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Investigation of friction between concrete slabs and subgrades

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Investigation of friction between concrete slabs and subgrades

Undersökning av friktion mellan betong och mark

By:

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Abstract Concrete slabs were cast on different ground materials to investigate friction between the concrete and the ground. Friction curves are non-linear and bilinear approximations were used to simplify them. This study shows different methods to create the bilinear approximation. Coefficient of friction depends upon the normal load and the ground materials.

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Contents

Preface

This master thesis has been performed for the Department of Structural Engineering at Lund Institute of Technology under control of my supervisor: Dan Pettersson.

The aim of this rapport is to study friction for slabs cast on different types of ground material and to simplify the non-linear behavior of friction curves to an approximate bilinear curve, which will be used as input in FE-calculations.

I give my thanks to my supervisor Dan Pettersson and Professor Sven Thelandersson, head of Division.
I also give my thanks to Mr. Muhammad Alhasani, doctoral student, Mr. Per-Olof Rosenkvist and Mr. Bo Johansson for their kindness and valuable help during my work.

Lund, March 1999

Ali Kawa Kader

Summary

Better knowledge of the frictional behavior for concrete structure on ground is necessary to improve the technique for control of cracking. For this reason friction tests were performed and evaluated. The results will be used in other projects as input values in FE-calculations, e.g. to estimate the restrain in structures.

The friction tests were performed by casting concrete slabs 1,2 m long, 0.8 m wide and 0.2 m high in a mould placed on different ground materials. For example compacted sand and crushed aggregate were used as ground materials. In some tests plastic sheeting were used to cover the compacted sand and crushed aggregate. Tests were also made for slabs on a flat concrete ground covered by asphalt paper and VolClay. Three different values of the uniformly distributed loads were used in the tests.

Friction curves are non-linear and depends on several parameters and the variation seems to be high. The simplest way to describe friction between concrete and ground materials is by the coefficient of friction and this is the most common in literature. The coefficient of friction, μ, has a top value for compacted materials. This top value of frictional force seems to consist of a friction part and a cohesive part. This gives higher coefficient of friction for lower load. The maximum coefficient of friction μ_{max} is between 0.85 to 1.2 for sand, between 1.5 to 3.0 for crushed aggregate, between 1.0 to 8.0 for concrete ground covered by asphalt paper and between 0.8 to 2.5 for concrete ground covered by VolClay.

The growing part of the curves are harder to characterize. Here we are interested in linear approximation. Different methods are used to estimate the value of friction stiffness K_s, such as the secant inclination, the tangent inclination, the mean value from secant and tangent inclination and the mean value from unloading and reloading of the test specimen. K_s is also calculated by equalizing the area under the non-linear curve and the bilinear curve. Most estimation of K_s is laying in the interval 5 to 124 MN/m^3.

This study shows different methods to create the bilinear curve which represent the non-linear friction curve and how to estimate the coefficient of friction, μ, and the friction stiffness, K_s. The coefficient of friction, μ, depends upon the slab thickness (or external load) and the ground materials.

Sammanfattning

Bättre kunskap om friktionsbeteendet för konstruktioner på mark behövs för att förbättra sprickkontrollen för betong. I detta examensarbete har friktionsförsök genomförts och utvärderats. Resultaten används i andra projekt, t ex som indata i FE-beräkningar där tvångsspänningar uppskattas.

Friktionsförsöken utfördes genom att betongplattor med längd 1.2m, bredden 0.8m och höjden 0.2m gjöts på olika grundmaterial. Som grundmaterial användes t ex kompakterad sand och makadam. I vissa av testerna täcktes grundmaterialet med plastfolie. Försök utfördes också med betong täckt med asfaltpapper eller VolClay som grundmaterial. Tre olika nivåer användes på den jämt utbredda lasten i vertikalriktningen.

Friktionskurvorna är olinjära med stor variation och beror av flera parametrar. Det enklaste sättet att beskriva friktion mellan betong och grundmaterial är med en friktionskoefficient, vilket också är det vanligaste i litteraturen. Friktionskoefficienten μ, har ett maxvärde för kompakterat material. Detta värde består av en friktionsdel och en kohesionsdel, vilket ger högre friktionskoefficienter för lägre vertikalbelastningar. Uppmätta värden för maximal friktionskoefficient, μ_{max}, är 0,85 till 1,2 för sand, mellan 1,5 och 3,0 för makadam, mellan 1,0 och 8.0 för betong täckt med asfaltpapper och mellan 0,8 och 2,5 för betong täckt med VolClay.

Den växande delen av friktionskurvan är svår att karakterisera. I detta projekt var målet att skapa linjära approximationer. Olika metoder har använts för att uppskatta friktionsstyvheten, K_s, t ex sekanten och tangenten till kurvorna samt lutningen på kurvan vid av och pålastning. K_s uppskattades också genom att arean under kurvan användes för att skapa en ekvivalent bilinjär kurva. De flesta av uppskattningarna av K_s ligger i intervallen 5 till 124 MN/m^3.

Detta projekt visar att man kan approximera friktionskurvorna med bilinjära kurvor som karakteriseras av friktionskoefficient, μ, och friktionsstyvheten, K_s. De både parametrarna beror bl a på grundmaterial och belastning i vertikalriktningen. Utvärderingen av försöksresultaten kan användas som vägledning för val av parametrarna.

1 Introduction

1.1General

Concrete structures on ground are drying and they contract because of shrinkage, change in daily temperature or of the combination of these cases. The movement is resisted by the restrain from the ground, which can be described as friction for the horizontal displacement of the structure relative to the ground. This produces restraint stress in the concrete structure.

When the tensile stress is higher than the ultimate tensile strength of concrete, the structure will crack. The objective may be either to prevent cracking or to limit the width of the cracks. To entirely prevent cracking can in some cases be very expensive. For these cases cracks may be acceptable but must be limited and well distributed with reinforcement, (Betong handbok).

Restraint stresses can be studied by FE-calculations, which gives the possibility to predict both the maximum tensile stress and its position in the structure. To produce input data to the FE-calculation, friction tests were performed. For this reason concrete slabs were cast on ground and subjected to the horizontal force (P) and the normal load (q), which is shown in figure (1.1). Friction forces were produced between the slab and the ground because of the horizontal force (P) and its amount depends on the coefficient of friction μ and on the uniformly distributed load q, including self-weight.

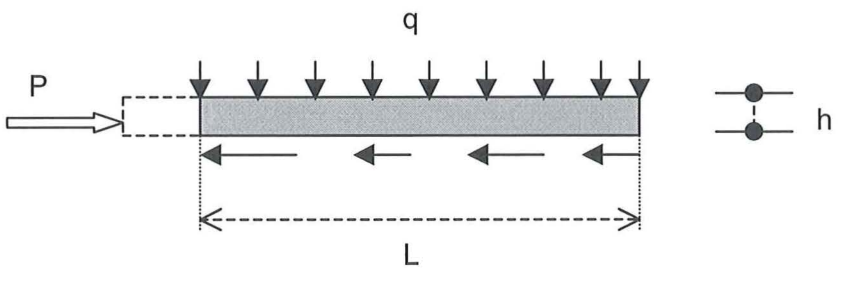


Figure (1.1) the effect of forces on the test spacemen.

$$P = \mu \cdot q \cdot L \qquad (1.1)$$

Where: P = Horizontal force (KN).

μ = Coefficient of friction.

L = Length (m).

q = Normal pressure (KN / m)

h = thickness of the slab (m).

1.2 Scope of this study

The purpose of this study is to:

- Investigate friction between concrete slabs and subgrades of different ground materials.
- Estimate the coefficient of friction.
- Study the friction curves and simplify them.
- Study the effect of uniformly distributed load and different ground materials.

1.3 The contents of this report.

Chapter (2) gives the friction theory and literature review of concrete structure on ground.

Chapter (3) gives the laboratory work, friction test and the testing program.

Chapter (4) reviews the result, curves and the approximation formula for different cases.

Chapter (5) gives the conclusions.

2

Friction theory & literature review

2.1 Friction theories

There are three groups of friction [11]:

1. Dry friction.
2. Fluid friction
3. Internal friction

When surfaces of two solids are in contact the dry friction is produced under condition of sliding or tendency to slide. When close layers in a fluid are moving at different velocities, fluid friction is developed and is not of interest here. Due to the soil deformation and when the soil particles were redistributed again, internal frictions are produced. In case of slabs on ground, the friction will be a combination of dry and internal friction.

2.1.1 Dry friction

At dry friction the resisting force (F) is developed, it is also called Coulomb friction[11]. The friction force (F) is shown in the figure (2.1). When the force (P) is applied to an object, resistance to the movement is produced such as the opposite acting force (F). Up to the sliding starts, the force (P) is growing to a certain limit and then the force (P) is reduced just after the sliding has started.

From figure (2.1), μ, is a coefficient of friction and is generally defined as:

$$\mu = \frac{F}{N}$$

Where:
F = tangential force.
N = normal force from e.g. self-weight.

The coefficient of friction, μ_s, is a static limit before sliding and, μ_k, is the kinetic coefficient of frictions value just after sliding. Both of, μ_s, and, μ_k, can be regarded as constants for a certain combination of materials.

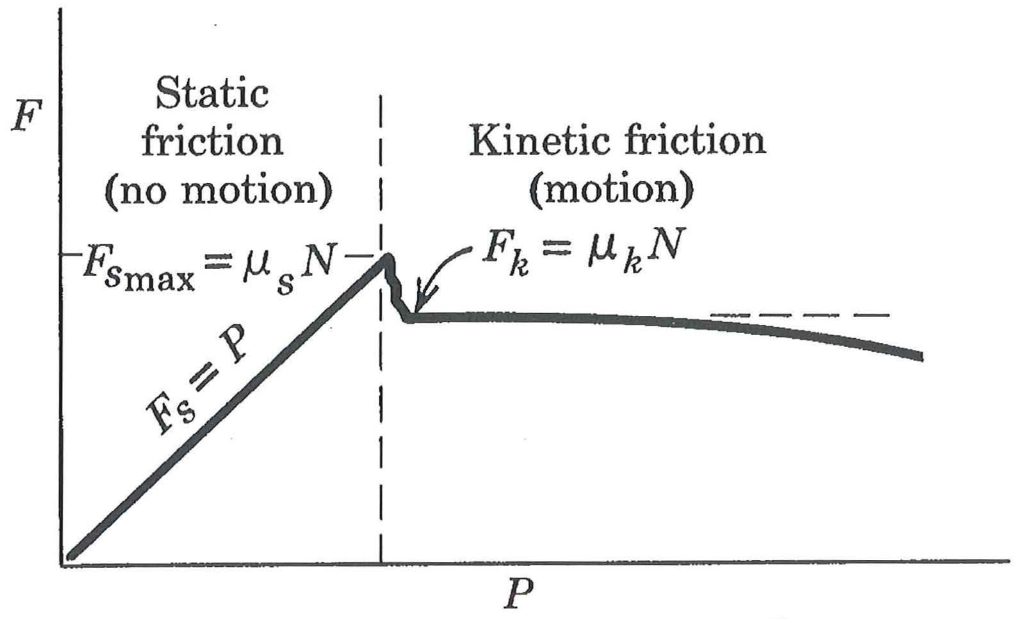


Figure (2.1) Frictional curve for dry friction. Taken from Meriam (1980)[11].

In geotechnical purposes the frictional angle φ are commonly used as an alternative to the coefficient of friction. The frictional angle is defined by:

$$\tan \varphi = \frac{F}{N}$$

2.1.2 Internal friction for ground material

Internal friction is produced when the soil or ground particles are redistributed. The typical curves from shear tests of ground materials are shows in Figure (2.2).

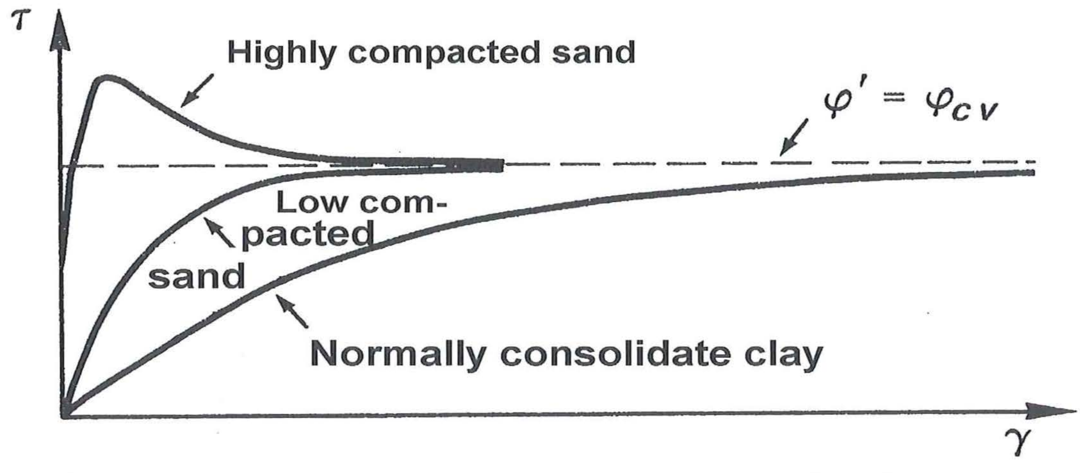


Figure (2.2) internal coefficient of friction as a function of the deformation in Principle, given by shear tests. (Handboken Bygg Geoteknik (1984))[9].

The force will have a peak value for a heavily compacted material, which then drops to a constant value. When the same material is low compacted, the force will raise slowly up to approximately the same constant value, φ_{cv}. Values for internal friction can be found in geotechnical literature. The values in Table (2.1) are taken from Handboken Bygg, Geoteknik (1984) Table G04: 5. They are translated to coefficients of friction and divided into low and high compacted material.

Table (2.1) Internal coefficient of friction taken from Handbook Bygg Geoteknik (1984)[9].

Soil type	Sand	Gravel	Sand moraine	Gravel moraine	Crushed aggregate	Blasted stone
Low compaction	0,53	0,58	0,7	0,78	0,58	0,84
High compaction	0,7	0,75	0,9	1	0,78	1

2.2 Literature review of concrete structures on ground

One of the purposes for measuring the friction between the structure and the ground is to find reasonable values for calculation of stresses in structures. Normally a low value of the friction is desirable for these purposes. A high value is desirable when the movement is not allowed.

Here in this project, the ground is the surface and the concrete structure is the object. Literature dealing with concrete and friction mostly concerns the behavior of the concrete surfaces, e.g. studying the skidding risk and how to prevent slippery road-surfaces, certain need to be sure that a structure can resist a horizontal force without movement.

Coefficient of friction is the simplest way to describe friction between concrete and ground material and this is also the most common in literature. Information about ground material is needed, such as compacting. The effect from the load level must be considered which is shown by friction tests. Normally the value of the coefficient of friction is given as the high value for non-sliding but it can also be the lower sliding friction.

A simplified bilinear curve is used to evaluate the non-linear friction curves given in literature and it is also used for FE-calculations in other projects. To simplify the curves the following guidelines are used. The coefficient of friction for the constant part of the curve, μ_{max}, is chosen as the maximum coefficient of friction. The non-liner growing part is replaced by a linear one, here expressed as modules of subgrade reaction for tangential direction, K_s.

Three extreme ways to express these modules are shown in Figure (2.3). K_{s1} gives a minimum value, which is the secant between the starting point and the point for the maximum friction. The other two module, K_{s2} and K_{s3}, can be regarded as maximum values. K_{s2} is shown approximately as a tangent for the start of the curve. K_{s3} is created for a structure loaded to sliding, the unloaded and reloaded again. This gives the same effect as compaction of the material.

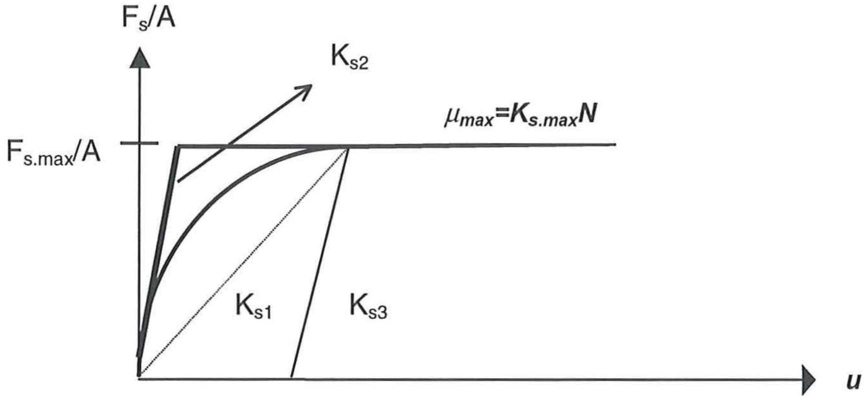


Figure (2.3) Three ways to express the modules of subgrade reaction (F_s =tangential force, A= area for the contact zone, u= displacement).

Some values of coefficient of friction are given In Betonghandbok Material (1994)[6] and Betonghandbok Arbetsutförande (1994)[5]. For plastic sheeting on sand μ_{max}=0.75 is assumed, for crushed aggregate μ_{max} is assumed to be greater than 2.0.

Friction curves are shown by Dorell & Nordberg (1993)[8], where gravel 16 - 32 mm was used as ground material. The curves have the same appearance as for compacted materials in Figures (4.1-4.6). The tests were performed with slabs with a contact area of approximately 1 m^2. The ground pressure was about 4.5 kpa. The deformation for the maximum friction was between 0.5 and 10 mm. For gravel, μ_{max} was between 1.5 and 2.9. K_s can be approximated with maximum and minimum values according to Figure (2.3) to be 1 - 10 MN/m^3. For plastic sheeting on gravel μ_{max} was 0.9 and K_s 0.7 - 3 MN/m^3.

In Denmark a full-scale test was performed, when the bridge across Lillebælt was built[4]. Compacted gravel was used as ground material. The contact area was about 41 m^2 and the structure was 3,5 m thick. The ground pressure was between 80 and 160 kpa. The coefficient of friction was growing non-linearly up to a maximum value of about 0.9. It was reached for deformations of about 20 mm. It is interesting to notice that the top value is not pronounced even though the ground material was compacted. This can be an effect from the high load level. The modules for subgrade reaction, K_s, estimated according to Figure (2.3), was 5 - 30 MN/m^3.

The coefficient of friction (given by Bergström)[13] was assumed to be between 3.6 for slabs with thickness of 50 mm and 2.3 for thickness of 200 mm in Bergström (1950)[13]. The type of ground material was not given. The ground pressure was low, about 1.2 and 4.8 kpa and this can explain the large variation of the friction. The same values were used by Losberg (1960)[10] in his calculations. The values were taken from Americans tests.

The ACI Committee 325 (1956-57)[2] gives American friction tests for pavement from the 20´ies to 50´ies. Figure (2.4) shows a simplified friction curves. The non-sliding friction coefficient is growing for displacement up to u_s, which was shown to be 0.5 - 2.5 mm. The limit for the non- sliding friction, μ_s, is higher for thinner slabs. For the coefficient for sliding friction, μ_k, a value of 1.5 was recommended.

The proportional constant k, given in Figure (2.4), varies between 9 and 45 $mm^{-0.5}$. For thinner slabs k is higher. The ground material also influences the value of k. Average silt loam and clay subgrade gives higher values than sand. Higher values are also given for highly cohesive subgrades and stiff compacted gravel. According to the given information and Figure (2.3) K_s, is supposed to have values in range 2 - 50 MN/m^3.

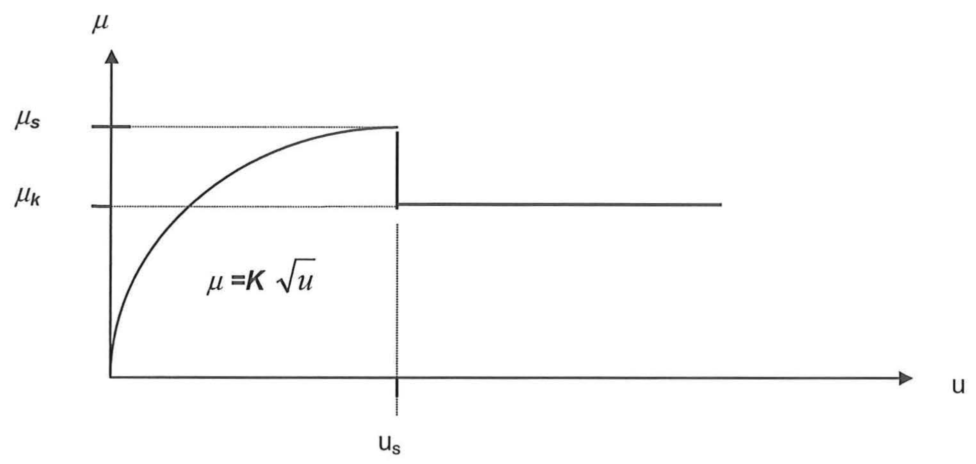


Figure (2.4) Simplified friction curve according to ACI Committee 325(1956- 57)[2].

Friction is treated as a coefficient of friction in ACI Committee 360 (1992) regarding design for slabs on grade.

Different ways to describe the friction curve are given by Schütte (1996)[3]. Bilinear curve is the simplest form. Polygon with three parts is another way to describe the curve. First a linearly increasing part, then a constant part and finally a decreasing part. Each part can be divided into subparts.

In (1988) extensive research on friction for structure on ground made by Kolb[15]. The non-dimensional friction curve given in Figure (2.5) was suggested. This valid for sand and gravel with smaller particles. The curve is based on friction tests with smaller and larger specimens by the author and others.

Actual coefficient of friction versus maximum coefficient of friction (μ/μ_{max}) is expressed as the y-axis and actual displacement versus a reference displacement (u/d_v) as the x-axis. The maximum coefficient of friction, μ_{max}, is given by:

$$\mu_{max} = 0.59 + 0.09.\ln\left(\frac{d_{50}}{d_v}\right) + 0.0372.\ln\left(R_R\right) + 0.561.D - 0.108\left(\frac{p}{p_a}\right)$$

Where:

d_{50} = mean size of the ground material given in mm
d_v = referenda value, 1 mm
R_R = relative coarseness = 0.1 smooth, = 1.0 high coarseness
D = compaction of the ground material (0.3 to 0.7 were recommended)
p = ground pressure
p_a = reference value , 100 kpa

The sliding friction for large displacement will reach the value given by:

$$\mu_{sl}=\left(1-\frac{\sqrt{D}}{4}\right)\cdot\ \mu_{max}$$

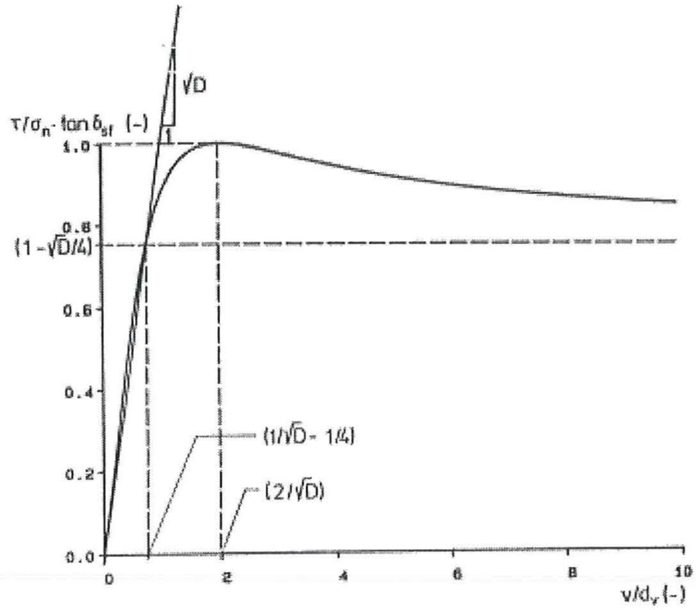


Figure 2.5 Non - dimensional friction curve according to Kolb (1988)[15].

For slabs on sand with ground pressure of 4.8 and 24 kpa with high coarseness, the values for μ_{max} are given between 0.73 and 0.98. The modules of subgrade reaction in the tangential direction, K_s, estimated for sand with the non-dimensional curve gives values between 5 - 16 MN/m^3. The same assumptions as for the coefficient of friction were used.

3 Laboratory work

3.1 Laboratory work and friction tests

To increase the knowledge of the friction behavior and the parameters, which influence them, friction tests were performed. All slabs for the performed tests have the same size, and were molded on different ground material. The size-effect on the curve from the ground pressure was an important parameter to study. The number of tests was limited to a few tests for each ground material. I also used some test results, which are done by Dan Pettersson[1].

Because of limiting number of tests no exact values for the characteristic parameters are recommended. The result can just be regarded as an indication of the size of the parameters.

The test equipment, the ground materials, the concrete used for slabs and the testing program will be describe in this capital.

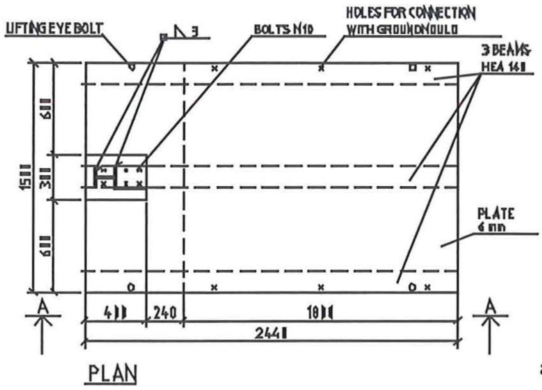
3.2 Test equipment

The test equipment, which is ready for test, is shown in figure (3.1). A box of length (1.4m), width (1.0m) and height (0.2m) was loaded with ground material and it was thoroughly compacted. A concrete slab (1.2m) long,(0.8m) wide and (0.2m) high was cast in a mold placed on the ground material. A horizontal compression force was applied to the hardened slab after one week by means of a hydraulic jack in a rate of 3.7 KN/sec. The force and deformations were measured.

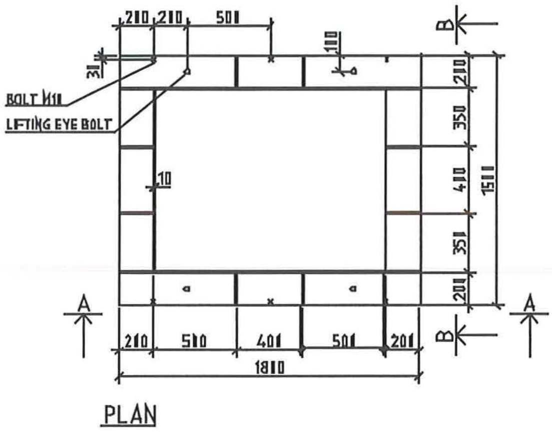
Figure (3.1) Test equipment prepared for testing.

Two plans and one section of the test equipment are shown in figure (3.2):

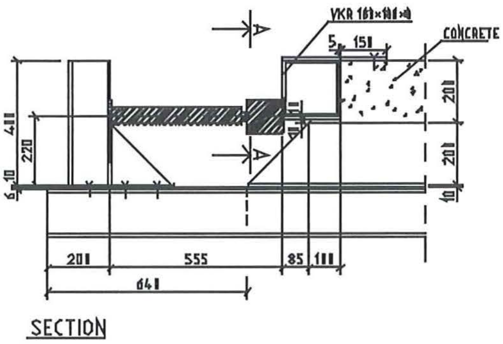
- Plan (a) shows the bottom board with the fixed holding-on tool where the hand operated jack is placed. A box containing the ground material is also fastened on this bottom board which is shown in the second plan. Because it is not fixed it can be turned around and tests with reverse direction of the force can be made.
- Plan (b) shows a casting model, which is placed on the ground model when the slab was cast.
- Section (c) shows the test equipment ready for testing. The jack is placed at the holding-on tool, the horizontal force is transmitted to the slab by the arrangement shown in the figure.



a)



b)



c)

Figure (3.2) Plans and section for the test equipment[1] a) bottom board b) Box for ground material. c) Prepared for testing

The measuring points for force and the different displacement are shown in Figure (3.3):

- Channel (1) measures the force
- Channels (2-7) measure different deformation.

The different deformations give an overview of the possible and unwanted rotation.

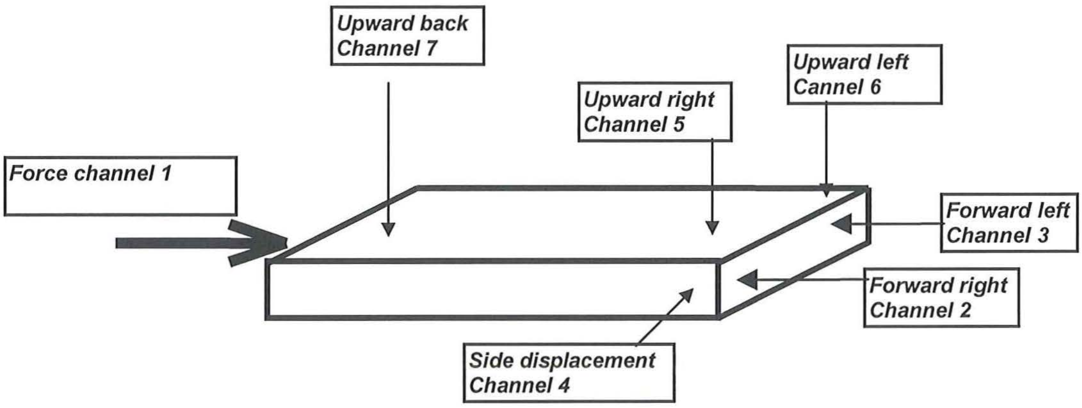


Figure (3.3) Measuring points for force and different displacement.

The displacement in the direction of the force is used for creating the curve for friction as function of the displacement and is thereby the most interesting displacement. Channel (2) and (3) measures these displacements and the mean value is used for the friction curves. It was found in all tests that there is no significant difference between the two channels. The side displacement measured in channel (4) gives very low values for all the cases, which also indicate that no significant side rotation will influence the result.

Channel (5, 6 and 7) measure the displacements upward in different position at top of the slab. The values are normally low except for tests with ground material with larger particles as crushed aggregate, which gives a nearly uniform lifting of the slab. None of the tests give any significant rotation. The unwanted displacement was low for all the tests.

3.3 Ground materials

The tests were concentrated on:
1. Compacted sand.
2. Compacted crushed aggregate.
3. Flat concrete ground covered with asphalt paper.
4. Flat concrete ground covered with VolClay.

The compacting was made in the same way for all the tests for sand and crushed aggregate. Sand was chosen because it is a commonly used ground material and the literature gives possibility for comparison. Crushed aggregate is also a common material but the information of the friction curve is almost non-existing. A plastic foil over ground material (sand and crushed aggregate) was used in some of the tests. Another ground material is concrete covered by asphalt paper which is commonly used to prevent the moisture and concrete covered by VolClay. Fiber foil is also a common foil, but it was not tested in this limited testing program. The influence from the two types of foil was regarded to be almost the same.

The sand used was a mineral soil with coarse sand and medium sand as the dominating parts. This means particles from (0.2 to 2 mm). The macadam consists of mineral soil of quartzite. The particles can be described as eve-grained soil of medium gravel (11 to 16 mm).
The compacting was made according to MarkAMA 83, with a vibrating tamper of weight about (70 kg). The ground material was run over (3 times) to be compacted according to table C/5.

3.4 The concrete used for slabs

The concrete recipe was chosen to correspond to normal concrete for slabs regarding strength and consistency, and the same recipe was used for all the tests. The consistency is important when casting on ground material with larger particles. A floating consistency gives a thick mix of concrete and ground material, and a more solid consistency gives a thinner mixed layer. The used concrete recipe gives a viscous mix measured according to table 8.5:2 in Betonghandbok Material (1994). The recipe used was:

Standard Portland Cement:		33.48 kg
Water:		23.40 kg
Gravel	0-8 mm	112.20 kg
Macadam	8-12 mm	56.04 kg
Macadam	12-16	56.04 kg

The starting point was to create a concrete corresponding to K30 in Swedish standard. Test with specimen show that the tensile strength was at least as needed to correspond to K30.The concrete was casted in a mold and sealed under hardening to prevent shrinkage with a gradient.
The tests were performed after 7 days of hardening. The strength was about (80%) of the (28-days) strength. The influence on the friction curve from the strength can be regarded as very low.

3.5 The testing program

Eighteen test specimens (slabs) and the performed types of testing are given in table (3.1). Explanation for some word in table (3.1) are given:

- Compacted means that this is the first test for the specimen after casting on the compacted ground material.
- "Reverse direction " means that the ground box is turned around after a complete test and a test with reverse direction of the force is performed. The ground material then acts as a low compacted material.
- For the " 2nd reverse direction " the procedure with turning the ground box is repeated and the material again acts as a low compacted under testing.
- Reloading means that unloading is done after the full horizontal force is reached and then reloading is done. This gives the possibility to measure the modules of subgrades reaction in the tangential defined as K_{s3} in Figure (2.3).
- 1*SW means that the vertical load is just the self-weight from the slab. In analogy 3*SW means that the vertical load is three times the self-weight of the slab.

The type of ground material is given in the Table as well as if plastic sheeting is used.

Table (3.1) the testing program.

Ground material	Compacted		Reverse direction		2^{nd} reveres direction	
	(Fwd)		(Bwd)		(2^{nd} Fwd)	
	Loading	Reloading	Loading	Reloading	Loading	Reloading
Crushed aggregate						
1*SW *	●					
3*SW *	●	●	●	●	●	●
5*SW	●	●	●	●	●	●
Crushed aggregate						
+Plastic sheeting						
1*SW *	●	●				
3*SW *	●	●	●	●	●	●
5*SW	●	●	●	●	●	●
Sand						
1*SW *	●	●	●			
3*SW *	●	●	●	●		
5*SW	●	●	●	●	●	●
Sand+ Plastic sheeting						
1*SW *	●		●	●	●	●
3*SW *	●	●	●	●	●	●
5*SW	●	●	●	●	●	●
Asphalt paper						
1*SW	●	●	●	●	●	●
3*SW	●	●	●	●	●	●
5*SW	●	●	●	●	●	●
VolClay						
1*SW	●	●	●	●	●	●
3*SW	●	●	●	●	●	●
5*SW	●	●	●	●	●	●

*I used the test results from eight tests, which was done by Dan Pettersson[1]. These tests are as follow:

- Compacted crushed aggregate with 1*sw and 3*sw
- Compacted crashed aggregate covered with plastic sheeting with 1*sw and 3*sw
- Compacted sand with 1*sw and 3*sw
- Compacted sand covered with plastic sheeting with 1*sw and 3*sw.

4 Results

4.1 Test results and curves

In this section the friction curves are plotted for three different amounts of loading on different ground materials and the conclusions are given for each case. Bilinear description of friction curves has been declared and different ways used to approximate friction curves and the bilinear curves are shown. Approximation values for friction stresses are described and all diagrams between shear stresses and normal pressure are plotted. The influences of different parameters such as loading and different ground material are studied.

4.2 Friction curves and the coefficient of friction in general

The Figures (4.1) to (4.6) show curves for each specific ground material, such as sand, crushed aggregate, sand and crushed aggregate covered by plastic sheeting, concrete ground covered by asphalt paper and concrete ground covered by VolClay. Three amount of normal loads were used:

- 1*SW, using only slabs self weight.
- 3*SW, using three times slabs self weight.
- 5*SW, using five times slabs self weight.

4.2.1 Friction curves for sand

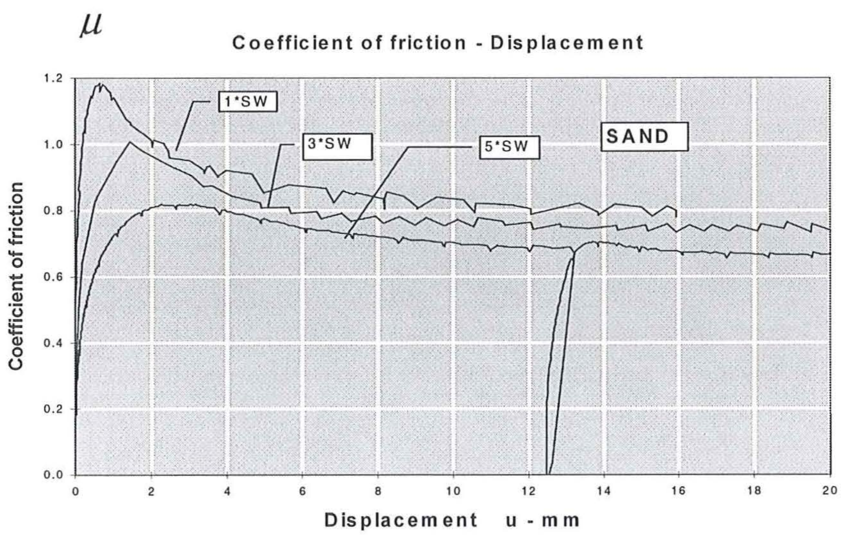


Figure (4.1) Friction curves for test specimens on sand.

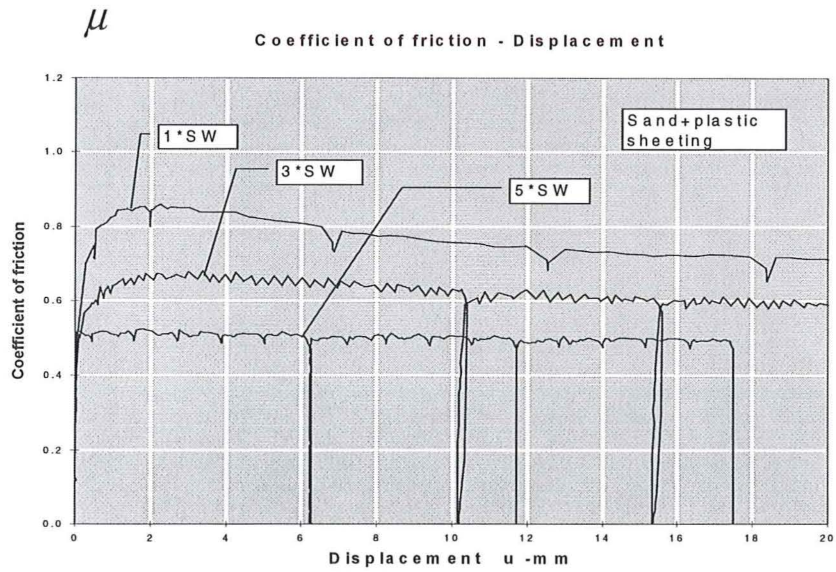


Figure (4.2) Friction curve for test specimens on sand covered with plastic sheeting.

Conclusions from Figures 4.1 and 4.2:

- When the ground material is sand, the coefficient of friction is higher for lower ground pressure, which means that when we increase the normal load the coefficient of friction decreases.
- The coefficient of friction is lower when plastic sheeting covers the sand.
- The coefficient of friction at large displacements for sand is more or less independent of the normal pressure. This tendency is less pronounced for sand covered by plastic sheeting.

For low compacted materials the coefficient of friction grows slowly with increasing displacement to an approximately constant value which was clear when the slab were loaded in reverse direction but it is not shown here. This is in agreement with Figure (2.2).

4.2.2 Friction curves for crushed aggregate

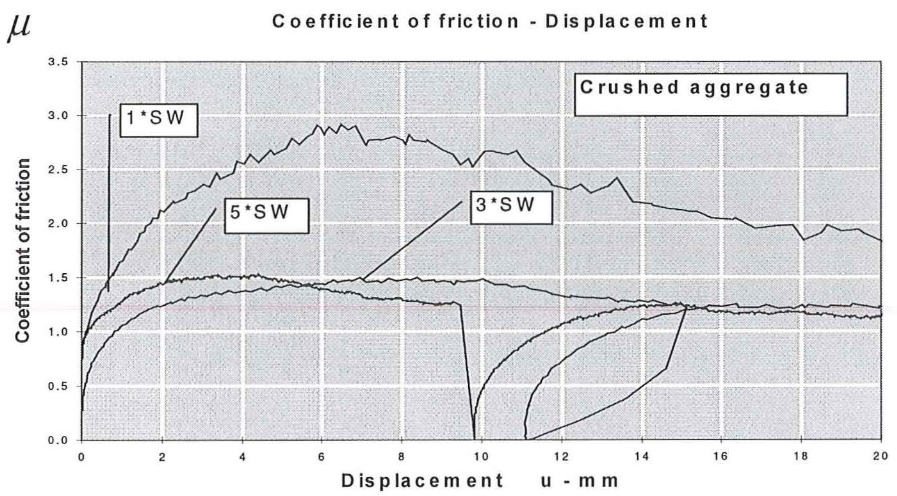


Figure (4.3) Friction curve for test specimens on crushed aggregate.

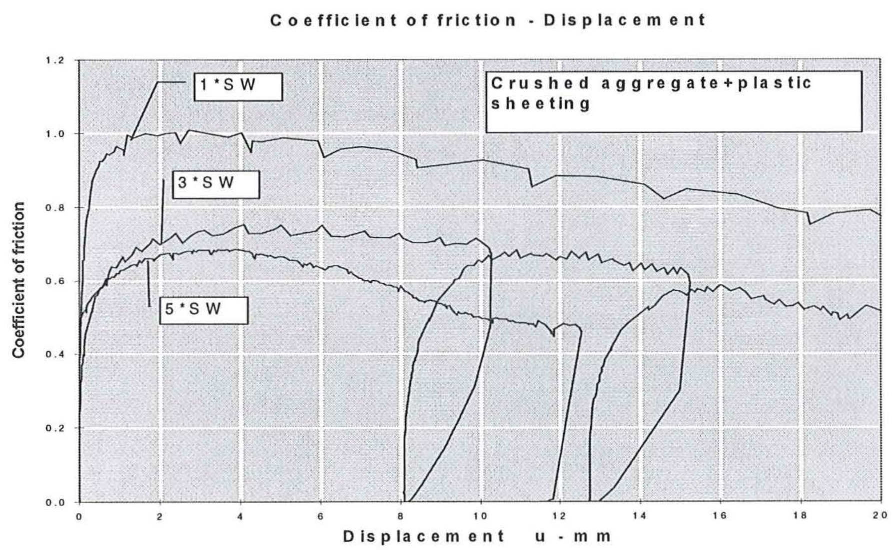


Figure (4.4) Friction curve for test specimens on crushed aggregate covered with plastic sheeting.

Conclusions from Figures 4.3 and 4.4:

- The coefficient of friction for crushed aggregate is higher than for sand, which means that the coefficient of friction increases when the size of the particles of the ground material increases.
- Larger displacement is needed to reach this maximum value of the coefficient of friction, but in case of plastic cover on crushed aggregate, the displacement needed is lower.
- The coefficient of friction for crushed aggregate is influenced significantly from the ground pressure.
- The plastic cover decreases the coefficient of friction.

4.2.3 Friction curves for concrete ground covered by asphalt paper

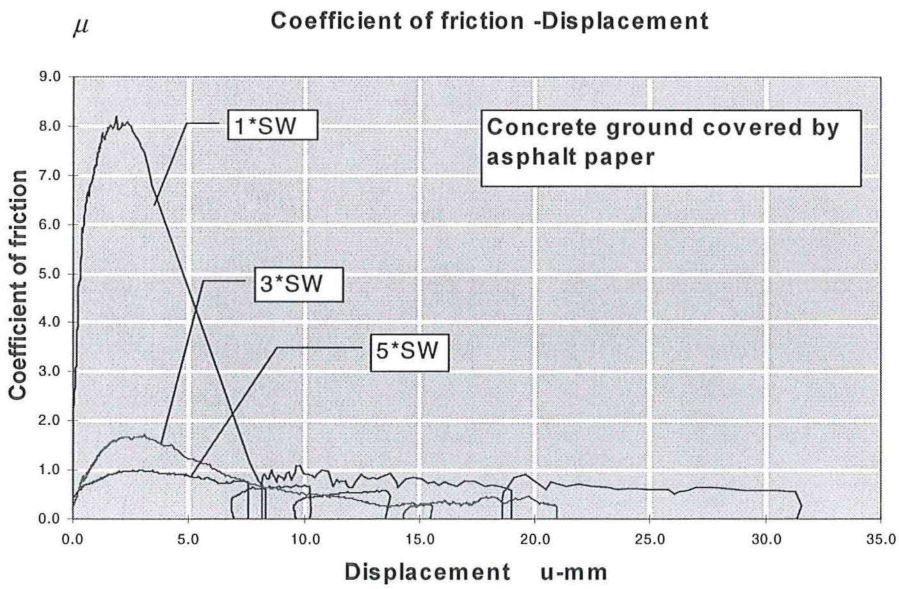


Figure (4.5) Friction curves for test specimens on concrete ground covered by asphalt paper.

Conclusions from Figures 4.5:

- With increasing the ground pressure the coefficient of friction decreases.
- The behavior of the curve for 1*SW is different with comparison to the curves for the other ground materials because of the high value of friction coefficient. May be it is because of the property of asphalt, which is sticky material, and try to bind the two surfaces. Also may be, it is because of the rate of loading for the case of 1*SW was about three time faster than for the other cases. Therefor it needs more tests on asphalt paper to reach a reasonable results.

4.2.4 Friction curves for concrete ground covered by VolClay:

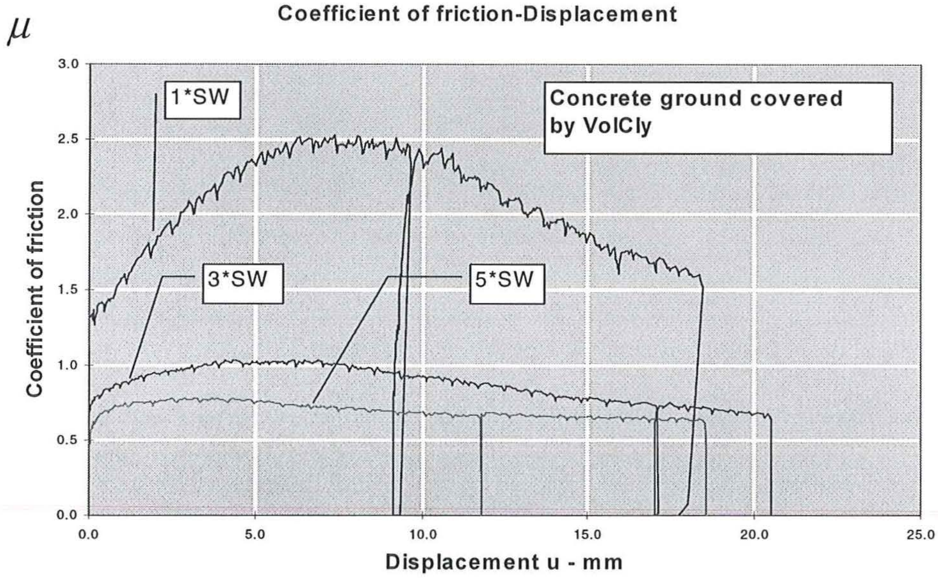


Figure (4.6) Friction curves from test specimens on concrete ground covered by VolClay.

Conclusions from Figures 4.6:

- The coefficient of friction increases with decreasing ground pressure.
- It needs larger displacement with respect to sand and asphalt paper to reach the maximum coefficient of friction.
- For large displacement the coefficient of friction decreases, which is different to the other cases. It needs more tests to reach a reasonable value.

4.2.5 The Conclusions in general:

- Compacted materials have a pronounced peak value for the coefficient of friction.
- Increasing ground pressure decrease the maximum coefficient of friction, which means that it can not be regarded as a material parameter.
- Increasing size of the ground particles increases the coefficient of friction.
- Plastic sheeting between the ground material and the slab decreases the coefficient of friction and almost eliminate the influence of the size of the particles.
- For large displacement the coefficient of friction almost reaches a constant value.

The tests show that the maximum coefficient of friction cannot in general be described as a material parameter, especially for crushed aggregate and VolClay. For large displacement for sand can be regarded the coefficient of friction as a material parameter, but for the other cases there is more or less shown tendency that the coefficient of friction depended on the normal pressure. This can probably be explained by cohesive forces caused by casting the concrete directly on the ground material.

4.3 Effect of the normal load on coefficient of friction

In this section the effect of the normal load and the ground materials on the coefficient of friction are described.

The relation between shear stresses from test results for different ground materials and the normal loads for the three loading cases are shown in figures (4.7-4.12). This relation can be expressed as a straight line. The inclination of this line, μ_0, is equal to the coefficient of friction due to the external load (pure coefficient of friction). The total friction stress is equal to the summation of cohesive shear stress, τ_0, and the shear stress from the movement of the slab on the ground. From this relation and the diagrams in the Figures (4.7-4.12), we come to an approximation in general such as:

$$\tau = \tau_0 + \mu_0 \cdot \gamma \cdot h \cdot f \qquad (4.1)$$

Where:

τ = Friction shear stress
τ_0 = Cohesive shear stress
μ_0 = Pure coefficient of friction.
γ = Concrete unit weight (= 0.024 MN/m^3)
h = Slabs high
f = weight factor depending on the number of slabs which used
γ. h. f= Normal weight of concrete slabs

From tests we have: $$\tau = \frac{F_{est.}}{A} = \mu_{est.} \cdot f \cdot \gamma \cdot h \qquad (4.2)$$

Where:

$F_{set.}$ = Estimated friction force (KN.)
$\mu_{est.}$ = Estimated coefficient of friction
A = Slabs area (KN/m^2)

From (4.1) and (4.2) we have: $$\mu_{est.} = \frac{\tau_0}{f \cdot \gamma \cdot h} + \mu_0 \qquad (4.3)$$

This equation gives the relation between the estimated total coefficient of friction, which equal to the summation of cohesive friction and friction from external load. Here $\gamma \cdot h$ is constant, which means that the total coefficient of friction depend upon the μ_0 and $\frac{1}{f}$.

This means that the total coefficient of friction ($\mu_{est.}$) increases if the slab thickness decreases. The values for, τ_0, and, μ_0, are summarized in table (4.1). From equation (4.3) and table (4.1) we can estimate total value of coefficient of friction, μ_{est}, for different ground materials.

Table (4.1) approximate value for initial friction stresses and coefficient of friction

Case	τ_0 initial friction stresses	μ_0 coefficient of friction
Crushed aggregate	8.9738	1.1264
Crushed agg. + plastic sheeting	2.1275	0.6125
Sand	2.6189	0.7058
Sand + plastic sheeting	2.6418	0.4223
Asphalt paper	38.337	-0.8017
VolClay	10.02	0.3578

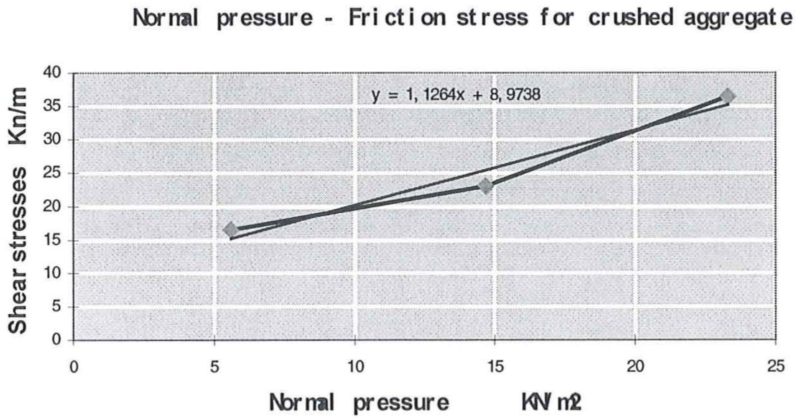


Figure (4.7) Relation between shear stress and normal pressure for crushed aggregate

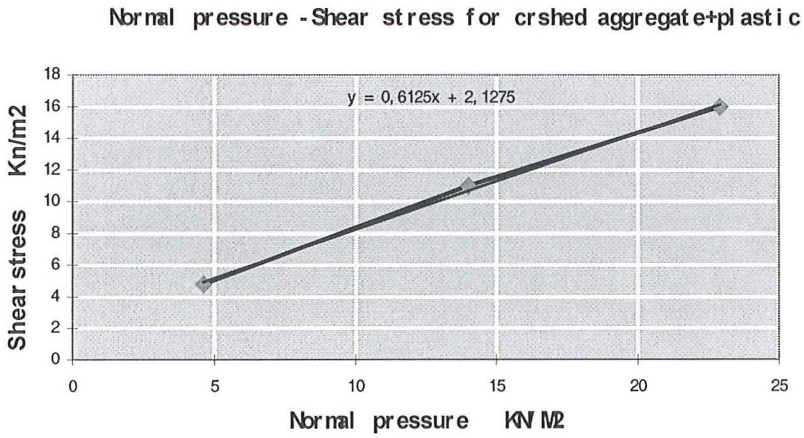


Figure (4.8) Relation between shear stress and normal pressure for crushed aggregate covered with plastic sheeting

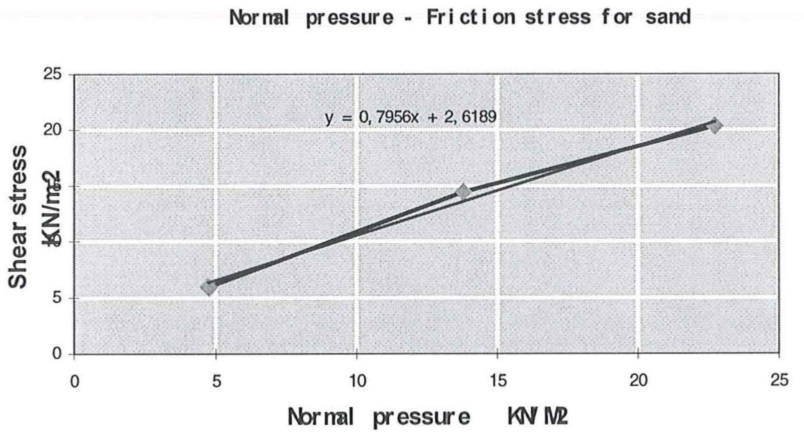


Figure (4.9) Relation between shear stress and normal pressure t for sand

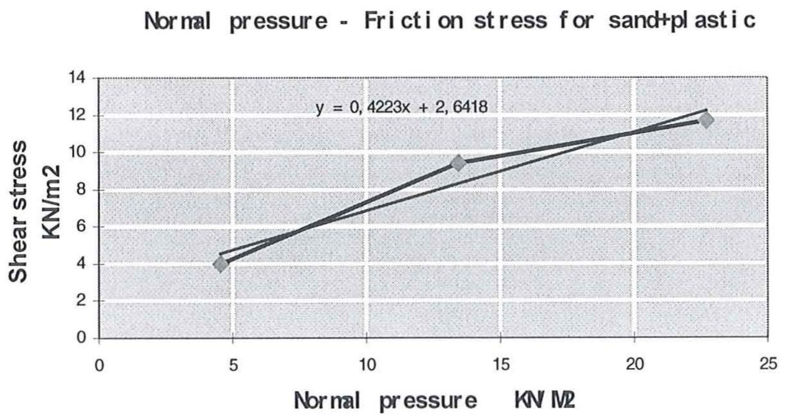


Figure (4.10) Relation between shear stress and normal pressure for sand covered with plastic sheeting.

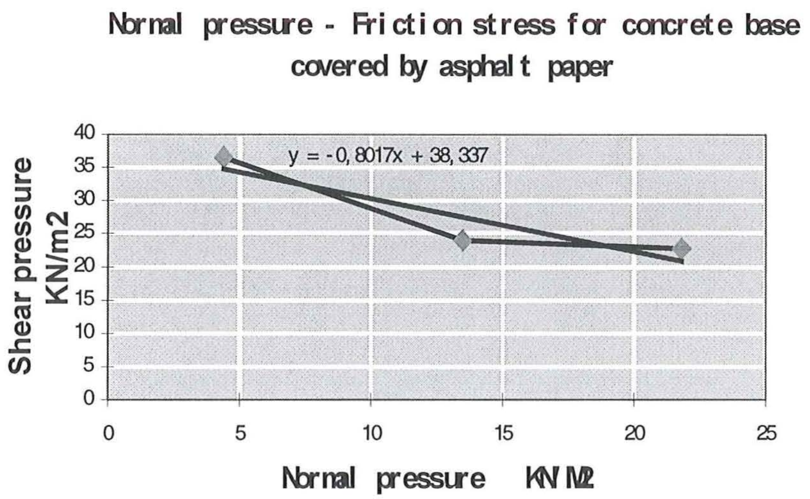


Figure (4.11) Relation between shear stress and normal pressure for concrete base covered by asphalt paper

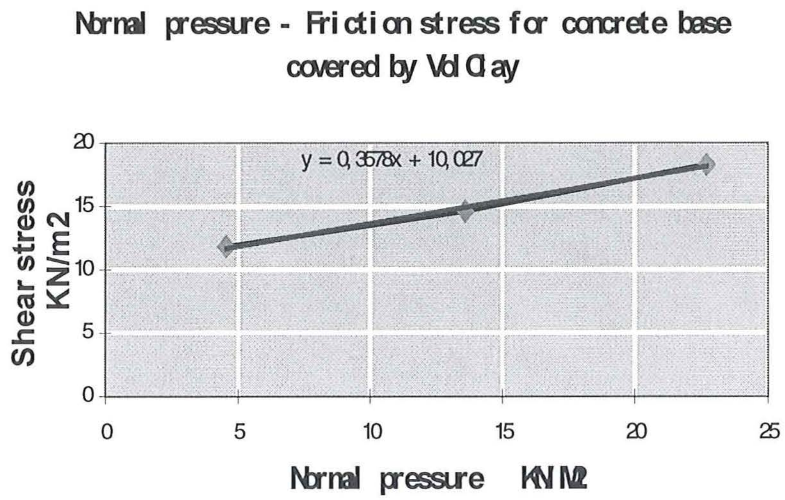


Figure (4.12) Relation between shear stress and normal pressure for concrete base covered with VolClay.

Figure (4.7) to (4.12) represent an approximation relation between friction shear stresses and normal pressure for each specified ground material

From the figures above we reach also to a conclusion that we can depend upon the approximation value of shear stresses, because they are to close to the real value of shear stresses from the tests. This means that we can used the approximation formula to calculate shear stresses for any thickness of the concrete slab, which it gives as a good estimation to the friction stresses.

4.4 Bilinear description of the friction curves

Friction curves can be approximated in different ways. Figure (4.13) shows non-linear and linear approximation for the growing part of the friction curve for the ground material. The non-linear approximations are given in the form of a general equation $F = K_1 \cdot U^b$ and a square root curve with equation $F = K_2 \cdot \sqrt{u}$ "u" is the displacement.

The first non-linear curve makes it possible to find close approximations, but two parameters are needed to describe the curve and they can not be regarded as material constants, since both vary strongly with ground pressure. The square root curve is recommended in ACI Committee 325 (1956-57), but the agreement between the test results and the approximation is not very good, even though it is better than for a linear approximation. Also for this case the parameter describing the curve can not be regarded as a material constant.

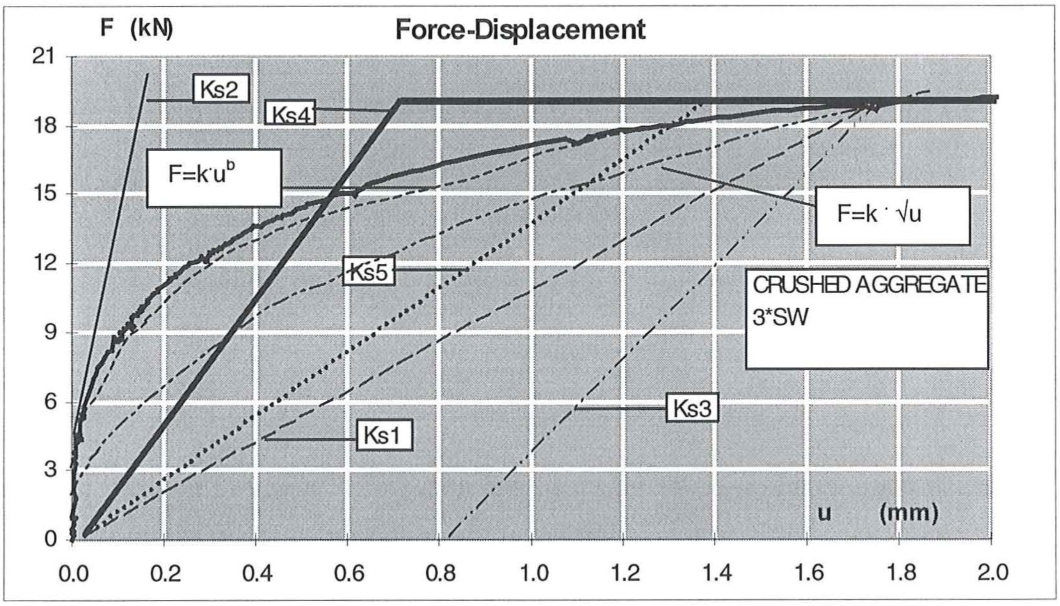


Figure (4.13) linear approximations for the friction curves according to Figure (2.3).

Linear approximations used for the test results are shown in table (4.2) and shown for one example in figure (4.13). The linear approximations are made according to Figure (2.3). The inclination K_{s1} corresponds to a secant, K_{s2} is the tangent in the starting point and K_{s3} is the mean value from unloading and reloading the test specimen. In Figure (4.13) are also given possible bilinear approximation, describing the full curve.

They are created with K_{s4} $\{(=K_{s1}+K_{s2})/2\}$ for the growing part and the μ_{max} for the constant part.
Another approximation to describe the growing part of the curve is by K_{s5}, which is created from the approximation equality between the area under the non-linear curve and the area under the bilinear approximation of friction curve, which is shown in the Figure (4.8).

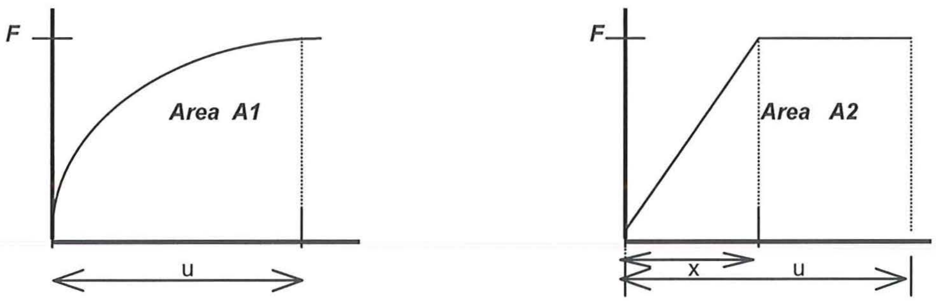


Figure (4.14) Approximation for friction curve to calculate K_{s5}.

Where:

u= horizontal displacement to the maximum value of friction force.

$A_1 = A_2$

A1 can be calculated from friction curve:

$A_1 = \frac{1}{2} * X * F + (u - X) * F$

The only unknown in the equation above is X. Knowing the value of X we can find K_{s5} from the equation:

$K_{s5} = \frac{F}{X}$

Table (4.2) gives that there is no distinct material parameter describing the growing part of the friction curve. The displacement varies in a high range. For practical cases Table (4.2) gives some guidance of the interval for the parameters. The values for sand are most reliable.

All friction stiffness values, which describe the growing part of the approximation bilinear curve, are shown in table (4.2). If we want to be in the safe side to estimate the restraint stresses for slabs on ground, it will be better to choose a higher value of K_s. For higher value of K_s, smaller displacement is needed to reach the full friction.

Table (4.2) Approximations for the friction curve given by the test result.

Case	μ_{max}	Displacement u (mm)	K_{s1}	K_{s2}	K_{s3}	K_{s4}	K_{s5}
Sand 1*SW	1.2	0.6	9.7	85.4	---	47.5	74.2
Sand 3*SW	1.0	1.4	10.3	56.5	---	33.4	46.4
Sand 5*SW	0.85	2.5	8.1	53.8	26.3	31	39.0
Sand+plastic,1*SW	0.86	1.2	3.3	24.0	---	13.6	19.8
Sand+plastic,3*SW	0.67	1.2	4.5	30.0	35.5	23.0	28.2
Sand+plastic,5*SW	0.5	0.04	299	1150	1120	724.5	---
Crushed agg.,1*SW	2.9	5.8	2.8	7.8	---	5.3	5.8
Crushed agg.,3*SW	1.5	6.7	4.4	62.5	4.48	33.4	11.5
Crushed agg.,5*SW	1.5	3.1	11.3	363	14.9	187.2	64.4
Crushed agg.+plastic,1*SW	1.0	1.2	3.2	15.6	---	9.4	
Crushed agg.+plastic,3*SW	0.75	2.4	4.4	34.9	3.9	19.7	24.4
Crushed agg.+plastic,5*SW	0.68	3.0	5.4	55.2	14.9	30.3	36.0
Asphalt paper 1*SW	8.0	2.0	17.5	31.3	14.1	24.4	84.4
Asphalt paper 3*SW	1.7	3.0	9.0	23.1	15.6	16.1	19.6
Asphalt paper 5*SW	1.0	3.0	8.6	12.8	14.2	10.7	12.7
VolClay 1*SW	2.5	6.5	1.6	15.0	11.2	2.9	5.12
VolClay 3*SW	1.03	4.0	3.5	26.7	28.0	2.8	20.4
VolClay 5*SW	0.77	3.0	5.9	150	85.0	6.5	124

From the values in the table (4.2), we reach to:

- The secant friction stiffness K_{s1} too small.
- The tangent friction stiffness K_{s2} is probably too high and is also too hard to estimate.
- K_{s3} is also not easy to estimate and its value is not high.
- K_{s4} is the mean value for K_{s1} and K_{s2}.
- K_{s5}, gives comparatively higher value. So it is suitable to use K_{s5} to represent the growing part of the bilinear curve and to be one of the input values to the FE-calculation.

For stress calculation a higher value of friction stiffness K_s is recommended.

4.5 The sliding surface for concrete structures on ground

Different locations of the sliding surface for a concrete structure on ground are possible. Sliding can start in the contact area between the slab and the ground material, or it can start in the ground material just under the contact area. If the ground material contains larger particles there will be no clear contact area because of the mix of concrete and ground material in the bottom of the slab. All these types of sliding surfaces were found in the performed tests.

Figures (4.15-4.17) shows the bottom surface of slabs with different locations of sliding surface. The slab in Figure (4.15) was cast on sand and the sliding surface is located just under the contact area. Sand with thickness of about 5 mm is fixed to the bottom of the slab. Figure (4.16) shows a mix of concrete and ground particles in the case of crushed aggregate, mixed concrete and crushed aggregate particles of about 30 mm are shown. Figure (4.17) shown if plastic sheeting is used between the slab and the ground, the location of the sliding surface is at the plastic sheeting and no ground material is fixed to it. The location of the sliding is at the asphalt paper and the VolClay.

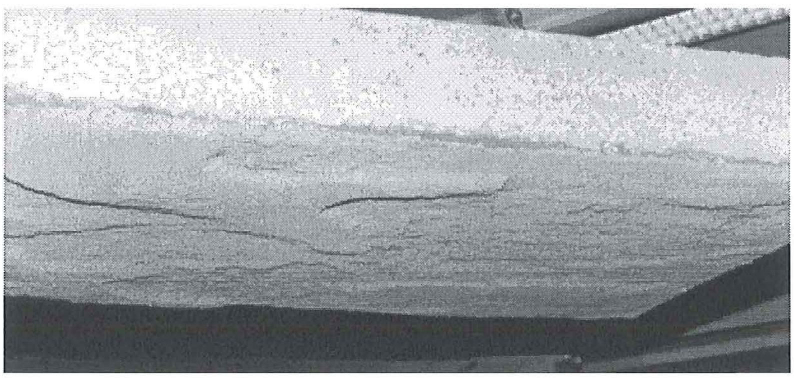


Figure (4.15) Bottom surface of the slab after performed test on sand.

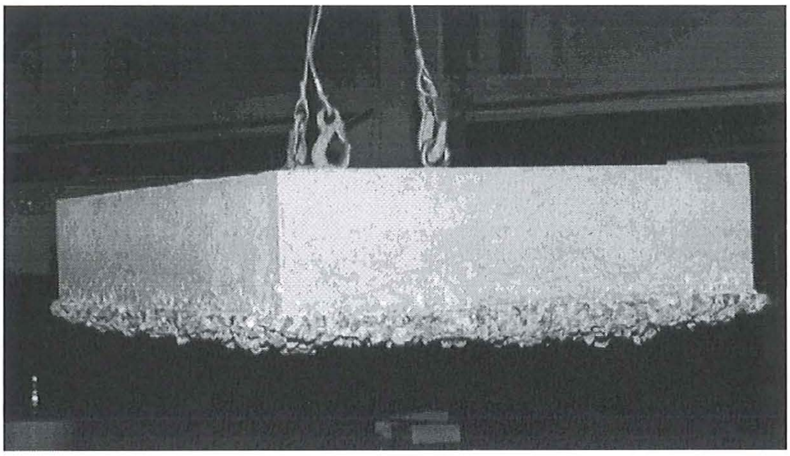


Figure (4.16) Bottom surface of the slab after performed test on crushed aggregate.

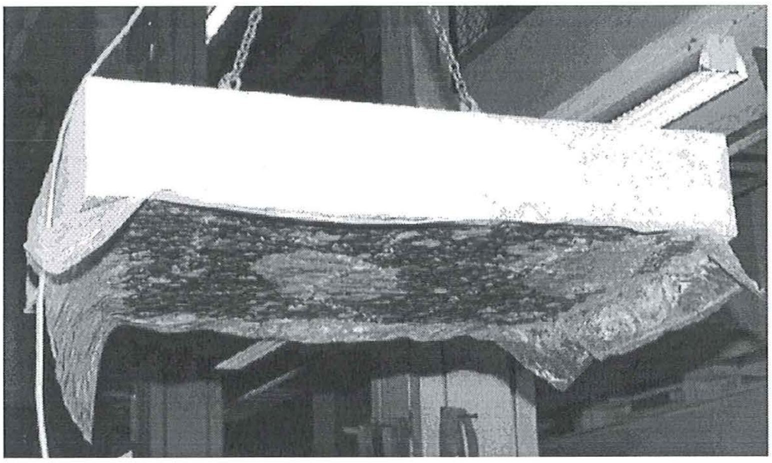


Figure (4.17) Bottom surface of the slab after performed test on ground covered by plastic sheeting.

5 Conclusions

Conclusions

The following conclusions are presented in this paper:

- Friction curves are non-linear curves.
- Coefficient of friction, μ, is depend upon the normal load (thickness of the slab) and the cohesive of the ground materials.
- The bilinear approximation of the friction curve is described by the coefficient of friction, μ, and friction stiffness, $K_{s.}$
- Different values of friction stiffness, K_s, were calculated to choose a better value of, K_s, which is used to simplify the friction curve. Large value of friction stiffness, K_s, is recommended some input value to FE-calculation.

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