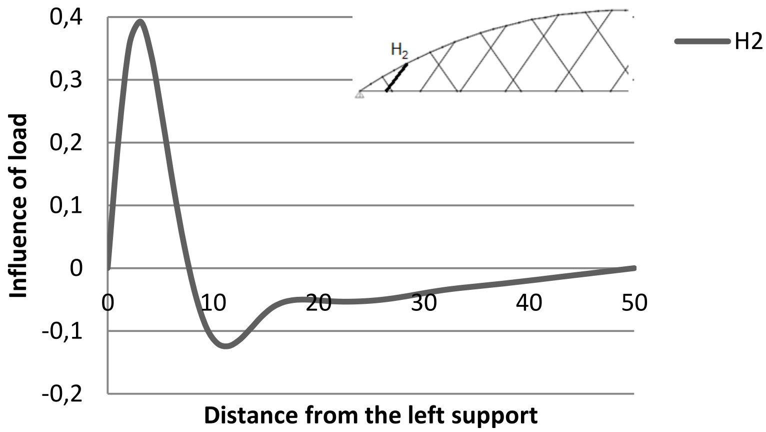


Figure 45 - Influence line of the axial force in a hanger for a network arch with a hanger inclination of 55 degrees. This is the hanger most prone to relaxation.

The bridge is loaded to try to make the hanger relax (to achieve maximum negative load). The loads used have been defined in section 7.2 and the load case used is 6.10b. The calculation was done in the ultimate limit state (ULS) and since the permanent load prevents relaxation it was assumed as a favorable load.

The calculations showed that all the three different network arches that had been modeled had big problems with relaxations in the ULS. A second calculation was done for the serviceability limit state (SLS) but also in this calculation the bridges had big problems with relaxation.

A study was made to detect what parameters could be modified to lessen the risk of relaxation.

8.6.1 Effect of hanger stiffness

The hanger stiffness had almost no effect on the risk for relaxation which can be seen by studying the graph below (see figure 46). (M.nr is a manufacturing term for the dimension of the hanger, for example M48 => $\emptyset = 48$ mm). The small positive effect of the increased peak on the positive side is canceled out by the increased peak on the negative side.

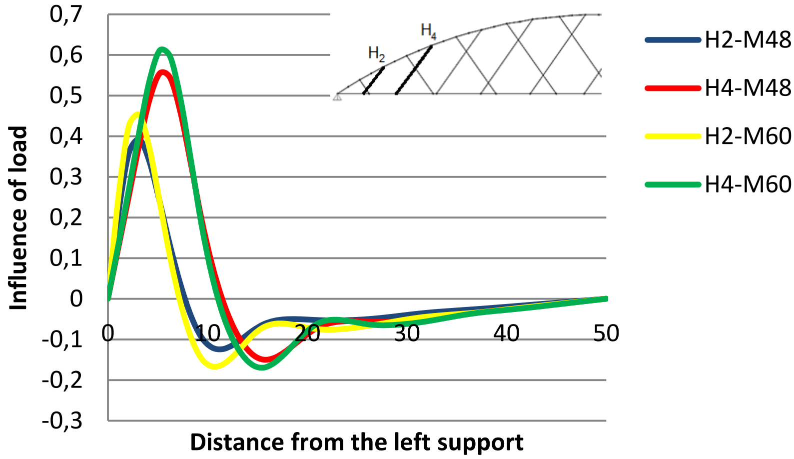


Figure 46 - Showing the influence lines of the axial force for the two hangers most prone to relax and the effect of changing the stiffness of the hangers. The graph shows that the stiffness of the hangers has little effect on the relaxation of the hanger.

8.6.2 Effect of arch stiffness

The study of the effect on relaxation due to the stiffness of the arch yielded almost the exact same result as the stiffness of the hangers. This proved that a change of the stiffness of the arch can't change the risk for relaxation.

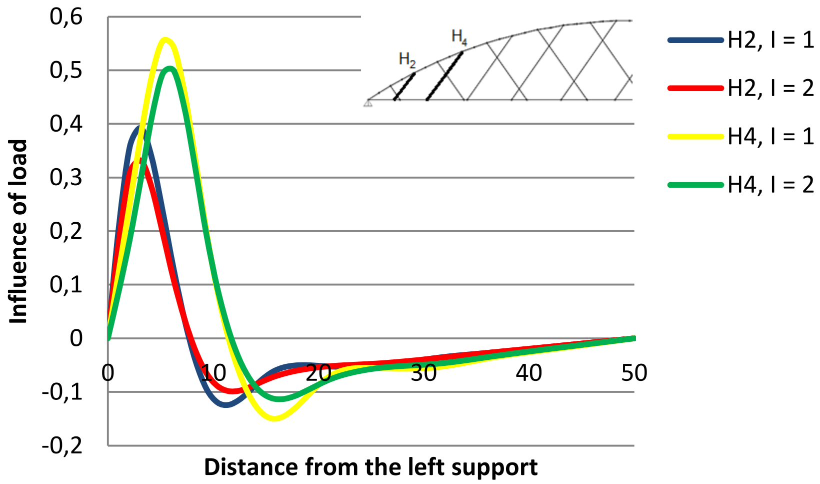


Figure 47 - Showing the influence lines of the axial force for the two hangers most prone to relax and the effect of changing the stiffness of the arch. The graph shows that the stiffness of the arch has little effect on the relaxation of the hanger.

8.6.3 Effect of deck stiffness

The stiffness of the deck proved to have a negative effect on the relaxation of the hangers. The stiffer the deck, the higher the risk of relaxation. This is not that hard to comprehend, since a stiffer deck will not deflect as much as a weaker deck, this means that even a very small shortening of the distance between the nodes for the hangers will relax it. This is proven in figure 48.

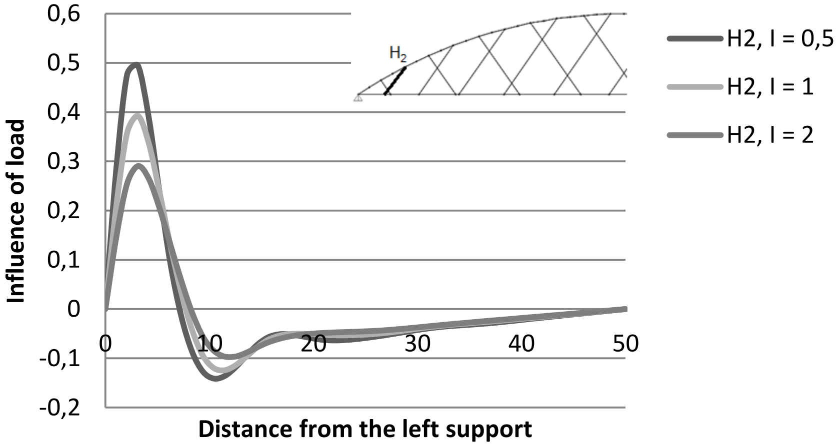


Figure 48 - Showing the influence lines of the axial force for the hanger that is most prone to relax and the effect of changing the stiffness of the deck. The graph shows that the stiffness of the deck has a reversed effect on the risk for relaxation (a stiffer deck = greater risk of relaxation).

8.6.4 Effect of hanger inclination

The hanger inclination should be directly linked to the relaxation according to Tveit. A hanger with a more horizontal hangar inclination should be more prone to relaxation than a more vertical one. Graphs have been constructed comparing the influence lines for the two hangers most prone to relax for the different hanger inclinations (see figure 49 – 50).

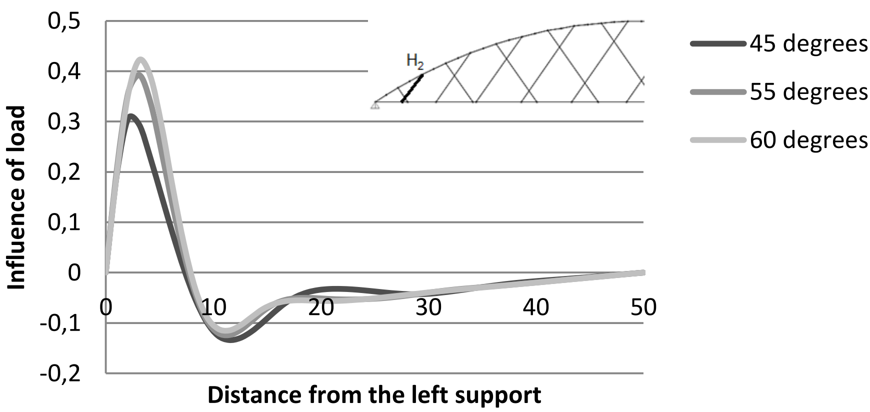


Figure 49 - Showing the influence lines of the axial force for the hanger H2 and the effect of changing the inclination of the hangars. The graph shows that the inclination of the hangars has an effect on the risk for relaxation (a more horizontal hanger = greater risk of relaxation).

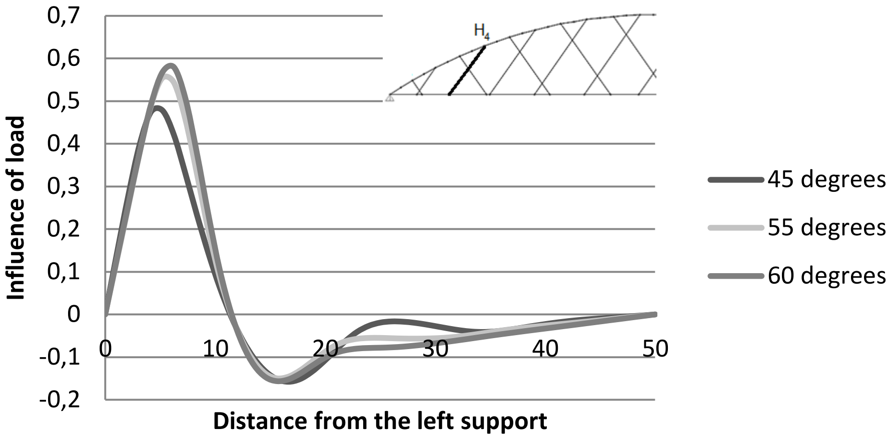


Figure 50 - Showing the influence lines of the axial force for the hanger H4 and the effect of changing the inclination of the hangars. The graph shows that the inclination of the hangars has an effect on the risk for relaxation (a more horizontal hanger = greater risk of relaxation).

The graphs above prove that a more horizontal hangar inclination yields a greater risk of relaxation just like the theory suggested.

A study has also been made to try to lessen the risk of relaxation by prestressing the hangers. If all the hangers are prestressed the same result is achieved as no prestress. By just prestressing the hanger most prone to relaxation it will give a positive response for that hanger. Unfortunately this increases the risk of relaxation for the hangers around it. This showed that prestress can't be used to lessen the risk of relaxation.

The study of relaxation showed that to lessen the risk of hangar relaxation you have to have a greater hanger inclination (more vertical hangers) or a greater permanent load. Since the problem with relaxation is most dominant near the edges of the bridge and only for the hangers inclined towards the supports, it would probably be a good idea to use a different type of hangar constellation. The use of a hanger constellation with a constant or parabolic change of the inclination is probably a better suggestion.

Brunn & Schanack, 2003, argue that the best hangar constellation is the use of a constant parabolic change of the inclination. The picture below shows a tied arch with one set of hangers using the parabolic change of the inclination (see figure 51).

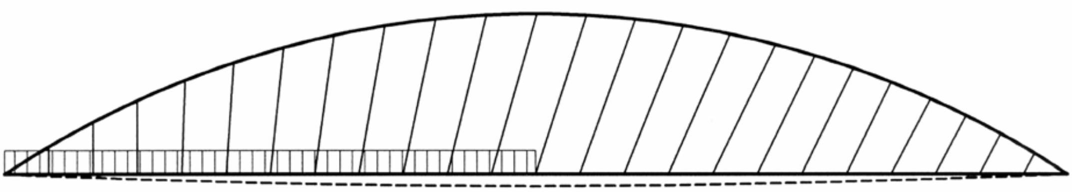


Figure 51 - Tied arch with one set of inclined hangers, using the parabolic change of inclination. (Brunn & Schanack, 2003).

8.7 Buckling

8.7.1 In plane buckling

The last part of the parametric study was the different bridges resistance to in plane buckling. The network arch should have an increased restrain against in plane buckling according to Tveit. The tied arch with a hanger inclination of 55 degrees was compared to the reference bridge and to a pure three hinged arch (see figure 52 – 54).

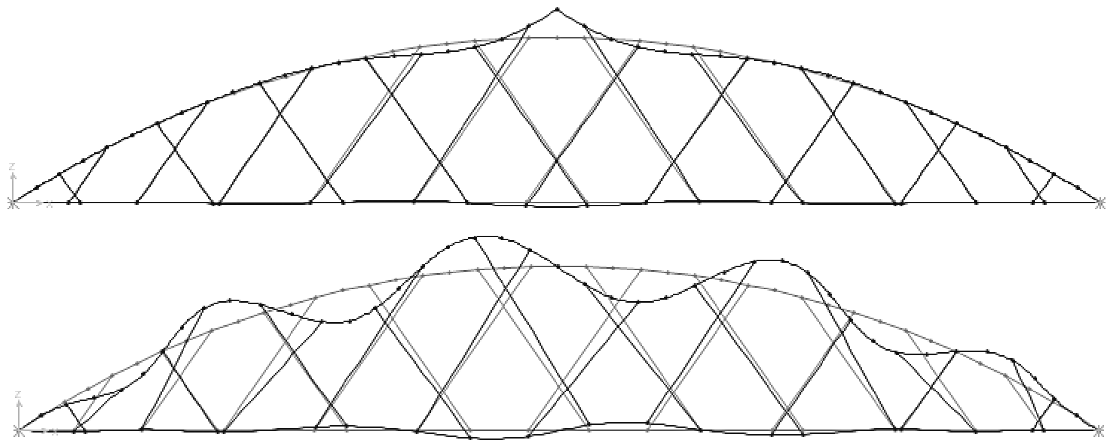


Figure 52 - First and second buckling mode for the network arch with a hanger inclination of 55 degrees. The buckling modes have been calculated using a nonlinear buckling analysis. The undeformed shape is shown in the background as a reference.

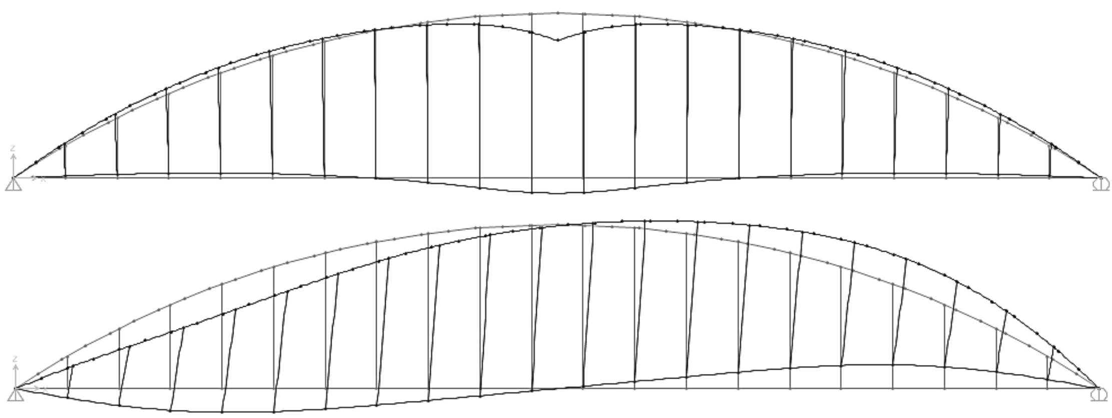


Figure 53 - First and second buckling mode for the reference bridge. The buckling modes have been calculated using a nonlinear buckling analysis. The undeformed shape is shown in the background as a reference.

These buckling modes seem reasonable since they are a match to the buckling modes calculated by (Bell & Wollebæk, 2011).

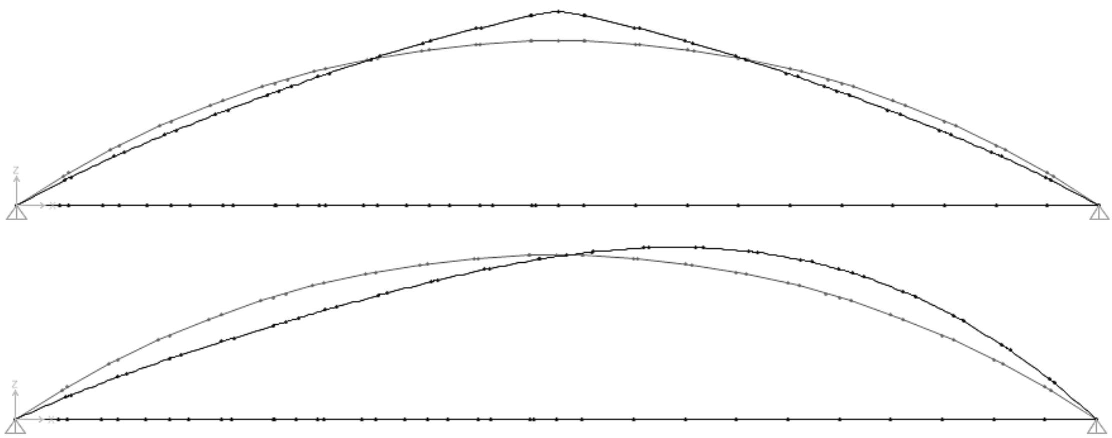


Figure 54 - First and second buckling mode for a pure three hinged arch. The buckling modes have been calculated using a nonlinear buckling analysis. The undeformed shape is shown in the background as a reference.

The calculation gives us the critical load (q_{crit}) that would cause a buckling behavior. By applying this load to the model the critical axial force for a chosen section can be achieved. The chosen section is the critical section for bending. The critical axial force for the second mode can't be calculated since the model becomes unstable after the first mode.

According to the theory displayed by (Timoshenko & Gere, 1963) the critical load should follow the equation:

$$q_{crit} = \gamma_4 \cdot \frac{EI}{l^3} \tag{17}$$

Where $\gamma_4 \approx 31$for a three hinged arch.

Table 8 - The different resistance against in plane buckling. The higher Ncrit the lesser the risk of buckling. (Computed with the use of SAP2000 except for the theoretical arch.)

	Mode 1 (q_{crit}) [kN/m]	Mode 2 (q_{crit}) [kN/m]	Mode 1 (N_{crit}) [kN]
Network arch (55°)	855	2100	36800
Reference bridge	910	2100	41600
Pure three hinged arch	48	60	1950
Theoretical arch	48	-	-

Table 8 shows that the restrain against buckling is very similar for the bridges, with a small advantage for the reference bridge. This is in contrast with Tveits prediction, but in his prediction he claims it will be better than a "normal" arch bridge with strait hangers. The arch bridge used as a reference in this study is using more hangers than "normal", which gives it a much higher restraint against buckling.

When comparing the bridges with the pure three hinged arch it is easy to see the positive effect of the hangers when it comes to buckling restraint.

To prove this method of calculating the buckling modes a simple hand calculation has been made. The calculation proved the method. For the full calculation see section 10.2.

An attempt was also done to determine the critical load using the second order analysis (described in SS-EN 1995) with the use of imperfections. But since the program SAP2000 is unable to define timber as a specific material and has a problem with defining deformations as loads this study was unsuccessful.

8.7.2 Out of plane buckling

A small study was also made for the out of plane buckling. But since the out of plane buckling is governed by the bracing between the arches no significant difference was found for the network arch typ.

8.8 Axial forces

The distribution of the axial forces in the arches has been compared. The study showed that the maximum axial forces for the reference bridge was located near the supports just as suspected according to normal arch theory (see figure 56). The network arch on the other hand had its maximum axial force in a section much higher up on the arch as can be seen in figure 55. This is due to the fact that the inclination of the hangers will induce a normal force to the arch in the sections where the hangers pull in opposite directions.

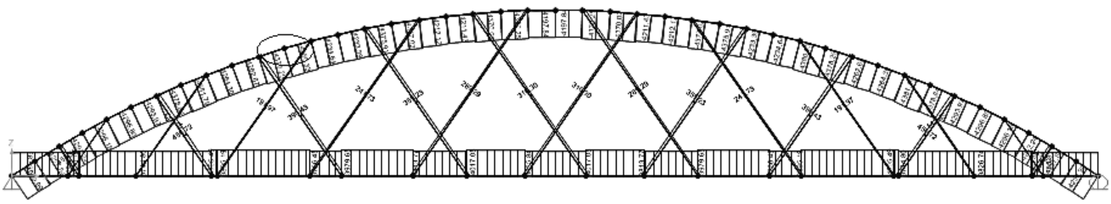


Figure 55 - The distribution of axial forces for a network arch subjected to an evenly distributed load. The maximum load will occur in the circled section between P5 and P6.

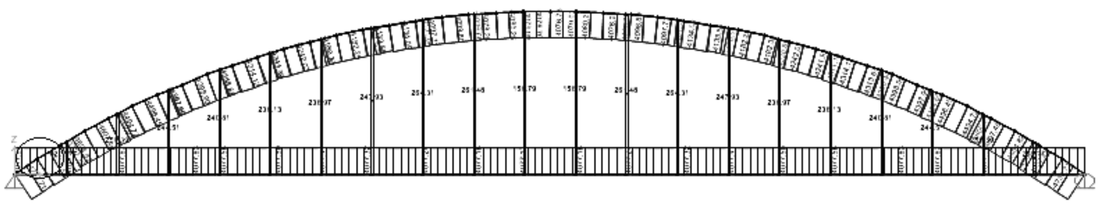


Figure 56 - The distribution of axial forces for the reference bridge subjected to an evenly distributed load. The maximum load will occur in the circled section close to the supports.

9 Designing the Bridge

The second part of this thesis is the design of a network arch. A reference arch will also be designed to determine the feasibility of the design. The network arch that has been decided to be designed is the one with a hanger inclination of 55 degrees see figure 19 on page 27. The reference arch is shown in figure 21 on page 27.

The bridge setup will follow the assumptions made in section 4. The bridge will have a length of 50 m and a width of 10 m. The loads that will be used have been calculated in section 7.2. The bridges will be designed in the ultimate limit state using the load case 6.10b, since the live loads are the determining loads.

In section 8.6 it was determined that a hanger constellation with the same angle for all of the hangers will have a problem with relaxation. For the sake of this design this will not be dealt with here.

The design will be done using the 2D-models used in the parametric study and will be verified using a final 3D-model with a more accurate load placement.

All of the influence lines for the two bridges (all hangers and critical sections) can be seen in Appendix H and Appendix I.

The two arches of the bridge are joined together in the top with three braces to make it less vulnerable for out of plane buckling. This bracing will not be designed but to lessen the demands it will follow some special considerations. The minimum distance between the bracing and the carriageway is 6 m which means that it will not have to be designed for any accidental loading, according to SS-EN 1991-1-7 and TVRK bro 11.

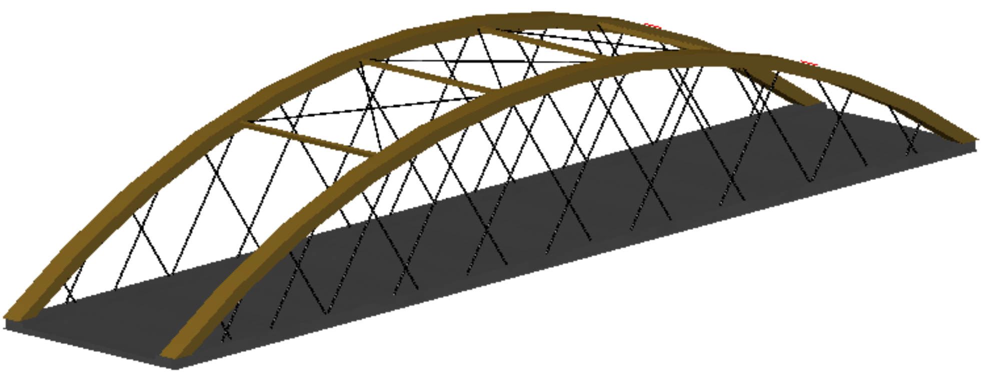


Figure 57 - A 3D-model of a network arch with a hanger inclination of 55 degrees.

The design and the equations used will follow the regulations according SS-EN 1995-1-1.

9.1 Design

9.1.1 Hanger design

The design starts by confirming that the strength of the hanger is sufficient. This is done by loading the model according to the influence lines for the hangers with the purpose the get the maximum axial tension force in the hangars. This is then compared with the capacity of the hangars shown in table 4. The fatigue of the hangers has not been taken into consideration in this study.

9.1.2 Bending

The models are then loaded to achieve the maximum bending moment in the arch. From this critical section the shear- and axial force are also acquired.

The bending moment is then checked using the following calculations:

$$\sigma_{md} \leq k_r \cdot f_{md} \tag{18}$$

$$k_r = \begin{cases} 1 & \text{If } \frac{r_{in}}{t} \geq 240 \\ 0{,}76 + 0{,}001\frac{r_{in}}{t} & \text{If } \frac{r_{in}}{t} < 240 \end{cases} \tag{19}$$

$$r_{in} = r - \frac{h}{2} \tag{20}$$

Where t is the thickness of the lamella and r_{in} is the inner radius of the arch.

$$\sigma_{md} = k_l \frac{6M_d}{b \cdot h^2} \tag{21}$$

$$k_l = k_1 + k_2\left(\frac{h}{r}\right) + k_3\left(\frac{h}{r}\right)^2 + k_4\left(\frac{h}{r}\right)^3 \tag{22}$$

$$k_1 = 1 + 1{,}4 \cdot \tan\alpha_{ap} + 5{,}4 \cdot \tan^2\alpha_{ap} \tag{23}$$

$$k_2 = 0{,}35 - 8 \cdot \tan\alpha_{ap} \tag{24}$$

$$k_3 = 0{,}6 + 8{,}3 \cdot \tan\alpha_{ap} - 7{,}8 \cdot \tan^2\alpha_{ap} \tag{25}$$

$$k_4 = 6 \cdot \tan^2\alpha_{ap} \tag{26}$$

For curved beams such as arches, $\alpha_{ap} = 0$.

9.1.3 Buckling

The next step will be to achieve the critical stress due to in plane and out of plane buckling. The critical stress is acquired. To achieve the N_{crit} the method discussed in section 8.7 is used. The N_{crit} is then recalculated to σ_{crit} by dividing it by the area of the arch. The designing compressive stress is calculated using the maximum axial load.

$$\sigma_{crit} = \frac{N_{crit}}{A} \tag{27}$$

$$\sigma_{cd} = \frac{N}{A} \tag{28}$$

The risk for out of plane buckling is then checked with the following equations, where σ_{crit} for out of plane buckling is used:

$$\frac{\sigma_{cd}}{k_{cz} \cdot f_{cd}} \leq 1{,}0 \tag{29}$$

$$k_{cz} = \frac{1}{k_c + \sqrt{k_c^2 - \lambda_{rel}^2}} \tag{30}$$

$$\lambda_{rel} = \sqrt{\frac{f_{ck}}{\sigma_{crit}}} \tag{31}$$

$$k_c = 0{,}5 \cdot (1 + \beta \cdot (\lambda_{rel} - 0{,}3) + \lambda_{rel}^2) \tag{32}$$

9.1.4 Combined bending and axial force

The combined bending and axial force was calculated and checked using the following equations, where σ_{crit} for in plane buckling is used:

$$\left(\frac{\sigma_{cd}}{k_{cy} \cdot f_{cd}}\right)^2 + \frac{\sigma_{md}}{f_{md}} \leq 1{,}0 \qquad \text{if } \lambda_{rel} \leq 0{,}3 \tag{33}$$

$$\frac{\sigma_{cd}}{k_{cy} \cdot f_{cd}} + \frac{\sigma_{md}}{f_{md}} \leq 1{,}0 \qquad \text{if } \lambda_{rel} > 0{,}3 \tag{34}$$

$$k_{cy} = \frac{1}{k_c + \sqrt{k_c^2 - \lambda_{rel}^2}} \tag{35}$$

$$\lambda_{rel} = \sqrt{\frac{f_{ck}}{\sigma_{crit}}} \tag{36}$$

$$k_c = 0{,}5 \cdot (1 + \beta \cdot (\lambda_{rel} - 0{,}3) + \lambda_{rel}^2) \tag{37}$$

β = 0,1 according to section 6.3.2 in SS-EN 1995-1-1

9.1.5 Combined tension perpendicular to the grain and shear

The combined tension perpendicular to the grain and shear was calculated and checked using the following calculations:

$$\frac{\tau_d}{f_{vd}} + \frac{\sigma_{t90d}}{k_{vol} \cdot k_{dis} \cdot f_{t90d}} \leq 1{,}0 \tag{38}$$

$$\tau_d = \frac{1{,}5 \cdot V_d}{b \cdot h} \tag{39}$$

$$\sigma_{t90d} = k_p \cdot \frac{6 \cdot M_d}{b \cdot h^2} \tag{40}$$

$$k_l = k_5 + k_6 \left(\frac{h}{r}\right) + k_7 \left(\frac{h}{r}\right)^2 \tag{41}$$

$$k_5 = 0{,}2 \cdot \tan \alpha_{ap} \tag{42}$$

$$k_6 = 0{,}25 - 1{,}5 \cdot \tan \alpha_{ap} + 2{,}6 \cdot \tan^2 \alpha_{ap} \tag{43}$$

$$k_7 = 2{,}1 \cdot \tan \alpha_{ap} - 7{,}8 \cdot \tan^2 \alpha_{ap} \tag{44}$$

$$k_{vol} = \left(\frac{V_0}{V}\right)^{0{,}2} \tag{45}$$

$k_{dis} = 1{,}4$ $\quad V_0 = 0{,}01$ $\quad$ according to SS-EN 1995-1-1

When calculating k_{vol} the V has to be decided. V is the arch's loaded volume. When looking at the figures below it is easy to see that the loaded volume is very different for the two bridges. This had to be considered when deciding V.

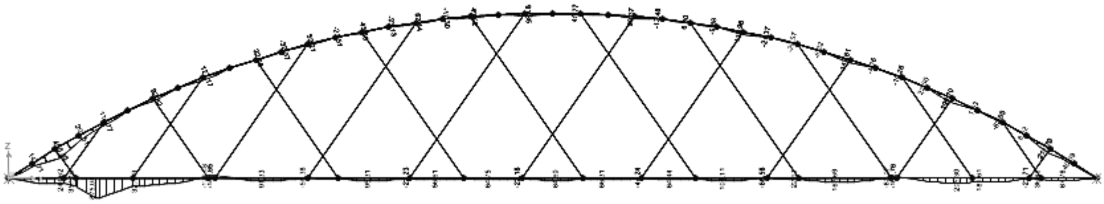


Figure 58 - Bending moment distribution for the maximum bending moment in the arch for the network arch. Only a small part of the arch segment is loaded with a positive bending moment.

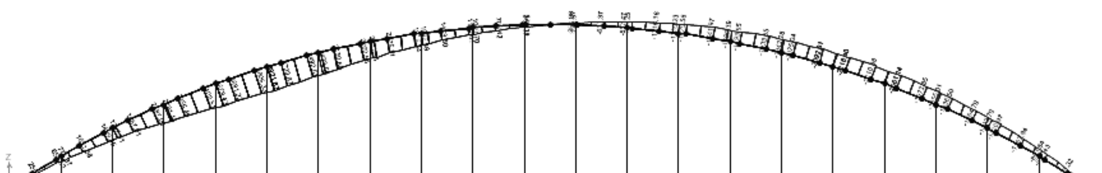


Figure 59 - Bending moment distribution for the maximum bending moment in the arch for the reference bridge. Almost the entire arch segment is loaded with a positive bending moment.

9.1.6 Accidental loading

Accidental loading means what would happen if an accident happened, for example if a vehicle drives in to the hangers.

When it comes to the hangers it is defined in TVRK bro 11 that a hanger collision should be avoided by a significant railing. If this is not possible then it should be assumed that a collision will render one hanger unusable. When calculating for accidental loading the load case 6.11a (described in section 7.1) has been used.

Influence lines have been examined for the removal of a hanger. The designing ones are displayed in the appendix together with the influence lines for the arches (Appendix H and I). The loads were placed according to these influence lines and bending moment, shear force and axial force for the arches were achieved. New axial forces for the hangers were also achieved.

New checks are made if the forces achieved in this way were bigger than the previous designing forces.

Accidental loading on the arches is defined in SS-EN 1991-1-7. It implies that if a loadbearing column (arch in this case) is in the risk of getting hit by a vehicle it should be able to withstand an accidental load of 750 kN in the x direction of the carriageway and a load of 375 kN in the y direction. Since a direct hit is hard to achieve in the x direction due to the angled shape of the arch at the abutments, this load should be reduced. No actual check has been done for this type of accidental loading for either of the bridges but looking on the size of the loads this should not be a problem.

9.1.7 3D-model

As a final check the loads were placed on a full 3D-model. This model should give smaller forces since it takes in to consideration the load distribution in the deck. This will only be used as a last check to see if the model is reasonable. No design will use the forces acquired from this calculation.

9.1.8 Conclusion

The design has been made by using an iterated method and arch and hangar dimensions have been decided that passed the checks.

Network arch 55°

Arch (b x h)	=	1000 x 600 mm^2
Hanger	=	M48

Reference bridge

Arch (b x h)	=	700 x 1600 mm^2
Hanger	=	M48

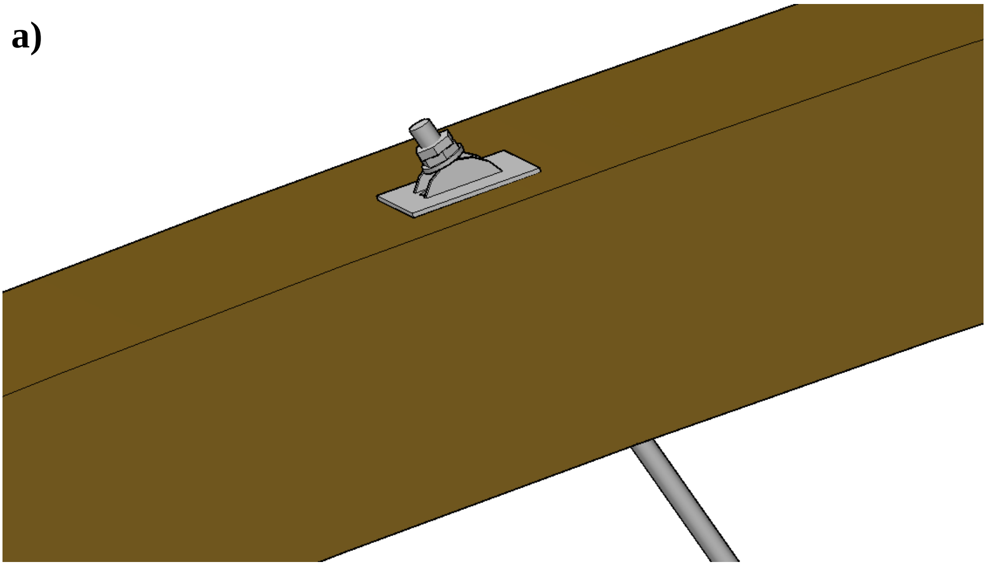
This means that the cross-sectional area of the reference bridge arch is 186 % of the area of the Network arch.
The network arch passed all of the checks while the reference bridge did not pass the check for combined tension perpendicular to the grain and shear. This has to be reinforced. To see the full calculations see section 10.3.

The dimensions of the arch for the reference bridge can be compared with a study done by (Torkkeli, Rautakorpi & Jutila, 1999) where they designed a timber arch bridge with vertical hangers for road traffic. The bridge was 40 m long and the cross-section of the arch was 1075x1360 mm^2.

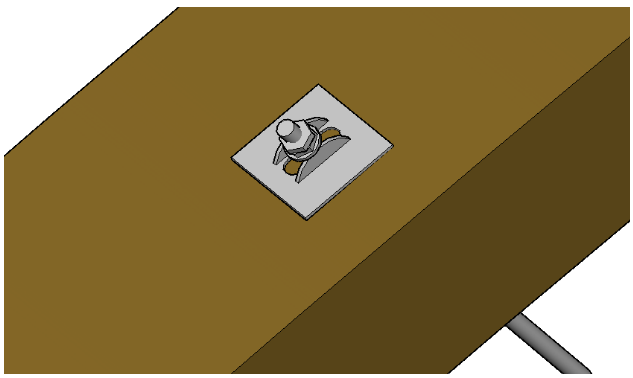
9.2 Details

A feasible solution to the detail between the arch and the hangers has been suggested. The detail should be relatively easy to use and you should be able to use it for all the hangers. To lessen the risk of bending the hangers due to relaxations this detail allows the hangers to slide. The direct fastening to the arch lessens the risk of involuntary extra bending moments. Drawings are shown in appendix G.

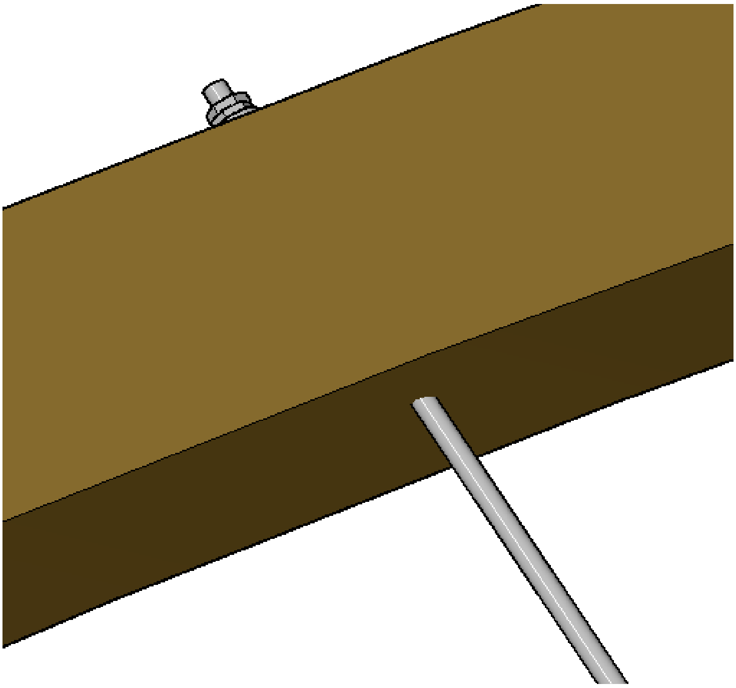
a)



b)



c)



d)

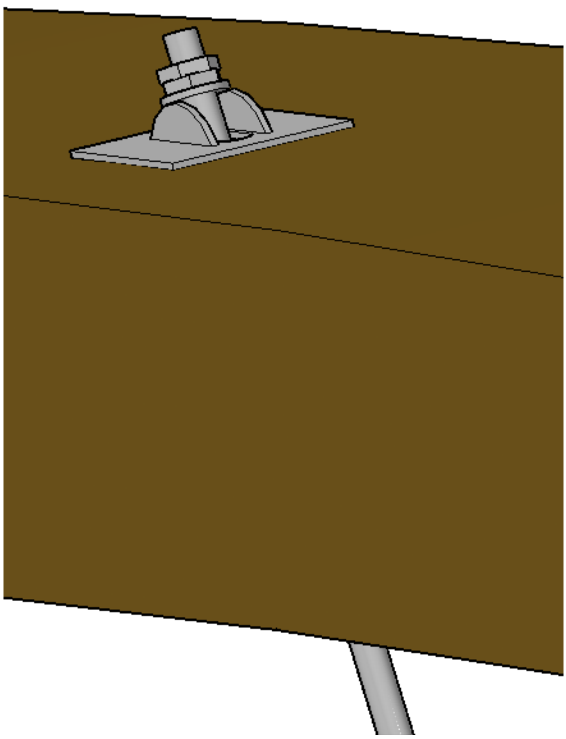


Figure 60 – a-d are different angles of a 3D-model showing the a possible solution to the detail of the hanger and arch connection.

10 Calculation

10.1 Traffic loading – load distribution

The load distribution is made to determine the loads subjecting the maximum loaded side.

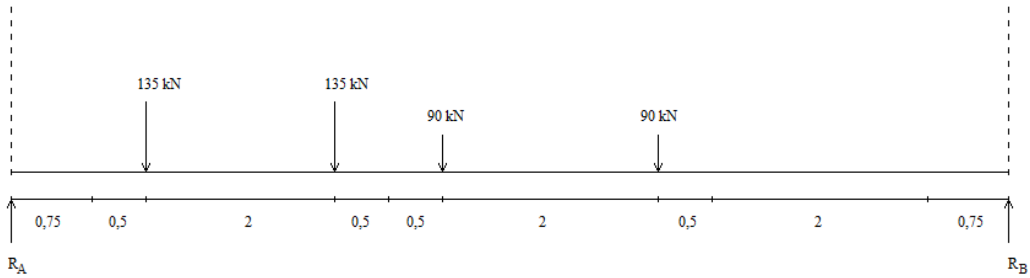


Figure 61 - Load distribution according to figure 16 and appendix A due to the axel loads. (The picture is of the bridge in the longitudinal direction, the dotted lines symbolize the hangers.)

Moment around R_B:

$$R_A \cdot 9{,}5 = 135 \cdot 8{,}25 + 135 \cdot 6{,}25 + 90 \cdot 5{,}25 + 90 \cdot 3{,}25 \rightarrow \tag{46}$$

$$\rightarrow R_A = \frac{2722{,}5}{9{,}5} = 286{,}6\ kN/axel \rightarrow Q = 573{,}2\ kN$$

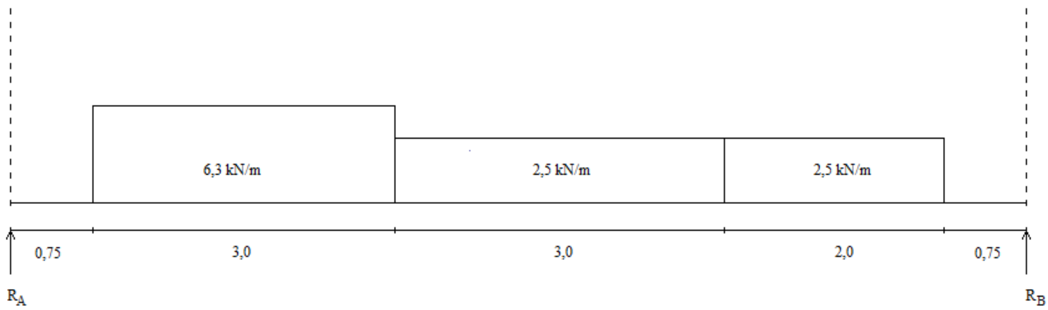


Figure 62 - Load distribution according to figure 16 and appendix A due to the distributed loads. (The picture is of the bridge in the longitudinal direction, the dotted lines symbolize the hangers.)

Moment around R_B:

$$R_A \cdot 9{,}5 = 6{,}3 \cdot 3 \cdot 7{,}25 + 2{,}5 \cdot 3 \cdot 4{,}25 + 2{,}5 \cdot 2 \cdot 2{,}25 \rightarrow \tag{47}$$

$$\rightarrow R_A = \frac{180{,}15}{9{,}5} = 19{,}0\ kN \rightarrow q = 19{,}0\ kN/m$$

10.2 Verification of buckling calculation

The pure three hinged arch from the study of buckling (section 8.7) has been subjected to its critical buckling load q_{crit} that was calculated using a nonlinear buckling analysis with SAP2000. To prove the analysis method the conversion factor for the length β should be a little less than 1,2 according to the theory.

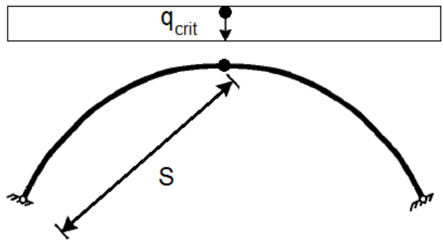


Figure 63 - A pure three hinged arch subjected to the uniformly distributed load qcrit.

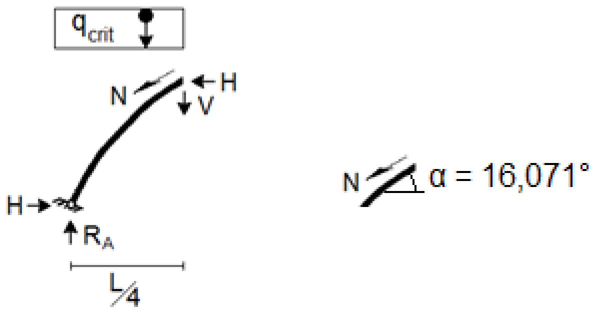


Figure 64 - A section of the arch with the forces displayed. The angle between the inclination of the arch in the section point.

$$R_A = R_B = \frac{50 \cdot q_{crit}}{2} = \frac{50 \cdot 48}{2} = 1200\ kN \tag{48}$$

$$H = \frac{q_{crit} \cdot L^2}{8 \cdot f} = \frac{48 \cdot 50^2}{8 \cdot 7{,}5834} = 1978\ kN \tag{49}$$

$$V = R_A - q_{crit} \cdot \frac{L}{4} = 1200 - 48 \cdot \frac{50}{4} = 600\ kN \tag{50}$$

$$N_{crit}\ (H) = H \cdot \cos\alpha = 1978 \cdot \cos(16{,}071) = 1900{,}7\ kN \tag{51}$$

$$N_{crit}\ (V) = V \cdot \sin\alpha = 600 \cdot \sin(16{,}071) = 166{,}1\ kN \tag{52}$$

$$N_{crit} = N_{crit}\ (H) + N_{crit}\ (V) = 1900{,}7 + 166{,}1 = 2066{,}8\ kN \tag{53}$$

According to the theory for buckling loads for a three hinged arch:

$$N_{crit} = \frac{\pi^2 \cdot EI}{L_k^2} \qquad\qquad L_k = \beta \cdot S \tag{54}$$

$$L_k = \pi \cdot \sqrt{\frac{EI}{N_{crit}}} = \pi \cdot \sqrt{\frac{11100000 \cdot 0{,}018}{2066{,}8}} = 30{,}89 \tag{55}$$

$$\beta = \frac{L_k}{S} = \frac{30{,}89}{26{,}12} = 1{,}18 \tag{56}$$

This proves the use of the SAP2000 nonlinear buckling analysis.

10.3 Designing the bridge

10.3.1 Network arch 55°

The network arch has been designed to have an arch of (b x h) 1000 x 600 mm^2. The hangers should be made of M48.

10.3.1.1 Hanger

Load placement according to the influence line for H7 gives the maximum tension force in the hangars. A SAP2000 calculation using the design loads => N_{max} = 680 kN.
The design capability of the M48 hanger is given by table 4 on page 22.

$$N_{max} = 680 < 795 \rightarrow OK! \tag{57}$$

10.3.1.2 Bending moment

A SAP2000 calculation using the design loads with the load placement according to the influence line for P1 gave the maximum bending moment in the arch. The shear and axial forces in the section were also acquired.
$M_{max} = 450{,}9\ kNm$ $V = 220{,}2\ kN$ $N = 4834{,}5\ kN$

$$\sigma_{md} \leq k_r \cdot f_{md} \tag{58}$$

$$\frac{r_{in}}{t} = \frac{r - \frac{h}{2}}{t} = \frac{45000 - \frac{600}{2}}{45} = 993 > 240 \rightarrow k_r = 1 \tag{59}$$

$$k_1 = 1 + 1{,}4 \cdot \tan 0 + 5{,}4 \cdot \tan^2 0 = 1 \tag{60}$$

$$k_2 = 0{,}35 - 8 \cdot \tan 0 = 0{,}35 \tag{61}$$

$$k_3 = 0{,}6 + 8{,}3 \cdot \tan 0 - 7{,}8 \cdot \tan^2 0 = 0{,}6 \tag{62}$$

$$k_4 = 6 \cdot \tan^2 0 = 0 \tag{63}$$

$$k_l = 1 + 0{,}35\left(\frac{600}{45000}\right) + 0{,}6\left(\frac{600}{45000}\right)^2 + 0\left(\frac{600}{45000}\right)^3 = 1{,}005 \tag{64}$$

$$\sigma_{md} = k_l \frac{6M_d}{b \cdot h^2} = 1{,}005 \cdot \frac{6 \cdot 533{,}1 \cdot 10^3}{1 \cdot 0{,}6^2} = 8{,}93\ MPa \tag{65}$$

$$\sigma_{md} \leq k_r \cdot f_{md} \rightarrow 8{,}93 \leq 1 \cdot 17{,}92 \rightarrow OK! \tag{66}$$

10.3.1.3 Buckling

SAP2000 buckling analysis has been used to calculate the critical load and recalculate it to a critical axial load.

In plane buckling:
$q_{crit} = 855\ kN/m$ $\qquad$ $N_{crit} = 36800\ kN$

$$\sigma_{crit} = \frac{N_{crit}}{A} = \frac{36800}{1 \cdot 0{,}6} = 60{,}50\ MPa \tag{67}$$

Out of plane buckling:
$q_{crit} = 407\ kN/m$ $\qquad$ $N_{crit} = 17390\ kN$

$$\sigma_{crit} = \frac{N_{crit}}{A} = \frac{17390}{1 \cdot 0{,}6} = 28{,}98\ MPa \tag{68}$$

Maximum axial load in the arch is achieved between P7 and P8. Load placement according to the influence line of this frame gave N_{max} = 5140 kN

$$\sigma_{cd} = \frac{N_{max}}{A} = \frac{5140}{1 \cdot 0{,}6} = 8{,}57\ MPa \tag{69}$$

$$\frac{\sigma_{cd}}{k_{cz} \cdot f_{cd}} \leq 1{,}0 \tag{70}$$

$$\lambda_{rel} = \sqrt{\frac{26{,}5}{28{,}98}} = 0{,}95 \tag{71}$$

$$k_c = 0{,}5 \cdot (1 + 0{,}1 \cdot (0{,}95 - 0{,}3) + 0{,}95^2) = 0{,}99 \tag{72}$$

$$k_{cz} = \frac{1}{0{,}99 + \sqrt{0{,}99^2 - 0{,}95^2}} = 0{,}80 \tag{73}$$

$$\frac{8{,}57}{0{,}80 \cdot 14{,}84} \leq 1{,}0 \rightarrow\ 0{,}72 \leq 1{,}0 \rightarrow OK! \tag{74}$$

The model used a pinned connection at the supports for the out of plane buckling. This is a very conservative assumption since this connection should be almost rigid. If there would have been a problem with the out of plane buckling this connection could have been modeled as a spring.

10.3.1.4 Combined bending and axial force

The critical stress for in plane buckling will be used in these calculations. Since the combined bending and axial force will happen in the same point the axial force should be taken from the same critical section.

$M_{max} = 450{,}9\ kNm$ $\sigma_{crit} = 60{,}50\ MPa$ $N = 4834{,}5\ kN$

$$\sigma_{cd} = \frac{N}{A} = \frac{4834{,}5}{1 \cdot 0{,}6} = 8{,}06\ MPa \tag{75}$$

$$\lambda_{rel} = \sqrt{\frac{26{,}5}{60{,}50}} = 0{,}66 \ \rightarrow \lambda_{rel} > 0{,}3 \tag{76}$$

$$\frac{\sigma_{cd}}{k_{cy} \cdot f_{cd}} + \frac{\sigma_{md}}{f_{md}} \leq 1{,}0 \tag{77}$$

$$k_c = 0{,}5 \cdot (1 + 0{,}1 \cdot (0{,}66 - 0{,}3) + 0{,}66^2) = 0{,}74 \tag{78}$$

$$k_{cy} = \frac{1}{0{,}74 + \sqrt{0{,}74^2 - 0{,}66^2}} = 0{,}94 \tag{79}$$

$$\frac{8{,}06}{0{,}94 \cdot 14{,}84} + \frac{7{,}55}{17{,}92} \leq 1{,}0 \rightarrow 0{,}998 \leq 1{,}0 \rightarrow OK! \tag{80}$$

10.3.1.5 Accidental loading

The accidental loading will be checked for the case that one hanger has been rendered unusable.

The designing axial force of the hangers was achieved using the load placement according to the influence line of H7 if H5 was broken.

$$N_{max} = 732{,}9\ kN < 795\ kN \rightarrow OK! \tag{81}$$

The designing bending moment and critical section was achieved using the load placement according to the influence line of P2 if H5 was broken.

$M_{max} = 533{,}1\ kNm$ $\sigma_{crit} = 63{,}61\ MPa$ $N = 3676{,}6\ kN$

$V = 333{,}9\ kN$

The combined bending and axial force is recalculated using the new forces.

$$\sigma_{md} = k_l \frac{6M_d}{b \cdot h^2} = 1{,}005 \cdot \frac{6 \cdot 533{,}1 \cdot 10^3}{1 \cdot 0{,}6^2} = 8{,}93\ MPa \tag{82}$$

$$\sigma_{cd} = \frac{N}{A} = \frac{3676{,}6}{1 \cdot 0{,}6} = 6{,}13\ MPa \tag{83}$$

$$\lambda_{rel} = \sqrt{\frac{26{,}5}{63{,}61}} = 0{,}65 \ \rightarrow \lambda_{rel} > 0{,}3 \tag{84}$$

$$\frac{\sigma_{cd}}{k_{cy} \cdot f_{cd}} + \frac{\sigma_{md}}{f_{md}} \leq 1{,}0 \tag{85}$$

$$k_c = 0{,}5 \cdot (1 + 0{,}1 \cdot (0{,}65 - 0{,}3) + 0{,}65^2) = 0{,}73 \tag{86}$$

$$k_{cy} = \frac{1}{0{,}73 + \sqrt{0{,}73^2 - 0{,}65^2}} = 0{,}95 \tag{87}$$

$$\frac{6{,}13}{0{,}94 \cdot 14{,}84} + \frac{8{,}93}{17{,}92} \leq 1{,}0 \rightarrow 0{,}93 \leq 1{,}0 \rightarrow OK! \tag{88}$$

10.3.1.6 Combined tension perpendicular to the grain and shear

Since this calculation is dependent on the bending moment and the shear force and not the axial force it is easy to see that we should use the designing forces that were achieved at the accidental loading.
$M_{max} = 533{,}1\ kNm$ $\qquad V = 333{,}9\ kN$

$$\frac{\tau_d}{f_{vd}} + \frac{\sigma_{t90d}}{k_{vol} \cdot k_{dis} \cdot f_{t90d}} \leq 1{,}0 \tag{89}$$

$$\tau_d = \frac{1{,}5 \cdot 333{,}9}{1 \cdot 0{,}6} = 834{,}75\ kPa \tag{90}$$

$$k_5 = 0{,}2 \cdot \tan 0 = 0 \tag{91}$$

$$k_6 = 0{,}25 - 1{,}5 \cdot \tan 0 + 2{,}6 \cdot \tan^2 0 = 0{,}25 \tag{92}$$

$$k_7 = 2{,}1 \cdot \tan 0 - 7{,}8 \cdot \tan^2 0 = 0 \tag{93}$$

$$k_p = 0 + 0{,}25 \left(\frac{600}{45000}\right) + 0 \left(\frac{600}{45000}\right)^2 = 0{,}003 \tag{94}$$

$$\sigma_{t90d} = k_p \cdot \frac{6 \cdot M_d}{b \cdot h^2} = 0{,}003 \cdot \frac{6 \cdot 533{,}1}{1 \cdot 0{,}6^2} = 29{,}62\ kPa \tag{95}$$

V is the loaded volume of the arch. By looking at the moment distribution in figure 65, V has been chosen to be approximately 1/5 of the total volume of the arch segment.

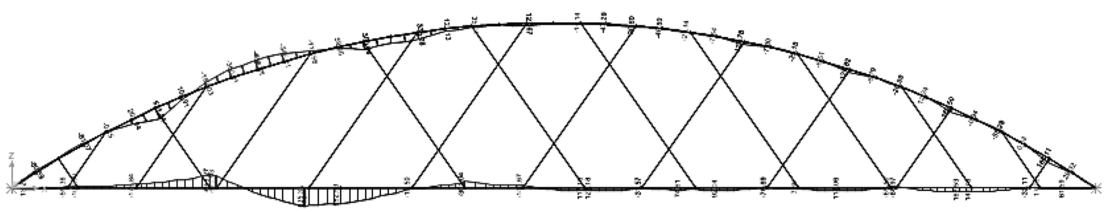


Figure 65 - The moment distribution in the network arch loaded according to the influence line for P2 if H5 is broken. Used as the foundation in the choice of the loaded volume of the arch (V).

$$V \approx 26{,}5 \cdot 1 \cdot 0{,}6 \cdot \frac{1}{5} = 3{,}2\ m^3 \tag{96}$$

$$k_{vol} = \left(\frac{0{,}01}{3{,}2}\right)^{0{,}2} = 0{,}32 \tag{97}$$

$$\frac{834{,}75}{1792} + \frac{29{,}62}{0{,}32 \cdot 1{,}4 \cdot 252} \leq 1{,}0 \rightarrow 0{,}73 \leq 1{,}0 \rightarrow OK! \tag{98}$$

The network arch with a hanger inclination of 55°, a cross-section of the arch of 1000x600 mm^2 and a hanger diameter of 48 mm completes all of the checks.

10.3.1.7 3D-model

A 3D-model was constructed and loaded according to the influence lines of the critical section for bending moment (see figure 66 – 67). The results of the maximum bending moment and the related shear- and axial force for the 2D- and 3D-model has been compared in table 9.

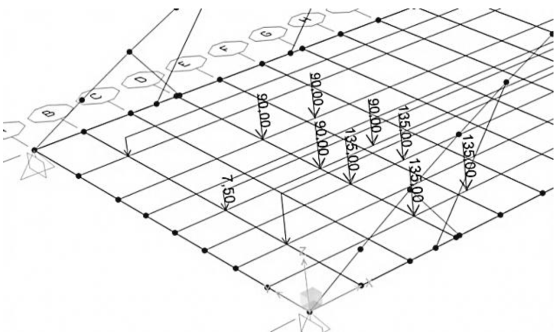


Figure 66 - Load placement in the 3D-model

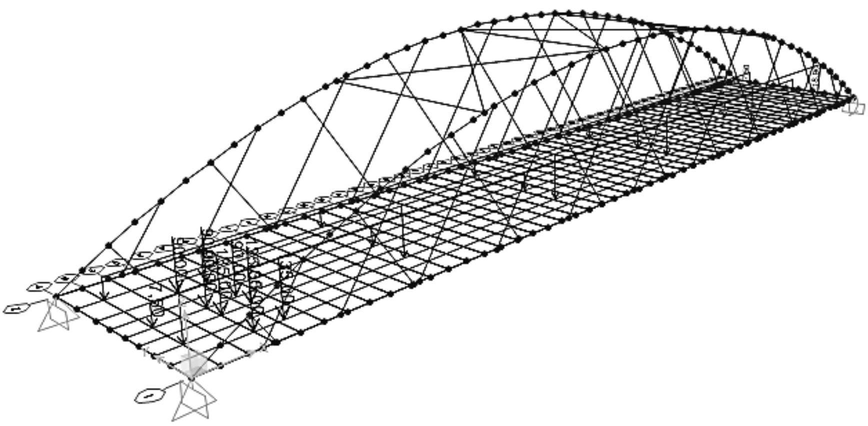


Figure 67 - The full 3D-model

Table 9 - A comparison of the forces in the critical section for bending using a 2D-model or a 3D-model.

	M_{max} [kNm]	V [kN]	N [kN]
2D-model	450,9	220,2	4834,5

3D-model	414,0	216,0	4522

10.3.2 Reference bridge

The reference bridge has been designed to have an arch of 700 x 1600 mm^2. The hangers should be made of M48.

10.3.2.1 Hanger

Load placement according to the influence line for H1 gives the maximum tension force in the hangars. N_{max} = 721 kN.
The design capability of the M48 hanger is given by table 4 on page 22.

$$N_{max} = 721 < 795 \rightarrow OK! \quad (99)$$

10.3.2.2 Bending moment

Load placement according to the influence line for the 1/4 -point gave the maximum bending moment in the arch. The shear and axial forces in the section was also acquired.
$M_{max} = 3921{,}9\ kNm$ $\quad V = 283{,}8\ kN$ $\quad N = 3544{,}1\ kN$

$$\sigma_{md} \leq k_r \cdot f_{md} \quad (100)$$

$$\frac{r_{in}}{t} = \frac{r - \frac{h}{2}}{t} = \frac{45000 - \frac{1600}{2}}{45} = 982 > 240 \rightarrow k_r = 1 \quad (101)$$

$$k_1 = 1 + 1{,}4 \cdot \tan 0 + 5{,}4 \cdot \tan^2 0 = 1 \quad (102)$$

$$k_2 = 0{,}35 - 8 \cdot \tan 0 = 0{,}35 \quad (103)$$

$$k_3 = 0{,}6 + 8{,}3 \cdot \tan 0 - 7{,}8 \cdot \tan^2 0 = 0{,}6 \quad (104)$$

$$k_4 = 6 \cdot \tan^2 0 = 0 \quad (105)$$

$$k_l = 1 + 0{,}35\left(\frac{1600}{45000}\right) + 0{,}6\left(\frac{1600}{45000}\right)^2 + 0\left(\frac{1600}{45000}\right)^3 = 1{,}013 \quad (106)$$

$$\sigma_{md} = k_l \frac{6M_d}{b \cdot h^2} = 1{,}013 \cdot \frac{6 \cdot 3921{,}9 \cdot 10^3}{0{,}7 \cdot 1{,}6^2} = 13{,}30\ MPa \quad (107)$$

$$\sigma_{md} \leq k_r \cdot f_{md} \rightarrow 13{,}30 \leq 1 \cdot 17{,}92 \rightarrow OK! \quad (108)$$

10.3.2.3 Buckling

SAP2000 buckling analysis has been used to calculate the critical load and recalculate it to a critical axial load.

In plane buckling:
$q_{crit} = 2550\ kN/m$ $N_{crit} = 97700\ kN$

$$\sigma_{crit} = \frac{N_{crit}}{A} = \frac{97700}{0{,}7 \cdot 1{,}6} = 87{,}50\ MPa \tag{109}$$

Out of plane buckling:
$q_{crit} = 1405\ kN/m$ $N_{crit} = 55800\ kN$

$$\sigma_{crit} = \frac{N_{crit}}{A} = \frac{55800}{0{,}7 \cdot 1{,}6} = 49{,}82\ MPa \tag{110}$$

Maximum axial load in the arch is achieved by loading the full span of the bridge with the point load in the middle. $N_{max} = 5310$ kN

$$\sigma_{cd} = \frac{N_{max}}{A} = \frac{5310}{0{,}7 \cdot 1{,}6} = 4{,}74\ MPa \tag{111}$$

$$\frac{\sigma_{cd}}{k_{cz} \cdot f_{cd}} \leq 1{,}0 \tag{112}$$

$$\lambda_{rel} = \sqrt{\frac{26{,}5}{49{,}82}} = 0{,}73 \tag{113}$$

$$k_c = 0{,}5 \cdot (1 + 0{,}1 \cdot (0{,}73 - 0{,}3) + 0{,}73^2) = 0{,}79 \tag{114}$$

$$k_{cz} = \frac{1}{0{,}79 + \sqrt{0{,}79^2 - 0{,}73^2}} = 0{,}92 \tag{115}$$

$$\frac{4{,}74}{0{,}92 \cdot 14{,}84} \leq 1{,}0 \rightarrow\ 0{,}35 \leq 1{,}0 \rightarrow OK! \tag{116}$$

The model used a pinned connection at the supports for the out of plane buckling. This is a very conservative assumption since this connection should be almost ridged. If there would have been a problem with the out of plane buckling this connection could have been modeled as a spring.

10.3.2.4 Combined bending and axial force

The critical stress for in plane buckling will be used in these calculations. Since the combined bending and axial force will happen in the same point the axial force should be taken from the same critical section.
$M_{max} = 3921{,}9\ kNm$ $\sigma_{crit} = 87{,}50\ MPa$ $N = 3544{,}1\ kN$

$$\sigma_{cd} = \frac{N}{A} = \frac{3544{,}1}{0{,}7 \cdot 1{,}6} = 3{,}16\ MPa \tag{117}$$

$$\lambda_{rel} = \sqrt{\frac{26,5}{87,50}} = 0,55 \ \rightarrow \lambda_{rel} > 0,3 \tag{118}$$

$$\frac{\sigma_{cd}}{k_{cy} \cdot f_{cd}} + \frac{\sigma_{md}}{f_{md}} \leq 1,0 \tag{119}$$

$$k_c = 0,5 \cdot (1 + 0,1 \cdot (0,55 - 0,3) + 0,55^2) = 0,66 \tag{120}$$

$$k_{cy} = \frac{1}{0,66 + \sqrt{0,66^2 - 0,55^2}} = 0,97 \tag{121}$$

$$\frac{3,16}{0,97 \cdot 14,84} + \frac{13,30}{17,92} \leq 1,0 \rightarrow 0,96 \leq 1,0 \rightarrow OK! \tag{122}$$

10.3.2.5 Accidental loading

The accidental loading will be checked for the case that one hanger has been rendered unusable.

The designing axial force of the hangers was achieved using the load placement according to the influence line of H1 if H2 was broken.

$$N_{max} = 718 \ kN < 795 \ kN \rightarrow OK! \tag{123}$$

The influence lines for bending moment remained the same after any of the hangers were removed. This means the accidental loading will not yield any new designing forces.

10.3.2.6 Combined tension perpendicular to the grain and shear

This calculation is dependent on the bending moment and the shear force which has been provided earlier.
$M_{max} = 3921,9 \ kNm$ $\qquad V = 283,8 \ kN$

$$\frac{\tau_d}{f_{vd}} + \frac{\sigma_{t90d}}{k_{vol} \cdot k_{dis} \cdot f_{t90d}} \leq 1,0 \tag{124}$$

$$\tau_d = \frac{1,5 \cdot 283,8}{0,7 \cdot 1,6} = 380,1 \ kPa \tag{125}$$

$$k_5 = 0,2 \cdot \tan 0 = 0 \tag{126}$$

$$k_6 = 0,25 - 1,5 \cdot \tan 0 + 2,6 \cdot \tan^2 0 = 0,25 \tag{127}$$

$$k_7 = 2,1 \cdot \tan 0 - 7,8 \cdot \tan^2 0 = 0 \tag{128}$$

$$k_p = 0 + 0,25 \left(\frac{1600}{45000}\right) + 0 \left(\frac{1600}{45000}\right)^2 = 0,009 \tag{129}$$

$$\sigma_{t90d} = k_p \cdot \frac{6 \cdot M_d}{b \cdot h^2} = 0,009 \cdot \frac{6 \cdot 3921,9}{0,7 \cdot 1,6^2} = 116,7 \ kPa \tag{130}$$

V is the loaded volume of the arch. By looking at the moment distribution in the figure below V has been chosen to be approximately 1/2 of the total volume of the arch segment.

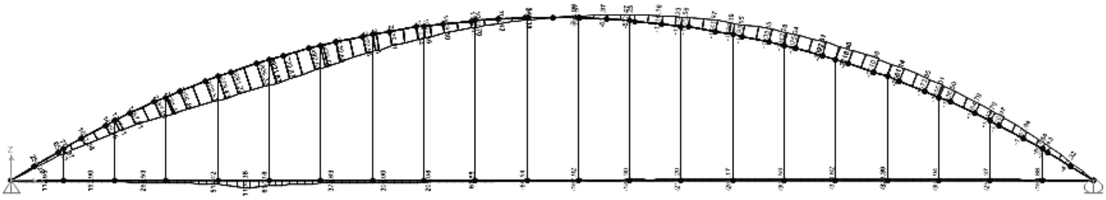


Figure 68 - The moment distribution in reference bridge loaded according to the influence line for the ¼ - point. Used as the foundation in the choice of the loaded volume of the arch (V).

$$V \approx 26{,}5 \cdot 0{,}7 \cdot 1{,}6 \cdot \frac{1}{2} = 14{,}8\ m^3 \tag{131}$$

$$k_{vol} = \left(\frac{0{,}01}{14{,}8}\right)^{0{,}2} = 0{,}23 \tag{132}$$

$$\frac{380{,}1}{1792} + \frac{116{,}7}{0{,}23 \cdot 1{,}4 \cdot 252} \leq 1{,}0 \rightarrow 1{,}64 > 1{,}0 \rightarrow NOT\ OK! \tag{133}$$

The reference arch has a big problem with the combined tension perpendicular to the grain and shear. It would probably be a possibility to reinforce the arch with fully threaded screws to minimize this problem.

The reference bridge with a cross-section of the arch of 700x1600 mm^2 and a hanger diameter of 48 mm completes all of the other checks.

This dimension seems reasonable compared to the arch designed in (Torkkeli, Rautakorpi & Jutila, 1999) where they designed a timber arch bridge for road traffic. The bridge was 40 m long and the cross-section of the arch was 1075x1360 mm^2.

10.3.2.7 3D-model

A 3D-model was constructed and loaded according to the influence lines of the critical section for bending moment. The results of the maximum bending moment and the related shear- and axial force for the 2D- and 3D-model has been compared in table 10.

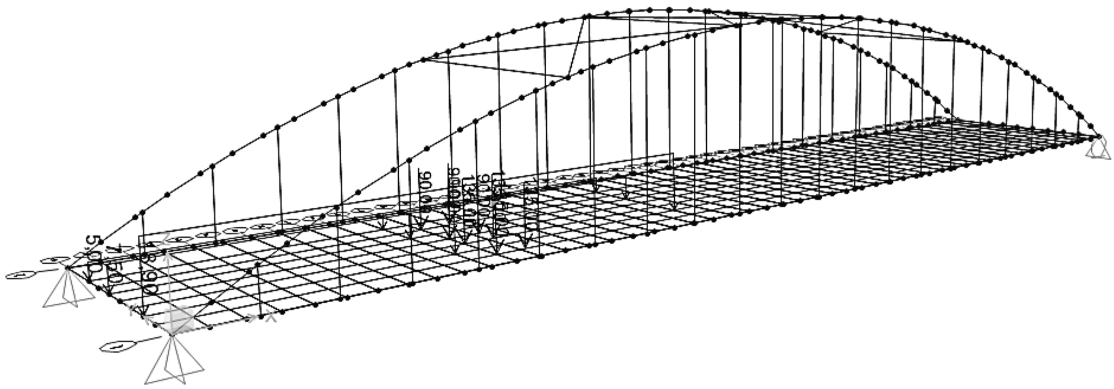


Figure 69 - The full 3D-model

Table 10 - A comparison of the forces in the critical section for bending using a 2D-model or a 3D-model.

	M_{max} [kNm]	V [kN]	N [kN]
2D-model	3921,9	283,8	3544,1
3D-model	3089,7	211,0	3600

11 Results

The parametric study showed that the most important parameters to consider are the stiffness of the hangers and the stiffness of the arch, both of which not only change the magnitude of the bending moment in the arch but also the critical section for bending moment. The stiffness of the hangers also has the capacity to change the shape of the influence line. If the stiffness of the hangers is very small the network arch will work more like a normal arch with straight hangers. This shows that it is a mistake to use high strength cables in the design.

The study also showed that the network arches with the same inclination for all of the hangers have a big problem with relaxation of hangers. A series of tests were performed to see which parameters would lessen the risk of relaxation.
It was shown that the stiffness of the hangers and arch had little effect on the risk of relaxation. The deck stiffness on the other hand had a reversed effect on the relaxation. A stiffer deck increased the risk of relaxation.
The major ways to lessen the relaxation is to use a heavier permanent load that will work as a prestress for the hangers or changing the hanger inclination. Just as Tveits theory suggested it was shown that the more horizontal the hangers were the greater the risk of relaxation.
Since the risk of relaxation is greatest close to the supports and only for some of the hangers it is probably a better idea to use a different hanger constellation. A hanger constellation with a constant or parabolic change in hanger inclination will probably lessen the risk of hanger relaxation.

In the design part of this thesis a 50 m long and 10 m wide network arch with a hanger inclination of 55° was designed. To determine the success of the design a reference bridge with vertical hangers was also designed. The problem with relaxation was not considered in this design.

The two bridges were designed using the Eurocodes and the results were:

Table 11 - The dimensions of the designed arches

	Network arch (55°) [mm]	Reference bridge [mm]
Hanger diameter	48	48
Width of the arch	1000	700
Height of the arch	600	1600

The area of the cross-section of the reference arch was 186 % bigger than the cross-section of the network arch. This showed that if you could accept the relaxations of the hangers or come up with a similar design without relaxation of the hangers, this is definitely a good way of designing timber glulam arch bridges for road traffic.

12 Discussion and Conclusion

The author think that looking at the picture below is evidence enough to prove the success of the network arch. The cross-section of the network arch is about half as big as for the conventional design and the bending moment for the conventional design is almost nine times bigger.

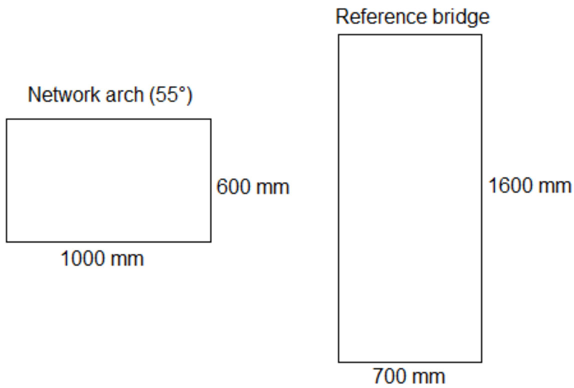


Figure 70 – A comparison between the cross-sections of the network arch and the reference bridge that have been designed.

These designs might be a little misguiding though, since the designs were made disregarding the relaxation. On the other hand, the reference bridge was unable to pass the check for combined tension perpendicular to the grain and shear.

The problems with relaxations were much bigger than the author first suspected. In the report written by Tveit (Tveit, 2011) is it easy to get a feeling the relaxations wouldn't be a problem. When looking at the influence lines and the response when loading the models it appears impossible to completely disregard this effect. A hanger constellation with the same angle for all of the hangers will have a problem with relaxation. Using a hanger constellation with a constant or parabolic change of the hanger inclination is probably a better solution just like (Brunn & Schanack, 2003) say in their report. In the report of (Bell & Karlsrud, 2001) they are using a network arch where the hanger nodes have been equidistantly distributed along both the arch and the deck. This should, according to the findings in this report, give an even greater problem with relaxation.

If someone would like to continue the research of this type of bridge the author would advise them to look in to a design using a hanger constellation with a constant or parabolic change of the hanger inclination.

But is the relaxation really that big of a problem? The esthetic problem with the hanger getting bent due to the shortening of the distance between the nodes could be fixed using a hanger connection that allows the hanger to slide. The problem with increased loading on surrounding hangers and the extra distance for the load carrying deck is probably not such a big problem either. Since the hanger only relaxes if the loading near it is small. This means that the hangers that would have to take the extra load would be minimally loaded anyway and the load on the deck would be very small as well. The only real problem that the author can see is the fact that the deck in these points would go from negative to positive bending moment due to the loss of a support. This would increase the demands on the decks. But since the network arch apparently has the ability to drastically lessen the bending moment in the arch this might be something we can live with.

One of the most confusing discoveries of this thesis was the fact that the bending moment was actually smaller for the network arch with a hanger inclination of 60° than the one with an inclination of 45°. According to Tveit should the bending moment decrease when the hangers become more horizontal, but the findings in this thesis did the opposite. This could be because Tveit has only worked on two hinged arches and here we are working with three hinged arches. It can also be because it is not just the inclination between the deck and the hanger that is important. The angle between the arch and the hanger might be more important than assumed. A more perpendicular angle might give a better utilization of the hanger.

The use of a linear calculation when deciding the influence lines is probably not ideally, since this method uses the assumption that no hanger will relax, and as we have already discussed this is not the case. A more correct influence line would probably be achieved by using a linear one as a starting point, then load the model and see if there are any relaxations. If there are, remove those hangers from the model and calculate a new linear influence line. This would have to be repeated until the relaxing hangers in the loaded model is the same as the removed hangers in the linear influence line. This has not been tried since it would be a very time consuming method.

The most surprising result was the effect of the hanger stiffness to the bending moment for the network arch. A change of the stiffness could change not only the magnitude of the bending moment but also the point of the critical section and the shape of the entire influence line. A small stiffness of the hangers gave an appearance of the influence line that was similar to the once for the conventional arch. My personal guess for this behavior is that the low stiffness of the hanger increases the strain of the hanger before it reaches its full strength. This gives the arch some extra “wiggle room”. If this extra space to move is big the network arch will behave similar to a conventional arch. This means that it is a bad idea to use high strength hangers, since these have a much smaller stiffness for the same strength, due to the decreased area needed.

When the forces were compared between using the 2D-model and the 3D-model in the design, the 3D- model gave much lower forces. This is probably due to the distributing effect of the deck and the more precise load placement of the 3D-model. The comparison also showed that the difference between the forces were bigger for the reference bridge. This is due to the fact that the point loads have a bigger contribution for the reference bridge which can be seen by studying the influence lines.

One of the biggest problems with this design is the fact that it is hard to get a template for all bridges. There are no simple calculations to get the arch design and hanger constellation that you need, and most starting setups has to be made by trial and error.

As a whole the author think this is a system with a great potential that should be used in a real design, even if some more research might be needed.

13 References

Carling O., 2008, *Limträ Handbok*, Svenskt Limträ AB, Stockholm.

Lorentsen M. & Sundquist H., 1995, *Bågkonstruktioner*, Kungliga Tekniska Högskolan, Stockholm.

Torkkeli M., Rautakorpi H. & Jutila A., 1999, *Arch Bridges for Road Traffic*, Nordic Timber Council AB, Stockholm.

Isaksson T. & Mårtensson A., 2008, *Byggkonstruktion Regel- och formelsamling*, Studentlitteratur AB, Lund.

Timoshenko S. & Gere J., 1963, *Theory of elastic stability*, McGraw-Hill international book company, London.

Tveit, P., 2011, 'The Network Arch. Bits of Manuscript in February 2011 after Lectures in 50 Countries', *http://home.uia.no/pert/index.php/The_Network_Arch*

Brunn B. & Schanack F., 2003, 'Calculation of a double track railway network arch bridge applying the European standards', *http://home.uia.no/pert/index.php/Masters_Theses*

Teich S. & Wendelin S., 2001, 'Vergleichsrechnung einer Netzwerkbogenbrücke unter Einsatz des Europäischen Normenkonzeptes', *http://home.uia.no/pert/index.php/Masters_Theses*

Bell K. & Wollebæk L., 2011, 'Stability of glulam arches', *http://www.ewpa.com/Archive/2004/jun/Paper_011.pdf*

Bell K., 2010, 'Structural systems for glulam arch bridge', ICTB2010, 49-66.

Bell K. & Karlsrud E., 2001, 'Large Glulam Arch Bridges - A Feasibility Study', IABSE2001, 31-36.

Pretec, 2011, ASDO hängarsystem, *http://www.pretec.se/sidor/Broschyrer.php*

13.1 Oral and E-mail references

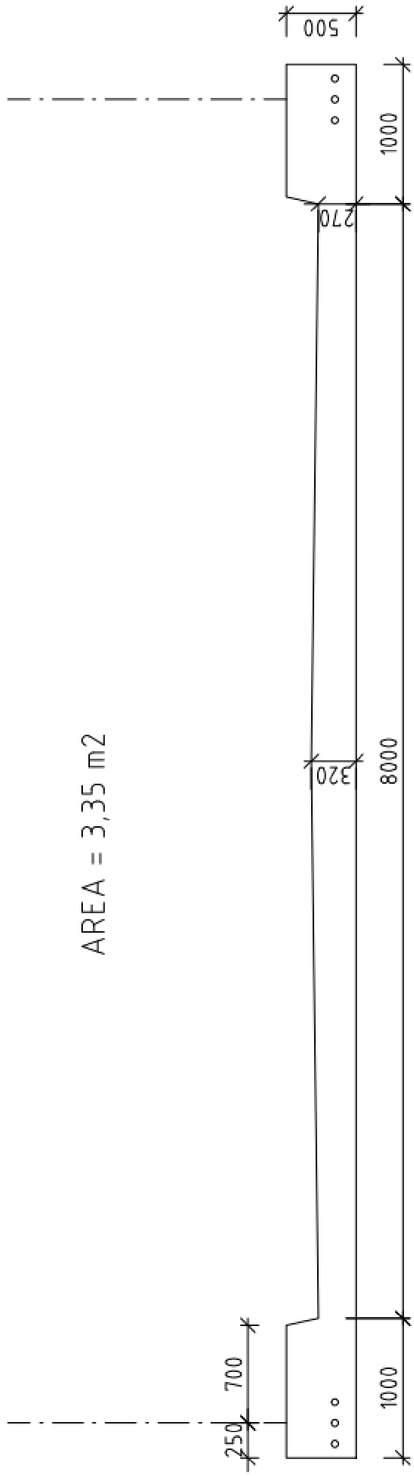
Tveit P., professor emeritus at Agders University in Norway.

Crocetti R., professor at LTH in Sweden.

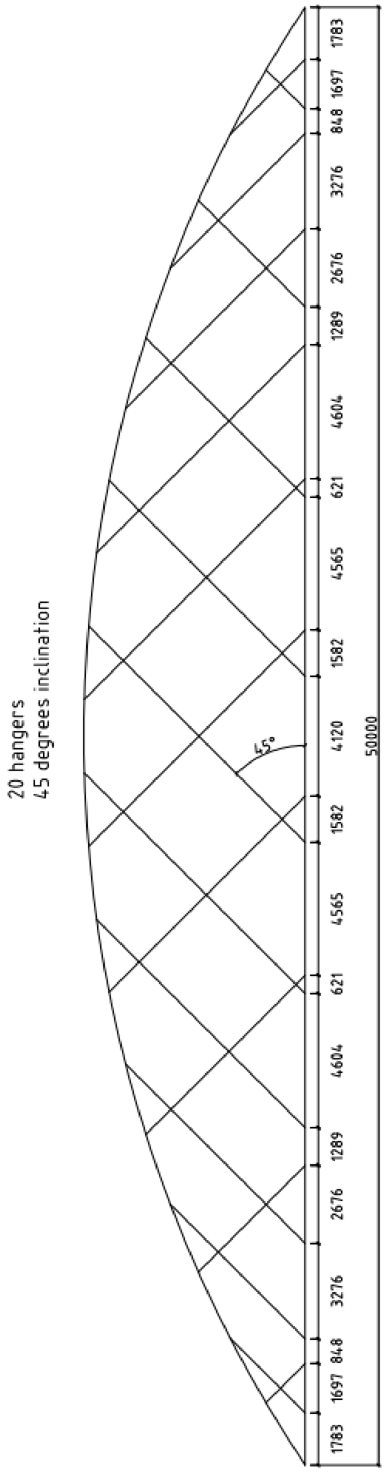
Hagemann A., engineer at Anker Schroeder in Germany.

Asp A., Pretec in Sweden.

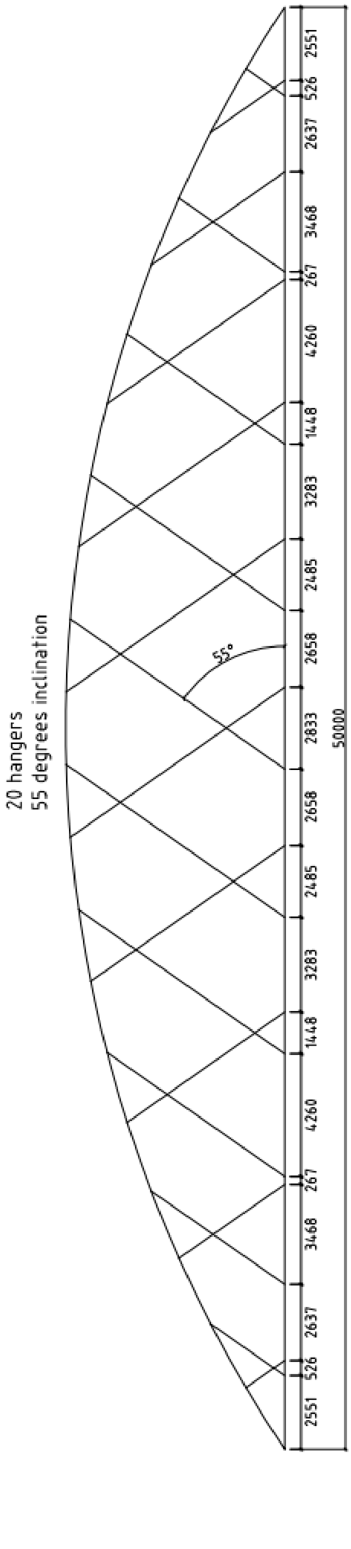
Appendix A – a scaled model of the concrete deck. (The hangers are shown by the dotted line).



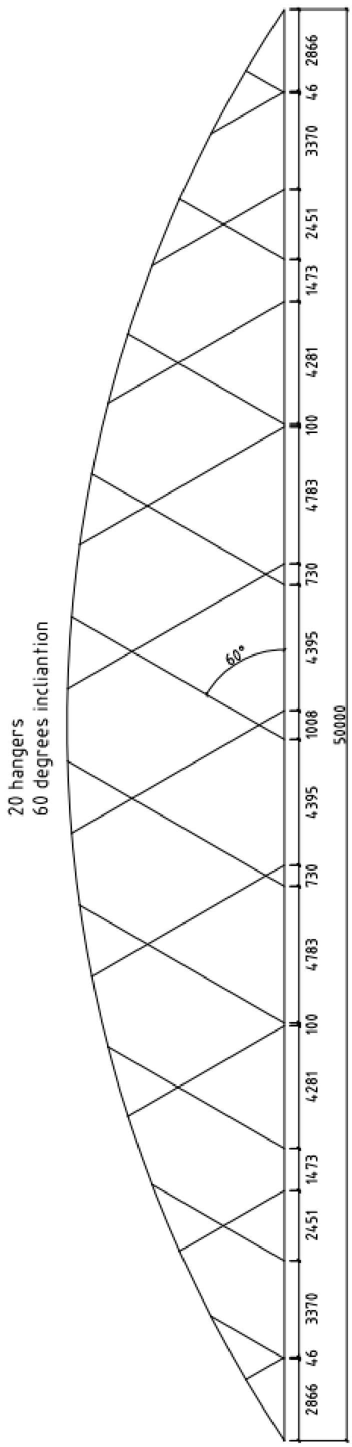
Appendix B – a section sketch of the network arch with a hanger inclination of 45 degrees.



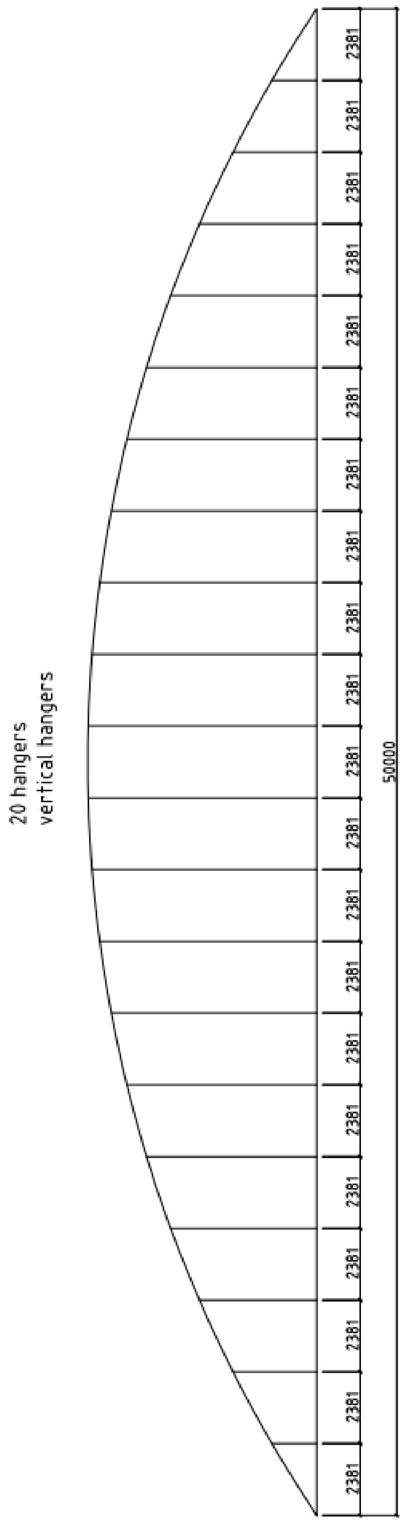
Appendix C – a section sketch of the network arch with a hanger inclination of 55 degrees.



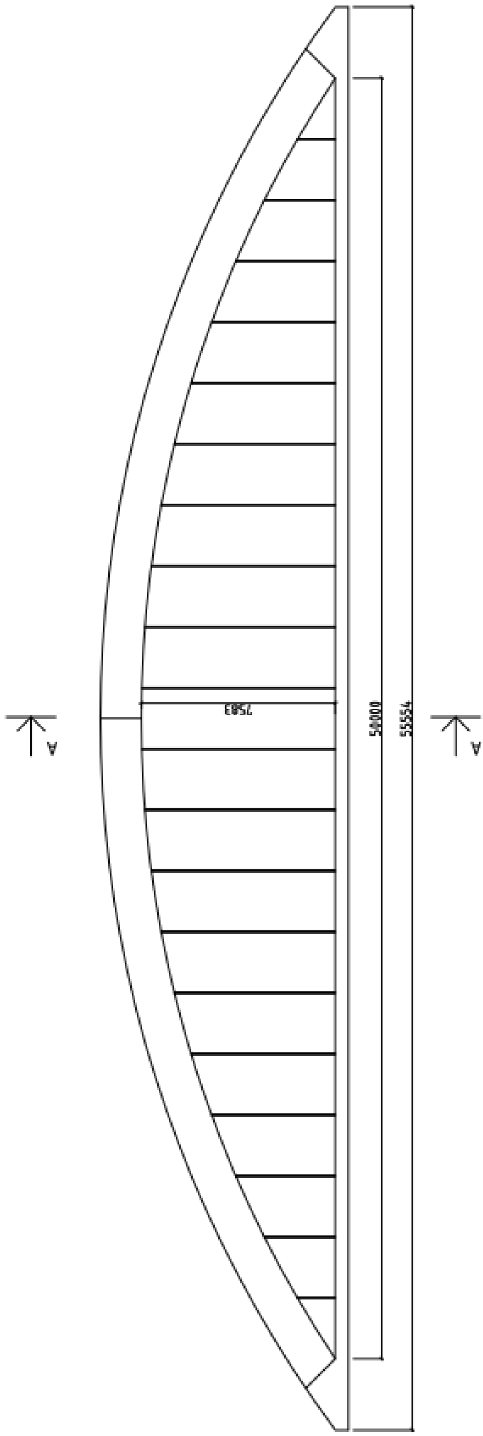
Appendix D – a section sketch of the network arch with a hanger inclination of 60 degrees.

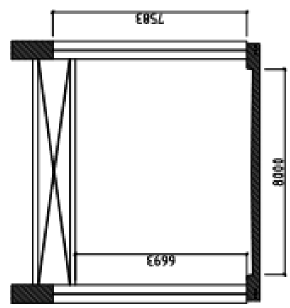


Appendix E – a section sketch of the reference bridge.



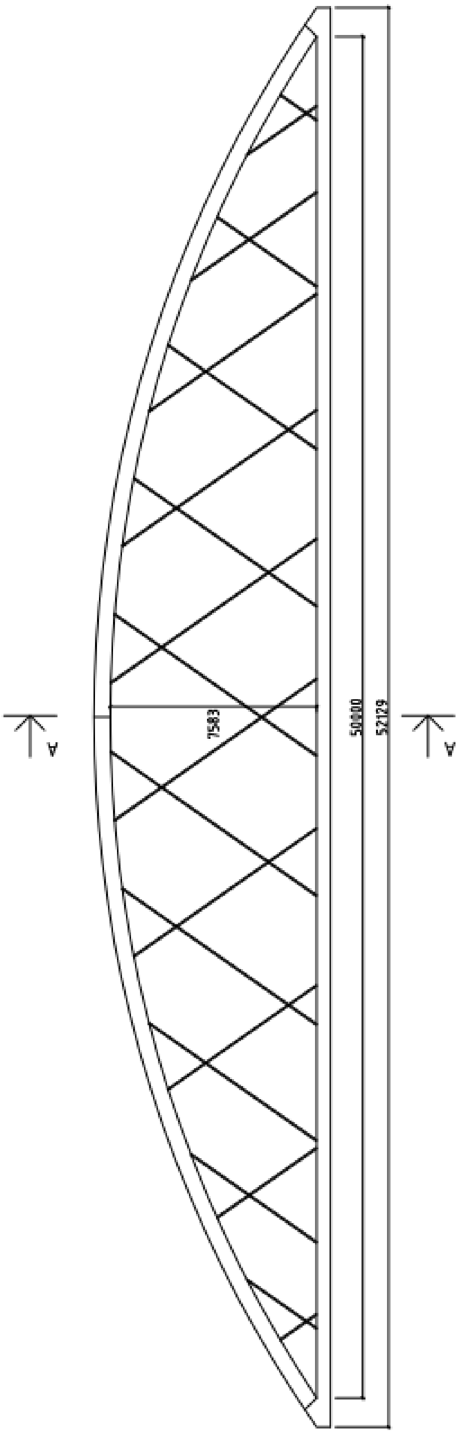
Appendix F – the elevation and the cross-section of the reference bridge that was designed.

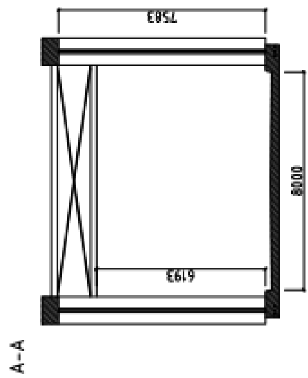




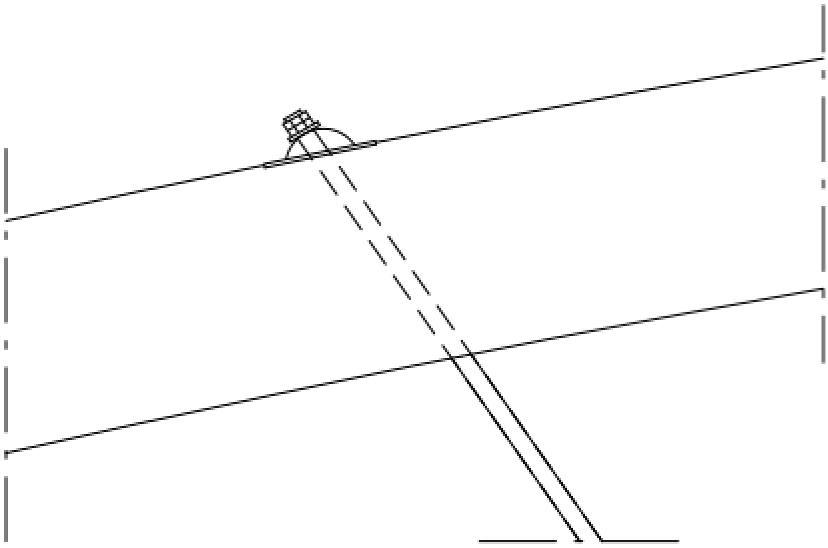
A-A

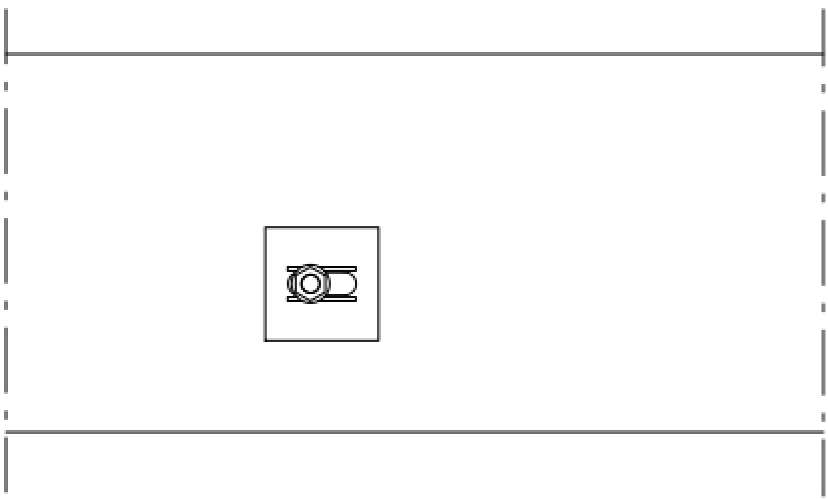
Appendix G – the elevation and the cross-section of the network arch that was designed.





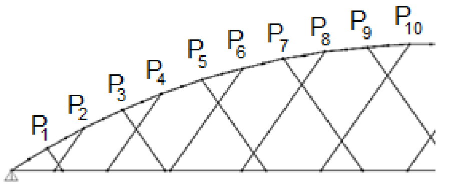
Appendix G – side and top view of the suggested detail between the hanger and the arch



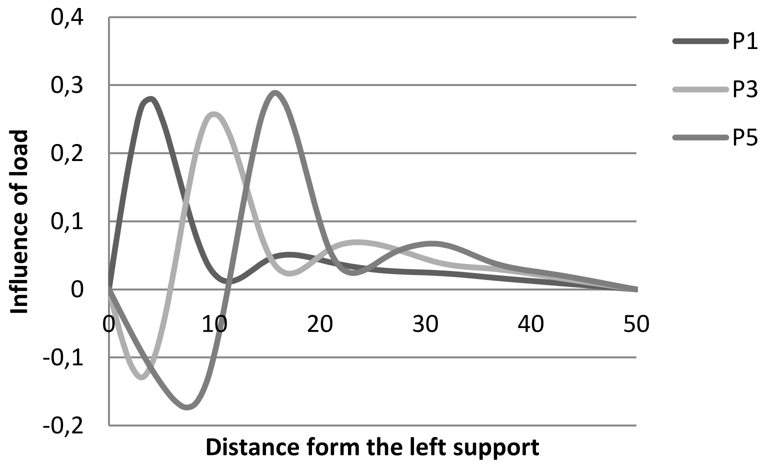


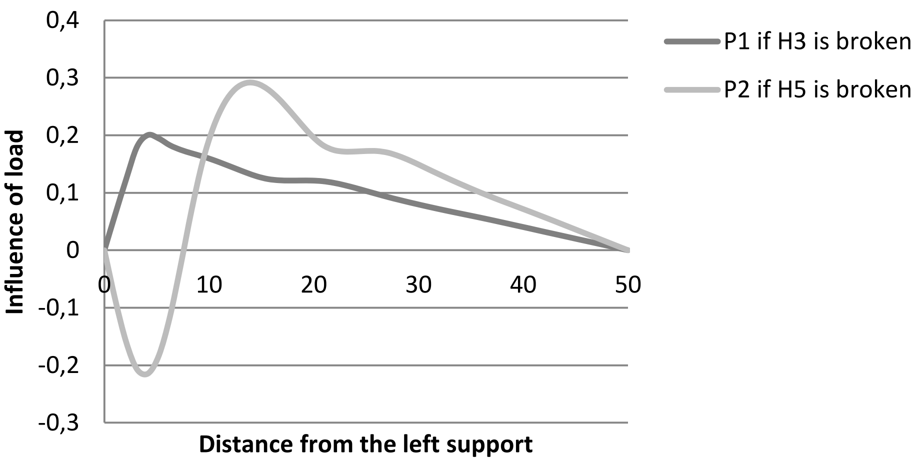
Appendix H – influence lines for a network arch with a hanger inclination of 55 degrees. The arch has the dimensions 1000 x 600 mm^2 and the diameter of the hanger is 48 mm.

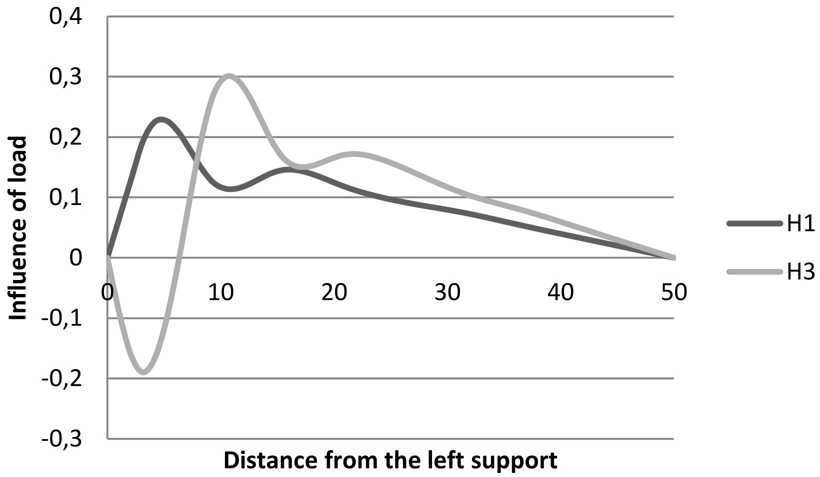
The first two graphs show the influence lines for bending and the rest shows the influence lines for axial force. The influence lines for the hangers have been shown with two influence lines per graph collected so they can be easily compared.

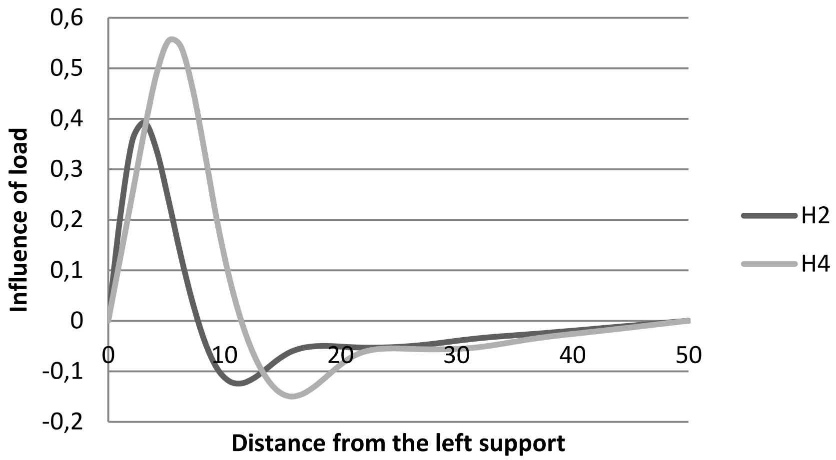


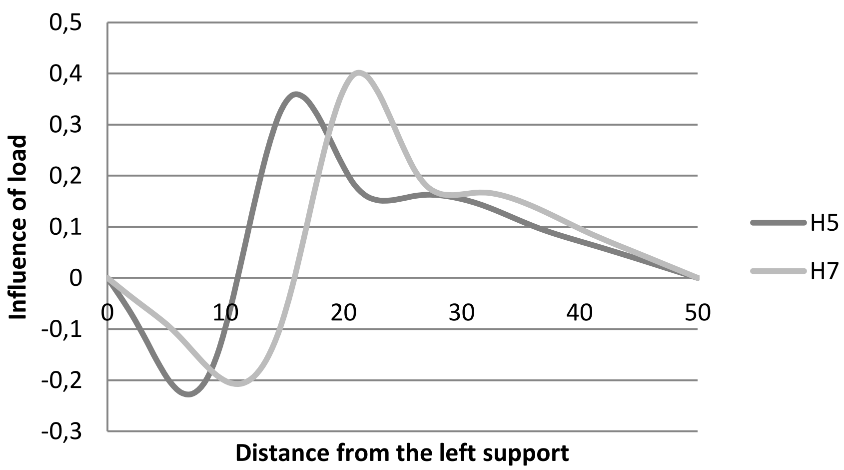
Figur 71 - The annotations used will follow this model, Px is the point of the arch where a hanger connects. Hangers will be similarly named after their connection point (Hx where x is the conection point).

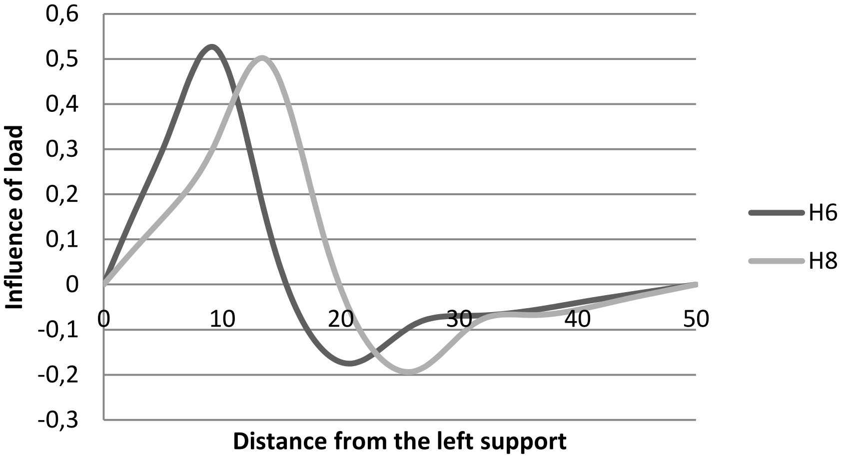


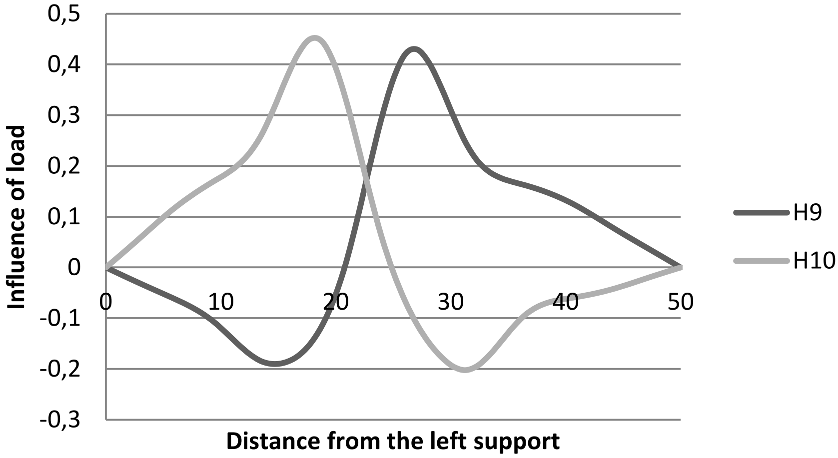


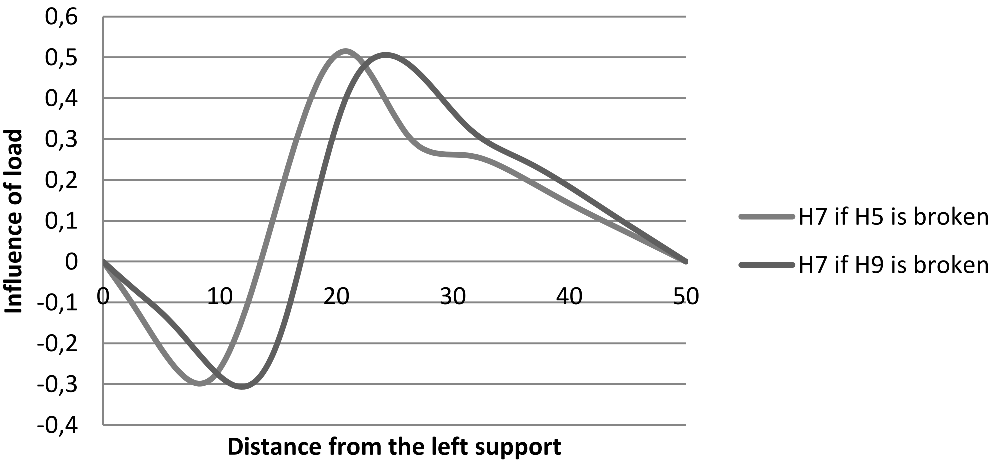

0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
Influence of load
0
10
20
30
40
50
Distance from the left support
H1
H3


0,6
0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
Influence of load
0
10
20
30
40
50
Distance from the left support
H2
H4


0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
Influence of load
0
10
20
30
40
50
Distance from the left support
H5
H7

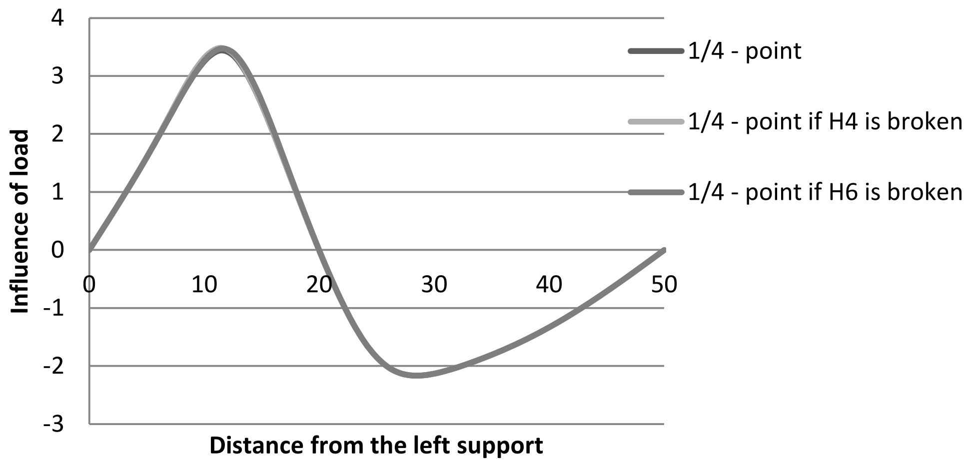

0,6
0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
0
10
20
30
40
50
Influence of load
Distance from the left support
H6
H8

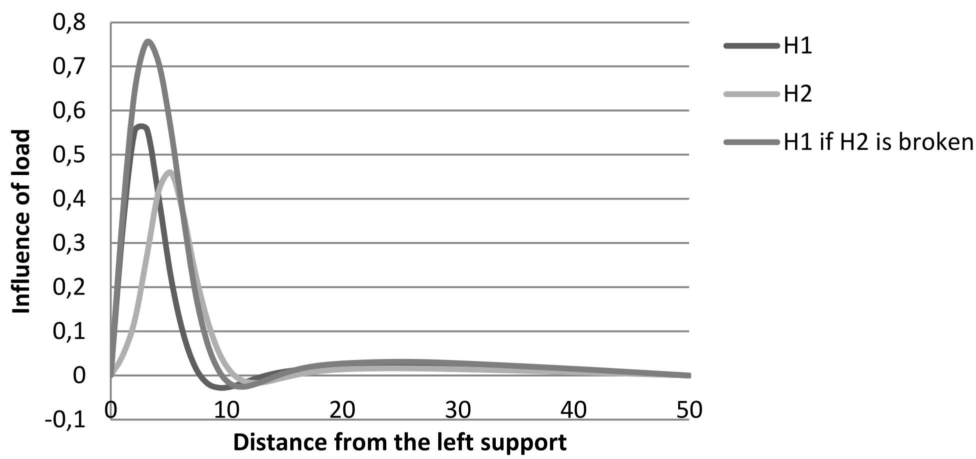

0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
0
10
20
30
40
50
Influence of load
Distance from the left support
H9
H10

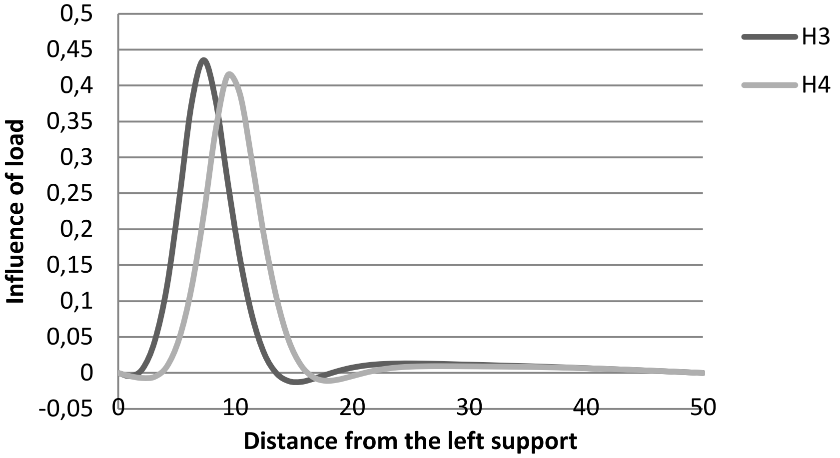

0,6
0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
-0,4
0
10
20
30
40
50
Influence of load
Distance from the left support
H7 if H5 is broken
H7 if H9 is broken

Appendix I – influence lines for reference bridge. The arch has the dimensions 700 x 1600 mm2 and the diameter of the hanger is 48 mm.

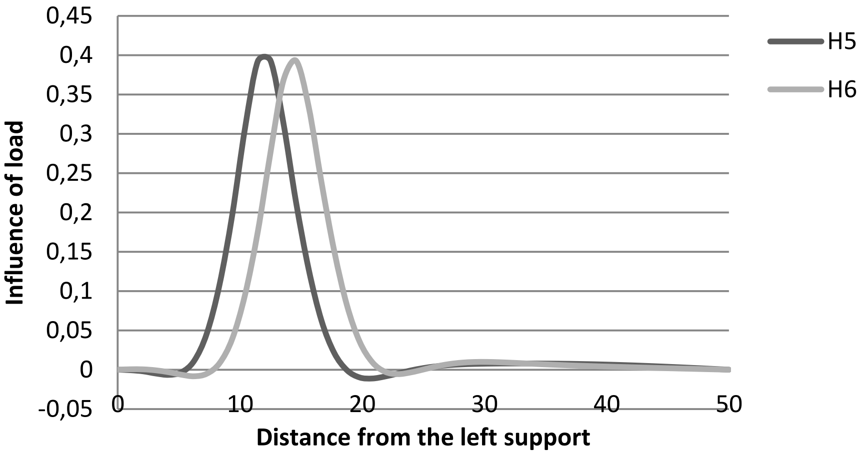
The first graph shows the influence lines for bending and the rest shows the influence lines for axial force. The graphs for the hangers have been shown with two influence lines per graph collected so they can be easily compared.



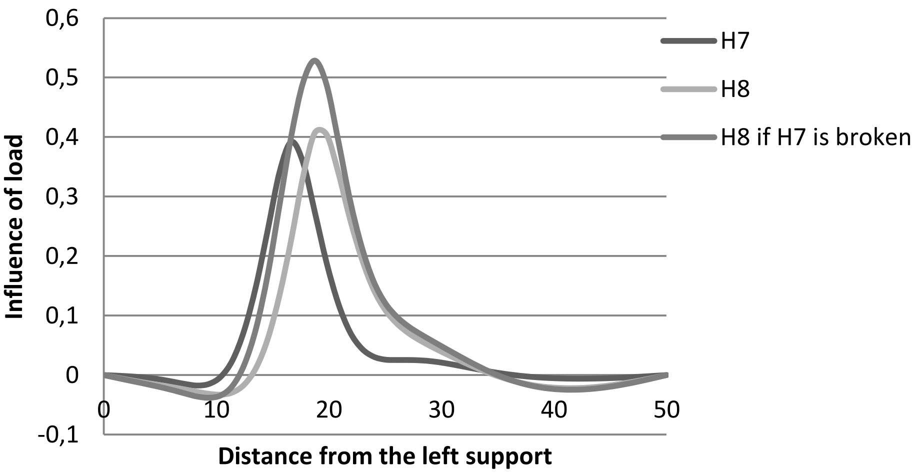




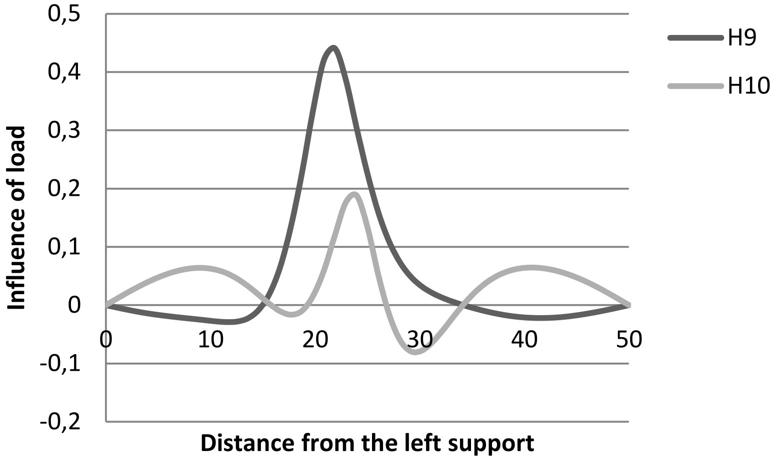
0,5
0,45
0,4
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0
-0,05
0
10
20
30
40
50
Influence of load
Distance from the left support
H3
H4



0,45
0,4
0,35
0,3
0,25
0,2
0,15
0,1
0,05
0
-0,05
0
10
20
30
40
50
Influence of load
Distance from the left support
H5
H6



0,6
0,5
0,4
0,3
0,2
0,1
0
-0,1
0
10
20
30
40
50
Influence of load
Distance from the left support
H7
H8
H8 if H7 is broken


0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
0
10
20
30
40
50
Influence of load
Distance from the left support
H9
H10